FORMULA REVISION

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$$a : b = \frac{a}{b}\begin{array}{l}\text{— antecedent}\\ \text{— consequent}\end{array}$$

- **The ratio compounded of the two ratios**

  a : b and c : d is ac: bd.

PROPERTIES OF RATIO

$a^2 : b^2$ is the duplicate ratio of $a : b$.

$a^3 : b^3$ is the triplicate ratio of $a : b$

For example

- duplicate ratio of $2 : 3$ is $4 : 9$.
- Triplicate ratio of $2 : 3$ is $8 : 27$.

$\sqrt{a} : \sqrt{b}$ is the Sub – duplicate ratio of $a : b$.

$\sqrt[3]{a} : \sqrt[3]{b}$. is the Sub – triplicate ratio of $a : b$

For example

- Sub – duplicate ratio of $4 : 9$ is $2 : 3$
- Sub – triplicate ratio of $8 : 27$ is $2 : 3$

PROPORTION

- An equality of two ratios is called a proportion.
- Four quantities a, b, c, d are said to be in proportion

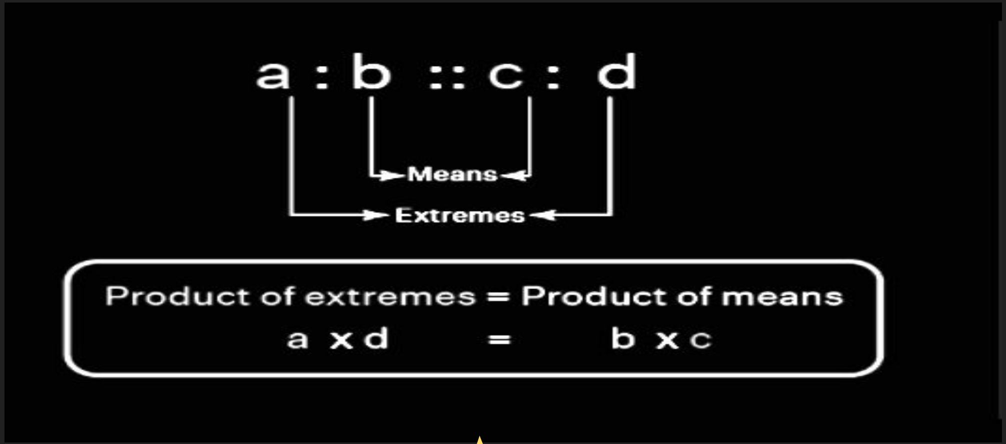
if , $a : b = c : d$ (also written as $a : b :: c : d$)

a = first term / first proportional

b = second term / second proportional

c = third term / third proportional

d = fourth term / fourth proportional



Four numbers a , b ,c ,d are in Proportion

Four Quantities need not be of same kind

first two quantities should be of the same kind and last two quantities should be of the same kind.

CONTINUOUS PROPORTION

- Three quantities a, b, c of the same kind (in same units) are said to be in continuous proportion if

$$a : b = b : c$$

i.e. $a/b = b/c$

$$b^2 = ac$$

$$\therefore b = \sqrt{ac}$$

here ,

a = first proportional

c = third proportional

b = mean proportional

PROPERTIES OF PROPORTION

INVERTENDO

If a : b = c : d, then b : a = d : c

$$\text{If } \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}.$$

ALTERNENDO

If a : b = c : d, then a : c = b : d

OR d : b = c : a

$$\text{If } \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d} \text{ Or } \frac{d}{b} = \frac{c}{a}.$$

PROPERTIES OF PROPORTION

COMPONENDO

If a : b = c : d, then a + b : b = c + d : d

$$\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a+b}{b}=\frac{c+d}{d}$$

DIVIDENDO

If a : b = c : d, then a - b : b = c - d : d

$$\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a-b}{b}=\frac{c-d}{d}$$

PROPERTIES OF PROPORTION

COMPONENDO AND DIVIDENDO

5 .If a : b = c : d, then a + b : a - b = c + d : c - d

$$\text{If } \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}.$$

PROPERTIES OF PROPORTION → **ADDENDO**

6. If a : b= c : d = e : f = ..., then each of these ratios is equal

(a + c + e + ...) : (b + d + f + ...)

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots\ldots$ then each of these ratios is equal to $\frac{a+c+e\ldots\ldots}{b+d+f\ldots\ldots}$

i.e ,

$\frac{a}{b} = \frac{a+c+e\ldots\ldots}{b+d+f\ldots\ldots}$, $\frac{c}{d} = \frac{a+c+e\ldots\ldots}{b+d+f\ldots\ldots}$, $\frac{e}{f} = \frac{a+c+e\ldots\ldots}{b+d+f\ldots\ldots}$

PROPERTIES OF PROPORTION

SUBTRAHENDO

7. If a : b= c : d = e : f = ..., then each of these ratios is equal

(a - c -e + ...) : (b -d - f - ....)

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots\ldots$ **then each of these ratios is equal to** $\frac{a-c-e\ldots}{b-d-f\ldots}$

i.e ,

$\frac{a}{b}=\frac{a-c-e\ldots}{b-d-f\ldots}$, $\frac{c}{d}=\frac{a-c-e\ldots}{b-d-f\ldots}$, $\frac{e}{f}=\frac{a-c-e\ldots}{b-d-f\ldots}$

For any real number a

- $a^n = a \times a \times a$..............to n factors.
- **Example :** $3^4 = 3 \times 3 \times 3 \times 3$

- $a^0 = 1$
- **Example:** $3^0 = 1$

- $a^{-n} = \frac{1}{a^n}$

- **Example:** $2^{-5} = 1/2^5$
- **Example:** $1/2^{-5} = 2^5$

LAW OF INDICES

LAW 1

$$a^m \times a^n = a^{m+n}$$

For example :

$$3^4 \times 3^5 = 3^9$$

LAW 2

$$a^m / a^n = a^{m-n}$$

For example

$$2^7 / 2^4 = 2^3$$

LAW 3

$$(a^m)^n = a^{mn}$$

For example

$$(2^4)^3 = 2^{12}$$

LAW 4

$$(ab)^n = a^n b^n$$

For Example

$$(2 \times 3)^3 = 2^3 \times 3^3$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

For Example

$$(5/3)^4 = 5^4/3^4$$

LAW 5

$$\sqrt[n]{a^m} = a^{m/n}$$

For Example

$$\sqrt{9} = 9^{1/2}$$

$$\sqrt[5]{9^2} = 9^{2/5}$$

REMARK

- If $a^x = a^y$, then $x = y$
- If $x^a = y^a$, then $x = y$
- If $a^m = k$, then $a = k^{1/m}$
- $\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$

LOGARITHM

If $a^x = n$

then

$x = \log_a n$

logarithm of n to the base a is x

CONDITIONS

- $n > 0$
- $a > 0$
- $a \neq 1$

Examples

$2^4 = 16$ $\quad 4 = \log_2 16$

$10^3 = 1000$ $\quad 3 = \log_{10} 1000$

$5^{-3} = 1/125$ $\quad -3 = \log_5 (1/125)$

$2^3 = 8$ $\quad 3 = \log_2 8$

PROPERTIES OF LOGARITHM

• $\log_a 1 = 0$ • $\log_a a = 1$	$\log_b m = \dfrac{\log_a m}{\log_a b}$
• $\log_a m + \log_a n = \log_a (mn)$	• $\log_b a \times \log_a b = 1$ • $\log_b a = 1 / \log_a b$
• $\log_a m - \log_a n = \log_a \dfrac{m}{n}$	$\log_{b^n} a^m = \dfrac{m}{n} \log_b a$
$\log_a m^n = n \log_a m$	$a^{\log_a n} = n$

Calculator trick to find LOGARITHM

- **FIND $\log_{10} x$**
- Type x
- Press √ for 13 times
- Subtract 1
- Multiply 3558

QUADRATIC FORMULA

$$ax^2 + bx + c = 0$$

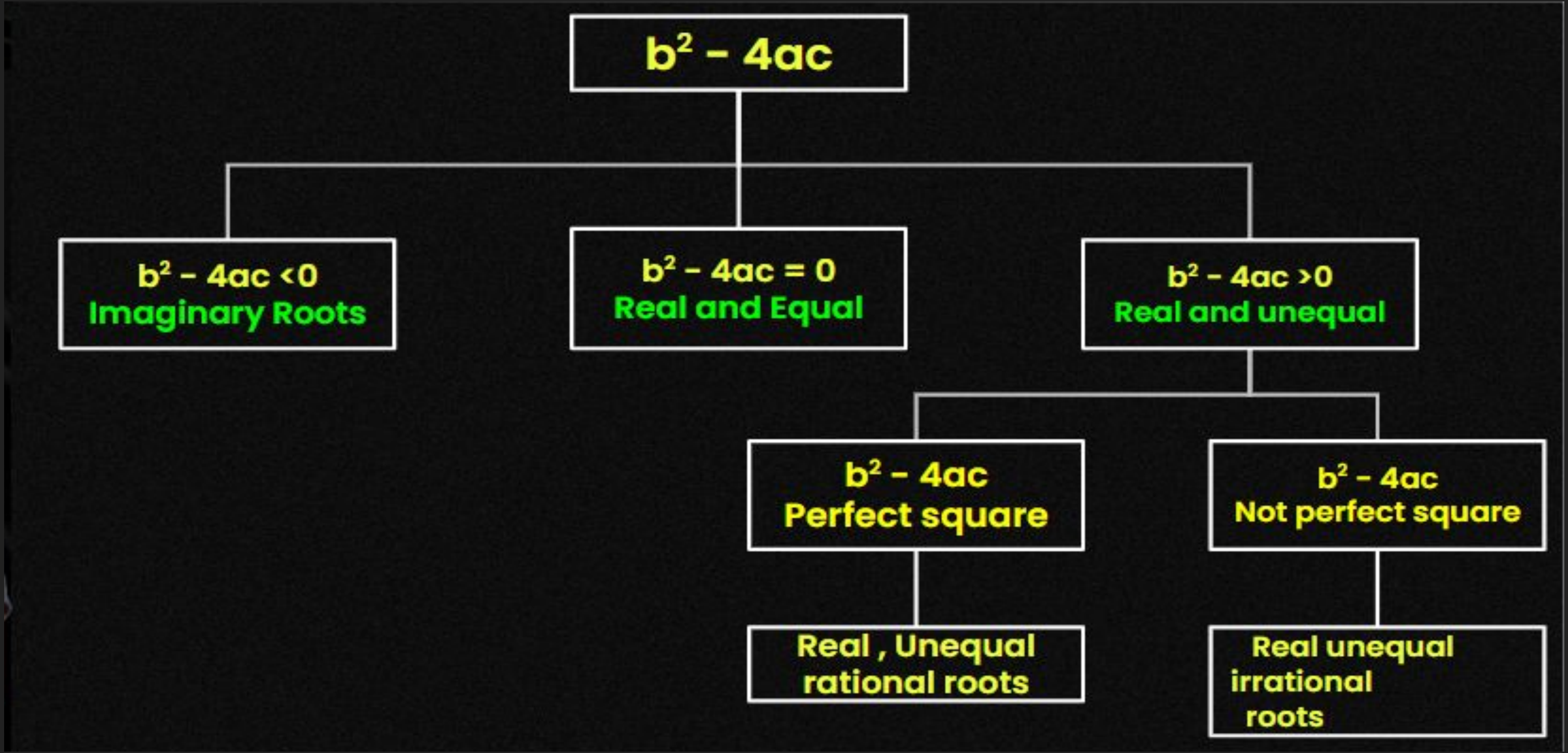
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Let one root be α and the other root be β

