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› **Class Limits (CL)** : CL is the minimum and maximum value the class interval may contain

$$CL = UCL - LCL$$

UCL : Upper Class Limit

LCL : Lower Class Limit

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Class Boundary (CB) : These are **actual class limits** of a class interval

› **In case of Mutually Exclusive Classification**

Class Boundary = Class Limit

$$UCB = UCL \text{ and } LCB = LCL$$

› **In case of Mutually Inclusive Classification**

$$UCB = UCL + 0.5$$
$$LCB = LCL - 0.5$$

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Mid Point of Class Interval : average of LCL and UCL or average of LCB and UCB

$$\frac{LCL+UCL}{2}$$

$$\frac{LCB+UCB}{2}$$

Mid point is also called as **Mid Value of a class or Class Mark**

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Class Length / Width of Class / Size of Class

UCB - LCB

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Frequency Density

$$\frac{\text{Frequency of a class}}{\text{Class length of that class}}$$

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Relative Frequency

$$\frac{\text{Frequency of class}}{\text{Total Frequency of distribution}}$$

Percentage Frequency

$$\frac{\text{Frequency of class}}{\text{Total Frequency of distribution}} \times 100$$

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Arithmetic Mean of Discrete Distribution:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

$$\bar{x} = \frac{\sum x}{n}$$

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Arithmetic Mean in case of Frequency Distribution :

$$\overline{x} = \frac{\sum fx}{N}$$

Here,

x will be equal to mid point of class interval when frequency distribution is grouped.

N is number of observations which is also equal to total of all frequencies.

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AM using Assumed Mean / Step Deviation Method

$$\bar{x} = A + \frac{\sum fd}{N} \times C$$

where $d = \frac{x - A}{C}$

A = assumed mean, C = class length

Property of AM

the algebraic sum of deviations of a set of observations from their AM is zero

$$\sum(x-\overline{x})=0$$

Property of AM

Combined AM

- If there are two groups containing n_1 & n_2 observations with AMs as $\overline{x_1}$ & $\overline{x_2}$ respectively then the combined AM is given by

$$\overline{x}_c = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}$$

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Median in case of discrete distribution

- *If n = odd, then middle term*
- *If n = even, average of two middle terms*

Same formula will be used for ***Simple Frequency distribution***

Median in case of Grouped Frequency Distribution

- **Step 1**: Prepare a less than type cumulative frequency distribution.
- **Step 2:** Calculate N/2 and check between which class boundaries it falls and call it as Median Class.
- **Step 4:** *Find l_1 = LCB of Median Class, N_u as Cum Freq. of Median Class,*
  N_l as Cum. Freq. of Pre Median Class, C = Class length of Median Class
- **Step 3**: Apply the below formula

$$Me = l_1 + \left(\frac{\frac{N}{2} - N_l}{N_u - N_l} \right) \times C$$

Property of Median

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*For a set of observations, the sum of **absolute** deviations is **minimum**, when the deviations are taken from the median.*

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Quartiles (in case of discrete observations)

Quartiles
$Q_1 = \left((n+1)\times\frac{1}{4}\right)^{th}\ term$
$Q_2 = \left((n+1)\times\frac{2}{4}\right)^{th}\ term$
$Q_3 = \left((n+1)\times\frac{3}{4}\right)^{th}\ term$

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Deciles (in case of discrete observations)

Deciles
$D_1 = \left((n+1)\times\frac{1}{10}\right)^{th}$ *term*
$D_2 = \left((n+1)\times\frac{2}{10}\right)^{th}$ *term*
$Q_9 = \left((n+1)\times\frac{9}{10}\right)^{th}$ *term*

Percentiles (in case of discrete observations)

Percentiles
$P_1 = \left((n+1)\times\frac{1}{100}\right)^{th}$ term
$P_2 = \left((n+1)\times\frac{2}{100}\right)^{th}$ term
$P_{99} = \left((n+1)\times\frac{99}{100}\right)^{th}$ term

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Quartiles (Grouped FD)

- For first Quartile Q_1
  - *Find Q_1 class*
  - *Use Formula :*

$$Q_1 = l_1 + \left(\frac{\frac{N}{4} - N_l}{N_u - N_l} \right) \times C$$

- For third Quartile Q_3
  - *Find Q_3 class*
  - *Use Formula :*

$$Q_3 = l_1 + \left(\frac{\frac{3N}{4} - N_l}{N_u - N_l} \right) \times C$$

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Deciles (Grouped FD)

