Part - A

Business Mathematics

CHAPTER

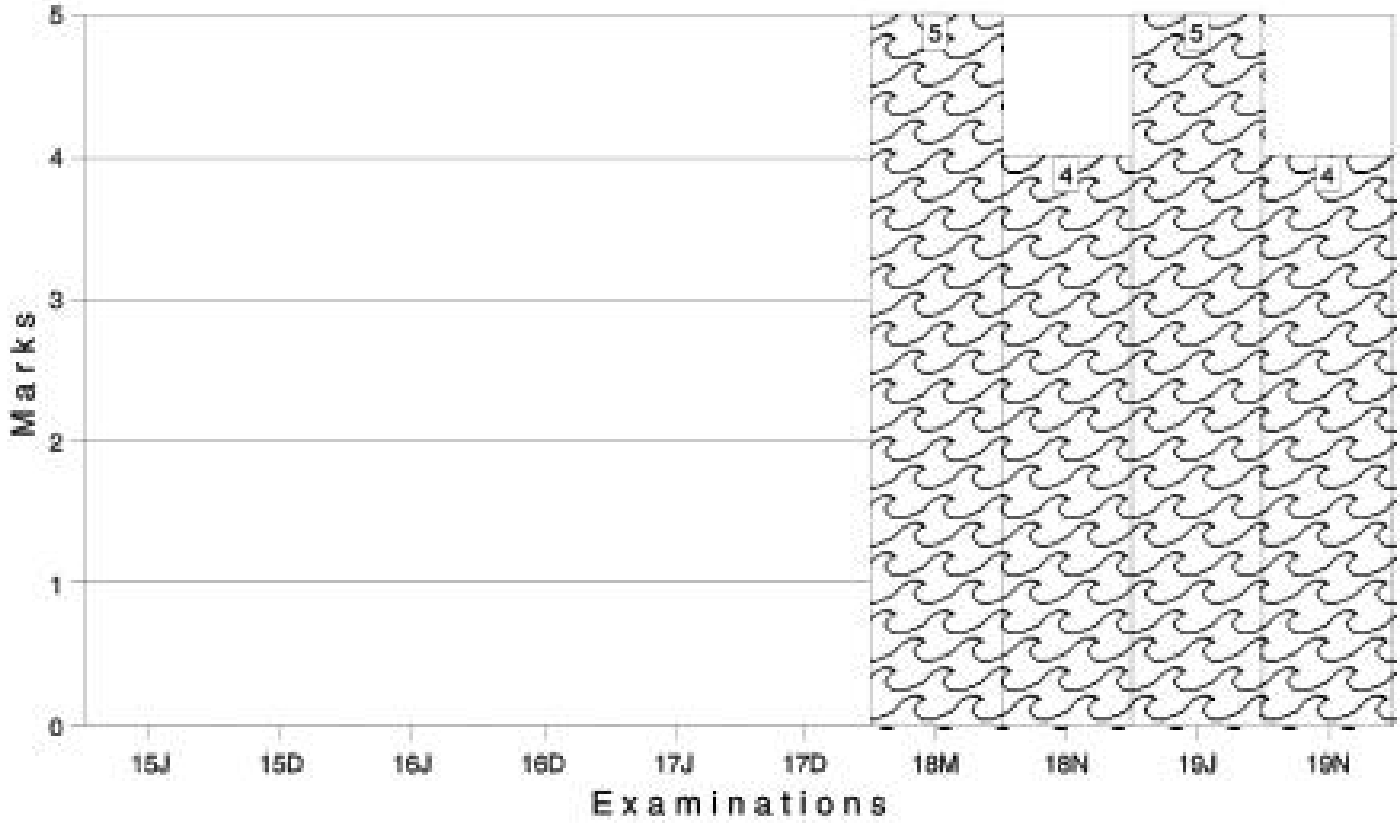
1

RATIO AND PROPORTION, INDICES AND LOGARITHMS

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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MULTIPLE CHOICE QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] Two numbers are in the ratio 2 : 3 and the difference of their squares is 320. The numbers are:

(a) 12, 18 (b) 16, 24

(c) 14, 21 (d) None. (1 mark)

Answer:

(b) Let numbers be $2x$ and $3x$.

Therefore, $(3x)^2 - (2x)^2 = 320$

$$9x^2 - 4x^2 = 320$$
$$5x^2 = 320$$
$$x^2 = 64$$
$$x = 8$$

$\therefore$ Numbers are: $2x = 2 \times 8 = 16$

$$3x = 3 \times 8 = 24$$

[2] If p : q is the sub-duplicate ratio of $p - x^2 : q - x^2$, then x^2 is :

(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$

(c) $\frac{qp}{p-q}$ (d) None. (1 mark)

Answer:

(d) As per the given information:

$$\frac{p - x^2}{q - x^2} = \frac{P^2}{q^2}$$

$$q^2 (p - x^2) = P^2(q - x^2)$$
$$pq^2 - x^2 q^2 = p^2 q - p^2 x^2$$
$$x^2 (p^2 - q^2) = pq(p - q)$$

$$x^2 = \frac{pq\ (p - q)}{p^2 - q^2}$$

$$x^2 = \frac{pq}{p + q}$$

[3] An alloy is to contain copper and zinc in the ratio 9 : 4. The zinc required to melt with 24 kg of copper is :

(a) $10\frac{2}{3}$kg (b) $10\frac{1}{3}$kg

(c) $9\frac{2}{3}$kg (d) 9kg (1 mark)

Answer:

(a) Let the quantity of copper and zinc in an alloy be 9x kg. and 4x kg.

Therefore, $9x = 24$

$$x = \frac{24}{9} = \frac{8}{3} = 2\frac{2}{3}\ kg.$$

So zinc $= 4x = 4 \times \frac{8}{3}\ kg.$

$$= 10\ \frac{2}{3}\ kg.$$

[4] $7 \log \left(\frac{16}{15}\right) + 5 \log \left(\frac{25}{24}\right) + 3 \log \left(\frac{81}{80}\right)$ is equal to :

(a) 0 (b) 1

(c) log 2 (d) log 3

Answer:

(c) $7\ \text{Log} \left(\frac{16}{15}\right) + 5\ \text{Log} \left(\frac{25}{24}\right) + 3\ \text{Log} \left(\frac{81}{80}\right)$

$= 7(\log 16 - \log 15) + 5(\log 25 - \log 24) + 3 \log (\log 81 - \log 80)$

$= 7\ [4 \log 2 - (\log 3 + \log 5)] + 5\ [2 \log 5 - (3 \log 2 + \log 3)]$
$+ 3\ [4 \log 3 - (4 \log 2 + \log 5)]$

$= 28 \log 2 - 7 \log 3 - 7 \log 5 + 10 \log 5 - 15 \log 2 - 5 \log 3$
$+ 12 \log 3 - 12 \log 2 - 3 \log 5 = \log 2$

2007 - FEBRUARY

[5] Two numbers are in the ratio 7 : 8. If 3 is added to each of them, their ratio becomes 8 : 9. The numbers are :

(a) 14,16 (b) 24, 27

(c) 21, 24 (d) 16,18 (1 mark)

Answer:

(c) Let the numbers be $7x$ and $8x$.

So, $\frac{7x + 3}{8x + 3} = \frac{8}{9}$

$9 (7x + 3) = 8 (8x + 3)$

$63x + 27 = 64x + 24$

$x = 3$

Numbers are : $7x = 7 \times 3 = 21$

$8x = 8 \times 3 = 24$

[6] A box contains ₹ 56 in the form of coins of one rupee, 50 paise and 25 paise. The number of 50 paise coin is double the number of 25 paise coins and four times the numbers of one rupee coins. The numbers of 50 paise coins in the box is :

(a) 64 (b) 32

(c) 16 (d) 14 (1 mark)

Answer:

(a) Let the number of one – rupee coins be x.

Then, number of 50 paise coins is 4x

and number of 25 – paise coins is 2x

So,

$x + \frac{4x}{2} + \frac{2x}{4} = 56$

$4x + 8x + 2x = 56 \times 4$

$14x = 224$

$x = \frac{224}{14} = 16$

Number of 50 paise coins is $4 \times 16 = 64$

[7] Value of $(a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$ is :

(a) $a + \frac{1}{a}$ (b) $a - \frac{1}{a}$

(c) $a^2 + \frac{1}{a^2}$ (d) $a^2 - \frac{1}{a^2}$ (1 mark)

Answer:

(b) $(a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$

$= (a^{1/4} - a^{-1/4})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$

[using $(a^2 - b^2) = (a - b)(a + b)$]

$= (a^{1/2} - a^{-1/2})(a^{1/2} + a^{-1/2})$

$= a^1 - a^{-1}$

$= a - \frac{1}{a}$

[8] The value of the expression :

$a^{\log_a b \cdot \log_b^c \cdot \log_c^d \cdot \log_d t}$

(a) t (b) abcdt

(c) (a + b + c + d + t) (d) None. (1 mark)

Answer:

(a) $a^{\log_a^b \cdot \log_b^c \cdot \log_c^d \cdot \log_d^t}$

$a^{\frac{\log b}{\log a} \times \frac{\log c}{\log b} \cdot \frac{\log d}{\log c} \cdot \frac{\log t}{\log d}} = \left[\text{using } \log a^b = \frac{\log^b}{\log^a}\right]$

$= a^{\frac{\log t}{\log a}}$

$= a \log_a^t$

$= t$ [using $a^{\log a^m} = m$]

[9] If $\log_{10000} x = \frac{-1}{4}$, then x is given by:

(a) $\frac{1}{100}$ (b) $\frac{1}{10}$

(c) $\frac{1}{20}$ (d) None of these. (1 mark)

Answer:

(b) $\text{Log}_{1000}\, x = -\frac{1}{4}$

$(10,000)^{-1/4}$ x = [using log $a^b = x$, = $a^x = b$

$\frac{1}{(10,000)^{1/4}}$ $= x$

$= \frac{1}{10} = x$

2007 - MAY

[10] Eight people are planning to share equally the cost of a rental car. If one person withdraws from the arrangement and the others share equally entire cost of the car, then the share of each of the remaining persons increased by :

(a) 1/9 (b) 1/8

(c) 1/7 (d) 7/8 (1 mark)

Answer:

(c) When number of people = 8

then, the share of each person $= \frac{1}{8}$ of the total cost.

When number of people = 7

then, the share of each person $= \frac{1}{7}$ of the total cost

Increase in the share of each person = i.e.

Answer:

(a) Let the number of coins be 3x,4x,and 5x.

Then, $3x + \frac{4x}{2} + \frac{5x}{10} = 187$

$30x + 20x + 5x = 187 \times 10$

$55x = 1870$

$x = \frac{1870}{55} = 34$

Number of coins:

One rupee = $3x = 3 \times 34 = 102$

50 paise = $4x = 4 \times 34 = 136$

10 paise = $5x = 5 \times 34 = 170$

[12] Simplification of $\frac{x^{m+3n} \cdot x^{4m-9n}}{x^{6m-6n}}$ is :

(a) x^m (b) x^{-m}

(c) x^n (d) x^{-n} (1 mark)

Answer:

(b) $\frac{x^{m-3n} \cdot x^{4m-9n}}{x^{6m-6n}}$

$= \frac{x^{m+3n-4m-9n}}{x^{6m-6n}}$ $\left[\text{using } \frac{x^a - x^b}{x^{a+b}}\right]$

$= \frac{x^{5m-6n}}{x^{6m-6n}}$

$= x^{5m-6n-6m+6n}$ $\left[\text{using } \frac{x^a}{x^b} = x^{a-b}\right]$

$= x^{-m}$

[13] If $\log(2a - 3b) = \log a - \log b$, then a = :

(a) $\frac{3b^2}{2b-1}$ (b) $\frac{3b}{2b-1}$

(c) $\frac{b^2}{2b+1}$ (d) $\frac{3b^2}{2b+1}$ (1 mark)

Answer:

(a) $\text{Log}\,(2a - 3b) = \log a - \log b$

$$\log (2a - 3b) = \log \left(\frac{a}{b}\right)$$

$$2a - 3b = \frac{a}{b}$$

$$2ab - 3b^2 = a$$

$$2ab - a = 3b^2$$

$$a(2b - 1) = 3b^2$$

$$a = \frac{3b^2}{2b - 1}$$

2007 - AUGUST

[14] On simplification $\frac{1}{1+z^{a-b}+z^{a-c}} + \frac{1}{1+z^{b-c}+z^{b-a}} + \frac{1}{1+z^{c-a}+z^{c-b}}$ reduces to:

(a) $\frac{1}{z^{2(a+b+c)}}$ (b) $\frac{1}{z^{(a+b+c)}}$

(c) 1 (d) 0 (1 mark)

Answer:

(c) $$\frac{1}{1+z^{a-b}+z^{a-c}} + \frac{1}{1+z^{b-c}+z^{b-a}} + \frac{1}{1+z^{c-a}+z^{c-b}}$$

$$= \frac{1}{1+\frac{z^{-b}}{z^{-a}}+\frac{z^{-c}}{z^{-a}}} + \frac{1}{1+\frac{z^{-c}}{z^{-b}}+\frac{z^{-a}}{z^{-b}}} + \frac{1}{1+\frac{z^{-a}}{z^{-b}}+\frac{z^{-b}}{z^{-c}}}$$

$$= \frac{z^{-a}}{z^{-a}+z^{-b}+z^{-c}} + \frac{z^{-b}}{z^{-b}+z^{-c}+z^{-a}} + \frac{z^{-c}}{z^{-c}+z^{-a}+z^{-b}}$$

$$= \frac{z^{-a}+z^{-b}+z^{-c}}{z^{-a}+z^{-b}+z^{-c}}$$

$$= 1$$

[15] Ratio of earnings of A and B is 4 : 7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earning becomes 8 : 7. What is A's earning ?

(a) ₹ 21,000 (b) ₹ 26,000
(c) ₹ 28,000 (d) Data inadequate. (1 mark)

Answer:

(d) Let the earning of A and B be 4x and 7x respectively.
New earning of A = 4x × 150% = 6x
New earning of B = 7x × 75% = 5.25

Then, $\frac{6x}{5.25x} = \frac{8}{7}$

This does not give the value of x
So, the given data is inadequate.

[16] P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 12 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is :

(a) 22 : 27 (b) 27 : 22
(c) 32 : 33 (d) None. (1 mark)

Answer:

(b) $\frac{P}{Q} = \frac{11}{12}$ and $\frac{P}{R} = \frac{9}{8}$

$\frac{P}{Q} = \frac{11 \times 9}{12 \times 9} = \frac{99}{108}$ and $\frac{P}{R} = \frac{9 \times 11}{8 \times 11} = \frac{99}{88}$

Therefore, $\frac{Q}{R} = \frac{108}{88} = \frac{27}{22}$

So, Q : R = 27:22

[17] $\frac{1}{\log_{ab}(abc)} + \frac{1}{\log_{bc}(abc)} + \frac{1}{\log_{ca}(abc)}$ is equal to :

(a) 0 (b) 1
(c) 2 (d) – 1 (1 mark)

Answer:

(c) $\frac{1}{\log_{ab}^{(abc)}} + \frac{1}{\log_{bc}^{(abc)}} + \frac{1}{\log_{ca}^{(abc)}}$

$$= \frac{1}{\frac{\log(abc)}{\log(ab)}} + \frac{1}{\frac{\log(abc)}{\log(bc)}} + \frac{1}{\frac{\log(abc)}{\log(ca)}}$$

$$\left[\text{using } \log_a b = \frac{\log b}{\log a}\right]$$

$$= \frac{\log(ab)}{\log(abc)} + \frac{\log(bc)}{\log(abc)} + \frac{\log(ca)}{\log(abc)}$$

$$= \frac{\log\ (ab \times bc \times ca)}{\log a\, b\, c}$$

$$= \frac{\log\ a^2 b^2 c^2}{\log(abc)}$$

$$= \frac{\log(abc)^2}{\log abc} = \frac{2\ \log(abc)}{\log(abc)} = 2$$

[18] Number of digits in the numeral for 2^{64}. [Given log 2 = 0.30103] :

(a) 18 digits (b) 19 digits

(c) 20 digits (d) 21 digits. (1 mark)

Answer:

(c) 2^{64}

$= 64 \log 2$

$= 64 \times 0.30103$

$= 19.26592$

Number of digit in $2^{64} = 20$.

(a) ₹ 165, ₹ 132, ₹ 110 (b) ₹ 165, ₹ 110, ₹ 132
(c) ₹ 132, ₹ 110, ₹ 165 (d) ₹ 110, ₹ 132, ₹ 165 (1 mark)

Answer:

(a) The ratio of share of A, B and C

$= \frac{1}{4} : \frac{1}{5} : \frac{1}{6}$

$= \frac{15:12:10}{60} = 15:12:10$

Therefore, A's share = $407 \times \frac{15}{37} = ₹165$

B's share $= 407 \times \frac{12}{37} = ₹132$

C's share $= 407 \times \frac{10}{37} = ₹110$

[20] The incomes of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5 : 3. If each saves ₹ 1,500, then B's income is :

(a) ₹ 6,000 (b) ₹ 4,500
(c) ₹ 3,000 (d) ₹ 7,500 (1 mark)

Answer:

(a) Let the income of A and B be $3x$ and $2x$ respectively and expenditures of A and B be 5y and 3y respectively.

Therefore, $3x - 5y = 1500$ ..... (i)

$2x - 3y = 1500$ ..... (ii)

Solving (i) and (ii) Simultaneously

We get $x = 3000$ and $y = 1500$

Therefore, B's income = $2x = 2 \times 3{,}000 = ₹ 6{,}000$

[21] If $4^x = 5^y = 20^z$ then z is equal to :

(a) xy (b) $\frac{x+y}{xy}$

(c) $\frac{1}{xy}$ (d) $\frac{xy}{x+y}$ (1 mark)

Answer:

(d) $4^x = 5^y = 20^z = k$ (say)

$4 = k^{1/x}$

$5 = k^{1/y}$

$20 = k^{1/z}$

$4 \times 5 = 20$

$k^{1/x} \times k^{1/y} = k^{1/z}$

$k^{1/x + 1/y} = k^{1/z}$ ($x^m \times x^n = x^{m+n}$)

$$= 3^{\frac{-15}{4}} \left(3^{\frac{1}{2}}\right)^{\frac{7}{2}} \times 3^2$$

$$= \left(\frac{3}{3^{3/2}}\right)^{7/2} 3^{\frac{-15}{4}} \times 3^{\frac{7}{4}} \times 3^2$$

$$= 3^{\frac{-15}{4} + \frac{7}{4} + 2}$$

$$= 3^{-2+2} = 3^0 = 1$$

[23] The value $\frac{\log_3 8}{\log_9 16 . \log_4 10}$ is :

(a) $3 \log_{10} 2$ (b) $7 \log_{10} 3$
(c) $3 \log_e z$ (d) None. (1 mark)

Answer:

(a) $\frac{\log_3^8}{\log_9^{16} - \text{Log}_4^{10}}$

$$= \log_3^8 . \log_{16}^9 . \log_{10}^4$$

$$= \log_3^{2^3} . \text{Log}_{4^2}^{3^2} . \log_{10}^{2^2}$$

$$= 3\log_3^2 \frac{2}{4} \log_2^3 2 \log_{10}^2$$

$$= \frac{3\log 2}{\log 3} . \frac{1\log 3}{2\log 2} . \frac{2\log 2}{\log 10}$$

$$= \frac{3\log 2}{\log 10}$$

$$= 3 \log_{10}^2$$

2008 - FEBRUARY

[24] In 40 litres mixture of glycerine and water, the ratio of glycerine and water is 3:1. The quantity of water added in the mixture in order to make this ratio 2:1 is:

(a) 15 litres (b) 10 litres
(c) 8 litres (d) 5 litres. (1 mark)

Answer:

(d) Quantity of glycerine = $40 \times \frac{3}{4} = 30$ litres

Quantity of water = $40 \times \frac{1}{4} = 10$ litres

Let x liters of water be added to the mixture.
Then, total quantity of mixture = (40 + x) litres
total quantity of water in the mixture = (10 + x) litres.

So, $\frac{30}{10+x} = \frac{2}{1}$

$30 = 20 + 2x$

$2x = 10$

$x = 5$ litres

Therefore, 5 litres of water must be added to the mixture.

[25] The third proportional between $(a^2 - b^2)$ and $(a + b)^2$ is :

(a) $\frac{a+b}{a-b}$ (b) $\frac{a-b}{a+b}$

(c) $\frac{(a-b)^2}{a+b}$ (d) $\frac{(a+b)^3}{a-b}$ (1 mark)

Answer:

(d) Let the third proportional be *x*.

=

Answer:

(c) $2^x - 2^{x-1} = 4$

$$2^x - \frac{2^x}{2} = 4$$

$$2^x\left[1 - \frac{1}{2}\right] = 4$$

$$2^x\left[\frac{1}{2}\right] = 4$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

$$\frac{1+x}{1-x} = e^{2n} \quad \frac{1+x}{1-x} = 2n$$

$$\text{Log}\left(\qquad = 2n, \; n = \qquad \text{Log } e \right.$$

[30] The ages of two persons are in the ratio 5:7. Eighteen years ago their ages were in the ratio of 8:13, their present ages (in years) are :

(a) 50, 70 (b) 70, 50

(c) 40, 56 (d) None. (1 mark)

Answer:

(a) Let the present ages of persons be 5x & 7x .

Eighteen years ago, their ages = 5x – 18 and 7x – 18.

According to given:

$$\frac{5x-18}{7x-18}=\frac{8}{13}$$

$65x - 234 = 56x - 144$

$9x = 90$

$x = 10$

Their present ages are $5x = 5 \times 10 = 50$ years

$7x = 7 \times 10 = 70$ years.

[31] If $x = y^a$, $y = z^b$ and $z = x^c$ then abc is:

(a) 2 (b) 1

(c) 3 (d) 4 (1 mark)

Answer:

(b) $Z = x^c$

$Z = (y^a)^c$($y^a = x$)

2008 - DECEMBER

[33] If $\log\left(\frac{a+b}{4}\right)=\frac{1}{2}(\log a+\log b)$ then: $\frac{a}{b}+\frac{b}{a}$

(a) 12 (b) 14
(c) 16 (d) 8 (1 mark)

Answer:

(b) $\text{Log}\left(\frac{a+b}{4}\right)=\frac{1}{2}(\text{Log } a+\text{Log } b)$

$\text{Log}\left(\quad\right)=\text{Log}\,(ab)^{1/2}$

Profit (at year end) = ₹ 2,42,000 gives

A's Share = × 2,42,000 = ₹ 72,600

$$= \frac{\frac{-4q+3q}{3}}{\frac{-4q-3q}{3}}$$

$$= \frac{-q}{3} \times \frac{3}{-7q}$$

$$= \frac{1}{7}$$

[36] Fourth proportional to x, 2x, (x+1) is:

(a) (x + 2) (b) (x – 2)

(c) (2x + 2) (d) (2x – 2) (1 mark)

Answer:

(c) Let the fourth proportional to x, 2x, (x + 1) be t, then,

$$\frac{x}{2x} = \frac{x+1}{t}$$

$$\frac{1}{2} = \frac{x+1}{t}$$

$$t = 2x + 2$$

Fourth proportional to x, 2x, (x + 1) is (2x + 2)

$$x^3 = 3+\frac{1}{3}+3x \quad \text{[Using (1)]}$$

$$x^3 - 3x = \frac{9+1}{3}$$

$$3(x^3 - 3x) = 10$$

$$3x^3 - 9x = 10$$

Answer:

(a) $\log (m + n) = \log m + \log n$

$\log (m + n) = \log (m\, n)$ [$\log (ab) = \log a + \log b$]

2009 - DECEMBER

[41] $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

(a) ½ (b) 3/2
(c) 2/3 (d) 1/3 (1 mark)

Answer:

(b) $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

$= 2^n (1 + \frac{1}{2})$

$= 2n (2 - 1)$

$= \frac{3}{2} = \frac{3}{2}$

$= 1$

[42] If $2^x \times 3^y \times 5^z = 360$ Then what is the value of x, y, z,?

(a) 3, 2, 1 (b) 1, 2, 3
(c) 2, 3, 1 (d) 1, 3, 2 (1 mark)

Answer:

(a) $2^x \times 3^y \times 5^z = 360$. .....(1)

The factors of 360 are:

$2^3 \times 3^2 \times 5$.

$2^3 \times 3^2 \times 5^1 = 360$.....(2)

Answer:

(c) $[\log_{10} \sqrt{25} - \log_{10}(2^3) + \log_{10} (4^2)]^x$

$= [\log_{10} 5 - 3 \log_{10} 2 + \log_{10} (2^4)]^x$

$= [\log_{10} 5 - 3 \log_{10} 2 + 4 \log_{10} 2]^x$

$= [\log_{10} 5 + \log_{10} 2]^x$

$= [\log_{10} (5 \times 2)]^x$ [$\log (mn) = \log m + \log n$]

[46] The students of two classes are in the ratio 5 : 7, if 10 students left from each class, the remaining students are in the ratio of 4 : 6 then the number of students in each class is:

(a) 30, 40 (b) 25, 24

(c) 40, 60 (d) 50, 70 (1 mark)

Answer:

(d) Let the ratio be 5x : 7x

If 10 student left, Ratio became 4 : 6

$$\frac{5x - 10}{7x - 10} = \frac{4}{6}$$

$30x - 60 = 28x - 40$

$2x = 20$

$x = 10$

No. of students in each class is 5x and 7x

[48] The recurring decimal 2.7777..... can be expressed as:

(a) 24/9 (b) 22/9

(c) 26/9 (d) 25/9 (1 mark)

Answer:

(d) 2.7777

$2 + 0.7 + 0.07 + 0.007 + ...$

$2 + \left(\frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + ...\right)$

$2 + 7\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + ...\right)$

$2 + 7\left(\frac{1/10}{1 - 1/10}\right)$

$= 2 + 7 \times \frac{1}{9}$

$= 2 + \frac{7}{9}$

$= \frac{18 + 7}{9}$

$= \frac{25}{9}$

[49] Solve : $\left(\frac{\log x_{10} - 3}{2}\right) + \left(\frac{11 - \log x_{10}}{3}\right) = 2$

(a) 10^{-1} (b) 10^{2}

(c) 10 (d) 10^{3} (1 mark)

Answer:

(a) $\left(\frac{\log_{10} x - 3}{2}\right) + \left(\frac{11 - \log_{10} x}{3}\right) = 2$

$3 \log_{10} x - 9 + 22 - 2 \log_{10} x = 12$

$\log_{10} x + 13 = 12$

$\log_{10} x = -1$

$x = 10^{-1}$

[50] If A:B = 2:5, then (10A + 3B):(5A + 2B) is equal to:

(a) 7 : 4 (b) 7 : 3

(c) 6 : 5 (d) 7 : 9 (1 mark)

Answer:

(a) $\frac{A}{B} = \frac{2}{5} = \frac{2k}{5k}$

$$\frac{10A + 3B}{5A + 2B} = \frac{20k + 15k}{10k + 10k} = \frac{35k}{20k}$$

$$= \frac{35}{20}$$

$$= \frac{7}{4}$$

2011 - JUNE

[51] If n = m! where ('m' is a positive integer > 2) then the value of :

$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots\ldots\ldots + \frac{1}{\log_m n}$$

(a) 1 (b) 0

(c) -1 (d) 2 (1 mark)

Answer:

(a) Given : n = M ! for M $\geq$ 2

$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots\ldots\ldots\ldots + \frac{1}{\log_m n}$$

or, $= \log_n 2 + \log_n 3 + \log_n 4 + \ldots\ldots\ldots\ldots + \log_n m$ $\left(\because \log_b a = \frac{1}{\log_a b}\right)$

$= \log_n (2 \times 3 \times 4 \times \ldots\ldots \times m)$ $(\because \log(mn) = \log m + \log n)$

$= \log_n (m!)$

$= \log_n n$

$= 1$

[52] In a film shooting, A and B received money in a certain ratio and B and C also received the money in the same ratio. If A gets ₹ 1,60,000 and C gets ₹ 2,50,000. Find the amount received by B ?

(a) ₹ 2,00,000 (b) ₹ 2,50,000

(c) ₹ 1,00,000 (d) ₹ 1,50,000 (1 mark)

Answer:

(a) Given : A : B = B : C

$\Rightarrow B^2 = A \times C$

or $B = \sqrt{A \times C}$

& A = 1,60,000 ; C = 2,50,000

B =

$$\sqrt{a} = \frac{8 \times 5 \times 3}{15 \times 4}$$

$$\sqrt{a} = 2$$

On squaring $(\sqrt{a})^2 = 2^2$

$$a = 4$$

[54] If $\log_2 x + \log_4 x = 6$, then the Value of x is :

(a) 16 (b) 32

(c) 64 (d) 128 (1 mark)

Answer:

(a) If $\log_2 x + \log_4 x = 6$

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 4} = 6$$

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 2^2} = 6$$

$$\frac{\log x}{\log 2} + \frac{\log x}{2\log 2} = 6$$

$$\frac{\log x}{\log 2}\left[1 + \frac{1}{2}\right] = 6$$

$$\frac{\log x}{\log 2} \times \frac{3}{2} = 6$$

$$\frac{\log x}{\log 2} = 6 \times \frac{2}{3}$$

$$\frac{\log x}{\log 2} = 4$$

$$\log x = 4 \log 2$$

$$\log x = \log 2^4$$

$$x = 2^4$$

$$x = 16$$

[55] If X Varies inversely as square of Y and given that Y = 2 for X = 1, then the Value of X for Y = 6 will be:

(a) 3 (b) 9

(c) 1/3 (d) 1/9 (1 mark)

Answer:

(d) Given x varies inversely as square of y

i. e. $x \propto \frac{1}{y^2}$

$x = k \frac{1}{y^2}$

$x = \frac{k}{y^2}$.....(1)

Given x = 1, y = 2 then

$1 = \frac{k}{(2)^2}$ $k = 1 \times 4 = 4$

Now putting y = 6, k = 4 in equation (1)

$x = \frac{4}{6^2}$

$x = \frac{4}{36} = \frac{1}{9}$

2012 - JUNE

[56] The value of $\frac{(3^{n+1} + 3^n)}{(3^{n+3} - 3^{n+1})}$ is equal to:

(a) 1/5 (b) 1/6

(c) 1/4 (d) 1/9 (1 mark)

Answer:

(b) $\frac{3^{n+1} + 3^n}{3^{n+3} - 3^{n+1}} = \frac{3^n . 3^1 + 3^n}{3^n . 3^3 - 3^n . 3^1}$

$= \frac{3^n (3^1 + 1)}{3^n (3^3 - 3)}$

$= \frac{(3 + 1)}{(27 - 3)}$

$= \frac{4}{24}$

$= \frac{1}{6}$

[57] If $\log x\ y = 100$ and $\log_2 x = 10$, then the value of 'y' is :

(a) 2^{10} (b) 2^{100}

(c) $2^{1,000}$ (d) $2^{10,000}$ (1 mark)

Answer:

(c) Given $\log_x y = 100$ .....(1)

$\log_2 x = 10$.....(2)

Multiply eq (1) & (2)

$$\log_x y \cdot \log_2 x = 100 \times 10$$

$$\frac{\log y}{\log x} \times \frac{\log x}{\log 2} = 1{,}000$$

$$\log y = 1{,}000 \log 2$$

$$\log y = \log 2^{1,000}$$

$$\Rightarrow y = 2^{1,000}$$

[58] Which of the numbers are not in proportion ?

(a) 6, 8, 5, 7 (b) 7, 3, 14, 6

(c) 18, 27, 12, 18 (d) 8, 6, 12, 9 (1 mark)

Answer:

(a) If say a, b, c, d are in proportion they bear a common ratio that is

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

Option (A) $\frac{6}{8} \neq \frac{5}{7}$

Option (B) $\frac{7}{3} = \frac{14}{6}$

Option (C) $\frac{18}{27} = \frac{12}{18}$

Option (D) $\frac{8}{6} = \frac{12}{9}$

2012 - DECEMBER

[59] Find the value of x, if $x\,(x)^{1/3} = (x^{1/3})^x$

(a) 3 (b) 4

(c) 2 (d) 6 (1 mark)

Answer:

(b) If $x^1\,(x)^{1/3} = (x^{1/3})^x$

$$x^{1+1/3} = x^{\frac{1}{3}x}$$

$$\Rightarrow x^{4/3} = x^{\frac{1}{3}x}$$

on comparing

$$\frac{4}{3} = \frac{x}{3}$$

$3x = 12 \Rightarrow x = 4$

[60] Which of the following is true.

If $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = \frac{1}{abc}$

(a) $\log (ab + bc + ca) = abc$ (b) $\log\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = abc$

(c) $\log (abc) = 0$ (d) $\log (a + b + c) = 0$ (1 mark)

Answer:

(d) Given

$$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = \frac{1}{abc}$$

$$\frac{c + a + b}{abc} = \frac{1}{abc}$$

$a + b + c = 1$

taking log on both side

$\log (a + b + c) = \log 1$

$\log (a + b + c) = 0$

[61] Find two numbers such that mean proportional between them is 18 and third proportional between them is 144

(a) 9, 36 (b) 8, 32

(c) 7, 28 (d) 6, 24 (1 mark)

Answer:

(a) Let two Nos. be x and y

Mean proportion between x and y is 18

So, x, 18, y are in proportion

$x : 18 :: 18 : y$

$$\frac{x}{18} = \frac{18}{y}$$

$$xy = 324$$

$$x = \frac{324}{y} \quad \text{_____ (1)}$$

If third proportion between x & y be 144

So, x, y, 144 are in proportion

$x : y :: y : 144$

$$\frac{x}{y} = \frac{y}{144}$$

$$y^2 = 144x \quad \text{_____ (2)}$$

Putting the value of x in equation (2)

$$y^2 = 144 \times \frac{324}{y}$$

$$y^3 = 144 \times 324$$

$$y = \sqrt[3]{144 \times 324}$$

$$y = \sqrt[3]{3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$$

$$y = \sqrt[3]{6 \times 6 \times 6 \times 6 \times 6 \times 6}$$

$$y = 6 \times 6$$

$$y = 36$$

Putting y = 36 in equation (1)

$$x = \frac{324}{36} = 9$$

$x = 9, y = 36$

2013 - JUNE

[62] For what value of x, the equation $(\log_{\sqrt{x}} 2)^2 = \log_x^2$ is true?

(a) 16 (b) 32
(c) 8 (d) 4 (1 mark)

Answer:

(a) Given

$$\left(\log_{\sqrt{x}} 2\right)^2 = \log_x 2$$

$$\left(\frac{\log 2}{\log \sqrt{x}}\right)^2 = \left(\frac{\log 2}{\log x}\right)$$

$$\left(\frac{\log 2}{\log x^{1/2}}\right)^2 = \frac{\log 2}{\log x}$$

$$\left(\frac{\log 2}{\frac{1}{2}\log x}\right)^2 = \frac{\log 2}{\log x}$$

$$\left(\frac{2\log 2}{\log x}\right)^2 = \left(\frac{\log 2}{\log x}\right)$$

$$4\left(\frac{\log 2}{\log x}\right)^2 = \left(\frac{\log 2}{\log x}\right)^1$$

$$4\,\frac{\log 2}{\log x} = 1$$

$$4\log 2 = \log x$$

$$\log 2^4 = \log x$$

$$\Rightarrow \quad 2^4 = x \Rightarrow x = 16$$

[63] The mean proportional between 24 and 54 is :

(a) 33 (b) 34
(c) 35 (d) 36 (1 mark)

Answer:

(d) Mean Proportion $= \sqrt{24 \times 54}$

$= \sqrt{1296}$

$= 36$

[64] The triplicate ratio of 4 : 5 is:

(a) 125 : 64 (b) 16 : 25

(c) 64 : 125 (d) 120 : 46 (1 mark)

Answer:

(c) The triplicate Ratio of $4 : 5 = 4^3 : 5^3$

$= 64 : 125$

2013 - DECEMBER

[65] If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c}$ then the value of $\left(\frac{a+b+c}{3}\right)^3$

(a) abc (b) 9abc

(c) $\frac{1}{abc}$ (d) $\frac{1}{9abc}$ (1 mark)

Answer:

(a) If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$

$a^{1/3} + b^{1/3} + c^{1/3} = 0$

$a^{1/3} + b^{1/3} = -c^{1/3}$ ..... (i)

Cube on both side

$(a^{1/3} + b^{1/3})^3 = (-c^{1/3})^3$

$(a^{1/3})^3 + (b^{1/3})^3 + 3 \cdot a^{1/3} \cdot b^{1/3} (a^{1/3} + b^{1/3}) = -c$

$a + b + 3a^{1/3} \cdot b^{1/3} \cdot (-c^{1/3}) = -c$

$a + b - 3a^{1/3} \cdot b^{1/3} \cdot c^{1/3} = -c$

$a + b + c = 3a^{1/3} \cdot b^{1/3} \cdot c^{1/3}$

$\left(\frac{a+b+c}{3}\right) = \frac{3a^{1/3} \cdot b^{1/3} \cdot c^{1/3}}{3}$

$\left(\frac{a+b+c}{3}\right)^3 = (a^{1/3} \cdot b^{1/3} \cdot c^{1/3})^3 = abc$

[66] Find three numbers in the ratio 1 : 2 : 3, so that the sum of their squares is equal to 504

(a) 6, 12, 18 (b) 3, 6, 9

(c) 4, 8, 12 (d) 5, 10, 15 (1 mark)

Answer:

(a) Since Ratio of three Number is 1 : 2 : 3

First No. = x

Second No. = 2x

Third No. = 3x

Sum of squares of numbers = 504

$$(x)^2 + (2x)^2 + (3x)^2 = 504$$

$$x^2 + 4x^2 + 9x^2 = 504$$

$$14x^2 = 504$$

$$x^2 = \frac{504}{14}$$

$$x^2 = 36$$

$$x = 6$$

First No. = x = 6

Second No. = 2x = 2 × 6 = 12

Third No. = 3x = 3 × 6 = 18

[67] The value of $\log_4 9 \cdot \log_3 2$ is:

(a) 3 (b) 9

(c) 2 (d) 1 (1 mark)

Answer:

(d) $\log_4 9 \cdot \log_3 2$

$$= \frac{\log 9}{\log 4} \cdot \frac{\log 2}{\log 3}$$

$$= \frac{\log 3^2}{\log 2^2} \cdot \frac{\log 2}{\log 3}$$

$$= \frac{2\log 3}{2\log 2} \cdot \frac{\log 2}{\log 3}$$

$$= 1$$

[68] The value of $(\log_y x \cdot \log_z y \cdot \log_x z)^3$ is

(a) 0 (b) - 1

(c) 1 (d) 3 (1 mark)

Answer:

(c) $(\log_y x \cdot \log_z y \cdot \log_x z)^3$

$$= \left(\frac{\log x}{\log y} \cdot \frac{\log y}{\log z} \cdot \frac{\log z}{\log x}\right)^3$$

$$= (1)^3$$

$$= 1$$

[69] Divide 80 into two parts so that their product is maximum, then the numbers are:

(a) 25, 55 (b) 35, 45

(c) 40, 40 (d) 15, 65 (1 mark)

Answer:

(c) The sum of two No. = 80

First No. = x

Second No. = (80 – x)

Product two No = x. (80 – x)

$P = 80x - x^2$ ..... (1)

w.r.f. (x)

$$\frac{dp}{dx} = 80 - 2x \quad \text{..... (2)}$$

$$\frac{d^2p}{dx^2} = -2 \quad \text{..... (3)}$$

For max/minima

$$\frac{dp}{dx} = 0$$

$80 - 2x = 0$

$2x = 80$

$x = 40$

x = 40 in equation (iii)

$\frac{d^2p}{dx^2} = -2$ (Negative)

function is maximum at x = 40

Numbers are 40, (80 – 40)

= 40, 40

2014 - JUNE

[70] If x : y = 2:3, then (5x+2y):(3x-y)= ____

(a) 19 : 3 (b) 16 : 3
(c) 7 : 2 (d) 7 : 3 (1 mark)

Answer:

(b) Given,

$x : y = 2 : 3$

Let $x = 2k$, $y = 3k$

$(5x + 2y) : (3x - y)$

$= \frac{(5x + 2y)}{(3x - y)}$

$= \frac{5 \times 2k + 2 \times 3k}{3 \times 2k - 3k}$

$= \frac{10k + 6k}{6k - 3k}$

$= \frac{16k}{3k}$

$= 16 : 3$

[71] If $(25)^{150} = (25x)^{50}$; then the value of x will be :

(a) 5^3 (b) 5^4
(c) 5^2 (d) 5 (1 mark)

Answer:

(b) If $(25)^{150} = (25x)^{50}$

$25^{150} = 25^{50} \cdot x^{50}$

$\Rightarrow \frac{25^{150}}{25^{50}} = x^{50}$

$\Rightarrow 25^{100} = x^{50}$

$\Rightarrow (5^2)^{100} = x^{50}$

$\Rightarrow 5^{200} = x^{50}$

$\Rightarrow (5^4)^{50} = x^{50}$

$\Rightarrow 5^4 = x$

$\Rightarrow x = 5^4$

[72] The value of $\left(\frac{y^a}{y^b}\right)^{a^2+ab+b^2} \times \left(\quad \times \quad \right)$ is equal to ___ .

Answer:

(b) If $x^2 + y^2 = 7xy$

$x^2 + y^2 + 2xy = 7xy + 2xy$

$(x + y)^2 = 9xy$

taking log on both side

$\log (x + y)^2 = \log 9xy$

$2 \log (x + y) = \log 9 + \log x + \log y$

$2 \log (x + y) = \log 3^2 + \log x + \log y$

$2 \log (x + y) = 2 \log 3 + \log x + \log y$

$2 \log (x + y) - 2 \log 3 = \log x + \log y$

$2 \left[\log \frac{(x + y)}{3}\right] \left[\log \frac{(x + y)}{3}\right]$

$= \log x + \log y$

$\log \frac{(x + y)}{3} = \frac{1}{2} [\log x + \log y]$

[75] A person has assets worth ₹ 1,48,200. He wish to divide it amongst his wife, son and daughter in the ratio 3 : 2 : 1 respectively. From this assets, the share of his son will be:

(a) ₹ 24,700 (b) ₹ 49,400

(c) ₹ 74,100 (d) ₹ 37,050 (1 mark)

Answer:

(b) A person has Assets worth = ₹ 1,48,200

Ratio of share of wife, son & daughter

= 3 : 2 : 1

Sum of Ratio = 3 + 2 + 1 = 6

Share of Son = $\frac{2}{6} \times 1,48,200$

= 49,400

[76] If $x = \log_{24}12$, $y = \log_{36}24$ and $z = \log_{48}36$, then $xyz + 1 =$ ____

(a) 2xy (b) 2xz

(c) 2yz (d) 2 (1 mark)

Answer:

(c) If $x = \log_{24}12$, $y = \log_{36}24$ and $z = \log_{49}36$ then

XYZ + 1

$$= \log_{24}12 \times \log_{36}24 \times \log_{48}36 + 1$$
$$= \frac{\log 12}{\log 24} \cdot \frac{\log 24}{\log 36} \cdot \frac{\log 36}{\log 48} + 1$$
$$= \frac{\log 12}{\log 48} + 1$$
$$= \frac{\log 12 + \log 48}{\log 48}$$
$$= \frac{\log(12 \times 48)}{\log 48}$$
$$= \frac{\log(576)}{\log 48}$$
$$= \frac{\log 24^2}{\log 48}$$
$$= \frac{2\log 24}{\log 48}$$
$$= 2 \cdot \frac{\log 24}{\log 36} \cdot \frac{\log 36}{\log 48}$$
$$= 2.\log_{36}24 \cdot \log_{48}36$$
$$= 2\ y\ z$$

2014 - DECEMBER

[77] If $\log x = a + b$, $\log y = a - b$ then the value of $\log \frac{10x}{y^2}$ = _____.

(a) $1 - a + 3b$ (b) $a - 1 + 3b$
(c) $a + 3b + 1$ (d) $1 - b + 3a$ (1 mark)

Answer:

(a) Given $\log x = a + b$, $\log y = a - b$

$$\log\left(\quad\right) = \log 10x - \log y^2$$

[78] If $x = 1 + \log_p qr$, $y = 1 + \log_q rp$ and $z = 1 + \log_r pq$ then the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} =$ _____

(a) 0 (b) 1
(c) -1 (d) 3 (1 mark)

Answer:

(b) If $x = 1 + \log_p qr$, $y = 1 + \log_q rp$, $z = 1 + \log_r pq$

$$x = 1 + \frac{\log qr}{\log p}$$

$$x = \frac{\log p + \log qr}{\log p}$$

$$x = \frac{\log pqr}{\log p}$$

$$\frac{1}{x} = \frac{\log p}{\log pqr}$$

Similarly

$$\frac{1}{y} = \frac{\log q}{\log pqr}$$

$$\frac{1}{z} = \frac{\log r}{\log pqr}$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{\log p}{\log pqr} + \frac{\log q}{\log pqr} + \frac{\log r}{\log pqr}$$

$$= \frac{\log p + \log q + \log r}{\log pqr}$$

$$= \frac{\log pqr}{\log pqr}$$

$$= 1$$

[79] For three months, the salary of a person are in the ratio 2 : 4 : 5. If the difference between the product of salaries of the first two months and last two months is ₹ 4,80,00,000; then the salary of the person for the second month will be:

(a) ₹ 4,000 (b) ₹ 6,000
(c) ₹ 8,000 (d) ₹ 12,000 (1 mark)

Answer:

(c) Ratio of the salary of a person in three months = 2 : 4 : 5

Let, Salary of Ist month = 2x
Salary of IInd month = 4x
Salary of IIIrd month = 5x

Given

(Salary of Product of last two months) – (Salary of Product Ist two months)

$= 4,80,00,000$

$(4x.5x) - (2x.4x) = 4,80,00,000$

$20x^2 - 8x^2 = 4,80,00,000$

$12x^2 = 4,80,00,000$

$x^2 = 40,00,000$

$x = 2,000$

Salary of the person for second month = $4x = 4 \times 2,000 = 8,000$

2015 - JUNE

[80] A dealer mixes rice costing ₹ 13.84 per Kg. with rice costing ₹ 15.54 and sells the mixture at ₹ 17.60 per Kg. So, he earns a profit of 14.6% on his sale price. The proportion in which he mixes the two qualities of rice is:

(a) 3 : 7 (b) 5 : 7
(c) 7 : 9 (d) 9 : 11 (1 mark)

Answer:

(a) Let SP of mixture is ₹ 100

Then Profit = 14.6% of 100
= 14.6

CP of mixture = (100 – 14.6)
= 85.4

If SP is ₹ 100 then CP = 85.4

∴ If SP is ₹ 17.60 then CP $= \frac{85.4}{100} \times 17.60$

$= 15.0304$

CP of the Mixture per kg = ₹ 15.0304

[83] If $15(2p^2 - q^2) = 7pq$, where p and q are positive, then p : q will be:

(a) 5 : 6 (b) 5 : 7

(c) 3 : 5 (d) 8 : 3 (1 mark)

Answer:

(a) If $15(2p^2 - q^2) = 7pq$

$30p^2 - 15q^2 = 7pq$

$30p^2 - 7pq - 15q^2 = 0$

$30p^2 - 25pq + 18pq - 15q^2 = 0$

$5p(6p - 5q) + 3q(6p - 5q) = 0$

$(6p - 5q)(5p + 3q) = 0$

If $6p - 5q = 0$ and $5p + 3q = 0$

$6p = 5q \quad 5p = -3q$

$\frac{p}{q} = \frac{5}{6} = p : q = 5 : 6 \quad \frac{p}{q} = \frac{-3}{5}$

(not possible)

2015 - DECEMBER

[84] The ratio of third proportion of 12, 30 to the mean proportion of 9, 25 is:

(a) 2:1 (b) 5:1

(c) 7:15 (d) 3:5 (1 mark)

Answer:

(b) The third proportion of 12,30

$c = \frac{b^2}{a} = \frac{(30)^2}{12} = \frac{900}{12} = 75$

The Mean proportion of 9,25

$b = \sqrt{ac} = \sqrt{9 \times 25} = \sqrt{225} = 15$

Ratio of third proportion of 12, 30 and Mean proportion of 9, 25 = 75:15

= 5:1

[85] The value of $\log_5 3 \times \log_3 4 \times \log_2 5$.

(a) 0 (b) 1

(c) 2 (d) $\frac{1}{2}$ (1 mark)

Answer:

(c) $\log_5 3 \times \log_3 4 \times \log_2 5$

$= \frac{\log 3}{\log 5} \times \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 2}$

$= \frac{\log 4}{\log 2}$

$= \frac{\log 2^2}{\log 2}$

$= \frac{2 \log 2}{\log 2} = 2$

[86] What number must be added to each of the numbers 10, 18, 22, 38 to make the numbers is proportion?

(a) 2 (b) 4

(c) 8 (d) None of these. (1 mark)

Answer:

(a) Let x to be added

Then (10 + x), (18 + x), (22 + x), (38 + x) are in prop.

Product of Extremes = Product of Mean

$(10 + x)(38 + x) = (18 + x)(22 + x)$

$380 + 10x + 38x + x^2 = 396 + 18x + 22x + x^2$

$48x + 380 = 396 + 40x$

$48x - 40x = 396 - 380$

$8x = 16$

$x = 2$

[87] The value of $\frac{2^n + 2^{n-1}}{2^{n-1} - 2^n}$ is :

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) 2 (1 mark)

Answer:

(b) $$\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{2^n + 2^n.2^{-1}}{2^n.2^1 - 2^n}$$

$$= \frac{2^n(1 + 2^{-1})}{2^n(2^1 - 1)}$$

$$= \frac{\left(\frac{1}{1} + \frac{1}{2}\right)}{(2 - 1)}$$

$$= \frac{\left(\frac{2 + 1}{2}\right)}{1}$$

$$= \left(\right.$$

$$= \frac{x+y-z}{x+y+z} + \frac{y+z-x}{x+y+z} + \frac{z+x-y}{x+y+z}$$

$$= \frac{x+y-z+y+z-x+z+x-y}{x+y+z}$$

$$= \frac{x+y+z}{x+y+z} = 1$$

[90] X, Y, Z together starts a business. If X invests 3 times as much as Y invests and Y invests two third of what Z invests, then the ratio of capitals of X, Y, Z is:

(a) 3:9:2 (b) 6:3:2
(c) 3:6:2 (d) 6:2:3 (1 mark)

Answer:

(d) Given $x = 3y$ and $y = \frac{2}{3}z$

$$\frac{x}{y} = \frac{3}{1} \text{ and } \frac{y}{z} = \frac{2}{3}$$

$x : y = 3 : 1$ and $y : z = 2 : 3$

$= 3 \times 2 : 1 \times 2$

$= 6 : 2$

$x : y : z = 6 : 2 : 3$

[91] If $\log_4(x^2 + x) - \log_4(x + 1) = 2$, then the value of X is:

(a) 2 (b) 3
(c) 16 (d) 8 (1 mark)

Answer:

(c) If $\log_4 (x^2 + x) - \log_4 (x + 1) = 2$

$$\Rightarrow \log_4 \left\{\frac{(x^2+x)}{(x+1)}\right\} = 2$$

$$\Rightarrow \log_4 \left\{ \qquad = 2\right.$$

[92] Value of $\frac{1}{\log_3^{60}}+\frac{1}{\log_4^{60}}+\frac{1}{\log_5^{60}}$ is :

(a) 0 (b) 1
(c) 5 (d) 60 (1 mark)

Answer:

(b) $\frac{1}{\log_3 60}+\frac{1}{\log_4 60}+\frac{1}{\log_5 60}$

$= \log_{60} 3 + \log_{60} 4 + \log_{60} 5$ $\because \left[\frac{1}{\log_a b} = \log_b a\right]$

$= \log_{60}(3 \times 4 \times 5)$

$= \log_{60} 60$

$= 1$

2016 - DECEMBER

[93] If $3^x = 5^y = 75^z$, then

(a) $x + y - z = 0$ (b) $\frac{2}{x}+\frac{1}{y}=\frac{1}{z}$
(c) $\frac{1}{x}+\frac{2}{y}=\frac{1}{z}$ (d) $\frac{2}{x}+\frac{1}{z}=\frac{1}{y}$ (1 mark)

Answer:

(c) If $3^x = 5^y = 75^z = k$ (let)
then $3^x = k, 5^y = k, 75^z = k$
$3 = k^{1/x}, 5 = k^{1/y}, 75 = k^{1/z}$
we know that
$75 = 3 \times 5 \times 5$
$k^{\frac{1}{z}} = k^{\frac{1}{x}} \cdot k^{\frac{1}{y}} \cdot k^{\frac{1}{y}}$
$k^{\frac{1}{z}} = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{y}}$

on comparing

$$\frac{1}{z} = \frac{1}{x} + \frac{1}{y} + \frac{1}{y}$$

$$\frac{1}{z} = \frac{1}{x} + \frac{2}{y}$$

$$\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$$

[94] If log 2 = 0.3010 and log 3 = 0.4771, then the value of log 24 is:

(a) 1.0791 (b) 1.7323

(c) 1.3801 (d) 1.8301 (1 mark)

Answer:

(c) If log 2 = 0.3010 and log 3 = 0.4771

then log 24 = log (2 × 2 × 2 × 3)

= log 2 + log 2 + log 2 + log 3

= 3 log 2 + log 3

= 3 × 0.3010 + 0.4771

= 0.9030 + 0.4771

= 1.3801

[95] If abc = 2, then the value of $\frac{1}{1+a+2b^{-1}} + \frac{1}{1+\frac{1}{2}b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ is:

(a) 1 (b) 2

(c) 3 (d) $\frac{1}{2}$ (1 mark)

Answer:

(a) If abc = 2

$ab = \frac{2}{c} = 2c^{-1}$ $\quad a = \frac{2}{bc} = 2b^{-1}c^{-1}$

$bc = \frac{2}{a} = 2a^{-1}$ $\quad b = \frac{2}{ca} = 2c^{-1}a^{-1}$

$ca = \frac{2}{b} = 2b^{-1}$ $\quad c = \frac{2}{ab} = 2a^{-1}b^{-1}$

Given $\frac{1}{1+a+2b^{-1}}+\frac{1}{1+\frac{1}{2}b+c^{-1}}+\frac{1}{1+c+a^{-1}}$

$= \frac{1}{1+a+2b^{-1}}+\frac{2b^{-1}}{2b^{-1}(1+\frac{1}{2}b+c^{-1})}+\frac{a}{a(1+c+a^{-1})}$

$= \frac{1}{(1+a+2b^{-1})}+\frac{2b^{-1}}{2b^{-1}+1+2b^{-1}c^{-1}}+\frac{a}{a+ac+1}$

$= \frac{1}{1+a+2b^{-1}}+\frac{2b^{-1}}{2b^{-1}+1+a}+\frac{a}{a+2b^{-1}+1}$

$= \frac{1+2b^{-1}+a}{1+a+2b^{-1}}$

$= 1$

[96] There are total 23 coins of ₹ 1, ₹ 2 and ₹ 5 in a bag. If their value is ₹ 43 and the ratio of coins of ₹ 1 and ₹ 2 is 3:2. Then the number of coins of ₹ 1 is:

(a) 12 (b) 5

(c) 10 (d) 14 (1 mark)

Answer:

(a) Total no. of coins = 23

Ratio of ₹ 1 coin : ₹ 2 coins = 3 : 2

let No. of ₹ 1 coins = 3x

No. of ₹ 2 coins = 2x

No. of ₹ 5 coins = 23 – 3x – 2x

= 23 – 5x

Total value of all coins = 43

$3x \times 1 + 2x \times 2 + (23 - 5x)\,5 = 43$

$3x + 4x + 115 - 25x = 43$

$-18x = 43 - 115$

$-18x = -72$

$x = \frac{-72}{-18} = 4$

No. of ₹ 1 coins = $3x = 3 \times 4 = 12$

2017 - JUNE

[97] If a : b = 2 : 3, b : c = 4 : 5 and c : d = 6 : 7, then a : d is:

(a) 24 : 35 (b) 8 : 15

(c) 16 : 35 (d) 7 : 15 (1 mark)

Answer:

(c) $a : b = 2 : 3 \Rightarrow \frac{a}{b} = \frac{2}{3}$ _____(i)

$b : c = 4 : 5 \Rightarrow \frac{b}{c} = \frac{4}{5}$ _____(ii)

$c : d = 6 : 7 \Rightarrow \frac{c}{d} = \frac{6}{7}$ _____(iii)

Multiply equation (i) & (ii) & (iii)

$$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} = \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} = \frac{16}{35}$$

[98] The value of $\log (1^3 + 2^3 + 3^3 + ...n^3)$ is equal to:

(a) 3 log 1 + 3 log 2 + ... + 3 log n

(b) 2 log n + 2 log (n+1) – 2 log 2

(c) log n + log (n+1) + log (2n+1) – log 6 (1 mark)

(d) 1

Answer:

(b) $\log (1^3 + 2^3 + 3^3 + ------+ n^3)$

$= \log (\Sigma n^3)$

$= \log \left[\frac{n(n+1)}{2}\right]^2$

$= 2 \log \left[\frac{n(n+1)}{2}\right]$

$= 2 [\log n + \log (n + 1) - \log 2]$

$= 2 \log n + 2 \log (n + 1) - 2 \log 2$

[99] If $a = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ and $b = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ then the value of $\frac{1}{a^2} + \frac{1}{b^2}$ is equal to:

(a) 480 (b) 482

(c) 484 (d) 486 (1 mark)

Answer:

(b) If $a = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ and $b = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

$$a + b = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} + \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$$

$$= \frac{(\sqrt{6}+\sqrt{5})^2 + (\sqrt{6}-\sqrt{5})^2}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})}$$

$$= \frac{6 + 5 + \cancel{2\sqrt{30}} + 6 + 5 - \cancel{2\sqrt{30}}}{(\sqrt{6})^2 - (\sqrt{5})^2}$$

$$= \frac{22}{6-5} = \frac{22}{1} = 22$$

$$a \cdot b = \left(\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}\right)\left(\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}\right) = 1$$

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{b^2 + a^2}{a^2 b^2} = \frac{(a+b)^2 - 2ab}{(ab)^2}$$

$$= \frac{(22)^2 - 2 \times 1}{(1)^2} = \frac{484 - 2}{1} = 482$$

2017 - DECEMBER

[100] The ratio of the number of ₹ 5 coins and ₹ 10 coins is 8 : 15. If the value of ₹ 5 coins is ₹ 360, then the number of ₹ 10 coins will be:

(a) 72 (b) 120

(c) 135 (d) 185 (1 mark)

Answer:

(c) Ratio of ₹ 5 coins and ₹ 10 coins = 8 : 15

Let the No. of ₹ 5 coins = 8x

and the No. of ₹ 10 coins = 15x

The value of ₹ 5 coins = ₹ 5 × 8x

360 = 40x

$$x = \frac{360}{40}$$

$$x = 9$$

No. of ₹ 10 coins = 15x
= 15 × 9
= 135

[101] If $\log_3 [\log_4 (\log_2 x)] = 0$, then the value of 'x' will be:

(a) 4 (b) 8
(c) 16 (d) 32 (1 mark)

Answer:

(c) If $\log_3 [\log_4 (\log_2 x)] = 0$

$\log_4 (\log_2 x) = 3^0$ [$\log_a b = x$ $b = a^x$]

[103] If $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{x}$ are in proportion, then the value of 'x' will be:

(a) $\frac{15}{2}$ (b) $\frac{6}{5}$

(c) $\frac{10}{3}$ (d) $\frac{5}{6}$ (1 mark)

Answer:

(a) If $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{x}$ are in proportion

then, product of extremes = Product of means

$$\frac{1}{2} \times \frac{1}{x} = \frac{1}{3} \times \frac{1}{5}$$

$$\frac{1}{2x} = \frac{1}{15}$$

$2x = 15$

$x = 15/2$

2018 - MAY

[104] If p : q is the sub-duplicate ratio of $p - x^2 : q - x^2$, then x^2 is :

(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$

(c) $\frac{qp}{p-q}$ (d) None. (1 mark)

Answer:

(d) Sub duplicate ratio of $(p - x^2) : (q - x^2) = \sqrt{p - x^2} : \sqrt{q - x^2}$

$p:q = \sqrt{p - x^2} : \sqrt{q - x^2}$

$$\frac{P}{q} = \frac{\sqrt{p - x^2}}{\sqrt{q - x^2}}$$

an squaring both side

$$\frac{p^2}{q^2} = \frac{p - x^2}{q - x^2}$$

$$p^2(q - x^2) = q^2(p - x^2)$$
$$p^2q - p^2x^2 = q^2p - q^2x^2$$
$$p^2q - q^2p = p^2x^2 - q^2x^2$$
$$pq(p - q) = (p^2 + q^2)x^2$$
$$pq(p - q) = (p + q)(p - q)x^2$$
$$x^2 = \frac{pq(p - q)}{(p + q)(P - q)}$$
$$x^2 = \frac{pq}{(p + q)}$$

[105] The value of the expression :

$a^{\log_a b \cdot \log_b c \cdot \log_c d \cdot \log_d t}$

(a) t (b) abcdt
(c) (a + b + c + d + t) (d) None (1 mark)

Answer:

(a) $a^{\log_a^b \cdot \log_b^c \cdot \log_c^d \cdot \log_d^t}$

$$= a^{\frac{\log^b}{\log^a} \cdot \frac{\log^c}{\log^b} \cdot \frac{\log^d}{\log^c} \cdot \frac{\log^t}{\log^d}}$$
$$= a^{\frac{\log^t}{\log^a}}$$
$$= a^{\log_a^t} \quad [\because e^{\log_e^x} = x]$$
$$= t$$

[106] The mean proportional between 24 and 54 is:

(a) 33 (b) 34
(c) 35 (d) 36 (1 mark)

Answer:

(d) Mean proportion $b = \sqrt{ac}$

$= \sqrt{24 \times 54}$
$= \sqrt{1,296}$
$= 36$

[107] The value of $\log_4 9 . \log_3 2$ is:

(a) 3 (b) 2
(c) 9 (d) 1 (1 mark)

Answer:

(d) $\log_4 9 . \text{Log}_3 2 = \frac{\log 9}{\log 4} \cdot \frac{\log 2}{\log 3}$

$= \frac{\log 3^2}{\log 2^2} \cdot \frac{\log 2}{\log 3}$

$= \frac{2\log 3}{2\log 2} \cdot \frac{\log 2}{\log 3}$

$= 1$

[108] $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$ (1 mark)

Answer:

(b) $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{2^n + 2^n . 2^{-1}}{2^n . 2^{+1} - 2^n}$

$= \frac{2^n + (1 + 2^{-1})}{2^n . (2 - 1)}$

$= \frac{\left(1 + \frac{1}{2}\right)}{1}$

$= \frac{\frac{3}{2}}{1}$

$= \frac{3}{2}$

2018 - NOVEMBER

[109] $\frac{3x-2}{5x+6}$ is the duplicate ratio of $\frac{2}{3}$ then find the value of x:

(a) 2 (b) 6

(c) 5 (d) 9 (1 mark)

Answer:

(b) is the duplicate ratio of

[111] If $x : y : z = 7 : 4 : 11$ then $\frac{x + y + z}{z}$ is:

(a) 2 (b) 3
(c) 4 (d) 5 (1 mark)

Answer:

(a) If $x: y: z = 7: 4: 11$

Let $x = 7k, y = 4k, z = 11k$

$$\frac{x + y + z}{2} = \frac{7k + 4k + 11k}{11k} = \frac{22k}{11k} = 2$$

[112] $\log_2 \log_2 \log_2 16 = ?$

(a) 0 (b) 3
(c) 1 (d) 2 (1 mark)

Answer:

(c) $\log_2 \log_2 \log_2^{16}$

$= \log_2 \log_2 (\log_2^{24})$

$= \log_2 \log_2^4 \log_2^2$

$= \log_2 \log_2^4$ ($\log_2^2 = 1$)

Answer:

(a) Ratio of two Numbers = 7 : 11

Let Ist No = 7x

IInd No = 11x

Given Condition

$(7x + 7) : (11x + 7) = 2 : 3$

$$\frac{7x+7}{11x+7} = \frac{2}{3}$$

$21x + 21 = 22x + 14$

$21 - 14 = 22x - 21x$

$7 = x$

Ist No = $7x = 7 \times 7 = 49$

IInd No = $11x = 11 \times 7 = 77$

[114] If $2^{x^2} = 3^{y^2} = 12^{z^2}$ then

(a) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$

(b) $\frac{1}{x^2} + \frac{2}{y^2} = \frac{1}{z^2}$

(c) $\frac{2}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$

(d) None (1 mark)

Answer:

(c) If $2^{x^2} = 3^{y^2} = 12^{z^2} = K$

$2^{x^2} = K, 3^{y^2} = K, 12^{z^2} = K$

$2 = K^{\frac{1}{x^2}}, 3 = K^{\frac{1}{y^2}}, 12 = K^{\frac{1}{z^2}}$

Now,

$12 = 2 \times 2 \times 3$

$K^{\frac{1}{z^2}} = K^{\frac{1}{x^2}} \times K^{\frac{1}{x^2}} \times K^{\frac{1}{y^2}}$

$K^{\frac{1}{z^2}} = K^{\frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{y^2}}$

On comparing

$$\frac{1}{z^2} = \frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{y^2}$$

$$\frac{1}{z^2} = \frac{2}{x^2} + \frac{1}{y^2}$$

[115] The value of

$$\log_5 \left(1 + \frac{1}{5}\right) + \log_5 \left(1 + \frac{1}{6}\right) + \text{----------} + \log_5 \left(1 + \frac{1}{624}\right)$$

(a) 2
(b) 3
(c) 5
(d) 0 (1 mark)

Answer:

(b) If $\log_5 \left(1 + \frac{1}{5}\right) + \log_5 \left(1 + \frac{1}{6}\right) + \ldots\ldots\ldots\ldots + \log_5 \left(1 + \frac{1}{624}\right)$

$= \log \left(\frac{6}{5}\right) + \log \left(\frac{7}{6}\right) \log \left(\frac{8}{7}\right) + \ldots\ldots\ldots\ldots + \log \left(\frac{625}{624}\right)$

$= \log_5 \left(\frac{\cancel{6}}{5} \times \frac{\cancel{7}}{\cancel{6}} \times \frac{\cancel{8}}{\cancel{7}} \times \ldots\ldots\ldots \frac{\cancel{624}}{\cancel{623}} \times \frac{625}{\cancel{624}}\right)$

$= \log_5 \left(\frac{625}{5}\right)$

$= \log_5 (125) = \log_5 5^3 = 3 \log_5 5$

$= 3 \times 1$

$= 3$

[116] $\log_{2\sqrt{2}}(512) : \log_{3\sqrt{2}} 324 =$

(a) 128 : 81
(b) 2 : 3
(c) 3 : 2
(d) None (1 mark)

Answer:

(c) $\log_{2\sqrt{2}} 512 : \log_{3\sqrt{2}} 324$

$= \frac{\log 512}{\log 2\sqrt{2}} : \frac{\log 324}{\log 3\sqrt{2}}$

$$= \frac{\log (8)^3}{\log \sqrt{2\times2\times2}} : \frac{\log 18^2}{\log \sqrt{3\times3\times2}}$$

$$= \frac{\log (8)^3}{\log (8)^{1/2}} : \frac{\log (18)^2}{\log (18)^{1/2}}$$

$$= \frac{3 \log 8}{1/2 \log 8} : \frac{2 \log 18}{1/2 \log 18}$$

$$(3 \times 2) : (2 \times 2)$$

$$= 6 : 4$$

$$= 3 : 2$$

[117] If $P = x^{1/3} + x^{-1/3}$ then $P^3 = 3P =$

(a) 3

(b) $\frac{1}{2}\left(x + \frac{1}{x}\right)$

(c) $\left(x + \frac{1}{x}\right)$

(d) $2\left(x + \frac{1}{x}\right)$ (1 mark)

Answer:

(c) If $P = x^{1/3} + n^{-1/p}$ then $P^3 - 3P$

Given $P = x^{1/3} + x^{-1/p}$ .....(1)

Cube on both side

$P^3 = (x^{1/3} + x^{-1/p})^3$

$P^3 = (x^{1/3})^3 + (x^{-1/p})^3 + 3x^{1/3} \cdot x^{-1/3} (x^{1/3} + x^{-1/3})$

$= x + x^{-1} + 3 \times 1 \times P$

$P^3 = x + \frac{1}{x} + 3P$

$P^3 - 3P = x + \frac{1}{x}$

2019 - NOVEMBER

[118] The ratio of two numbers are 3 : 4. The difference of their squares is 28 Greater no. is:

(a) 8
(b) 12
(c) 24
(d) 64. (1 mark)

Answer:

(a) Let the two numbers be x and y

Greater no. y

Smaller no. x

According to question,

$\frac{x}{y} = \frac{3}{4}$ — Eq 1 and $y^2 - x^2 = 28$ — Eq 2

Further solving Eq 1

$x = \frac{3}{4}y$ — Eq 3

Put Eq 3 in Eq 2

$y^2 - \left(\frac{3}{4}y\right)^2 = 28$

$\frac{y^2}{1} - \frac{9y^2}{16} = 28$

$\frac{7y^2}{16} = 28$

$y^2 = \frac{28 \times 16}{7}$

$y^2 = 64$

$y = 8$ {square root both sides}

So, the greater number i.e. y is equal to 8.

