Index : Mathematics important points revision

1. RATIO AND PROPORTION

WHAT IS RATIO?

→ A Ratio is a comparision of the sizes of two or more.

→ If a and b are two quantities of the same kind (in same units), then the fraction a/b is called the ratio of a to b. It is written as a : b.

→ The quantities a and b are called the terms of the ratio.

→ 'a' is called FIRST TERM OR ANTECEDENT.
'b' is called SECOND TERM OR CONSEQUENT.

DIFFERENT KINDS OF RATIO

① **INVERSE RATIO** :– One Ratio is the Inverse of other If their **Product is one**. Thus, a : b is the Inverse of b : a and vice-versa.

eg: The Inverse Ratio of 11:15 is 15 : 11 .

<A> 15 : 11 <B> 11 : 11 <C> 15 : 15 <D> $\sqrt{11} : \sqrt{15}$

② **RATIO OF EQUALITY** :– A Ratio a : b is said to be ratio of greater inequality or equality or of less inequality according as If a > b or If a = b or If a < b.

eg: 5 : 3 is the ratio of greater Inequality
4 : 4 is the ratio of Equality
3 : 5 is the ratio of less Inequality

③ DUPLICATE RATIO :-

When two equal ratios are compounded, The resulting ratio is called the Duplicate ratio of the given ratio. If $a:b$ and $a:b$ are compounded then the ratio $a^2:b^2$ is called the duplicate ratio of $a:b$.

eg: $\frac{3x-2}{5x+6}$ is the duplicate ratio of $\frac{2}{3}$, then find the value of x?

$$\Rightarrow \frac{3x-2}{5x+6} = \frac{(2)^2}{(3)^2}$$

$$9(3x-2) = 4(5x+6)$$

$$27x - 18 = 20x + 24$$

$$7x = 42$$

$$\boxed{x = 6}$$

(4) SUB-DUPLICATE RATIO :- The sub-duplicate ratio of a:b is $\sqrt{a} : \sqrt{b}$

eg: sub-duplicate ratio of 16:9 is $\sqrt{16} : \sqrt{9} = 4:3$

→

(5) TRIPLICATE RATIO :- When three equal ratios are compounded, the new ratio is called the Triplicate ratio of the given ratios.

$a^3 : b^3$ is called the Triplicate ratio of a:b

eg: The Triplicate ratio of 4 : 7 is $(4)^3 : (7)^3 = 64 : 343$

(6) SUB-TRIPLICATE RATIO :- The Sub-Triplicate ratio of a : b is $\sqrt[3]{a} : \sqrt[3]{b}$.

eg: Sub-Triplicate ratio of 125 : 729 is $\sqrt[3]{125} : \sqrt[3]{729}$

$= 5 : 9$

(7) COMPOUND RATIO :- Ratios are compounded by multiplying the fractions which denote them.

eg: A sum of money is to be distributed among A, B, C and D in the proportion of 5 : 2 : 4 : 3. If C gets ₹ 1000 more than D. What is B's share?

<A> 500 <B> 1500 <(C)> 2000 <D> None of these.

⇒ let the common multiple be x

$A = 5x, \; B = 2x, \; C = 4x, \; D = 3x$

$4x = 1000 + 3x$

$x = 1000$

B's share $= 2 \times 1000 = 2000$

PROPORTIONS

→ An equality of two ratio is called a proportion.

→ four quantities a, b, c, d are said to be in proportion If $a : b = c : d$

→ first and fourth term are called "EXTREMES". second and third term are called "MEANS".

Four quantities are in proportional if and only if "Product of extremes is equal to product of means".

i.e $\frac{a}{b} = \frac{c}{d}$

$\therefore ad = bc$

[It is also called as "Product Rule"]

CONTINUED PROPORTION

→ Three Quantities are said to be in continued proportion if $a:b = b:c$, then $\frac{a}{b} = \frac{b}{c}$ then $b^2 = ac$, where b is Mean proportional between a and c i.e. $b = \sqrt{ac}$.

SOME IMPORTANT PROPORTIONS OF FOUR QUANTITIES

1. INVERTENDO :– If $a:b = c:d$ then $b:a = d:c$
2. ALTERNENDO :– If $a:b = c:d$ then $a:c = b:d$
3. COMPONENDO :– If $a:b = c:d$ then $\frac{a+b}{b} = \frac{c+d}{d}$
4. DIVIDENDO :– If $a:b = c:d$ then $\frac{a-b}{b} = \frac{c-d}{d}$

(5) **COMPONENDO – DIVIDENDO** :– If $a : b = c : d$, then

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

(6) If $a : b : c : d : e : f =$ ________ then,

(a) **Addendo** :– $\frac{a+c+e+\ldots\ldots}{b+d+f+\ldots\ldots}$ = each of the ratios $a:b$, $c:d$, $e:f$ ......

(b) **Subtrahendo** :– $\frac{a-c-e\ldots\ldots}{b-d-f\ldots\ldots}$ = each of the ratios $a:b$, $c:d$, $e:f$ ......

2. INDICES

Important points:

1) $a^m \times a^n = a^{m+n}$

eg: $2^3 \times 2^2 = 2^5$

$3^3 \times 3^{-2} = 3^1$

2) $\dfrac{a^m}{a^n} = a^{m-n}$

eg: $\dfrac{3^5}{3^2} = 3^3$ $\dfrac{5^2}{5^{-4}} = 5^6$

3) $(a^m)^n = a^{mn}$

4) $(ab)^n = a^n b^n$

5) $(a)^0 = 1$

6) $(a)^1 = a$

7) $(a)^{-m} = \dfrac{1}{a^m}$

8)

$$\sqrt{ab} = (ab)^{\frac{1}{2}}$$

$$\sqrt[3]{ab} = (ab)^{1/3}$$

$$\sqrt[4]{ab} = (ab)^{1/4}$$

$$\sqrt[q]{(ab)^p} = (ab)^{p/q}$$

9)

$$if\ a^x = a^y, then\ x = y$$

$$if\ x^a = y^a, then\ x = y$$

10)

$$if\ a^x = p\ then\ a = p^{\frac{1}{x}}$$

$$if\ a^{\frac{1}{x}} = p\ then\ a = p^x$$

Identities:

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a+b)(a-b)$
4. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
5. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

3. LOGARITHMS

→ If $a^x = m$ [a = base, x = Power, m = number] then it's logarithmic form is given by $\log_a m = x$ [log m to the base a equals 'x']

→ $a^x = m$ [exponential form]
$\log_a m = x$ [logarithmic form]

EXAMPLE

① $5^3 = 125$

⇒ It is of the form $a^x = m$

∴ $\log_a m = x$

$\log_5 125 = 3$

② $\log_2 8 = 3$

⇒ It is of the form $\log_a m = x$

∴ $a^x = m$

$2^3 = 8$

③ $3^{-1} = \frac{1}{3}$

⇒ It is of the form $a^x = m$

∴ $\log_a m = x$

$\log_3 \left(\frac{1}{3}\right) = -1$

TYPES OF LOGARITHMS

COMMON OR BRIGGS LOGARITHM
↓
logarithm to the base "10"

NATURAL OR NAPERIAN LOGARITHM
↓
logarithm to the base "e"
[$e = 2.7182$]

PROPERTIES OF LOGARITHM

① PRODUCT RULE :

$$\log_m (AB) = \log_m A + \log_m B$$

EXAMPLE:

$$\log_{10} 5 + \log_{10} 6 = \log_{10} (5 \times 6) = \log_{10} 30$$

$$\log 8 + \log 9 = \log_e (8 \times 9) = \log_e 72$$

② QUOTIENT RULE

$$\log_m\left(\frac{A}{B}\right) = \log_m A - \log_m B$$

EXAMPLE : ① $\log_{10} 8 - \log_{10} 2 = \log_{10}\left(\frac{8}{2}\right)$

② $\log 40 - \log 20 = \log_e\left(\frac{40}{20}\right)$

③ EXPONENT RULE

$$m \log_{10} A = \log_{10} (A)^m$$

EXAMPLE : ① $5 \log_{10} 3 = \log_{10} (3)^5$

② $7 \log 2 = \log_e (2)^7$

⚠ $(\log_{10} m)^n \neq n \log_{10} m$

④ $\log\left(\frac{AB}{CD}\right)$

$= \log A + \log B - \log C - \log D$

EXAMPLE :

$\log 3 + \log 5 - \log 2 - \log 7$

$= \log\left[\frac{3 \times 5}{2 \times 7}\right] = \log\left(\frac{15}{14}\right)$

⑤ CHANGE OF BASE LAW

$$\log_a m = \frac{1}{\log_m a}$$

EXAMPLE : $\log_3 5 = \frac{1}{\log_5 3}$

⑥ $\log_a m \times \log_m a = 1$

$(\text{eg: } \log_5 7 \times \log_7 5 = 1)$

⑦ $\log_b a = \frac{\log a}{\log b}$

⑧ $\log_a a = 1$ * $\log_1 \text{any no} = \text{N.D}$

⑨ $\log_a 1 = 0$ [log 1 to any base is zero]