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

NATURE OF ROOTS



IDENTITIES

- $(a+b)^2 = a^2+b^2+2ab$
- $(a-b)^2 = a^2+b^2-2ab$
- $(a+b)^2 = (a-b)^2+4ab$
- $(a+b)^3 = a^3+b^3+3ab(a+b)$
- $(a-b)^3 = a^3-b^3-3ab(a-b)$
- $a^2-b^2 = (a-b)(a+b)$

Sum and Product of the Roots:

$ax^2 + bx + c = 0$

sum of roots = $-\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coeffient of } x^2}$

product of the roots = $\frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

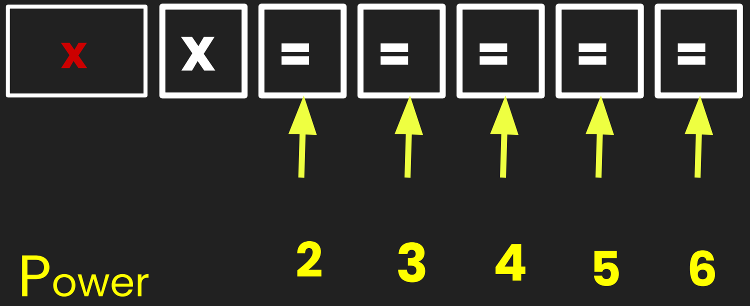
How To Construct A Quadratic Equation

Let one root be α and the other root be β

x^2 – (sum of the roots) x + Product of the roots = 0

$$x^2 - (\alpha + \beta)\,x + \alpha\beta = 0$$

SIMPLE POWER (Integer Power x^n)



EXAMPLE :

- 3^4
- $(0.9)^3$
- 6^4

To find nth root $(x^{1/n})$

STEPS

- Write x
- Press √ 12 times
- Subtract 1
- Divide by n
- Add 1
- Press x= 12 times

EXAMPLE :

- $27^{1/3}$
- $32^{1/5}$

Non - Integer POWER (x^n)

STEPS

- Write x
- Press [√] 12 times
- Subtract 1
- Multiply by n
- Add 1
- Press [x=] 12 times

Example :

$21^{1.2}$

SIMPLE INTEREST

Yearly Basis

$$SI = \frac{P.r.t}{100}$$

time period in years

$$\frac{\text{month}}{12}$$

computed on the principal for the entire period of borrowing

$$A = P + SI$$

$$A = P\left(1 + \frac{rt}{100}\right)$$

Value of interest remains constant for each year.

$A_2 - A_1$ = Interest of a year

COMPOUND INTEREST

$$A_n = P(1+i)^n$$

$$\text{where, } i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year}}$$

$$\text{Interest} = A_n - P = P(1+i)^n - P$$

$$= P\left[(1+i)^n - 1\right]$$

n is total conversions i.e. t x no. of conversions per year

CALCULATOR TRICK TO FIND AMOUNT

P	+	i	%	+	i	%	...

COMPOUND INTEREST

- The period at the end of which the interest is compounded is called conversion period.

Conversion period	Description	Number of conversion period in a year
1 day	Compounded daily	365
1 month	Compounded monthly	12
3 months	Compounded quarterly	4
6 months	Compounded semi annually	2
12 months	Compounded annually	1

EFFECTIVE RATE OF INTEREST

The effective interest rate can be computed directly by following formula:

$$E = (1 + i)^n - 1$$

Where E is the effective interest rate

i = actual interest rate in decimal

n = number of conversion period

DEPRECIATION

Depreciation is the fall in the value of an asset due to wear and tear , efflux of time , obsolescence .

$$A = P(1 - i)^n$$

Where ,

P = historical cost of asset
A = Scrap value / residual value
n = no . of periods
i = depreciation

SINGLE CASH FLOW

If single amount is paid or received initially and then direct finally at the end .

FUTURE VALUE : SINGLE CASH FLOW

- Future value is the cash value of an investment at some time in the future.
- $F = C.F. (1 + i)^n$

PRESENT VALUE : SINGLE CASH FLOW

- Present value is today's value of tomorrow's money discounted at the interest rate.
- $P = \frac{C.F.}{(1+i)^n}$

TYPES ANNUITY

- Amount paid (or received) must be constant over the period of annuity
- Time interval between two consecutive payments (or receipts) must be the same.

NOT MENTIONED

Annuity

- Annuity regular
  - First payment/receipt at the end of the period
- Annuity due or annuity immediate
  - First payment/receipt at the beginning of the period

FUTURE VALUE OF ANNUITY

FUTURE VALUE OF ANNUITY REGULAR

$$FVAR = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Where ,

A = periodic payments

FUTURE VALUE OF ANNUITY DUE

$$FVAD = A\left[\frac{(1+i)^n - 1}{i}\right] \times (1 + i)$$

Where ,

A = periodic payments

- **Size of the sinking fund deposit is same as future value of Annuity .**

PRESENT VALUE OF ANNUITY

PRESENT VALUE OF ANNUITY REGULAR

$$PVAR = A \times PVAF(n, i)$$

$$PVAR = \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

PRESENT VALUE OF ANNUITY DUE

$$PVAD = A \times PVAF\{(n-1), i\} + A$$

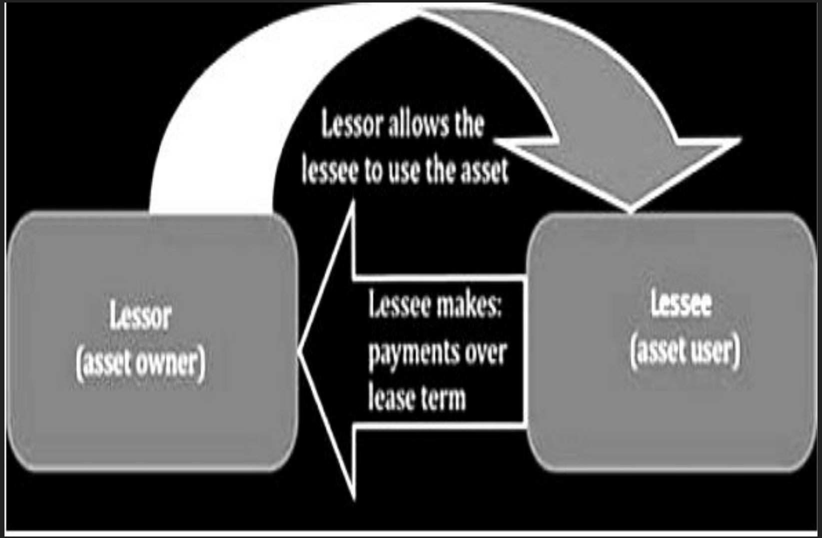
Where ,

A = periodic payments

PVAF CALCULATOR TRICK :

1 + i	÷	=	=	=	= ..... n times	GT

LEASING



- **Leasing is a financial arrangement under which the owner of the asset (lessor) allows the user of the asset (lessee) to use the asset for a defined period of time(lease period) for a consideration (lease rental) payable over a given period of time. This is a kind of taking an asset on rent.**

HOW TO SOLVE QUESTION ?

Present value of Lease rentals are compared with asset cash down price to decide if leasing is preferable or not .

CAPITAL EXPENDITURE (INVESTMENT DECISION)

- Capital expenditure means purchasing an asset (which results in outflows of money) today in anticipation of benefits (cash inflow) which would flow across the life of the investment.

How will decision be taken ?

- **Compare purchase value of asset with the present value of future benefits .**
- **If present value of future benefit is greater than purchase value of asset ,decision should be in the favour of investment**

PERPETUITY

- Perpetuity is an annuity in which the periodic payments or receipts begin on a fixed date and continue indefinitely or perpetually .
- We can calculate PV of Perpetuity .
- FV of Perpetuity is not defined .

$$PVP = \frac{A}{i}$$

PVP = Present Value of Perpetuity

A = Installment (Annuity Value)

i = adjusted interest rate

GROWING PERPETUITY

- A stream of cash flows that grows at a constant rate forever is known as growing perpetuity.

$$PVGP = \frac{A}{i - g}$$

PVGP = Present Value of Growing Perpetuity

A = Installment (Annuity Value)

i = adjusted interest rate

g = growth rate

NET PRESENT VALUE

Net present value (NPV) = Present value of cash inflow – Present value of cash outflow

Decision Rule:

If NPV > 0 Accept the Proposal

If NPV < 0 Reject the Proposal

REAL RATE OF RETURN

- Real Rate of Return = Nominal Rate of Return – Inflation

FACTORIAL

$$n! = n(n-1)(n-2) \ldots\ldots\ldots 3.2.1$$

- $4! = 4 \times 3 \times 2 \times 1$
- $3! = 3 \times 2 \times 1$
- $2! = 2 \times 1$
- $1! = 1$

For a natural number n	EXAMPLE
$n! = n(n-1)!$	$5! = 5 \times 4 \times 3 \times 2 \times 1$
$n! = n(n-1)(n-2)!$	$5! = 5 \times 4!$
	$5! = 5 \times 4 \times 3!$

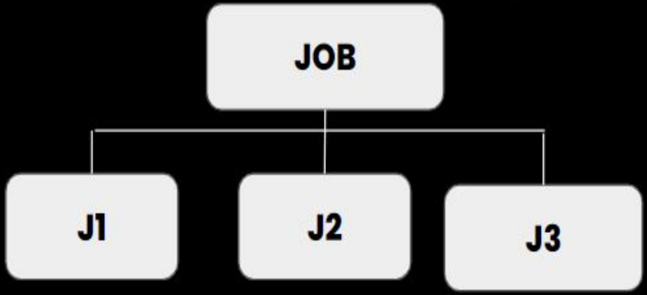
RESULT :

$$(n+1)! - n! = \Rightarrow n.n!$$

Fundamental Principles of Counting

AND

Multiplication Principle



m ways of doing

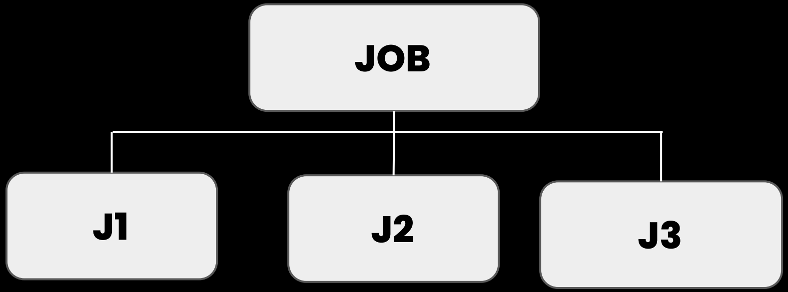
n ways of doing

p ways of doing

No . of ways of doing the job = m x n x p

OR

Addition Principle



m ways of doing

n ways of doing

p ways of doing

No . of ways of doing the job = m + n + p

PERMUTATIONS

- A permutation is an **arrangement in a definite order** of a number of objects taken **some or all at a time** .