- For first Decile D_1
  - *Find D_1 class*
  - *Use Formula:*

$$D_1 = l_1 + \left(\frac{\frac{N}{10} - N_l}{N_u - N_l} \right) \times C$$

- For ninth Decile D_9
  - *Find D_9 class*
  - *Use Formula:*

$$D_9 = l_1 + \left(\frac{\frac{9N}{10} - N_l}{N_u - N_l} \right) \times C$$

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Percentiles (Grouped FD)

- For first Percentile P_1
  - *Find P_1 class*
  - *Use Formula:*

$$P_1 = l_1 + \left(\frac{\frac{N}{100} - N_l}{N_u - N_l} \right) \times C$$

- For 99th Percentile P_{99}
  - *Find P_{99} class*
  - *Use Formula:*

$$P_{99} = l_1 + \left(\frac{\frac{99N}{10} - N_l}{N_u - N_l} \right) \times C$$

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Mode in case of discrete observation

Observation repeating for maximum no. of times/ max frequency

- *When all observations have same frequency :* ***No Mode***
- *When two observations have max frequency :* ***Bimodal Observation***
- *In general, if we have two or more modes :* ***Multi Modal Observation***

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- Mode in case of grouped frequency distribution
  - *First find modal class i.e. class with highest frequency*
  - *Now, Find*
    - $f_0 =$ *frequency of the modal class,*
    - $f_{-1} =$ *frequency of pre modal class,*
    - $f_1 =$ *frequency of the post modal class*
  - Apply formula

$$Mo = l_1 + \left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \right) \times C$$

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Relationship between Mean, Median and Mode

In case of symmetrical Distribution

Mean = Median = Mode

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Relationship between Mean, Median and Mode

In case of moderately skewed distribution

Mean – Mode = 3 (Median – Median)

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Geometric Mean (in case of discrete positive observations)

$$G = \left(x_1 \times x_2 \times \ldots \times x_n \right)^{1/n}$$

Geometric Mean (in case of frequency distribution)

$$G = ({x_1}^{f_1} \times {x_2}^{f_2} \times {x_3}^{f_3} \times \cdots \times {x_n}^{f_n})^{1/N}$$

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Harmonic Mean (in case of discrete observations)

$$H = \frac{n}{\Sigma\left({}^{1}/_{x}\right)}$$

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Harmonic Mean (in case of frequency distribution)

$$H = \frac{N}{\Sigma\left(f/_{x}\right)}$$

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Combined HM

$$\bar{x}_c = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$$

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› Relationship between AM, GM and HM

When to use	Relationship
When all the observations are same	AM = GM = HM
When all the observations are distinct	AM > GM > HM
In General	AM ≥ GM ≥ HM

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Range (in case of Discrete Observations)

$$L - S$$

L = Largest Observation

S = Smallest Observation

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Range (in case of Grouped FD)

$$L - S$$

L = UCB of last class interval

S = LCB of first class interval

Coefficient of Range

$$\frac{L-S}{L+S}\times 100$$

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Mean Deviation (in case of discrete observations)

$$MD_A = \frac{1}{n}\Sigma|x - A|$$

Here A is mean or median or mode as given in question

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Mean Deviation (in case of grouped frequency distributions)

$$MD_A = \frac{1}{N}\Sigma f|x - A|$$

Here A is mean or median or mode as given in question

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Coefficient of Mean Deviation

$$\frac{\text{Mean Deviation about A}}{\text{A}} \times 100$$

π

Standard Deviation (in case of discrete observations)

$$\sigma_x = SD_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

$$\sigma_x = SD_x = \sqrt{\frac{\sum x^2}{n} - \left(\bar{x}\right)^2}$$

π

Standard Deviation (in case of grouped frequency observations)

$$\sigma_x = SD_x = \sqrt{\frac{\sum f(x-\bar{x})^2}{N}}$$

$$\sigma_x = SD_x = \sqrt{\frac{\sum fx^2}{N} - (\bar{x})^2}$$

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Coefficient of Variation

$$\frac{SD_x}{\bar{x}} \times 100$$

For any two numbers, SD = half of range

$$SD = \frac{|a-b|}{2}$$

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- Standard Deviation of first n natural numbers

$$s = \sqrt{\frac{n^2 - 1}{12}}$$

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Combined SD

$$SD_c = \sqrt{\frac{n_1 s_1{}^2 + n_2 s_2{}^2 + n_1 d_1{}^2 + n_2 d_2{}^2}{n_1 + n_2}}$$

$$d_1 = \bar{x}_c - \bar{x}_1$$

$$d_2 = \bar{x}_c - \bar{x}_2$$

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Property of SD

If all the observations are constant, SD is Zero

Property of SD

- *No effect of change of origin but affected by change of scale in the magnitude (ignore sign)*

