CHAPTER - 6

SEQUENCE AND SERIES

ARITHMETIC AND GEOMETRIC PROGRESSION

SEQUENCE :

* An ordered collection of numbers $a_1, a_2, a_3, a_4, \ldots\ldots, a_n, \ldots\ldots$ is a sequence if according to some definite rule or law, there is a definite value of a_n called the term or element of the sequence, corresponding to any value of the natural number n.

Eg: 1, 2, 3, 4, 5, 6, ...

20, 18, 16, 14, 12, 10, ...

* nth term of a sequence is a function of the positive integer n.
* The n^{th} term is also called the general term of the sequence.
* A sequence may be finite or infinite.
* If the number of elements in a sequence is finite, the sequence is called finite sequence and it is denoted by $\{a_i\}_{i=1}^{n}$.
* If the number of elements is unending, the sequence is infinite and is denoted by $\{a_n\}_{n=1}^{\infty}$

SERIES :

* An expression of the form $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ which is the sum of the elements of the sequences $\{a_n\}$ is called a series.
* If the series contains a finite number of elements, it is called a finite series, otherwise called an infinite series.
* If $S_n = u_1 + u_2 + u_3 + u_4 + \cdots + u_n$, then S_n is called the sum to n terms (or the sum of first n terms) of the series and is denoted by the Greek letter Σ.
* Thus, $S_n = \sum_{r=1}^{n} u_r$ or simply by Σu_n

SUM OF IST n NATURAL NUMBERS:

$$S = \frac{n(n+1)}{2}$$

SUM OF SQUARES OF THE FIRST n NATURAL NOS:

$$S = \frac{n(n+1)(2n+1)}{6}$$

SUM OF CUBES OF THE FIRST n NATURAL NOS:

$$S = \left[\frac{n(n+1)}{2}\right]^2$$

SUM OF IST n ODD NUMBERS:

$$S = n^2$$

BASIS	ARITHMETIC PROGRESSION	GEOMETRIC PROGESSION
MEANING	AP is a sequence in which each term is obtained by adding a constant d to the preceding term.	GP is a sequence in which each term is obtained by multiplying a constant r to the preceding term.
GENERAL FORM	$a, a+d, a+2d, a+3d, \ldots$	$a, ar, ar^2, ar^3, \ldots$
CONDITION	$d = a_2 - a_1 = a_3 - a_2$ $d = a_n - a_{n-1}$	$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3}$ $r = \frac{a_n}{a_{n-1}}$
GENERAL TERM	$t_n = a + (n-1)d$	$t_n = ar^{n-1}$
SUM OF TERMS	$S = \frac{n(a+l)}{2}$ (or) $S = \frac{n}{2}\{2a+(n-1)d\}$	$S_n = \frac{a(1-r^n)}{1-r}$ when $r<1$ $S_n = \frac{a(r^n-1)}{r-1}$ when $r>1$ $S_\infty = \frac{a}{1-r}$ if $-1<r<1$
THREE CONSECUTIVE TERMS	$(a-d), a, (a+d)$	$\frac{a}{r}, a, ar.$