- Arrangements made with letters

  a , b , c taking all at a time

  abc, acb , bac , bca , cab , cba

- Arrangements made with letters

  a , b , c taking two at a time

  ab , ba , ac, ca, bc, cb

The number of permutations of n things when r are chosen at a time

$$^nP_r = n(n-1)(n-2)\ldots(n-r+1)$$

where the product has **exactly r factors.**

$$^{n}P_{r} = \frac{n!}{(n-r)!}, 0 \le r \le n$$

$$^{n}P_{n} = n!$$

TYPES

- The number of permutations of n **different** objects taken r at a time and objects do not repeat is $^{n}P_{r} = \frac{n!}{(n-r)!}, 0 \le r \le n$

- The number of permutations of n **different** objects taken all at a time is $^{n}P_{n} = n!$

- The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is

$$\frac{n!}{p_1!\, p_2! \ldots p_k!}$$

Types

- ***Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is***

$$^{n-1}P_r$$

- ***Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement***

$$r \cdot {}^{n-1}P_{r-1}$$

Types

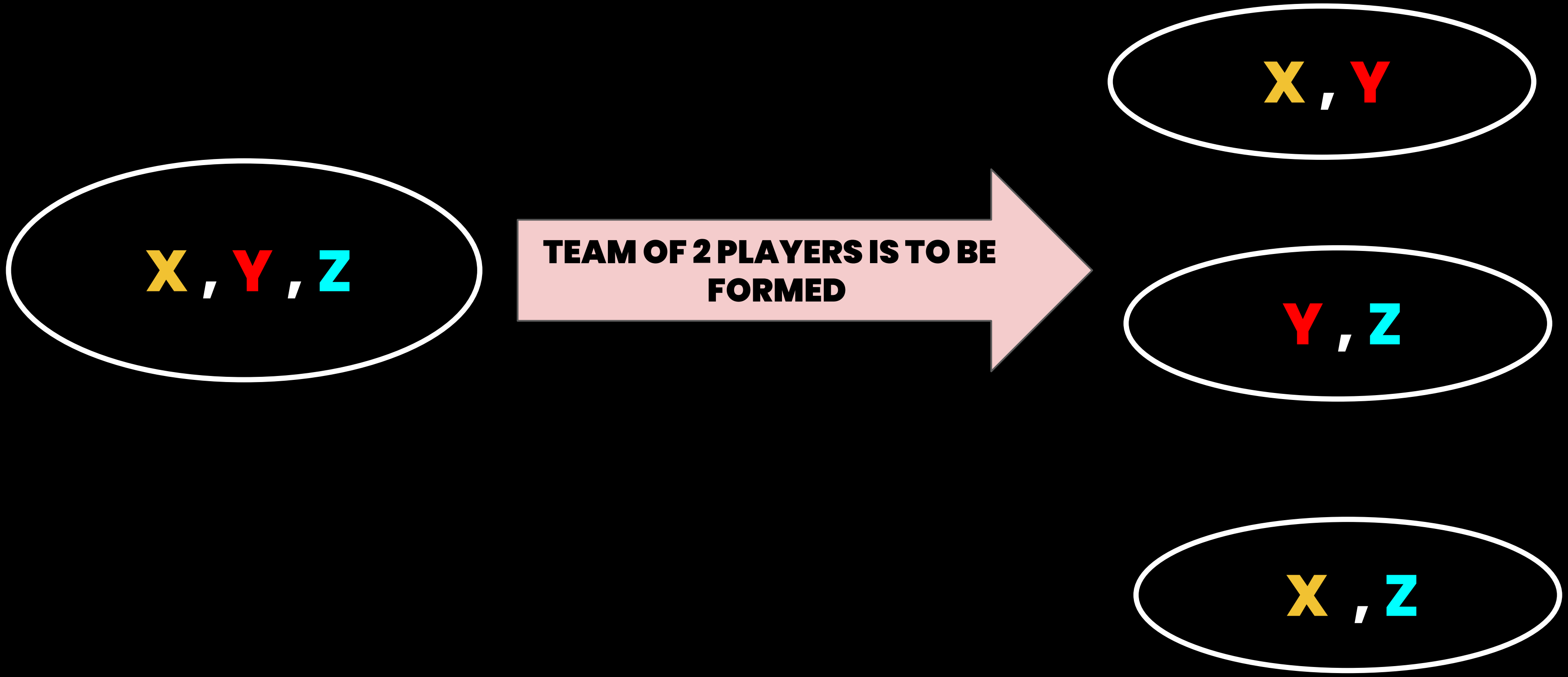
- **The number of circular permutations of n different things chosen all at a time is**

  $(n-1)!$

- **The number of ways of arranging n persons along a round table so that no person has the same two neighbours is** $=\frac{1}{2}\lfloor n-1$

- ***The number of necklaces formed with n beads of different*** $=\frac{1}{2}\lfloor n-1$

COMBINATIONS



$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!},\ 0 \le r \le n.$$

- $^{n}C_{0} = 1$

- $^{n}C_{n} = 1$

- $^{n}C_{r} = {}^{n}C_{n-r}$

$$^nP_r = {}^nC_r \; r!,$$

$$^nC_a = {}^nC_b \Rightarrow a = b \text{ or } n = a + b$$

$$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

Combinations of n different things taking **one or more** out of n things at a time $2^n - 1$

- Write the given number say N in the form $N = p^a . q^b . r^c$

  where p, q, r are prime factors of the number N .

- Number of factors $N = (a + 1)(b + 1)(c + 1)$

Number of straight lines with the given n points	${}^{n}C_{2}$
Number of Straight lines with the given n points where m points are collinear	${}^{n}C_{2} - {}^{m}C_{2} + 1$
Number of triangles with the given n points	${}^{n}C_{3}$
Number of triangles with the given n points where m points are collinear	${}^{n}C_{3} - {}^{m}C_{3}$

Number of parallelogram with the given one set of m parallel lines and another set of n parallel lines	${}^{m}C_2 \times {}^{n}C_2$

Number of Diagonals with n sides	${}^{n}C_2 - n$

The maximum number of points of intersection of n circles will be	$n(n-1)$

AP	GP
A sequence $a_1, a_2, a_3, \ldots, a_n$ is called an Arithmetic Progression (A.P.) when $a_2 - a_1 = a_3 - a_2 = a_n - a_{n-1}$	A sequence $a_1, a_2, a_3, \ldots, a_n$ is called geometric progression, if each term is nonzero and $\frac{a_{k+1}}{a_k} = r$ (constant) for all $k \geq 1$
This constant 'd' is called the *common difference* of the A.P	The constant ratio is called its common ratio
d can be positive , negative, zero	

GENERAL FORM OF AP

$$a,\ a+d,\ a+2d \ldots\ldots\ldots\ldots$$

$$a_n = a + (n-1)\,d$$

| a | + | d | = | = | = |

2nd term, 3rd term, 4th term, ...so on

- If d is positive take + or if d is negative take –

Here, a is first term
d is common difference

nth term of a GP

$$a,\ ar,\ ar^2,\ ar^3,\ldots\ldots\ldots\ldots\ ar^{n-1}$$

$$a_n = ar^{n-1}$$

| r | x | a | = | = | = |

2nd term, 3rd term, 4th term, ...so on

AP

- If 3 numbers a, b, c are in A.P., we say

  $b - a = c - b$ or $a + c = 2b$;

  b is called the arithmetic mean between a and c.

GP

If a, b, c are in G.P

$b/a = c/b \Rightarrow b^2 = ac$,

$b = \sqrt{ac}$

b is called the **geometric mean** between **a** and **c** .

- *THREE NUMBERS IN AP*
- $(a - d), a, (a + d)$

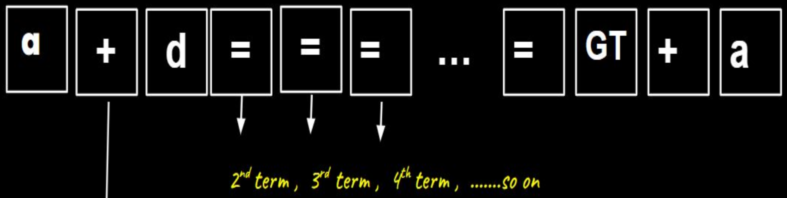
- *3 Numbers in GP*

$\frac{a}{r}, a, ar$

SUM OF First n terms of AP

$$S = \frac{n}{2}\{2a + (n-1)d\}$$

- $S = n(a + l)/2$



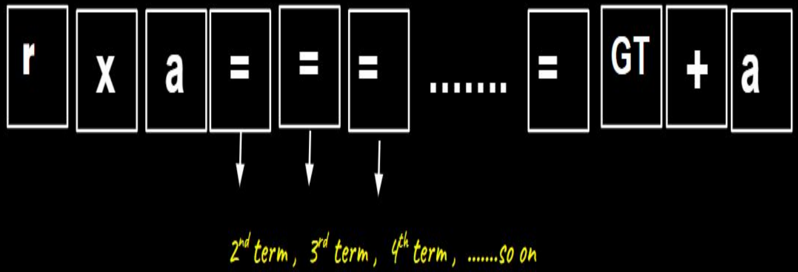
- *If d is positive take + or if d is negative take –*

Here, a is first term

d is common difference

SUM OF n TERMS OF A GP

$$S_n = \begin{cases} na, \text{ when } r = 1; \\ \dfrac{a(1 - r^n)}{(1 - r)}, \text{ when } r < 1; \\ \dfrac{a(r^n - 1)}{(r - 1)}, \text{ when } r > 1. \end{cases}$$



RESULTS

$$1 + 2 + 3 + \dots\dots + n = \frac{n(n+1)}{2}.$$

$$1 + 3 + 5 + \dots + (2n - 1) = n^2$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

TRICKS

- How many 3 digit numbers are divisible by 7?

Ans : 128

STEP 1: Write first and last 3 digit number

100 and 999

Step 2 : Divide both by 7

100/7 = 14.2857

999/7 = 142.714

Step 3 : Avoid decimal and Subtract

142 - 14 = 128

TRICKS

If the ratio between the sum of n terms of two AP is (7n +1) : (4n + 27) , find the ratio of their 11th term

Sol : Ratio of their 11 th term is

$$\frac{7(21) + 1}{4(21) + 27} = \frac{148}{111}$$

we need to find ratio of 11th term

Step 1 : multiply 11 by 2 and subtract 1

$2 \times 11 - 1 = 21$

Step 2 : In the given ratio substitute your n by 21 and get your answer

TRICKS

If 9 times 9^{th} term of an AP is equal to 13 times the 13^{th} term then the 22^{nd} term of an AP is ?

Ans : 0

If m times the mth term of an AP is equal to n times the n^{th} term, then $(m + n)^{th}$ term of the AP is 0

$$m\,a_m = n\,a_n$$

$$a_{m+n} = 0$$

TRICKS

If $a_{64} = 13$ and $a_{13} = 64$, find $a_{101} = ?$

Ans : - 24

If m^{th} term of a given AP is n and its n^{th} term is m then its pth term is (n + m - p)

$a_m = n$

$a_n = m$

$a_p = (n+m-p)$

GEOMETRIC PROGRESSIONS (AP)

SUM OF INFINITE GEOMETRIC SERIES

$$S_{\infty} = \frac{a}{1-r}, \quad \text{if } -1<r<1.$$

Type of Sets	Definition	Example
Empty Set	A set containing no element at all is called the empty set or the null set or the void set, denoted by ϕ. or { }.	$\{x : x \in N \text{ and } 2 < x < 3\} = \phi$
Singleton Set	A set containing exactly one element is called a singleton set.	$\{x : x \in Z \text{ and } x + 4 = 0\} = \{-4\}$,
Finite sets	• An empty set or a non-empty set in which the process of counting of elements surely comes to an end is called a finite set. • The number of distinct elements contained in a finite set A is denoted by n(A).	Let A = { 2, 4, 6, 8, 10, 12 }. n(A) = 6.