[119] The price of scooter and moped are in the ratio 7 : 9. The price of moped is ₹ 1,600 more than that of scooter. Then the price of moped is:

(a) ₹ 7,200
(b) ₹ 5,600
(c) ₹ 800
(d) ₹ 700 (1 mark)

Answer:

(a) $\frac{\text{Price of scooter}}{\text{Price of Moped}} = \frac{7}{9}$

Let; the price of scooter = 7x
and price of moped = 9x
According to question

$9x = 7x + 1600$
$2x = 1600$
$x = ₹\ 800$

So, price of moped = 9x = 9 (800) = ₹ 7200

[120] $\log_{0.01} 10{,}000 = ?$

(a) 2
(b) –2
(c) 4
(d) –4 (1 mark)

Answer:

(b) $\log_{0.01} 10{,}000$

$\frac{\log 10{,}000}{\log 0.01}$ Since $\log_a b = \frac{\log b}{\log a}$

$\frac{\log (10)^4}{\log\left(\frac{1}{100}\right)}$ $\log a^n = n \log a$

$\frac{4 \times \log 10}{\log 1 - \log 100}$ $\log\left(\frac{b}{a}\right) = \log b - \log a$

$\frac{4 \times 1}{0 - \log (10)^2}$ $\log 10 = 1$, $\log 1 = 0$

$\frac{4}{-4} = \frac{4}{-4} = -2$

[121] Value of $\left[9^{n+\frac{1}{4}} \cdot \frac{\sqrt{3.3^n}}{3.\sqrt{3^{-n}}}\right]^{\frac{1}{n}}$

(a) 9
(b) 27
(c) 81
(d) 3 (1 mark)

Answer:

(b) $= \left[\frac{9^{n+\frac{1}{4}} \sqrt{3^{(n+1)}}}{3.\sqrt{3^{-n}}}\right]^{\frac{1}{n}}$

$= \left[\frac{3^{2n+\frac{1}{2}} . 3^{\frac{(n+1)}{2}}}{3.\ 3^{-n/2}}\right]^{\frac{1}{n}}$

$= \left[\frac{3^{2n+\frac{1}{2}+\frac{n}{2}+\frac{1}{2}}}{3^{1-n/2}}\right]^{\frac{1}{n}}$

$= \left[(3)^{\frac{5n}{2}+1-1+\frac{n}{2}}\right]^{\frac{1}{n}}$

$= \left[(3)^{\frac{6n}{2}}\right]^{\frac{1}{n}}$

$= (3)^3$

$= 27$

CHAPTER

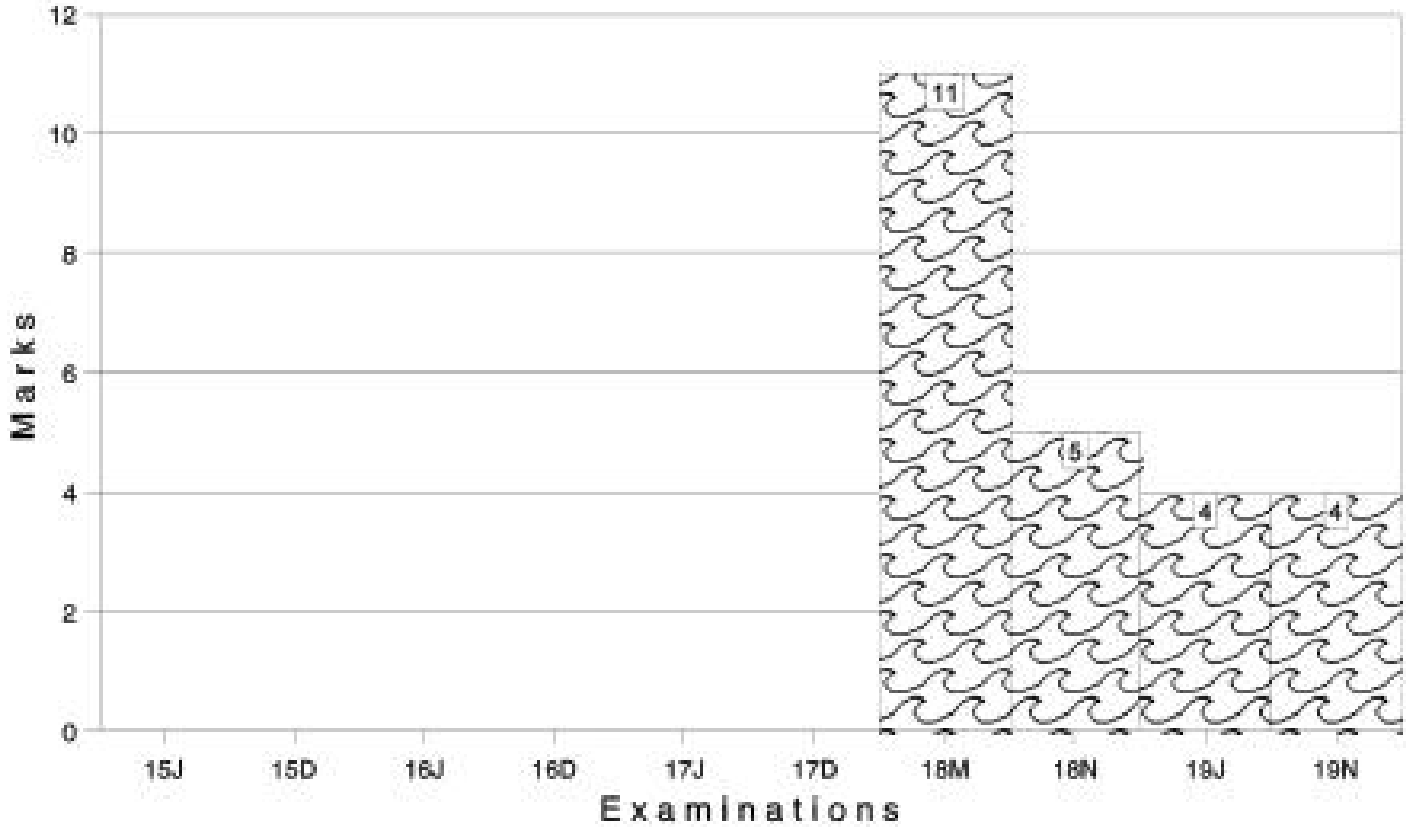
2

EQUATIONS AND MATRICES

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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MATRICES

2.2.1 Introduction:

Matrices are one of he most powerful tools in mathematics. Matrices applications are used in Business, Finance, Economics, genetics, Sociology, Modern Psychology and Industrial Management.

Matrix: A matrix is an ordered rectangular array of numbers (real or complex) or functions. The numbers or functions are called the elements or entries of the matrix. We denote matrices by capital letters.

$$A = \begin{bmatrix} 4 \\ 9 \\ 12 \end{bmatrix}; \quad B = \begin{bmatrix} 4 & 2 \\ 6 & 3 \\ 9+i & 4 \end{bmatrix}; \quad C = \begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 6 \\ 9 & 3 & 5 \end{bmatrix}$$

It is to be noted that a matrix is just an arrangement of elements with out any value in rows and columns.

Example : Consider the following information regarding the number of men and women workers in three factories I, II and III

	Men Workers	**Women Workers**
I	30	25
II	25	31
III	27	26

Represent the above information in the form of a 3 × 2 matrix. What does the entry in the third row and second column represent?

Answer:

The information is represented in the form of a 3 × 2 matrix as follows:

$$A = \begin{bmatrix} 30 & 25 \\ 25 & 31 \\ 27 & 26 \end{bmatrix}$$

The entry in the third row and second column represents the number of women workers in factory III.

Order of a Matrix: A matrix having m rows and n columns is called a matrix of order m × n or simply m × n matrix (read as an m by n matrix).
Order of matrix m × n means there are m (rows) and n (= columns). The number of elements in an m × n matrix will be equal to m.n.

e.g. (i) $A = \begin{bmatrix} 2 & 1 \\ 3 & 6 \end{bmatrix}_{2\times 2}$

The order of this matrix is 2 × 2 (2 rows and 2 columns). Total number of elements will be 4. (= 2 × 2)

(ii) $B \begin{bmatrix} 5 & 6 & 9 \\ 3 & 2 & 12 \\ 4 & 3 & 9 \end{bmatrix}_{3\times 3}$

The order of this matrix is 3 × 3 (3 rows and 3 columns). Total number of elements are 9. (= 3 × 3).

2.2.2 Types of Matrices

(i) **Column Matrix:** A matrix is said to be a column matrix or column vector if it has only one column. The order of column matrix is m × 1

e.g. $A = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}_{3\times 1}$; $B = \begin{bmatrix} 9 \\ 12 \end{bmatrix}_{2\times 1}$

(ii) **Row Matrix:** A matrix is said to be a row matrix or row vector if it has only one row. The order of row matrix is 1 × m

e.g. $A = \begin{bmatrix} 2 & 4 & 6 \end{bmatrix}_{1\times 3}$ $B = \begin{bmatrix} 9 & 15 \end{bmatrix}_{1\times 2}$

(iii) **Square Matrix:** A matrix in which the number of rows are equal to number of column is said to be a square matrix. Thus an m × n matrix is said to be a square matrix if m = n and is known as a square matrix of order n.

e.g. $A \begin{bmatrix} 4 & 3 \\ 5 & 9 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

(iv) **Diagonal Matrix:** A square matrix is said to be a diagonal matrix if all its non diagonal elements are zero.

e.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$; $B = \begin{bmatrix} \\ \\ \end{bmatrix}$

(x) **Lower triangular Matrix:** A matrix is known as lower triangular matrix if all the elements above the leading diagonal or principal diagonal are zero.

e.g. $A = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 3 & 0 \\ 4 & 2 & 8 \end{bmatrix}$

(xi) **Sub Matrix:** The matrix obtained by deleting one or more rows or columns or both of a matrix is called its sub matrix.

e.g. $A = \begin{bmatrix} 7 & 8 & 9 \\ 3 & 2 & 3 \\ 5 & 4 & 6 \end{bmatrix}$

The sub matrix is obtained by deleting 3rd Row and 1st Column.

$B = \begin{bmatrix} 8 & 9 \\ 2 & 3 \end{bmatrix}$

(xii) **Equal Matrices:** Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if they full fill two conditions.

(a) The order of both the matrices is same

(b) Corresponding elements in both the matrices are equal, that is $a_{ij} = b_{ij}$ for all i and j Symbolically, if two matrices A and B are equal, we write A = B

e.g. (i) $A = \begin{bmatrix} 4 & 5 \\ 6 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 5 \\ 6 & 9 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 1 & 4 \\ 9 & 6 \\ 0 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 4 \\ 9 & 6 \\ 0 & 6 \end{bmatrix}$

2.2.3 Algebra of Matrices:

In this Section, we shall introduce certain operations on matrices, namely, addition and subtraction of matrices, multiplication of a matrix by a scalar and multiplication of matrices.

Addition of Matrices:

(i) Order of the matrices should be same.

(ii) Add corresponding elements.

(iii) Order of resultant matrix will be same as original matrices.

e.g. $A = \begin{bmatrix} 2 & 5 \\ 1 & 6 \\ 4 & 3 \end{bmatrix}_{3\times 2}$; $B = \begin{bmatrix} 3 & 4 \\ 2 & 9 \\ 1 & 6 \end{bmatrix}_{3\times 2}$

$$A + B = \begin{bmatrix} 2 & 5 \\ 1 & 6 \\ 4 & 3 \end{bmatrix}_{3\times 2} + \begin{bmatrix} 3 & 4 \\ 2 & 9 \\ 1 & 6 \end{bmatrix}_{3\times 2} = \begin{bmatrix} 5 & 9 \\ 3 & 15 \\ 5 & 9 \end{bmatrix}_{3\times 2}$$

We add Corresponding elements.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} ; \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}_{2\times 3}$$

$$A + B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{bmatrix}_{2\times 3}$$

Subtraction of Matrices will follow the same process as Addition of the matrices:

$$A = \begin{bmatrix} 4 & 2 & 9 \\ 6 & 3 & 7 \end{bmatrix}_{2\times 3} ; B = \begin{bmatrix} 5 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix}_{2\times 3}$$

$$A - B = \begin{bmatrix} 4 & 2 & 9 \\ 6 & 3 & 7 \end{bmatrix}_{2\times 3} - \begin{bmatrix} 5 & 1 & 6 \\ 3 & 2 & 7 \end{bmatrix}_{2\times 3}$$

$$A - B = \begin{bmatrix} -1 & 1 & 3 \\ 3 & 1 & 0 \end{bmatrix}_{2\times 3}$$

→ **Negative of a Matrix:** The negative of a matrix is denoted by – A.

We define = – A = (– 1) A.

e.g. $A = \begin{bmatrix} 5 & 1 \\ -8 & y \end{bmatrix}$, then – A is

$$-A = (-1)\,A = (-1)\begin{bmatrix} 5 & 1 \\ -8 & y \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 8 & -y \end{bmatrix}$$

Properties of matrix addition:

(i) **Commutative Law:** If A and B are matrices of the Same order, then
$A + B = B + A$

(ii) **Associative Law:** If A, B and C of the same order, then
$(A + B) + C = A + (B + C)$

(iii) **Existence of additive identity:** Let A and O of the same order matrix (O is zero matrix), then
$A + O = O + A = A.$
In other words, O is the additive identity for matrix addition.

(iv) **The existence of additive inverse:** Let A by any matrix, then we have another, matrix as $-A$ (having same order as A).
Such that $A + (-A) = (-A) + A = O$. So $-A$ is the additive inverse of A or negative of A.

Multiplication of a matrix by a scalar:

In general, we may define multiplication of a matrix by a scalar as follows. If A is a matrix and K is a scalar, then KA is another matrix which is obtained by multiplying each element of A by the scalar K.

e.g. Let $A = \begin{bmatrix} 2 & 1 & 4 \\ 6 & 3 & 2 \end{bmatrix}$; K = 3 (Scalar)

$$3A = 3\begin{bmatrix} 2 & 1 & 4 \\ 6 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 12 \\ 18 & 9 & 6 \end{bmatrix}$$

Properties of Scalar Multiplication of a matrix:

If A and B be two matrices of the same order, and K and L are scalars, then

(i) $K(A + B) = KA + KB$

(ii) $(K + L)A = KA + LA$

Multiplication of two matrices:

The product AB of two matrices A and B defined only if the number of columns in matrix A is equal to the number of rows in matrix B. (Conformability of multiplication).

Let A be an m × n matrix and B be an n × p matrix, then

$$(A_{m\times n})\,(B_{n\times p}) = (C_{m\times p})$$

(is resultant matrix whose order is m × p)

e.g. Find AB, If $A = \begin{bmatrix} 6 & 9 \\ 2 & 3 \end{bmatrix}_{2\times 2}$ and $B = \begin{bmatrix} 2 & 6 & 0 \\ 7 & 9 & 8 \end{bmatrix}_{2\times 3}$

the matrix A has 2 columns which is equal to the number of row of B.

So AB is defined,

$$AB = \begin{bmatrix} 6(2)+9(7) & 6(6)+9(9) & 6(0)+9(8) \\ 2(2)+3(7) & 2(6)+3(9) & 2(0)+3(8) \end{bmatrix}$$

$$AB = \begin{bmatrix} 12+63 & 36+81 & 0+72 \\ 4+21 & 12+27 & 0+24 \end{bmatrix}$$

$$AB = \begin{bmatrix} 75 & 117 & 72 \\ 25 & 34 & 24 \end{bmatrix}_{2\times 3}$$

Properties of Matrix Multiplication:

(i) Matrix multiplication is not commutative in general i.e. AB BA

e.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

then $AB = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, clearly AB BA

Note: Diagonal matrices of same order will be commutative

e.g. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$

then $AB = BA \begin{bmatrix} 3 & 0 \\ 0 & 8 \end{bmatrix}$

(ii) **Associative Law:** For any three matrices A, B and C, we have
(AB) C = A (BC)

(iii) **Distributive Law:** For the matrices A, B and C
(a) A (B + C) = AB + AC
(b) (A + B) C = AC + BC

(iv) **Existence of Multiplicative identity:**
For every square matrix A, there exist an identity matrix of same order such that IA = AI = A.

(v) If A, B and C are three matrices such that AB = AC, then the general B C

(vi) If A is m x n matrix and O is an n x p null matrix, then AO = O, A = O

Note: The product of two non-zero matrices may be zero matrix

e.g. If $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Example : If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$

Calculate AC, BC and (A + B)C. Also, verify that (A + B)C = AC + BC

Answer:

Now, $A + B = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix}$

So $(A+B)\,C = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 & -14 & +24 \\ -10 & +0 & +30 \\ 16 & +12 & +0 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$

Further $AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 & -12 & +21 \\ -12 & +0 & +24 \\ 14 & +16 & +0 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$

and $BC = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 & -2 & +3 \\ 2 & +0 & +6 \\ 2 & -4 & +0 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$

So $AC+BC = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix} + \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$

Clearly, (A + B)C = AC + BC

Example : If $A \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = 0$

Answer:

We have $A^2 = A \cdot A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$

So, $A^3 = A \ A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$

Now,

$$A^3 - 23A - 40I = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23 \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} + \begin{bmatrix} -23 & -46 & -69 \\ -69 & 46 & -23 \\ -92 & -46 & -23 \end{bmatrix} + \begin{bmatrix} -40 & 0 & 0 \\ 0 & -40 & 0 \\ 0 & 0 & -40 \end{bmatrix}$$

$$= \begin{bmatrix} 63-23-40 & -46-46+0 & 69-69+0 \\ 69-69+0 & -6+46-40 & 23-23+0 \\ 92-92+0 & 46-46+0 & 63-23-40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Example : In a legislative assembly election, a political group hired a public relations firm to promote its candidate in three ways: telephone, house calls, and letters. The cost per contract (in paise) is given in matrix A as

Cost per contract

$$A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \text{Telephone} \\ \text{Housecall} \\ \text{Letter} \end{matrix}$$

The number of contracts of each type made in two cities X and Y is given by

$$B = \begin{matrix} \text{Telephone} & \text{Housecall} & \text{Letter} \\ \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10,000 \end{bmatrix} & & \begin{matrix} \rightarrow X \\ \rightarrow Y \end{matrix} \end{matrix}$$

Find the total amount spent by the group in the two cities X and Y.

Answer:

We have

$$BA = \begin{bmatrix} 40,000 + 50,000 + 2,50,000 \\ 120,000 + 100,000 + 5,00,000 \end{bmatrix} \begin{matrix} \rightarrow X \\ \rightarrow Y \end{matrix}$$

$$= \begin{bmatrix} 3,40,000 \\ 7,20,00 \end{bmatrix} \begin{matrix} \rightarrow X \\ \rightarrow Y \end{matrix}$$

So the total amount spent by the group in the two cities in 3,40,000 paise and 7,20,000 paise, i.e., ₹ 3,400 and ₹ 7,200, respectively.

Transpose of Matrix:

Major points of this topic are transpose of matrix, symmetric and skew symmetric matrix.

→ If A = [aij] be an m x n matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A′ or [A^T].

Symbolically

$$A = [aij]_{m\times n},\ A' = [aji]_{n\times m}$$

e.g. $A = \begin{bmatrix} 1 & 6 \\ 2 & 2 \\ 4 & 3 \end{bmatrix}_{3\times 2}$

$$A' = A^T = \begin{bmatrix} 1 & 2 & 4 \\ 6 & 2 & 3 \end{bmatrix}_{2\times 3}$$

Example: If $A=\begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix}$ and $B=\begin{bmatrix}2 & -1 & 2\\ 1 & 2 & 4\end{bmatrix}$, verify that

(i) $(A')' = A$, (ii) $(A + B)' = A' + B'$,

(iii) $(kB)' = kB'$, where k is any constant.

Answer:

(i) We have

$$A = \begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix} \rightarrow A'=\begin{bmatrix}3 & 4\\ \sqrt{3} & 2\\ 2 & 0\end{bmatrix} \rightarrow (A')'=\begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix} = A$$

Thus $(A')' = A$

(ii) We have

$$A=\begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix}, B=\begin{bmatrix}2 & -1 & 2\\ 1 & 2 & 4\end{bmatrix} \Rightarrow A+B=\begin{bmatrix}5 & \sqrt{3-1} & 4\\ 5 & 4 & 4\end{bmatrix}$$

Therefore $(A + B)' = \begin{bmatrix}5 & 5\\ \sqrt{3-1} & 4\\ 4 & 4\end{bmatrix}$

Now $A' = \begin{bmatrix}3 & 4\\ \sqrt{3} & 2\\ 2 & 0\end{bmatrix}, B' = \begin{bmatrix}2 & 1\\ -1 & 2\\ 2 & 4\end{bmatrix}$

So $A'+ B' = \begin{bmatrix}5 & 5\\ \sqrt{3-1} & 4\\ 4 & 4\end{bmatrix}$

Thus $(A + B)' = A'+B'$

(iii) We have

$$kB = K\begin{bmatrix}2 & -1 & 2\\ 1 & 2 & 4\end{bmatrix} = \begin{bmatrix}2k & -k & 2k\\ k & 2k & 4k\end{bmatrix}$$

Then $(kB)' = \begin{bmatrix}2k & k\\ -k & 2k\\ 2k & 4k\end{bmatrix} = k\begin{bmatrix}2 & 1\\ -1 & 2\\ 2 & 4\end{bmatrix} = kB'$

Thus $(kB)' = kB'$

Symmetric Matrices:

A square matrix A = [aij] is said to be symmetric if A′ = A, that is [aij] = [aji], for all possible values of i and j.

e.g. $A = \begin{bmatrix} 3 & 2 & 5 \\ 2 & 4 & -1 \\ 5 & -1 & 9 \end{bmatrix}$; $A' = \begin{bmatrix} 3 & 2 & 5 \\ 2 & 4 & -1 \\ 5 & -1 & 9 \end{bmatrix}$

Skew Symmetric Matrices: A square matrix A = [aij] is said to be skew symmetric matrix if A′ = – A,
that is aji = – aij for all possible values of i and j.
Now if we put i = j, we have

$$a_{ii} = -a_{ii}$$
$$\therefore 2a_{ii} = 0$$
$$\text{or } a_{ii} = 0 \text{ for all i' s.}$$

This shows that all the diagonal elements of a skew symmetric matrix are zero.

Note: For any square matrix A with real number entries, A + A′ is a symmetric matrix and A – A′ is a skew symmetric matrix.

2.2.4 Determinants

Determinants have wide applications in Engineering, Science, Economics, Social Science, etc.
Determinants are quite useful to solving a system of linear equations. They are also helpful in expressing certain formulas.
Every square matrix $A = [a_{ij}]$ of order n, we can associate a number (real or complex) called determinant of the square matrix A, denoted by | A | or det A or Δ

(i) For matrix A, | A | is read as determinant of A and not modulus of A.
(ii) Only square matrices have determinants.

→ **Determinant of a Matrix of Order One:**

Let A = [a] be the matrix of order 1, then determinant of A is defined to be equal to a

→ **Determinant of a Matrix of Order Two:**

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of order 2 × 2, then the determinant of A is defined as :

$$\det(A) = |A| = \Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\, a_{22} - a_{21}\, a_{12}$$

Example: Evaluate = $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$

Answer:

We have $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} = 2\,(2) - 4\,(-1) = 4 + 4 = 8.$

Example: Evaluate $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$

Answer:

We have

$$\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix} = x\,(x) - (x+1)\,(x-1)$$

$$= x^2 - (x^2 - 1) = x^2 - x^2 + 1 = 1$$

Determinant of a Matrix of Order 3 × 3

determinant of a matrix of order three can be determined by expressing it in terms of second order determinants. This is known as expansion of a determinant along a row (or a column). There are six ways of expanding a determinant of order 3 corresponding to each of three rows (R_1, R_2 and R_3) and three columns (C_1, C_2 and C_3) giving the same value as shown below. Consider the determinant of square matrix $A = [a_{ij}]_{3\times3}$

i.e., $$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Expansion along first Row (R_1)

Step 1: Multiply first element a_{11} of R_1 by $(-1)^{(1+1)}$ $[(-1)^{\text{sum of suffixes in } a_{11}}]$ and with the second order determinant obtained by deleting the elements of first row (R_1) and first column (C_1) of $|A|$ as a_{11} lies in R_1 and C_1,

i.e., $$(-1)^{1+1}\, a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

Step 2: Multiply 2nd element a_{12} of R_1 by $(-1)^{1+2}$ $[(-1)^{\text{sum of suffixes in } a_{12}}]$ and the second order determinant obtained by deleting elements of first row (R_1) and 2nd column (C_2) of $|A|$ as a_{12} lies in R_1 and C_2,

i.e., $(-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

Step 3: Multiply third element a_{13}, of R_1 by $(-1)^{1+3}$ $[(-1)^{\text{sum of suffixes in } a_{13}}]$ and the second order determinant obtained by elements of first row (R_1) and third column (C_3)of $|A|$ as a_{13} lies in R_1 and C_3,

i.e., $(-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Step 4: Now the expansion of determinant of A, that is $|A|$ written as sum of all three terms obtained in step 1, 2 and 3 above is given by

$$\det A = |A| = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Or $$|A| = a_{11}(a_{22} a_{33} - a_{32} a_{23}) - a_{12}(a_{21} a_{33} - a_{31} a_{23}) + a_{13}(a_{21} a_{32} - a_{31} a_{22})$$

$$= a_{11} a_{22} a_{33} - a_{11} a_{32} a_{23} - a_{12} a_{21} a_{33} + a_{12} a_{31} a_{23} + a_{13} a_{21} a_{32} - a_{13} a_{31} a_{22} \quad \text{---------- (1)}$$

Minors and Cofactors:

In this section, we will learn to write the expansion of a determinant in compact form using minors and cofactors.

Definition 1: Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.

Remark: Minor of an element of a determinant of order $n (n \geq 2)$ is a determinant of order $n - 1$.

Example: Find the minor of element 6 in the

$$\text{determinant } \Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Answer:

Since 6 lies in the second row and third column, its minor M_{23} is given by

$M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = 8 - 14 = -6$ (obtained by deleting R_2 and C_3 in Δ).

Definition 2 Cofactor of an element a_{ij}, denoted by A_{ij} is defined by

$A_{ij} = (-1)^{i+j} M_{ij}$ is minor of a_{ij}.

Example : Find minors and cofactors of all the elements of the determinant $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$.

Answer:

Minor of the element a_{ij} is M_{ij}

Here $a_{11} = 1$. So M_{11} Minor of $a_{11} = 3$

M_{12} = Minor of the element $a_{12} = 4$

M_{21} = Minor of the element $a_{21} = -2$

M_{22} = Minor of the element $a_{22} = 1$

Now, cofactor of a_{ij} is A_{ij}. So

$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (3) = 3$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (4) = -4$

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-2) = 2$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (1) = 1$

Properties of Determinants:

(i) The value of the determinant remains unchanged if its row and columns are interchanged.

Example: Verify Property 1 for $\Delta = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

Answer:

Expanding the determinant along first row, we have

$$\Delta = 2\begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} - (-3)\begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} + 5\begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix}$$

$$= 2(0 - 20) + 3(-42 - 4) + 5(30 - 0)$$

$$= -40 - 138 + 150 = -28$$

By interchanging rows and columns, we get

$$\Delta_1 = \begin{vmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{vmatrix}$$ (Expanding along first column)

$$= 2\begin{vmatrix} 0 & 5 \\ 4 & -7 \end{vmatrix} - (-3)\begin{vmatrix} 6 & 1 \\ 4 & -7 \end{vmatrix} + 5\begin{vmatrix} 6 & 1 \\ 0 & 5 \end{vmatrix}$$

$$= 2(0 - 20) + 3(-42 - 4) + 5(30 - 0)$$

$$= -40 - 138 + 150 = -28$$

Clearly $\Delta = \Delta_1$

Hence, Property 1 is verified.

(ii) If any two rows or columns of a determinant are interchanged, then sign of determinant changes

Example : Verify Property 2 for $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

Answer:

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = -28$$ (See Example 6)

Interchanging rows R_2 and R_3 i.e., $R_2 \Leftrightarrow R_3$, we have

$$\Delta_1 = 2\begin{vmatrix} 2 & -3 & 5 \\ 1 & 5 & -7 \\ 6 & 0 & 4 \end{vmatrix}$$

Expanding the determinant Δ_1 along first row, we have

$$\Delta_1 = 2\begin{vmatrix} 5 & -7 \\ 0 & 4 \end{vmatrix} - (-3)\begin{vmatrix} 1 & -7 \\ 6 & 4 \end{vmatrix} + 5\begin{vmatrix} 1 & 5 \\ 6 & 0 \end{vmatrix}$$

$$= 2(20 - 0) + 3(4 + 42) + 5(0 - 30)$$

$$= 40 + 138 - 150 = 28$$

Clearly $\Delta_1 = -\Delta$

Hence, Property 2 is verified.

(iii) If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then value of determinant is zero.

e.g. $|A| = \begin{bmatrix} 6 & 2 & 3 \\ 7 & 2 & 3 \\ 9 & 2 & 3 \end{bmatrix}$

We get

$|A| = 0$

C_2 and C_3 are identical.

(iv) If each element of a row (or a column) of a determinant is multiplied by a constant K, then its value gets multiplied by K.

e.g. $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 5 & -7 \\ 6 & 0 & 4 \end{vmatrix} = 28$

$|B| = 3\begin{vmatrix} 2 & -3 & 5 \\ 1 & 5 & -7 \\ 6 & 0 & 4 \end{vmatrix} = 28 \times 3 = 84$

(v) If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

e.g. $\begin{bmatrix} 2 & 3 & 4 \\ a+2x & b+2y & c+2z \\ M & N & O \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ a & b & c \\ M & N & O \end{bmatrix} + \begin{bmatrix} 2 & 3 & 4 \\ 2x & 2y & 2z \\ M & N & O \end{bmatrix}$

(vi) Property 6 (with verification and remark)

If to each element of any row or column of a determinant, the equimultiples of corresponding elements of other row (or column) are added, then value of determinant remains the same, i.e., the value of determinant remain same if we apply the operation

$R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Verification :

Let $\Delta = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ and $\Delta_1 = \begin{bmatrix} a_1 + kc_1 & a_2 + kc_2 & a_3 + kc_3 \\ b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$,

where Δ_1 is obtained by the operation $R_1 \to R_1 + kR_3$.

Here, we have multiplied the elements of the third row (R_3) by a constant k and added them to the corresponding elements of the first row (R_1)

Symbolically, we write this operation as $R_1 \to R_1 + kR_3$.

Now, again

$$\Delta_1 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} kc_1 & kc_2 & kc_3 \\ b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$ (Using Property 5)

$= \Delta + 0$ (since R_1 and R_3 are proportional)

Hence, $\Delta = \Delta_1$

Remarks:

(i) If Δ_1 is the determinant obtained by applying $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$ to the determinant Δ, then $\Delta_1 = k\,\Delta$.

(ii) If more than one operation like $R_i \rightarrow R_i + k\,R_j$ is done in one step, care should be taken to see that a row that is affected in one operation should not be used in another operation. A similar remark applies to column operations.

Note:

(i) If $|A| = 0$, square matrix A is singular matrix
If $|A| \neq 0$, square matrix A is non-singular matrix

(ii) A square matrix A has inverse if
A is non-singular.

2.2.5 Inverse of a Matrix:

If A is a square matrix and $|A| \neq 0$
then,

$$A^{-1} = \frac{1}{|A|}\,(\text{Adjoint of A}) = \frac{\text{Adj (iA)}}{|A|}$$

Adjoint of a Matrix:

The adjoint of a square matrix $A = [a_{ij}]_{n\times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n\times n}$ where A_{ij} is the co-factor of the element a_{ij}

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

then adj A = Transpose of $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

Ex:1 Use Cramer's Rule to solve

$$2x + 3y - z = 1$$
$$4x + y - 3z = 11$$
$$3x - 2y + 5z = 21$$

Solution:

Find the determinant D, by using x, y and z values

$$D = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 1 & -3 \\ 3 & -2 & 5 \end{bmatrix} = -78$$

Find D_x, by replacing x – values in 1st column with the values after the equal sign leaving the y and z columns unchanged.

$$D_x = \begin{bmatrix} 1 & 3 & -1 \\ 11 & -1 & -3 \\ 21 & -2 & 5 \end{bmatrix} = -312$$

Similarly one can find the value of D_y and D_z
(replacing y and z values in the 2nd and 3rd column)

$$D_y = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 11 & -3 \\ 3 & 21 & 5 \end{bmatrix} = 156; \quad D_z = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 1 & 11 \\ 3 & -2 & 21 \end{bmatrix} = -78$$

Now use Cramer's Rule.

$$x = \frac{D_x}{D} = \frac{-312}{-78} = 4$$

$$y = \frac{D_y}{D} = \frac{156}{-78} = -2$$

$$z = \frac{D_z}{D} = \frac{-78}{-78} = 1$$

Example: Find adj A for $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

Answer: We have $A_{11} = 4$, $A_{12} = -1$, $A_{21} = -3$, $A_{22} = 2$

Hence, adj $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

Example: If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then verify that A adj A = $|A| I$. Also find A^{-1}.

Answer:

We have $|A| = 1(16-9) - 3(4-3) + 3(3-4) = 1 \neq 0$

Now $A_{11} = 7$, $A_{12} = -1$, $A_{13} = -1$, $A_{21} = -3$, $A_{22} = 1$, $A_{23} = 0$, $A_{31} = -3$, $A_{32} = 0$, $A_{33} = 1$

Therefore $$\text{adj } A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Now $$A(\text{adj } A) = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7-3-3 & -3+3+0 & -3+0+3 \\ 7-4-3 & -3+4+0 & -3+0+3 \\ 7-3-4 & -3+3+0 & -3+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = (1)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |A| . I$$

Also $$A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{1}\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Example: If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$

Answers:

We have $AB = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$

Since, $|AB| = -11 \neq 0$, $(AB)^{-1}$ exists and is given by

$$(AB)^{-1} = \frac{1}{|AB|}\text{adj}(AB) = -\frac{1}{11}\begin{bmatrix}-14 & -5\\ -5 & -1\end{bmatrix} = \frac{1}{11}\begin{bmatrix}14 & 5\\ 5 & 1\end{bmatrix}$$

Further, $|A| = -11 \neq 0$ and $|B| = 1 \neq 0$. Therefore, A^{-1} and B^{-1} both exist and are given by

$$A^{-1} = -\frac{1}{11}\begin{bmatrix}-4 & -3\\ -1 & 2\end{bmatrix} B = \begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$$

Therefore, $B^{-1}A^{-1} = -\frac{1}{11}\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}\begin{bmatrix}-4 & -3\\ -1 & 2\end{bmatrix} = -\frac{1}{11}\begin{bmatrix}-14 & -5\\ -5 & -1\end{bmatrix} = \frac{1}{11}\begin{bmatrix}14 & 5\\ 5 & 1\end{bmatrix}$

Hence, $(AB)^{-1} = B^{-1}A^{-1}$.

Example: Show that the matrix $A = \begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ satisfies the equation $A^2 - 4A + I = 0$, where I is 2×2 identity matrix and O is 2×2 zero matrix. Using this equation, find A^{-1}.

Answer:

We have $A^2 = A.A = \begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix} = \begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}$

Hence, $A^2 - 4A + I = \begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix} - \begin{bmatrix}8 & 12\\ 4 & 8\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} = 0$

Now, $A^2 - 4A + I = 0$

Therefore, $AA - 4A = -I$

Or, $AA(A^{-1}) - 4AA^{-1} = -IA^{-1}$ (Post multiplying by A^{-1} because $|A| \neq 0$)

Or, $A(AA^{-1}) - 4I = -A^{-1}$

Or, $A^{-1} = 4I - A = \begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix} - \begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix} = \begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}$

2.2.6 Solution of linear Equations in three variables (Cramer's Rule)

(i) $\Delta \neq 0$, the system has a unique solution given by

$$x = \frac{\Delta x}{\Delta},\quad y = \frac{\Delta y}{\Delta},\quad z = \frac{\Delta z}{z}$$

(ii) $\Delta = 0$, and at least one of $\Delta_x, \Delta_y, \Delta_z \neq 0$.

The system has no solution. It is inconsistent

(iii) $\Delta = 0$, and $\Delta_x, \Delta_y, \Delta_z = 0$. The system may or may not have a solution. If it has a solution, the equations are dependent and there will be an infinite number of solutions. If it doesn't have a solution, the equations are inconsistent.

MULTIPLE CHOICE QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] On solving $\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$, we get one value of x as :

(a) $\frac{4}{13}$ (b) $\frac{1}{13}$

(c) $\frac{2}{13}$ (d) $\frac{3}{13}$ (1 mark)

Answer:

(a) $\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$

$= \sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}} = \frac{13}{6}$

Let $\sqrt{\frac{x}{1-x}} = y$

Then $y+\frac{1}{y} = \frac{13}{6}$

$\frac{y^2+1}{y} = \frac{13}{6}$

$6y^2 + 6 = 13y$

$6y^2 - 13y + 6 = 0$

$6y^2 - 9y - 4y + 6 = 0$

$3y(2y-3)-2(2y-3)=0$

$(3y-2)(2y-3)=0$

$y = \frac{2}{3}, \frac{3}{2}$

When $y = \frac{2}{3}$; Then

$$\sqrt{\frac{x}{1-x}} = \frac{2}{3}$$

Squaring both the sides,

$$\frac{x}{1-x} = \frac{4}{9}$$

$9x = 4 - 4x$

$13x = 4$

$x = \frac{4}{13}$

When $y = \frac{3}{2}$, Then

$$\sqrt{\frac{x}{1-x}} = \frac{3}{2}$$

On squaring both sides, we get

$$\frac{x}{1-x} = \frac{9}{4}$$

$4x = 9 - 9x$

$13x = 9$

$x = \frac{9}{13}$

Therefore, as per the given options, $x = \frac{4}{13}$

[2] Find the positive value of k for which the equations : $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots :

(a) 12 (b) 16

(c) 18 (d) 22 (1 mark)

Answer:

(b) For real roots, discriminant ≥ 0

$b^2 - 4ac \geq 0$

$k^2 - 4 \times 1 \times 64 \geq 0$
$k^2 - 256 \geq 0$
$k^2 \geq 256$
$k \geq 16$

for $x^2 - 8x + k = 0$
$(-8)^2 - 4 \times 1 \times 4 \geq 0$
$64 - 4k \geq 0$
$4k \leq 64$
$k \leq 16$

Hence, $k = 16$

2007 - FEBRUARY

[3] A man sells 6 radios and 4 televisions for ₹ 18,480. If 14 radios and 2 televisions are sold for the same amount, what is the price of a television?

(a) ₹ 1,848 (b) ₹ 840
(c) ₹ 1,680 (d) ₹ 3,360 (1 mark)

Answer:

(d) Let the price of 1 radio be ₹ x and television be ₹ y.

Then, $6x + 4y = 18,480$ ---------- (1)
$14x + 2y = 18,480$ ---------- (2)

Solving (1) & (2) simultaneously :

$6x + 4y = 18,480$
$28x + 4y = 36,960$
(–) (–) (–)
$-22x = -18,480$
$x = 840$

When $x = 240$, $6 \times 840 + 4y = 18,480$
$4y = 18,480 - 5,040$
$y = \frac{13,440}{4} = 3,360$

Therefore, Price of a television is ₹ 3,360

[4] If one root of a equation is $2 + \sqrt{5}$, then the quadratic equation is :

(a) $x^2 + 4x - 1 = 0$ (b) $x^2 - 4x - 1 = 0$

(c) $x^2 + 4x + 1 = 0$ (d) $x^2 - 4x + 1 = 0$ (1 mark)

Answer:

(b) If one root is $2 + \sqrt{5}$, then other root will be $2 - \sqrt{5}$, because irrational roots always occur in pairs.

Now, equation will be :

$[x - (2 - \sqrt{5})]\,[x - (2 - \sqrt{5})] = 0$

$x^2 - (2 + \sqrt{5})x - (2 - \sqrt{5})x + (2 + \sqrt{5})\,(2 - \sqrt{5}) = 0$

$x^2 - 2x - \sqrt{5}x - 2x + \sqrt{5}x + (4 - 5) = 0$

$x^2 - 4x - 1 = 0$

2007 - MAY

[5] A man starts his job with a certain monthly salary and earns a fixed increment every year. If his salary was ₹ 1,500 after 4 years of service and ₹ 1,800 after 10 years of service, what was his starting salary and what is the annual increment in rupees?

(a) ₹ 1,300, ₹ 50 (b) ₹ 1,100, ₹ 50

(c) ₹ 1,500, ₹ 30 (d) None. (1 mark)

Answer:

(a) Let the starting salary be ₹x and the annual increment be ₹y.

Then, $x + 4y = 1500$ .....(1)

$x + 10y = 1800$ .....(2)

Subtracting (1) from (2)

$x + 10y = 1800$

$x + 4y = 1500$

(–) (–) (–)

$6y = 300$

$y = ₹50$

Substituting $y = 50$ in (1), we get $x = 1,300$

Therefore, starting salary = x = ₹ 1,300

Annual increment = y = ₹ 50

2007 - AUGUST

[6] The sides of an equilateral triangle are shortened by 12 units, 13 units and 14 units respectively and a right angled triangle is formed. The side of the equilateral triangle is :

(a) 17 units (b) 16 units
(c) 15 units (d) 18 units. (1 mark)

Answer:

(a) Let the side of the equilateral triangle be x units.

Then, side 1 = (x – 12) units
side 2 = (x – 13) units
side 3 = (x – 14) units

Since, the new triangle formed is a right angled triangle, therefore by Pythagoras theorem :

$(\text{side } 1)^2 = (\text{side } 2)^2 + (\text{side } 3)^2$

$(x - 12)^2 = (x - 13)^2 + (x - 14)^2$

$x^2 + 144 - 24x = x^2 - 169 - 26x + x^2 + 196 - 28x$

$x^2 - 24x + 144 = 2x^2 - 54x + 365$

$2x^2 - x^2 - 54x + 24x + 365 - 144 = 0$

$x^2 - 30x + 221 = 0$

$x^2 - 17x - 13x + 221 = 0$

$x(x - 17) - 13(x - 17) = 0$

$(x - 13)(x - 17) = 0$

$x = 13$ and 17

$x = 13$ is not possible because if $x = 13$ then the length of side 2 = 0 which is not possible therefore $x = 17$ units.

[7] The value of $\sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\ldots \infty}}}$ is :

(a) – 3 (b) 2
(c) 3 (d) 4 (1 mark)

Answer:

(c) Let $\sqrt{6 + \sqrt{6 + \sqrt{6 \ldots\ldots \infty}}} = y$ ..... (1)

on squaring both sides, we get

$6 + y = y^2$ [From (1)]
$y^2 - y - 6 = 0$
$y^2 - 3y + 2y - 6 = 0$
$y(y - 3) + 2(y - 3) = 0$
$(y + 2)(y - 3) = 0$
$y = -2, 3$
$y = -2$ is not possible, therefore $y = 3$

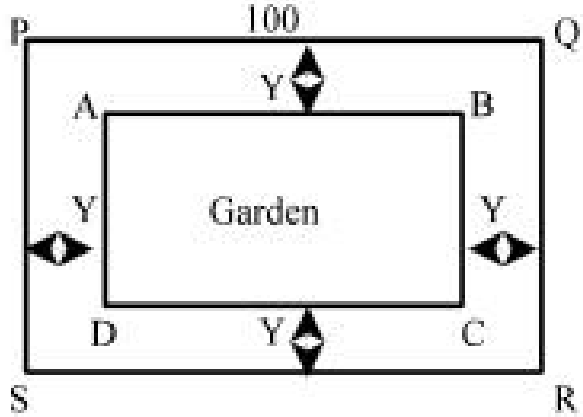
2007 - NOVEMBER

[8] Area of a rectangular garden is 8000 square metres. Ratio in length and breadth is 5:4. A path of uniform width, runs all round the inside of the garden. If the path occupies 3200 m^2, what is its width?

(a) 12m (b) 6m
(c) 10m (d) 4m. (1 mark)

Answer:

(c) Let the length and breadth of a rectangular garden be 5x and 4x.
Area = Length × breadth
$8000 = 5x \times 4x$
$8000 = 20x^2$
$x^2 = 400$
$x = 20$
Length = $5x = 5 \times 20 = 100$m
Breadth = $4x = 4 \times 20 = 80$m.
Let the width of the path be y m.



Then, Length = (100 – 2y)m

Area of PQRS = 8000
Area of width = Area of PQRS – Area of ABCD
$3200 = 8000 - [(100 - 2y)(80 - 2y)]$
$3200 = 8000 - (8000 - 160y - 200y + 4y^2)$
$3200 = 8000 - 8000 + 360y - 4y^2$
$4y^2 - 360y + 3200 = 0$
$y^2 - 90y + 800 = 0$
$y^2 - 80y - 10y + 800 = 0$
$y(y - 80) - 10(y - 80) = 0$
$(y - 80)(y - 10) = 0$
$y = 80 = 10$
Width = 80 m is not possible therefore, width of the path = 10m.

2008 - FEBRUARY

[9] A man went to the Reserve Bank of India with ₹ 1,000. He asked the cashier to give him ₹ 5 and ₹ 10 notes only in return. The man got 175 notes in all. Find how many notes of ₹ 5 and ₹ 10 did he receive?

(a) (25, 150) (b) (40, 110)
(c) (150, 25) (d) None. (1 mark)

Answer:

(c) Let the number of notes of ₹ 5 be x and notes of ₹10 be y.
Then, $x + y = 175$ .....(1)
$5x + 10y = 1000$ .....(2)
Solving (1) and (2) simultaneously. we get :

$$\begin{array}{rrr} 5x + 5y & = & 875 \\ 5x + 10y & = & 1000 \\ \hline (-) \quad (-) & & (-) \\ \hline -5y & = & -125 \end{array}$$

$y = 25$
When $y = 25$ then $x + y = 175$
$x + 25 = 175$
$x = 50$
therefore, No. of notes of ₹ 5 = 150
No. of notes of ₹ 10 = 25

2008 - JUNE

[10] A man rowing at the rate of 5 km in an hour in still water takes thrice as much time in going 40 km up the river as in going 40 km down. Find the rate at which the river flows :

(a) 9 km/hr (b) 2.5 km/hr

(c) 12 km/hr (d) None. (1 mark)

Answer:

(b) Let the speed of the river be x km/hr.

We know that speed = $\frac{\text{Distance}}{\text{Time}}$

$\Rightarrow$ Time = $\frac{\text{Distance}}{\text{Speed}}$

While going upstream, the speed of the river will reduce the speed of the boat. Hence, net speed of the boat while going upstream = (5 – x) km/hr.

Time to row upstream = $\frac{40}{5-x}$ hrs.

While going downstream, the speed of the river will increase the speed of the boat. Hence, net speed of the boat while going downstream (5 + x) km/hr.

Time to row downstream = $\frac{40}{5+x}$ hrs.

Now, go by options:

Obviously if the speed of the river is more than the speed of the boat, then rowing upstream is not possible. Therefore, we can easily rule out options (a) and (c) as the speed given in these options is more than the speed of the boat.

Option (b) → 2.5 km/hr. → If the speed of the river is 2.5 km/hr.

Time to row upstream = $\frac{40}{5-2.5} = \frac{40}{2.5}$ = 16 hrs.

Time to row downstream = $\frac{40}{5+2.5} = \frac{40}{7.5}$ = 5.33 hrs.

As per the question.

Time to row upstream = 3 × Time to row downstream

[11] The value of

$$2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\ldots\infty}}}}$$ is :

(a) $1+\sqrt{2}$ (b) $2\pm\sqrt{5}$

(c) $2\pm\sqrt{3}$ (d) None. (1 mark)

Answer:

(a)

$$\text{Let } 2+\cfrac{1}{2+\cfrac{1}{2+\ldots\ldots\infty}} = y \quad (1)$$

$2 + \frac{1}{y} = y \rightarrow$ From (1)

$2y + 1 = y^2$

$y^2 - 2y - 1 = 0$

By Shri Dharacharya,

$$y = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{4+4}}{2}$$

$$= 2 \pm \frac{\sqrt{8}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$= \frac{2(1 \pm \sqrt{2})}{2}$$

$\because$ But answer i.e. value of y will be grater than 2 because.

$$y = 2 + \frac{1}{2} + \ldots\ldots$$

So, $y = 1 + \sqrt{2}$ & $y \neq 1 - \sqrt{2}$

2008 - DECEMBER

[12] If $x^3 - 6x^2 + 11x - 6 = 0$ then find the value of $(3x - 4)$.

(a) (1, 2, 3) (b) (– 1, 2, 5)

(c) (– 1, 3, 5) (d) (2, 3, 5) (1 mark)

Answer:

(b) $x^3 - 6x^2 + 11x - 6 = 0$

Put $x = 1$, which satisfies the given equation.

$\therefore$ $(x - 1)$ is one of the factors of the above equation.

To find the other two roots, we shall proceed with the division method.

$$\begin{array}{r|l} & x^2 - 5x + 6x - 1 \\ \hline x^3 - 6x^2 + 11x - 6 & \\ x^3 - x^2 & \\ (-) \quad (+) & \\ \hline x \quad -5x^2 + 11x & \\ -5x^2 + 5x & \\ (+) \quad (-) & \\ \hline x \quad 6x - 6 & \\ 6x - 6 & \\ (-) \quad (+) & \\ \hline \times \quad \times & \end{array}$$

Factorising, $x^2 - 5x + 6 = (x - 2)(x - 3)$

$\therefore$ Roots of given equation are (1,2,3)

Now, values of $(3x - 4)$

$\Rightarrow$ $3 \times 1 - 4 = -1$

$3 \times 2 - 4 = 2$

$3 \times 3 - 4 = 5$

[13] If $(2+\sqrt{3})$ is a root of a quadratic equation $x^2 + p_x + q = 0$ then find the value of p and q.

(a) (4,– 1) (b) (4,1)

(c) (– 4,1) (d) (2,3) (1 mark)

Answer:

(c) If one of the roots of the equation is $2+\sqrt{3}$, then other root is $2-\sqrt{3}$

$\therefore$ Sum of roots $= 2+\sqrt{3}+2-\sqrt{3} = 4$

Product of roots $= (2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$

$\therefore$ Required equation is :

x^2 – (sum of roots) x + product of roots = 0

Or $x^2 - 4x + 1 = 0$

Now comparing with $x^2 + px + q = 0$

we get, p = – 4 and q = 1

Required answer is (– 4,1)

[14] If area and perimeter of a rectangle is 6000 cm^2 and 340 cm respectively, then the length of rectangle is :

(a) 140 (b) 120

(c) 170 (d) 200 (1 mark)

Answer:

(b) Given :

Area = 6000 cm^2, Perimeter = 340cm

As we know,

$\ell \times b = 6000$ .....(1)

$2(\ell + b) = 340$ .....(2)

Where, ℓ = length and b = breadth

2009 - June

[15] One root of the equation :
$x^2 - 2(5 + m)x + 3(7 + m) = 0$ is reciprocal of the other.
Find the value of M.

(a) -7 (b) 7
(c) 1/7 (d) $-1/7$ (1 mark)

Answer:

(a) If one root of the equation is reciprocal of the other then the product of the roots is 1.

Now, $x^2 - 2(5 + m)x + 3(7 + m) = 0$

$x^2 - (10 + 2m)x + (21 + 3m) = 0$

Since the roots of the equation aforementioned are reciprocal of each other.

Therefore, product of the roots = 1

$$\frac{c}{a} = 1$$

$$\frac{21 + 3m}{1} = 1$$

$21 + 3m = 1$

$3m = -20$

$m = 20/3 = -6.6$ or -7

Therefore, the value of m is 7.

2009 - December

[16] If the length of a rectangle is 5 cm more than the breadth and if the perimeter of the rectangle is 40 cm, then the length & breadth of the rectangle will be :

(a) 7.5 cm, 2.5 cm (b) 10 cm, 5 cm
(c) 12.5 cm, 7.5 cm (d) 15.5 cm, 10.5 cm. (1 mark)

Answer:

(c) Let the breadth of the rectangle be x cm. Therefore length = $(x+5)$ cm.

Now, Perimeter = $2(l + b)$

$40 = 2[(x + 5) + x]$

$20 = x + 5 + x$

$20 = 2x + 5$

$2x = 20 - 5$

$2x = 15$

$x = \frac{15}{2}$

$x = 7.5$

So, breadth = x = 7.5 cm and length = $x + 5 = 7.5 + 5 = 12.5$ cm.

2010 - June

[17] Roots of the equation $3x^2 - 14x + k = 0$ will be reciprocal of each other if :

(a) $k = -3$ (b) $k = 0$

(c) $k = 3$ (d) $k = 14$. (1 mark)

Answer:

(c) Since roots are reciprocal of each other,

Roots of equation will be $\alpha, \frac{1}{\alpha}$

Product of Roots will be $1 = + \frac{c}{a}$

$\frac{k}{3} = 1$

$\therefore k = 3$

2010 - December

[18] Positive value of 'k' for which the roots of equation $12x^2 + kx + 5 = 0$ are in ratio 3:2, is:

(a) 5/12 (b) 12/5

(c) $\frac{5\sqrt{10}}{2}$ (d) $5\sqrt{10}$ (1 mark)

Answer:

(d) $12x^2 + kx + 5 = 0$

Here, $a = 12$; $b = k$; $c = 5$

Let the roots be 3y and 2y respectively.

We know that:

Sum of Roots = $\frac{-b}{a}$

$3y + 2y = \frac{-k}{12}$

$\Rightarrow 5y = \frac{-k}{12}$

$\Rightarrow y = \frac{-k}{12 \times 5}$

$\Rightarrow y = \frac{-k}{60}$ _____ eq. (1)

Also, we know that:

Product of Roots = $\frac{c}{a}$

$\Rightarrow 3y \times 2y = \frac{5}{12}$

$\Rightarrow 6y^2 = \frac{5}{12}$

$\Rightarrow y^2 = \frac{5}{12 \times 6}$

$\Rightarrow y^2 = \frac{5}{72}$ _____ eq. (2)

Putting the value of $y = \frac{-k}{60}$ in the above equation:

$$\left(-\frac{k}{60}\right)^2 = \frac{5}{72}$$

$$\Rightarrow \quad \frac{k^2}{3,600} = \frac{5}{72}$$

$$\Rightarrow \quad k^2 = \frac{5 \times 3,600}{72}$$

$$\Rightarrow \quad k = \sqrt{\frac{5 \times 3,600}{72}}$$

$$\Rightarrow \quad k = 15.811$$

On going through the options on calculator, we find that $5\sqrt{10} = 15.811$. Therefore, answer is option (d) → $5\sqrt{10}$.

[19] If one root of the equation $x^2 - 3x + k = 0$ is 2, then value of k will be:

(a) – 10 (b) 0

(c) 2 (d) 10 (1 mark)

Answer:

(c) $x^2 - 3x + k = 0$

$\because$ one root = 2 ⇛ it will satisfy the e.g :

putting x = 2, we get

$(2)^2 - 3(2) + k = 0$

⇛ $4 - 6 + k = 0$

$\therefore$ $k = 2$

2011 - June

[20] If the ratio of $(5x - 3y)$ and $(5y - 3x)$ is 3 : 4, then the value of x : y is :

(a) 27 : 29 (b) 29 : 27

(c) 3 : 4 (d) 4 : 3 (1 mark)

Answer:

(a) Given : $\frac{5x - 3y}{5y - 3x} = \frac{3}{4}$

$4(5x - 3y) = 3(5y - 3x)$

$\Rightarrow$ $20x - 9x = 15y - 12y$

$\therefore$ $29x = 27y$

$\therefore$ $\frac{x}{y} = \frac{27}{29}$

or, $x : y = 27 : 29$

[21] If roots of equation $x^2 + x + r = 0$ are 'α' and 'β' and $\alpha^3 + \beta^3 = -6$. Find the value 'r' ?