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Quartile Deviation

$$QD_x = \frac{Q_3 - Q_1}{2}$$

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Coefficient of Quartile Deviation

$$\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

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Relationship between SD, MD, QD

For a moderately skewed distribution

$$4SD = 5MD = 6QD$$

$$SD : MD : QD = 15 : 12 : 10$$

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Basic formula of Probability

$$P(A) = \frac{(no.of\ favorable\ events\ to\ A)}{Total\ no.of\ events}$$

Odds in Favor of Event A

$$\frac{\text{no. of favorable events}}{\text{no. of unfavorable events}}$$

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- Odds against an Event

$$\frac{\text{no. of unfavorable events}}{\text{no. of favorable events}}$$

π

› No. of total outcome of a random experiment

If an experiment results in p outcomes and if it is repeated q times then

Total number of outcomes is

$$p^q$$

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Relative Frequency Probability

$$\text{Relative Frequency} = \frac{\text{no. of times the event occurred during experimental trials}}{\text{total no. of trials}} = \frac{f_A}{n}$$

Probability by this method is defined as

$$P(A) = \lim_{n \to \infty} \frac{f_A}{n}$$

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› Set Based Probability

$$P(A) = \frac{\text{no.of sample points in A}}{\text{no.of sample points in S}} = \frac{n(A)}{n(S)}$$

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Addition Theorem 1

In case of two mutually exclusive events A and B

$$P(A \cup B) = P(A+B) = P(A \text{ or } B) = P(A) + P(B)$$

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Addition Theorem 2

In case of two or more mutually exclusive events

$$P\,(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \ldots$$

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Addition Theorem 3

For any two events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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Addition Theorem 4

In case of any three events

$$\boldsymbol{P\,(A \cup B \cup C)}$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$$
$$- P(A \cap C) + P(A \cap B \cap C)$$

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Conditional Probability of Event B knowing that Event A is already occurred

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$

$$provided\ P(A) \neq 0$$

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Conditional Probability of Event A knowing that Event B is already occurred

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$provided\ P(B) \neq 0$$

Compound Theorem

In case of two dependent events

$$P(A \cap B) = P(B) \times P(A/B) \text{ or}$$

$$P(A \cap B) = P(A) \times P(B/A)$$

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Compound Theorem

In case of two independent events

$$P(A \cap B) = P(A) \times P(B)$$

Same thing applies for two or more independent events

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Expected value of a Probability Distribution

$$E(x) = \sum p_i x_i$$

$$E(x) = \mu$$

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Variance of Probability Distribution

$$V(x) = \sigma^2$$

$$E(x-\mu)^2 = E(x)^2 - \mu^2$$

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Probability Mass Function in case of Binomial Distribution

$$f(x) = P(X = x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

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Mean and Variance in case of Binomial Distribution

$$\mu = np$$

$$\sigma^2 = npq$$

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Mode in case of Binomial Distribution

Calculate $(n+1)p$

if the resulting value is integer then Bi-modal	$\mu_0 = (n+1)p$ *and* $[(n+1)p - 1]$
If the resulting value is non-integer then Uni-modal	$\mu_0 =$ *largest integer contained in* $(n+1)p$

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Probability Mass Function in case of Poisson Distribution

$$f(x) = P(X = x) = \frac{(e^{-m}.m^{x})}{x!}$$

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Mean, Variance and SD in case of Poisson Distribution

$$\mu = m$$

$$\sigma^2 = m$$

$$\sigma = \sqrt{m}$$

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Mode in case of Poisson Distribution

if the value of m is integer then Bi-modal	$\mu_0 = m$ *and* $[m-1]$
If the value of m is non-integer then Uni-modal	$\mu_0 =$ *largest integer contained in* m

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Probability Density Function in case of Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{-(\frac{x-\mu}{\sigma})^2 \times \frac{1}{2}}$$

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Mean Deviation in case of Normal Distribution

$$MD = 0.8\,\sigma$$

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Quartiles in case of Normal Distribution

$$Q_1 = \mu - 0.675\sigma$$

$$Q_3 = \mu + 0.675\sigma$$

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Quartile Deviation in case of Normal Distribution

$$QD = 0.675\sigma$$

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Points of Inflexion

$$\mu - \sigma \quad \& \quad \mu + \sigma$$

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Ratio between

QD : MD : SD = 10:12:15

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Conditions of Standard Normal Distribution