Infinite Sets	A set which is not finite is called an infinite set .	ii. N : the set of all natural numbers . iii. Z : the set of all integers.
Equal Set	Two non-empty sets A and B are said to be equal, if they have exactly the same elements and we write A = B.	Let A = Set of letters in the word 'follow' B = Set of letters in the word 'wolf ' Here , A= B
Equivalent Set	• Two finite sets A and B are said to be equivalent, if n(A) = n(B). • Equal sets are always equivalent. But, equivalent sets need not be equal.	i. Let A = {1, 3, 5} and B = {2, 4, 6}. Then, n(A) = n(B) = 3. So, A and B are equivalent.

- No. of **possible subsets** of set containing **n elements**

  $$2^n$$

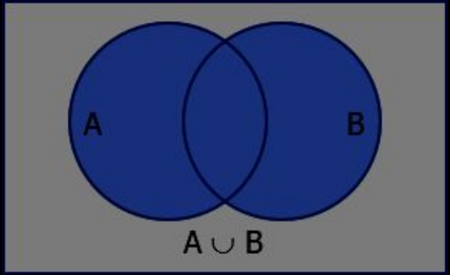
- Every set is a subset of itself.
- The empty set is a subset of every set .
- No. of **proper subsets** of set containing **n elements**

  $$2^n - 1$$

OPERATIONS ON SETS

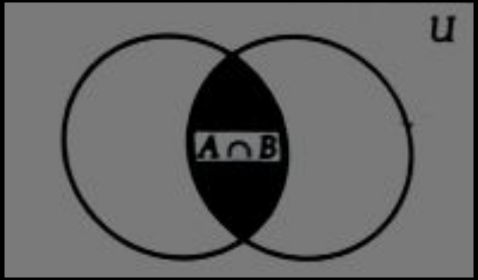
UNION

$A \cup B = \{x : x \in A \text{ or } x \in B\}$.



INTERSECTION

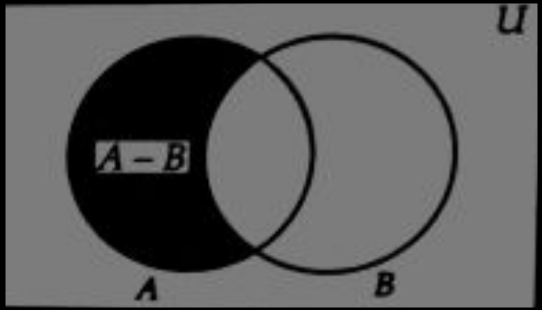
$A \cap B = \{x : x \in A \text{ and } x \in B\}$



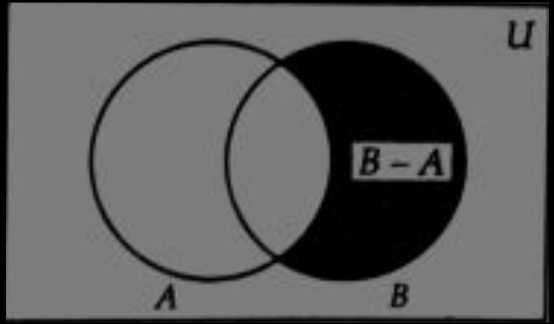
OPERATIONS ON SETS

DIFFERENCE OF SETS

A - B = {x : x ∈ A and x ∉ B}

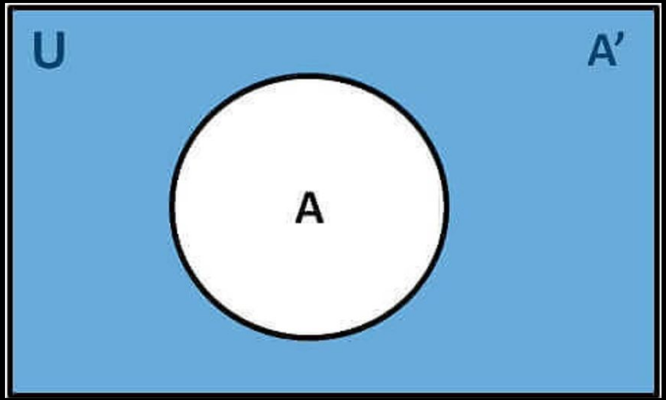


B - A = {x ∈ B and x ∉ A}



COMPLEMENT OF A SET

- Let U be the universal set and let A be a set such that A ⊂ U. Then, the complement of A with respect to U is denoted by A' or A^c or U - A and is defined the set of all those elements of U which are not in A



DE MORGAN'S LAW

For any two sets A and B,

(i) $(A \cup B)' = (A' \cap B')$

(ii) $(A \cap B)' = (A' \cup B')$

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

- $n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(P \cap R) + n(P \cap Q \cap R)$

If A and B are finite sets consisting of m and n elements respectively then A × B has mn

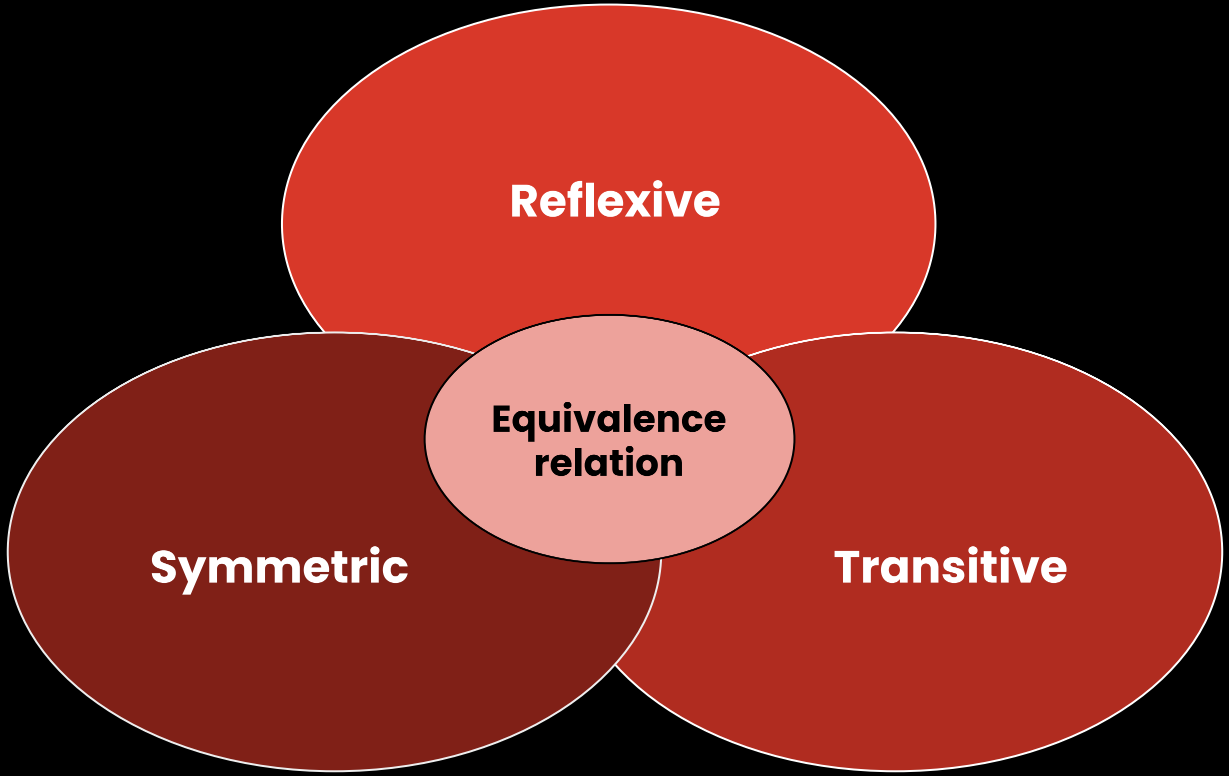
total number of relations from A to B is 2^{mn}.

DOMAIN ,RANGE , CODOMAIN OF A RELATION

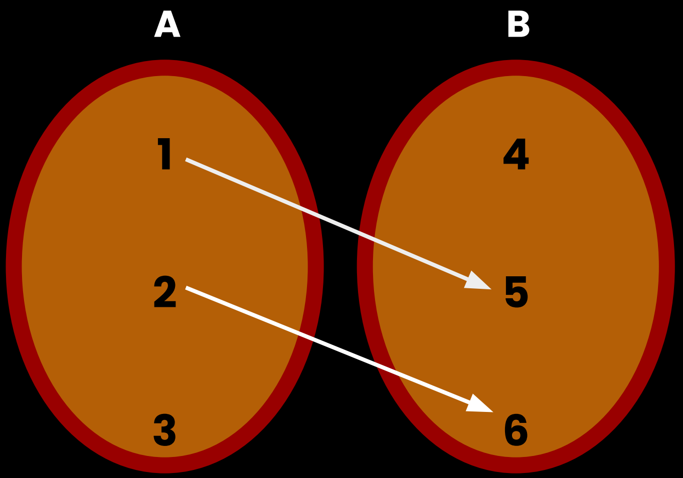
- If A = {1, 3, 5, 7}

B = {2, 4, 6, 8, 10} and R is relation from A to B

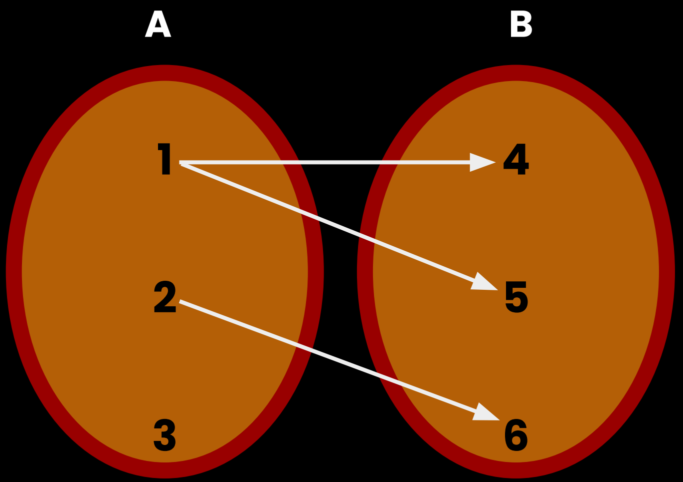
R = {(1, 8), (3, 6), (5, 2), (1, 4)}


Reflexive
Equivalence relation
Symmetric
Transitive

Que. Identity which of them is a function from A to B ?

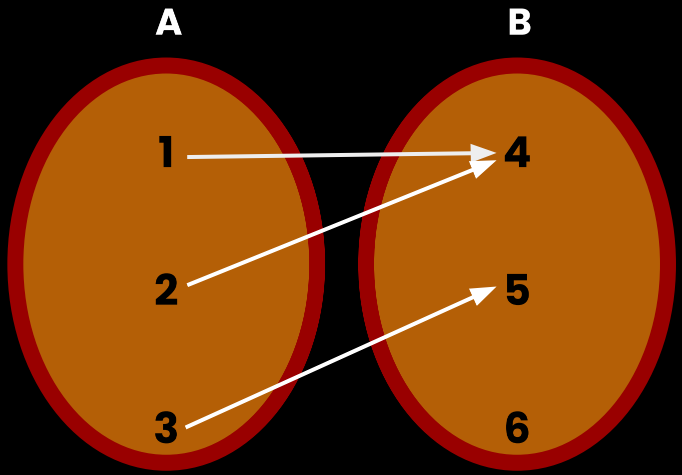


Not a Function

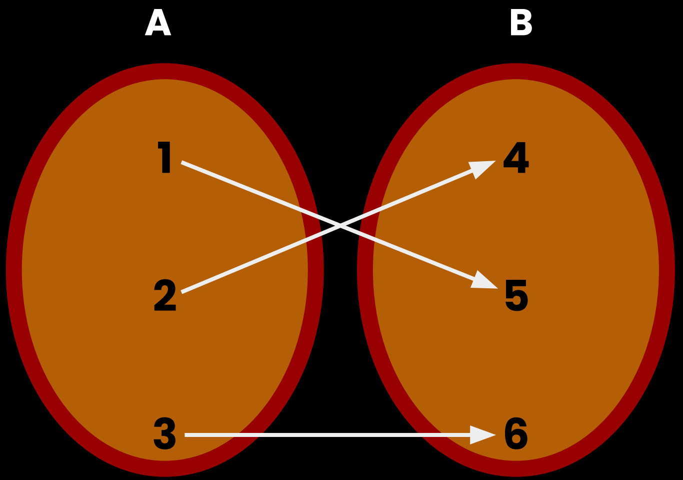


Not a Function

Que. Identity which of them is a function from A to B ?