(a) $\frac{-5}{3}$ (b) $\frac{7}{3}$

(c) $\frac{-4}{3}$ (d) 1 (1 mark)

Answer:

(a) Given : 'α' and 'β' are roots of $x^2 + x + r = 0$
and $\alpha^3 + \beta^3 = -6$
Quadratic Equation : $x^2 + x + r = 0$
here $a = 1$, $b = 1$ and $c = r$

$\therefore$ Sum of Roots : $\alpha + \beta = \frac{-b}{a} = -1$

and Product of Roots : $\alpha - \beta = \frac{c}{a} = r$

also,

$\because$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)$

on putting the values,

$-6 = (-1)^3 - 3r\,(-1)$

$\Rightarrow r = -5/3$

2011 - December

[22] If one root of the Equation $px^2 + qx + r = 0$ is r then other root of the Equation will be:

(a) 1/q (b) 1/r

(c) 1/p (d) $\frac{1}{p+q}$ (1 mark)

Answer:

(c) The Roots of the equation

$px^2 + qx + r = 0$ are α & β

given $\alpha = r$ then,

Sum of Roots $\alpha + \beta = \frac{-b}{a}$

$$r + \beta = \frac{-q}{p} \quad \text{.....(1)}$$

Product of Roots

$$\alpha.\beta = \frac{c}{a}$$

$$r.\beta = \frac{r}{p}$$

$$\boxed{\beta = \frac{1}{p}}$$

[23] If the ratio of the roots of the Equation $4x^2 - 6x + p = 0$ is 1:2 then the value of p is:

(a) 1 (b) 2

(c) -2 (d) -1 (1 mark)

Answer:

(b) Let the roots of Q. E $4x^2 - 6x + P = 0$ is α, β

Here

$a = 4, b = -6, c = p$

$\alpha : \beta = 1:2$

$\alpha = k, \beta = 2k$

$$\alpha + \beta = \frac{-b}{a}$$

$$k + 2k = -\left(\frac{-6}{4}\right)$$

$$3k = \frac{3}{2}$$

$$k = \frac{1}{2} \text{.....(1)}$$

and $\alpha.\beta = \frac{c}{a}$

$k.\ 2k = \frac{p}{4}$

$2k^2 = \frac{p}{4}$

$k = ½$

$\because \quad 2.\left(\frac{1}{2}\right)^2 = \frac{p}{4}$

$p = 2 \times \frac{1}{4} \times 4$

$p = 2$

[24] If p & q are the roots of the Equation $x^2 - bx + C = 0$, then what is the Equation whose roots are $(pq + p + q)$ and $(pq - p - q)$?

(a) $x^2 - 2cx + C^2 - b^2 = 0$ (b) $x^2 - 2bx + C^2 + b^2 = 0$

(c) $8cx^2 - 2(b + c)x + C^2 = 0$ (d) $x^2 + 2bx - (C^2 - b^2) = 0$ (1 mark)

Answer:

(a) If p and q are the roots of the Equation $x^2 - bx + c = 0$

Then

$p + q = \frac{-b}{a} = -\frac{(-b)}{1} = b$

$p.q = \frac{c}{a} = \frac{c}{1} = c$

New Roots are $(pq + p + q)$ & $(pq - p - q)$

Sum of New Roots (s) $= (pq + p + q) + (pq - p - q)$

$= pq + p + q + pq - p - q$

$= 2pq$

$= 2c$

Product of Roots (p) $= (pq + p + q)(pq - p - q)$

$= [pq + (p + q)][pq - (p + q)]$

$= (pq)^2 - (p + q)^2$

$= (c^2 - b^2)$

Now Quadratic Equation is

$x^2 - sx + p = 0$

$x^2 - 2cx + c^2 - b^2 = 0$

2012 - June

[25] If one of the roots of the equation $x^2 + px + a$ is $\sqrt{3} + 2$, then the value of 'p' and 'a' is:

(a) - 4, -1 (b) 4, - 1

(c) - 4, 1 (d) 4, 1 (1 mark)

Answer:

(c) The one Roots of Quadratic Equation is = $(2 + \sqrt{3})$

Then other Roots is = $(2 - \sqrt{3})$

The sum of Roots (S) = $2 + \sqrt{3} + 2 - \sqrt{3}$

= 4

The product of Roots (P) = $(2 + \sqrt{3})(2 - \sqrt{3})$

= 4 - 3

= 1

Quadratic Equation is

$x^2 - Sx + P = 0$

$x^2 - 4x + 1 = 0$

on comparing

$x^2 + Px + q = 0$

we get $P = -4, \quad q = 1$

2012 - December

[26] If $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$ then, the value of x = _____.

(a) 7 (b) 3

(c) 5 (d) 10 (1 mark)

Answer:

(b) If $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$

$\log_{10} \{5.(5x + 1)\} = \log_{10} (x + 5) + \log_{10} 10$

$\Rightarrow \quad \log_{10} \{5.(5x + 1)\} = \log_{10} \{(x + 5).\ 10\}$

on comparing,

$$\Rightarrow \quad 5(5x + 1) = (x + 5)\,10$$
$$5x + 1 = (x + 5)\,2$$
$$5x + 1 = 2x + 10$$
$$5x - 2x = 10 - 1$$
$$3x = 9$$
$$\boxed{x = 3}$$

[27] If $|x - 2| + |x - 3| = 7$ then, 'x' will be equal to

(a) 6 (b) -1

(c) 6 and -1 (d) None of the above. (1 mark)

Answer:

(c) If $|x - 2| + |x - 3| = 7$

If $x - 2 \geq 0$ and $x - 3 \geq 0$

$$(x - 2) + (x - 3) = 7$$
$$x - 2 + x - 3 = 7$$
$$2x = 7 + 2 + 3$$
$$2x = 12$$
$$x = 6$$

If $x - 2 < 0$ & $x - 3 < 0$

$$-(x - 2) - (x - 3) = 7$$
$$-x + 2 - x + 3 = 7$$
$$-2x = 2 \quad x = -1$$

[28] Roots of equation $2x^2 + 3x + 7 = 0$ are α and β. The value of $\alpha\beta^{-1} + \beta\alpha^{-1}$ is

(a) 2 (b) 3/7

(c) 7/2 (d) $-19/14$ (1 mark)

Answer:

(d) Given equation

$$2x^2 + 3x + 7 = 0$$

on comparing with $ax^2 + bx + c = 0$

we get, $a = 2, b = 3, c = 7$

If α, β are the roots of Q.E then

$$\alpha + \beta = \frac{-b}{a} = \frac{-3}{2}$$

$$\alpha\beta = \frac{c}{a} = \frac{7}{2}$$

$$\alpha\beta^{-1} + \beta\alpha^{-1} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(\frac{-3}{2}\right)^2 - 2\left(\frac{7}{2}\right)}{\frac{7}{2}}$$

$$= \frac{-19}{14}$$

[29] The quadratic equation $x^2 - 2kx + 16 = 0$ will have equal roots when the value of 'k' is _____.

(a) ± 1 (b) ± 2

(c) ± 3 (d) ± 4

Answer:

(d) Given, Quadratic Equation

$$X^2 - 2kx + 16 = 0$$

Comparing from $ax^2 + bx + c = 0$

We get

$$a = 1, b = -2k, c = 16$$

If Roots of Quadratic Equation are equal

So, $D = b^2 - 4ac = 0$

$\Rightarrow (-2k)^2 - 4 \times 1 \times 16 = 0$

$\Rightarrow 4k^2 - 64 = 0$

$\Rightarrow 4k^2 = 64$

$K^2 = \frac{64}{4} 16$

$k = \pm 4$

2013 - June

[30] If α and β are the roots of the equation $x^2 + 7x + 12 = 0$, then the equation whose roots $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$ will be:

(a) $x^2 - 14x + 49 = 0$ (b) $x^2 - 24x + 144 = 0$

(c) $x^2 - 50x + 49 = 0$ (d) $x^2 - 19x + 144 = 0$ (1 mark)

Answer:

(c) Given Equation

$x^2 + 7x + 12 = 0$

$(x + 3)(x + 4) = 0$

$x = -3, -4$

$\alpha = -3$ and $\beta = -4$

New Roots are $(\alpha + \beta)^2 = (-3 - 4)^2 = 49$

and $(\alpha - \beta)^2 = (-3 + 4)^2 = 1$

Sum of Roots (S) $= (\alpha + \beta)^2 + (\alpha - \beta)^2$

$= 49 + 1$

$= 50$

Product of Roots (P) $= (\alpha + \beta)^2 . (\alpha - \beta)^2$

$= 49 \times 1$

$= 49$

Q. E. is

$x^2 - 5x + P = 0$

$x^2 - 50x + 49 = 0$

2013 - December

[31] If $b^2 - 4ac$ is a perfect square but not equal to zero than the roots are:

(a) real and equal (b) real, irrational and equal

(c) real, rational and unequal (d) Imaginary. (1 mark)

Answer:

(c) If $b^2 - 4ac \neq 0$ and have perfect square i.e. $D > 0$ and have perfect square

So, Roots of Q.E are **real unequal** and **Rational**

[32] A seller makes an offer of selling certain articles that can be described by the equation $x = 25 - 2y$ where 'x' is the price per unit and 'y' denotes the number of unit. The cost price of the article is ₹ 10 per unit. The maximum quantity that can be offered in a single deal to avoid loss is _____.

(a) 6 (b) 7
(c) 8 (d) 9 (1 mark)

Answer:

(b) Go by options:

Option (a) → 6. If the No. of units sold are 6, the
Selling Price p.u. $(x) = 25 - 2 \times 6$
$= 25 - 12$
$= 13$

Option (b) → 7. If the No. of units sold are 7, the
Selling Price p.u. $(x) = 25 - 2 \times 7$
$= 25 - 14$
$= 11$

Option (c) → 8. If the No. of units sold are 8, the
Selling Price p.u. $(x) = 25 - 2 \times 8$
$= 25 - 16$
$= 9$

Option (d) → 9. If the No. of units sold are 9, the
Selling Price p.u. $(x) = 25 - 2 \times 9$
$= 25 - 18$
$= 7.$

Clearly, Options (c) and (d) cannot be the answer as the Selling Price is coming out to be less than ₹ 10 p.u., and hence there will be a loss.

There is profit is option (a) as well as in option (b). Since the question has asked the "MAXIMUM" quantity that can be sold without loss, the answer is option (b), i.e. 7 units.

[33] If $kx - 4 = (k - 1)x$, then which of the following is true?

(a) $x = -5$ (b) $x = -4$
(c) $x = -3$ (d) $x = +4$ (1 mark)

Answer:

(d) If $kx - 4 = (k - 1)x$

$kx - 4 = kx - x$

$\boxed{x = 4}$

[34] The value of 'K' for which the system of equations $kx + 2y = 5$ and $3x + y = 1$ has no solution is:

(a) 5 (b) $\frac{2}{3}$

(c) 6 (d) $\frac{3}{2}$ (1 mark)

Answer:

(c) $Kx + 2y = 5$

$3x + y = 1$

The coefficient matrix is given by:

$$\begin{bmatrix} k & 2 \\ 3 & 1 \end{bmatrix}$$

For the system of equations to have no solution, the determinant of the coefficient matrix must be zero.

$(k \times 1 - (3 \times 2) = 0$

$\Rightarrow \quad k - 6 = 0$

$\Rightarrow \quad k = 6.$

2014 - June

[35] The roots of the equation $y^3 + y^2 - y - 1 = 0$ are:

(a) (1, 1, -1) (b) (-1, -1, 1)

(c) (1, 1, 1) (d) None of these (1 mark)

Answer:

(b) Given Equation

$y^3 + y^2 - y - 1 = 0$

$y^2 (y + 1) - 1(y + 1) = 0$

$(y + 1) (y^2 - 1) = 0$

$(y + 1) (y + 1) (y - 1) = 0$

then $y = -1, -1, 1$

2014 - December

[36] The equation $x + 5y = 33$; $\frac{x+y}{x-y} = \frac{13}{3}$ has the solution (x, y) as:

(a) (4, 8) (b) (8, 5)
(c) (4, 16) (d) (16, 4) (1 mark)

Answer:

(b) $x + 5y = 33$ _____ (1)

$\frac{x+y}{x-y} = \frac{13}{3}$ _____ (2)

$3(x + y) = 13(x - y)$
$3x + 3y = 13x - 13y$
$3y + 13y = 13x - 3x$
$16y = 10x$
$y = \frac{10}{16}x$
$y = \frac{5}{8}x$ _____ (3)

$y = \frac{5}{8}x$ in eq (1)

$x + 5 \times \frac{5}{8}x = 33$

$\frac{8x + 25x}{8} = 33$

$33x = 33 \times 8$

$x = \frac{33 \times 8}{33}$

$x = 8$ in equation (1)

$y = \frac{5}{8}x = \frac{5}{8} \times 8 = 5$

$x = 8, y = 5$

2015 - June

[37] The number of students in each section of a school is 36. After admitting 12 new students, four new sections were started. If total number of students in each section now is 30, than the number of sections initially were.

(a) 6 (b) 10

(c) 14 (d) 18 (1 mark)

Answer:

(d) Go by options.

Option (a) → 6. If there were 6 sections initially, the total number of students = 6 × 36 = 216.

Now, 12 new students are admitted and 4 new sections are started.

Therefore, total no. of students now = 216 + 12 = 228.

Also, total no. of sections now = 6 + 4 = 10

$$\text{Total no. of students per section} = \frac{228}{10} = 22.8$$

The question clearly mentions that no. of students per section now is 30. Therefore, option (a) can't be the answer.

Repeat the same procedure with all the options.

As per option (d) the initial no. of sections is 18.

No. of students initially = 18 × 36 = 648

No. of students now = 648 + 12 = 660

No. of sections now = 18 + 4 = 22

$\therefore$ No. of students per section $= \frac{660}{22} = 30$

$\therefore$ Option (d) is the answer.

[38] If α and β be the roots of the quadratic equation $2x^2 - 4x = 1$, the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is _____.

(a) -11 (b) 22

Answer:

(c) Given Q.E.

$2x^2 - 4x = 1$

$2x^2 - 4x - 1 = 0$

Comparing from

$ax^2 + bx + c = 0$

We get $a = 2, b = -4, c = -1$

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{2} = 2$$

$$\alpha.\beta = \frac{c}{a} = \frac{-1}{2}$$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}$$

$$= \frac{(2)^3 - 3 \times \left(\frac{-1}{2}\right)(2)}{(-1/2)}$$

$$= \frac{8 + 3}{(-1/2)} = \frac{11}{-1/2} = -11 \times 2 = -22$$

[39] A person on a tour has ₹ 9,600 for his expenses. If his tour is extended by 16 days, he has to cut down his daily expenses by ₹ 20, his original duration of tour had been.

(a) 48 days (b) 64 days

(c) 80 days (d) 96 days (1 mark)

Answer:

(c) Go by options:

Option (a) 48 days.

If the person's initial tour was for 48 days, he had $\frac{₹9,600}{48} = ₹ 200$ to spend every day.

Since his trip has extended by 16 days, and he has to cut his daily expenses by ₹ 20, he now has ₹ 200 - ₹ 20 = ₹ 180 to spend daily.

∴ His total expenses would be 64 × ₹ 180 = ₹ 11,520

This is obviously wrong, since his total expenses cannot exceed ₹ 9,600.

Option (b) 64 days

Initial expenses per day = $\frac{₹9,600}{64}$ = ₹ 150

After cutting ₹ 20 short, expenses per day = ₹ 150 - ₹ 20 = ₹ 130

New tour duration = 64 + 16 = 80 days.

Total expenses = 80 × ₹ 130 = ₹ 10,400 Not Possible

Option (c) 80 days

Initial expenses per day = $\frac{₹9,600}{80}$ = ₹ 120

After cutting ₹ 20 short, expenses per day = ₹ 120 - ₹ 20 = ₹ 100

New tour duration = 80 + 16 = 96 days.

Total expenses = 96 × ₹ 100 = ₹ 9,600.

Therefore, option (c) is the answer

[40] The present age of a man is 8 years more than thrice the sum of the ages of his two grandsons who are twins. After 8 years, his age will be 10 years more than twice the sum of the ages of his grandsons. The age of a man when his grandsons were born was:

(a) 86 years (b) 73 years

(c) 68 years (d) 63 years (1 mark)

Answer:

(b) Let the present Age of Grand Father = x

The present Age of Sum of Grand Sons = y

Ist Condition:

$x = 3y + 8$ ..... (1)

After 8 years

The Age of Grand Father = x + 8

The Age of Sum of Grand Sons = y + 16

IInd Condition:

$(x + 8) = 2(y + 16) + 10$ ..... (2)

$3y + 8 + 8 = 2y + 32 + 10$ [from equation (1)]

$y = 42 - 16$

$y = 26$ in equation (1)

$x = 3 \times 26 + 8$

$= 78 + 8$

$= 86$

Both grand son's are twins so their Age = $\frac{26}{2}$ = (13)

The Age of Grand Father when his Grand Sons was born = 86 - 13 = 73

[41] The roots of the cubic equation $x^3 - 7x + 6 = 0$ are:

(a) 1, 2 and 3 (b) 1, -2 and 3

(c) 1, 2 and - 3 (d) 1, -2 and - 3 (1 mark)

Answer:

(c) $x^3 - 7x + 6 = 0$

$x^3 - x^2 + x^2 - 7x + 6 = 0$

$x^2 (x - 1) + x^2 - x - 6x + 6 = 0$

$x^2 (x - 1) + x (x - 1) - 6 (x - 1) = 0$

$(x - 1) (x^2 + x - 6) = 0$

$(x - 1) (x^2 + 3x - 2x - 6) = 0$

$(x - 1) [x (x + 3) - 2(x + 3)] = 0$

$(x - 1) (x + 3) (x - 2) = 0$

If $x - 1 = 0$ If $x + 3 = 0$ If $x - 2 = 0$

$\boxed{x = 1}$ $\boxed{x = -3}$ $\boxed{x = 2}$

2015 - December

[42] If the roots of the equation $4x^2 - 12x + k = 0$ are equal, then the value of k is:

(a) -3 (b) 3

(c) -9 (d) 9 (1 mark)

Answer:

(d) Given Quadratic Equation

$4x^2 - 12x + k = 0$

Comparing from
$ax^2 + bx + c = 0$
we get $a = 4, b = -12, c = k$
Since Roots are equal
$D = 0$
$b^2 - 4ac = 0$
$b^2 = 4ac$
$(-12)^2 = 4 \times 4 \times k$
$144 = 16K$
$K = 9$

[43] If $\alpha + \beta = -2$ and $\alpha\beta = -3$, then α, β are the roots of the equation, which is:

(a) $x^2 - 2x - 3 = 0$ (b) $x^2 + 2x - 3 = 0$
(c) $x^2 + 2x + 3 = 0$ (d) $x^2 - 2x + 3 = 0$ (1 mark)

Answer:

(b) Given $\alpha + \beta = -2$, and $\alpha\beta = -3$
Q.E. is
$x^2 - (\text{Sum of Roots})x + \text{Product of Roots} = 0$
$x^2 - (\alpha + \beta)x + \alpha\beta = 0$
$x^2 - (-2)x + (-3) = 0$
$x^2 + 2x - 3 = 0$

2016 - June

[44] Let E_1 and E_2 are two linear equations in two variables x and y. (0,1) is a solution of both equations E_1 and E_2. (2, -1) is a solution of equation E_1 only and (-2, -1) is solution of E_2 only then E_1 and E_2 are _____.

(a) $X = 0, y = 1$ (b) $2x - y = -1, 4x + y = 1$
(c) $x + y = 1, x - y = -1$ (d) $x + 2y = 2, x + y = 1$ (1 mark)

Answer:

(c) HFTS/TRIAL

$x + y = 1$ _____ (1) ⇒ (E_1)
$x - y = -1$ _____ (2) ⇒ (E_2)

Solving these equation we get

$x = 0, y = 1$

Solution of E_1 $(2, -1)$ is satisfied and solution of E_2 $(-2, -1)$ is also satisfied.

So, option (c) is correct.

[45] If difference between the roots of the equation $x^2 - kx + 8 = 0$ is 4, then the value of K is:

(a) 0 (b) ± 4

(c) $\pm 8\sqrt{3}$ (d) $\pm 4\sqrt{3}$ (1 mark)

Answer:

(d) Given Q.E

$x^2 - kx + 8 = 0$

If α and β are the roots of Q.E

$$\alpha + \beta = -b/a = -\frac{(-k)}{1} = k$$

$\alpha + \beta = k$ _____(1)

$$\alpha . \beta = \frac{c}{a}$$

$$\alpha . \beta = \frac{8}{1}$$

$\alpha . \beta = 8$ _____(2)

Given $\alpha - \beta = 4$ _____ (3)

Adding (1) & (3)

$\alpha + \beta = k$

$\alpha - \beta = 4$

$2\alpha = k + 4$

$\alpha = \frac{(k+4)}{2}$ in eg. (1)

$$\frac{k+4}{2} + \beta = k$$

$$\beta = k - \frac{(k+4)}{2}$$

$$\beta = \frac{2k - k - 4}{2}$$

$$\beta = \frac{k - 4}{2}$$

Putting the value of α and β in equation (2)

$$= 8$$

$$1+y = \frac{3}{2}$$

$$y = \frac{3}{2} - 1 = \frac{1}{2}$$

$$x = 1, y = \frac{1}{2}$$

2016 - December

[47] A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of each article (in ₹) was 2 more than thrice the number of articles produced on that day. If the total cost of production on that day was ₹ 800, the number of articles produced was

(a) 14 (b) 16
(c) 12 (d) 18 (1 mark)

Answer:

(b) Let No. of articles = N
cost of 1 articles (x) = (3 N + 2)
Total cost of production on that day = 800
$N x = 800$
$N (3N + 2) = 800$
$3N^2 + 2N - 800 = 0$
$3N^2 + 50N - 48N - 800 = 0$
$N (3N + 50) - 16 (3N + 50) = 0$
$(3N + 50) (N - 16) = 0$
If $3N + 50 = 0$ and $N - 16 = 0$
$N = -\frac{50}{3}$ (Impossible) N=16

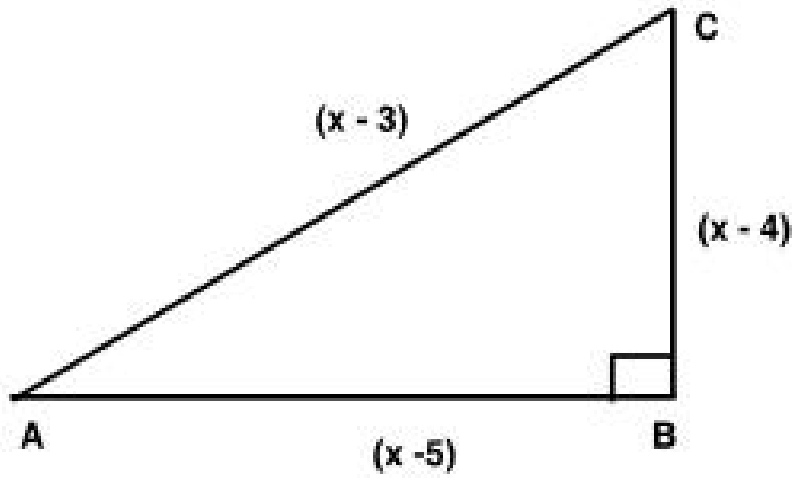
2017 - June

[48] If the sides of an equilateral triangle are shortened by 3 units, 4 units and 5 units respectively and a right triangle is formed, then the side of

(a) 6 units
(b) 7 units
(c) 8 units
(d) 10 units. (1 mark)

Answer:

(c) Let the side of equilateral triangle is x units. If the sides of an equilateral triangle are shortened by 3 units, 4 units and 5 units respectively and a right angle triangle to formed then hypotenuse = (x - 3) units Perpendicular = (x - 4) units Base = (x- 5) units.



In Δ ABC

$AC^2 = AB^2 + BC^2$

$(x - 3)^2 = (x - 4)^2 + (x - 5)^2$

$x^2 + 9 - 6x = x^2 + 16 - 8x + x^2 + 25 - 10x$

$x^2 - 12x + 32 = 0$

$x^2 - 8x - 4x + 32 = 0$

$x(x - 8) - 4(x - 8)$

$(x - 8)(x - 4) = 0$

If $x - 8 = 0$ $\boxed{x = 8}$

If $x - 4 = 0$ $\boxed{x = 4}$ (Impossible)

Answer: Side of Equilateral triangle is 8 units.

[49] If α, β are the roots of the equation $x^2 + x + 5 = 0$ then $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is equal to

(a) $\frac{16}{5}$ (b) 2

(c) 3 (d) $\frac{14}{5}$ (1 mark)

Answer:

(d) Given Quadratic Equation

$x^2 + x + 5 = 0$

If α & β are the Roots of Quadratic Equation

$\alpha + \beta = \frac{-b}{a} = \frac{-1}{1} = -1$

$\alpha \cdot \beta = \frac{c}{a} = \frac{5}{1} = 5$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{(-1)^3 - 3 \times 5 \times (-1)}{5}$$

$$= \frac{-1 + 15}{5}$$

$$= \frac{14}{5}$$

[50] If $\frac{3}{x+y} + \frac{2}{x-y} = -1$ and $\frac{1}{x+y} - \frac{1}{x-y} = \frac{4}{3}$ then (x, y) is :

(a) (2, 1) (b) (1, 2)

(c) (– 1, 2) (d) (– 2, 1) (1 mark)

Answer:

(b) $\frac{3}{x+y} + \frac{2}{x-y} = -1$ _____(1) and $\frac{1}{x+y} - \frac{1}{x-y} = \frac{4}{3}$ _____(2)

Solving (1) & (2), we get (1, 2) By (Hits/Trial method)

2017 - December

[51] The roots of the cubic equation $x^3 + 7x^2 - 21x - 27 = 0$ are

(a) $-1, 3, 9$ (b) $1, -3, 9$

(c) $-1, 3, -9$ (d) $-1, -3, 9$ (1 mark)

Answer:

(c) Given Cubic Equation

$x^3 + 7x^2 - 21x - 27 = 0$

By Hits/trial $-1, 3, -9$ are Satisfied Cubic Equation

So, Roots of Cubic Equation are $-1, 3, -9$.

[52] The difference between the roots of the equation $x^2 - 7x - 9 = 0$ is :

(a) 7 (b) $\sqrt{85}$

(c) 9 (d) $2\sqrt{85}$ (1 mark)

Answer:

(b) If α, β are the Roots of Q.E

$$x^2 - 7x - 9 = 0$$

Comparing from $ax^2 + bx + c = 0$

We get $a = 1, b = -7, c = -9$

then $$\alpha + \beta = \frac{-b}{a} = \frac{-(-7)}{1} = 7$$

$$\alpha.\beta = \frac{c}{a} = \frac{-9}{1} = -9$$

$$\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$= \sqrt{(7)^2 - 4 \times (-9)}$$

$$= \sqrt{49 + 36}$$

$$= \sqrt{85}$$

[53] If the sum of two numbers is 13 and the sum of their squares is 85, then the numbers will be:

(a) 3, 10 (b) 5, 8

(c) 4, 9 (d) 6, 7 (1 mark)

Answer:

(d) Let two numbers are x & y

Given $x + y = 13$ ——— (1)

$x^2 + y^2 = 85$ ——— (2)

from equation (1)

$x + y = 13$

putting $y = 13 - x$ in equation (2)

$x^2 + (13-x)^2 = 85$

$x^2 + 169 + x^2 - 26x = 85$

$2x^2 - 26x + 169 - 85 = 0$

$2x^2 - 26x + 84 = 0$

$2(x^2 - 13x + 42) = 0$

$x^2 - 13x + 42 = 0$

$x^2 - 7x - 6x + 42 = 0$

$x(x - 7) - 6(x - 7) = 0$

$(x - 7)(x - 6) = 0$

If $x-7 = 0$ if $x - 6 = 0$

$x = 7$ $x = 6$

putting $x = 7$ in equation (1) we get $y = 6$

putting $x = 6$ in equation (2) we get $x = 7$

[54] If $u^{5x} = v^{5y} = w^{5z}$ and $u^2 = vw$, then the value of $xy + xz - 2yz$ will be:

(a) 5 (b) 2

(c) 1 (d) 0

Answer:

(d) If $u^{5x} = v^{5y} = w^{5z} = k$ (let)

$u^{5x} = k$, $v^{5y} = k$ and $w^{5z} = k$

$u = k^{1/5x}$ $v = k^{1/5y}$, $w = k^{1/5z}$

Given $u^2 = vw$

$(k^{1/5x})^2 = k^{1/5y}\ k^{1/5z}$

$K^{\frac{2}{5x}} = K^{\frac{1}{5y}+\frac{1}{5z}}$

on comparing

$\frac{2}{5x} = \frac{1}{5y} + \frac{1}{5z}$ $\frac{2}{5x} = \frac{y+z}{5yz}$

$xy + xz = 2yz$

2018 - May

[55] If $|A| = 0$, then A is:

(a) 0 (b) uro matrix

(c) singular matrix (d) non-singular matrix (1 mark)

Answer:

(c) if $|A| = 0$ then A is singular Matrix.

[56] If $A = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$ then $|A| = ?$

(a) 2 (b) 3

(c) 4 (d) 5 (1 mark)

Answer:

(b) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|A| = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$= 2i \cdot (-i) - i \cdot i$

$= -2i^2 - i^2$ $[\because i^2 = -1]$

$= -2 \times (-1) - (-1)$

$= 2 + 1 = 3$

[57] If A and B are matrices then which from the following is true?

(a) $A+B \neq B+A$ (b) $(At)^t \neq A$

(c) $AB \neq BA$ (d) all are true (1 mark)

Answer:

(c) If A & B are two matrices

Then $AB \neq BA$

[58] Transpose of a rectangular matrix is a

(a) Rectangular matrix (b) Diagonal matrix

(c) Square matrix (d) Scalar matrix (1 mark)

Answer:

(a) Transpose of a rectangular matrix is a rectangular matrix.

[59] If $\alpha + \beta = -2$ and $\alpha\beta = -3$, then α, β are the roots of the equation, which is:

(a) $x^2 - 2x - 3 = 0$ (b) $x^2 + 2x - 3 = 0$

(c) $x^2 + 2x + 3 = 0$ (d) $x^2 - 2x + 3 = 0$ (1 mark)

Answer:

(b) If $\alpha + \beta = -2$ & $\alpha.\beta = -3$

Q.E. is

$x^2 - (\alpha + \beta)x + \alpha.\beta = 0$

$x^2 - (-2)x + (-3) = 0$

$x^2 + 2x - 3 = 0$

[60] If $2^{x+y} = 2^{2x-y} = \sqrt{8}$, then the respective values of x and y are _____

(a) $1, \frac{1}{2}$ (b) $\frac{1}{2}, 1$

(c) $\frac{1}{2}, \frac{1}{2}$ (d) None of these (1 mark)

Answer:

(a) If $2^{x+y} = 2^{2x-y} = \sqrt{8}$

$2^{x+y} = \sqrt{8}$

$2^{x+y} = (2^3)^{1/2}$

$2^{x+y} = 2^{3/2}$

on comparing

$x + y = \frac{3}{2}$ ---------- (1)

and $2^{2x-y} = \sqrt{8}$

$2^{2x-y} = (2^3)^{1/2}$

$2^{2x-y} = 2^{3/2}$

on comparing

$2x - y = \frac{3}{2}$ ---------- (2)

Add : (1) & (2)

$x + y = \frac{3}{2}$ ---------- (1)

$2x - y = \frac{3}{2}$ ---------- (2)

$3x = 3$

$x = 1$

Putting $x = 1$ in equation (1) weg.

$x + y = \frac{3}{2}$

$$1 + y = \frac{3}{2} \Rightarrow y = \frac{1}{2}$$

$$\boxed{x = 1},\ y = \frac{1}{2}$$

[61] If $\frac{3}{x+y} + \frac{2}{x-y} = -1$ and $\frac{1}{x+y} - \frac{1}{x-y} = \frac{4}{3}$ then (x, y) is:

(a) (2,1) (b) (1,2)
(c) (-1, 2) (d) (-2,1) (1 mark)

Answer:

(b) if $\frac{3}{x+y} + \frac{2}{x-y} = -1$ and $\frac{1}{x+y} - \frac{1}{x-y} = \frac{4}{3}$

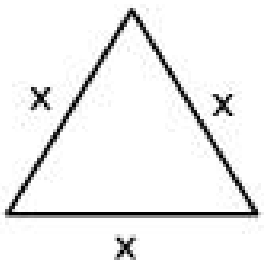
By Hits and trial (1, 2) satisfied both equation
so answer is (1, 2)

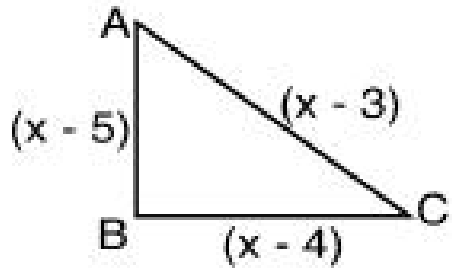
[62] If the sides of an equilateral triangle are shortened by 3 units, 4 units and 5 units respectively and a right triangle is formed then the sides of an equilateral triangle is:

(a) 6 units (b) 7 units
(c) 8 units (d) 10 units (1 mark)

Answer:

(c) Let the side of equilateral triangle is x





in Δ ABC

$(\text{hypo})^2 = (\text{Base})^2 + (\text{per})^2$

$(x - 3)^2 = (x - 4)^2 + (x - 5)^2$

$x^2 + 9 - 6x = x^2 + 16 - 8x + x^2 + 25 - 10x$

$x^2 - 18x + 41 + 6x - 9 = 0$

$x^2 - 12x + 32 = 0$

$x^2 - 8x - 4x + 32 = 0$

$x(x-8)-4(x-8)=0$
$(x-8)(x-4)=0$
$x-8=0$ if $x-4=0$
$\boxed{x=8}$ and $\boxed{x=4}$ Impossible
Side of the triangle is 8.

[63] If α, β are the roots of the equation $x^2+x+5=0$ then $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ is equal to

(a) $\frac{16}{5}$ (b) 2

(c) 3 (d) $\frac{14}{5}$ (1 mark)

Answer:

(d) Given Q.E.
$x^2+x+5=0$
$a=1, b=1, c=5$
if α & β are the root of Q.E.

$$\alpha+\beta=\frac{-b}{a}=\frac{-1}{1}=-1$$

$$\alpha.\beta=\frac{c}{a}=\frac{5}{1}=5$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{(-1)^3-3\times5\times(-1)}{5}$$

$$=\frac{-1+15}{5}$$

$$=\frac{14}{5}$$

2018 - November

[64] Let α and β be the roots of $x^2+7x+12=0$. Then the value of $\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)$ will be

(a) $\frac{7}{12} + \frac{12}{7}$ (b) $\frac{49}{144} + \frac{144}{49}$

(c) $-\frac{91}{12}$ (d) None of the above. (1 mark)

Answer:

(c) If α & β are the Roots of Q. E.

$x^2 + 7x + 12 = 0$

then $\alpha + \beta = \frac{-b}{a} = \frac{-7}{1} = -7$

$\alpha \times \beta = \frac{c}{a} = \frac{12}{1} = 12$

$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$

$= \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}$

$= \frac{(-7)^3 - 3 \times 12 \times (-7)}{12}$

$= \frac{-343 + 252}{12}$

$= \frac{-91}{12}$

[65] If $A = \begin{bmatrix} -5 & 2 \\ 1 & -3 \end{bmatrix}$, then adj A is

(a) $\begin{bmatrix} -3 & -2 \\ -1 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -2 \\ -1 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 1 \\ 2 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$ (1 mark)

Answer:

(a) Given $A = \begin{bmatrix} -5 & 2 \\ 1 & -3 \end{bmatrix}$

The co-factor of A

$A_{11} = (-1)^{1+1}.(-3) = (-1)^2.(-3) = -3$

$A_{12} = (-1)^{1+2}.(1) = (-1)^3.(1) = -1$

$A_{21} = (-1)^{2+1}.(2) = (-1)^3.(2) = -2$

$A_{22} = (-1)^{2+2}.(-5) = (-1)^4 \times (-5) = -5$

Matrix made by co-factor of A

$$B = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ -2 & -5 \end{bmatrix}$$

$$\text{Adj } A = B^T$$

$$= \begin{bmatrix} -3 & -1 \\ -2 & -5 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -2 \\ -1 & -5 \end{bmatrix}$$

[66] If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$, then

(a) $x = 0, y = 5$ (b) $x + y = 5$

(c) $x = y$ (d) None of the above (1 mark)

Answer:

(c) If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$

Then, $A^T = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$

Given $A = A^T$

$$\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$$

on comparing we get

$$x = y$$

[67] Let A^T be the transpose of matrix A having order $m \times n$, then $A^T A$ is a matrix of order

(a) $m \times m$ (b) $n \times n$

(c) $m \times n$ (d) $n \times m$ (1 mark)

Answer:

(a) If A^T be the transpose of matrix A having order $(m \times n)$

i.e. order of A^T $= m \times n$

order of matrix of A $= n \times m$

than order of $A^T . A$ $= (m \times n)(n \times m)$

$= (m \times m)$

[68] When two roots of quadratic equation are α, $\frac{1}{\alpha}$ then what will be the quadratic equation:

(a) $\alpha x^2 - (\alpha^2 + 1)x + \alpha = 0$ (b) $\alpha x^2 - \alpha^2 x + 1 = 0$

(c) $\alpha x^2 - (\alpha^2 + 1)x + 1 = 0$ (d) None of these (1 mark)

Answer:

(a) If Roots of Q.E. are α, & $\frac{1}{\alpha}$ $\alpha = \alpha, \beta = \frac{1}{\alpha}$

Sum of Roots (S) $= \alpha + \beta$

$= \alpha + \frac{1}{\alpha}$

$= \left(\frac{\alpha^2 + 1}{\alpha}\right)$

Product of Roots (P) $= \alpha \cdot \beta$

$= \alpha \cdot \frac{1}{\alpha} = 1$

Q. E is given by

$x^2 - 5x + p = 0$

$x^2 - \left(\frac{\alpha^2 + 1}{\alpha}\right)x + 1 = 0$

$\alpha x^2 - (\alpha^2 + 1)x + \alpha = 0$

2019 - June

[69] A number consists of two digits such that the digit in one's place in thrice the digit in ten's place. If 36 be added then the digits are reversed. Find the number _____.

(a) 62

(b) 26

(c) 39

(d) None of these (1 mark)

Answer:

(b) Let ten's place digit = x

one's place digit = y

then two digit No = 10x + y

after reversing the digit then New No. = 10y + x

I[st] condition

$y = 3x$ _____ (i)

$(10x + y) + 36 = (10y + x)$

$10x + y + 36 = 10y + x$

$10x + y - 10y - x = -36$

$9x - 9y = -36$

Putting $y = 3x$

$9x - 9 \times 3x = -36 \rightarrow 18x = -36$

$\boxed{x = 2}$

$x = 2$ in eq (i) we get $y = 3 \times 2 = 6$

original No = $10x + y = 10x\ 2 + 6 = 26$

[70] If $\begin{pmatrix} x+y & 1 \\ 1 & x-y \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 2 & -4 \end{pmatrix} = \begin{pmatrix} 12 & 4 \\ 3 & 0 \end{pmatrix}$ then

(a) $x = 7, y = -3$

(b) $x = -7, y = -3$

(c) $x = -7, y = 3$

(d) $x = 7, y = 3$ (1 mark)

Answer:

(d) If $\begin{pmatrix} x+y & 1 \\ 1 & x-y \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 2 & -4 \end{pmatrix} = \begin{pmatrix} 12 & 4 \\ 3 & 0 \end{pmatrix}$

$\begin{pmatrix} x+y+2 & 1+3 \\ 1+2 & x-y-4 \end{pmatrix} = \begin{pmatrix} 12 & 4 \\ 3 & 0 \end{pmatrix}$

On Comparing

$x + y + 2 = 12 \Rightarrow x + y = 10$ _____ (1)

And $x - y - 4 = 0 \Rightarrow x - y = 4$ _____ (2)

Solving (1) and (2) We get

$x = 7, y = 3$

[71] Find the condition that one roots is double the other of $ax^2+bx+c = 0$

(a) $2b^2 = 3ac$

(b) $b^2 = 3ac$

(c) $2b^2 = 9ac$

(d) $2b^2 > 9ac$ (1 mark)

Answer:

(c) Here, given Q.C. is

$$ax^2 + bx + c = 0$$

Given, $\beta = 2\alpha$

$\rightarrow n = 2$

then condition is

$$\frac{b^2}{ac} = \frac{(n+1)^2}{n}$$

$$\frac{b^2}{ac} = \frac{(2+1)^2}{2}$$

$$\frac{b^2}{ac} = \frac{9}{2} \Rightarrow 2b^2 = 9ac$$

[72] $[1, 2, 3]\begin{bmatrix}\log_{10} & 2\\ \log_{10} & 3\\ \log_{10} & 4\end{bmatrix} =$

(a) $\log_{10}(1521)$

(b) $\log_{10}(1152)$

(c) $\log_{10}(5211)$

(d) $\log_{10}(2151)$ (1 mark)

Answer:

(b) $[1, 2, 3]\begin{bmatrix}\log_{10} 2\\ \log^{10} 3\\ \log_{10} 4\end{bmatrix}$

$$= 1 \times \log_{10} 2 + 2\log_{10} 3 + 3 \times \log_{10} 4$$

$$= \log_{10} 2 + \log_{10} 3^2 + \log_{10} 4^3$$

$$= \log_{10} 2 + \log_{10} 9 + \log_{10} 64$$

$$= \log_{10}(2 \times 9 \times 64)$$

$$= \log_{10}(1152)$$

2019 - November

[73] Roots of the equation $x^3 + 9x^2 - x - 9 = 0$.

(a) 1, 2, 3
(b) 1, – 1, – 9
(c) 2, 3, – 9
(d) 1, 3, 9 (1 mark)

Answer:

(b) $x^3 + 9x^2 - x - 9 = 0$

By factorization method

$x^2 (x + 9) - 1 (x + 9) = 0$

$(x^2 - 1) (x + 9) = 0$

$(x + 1) (x - 1) (x + 9) = 0$ [$a^2 - b^2 = (a + b) (a - b)$]

$(x + 1) = 0$ $(x - 1) = 0$ $(x + 9) = 0$

$x = - 1$ $x = 1$ $x = - 9$

[74] $\frac{2x + 5}{10} + \frac{3x + 10}{15} = 5$

(a) 10.58
(b) 9.58
(c) 9.5
(d) None (1 mark)

Answer:

(b) $\frac{2x + 5}{10} + \frac{3x + 10}{15} = 5$

$\frac{15(2x + 5) + 10(3x + 10)}{150} = 5$

$30x + 75 + 30x + 100 = 750$

$60x = 575$

$x = \frac{575}{60}$

$x = 9.58$ (approx)

[75] Find value of $x^2 - 10x + 1$ if $x = \frac{1}{5 - 2\sqrt{6}}$

(a) 25
(b) 1
(c) 0
(d) 49 (1 mark)

Answer:

(c) $x^2 - 10x + 1 = 0$ given equation

$$x = \frac{1}{5-2\sqrt{6}}$$

Multiplying by conjugate

$$x = \frac{1}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}$$

$$x = \frac{5+2\sqrt{6}}{(5)^2 - (2\sqrt{6})^2} \quad (a+b)(a-b) = a^2 - b^2$$

Answer:

(a) $3x^2 - 2kx + 5 = 0$ {given equation} as it is given $x = 2$

then put in place of $x = '2'$

$3 \times (2)^2 - 2k(2) + 5 = 0$

$3 \times 4 - 2k(2) + 5 = 0$

$12 - 4k + 5 = 0$

$-4k = -12 - 5$

$-4k = -17$

$k = \frac{17}{4}$

PRACTICE QUESTIONS

[1] If the order of matrix A is m × p and the order of B is p × n. Then the order of matrix AB is ?

(a) m × n (b) n × m

(c) n × p (d) m × p

[2] If A and B are matrices, then which from the following is true?

(a) $A + B \neq B + A$ (b) $(A')' \neq A$

(c) $AB \neq BA$ (d) all are true

[3] What is a, if $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix ?

(a) 5 (b) 6

(c) 7 (d) 8

[4] If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $A = ?$

(a) 2 (b) 3

(c) 4 (d) 5

[5] If $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} A = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$ then order of matrix A = ?

(a) 2 × 2 (b) 2 × 3

[6] $(AB)^t = ?$

(a) $B^t A^t$ (b) $A^t B^t$

(c) AB (d) BA

[7] The matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & 5 \end{bmatrix}$ is a ?

(a) symmetric (b) skew-symmetric

(c) hermitian (d) skew-hermitian

[8] If $A = \begin{bmatrix} 5 & 3 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ then $|A| = ?$

(a) 30 (b) 40

(c) 50 (d) 60

[9] The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a ?

(a) even matrix (b) odd matrix

(c) scalar matrix (d) identity matrix

[10] The transpose of a rectangular matrix is a

(a) rectangular matrix (b) diagonal matrix

(c) square matrix (d) scaler matrix

[11] The transpose of a column matrix is

(a) zero matrix (b) diagonal matrix

(c) column matrix (d) row matrix

[12] Two matrices A and B are multiplied to get AB if

(a) both are rectangular

(b) both have same order

(c) no of columns of A is equal to no. of rows of B

(d) no of rows of A is equal to no. of columns of B

[13] If $|A| = 0$, then A is

(a) zero matrix (b) singular matrix

(c) non-singular matrix (d) 0

[14] If A is a symmetric matrix, then A^t =
(a) A (b) A
(c) 0 (d) diagonal matrix

[15] In a matrix multiplication for A and B, $(AB)^t$
(a) $A^t B^t$ (b) $B^t A^t$
(c) 1/AB (d) AB

[16] For a non-trivial solution A is
(a) $|A| > 0$ (b) $|A| < 0$
(c) $|A| = 0$ (d) $|A| \neq 0$

[17] Two matrices A and B are multiplied to get BA if
(a) both are rectangular
(b) both have same order
(c) no of columns of B is equal to rows of A
(d) both are square matries

[18] A matrix having m rows and n columns with $m \neq n$ is said to be a
(a) rectangular matrix (b) square matrix
(c) identity matrix (d) scaler matrix

[19] [a b c] is a
(a) zero matrix (b) diagonal matrix
(c) column matrix (d) row matrix

[20] Two matrices A and B are added if
(a) both are rectangular
(b) both have same order
(c) no of columns of A is equal to columns of B
(d) no of rows of A is equal to no of columns of B

[21] The transpose of a row matrix is
(a) zero matrix (b) diagonal matrix
(c) column matrix (d) row matrix

[22] Matrices obtained by changing rows and columns is called
(a) rectangular matrix (b) transpose
(c) symmetric (d) None of Above

[23] [0 0 0] is
(a) Scalar matrix (b) diagonal matrix
(c) identity matrix (d) null matrix

[24] If A is a matrix of order m × n and B is a matrix of order n × p then the order of AB is
(a) p × m (b) p × n
(c) n × p (d) m × p

[25] The transpose of a square matrix is a
(a) rectangular matrix (b) diagonal matrix
(c) square matrix (d) scalar matrix

[26] If $|A| \neq 0$, then A is
(a) zero matrix (b) singular matrix
(c) non-singular matrix (d) diagonal matrix

[27] If AB exists, then $(AB)^{-1}$
(a) $A^{-1} B^{-1}$ (b) $B^{-1} A^{-1}$
(c) AB (d) None of Above

[28] If A a skew symmetric matrix, then A^t
(a) – A (b) A
(c) 0 (d) diagonal matrix

[29] Two matrices A and B are equal if
(a) both are rectangular
(b) both have same order
(c) no of columns of A is equal to columns of B
(d) both have same order and equal corresponding elements

[30] The order of a matrix [2 5 7] is
(a) 3 × 3 (b) 1 × 1
(c) 3 × 1 (d) 1 × 3

[31] A matrix having m rows and n columns with m = n is said to be a
(a) rectangular matrix (b) square matrix
(c) identity matrix (d) scalar matrix

[32] Equations having a common solution are called
(a) linear equations
(b) homogeneous equations
(c) simultaneous equations
(d) None of Above

[33] If a matrix has m rows and n columns then order is
(a) m + n (b) n x n
(c) m x m (d) m x n

[34] The transpose of a column matrix is a :
(a) null matrix (b) row matrix
(c) scalar matrix (d) column matrix

[35] This matrix is a _____ .

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

(a) scalar matrix (b) identity matrix
(c) diagonal matrix (d) triangular matrix

[36] This matrix is a _____.

$$\begin{bmatrix} 6 & 1 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & 9 \end{bmatrix}$$

(a) scalar matrix (b) identity matrix
(c) diagonal matrix (d) triangular matrix

[37] A matrix whose each element is zero is called a :
(a) null matrix (b) scalar matrix
(c) identity matrix (d) symmetric matrix

[38] The transpose of a square matrix is a :
(a) null matrix (b) row matrix

[39] A square matrix all of whose elements except the main diagonal are zeros is called a :

(a) null matrix (b) singular matrix
(c) diagonal matrix (d) symmetric matrix

[40] Square matrix A for which $A^T = -A$ is called a :

(a) row matrix (b) column matrix
(c) symmetric matrix (d) skew-symmetric matrix

[41] Square matrix A for which $A^T = A$ is called a :

(a) row matrix (b) column matrix
(c) symmetric matrix (d) skew-symmetric matrix

[42] A square matrix A with complex entries for which $(A)^T = -A$ is called :

(a) identity matrix (b) hermitian matrix
(c) symmetric matrix (d) skew-hermitian matrix

[43] This matrix is a _____ .

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) null matrix (b) row matrix
(c) square matrix (d) symmetric matrix

[44] If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then A^T is :

(a) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 6 \\ 9 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & 9 \\ 6 & 4 \end{bmatrix}$

[45] This matrix is a _____.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) unit matrix (b) null matrix
(c) triangular matrix (d) rectangular matrix

[46] If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $|A|$ is :

(a) 0 (b) 1
(c) 2 (d) – 2

[47] This matrix is a _____ .

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(a) null matrix (b) zero matrix
(c) scalar matrix (d) column matrix

[48] Matrices are represented by :

(a) small letters (b) real numbers
(c) capital letters (d) natural numbers

[49] If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|\text{adj } A|$ is

(a) a^{27} (b) a^9
(c) a^6 (d) a^2

[50] If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det (adj (adj A)) is

(a) 14^4 (b) 14^3
(c) 14^2 (d) 14

[51] For any 2 × 2 matrix, if A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then A is equal to

(a) 20 (b) 100
(c) 10 (d) 0

[52] If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then A^5 =

(a) 5A (b) 10A
(c) 16A (d) 32A

[53] If A is a matrix of order 3 and $|A| = 8$, then $|\text{adj } A| =$

(a) 1 (b) 2

(c) 2^3 (d) 2^6

[54] Let $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and X be a matrix such that A = BX, then X is equal to

(a) $\frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (d) none of these

[55] If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that A^{-1} kA, then k equals

(a) 19 (b) 1/19

(c) – 19 (d) – 1/19

[56] The system of equation x+y+z = 2, 3x – y + 2z = 6 and 3x + y + z = – 18 has

(a) a unique solution

(b) no solution

(c) an infinite number of solutions

(d) zero solution as the only solution

[57] Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$. If AX = B, then X is equal to

(a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$

(c) $\begin{bmatrix} & \end{bmatrix}$ (d)

[58] If the system of equations $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x+3y-4z = 0$ has non trivial solution, then $\frac{xy}{z^2}$ = ...

(a) $\frac{5}{6}$ (b) $-\frac{5}{6}$

(c) $\frac{6}{5}$ (d) $-\frac{6}{5}$

[59] Let $A = \begin{bmatrix} 4 & 4k & k \\ 0 & k & 4k \\ 0 & 0 & 4 \end{bmatrix}$. If $\det(A^2) = 16$ then $|k|$ is ...

(a) 1 (b) $\frac{1}{4}$

(c) 4 (d) 4^2

[60] The value of $\begin{bmatrix} \end{bmatrix}_x$ = ...

[64] If the equations $x+3y+z = 0$, $2x - y - z = 0$, $kx + 2y + 3z = 0$ have non-trivial solution then $k = ...$

(a) $\frac{13}{2}$ (b) $\frac{9}{2}$

(c) $-\frac{15}{2}$ (d) $-\frac{13}{2}$

[65] If the equations $ax + by + cz = 0$, $4x + 3y + 2z = 0$, $x + y + z = 0$ have non -trivial solution, then a, b, c are in ...

(a) A. P. (b) G. P.

(c) Increasing sequence (d) decreasing sequence

[66] If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then $A^n = ...$

(a) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ (b) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$

(c) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$ (d) $\begin{bmatrix} 2n+1 & -4n \\ n & 1-2n \end{bmatrix}$

[67] If $A = \begin{bmatrix} \alpha & 0 \\ 2 & 3 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A^2 = 91$ for

(a) $\alpha = 4$ (b) $\alpha = 3$

(c) $\alpha = -3$ (d) no $= \alpha$

ANSWER

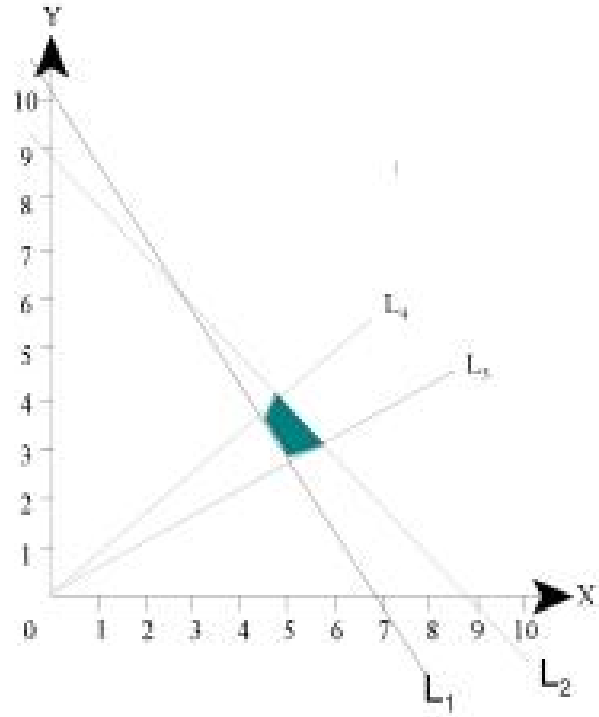
1	(a)	**2**	(c)	**3**	(d)	**4**	(b)	**5**	(b)	**6**	(a)
7	(a)	**8**	(d)	**9**	(c)	**10**	(a)	**11**	(d)	**12**	(c)
13	(b)	**14**	(a)	**15**	(b)	**16**	(c)	**17**	(d)	**18**	(a)
19	(d)	**20**	(b)	**21**	(c)	**22**	(b)	**23**	(d)	**24**	(d)
25	(b)	**26**	(c)	**27**	(b)	**28**	(a)	**29**	(d)	**30**	(d)

31	(b)	**32**	(c)	**33**	(d)	**34**	(b)	**35**	(c)	**36**	(d)
37	(a)	**38**	(c)	**39**	(c)	**40**	(d)	**41**	(d)	**42**	(d)
43	(a)	**44**	(a)	**45**	(a)	**46**	(d)	**47**	(d)	**48**	(c)
49	(c)	**50**	(a)	**51**	(c)	**52**	(c)	**53**	(d)	**54**	(a)
55	(b)	**56**	(a)	**57**	(d)	**58**	(b)	**59**	(b)	**60**	(b)
61	(d)	**62**	(d)	**63**	(b)	**64**	(c)	**65**	(a)	**66**	(d)
67	(c)										

MULTIPLE CHOICE QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] Graphs of Inequations are drawn below :



$L_2 : 5x + 3y = 30$ $L_1 : x + y = 9$

$L_3 : y = \frac{x}{3}$ $L_4 : y = \frac{x}{2}$

The common region (shaded part) shown in the diagram refers to the inequalities :

(a) $5x + 3y \leq 30$

$x + y \leq 9$

$y \leq \frac{1}{2}x$

$y \leq x/2$

$x \geq 0, y \geq 0$

(b) $5x + 3y \geq 30$
$x + y \leq 9$
$y \geq x/3$
$y \leq x/2$
$x \geq 0, y \geq 0.$

(c) $5x + 3y \geq 30$
$x + y \geq 9$
$y \leq x/3$
$y \geq x/2$
$x \geq 0, y \geq 0.$

(d) $5x + 3y > 30$
$x + y < 9$
$y \geq 9$
$y \leq x/2$
$x \geq 0, y \geq 0.$ (1 mark)

Answer:

(b) Region represented by L_2 i.e. $5x + 3y = 30$. The, line $5x + 3y = 30$ meets the coordinate axes at (7,0) and (0,10) joining these points, we can get the graph of the equation. Since the shaded region is towards the right side of L_1, therefore it is represented by greater than or equal to sign i.e. $5x + 3y \geq 30$.

Similarly for the in equation $x + y = 9$ the shaded region is to the left of L_1.Hence it is represented by $\leq$ sign i.e. $x + y \leq 9$.

And as for the line $y = \frac{x}{3}$ is concerned, it passes through the origin and the shaded region lies upward of the line L_3, it represents greater than or equal sign ($\geq$) i.e. $y \geq \frac{x}{3}$.

And similarly for L_4 also.

[2] If $\left|x+\frac{1}{4}\right| > \frac{7}{4}$, then :

(a) $x < \frac{-3}{2}$ or $x > 2$

(b) $x < -2$ or $x > \frac{3}{2}$

(c) $-2 < x < \frac{3}{2}$

(d) None of these. (1 mark)

Answer:

(b) $\left|x+\frac{1}{4}\right| > \frac{7}{4}$

when $\left|x+\frac{1}{4}\right| = x+\frac{1}{4}$

$x+\frac{1}{4} > \frac{7}{4}$

$x > \frac{6}{4}$ i.e. $x > \frac{3}{2}$

when $\left|x+\frac{1}{4}\right| = -\left(x+\frac{1}{4}\right)$

$-($ $>$

2007 - FEBRUARY

[3] If $\left|\frac{3x-4}{4}\right| \leq \frac{5}{12}$, the solution set is :

(a) $\left\{x : \frac{19}{18} \leq x \leq \frac{29}{18}\right\}$ (b) $\left\{x : \frac{7}{9} \leq x \leq \frac{17}{9}\right\}$

(c) $\left\{x : \frac{-29}{18} \leq x \leq \frac{-19}{18}\right\}$ (d) None of these. (1 mark)

Answer:

(b) $\left|\frac{3x-4}{4}\right| \leq \frac{5}{12}$

when $\left|\frac{3x-4}{4}\right| = \left(\frac{3x-4}{4}\right)$

$\frac{3x-4}{4} \leq \frac{5}{12}$

$3x - 4 \leq \frac{5}{3}$ (Multiplying both sides by 4)

$3x \leq \frac{5}{3} + 4$

$3x \leq \frac{17}{3}$

$x \leq \frac{17}{9}$

When $\left|\frac{3x-4}{4}\right| = -\left(\frac{3x-4}{4}\right)$

$-\left(\frac{3x-4}{4}\right) \leq \frac{5}{12}$

Multiplying both sides with – 1

$\frac{3x-4}{4} \geq -\frac{5}{12}$

$3x - 4 \geq -\frac{5}{3}$

$9x - 12 \geq -15$

$9x \geq -7$

$x \geq \frac{7}{9}$

Therefore , $\frac{7}{9} \leq x \leq \frac{17}{9}$

[4] On solving the inequalities $6x + y \geq 18$, $x + 4y \geq 12$, $2x + y \geq 10$, we get the following situation:

(a) (0, 18), (12, 0), (4, 2) & (2, 6)

(b) (3, 0), (0, 3), (4, 2), & (7, 6)

(c) (5, 0), (0, 10), (4, 2) & (7, 6)

(d) (0, 18), (12, 0), (4, 2), (0, 0) and (7, 6) (1 mark)

Answer:

(a) We draw the graph of $6x + y \geq 18$, $x + 4y \geq 12$, and $2x + y \geq 10$ in the same plane. The solution set of system is that portion of the graphs of the given inequality which is represented by the intersection of the above three equations.

For this purpose, we replace, the inequalities respectively by $6x+y=18$, $x + 4y = 12$ and $2x + y = 10$

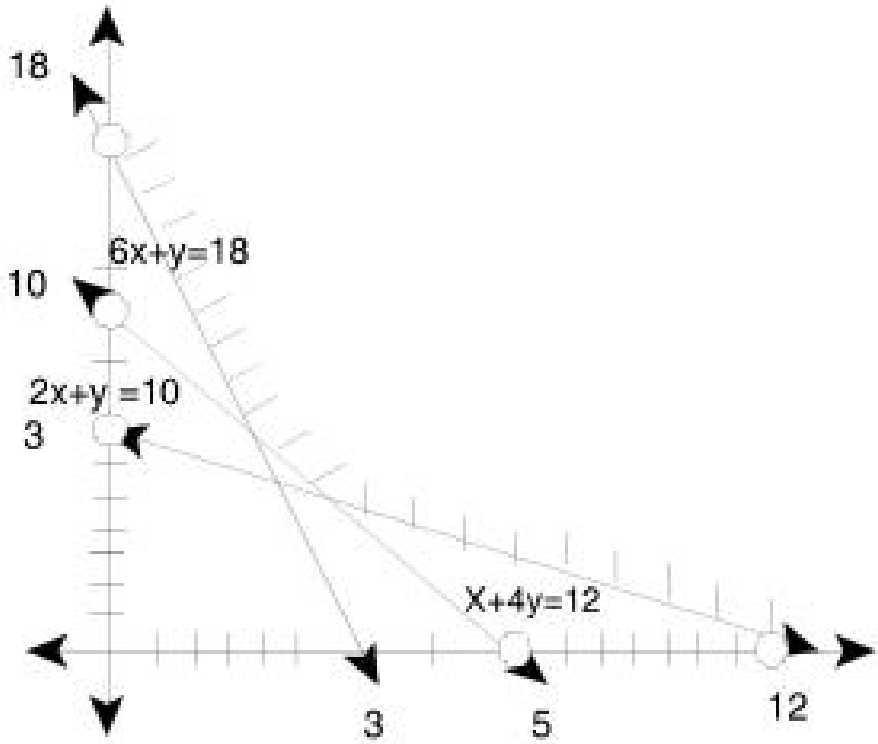
For $6x + y = 18$, For $x + 4y = 12$

x	0	3
y	18	0

x	0	12
y	3	0

For $2x + y = 10$

x	0	5
y	10	0



Hence, the common region is bounded by (0, 18) (12, 0) (4, 2) and (2,6) which satisfies the given inequalities.