⑩ If $\log_a m = \log_a n \Rightarrow$ $m = n$

If $\log_a m = \log_b m \Rightarrow$ $a = b$

⑪ If $\log_b a = x$, then

***

① $\log_{1/b} a = -x$

2 $\log_b \frac{1}{a} = -x$

3 $\log_{1/b} 1/a = x$

⑫ $m^{\log_m x} = x$

Eg: $3^{2\log_3 x} = 3^{\log_3 x^2}$

$= x^2$

log → √‾‾‾ 19 times

→ −1 =

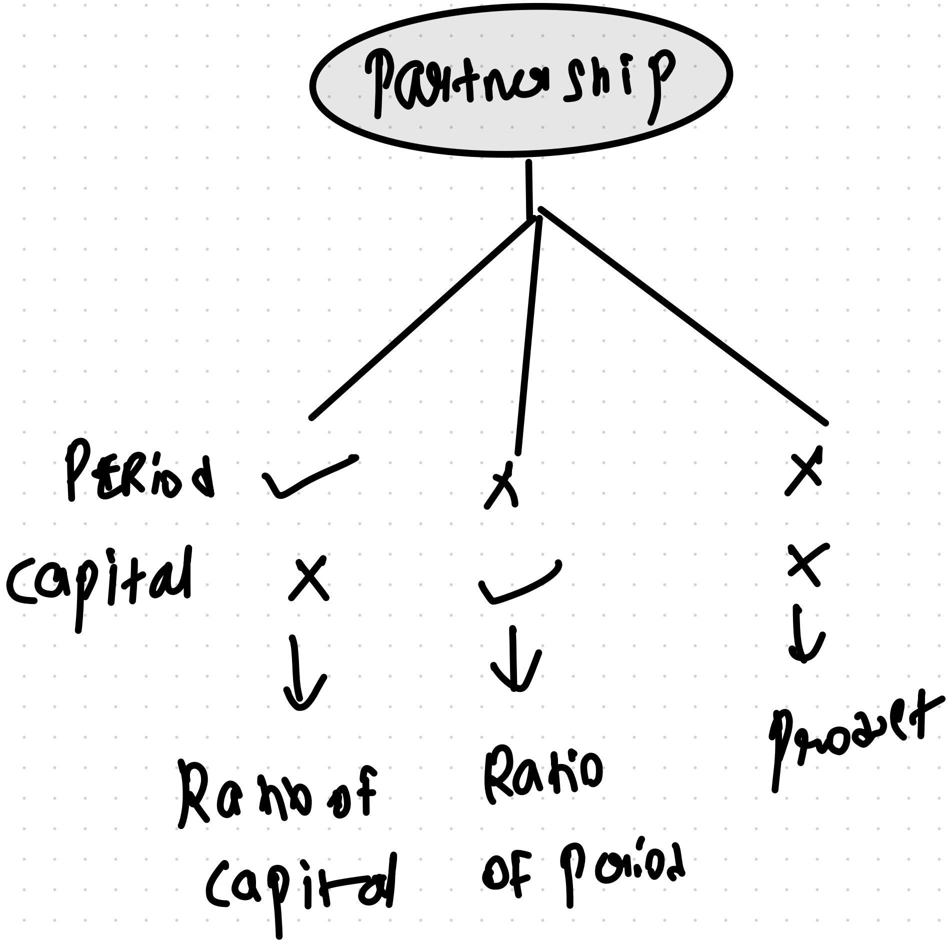
→ × 227695 = → n − 1

characterisn

mantissa

log 22 = (1). ____

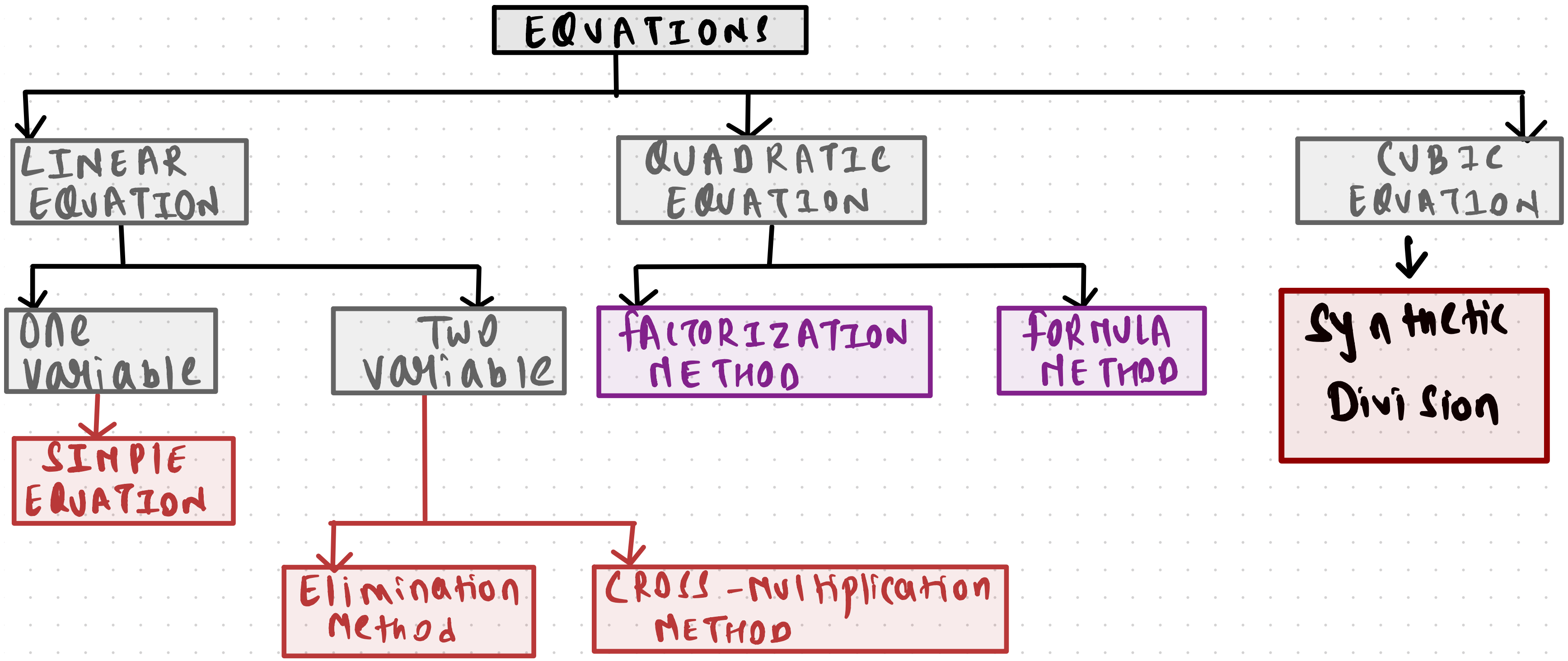
log 232 = 2.



Partnership
Period ✓ ✗ ✗
Capital ✗ ✓ ✗
Ratio of capital
Ratio of period
Product

4. EQUATIONS

OVERVIEW :



WHAT IS EQUATIONS ?

→ It is defined as "MATHEMATICAL STATEMENT OF EQUALITY".

TYPES OF EQUATIONS :-

1. LINEAR EQUATION :- Equation having degree 1.
2. QUADRATIC EQUATION :- Equation having degree 2.
3. CUBIC EQUATION :- Equation having degree 3.

EQUATION IN ONE VARIABLE :

⇒ **SIMPLE EQUATION** :- A simple equation in one unknown x is in the form $ax + b = 0$ where a, b are known constants and $a \neq 0$.

EQUATION IN TWO VARIABLE :

→ The linear equations in two unknowns x and y is $ax + by + c = 0$, where a, b are non-zero coefficients and c is a constant.

→ Two such equations $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ forms a pair of simultaneous equations in x and y.

QUADRATIC EQUATIONS

→ An equation of the form $ax^2 + bx + c = 0$, where x is a variable and a, b, c are constants with $a \neq 0$ is called a Quadratic equation or equation of the second degree.

* When $b = 0$, The equation is called a "Pure Quadratic Equations".

* When $b \neq 0$, The equation is called an "Affected Quadratic equation".

⇒ FORMULA METHOD :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Nature of Roots

→ It depends on "value of Discriminant" i.e. $D = b^2 - 4ac$.

Equations	$D = b^2 - 4ac$	Discriminant	Nature of Roots
① $x^2 - 6x + 9 = 0$	$D = 36 - 36$ $= 0$	$D = 0$	Roots are real and equal
② $x^2 - 6x - 16 = 0$	$D = 36 + 64$ $= 100$	$D > 0$ (Perfect square)	Roots are REAL, RATIONAL and UNEQUAL
③ $x^2 - 6x + 7 = 0$	$D = 36 - 28$ $= 8$	$D > 0$ (Not a perfect square)	REAL, IRRATIONAL and unequal (conjugate roots)
④ $x^2 - 6x + 13 = 0$	$D = 36 - 52$ $= -16$	$D < 0$ ($i^2 = -1$)	Roots are imaginary (conjugate complex surds)

NOTE :

1. If $P + \sqrt{Q}$ is a root, then $P - \sqrt{Q}$ is also a root.

2. If $P + iQ$ is a root, then $P - iQ$ is also a root. (where $i^2 = -1$)

3. Sum of roots : $\alpha + \beta = -\frac{b}{a}$

4. Product of roots : $\alpha\beta = \frac{c}{a}$

5. An equation with roots α and β is given by $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

i.e. $x^2 -$ (sum of roots) x + Product of roots $= 0$.

6. If one root is reciprocal of the other root $\left(\alpha, \frac{1}{\alpha}\right)$, then Product is 1 also $a = c$

7. If roots are equal in magnitude but opposite in sign $(\alpha, -\alpha)$, then sum of roots $= 0$.

8. If $a+b+c=0$ then one of the roots $= 1$ and other roots $= \frac{c}{a}$. eg: $x^2+5x-6=0$

9. If $a-b+c=0$, then one of the roots $= -1$ and other roots $= -\frac{c}{a}$. eg: $x^2+6x+5=0$.

(10) If α, β are roots of $ax^2+bx+c=0$ then

$\frac{1}{\alpha}, \frac{1}{\beta}$ will be roots of $cx^2+bx+a=0$.

→ Replace 'a' by 'c'.

(11) $$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$$

(12) $$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$$

(13) $$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

(14) $$\alpha^3-\beta^3 = (\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$$

CUBIC EQUATIONS

→ An Equation of the form $ax^3 + bx^2 + cx + d = 0$, $a \neq 0$ where a, b, c, d are all real numbers is called a cubic Equations.

→ It has 3 roots (α, β, γ)

→ Either all the three root are real or one.

⊛ RELATION BETWEEN ROOTS AND CO-EFFICIENT :

① $\alpha + \beta + \gamma = -\frac{b}{a}$ [SUM OF ROOTS]

② $\alpha\beta\gamma = -\frac{d}{a}$ [PRODUCT OF ROOTS]

③ $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$ (sum of roots taken two at a time)

SOLUTION OF CUBIC EQUATION

→ On Trial basis, putting some value of x to check, whether L.H.S is zero then to get a factor.

→ For other factor, use synthetic division.

eg: Solve : $x^3 - 7x + 6 = 0$

$\Rightarrow$ Sum of coefficient $= 1 - 7 + 6 = 0$

$x = 1$ is one of the factor

1	1	0	-7	6
	↓	1	1	-6
	1	1	-6	0

$x^2 + x - 6 = 0$

6 → +3, -2

$x + 3 = 0$ or $x - 2 = 0$

$x = -3$ $\quad$ $x = 2$

5. LINEAR INEQUALITIES

→ **GENERAL EQUATION OF A LINEAR EQUATION :**

$$ax + by + c = 0$$

→ Inequation contains $>$, $<$, $\geq$ or $\leq$

i.e $ax + by \geq c$ or $ax + by \leq c$.

→ **INTERVALS :**

(1) Open Interval ( ) : $x \in (a, b)$ means x is lying between a and b excluding both.

$$a < x < b$$

② **Closed Interval** [ ] :- $x \in [a, b]$ means x is lying between a and b both Including.

$a \leq x \leq b$

③ **Semi-closed Interval**

[ ) or ( ]

Semi-left closed Interval	Semi-Right closed Interval
$x \in [a, b)$	$x \in (a, b]$
$a \leq x < b$	$a < x \leq b$

NOTE :

> Open Interval means unbounded.
> Closed Interval means bounded.