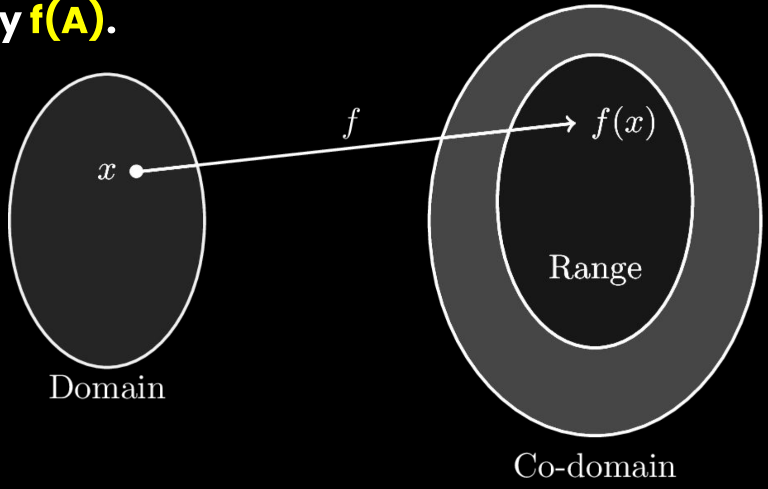
Function



Function

Domain, Codomain and Range of Function

- If $f : A \longrightarrow B$, the set A is known as the **domain** of f and the set B is known as the **co-domain** of f.
- The set of all f-images of elements of A is known as **the range of f or image set of A under f** and is denoted by **f(A)**.



IDENTITY FUNCTION

- The function $f : R \longrightarrow R$

$$f(x) = x$$

- Dom (f) = R and Range (f) = R

CONSTANT FUNCTION

$f : R \longrightarrow R :$

$$f(x) = k$$

- Dom (f) = R and
- Range (f) is the singleton set $\{ k \}$

COMPOSITION OF FUNCTION

- $fog(x) = f(g(X))$
- $gof(x) = g(f(x))$

INVERSE OF FUNCTION

- **ALGORITHM**

Let $f : A \longrightarrow B$ be a bijection . To find the inverse of f we follow the following steps :

STEP 1: Put $f(x) = y$

STEP 2: Solve $f(x) = y$ to obtain x in terms of y

STEP 3: In the relation obtained in Step 2 replace x by $f^{-1}(y)$ to obtain the required inverse

Language	Word
LATIN	STATUS
ITALIAN	STATISTA
GERMAN	STATISTIK
FRENCH	STATISTIQUE

PRIMARY

The data which are collected for the first time by an investigator or agency

SECONDARY

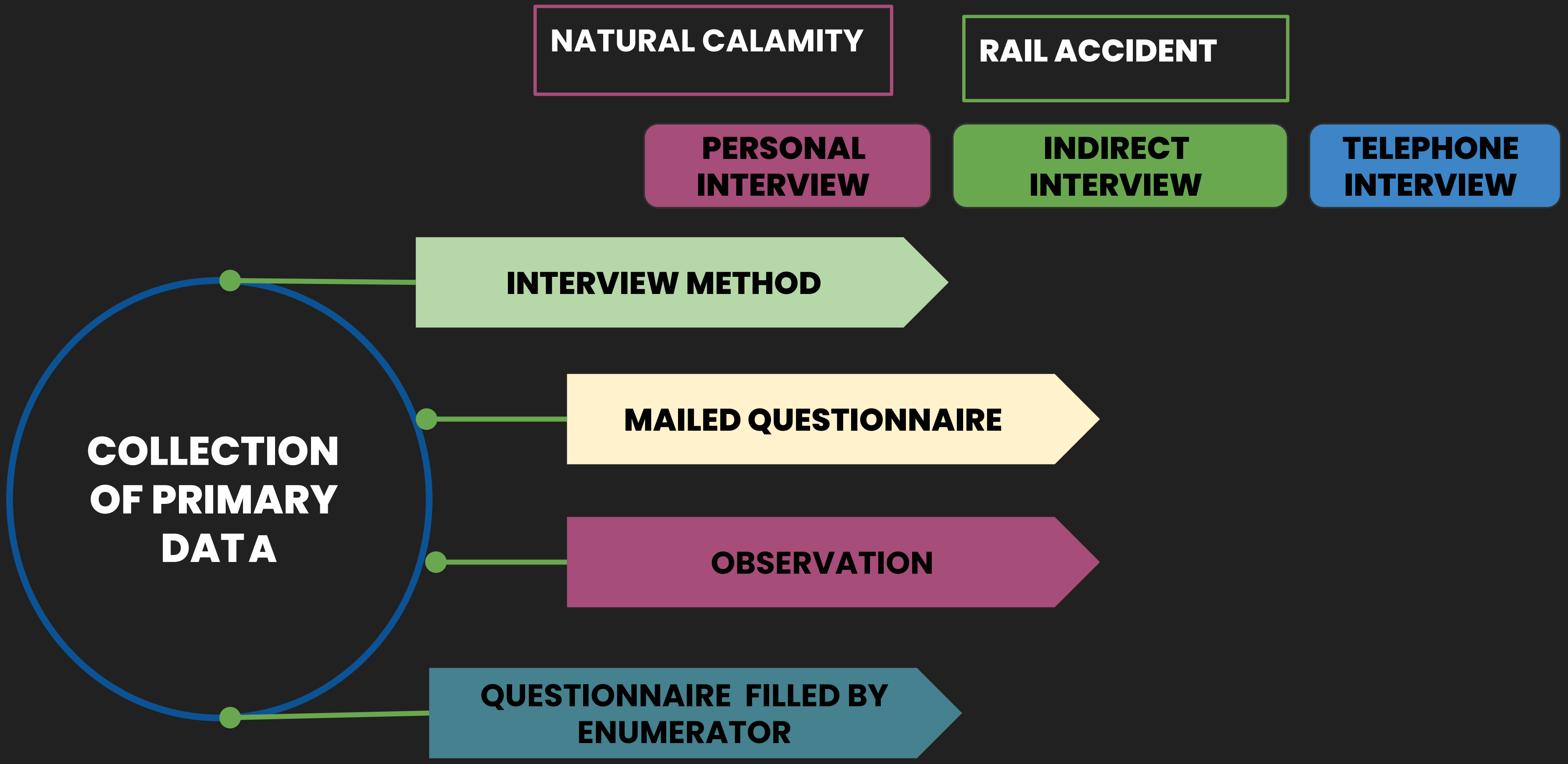
collected data used by a different person or agency.

DISCRETE VARIABLE

- Annual income of a person
- Marks of a student
- The distribution of shares

CONTINUOUS VARIABLE

- Age of a person
- The distribution of profits of a blue-chip company


NATURAL CALAMITY
RAIL ACCIDENT
PERSONAL INTERVIEW
INDIRECT INTERVIEW
TELEPHONE INTERVIEW
INTERVIEW METHOD
COLLECTION OF PRIMARY DATA
MAILED QUESTIONNAIRE
OBSERVATION
QUESTIONNAIRE FILLED BY ENUMERATOR

TABULAR PRESENTATION / TABULATION

BOX HEAD : entire upper part of the table which includes columns and sub-column numbers, unit(s) of measurement along with caption.

CAPTION : the upper part of the table, describing the columns and sub-columns,

STUB : left part of the table providing the description of the rows.

BODY : main part of the table that contains the numerical figures.

FOOTNOTE : source of the data at the bottom of table

Pie chart

It is used for circular presentation of relative data

Segment angle = $$\frac{(\text{segment value} \times 360^0)}{(\text{total value})}$$

Class length = UCB - LCB

$$\text{mid-point} = \frac{LCL + UCL}{2} = \frac{LCB + UCB}{2}$$

No. of class interval × class lengths = Range

Frequency Density

$$\text{Frequency Density} = \frac{\text{Class Frequency}}{\text{Class Length of Class}}$$

Relative Frequency

$$\text{Relative frequency} = \frac{\text{Class Frequency}}{\text{Total Frequency}}$$

Relative frequencies add up to unity

Relative frequency for a particular class Lies between 0 and 1

Percentage Frequency

$$\text{Percentage Frequency} = \frac{\text{Class Frequency}}{\text{Total Frequency}} \times 100$$

percentage frequencies add up to one hundred.

HISTOGRAM / AREA DIAGRAM

- This is a very convenient way to represent a frequency distribution.
- Comparison between frequency of two different classes are possible
- It is used to calculate **MODE.**

OGIVES / CUMULATIVE FREQUENCY GRAPH

- quartiles , median

By plotting cumulative frequency against the respective class boundary, we get ogives

TWO TYPES OF OGIVES

Less than type Ogives

- less than type ogives, obtained by taking less than cumulative frequency on the vertical axis

More than type Ogives

- more than type ogives by plotting more than type cumulative frequency on the vertical axis

- Starting with a population of N units, we can draw many a sample of a fixed size n.

- In case of sampling with replacement, the total number of samples that can be drawn is N^n

- When it comes to sampling without replacement, the total number of samples that can be drawn is ${}^{N}C_{n}$

ARITHMETIC MEAN

Discrete Observation

$$\bar{X} = \frac{x_1 + x_2 + x_3 + \dots\dots\dots\dots + x_n}{n}$$

$$= \frac{\sum_{i=1}^{n} x_i}{n}$$

Simple Frequency Distribution

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \dots\dots\dots\dots + f_nx_n}{f_1 + f_2 + f_3 + \dots\dots\dots\dots + f_n}$$

$$= \frac{\sum f_ix_i}{\sum f_i}$$

$$\bar{X} = \frac{\sum f_ix_i}{N}$$

where ,

$N = \Sigma f_i$.

Grouped Frequency Distribution

$$\bar{X} = \frac{\sum f_ix_i}{N}$$

where ,

x_i= mid point of class interval

$N = \Sigma f_i$.

GEOMETRIC MEAN

Average Rates , percentages

- For a given set of n positive observations, the geometric mean is defined as the n-th root of the product of the observations.

Discrete Observation

$$G = (x_1 \times x_2 \times x_3 \ldots\ldots\ldots\ldots \times x_n)^{1/n}$$

Frequency Distribution

$$G = (x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \ldots\ldots\ldots\ldots\ldots\ldots \times x_n^{f_n})^{1/N}$$

HARMONIC MEAN

Average Rates ,SPEED

- For a given set of non-zero observations, harmonic mean is defined as the reciprocal of the AM of the reciprocals of the observation.