2007 - MAY

[5] A car manufacturing company manufactures cars of two types A and B. Model A requires 150 man-hours for assembling, 50 man-hours for painting and 10 man-hours for checking and testing. Model B requires 60 man-hours for assembling, 40 man-hours for painting and 20 man-hours for checking and testing. There are available 30 thousand man-hours for assembling, 13 thousand man-hours for painting and 5 thousand man-hours for checking and testing. Express the above situation using linear inequalities. Let the company manufacture x units of type A model of car and y units of type B model of car. Then, the inequalities are:

(a) $5x + 2y \geq 1000$; $5x + 4y \geq 1300$,
$x + 2y \leq 500$; $x \geq 0$, $y \geq 0$,

(b) $5x + 2y \leq 1000$, $5x + 4y \leq 1300$,
$x + 2y \geq 500$; $x \geq 0$, $y \geq 0$.

(c) $5x + 2y \leq 1,000$, $5x + 4y \leq 1300$,
$x + 2y \leq 500$; $x \geq 0$, $y \geq 0$.

(d) $5x + 2y = 1000$, $5x + 4y \geq 1300$,
$x + 2y = 500$; $x \geq 0$, $y \geq 0$. (1 mark)

Answer:

(c) Let x units of cars of model A and y units of Cars of model B are manufactured. Then

	ASSEMBLING	PAINTING	CHECKING & TESTING
Model A :	150 hrs	50 hrs	10 hrs
Model B :	60 hrs	40 hrs	20 hrs
Available time (in hrs)	30,000	13,000	5,000

According to given information,

$150x + 60y \leq 30,000$

$50x + 40y \leq 13,000$

$10x + 20y \leq 5000$

$x \geq 0$, $y \geq 0$ (non- negative restrictions)

Therefore,

$5x + 2y \leq 1000$, $5x + 4y \leq 13,000$

$x + 2y \leq 500$, $x \geq 0$, $y \geq 0$

2007 - AUGUST

[6] The rules and regulations demand that the employer should employ not more than 5 experienced hands to 1 fresh one and this fact is represented by : (Taking experienced person as x and fresh person as y)

(a) $y \geq \frac{x}{5}$ (b) $5y \leq x$

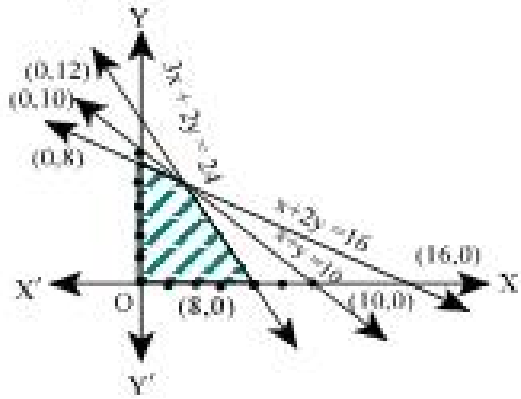
Answer:

(a) For the given question, we have to find the ratio of x and y (i.e. $\frac{x}{y}$) Here, x implies 5 fresh hands and y implies experienced hand.

$$\frac{x}{y} \leq \frac{5}{1}$$

$$y \geq \frac{x}{5}$$

[7] The shaded region represents :



(a) $3x + 2y \leq 24, x + 2y \geq 16, x + y \leq 10x, x \geq 0, y \geq 0$
(b) $3x + 2y \leq 24, x + 2y \leq 16, x + y \geq 10, x \geq 0, y \geq 0$
(c) $3x + 2y \leq 24, x + 2y \leq 16, x + y \leq 10, x \geq 0, y \geq 0$
(d) None of these. (1 mark)

Answer:

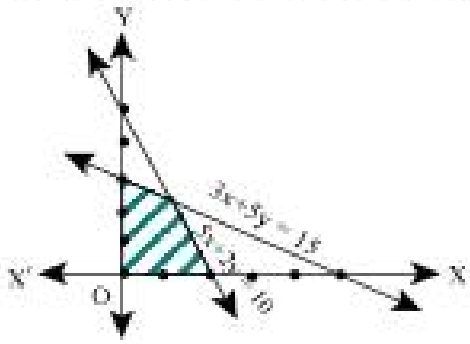
(c) Region represented by the line $3x + 2y = 24$ meets the coordinate axes at (8,0) and (0,12). Since the shaded region lies below the line $3x + 2y = 24$ therefore it is represented by less than or equal to sign i.e. $3x + 2y \leq 24$.

Similarly for the line $x + 2y = 16$ the shaded region lies below the line therefore it is represented by less than or equal to sign i.e. $x + 2y \leq 16$

Clearly $x \geq 0$ and $y \geq 0$ represents the region lying on the right side of y axis and above x axes.

2008 - NOVEMBER

[8] The shaded region represents :



(a) $3x + 5y \leq 15, 5x + 2y \geq 10, x, y \geq 0$
(b) $3x + 5y \leq 15, 5x + 2y \leq 10, x, y \geq 0$
(c) $3x + 5y \geq 15, 5x + 2y \geq 10, x, y, \geq 0$
(d) None of these. (1 mark)

Answer:

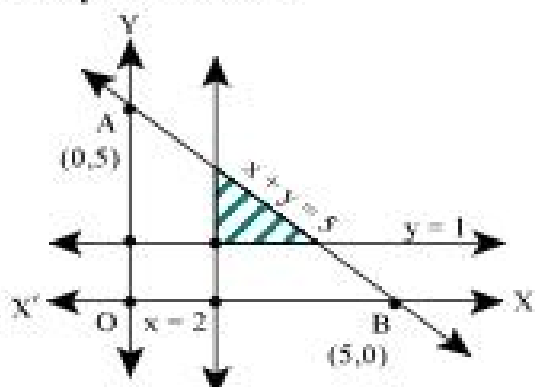
(b) The shaded region lies to the left of the equation $3x + 5y = 15$ therefore it is represented by less than or equal sign i.e. $3x + 5y \leq 15$.

Similarly for the equation $5x + 2y \leq 10$

Clearly $x \geq 0$ and $y \geq 0$ represents the region lying on the right side of y axes and above x axes .

2008 - FEBRUARY

[9] The shaded region represents :



(a) $x + y \leq 5, x \geq 2, y \leq 1$ (b) $x + y \leq 5, x \geq 2, y \geq 1$
(c) $x + y \geq 5, x \geq 2, y \geq 1$ (d) None of these. (1 mark)

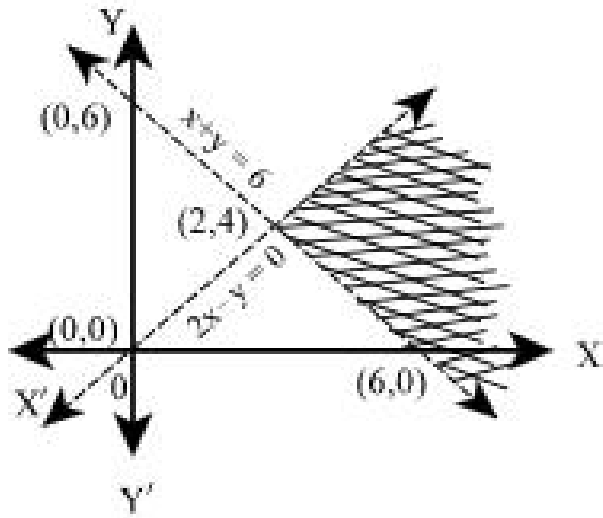
Answer:

(b) Region represented by the line $x + y=5$ touch the coordinate axes at (5, 0) and (0, 5) Since the shaded region lies below the line $x +y=5$. Hence it is represented by the inequation $x +y \leq 5$.

And for the equation $x = 2$ we find that the point (0,0) does not satisfy the in equation $x \geq 2$ and also the shaded region lies to the right of $x = 2$. Hence it is represented by inequation $x \geq 2$. Similarly for the equation $y = 1$ it is represented by the inequation $y \geq 1$.

2008 - JUNE

[10] The shaded region represents :



(a) $x + y > 6, 2x - y > 0$ (b) $x + y < 6, 2x - y > 0$
(c) $x + y > 6, 2x - y < 0$ (d) None of these (1 mark)

Answer:

(a) The shaded region lies above the equation $x +y = 6$, therefore it is represented by the in equation $x + y > 6$. Similarly for the equation $2x - y = 0$ since the shaded region lies to the right of the equation. Hence it is represented by the in equation $2x - y$

[11] If a >0 and b <0, it follows that :

(a) $\frac{1}{a} > \frac{1}{b}$ (b) $\frac{1}{a} < \frac{1}{b}$

(c) $\frac{1}{a} = \frac{1}{b}$ (d) None of these. (1 mark)

Answer:

(a) (∵ given that a > 0)

Since a is a positive number therefore its reciprocal $\left(\text{i.e. } \frac{1}{a}\right)$ will also be positive.

Similarly, since b is a negative number (∵ given that b < 0), therefore its reciprocal $\left(\text{i.e. } \frac{1}{b}\right)$ will also be negative.

So, we can conclude that $\frac{1}{a} > \frac{1}{b}$.

2008 - DECEMBER

[12] The Linear relationship between two variables in an inequality :

(a) $ax + by \leq c$ (b) $ax\ by \leq c$

(c) $axy + by \leq c$ (d) $ax + bxy \leq c$ (1 mark)

Answer:

(a) The linear relationship between two variables in an inequality is given by $a\,x + by \leq c$

Any linear function that involves an inequality sign is a linear inequality It may be of one variable, or, of more than one variable.

Ex : $3x + y < 6$, $x - y \leq -2$, etc.

2010 - JUNE

[13] The solution of the inequality $\frac{(5 - 2x)}{3} \leq \frac{x}{6} - 5$ is

(a) $x \geq 8$ (b) $x \leq 8$

(c) $x = 8$ (d) None of these. (1 mark)

Answer:

(a) $\frac{5 - 2x}{3} \leq \frac{x}{6} - 5$

$\frac{5 - 2x}{3} \leq \frac{x - 30}{6}$

$10 - 4x \leq x - 30$

$40 \leq 5x$

$\therefore \quad x \geq 8$

2010 - DECEMBER

[14] On the average an experienced person does 7 units of work while a fresh one work 5 units of work daily but the employer has to maintain an output of atleast 35 units of work per day. The situation can be expressed as :

(a) $7x + 5y < 35$ (b) $7x + 5y \leq 35$

(c) $7x + 5y > 35$ (d) $7x + 5y \geq 35$ (1 mark)

Answer:

(d) Let no. of experienced worker be x

Let no. of fresh worker be y

$\therefore$ According to the given condition :

$7x + 5y \geq 35$

2011 - JUNE

[15] Solution space of the inequalities $2x + y \leq 10$ and $x - y \leq 5$:
(i) includes the origin.
(ii) includes the point (4, 3)
which one is correct ?
(a) Only (i) (b) Only (ii)
(c) Both (i) and (ii) (d) None of the above. (1 mark)

Answer:

(a) Given : $2x + y \leq 10$ ____(1)
$x - y \leq 5$ ____(2)
Adding (1) & (2) we get
$3x \leq 15 \rightarrow x \leq 5$
putting $x \leq 5$ in eq^n(1) we get
$y \leq 0$
hence $x \leq 5$ and $y \leq 0$ is the solution space.

2011 - DECEMBER

[16] On an average, experienced person does 5 units of work while a fresh person does 3 units of work daily but the employer has to maintain the output of atleast 30 units of work per day. The situation can be expressed as.
(a) $5x + 3y \leq 30$ (b) $5x + 3y \geq 30$
(c) $5x + 3y > 30$ (d) $5x + 3y = 30$ (1 mark)

Answer:

(b) $5x + 3y \geq 30$

2012 - JUNE

[17] Find the range of real values of x satisfying the inequalities $3x - 2 > 7$ and $4x - 13 > 15$

(a) $x > 3$ (b) $x > 7$

(c) $x < 7$ (d) $x < 3$ (1 mark)

Answer:

(b) Given $3x - 2 > 7$

$\rightarrow 3x > 9$

or $x > 3$.....(1)

Also $4x - 13 > 15$

$\rightarrow 4x > 28$

or $x > 7$.....(2)

hence, condition satisfying both (1) & (2) with 'x' satisfying both the roots is the common region on the above two number lines as $x > 7$ sharing the same region requiring (+) ve sign.

2012 - DECEMBER

[18] On the average, experienced person does 5 units of work while a fresh one 3 units work daily but the employer have to maintain the output of at least 30 units of work per day. The situation can be expressed as.

(a) $5x + 3y \leq 30$ (b) $5x + 3y \geq 30$

(c) $5x + 3y = 30$ (d) None of these. (1 mark)

Answer:

(b) Let Experience Person = X unit work per day

Fresh one = Y unit work per day

So situation is

2013 - JUNE

[19] The union forbids employer to employ less than two experienced person (x) to each fresh person (y). This situation can be expressed as:

(a) $x \leq y/2$ (b) $y \leq x/2$

(c) $y \geq x/2$ (d) None of these. (1 mark)

Answer:

(b) Let "x" & "y" be the no. of experienced and fresh hands respectively.

In this problem, the word "forbid' plays an important role.

Meaning of "Forbid" is "Not allowed"

The union forbids the employer to employ less than 2 experienced hands. That is, the union does not allow the employer to employ less than 2 experienced hands.

Therefore, the employer should employ 2 or more than 2 experienced hands.

So, we have $x \geq 2$ or $x/2 \geq 1$ ----------(1)

And also, no. of fresh persons to be employed is equal to 1

So, we have $y = 1$

In (1), replacing 1 by "y", we get $x/2 \geq y$ or $y \leq x/2$

2013 - DECEMBER

[20] The solution of the inequality $8x + 6 < 12x + 14$ is:

(a) (- 2, 2) (b) (0, - 2)

(c) (2, ∞) (d) (- 2, ∞) (1 mark)

Answer:

(d) Given $8x + 6 < 12x + 14$

$6 - 14 < 12x - 8x$

$-8 < 4x$

$$4x > -8$$

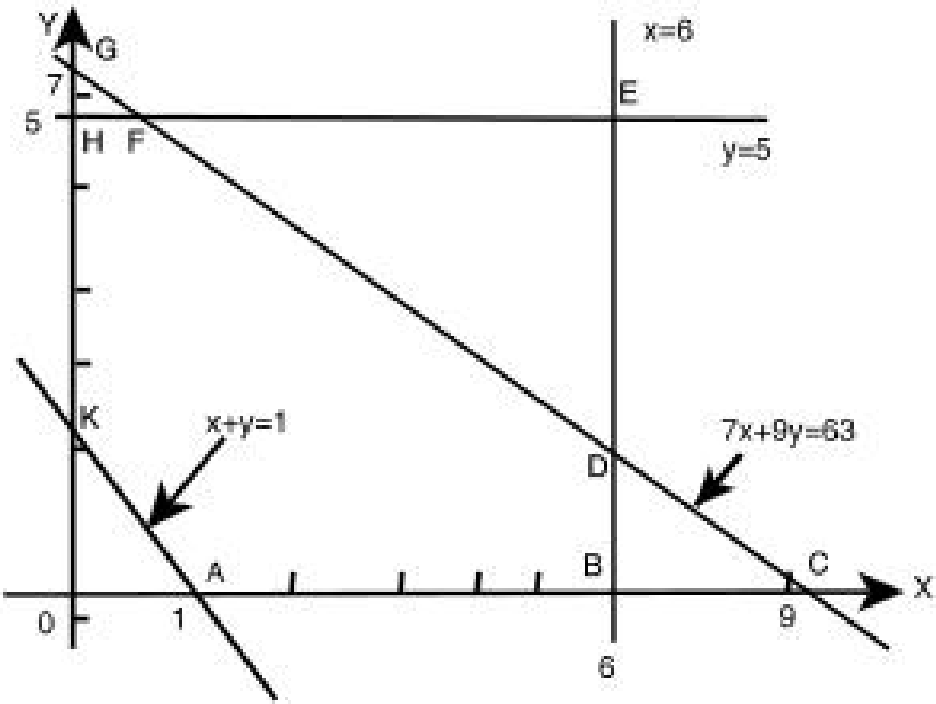
$$x > \frac{-8}{4}$$

$$x > -2$$

Solution set = $(-2, \infty)$

2014 - JUNE

[21] The graph of linear inequalities $7x + 9y \leq 63$, $x + y \geq 1$, $0 \leq x \leq 6$ and



Common region of the inequalities is:

(a) BCDB and DEFD (b) Unbounded

(c) HFGH (d) ABDFHKA (1 mark)

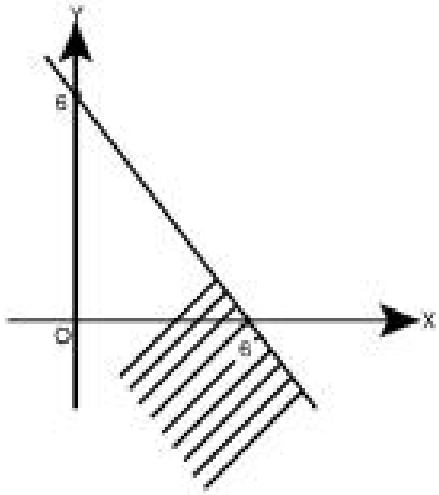
Answer:

(d) Common Region of the inequalities is ABDFHKA.

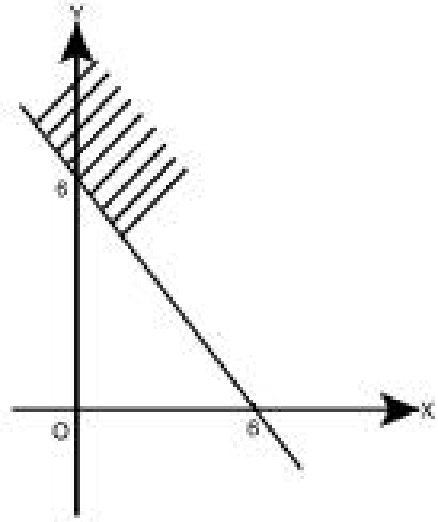
2014 - DECEMBER

[22] The graph to express the inequality $x + y \leq 6$ is:

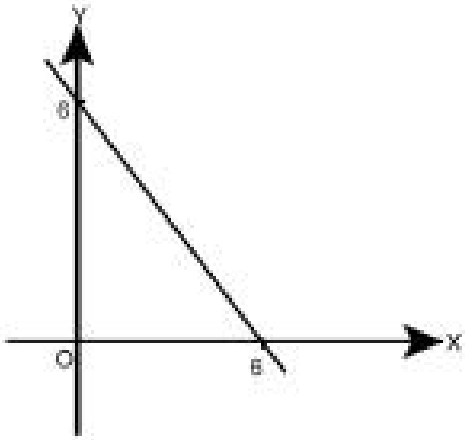
(a)



(b)



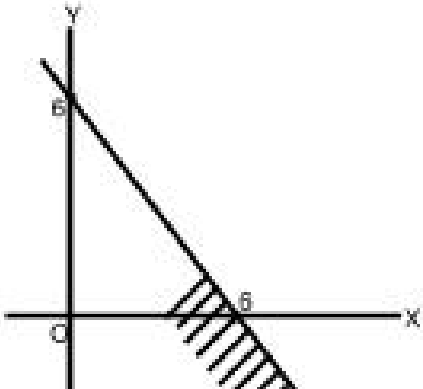
(c)



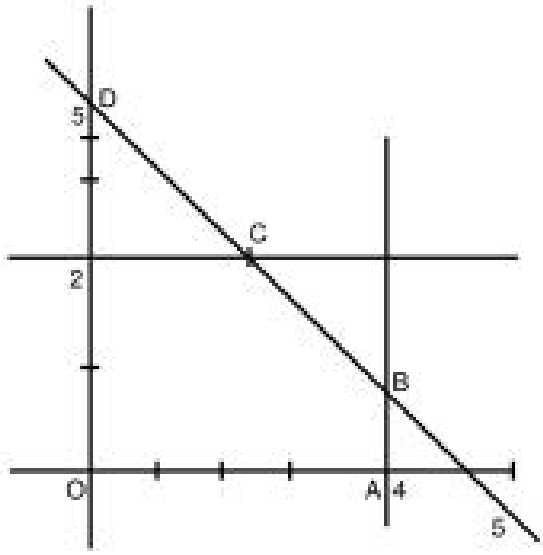
(d) None of these. (1 mark)

Answer:

(a) $x + y \leq 6$ is graphically represent by



[23] The graph of linear inequalities $x + y \geq 5$; $x + y \leq 5$; $0 \leq x \leq 4$ and $0 \leq y \geq 2$ is given below:



The common region of the inequalities will be:

(a) OABCEO (b) ECDE

(c) Line Segment DC (d) Line Segment BC (1 mark)

Answer:

(c) Line segment DC.

2015 - JUNE

[24] The common region in the graph of linear inequalities $2x + y \geq 18$, $x + y \geq 12$ and $3x + 2y \leq 34$ is:

(a) unbounded (b) infeasible

(c) feasible and bounded (d) feasible and unbounded

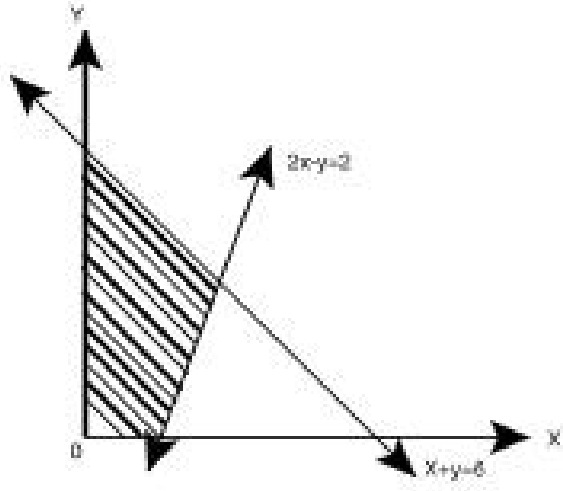
(1 mark)

Answer:

(c) Feasible and bounded

2015 - DECEMBER

[25] The common shaded region in the graph represents the linear inequalities as:



(a) $x + y \geq 6$
$2x - y - 2 \geq 0$
$x, y \geq 0$

(b) $x + y \geq 6$
$2x - y - 2 \leq 0$
$x, y \geq 0$

(c) $x + y \leq 6$
$2x - y - 2 \leq 0$
$x, y \geq 0$

(d) $x + y \leq 6$
$2x - y - 2 \geq 0$
$x, y \geq 0$ (1 mark)

Answer:

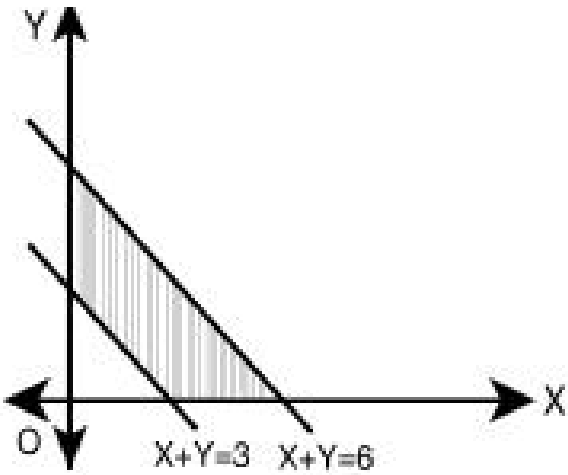
(c) The common shaded region in the graph represents the linear inequalities are

$x + y \leq 6$
$2x - y - 2 \leq 0$

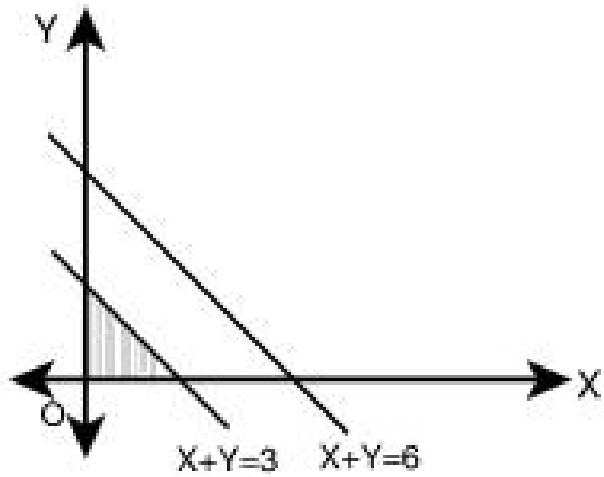
2016 - JUNE

[26] The common region of $x + y \leq 6$; $x + y \geq 3$; $x \geq 0$; $y \geq 0$, is (as shown by shaded region):

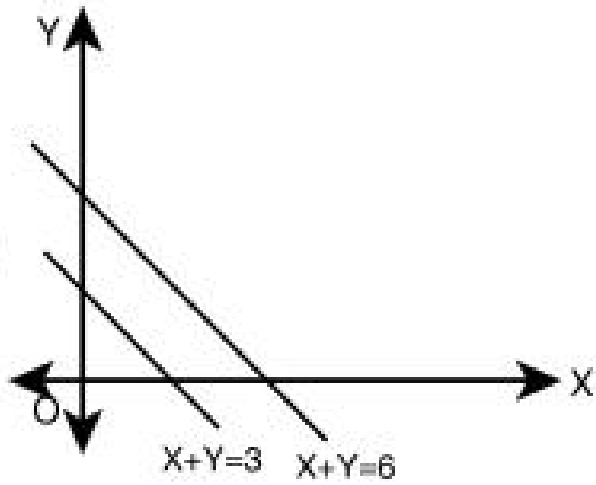
(a)



(b)



(c)



(d) None the these. (1 mark)

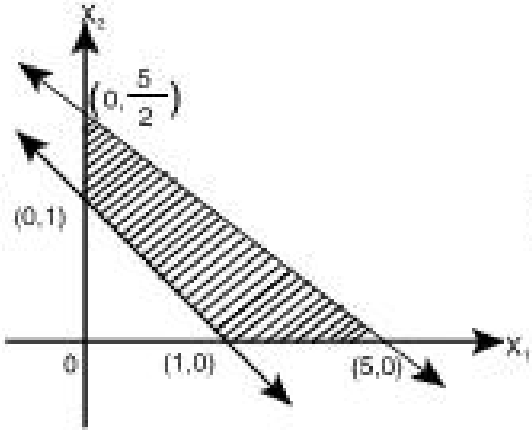
Answer:

(a) The common region required to satisfy above equations is depicted in option (A) as shown in its graph.

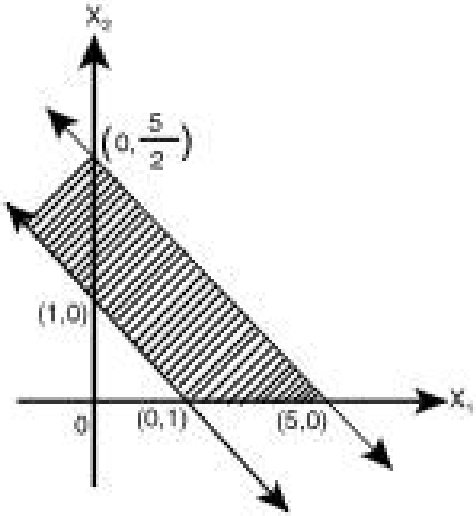
2016 - DECEMBER

[27] The common region by the inequalities $x_1 + 2x_2 \leq 5$, $x_1 + x_2 \geq 1$, $x_1 \geq 0$, $x_2 \geq 0$ is given as shaded portion in:

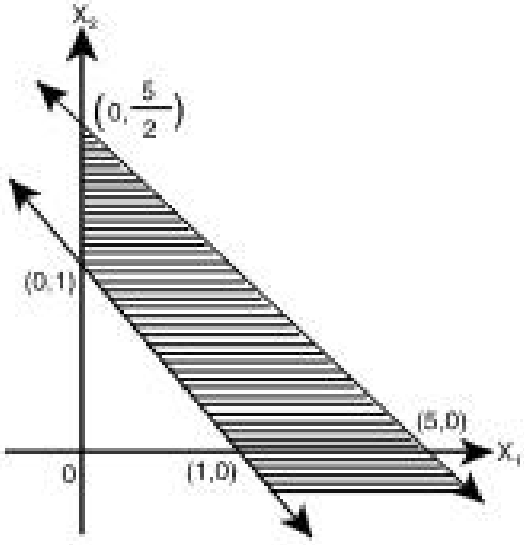
(a)



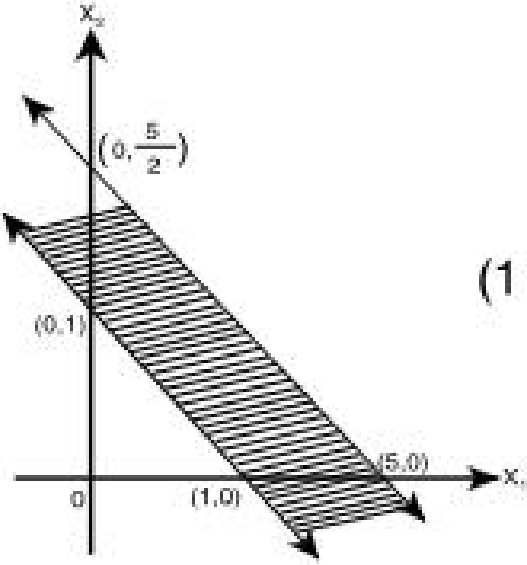
(b)



(c)



(d)



(1 mark)

Answer:

(a)

Given $x_1 + 2x_2 \leq 5$

and $x_1 + x_2 \geq 1$

$x_1 \geq 0$ and $x_2 \geq 0$

x_1	0	5
x_2	5/2	0

x_1	0	1
x_2	1	0

2017 - JUNE

[28] A dietician wishes to mix together two kinds of food so that the vitamins content of the mixture is atleast 9 units of vitamin A, 7units of vitamin B, 10 units of vitamin C, 12 units of vitamin D. The vitamin content per kg. of each food is shown in table. Assuming 'x' units of food I is to be mixed with 'y' units of food II the situation can be expressed as:

	A	B	C	D
Food I	2	1	1	2
Food II	1	1	2	3

(a) $2x + y \leq 9$
$x + y \leq 7$
$x + 2y \leq 10$
$2x + 3y \leq 12$
$x > 0, y > 0$

(b) $2x + y \geq 30$
$x + y \leq 7$
$x + 2y \geq 10$
$2x + 3y \geq 12$
$x > 0, y > 0$

(c) $2x + y \geq 9$
$x + y \geq 7$
$x + 2y \leq 10$
$x + 3y \leq 12$
$x \geq 0, y > 0$

(d) $2x + y \geq 9$
$x + y \geq 7$
$x + 2y \geq 10$
$2x + 3y \geq 12$

Answer:

(d) At least means $\geq$

Than $2x + y \geq 9$

$x + y \geq 7$

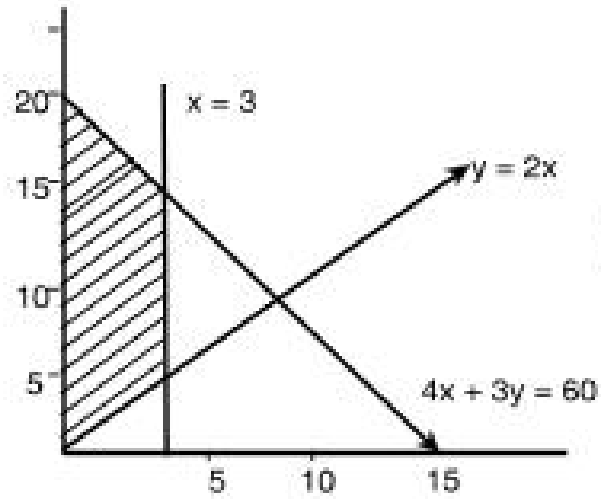
$x + 2y \geq 10$

$2x + 3y \geq 12$

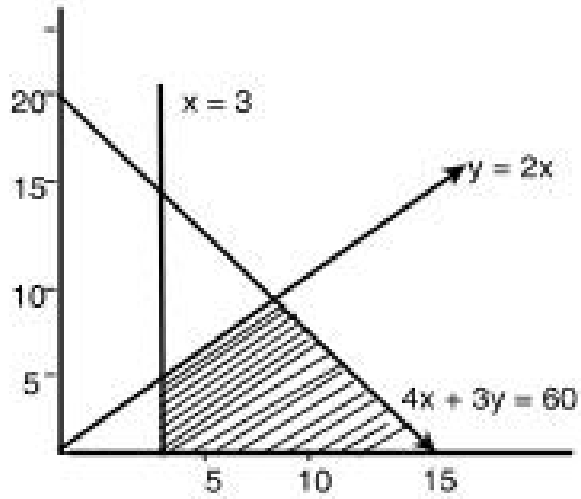
$x \geq 0, y \geq 0$ **'d'** option is correct.

[29] The common regions by the inequalities $4x + 3y \leq 60$; $y \geq 2x$; $x \geq 3$, $x \geq 0$ and $y \geq 0$ is

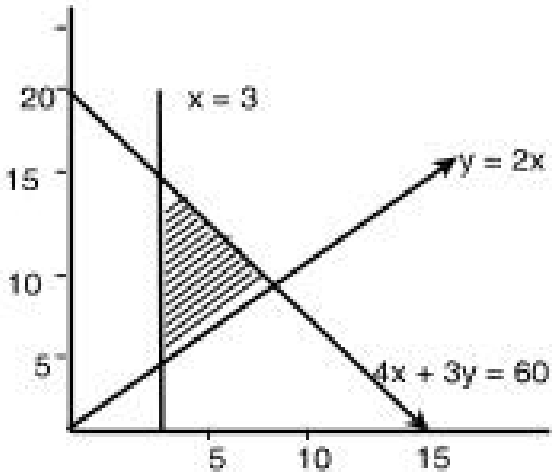
(a)



(b)



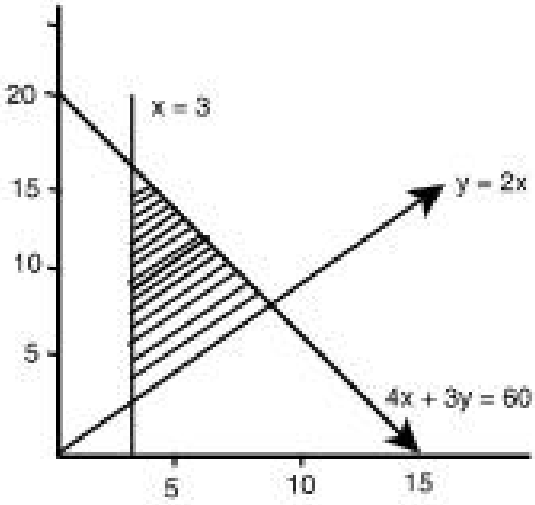
(c)



(d) None of these. (1 mark)

Answer:

(c)



2018 - MAY

[30] The linear relationship between two variables in an inequality:

(a) $ax + by \leq c$ (b) $ax.by \leq c$

(c) $axy + by \leq c$ (d) $ax + bxy \leq c$ (1 mark)

Answer:

(a) The linear relationship b/w two variables in an inequality $ax + by \leq c$

2018 - NOVEMBER

[31] On Solving the Inequalities $5x + y \leq 100$, $x + y \leq 60$, $x \geq 0$, $y \geq 0$, we get the following solution:

(a) (0, 0), (20, 0), (10, 50) & (0,60)

(b) (0, 0), (60, 0), (10, 50) & (0,60)

(c) (0, 0), (20, 0), (0, 100) & (10,50)

(d) None of these (1 mark)

Answer:

(a) On Solving the Inequalities $5x + y \leq 100$, $x + y \leq 60$, $x \geq 0$, $y \geq 0$, we get

(0, 0), (20, 0), (10, 50) & (0,60) all satisfied above Inequalities.

2019 - JUNE

[32] An employer recruits experienced (x) and fresh workmen (y) for his under the condition that he cannot employ more than 11 people. x and y can related by the inequality.

(a) $x + y \neq 11$

(b) $x + y \leq 11, x \geq 0, y \geq 0$

(c) $x + y \geq 11, x \geq 0, y \geq 0$

(d) None of these (1 mark)

Answer:

(b) $x + y \leq 11, x \geq 0, y \geq 0$

[33] The solution set of the in equations $x + 2 > 0$ and $2x - 6 > 0$ is

(a) $(-2, \infty)$

(b) $(3, \infty)$

(c) $(-\infty, -2)$

(d) $(-\infty, -3)$ (1 mark)

Answer:

(b) Given

$x + 2 > 0$

$x > -2$

$x = \{-1, 0, 1, \ldots\ldots\ldots\ldots \infty\}$

and

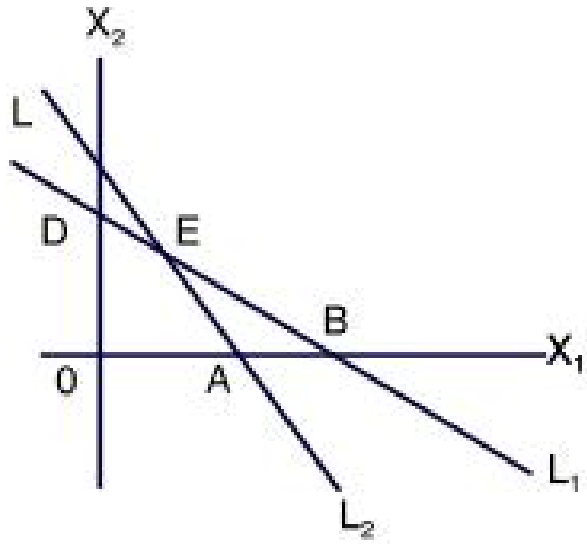
$2x - 6 > 0$

$2x > 6$

$x > 3$

$x = \{4, 5, 6, \ldots\ldots\ldots \infty\}$

[34] The common region represented by the following in equalities
$L_1 = X_1 + X_2 < 4$; $L_2 = 2X_1 - X_2 > 6$



(a) OABC
(b) Outside of OAB
(c) Δ BCE
(d) Δ ABE (1 mark)

Answer:

(d) Common Region represented by the
In equalities
$4 = x_1 + x_2 < 4$; $L_2 = 2x_1 - x_2 > 6$
is (Δ ABE)

2019 - NOVEMBER

[94] $6x + y \geq 18$, $x + 4y \geq 12$, $2x + y \geq 10$
On solving the inequalities; we get
(a) (0, 18), (12, 0), (4, 2), & (7, 6)
(b) (3, 0), (0, 3), (4, 2) & (7, 6)
(c) (5, 0), (0, 10), (4, 2) & (7, 6)
(d) (0, 18), (12, 0), (4, 2), (0, 0) and (7, 6) (1 mark)

Answer:

(a) We draw the graph of $6x + y \geq 18$, $x + 4y \geq 12$ and $2x + y \geq 10$ in the same plane. The solution set of system is that portion of the graphs of the given inequality which is represented by the intersection of the above three equations.

For this purpose, we replace, the inequalities respectively by $6x+y=18$, $x + 4y = 12$ and $2x + y = 10$

For $6x + y = 18$,

x	0	3
y	18	0

For $x + y = 12$

x	0	12
y	3	0

For $2x + y = 10$

x	0	5
y	10	0

CHAPTER

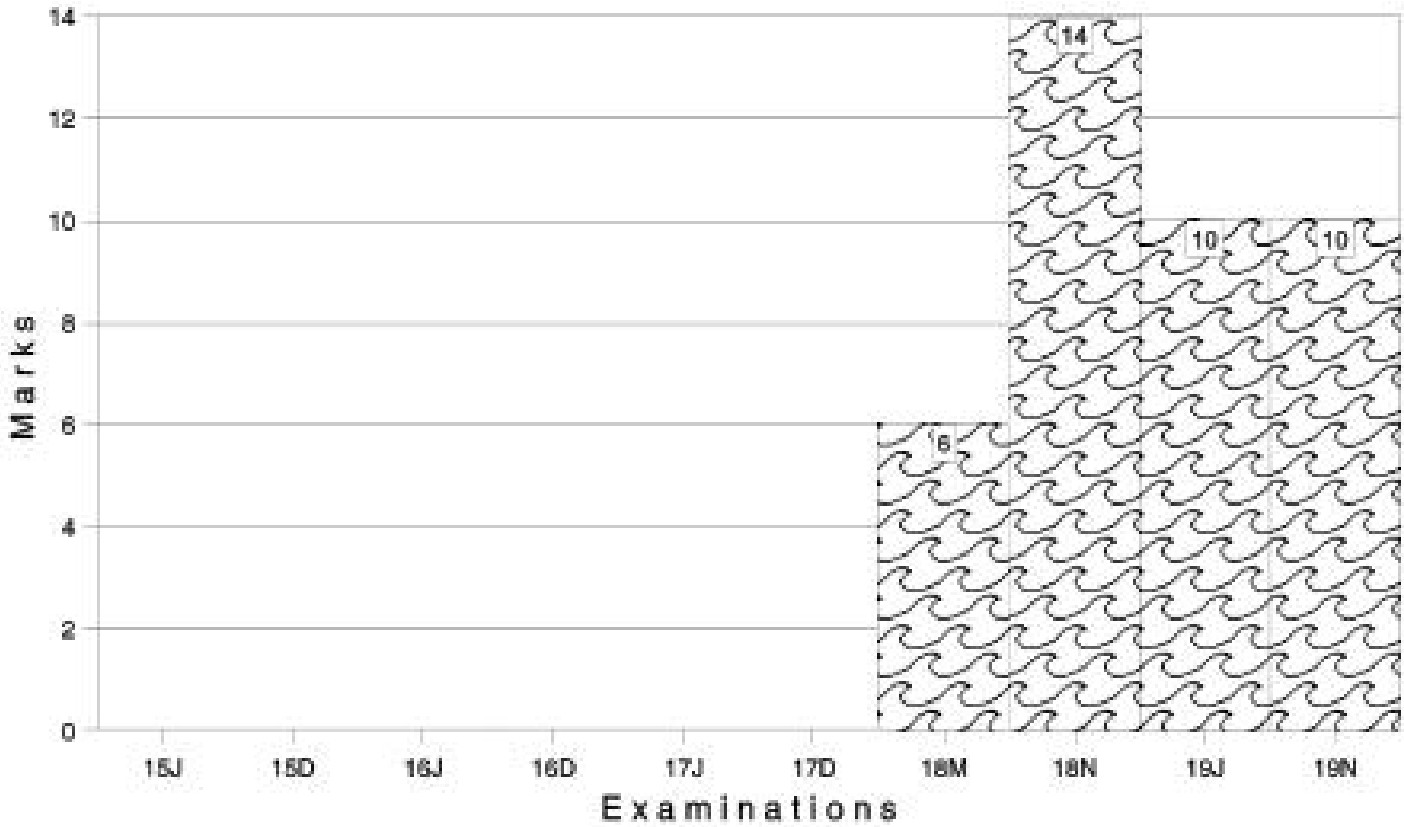
4

TIME VALUE OF MONEY

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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PAST YEAR QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] ₹ 8,000 becomes ₹ 10,000 in two years at simple interest. The amount that will become ₹ 6,875 in 3 years at the same rate of interest is :

(a) ₹ 4,850 (b) ₹ 5,000

(c) ₹ 5,500 (d) ₹ 5,275 (1 mark)

Answer:

(b) $A = P\left[1 + \frac{rt}{100}\right]$

$10,000 = 8,000\left[1 + \frac{r \times 2}{100}\right]$

$\frac{10,000}{8,000} = \frac{100 + 2r}{100}$

$100 + 2r = \frac{10 \times 100}{8}$

$2r = 125 - 100$

$r = \frac{25}{2} = 12.5\%$ p.a.

Let the amount which will become ₹ 6,875 be P. Then,

$6,875 = P\left(\quad\right)$

[2] The difference between the simple and compound interest on a certain sum for 3 year at 5% p.a. is ₹ 228.75. The compound interest on the sum for 2 years at 5% p.a. is :

(a) ₹ 3,175 (b) ₹ 3,075
(c) ₹ 3,275 (d) ₹ 2,975. (1 mark)

Answer:

(b) Let the sum be ₹ P.

S.I = PRT/100

$$S.I. = \frac{P \times 5 \times 3}{100} = \frac{15P}{100} = 0.15P$$

$$C.I. = P[(1+i)^n - 1]$$
$$= P[(1+0.05)^3 - 1]$$
$$= 0.157625$$

Diff. between C.I. and S.I. = ₹ 228.75

$0.157625 - 0.15\,P = 228.75$

$0.007625p = 228.75$

$$P = \frac{228.75}{0.007625} = ₹\ 30,000$$

C.I. on ₹ 30,000 @ 5% p.a for 2 years.

$$C.I. = 30,000[(1+0.05)^2 - 1]$$
$$= ₹\ 3,075$$

[3] Mr. X Invests ₹ 10,000 every year starting from today for next 10 years suppose interest rate is 8% per annum compounded annually. Calculate future value of the annuity:

(Given that $(1 + 0.08)^{10} = 2.15892500$]

(a) ₹ 1,56,454.88 (b) ₹ 1,44,865.625
(c) ₹ 1,56,554.88 (d) None of these (1 mark)

Answer:

(a) Future value of annuity due

$$= \frac{a(1+i)}{i}[(1+i)^n - 1]$$

$$= \frac{10,000}{0.08}(1+0.08)[(1+0.08)^{10} - 1]$$

= 1,35,000 [1.15892500]
= ₹ 1,56,454.875
= ₹ 1,56,454.88 (approx.)

[4] The present value of an annuity of ₹ 3,000 for 15 years at 4.5% p.a. C.I. is :

[Given that $(1.045)^{15} = 1.935282$]

(a) ₹ 23,809.67 (b) ₹ 32,218.67
(c) ₹ 32,908.67 (d) None of these (1 mark)

Answer:

(b) $P.V. = \frac{a}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right]$

$= \frac{3000}{0.045}\left[\frac{(1+0.045)^{15} - 1}{(1+0.045)^{15}}\right]$

= ₹ 32,218.67 (approx.)

2007 - FEBRUARY

[5] The rate of simple interest on a sum of money is 6% p.a. for first 3 years, 8% p.a. for the next five years and 10% p.a. for the period beyond 8 years. If the simple interest accrued by the sum for a period for 10 years is ₹ 1,560. The sum is :

(a) ₹ 1,500 (b) ₹ 2,000
(c) ₹ 3,000 (d) ₹ 5,000 (1 mark)

Answer:

(b) Let the sum invested be ₹ P. Then,

S.I = ₹1,560

$\frac{P \times 6 \times 3}{100} + \frac{P \times 8 \times 5}{100} + \frac{P \times 10 \times 2}{100} = 1,560$

$P\left(\frac{18}{100} + \frac{40}{100} + \frac{20}{100}\right) = 1,560$

$$\frac{78P}{100} = 1,560$$

$$P = \frac{1560 \times 100}{78} = ₹\ 2,000$$

[6] A sum of money doubles itself in 10 years. The number of years it would treble itself is :

(a) 25 years (b) 15 years
(c) 20 years (d) None. (1 mark)

Answer:

(c) Let the sum of money invested be ₹ P.

Then, Amount = ₹ 2P.

$$A = P\left[1 + \frac{rt}{100}\right]$$

$$2P = P\left[1 + \frac{r \times 10}{100}\right]$$

$$2 = \frac{100 + 10r}{100}$$

$10r = 100$

$r = 10\%$ p.a.

New amount = 3p.

$$3P = P\left[1 + \frac{10 \times t}{100}\right]$$

$$3 = \frac{100 + 10t}{100}$$

$10t = 200$

$t = 20$ years.

[7] In what time will ₹ 3,90,625 amount to ₹ 4,56,976 at 8% per annum, when the interest is compounded semi-annually?

[Given : $(1.04)^4 = 1.16986$]

(a) 2 years (b) 4 years
(c) 5 years (d) 7 years (1 mark)

Answer:

(a) P = ₹ 3,90,625

A = ₹ 4,56,976

$i = \frac{8}{100} \times \frac{1}{2} = 0.04$

time = n

$A = P(1+i)^n$

$4,56,976 = 3,90,625\ (1+0.04)^n$

$\frac{456976}{390625} = (1.04)^n$

$1.16986 = (1.04)^n$

$(1.04)^4 = (1.04)^n$

$n = 4$

time = $\frac{n}{2} = \frac{4}{2}$ = 2 years.

[8] A machine can be purchased for ₹ 50,000. Machine will contribute ₹ 12,000 per year for the next five years. Assume borrowing cost is 10% per annum. Determine whether machine should be purchased or not :

(a) Should be purchased

(b) Should not be purchased

(c) Can't say about purchase

(d) None of the above (1 mark)

Answer:

(b) Present value (P.V.) of outflow of machines - ₹ 50,000

Prevent value of inflow of machine

$= \frac{A}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right]$

$= \frac{12000}{0.10}\left[\frac{(1+0.10)^5 - 1}{(1+0.10)^5}\right]$

= ₹ 45,489.44 (approx)

Since, P.V. of Inflows is less as compared to P.V. of outflow, hence machine should not be purchased.

[9] How much amount is required to be invested every year so as to accumulate ₹ 3,00,000 at the end of 10 years, if interest is compounded annually at 10%?

[Give $(1.1)^{10} = 2.5937$]

(a) ₹ 18,823.65 (b) ₹ 18,828.65
(c) ₹ 18,832.65 (d) ₹ 18,882.65 (1 mark)

Answer:

(a) F.Y. = ₹ 3,00,000

Let ₹ a is invested annually.

$$F.V. = \frac{a}{i}[(1+i)^n - 1]$$

$$3,00,000 = \frac{a}{0.10}[(1+0.10)^{10} - 1]$$

$$30,000 = a\,[2.5937 - 1]$$

$$30,000 = 1.5937a$$

$$a = \frac{30,000}{1.5937}$$

a = ₹ 18,823.65 (approx.)

2007 - MAY

[10] A certain sum of money amounts to ₹ 6,300 in two years and ₹ 7,875 in three years nine months at simple interest. Find the rate of interest per annum :

(a) 20% (b) 18%
(c) 15% (d) 10% (1 mark)

Answer:

(a) Let the invested sum be ₹ P.

$$\text{Amount} = P\left[1 + \frac{rt}{100}\right]$$

$$6,300 = P\left[1 + \frac{r \times 2}{100}\right]$$

$$6,300 = P\left(\frac{100 + 2r}{100}\right) \quad \text{.....(1)}$$

$$7,875 = P\left[1 + \frac{3.75 \times r}{100}\right]$$

$$7,875 = P\left(\frac{100 + 3.75r}{100}\right) \quad \text{..... (2)}$$

Dividing (2) by (1)

$$\frac{7,875}{6,300} = \frac{P\left(\frac{100 + 3.75r}{100}\right)}{P\left(\frac{100 + 2r}{100}\right)}$$

$$1.25 = P\ \frac{100 + 3.75r}{100 + 2r}$$

$125 + 2.5r = 100 + 3.75r$

$3.75r - 2.5 = 125 - 100$

$1.25\ r = 25$

$$r = \frac{25}{1.25} = 20\% \text{ p.a.}$$

[11] How long will ₹ 12,000 take to amount to ₹ 14,000 at 5% p.a. converted quarterly ?

[Given : $(1.0125)^{12.4} = 1.1666$]

(a) 3 years (b) 3.1 years

(c) 13.5 years (d) 12.4 years. (1 mark)

Answer:

(b) P = ₹ 12,000

A = ₹ 14,000

$$i = \frac{5}{4}\% = 1.25\% = 0.0125$$

No. of conversion periods = n

$A = P\ (1 + i)^n$

$14,000 = 12,000\ (1 + 0.0125)^n$

$$\frac{14,000}{12,000} = (1.0125)^n$$

$$1.1666 = (1.0125)^n$$

$$(1.0125)^{12.4} = (1.0125)^n \text{ (given)}$$

$$\therefore n = 12.4$$

$$\text{Time} = \frac{n}{4} = \frac{12.4}{4} = 3.1 \text{ years}$$

[12] A company is considering proposal of purchasing a machine either by making full payment of ₹ 4,000 or by leasing it for four years at an annual rate of ₹ 1,250. Which course of action is preferable, if the company can borrow money at 14% compounded annually?

[Given : $(1.14)^4 = 1.68896$]

(a) Leasing is preferable (b) Should be purchased
(c) No difference (d) None of these (1 mark)

Answer:

(a) P.V. of outflows i.e. annuity paid $= \frac{a}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right]$

$$= \frac{1250}{0.14}\left[\frac{(1+0.14)^4 - 1}{(1+0.14)^4}\right]$$

$$= 8928.57 \times 0.4079$$

$$= ₹\ 3642.14 \text{ (approx)}$$

Purchase price of machine = ₹ 4,000 Since, purchase price of machine is more than the P.V. of outflows of annuity, hence, leasing is preferable.

[13] Vipul purchases a car for ₹ 5,50,000. He gets a loan of ₹ 5,00,000 at 15% p.a. from a Bank and balance ₹ 50,000 he pays at the time of purchase. He has to pay the whole amount of loan in 12 equal monthly instalments with interest starting from the end of the first month. The money he has to pay at the end of every month is :

[Given $(1.0125)^{12} = 1.16075452$]

(a) ₹ 45,130.43 (b) ₹ 45,230.43
(c) ₹ 45,330.43 (d) None of these (1 mark)

Answer:

(a) Loan amount (i.e. P.V.) = ₹ 5,00,000.

$$i = \frac{15}{100} \times \frac{1}{12} = 0.0125$$

$n = 1 \text{ year} = 1 \times 12 = 12$

P.V. = 5,00,000

$$\frac{a}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right] = 5,00,000$$

$$\frac{a}{0.0125}\left[\frac{(1+0.0125)^{1}2 - 1}{(1+0.0125)^{1}2}\right] = 5,00,000$$

$$\frac{a}{0.0125}(0.13849) = 5,00,000$$

$$a = \frac{5,00,000 \times 0.0125}{0.13849}$$

a = ₹ 45,130.43 (approx.)

2007 - AUGUST

[14] If ₹ 1,000 be invested at interest rate of 5% and the interest be added to the principal every 10 years, then the number of years in which it will amount to ₹ 2,000 is :

(a) $16\frac{2}{3}$ years (b) $6\frac{1}{4}$ years

(c) 16 years (d) $6\frac{2}{3}$ years. (1 mark)

Answer:

(a) $\text{Amount} = P\left[1 + \frac{rt}{100}\right]$

$$\text{Amount after 10 years} = 1,000\left[1 + \frac{5 \times 10}{100}\right] = ₹\ 1,500$$

Let the time be t years when ₹1,500 amount to ₹ 2,000

$$2,000 = 1,500\left[1 + \frac{5 \times t}{100}\right]$$

$$\frac{2,000}{1,500} = \left(\frac{100 + 5t}{100}\right)$$

$$1.33 = \left(\frac{100 + 5t}{100}\right)$$

$$100 + 5t = 1.33 \times 100$$

$$5t = 133 - 100$$

$$t = \frac{33}{5} = 6.67 \text{ years}$$

$$= 6\frac{2}{3} \text{ years.}$$

Therefore, total time taken $= \left[10 + \frac{62}{3}\right]$ years.

$$= 16\ \frac{2}{3} \text{ years}$$

[15] The annual birth and death rates per 1000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is :

(a) 35 years (b) 30 years

(c) 25 years (d) None of thee (1 mark)

Answer:

(a) Annual growth rate = Annual birth rate – Annual death rate

= 39.4 – 19.4

= 20 per 1000

$\therefore$ Rate $= \frac{20}{1000} \times 100 = 2\%$

Let the original population be P.

Therefore, $A = P(1+ i)^n$

$2P = P(1+ 0.02)^n$

$2 = (1.02)^n$

$(1.02)^{35} = (1.02)^n$

$n = 35$ years

[16] The effective rate equivalent to nominal rate of 6% compounded monthly is :

(a) 6.05 (b) 6.16

(c) 6.26 (d) 6.07 (1 mark)

Answer:

(b) $i = 6\%$ p.a. $= \frac{6}{100} \times \frac{1}{12} = 0.005$

$n = 1 \text{ year} = 1 \times 12 = 12$ converted period

$E = (1+i)^n - 1$

$= (1 + 0.005)^{12} - 1$

$= 0.0616$ or 6.16%

[17] A company establishes a sinking fund to provide for the payment of ₹ 2,00,000 debt maturing in 20 years. Contributions to the fund are to be made at the end of every year. Find the amount of each annual deposit if interest is 5% per annum :

(a) ₹ 6,142 (b) ₹ 6,049

(c) ₹ 6,052 (d) ₹ 6,159 (1 mark)

Answer:

(b) Let the annual deposit be ₹ a.

$$F.Y. = \frac{a}{i}[(1+i)^n - 1]$$

$$2,00,000 = \frac{a}{0.05}[(1+0.05)^{20} - 1]$$

$$10,000 = a\,(1.6533)$$

$$a = \frac{10,000}{1.6533}$$

$a =$ ₹ 6,049 (approx)

2007 - NOVEMBER

[18] A person borrows ₹ 5,000 for 2 years at 4% p.a. simple interest . He immediately lends to another person $6\frac{1}{4}$% p.a. simple interest for 2 years. Find his gain in the transaction per year :

(a) ₹ 112.50 (b) ₹ 125

(c) ₹ 225 (d) ₹ 167.50 (1 mark)

Answer:

(a) S.I. paid on money borrowed

$$= \frac{Prt}{100} = \frac{5000 \times 2 \times 4}{100}$$

= ₹ 400

S.I. received on money lended

$$= \frac{Prt}{100} = \frac{5000 \times 25 \times 2}{4 \times 100}$$

= ₹ 6.25

Total Gain in the transaction for 2 years.

= S.I. received – S.I. Paid

= ₹ 625 – ₹ 400

= ₹ 225.

Gain per year = $\frac{225}{2}$ = ₹ 112.50

[19] A person deposited ₹ 5,000 in a bank. The deposit was left to accumulate at 6% compounded quarterly for the first five years and at 8% compounded semi-annually for the next eight years. The compound amount at the end of 13 years is :

(a) ₹ 12621.50 (b) ₹ 12613.10

(c) ₹ 13613.10 (d) None. (1 mark)

Answer:

(b) Amount accumulated in first 5 years :

$P = ₹ 5,000$

$i = \frac{6}{4 \times 100} = 0.015$

$n = 5 \times 4 = 20$

$A = P(1+i)^n$

$= 5,000 (1+0.015)^{20}$

$= 5,000 \times 1.3469$

$= ₹ 6,734.28$

Amount accumulated after 13 years.

$P = ₹ 6,734.28$

$i = \frac{8}{2 \times 100} = 0.04$

$n = 8 \times 2 = 16$

$A = P(1+i)^n$

$= 6734.28 (1 + 0.04)^{16}$

$= 6,734.28 \times 1.873$

$= ₹ 12,613.10$ (approx.)

[20] Raja aged 40 wishes his wife Rani to have ₹ 40 lakhs at his death. If his expectation of life is another 30 years and he starts making equal annual investments commencing now at 3% compound interest p.a. How much should he invest annually?

(a) ₹ 84,077 (b) ₹ 81,628

(c) ₹ 84,449 (d) ₹ 84,247 (1 mark)

Answer:

(a) Future Value = ₹ 40 Lakhs. [∵ mentioned in ques. commencing from now so ques. of annuity regular]

$n = 30$ years

$i = 0.03$

Future value of annuity regular

$$= \frac{A}{i}[(1 + i)^n - 1]$$

$$\Rightarrow 40,00,000 = \frac{A}{0.03}[(1+ 0.03)^{30} - 1]$$

$$\Rightarrow A = ₹ 84,077 \text{ (approx).}$$

2008 - FEBRUARY

[21] Two equal sums of money were lent at simple interest at 11% p.a. for $3\frac{1}{2}$ years and $4\frac{1}{2}$ years respectively.

If the difference in interests for two periods was ₹ 412.50, then each sum is :

(a) ₹ 3,250 (b) ₹ 3,500

(c) ₹ 3,750 (d) ₹ 4,350 (1 mark)

Answer:

(c) Let the, sum invested be ₹ P.

$$\text{S.I.} = \frac{Prt}{100}$$

$$\left(\frac{P \times 11 \times 9}{100 \times 2}\right) - \left(\frac{P \times 11 \times 7}{100 \times 2}\right) = 412.50$$

$$P\left[\frac{99}{200} - \frac{77}{200}\right] = 412.50$$

$$\frac{22P}{200} = 412.50$$

$$P = \frac{412.50 \times 200}{22}$$

$$P = ₹ 3,750$$

[22] Anshul's father wishes to have ₹ 75,000 in a bank account when his first college expenses begin. How much amount his father should deposit now at 6.5% compounded annually if Anshul is to start college in 8 years hence from now ?

(a) ₹ 45,360 (b) ₹ 46,360
(c) ₹ 55,360 (d) ₹ 48,360. (1 mark)

Answer:

(a) Amount = ₹ 75,000

$P(1+i)^n = 75,000$

$P(1 + 0.065)^8 = 75,000$

$P = \frac{75000}{1.655}$

= ₹ 45,317 (approx.)

as per the options given the nearest option is ₹ 45,360.

[23] A company may obtain a machine either by leasing it for 5 years (useful life) at an annual rent of ₹ 2,000 or by purchasing the machine for ₹ 8,100. If the company can borrow money at 18% per annum, which alternative is preferable ?

(a) Leasing (b) Purchasing
(c) Can't say (d) None of these (1 mark)

Answer:

(a) Purchase cost of machine at present = ₹ 8,100

Present value of the lease rental $= \frac{a}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right]$

$= \frac{2000}{0.18}\left[\frac{(1+0.18)^5 - 1}{(1+0.018)^5}\right]$

= 11,111 × 0.5629

= ₹ 6,254.34 (approx)

Which is lesser than the initial cost of the asset and consequently leasing is favourable to the lessor.

2008 - JUNE

[24] In how much time would the simple interest on a certain sum be 0.125 times the principal at 10% per annum?