Mean μ	0
Standard Deviation σ	1

Z Value

$$Z = \frac{x - \mu}{\sigma}$$

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Area under Normal Curve

From	To	Area/Probability
μ	+σ	34.135%
+σ	+2σ	13.59%
+2σ	+3σ	2.14%
3σ	∞	0.135%

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For a bivariate frequency distribution

$$No.\ of\ marginal\ distributions = 2$$

$$No.\ of\ conditional\ distributions = m + n$$

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Karl Pearson's Product Moment Correlation Coefficient

$$r_{xy} = \frac{Cov(x, y)}{\sigma_x . \sigma_y}$$

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Covariance

$$Cov(x, y) = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{n} \; or \; \frac{\Sigma xy}{n} - \bar{x}.\bar{y}$$

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Spearman's Rank Correlation Coefficient

$$r_R = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

here d means difference in ranks

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Adjustment value in case of Tie

$$\Sigma \frac{(t^3 - t)}{12}$$

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Coefficient of Concurrent Deviations

$$r_c = \pm\sqrt{\pm\frac{2c-m}{m}}$$

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Regression Coefficients

$$b_{yx} = r.\frac{\sigma_y}{\sigma_x} \text{ or } b_{yx} = \frac{Cov(x,y)}{{\sigma_x}^2}$$

$$b_{xy} = r.\frac{\sigma_x}{\sigma_y} \text{ or } b_{xy} = \frac{Cov(x,y)}{{\sigma_y}^2}$$

π

Regression Property

$$r = \pm\sqrt{b_{xy} \times b_{yx}}$$

b_{xy} , b_{yx} *and r all will have same sign*

Regression Property

- **Change of Origin and Scale**
  - *Origin: No Impact*
  - *Scale: If original pair is x, y and modified pair is u, v*

then,

$$b_{vu} = b_{yx} \times \frac{change\ of\ scale\ of\ y}{change\ of\ scale\ of\ x}$$

$$b_{uv} = b_{xy} \times \frac{change\ of\ scale\ of\ x}{change\ of\ scale\ of\ y}$$

Regression Property

- Intersection of two regression lines:
  - *Two regression lines (if not identical) will intersect at the point [means]*

$$\left(\bar{x}, \bar{y}\right)$$

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Coefficient of determination

Explained Variance

Accounted Variance

$$r^2$$

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Coefficient of non-determination

Unexplained Variance

Unaccounted Variance

$$1 - r^2$$

Probable Error

$$0.6745 \times \frac{1-r^2}{\sqrt{N}}$$

- *where,* r *= correlation coefficient,* N *= no. of pairs of observation*

Limits of Population Correlation Coefficient

$$r \pm PE$$

Index Relatives

Price Relative	*Price Relative* $= \frac{P_n}{P_0}$
Quantity Relative	*Quantity Relative* $= \frac{Q_n}{Q_0}$
Value Relative	*Value Relative* $= \frac{V_n}{V_0} = \frac{P_n Q_n}{P_0 Q_0}$
Link relative	$\frac{P_1}{P_0}, \frac{P_2}{P_1}, \frac{P_3}{P_2}, \dots, \frac{P_n}{P_{n-1}}$ *Same can be created for quantities also*
Chain relatives	*When the above relatives are in respect to a fixed base period these are also called the chain relatives* $\frac{P_1}{P_0}, \frac{P_2}{P_0}, \frac{P_3}{P_0}, \dots, \frac{P_n}{P_0}$

Simple Aggregative Method of Index

Price Index is expressed as total of commodity prices in a given year as a percentage of total of commodity prices in the base year

- Formula

$$\frac{\Sigma P_n}{\Sigma P_0} \times 100$$

Simple Average of Relatives – Method of Index

- *Under this method, we invert the actual price for each variable into percentage of the base period. These percentages are called relatives.*
- Formula

$$\frac{\Sigma \frac{P_n}{P_0}}{N}$$

Weighted Index Methods

› *Laspeyres' Index*

$$\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times 100$$

Weighted Index Methods

› *Passche's Index*

$$\frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n} \times 100$$

Weighted Index Methods

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› *Marshall-Edgeworth Index*

$$\frac{\Sigma P_n(Q_0 + Q_n)}{\Sigma P_0(Q_0 + Q_n)} \times 100$$

Weighted Index Methods

› *Fisher's Index*

(GM of Laspeyres' and Paasche's)

$$\sqrt{\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n}} \times 100$$

Deflated Value

$$\text{Deflated Value} = \frac{\text{Current Value}}{\text{Price Index of the current year}}$$