Discrete Observation

$$H = \frac{n}{\Sigma(1/x_i)}$$

Frequency Distribution

$$H = \frac{N}{\Sigma\left[\frac{f_i}{x_i}\right]}$$

COMBINED ARITHMETIC MEAN

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

COMBINED HARMONIC MEAN

$$\frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$$

RELATION BETWEEN AM ,GM ,HM

When all the observations are distinct → AM > GM > HM

When all the observations are same → AM = GM = HM

When nothing is mentioned → AM ≥ GM ≥ HM

RELATION BETWEEN AM ,GM ,HM

$$GM^2 = AM \times HM$$

This result holds for only two positive observations

- If all the values assumed by a variable are constant , say k , then the AM ,GM HM is also k .

HM of 1,1/2, 1/3,.................1/n is given by

$$\frac{2}{(n+1)}$$

- To calculate Average speed , use Harmonic Mean .

  The harmonic mean of two numbers x and y is given by

$$\frac{2xy}{x+y}$$

MEDIAN – PARTITION VALUE

FOR DISCRETE OBSERVATION

$$\text{Median} = \begin{cases} \left(\dfrac{n+1}{2}\right)\text{th observation, if } n \text{ is odd} \\[2ex] \dfrac{\left(\dfrac{n}{2}\right)\text{th observation} + \left(\dfrac{n}{2}+1\right)\text{th observation}}{2}, \text{ if } n \text{ is even} \end{cases}$$

FOR SIMPLE FREQUENCY DISTRIBUTION

- Arrange the series into ascending or descending order.
- Calculate cumulative frequency .
- Calculate $\dfrac{N+1}{2}$
- Check cumulative frequency which is greater than $\dfrac{N+1}{2}$
- The value of x corresponding to this cumulative frequency would be the median .

FOR GROUPED FREQUENCY DISTRIBUTION

Compute the median using the formula:

$$\text{Median, } M_e = l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}, \text{ where}$$

l = lower limit of median class;

h = width of median class;

f = frequency of median class;

cf = cumulative frequency of the class preceding the median class;

$N = \Sigma f_i$.

PARTITION VALUE

DISCRETE OBSERVATIONS

$$(n + 1)\, p^{th} \text{ term}$$

Where

n denotes the total number of observations

- $p = 1/4, 2/4, 3/4$ for Q_1, Q_2 and Q_3 respectively.
- $p = 1/10, 2/10, \ldots\ldots\ldots\ldots 9/10$. For D_1, $D_2, \ldots\ldots, D_9$ respectively.
- $p = 1/100, 2/100, \ldots\ldots, 99/100$ for P_1, P_2, $P_3 \ldots\ldots P_{99}$ respectively.

MODE

Find the class interval with the highest frequency

•

This class interval is called MODAL CLASS

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where l = lower limit of the modal class,

[redacted]ning all class sizes to be equal),

f_1 = frequency of the modal class,

f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class succeeding the modal class.

RELATIONSHIP BETWEEN MEAN , MODE AND MEDIAN

FOR SYMMETRIC DATA

Mean = Median = Mode

In case of MODERATELY SKEWED DISTRIBUTION (EMPIRICAL RELATIONSHIP)

Mean – Mode = 3(Mean – Median)

Or

Mode = 3 Median – 2 Mean

the algebraic sum of deviations of a set of observations from their AM is zero

i.e. for unclassified data, $\sum (x_i - \bar{x}) = 0$ ....(14.1.4)

- **For a set of observations, the sum of absolute deviations is minimum when the deviations are taken from the median.**

  $\Sigma |x_i - A|$ is minimum if we choose A as the median.

- **AM is affected both due to change of origin and scale.**

  If $y = a + bx$ then $\bar{y} = a + b\bar{x}$.

- **If x and y are two variables, to be related by $y = a + bx$ for any two constants a and b, then the median of y is given by**

  $y_{me} = a + b\,x_{me}$

- **Mode is affected due to change in scale and due to change in origin .**

  if $y = a + bx$, then $y_{mo} = a + bx_{mo}$

RANGE

Discrete Observation

Range = L – S

Where ,

L : largest observations

S : smallest observations

COEFFICIENT OF RANGE

$$\text{Coefficient of range} = \frac{L-S}{L+S} \times 100$$

RANGE

Grouped Frequency distribution

Range = Uppermost Class Boundary – Lowermost Class Boundary

COEFFICIENT OF RANGE

$$\frac{\text{Uppermost class boundary} - \text{Lowermost class boundary}}{\text{Uppermost class boundary} + \text{Lowermost class boundary}} \times 100$$

MEAN DEVIATION

Discrete Observation

$$MD_A = \frac{1}{n}\Sigma|x_i - A|$$

Frequency Distribution

$$MD_A = \frac{1}{N}\Sigma f\,|x - A|$$

COEFFICIENT OF Mean deviation

$$\text{Coefficient of mean deviation} = \frac{\text{Mean deviation about A}}{\text{A}} \times 100$$

- **Mean Deviation takes its minimum value when deviations are taken from Median**

STANDARD DEVIATION

DISCRETE OBSERVATION

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

Or

$$s = \sqrt{\frac{\sum x_i^{\,2}}{n} - \bar{x}^2}$$

FREQUENCY DISTRIBUTION

$$s = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{N}}$$

Or

$$\sqrt{\frac{\sum f_i x_i^{\,2}}{N} - \bar{x}^2}$$

$$\text{Coefficient of Variation (CV)} = \frac{SD}{AM} \times 100$$

SD for any two numbers

$$S = \frac{\text{Range}}{2}$$

SD for first n natural numbers

$$\sqrt{\frac{n^2 - 1}{12}}.$$

QUARTILE DEVIATION

- Another measure of dispersion is provided by quartile deviation or semi-inter - quartile range which is given by

$$Q_d = \frac{Q_3 - Q_1}{2}$$

$$\text{Coefficient of quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

RELATIONSHIP BETWEEN SD, MD AND QD

4 SD = 5 MD = 6 QD

Or

SD : MD : QD = 15 : 12 : 10

If all the observations are constant i.e. equal, then the range ,MD ,SD , is zero.

Range , MD ,SD ,QD remains unaffected due to a change of origin but affected in the same ratio due to a change in scale .

$y = a + bx$,

- $R_y = |b| \times R_x$
- $MD_Y = |b| \times MD_X$
- $S_y = |b| S_x$
- $QD_Y = |b| \times QD_X$

DIVISIONS OF PROBABILITY

SUBJECTIVE PROBABILITY

OBJECTIVE PROBABILITY

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

COMPOSITE / COMPOUND EVENT

Event that can be subdivided into further events

The probability of an event lies between 0 and 1, both inclusive.

i.e. $0 \leq P(A) \leq 1$

SURE EVENT

- If probability of occurrence of an event is 1

IMPOSSIBLE EVENT

- If probability of occurrence of an event is 0

$$P(A) + P(A') = 1$$

If more than one object is to be selected

Use combination to calculate favourable outcome and total outcome

ODDS IN FAVOUR

Odds in favour of an event A

$$= \frac{\text{no of favorable events to A}}{\text{no of unfavorable events to A}}$$

ODDS AGAINST AN EVENT

Odds against an event A

$$= \frac{\text{no of unfavourable events to A}}{\text{no of favourable events to A}}$$

PROBABILITY OF AN EVENT

$$P(A) = \frac{\text{no of favourable events to A}}{\text{no of favourable + no of unfavourable}}$$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- If A and B are mutually exclusive

$$P(A \cup B) = P(A) + P(B)$$

- If A and B are mutually exclusive then $A \cap B = \Phi$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

- If A ,B and C are mutually exclusive

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

- Two events A and B are exhaustive if

$$P(A \cup B) = 1$$

- Three events A, B and C are exhaustive if

$$P(A \cup B \cup C) = 1$$

- Events whose union is equal to sample space

- Three events A, B and C are equally likely if

$$P(A) = P(B) = P(C)$$

- If A, B and C are mutually exclusive and exhaustive events,

then , $P(A) + P(B) + P(C) = 1$

RESULT

- **Probability that only event A occurs**

$$P(A-B) = P(A \cap B') = P(A) - P(A \cap B)$$

- **Probability that only event B occurs**

$$P(B-A) = P(B \cap A') = P(B) - P(A \cap B)$$

COMPOUND PROBABILITY / JOINT PROBABILITY

WITHOUT REPLACEMENT
(DEPENDENT EVENT)

$$P(A \cap B) = P(A) \,.\, P(B \mid A)$$

WITH REPLACEMENT
(INDEPENDENT EVENT)

$$P(A \cap B) = P(A) \,.\, P(B)$$

It is used when we have to find simultaneous occurrence of two or more events

CONDITIONAL PROBABILITY

Event for which we are finding Conditional Probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Event which is occured

RANDOM VARIABLE

If a coin is tossed three times

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

X denotes the number of heads , then X is a random variable variable.

X = { 0 , 1, 2 ,3 }

PROBABILITY DISTRIBUTION

X	0	1	2	3
P	1/8	3/8	3/8	1/8

(i) $p_i \geq 0$ for every i

(ii) $\Sigma p_i = 1$ (over all i)

RANDOM VARIABLE / PROBABILITY DISTRIBUTION

X	0	1	2	3
P	1/8	3/8	3/8	1/8

Expected Value $\mu = E(x) = \sum p_i x_i$

$$E(X) = 0 \times 1/8 + 1 \times 3/8 + 2 \times 3/8 + 3 \times 1/8$$

$$= 12/8$$

$$= 1.5$$

RANDOM VARIABLE / PROBABILITY DISTRIBUTION

X	0	1	2	3
P	1/8	3/8	3/8	1/8

Variance of x, to be denoted by , σ^2 is given by

$$V(x) = \sigma^2 = E(x-\mu)^2$$

$$= E(x^2) - \mu^2$$

$E(X^2) = \frac{1}{8} \times 0^2 + \frac{3}{8} \times 1^2 + \frac{3}{8} \times 2^2 + \frac{1}{8} \times 3^2 = 3$

$E(x) = 1.5$

$v(x) = 0.75$

$SD = \sqrt{0.75}$

COINS

- Total number of elements in sample space while tossing a coin is given by 2^n

1. If a coin is tossed once $2^1 = 2$

   {H, T}

1. If two coins are tossed once or one coin tossed twice $2^2 = 4$

   {HH, HT, TH, TT}

1. If three coins are tossed once or one coin is tossed thrice

   $2^3 = 8$

   {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

DICE

- **Total number of elements in sample space while tossing a dice is given by 6^n**

1. **If a dice is rolled once $6^1 = 6$**

   {1, 2, 3, 4, 5, 6}

DICE

2. If two die is rolled once or one dice is rolled twice

$6^2 = 36$

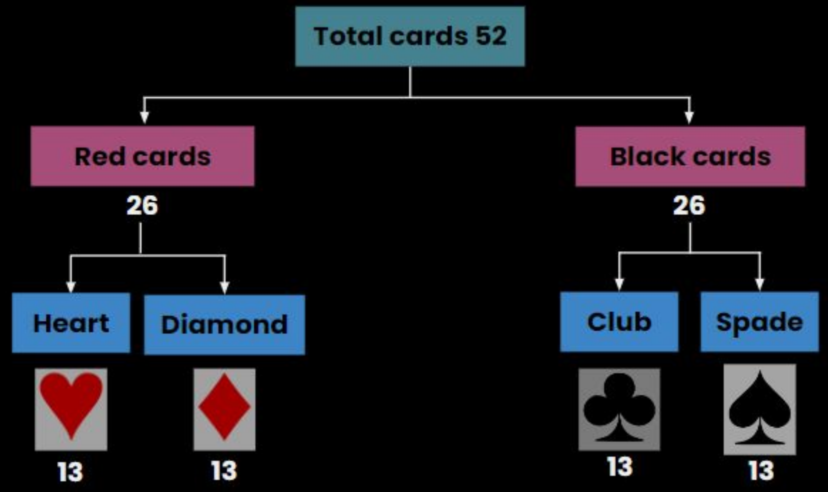
	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