(a) $1\frac{1}{4}$ years (b) $1\frac{3}{4}$ years

(c) $2\frac{1}{4}$ years (d) $2\frac{3}{4}$ years (1 mark)

Answer:

(a) Let the principal be ₹ P.

$$\text{S.I.} = \frac{Prt}{100}$$

$$0.125p = \frac{P \times 10 \times t}{100}$$

$$0.125 = \frac{10t}{100}$$

$$t = \frac{0.125 \times 100}{10}$$

$$t = 1.25 = 1\frac{1}{4} \text{ years.}$$

[25] The difference between compound interest and simple interest on a certain sum for 2 years @ 10% p.a. is ₹ 10. Find the sum :

(a) ₹ 1,010 (b) ₹ 1,095

(c) ₹ 1,000 (d) ₹ 990 (1 mark)

Answer:

(c) Let the sum be ₹ P.

C.I. – S.I. = ₹ 10

$$P[(1+i)^n - 1] - \frac{Prt}{100} = 10$$

$$P[(1+0.10)^2 - 1] - \frac{P \times 10 \times 2}{100} = 10$$

$$0.21P - 0.2P = 10$$

$0.01P = 10$

$P = \frac{10}{0.01}$

$P = ₹\ 1,000$

[26] A machine worth ₹ 4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹ 2,00,000 :

(a) 5 years 6 months (b) 5 years 7 months

(c) 5 years 5 months (d) None. (1 mark)

Answer:

(a) Amount = 2,00,000

In case of depreciation

$A = P(1 - i)^n$

$2,00,000 - 4,90,740\ (1 - 0.15)^n$

$0.4075 = (0.85)^n$

$(0.85)^{5.5} = (0.85)^n$

$n = 5.5$ or 5 years 6 months (approx).

[27] A sinking fund is created for redeeming debentures worth ₹ 5 lacs at the end of 25 years. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a.?

(a) ₹ 12,006 (b) ₹ 12,040

(c) ₹ 12,039 (d) ₹ 12,035 (1 mark)

Answer:

(a) Let the annual investment be ₹ a

Then, Amount $= \frac{a}{i}[(1+i)^n - 1]$

$5,00,000 = \frac{a}{0.04}[(170.04)^{25} - 1]$

$20,000 = a \times 1.6658$

$a = \frac{20,000}{1.6658}$

$a = ₹\ 12,006$ (approx)

2008 - DECEMBER

[28] If the difference between simple interest and compound interest is ₹ 11 at the rate of 10% for two years, then find the sum.

(a) ₹ 1,200 (b) ₹ 1,100

(c) ₹ 1,000 (d) None of these (1 mark)

Answer:

(b) Difference between S.I. and C.I. = ₹ 11

Now, Rate = 10%, i = 0.1, t = 2

$P[(1+i)^n - 1] - P \times i \times t = 11$

$P[(1+0.1)^2 - 1 - 0.1 \times 2] = 11$

$0.01P = 11$

$\therefore P = ₹ 1,100$

[29] Future value of an ordinary annuity:

(a) $A(n, i) = A\left[\frac{(1+i)^n - 1}{i}\right]$ (b) $A(n, i) = A\left[\frac{(1+i)^n + 1}{i}\right]$

(c) $A(n, i) = A\left[\frac{1-(1+i)^n}{i}\right]$ (d) $A(n, i) = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$ (1 mark)

Answer:

(a) Future value of an ordinary annuity is given by:

$$A(n,i) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

[30] Find the numbers of years in which a sum doubles itself at the rate of 8% per annum.

(a) $11\frac{1}{2}$ (b) $12\frac{1}{2}$

(c) $9\frac{1}{2}$ (d) $13\frac{1}{2}$ (1 mark)

Answer:

(b) Let the principal be ₹ x

$\therefore 2x = x [1+ it]$ ($\because$ Amount = 2x)

$2x = x [1+0.08t]$

$\Rightarrow t = 12\frac{1}{2}$ years.

2009 - JUNE

[31] In how many years, a sum will become double at 5% p.a. compound interest.

(a) 14.0 years (b) 15 years

(c) 16 years (d) 14.3 years (1 mark)

Answer:

(d) Let the principal amount be ₹ P.

then, Amount = ₹ 2P.

Rate of interest = 5 %

$\therefore i = 0.05$

Number of years = n

Amount = Principal $(1 + i)^n$

$2 P = P (1 + i)^n$

$2 P = P (1 + 0.05)^n$

$2 = (1.05)^n$

$(105)^{143} = (105)^n$

$\therefore n = 14.3$

So, the number of years = 14.3 years in which the sum will double itself at 5% p.a. C. I.

[32] The time by which a sum of money is 8 times of itself if it doubles itself in 15 years interest compounded annually.

(a) 42 years (b) 43 years

(c) 45 years (d) 46 years (1 mark)

Answer:

(c) Let the principal amount be ₹ P.

Rate of interest = r % p.a.= *i*

According to given,

Amount = Principal $(1 + i)^n$

$2P = P\ (1 + i)^n$

$2P = P\ (1 + i)^{15}$

$2 = (1 + i)^{15}$

$(1.0473)^{15} = (1 + i)^{15}$

$\therefore 01 + i = 1.0473$

$i = 0.0473$

Now, $8P = P\ (1 + i)^n$

$8 = (1.0473)^n$

$(1.0473)^{45} = (1.0473)^n$

$\therefore 45 = n$

So, a sum of money will be 8 times of itself in 45 years.

[33] What is the rate of simple interest if a sum of money amounts to ₹ 2,784 in 4 years and ₹ 2,688 in 3 years ?

(a) 1% p.a. (b) 4% p.a.

(c) 5% p.a. (d) 8% p.a. (1 mark)

Answer:

(b) $S.I. = \frac{P \times R \times T}{100}$

$A = P + S.I$

$A = P + \frac{P \times R \times T}{100}$

$A = P\left(1 + \frac{RT}{100}\right)$

$A = P\left(1 + \frac{4R}{100}\right)$

$\therefore 2,784 = P\left(1 + \frac{4R}{100}\right)$

$2,78,400 = 100\ P + 4\ PR$ .....(1)

and, $2,688 = P\left(1 + \frac{3R}{100}\right)$

$2,68,800 = 100\ P + 3\ PR$ .....(2)

subtracting (2) from (1), we get

$2,78,400 = 100\ P + 4\ P\ R$

$2,68,800 = 100\ P + 3\ PR$

(-) (-) (-)

$9,600 = PR$

Substituting $PR = 9,600$ in (1)

$2,78,400 = 100\ P + 4 \times 9,600$

$2,78,400 = 100\ P + 38,400$

$2,78,400 - 38400 = 100\ P$

$2,40,000 = 100\ P$

$P = ₹\ 2400$

Now, $PR = 9600$ & $P = ₹\ 2400$

$2400\ R = 9600$

$R = \frac{96,000}{2,400} = 4\ \%$ p.a.

[34] A sum amount to ₹ 1,331 at a principal of ₹ 1,000 at 10 % compounded annually. Find the time.

(a) 3.31 years (b) 4 years

(c) 3 years (d) 2 years (1 mark)

Answer:

(c) $P = ₹\ 1,000$

$A = ₹\ 1,331$

$i = 0.10$

Time = n years

$A = P\ (1 + i)^n$

$1331 = 1000\ (1 + 010)^n$

$1.331 = (1.10)^n$

$(1.10)^3 = (1.10)^n$

$\therefore\ n = 3$

[35] Paul borrows ₹ 20,000 on condition to repay it with compound interest at 5% p.a. in annual instalment of ₹ 2,000 each. Find the number of years in which the debt would be paid off.

(a) 10 years (b) 12 years

(c) 14 years (d) 15 years (1 mark)

Answer:

(d) Present Value (P. V.) = ₹ 20,000

Annuity (A) = ₹ 2,000

r = 5 % p.a. or i = 0.05

Number of years = n

$$P.V. = \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$20,000 = \frac{2,000}{0.05}\left[1 - \frac{1}{(1+0.05)n}\right]$$

$$\frac{20,000 \times 0.05}{2,000} = \left[1 - \frac{1}{(1.05)^n}\right]$$

$$0.5 = 1 - \frac{1}{(1.05)^n}$$

$$\frac{1}{(1.05)^n} = 1 - 0.5$$

$$\frac{1}{(1.05)^n} = 0.5$$

$$(1.05)^n = \frac{1}{0.5}$$

$$(1.05)^n = 2$$

n = 15 years (approx.)

[NOTE : Students may get the value of n by substituting the value of n from the options. The option giving the most nearest answer will be taken as the correct answer].

2009 - DECEMBER

[36] In how many years, a sum of ₹ 1,000 compounded annually @ 10%, will amount to ₹ 1,331?

(a) 6 years (b) 5 years
(c) 4 years (d) 3 years (1 mark)

Answer:

(d) A = ₹1,331
P = ₹ 1,000
i = 0.10
n = ?
$A = P(1+i)^n$
$1331 = 1000(1 + 0.10)^n$
$\frac{1,331}{1,000} = (1.10)^n$
$1.331 = (1.1)^n$
$(1.1)^3 = (1.1)^n$
Therefore, n = 3 years

[37] The compound interest for a certain sum @ 5% p.a. for first year is ₹ 25. The S-I for the same money @ 5% p.a. for 2 years will be.

(a) ₹ 40 (b) ₹ 50
(c) ₹ 60 (d) ₹ 70 (1 mark)

Answer:

(b) C.1 = ₹25
i = 0.05
n = 1
P = ?
$C.1 = P[(1+i)^n - 1]$
$25 = P[(1 + 0.05)^1 - 1]$
$25 = P[(1.05)^1 - 1]$
$25 = P[0.05]$

$$P = \frac{25}{0.05}$$

$\therefore$ P = ₹ 500.

Now, P = ₹ 500

r = 5%

t = 2 years

S.1 = ?

$$S.1 = \frac{P \times r \times t}{100}$$

$$S.1 = 500 \times \frac{5}{100} \times 2$$

$\therefore$ S.1 = ₹ 50.

2010 - JUNE

[38] At what % rate of compound interest (C.I) will a sum of money become 16 times in four years, if interest is being calculated compounding annually:

(a) r = 100% (b) r = 10%

(c) r = 200% (d) r = 20% (1 mark)

Answer:

(a) Let the Principal be P

$\therefore$ Amount = 16P

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$16P = P\left(1 + \frac{R}{100}\right)^4$$

$$16 = \left(1 + \frac{R}{100}\right)^4 = 2^4$$

$$\therefore \quad 1 + \frac{R}{100} = 2$$

$$\frac{R}{100} = 1$$

$$R = 100\%$$

[39] Find the present value of an annuity of ₹ 1,000 payable at the end of each year for 10 years. If rate of interest is 6% compounding per annum (given $(1.06)^{-10} = 0.5584$) :

(a) ₹ 7,360 (b) ₹ 8,360

(c) ₹ 12,000 (d) None of these. (1 mark)

Answer:

(a) $V = \frac{A}{r}[1 - (1 + i)^{-n}]$

$= \frac{1,000}{0.06}[1 - (1.06)^{-10}]$

$= \frac{1000}{0.06}[1 - .05584]$

= ₹ 7,360

[40] If the simple interest on a sum of money at 12% p.a. for two years is ₹ 3,600. The compound interest on the same sum for two years at the same rate is :

(a) ₹ 3,816 (b) ₹ 3,806

(c) ₹ 3,861 (d) ₹ 3,860 (1 mark)

Answer:

(a) $SI = \frac{PRT}{100}$

$3,600 = \frac{P \times 12 \times 2}{100}$

P = ₹ 15,000

$$= \left[\left(1 + \frac{R}{100}\right)^n - 1\right]$$

$$= 15{,}000\left[\left(1 + \frac{12}{100}\right)^2 - 1\right]$$

$$= ₹\ 3{,}816$$

2010 - DECEMBER

[41] The future value of an annuity of ₹ 5,000 is made annually for 8 years at interest rate of 9% compounded annually [Given that $(1.09)^8 = 1.99256$] is_____

(a) ₹ 55,142.22 (b) ₹ 65,142.22

(c) ₹ 65,532.22 (d) ₹ 57,425.22 (1 mark)

Answer:

(a) $F.V. = P\left[\frac{(1+i)^n - 1}{i}\right]$

$P = 5{,}000$ $i = 9\%$

$n = 8$

$F.V. = 5{,}000\left[\frac{(1 + 9/100)^8 - 1}{0.09}\right]$

$F.V. = 5{,}000\left[\frac{1.99256 - 1}{0.09}\right]$

$F.V. = 55{,}142.22$

[42] The effective annual rate of interest corresponding to nominal rate 6% p.a. payable half yearly is

(a) 6.06% (b) 6.07%

(c) 6.08% (d) 6.09% (1 mark)

Answer:

(d) Nominal rate = 6% p.a.

Effective rate = $(1 + i)^n - 1$

$i = 3\%$

$n = 2$

Effective rate $= \left(1 + \frac{3}{100}\right)^2 - 1$

$= 0.0609$

$= 6.09\%$

[43] The cost of Machinery is ₹1,25,000/- If its useful life is estimated to be 20 years and the rate of depreciation of its cost is 10% p.a., then the scrap value of the Machinery is

[given that $(0.9)^{20} = 0.1215$]

(a) ₹ 15,187 (b) ₹ 15,400

(c) ₹ 15,300 (d) ₹ 15,250 (1 mark)

Answer:

(a) Scrap value $(v) = P(1 - i)^n$

$P = 1,25,000$

$i = 10\%$

$n = 20$

$V = 1,25,000 \left[1 - \frac{10}{100}\right]^{20}$

$= 1,25,000 \times 0.1215$

$V = 15,187$

[44] Mr. X invests 'P' amount at Simple Interest rate 10% and Mr. Y invests 'Q' amount at Compound Interest rate 5% compounded annually. At the end of two years both get the same amount of interest, then the relation between two amounts P and Q is given by :

(a) $P = \frac{41Q}{80}$ (b) $P = \frac{41Q}{40}$

(c) $P = \frac{41Q}{100}$ (d) $P = \frac{41Q}{200}$ (1 mark)

Answer:

(a) $S1 = P \times \frac{10}{100} \times 2 = \frac{P}{5}$

$C1 = Q[(1 + 5/100)^2 - 1] = 0.1025Q$

$\frac{P}{5} = 0.1025Q$

$$\frac{P}{5} = \frac{1,025}{10,000}Q$$

$$\frac{P}{5} = \frac{41}{400}Q$$

$$P = \frac{41}{80}Q$$

2011 - JUNE

[45] If the difference of S.I and C.I is ₹ 72 at 12% for 2 years. Calculate the amount.

(a) ₹ 8,000 (b) ₹ 6,000

(c) ₹ 5,000 (d) ₹ 7,750 (1 mark)

Answer:

(c) Let the Principal be Rs 'x'

Given : Rate (R) = 12% p.a.

Time (T) = 2 years

∴ Simple Interest (S.I.) for 2 years :

$$S.I = \frac{P \times R \times T}{100} = \frac{x \times 12 \times 2}{100}$$

$$\therefore \quad S.I = \frac{24x}{100}$$

And, Compound Interest (C.I) for 2 years :

$$C.I = P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$$

$$\therefore \quad C.I = x\left[\left(1 + \frac{12}{100}\right)^2 - 1\right]$$

$$C.I = \frac{159x}{625}$$

by Question : Difference of C.I & S.I is ₹ 72

$$\therefore \quad \frac{159x}{625} - \frac{24x}{100} = 72$$

On Solving, we get x = 5,000

[46] If a simple interest on a sum of money at 6% p.a. for 7 years is equal to twice of simple interest on another sum for 9 years at 5% p.a.. The ratio will be :

(a) 2 : 15 (b) 7 : 15
(c) 15 : 7 (d) 1 : 7 (1 mark)

Answer:

(c) Let the Principals be 'x' and 'y'

Case : I $(S.I)_1 = \frac{x \times 6 \times 7}{100}$

Case : II $(S.I)_2 = \frac{y \times 5 \times 9}{100}$

Given :

$(S.I)_1 = 2 \times (S.I)_2$

$\frac{x \times 6 \times 7}{100} = \frac{2 \times y \times 5 \times 9}{100}$

$\Rightarrow \frac{x}{y} = \frac{15}{7}$

$\therefore x : y = 15 : 7$

[47] By mistake a clerk, calculated the simple interest on principal for 5 months at 6.5% p.a. instead of 6 months at 5.5% p.a. If the error in calculation was ₹ 25.40. The original sum of principal was_____.

(a) ₹ 60,690 (b) ₹ 60,960
(c) ₹ 90,660 (d) ₹ 90,690 (1 mark)

Answer:

(b) Let the Principal be Rs 'x'

Incorrect S.I $= \frac{x \times 6.5 \times 5}{100 \times 12} = \frac{32.5x}{1,200}$

Correct S.I $= \frac{x \times 5.5 \times 6}{100 \times 12} = \frac{33x}{1,200}$

$\therefore$ Error in calculation = ₹ 25.40

Correct S.I – Incorrect S.I = ₹ 25.40

$\frac{33x}{1,200} - \frac{32.5x}{1,200} = 25.40$

x = ₹ 60,960

2011 - DECEMBER

[48] If the Simple Interest on ₹ 1,400 for 3 years is less than the simple interest on ₹ 1,800 for the same period by ₹ 80, then the rate of interest is

(a) 5.67% (b) 6.67%
(c) 7.20% (d) 5.00% (1 mark)

Answer:

(b) CASE 1 Principal (P) = ₹ 1,400
(T) = 3 yrs
Let rate (R) = x %

$$(S.I)_1 = \frac{PRT}{100} = \frac{1,400 \times x \times 3}{100} = 42x$$

CASE - 2 Principal (P) = ₹ 1,800
Let rate (R) = x%
(T) = 3 yrs

$$(S.I)_2 = \frac{PRT}{100} = \frac{1,800 \times x \times 3}{100} = 54x$$

Given

$(S.I)_2 - (S.I)_1 = 80$

$54x - 42x = 80$

$12x = 80$

$x = \frac{80}{12}$

$x = 6.67\%$

Rate (R) = 6.67%

[49] Nominal rate of interest is 9.9% p.a. If interest is Compounded monthly, What will be the effective rate of interest

(given $\left(\frac{4033}{4000}\right)^{12} = 1.1036$ (approx))?

(a) 10.36% (b) 9.36%
(c) 11.36% (d) 9.9% (1 mark)

Answer:

(a) R = 9.9% p.a.

If Interest is paid Compounded Monthly

$$i = \frac{9.9}{12 \times 100}$$

$$i = \frac{99}{12 \times 100 \times 10} = \frac{33}{4,000}$$

And n = 12

$$\text{Effective Rate} = (1 + i)^n - 1$$

$$= \left(1 + \frac{33}{4,000}\right)^{12} - 1$$

$$= \left(\frac{4,033}{4,000}\right)^{12} - 1$$

$$= 1.1036 - 1$$

$$= 0.1036 = 10.36\%$$

2012 - JUNE

[50] The S.I. on a sum of money is $\frac{4}{9}$ of the principal and the no. of years is equal to the rate of interest per annum. Find the rate of interest per annum ?

(a) 5% (b) 20/3%

(c) 22/7% (d) 6% (1 mark)

Answer:

(b) Given Say Principal = P

$$\therefore \text{S.I.} = \frac{4}{9}P$$

No. of years (time) T= Rate of interest (R)

$$\therefore \text{S.I.} = \frac{PTR}{100}$$

$$\therefore \frac{4}{9}P = \frac{P \times R \times R}{100}$$

$$\rightarrow \quad R^2 = \frac{400}{9}$$

$$\therefore \quad R = \frac{20}{3}\%$$

[51] Simple interest on ₹ 2,000 for 5 months at 16% p.a. is _____.

(a) ₹ 133.33 (b) ₹ 133.26

(c) ₹ 134.00 (d) ₹ 132.09 (1 mark)

Answer:

(a) Given : Principal = ₹ 2,000

Time $= \frac{5}{12}$ yrs. (i.e. 5 months)

Rate = 16% p.a.

$\therefore S.I = \frac{P \times T \times R}{100}$

$= \frac{2,000 \times 5 \times 16}{12 \times 100}$

S.I = ₹ 133.33

2012 - DECEMBER

[52] How much investment is required to yield an Annual income of ₹ 420 at 7% p.a. Simple interest.

(a) ₹ 6,000 (b) ₹ 6,420

(c) ₹ 5,580 (d) ₹ 5,000 (1 mark)

Answer:

(a) Given S.I = 420, T = 1 yrs, R = 7% p.a.

Let Investment (P) = ₹ x

$P = \frac{S.I \times 100}{R \times T} = \frac{420 \times 100}{7 \times 1} = ₹ 6,000$

[53] Mr. X invests ₹ 90,500 in post office at 7.5% p.a. simple interest. While calculating the rate was wrongly taken as 5.7% p.a.
The difference in amounts at maturity is ₹ 9,774. Find the period for which the sum was invested:

(a) 7 years (b) 5.8 years
(c) 6 years (d) 8 years (1 mark)

Answer:

(c) Principal (P) = ₹ 90,500
Wrong Rate (R_1) = 5.7% p.a.
Correct Rate (R_2) = 7.5% p.a.
T = ?
diff in Amount = 9,774

$$\text{Diff in Amount} = \left[P\left(1+\frac{R_2T}{100}\right) - P\left(1+\frac{R_1T}{100}\right)\right]$$

$$9{,}774 = P\left[1+\frac{R_2T}{100} - 1 - \frac{R_1T}{100}\right]$$

$$9{,}774 = 90{,}500\left[\frac{7.5T}{100} - \frac{5.7T}{100}\right]$$

$$9{,}774 = 90{,}500 \times \frac{1.8T}{100}$$

$$T = \frac{9{,}774 \times 1{,}000}{90{,}500 \times 18} = 6$$

2013 - JUNE

[54] The difference between compound and simple interest on a certain sum of money for 2 years at 4% p.a. is ₹ 1. The sum (in ₹) is:

(a) 625 (b) 630
(c) 640 (d) 635 (1 mark)

Answer:

(a) Let principal (p) = 100, R = 4%, T = 2 yrs

$$S.I. = \frac{PRT}{100} = \frac{100 \times 4 \times 2}{100} = ₹\ 8$$

$$C.I. = P\left[\left(1 + \frac{R}{100}\right)^T - 1\right] = 100\left[\left(1 + \frac{4}{100}\right)^2 - 1\right]$$

$$= 100\,[(1 + 0.04)^2 - 1]$$
$$= 100\,[(1.04)^2 - 1]$$
$$= 100 \times (1.04 + 1)\,(1.04 - 1)$$
$$= 100 \times 2.04 \times 0.04$$
$$= 8.16$$

difference b/w C.I. & S.I. = 8.16 – 8 = 0.16

∵ If difference is ₹ 0.16 then principal = 100

$$\therefore \quad ₹\ 1 = \frac{1000}{0.16}$$

$$\therefore \quad = ₹\ 625$$

[55] A sum of money compounded annually becomes ₹ 1,140 in two years and ₹ 1,710 in three years.
Find the rate of interest per annum.

(a) 30% (b) 40%
(c) 50% (d) 60% (1 mark)

Answer:

(c) Let principal be P and Rate is R % p.a.

Case - 1

$$A = P\left(1 + \frac{R}{100}\right)^T$$

$$1140 = P\left(1 + \frac{R}{100}\right)^2 \quad \text{.....(1)}$$

Case - 2

$$A = P\left(1 + \frac{R}{100}\right)^T$$

$$1710 = P\left(1 + \frac{R}{100}\right)^3 \quad \text{.....(2)}$$

Divide by (2)/ eq (1)

$$\frac{1710}{1140} = \frac{P\left(1+\frac{R}{100}\right)^2}{P\left(1+\frac{R}{100}\right)^2}$$

$$\frac{3}{2} = \left(1+\frac{R}{100}\right)$$

$$\frac{3}{2} - 1 = \frac{R}{100}$$

$$\frac{1}{2} = \frac{R}{100} \Rightarrow 2R = 100$$

$$R = 50\%$$

2013 - DECEMBER

[56] On what sum difference between compound interest and simple interest for two years at 7% p.a. interest is ₹ 29.4

(a) ₹ 5,000 (b) ₹ 5,500

(c) ₹ 6,000 (d) ₹ 6,500 (1 mark)

Answer:

(c) Let the sum (P) ₹ = x Given R = 7% p.a., T = 2yrs

$$S.I. = \frac{PRT}{100} = \frac{x.7.2}{100} = \frac{14x}{100} = 0.14x$$

$$C.I. = P\left[\left(1+\frac{R}{100}\right)^T - 1\right]$$

$$= x\left[\left(1+\frac{7}{100}\right)^2 - 1\right]$$

$$= x\,[(1.07)^2 - 1]$$

$$C.I. = x\,[1.1449 - 1]$$

$$= x \times 0.1449$$

$$= 0.1449x$$

difference b/w C.I & S.I. = ₹ 29.4

$$\begin{aligned} \text{C.I - S.I.} &= 29.4 \\ 0.1449x - 0.14x &= 29.4 \\ 0.0049x &= 29.4 \\ x = \frac{29.4}{0.0049} &= ₹\ 6{,}000 \end{aligned}$$

[57] In what time will a sum of money double itself at 6.25% p.a. simple interest?

(a) 5 years (b) 8 years
(c) 12 years (d) 16 years (1 mark)

Answer:

(d) Let Principal (P)= 100, R = 6.25% p.a.
Amount (A) = 200 T = ?

S.I. = A - P
= 200 - 100
= 100

$$T = \frac{S.I. \times 100}{P \times R} = \frac{\cancel{100} \times 10000}{\cancel{100} \times 625} = 16 \text{ yrs}$$

[58] What principal will amount to ₹ 370 in 6 years at 8% p.a. at simple interest?

(a) ₹ 210 (b) ₹ 250
(c) ₹ 310 (d) ₹ 350 (1 mark)

Answer:

(b) Given Amount (A) = ₹ 370, T = 6 yrs, R = 8% p.a.

Let P = ₹ x

$$S.I = \frac{PRT}{100} = \frac{x \times 8 \times 6}{100} = \frac{48x}{100}$$

$$A = P + S.I = x + \frac{48x}{100}$$

$$370 = \frac{148x}{100}$$

$$x = \frac{370 \times 100}{148} = ₹\ 250$$

2014 - JUNE

[59] The partners A and B together lent ₹ 3,903 at 4% per annum interest compounded annually. After a span of 7 years, A gets the same amount as B gets after 9 years. The share of A in the sum of ₹ 3,903 would have been:

(a) ₹ 1,875 (b) ₹ 2,280
(c) ₹ 2,028 (d) ₹ 2,820 (1 mark)

Answer:

(c) Let Principal of A (P_1) = ₹ x
Rate R_1 = 4% p.a.
T_1 = 7 years
Principal of B (P_2) = ₹ (3,903 – x)
R_2 = 4%
T_2 = 9 years
Given $A_1 = A_2$

$$P_1\left(1+\frac{R_1}{100}\right)^{T_1} = P_2\left(1+\frac{R_2}{100}\right)^{T_2}$$

$$x\left(1+\frac{4}{100}\right)^{7} = (3,903-x)\left(1+\frac{4}{100}\right)^{9}$$

$$x\,(1.04)^7 = (3,903-x)\,(1.04)^9$$

$$\frac{x}{3,903-x} = \frac{(1.04)^9}{(1.04)^7}$$

$$\frac{x}{(3,903-x)} = (1.04)^2$$

$$\frac{x}{(3,903-x)} = 1.0816$$

$$x = 1.0816\,(3,903-x)$$

$$x = 4,221.4848 - 1.0816x$$

$$x + 1.0816x = 4,221.4848$$
$$2.0816x = 4,221.4848$$
$$x = \frac{4,221.4848}{2.0816}$$
$$x = 2,028$$

The share of A = 2,028

[60] If a sum triples in 15 years at simple rate of interest, the rate of interest per annum will be:

(a) 13.0% (b) 13.3%
(c) 13.5% (d) 18.0% (1 mark)

Answer:

(b) Let Principal P = P
Amount A = 3P
T = 15 years
S.I. = A – P
= 3P – P
= 2P

$$R = \frac{S.I. \times 100}{P \times T}$$
$$= \frac{2P \times 100}{P \times 15}$$
$$= \frac{200}{15}$$
$$R = \frac{40}{3}$$
$$= 13.3\%$$

[61] How much amount is required to be invested every year as to accumulate ₹ 6,00,000 at the end of 10 years, if interest is compounded annually at 10% rate of interest [Given: $(1.1)^{10} = 2.59374$].

(a) ₹ 37,467 (b) ₹ 37,476
(c) ₹ 37,647 (d) ₹ 37,674 (1 mark)

Answer:

(c) Given A(n, i) = ₹ 6,00,000

Annual Installment (A) = ?

$$R = 10\% \text{ p.a. } i = \frac{R}{100} = \frac{10}{100} = 0.1$$

$$n = 10$$

Future value

$$A(n, i) = \frac{A}{i}[(1 + i)^n - 1]$$

$$6,00,000 = \frac{A}{0.1}[(1 + 0.1)^{10} - 1]$$

$$6,00,000 \times 0.1 = A[(1.1)^{10} - 1]$$

$$60,000 = A[2.59374 - 1]$$

$$60,000 = A \times 1.59374$$

$$A = \frac{60,000}{1.59374}$$

$$A = 37,647$$

2014 - DECEMBER

[62] The future value of an annuity of ₹ 1,000 made annually for 5 years at the interest of 14% compounded annually is:
(Given $(1.14)^5 = 1.92541$)

(a) ₹ 5,610 (b) ₹ 6,610
(c) ₹ 6,160 (d) ₹ 5,160 (1 mark)

Answer:

(b) Annuity (A) = ₹ 1,000

$$n = 5, R = 14\%$$

$$i = \frac{R}{100} = \frac{14}{100} = 0.14$$

Future value

$$A(n, i) = \frac{A}{i}[(1 + i)^n - 1]$$

$$= \frac{1,000}{0.14}[(1 + 0.14)^5 - 1]$$

$$= \frac{1,000}{0.14}[(1.14)^5 - 1]$$

$$= \frac{1,000}{0.14} \times [1.92541 - 1]$$

$$= \frac{1,000}{0.14} \times 0.92541 = ₹\ 6,610$$

[63] A sum of money invested of compound interest doubles itself in four years. It becomes 32 times of itself at the same rate of compound interest in

(a) 12 years (b) 16 years

(c) 20 years (d) 24 years (1 mark)

Answer:

(c) CASE → I

Let Principal (P) = 100, T = 4 years

(A) = 200 R = ?

$$A = P\left(1 + \frac{R}{100}\right)^T$$

$$200 = 100\left(1 + \frac{R}{100}\right)^4$$

$$\frac{200}{100} = \left(1 + \frac{R}{100}\right)^4$$

$$2 = \left(1 + \frac{R}{100}\right)^4$$

$$\left(1 + \frac{R}{100}\right) = 2^{1/4}$$

CASE → II Let P = 100

A = 3,200

$\left(1 + \frac{R}{100}\right) = 2^{1/4}$, T = ?

$$A = P\left(1 + \frac{R}{100}\right)^T$$

$$3,200 = 100\left(1 + \frac{R}{100}\right)^T$$

$$\frac{3,200}{100} = \left(1 + \frac{R}{100}\right)^T$$

$$32 = (2^{1/4})^T$$

$$2^5 = 2^{\frac{T}{4}} \Rightarrow 5 = \frac{T}{4}$$

$$\Rightarrow T = 20 \text{ years}$$

[64] A certain sum of money was invested at simple rate of interest for three years. If the same has been invested at a rate that was seven percent higher, the interest amount would have been ₹ 882 more. The amount of sum invested is:

(a) ₹ 12,600 (b) ₹ 6,800

(c) ₹ 4,200 (d) ₹ 2,800 (1 mark)

Answer:

(c) Let certain sum $(P_1) = P_2 = P$

CASE → 1

Principal $(P_1) = P$

$(R_1) = R\%$ $\quad (S.I.)_1 = \frac{P_1R_1T_1}{100}$

$T_1 = 3$ yrs $\quad = \frac{PR3}{100}$

CASE → 2

Principal $(P_2) = P$

$(R_2) = (R + 7)\%$ $\quad (S.I.)_2 = \frac{P_2R_2T_2}{100}$

$(T_2) = 3$ yrs $\quad = \frac{P(R+7)3}{100}$

Given, $(S.I.)_2 - (S.I.)_1 = 882$

$$\frac{P(R+7)3}{100} - \frac{PR3}{100} = 882$$

$$\frac{3PR + 21P - 3PR}{100} = 882$$

$$21P = 882 \times 100$$
$$P = \frac{882 \times 100}{21}$$
$$P = 42 \times 100$$
$$P = 4,200$$

2015 - JUNE

[65] A sum of money doubles itself in 8 years at simple interest. The number of years it would triple itself is _____.

(a) 20 years (b) 12 years
(c) 16 years (d) None of these. (1 mark)

Answer:

(c) Case - 1

Let $P = 100$
$A = 200$
$S.I. = A - P$
$= 200 - 100$
$= 100$

$$R = \frac{S.I. \times 100}{P \times T}$$
$$= \frac{100 \times 100}{100 \times 8}$$

$R = 12.5\%$.

Case - 2

$P = 100, R = 12.5\%$
$A = 300$
$S.I. = 300 - 100 = 200$

$$T = \frac{S.I. \times 100}{P \times R}$$
$$= \frac{200 \times 100}{100 \times 12.5}$$
$$= 2 \times 8$$
$$= 16 \text{ yrs.}$$

[66] A sum of ₹ 44,000 is divided into three parts such that the corresponding interest earned after 2 years, 3 years and 6 years may be equal. If the rates of simple interest are 6% p.a., 8% p.a. and 6% p.a. respectively, then the smallest part of the sum will be:

(a) ₹ 4,000 (b) ₹ 8,000

(c) ₹ 10,000 (d) ₹ 12,000 (1 mark)

Answer:

(b) Total Sum = ₹ 44,000

Let Ist part = ₹ x

IInd part = ₹ y

IIIrd part = ₹ z

Case - 1 Principal (P_1) = ₹ x

Rate (R_1) = 6% p.a.

(T_1) = 2 yrs.

$$(S.I.)_1 = \frac{P_1 R_1 T_1}{100}$$

$$(S.I.)_1 = \frac{x \times 6 \times 2}{100}$$

$$(S.I.)_1 = \frac{12x}{100}$$

Case - 2 Principal (P_2) = ₹ y

(R_2) = 8% p.a.

(T_2) = 3 yrs.

$$(S.I.)_2 = \frac{P_2 R_2 T_2}{100} = \frac{y \times 8 \times 3}{100}$$

$$(S.I.)_2 = \frac{24y}{100}$$

Case - 3 Principal (P_3) = ₹ z

R_3 = 6% p.a.

T_3 = 6 yrs.

$$(S.I.)_3 = \frac{P_3 R_3 T_3}{100} = \frac{z \times 6 \times 6}{100}$$

$$(S.I.)_3 = \frac{36z}{100}$$

Given: $(S.I.)_1 = (S.I.)_2 = (S.I.)_3$

$$\frac{12x}{100} = \frac{24y}{100} = \frac{36z}{100}$$

or $x = 2y = 3z = k$ (let)

$x = k$, $2y = k$, $3z = k$

$y = k/2$ $z = k/3$

$$x : y : z = \frac{k}{1} : \frac{k}{2} : \frac{k}{3}$$

$$= 6k : 6 \times \frac{k}{2} : 6 \times \frac{k}{3}$$

$$= 6 : 3 : 2$$

$$\text{Smallest Sum} = \frac{2}{6+3+2} \times 44{,}000$$

$$= \frac{2}{11} \times 44{,}000$$

$$= ₹\ 8{,}000$$

2015 - DECEMBER

[67] Suppose your parent decides to open a PPF (Public Provident Fund) account in a bank towards your name with ₹ 10,000 every year starting from today for next 16 years. When you receive and get 8.5% per annum interest rate compounded annually. What is the present value of this annuity? (Give answer in ₹ without any fraction.)
(Given P (15,0.085) = 8.304236576)

(a) 83,042 (b) 1,66,084

(c) 93,042 (d) 8,30,423 (1 mark)

Answer:

(c) Annual Installment (A) = ₹ 10,000

n = 16 years

R = 8.5% p.a.

$$i = \frac{8.5}{100} = 0.085$$

Present value = A. P(n-1, i) + A
= 10,000.P (15, 0.085) + 10,000
= 10,000 × 8.304236576 + 10,000
= 83,042.36576 + 10,000
= ₹ 93,042

[68] In how many years will a sum of money become four times at 12% p.a. simple interest?

(a) 18 years (b) 21 years
(c) 25 years (d) 28 years (1 mark)

Answer:

(c) Let Principal P = 100
A = 400
S. I. = A-P
= 400 - 100
= 300
R = 12%, T=?

$$S.I. = \frac{PRT}{100}$$

$$T = \frac{S.I. \times 100}{PR} = \frac{300 \times 100}{100 \times 12} = 25 \text{ years}$$

[69] The simple interest for a certain sum for 2 years at 10% per annum is ₹ 90. The corresponding compound interest is (In ₹):

(a) 99 (b) 95.60
(c) 94.50 (d) 108 (1 mark)

Answer:

(c) Let Principal (P) = ₹ x
T = 2 years , R = 10% p.a.
S.I. = ₹ 90

$$P = \frac{S.I. \times 100}{R \times T} = \frac{90 \times 100}{10 \times 2} = ₹ 450$$

Now

$$C.I. = P\left[\left(1+\frac{R}{100}\right)^{T}-1\right]$$

$$= 450\Big[\qquad\qquad\Big]$$

[71] If an amount is kept at simple interest, it earns an interest of ₹ 600 in first two years but when kept at compound interest it earns an interest of ₹ 660 for the same period, then the rate of interest and principal amount respectively are:

(a) 20%, ₹ 1,200 (b) 10%, ₹ 1,200

(c) 20%, ₹ 1,500 (d) 10%, ₹ 1,500 (1 mark)

Answer:

(c) For 2 years

S.I. = 600 and C.I. = 660

$$S.I. = \frac{PRT}{100}$$

$$600 = \frac{PR \times 2}{100}$$

$$PR = \frac{600 \times 100}{2}$$

$$PR = 30000 \Rightarrow P = \frac{30000}{R} \quad \text{(1)}$$

$$C.I. = P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$$

$$660 = \frac{30000}{R}\left[\left(1 + \frac{R}{100}\right)^2 - 1^2\right]$$

$$660 = \frac{30000}{R}\left[\left(1 + \frac{R}{100} + 1\right)\left(1 + \frac{R}{100} - 1\right)\right]$$

$$660 = \frac{30000}{R}\left[\left(2 + \frac{R}{100}\right)\left(\frac{R}{100}\right)\right]$$

$$660 = 30000\left(\frac{200 + R}{100}\right).\frac{1}{100}$$

$$660 \times 100 \times 100 = 30000\,(200 + R)$$

$$200 + R = \frac{660 \times 100 \times 100}{30000}$$

$$200 + R = 220$$

$$R = 20\%$$

R = 20% in equation (1)

$$P = \frac{30000}{20} = 1500$$

2016 - DECEMBER

[72] The sum invested at 4% per annum compounded Semi-annually amounts to ₹ 7,803 at the end of one year, is:

(a) ₹ 7,000 (b) ₹ 7,500

(c) ₹ 7,225 (d) ₹ 8,000 (1 mark)

Answer:

(b) Let, sum (Principal)

P = ₹ x T = 1 year

R = 4% p.a. C.I.

A = ₹ 7,803

Interest is compounded half yearly (Semi Annually)

then $R = \frac{4}{2}\% = 2\%$

$T = 1 \times 2 = 2$ half yearly

Amount after T years

$$A = P\left(1 + \frac{R}{100}\right)^T$$

$$7,803 = x\left(1 + \frac{2}{100}\right)^2$$

$$7,803 = x\,(1.02)^2$$

$$7,803 = x \times 1.0404$$

$$x = \frac{7,803}{1.0404} = 7,500$$

Sum = ₹ 7,500

[73] A compound interest on a sum for 2 years is ₹ 30 more than the simple interest at the rate of 5% per annum then the sum is

(a) ₹ 11,000 (b) ₹ 13,000

(c) ₹ 12,000 (d) ₹ 15,000 (1 mark)

Answer:

(c) Given C.I. – S.I. = 30

T = 2 years

R = 5%

$$C.I. - S.I = P\left(\frac{R}{100}\right)^2$$

$$30 = P\left(\frac{5}{100}\right)^2$$

$$30 = P(0.05)^2$$

$$30 = P(0.0025)$$

$$P = \frac{30}{0.0025} = ₹\ 12,000$$

[74] A person lends ₹ 6,000 for 4 years and ₹ 8,000 for 3 years at simple interest. If he gets ₹ 2,400 as total interest, the rate of interest is:

(a) 5% (b) 4%

(c) 6% (d) 7% (1 mark)

Answer:

(a) Given P_1 ₹ 6,000 P_2 = ₹ 8,000, Total S.I. = 2,400

$R_1 = R\%$ $R_2 = R\%$

$T_1 = 4$ years, $T_2 = 3$years

Total Interest $= (S.I)_1 + (S.I)_2$

$$2,400 = \frac{P_1 R_1 T_1}{100} + \frac{P_2 R_2 T_2}{100}$$

$$2,400 = \frac{6,000 \times R \times 4}{100} + \frac{8,000 \times R \times 3}{100}$$

$$2,400 = 240\ R + 240\ R$$

$$2,400 = 480R \rightarrow R = \frac{2,400}{480} = 5\%$$

2017 - JUNE

[75] The future value of an annuity of ₹ 1,500 made annually for five years at interest rate 10% compounded annually is (Given that $(1.1)^5 = 1.61051$):

(a) ₹ 9,517.56 (b) ₹ 9,157.65

(c) ₹ 9,715.56 (d) ₹ 9,175.65 (1 mark)

Answer:

(b) Annual Installment (Annuity)A= ₹ 1,500

$n = 5$

$R = 10\%$

$i = \frac{10}{100} = 0.10$

Future Value $A_{(n, i)} = \frac{A}{i}[(1 + i)^n - 1]$

$= \frac{1,500}{0.10}[(1 + 0.10)^5 - 1]$

$= \frac{1,50,000}{10}[(1 + 0.10)^5 - 1]$

$= 15000\ [(1.1)^5 - 1]$

$= 15000 \times [1.61051 - 1]$

$= 15000 \times 0.61051$

$= 9,157.65$

[76] The difference between the Compound interest and Simple interest at 10% per annum for 4 years on ₹ 10,000 is ₹ _____.

(a) 650 (b) 640

(c) 641 (d) 600 (1 mark)

Answer:

(c) Principal (P) = 10,000, Rate (R) = 10%, T = 4 years

For S. I

$S.I = \frac{PRT}{100} = \frac{10,000 \times 10 \times 4}{100} = 4,000$

For C. I

$C.I = P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$

$= 10,000\left[\left(1 + \frac{10}{100}\right)^4 - 1\right] = 10,000\ [(1.1)^4 - 1]$

$= 10,000\ (1.4641 - 1)$

$= 10,000 \times 0.4641 = 4,641$

C.I - S.I = 4,641 - 4,000 = 641

[77] How much amount is required to be invested every year as to accumulate ₹ 7,96,870 at the end of 10 years, if interest compounded annually at 10% given that A(10, 0.1) = 15.9374?

(a) ₹ 40,000 (b) ₹ 4,50,000

(c) ₹ 48,000 (d) ₹ 50,000 (1 mark)

Answer:

(d) Future Value $A_{(n, i)}$ = 7,96,870

n = 10, R = 10%

$i = \frac{10}{100} = 0.1$

Future Value

$$A_{(n, i)} = \frac{A}{i}[(1 + i)^n - 1]$$

$$7,96,870 = \frac{A}{0.1}[(1 + 0.1)^{10} - 1]$$

$$7,96,870 = \frac{A}{0.1}[(1.1)^{10} - 1]$$

$$7,96,870 = \frac{A}{0.1}[2.59374 - 1]$$

$$7,96,870 = \frac{A}{0.1}[1.59374]$$

$$\frac{0.1 \times 7,96,870}{1.59374} = A$$

50,000 = A

2017 - DECEMBER

[78] If compound interest on any sum at the rate of 5% for two years is ₹ 512.50 then the sum would be:

(a) ₹ 3,000 (b) ₹ 4,000

(c) ₹ 5,000 (d) ₹ 6,000 (1 mark)

Answer:

(c) Given

R = 5%,

T = 2 years,

C.I. = ₹ 512.50

P = ?

$$C.I = P\left[\left(1+\frac{R}{100}\right)^{T}-1\right]$$

$$512.50 = P\left[\left(1+\frac{5}{100}\right)^{2}-1\right]$$

$$512.50 = P\left[(1.05)^2 - 1\right]$$

$$512.50 = P\left[1.1025 - 1\right]$$

$$512.50 = P \times 0.1025$$

$$P = \frac{512.50}{0.1025}$$

$$= ₹\ 5{,}000$$

[79] The effective rate of interest equivalent to the nominal rate of 7% converted monthly:

(a) 7.26% (b) 7.22%

(c) 7.02% (d) 7.20% (1 mark)

Answer:

(b) $R = \frac{7}{12}\%$ monthly

$$i = \frac{7}{12\times 100} = 0.00583$$

n = 1 year

= 1× 12 months

= 12

Effective Rate (E) = $[(1+i)^n - 1] \times 100\%$

$= [(1 + 0.00583)^{12} - 1] \times 100\%$

$= [(1.00583)^{12} - 1] \times 100\%$

$= [1.0722 - 1] \times 100\%$

$= 0.0722 \times 100\%$

$= 7.22\%$

2018 - MAY

[80] Mr. X invest ₹ 10,000 every year starting from today for next: 10 years suppose interest rate is 8% per annual compounded annually. Calculate future value of the annuity.

(a) ₹ 1,56,454.88 (b) ₹ 1,56,554.88

(c) ₹ 1,44,865.625 (d) None of these (1 mark)

Answer:

(a) Annual Installment (A) = 10,000, R = 8% p. a. c. i.

n = 10 years $A_{(n, i)}$ = ?

$i = \frac{8}{100} = 0.08$

Future Value of Annuity due

$$A_{(n, i)} = \frac{A}{i}\left[(1+i)^n - 1\right](1+i)$$

$$= \frac{10,000}{0.08}\left[(1+0.08)^{10} - 1\right](1+0.08)$$

$$= \frac{10,000}{0.08}\left[(1.08)^{10} - 1\right](1.08)$$

$$= 1,56,454.88$$

[81] How much amount is required to be invested every year so as to accumulate ₹ 3,00,000 at the end of 10 years, if interest is compounded annually at 10%?

(a) ₹ 18,823.65 (b) ₹ 18

(c) ₹ 18,828.65 (d) ₹ 18,882.65 (1 mark)

Answer:

(a) Annuity (Annual Installment) = A

Future Value $A_{(n, i)}$ = 3,00,000

R = 10% , n = 10 years

$i = \frac{R}{100} = \frac{10}{100} = 0.1$

$$A_{(n,i)} = \frac{A}{i}[(1+i)^n - 1]$$

$$3,00,000 = \frac{A}{0.1}[(1+0.1)^{10} - 1]$$

$$3,00,000 = \frac{A}{0.1}[(1.1)^{10} - 1]$$

$$3,00,000 = \frac{A}{0.1}[2.59374 - 1]$$

$$= \frac{A}{0.1} \times 1.59374 \times 10$$

$$3,00,000 = A \times 15.9374$$

$$A = \frac{3,00,000}{15.9374} = ₹\ 18,823.65$$

[82] If ₹ 1,000 be invested at interest rate of 5% and the interest be added to the principal every 10 years, than the number of years in which it will amount to ₹ 2,000 is:

(a) $16\frac{2}{3}$ years (b) $6\frac{1}{4}$ years

(c) 16 years (d) $6\frac{2}{3}$ years (1 mark)

Answer:

(a) P = 1,000, R = 5% p.a. s.i, T = 10 years

$$SI = \frac{PRT}{100} = \frac{1000 \times 5 \times 10}{100} = 500$$

Amount after 10 years

A = P + S. I. = 1,000 + 500 = 1,500

Now after 10 years

P = 1,500 , R = 5%

A = 2,000 , T = ?

S.I. = A–P

= 2,000 – 1,500

= 500

$$T = \frac{S.I. \times 100}{P \times R} = \frac{500 \times 100}{1500 \times 5} = \frac{20}{3} = 6\frac{2}{3} \text{ years}$$

Total time taken = 10 years + $6\frac{2}{3}$ years

= $16\frac{2}{3}$ years.

[83] A person borrows ₹ 5,000 for 2 years at 4% per annual simple interest. He immediately lends to another person at $6\frac{1}{4}$%. Per annual for 2 years find his gain in the transaction for year:

(a) ₹ 112.50 (b) ₹ 225

(c) ₹ 125 (d) ₹ 107.50 (1 mark)

Answer:

(b) **Case – 1**

P = 5,000

R = 4% p.a.s.I.

T= 2 years

$$S.I = \frac{PRT}{100} = \frac{5,000 \times 4 \times 2}{100} = 400$$

Case – 2

P = 5,000

$R = 6\frac{1}{4}\% = \frac{25}{4}\%$ p.a.s.i.

T= 2 years

$$S.I = \frac{PRT}{100} = \frac{5,000 \times 25}{100 \times 4} \times 2 = ₹\ 625$$

his gain = 625 – 400 = 225

[84] If an amount is kept at S.I. it earns an interest of ₹ 600 in first two years but when kept at compound interest it earns an interest of ₹ 660 for the same period, then the rate of interest and principal amount respectively are:

(a) 20%., ₹ 1,200 (b) 20%, ₹ 1,500

(c) 10%, ₹ 1,200 (d) 10%., ₹ 1,500 (1 mark)

Answer:

Case -1

Let P = x, R = R, T= 2, S.I. = 600

$$S.I. = \frac{PRT}{100}$$

$$600 = \frac{XR2}{100}$$

$$XR = \frac{600 \times 100}{2}$$

$$XR = 30,000$$

$$X = \left(\frac{(30,000}{R}\right) \text{——} (1)$$

Case - 2

P = x, R = R, T= 2, C.I = 660

$$C.I = P\left[\left(1 + \frac{R}{100}\right)^2 - 1\right]$$

$$660 = \left(\frac{(30,000}{R}\right)\left[\left(1 + \frac{R}{100}\right)^2 - (1)^2\right]$$

$$660 = \frac{30,000}{R}\left[\left(1 + \frac{R}{100} + 1\right)\left(1 + \frac{R}{100}\right) - 1\right]$$

$$660 = \frac{30,000}{R} \times \left(2 + \frac{R}{100}\right) \times \frac{R}{100}$$

$$\frac{660}{300} = 2 + \frac{R}{100}$$

$$\frac{R}{100} = \frac{660}{300} - 2$$

$$\frac{R}{100} = \frac{660 \times 600}{300}$$

$$\frac{R}{100} = \frac{60}{300}$$

$$R = \frac{60 \times 100}{300} = 20\%$$

Putting R = 20% in Eq.....(1)

$$X = \frac{30,000}{20}$$

$$X = ₹\ 1,500$$

Answer: P = x = ₹ 1500

R = 20% p.a.

[85] The future value of an annuity of ₹ 1,000. made annually for 5 years at the interest of 14% compounded annually is:

Given $(1.14)^5$ 1.92541)

(a) ₹ 5,610 (b) ₹ 6,610

(c) ₹ 6,160 (d) ₹ 5,160 (1 mark)

Answer:

(b) Given, Annuity (A) = ₹ 1,000

R = 14%

$$i = \frac{14}{100} = 0.14$$

Future Value n = 5

$$A_{(n,i)} = \frac{A}{i}[(1+i)^n - 1]$$

$$= \frac{1,000}{0.14}[(1+0.14)^5 - 1]$$

$$= \frac{1,000}{0.14}[1.92541 - 1]$$

$$= \frac{1,00,000}{0.14}[0.92541]$$

$$= ₹\ 6,610$$

2018 - NOVEMBER

[86] If ₹ 10,000 is invested at 8% per year compound quarterly, then the value of the investment after 2 years is

[given $(1 + 0.2)^8 = 1.171659$]

(a) ₹ 11,716.59 (b) ₹ 10,716.59
(c) ₹ 117.1659 (d) None of the above (1 mark)

Answer:

(a) Given P = ₹ 10,000 , $R = \frac{8\%}{4}$

R = 2% Quarterly
T = 2 × 4 = 8 Quarter
Value of Investment after 'T' years

$$A = P\left(1+\frac{R}{100}\right)^T$$

$$= 10{,}000\left(1+\frac{2}{100}\right)^8$$

$$= 10{,}000\,(1+0.02)^8$$

$$= 10{,}000 \times (1.02)^8$$

$$= 10{,}000 \times 1.171659$$

$$= 11{,}716.59$$

[87] A bank pays 10% rate of interest compounded annually. A sum of ₹ 400 is deposited in the bank. The amount at the end of 1 year will be
(a) ₹ 440 (b) ₹ 439
(c) ₹ 441 (d) ₹ 442 (1 mark)

Answer:

(a) Given Principal (P) = 400
R = 10% p. a.
T = 1 years
Amount after T years

$$A = P\left(1+\frac{R}{100}\right)^T$$

$$= 400\left(1+\frac{10}{100}\right)^1$$

$$= 400\,(1.1)$$

$$= 440$$

[88] A certain money doubles itself in 10 years when deposited on simple interest. It would triple itself in

(a) 20 years (b) 15 years

(c) 25 years (d) 30 years (1 mark)

Answer:

(a) Case - I

Let Principal (P) = 100

Amount (A) = 200

R = ?

T = 10 years

S. I. = A – P

= 200 – 100

= 100

$$R = \frac{S.I. \times 100}{P \times T}$$

$$= \frac{100 \times 100}{100 \times 10}$$

R = 10%

Case - II

Let Principal (P) = 100

Amount (A) = 300

(T) = ?

R = 10%

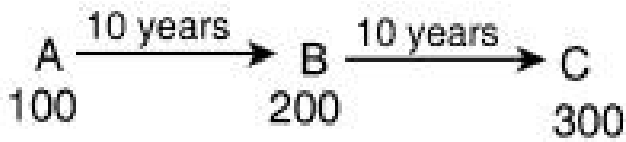
S. I. = A – P

= 300 – 100 = 200

$$T = \frac{S.I. \times 100}{P \times R}$$

$$= \frac{200 \times 100}{100 \times 10} = 20 \text{ years}$$

SHORT CUT



Total time = 10 years + 10 years = 20 years.

[89] A man deposited ₹ 8,000 in a bank for 3 years at 5% per annum compound interest, after 3 years he will get

(a) ₹ 8,800 (b) ₹ 9,261

(c) ₹ 9,200 (d) ₹ 9,000 (1 mark)

Answer:

(b) Given P = 8,000

R = 5% p.a.

T = 3 years

Amount after 'T' years

$$A = P\left(1 + \frac{R}{100}\right)^T$$

$$= 8{,}000\left(1 + \frac{5}{100}\right)^3$$

$$= 8{,}000\ (1.05)^3$$

$$= 8{,}000 \times 1.05 \times 1.05 \times 1.05$$

$$= 9{,}261$$

[90] If in two years time a principal of ₹ 100 amounts to ₹ 121 when the interest at the rate of r % is compounded annually, then the value of r will be

(a) 10.5 (b) 10%

(c) 15 (d) 14 (1 mark)

Answer:

(b) Given, Principal (P) = ₹ 100

Amount (A) = ₹ 121

Rate R = r % p.a.

Time T = 2 years

The Amount after 'T' years

$$A = P\left(1 + \frac{R}{100}\right)^T$$

$$121 = 100\left(\quad\right)$$

$$\frac{121}{100} = \left(1+\frac{r}{100}\right)^2$$

$$\left(\frac{11}{10}\right)^2 = \left(1+\frac{r}{100}\right)^2$$

on comparing

$$\frac{11}{10} = 1+\frac{r}{100}$$

$$\frac{11}{10}-1 = \frac{r}{100}$$

$$\frac{11-10}{10} = \frac{r}{100}$$

$$\frac{1}{10} = \frac{r}{100}$$

$$r = \frac{100}{10}$$

$$r = 10\%$$

[91] A certain sum of money Q was deposited for 5 year and 4 months at 4.5% simple interest and amounted to ₹ 248, then the value of Q is

(a) ₹ 200 (b) ₹ 210

(c) ₹ 220 (d) ₹ 240 (1 mark)

Answer:

(a) Given Principal (P) = x

R = 4.5%

T = 5 years 4 month

$= 5 \text{ years} + \frac{4}{12} \text{ years}$

$= 5 \text{ years} + \frac{1}{3} \text{ years}$

$= 5\frac{1}{3} \text{ years}$

$= \frac{16}{3} \text{ years}$

Amount after T years

$$A = P + S.I.$$

$$A = P + \frac{PRT}{100}$$

$$A = x + \frac{x \times 15 \times 16}{1000 \times 8}$$

$$248 = x + \frac{240x}{1000}$$

$$248 = x + \frac{24x}{100}$$

$$\frac{248}{1} \quad \frac{100x + 24x}{100}$$

$$124x = 24{,}800$$

$$x = \frac{24{,}800}{124} = 200$$

[92] If compound interest on a sum for 2 years at 4% per annum is ₹ 102, then the simple interest on the same sum for the same period at the same rate will be

(a) ₹ 99 (b) ₹ 101

(c) ₹ 100 (d) ₹ 95 (1 mark)

Answer:

(c) Given T = 2 Years

C.I. = ₹ 102

R = 4%

P = ?

$$C.I. = P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$$

$$102 = P\left[\left(1 + \frac{4}{100}\right)^2 - 1\right]$$

$$102 = P\left[(1.04)^2 - 1\right]$$

$$102 = P\left[1.0816 - 1\right]$$

$102 = P\ (0.0816)$

$P = \frac{102}{0.0816} = 1,250$

Now S.I. $= \frac{PRT}{100} = \frac{1,250 \times 4 \times 2}{100} = \frac{10,000}{100} = ₹\ 100$

[93] A man invests an amount of ₹ 15,860 in the names of his three sons A, B and C in such a way that they get the same amount after 2,3 and 4 years respectively. If the rate of interest is 5%, then the ratio of amount invested in the name of A, B and C is

(a) 6: 4: 3 (b) 3: 4: 6

(c) 30: 12: 5 (d) None of the above (1 mark)

Answer:

(a) Total Amount invested = ₹ 15,860

Amount Invested into three persons (Son's) A, B, C.

Let,

Amount Invest in the Name of A = ₹ x

Amount Invest in the Name of B = ₹ y

Amount Invest in the Name of C = ₹ z

Then

Case – 1 For A

$P_1 = ₹\ x,\ R_1 = 5\%,\ T_1 = 2$ Years

$(S.I.)_1 = \frac{P_1 R_1 T_1}{100} = \frac{x \times 5 \times 2}{100} = \frac{10x}{100}$

Case – 2 For B

$P_2 = ₹\ Y,\ R_2 = 5\%,\ T_2 = 3$ Years

$(S.I.)_2 = \frac{P_2 R_2 T_2}{100} = \frac{x \times 5 \times 3}{100} = \frac{15x}{100}$

Case – 3 For C

$P_3 = ₹\ z,\ R_3 = 5\%,\ T_3 = 4$ Years

$(S.I.)_3 = \frac{P_3 R_3 T_3}{100} = \frac{z \times 5 \times 4}{100} = \frac{20z}{100}$

Given $(S.I.)_1 = (S.I.)_2 = (S.I.)_3$

$\frac{10x}{100} = \frac{15y}{100} = \frac{20z}{100}$

$10x = 15y = 20z = k$

$10x = k,\ 15y = k,\ 20z = k$

$x = \frac{k}{10},\ y = \frac{k}{15}\ z = \frac{k}{20}$

$x: y: z = \frac{k}{10} : \frac{k}{15} : \frac{k}{20}$

$= \frac{1}{10} : \frac{1}{15} : \frac{1}{20} = 60 \times \frac{1}{10} : 60 \times \frac{1}{15} : 60 \times \frac{1}{20}$

$= 6: 4: 3$

[94] If the difference between the compound interest compounded annually and simple interest on a certain amount at 10% per annum for two years is ₹ 372, then the principal amount is

(a) ₹ 37,200 (b) ₹ 37,000

(c) ₹ 37,500 (d) None of the above (1 mark)

Answer:

(a) For two year

$C.I. - S.I. = P\left(\frac{R}{100}\right)^2$

$372 = P\left(\quad\right)$

$$2,00,000 = P\left(1+\frac{5}{100}\right)^2$$

$$2,00,000 = P\,(1.05)^2$$

$$P = \frac{2,00,000}{(1.05)^2}$$

$$= \frac{2,00,000}{1.1025} = 1,81,405.896$$

$$= 1.81 \text{ Lakhs}$$

[96] The effective rate of interest for one year deposit corresponding to a nominal 7% rate of interest per annum convertible quarterly is

(a) 7% (b) 7.5%

(c) 7.4% (d) 7.18% (1 mark)

Answer:

(d) Given $R = \frac{7}{4}\%$ Quarterly $= 1.75\%$

$$T = 1 \times 4 \text{ Quarter}$$

$$= 4 \text{ Quarter}$$

$$\text{Effective Rate (E)} = \left[\left(1+\frac{R}{100}\right)^T - 1\right] \times 100\%$$

$$= \left[\left(1+\frac{1.75}{100}\right)^4 - 1\right] \times 100\%$$

$$= [(1+0.0175)^4 - 1] \times 100\%$$

$$= [(1.0175)^4 - 1] \times 100\%$$

$$= [1.07185 - 1] \times 100\%$$

$$= 0.0718 \times 100\%$$

$$= 7.18\%$$

[97] How much will ₹ 25,000 amount to in 2 years at compound interest if the rates for the successive years are 4% and 5% per year

(a) ₹ 27,300 (b) ₹ 27,000

(c) ₹ 27,500 (d) ₹ 27,900 (1 mark)

Answer:

(a) Given Principal (P) = 25,000

R_1 = 4%

R_2 = 5%

T = 2 years

Amount after '₹' Years

$$A = P\left(1+\frac{R_1}{100}\right)^1\left(1+\frac{R_2}{100}\right)^1$$

$$A = 25{,}000\left(1+\frac{4}{100}\right)^1\left(1+\frac{5}{100}\right)^1$$

$$= 25{,}000\left(1+\frac{1}{25}\right)\left(1+\frac{1}{20}\right)$$

$$= 25{,}000\left(\frac{26}{25}\right)\times\left(\frac{21}{20}\right)$$

$$= 27{,}300$$

[98] ₹ 8,000/- at 10% per annum interest compounded half yearly will become at the end of one year

(a) ₹ 8,800 (b) ₹ 8,820

(c) ₹ 8,900 (d) ₹ 9,600 (1 mark)

Answer:

(b) Given P = 8,000, $R = \frac{10}{2}\% = 5\%$, T = 1 × 2 h. y, T = 2

$$A = P\left(1+\frac{R}{100}\right)^T$$

$$= 8{,}000\left(1+\frac{5}{100}\right)^2$$

$$= 8{,}000\left(\frac{21}{20}\right)^2$$

$$= 8{,}000\times\frac{21}{20}\times\frac{21}{20}$$

$$= 20\times 21\times 21$$

A = ₹ 8,820

[99] The value of furniture depreciates by 10% a year, it the present value of the furniture in an office is ₹ 21,870, calculate the value of furniture 3 years ago

(a) ₹ 30,000 (b) ₹ 35,000
(c) ₹ 40,000 (d) ₹ 50,000 (1 mark)

Answer:

(a) Present Value of Furniture (A) = ₹ 21,870
Rate of Depreciation (R) = 10%
Time T = 3 Year ago
Value of Furniture 3 Year ago = P.
Scrap Value after T Years

$$A = P\left(1 - \frac{R}{100}\right)^T$$

$$21,870 = P\left(1 - \frac{10}{100}\right)^3$$

$$21,870 = P\,(0.9)^3$$

$$P = \frac{21,870}{0.729} = 30,000$$

2019 - JUNE

[100] The certain sum of money became ₹ 692/- in 2 yrs and ₹ 800/- in 5 yrs then the principle amount is _____

(a) ₹ 520
(b) ₹ 620
(c) ₹ 720
(d) ₹ 820 (1 mark)

Answer:

(b) ∵ The amount of any sum in 5 years = ₹ 800
The amount of same sum in 2 years = ₹ 692
S.I of 3 years = 108

$\therefore$ S.I of 1 year $= \frac{108}{3} = 36$

$\therefore$ S.I of 2 years $= 36 \times 2 = 72$

For 2 years

Amount (A) = ₹ 692

S.I = ₹ 72

P = A – S.I

= 692 – 72

= 620

[101] A sum of money amount to ₹ 6,200 in 2 years and ₹ 7,400 in 3 years as per S.I. then the principal is

(a) ₹ 3,000

(b) ₹ 3,500

(c) ₹ 3,800

(d) None (1 mark)

Answer:

(c) The amount of any sum in 3 years = ₹ 7,400

The amount of same sum in 2 years = ₹ 6,200

S.I of 1 year = ₹ 1,200

S.I of 2 years = ₹ 1,200 × 2

= ₹ 2,400

For 2 years

Amount (A) = ₹ 6,200

S.I = ₹ 2,400

P = A – S.I

= ₹ 6,200 – 2,400

= ₹ 3,800

[102] A sum was invested for 3 years as per C.I. and the rate of interest for first year is 9%, 2nd year is 6% and 3rd year is 3% p.a. respectively. Find the sum if the amount in three years is ₹ 550?