→ To solve a linear Inequation in one variable :

(1) While Multiplying or dividing by a negative no,

> the sign of Inequation will change

eg: $-3x \leq 15$

$$x \geq \frac{15}{-3} \quad \text{(Dividing by "}-3\text{")}$$

$$\boxed{x \geq -5}$$

② For MODULUS (Absolute) value of $|x|$ or $|x \pm k|$,

Remove the modulus sign and keep the variable between the positive and negative integer value given.

eg: ① $|x| < 4$

$\Rightarrow$ $-4 < x < 4$

② $|2x+7| \geq 25$

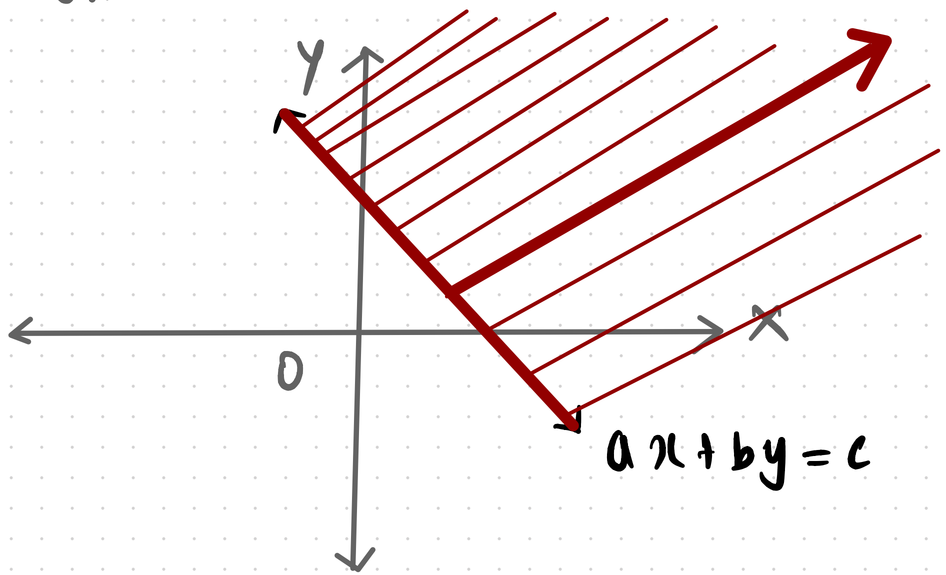
(Positive value ⇒ same sign
negative value ⇒ change the sign)

$2x + 7 \geq 25$ or $2x+7 \leq -25$

→ LINEAR INEQUATION IN 2 VARIABLES :

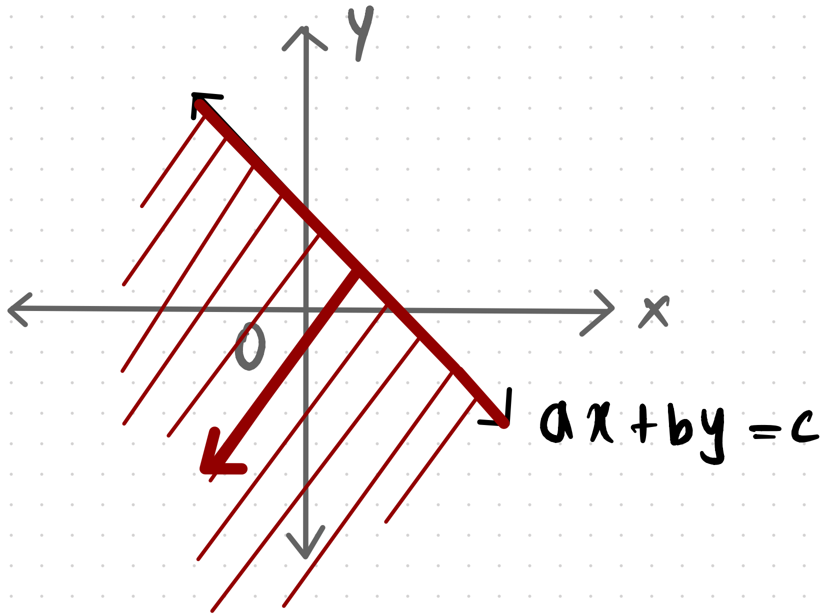
① $ax + by \geq c$ (positive constant)

non-Origin side of the line Including the points on the line.



② $ax + by \leq c$

Origin side of the line Including all the points on the line.



NOTE

① Greater than $\geq$ → non – origin side

② less than $\leq$ → Origin side

③ $x = k$ → Line ∥ y-axis

④ $y = k$ → Line ∥ x-axis

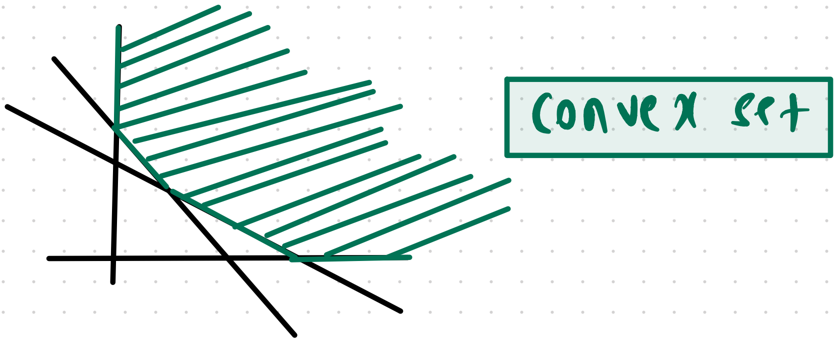
⑤ for $ax + by \geq 0$ or $ax + by \leq 0$

Plot the line (passing through origin) and then check for the point (1, 0).

For a system of linear Inequations,

(1) The **common region** of all the Inequations in a given system is called **feasible region** or **common solution** of the system of Inequations. ***

(The feasible region may be a bounded region or unbounded region but always a convex set.)



→ LINEAR PROGRAMMING PROBLEMS

Linear programming means planning a certain problem to optimize its objective function within the given constraints.

BASIC CONCEPT

① The variable involved in LPP are called "Decision Variable"

② Objective function : The aim of the problem

(Profit → To be maximized
cost → To be minimized) is called the objective function and is denoted by $z = ax + by$.

③ Constraints :

The limitations or restrictions involved in LPP are called as constraints which are of the form

$$ax + by \geq c \quad (\text{Atleast})$$

$$ax + by \leq c \quad (\text{Atmost})$$

6. TIME VALUE OF MONEY

BASICS :

- The sum of Money received in future is less valuable then It is today.
- RS.100 note given today is More valuable than Rs. 100 note given a year later due to following reasons:
  - Risk factor
  - liquidity Preference
  - Inflation
  - opportunity cost

(1) INTEREST

SIMPLE INTEREST

→ Interest is calculated Uniformly on original Amount

→ Here, Principal Remains constant

COMPOUND INTEREST

→ Interest is calculated on new Principal i.e (P+i) every year.

→ Here, Principal keeps on changing every year.

(A) SIMPLE INTEREST

1. $S.I = P \times r \times t$
2. $A = P + S.I$
3. $A = P(1 + rt)$

WHERE,

P = Principal

r = Rate of interest (in Decimal)

t = No of years

A = Amount

SHORTCUTS FOR SIMPLE INTEREST

① If 'r' is the simple rate of Interest, then Amt becomes double of itself in $n = \frac{100}{r}$ years.

eg: In how many years a sum of money becomes double of itself at 10% S.I?

→ $n = \frac{100}{r} = \frac{100}{10} = 10$ yrs

② If 'r' is the simple rate of Interest, then Amt becomes triple of itself in $n = \frac{200}{r}$ years.

eg: A sum of Money double itself in 10 years. The no. of years it would triple itself is?

→ $n = \frac{100}{r}$ | $10 = \frac{100}{r}$ | $r = 10\%$ | $\therefore n = \frac{200}{r} = \frac{200}{10}$

$n = 20$ yr

③ If a Sum Of Money deposited in a bank becomes A_1 in t_1 Years and A_2 in t_2 years then the Amount deposited initially is

$$P = \frac{A_1 t_2 - A_2 t_1}{t_2 - t_1}$$

→ A certain sun of money at Simple Interest Amounts to Rs. 2520 in 2 yrs and Rs. 2700 in 5 yrs. What is the amount deposited Initially?

$A_1 = 2520$, $t_1 = 2$

$A_2 = 2700$, $t_2 = 5$

$$P = \frac{A_1 t_2 - A_2 t_1}{t_2 - t_1}$$

$P = 2400$

2520 x 5 M+
2700 x 2 M-
MRC ÷ 3 =

4 If sum of Money becomes 'n' times in 't' years, then It will become 'm' times in $\left(\frac{m-1}{n-1}\right) \times t$ yrs

eg: If a certain sum of Money is double in 8 years at a given simple Interest, In how many years It will be four times?

→ $n = 2$, $m = 4$, $t = 8$ $\quad \left(\frac{4-1}{2-1}\right) \times 8 = 24$

(B) COMPOUND INTEREST

① $C.I = P\left[(1+i)^n - 1\right]$

② $A = P(1+i)^n$

where,

$i = \frac{r}{100 \times m}$ and $n = t \times m$

m = no. of times compounding done in a year.

NOTE : While calculating C.I, we need to adjust Interest Rate and time period according to compounding Frequency

① If compounded annually, $m = 1$

② If compounded semi-annually, $m = 2$

③ If compounded quarterly, $m = 4$

④ If compounded monthly, $m = 12$.

⑤ If compounded daily, $m = 365$.

SHORTCUTS FOR COMPOUND INTEREST

1) If 'r' is the compound rate of Interest, then Amount becomes double of Itself in $n = \frac{69}{r} + 0.35$ yrs

eg: In what time period, Amt deposited of Rs. 12,000 will become double at 10% p.a compounded annually?

$$n = \frac{69}{r} + 0.35$$

$$n = \frac{69}{10} + 0.35$$

$n = 7.25$ yrs

7 yrs, 3 months

(0.25×12)

2) If 'r' is the compound rate of Interest, then Amount becomes Triple of Itself in $n = \frac{111.444}{r} + 0.35$ yrs

eg: In what time period, Amt deposited of Rs. 8000 will become Triple at 15% p.a compounded annually?

$\rightarrow$ $n = \frac{111.444}{r} + 0.35$

$n = \frac{111.444}{15} + 0.35$

$n = 7.7796$ yrs

OR

= 7 yrs, 10 Months

3) If 'd' is the difference between S.I and C.I is given for 2 yrs, then In such a case 'P' will be :-

$$P = \frac{d \times (100)^2}{(r)^2}$$

eg: If the difference between C.I and S.I on a certain sum of money at 5% P.a for 2 years is ₹1.50 find the sum of money?

$$\Rightarrow P = \frac{1.50 \times 10000}{25} = 600$$

4) If 'd' is the difference between S.I and C.I is given for 3 yrs, then In such a case 'P' will be :-

$$P = \frac{d \times (100)^3}{r^2 (r+300)}$$

eg: On what sum, will the difference between C.I and S.I for 3 yrs at 6% p.a amounts to Rs. 13.77?

$$\Rightarrow P = \frac{13.77 \times 1000000}{36 \times 306}$$

$P = 1250$

$13.77 \times 1000000 \div 36 \div 306 =$

EFFECTIVE RATE OF INTEREST

$\left[(1+i)^n - 1 \right] \rightarrow$ Principal ×

→ Annual Compound Interest rate is called as nominal Interest rate.

→ But, If Interest is compounded More than once i.e twice, Quarterly, monthly then the Actual % Of Interest per year is called effective rate OF Interest.