DICE

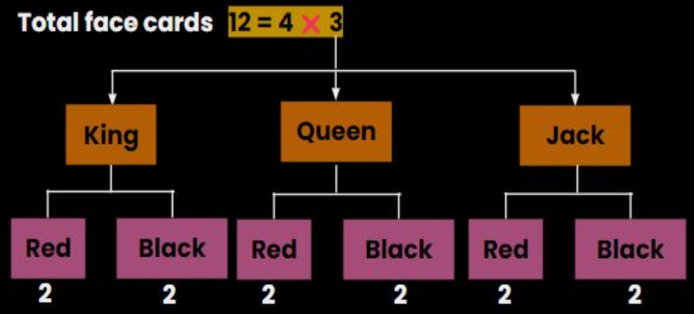
3. If three dice are rolled once or one dice is rolled thrice

$6^3 = 216$

CARDS


Total cards 52
Red cards
26
Black cards
26
Heart
13
Diamond
13
Club
13
Spade
13

CARDS


Total face cards 12 = 4 × 3
King
Queen
Jack
Red
Black
Red
Black
Red
Black
2
2
2
2
2
2


K
K


Q
Q
Q
Q


J
J

BINOMIAL DISTRIBUTION

(bi - parametric discrete probability distribution)

- A **discrete random variable X** is defined to follow binomial distribution with parameters n and p,

$$X \sim B(n, p),$$

Probability Mass Function

$$f(x) = p(X = x) = {}^{n}c_{x}\, p^{x}\, q^{n-x} \quad \text{for } x = 0, 1, 2, \ldots, n$$

BINOMIAL DISTRIBUTION

(bi - parametric discrete probability distribution)

Mean

$$\mu = n\,p$$

Variance

- The variance of the binomial distribution is given by

$$\sigma^2 = n\,p\,q$$

Variance of a binomial variable is always less than its mean.

Variance of X attains its maximum value at $p = q = 0.5$ and this maximum value is $n/4$.

MODE

$(n+1)p$

INTEGER

- $\mu_0 = (n+1)p$
- $\mu_0 = (n+1)p - 1$

Bi - Modal

NON - INTEGER

μ_0 = the largest integer contained in $(n+1)p$

Uni- Modal

POISSON DISTRIBUTION

(UNI- parametric discrete probability distribution)

- **Poisson distribution is applied when the total number of events is pretty large but the probability of occurrence is very small.**

- **A discrete random variable X that follows Poisson Distribution denoted as**

$$X \sim P(m)$$

POISSON DISTRIBUTION

- A discrete random variable X that follows Poisson Distribution denoted as

$$X \sim P(m)$$

Probability Mass Function

$$f(x) = P(X = x) = \frac{e^{-m} . m^{x}}{x!} \text{ for } x = 0, 1, 2, \ldots \infty$$

where ,

$e = 2.71828$

$m = n p$

POISSON DISTRIBUTION

Mean

- The mean of Poisson distribution is given by

$$\mu = m$$

Variance

- The variance of Poisson distribution is given by

$$\sigma^2 = m$$

Standard Deviation

$$\sqrt{m}$$

POISSON DISTRIBUTION

MODE

m

INTEGER

- $\mu_0 = m$
- $\mu_0 = m - 1$

Bi - Modal

NON - INTEGER

μ_0 = the largest integer contained in m

Uni- Modal

NORMAL DISTRIBUTION

(BI - parametric CONTINUOUS probability distribution)

- A **continuous random variable x** is defined to follow normal distribution with parameters μ and σ^2, to be denoted by

$$X \sim N(\mu, \sigma^2)$$

NORMAL DISTRIBUTION

(BI - parametric CONTINUOUS probability distribution)

Probability Density Function

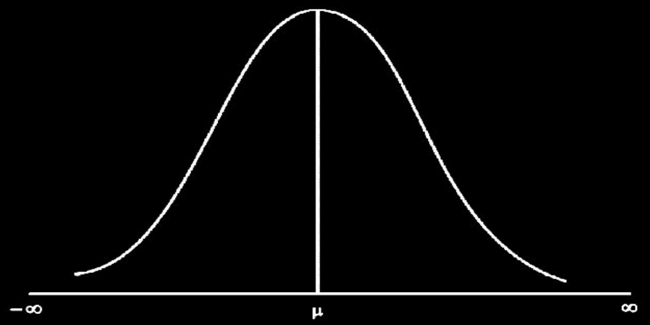
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} . e^{-(\bar{x}-u)^2/2\sigma^2}$$

for $-\infty < x < \infty$.

- e = 2.71828
- X = random variable
- **μ = mean of normal random variable x**
- **σ = standard deviation of the given normal distribution**

NORMAL CURVE

- The normal curve is bell shaped .
- The line drawn through x = μ has divided the normal curve into two parts which are equal in all respect.
- Normal distribution is symmetrical about x = μ. As such, its skewness is zero
- The two tails of the normal curve extend indefinitely on both sides of the curve and both the left and right tails never touch the horizontal axis.
- The total area of the normal curve or for that any probability curve is taken to be unity i.e. one.



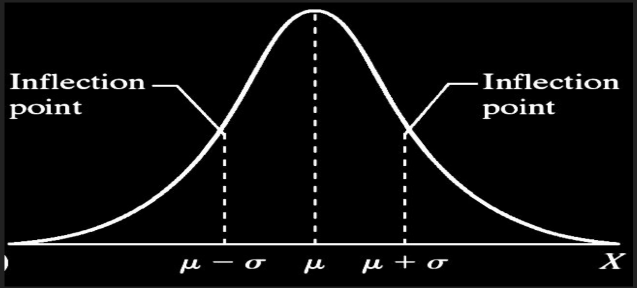
Normal curve /probability curve,

The area under this curve gives us the probability .

The area between −∞ to μ = the area between to μ to ∞ = 0.5

NORMAL DISTRIBUTION

MEAN = MEDIAN = MODE = μ (Symmetric distribution)

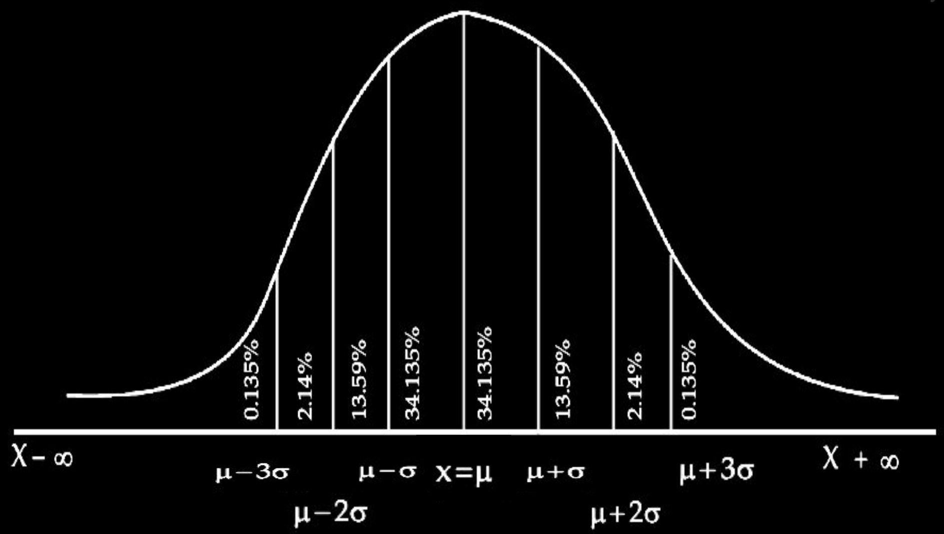


Two points of inflexion

- $\mu - \sigma$ and $\mu + \sigma$

VARIANCE	σ^2 (given in question)
Standard deviation	σ
Mean deviation	$0.8\ \sigma$
Quartile Deviation	$0.675\ \sigma$
Quartiles	$Q_1 = \mu - 0.675\sigma$ $Q_3 = \mu + 0.675\sigma$

NORMAL CURVE



$$P(\mu - \sigma < x < \mu + \sigma) = 0.6828$$

$$P(\mu - 2\sigma < x < \mu + 2\sigma) = 0.9546$$

$$P(\mu - 3\sigma < x < \mu + 3\sigma) = 0.9973$$

NORMAL CURVE

- If x and y are independent normal variables with means and standard deviations as μ_1 and μ_2 and σ_1 and σ_2, respectively, then z = x + y also follows normal distribution with

  $SD = \sqrt{\sigma_1^2 + \sigma_2^2}$ respectively.

- mean $(\mu_1 + \mu_2)$ and

STANDARD NORMAL DISTRIBUTION

- If we take $\mu = 0$ and $\sigma = 1$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad \text{for } -\infty < z < \infty$$

- The random variable z is known as standard normal variate (or variable) or standard normal deviate.

- It is given by $z = \frac{x - \mu}{\sigma}$

IMPORTANT RESULTS of STANDARD NORMAL DISTRIBUTION

- Mean = Median = Mode= 0
- The standard normal distribution is symmetrical about z = 0
- Variance = 1
- Standard deviation = 1
- Point of Inflexion = -1 and 1
- Mean deviation = 0.8
- Quartile deviation = 0.675

Cumulative Distribution Function

$P(z \le k) = \phi(k)$

$$P(x < a) = P\left[\frac{x-\mu}{\sigma} < \frac{a-\mu}{\sigma}\right]$$

$$= P(z < k),\ (k = a - \mu/\sigma)$$

$$= \phi(k) \quad \ldots\ldots\ldots\ldots (16.27)$$

Also $P(x \le a) = P(x < a)$ as x is continuous.

$\phi(-k) = 1 - \phi(k)$

$$P(x > b) = 1 - P(x \le b)$$

$$= 1 - \phi(b - \mu/\sigma) \quad \ldots\ldots\ldots\ldots (16.28)$$

$$P(a < x < b) = \phi(b - \mu/\sigma) - \phi(a - \mu/\sigma)$$

- $\phi(k)$ gives the area from $-\infty$ to the point K
- Z table gives us the probability of values z = 0 to any value of z