(a) ₹ 250

(b) ₹ 300

(c) ₹ 462.16

Answer:

(c) $A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

$R_1 = 9\%, R_2 = 6\%, R = 3\%, A = 550$

$550 = P\left(1+\frac{9}{100}\right)\left(1+\frac{6}{100}\right)\left(1+\frac{3}{100}\right)$

$550 = P\,(1.09)\,(1.06)\,(1.03)$

$P = \frac{550}{1.09 \times 1.06 \times 1.03}$

$P = \frac{550}{1.190062} = 462.16$

[103] P = ₹ 5,000 R = 15% T = $4^{1/2}$ using $I = \frac{PTR}{100}$ then I will be

(a) ₹ 3,375
(b) ₹ 3,300
(c) ₹ 3,735
(d) None (1 mark)

Answer:

(a) P = ₹ 5,000, R = 15%, $T = 4\frac{1}{2}$ years

$= \frac{9}{2}$ years

$I = \frac{P.R.T}{100} = \frac{5,000 \times 15 \times 9/2}{100}$

= ₹ 3,375

[104] The effective rate of interest does not depend upon

(a) Amount of Principal
(b) Amount of Interest
(c) Number of Conversion Periods
(d) None of these (1 mark)

Answer:

(a) The Effective Rate of Interest does not depends upon principal.

[105] A person wants to lease out a machine costing ₹ 5,00,000 for a 10 year period. It has fixed a rental of ₹ 51,272 per annum payable annually starting from the end of first year. Suppose rate of interest is 10% per annum compounded annually on which money can be invested. To whom this agreement is favourable?

(a) Favour of Lessee
(b) Favour of Lessor
(c) Not for both
(d) Can't be determined (1 mark)

Answer:

(a) Let's analyse this problem from the point of view of the lessor.
If he sells the machine today, he would receive ₹ 5,00,000.
If he leases this machine out, he would receive ₹ 51,272 per year for 10 years.
PV = Annuity × Sum of Discounting Factors
We have -
Annuity (A) = ₹ 51,272
i = 0.10
t = 10 years
No. of Conversion Periods Per Year (NOCPPY) = 1
i / NOCPPY = 0.10/1 = 0.10
n = t × NOCPPY = 10 × 1 = 10
Discount Rate = 0.10
Factor of Discount Rate = 1.10

$$PV = \text{Annuity} \times \text{Sum of Discounting Factors}$$

$$= \text{Annuity} \times \frac{(\text{Factor of Discount Rate})^n - 1}{\text{Discount Rate} \times (\text{Factor of Discount Rate})^n}$$

$$= 51{,}272 \times \frac{(1.10)^{10} - 1}{0.10 \times (1.10)^{10}} = ₹\ 3{,}15{,}044$$

Clearly, leasing is not favourable to lessor. If the lessee purchases this machine today, his cash outflow would be ₹ 5,00,000.
If he leases it, his cash outflow every year would be ₹ 51,272.
PV = ₹ 3,15,044
Therefore clearly, the lease is favourable to lessee.

[106] Let a person invest a fixed sum at the end of each month in an account paying interest 12% per year compounded monthly. If the future value of this annuity after the 12th payment is ₹ 55,000 then the amount invested every month is?

(a) ₹ 4,837
(b) ₹ 4,637
(c) ₹ 4,337
(d) ₹ 3,337 (1 mark)

Answer:

(c) Let A = A

$A_{(n, i)}$ = ₹ 55,000

$$R = \frac{12\%}{12} \text{ p.a.} = 1\% \text{ per month}$$

$$n = 12 , i = \frac{R}{100} = \frac{1}{100} = 0.01$$

Future Value

$$A_{(n, i)} = \frac{A}{i}[(1 + i)^n - 1]$$

$$55,000 = \frac{A}{0.01}[(1 + 0.01)^{12} - 1]$$

$$55,000 = \frac{A}{0.01}[(1.01)^{12} - 1]$$

$$55,000 = \frac{A}{0.01}[1.126825 - 1]$$

$$55,000 = A \times 12.6825$$

$$A = \frac{55,000}{12.6825}$$

$$A = 4336.68 = ₹ 4,337$$

[107] If P^{12} = ₹ 96, and R = 8% compounded annually then P =

(a) ₹ 14,000
(b) ₹ 15,000
(c) ₹ 16,000
(d) ₹ 17,000 (1 mark)

Answer:

(b) if P i^2 = 96 and R = 8% Compound Annually, P = ?

$$P\,i^2 = 96$$

$$P\left(\frac{R}{100}\right)^2 = 96$$

$$P\left(\frac{8}{100}\right)^2 = 96$$

$$P \times (0.08)^2 = 96$$

$$P \times 0.0064 = 96$$

$$P = \frac{96}{0.0064} = ₹\ 15{,}000$$

[108] Determine the present value of perpetuity of ₹ 50,000 per month @ rate of interest 12% p.a. is _____

(a) ₹ 45,00,000

(b) ₹ 50,00,000

(c) ₹ 55,00,000

(d) ₹ 60,00,000 (1 mark)

Answer:

(b) Given, Annual Installment (A) = ₹ 50,000

$$R = \frac{12}{12}\% \text{ monthly}$$

$$= 1\% \text{ monthly}$$

$$i = \frac{R}{100} = \frac{1}{100} = 0.01$$

For Perpetuity n = ∞

$$V = \frac{A}{i}\left[1 - (1+i)^{-n}\right]$$

$$V = \frac{A}{i}\left[1 - (1+i)^{-\infty}\right]$$

$$V = \frac{A}{i}\left[\because (1+i)^{-\infty} = 0\right]$$

$$V = \frac{50{,}000}{0.01} = ₹\ 50{,}00{,}000$$

[109] In simple interest if the principal is ₹ 2,000 and the rate and time are the roots of the equation $x^2 - 11x + 30 = 0$ then simple interest is

(a) ₹ 500
(b) ₹ 600
(c) ₹ 700
(d) ₹ 800 (1 mark)

Answer:

(b) Here, principal (P) = ₹ 2,000

given Equ.

$x^2 - 11x + 30 = 0$

$x^2 - 6x - 5x + 30 = 0$

$x(x - 6) - 5(x - 6) = 0$

$(x - 6)(x - 5) = 0$

If $x - 6 = 0$ if $x - 5 = 0$

$x = 6, x = 5$

Rate (R) = 6%, Time (T) = 5 years

$$S.I. = \frac{P.R.T.}{100} = \frac{2,000 \times 6 \times 5}{100}$$

= ₹ 600

2019 - NOVEMBER

[110] A man invests ₹ 12,000 at 10% p.a. and another sum of money at 20% p.a. for one year. The total investment earns at 14% p.a. simple interest the total investment is:

(a) ₹ 8,000
(b) ₹ 20,000
(c) ₹ 14,000
(d) ₹ 16,000 (1 mark)

Answer:

(b) Let the another sum of money be ₹ x

So total investment ₹ (12,000 + x)

$$\because SI = \frac{P \times R \times T}{100}$$

According to ques,

$$\frac{12,000 \times 10 \times 1}{100} + \frac{x \times 20 \times 1}{100} = (12,000 + x) \times \frac{14}{100} \times 1$$

$$1,20,000 + 20x = 1,68,000 + 14x$$

$$6x = ₹\ 48,000$$

$$x = ₹\ 8,000$$

So total investment

$$= ₹\ (12,000 + x)$$

$$= ₹\ (12,000 + 8,000)$$

$$= ₹\ 20,000$$

[111] The difference in simple interest of a sum invested of ₹ 1,500 for 3 years is ₹ 18. The difference in their rates is:

(a) 0.4

(b) 0.6

(c) 0.8

(d) 0.10 (1 mark)

Answer:

(a) Let the two rates of interest be $r_1\%$, $r_2\%$

$$SI = \frac{P \times R \times T}{100}$$

According to ques,

$$(SI)_1 - (SI)_2 = 18$$

$$1500 \times \frac{r_1}{(100)} \times 3 - 1500 \times \frac{r_2}{(100)} \times 3 = 18$$

$$\frac{4500}{(100)} (r_1 - r_2) = 18$$

$$(r_1 - r_2) = 0.4$$

So, the difference in their rates is 0.4.

[112] Find the effective rate of interest on ₹ 10,000 on which interest is payable half yearly at 5% p.a.

(a) 5.06%
(b) 4%
(c) 0.4%
(d) 3% (1 mark)

Answer:

(a) Here, R = 5% T = 1 yr

Since interest is payable half yearly

$R = \frac{5}{2}\%$ and $T = 1 \times 2 = 2$ year

$$= \left[\left(1 + \frac{R}{100}\right)^{T} - 1\right] \times 100$$

$$= \left[\left(1 + \frac{5}{2 \times 100}\right)^{2} - 1\right] \times 100$$

$$= [(1.025)^2 - 1] \times 100$$

$$= [0.050625] \times 100$$

$$= 5.0625\%$$

$$= 5.06\% \text{ (approx)}$$

[113] Find the effective rate of interest at 10% p.a. when interest is payable quarterly.

(a) 10.38%
(b) 5%
(c) 5.04%
(d) 4% (1 mark)

Answer:

(a) Here; R = 10% T = 1 year

Since interest is payable quarterly

$R = \frac{10\%}{4}$ $T = 1 \times 4$ years

$$= \left[\left(1 + \frac{R}{100}\right)^{T} - 1\right] \times 100$$

$$= \left[\left(1 + \frac{10}{4 \times 100}\right)^4 - 1\right] \times 100$$

$$= [(1.025)^4 - 1] \times 100$$

$$= 10.38\%$$

[114] What will be the population after 3 years when present population is ₹ 25,000 and population increases at the rate of 3% in I year, at 4% in II year and at 5% in III year?

(a) ₹ 28,119
(b) ₹ 29,118
(c) ₹ 27,000
(d) ₹ 30,000 (1 mark)

Answer:

(a) When population increases at the rate of r_1% in Ist year, r_2% in IInd year and r_3% in IIIrd year.

Population after 't' years is given by

$$A = P\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)\left(1 + \frac{r_3}{100}\right)$$

Here, P = ₹ 25,000

$r_1 = 3\%$, $r_2 = 4\%$, $r_3 = 5\%$

$$\text{Population after 3 years} = 25{,}000\left(1 + \frac{3}{100}\right)\left(1 + \frac{4}{100}\right)\left(1 + \frac{5}{100}\right)$$

$$= 25000\ (1.03)\ (1.04)\ (1.05)$$

$$= 28119$$

[115] The value of scooter is ₹ 10,000 find its value after 7 years if rate of depreciation is 10% p.a.

(a) ₹ 4,782.96
(b) ₹ 4,278.69
(c) ₹ 42,079
(d) ₹ 42,000 (1 mark)

Answer:

(a) We know,

$$A = P\left(1 - \frac{R}{\ldots}\right)^T$$

Where, A scrap value
P Present value
R Rate of depreciation
T Time

Here,

$P = ₹ 10,000, R = 10\%, T = 7$ years

$$A = 10,000 \left(1 - \frac{10}{100}\right)^7$$

$$A = 10,000 (0.9)^7$$

$$A = 4782.96$$

So value of scooter is ₹ 4782.96 after 7 years

[116] SI = 0.125P at 10% p.a. Find time.

(a) 1.25 years
(b) 25 years
(c) 0.25 years
(d) None (1 mark)

Answer:

(a) We know,

$$SI = \frac{P \times R \times T}{100}$$

Here, $SI = 0.125\ P$ $R = 10\%$

Put these values in the above formula

$$0.125\ P = P \times \frac{10}{100} \times T$$

$$T = \frac{0.125P \times 100}{10 \times R}$$

$T = 1.25$ years

[117] Scrap value of a machine valued at ₹ 10,00,000, after 10 years within depreciation at 10% p.a.:

(a) ₹ 3,48,678.44
(b) ₹ 3,84,679.45
(c) ₹ 4,00,000
(d) ₹ 3,00,000 (1 mark)

Answer:

(a) We know,

$$A = P\left(1 - \frac{R}{100}\right)^{T}$$

Where A ⇒ scrap value after 't' years.

P ⇒ Present value R ⇒ Rate of depreciation

Here, P = ₹ 10,00,000, R = 10%, T = 10 years

$$A = 10,00,000\left(1 - \frac{10}{100}\right)^{10} = ₹\ 348678.44$$

So value of machine after 10 year will be ₹ 348678.44

[118] The difference between CI and SI for 2 years, is 21. If rate of interest is 5% find principal

(a) ₹ 8,400

(b) ₹ 4,800

(c) ₹ 8,000

(d) ₹ 8,200 (1 mark)

Answer:

(a) Method 1

Difference between SI and CI for 2 years is given by

$$CI - SI = P\left(\frac{R}{100}\right)^{2}$$

$$21 = P\left(\frac{5}{100}\right)^{2}$$

$$P = \frac{21 \times 100 \times 100}{5 \times 5} = ₹\ 8400$$

Method 2

$$CI = P\left[\qquad\right]$$

SI =

CI = P (0.1025) SI = P(0.1)

CI - SI = 0.1025 P - 0.1P

21 = 0.0025 P

$P = ₹ \frac{21}{0.0025} = ₹ 8400$

So principal is ₹ 8400

[119] Present value of a scooter is ₹ 7,290 if its value decreases every year by 10% then its value before 3 years is equal to:

(a) 10,000

(b) 10,500

(c) 20,000

(d) 20,500 (1 mark)

Answer:

(a) Let the value of the scooter be ₹ x. before 3 years

Before three years,

A (scrap value after 3 year) = ₹ 7,290

R = 10% (dep Rate)

T = 3 years

$A = P\left(1 - \frac{R}{100}\right)^T$

$7,290 = x\left(1 - \frac{10}{100}\right)^3$

x = ₹ 10,000

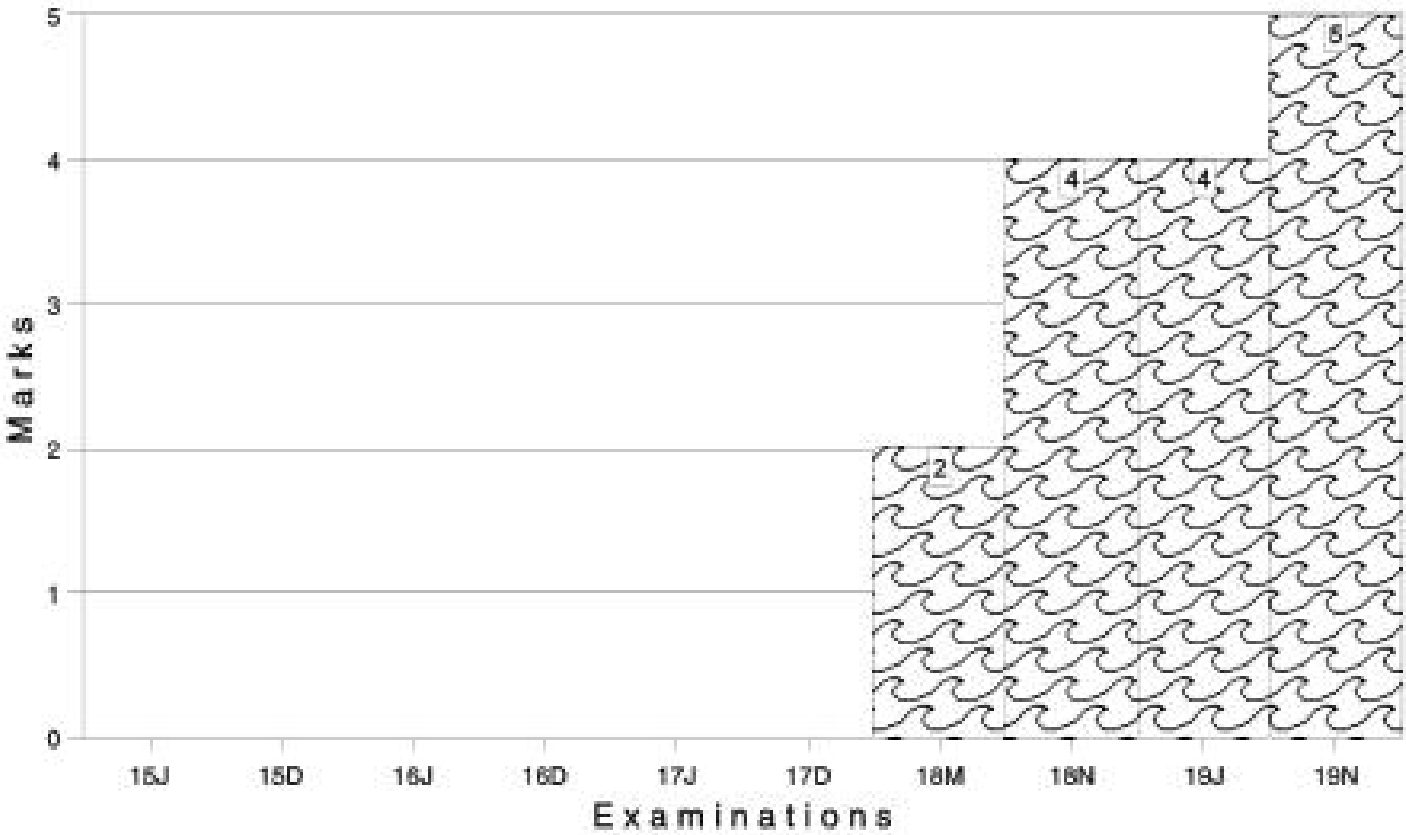
CHAPTER

5 PERMUTATIONS AND COMBINATIONS

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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MULTIPLE CHOICE QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] The number of triangles that can be formed by choosing the vertices from a set of 12 points, seven of which lie on the same straight line, is:

(a) 185 (b) 175

(c) 115 (d) 105 (1 mark)

Answer:

(a) The number of triangles that can be formed from a set of 12 points $= {}^{12}C_3 = 220$

Since seven points are on the same line, therefore no triangle can be formed from these points.

i.e. Numbers of triangles

$= {}^{12}C_3 - {}^{7}C_3 = 220 - 35 = 185$

[2] A code word is to consist of two English alphabets followed by two distinct numbers between 1 and 9. How many such code words are there?

(a) 6,15,800 (b) 46,800

(c) 7,19,500 (d) 4,10,800 (1 mark)

Answer:

(b) The number of ways of filling the first two places with English alphabets

$= 26 \times 25 = 650$

The number of ways of filling the last two places with distinct numbers

$= 9 \times 8 = 72$

∴ The number of code words that can be formed are

$= 650 \times 72$

$= 46,800$

[3] A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics part-II unless Mathematics part-I is also borrowed? In how many ways can he choose the three books to be borrowed?

(a) 41 (b) 51
(c) 61 (d) 71 (1 mark)

Answer:

(a) There are two cases possible :

Case -1 When Mathematics Part - II is borrowed (i.e. it means Mathematics Part-I has also been borrowed).

Number of ways = ${}^{6}C_{1} = 6$ ways

Case -2 When Mathematics part-II is not borrowed (i.e. 3 books are to be selected out of 7).

Number of ways = ${}^{7}C_{3} = 35$ ways

Therefore, total number of ways

$= 35 + 6 = 41$ ways.

2007 - FEBRUARY

[4] An examination paper consists of 12 questions divided into two parts A and B. Part A contains 7 questions and part B contains 5 questions. A candidate is required to attempt 8 questions selecting at least 3 from each part. In how many maximum ways can the candidate select the questions?

(a) 35 (b) 175
(c) 210 (d) 420 (1 mark)

Answer:

(d) The candidate can select 8 questions by selecting at least three from each part in the following ways :

(a) 3 questions from part A and 5 questions from part B

$= {}^{7}C_{3} \times {}^{5}C_{5} = 35$ ways

(b) 4 questions from part A and part B each

(c) 5 questions from part A and 3 questions from part B $= {}^{7}C_5 \times {}^{5}C_3 = 210$ ways.

Hence, the total number of ways in which the candidate can select the question will be = 35 + 175 + 210 = 420 ways.

[5] A Supreme Court Bench consists of 5 judges. In how many ways, the bench can give a majority decision?

(a) 10 (b) 5

(c) 15 (d) 16 (1 mark)

Answer:

(d) Majority decision can be taken by a bench of 5 judges in

$= ({}^{5}C_3 + {}^{5}C_4 + {}^{5}C_5)$ ways

$= 10 + 5 + 1 = 16$ ways

[6] Given : $P(7, k) = 60\ P(7, k-3)$. Then :

(a) $k = 9$ (b) $k = 8$

(c) $k = 5$ (d) $k = 0$ (1 mark)

Answer:

(c) $p(7, k) = 60\ P(7, k-3)$

$= {}^{7}p_k = 60\ {}^{7}p_{k-3}$

$$\frac{7!}{(7-k)!} = 60 \times \frac{7!}{[7-(k-3)]!} = \frac{1}{(7-k)!} = 60 \times \frac{1}{(7-k+3)!} = \frac{(10-k)!}{(7-k)!} = 60$$

$$\frac{(10-k)(9-k)(8-k)(7-k)!}{(7-k)!} = 60$$

$(10-k)(9-k)(8-k) = 60$

$720 - 242k + 27k^2 - k^3 = 60$

$660 - 242k + 27k^2 - k^3 = 0$

$k^3 - 27k^2 + 242k - 660 = 0$

$(k-5)(k^2 - 22k + 132) = 0$

$k = 5$, since $k^2 - 22k + 132 = 0$ gives imaginary roots.

[7] The number of ways in which n books can be arranged on a shelf so that two particular books are not together is :

(a) $(n-2) \times (n-1)!$ (b) $(n-2) \times (n+1)!$

Answer:

(a) We first find the total number of arrangements in which all n books can be arranged on the shelf without any restriction. This number is ${}^{n}p_{n} = n!$

Then, we find the total number of arrangements in which two particular books are together.

The books can be together in ${}^{2}p_{2} = 2! = 2$ ways Now, we consider those two books which are kept together as one composite book and with the rest of the (n – 2) books form (n – 1) books; which are to be arranged on the shelf, the number of arrangements is ${}^{n-1}P_{n-1}$ = (n – 1)! ways. The total number of arrangements on which the two particulars books are together = 2x (n – 1)!

Hence, the required number of arrangements of n books on a shelf so that two particular books are not together = n! – 2 × (n – 1)!

= n (n – 1)! – 2(n – 1)!

= (n – 2)(n – 1)!

2007 - MAY

[8] In how many ways can the letters of the word FAILURE be arranged so that the consonants may occupy only odd positions?

(a) 576 (b) 476

(c) 376 (d) 276 (1 mark)

Answer:

(a) The word FAILURE have 7 letters, out of which 3 letters, F, L and R are consonants and 4 letters namely A, I, U and E are vowels.

The four positions to be filled up with consonants are indicated below:

Since, there are only 3 consonant, the total number of

Again, for the arrangement described above, the four vowels can occupy the four remaining positions not occupied by consonants = 4! = 24 ways.

Hence the total number of arrangements are :

$= 24 \times 24 = 576$

[9] Five bulbs of which three are defective are to be tried in two lights-points in a dark-room. In how many trials the room shall be lighted?

(a) 10 (b) 7

(c) 3 (d) None of these (1 mark)

Answer:

(b) Total number of trials = ${}^5C_2 = 10$ ways No. of trials for no light in the room = ${}^3C_2 = 3$

∴ The room shall be lighted in $= 10 - 3 = 7$ ways.

[10] In how many ways can a party of 4 men and 4 women be seated at a circular table, so that no two women are adjacent?

(a) 164 (b) 174

(c) 144 (d) 154 (1 mark)

Answer:

(c) The number of ways in which 4 men can be seated at the circular table so that there is a vacant seat between every pair of men is $= (4 - 1)! = 3! = 6$ ways

Therefore, Number of ways in which 4 vacant seats can be occupied by 4 women = 4! = 24 ways.

Required number of ways = $6 \times 24 = 144$ ways.

2007 - AUGUST

[12] If ${}^6P_r = 24\ {}^6C_r$, then find r :

(a) 4 (b) 6

(c) 2 (d) 1 (1 mark)

Answer:

(a) ${}^6P_r = 24\ {}^6C_r$

$$\frac{6!}{(6-r)!} = 24\ \frac{6!}{r!\times(6-r)!}$$

$$! = \frac{24}{r!}$$

$$r! = \frac{24}{4!}$$

$r! = 4!$ Therefore, $r = 4$

[13] Find the number of combinations of the letters of the word COLLEGE taken four together :

(a) 18 (b) 16

(c) 20 (d) 26 (1 mark)

Answer:

(a) There are 7 letters of five different kinds C, O, (L, L), (E, E), G. Thus, following cases arise :

(i) All the four letters are different :
The required number of combination = ${}^5C_4 = 5$.

(ii) 2 letters are alike and 2 are different; There are 2 pairs of alike letters, viz, (L, L),(E, E).One pair can be chosen in ${}^2C_1$ ways. Remaining 2 different letters can be selected from remaining different letters in ${}^4C_2$ ways. Therefore the number of combinations = ${}^2C_1 \times {}^4C_2 = 2 \times 6 = 12$

(iii) 2 Letters are alike of one kind, and 2 are alike of other kind : Two pairs of similar letter can be chosen in ${}^{2}C_{2} = 1$ way.

Hence, the total number of required combination is = 5 + 12 + 1 = 18 ways.

[14] How many words can be formed with the letters of the word 'ORIENTAL' so that A and E always occupy odd places:

(a) 540 (b) 8640

(c) 8460 (d) 8450 (1 mark)

Answer:

(b) There are 4 odd places and 2 letters, hence this can be done in $= {}^{4}P_{2} = 12$ ways. The remaining 6 letters i.e. O, R, I, N, T, L, Can be arranged in 6! ways.

Hence, the total number of arrangements = 6! × 12 = 8640

2007 - NOVEMBER

[15] If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$, find x :

(a) 999 (b) 998

(c) 997 (d) 1000 (1 mark)

Answer:

(a) ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$

Since ${}^{n+1}C_{r} = {}^{n}C_{r} + {}^{n}C_{r-1}$

${}^{999+1}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$

${}^{999}C_{98} + {}^{999}C_{97} = {}^{999}C_{97} + {}^{x}C_{901}$

${}^{999}C_{98} + {}^{999}C_{97} = {}^{999}C_{97} + {}^{x}C_{98}$ $(\because {}^{n}C_{r} = {}^{n}C_{n-r})$

This implies that x = 999

[16] How many numbers greater than a million can be formed with the digits 4, 5, 5, 0, 4, 5, 3 ?

(a) 260 (b) 360

(c) 280 (d) 380 (1 mark)

Answer:

(b) Total number of numbers that can be formed from the digits

$4,5,5,0,4,5,3 = \frac{7!}{2! \times 3!} = 420$ ways

Out of these 420 numbers, some will begin with 0 and are less than one million, so they are to be rejected.

∴ Number of numbers beginning with

$0 = \frac{6!}{2! \times 3!} = 60$ ways

Hence required numbers of numbers

$= 420 - 60 = 360$ ways.

[17] A building contractor needs three helpers out of ten men supply. In how many ways can these selections take place?

(a) 36 (b) 15

(c) 150 (d) 120 (1 mark)

Answer:

(d) Since, there is no regard for order, the contractor can select any of the three helpers out of 10 men.

This can be done in ${}^{10}C_3$ ways i.e. 120 ways.

2008 - FEBRUARY

[18] There are three blue balls, four red balls and five green balls. In how many ways can they be arranged in a row?

(a) 26,720 (b) 27,720

(c) 27,820 (d) 26,620 (1 mark)

Answer:

(b) Number of ways in which these balls can be arranged

$= \frac{12!}{3! \times 4! \times 5!}$

$= 27,720$ ways.

[19] If C(n, r) : C(n, r +1) = 1 : 2 and C (n, r + 1): C (n, r + 2) = 2:3, determine the value of n and r :

(a) (14, 4) (b) (12, 4)

(c) (14, 6) (d) None. (1 mark)

Answer:

(a) $\frac{C(n,r)}{C(n,r,+1)} = \frac{1}{2}$

$$\frac{\frac{n!}{r! \times (n-r,)!}}{} = \frac{1}{2}$$

$$= \frac{n!}{(r+1)! \times (n-r-1)!}$$

$$= \frac{n!}{r! \times (n-r,)!} \times \frac{(r+1)! \times (n-r-1)!}{n!} = \frac{1}{2}$$

$$= \frac{(r+1)! \times (n-r-1)!}{r! \times (n-r)!} = \frac{1}{2}$$

$$\frac{(r+1) \times r! \times (n-r-1)!}{r! \times (n-r) \times (n-r-1)!} = \frac{1}{2}$$

$$\frac{r+1}{n-r} = \frac{1}{2}$$

$$2r + 2 = n - r$$

$$n - 3r = 2 \ldots\ldots\ldots\ldots (1)$$

$$\frac{C(n,r+1)}{C(n,r+2)} = \frac{2}{3}$$

$$\frac{\frac{n!}{(r+1)! \times (n-r-1)!}}{} = \frac{2}{3} \frac{n!}{(r+2)! \times (n-r-2)!}$$

$$= \frac{n!}{(r+1)! \times (n-r-1)!} \times \frac{(n+2)! \times (n-r-2)!}{n!} = \frac{2}{3}$$

$$= \frac{(r+2) \times (r+1)! \times (n-r-2)!}{(r+1)! \times (n-r-1) \times (n-r-2)!} = \frac{2}{3}$$

$= \frac{r+2}{n-r-1} = \frac{2}{3}$

$= 3r + 6 = 2n - 2r - 2$

$= 2n - 5r = 8$.....(2)

solving (1) and (2) simultaneously.

$2n - 5r = 8$

$3n - 6r = 4$

(–) (+) (–)

$r = 4$

substituting $r = 4$ in (1)

$n - 3 \times 4 = 2$ or $n = 2 + 12 = 14$

Therefore, $n = 14$ and $r = 4$

2008 - JUNE

[20] Six seats of articled clerks are vacant in a 'Chartered Accountant Firm'. How many different batches of candidates can be chosen out of ten candidates?

(a) 216 (b) 210

(c) 220 (d) None (1 mark)

Answer:

(b) The number of ways in which 6 articled clerks can be selected out of 10 candidates = ${}^{10}C_6 = 210$ ways.

[21] Six persons A, B, C, D, E and F are to be seated at a circular table. In how many ways can this be done, if A must always have either B or C on his right and B must always have either C or D on his right?

(a) 3 (b) 6

(c) 12 (d) 18 (1 mark)

Answer:

(d) Using the given restrictions, we must have AB or AC and BC or BD. Therefore, we have the following alternatives :

(i) ABC, D, E, F which gives (4 – 1)! or 3! ways.

(ii) ABD, C, E, F which gives (4 – 1)! or 3! ways.

(iii) AC, BD, E, F which gives (4 – 1)! or 3! ways.

Hence, the total number of ways are

= 3! + 3! + 3!

= 6 + 6 + 6 = 18 ways

2008 - DECEMBER

[22] If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$ then find the value of 'n'

(a) 2 (b) 3

(c) 4 (d) 5 (1 mark)

Answer:

(b) ${}^nP_r = {}^nP_{r+1}$ (Since, when ${}^nP_r = {}^nP_s$ then r + s = 2n – 1)

$\Rightarrow$ n = r + 1.....(1)

and

${}^nC_r = {}^nC_{r-1}$

$\Rightarrow$ n = 2r – 1.....(2) (Since, when ${}^nC_r = {}^nC_s$, then r + s = n or r = s)

From (1) and (2), we have

r + 1 = 2r – 1

r = 2 and n = 2 + 1 = 3 (from 1 above)

[23] How many six digit telephone numbers can be formed by using 10 distinct digits?

(a) 10^6 (b) 6^{10}

(c) ${}^{10}C_6$ (d) ${}^{10}P_6$ (1 mark)

Answer:

(d) 10 distinct digits can be from 0 – 9

To form a 6 – digit telephone number, at each place a distinct no. is to be placed.

∴ Therefore, it can be done in

10 × 9 × 8 × 7 × 6 × 5 or simply ${}^{10}P_6$

[24] In how many ways a committee of 6 members can be formed from a group of 7 boys and 4 girls having at least 2 girls in the committee.

(a) 731 (b) 137

(c) 371 (d) 351 (1 mark)

Answer:

(c) The committee of 6 members is to include at least 2 girls. This can be constituted as follows :

(i) 4 **boys** and 2 **girls,** It can be done is ${}^{7}C_4 \times {}^{4}C_2$ ways

(ii) 3 **boys** and 3 **girls,** It can be done is ${}^{7}C_3 \times {}^{4}C_3$ ways

(iii) 2 **boys** and 4 **girls,** It can be done in ${}^{7}C_2 \times {}^{4}C_4$ ways

Thus, the total number of ways of selecting the committee

$$= {}^{7}C_4 \times {}^{4}C_2 + {}^{7}C_3 \times {}^{4}C_3 + {}^{7}C_2 \times {}^{4}C_4$$

$$= \frac{7!}{4! \times 3!} \times \frac{4!}{2! \times 2!} + \frac{7!}{3! \times 4!} \times \frac{4!}{3! \times 1!} + \frac{7!}{2! \times 5!} \times \frac{4!}{4! \times 0!}$$

$$= 35 \times 6 + 35 \times 4 + 21 \times 1$$

$$= 371$$

2009 - JUNE

[25] Number of ways of painting a face of a cube by 6 colours is_____

(a) 36 (b) 6

(c) 24 (d) 1 (1 mark)

Answer:

(b) Number of ways of painting a face of a cube by 6 colours is 6, since any of the six colours can be used to paint the face of the cube.

[26] If _____ ${}^{18}C_r = {}^{18}C_{r+2}$ find the value of ${}^{r}C_5$.

(a) 55 (b) 50

(c) 56 (d) None of these (1 mark)

Answer:

(c) ${}^{18}C_r = {}^{18}C_{r+2}$

Since, ${}^{n}C_x = {}^{n}C_y$

$n = x + y$

So here

$18 = 2r + 2$

$\boxed{r = 8}$

i.e. $r = 8$

Now, ${}^{r}C_5$

$= {}^{8}C_5$

$= \frac{8!}{5!(8-5)!}$ $\left[\text{since } {}^{n}C_r \frac{n!}{r!(n-r)!}\right]$

$= \frac{40320}{120 \times 6}$

$\therefore {}^{8}C_5 = 56$

[27] 7 books are to be arranged in such a way so that two particular books are always at first and last place. Final the number of arrangements.

(a) 60 (b) 120

(c) 240 (d) 480 (1 mark)

Answer:

(c) Since 2 particular books are to be kept always at the first and last place, so if we fix their places, the remaining 5 books can be arranged in ${}^{5}P_5$ i.e. 5 ! ways.

Those, 2 books can also change their places in ${}^{2}P_2$ i.e. 2! ways.

$\therefore$ The total number of arrangements are

$= 5! \times 2!$

$= 120 \times 2$

$= 240$ ways.

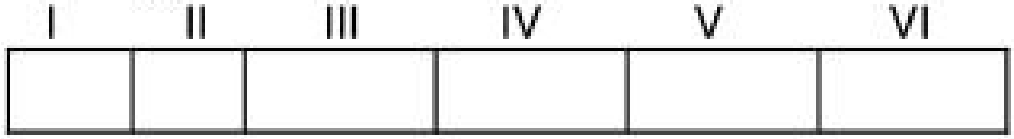
[28] Find the number of arrangements in which the letters of the word 'MONDAY' be arranged so that the words thus formed begin with 'M' and do not end with 'N'.

(a) 720 (b) 120

(c) 96 (d) None. (1 mark)

Answer:

We have to arrange 6 letters.



In the first place only M can come. Therefore no. of ways of filling first place is 1! = 1 way.

In the last place, any letter other than 'M' and 'N' can come. Therefore, no. of ways in which last place can be filled is 4 ways i.e. (6-2) ways.

The remaining 4 places have no restriction. They can be filled by any of the remaining 4 letters. So, the no. of ways of filling those 4 place are ${}^4P_4$ i.e. 4 ! ways.

So, the total no. of words that can be formed are = (1 × 4 × 4!) ways
= 4 × 24 = 96 ways.

[29] In how many ways can 17 billiard balls be arranged if 7 of them are black, 6 red and 4 white ?

(a) 4084080 (b) 1
(c) 8048040 (d) None of these (1 mark)

Answer:

(a) Number of ways of arranging 17 billiard balls are = ${}^{17}P_{17}$ i.e. 17 ! Ways.

But 7 of them are black, 6 red and 4 white, therefore, the number of ways of arranging the balls are = $\frac{17!}{7! \times 6! \times 4!}$ = 40,84,080.

2009 - DECEMBER

[30] (n + 1) ! = 20 (n – 1) !, find n

(a) 6 (b) 5
(c) 4 (d) 10 (1 mark)

Answer:

(c) (n+1) ! = 20(n –1) !
(n+1) (n) (n – 1) ! = 20 (n –1) !
n (n + 1) = 20
$n^2 + n = 20$
$n^2 + n - 20 = 0$
$n^2 + 5n - 4n - 20 = 0$
n(n +5)– 4 (n+5) = 0
(n – 4) (n +5) = 0
n = 4 and n = – 5
Since n cannot be negative, therefore n = 4

[31] Out of 4 gents and 6 ladies, a committee is to be formed. Find the number of ways the committee can be formed such that it comprises of at least 2 gents and the number of ladies should at least be double

(a) 94 (b) 132
(c) 136 (d) 104 (1 mark)

Answer:

(c) There are 4 gents and 6 ladies.

Method 1 ⇒ 2 gents are selected.

No. of ways 2 gents can be selected - $^4C_2$

Now, ladies could either be 4, or 5, or 6.

No. of ways of selecting 4 ladies OR 5 ladies OR 6 ladies = $^6C_4 + {}^6C_5 + {}^6C_6$

∴ No. of ways = $^4C_2 \times (^6C_4 + {}^6C_5 + {}^6C_6)$

Method 2 - 3 gents are selected.

No. of ways 3 gents can be selected = $^4C_3$

Now, there should be 6 ladies.

No. of ways 6 ladies can be selected = $^6C_6$

∴ Total No. of ways = $^4C_3 \times {}^6C_6$

Therefore total ways = $\{^4C_2 \times (^6C_4 + {}^6C_5 + {}^6C_6)\} + (^4C_3 \times {}^6C_6)$

= 132 + 4 = 136

2010 - JUNE

[32] Six points are on a circle. The number of quadrilaterals that can be formed are:

(a) 30 (b) 360
(c) 15 (d) None of the above (1 mark)

Answer:

(c) No. of quadrilaterals that can be formed from 6 points on a circle

$= {}^6C_4$

$= \frac{6!}{2!4!}$

$= \frac{30}{2}$

$= 15$

[33] The number of ways of arranging 6 boys and 4 girls in a row so that all 4 girls are together is :

(a) 6!. 4! (b) 2 (7!. 4!)

(c) 7!. 4! (d) 2. (6!. 4!) (1 mark)

Answer:

(c) Let us assume 4 Girls to be seated together so they can be arranged in 4! ways.

Now, if we assume 4 Girls to be one single group,

Total number of ways of arranging Boys and Girls is 7!

Required numbers of ways are $7! \times 4!$

[34] How many numbers not exceeding 1000 can be made from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if repetition is not allowed.

(a) 364 (b) 585

(c) 728 (d) 819 (1 mark)

Answer:

(b) Total number of 2 digit number that can be formed $= 9 \times 8 = 72$

Total number of 3 digit number that can be formed $= 9 \times 8 \times 7 = 504$

Total number of 1 digit number $= 9$

$\therefore$ Total numbers formed $= 9 + 72 + 504 = 585$

2010 - DECEMBER

[35] A garden is having 6 tall trees in a row. In how many ways can 5 children stand, one in a gap between the trees in order to pose for a photograph?

(a) 24 (b) 120

(c) 720 (d) 30 (1 mark)

Answer:

(b) 5 places between the trees.

[36] ${}^{15}C_3 + {}^{15}C_{13}$ is equal to :

(a) 16_{c_3} (b) $30_{c_{15}}$

(c) $15_{c_{15}}$ (d) $15_{c_{15}}$ (1 mark)

Answer:

(a) ${}^{15}C_3 + {}^{15}C_{13}$

${}^{15}C_3 + {}^{15}C_2$ [$\because {}^nC_r = {}^nC_{n-r}$]

$= {}^{16}C_3$ [$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$]

[37] How many ways a team of 11 players can be made out of 15 players if one particular player is not to be selected in the team.

(a) 364 (b) 728

(c) 1,001 (d) 1,234 (1 mark)

Answer:

(a) If one particular player is not selected

No. of ways $= {}^{14}C_{11}$

$= {}^{14}C_3$

$= \frac{14 \times 13 \times 12}{3 \times 2 \times 1} = 364$

2011 - JUNE

[38] Find the number of arrangements of 5 things taken out of 12 things, in which one particular thing must always be included.

(a) 39,000 (b) 37,600

(c) 39,600 (d) 36,000 (1 mark)

Answer:

(c) No. of arrangement when one particular thing is always taken is given by r. ${}^{n-1}P_{r-1}$

here, $n = 12$ $r = 5$

$\therefore$ 5. ${}^{12-1}P_{5-1} = 39,600$

2011 - DECEMBER

[39] In how many ways 3 prizes out of 5 can be distributed amongst 3 brothers equally?

(a) 10 (b) 45

(c) 60 (d) 120 (1 mark)

Answer:

(c) No. of ways = ${}^{n}p_{r}$

Here n = 5, r = 3

$= {}^{5}P_{3}$

$= \frac{\lfloor 5}{\lfloor 5-3}$

$= \frac{5 \times 4 \times 3 \lfloor 2}{\lfloor 2}$

= 60

[40] There are 12 questions to be answered in Yes or No. How many ways can these be Answered?

(a) 1024 (b) 2048

(c) 4096 (d) None (1 mark)

Answer:

(c) No. of ways = $n^{r} = 2^{12} = 4,096$

2012 - JUNE

[41] The letters of the word "VIOLENT" are arranged so that the vowels occupy even place only. The number of permutations is _____.

(a) 144 (b) 120

(c) 24 (d) 72 (1 mark)

Answer:

(a) In word "VIOLENT"

No. of vowel = 3 (I, O, E)

No. of consonant = 4 (V, L, N, T)

Total no of letter = 7

OEOEOEO O → Odd

E → Even

There are 3 Even places and 4 Odd places so 3 vowels can fill in 3 even places and 4 consonant can fill in 4 Odd places. Then total number of ways = ${}^3P_3 \ {}^4P_4$

$= 3! \times 4!$

$= 6 \times 24$

$= 144$

[42] If ${}^nP_4 = 20 ({}^nP_2)$ then the value of 'n' is _____.

(a) - 2 (b) 7

(c) - 2 and 7 both (d) None of these. (1 mark)

Answer:

(b) Given ${}^nP_4 = 20. {}^nP_2$

$$\rightarrow \frac{\lfloor n}{\lfloor n-4} = 20. \frac{\lfloor n}{\lfloor n-2}$$

or $(n - 2)(n - 3) = 20$

or $n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

$n(n - 7) + 2(n - 7) = 0$

$\rightarrow n = 7$ or $n = -2$ (not possible)

2012 - DECEMBER

[43] A man has 3 sons and 6 schools within his reach. In how many ways, he can send them to school, if no two of his sons are to read in the same school?

(a) ${}^6P_2$ (b) ${}^6P_3$

(c) 6^3 (d) 3^6 (1 mark)

Answer:

(b) Total school (n) = 6
No. of son (r) = 3
No. of ways = ${}^{n}P_{r}$ = ${}^{6}P_{3}$

[44] How many permutations can be formed from the letters of the word "DRAUGHT", if both vowels may not be separated ?

(a) 720 (b) 1,440
(c) 140 (d) 1,000 (1 mark)

Answer:

(b) Words 'DRAUGHT'
If both vowels may not be separated this mean that both vowels comes together

D	R	G	H	T	AU
6	5	4	3	2	1

No. of ways if two vowels comes together
$= 6 ! \times 2 ! = 720 \times 2 = 1,440$

[45] If ${}^{13}C_{6} + 2\ {}^{13}C_{5} + {}^{13}C_{4} = {}^{15}C_{x}$ then, x = _____.

(a) 6 (b) 7
(c) 8 (d) 9 (1 mark)

Answer:

(a) If ${}^{13}C_{6} + 2\ {}^{13}C_{5} + {}^{13}C_{4} = {}^{15}C_{x}$
$({}^{13}C_{6} + {}^{13}C_{5}) + ({}^{13}C_{5} + {}^{13}C_{4}) = {}^{15}C_{x}$
${}^{14}C_{6} + {}^{14}C_{5} = {}^{15}C_{x}$ $[\because {}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}]$
${}^{15}C_{6} = {}^{15}C_{x}$
on comparing, **x = 6**

2013 - JUNE

[46] A polygon has 44 diagonals then the number of its sides are:

(a) 8 (b) 9
(c) 10 (d) 11 (1 mark)

Answer:

(d) Let number of side of polygon is n.

No. of diagonal = $\frac{n(n-3)}{2}$

$$\frac{44}{1} = \frac{n^2 - 3n}{2}$$

$$88 = n^2 - 3n$$

$$n^2 - 3n - 88 = 0$$

$$(n - 11)(n + 8) = 0$$

If $n - 11 = 0$ If $n + 8 = 0$

$n = 11$ $n = -8$ (not valid)

[47] The number of words that can be formed out of the letters of the word "ARTICLE" so that vowels occupy even place is:

(a) 36 (b) 144

(c) 574 (d) 754 (1 mark)

Answer:

(b) In the word 'ARTICLE'

Total Vowels = A, I, E (3)

Total consonant = R, T, C, L (4)

✓ ✓ ✓

O E O E O E O

No. of ways = ${}^3P_3 \times 4!$

$= 3! \times 4!$

$= 6 \times 24$

$= 144$

[48] Number of ways of shaking hands in a group of 10 persons shaking hands to each other are:

(a) 45 (b) 54

(c) 90 (d) 10 (1 mark)

Answer:

(a) For shaking hand

No. of ways = ${}^{n}C_{2}$

$= {}^{10}C_{2}$

$= \frac{10 \times 9}{2 \times 1}$

$= 45$

2013 - DECEMBER

[49] If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, then 'r' is equal is

(a) 2 (b) 3

(c) 4 (d) 5 (1 mark)

Answer:

(b) Given ${}^{15}C_{3r} = {}^{15}C_{r+3}$ $\left[\because \text{If } {}^{n}C_{x} = {}^{n}C_{y} \text{ then } x + y = n\right]$

Then $3r + r + 3 = 15$

$4r = 15 - 3$

$4r = 12$

$r = 3$

[50] How many different words can be formed with the letters of the word "LIBERTY"

(a) 4050 (b) 5040

(c) 5400 (d) 4500 (1 mark)

Answer:

(b) No. of different word can be formed from the letters of 'LIBERTY' = 7!

$= 5040$

[51] In how many ways can a family consist of three children having different birthdays in a leap year

(a) ${}^{365}C_{3}$ (b) ${}^{366}C_{3} - 3$

(c) $366 \times 365 \times 364$ (d) ${}^{366}C$ (1 mark)

Answer:

(c) No. of way can a family consist of three children have different birthday in a leap years = $366 \times 365 \times 364$

2014 - JUNE

[52] If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$, then the value of x will be :

(a) 999 (b) 998

(c) 997 (d) None of these. (1 mark)

Answer:

(a) If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$

By Properties ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

${}^{999}C_{97} + {}^{x}C_{901} = {}^{1000}C_{98}$ ..... (1)

${}^{999}C_{97} + {}^{999}C_{98} = {}^{999+1}C_{98}$

${}^{999}C_{97} + {}^{999}C_{98} = {}^{1000}C_{98}$

${}^{999}C_{97} + {}^{999}C_{999-98} = {}^{1000}C_{98}$

${}^{999}C_{97} + {}^{999}C_{901} = {}^{1000}C_{98}$ ..... (2)

Comparing (1) & (2) we get

$x = 999$

[53] If six times the number of permutations of 'n' items taken 3 at a time is equal to seven times the number of permutation of (n-1) items taken 3 at a time, then the value of 'n' will be:

(a) 7 (b) 9

(c) 13 (d) 21 (1 mark)

Answer:

(d) The no. of permutation of 'n' item taken 3 at a time $= {}^{n}P_{3}$

The no. of permutation of 'n-1' item taken 3 at a time $= {}^{n-1}P_{3}$

Given

$6.\ {}^{n}P_3 = 7\ {}^{n-1}P_3$

$6.\ \frac{\lfloor n}{\lfloor n-3} = 7\ \frac{\lfloor n-1}{\lfloor n-1-3}$

$6x = \frac{n\lfloor n-1}{n-3\lfloor n-4} 7\ \frac{\lfloor n-1}{\lfloor n-4} \Rightarrow 6n = 7n - 21$

$\rightarrow n = 21$

2014 - DECEMBER

[54] If ${}^{6}P_r = 360$, then the value of 'r' is:

(a) 5 (b) 3

(c) 4 (d) None of these. (1 mark)

Answer:

(c) If ${}^{6}p_r = 360$

$$\frac{6!}{(6-r)!} = \frac{360}{1}$$

$360.\ (6-r)! = 6!$

$360.\ (6-r)! = 720$

$$(6-r)! = \frac{720}{360}$$

$(6-r)! = 2$

$(6-r)! = 2!$

On comparing $6 - r = 2$

$\boxed{r = 4}$

[55] There are 5 books on English, 4 Books on Tamil and 3 books on Hindi. In how many ways can these books be placed on a shelf if the books on the same subjects are to be together?

(a) 1,36,800 (b) 1,83,600

(c) 1,03,680 (d) 1,63,800 (1 mark)

Answer:

(c) No. of English Book = 5
No. of Tamil Book = 4
No. of Hindi Book = 3

$E_1\ E_2\ E_3\ E_4\ E_5$	$T_1\ T_2\ T_3\ T_4$	$H_1\ H_2\ H_3$
1	2	3

No. of ways = $5! \times 4! \times 3! \times 3! = 1,03,680$

[56] 5 Men and 4 Women to sit in a row in such a manner that the women always occupy the even places. The number of such arrangement will be:

(a) 126 (b) 1056
(c) 2080 (d) 2880 (1 mark)

Answer:

(d) Total men = 5, Total women = 4

O E O E O E O E O
1 2 3 4

No. of ways $= {}^4p_4 \times 5!$ [${}^np_n = n!$]
$= 4! \times 5!$
$= 24 \times 120$
$= 2,880$

2015 - JUNE

[57] The four digit numbers that can be formed out of the seven digits 1, 2, 3, 5, 7, 8, 9 such that no digit is repeated in any number and are greater than 3000 are:

(a) 120 (b) 480
(c) 600 (d) 840 (1 mark)

Answer:

(c)

		Th		H		T		U	
1	(3)	1	×	6	×	5	×	4	= 120
2 3	(5)	1	×	6	×	5	×	4	= 120
5	(7)	1	×	6	×	5	×	4	= 120
7 8	(8)	1	×	6	×	5	×	4	= 120
9	(9)	1	×	6	×	5	×	4	= 120
Total No. of ways									= 600

[58] A person has ten friends of whom six are relatives. If he invites five guests such that three of them are his relatives, then the total number of ways in which he can invite them are:

(a) 30 (b) 60

(c) 120 (d) 75 (1 mark)

Answer:

(c) Total Friend = 10

No. of Relative = 6

No. of Friend = 4

No. of ways to invite five guest such that three of them are his relatives.

$= {}^{6}C_{3} \times {}^{4}C_{2}$

$= \frac{6\times5\times4}{3\times2\times1} \times \frac{4\times3}{2\times1}$

$= 20 \times 6$

$= 120$

[59] A student has three books on computer, three books on Economics and five books on Commerce. If these books are to be arranged subject wise, then these can be placed on a shelf in the number of ways:

(a) 25290 (b) 25920

(c) 4230 (d) 4320 (1 mark)

Answer:

(b) No. of ways $= 3!\ 3!\ 5!\ 3!$

$= 6 \times 6 \times 120 \times 6$

$= 216 \times 120$

$= 25,920$

2015 - DECEMBER

[60] An examination paper with 10 questions consists of 6 questions in mathematics and 4 questions in statistic part. At least one question from each part is to be attempted in how many ways can this be done?

(a) 1024 (b) 945

(c) 1005 (d) 1022 (1 mark)

Answer:

(b) Total question =10

No. of Mathematics questions = 6

No. of Statistics questions = 4

No. of ways at least one question of Mathematics

$= (2^6 - 1) = (64 - 1) = 63$

No. of ways at least one question of statistics

$= (2^4 - 1) = (16 - 1) = 15$

Total no. of ways $= 63 \times 15 = 945$

[61] If ${}^nP_r = 720$ and ${}^nC_r = 120$, then value of 'r' is:

(a) 4 (b) 5

(c) 6 (d) 3 (1 mark)

Answer:

(d) ${}^nP_r = 720$ and ${}^nC_r = 120$

we know that

$$\frac{{}^nC_r}{{}^nP_r} = \frac{1}{\lfloor r}$$

$$\frac{120}{720} = \frac{1}{\lfloor r}$$

$$\frac{1}{6} = \frac{1}{\lfloor r}$$

$$\lfloor r = 6$$

$$\lfloor r = \lfloor 3 \qquad [\because \lfloor 3 = 3.2.1 = 6]$$

$$\boxed{r = 3}$$

[62] There are 6 men and 4 women in a group, then the number of ways in which a committee of 5 persons can be formed of them, if the committee is to include at least 2 women are:

(a) 180 (b) 186

(c) 120 (d) 105 (1 mark)

Answer:

(b)

Men	Women
6	4

taken member at a time = 5

at least 2 women are taken then it may be following ways

(a) 3 Men and 2 Women = ${}^6c_3 \times {}^4c_2 = 20 \times 6$

(b) 2 Men and 3 Women = ${}^6c_2 \times {}^4c_3 = 15 \times 4$

(c) 1 Men and 4 Women = ${}^6c_1 \times {}^4c_4 = 6 \times 1$

Total ways $= 20 \times 6 + 15 \times 4 + 6 \times 1$

$= 120 + 60 + 6$

$= 186$

2016 - JUNE

[63] In how many ways can a selection of 6 out of 4 teachers and 8 students be done so as to include at least two teachers?

(a) 220 (b) 672

(c) 596 (d) 968 (1 mark)

Answer:

(b)

Teachers (T)	Students (S)
4	8

Ⓢ

If taken at least two teacher. It may be following ways

(i) 2T and 4S = $^4C_2 \times {}^8C_4 = 6 \times 70 = 420$

(ii) 3T and 3S = $^4C_3 \times {}^8C_3 = 4 \times 56 = 224$

(iii) 4T and 2S = $^4C_4 \times {}^8C_2 = 1 \times 28 = 28$

Total no of ways = 420 + 224 + 28

= 672

[64] There are 10 students in a class including 3 girls. The number of ways to arrange them in a row when any two girls out of three never comes together:

(a) $^8P_3 \lfloor 7$ (b) $^3P_3 \lfloor 7$

(c) $^8P_3 \lfloor 10$ (d) None of these (1 mark)

Answer:

(a) There are 10 students in the class

No. of girls = 3

No. of boys = 7

× B × B × B × B × B × B × B ×

If any two girls never comes together

No of ways = $^8P_3 \cdot 7!$

[65] The maximum number of points of inter section of 10 circles will be:

(a) 2 (b) 20

(c) 90 (d) 180 (1 mark)

Answer:

(c) The maximum number of points of intersection of n circle

$= {}^nP_2$

$= {}^{10}P_2$

$= 10 \times 9$

$= 90$

2016 - DECEMBER

[66] If ${}^{n+1}C_{r+1} : {}^{n}C_{r} : {}^{n-1}C_{r-1} = 8 : 3 : 1$, then n is equal to:

(a) 20 (b) 16

(c) 10 (d) 15 (1mark)

Answer:

(d) If ${}^{n+1}C_{r+1} : {}^{n}C_{r} : {}^{n-1}C_{r-1} = 8 : 3 : 1$

Now ${}^{n+1}C_{r+1} : {}^{n}C_{r} = 8 : 3$

$$\frac{\lfloor n+1}{\lfloor r+1 \lfloor n+1-r-1} : \frac{\lfloor n}{\lfloor r \lfloor n-r} = 8 : 3$$

$$\frac{(n+1)\lfloor n}{(r+1)\lfloor r \lfloor n-r} : \frac{\lfloor n}{\lfloor r \lfloor n-r} = 8 : 3$$

$$\frac{(n+1)\lfloor \cancel{r}}{(r+1)\lfloor \cancel{r} \lfloor \cancel{n-r}} \times \frac{\lfloor \cancel{r} \lfloor \cancel{n-r}}{\lfloor \cancel{r}} = \frac{8}{3}$$

$$\frac{n+1}{r+1} = \frac{8}{3}$$

$3n + 3 = 8r + 8$

$3n - 8r = 8 - 3$

$3n - 8r = 5$ _____ (1)

and ${}^{n}C_{r} : {}^{n-1}C_{r-1} = 3 : 1$

$$\frac{\lfloor n}{\lfloor r \lfloor n-r} : \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-\cancel{1}-r+\cancel{1}} = \frac{3}{1}$$

$$\frac{n\lfloor n-1}{r\lfloor r-1 \lfloor n-r} : \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r} = \frac{3}{1}$$

$$\frac{n\,\lfloor n-1}{r\,\lfloor r-1\rfloor n-r} \times \frac{\lfloor r\rfloor n-r}{\lfloor n-r} = \frac{3}{1}$$

$$\frac{n}{r} = \frac{3}{1}$$

$$n = 3r \qquad \text{(2)}$$

Putting n= 3r in equation (1) we get

$3 \times 3r - 8r = 5$

$9r - 8r = 5$

Putting $\boxed{r = 5}$ in equation (2)

$n = 3r = 3 \times 5 = 15$

[67] The number of numbers between 1,000 and 10,000, which can be formed by the digits 1, 2, 3, 4, 5, 6 without repetition is:

(a) 720 (b) 180

(c) 360 (d) 540 (1 mark)

Answer:

(c) I[st] method

1	TH	H	T	U		
2	6 ×	5	× 4	× 3	=	360
3						
4						
5						
6						

No. of Numbers between 1,000 and 10,000 using the digit 1,2,3,4,5,6 = 360.