Tricks to calculate effective Rate of Interest : ☺

① **If Half-yearly:**

$$\frac{R}{200} + 1 \times 100 \; (=) \; - 100 = \text{Effective Rate.}$$

(=) → 2 times

② **IF Quarterly:**

$$\frac{R}{400} + 1 \times 100 \; (=) \; - 100 = \text{Effective Rate.}$$

(=) → 4 times

Depreciation : $P.V = V(1 - r\%)^n$

$= (1 - r\%) \times V$ = no. of life => scrap valu

$P.V$ = P.V of Asset

V = original value of Asset

r = rate of depreciation

n = tenure

Relationship between present value and future value

→ $A = P(1+i)^n$

→ $P = \dfrac{A}{(1+i)^n}$

where,

A = Accumulated value / future value / Sum Due

P = Present value / Cash value / Principal

eg: Rs. 5000 deposited today @ 10% P.a for 3 years will amount to ?

$\Rightarrow A = P(1+i)^n$

$= 5000(1.1)^3$

A = ₹ 6655

Annuity

→ when a fixed amount OF money is Invested for a Regular Interval OF time, It is said to be annuity.

eg: Anil Invested Rs.8,000 every Half-Yearly @ 10% p.a for 3 years.

Annuity

- **Ordinary annuity/annuity regular**
  ↓
  Payment is made at the "end"
- **Immediate annuity/Annuity Due**
  ↓
  payment is Made at the "beginning"

→ Annuity can be calculated Of

PRESENT VALUE

↓

Money's Todays worth

eg: Bank loan

FUTURE VALUE

↓

Money's future Worth

eg: Sinking fund/ Investments.

SOME NOTATIONS

1. 'A' stands for Accumulated value.
2. 'C' Stands for Amount Invested every monthly/ Quarterly/ Half-yearly / yearly.
3. 'i' stands for Interest rate as per plan.
4. 'm' stands for no. of times interest is given in a year.
5. 'n' stands for Total no. of times Interest to be given in tenure.

Formulas:

1) Future value of ordinary annuity (FVA):

$$fva = \frac{c}{i}\left((1+i)^n - 1\right)$$

2) future value of Annuity Immediate/Due (fvA')

$$fva' = \frac{c\ (1+i)}{i}\left((1+i)^n - 1\right)$$

3) Present value of Ordinary Annuity (PVA)

$$pva = \frac{c}{i}\left(1 - (1+i)^{-n}\right)$$

4) Present value of Immediate Annuity / Due

$$pva' = \frac{c\,(1+i)}{i}\left(1 - (1+i)^{-n}\right)$$

→ $i = \frac{r}{100 \times m}$, $n = t \times m$

① If Annually, $i = \frac{r}{100}$, $n = t \times 1$

② If Half-yearly, $i = \frac{r}{200}$, $n = t \times 2$

③ If Quarterly, $i = \frac{r}{400}$, $n = t \times 4$

④ If Monthly, $i = \frac{r}{1200}$, $n = t \times 12$

NOTE : IF QUESTION SILENT, ASSUME PAYMENT AT END. (Ordinary / Regular)

SINKING FUND

→ Money required for future purpose.

Deposit is done only once

↓

sinking fund = Sum due (A)

$A = P(1+i)^n$

Deposit is done Regularly for a specific period (Annuity)

↓

Sinking fund = f·v·A

$f \cdot v \cdot A = \frac{c}{i}\left[(1+i)^n - 1\right]$

BANK LOAN TAKEN

If Repaid in "LUMPSUM"

Loan Taken = Principal (P)

$$P = \frac{A}{(1+i)^n}$$

IF Repaid in "Installments"

Loan Taken = P.V.A

$$P.V.A = \frac{C}{i}\left[1 - (1+i)^{-n}\right]$$

APPLICATION OF PRESENT VALUE

1. Capital Expenditure (Investment decision)

→ It Means purchasing an asset today in anticipation OF benefits which would flow across the life OF the Investment.

→ IF the Pv of cash Inflows is greater than Pv OF the cash outflows, then the decision should be in favour of Investment.

Eg: Machine A costs Rs. 10,000 and has a useful life of 8 years. Machine B Costs Rs. 8000 and has a useful life of 6 Years. Suppose, Machine A generated an annual savings OF Rs.2000 and Machine B generated an annual savings OF Rs. 1800. Assuming time value of money is 10% P.a, which Machine is Preferable ?

② LEASING :-

→ It is a **financial arrangement** under which the owner of the assets allows the user of the asset to the use the asset For a defined period of time for a consideration payable over a given period of time.

Eg: A company is considering proposal of purchasing a Machine either by Making full payment OF Rs. 4000 Or by leasing it for 4 years at an annual rate of Rs. 1250. Which course of Action is preferable, IF the Company Can borrow money @ 14% p.a compounded annually?

→ $PVA = \frac{1250}{0.14}\left(1 - (1.14)^{-4}\right)$

$= ₹\ 3642.14$

CASH VALUE = 4000

∴ PVA < CASH VALUE

∴ LEASING IS PREFERABLE.

3.PERPETUITY

Growing perpetuity = $\frac{c}{i-g}$

→ It is an annuity in which the periodic payment or receipts begin on a fixed date and continue Indefinitely or perpetually.

$$PVA = \frac{c}{i}$$

NOTE: Both payment and Interest rate should be adjusted according to scheme.

(52) Determine the present value of perpetuity ₹ 10 per month for infinite period at an effective rate of interest of 14% p.a.?

LEQ

(a) ₹ 657 (b) ₹ 757 (c) ₹ 857 (d) ₹ 957

$c = 120 \ (10 \times 12)$

$i = 0.14$

$$PVA = \frac{120}{0.14} = \boxed{857}$$

4. VALUATION OF BOND

→ A bond is a debt security in which the company issuing it owes the holder a debt and is obliged to repay the principal and Interest.

→ FORMULA : Value of bond = P.V of Interest + P.V of Maturity Value

P.V of Interest → USE PVAI

P.V of Maturity Value → Relation → USE (1+i) × (=) no. of times then ÷ = then multiply by Principal

NOTE : Suppose bond of £ 1000 is Matured at premium of 10%, then Principal value for maturity to be taken as £ 1100

5. RATE OF RETURN

Nominal rate of Return

→ Inflation is Ignored

→ FORMULA

$$NRR = \left[\frac{\text{Present value of Investment} - \text{Original value of Investment}}{\text{Original value of Investment}} \times 100 \right]$$

Real Rate Of Return

→ Inflation is considered

→ FORMULA

$$\left[\frac{1 + NRR}{1 + IR} - 1 \right] \times 100$$

6. Compounded Annual growth Rate model[CAGR]

→ It Measures growth rate over the period Of time.

→ **FORMULA FOR CALCULATING CAGR** :

$$CAGR = \left[\frac{\text{value at the end}}{\text{value at the beginning}}\right]^{\frac{1}{\text{end year} - \text{beginning year}}} - 1 \times 100.$$

eg:

YEAR	2013	2014	2015	2016	2017
REVENUE	100	120	160	210	250

$$CAGR = \left[\frac{250}{100}\right]^{\frac{1}{4}} - 1 \times 100 = 25.74\%$$

$\sqrt{}$ 7 12 times

$-1 =$

$\times 1 \quad \div 3 \quad =$

$+1 =$

$x =$ 12 times

$(27)^{1/3} = 3$

8. SEQUENCE AND SERIES

① SEQUENCE:

→ A Sequence is a collection of numbers arranged in a definite order and Obtained in succession according to some definite rule.

EXAMPLE:

1) 1, 2, 3, 4, 5 ......
2) 1, 3, 5, 7, 9........
3) 2, 3, 5, 8, 12 ....
4) 3, 9, 27, 81.....

② PROGRESSION :

→ A sequence that follows a specific Patterns are called progressions.

Ex: 2,4, 6, 8, 10 .... , 3, 9, 27, 81 ....

3. TYPES OF PROGRESSION

- **ARITHMETIC PROGRESSION (AP)** → Common Difference is constant (C.d)
- **GEOMETRIC PROGRESSION (G.P)** → Common Ratio is constant (r)
- **HARMONIC PROGRESSION (HP)** → Reciprocal of A.P (Not in syllabus)

4. ARITHMETIC PROGRESSIONS (AP): -

→ when the Difference between every 2 consecutive terms is constant, The Sequence is called A. P.

EG: 3, 5, 7, 9, 11 ... is a sequence of AP.

→ THE first term is denoted by 'a' (t_1), common difference is denoted by 'd' and THE last term is denoted by '(t_n)'.

→ Common difference (d) = $t_n - t_{n-1}$

→ GENERAL SEQUENCE OF A.P

$a,\ a+d,\ a+2d,\ a+3d \ldots\ldots a+(n-1)d$

→ n^{th} TERM Formula

$$t_n = a + (n-1)d$$

$t_5 = a + 4d$

$t_9 = a + 8d$

PROPERTIES OF A.P

→ IF $t_1, t_2, t_3 \ldots\ldots t_n$ are terms in AP, then

① $t_1 \pm k, t_2 \pm k, t_3 \pm k \ldots\ldots t_n \pm k$ are in A.P, **C.d = d**

② $t_1 k, t_2 k, t_3 k \ldots\ldots\ldots t_n k$ are in A.P, **C.d = Kd**

③ $\frac{t_1}{k}, \frac{t_2}{k}, \frac{t_3}{k} \ldots\ldots\ldots \frac{t_n}{k}$ are in A.P, **$C.d = \frac{d}{K}$**

Sum of 'n' Terms In A.P

$$S_n = \frac{n}{2}[t_1 + t_n]$$

or

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

→ SUM OF 1st 'n' even natural no

$$S_n = n(n+1)$$

→ SUM OF 1st 'n' odd natural no

$$S_n = n^2$$

→ IF S_n is known and t_n is to be obtained, then use formula

$$t_n = S_n - S_{n-1}$$

eg: $t_3 = S_3 - S_2$

$S_2 = t_1 + t_2$

$S_3 = t_1 + t_2 + t_3$

$S_3 = S_2 + t_3$

$t_3 = S_3 - S_2$

TERMS IN A.P

1. 3 TERMS IN A.P : $a-d, a, a+d$
2. 4 TERMS IN A.P : $a-3d, a-d, a+d, a+3d$.
3. 5 TERMS IN A.P : $a-2d, a-d, a, a+d, a+2d$.

SHORT TRICKS

① If $m t_m = n t_n$ then $t_{m+n} = 0$.

② If $t_p = \frac{1}{q}$, $t_q = \frac{1}{p}$ then $t_{pq} = 1$ and $S_{pq} = \frac{1}{2}(pq+1)$

③ If $t_p = q$, $t_q = p$ then $t_r = q + p - r$

④ If $S_p = q$, $S_q = p$, then $S_{p+q} = -(p+q)$

⑤ If a, b, c are terms in A.P, then

$$2b = a + c$$

GEOMETRIC PROGRESSION

→ When the **ratio** between every 2 consecutive terms is constant, the sequence is in G.P.

eg: 2, 4, 8, 16, 32 ......

