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Measures of Central Tendency

- Central Tendency is the tendency of a given set of observations to cluster around a single central or middle value.
- The single value that represents the given set of observations is described as a measure of central tendency.

Arithematic Mean ($\bar{X}$)

Discrete Observation

$\bar{X} = \frac{\Sigma X}{n}$ (Sum of obser) / no of observation

Simple frequency Distribution

$\bar{X} = \frac{\Sigma fx}{N}$

N (Sum of all freq.)

Grouped frequency Distribution

$\bar{X} = \frac{\Sigma fx}{N}$

X = mid point

Mid point $= \frac{Upp + Low}{2}$

~~Assumed Mean~~ or ~~Step deviation~~ Method

$\bar{X} = A + \frac{\Sigma fd}{N} \times C$ where $d = \frac{x - A}{C}$

i.e. C = class length

Median (Also called as positional Average)

- Discrete observations : If n = odd, then middle value; If n = even, then average of two middle terms
- Grouped frequency distribution

$Me = l_1 + \left(\frac{\frac{N}{2} - N_L}{\underbrace{N_U - N_1}_{f}}\right) \times C$ where, l_1 = LCB of Median class

N_U = Cum freq. of Median class

N_1 = cum freq of Pre-median class

C = class length of Median class

Root $\sqrt{(20736)^{\frac{1}{4}}}$

$\sqrt{}$ 20736 then $\sqrt{}$ it 12 times ÷ by 4 + 1 then x = 12 times

Mode :-

$$\cancel{M_O = l_1 + \left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1}}\right)}$$

$$M_O = l_1 + \left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1}\right) \times C$$

Relation b/w Mean, Median, Mode

- In case of symmetric distribution :- Mean = Median = Mode
- In case of Moderately skewed distribution (empirical relation)

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$

$$\text{Mode} = \overset{3\text{Median}}{\cancel{2\text{Median}}} - 2\text{Mean}$$

Geometric Mean

- Discrete :-

$$G = (x_1 \times x_2 \times x_3 \times \cdots x_n)^{1/n}$$

- freq. distr :- G

$$G = (x_1^{f_1} \times x_2^{f_2} \times \cdots \times x_n^{f_n})^{1/N}$$

Harmonic Mean

- Discrete

$$H = \frac{n}{\Sigma\left(\frac{1}{x}\right)}$$

- freq. distribution

$$H = \frac{N}{\Sigma\left(\frac{f}{x}\right)}$$

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- Combined HM :- $\dfrac{n_1 + n_2}{\dfrac{n_1}{H_1} + \dfrac{n_2}{H_2}}$

- Special relation when there are only two observations
$$AM \times HM = (GM)^2$$

Weighted Average :-

Weighted AM $\Rightarrow$ $\dfrac{\Sigma wx}{\Sigma w}$

Weighted GM $\Rightarrow$ $\left(x_1^{w_1} \times x_2^{w_2} \times \cdots \times x_n^{w_n}\right)^{\frac{1}{\Sigma w}}$

Weighted HM $\Rightarrow$ $\dfrac{\Sigma w}{\Sigma\left(\dfrac{w}{x}\right)}$

Dispersion

Dispersion for a given set of observations may be defined as the amount of deviations of the observations, usually, from an appropriation measure of central tendency.

Types

- Absolute Measure : Range, Mean Deviation, Standard deviation, Quartile deviation. Not useful for comparison of variables with different units.
- Relative Measure :- Coeff. of range, coeff of MD, Coeff of SD, Coeff of QD. Useful for comparison of two variables with different units

* Range :

Discrete / Simple FD

- $L - S$
- where, $L \Rightarrow$ Largest observation , $S \Rightarrow$ Smallest Observation

formula ⟶ Grouped FD

- $L - S$
- where $L \Rightarrow$ UCB of last interval , $S \Rightarrow$ LCB of first class interval

Coefficient of Range :- $\frac{L-S}{L+S} \times 100$

* Mean Deviation :

Discrete $\Rightarrow$ $MD_A = \frac{1}{n} \Sigma |X - A|$ where $A \Rightarrow$ Appropriate central Tendency.

frequency distribution $\Rightarrow$ $MD_A = \frac{1}{N} \Sigma f |X - A|$

Coefficient of Mean Deviation $\Rightarrow$ $\frac{\text{Mean Deviation about } A}{A} \times 100$

* Standard Deviation :

- Discrete

$$\sigma_x = SD_x = \sqrt{\frac{\Sigma (X - \bar{X})^2}{n}}$$

$$= \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$$

- FD

$$\sigma_x = SD_x = \sqrt{\frac{\Sigma f (X - \bar{X})^2}{N}}$$

$$= \sqrt{\frac{\Sigma f X^2}{N} - (\bar{X})^2}$$

- Coefficient of Variation $\Rightarrow \frac{SD_x}{\bar{x}} \times 100$
- SD for any two no. $\Rightarrow$ $SD = \frac{\text{Range}}{2}$
- SD for first n natural no. $\Rightarrow$ $s = \sqrt{\frac{n^2-1}{12}}$
- Combined SD :

$$SD_c = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

where, $d_1 = \bar{x}_c - \bar{x}_1$, $d_2 \Rightarrow \bar{x}_c - \bar{x}_2$

* Quartile Deviation

- $QD = QD_x = \frac{Q_3 - Q_1}{2}$
- Coefficient of QD $\Rightarrow \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

* Relation b/w SD, MD & QD

$4\,SD = 5MD = 6QD$

★ $QD = \frac{1}{2}$ Inter Quartile Range

So, Inter Quartile Range $= 2QD$