II[nd] Method

OR

No. of ways = ${}^{n}p_{r}$ ${}^{6}P_{4} = \frac{\lfloor 6}{\lfloor 6-4}$

$= \frac{\lfloor 6}{\lfloor 2}$

$= \frac{720}{2}$ $= 360$

[68] The number of ways in which 4 persons can occupy 9 vacant seats is:

(a) 6048 (b) 3024

(c) 1512 (d) 4536 (1 mark)

Answer:

(b) No. of ways in which 4 persons can occupy 9 vacant seats $= {}^{9}p_{4}$

$= \frac{\lfloor 9}{\lfloor 9-4}$

$= \frac{\lfloor 9}{\lfloor 5}$

$= \frac{9 \times 8 \times 7 \times 6 \times \lfloor 5}{\lfloor 5}$

$= 9 \times 8 \times 7 \times 6$

$= 3,024$

2017 - JUNE

[69] If ${}^{10}C_{3} + 2.\ {}^{10}C_{4} + {}^{10}C_{5} = {}^{n}C_{5}$ then value of n is:

(a) 10 (b) 11

(c) 12 (d) 13 (1 mark)

Answer:

(c) If ${}^{10}C_3 + 2\,{}^{10}C_4 + {}^{10}C_5 = {}^{n}C_5$

$${}^{10}C_3 + {}^{10}C_4 + {}^{10}C_4 + {}^{10}C_5 = {}^{n}C_5$$

$$({}^{10}C_5 + {}^{10}C_4) + ({}^{10}C_4 + {}^{10}C_3) = {}^{n}C_5$$

$${}^{10+1}C_5 + {}^{10+1}C_4 = {}^{n}C_5 \qquad (\because {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r)$$

$${}^{11}C_5 + {}^{11}C_4 = {}^{n}C_5$$

$${}^{11+1}C_5 = {}^{n}C_5$$

$${}^{12}C_5 = {}^{n}C_5$$

on comparing $\boxed{n = 12}$

[70] The number of parallelograms, formed from a set of six parallel lines intersecting another set of four parallel lines is:

(a) 360 (b) 90

(c) 180 (d) 45 (1 mark)

Answer:

(b) No. of Parallelograms are formed from a set of m parallel lines intersecting another set of n

Parallel lines $= {}^{m}C_2 \times {}^{n}C_2$

Here m $= 6,\ n = 4$

$$= {}^{6}C_2 \times {}^{4}C_2$$

$$= \frac{6\times5}{2\times1} \times \frac{4\times3}{2\times1}$$

$$= 15 \times 6$$

$$= 90$$

[71] The number of words which can be formed by letters of the word 'ALLAHABAD' is:

(a) 7560 (b) 3780

(c) 30240 (d) 15120 (1 mark)

Answer:

(a) Given word → 'ALLAHABAD'

$$\text{No. of ways} = \frac{n!}{p!q!}$$

Total No. of letter (n) = 9

No. of 'A' 'p' = 4
No. of 'L' 'q' = 2

$$\text{No. of words} = \frac{9!}{4!2!} = \frac{9\times8\times7\times6\times5\times\cancel{4!}}{\cancel{4!}\times2!}$$

$$= \frac{9\times8\times7\times\cancel{6^3}\times5}{\cancel{2}\times1}$$

$$= 7560$$

2017 - DECEMBER

[72] If ${}^{n}P_{13} : {}^{n+1}P_{12} = 3:4$, then the value of 'n' will be:

(a) 13 (b) 15
(c) 18 (d) 31 (1 mark)

Answer:

(b) Given ${}^{n}P_{13} : {}^{n+1}P_{12} = 3 : 4$

$$\frac{\lfloor n}{\lfloor n-13} : \frac{\lfloor n+1}{\lfloor n+1-12} = 3 : 4$$

$$\frac{\lfloor n}{\lfloor n-13} \times \frac{\lfloor n-11}{\lfloor n+1} = 3 : 4$$

$$\frac{\lfloor n}{\lfloor n-13} \times \frac{(n-11)\,(n-12)\lfloor n-13}{(n+1)\lfloor n} = \frac{3}{4}$$

$$\frac{(n-11)\,(n-12)}{(n+1)} = \frac{3}{4}$$

$$\frac{n^2-12n-11n+132}{n+1} = \frac{3}{4}$$

$$4(n^2-23n+132) = 3\,(n+1)$$

$$4n^2-92n+528 = 3n+3$$

$$4n^2-92n-3n+528-3 = 0$$

$$4n^2-95n+525 = 0$$

$$4n^2-60n-35n+525 = 0$$

$$4n\,(n-15)-35\,(n-15) = 0$$

$(n - 15)(4n - 35) = 0$

if $n - 15 = 0$ → $n = 15$ if $4n - 35 = 0$ $n = \frac{35}{4}$ (Impossible)

[73] If 3 books on computer, 3 books on commerce, and 5 books on economics are arranged in such away that the books of same subject are kept together, then the number of ways in which this can be done are:

(a) 4320 (b) 35820
(c) 35920 (d) 25920 (1 mark)

Answer:

(d) No. of ways = $3! \times 3! \times 3! \times 5!$
$= 6 \times 6 \times 6 \times 120$
$= 216 \times 120$
$= 25,920$

2018 - MAY

[74] The number of triangle that can be formed by choosing the vertices from a set of 12 points, seven of which lie on the same straight line, is:

(a) 185 (b) 175
(c) 115 (d) 105 (1 mark)

Answer:

(a) Here $n = 12$, $k = 7$
No. of triangle are formed from 'n' point
In which (k) points are collinear = ${}^{n}C_3 - {}^{k}C_3$
$= {}^{12}C_3 - {}^{7}C_3$
$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} - \frac{7 \times 6 \times 5}{3 \times 2 \times 1}$
$= 220 - 35$
$= 185$

[75] If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$, find x:

(a) 999 (b) 998

(c) 997 (d) 1,000 (1 mark)

Answer:

(a) If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$

$\because {}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

then x = 999 $[{}^{999}C_{901} = {}^{999}C_{98}]$

2018 - NOVEMBER

[76] A bag contains 4 red, 3 black and 2 white balls. In how many ways 3 balls can be drawn from this bag so that they include at least one black ball?

(a) 64 (b) 46

(c) 85 (d) None of the above (1 mark)

Answer:

(a) No. of Total Balls = 4 Red + 3 Black + 2 white

= 9 Balls.

If 3 balls are drawn from this bag getting at least one black balls. It may be following cases

(a) 1 B & 2 other = ${}^{3}C_{1} \times {}^{6}C_{2} = 3 \times 15 = 45$

(b) 2 B & 1 other = ${}^{3}C_{2} \times {}^{6}C_{1} = 3 \times 6 = 18$

(c) 3 B & 0 other = ${}^{3}C_{3} \times {}^{6}C_{0} = 1 \times 1 = 1$

Total ways = 45 + 18 + 1

= 64

[77] The number of words from the letters of the word BHARAT, in which B and H will never come together, is

(a) 360 (b) 240

(c) 120 (d) None of the above (1 mark)

Answer:

(b) Given Word

'B H A R A T'

1 2 3 4 5 6

Total No of ways arrange the letter of the word = $\frac{6!}{2!} = \frac{720}{2} = 360$

If Letter 'B' and 'H' are always together

Then No. of ways = $\frac{5! \times 2!}{2!} = \frac{120 \times 2}{2} = 60 \times 2 = 120$

Then No. of ways If 'B' and 'H' are never taken together

= 360 – 120

= 240.

[78] The value of *N* in $\frac{1}{7!} + \frac{1}{8!} = \frac{N}{9!}$ is

(a) 81 (b) 78

(c) 89 (d) 64 (1 mark)

Answer:

(a) If $\frac{1}{7!} + \frac{1}{8!} = \frac{N}{9!}$

$\frac{9 \times 8 \times 1}{9 \times 8 \times 7!} + \frac{9 \times 1}{9 \times 8!} = \frac{N}{9!}$

$\frac{72}{9!} + \frac{9}{9!} = \frac{N}{9!}$

$\frac{81}{9!} = \frac{N}{9!}$

N = 81

[79] If ${}^{n}P_{r} = 720$ and ${}^{n}C_{r} = 120$, then r is

(a) 3 (b) 4

(c) 5 (d) 6 (1 mark)

Answer:

(a) Given ${}^nP_r = 720$, ${}^nC_r = 120$

We know that

$$\frac{{}^nC_r}{{}^nP_r} = \frac{1}{\lfloor r}$$

$$\frac{120}{720} = \frac{1}{\lfloor r}$$

$$\frac{1}{6} = \frac{1}{\lfloor r}$$

$$\lfloor r = 6$$

$$\lfloor r = 3 \times 2 \times 1$$

$$\lfloor r = \lfloor 3$$

2019 - JUNE

[80] Which of the following is a correct statement.

(a) ${}^np_n = {}^np_{n-1}$

(b) ${}^np_n = {}^{2n}p_{n-2}$

(c) ${}^np_n = {}^{3n}p_{n-3}$

(d) ${}^np_n = {}^{n.(n-1)}p_{n-1}$ (1 mark)

Answer:

(a) ${}^nP_n = {}^nP_{n-1}$ is correct statement

$$\text{L.H.S} = {}^nP_n = \frac{\lfloor n}{\lfloor n-n} = \frac{\lfloor n}{\lfloor 0} = \frac{\lfloor n}{1} = \lfloor n$$

$$\text{R.H.S} = {}^nP_{n-1} = \frac{\lfloor n}{\lfloor n-n+1} = \frac{\lfloor n}{\lfloor 1} = \frac{\lfloor n}{1} = \lfloor n$$

[81] If these are 40 guests in a party. If each guest takes a shake hand with all the remaining guests. Then the total number of hands shake is _____ :

(a) 780
(b) 840
(c) 1,560
(d) 1,600 (1 mark)

Answer:

(a) For shaking hands

No. of ways = ${}^{n}C_{2}$

Here, n = 40

No. of ways = ${}^{40}C_{2}$

$$= \frac{40 \times 39}{2 \times 1}$$

$$= 20 \times 39$$

$$= 780$$

[82] If ${}^{11}C_{x} = {}^{11}C_{2x-4}$ and $x \neq 4$ than the value of ${}^{7}C_{x}$ =

(a) 20
(b) 21
(c) 22
(d) 23 (1 mark)

Answer:

(b) If ${}^{11}C_{x} = {}^{11}C_{2x-4}$

If ${}^{n}C_{x} = {}^{n}C_{y}$ then $n = x + y$

$x + (2x - 4) = 11$

$x + 2x - 4 = 11$

$3x = 11 + 4$

$3x = 15$

$x = 5$

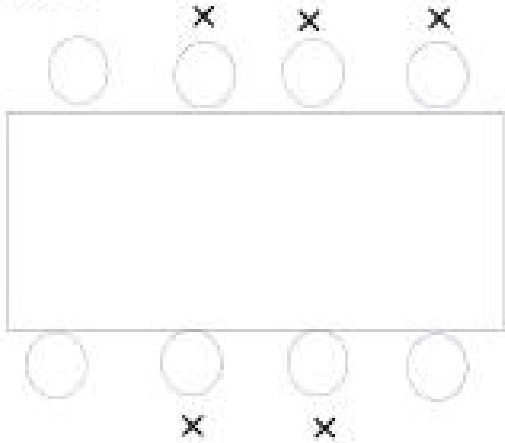
$${}^{7}C_{x} = {}^{7}C_{5} = {}^{7}C_{2} = \frac{7 \times 6}{2 \times 1} = 21$$

[83] In how many ways can the crew of an eight oared boat be arranged so that 3 of crew can row only on a stroke side and 2 row on the other side?

(a) 1,728
(b) 256
(c) 164
(d) 126 (1 mark)

Answer:

(a)



Since it is an eight-oared boat, it is safe to assume that there are 8 rowers. First, lets arrange the 2 persons who will row on one side. There are 4 seats, and 2 persons are to be arranged. This can be done in ${}^4P_2$ ways.

Now, lets arrange the 3 rowers for the other side. There are 4 seats and 3 persons are to be arranged. This can be done in ${}^4P_3$ ways.

Now 3 rowers and 3 seats remain. These 3 persons can be arranged in there 3 seats in ${}^3P_3$ ways.

Therefore, total no. of ways =

$= {}^4P_2 \times {}^4P_3 \times {}^3P_3$

$= 12 \times 24 \times 6$

$= 1,728$

2019 - NOVEMBER

[84] Three girls and five boys are to be seated in a row so that no two girls sit together. Total no. of ways of this arrangement are:

(a) 14,400

(c) ${}^5p_3$

(d) $3! \times 5!$ (1 mark)

Answer:

(a) Required arrangement

$X\ B_1\ X\ B_2\ X\ B_3\ X\ B_4\ X\ B_5\ X$

No. of ways of arranging 3 girls in 6 places

$= {}^6P_3$

No. of ways of arranging 5 boys in 5 places

$= {}^5P_5$

Total ways $= {}^6P_3 \times {}^5P_5$

$$= \frac{6!}{(6-3)!} \times 5!$$

$$= \frac{6 \times 5 \times 4 \times 3!}{3!} \times 120 = ₹\ 14{,}400$$

[85] How many numbers can be formed with the help of 2, 3, 4, 5, 6, 1 which is not dimisible by 5, given that it is a five digit no. and digits are not repeating?

(a) 600

(b) 400

(c) 1200

(d) 1400 (1 mark)

Answer:

(a) No's 2, 3, 4, 5, 6, 1

A no. is divisible by 5 when it ends with 0 or 5

T Th	Th	H	T	O

No. of ways of filling one's digit = 5 {all except 5}

No. of ways of filling ten's digit = 5

No. of ways of filling thousand place = 4

No. of ways of filling ten thousand place = 3

No. of ways of filling hundred's place = 2

Total ways $= 5 \times 5 \times 4 \times 3 \times 2$

$= 600$ ways

[86] How many different groups of 3 people can be formed from a group of 5 people?

(a) 5
(b) 6
(c) 10
(d) 9 (1 mark)

Answer:

(c) We know,

No. of ways to choose r objects out of n objects is ${}^{n}C_{r}$

Using this formula,

choosing 3 distinct objects (groups) from

$$5 = {}^{5}C_{3} = \frac{5!}{(5-3)! \times 3!}$$

$$= \frac{5!}{2! \times 3!}$$

$$= \frac{5 \times 4 \times 3!}{2 \times 3!}$$

= 10 ways

[87] In how many ways can 4 people be selected at random from 6 boys and 4 girls if there are exactly 2 girls?

(a) 90
(b) 360
(c) 92
(d) 480 (1 mark)

Answer:

(a) Boys (6) Girls (4)

2 2

No. of ways of selecting 2 boys out of 6 = ${}^{6}C_{2}$

No. of ways of selecting 2 girls out of 4 = ${}^{4}C_{2}$

Total ways = ${}^{6}C_{2} \times {}^{4}C_{2}$

$$= \frac{6!}{(6-2)! \times 2!} \times \frac{4!}{21 \times (4-2)!}$$

$$= \frac{6 \times 5 \times 4!}{4! \times 2} \times \frac{4 \times 3 \times 2!}{2! \times 2}$$

$= 15 \times 6 = 90$ ways

[88] ${}^{n}P_3 : {}^{n}P_2 = 2 : 1$

(a) 4

(b) 7/2

(c) 5

(d) 2/7

Answer:

(a) ${}^{n}P_r = \frac{n!}{(n-r)!}$

${}^{n}P_3 : {}^{n}P_2 = 2 : 1$

$$\frac{n!}{(n-3)!} : \frac{n!}{(n-2)!} = \frac{2}{1}$$

$$\frac{n!}{(n-3)!} \times \frac{(n-2)(n-3)!}{n!} = \frac{2}{1}$$

$(n - 2) = 2$

$n = 4$

CHAPTER

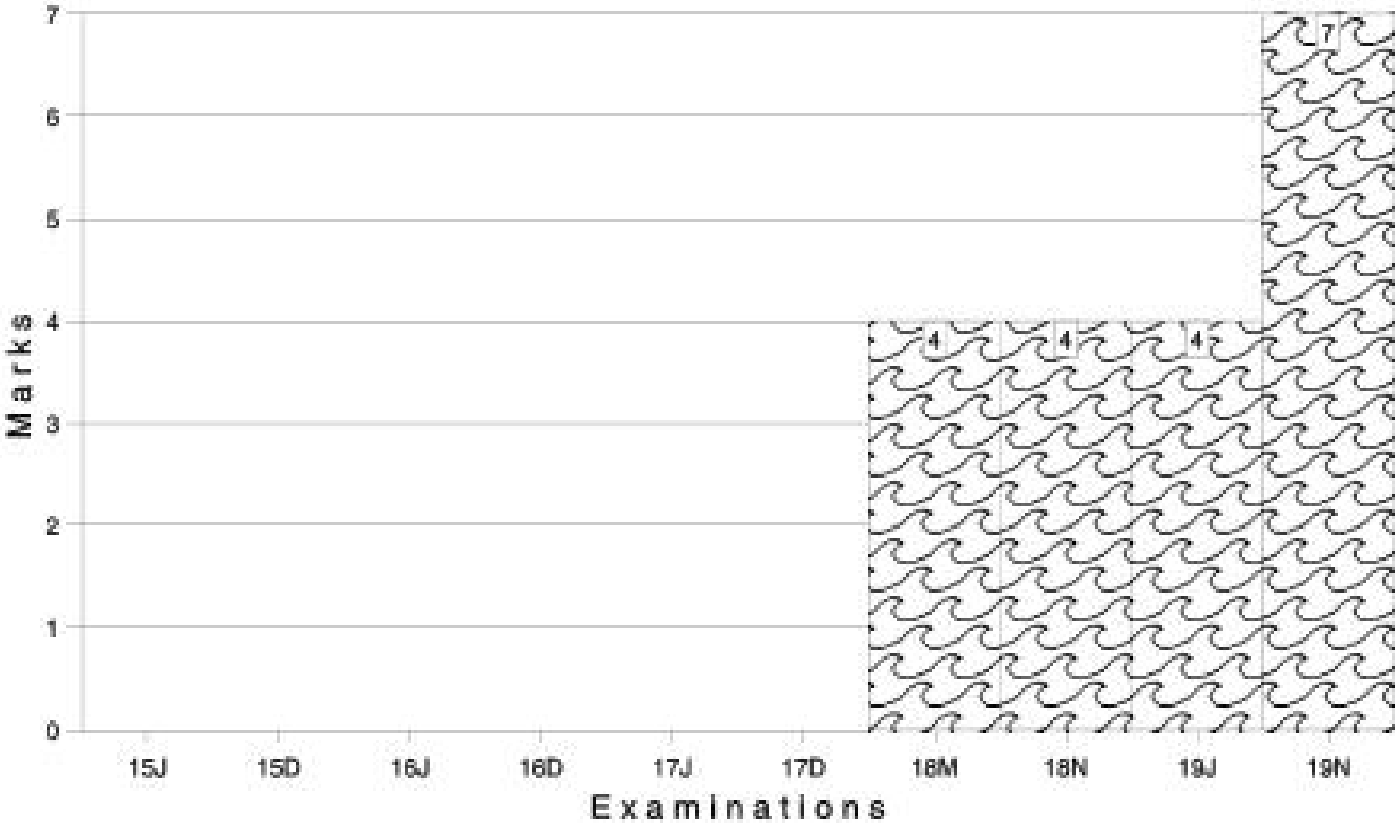
6

SEQUENCE AND SERIES

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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PAST YEAR QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] The sum of all natural numbers between 100 and 1000 which are multiple of 5 is:

(a) 98,450 (b) 96,450
(c) 97,450 (d) 95,450 (1 mark)

Answer:

(a) The required series will be :

105, 110, 115, ...995.

Therefore, last term = 995

$a + (n - 1)d = 995$

$105 + (n - 1)5 = 995$

$(n - 1)5 = 995 - 105$

$n - 1 = \frac{890}{5}$

$n = 178 + 1 = 179$

Sums of the series $= \frac{n}{2}[a + \ell]$

$= \frac{179}{2}[105 + 995]$

$= \frac{179}{2} \times 1{,}100 = 98{,}450$

[2] Find n such that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b:

(a) ½ (b) 1
(c) – ½ (d) 0 (1 mark)

Answer:

(c) G.M. of a and b = $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$\sqrt{ab} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ (∵ G.M. of x and y is $\sqrt{xy}$)

$(ab)^{1/2} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$a^{1/2}.b^{1/2} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$a^{1/2}.b^{1/2}(a^n+b^n) = a^{n+1}+b^{n+1}$

$a^{n+1/2}.b^{1/2}+a^{1/2}.b^{n+1/2} = a^{n+1}+b^{n+1}$

$a^{n+1}-a^{n+1/2}.b^{1/2} = a^{1/2}.b^{n+1/2}-b^{n+1}$

$a^{n+1/2}(\sqrt{a}-\sqrt{b}) = b^{n+1/2}(\sqrt{a}-\sqrt{b})$

$a^{n+1/2} = b^{n+1/2}$

$\frac{a^{n+1/2}}{b^{n+1/2}} = 1$

$\left(\frac{a}{b}\right)^{n+\frac{1}{2}} = \left(\frac{a}{b}\right)^0$

$n+\frac{1}{2} = 0$ [∵ when $x^m = x^n$ then m = n]

$n = -\frac{1}{2}$

[3] The sum of an A P, whose first term is - 4 and last term is 146 is 7171. Find the value of n.

(a) 99 (b) 100

(c) 101 (d) 102 (1 mark)

Answer:

(c) a = – 4 and ℓ = 146

Let the number of terms be n

Sum = 7171

$$\frac{n}{2}[a + \ell] = 7171$$

$$\frac{n}{2}[-4 + 146] = 7171$$

$$142\,n = 14342$$

$$n = \frac{14342}{142}$$

$$n = 101$$

[4] If the first term of a G.P exceeds the second term by 2 and the sum to infinity is 50, the series is :

(a) 10, 8, $\frac{32}{5}$, ... (b) 10, 8, $\frac{5}{2}$,.....

(c) 10, $\frac{10}{3}, \frac{10}{9}$,..... (d) None (1 mark)

Answer:

(a) Let the first terms of GP be a, then, its second term = a – 2

Common ratio i.e. $r = \frac{a-2}{a}$

sum to infinity = 50

$$\frac{a}{1-r} = 50$$

$$\frac{a}{1-\frac{a-2}{a}} = 50$$

$$\frac{a}{\frac{a-a+2}{a}} = 50$$

$$\frac{a^2}{2} = 50$$

$$a^2 = 100$$

$a = 10$

$r = \frac{10-2}{10} = \frac{8}{10} = \frac{4}{5}$

Therefore, the required series is $10, 8, \frac{32}{5}, \ldots\ldots\ldots\ldots$

2007 - FEBRUARY

[5] Σn^2 defines :

(a) $\frac{n(n+1)(2n+1)}{6}$ (b) $\frac{n(n+1)}{2}$

(c) $\left[\frac{n(n+1)}{2}\right]^2$ (d) None of these (1 mark)

Answer:

(a) $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$

This can be proved by the principle of Mathematical Induction.

Let P (n) be the statement given by:

$P(n) : 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Step-I: We have $P(1) : 1^2 = \frac{1(1+1)(2\times1+1)}{6}$

$\therefore 1^2 = 1 = \frac{(1+1)(2\times1+1)}{6}$

P(1) is true.

Now, $1^2 + 2^2 + 3^2 + ...+ m^2 + (m + 1)^2$

$= [1^2 + 2^2 + 3^2 + ...+ m^2] + (m + 1)^2$

$= \frac{m(m + 1)(2m + 1)}{6} + (m + 1)^2$ [Using (1)]

$= \frac{1}{6}(m + 1)\{m(2m + 1) + 6(m + 1)\}$

$= \frac{1}{6}(m + 1)\{2m^2 + 7m + 6\}$

$= \frac{1}{6}(m + 1)(m + 2)(2m + 3)$

$= \frac{1}{6}(m + 1)((m + 1)+1)\ (2(m + 1)+1)$

$\therefore$ P (m + 1) is true

Thus, P (m) is true and P (m + 1) is true

Hence, by the principle of mathematical induction, the given result is true for all n $\sum$ N.

Note:

However, the derivation of the required result given above is not covered under the purview of CPT syllabus. It has just been provided for your knowledge. It is suggested for the students to learn the result directly.

[6] Divide 30 into five parts in A.P., such that the first and last parts are in the ratio 2 :3 :

(a) $\frac{24}{5}, \frac{27}{5}, 6, \frac{33}{5}, \frac{36}{5}$ (b) $6, \frac{36}{5}, \frac{33}{5}, \frac{24}{5}, \frac{27}{5}$

(c) $\frac{27}{5}, \frac{24}{4}, \frac{36}{5}, \frac{33}{5}, 6$ (d) $6, \frac{24}{5}, \frac{27}{5}, \frac{33}{5}, \frac{36}{5}$ (1 mark)

Answer:

(a) Let the five parts be a – 2d, a – d, a, a + d, a + 2d.

Then, (a – 2d) + (a – d) + a + (a + d) + (a + 2d) = 30

5a = 30

a = 6

Further,

$$\frac{a-2d}{a+2d}=\frac{2}{3}$$

$$\frac{6-2d}{6+2d}=\frac{2}{3}$$

$18 - 6d = 12 + 4d$

$6 = 10d$

$$d = \frac{6}{10} = \frac{3}{5}$$

Substituting the values of a and d, the five parts are :

$$6-\frac{2\times3}{5}, 6-\frac{3}{5}, 6, 6+\frac{3}{5}, 6+\frac{2\times3}{5}$$

$$=\frac{24}{5}, \frac{27}{5}, 6, \frac{33}{5}, \frac{36}{5}$$

[7] If $a^{1/x} = b^{1/y} = c^{1/z}$ and a, b, c are in G.P; the x, y, z are in :

(a) A.P. (b) G.P.

(c) Both (a) & (b) (d) None (1 mark)

Answer:

(a) Let $a^{1/x} = b^{1/y} = c^{1/z}, = k$

Then, $a = k^x$, $b= k^y$, $c = k^z$

Since, a, b, c are in G.P.

Then $b^2 = ac$

$(k^y)^2 = k^x k^z$

$k^{2y} = k^{x+z}$

$2y = x + z$ ($\because$ when $x^m = x^n$ then m=n)

$$y = \frac{x+z}{2}$$

This shows that x, y, z are in A.P.

[8] Find the sum to n terms of the series : 7 + 77 + 777 + ... to n terms :

(a) $\frac{7}{9}(10^{n+1}-10)-\frac{7n}{9}$ (b) $\frac{7}{9}(10^{n+1}-10)+\frac{7n}{9}$

(c) $\frac{7}{81}(10^{n+1}-10)-\frac{7n}{9}$ (d) $\frac{7}{81}(10^{n+1}-10)+\frac{7n}{9}$ (1 mark)

Answer:

(c) $7 + 77 + 777 +$ ..... to n terms

$7\,(1 + 11 + 111 +$ ..... to n terms)

Multiplying and dividing by 9

$\frac{7}{9}$ $(9 + 99 + 999 +$..... to n terms)

$\frac{7}{9}$ $[(10 - 1) + (100 - 1) + (1000 - 1) +$..... to n terms)

$\frac{7}{9}$ $[(10 + 10^2 + 10^3 +$.....to n terms) + $(1 + 1 + 1 +$.....to n terms)

$$\frac{7}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right]$$

$$\left[\because \text{sums of n terms of G.P.} = \frac{a(r^n - 1)}{r - 1}, r > 1\right]$$

$$\frac{7}{9}\left[\frac{10^{n+1} - 10}{9} - n\right]$$

$$\frac{7}{81}(10^{n+1} - 10) - \frac{7n}{9}$$

2007 - MAY

[9] Find the sum of all natural numbers between 250 and 1,000 which are exactly divisible by 3 :

(a) 1,56,375 (b) 1,56,357

(c) 1,65,375 (d) 1,65,357 (1 mark)

Answer:

(a) The series of the numbers between 250 and 1000 divisible by 3 is 252, 255, 258, ... 999.

Since, Last term = 999

$a + (n - 1)d = 999$

$252 + (n - 1)3 = 999$

$(n - 1)3 = 999 - 252$

$n - 1 = \frac{747}{3}$

$n = 249 + 1$

$n = 250$

$\text{sum} = \frac{n}{2}[a + \ell\,]$

$= \frac{250}{2}[252 + 999]$

$= 125 \times 1251$

$= 1,56,375.$

[10] If the p^{th} term of a G.P. is x and the qth term is y, then find the nth term :

(a) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]$ (b) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]^{(p-q)}$

(c) 1 (d) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]^{\frac{1}{p-q}}$ (1 mark)

Answer:

(d) Let a be the first term and r be the common ratio of G.P.

Then, $x = ar^{p-1}$ and $y = ar^{q-1}$

$\frac{x}{y} = \frac{ar^{p-1}}{ar^{q-1}} = \frac{r^{p-1}}{r^{q-1}}$

$\frac{x}{y} = r^{p-1-q+1}$

$\frac{x}{y} = r^{p-q}$

$r = \left(\frac{x}{y}\right)^{\frac{1}{p-q}}$

using this value of r in $x = ar^{p-1}$, we have

$x = a\left[\left(\frac{x}{y}\right)1/p - q\right]^{p-1}$

$$x = a\left[\left(\frac{x}{y}\right)p-1/p-q\right]$$

$$a = x\left[\left(\frac{x}{y}\right)p-1/p-q\right]$$

$$\therefore tn = ar^{n-1}$$

$$= x\left[\left(\frac{y}{x}\right)p-1/p-q\right]\left[\left(\frac{x}{y}\right)1/p-q\right]^{n-1}$$

$$= \left[x^{(p-q)} \times \left[\frac{y}{x}\right]^{p-1} \times \left(\frac{x}{y}\right)^{n-1}\right]^{\frac{1}{p-q}}$$

$$= \left[\frac{x^{(p-q)-(p-1)+(n-1)}}{y^{(n-1)-(p+1)}}\right]^{\frac{1}{p-q}}$$

$$= \left[\frac{x^{(p-q-p+1+n-1)}}{y^{(n-1-p+1)}}\right]^{\frac{1}{p-q}}$$

$$= \left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]^{\frac{1}{p-q}}$$

[11] A person pays ₹ 975 in monthly instalments, each instalment is less than former by ₹ 5. The amount of first instalment is ₹ 100. In what time will the entire amount be paid?

(a) 26 months (b) 15 months
(c) Both (a) & (b) (d) 18 months (1 mark)

Answer:

(b) **a =** ₹ 100, $d = -5$, $S_n = 975$

Let the time taken to pay ₹ 975 be n months;

$S_n = 975$

$$\frac{n}{2}[2a + (n-1)d] = 975$$

$$\frac{n}{2}[2 \times 100 + (n-1)(-5)] = 975$$

$$n[200 - 5n + 5] = 975 \times 2$$

$$n[205 - 5n] = 1950$$

$$205n - 5^n = 1950$$

$5n^2 - 205n + 1950 = 0$
$n^2 - 41n + 390 = 0$
$n^2 - 15n - 26n + 390 = 0$
$n(n - 15) - 26(n - 15) = 0$
$(n - 26)(n - 15) = 0$
$n = 26$, or $n = 15$
We reject $n = 26$, because $t_{26} = 100 + (26 - 1)(-5) = -25$ which is not acceptable. Hence, the entire amount is to be paid in 15 months.

2007 - August

[12] If the sum of n terms of an A.P. is $(3n^2 - n)$ and its common difference is 6, then its first term is :

(a) 3 (b) 2
(c) 4 (d) 1 (1 mark)

Answer:

(b) Given : $S_n = 3n^2 - n$
$S1 = 3 \times 1^2 - 1 = 3 - 1 = 2$
$\because\ s_1 = t_1$
$\therefore\ t_1 = 2$

[13] Find the sum of the series :
$2 + 7 + 12 + \ldots\ldots\ldots\ldots 297.$

(a) 8970 (b) 8870
(c) 7630 (d) 9875 (1 mark)

Answer:

(a) Since the series is an AP with $a = 2$, $d = 5$
$\ell = 297$
$a + (n - 1)d = 297$
$2 + (n - 1) \times 5 = 297$
$(n - 1) \times 5 = 297 - 2$

$$n - 1 = \frac{295}{5}$$

$$n = 59 + 1$$

$$n = 60$$

$$\text{Sum} = \frac{n}{2}[a + l]$$

$$= \frac{60}{2}[2 + 297]$$

$$= 30 \times 299$$

$$= 8970$$

[14] A certain ball when dropped to the ground rebounds to $\frac{4}{5}^{th}$ of the height from which it falls; it is dropped from a height of 100 metres find the total distance it travels before finally coming to rest :

(a) 600m (b) 700m

(c) 900m (d) 200m (1 mark)

Answer:

(c) Since the ball was dropped from a height of 100 m, so it travelled 100 m first, Then, it went up to $100 \times \frac{4}{5} = 80$ m and dropped. So it travelled 2×80 m. In this way the total distance travelled would be :

$$100 + 2\left[80 + 80\left(\frac{4}{5}\right) + 80\left(\frac{4}{5}\right)^2 + 80\left(\frac{4}{5}\right)^3 + \ldots\ldots\ldots\ldots\right]$$

$$= 100 + 2\left[\frac{80}{1 - 4/5}\right]$$

[Since, sum of infinite G.P. $= \frac{a}{1 - r}$

$$= 100 + 2\left[\frac{80 \times 5}{5 - 4}\right]$$

$$= 100 + (2 \times 400)$$

$$= 900 \text{ m.}$$

[15] In a G.P if the $(p + q)^{th}$ term is m and $(p - q)^{th}$ term is n, then the pth term is :

(a) mn (b) $\sqrt{mn}$

(c) m^2 (d) n^2 (1 mark)

Answer:

(b) Let a, ar, ar^2,..... be the G.P. then

$T_{p+q} = m = ar^{p+q-1}$ and --------- (1)

$T_{p-q} = n = ar^{p-q-1}$ and --------- (2)

Dividing (1) by (2)

$$\frac{m}{n} = \frac{ar^{p+q-1}}{ar^{p-q-1}}$$

$$\frac{m}{n} = \frac{r^{p+q-1}}{r^{p-q-1}} s$$

$$\frac{m}{n} = r^{(p+q-1)-(p-q-1)}$$

$$\frac{m}{n} = r^{p+q-1-p+q+1}$$

$$\frac{m}{n} = r^{2q}$$

$$r = \left(\frac{m}{n}\right)^{\frac{1}{2q}}$$

Let this value of r be substituted in equation 1.

$$\therefore \quad M = a\left(\frac{m}{n}\right)^{\frac{p+q-1}{2q}}$$

$$a = \frac{m}{\left(\frac{m}{n}\right)^{\frac{p+q-1}{2q}}}$$

Now, the p^{th} term will be

$$p^{th} = \frac{m}{\left(\frac{m}{n}\right)^{\frac{p+q-1}{2q}}} \times \left[\left(\frac{m}{n}\right)^{\frac{1}{2q}}\right]^{P-1}$$

$$= \frac{m}{\left(\frac{m}{n}\right)^{\frac{p+q-1}{2q}}\left(\frac{m}{n}\right)^{\frac{q}{2q}}} \times \left[\left(\frac{m}{n}\right)^{\frac{1}{2q}}\right]^{P-1}$$

$$= \frac{m}{\left(\frac{m}{n}\right)^{\frac{1}{2}}} = \sqrt{mn}$$

2007 - NOVEMBER

[16] The sum of the series :
$0.5 + 0.55 + 0.555 +$ ..... to n terms is :

(a) $\frac{5n}{9} + \frac{5}{9}[1 - (0.1)^n]$ (b) $\frac{5n}{9} - \frac{5}{81}[1 - (0.1)^n]$

(c) $\frac{5n}{9} + \frac{5}{81}[1 - (0.1)^n]$ (d) $\frac{5n}{9} + \frac{5}{81}[1 + (0.1)^n]$ (1 mark)

Answer:

(b) $0.5 + 0.55 + 0.555 +$ ..... to n term

$5(0.1 + 0.11 + 0.111 +$..... to n term)

Multiplying and dividing by 9

$\frac{5}{9}(0.9 + 0.99 + 0.999 +$..... to n terms)

$\frac{5}{9}[(1 - 0.1) + (1 - 0.01) + (1 - 0.001) +$..... to n terms)

$\frac{5}{9}[(1 + 1 + 1$..... to n terms) $- (0.1 + 0.01 + 0.001 +$ .... to n terms)

$$\frac{5}{9}\left[n - \frac{(0.1)[1-(0.1)^n]}{1-0.1}\right]$$

$\left(\because \text{ sum of n terms of G.P. } = \frac{a(1-r^n)}{1-r}, r < 1\right)$

$$\frac{5n}{9} - \frac{5}{9}\left(\frac{0.1\times(1-(0.1)^n)}{0.9}\right)$$

$$\frac{5n}{9} - \frac{5}{9}\left(\frac{(1-(0.1)^n)}{9}\right)$$

$$\frac{5n}{9} - \frac{5}{81}[1-(0.1)^n]$$

[17] A contractor who fails to complete a building in a certain specified time is compelled to forfeit ₹ 200 for the first day of extra time required and thereafter forfeited amount is increased by ₹ 25 for every day. If he loses ₹ 9,450, for how many days did he over-run the contract time?

(a) 19 days (b) 21 days

(c) 23 days (d) 25 days (1 mark)

Answer:

(b) Here a = ₹ 200 and d = ₹ 25. Also S_n = ₹ 9450 Assume that the contract time is over run for n days. Then ,

$$s_n = \frac{n}{2}[2a + (n-1)d]$$

$$9450 = \frac{n}{2}[2 \times 200 + (n-1)25]$$

$$18900 = n(400 + 25n - 25)$$

$$18900 = n(375 + 25n)$$

$$18900 = 375n + 25n^2$$

$$25n^2 + 375n - 18900 = 0$$

$$n^2 + 15n - 756 = 0$$

$$n^2 + 36n - 21n - 756 = 0$$

$$n(n+36) - 21(n+36) = 0$$

$$(n-21)(n+36) = 0$$

$$n = 21 \text{ or } n = -36$$

Since, the number of days over run can never be negative, rejecting $n = -36$, we have $n = 2$. Hence, the contractor over-run the contract by 21 days.

[18] The first, second and seventh term of A.P. are in G.P. and the common difference is 2, the 2nd term of A.P. is :

(a) 5/2 (b) 2

(c) 3/2 (d) ½ (1 mark)

Answer:

(a) $T_1 = a$

$T_2 = a + (2 - 1)d = a + d$

$T_3 = a + (7 - 1)d = a + 6d$

Since T_1, T_2 and T_7 are in G.P.

$$\therefore \frac{a+d}{a} = \frac{a+6d}{a+d}$$

$$\frac{a+2}{a} = \frac{a+12}{a+2} \quad (\because d = 2)$$

$(a + 2)^2 = a(a + 12)$

$a^2 + 4a + 4 = a^2 + 12a$

$12a - 4a = 4$

$8a = 4$

$$a = \frac{1}{2}$$

Therefore, second term $\frac{1}{2} + 2 = \frac{5}{2}$

2008 - FEBRUARY

[19] A man employed in a company is promised a salary of ₹ 3,000 every month for the first year and an increment of ₹ 1,000 in his monthly salary every succeeding year. How much does the man earn from the company in 20 years?

(a) ₹ 30,00,000 (b) ₹ 27,50,000
(c) ₹ 19,10,000 (d) ₹ 7,90,000 (1 mark)

Answer:

(a) We are given

$a = ₹\ 3,000 \times 12 = ₹\ 36,000$

$n = 20$

$d = ₹\ 1,000 \times 12 = ₹\ 12,000$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2 \times 36,000\ (20-1)12,000]$

$= 10\ [72,000 + 2,28,000]$

$= 10 \times 3,00,000$

$= ₹\ 30,00,000$

[20] If a, b, c are in A.P. and x, y, z are in G.P, then the value of $x^{(b-c)} . y^{(c-a)} . z^{(a-b)}$ is :

(a) 1 (b) 0
(c) b(c - a) (d) None (1 mark)

Answer:

(a) Let d be the common difference and r be the common ratio of the given A.P. and G.P. respectively. Then,

$b = a + d$, $c = a + 2d$ and $y = xr$, $z = xr^2$

$\therefore x^{(b-c)} . y^{(c-a)} . z^{(a-b)}$

$= x^{(a+d-a-2d)} \times y^{2d} \times z^{(a-d-d)}$

$= x^{(-d)} \times (xr)^{2d} \times (xr^2)^{(-d)}$

$= [x^{-d} \times x^{2d} \times x^{-d}]\ [r^{2d} \times r^{-2d}]$

$= x^{2d-2d} \times r^{2d-2d}$

$= x^0 \times r^0$

$= 1 \times 1$

$= 1$

[21] Insert 4 A.M.'s between 3 and 18 :

(a) 12, 15, 9, 6 (b) 6, 9, 12, 15
(c) 9, 6, 12, 15 (d) 15, 12, 9, 6 (1 mark)

Answer:

(b) Let A_1, A_2, A_3, A_4 are the four A.M's between 3 and 18. Then, the numbers 3, A_1, A_2, A_3, A_4 18 from an A.P. whose first term is 3 and 6^{th} term is 18

$$\therefore T_n = a + (n-1)d$$
$$18 = 3 + (6-1)d$$
$$5d = 18 - 3$$
$$d = \frac{15}{5} = 3$$

Thus, $A_1 = 3 + 3 = 6$
$A_2 = 6 + 3 = 9$
$A_3 = 9 + 3 = 12$
$A_4 = 12 + 3 = 15$

2008 - JUNE

[22] If $x = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\ldots\ldots\ldots \infty$

$y = 1 + \frac{1}{4} + \frac{1}{4^2} + \ldots\ldots\ldots\ldots \infty$

Find xy.

(a) 2 (b) 1
(c) 8/9 (d) ½ (1 mark)

Answer:

(a) $x = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\ldots\ldots\ldots \infty$

$x = \frac{1}{1 - \frac{1}{3}}$ ($\because$ sum to infinity in case of G.P. $= \frac{a}{1-r}$)

$x = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

$$y = 1 + \frac{1}{4} + \frac{1}{4^2} + \ldots\ldots\ldots\ldots\infty$$

$$y = \frac{1}{1 - \frac{1}{4}}$$

$$y = \frac{1}{3/4} = \frac{4}{3}$$

$$\therefore xy = \frac{3}{2} \times \frac{4}{3} = 2$$

[23] On 1st January every year a person buys National Saving Certificates of value exceeding that of his last year's purchase by ₹ 100. After 10 years, he finds that the total value of the certificates purchased by him is ₹ 54,500. Find the value of certificates purchased by him in the first year :

(a) ₹ 6,000 (b) ₹ 4,000

(c) ₹ 5,000 (d) ₹ 5,500 (1 mark)

Answer:

(c) The investment made by a person is forming an A.P. with d = 100 and $S_{10} = 54, 500$. If a be the first team, we have.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2a + (10-1)100]$$

$$S_{10} = 5(2a + 900)$$

$$\frac{54,500}{5} = 2a + 900$$

$$2a = 10,900 - 900$$

$$a = \frac{10,000}{2} = 5,000$$

Hence, value of certificates purchased in the first year will be of ₹ 5,000.

[24] Find three numbers in G.P. such that their sum is 21, and the sum of their squares is 189 :

(a) 5, 7, 9 (b) 3, 7, 11

(c) 3, 6, 12 (d) 4, 8, 9 (1 mark)

Answer:

(c) Let three numbers in G.P. be $\frac{a}{r}$, a and are then.

$$\frac{a}{r} + a + ar = 21$$

$$\frac{a}{r}(1 + r + r^2) = 21 \ldots\ldots\ldots \quad (1)$$

and $\frac{a^2}{r^2} + a^2 + a^2r^2 = 189$

$$\frac{a^2}{r^2}(1 + r^2 + r^4) = 189 \ldots\ldots (2)$$

Dividing (2) by square of (1), we shall have

$$\frac{1 + r^2 + r^4}{(1 + r + r^2)^2} = \frac{189}{21 \times 21} = \frac{3}{7}$$

$$3(1 + r + r^2)^2 = 7(1 + r + r^2)(1 - r + r^2)$$

$$3(1 + r + r^2) = 7(1 - r + r^2)$$

$$3 + 3r + 3r^2 = 7 - 7r + 7r^2$$

$$4r^2 - 10r + 4 = 0$$

$$2r^2 - 5r + 2 = 0$$

$$2r^2 - 4r - r + 2 = 0$$

$$2r(r - 2) - 1(r - 2) = 0$$

$$(r - 2)(2r - 1) = 0$$

$$r = 2 \text{ or } r = \frac{1}{2}$$

When $r = 2$ from (2) $a = 6$

Also when $r = \frac{1}{2}$ from (1) $a = 6$

Therefore, the required numbers are 3, 16, 12 and 12, 6, 3

2008 - DECEMBER

[25] Find the ninth term of the series :

$\sqrt{2}, 5\sqrt{2}, 9\sqrt{2}$ .....

(a) $25\sqrt{2}$ (b) $31\sqrt{2}$

(c) $33\sqrt{2}$ (d) $25\sqrt{2}$ (1 mark)

Answer:

(c) $a = \sqrt{2}$, $d = 4\sqrt{2}$

$t_n = a + (n - 1)d$

$t_9 = \sqrt{2} + (9 - 1)4\sqrt{2}$

$= \sqrt{2} + 36\sqrt{2} - 4\sqrt{2}$

$= 33\sqrt{2}$

[26] The sum of how many terms of the sequence 256, 128, 64,..... is 511.

(a) 8 (b) 9

(c) 7 (d) None of these. (1 mark)

Answer:

(b) Clearly, the given series is a G.P.

Given : $S_n = 511$, $a = 256$, $r = ½$

We know that, $Sn = \frac{a(1 - r^n)}{1 - r}$

$511 = 256$ ———

$$\left(\frac{1}{2}\right)^n = \frac{1}{512}$$

$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^9, \rightarrow \boxed{n = 9}$$

[27] (x + 1), 3x, (4x + 2) are in A.P. Find the value of x

(a) 2 (b) 3

(c) 4 (d) 5 (1 mark)

Answer:

(b) Since, (x + 1), 3x, (4x + 2) are in A.P.

$2 \times 3x = (x + 1) + (4x + 2)$ [Since, if a, b, c are in A.P, $2b = a + c$]

$6x = 5x + 3$

$x = 3$

[28] Find two numbers whose A.M. is 10 and G.M. is 8.

(a) [10,10] (b) [16, 4]

(c) [18, 2] (d) [14, 6] (1 mark)

Answer:

(b) Given : AM = 10 and GM = 8

$\rightarrow \frac{a+b}{2} = 10$ and $\sqrt{ab} = 8$

$\therefore$ $a + b = 20$..... (1) and

$ab = 64$..... (2)

$\rightarrow$ $b = 20 - a$ from (1)

Substitute b in (2)

$a(20 - a) = 64$

$\rightarrow$ $20a - a^2 - 64 = 0$

$\rightarrow$ $a^2 - 20a + 64 = 0$

$\rightarrow$ $a^2 - 16a - 4a + 64 = 0$

$\rightarrow$ $a(a - 16) - 4(a - 16) = 0$

$\rightarrow$ $(a - 16)(a - 4) = 0$

$\rightarrow$ Now, if $a = 16$, $b = 4$

or if $a = 4$, $b = 16$

$\therefore$ Numbers are [16, 4]

2009 - JUNE

[29] The sum of terms of an infinite GP is 15. And the sum of the squares of the term is 45. Find the common ratio.

(a) 3/2 (b) 1

(c) – 2/3 (d) 2/3 (1 mark)

Answer:

(d) Let the first term of G.P. be 'a' and common ratio be 'r'. Then,

sum of an infinite G.P. $= \frac{a}{1-r}$

$$15 = \frac{a}{1-r}$$

$$15 - 15r = a$$

$$a + 15r = 15$$

$$a = 15 - 15r$$

sum of the squares of terms of G.P. $= \frac{a}{1-r^2}$

$$45 = \frac{a^2}{1-r^2}$$

$$45 - 45\,r^2 = a^2$$

$$a^2 + 45r^2 = 45$$

$$(15 - 15r)^2 + 45r^2 = 45$$

$$225 + 225r^2 - 450r + 45r^2 = 45$$

$$270r^2 - 450r + 180 = 0$$

$$3r^2 - 5r + 2 = 0$$

$$3r^2 - 3r - 2r + 2 = 0$$

$$3r(r - 1) - 2(r - 1) = 0$$

$$(3r - 2)(r - 1) = 0$$

$$r = \frac{2}{3} \text{ or } r = 1$$

Since $r = 1$ is not possible therefore $r = \frac{2}{3}$

[30] If in an A.P., Tn represents nth term.

If $t_7 : t_{10} = 5:7$ then $t_8 : t_{11} =$ _____

(a) 13 : 16 (b) 17 : 23

(c) 14 : 17 (d) 15 : 19 (1 mark)

Answer:

(b) Let a be the first term of an A.P. and common difference be d

Then,

$$\frac{t_7}{t_{10}} = \frac{5}{7}$$

$$\frac{a+(7-1)d}{a+(10-1)d} = \frac{5}{7}$$

$$\frac{a+6d}{a+9d} = \frac{5}{7}$$

$$7a + 42d = 5a + 45d$$

$$2a = 3d$$

$$a = \frac{3d}{2} \quad \text{..... (1)}$$

Now, $\frac{t_8}{t_{11}} = \frac{a+(8-1)d}{a(11-1)d} = \frac{a+7d}{a+10d}$

Substituting the value of a from (1)

$$= \frac{\frac{3d}{2}+7d}{\frac{3d}{2}+10d} = \frac{3d+14d}{2} \times \frac{2}{3d+20d} = \frac{17d}{23d}$$

$$\frac{t_8}{t_{11}} = \frac{17}{23}$$

$\therefore t_8 : t_{11} = 17 : 23$

2009 - DECEMBER

[31] Find the sum to infinity of the following series:

$1 - 1 + 1 - 1 + 1 - 1 + ...\infty$

(a) 1 (b) ∞

Answer:

(c) $a = 1$

$r = -1$

Sum of the G.P. upto $\infty = \frac{a}{1-r} = \frac{1}{1-(-1)} = \frac{1}{1+1}$

$$\text{Sum} = \frac{1}{2}$$

2010 - JUNE

[32] Divide 144 into three parts which are in AP and such that the largest is twice the smallest, the smallest of three numbers will be :

(a) 48 (b) 36

(c) 13 (d) 32 (1 mark)

Answer:

(d) Let the three numbers in AP be

$a - d, a, a + d$

Given : $a + d = 2(a - d)$

$a - d + a + a + d = 144$

$\therefore 3a = 144$

$a = 48$

$48 + d = 2(48 - d)$

$48 + d = 96 - 2d$

$3d = 48$

$\therefore d = 16$

Smallest Number $= a - d$

$= 48 - 16$

$= 32$

[33] Sum of series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots\infty$ is

(a) 15/36 (b) 35/36

(c) 35/16 (d) 15/16 (1 mark)

Answer:

(c) Let $S = 1 + \frac{4}{} + \frac{7}{} + \frac{10}{} \ldots\ldots\ldots\ldots\infty$ (1)

$$\frac{1}{5}S = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \ldots\ldots\ldots\ldots\infty \qquad (2)$$

$$\rightarrow \left(1 - \frac{1}{5}\right)S = 1 + \frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \ldots\ldots\ldots\ldots\infty \text{ [From (1) and (2)]}$$

$$\frac{4}{5}S = 1 + 3 \left\{ \qquad = 1 +$$

$$= \frac{b-a}{ab(b-a)}$$
$$= \frac{1}{ab}$$
$$= \frac{1}{(\sqrt{ab})^2}$$
$$= \frac{1}{G^2}$$

2011 - JUNE

[35] If Sum (S_n) of 'n'- terms of an Arithmetic Progression is ($2n^2 + n$). What is the difference of its 10^{th} and 1^{st} term ?

(a) 207 (b) 36
(c) 90 (d) 63 (1 mark)

Answer:

(b) Given :- Sum of n - terms of an A.P

i.e $S_n = 2n^2 + n$

$\because$ $S_{n-1} = 2(n-1)^2 + (n-1)$

or $S_{n-1} = 2n^2 - 3n + 1$

$\because$ n^{th} Term of this A.P will be

$T_n = S_n - S_{n-1}$

$= (2n^2 + n) - (2n^2 - 3n + 1)$

$T_n = 4n - 1$

$\because$ First Term (T_1) $= 4(1) - 1 = 3$

& Tenth Term (T_{10}) $= 4(10) - 1 = 39$

$\because$ by question Required Difference

$= T_{10} - T_1$

$= 39 - 3$

$= 36$

[36] Find the product of :

$(243), (243)^{1/6}, (243)^{1/36}, \ldots\ldots \infty$

(a) 1,024 (b) 27
(c) 729 (d) 246 (1 mark)

Answer:

(c) $(243) \times (243)^{\frac{1}{6}} \times (243)^{\frac{1}{36}} \times \ldots \infty$

$= (243)^{1+\frac{1}{6}+\frac{1}{36}+\ldots\infty}$

The power $1+\frac{1}{6}+\frac{1}{36}+\ldots\infty$ is an infinite G.P. with a= 1, and r = $\frac{1}{6}$.

$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{6}} = \frac{1}{5/6} = \frac{6}{5}$

Therefore, we have $(243)^{\frac{6}{5}} = (3^5)^{\frac{6}{5}} = 3^{5\times\frac{6}{5}} = 3^6 = 729.$

[37] Insert two Arithmetic means between 68 and 260

(a) 132, 196 (b) 130, 194

(c) 70, 258 (d) None of the above. (1 mark)

Answer:

(a) Let A_1 A_2, be the two Arithmetic Means between 68 and 260

$\because$ 68, A_1, A_2, 260 are in A.P

here,

First Term (a) = 68

Last Term (l) = 260

Total no. of terms = 4

we know,

$l = a + (n-1)d$

$260 = 68 + (4-1)d$

$\because$ $d = 64$

$\because$ $A_1 = a + d = 68 + 64 = 132$

and $A_2 = a + 2d = 68 + 2 \times 64 = 196$

[38] Geometric Mean of $P, P^2, P^3 \ldots\ldots\ldots, P^n$ will be :

(a) P^{n+1} (b) $P^{\frac{1+n}{2}}$

(c) $P^{\frac{n(n+1)}{2}}$ (d) None of the above. (1 mark)

Answer:

(b) Given $P, P^2, P^3, \ldots\ldots\ldots P^n$

$$= [P^{(1+2+\cdots+n)}]^{1/n}$$
$$= \left[P^{\frac{n(n+1)}{2}}\right]^{1/n}$$
$$= P^{\frac{(n+1)}{2}}$$

2011 - DECEMBER

[39] Find the numbers whose arithmetic mean is 12.5 and geometric mean is 10.

(a) 20 and 5 (b) 10 and 5
(c) 5 and 4 (d) None of these (1 mark)

Answer:

(a) Let two numbers be a and b

$$A.M = \frac{a+b}{2} = 12.5$$
$$a + b = 25 \ldots\ldots\ldots\ldots (1)$$
$$G.M = \sqrt{ab} = 10$$
$$ab = 100 \ldots\ldots\ldots\ldots (2)$$
$$(a-b)^2 = (a+b)^2 - 4ab$$
$$= (25)^2 - 4 \times 100$$
$$= 625 - 400$$
$$(a-b)^2 = 225$$
$$(a-b) = \sqrt{225}$$
$$a - b = 15 \ldots\ldots\ldots\ldots (3)$$

Solving (1) & (3) we get

$$a = 20, b = 5$$

[40] If sum of 3 arithmetic means between "a" and 22 is 42, then "a" = _____

(a) 14 (b) 11
(c) 10 (d) 6 (1 mark)

Answer:

(d) Let the three Arithmetic mean between a and 22 are A_1 A_2 A_3. Then

Total term n = 5
First term (a) = a
l = 22
Let common diff. be d
l = a + (n – 1)d
22 = a + (5 – 1)d
22 = a + 4d
a + 4d = 22.....(1)
A_1 = a + d
A_2 = a + 2d
A_3 = a + 3d
Sum = 42
$A_1 + A_2 + A_3 = 42$
a + d + a + 2d + a + 3d = 42
3 (a + 2d) = 42
a + 2d = 14 .....(2)
eq (1) – eq (2)

$$\begin{array}{r} a + 4d = 22 \\ -a \pm 2d = -14 \\ \hline 2d = 8 \\ d = 4 \end{array}$$

Putting the valued of d in eq (1)
a + 4 × 4 = 22
a + 16 = 22
a = 22 – 16 = 6

[41] If each month ₹ 100 increases in any sum then find out the total sum after 10 months, if the sum of first month is ₹ 2,000.
(a) ₹ 24,500 (b) ₹ 24,000
(c) ₹ 50,000 (d) ₹ 60,000 (1 mark)

Answer:

(a) First term (a) = 2,000
Common diff (d) = 100
(n) = 10
S_n = ?
$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{10}{2}[2 \times 2{,}000 + (10 - 1)100]$

$= 5[4{,}000 + 900]$

$= 5 \times 4{,}900$

$= ₹\ 24{,}500$

[42] The sum of all two Digit odd numbers is

(a) 2475 (b) 2575

(c) 4950 (d) 5049 (1 mark)

Answer:

(a) All two digit No. are

11, 13, 15.....99

First term (a) = 11

Last term (l) = 99

Common difference (d) = 13 – 11 = 2

$n = ?$

$l = a + (n - 1)d$

$99 = 11 + (n - 1) \times 2$

$99 = 11 + 2n - 2$

$2n = 99 + 2 - 11$

$2n = 90 \Rightarrow n = 45$

Sum of all two digit odd numbers

$S_n = \frac{n}{2}[a + l]$

$= \frac{45}{2}[11 + 99]$

$= \frac{45}{2}[110]$

$= 45 \times 55$

$= 2{,}475$

[43] If 5^{th} term of a G.P. is $3\sqrt{3}$, then the product of first nine terms is

(a) 8 (b) 27

(c) 243 (d) 9 (1 mark)

Answer:

(b) Let the first term be a and the common ratio be r.

$t_n = ar^{n-1}$.

$t_5 = 3\sqrt{3}$

Product of the first 9 terms is given by $a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8$

$= a^9.r^{1+2+3+4+5+6+7+8}$

= The sum of n natural numbers is given by $\frac{n(n+1)}{2}$.

$a^9.r^{1+2+3+4+5+6+7+8} = a^9.r^{\frac{8(8+1)}{2}}$

$= a^9.r^{36}$

$= (ar^4)^9$

$= (3\sqrt{3})^9$

$= \left(3^{\frac{1}{3}}\right)^9$

$= 3^{\frac{1}{3}\times 9}$

$= 3^3$

$= 27$

[44] The sum of the third and ninth term of an A.P. is 8. Find the sum of the first 11 terms of the progression.

(a) 44 (b) 22

(c) 19 (d) 11 (1 mark)

Answer:

(a) Let the first term and Common difference are a and d respectively

n^{th} of A.P

$T_n = a + (n-1)\,d$

$T_3 = a + (3-1)\,d = a + 2d$

$T_9 = a + (9-1)\,d = a + 8d$

Given

$T_3 + T_9 = 8$

$a + 2d + a + 8d = 8$

$2a + 10d = 8$ .....(1)

Sum of n terms

$S_n = \frac{n}{2}[2a + (n-1)d]$

Sum of 11 terms

$S_{11} = \frac{11}{2}[2a + (11-1)d]$

$= \frac{11}{2}[2a + 10d]$

$= \frac{11}{2} \times 8$ [from eq (1)]

$S_{11} = 44$

2012 - JUNE

[45] If 8th term of an A.P is 15, then sum of its 15 terms is

(a) 15 (b) 0

(c) 225 (d) 225/2 (1 mark)

Answer:

(c) Let a be the first term of A.P and $T_n = n^{th}$ term of A.P and d be the common ratio of A.P series

$T_n = a + (n - 1)\, d$

$T_8 = a + (8 - 1)\, d = 15$

$a + 7d = 15$.....(i)

The sum of n term

$S_n = \frac{n}{2}[2a + (n - 1)\, d]$

$S_{15} = \frac{15}{2}[2a + (15 - 1)\, d]$

$= \frac{15}{2}[2a + 14d]$

$= \frac{15}{2} \times 2\,(a + 7d)$

$= 15 \times 15$ [from eq. (i)]

$= 225$

[46] Find the sum of the infinite terms $2, \frac{4}{y}, \frac{8}{y^2}, \frac{16}{y^3}$.....; if $y > 2$

(a) $\frac{2y}{y-2}$ (b) $\frac{4y}{y-2}$

(c) $\frac{3y}{y-2}$ (d) None of these. (1 mark)

Answer:

(a) Say

$$S_n = \frac{2}{y^0} + \frac{4}{y^1} + \frac{8}{y^2} + \frac{16}{y^3} + \ldots\ldots\ldots\ldots\ldots\ldots \frac{2^n}{y^{n-1}}$$

Multiplying its equation (1) by $\frac{2}{y}$ we get.....(1)

$$\frac{2S_n}{y} = \frac{4}{y} + \frac{8}{y^2} + \frac{16}{y^3} + \ldots\ldots\ldots\ldots\ldots\ldots + \frac{2^{n-1}}{y^n}$$

Subtracting e.g. (2) from (1) we get .....(2)

$$S_n\left[1 - \frac{2}{y}\right] = 2 - \frac{2^{n-1}}{y^n}$$

or $S_n = \left(\frac{y}{y-2}\right) \times 2\left[1 - \left(\frac{2}{y}\right)^n\right]$

$S_n = \left(\frac{2y}{y-2}\right)[$ (Given $y > 2$)

Solving (1) and (2) we get

$$2(2d - 1) - 3d = 0$$

$$\Rightarrow \quad d = 2$$

Putting d = 2 in (2) we get

$$a = 2(2) - 1$$

$$a = 3$$

[48] If arithmetic mean between roots of a quadratic equation is 8 and the geometric mean between them is 5, the equation is _____.

(a) $x^2 - 16x - 25 = 0$ (b) $x^2 - 16x + 25 = 0$

(c) $x^2 - 16x + 5 = 0$ (d) None of these. (1 mark)

Answer:

(b) Say, α, β are the roots of quadratic equation then as per given conditions, we have

A.M. $\Rightarrow \frac{\alpha + \beta}{2} = 8$

$\Rightarrow \alpha + \beta = 16$ (i.e. Sum of roots)

and given

G.M. $\Rightarrow \sqrt{\alpha.\beta} = 5$

$\Rightarrow \alpha.\beta = 25$ (i.e. Product of roots)

$\therefore$ required quadratic equation is :

x^2 - (Sum of roots) x + (Product of roots) = 0

$\Rightarrow x^2 - 16x + 25 = 0$

2012 - DECEMBER

[49] In an A.P., if common difference is 2, Sum of n terms is 49, 7^{th} term is 13 then n = _____.