→ **Common ratio** $(r) = \dfrac{t_{n+1}}{t_n}$

→ **GENERAL SEQUENCE OF G.P**

$a, ar, ar^2, ar^3 \ldots\ldots a \cdot (r)^{n-1}$

→ **GENERAL TERM FORMULA OR nth term formula**

$$t_n = a \cdot (r)^{n-1}$$

eg: $t_4 = a(r)^3$, $t_6 = a(r)^5$, $t_{11} = a(r)^{10}$, $t_{35} = a(r)^{34}$

→ Properties of G.P

a, b, c

$b^2 = ac$

If $t_1, t_2, t_3 \dots$ are terms in G.P

1. $t_1 k, t_2 k, t_3 k \dots$ are still in G.P, $C.r = r$

2. $\frac{t_1}{k}, \frac{t_2}{k}, \frac{t_3}{k} \dots$ are still in G.P, $C.r = r$

3. $t_1^k, t_2^k, t_3^k \dots$ are still in G.P, $C.r = r^k$

4. 2, 4, 8, 16, 32, 64, 128 …..

2, 16, 128 ….

If terms are selected at regular Interval in a G.P, The new sequence is also a G.P

(5) SURE QUESTION

If $t_1, t_2, t_3 \ldots$ are terms in G.P, then

$\log t_1, \log t_2, \log t_3 \ldots$ are terms in A.P

eg: $2, 4, 8, 16 \ldots$ is in G.P

$\log 2, \log 4, \log 8, \log 16$

$\log 2, 2\log 2, 3\log 2, 4\log 2$ is in A.P $(d = \log 2)$

S_n in a G.P $\Rightarrow$ S_n formula depends on "r"

If $r = 1$

$$S_n = na$$

If $r > 1$

$$S_n = a \cdot \frac{[(r)^n - 1]}{r - 1}$$

If $r < 1$

$$S_n = \frac{a[1 - (r)^n]}{1 - r}$$

TERMS in G.P

[$+ \rightarrow \times$ $d \rightarrow r$
$- \rightarrow \div$ $n0 \rightarrow$ power]

3 TERMS in G.P : $\frac{a}{r}, a, ar$

4 TERMS in G.P : $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

5 TERMS in G.P : $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

SHORTCUTS

(1) $a + aa + aaa + \ldots\ldots n$ terms

$$S_n = \frac{a}{81}\left[(10)^{n+1} - 10 - 9n\right]$$

② $0.9 + 0.99 + 0.999 + \dots$ n term

$$S_n = \frac{a}{81}\left[9n - 1 + (0.1)^n\right]$$

eg: $0.2 + 0.22 + 0.222 + \dots$ n terms

$$S_n = \frac{2}{81}\left[9n - 1 + (0.1)^n\right]$$

Sum to Infinity of a G.P (S_∞)

→ S_∞ of a G.P exists, only If $|r| < 1$

eg: 2, 4, 8, 16 ..... $r = 2$ (S_∞ dou not exist)

eg: $1 - 1 + 1 - 1 + 1 - 1 + \dots$

$r = -1$ $|r| = 1$ $1 < 1$ (It is not possible)

$\therefore S_\infty$ dos not exist.

If S_∞ exist, $S_\infty = \dfrac{a}{1-r}$, $|r| < 1$

eg: $-64, 32, -16, 8 \ldots$

$r = \dfrac{32}{-64}$

$r = -\dfrac{1}{2}$, $|r| = \dfrac{1}{2} < 1$ $\therefore S_\infty$ exist

$S_\infty = \dfrac{-64}{1+\frac{1}{2}} = \dfrac{-64}{3/2} = \boxed{\dfrac{-128}{3}}$

→ Sum of square of infinity of G.P

$a^2, a^2r^2, a^2r^4 \ldots\ldots$

$S_\infty = \dfrac{a^2}{1-r^2}$, $|r| < 1$

AGP

$S_\infty = \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}$

CONCEPT of A.M, G.M and H.M

① **Arithmetic mean (A.M)** :- The number "A" is said to be A.M between a and b is given by

$$A = \frac{a+b}{2}$$

Proof : a, A, b are in A.P

$$A - a = b - A$$

$$2A = a + b$$

$$A = \frac{a+b}{2}$$

Note: If 'n' A.M's are inserted between a and b then sum of n A.M's is given by $n\left(\frac{a+b}{2}\right)$

② GEOMETRIC MEAN (G.M) :-

The number "G" is said to be geometric mean of numbers a and b and is given by

$$G = \sqrt{ab}$$

Proof : a, G, b are in g.p

$$\frac{G}{a} = \frac{b}{G}$$

$$G^2 = ab$$

$$\therefore \quad G = \sqrt{ab}$$

Product of "n" G.M between a and b is given by

$$(\sqrt{ab})^n = (ab)^{n/2}$$

(3) Harmonic Mean (H.M) :- The number "H" is said to be H.M of 2 numbers a and b, given by

$$H = \frac{2ab}{a+b}$$

(4) Relationship between A.M, G.M and H.M

→ If A, G, and H are Arithmetic mean, geometric mean and Harmonic mean of 2 numbers a and b respectively, then

① $G = \sqrt{AH}$

② $A.M \geq G.M \geq H.M$

SERIES

Sigma notation

→ Suppose, that a certain values x takes n values say $x_1, x_2, x_3 \dots x_n$.
The sum of n terms is $x_1 + x_2 + x_3 \dots + x_n$.

→ These sum is denoted by $\sum_{i=1}^{n} x_i \;;\; i = 1, 2, 3 \dots n$.

→ The notation "Σ" is called sigma notation.

Eg: 1. $$\sum_{i=1}^{10} a_i = a_1 + a_2 + a_3 + \dots + a_{10}$$

2. $$\sum_{i=1}^{10} (a)^i = a + a^2 + a^3 + \dots + a^{10}$$

FORMULAS :

① SUM OF FIRST "n" NATURAL NO

$$1+2+3+4+\dots+n \Rightarrow \sum_{n=1}^{n} n = \frac{n(n+1)}{2}$$

② SUM OF SQUARES OF FIRST "n" NATURAL NO

$$1^2+2^2+3^2+\dots+n^2 \Rightarrow \sum_{n=1}^{n} n^2 = \frac{n(n+1)(2n+1)}{6}$$

③ SUM OF CUBES OF FIRST "n" NATURAL NO

$$1^3+2^3+3^3+\dots+n^3 \Rightarrow \sum_{n=1}^{n} n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

④ $$\sum_{i=1}^{n} 1 = 1+1+1\dots \text{ n times} = n.$$

Permutations & combinations

1. $0! = 1$
2. $n! = n(n-1)(n-2)(n-3)\ldots\ldots 3.2.1$
3. $^nP_r = n(n-1)(n-2)(n-3)\ldots\ldots[n-(r-1)]$
4. $^nP_r = \dfrac{n!}{(n-r)!}, \quad n \geq r$ → arrangement
5. $^nP_0 = 1$
6. $^nP_1 = n$

⑦ $n = \dfrac{{}^{n}P_{r}}{{}^{(n-1)}P_{r-1}}$

Brads

$= \dfrac{(n-1)!}{2}$

⑧ $${}^{n}P_{r} = {}^{n-1}P_{r} + r \cdot {}^{n-1}P_{r-1}$$

$${}^{7}P_{4} \Rightarrow {}^{6}P_{4} + 4 \cdot {}^{6}P_{3}$$

⑨ Total number of Arrangement of n distinct objects is $n!$

⑩ Total number of Arrangement of n distinct Objects in a circular way is $(n-1)!$

11. Number of Permutation of 'n' distinct objects taken 'r' at a time

- when a particular object is not taken in any arrangement is ${}^{n-1}P_r$.
- when a particular object is always included in any arrangement is ${}^{n-1}P_{r-1}$.

12. Total number of Arrangement of 2 particular thing never occur together out of n things is

$(n-2)(n-1)!$ ways.

13. $${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}, \quad n \geq r$$ → Selection

14. ${}^{n}C_{0} = {}^{n}C_{n} = 1$.

15. ${}^{n}C_{r} = {}^{n}C_{n-r}$

16. ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{(n+1)}C_{r}$

$${}^{5}P_{3} = 5 \times 4 \times 3$$

$${}^{5}C_{3} = \frac{5 \times 4 \times 3}{3 \times 2 \times 1}$$

17. Relation between ${}^{n}P_{r}$ and ${}^{n}C_{r}$ is

$$r! = \frac{{}^{n}P_{r}}{{}^{n}C_{r}}$$

(18). If ${}^nC_x = {}^nC_y$ then either $x = y$ or $x + y = n$

(19) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots\dots + {}^nC_n = 2^n$

(20). ${}^nC_1 + {}^nC_2 + \dots\dots + {}^nC_n = 2^n - 1$

(21). ${}^nC_0 + {}^nC_2 + {}^nC_4 + \dots\dots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots = 2^{n-1}$

(22) No. of straight lines = no. of handshakes = ${}^nC_2$

(23). No. of Triangles = ${}^nC_3$

(24). No. of Diagnols = ${}^nC_2 - n$ or $\frac{n(n-3)}{2}$

(25) If there are 'n' distinct points, out of which k points are collinear, then

- No. of straight lines $= {}^{n}C_2 - {}^{k}C_2 + 1$
- No. of Triangles $= {}^{n}C_3 - {}^{k}C_3$

(26). No. of Parallelograms $= {}^{m}C_2 \cdot {}^{n}C_2$

(27). The number of ways in which 'n' distinct objects can be split into three groups containing r, s and t objects where $n = r + s + t$ is $\dfrac{n!}{r!\, s!\, t!}$

SETS, RELATIONS AND FUNCTIONS

PART 1 : SETS

① What is sets ?

→ A set is a "collection of well defined distinct objects".

→ Sets are usually denoted by **Capital Alphabets** such as X, Y, Z etc. and the elements of a set are denoted by **small alphabets** as x, y, z.

→ If X is a set and x is a member of set X i.e x is an element of X.

→ Symbolically,

$x \in X$	[x belongs to X]
$x \notin X$	[x does not belongs to X]

→ Eg: collection of first six even natural no
$S = \{2, 4, 6, 8, 10, 12\}$

② REPRESENTATION OF SET :

→ There are **2 ways** of describing a set

① Tabular form / Roster Method :

→ set is denoted by "**listing**" all it's elements, seperated by "**comma (,)**" within the "**braces { }**"

eg: $V = \{a, e, i, o, u\}$
$N = \{1, 2, 3, \ldots\}$

② SET-BUILDER METHOD / RULE METHOD :

→ Set is described by stating a "**Property**" which is satisfied by all it's elements.

eg: $X = \{x \,/\, x$ is number $1 < x < 40\}$.
$D = \{x \,/\, x$ is a divisor of $24\}$.

Types of sets

1) Singleton set : only 1 element

2) Empty set / null / void set : No element { }, ϕ (phi)

3) Equal set : Every element is same

$A = \{a, e, i, o, u\}$

$B = \{$ set of vowels in a, e, i, o, u $\}$

4) EQUIVALENT SET :- Two sets A and B are said to be equivalent sets, if their "cardinal number are same"

i.e $n(A) = n(B)$

eg: let $A = \{1, 2, 3\}$
$B = \{x, y, z\}$

$\therefore$ $n(A) = n(B)$

5) SUBSET : A set X is said to be a subset of the set Y "If every element of set X is also an element of set Y" i.e. $X \subseteq Y$, X is a subset of Y and Y is called the superset of X.

eg: ① If $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 5\}$ then B is a subset of A.