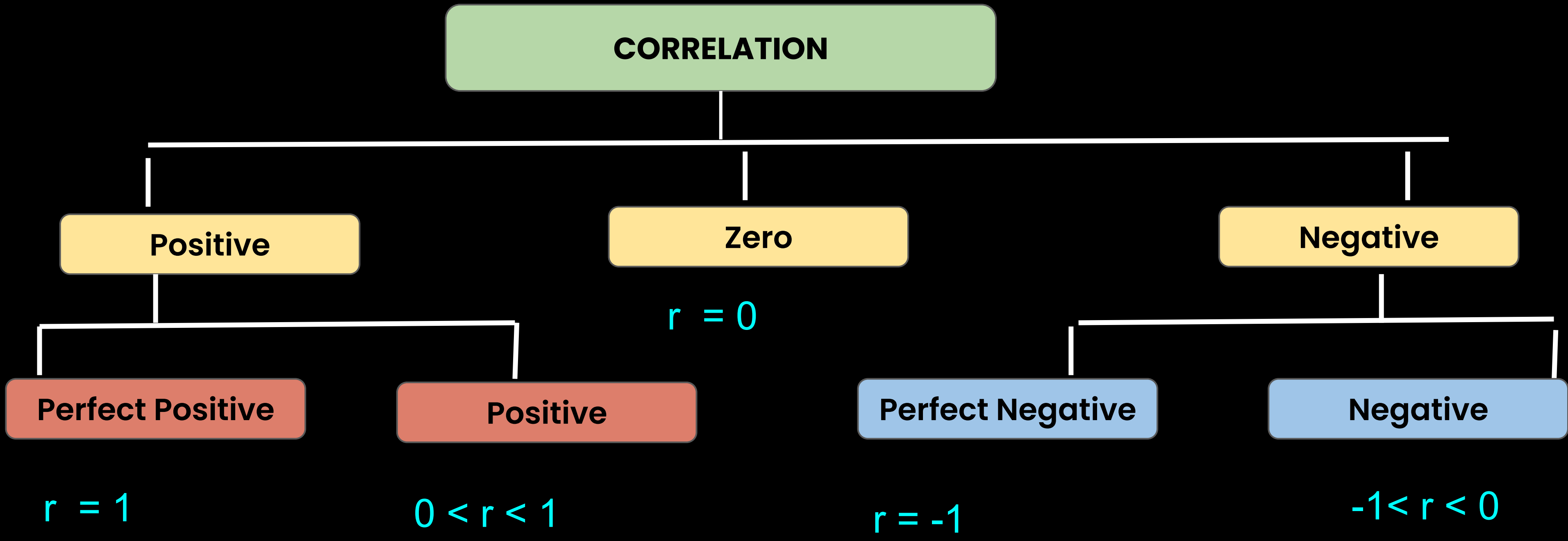
- No . of cells = m x n
  where ,
  m = no. of class interval of x
  n = no. of class interval of y

- No . of Marginal Distributions in Bivariate data = 2

- No . of Conditional Distributions = m +n
  where ,
  m = no. of class interval of x
  n = no. of class interval of y

Correlation

- **Correlation is expressed using r**
- **The value of correlation ranges from - 1 to 1 , both inclusive** $-1 \leq r \leq 1$.



KARL PEARSON'S PRODUCT MOMENT CORRELATION COEFFICIENT

$$r = r_{xy} = \frac{Cov(x, y)}{S_x \times S_y} \quad \text{..................................}$$

where

$$cov(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n} = \frac{\sum x_i y_i}{n} - \bar{x}\bar{y} \; \text{..}$$

$$S_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2} \quad \text{.................}$$

(i) The Coefficient of Correlation is a unit-free measure.

(ii) The coefficient of correlation always lies between –1 and 1, including both the limiting values $-1 \leq r \leq 1$.

(iii) If two variables are related by a linear equation , then correlation coefficient will always be perfect +1 or -1 depends on the sign of slope of equation .

PROPERTIES OF CORRELATION COEFFICIENT

- **Change of Origin : NO Impact**
- **Change of Scale : No Impact of value but affected by sign**

- **If sign of both change of scale are same**

$$r_{uv} = r_{xy}$$

- **If sign of both change of scale are different**

$$r_{uv} = -r_{xy}$$

SPEARMAN'S RANK CORRELATION COEFFICIENT

- **When we need finding correlation between two qualitative characteristics, say, beauty and intelligence, we take recourse to using rank correlation coefficient.**

$$r_R = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$$

COEFFICIENT OF CONCURRENT DEVIATIONS

- A very simple and casual method of finding correlation when we are not serious about the magnitude of the two variables .

$$r_c = \pm\sqrt{\pm\frac{(2c-m)}{m}}$$

where c is the number of concurrent deviations (same direction)

m is number of pairs compared , m = n-1

Estimation of Y when X is given → **Y on X**

Y : Dependent

X : Independent

$$y = a + bx$$

Estimation of X when Y is given → **X on Y**

X : Dependent

Y : Independent

$$x = a + by$$

REGRESSION

Estimation of Y when X is given

Regression line of Y on X

$$Y - \overline{Y} = b_{yx}(X - \overline{X})$$

Estimation of X when Y is given

Regression line of X on Y

$$X - \overline{X} = b_{xy}(Y - \overline{Y})$$

METHOD OF LEAST SQUARES

REGRESSION COEFFICIENT

Regression Coefficient of Y on X

$$b_{yx} = \frac{Cov(x,y)}{\text{Var of } x}$$

$$b_{yx} = r \cdot \frac{SD_y}{SD_x}$$

REGRESSION COEFFICIENT

Regression Coefficient of X on Y

$$b_{xy} = \frac{Cov(x,y)}{Var\ of\ y}$$

$$b_{xy} = r \cdot \frac{SD_x}{SD_y}$$

Example If the relationship between two variables x and u is $u + 3x = 10$ and between two other variables y and v is

$2y + 5v = 25$, and the regression coefficient of y on x is known as 0.80, what would be the regression coefficient of v on u?

The regression coefficients remain unchanged due to a shift of origin but change due to a shift of scale.

PROPERTIES REGRESSION LINES / COEFFICIENTS

(ii) The two lines of regression intersect at the point $(\bar{x}, \bar{y})$ mean where x and y are the variables under consideration.

According to this property, the point of intersection of the regression line of y on x and the regression line of x on y is $(\bar{x}, \bar{y})$ i.e. the solution of the simultaneous equations in x and y.

PROPERTIES REGRESSION LINES / COEFFICIENTS

(iii) The coefficient of correlation between two variables x and y is the simple geometric mean of the two regression coefficients. The sign of the correlation coefficient would be the common sign of the two regression coefficients.

$$r = \pm \sqrt{b_{yx} \times b_{xy}}$$

If both the regression coefficients are negative, r would be negative and if both are positive, r would assume a positive value.

NOTE

- Product of the regression coefficient must be numerically less than unity .
- The two lines of regression coincide i.e. become identical when $r = -1$ or 1 or in other words, there is a perfect negative or positive correlation between the two variables under discussion.
- If $r = 0$ Regression lines are perpendicular to each other

Coefficient of Determination / Explained Variance / Accounted Variance = r^2

Coefficient of non-determination = $(1 - r^2)$

$$\text{Simple aggregative price index} = \frac{\Sigma P_n}{\Sigma P_o} \times 100$$

$$\text{SIMPLE AVERAGE OF RELATIVES} = \frac{\Sigma\left(\frac{P_n}{P_o} \times 100\right)}{N}$$

WEIGHTED AGGREGATIVE INDEX

a. **Laspeyres' Index**: In this Index base year quantities are used as weights:

$$\text{Laspeyres Index} = \frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times 100$$

b **Paasche's Index:** In this Index current year quantities are used as weights:

$$\text{Passche's Index} = \frac{\Sigma P_n Q_n}{\Sigma P_o Q_n} \times 100$$

WEIGHTED AGGREGATIVE INDEX

C **The Marshall-Edgeworth index** uses this method by taking the **average** of the base year and the current year

$$\text{Marshall-Edgeworth Index} = \frac{\Sigma P_n (Q_o + Q_n)}{\Sigma P_o (Q_o + Q_n)} \times 100$$

d. **Fisher's ideal Price Index:** This index is the **geometric mean** of Laspeyres' and Paasche's.

$$\text{Fisher's Index} = \sqrt{\frac{\Sigma P_n Q_o}{\Sigma P_o Q_o} \times \frac{\Sigma P_n Q_n}{\Sigma P_o Q_n}} \times 100$$

WEIGHTED AGGREGATIVE INDEX

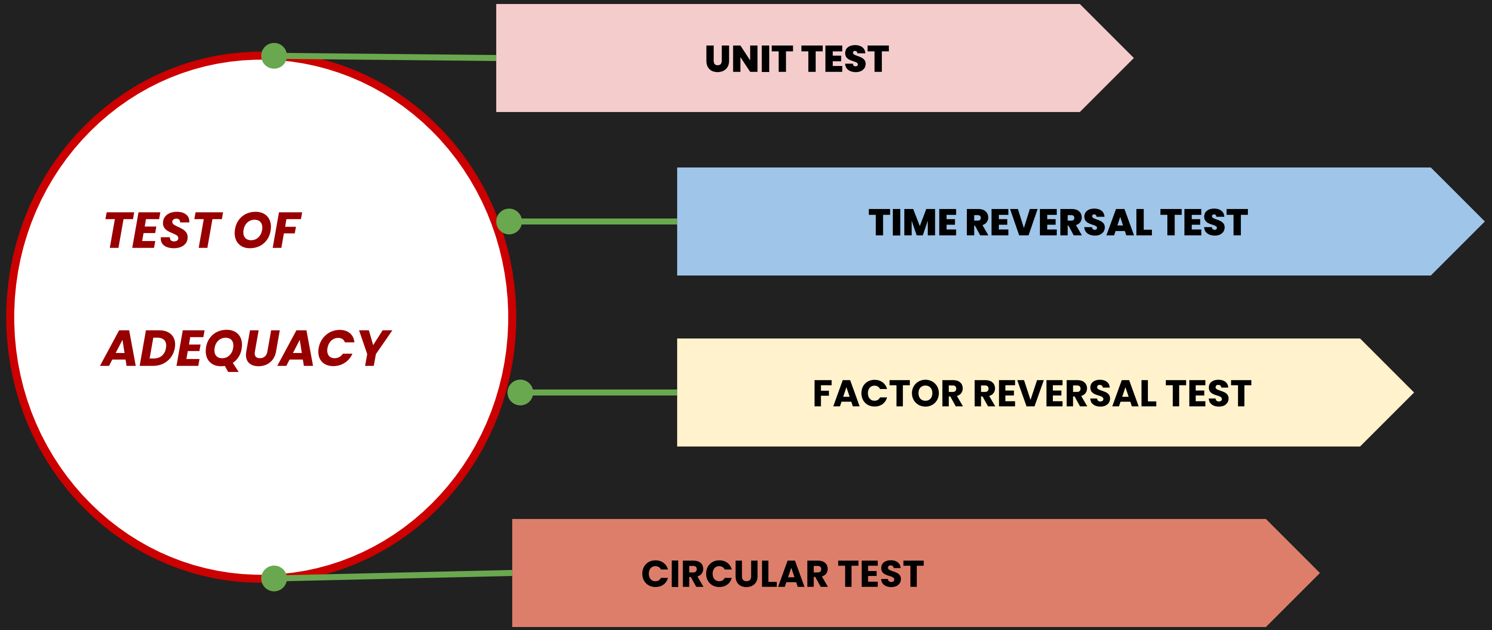
BOWLEY INDEX:

$$\frac{\text{Laspeyres' Index} + \text{Paasche's Index}}{2}$$

$$\text{Chain Index} = \frac{\text{Link relative of current year} \times \text{Chain Index of the previous year}}{100}$$

$$\text{Deflated Value} = \frac{\text{Current Value}}{\text{Price Index of the current year}}$$

$$\text{Shifted Price Index} = \frac{\text{Original Price Index}}{\text{Price Index of the year on which it has to be shifted}} \times 100$$


TEST OF ADEQUACY
UNIT TEST
TIME REVERSAL TEST
FACTOR REVERSAL TEST
CIRCULAR TEST

UNIT TEST

This test requires that the formula should be independent of the unit

Except for the simple (unweighted) aggregative index all other formulae satisfy this test

TIME REVERSAL TEST

$$P_{01} \times P_{10} = 1$$

Laspeyres' method and Paasche's method do not satisfy this test, but Fisher's Ideal Formula does.

FACTOR REVERSAL TEST

$$P_{01} \times Q_{01} = V_{01}$$

Fisher's Index satisfies Factor Reversal test

CIRCULAR TEST

$$P_{01} X P_{12} X P_{20} = 1$$

shiftability of base

This test is not met by Laspeyres, or Paasche's or the Fisher's ideal index.

simple geometric mean of price relatives and the weighted aggregative with fixed weights meet this test.