② If $A = \{1, 2, 3, 4, 5\}$ $B = \{2, 3, 4\}$, $C = \{1, 4, 5\}$ then $B \subseteq A$, $C \subseteq A$.

⑥ PROPER SUBSET :- X is a Proper subset of Y If "EVERY element of X belongs to Y but there is atleast one element of Y which is not in X" i.e. $X \subset Y$

⑦ DISJOINT SETS :- If two sets have no elements in common. i.e. $A \cap B = \phi$

⑧ POWER SET : Set of "all possible subsets" of the given set A and It is denoted by P(A). It has 2^n Elements

eg: $A = \{2, 3, 4\}$

$A = \{ \{\}, \{2\}, \{3\}, \{4\}, \{2, 3\}, \{3, 4\}, \{2, 4\}, \{2, 3, 4\} \}$

$n(A) = 8$

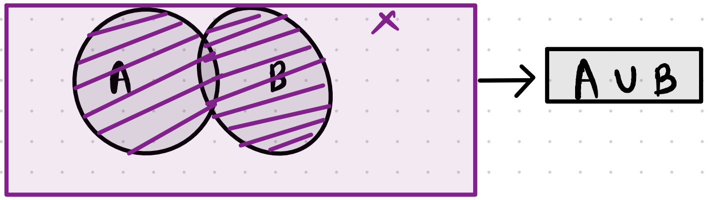
SOME IMP NOTE :-

1. Every set is a "subset" of Itself.
2. The "Empty set" is a subset of Every set.
3. A set containing "n" elements has 2^n subsets and $2^n - 1$ proper subsets.
4. All Equal sets are Equivalent sets but Vice-versa is not True.

4 VENN DIAGRAMS :-

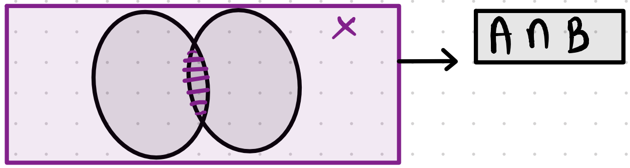
1. UNION OF TWO SETS :

[All, Atleast one also means union]

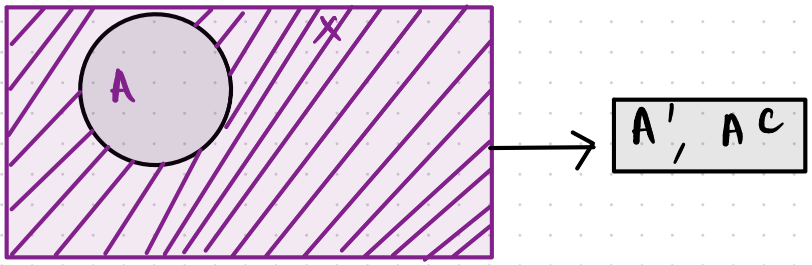


2. INTERSECTION OF TWO SETS :

[Both also mean Intersection]

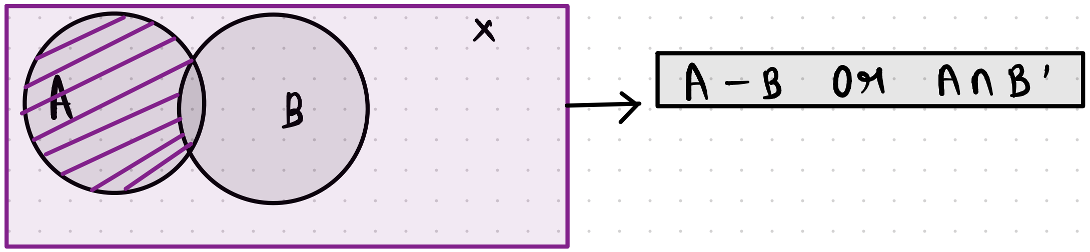


3. COMPLEMENT OF A SET :



(4) DIFFERENCE OF TWO SETS : Denoted by "A − B" or "A ∩ B'"

[All those elements of A which are not in B]



(5) SYMMETRIC DIFFERENCE OF TWO SETS :

$$A \Delta B = (A - B) \cup (B - A)$$

eg: $A = \{a, b, c\}$, $B = \{b, c, d\}$

$A - B = \{a\}$

$B - A = \{d\}$

$A \Delta B = \{a\} \cup \{d\}$

$= \{a, d\}$

⑤ IMPORTANT FORMULAE

① $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

② $n(A \cup B) = n(A) + n(B)$ ∴ [If Disjoint sets]

③ $n(A \cap B') = n(A) - n(A \cap B)$ [A but not B]

④ $n(B \cap A') = n(B) - n(A \cap B)$ [B but not A]

⑤ $n(A \Delta B) = n(A) + n(B) - 2\, n(A \cap B)$

⑥ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

⑦ $n(A \cup B \cup C) = n(A) + n(B) + n(C)$ ∴ [If Disjoint]

⑧ $n(A' \cup B') = n(A \cap B)' = n(X) - n(A \cap B)$
$n(A' \cap B') = n(A \cup B)' = n(X) - n(A \cup B)$ ⟹ De Morgan's law

IMP NOTE :

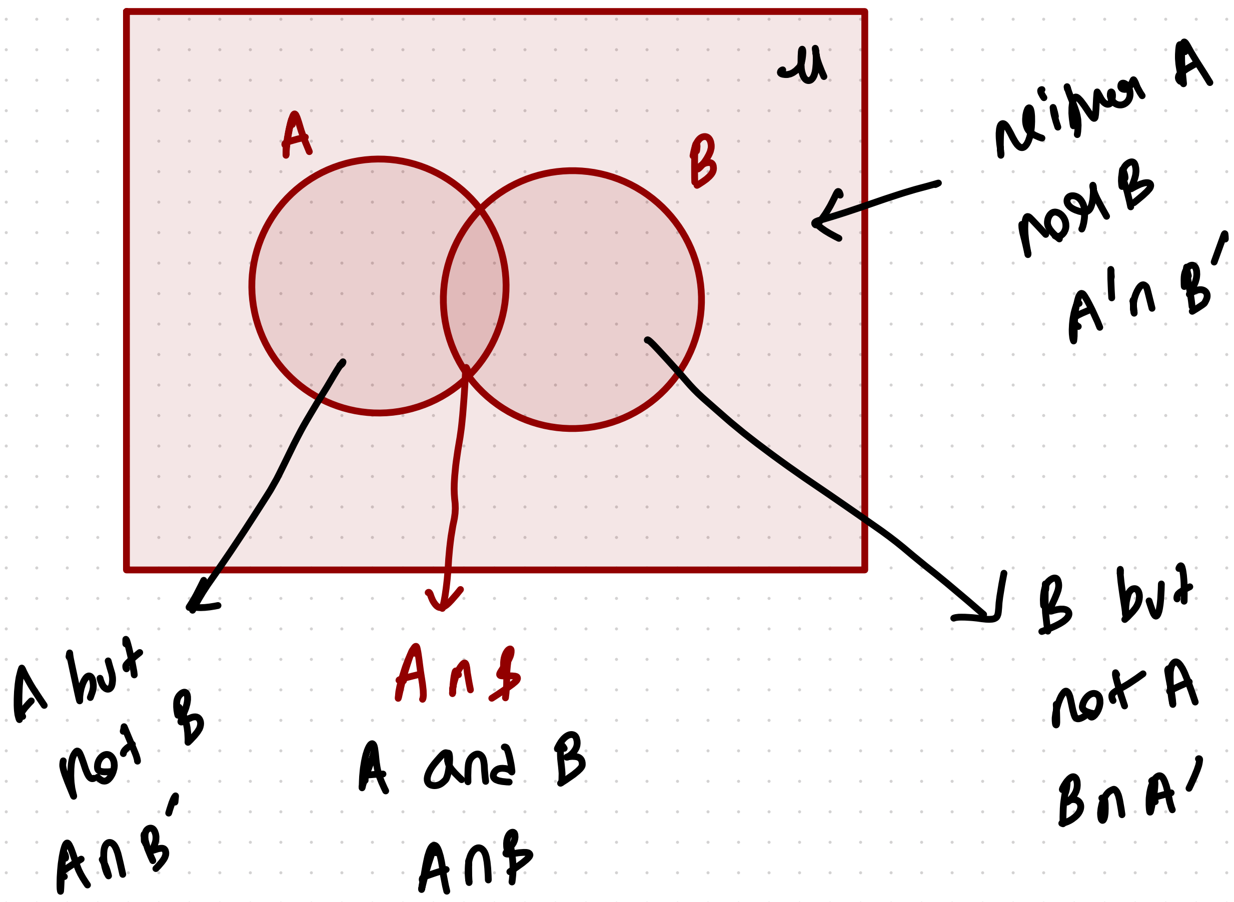
① No. of elements in exactly two of the three sets A, B, C

$$n(A \cap B) + n(B \cap C) + n(C \cap A) - 3\,n(A \cap B \cap C)$$

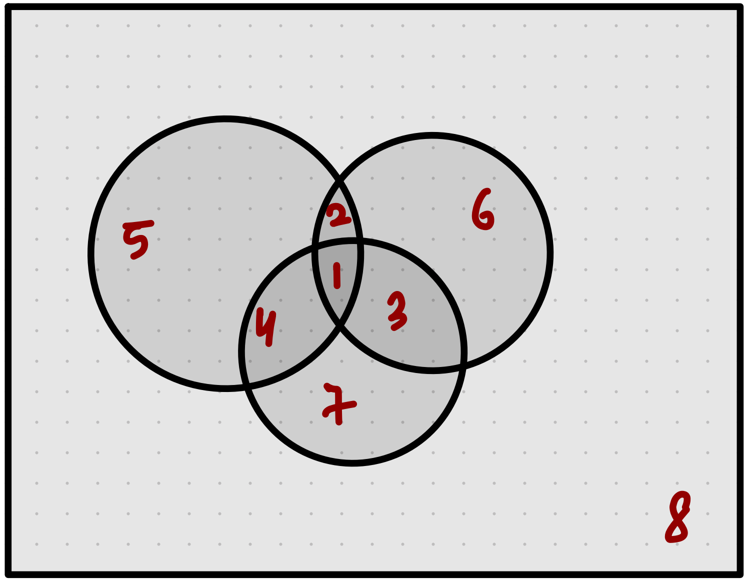
② No. of elements in exactly one of the three sets A, B, C

$$n(A) + n(B) + n(C) - 2\,[n(A \cap B) + n(B \cap C) + n(A \cap C)] + 3\,n(A \cap B \cap C)$$

2 set



3 set



1) common of 3 set
($A \cap B \cap C$)

2) $A \cap B \cap C'$

3) $B \cap C \cap A'$

4) $A \cap C \cap B'$

5) only A ($A \cap B' \cap C'$)

6) only B ($A' \cap B \cap C'$)

7) only C ($A' \cap B' \cap C$)

8) neither A, B, C
($A' \cap B' \cap C'$)

RELATIONS

→ given two finite sets A and B, If **atleast one element of 'A' is related to 'B'**

i.e "A × B" → Cartesian Product

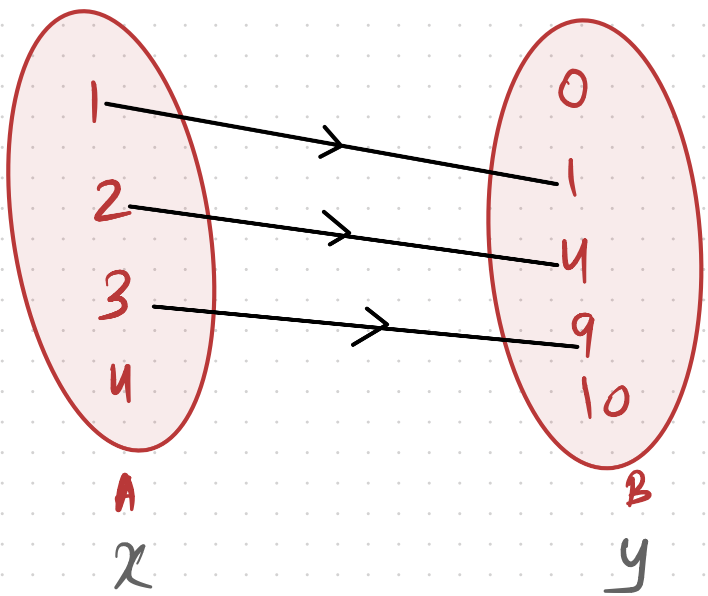
A → x, B → y

Cartesian Product ↓ (x, y)

For eg: $A = \{1, 2, 3, 4\}$ → x

$B = \{0, 1, 4, 9, 10\}$ → y

$A \times B = \{(1, 1)\ (2, 4)\ (3, 9)\}$



$$y = x^2$$

This is a relation from A to B and is denoted as "$A \xrightarrow{R} B$"

A (x) → Domain

B (y) → Co-domain

Range :- elements of y which is making relations

6. FUNCTIONS

1. FUNCTION (A ka image repeat nahi hona chahiye)

given two sets A and B, If **all** the elements of 'A' has an image in 'B' then the relation is called Function from A to B.

$$A \xrightarrow{f} B$$

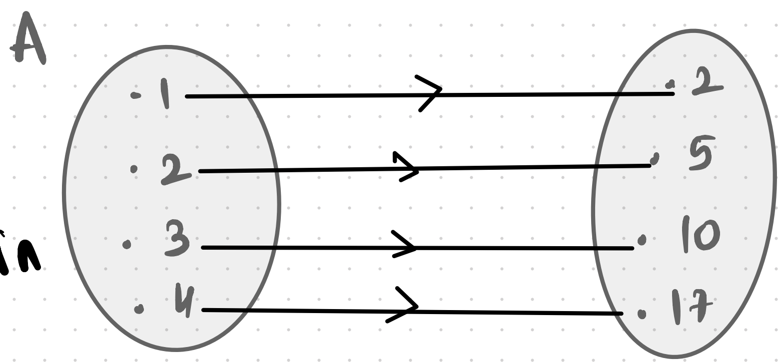
eg: $A = \{1, 2, 3, 4\}$

$B = \{2, 5, 10, 17\}$

$A \times B = \{(1,2)\ (2,5)\ (3,10)\ (4,17)\}$

(x, y)

x
Domain



y
Co-Domain

Many to one

Many element of A → same image B

" $y = x^2 + 1$ "

② Types of function

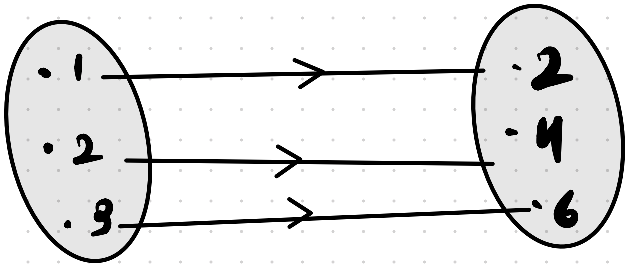
① **one-one function** : let $f : A \to B$ If different elements in A have different elements in B is said to be one-one or Injective function.

eg: $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$
$f : A \to B$, $f(x) = 2x$

$f(1) = 2$ $f(2) = 4$ $f(3) = 6$

One to many ⊗

Set of ordered pairs = {(1,2) (2,4) (3,6)}



eg: $A = \{1, 2, 3, 4\}$, $B = \{1, -1\}$

$f(x) = (-1)^x$,

→ $f(1) = -1$, $f(2) = 1$, $f(3) = -1$, $f(4) = 1$

Set of ordered pairs = {(1,−1), (2,1) (3,−1) (4,1)}

Many-one function

Not a one-one function.

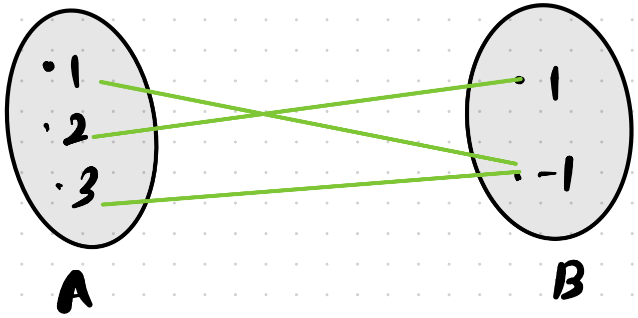
② ONTO OR SURJECTIVE FUNCTION (B set → no extra element)

Let $f : A \to B$, If every element in B has atleast one pre-image in A then function is said to be onto or surjective.

eg: $A = \{1, 2, 3\}$, $B = \{1, -1\}$

$f : A \to B$, $f(x) = (-1)^x$

Ordered Pairs $\equiv \{(1, -1)\ (2, 1)\ (3, -1)\}$

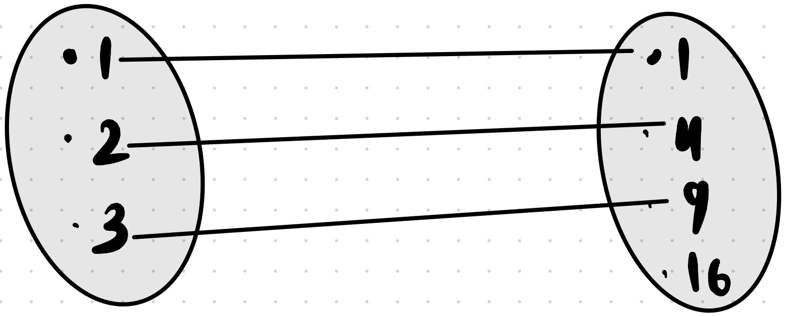


ONTO function

eg: $A = \{1, 2, 3\}$
$B = \{1, 4, 9, 16\}$

$f(x) = x^2$, $f : A \to B$

Ordered Pairs ≡ {(1,1) (2,4) (3,9)}



Not an onto function

If Range ≠ co-domain, It cannot be onto.

③ **BIJECTION function :-**

A function which is **one - one and onto** is called Bijection.

④ **Identity function :-** $f(x) = x$ (Input = output)

⑤ **Constant function :-** $f(x) = c$

⑥ **Equal function :-** $f(x) = g(x)$

eg: $f(x) = x^2 + 2x$
$g(x) = x(x+2)$ $\quad g(x) = x^2 + 2x$

⑦ **INTO function :-** let $f: A \rightarrow B$. If there exist a single element in B having no pre-image in A, then it is said to be Into function.

(B has one extra element)

(3) Value of a Polynomial

Polynomial

$P(x) = x^2 - 3x + 4$ (Quadratic)

$P(x) = x + 4$ (linear)

$P(x) = x^3 - 3x^2 + 4x + 5$ (cubic)

If $f(x)$ is any function of 'x' then $f(k)$ is the value of function when $x = k$.

eg: $F(2)$ is the value of function when $x = 2$

eg: If $f(x) = 3x^3 - 2x^2 + 4x - 10$
Find $f(0)$, $f(2)$, $f(-1)$

Solution : $f(x) = 3x^3 - 2x^2 + 4x - 10$

$f(0) = 3(0) - 2(0) + 4(0) - 10$

(Replace x with 0)

$$\boxed{f(0) = -10}$$

$f(2) = 3 \times 8 - 2 \times 4 + 8 - 10$

(Replace x with 2)

$= 24 - 8 + 8 - 10$

$$\boxed{f(2) = 14}$$

$f(-1) = 3(-1) - 2(1) - 4 - 10$

(Replace x with -1)

$= -3 - 2 - 4 - 10$

$= -19$

→ To Find the Domain and Range of function

Steps : ① Write the function in the form $y = f(x)$

② Domain = { All $x \in R$ for which the f(x) exist and is Meaningful }

> exclude the values of x for which f(x) is not real or Meaningless.

③ To find the Range, express 'x' in terms of 'y'

Range = { All values of $y \in R$ for which f(y) is exists and is meaningful }

COMPOSITE FUNCTIONS

If $y \xrightarrow{f} u \xrightarrow{f} x$

Composite function

If $f(x)$ and $g(x)$ are any functions of 'x' then

* $f[g(x)]$ is a composite function

  Substitute '$g(x)$' instead of 'x' in $f(x)$

* $g[f(x)]$ is a composite function

  Substitute '$f(x)$' instead of 'x' in $g(x)$

eg: If $f(x) = 3x + 5$

$g(x) = 2x + 4$

→ $f[g(x)] = 3[g(x)] + 5$

$= 3(2x + 4) + 5$

$= 6x + 12 + 5$

$= 6x + 17$

→ $g[f(x)] = 2f(x) + 4$

$= 2(3x + 5) + 4$

$= 6x + 10 + 4$

$= 6x + 14$

NOTE :

$$(f \circ g)(x) = f[g(x)]$$

$$(g \circ f)(x) = g[f(x)]$$

$$(f \circ f)(x) = f[f(x)]$$

$$(g \circ g)(x) = g[g(x)]$$

INVERSE OF A FUNCTION

→ If $y = f(x)$ is any function of x then $\boxed{x = f^{-1}(y)}$ is called the Inverse function.

Steps to find Inverse of any function

1. let $\boxed{y = f(x)}$
2. Now, simplify and find 'x' in terms of y i.e $\boxed{x = f(y)}$
3. Now, Replace 'y' by 'x', we get $\boxed{f^{-1}(x)}$

eg: $F(x) = x + 4$, $y = x + 4$ $\therefore x = y - 4$ $\therefore \boxed{f^{-1}(x) = x - 4}$

> Inverse of a function only exist If function is one-one and onto. i.e. Bijective.

TYPES OF RELATIONS

① **Reflexive Relation** : If R contains **all** the order pairs of the form (a, a) in product set, then relation is Reflexive.

Eg: If $A = \{1, 2, 3\}$

$A \times A = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$

$R_1 \rightarrow \{(1,1)\ (2,2)\ (3,3)\}$ R ✓

$R_2 \rightarrow \{(1,1)\ (2,2)\}$ R ×

☞ Product set ke same elements (a, a) form relation me hona chaiye

② SYMMETRIC RELATION :

If $(a,b) \in R$ then $(b,a) \in R$, then Relation is symmetric.

Eg: $R3 = \{(1,2)\ (1,3)\ (2,1)\ (3,1)\}$ S ✓

$R4 = \{(1,1)\ (2,2)\ (3,3)\}$ S ✓
R ✓

③ Transitive Relation

If $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$ then Relation is Transitive.

Eg: $R5 = \{(1,2)\ (2,3)\ (1,3)\}$ T ✓, S ✗, R ✓

$R6 = \{(1,2)\ (2,3)\ (1,1)\ (2,2)\ (3,3)\}$ T ✗, S ✗, R ✓

$R_7 = \{ (1,1)\ (2,2)\ (3,3) \}$ R ✓, S ✓, T ✓

4 EQUIVALENCE RELATION :

If a Relation is Reflexive, symmetric and Transitive then Relation is Equivalence.

eg: A Relation is parallel to set S

→ " is Parallel to " $A = \{a, b, c\}$

$$A \times B = \{ (a,a)\ (a,b)\ (a,c)\ (b,a)\ (b,b)\ (b,c)\ (c,a)\ (c,b)\ (c,c) \}$$

$R_8 = \{ (a,a)\ (b,b)\ (a,b)\ (b,c)\ (a,c)\ (b,a)\ (c,b) \}$

R ✗, S ✗, T ✗

is less than
R ✗ S ✗ T ✓

is ⊥r to
R ✗ S ✓ T ✗

is ∥ to
R ✓ S ✓ T ✓ E ✓

22/4/23

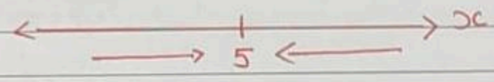
Basics of limits AND CONTINUITY FUNCTION

* Limits AND Continuity

1) $x \xrightarrow{\lim} a$ (limit x tends to a)
i.e. x is very close to a but $x \neq a$

eg : $x \xrightarrow{\lim} 5$

$x = 4.999$ $\quad$ $x = 5.0001$



2) left hand limit : $x \xrightarrow{\lim} a^-$, x close to a but $x < a$

3) Right hand limit : $x \xrightarrow{\lim} a^+$; x close to a but $x > a$

4) If $x \xrightarrow{\lim} a$ is given & [redacted]s
"Apply the limiting Value" means that put $x = a$ in given Question

5) $x \xrightarrow{\lim} a$ $f(x)$ will exist only if , L·H·L = R·H·L

$\therefore x \xrightarrow{\lim} a^- f(x) = x \xrightarrow{\lim} a^+ f(x)$

6)

6) For sums of form $\lim_{x \to a} \frac{f(x)}{g(x)}$.

Steps : 1) Apply the limiting value in the Denominator orally

2) If Denominator not equal to zero then, apply the limiting value to the entire Question and get the Answer. (Deno $\neq 0$)

3) If Deno $= 0$ then DO NOT apply the limiting value instead of that we can,

A) factorize (Quadratic / Cubic expression)

B) Rationalize (Square root wala terms in Addition or subtraction)

c) use standard formula.

7) Standard formula :

i) $\lim_{x \to a} \left[\frac{x^n - a^n}{x - a} \right] = n \cdot a^{n-1}$

in the Question Also, uppar ka power same, neeche ka power 1

Note : $\lim_{x \to a} \left[\frac{x^n - a^n}{x^m - a^m} \right] = \frac{n}{m} \cdot a^{n-m}$

ii) $\lim_{x \to 0} \left[\frac{a^x - 1}{x} \right] = \log_e a$

$\lim_{x \to 0} \left[\frac{e^x - 1}{x} \right] = 1$ (i.e. $\log_e e$)

Conditions : 1) limit should tend to 0
2) 1 should be subtracted from a^x or e^x
3) Denominator should be same as power of a OR e

iii] $\lim_{x \to 0} \left[\frac{\log(1+x)}{x} \right] = 1$

Conditions : 1) limit should tend to 0
2) Constant term is always 1
3) Whatever is added after 1, the same term should be present in Denominator.

iv] $\lim_{x \to 0} \left[1+x \right]^{1/x} = e$

Conditions : 1) limit should tend to 0
2) Constant term is always 1
3) Whatever is added after 1, the power should be its reciprocal.

v) $\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to \infty} \frac{1}{x^n} = 0$

(n is +ve no)

$$\left\{ \frac{1}{0} = \infty \quad \therefore \frac{1}{\infty} = 0 \right\}$$

vi) $\lim_{x \to \infty} \left[1 + \frac{1}{x}\right]^{x} = e$

* Shortcuts :

1) $\lim_{x \to 0} \left[1 + kx\right]^{1/x} = e^{k}$

$\lim_{x \to \infty} \left[1 + k \cdot \frac{1}{x}\right] = e^{k}$

2) $\lim_{x \to \infty} \left[\frac{\text{Polynomial}}{\text{Polynomial}}\right]$

- Degree of N = Deg of D ⇓ $\frac{\text{Co-efficient of highest power of } x}{\text{Co-efficient of highest power of } x}$
- Deg of N < Deg of D: Ans = 0
- Deg of N > Deg of D ⇓ undefined.

3) $\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right]$ → $\frac{0}{0}$, $\frac{\infty}{\infty}$ } Ans = $\lim_{x \to a} \left[\frac{f'(x)}{g'(x)}\right]$

classmate
Date
Page

* Representation of $|f(x)|$

Steps - 1) equate, $f(x) = 0$ and find Value of x
2) Suppose, $x = a$
3) $|f(x)|$ — $x > a$; $x < a$; $x = 0$, 0

* Continuity

1) To Check whether $f(x)$ is continuous at $x = a$

I

$f(x) =$ ___ , $x \neq a$
$=$ ___ , $x = a$

$\therefore \lim_{x \to a} f(x) = f(a)$

II

$f(x) =$ ___ , $x > a$
$=$ ___ , $x < a$
$=$ ___ , $x = a$

$\therefore \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

2) Continuity of some standard functions

a) Constant function : $f(x) = K$, it is continuous for all real values of x.

b) polynomial function : It is continuous for all real Values of x.

c) Rational function : It is $\frac{\text{polynomial}}{\text{polynomial}}$ and it is continuous for all real Values of x, except those Values of x where Denominator $= 0$.

d) exponential function : e^x or a^x is continuous for all real Values of x

e) log function : $f(x) = \log x$, it is continous for positive real Values of x.

f) modulus function : It is continuous for all real Values of x.

Differential Calculus

1. $\frac{d}{dx}(x^n) = n \cdot (x)^{n-1}$

2. $\frac{d}{dx}(e^x) = e^x$

3. $\frac{d}{dx}(a^x) = a^x \cdot \log a$

4. $\frac{d}{dx}(K) = 0$

5. $\frac{d}{dx}(\log x) = \frac{1}{x}$

6. $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

7. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

⑧ $$\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v \cdot \frac{du}{dx}$$

⑨ $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

⑩ If $f(x, y) = 0$ (Implicit function)

$$\frac{dy}{dx} = -\frac{[\text{Derivative w.r.t 'x' keeping 'y' constant}]}{\text{Derivative w.r.t 'y' keeping 'x' constant}}$$

(11) If $x^m \cdot y^n = (x+y)^{m+n}$, then

$$\frac{dy}{dx} = \frac{y}{x} \quad , \quad \frac{d^2y}{dx^2} = 0$$

(12) If $ax^2 + 2hxy + by^2 = 0$, then

$$\frac{dy}{dx} = \frac{y}{x} \quad , \quad \frac{d^2y}{dx^2} = 0$$

(13) If function $\left[\frac{x^n - y^n}{x^n + y^n}\right] = k$, then $\frac{dy}{dx} = \frac{y}{x}$

(14) slope of tangent = dy/dx (gradient)

(15) slope of normal = $-1/(dy/dx)$

(16) If $y = f(x)$, then $f'(x) > 0$, Increasing function
$f'(x) < 0$, Decreasing function

(17) **For minima/maxima:**

1. find $f(x)$
2. $f'(x)$
3. $f''(x)$
4. let $f'(x) = 0$ → $x =$ ___ , $x =$ ___
5. Substitute value of x in $f''(x)$
   If $f''(x) > 0$, minima
   $f''(x) < 0$, maxima

* Application of Derivatives

1) $C \longrightarrow$ Total Cost

C = Variable Cost + fixed Cost

Variable Cost ↓ $f(x)$ ↓ x : no. of items manufactured

fixed Cost ↓ Constant term ↓ Independent of x

Note : In 'C', if we put $x = 0$ then
C = fixed cost.

2) Avg. Cost $= \dfrac{C}{x}$

Marginal Cost $= \dfrac{d}{dx}[C]$

3] Total revenue $\Rightarrow R = P \times D$ < Qty Demanded / Qty manufactured

Avg revenue $= \dfrac{R}{D} = \dfrac{P \times D}{D} = P$

Marginal revenue = i) $\dfrac{dR}{dP}$ ii) $\dfrac{dR}{dD}$

4) Profit $= R - C$

Break even point $\Rightarrow R = C$

5) $f(x)$

↓

$f'(x)$

↙ ↘

$f'(x) > 0$ → $f(x)$ is Increasing

$f'(x) < 0$ → $f(x)$ is Decreasing.

Integral calculus

1. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

2. $\int 1 \, dx = x + c$

3. $\int e^x \, dx = e^x + c$

4. $\int e^{ax} \, dx = \frac{e^{ax}}{a} + c$

5. $\int \frac{dx}{x} = \log|x| + c$

6. $\int a^x \, dx = \frac{a^x}{\log a} + c$

7. $\int \frac{1}{\sqrt{x}} \, dx = 2\sqrt{x} + c$

(8) $$\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

(9) $$\int \frac{1}{a^2 - x^2}\, dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$$

(10) $$\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + c$$

(11) $$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right| + c$$

(12) $\int e^{x}[f(x)+f'(x)]\,dx = e^{x}.[f(x)]+c$

(13) $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)|+c$

(14) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log|x+\sqrt{x^2+a^2}|+c$

✗

(15) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log|x+\sqrt{x^2+a^2}|+c$

✗

(16) **Integration by Parts** (LAE)

$$\int u \cdot v \, dx = u \int v \, dx - \int \left[\frac{d}{dx}(u) \int v \, dx \right] dx$$

(17) $$\int_a^b f(x) \, dx = \int_a^b f(t) \, dt$$

(18) $$\int_a^b f(x) \, dx = - \int_b^a f(x) \, dx$$

(19) $$\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx, \quad a < c < b$$

(20) $$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx, \quad \text{If } f(-x) = f(x)$$

(even function)

$$= 0, \quad \text{If } f(-x) = -f(x)$$

(odd function)

(21) $$\int_{0}^{a} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} \Rightarrow \boxed{I = a/2}$$

(22) $$\int_{a}^{b} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}}\,dx \Rightarrow \boxed{I = \frac{b-a}{2}}$$