(a) 0 (b) 5

(c) 7 (d) 13 (1 mark)

Answer:

(c) In A.P, Common difference (d) = 2

Sum of n terms (S_n) = 49

We know that,

$T_n = a + (n-1)\,d$

$T_7 = a + (7-1) \times 2$

$T_7 = a + 6 \times 2$

$13 = a + 12$

$a = 13 - 12 = 1$

$S_n = \frac{n}{2}[2a + (n-1)\,d]$

$49 = \frac{n}{2}[2 \times 1 + (n-1)\,2]$

$49 = \frac{n}{2} \times 2[1 + n - 1]$

$49 = n^2$

$n = \sqrt{49} = 7$

[50] The first term of a G.P. where second term is 2 and sum of infinite term is 8 will be:

(a) 6 (b) 3

(c) 4 (d) 1 (1 mark)

Answer:

(c) Let first term of G.P. is 'a' and common Ratio is 'r'

$T_2 = ar^{2-1}$ [$\because T_n = ar^{n-1}$]

$2 = ar$ _____ (1)

$S\alpha = \frac{a}{1-r}$

$8 = \frac{a}{(1-r)}$

$a = 8(1-r)$ _____ (2)

Putting the value of a in eq (i)

$2 = 8(1-r).\,r$

$2 = 8r - 8r^2$

$8r^2 - 8r + 2 = 0$

$4r^2 - 4r + 1 = 0$

$4r^2 - 2r - 2r + 1 = 0$

$2r(2r-1) - 1(2r-1) = 0$

$(2r-1)(2r-1) = 0$

If $2r - 1 = 0$

$r = ½$ in eq (i)

$$2 = a \times \frac{1}{2} \Rightarrow a = 4$$

[51] If the sum of n terms of an A.P be $2n^2 + 5n$, then its 'n^{th}' term is:

(a) $4n - 2$ (b) $3n - 4$

(c) $4n + 3$ (d) $3n + 4$ (1 mark)

Answer:

(c) Given Sum of n term $(S_n) = 2n^2 + 5n$

$$S_{n-1} = 2(n-1)^2 + 5(n-1)$$
$$= 2(n^2 + 1 - 2n) + 5n - 5$$
$$= 2n^2 + 2 - 4n + 5n - 5$$
$$= 2n^2 + n - 3$$

n^{th} term $(T_n) = S_n - S_{n-1}$

$$= (2n^2 + 5n) - (2n^2 + n - 3)$$
$$= 2n^2 + 5n - 2n^2 - n + 3$$
$$= 4n + 3$$

2013 - JUNE

[52] If the sum of n terms of an A.P be $3n^2 - n$ and its common difference is 6, then its first term is :

(a) 2 (b) 3

(c) 4 (d) 5 (1 mark)

Answer:

(a) Given the sum of n term $S_n = 3n^2 - n$

$n = 1$ $S_1 = 3(1)^2 - 1$

$= 3 - 1 = 2$

First term $(T_1) = S_1 = 2$

[53] If the sum of the 4^{th} term and the 12^{th} term of an A.P. is 8, what is the sum of the first 15 terms of the progression?

(a) 60 (b) 120

Answer:

(a) Let first term and common difference of A.P. is a and d

Given $T_4 + T_{12} = 8$

$a + 3d + a + 11d = 8$

$2a + 14d = 8$

$S_{15} = \frac{15}{2}[2a + (15-1)d]$ $\{\because S_n = \frac{n}{2}[2a + (n-1)d]\}$

$S_{15} = \frac{15}{2}[2a + 14d] = \frac{15}{\cancel{2}} \times \cancel{8}^{4} = 60$

[54] If 'n' arithmetic means are inserted between 7 & 71 and 5th arithmetic mean is 27, then 'n' is equal to:

(a) 15 (b) 16

(c) 17 (d) 18 (1 mark)

Answer:

(a) If 'n' Arithmetic mean are inserted between 7 & 71 are A_1, A_2 ..... An

7 A_1, A_2, A_3 ..... An 71

$$d = \frac{l - a}{n+1} = \frac{71 - 7}{n+1} = \frac{64}{n+1}$$

Given $A_5 = 27$

$a + 5d = 27$

$7 + 5 \times \frac{64}{(n+1)} = 27$

$\frac{320}{n+1} = 20 \rightarrow (n+1) = \frac{320}{20}$

$n + 1 = 16 \Rightarrow \boxed{n = 15}$

[55] In a G.P. the sixth term is 729 and the common ratio is 3, then the first term of G.P. is:

(a) 2 (b) 3

(c) 4 (d) 7 (1 mark)

Answer:

(b) Let Ist term of G.P. is a and common Ratio is r

Given $T_6 = 729$, Common Ratio (r) = 3

$$ar^{6-1} = 729$$
$$ar^5 = 729$$
$$\rightarrow \quad a(3)^5 = 729$$
$$\Rightarrow \quad a \times 243 = 729$$
$$a = \frac{729}{243}$$
$$a = 3$$

2013 - DECEMBER

[56] An Arithmetic progression has 13 terms whose sum is 143. The third term is 5 so the first term is:

(a) 4 (b) 7
(c) 9 (d) 2 (1 mark)

Answer:

(d) Given $n = 13, S_{13} = 143, T_3 = 5, a = ?$

$T_3 = a + 2d = 5$ ..... (1)

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{13} = \frac{13}{2}[2a + (13 - 1)d] = 143$

$\frac{13}{2}[2a + 12d] = 143$

$\frac{13}{2} \times 2 (a + 6d) = 143$

$a + 6d = \frac{143}{13}$

$a + 6d = 11$ ..... (2)

Solving (1) & (2) we get $\boxed{a = 2}$

[57] If Geometric mean (G.M.) of a, b, c, d is 3, then G.M. of $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ will be:

(a) 1/3 (b) 3

Answer:

(a) G.M. of a, b, c, d

$(G.M) = (a \cdot b \cdot c \cdot d)^{1/4} = 3$

$a \cdot b \cdot c \cdot d = 3^4$

$a \cdot b \cdot c \cdot d = 81$ ..... (i)

G.M of $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d} = \left(\frac{1}{a} \cdot \frac{1}{b} \cdot \frac{1}{c} \cdot \frac{1}{d}\right)^{1/4}$

$= \left(\frac{1}{abcd}\right)^{1/4}$

$= (abcd)^{-1 \times \frac{1}{4}}$

$= (81)^{-1/4}$

$= 3^{4x-1/4}$

$= 3^{-1}$

$= \left(\frac{1}{3}\right)$

2014 - JUNE

[58] The sum to m terms of the series 1+11+111+.... upto m terms, is equal to:

(a) $\frac{1}{81}(10^{m+1} - 9m - 10)$ (b) $\frac{1}{27}(10^{m+1} - 9m - 10)$

(c) $10^{m+1} - 9m - 10$ (d) None of these (1 mark)

Answer:

(a) Given

I + II + III + ... m terms

$= \frac{1}{9}$ [9 + 99 + 999 + ... m terms]

$= \frac{1}{9}$ [(10 – 1) + (100 – 1) + (1000 – 1) + ... m terms]

$= \frac{1}{9}$ [(10 + 100 + 1000 + ...m terms) –

$$= \frac{1}{9}\left[\frac{10(10^m - 1)}{10 - 1} - m\right]$$

$$= \frac{1}{9}\left[\frac{10}{9}(10^m - 1) - m\right]$$

$$= \frac{1}{81}(10^{m+1} - 10 - 9m)]$$

$$= \frac{1}{81}(10^{m+1} - 9m - 10)$$

[59] The sum of the infinite G.P.

$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\ldots$ is equal to:

(a) 1.95 (b) 1.5

(c) 1.75 (d) None of these (1 mark)

Answer:

(b) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\ldots\ldots\ldots \infty$

$$a = 1, r = \frac{\frac{1}{3}}{1} = \frac{1}{3} < 1$$

$$S_\infty = \frac{a}{1 - r} = \frac{1}{1 - \frac{1}{3}} = \frac{1}{2} = \frac{3}{2} = 1.5$$

[60] The value of $1^3+2^3+3^3+4^3+\ldots\ldots+m^3$ is equal to:

(a) $\left[\frac{m(m+1)}{2}\right]^3$ (b) $\frac{m(m+1)(2m+1)}{6}$

(c) $\left[\frac{m(m+1)}{2}\right]^2$ (d) None of these. (1 mark)

Answer:

(c) $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots + m^3$

$= \Sigma m^3$

$= \left[\frac{m(m+1)}{2}\right]^2$

2014 - DECEMBER

[61] If x, y, z are the terms in G.P. then the terms $x^2 + y^2$, $xy + yz$, $y^2 + z^2$ are in:

(a) A.P. (b) G.P.
(c) H.P. (d) None of these. (1 mark)

Answer:

(b) If x, y, z are in G.P

then $x = 1, y = 2, z = 4$

$x^2 + y^2, xy + yz, y^2 + z^2$

$= 1^2 + 2^2, 1 \times 2 + 2 \times 4, 2^2 + 4^2$

$= 5, 10, 20$ are in G.P.

[62] If $S_n = n^2p$ and $S_m = m^2p$ $(m \neq n)$ is the sum of an A.P., then $S_p =$ _____

(a) p^2 (b) p^3
(c) $2p^3$ (d) p^4 (1 mark)

Answer:

(b) If $S_n = n^2p$ and $S_m = m^2p$ find $S_p = ?$

Now

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$n^2p = \frac{n}{2}[2a + (n-1)d]$$

$$\frac{2n^2p}{n} = 2a + nd - d$$

$$2a + nd - d = 2np \quad \text{_____ (1)}$$

$$\text{and } S_m = \frac{m}{2}[2a + (m-1)d]$$

$$m^2p = \frac{m}{2}[2a + md - d]$$

$$\frac{2m^2p}{m} = [2a + md - d]$$

Equation (1) – Equation (2)

$$2a + nd - d = 2np$$
$$2a + md - d = 2mp$$
$$- \quad - \quad + \quad -$$

$$nd - md = 2np - 2mp$$
$$d(n - m) = 2p(n - m)$$
$$d = 2p$$

$d = 2p$ in equation (1)

$$2a + m \cdot 2p - 2p = 2mp$$
$$2a = 2mp - 2mp + 2p$$
$$2a = 2p$$
$$a = p$$

The sum of p term of A.P.

$$S_p = \frac{p}{2}[2a + (p - 1)d]$$
$$= \frac{p}{2}[2.p + (p - 1)2p]$$
$$= \frac{p}{2}[2p + 2p^2 - 2p]$$
$$= \frac{p}{2} \cdot 2p^2$$
$$= p^3$$

[63] The arithmetic mean of the square of first 2n natural numbers is:

(a) $\frac{1}{6}(2n + 1)(4n - 1)$ (b) $\frac{1}{6}(2n - 1)(4n - 1)$

(c) $\frac{1}{6}(2n - 1)(4n + 1)$ (d) $\frac{1}{6}(2n + 1)(4n + 1)$ (1 mark)

Answer:

(d) We know,

Sum of squares of first, 'n' natural nos. is

$$S_n = \frac{n}{6}(n + 1)(2n + 1)$$

So for '2n' natural numbers.

Replacing n by 2n in above formula, we get,

$$S_{2n} = \frac{2n}{6}(2n + 1)(2(2n) + 1) = \frac{n}{3}(2n + 1)(4n + 1)$$

Mean of '2n' natural Nos.

$$P^2 = \frac{S^n}{R^n}$$

$$P^2 R^n = S^n$$

[66] The sum of the series 1 + 11 + 111 + ... to n terms is _____.

(a) $\frac{1}{27}(10^{n+1} - 9n - 10)$ (b) $10^{n+1} - 9n - 10$

(c) $\frac{1}{81}(10^{n+1} - 9n - 10)$ (d) None of these (1 mark)

Answer:

(c) Given series:

$1 + 11 + 111 + \ldots\ldots$ to n term

$= \frac{1}{9}[9 + 99 + 999 + \ldots\ldots$ to n terms]

$= \frac{1}{9}[(10 - 1) + (100 - 1) + (1000 - 1) \ldots\ldots$ to n terms]

$= \frac{1}{9}[(10 + 100 + 1000 + \ldots\ldots$ n terms) - (1 + 1 + 1 + ... n term)]

$= \frac{1}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right]$

$= \frac{1}{9}\left[\frac{10^{n+1} - 10}{9} - n\right]$

$= \frac{1}{9}\left[\frac{10^{n+1} - 10 - 9n}{9}\right]$

$= \frac{1}{81}[10^{n+1} - 9n - 10]$

[67] If third term and seventh term of an A.P are eighteen and thirty respectively, then sum of first twenty terms will be:

(a) 540 (b) 610

(c) 740 (d) 810 (1 mark)

Answer:

(d) Let 1st term of A.P. is a and common difference is d.

Given: $T_3 = 18$ and $T_7 = 30$

$a + 2d = 18$ _____(1) $a + 6d = 30$_____ (2)

eq (2) = eq (1)

$a + 6d = 30$
$a + 2d = 18$
$-\quad\quad -$

$4d = 12$

$d = 3$ in equation (1)

$a + 6 \times 3 = 30$

$a + 18 = 30$

$a = 30 - 18 = 12$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{20} = \frac{20}{2}[2 \times 12 + (20 - 1)3]$

$= 10\,[24 + 19 \times 3]$

$= 10\,[24 + 57]$

$= 10 \times 81 = 810$

2015 - DECEMBER

[68] If the sum of 'n' terms of an Arithmetic Progression (A.P) is $3x^2 + 5x$ and its m^{th} term is 164, then the value of m is:

(a) 27 (b) 28

(c) 24 (d) 26 (1 mark)

Answer:

(a) Given $S_n = 3n^2 + 5n$

putting n = 1, $S_1 = 3(1)^2 + 5(1) = 3 + 5 = 8$

n = 2, $S_2 = 3(2)^2 + 5(2) = 12 + 10 = 22$

$S_3 = 3(3)^2 + 5(3) = 27 + 15 = 42$

then,

$T_1 = S_1 = 8$

$T_2 = S_2 - S_1 = 22 - 8 = 14$

$T_3 = S_3 - S_2 = 42 - 22 = 20$

A.P. series is

8,14, 20,.....

$a = 8, d = 14 - 8 = 6, T_m = 164$

$T_m = a + (m - 1) d$

$164 = 8 + (m - 1) \times 6$

$164 = 8 + 6m - 6$

$6m = 164 + 6 - 8$

$6m = 162$

$m = \frac{162}{6}$

$m = 27$

[69] If a, b, c are in Arithmetic Progression (A.P.), then the value of a-b+c is:

(a) a (b) -b

(c) b (d) c (1 mark)

Answer:

(c) If a, b, c are in A.P.

then $a = 1, b = 2, c = 3$

$a - b + c = 1 - 2 + 3 = 4 - 2 = 2$

[70] Find the two numbers whose geometric mean is 5 and arithmetic mean in 7.5.

(a) 10 and 5 (b) 13.09 and 1.91

(c) 12 and 3 (d) None of the above (1 mark)

Answer:

(b) Let two Number a and b

$A.M. = \frac{a+b}{2}$

$7.5 = \frac{a+b}{2}$

$a+b = 15$ ____(1)

$G.M. = \sqrt{ab}$

$5 = \sqrt{ab}$

$25 = ab$ _____ (2) (on squaring both side)

Solving (1) and (2) we get

a = 13.09 and b = 1.91

2016 - JUNE

[71] The sum of n terms of the series $\log x + \log \frac{x^2}{y} + \log \frac{x^3}{y^2} +$ ..... is

(a) $\frac{n}{2}\left[2n \log\left(\frac{x}{y}\right) + \log xy\right]$ (b) $\frac{n}{2}\left[n \log xy + \log\left(\frac{x}{y}\right)\right]$

(c) $\frac{n}{2}\left[n \log\left(\frac{x}{y}\right) - \log xy\right]$ (d) $\frac{n}{2}\left[n \log\left(\frac{x}{y}\right) + \log xy\right]$ (1 mark)

Answer:

(d) Given Series

$\log x + \log \frac{x^2}{y} + \log \frac{x^3}{y^2} +$ ..... n terms

First term (a) = $\log x$

Common difference (d) = $T_2 - T_1$

$= \log\left(\frac{x^2}{y}\right) - \log x$

$= \log\left(\frac{x^2}{y.x}\right)$

$= \log\left(\frac{x}{y}\right)$

Sum of n term of A.P.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}\left[2.\log x + (n-1)\log\left(\frac{x}{y}\right)\right]$

$$= \frac{n}{2}\left[2\log x + n\log\left(\frac{x}{y}\right) - \log\left(\frac{x}{y}\right)\right]$$

$$= \frac{n}{2}\left[n\log\left(\frac{x}{y}\right) + 2\log x - \log\left(\frac{x}{y}\right)\right]$$

$$= \frac{n}{2}\left[n\log\left(\frac{x}{y}\right) + \log x^2 + \log\left(\frac{y}{x}\right)\right]$$

$$= \frac{n}{2}\left[n\log\left(\frac{x}{y}\right) + \log\left(x^2 \cdot \frac{y}{x}\right)\right]$$

$$= \frac{n}{2}\left[n\log\frac{x}{y} + \log xy\right]$$

[72] A G. P. (Geometric Progression) consists of 2n terms. If the sum of the terms occupying the odd places is S_1 and that of terms in the even places is S_2, the common ratio of the progression is:

(a) n (b) $2S_1$

(c) $\frac{S_2}{S_1}$ (d) $\frac{S_1}{S_2}$ (1 mark)

Answer:

(c) Given G.P. series consist of 2n terms

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 +$ ..... 2n term

Here

$S_1 = a_1 + a_3 + a_5 +$ ..... n terms

$= a + ar^2 + ar^4 +$ ..... n terms

$= a\,[1 + r^2 + r^4 +$ ..... n terms$]$

$= a, 1\left[\frac{(r^2)^n - 1}{r - 1}\right]$

$S_1 = a\left[\frac{r^{2n} - 1}{r - 1}\right]$ ..... (1)

and

$S_2 = a_2 + a_4 + a_6 +$ ..... n terms

$= ar + ar^3 + ar^5 +$ ..... n terms

$= ar\,[1 + r^2 + r^4 +$ ..... n terms$]$

$$= ar\left[\frac{1.(r^{2n}-1)}{r-1}\right]$$

$$= \frac{ar(r^{2n}-1)}{r-1} \quad \text{..... (2)}$$

equation (2) / equation (1)

$$\frac{S_2}{S_1} = \frac{\frac{ar(r^{2n}-1)}{r-1}}{\frac{a(r^{2n}-1)}{r-1}}$$

$$\frac{S_2}{S_1} = r$$

Common Ratio = $\frac{S_2}{S_1}$

[73] If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in arithmetic progression then a^2, b^2, c^2, are in

(a) Arithmetic Progression
(b) Geometric Progression
(c) Both in arithmetic and geometric Progression
(d) None of these (1 mark)

Answer:

(a) If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in arithmetic Progression then

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{b-c-c-a}{(c+a)(b+c)} = \frac{c+a-a-b}{(a+b)(c+a)}$$

$$\frac{(b-a)}{b+c} = \frac{(c-b)}{(b+a)}$$

$(b - a)(b + a) = (c - b)(c + b)$

$b^2 - a^2 = c^2 - b^2$

$b^2 + b^2 = c^2 + a^2$

$2b^2 = c^2 + a^2$

a^2, b^2, c^2 are in A.P.

Sum of n terms $S_n = 0$

$S_n = \frac{n}{2}[2a + (n-1)d]$ $\quad 0 = \frac{n}{2}[2 \times 50 + (n-1)(-5)]$

$\frac{2}{n} \times 0 = 100 - 5n + 5$

$0 = 105 - 5n$

$5n = 105$

$n = \frac{105}{5} = 21$

[76] The number 2.353535 _____ in $\frac{p}{q}$ form is:

(a) $\frac{235}{99}$ (b) $\frac{234}{99}$

(c) $\frac{230}{99}$ (d) $\frac{233}{99}$ (1 mark)

Answer:

(d) 2.35 35 35 ----------

$= 2.\overline{35}$

$= 2 + 0.\overline{35}$

$= 2 + \frac{35}{100-1}$ $\because$ [ ]

2017 - JUNE

[77] The sum of n terms of the series $1 + (1 + 3) + (1 + 3 + 5) +$ ..... is

(a) $\frac{n(n+1)(2n+1)}{6}$ (b) $\frac{n(n+1)(n+2)}{6}$

(c) $\frac{n(n+1)(2n+1)}{3}$ (d) None of these (1 mark)

Answer:

(a) Given Series

$1 + (1 + 3) + (1 + 3 + 5) +$ ----------n term

$= 1 + 4 + 9 +$ ----------n term

$= 1^2 + 2^2 + 3^2 +$ -------- $+ n^2$

$= \Sigma n^2$

$= \frac{n(n+1)(2n+1)}{6}$

[78] The sum of first 20 terms of a GP is 1025 times the sum of first 10 terms of same GP then common ratio is:

(a) $\sqrt{2}$ (b) 2

(c) $2\sqrt{2}$ (d) 1/2 (1 mark)

Answer:

(b) Let 1st term of G.P is a and Common Ratio of G.P is r.

Given, $S_{20} = 1025\ S_{10}$

$\frac{a(r^{20}-1)}{(r-1)} = 1025\ \frac{a(r^{10}-1)}{(r-1)}$

$r^{20} - 1 = 1025\ (r^{10} - 1)$

$(r^{10})^2 - (1)^2 = 1025\ (r^{10} - 1)$

$(r^{10} + 1)\ (r^{10} - 1) = 1025\ (r^{10} - 1)$

$r^{10} + 1 = 1025$

$r^{10} = 1025 - 1$

$r^{10} = 1024$

$(r^2)^5 = (4)^5 \quad r^2 = 4 \quad r = \pm 2$

$r = 2$

[79] The value C such that a, – 3, b, 5, c are in A.P. is:

(a) –7 (b) 1

(c) 13 (d) 9 (1 mark)

Answer:

(d) If a, -3, b, 5, c are in A.P.

Now -3, b, 5 are in A.P.

$b - (-3) = 5 - b$

$b + 3 = 5 - b$

$b + b = 2$

$2b = 2$

$\boxed{b = 1}$

and b, 5, c are in A.P.

$5 - b = c - 5$

$5 - 1 = c - 5$

$4 = c - 5$

$c = 4 + 5$

$\boxed{c = 9}$

2017 - DECEMBER

[80] The sum of all numbers between 100 and 1000 which are divisible by 11 will be:

(a) 44550 (b) 66770

(c) 55440 (d) 33440 (1 mark)

Answer:

(a) Number b/w 100 and 1000 which are divisible by 11 are

110,121,132, ----------,990

Here, a = 110 , d= 121-110 = 11, l = 990

then, $n = \frac{l - a + d}{d} = \frac{990 - 110 + 11}{11} = \frac{891}{11} = 81$

Sum of n term of A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{81} = \frac{81}{2}[2\times110 + (81-1)11]$$

$$= \frac{81}{2}[220 + 80 \times 11]$$

$$= \frac{81}{2}[220 + 880] = 44550$$

2018 - MAY

[81] The sum to m terms of the series 1+11+111+.....upto m terms, is equal to:

(a) $\frac{1}{81}(10^{m+1} - 9m - 10)$ (b) $\frac{1}{27}(10^{m+1} - 9m - 10)$

(c) $10^{m+1} - 9m - 10$ (d) None of these (1 mark)

Answer:

(a) Given series

$1 + 11 + 111 + \ldots\ldots\ldots\ldots\ldots$ m term

$= \frac{1}{9}[9 + 99 + 999 + \ldots\ldots\ldots$ m term]

$= \frac{1}{9}[(10-1) + (100-1) + (1000-1) + \ldots\ldots\ldots +$ m term]

$= \frac{1}{9}[(10 + 100 + 100 + \ldots\ldots\ldots$ m term) $-$ (1+1+1+..... m term)]

$= \frac{1}{9}\left[\frac{10.(10^m-1)}{10-1} - m\right]$

$$= \frac{1}{9}\left[\frac{10^{m+1}-10}{9} - m\right]$$

$$= \frac{1}{9}\left[\frac{10.10^{m+1}-10-9m)}{9} - m\right]$$

$$= \frac{1}{81}\left[10^{m+1} - 9m - 10)\right]$$

[82] A person pays ₹ 975 in monthly instalments, each instalment is less than former by ₹ 5. The amount of Ist instalment is ₹ 100. In what time will the entire amount be paid?

(a) 26 months (b) 15 months

(c) Both (a) & (b) (d) 18 months. (1 mark)

Answer:

(b) $S_n = 975$, $a = 100$, $d = -5$, $n = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$975 = \frac{n}{2}[2\times 100 + (n-1)(-5)]$$

$1,950 = n\,[200 - 5n + 5]$

$1,950 = n\,[205 - 5n]$

$1950 = 205n - 5n^2$

$5n^2 - 205n + 1950 = 0$

$5(n^2 - 41n + 390) = 0$

$n^2 - 41n + 310 = 0$

$n^2 - 26n - 15n + 390 = 0$

$n\,(n - 26) - 15\,(n - 26) = 0$

$(n - 26)\,(n - 15) = 0$

if $n - 15 = 0$ if $n - 26 = 0$

$n = 15$ $n = 26$

The entre amount will be paid in 15 months.

[83] If the sum of n terms of an AP is $(3n^2\text{-}n)$ and its common difference is 6, then its first term is:

(a) 3 (b) 2

Answer:

(b) Given,

$S_n = (3n^2 - n)$

$n = 1,$

$S_1 = 3(1)^2 - 1 = 3 - 1 = 2$

$n = 2, S_2 = 3(2)^2 - 1 = 12 - 1 = 11$

$n = 3, S_3 = 3(3)^2 - 1 = 27 - 1 = 26$

$T_1 = S_1 = 2$

$T_2 = S_2 - S_1 = 11 - 2 = 9$

$T_3 = S_3 - S_2 = 26 - 11 = 15$

First term of series $= T_1 = 2$

[84] Insert two arithmetic means between 68 and 260.

(a) 132, 196 (b) 130, 194

(c) 70, 258 (d) None of the above (1 mark)

Answer:

(a) Let, two A.M's between 68 and 260 are A_1, A_2

68, A_1, A_2,; 260

$d = \frac{b - a}{n + 1}$ Here, $a = 68, n = 2, b = 260$

$d = \frac{260 - 68}{2 + 1} = \frac{192}{3} = 64$

$A_1 = a + d = 68 + 64 = 132$

$A_2 = a + 2d = 68 + 2 \times 64 = 68 + 128 = 196$

2018 - NOVEMBER

[85] If the P^{th} term of an A.P. is 'q' and the q^{th} term is 'p', then its r^{th} term is

(a) $p + q - r$ (b) $P + q + r$

(c) $p - q - r$ (d) $p - q$ (1 mark)

Answer:

(a) Let, Ist term of AP is 'a'
and common difference is 'd'

Given $T_p = q$

$a + (p - 1)\,d = q$

$a + pd - d = q$ ——————————— (i)

and $T_q = P$

$a + (q - 1)d = p$

$a + qd - d = p$ ——————————— (ii)

equation (i) and equation (ii)

$$\begin{array}{r} a + pd - d = q \\ a + qd - d = p \\ \hline - \quad - \quad + \quad - \end{array}$$

$pd - qd = q - p$

$d(p - q) = -(p - q)$

$d = -1$

Putting $d = -1$ in equation (i)

$a + p(-1) - (-1) = q$

$a - p + 1 = q$

$a = p + q - 1$

Then, $T_r = a + (r - 1)\,d$

$= p + q - 1 + (r - 1)(-1)$

$= p + q - 1 - r + 1$

$= p + q - r$

[86] The 3rd term of a G.P. is $\frac{2}{3}$ and the 6th term is $\frac{2}{18}$, then the 1st term is

(a) 6 (b) $\frac{1}{3}$

(c) 9 (d) 2 (1 mark)

Answer:

(a) Let Ist term of G.P. is 'a' and common Ratio is 'r' then

Given $T_3 = \frac{2}{3}$ and $T_6 = \frac{2}{81}$

$ar^2 = \frac{2}{3}$ _____ (i) $ar^5 = \frac{2}{81}$ _____ (ii)

eq. (2)/eq. (1)

$$\frac{ar^5}{ar^2} = \frac{\frac{2}{81}}{\frac{2}{3}}$$

$$r^3 = \frac{2}{81} \times \frac{3}{2} \Rightarrow r^3 = \frac{1}{27} = r = \frac{1}{3}$$

Putting $r = \frac{1}{3}$ in equation (i)

$$ar^2 = \frac{2}{3}$$

$$a\left(\frac{1}{3}\right)^2 = \frac{2}{3} \rightarrow a \times \frac{1}{9} = \frac{2}{3}$$

$$\rightarrow a = \frac{2}{3} \times \frac{9}{1}$$

$$\rightarrow a = 6$$

[87] The sum of the series – 8, – 6, – 4, ...n terms is 52. The number of terms n is

(a) 11 (b) 12

(c) 13 (d) 10 (1 mark)

Answer:

(c) Given series

– 8, – 6, – 4, ... n term

Let term (a) = – 8

common difference (d) = (– 6) – (– 8)

= – 6 + 8

= 2

Sum of 'n' term (S_n) = 52, n = ?

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$52 = \frac{n}{2}[2\times(-8) + (n-1)(2)]$$

$$52 \times 2 = n(-16 + 2n - 2)$$
$$104 = n(2n - 18)$$
$$104 = 2n^2 - 18n$$
$$2n^2 - 18n - 104 = 0$$
$$n^2 - 9n - 52 = 0$$
$$(n - 13)(n + 4) = 0$$

If $n - 13 = 0 \rightarrow n = 13$ and $n + 4 = 0 \rightarrow n = -4$

[88] The value of K, for which the terms $7K + 3$, $4K - 5$, $2K + 10$ are in A.P., is

(a) 13 (b) – 13
(c) 23 (d) – 23 (1 mark)

Answer:

(d) If $7K + 3$, $4K - 5$, $2K + 10$ are in A.P.

Then,

$$(4K - 5) - (7K + 3) = (2K + 10) - (4K - 5)$$
$$4K - 5 - 7K - 3 = 2K + 10 - 4K + 5$$
$$-3K - 8 = -2K + 15$$
$$-8 - 15 = -2K + 3K$$
$$-23 = K$$

2019 - JUNE

[89] If the ratio of sum of n terms of two APs is (n+1):(n-1), then the ratio of their m^{th} terms is:

(a) (m +1) : 2m
(b) (m + 1) : (m - 1)
(c) (2m - 1) : (m + 1)
(d) m : (m - 1) (1 mark)

Answer:

(d) Let one AP be AP_1 with first term a_1 and common difference d_1. The sum (S_1) of n terms of this AP: $S_1 = \frac{n}{2} \times \{2a_1 + (n-1)d_1\}$. Let another AP be AP_2 with first term a_2 and common difference d_2. The sum (S_2) of n terms of this AP: $S_2 = \frac{n}{2} \times \{2a_2 + (n-1)d_2\}$.

Given: $\frac{S_1}{S_2} = \frac{n+1}{n-1}$

$$\frac{\frac{n}{2} \times \{2a_1 + (n-1)d_1\}}{\frac{n}{2} \times \{2a_2 + (n-1)d_2\}} = \frac{n+1}{n-1}$$

$$\frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{n+1}{n-1}$$

Taking out 2 common in both numerator as well as denominator on LHS:

$$\frac{2\left\{a_1 + \left(\frac{n-1}{2}\right)d_1\right\}}{2\left\{a_2 + \left(\frac{n-1}{2}\right)d_2\right\}} = \frac{n+1}{n-1}$$

$$\frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{n+1}{n-1} \quad \text{..... Eq. (1)}$$

Also, we know that the m^{th} term of $AP_1 = a_1 + (m-1)d_1$; the m^{th} term of $AP_2 = a_2 + (m-1)d_2$.

Therefore, for finding the m^{th} terms, $\left(\frac{n-1}{2}\right)$ has to be equated with (m - 1).

$$\frac{n+1}{2} = m - 1$$

$$n = 2(m-1) + 1$$

$$n = 2m - 2 + 1$$

$$n = 2m - 1$$

Therefore, replacing $\left(\frac{n-1}{2}\right)$ with (m – 1) on LHS, and n with (2m – 1) on RHS in Eq. (1), we'll get the ratio of the m^{th} terms of both the Aps.

$$\frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2} = \frac{(2m-1)+1}{(2m-1)-1}$$

$$\frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2} = \frac{2m}{2m-2}$$

$$\frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2} = \frac{2m}{2(m-1)}$$

Therefore, the ratio of the m^{th} terms of both the APs is m:(m-1)

[90] In a G.P. If the fourth term is '3' then the product of first seven terms is

(a) 3^5

(b) 3^7

(c) 3^6

(d) 3^8 (1 mark)

Answer:

(b) In G.P. $T_4 = ar^{4-1} = 3$

$ar^3 = 3$

Product of Ist seven terms

$= (a.\ ar\ .\ ar^2\ .\ ar^3\ .\ ar^4\ .\ ar^5\ .\ ar^6)$

$= a^7\ .\ r^{21}$

$= (ar^3)^7$

$= (3)^7$

[91] If $2 + 6 + 10 + 14 + 18 + \ldots\ldots + x = 882$ then the value of x

(a) 78

(b) 80

(c) 82

(d) 86 (1 mark)

Answer:

(c) If $2 + 6 + 10 + 14 + 18 + \text{--------} + x = 882$

$a = 2,\ d = 6 - 2 = 4,\ n = n$

$s_n = 882,\ l = x$

$S_n = \frac{n}{2}[2a + (n-1)\ d]$

$882 = \frac{n}{2}[2 \times 2 + (n-1)\ 4]$

$882 \times 2 = n(4 + 4n - 4)$

$882 \times 2 = 4n^2$

$n^2 = \frac{882 \times 2}{2 \times 2}$

$n^2 = 441 \Rightarrow n = 21$

Now, $l = a + (n-1)\ d$

$x = 2 + (21-1) \times 4$

$x = 2 + 80$

$\boxed{x = 82}$

[92] If $y = 1 + x + x^2 + \ldots\ldots\ldots\ldots\infty$ then x =

(a) $\frac{y-1}{y}$

(b) $\frac{y+1}{y}$

(c) $\frac{y}{y+1}$

(d) $\frac{y}{y-1}$ (1 mark)

Answer:

(a) If $y = 1 + x + x^2 + \text{---------} \infty$

$a = 1, r = \frac{x}{1} = x$

$y = \frac{1}{1-x} \left[\because S_\infty = \frac{a}{1-r}\right]$

In G.P.

$y(1-x) = 1$

$y - xy = 1$

$xy = y - 1$

$x = \frac{y-1}{y}$

2019 - NOVEMBER

[93] If $\frac{(b+c-a)}{a}, \frac{(c+a-b)}{b}, \frac{(a+b-c)}{c}$ are in AP then a, b, c are in:

(a) AP

(b) GP

(c) HP

(d) None (1 mark)

Answer:

(c) Since, $\frac{(b+c-a)}{a}, \frac{(c+a-b)}{b}, \frac{(a+b-c)}{c}$ are in AP

$$\frac{(c+a-b)}{b} - \frac{(b+c-a)}{a} = \frac{(a+b-c)}{c} - \frac{(c+a-b)}{b}$$

$$\frac{a(c+a-b)-b(b+c-a)}{ab} = \frac{b(a+b-c)-c(c+a-b)}{bc}$$

$$\frac{ac+a^2-ab-b^2-bc+ab}{a} = \frac{ab+b^2-bc-c^2-ac+bc}{c}$$

$$\frac{(a^2-b^2)+c(a-b)}{a} = \frac{b^2-c^2+ab-ac}{c}$$

$$\frac{(a+b)(a-b)+c(a-b)}{a} = \frac{(b+c)(b-c)+a(b-c)}{c}$$

$$\frac{(a-b)(a+b+c)}{a} = \frac{(b-c)(a+b+c)}{c}$$

$$ac - bc = ab - ac$$

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP

a, b, c are in HP

[94] Sum upto infinity of series.

$$\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \frac{1}{25} + \ldots\ldots\ldots$$

(a) 19/24
(b) 24/19
(c) 5/24
(d) None (1 mark)

Answer:

(a) We know,

$$S_\infty = \frac{a}{\quad}, r < 1$$

Here, $\frac{1}{2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}+\frac{1}{2^5}+\ldots$

$\left(\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}+\ldots\ldots\infty\right)+\left(\frac{1}{3^2}+\frac{1}{3^4}+\ldots\ldots\infty\right)$

$\left\{a=\frac{1}{2}, r=\frac{1}{4}<1\right\}\ ;\ \left\{a=\frac{1}{9}, r=\frac{1}{9}\right\}$

$\left(\frac{\frac{1}{2}}{1-\frac{1}{4}}\right)+\left(\frac{\frac{1}{9}}{1-\frac{1}{9}}\right)$ [Using above formula]

$\frac{\frac{1}{2}}{\frac{3}{4}}+\frac{\frac{1}{9}}{\frac{8}{9}}$

$\frac{1}{2}\times\frac{4}{3}+\frac{1}{9}\times\frac{9}{8}$

$\frac{2}{3}+\frac{1}{8}$

$\frac{19}{24}$

[95] Sum the series $\frac{1}{5}, \frac{1}{5^2}, \frac{1}{5^3}\ldots\ldots\ldots\ldots\frac{1}{5^n}$.

(a) $\frac{1}{4}\left[1-\left(\frac{1}{5}\right)^n\right]$

(b) $\frac{1}{5}\left[1-\left(\frac{1}{4}\right)^n\right]$

(c) both

(d) None (1 mark)

Answer:

(a) Series $\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\ldots\ldots\ldots\frac{1}{5^n}$

So, here $a = \frac{1}{5}$, $r = \frac{1}{5}$, $\frac{1}{5} < 1$

$$Sn = a\frac{(1 - r^n)}{(1 - r)}, r < 1$$

$$Sn = \frac{1}{5}\left[\frac{1 - \left(\frac{1}{5}\right)^n}{\left(1 - \frac{1}{5}\right)}\right]$$

$$Sn = \frac{1}{5} \times \frac{5}{4}\left[1 - \left(\frac{1}{5}\right)^n\right]$$

$$Sn = \frac{1}{4}\left[1 - \left(\frac{1}{5}\right)^n\right]$$

[96] Find the no. of terms of the series

$25, 5, 1 \ldots\ldots\ldots\ldots \frac{1}{3125}$

(a) 6
(b) 7
(c) 8
(d) 9 (1 mark)

Answer:

(c) Given series

$25, 5, 1 \ldots\ldots\ldots\ldots \frac{1}{3125}$

$a = 25$, or $= \frac{1}{5} < 1$

$T_n = a\,(r)^{n-1}$

$$\frac{1}{3125} = 25\left(\frac{1}{5}\right)^{n-1}$$

$$\frac{1}{78125} = \left(\frac{1}{5}\right)^{n-1}$$

$$\left(\frac{1}{5}\right)^7 = \left(\frac{1}{5}\right)^{n-1}$$

$n-1 = 7$

$n = 8$

[97] If the sum of fine terms of AP is 75. Find the third term of the series

(a) 35

(b) 30

(c) 15

(d) 20 (1 mark)

Answer:

(c) We know,

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$n = 5$ $S_5 = 75$

$$S_5 = \frac{5}{2}[2a + (5-1)d]$$

$$75 = \frac{5}{2}[2a + 4d]$$

$$75 = \frac{5 \times 2}{2}[a + 2d]$$

$15 = a + 2d$ — Eq (1)

$T_3 = a + (3-1)\,d$

$T_3 = a + 2d$

— From Eq (1)

$T_3 = 15$

[98] If the AM and GM of two numbers is 6.5 and 6 the no.'s are:

(a) 3 and 2

(b) 9 and 4

(c) 81 and 16

(d) None (1 mark)

Answer:

(d) Let the two nos. be 'a' and 'b'

$AM = \frac{a+b}{2}$; $GM = \sqrt{ab}$

$\sqrt{ab} = 6$

$a + b = 6.5$ On Squaring

$ab = 36$ — Equation (2)

$a + b = 13$

$a = 13 - b$ — Equation (1)

Put Eq (1) in Eq (2)

$b \times (13 - b) = 36$

$13b - b^2 = 36$

$b^2 - 13b + 36 = 0$

$b^2 - 9b - 4b + 36 = 0$

$b(b-9) - 4(b-9) = 0$

$b = 9$ $b = 4$

$a = 13 - 9$ $a = 13 - 4$

$a = 4$ $a = 9$

So the two numbers are 4 and 9

[99] If AM and HM for two numbers are 5 and 3 : 2, respectively. GM will be:

(a) 20

(b) 16

(c) 4

(d) 5 (1 mark)

Answer:

(c) We know that

$(GM)^2 = AM \times HM$

Here $(GM)^2 = 5 \times 3.2$

$(GM)^2 = 16$

$(GM) = 4$

PRACTICE QUESTIONS

[1] Periodic movements with duration longer than a year are called...
(a) Long-term Movements
(b) Random Movements
(c) Cyclical Movements
(d) Seasonal Movements

[2] Cyclical Variations are Caused by...
(a) Festivals
(b) Trade or business cycles
(c) Earthquakes
(d) None

[3] The Most important factor Causing Seasonal variations are
(a) Weather
(b) Social Customs
(c) Both (a) and (b)
(d) None

[4] Seasonal variations are Short-term variations with period..
(a) Less than one year
(b) More than one month
(c) More than twelve months
(d) Less than one month

[5] Using multiplicative model of time series, the time series values (y) are given by y = ...
(a) T + S + C + I
(b) (T × S) + (C × I)
(c) (T + S) × (C + I)
(d) T × S × C × I

[6] _____ is the overall tendency of the time series data to ___ or_____ over a _____ period of time.
(a) secular; increase; decrease; long
(b) trend, increase, decrease, long
(c) either (a) or (b)
(d) both (a) and (b)

[7] After deseasonalization, a time series can be represented as.
(a) $Y = S \times C \times I$
(b) $Y = T \times S \times C \times I$
(c) $Y = T \times C \times I$
(d) $Y = T \times S \times I$

[8] If the growth rate is constant, the trend is .....
(a) irregular
(b) cyclical
(c) linear
(d) random

[9] For the annual data _____ Component of time series is missing.....
(a) seasonal
(b) irregular
(c) chronological
(d) none

[10] The component of time series useful for long-term forecasting is _____
(a) Seasonal Variations
(b) Secular Trend
(c) Random Variations
(d) None

[11] The trend line $y = a + bx$ obtained by the Least Squares Method is Known as the line of .....
(a) freehand
(b) moving Averages
(c) best fit
(d) semi Averages

[12] The additive model of a time Series is a expressed as ...
(a) $Y = T + S + C + I$
(b) $S = T + Y + C + I$
(c) $T = Y + S + C + I$
(d) $I = T + S + C + I$

[13] A time series consists of data arranged
(a) Sequentially
(b) Chronologically
(c) Randomly

[14] An overall rise or fall in a time series is called.....
(a) fluctuations
(b) secular Trend
(c) random Variations
(d) business Cycles

[15] The most widely used method of measuring seasonal variation is:
(a) Ratio-to-trend method
(b) Ratio-to-Moving average method
(c) Link relative method
(d) None of these

[16] If the trend is absent in the time series, the seasonal indices are obtained by using :
(a) Method of simple averages
(b) Ratio-to-trend Method
(c) Ratio-Moving average Method
(d) Method of Least Squares

[17] Trend refer to a long term tendency to :
(a) Decrease only
(b) Either increase or decrease
(c) Increase only
(d) None of these

[18] An orderly set of data arranged in accordance with their time of occurrence is called.
(a) Arithmetic series
(b) Harmonic series
(c) Geometric series
(d) Time series

[19] A Time Series Consists of.
(a) Short-term Variations
(b) Long-term Variations
(c) Irregular Variations
(d) All of the above

[20] The graph of time series is called :
(a) Histogram
(b) Straight line
(c) Historigram

[21] Secular trend can be measured by :
(a) Two Methods
(b) Three Methods
(c) Four Method
(d) Five Methods

[22] The secular trend is measured by the method of Semi-averages when:
(a) Time Series based on yearly values
(b) Trend is linear
(c) Time Series Consists of even number of values
(d) None of them

[23] Increase in the number of patients in the hospital due to heat stroke is.
(a) Secular trend
(b) Irregular variation
(c) Seasonal Variation
(d) Cyclical variation

[24] Wheat crops badly damaged on account of rains is.
(a) Cyclical Movement
(b) Random Movement
(c) Secular Trend
(d) Seasonal Movement

[25] The Method of Moving average is used to find the
(a) Secular Trend
(b) Seasonal Variation
(c) Cyclical Variation
(d) Irregular Variation

[26] Most Frequency used Mathematical Model of a time series is
(a) Additive Model
(b) Mixed Model
(c) Multiplicative Model
(d) Regression Model

[27] In a straight line equation $y = a + b X$; a is the
(a) X-intercept
(b) Slope
(c) Y-intercept
(d) None of them

[28] In a Straight line equation y = a +b X; b is the
(a) Y-intercept
(b) Slope
(c) X-Intercept
(d) Trend

[29] Value of b in the trend line y = a + b X is.
(a) Always Negative
(b) Always Positive
(c) Always Zero
(d) Both Negative and Positive

[30] In fitting a straight line, the value of slope b remain unchanged with the change of
(a) Scale
(b) Origin
(c) Both (a) and (b)
(d) Neither (a) and (b)

[31] Moving average method is used for Measurement of trend when.
(a) Trend is linear
(b) Trend is non linear
(c) Trend is Curvilinear
(d) None of them

[32] Indicate which of the following an example of Seasonal Variations is.
(a) Death rate decreased due to advance in Science
(b) The Sale of air condition increases during Summer
(c) Recovery in business
(d) Sudden causes by wars.

[33] The most commonly used method for measuring the trend is.
(a) Moving average Method
(b) Semi average Method
(c) Method of least Squares
(d) None of them

[34] Decomposition of time series is called.
(a) Historigram
(b) Analysis of time series
(c) Histogram
(d) Detrending

[35] The fire in a factory is an example of.
(a) Secular Trend
(b) Seasonal Movements
(c) Cyclical Variations
(d) Irregular Variations

[36] Increased demand of admission in the subject of Computer in IIT is.
(a) Secular trend
(b) Cyclical trend
(c) Seasonal trend
(d) Irregular trend

[37] Damages due to floods, droughts, Strikes Fires and Political disturbances are.
(a) Trend
(b) Seasonal
(c) Cyclical
(d) Irregular

[38] The general pattern of increase or decrease in economics or Social Phenomena is shown by.
(a) Seasonal trend
(b) Cyclical trend
(c) Secular trend
(d) Irregular trend

[39] In Moving average method, we cannot find the trend values of some.
(a) Middle Periods
(b) End Period
(c) Starting Periods
(d) Between Extreme Periods

[40] The best fitting trend is one which the sum of squares of residuals is.
(a) Negative
(b) Least
(c) Zero
(d) Maximum

[41] Depression in business is.
(a) Secular trend
(b) Cyclical
(c) Seasonal
(d) Irregular

[42] Semi-averages method is used for measurement of trend when.
(a) Trend is linear
(b) Observed data contains yearly values
(c) The given time series contains odd number of values.
(d) None of them

[43] Moving-averages.
(a) Give the trend in a straight line
(b) Measure the seasonal variations
(c) Smooth-out the time series
(d) None of them

[44] The rise and fall of a time series over periods longer than one year is called.
(a) Secular trend
(b) Seasonal Variations
(c) Cyclical variations
(d) Irregular variation

[45] A Time series has
(a) Two Components
(b) Three Components
(c) Four Components
(d) Five Components

[46] The difference between the actual value of the Time Series and the Forecasted value is called.
(a) Residual
(b) Sum of variation
(c) Sum of Squares of residual
(d) All of the above

[47] A pattern that is repeated throughout a time series and has a recurrence period of at most one year is called.
(a) Cyclical variation
(b) Irregular variation
(c) Seasonal variation
(d) long term variation

[48] A business cycle has
(a) One stage
(b) Two stages
(c) Three stage
(d) Four stage

[49] When the production of a thing is maximum, this stage is called.
(a) Boom
(b) Recovery
(c) Recession
(d) Depression

[50] When the production of a thing is minimum, this stage is called.
(a) Prosperity
(b) Recession
(c) Recovery
(d) Depression

[51] When the production of thing is increasing towards prosperity, this stage is called as.
(a) Recession
(b) Recovery
(c) Boom
(d) Depression

[52] When the production of thing is decreasing, this stage is called.
(a) Recession
(b) Recovery
(c) Prosperity
(d) Depression

[53] The straight line is fitted to the time Series when the movements in the time Series are.
(a) Nonlinear
(b) Linear
(c) Irregular
(d) Upward

[54] Far a Time Series, Interval can be
(a) Year
(b) Month
(c) Week
(d) Any of these

[55] Change in Population growth represents.
(a) Secular trend
(b) Periodic variation
(c) Random variation
(d) None of these

[56] Seasonal and cyclic variations are the types of.
(a) Secular Trend
(b) Random Variations
(c) Irregular Variation
(d) Oscillatory Variations

[57] Time series can be analysis using.
(a) Additive model
(b) Multiplicative model
(c) Subtractive
(d) Both (a) and (b)

[58] Which of these is not a method of measurement of trend?
(a) Graphic Method
(b) Calculative Method

(c) Method of moving averages.
(d) Method of least squares

[59] The general form y = a + b, is of.
(a) Linear Trend
(b) Secular Trend
(c) Exponential Trend
(d) Parabolic Trend

[60] _____ is defined as the estimation or Prediction of future values.
(a) Forecasting
(b) Planning
(c) Expecting
(d) None of these

[61] Which of these is a method of forecasting?
(a) Opinion Polling
(b) Trend Projection
(c) Both (a) and (b)
(d) None of these

[62] The tendency of trend to increase or decrease or Stagnate over a long period of time is called
(a) Periodic Variation
(b) Cyclic Variation
(c) Secular Trend
(d) Random Variation

[63] The equation y = a + b x is used to get the value of
(a) Parabolic Trend
(b) Exponential Trend
(c) Linear Trend
(d) None of the above

[64] "Occurrence of Floods" falls under which type of variation
(a) Seasonal Variation
(b) Simple Variation
(c) Cyclic Variation
(d) Random Variation

[65] How the data is arranged in a Time Series. Analysis?

(a) In descending order of their Magnitude
(b) Arranged abruptly
(c) Arranged Chronologically
(d) In ascending order of their Magnitude

ANSWER

1	(c)	**2**	(b)	**3**	(c)	**4**	(a)	**5**	(d)	**6**	(d)
7	(c)	**8**	(c)	**9**	(a)	**10**	(b)	**11**	(c)	**12**	(a)
13	(b)	**14**	(b)	**15**	(b)	**16**	(a)	**17**	(b)	**18**	(d)
19	(d)	**20**	(a)	**21**	(c)	**22**	(b)	**23**	(c)	**24**	(b)
25	(a)	**26**	(c)	**27**	(c)	**28**	(b)	**29**	(d)	**30**	(b)
31	(a)	**32**	(b)	**33**	(c)	**34**	(b)	**35**	(d)	**36**	(a)
37	(d)	**38**	(c)	**39**	(d)	**40**	(b)	**41**	(a)	**42**	(a)
43	(c)	**44**	(c)	**45**	(c)	**46**	(a)	**47**	(c)	**48**	(d)
49	(a)	**50**	(d)	**51**	(b)	**52**	(a)	**53**	(b)	**54**	(d)
55	(a)	**56**	(d)	**57**	(d)	**58**	(b)	**59**	(a)	**60**	(a)
61	(c)	**62**	(c)	**63**	(c)	**64**	(d)	**65**	(c)		

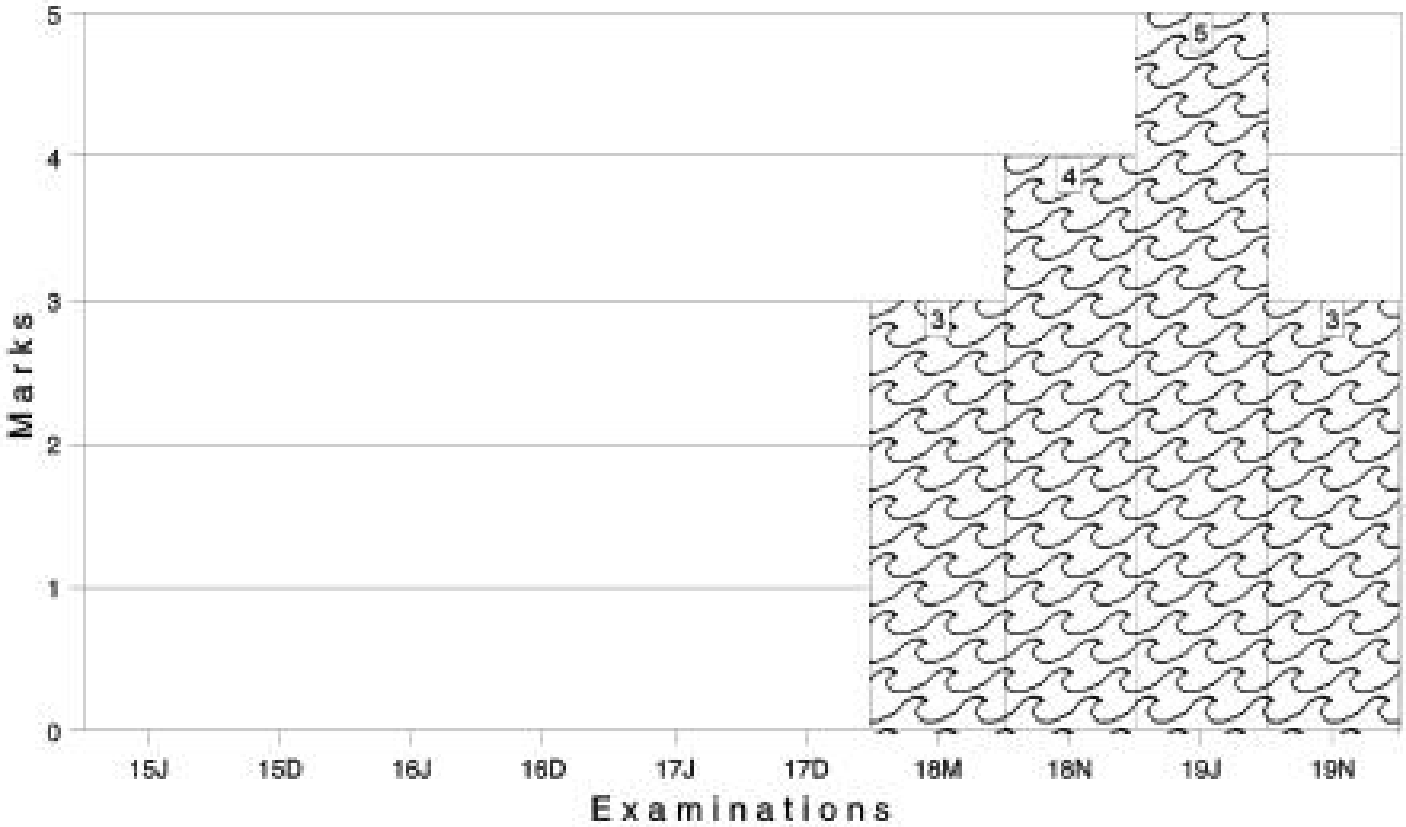
CHAPTER

7 SETS, RELATIONS AND FUNCTIONS

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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PAST YEAR QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] Out of 20 members in a family, 11 like to take tea and 14 like coffee. Assume that each one likes at least one of the two drinks. Find how many like both coffee and tea :

(a) 2 (b) 3
(c) 4 (d) 5 (1 mark)

Answer:

(d) Let T: set of people who like tea, and
C: set of people who like coffee.
Then n (T) = 11, n(C) = 14 and n(TUC) = 20

$\therefore$ n(TUC) = n (T) + n(C) – n(T$\cap$C)

n(T$\cap$ C) = 11 + 14 – 20 = 5

2007 - FEBRUARY

[2] In a group of 70 people, 45 speak Hindi, 33 speak English and 10 speak neither Hindi nor English. Find how many can speak both English as well as Hindi :

(a) 13 (b) 19
(c) 18 (d) 28 (1 mark)

Answer:

(c) Let H : set of those people who speak Hindi and
E : set of those people who speak English
So, n(H) = 45, n(E) = 33, n(E$\cup$H) = 70 – 10 = 60.

$\therefore$ n(E$\cup$H) = n(E) + n(H) – n(E$\cap$H)

60 = 45 + 33 – n(E$\cap$H)

n(E$\cap$H) = 45 + 33 – 60 = 18

[3] Let R is the set of real numbers, such that the function f : R → R and g: R → R are defined by $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$. Find (fog) :

(a) $4x^2 + 6x + 1$ (b) $x^2 + 6x + 1$

(c) $4x^2 - 6x + 1$ (d) $x^2 - 6x + 1$

Answer:

(c) $(fog)x = f\{g(x)\}$

$= f(2x - 3)$

$= (2x - 3)^2 + 3(2x - 3) + 1$

$= 4x^2 + 9 - 12x + 6x - 9 + 1$

$= 4x^2 - 6x + 1$

2007 - MAY

[4] In a survey of 300 companies, the number of companies using different media - Newspapers (N), Radio (R) and Television (T) are as follows :

$n(N) = 200$, $n(R) = 100$, $n(T) = 40$, $n(N \cap R) = 50$, $n(R \cap T) = 20$, $n(N \cap T) = 25$ and $n(N \cap R \cap T) = 5$.

Find the numbers of companies using none of these media :

(a) 20 companies (b) 250 companies

(c) 30 companies (d) 50 companies (1 mark)

Answer:

(d) $n(NURUT) = n(N) + n(R) + n(T) - n(N \cap R)$

$- n(N \cap T) - n(R \cap T) + (N \cap R \cap T)$

$= 200 + 100 + 40 - 50 - 25 - 20 + 5$

$= 250$

$\therefore$ Number of companies not using any media

$= n(S) - n(NURUT)$

$= 300 - 250$

$= 50$

[5] If R is the set of real numbers such that the function f : R → R is defined by $f(x) = (x + 1)^2$, then find (fof) :

(a) $(x + 1)^2 + 1$ (b) $x^2 + 1$

(c) $\{(x + 1)^2 + 1\}^2$ (d) None (1 mark)

Answer:

2007 - AUGUST

[6] If f : R → R, f(x) = 2x + 7, then the inverse of f is :

(a) $f^{-1}(x) = (x - 7)/2$ (b) $f^{-1}(x) = (x + 7)/2$

(c) $f^{-1}(x) = (x - 3)/2$ (d) None (1 mark)

Answer:

(a) Let X ∈ R (domain) and Y ∈ R (co – domain)

such that $f(x) = y$

and $2x + 7 = y$

$$x = \frac{y - 7}{2}$$

$$f^{-1}(y) = \frac{y - 7}{2}$$

Then f^{-1}: R and R such that

$$f^{-1}x = \frac{x - 7}{2} \text{ for all } X \in R.$$

2007 - NOVEMBER

[7] In a town of 20,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then the number of families which buy A only is:

(a) 6600 (b) 6300

(c) 5600 (d) 600 (1 mark)

Answer:

(a) n(s) = 20,000

n(A) = 40% of 20,000 = 8,000

n(B) = 20% of 20,000 = 4,000

n(C) = 10% of 20,000 = 2,000

n(A∩B) = 5% of 20,000 = 1,000

$n(B \cap C)$ = 3% of 20,000 = 600

$n(C \cap A)$ = 4% of 20,000 = 800

$n(A \cap B \cap C)$ = 2% of 20,000 = 400

Now, we, have to find $n(A \cap B' \cap 'C)$

$n(A \cap B' \cap C') = n(A \cap (B \cup C))$

$= n(A \cap (B \cup C))'$

$= n(A) - n[A \cap (B \cap C)]$

$= n(A) - n(A \cap B) \cap (A \cap C))$

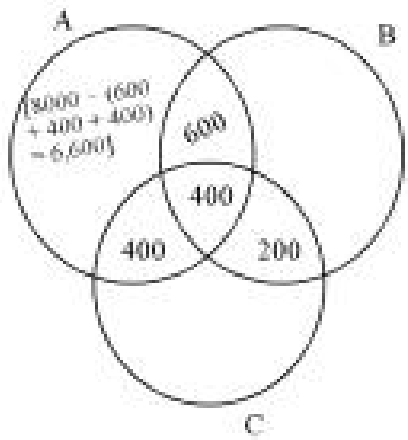
$= n(A) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$

$= 8,000 - (1,000 + 800 - 400)$

$= 8,000 - 1,400$

$= 6,600$

Alternatively, through Venn – diagram



[8] Let $f: R \to R$ be such that $f(x) = 2^x$, then $f(x + y)$ equals:

(a) $f(x) + f(y)$ (b) $f(x) \cdot f(y)$

(c) $f(x) \div f(y)$ (d) None of these (1 mark)

Answer:

(b) $f(x) = 2^x$

$f(x + y) = 2^{x \cdot Y}$

$= 2^x 2^y$

$= f(x) f(y)$

2008 - FEBRUARY

[9] Out of total 150 students, 45 passed in Accounts, 30 in Economics and 50 in Maths, 30 in both Accounts and Maths, 32 in both Maths and Economics, 35 in both Accounts and Economics, 25 students passed in all the three subjects. Find the numbers who passed at least in any one of the subjects :

(a) 63 (b) 53
(c) 73 (d) None (1 mark)

Answer:

(b) $n(A) = 45$

$n(M) = 50$

$n(E) = 30$

$n(A \cap M) = 30$

$n(M \cap E) = 32$

$n(A \cap E) = 35$

$n(A \cap M \cap E) = 25$

$n(A \cup M \cup E) = n(A) + n(M) + n(E) - n(A \cap M)$
$- n(A \cap E) - n(M \cap E) + n(A \cap M \cap E)$
$= 45 + 50 + 30 - 30 - 35 - 32 + 25$
$n(A \cup M \cup E) = 53$

2008 - JUNE

[10] If $f(x) = \frac{2 + x}{2 - x}$, then $f^{-1}(x)$:

(a) $\frac{2(x - 1)}{x + 1}$ (b) $\frac{2(x + 1)}{x - 1}$
(c) $\frac{x + 1}{x - 1}$ (d) $\frac{x - 1}{x + 1}$ (1 mark)

Answer:

(a) Let f (x) = y

$$\frac{2+x}{2-x} = y$$

$$2 + x = 2y - xy$$

$$x + xy = 2y - 2$$

$$x(1 + y) = 2(y - 1)$$

$$x = \frac{2(y-1)}{(y+1)}$$

$$f^{-7}(y) = \frac{2(y-1)}{y+1}$$

Therefore, $f^{-1}(x) = \frac{2(x-1)}{(x+1)}$

2008 - DECEMBER

[11] If A = {1, 2, 3, 4, }
B = {2, 4, 6, 8,}
f (1) = 2, f (2) = 4, f (3) = 6 and
f (4) = 8, And f : A→ B then f^{-1} is :
(a) {(2,1), (4, 2), (6, 3), (8, 4)}
(b) {(1, 2), (2, 4), (3, 6), (4, 8)}
(c) {(1, 4), (2, 2), (3, 6), (4, 8)}
(d) None of these (1 mark)

Answer:

(a) A = {1, 2, 3, 4}, B = {2, 4, 6, 8}
When f : A → B , f = {(1, 2), (2, 4), (3, 6), (4, 8)}
f^{-1} implies f : B→A
f^{-1} = {(2, 1), (4, 2), (6, 3), (8, 4)}

[12] If f (x) = x^2 + x– 1 and 4f (x) = f (2x) then find 'x'.
(a) 4/3 (b) 3/2
(c) – 3/4 (d) None of these (1 mark)

Answer:

(b) $f(x) = x^2 + x - 1$

$4\ f(x) = f(2x)$

$4\ [x^2 + x - 1] = (2x)^2 + (2x) - 1$

$\Rightarrow\ 4x^2 + 4x - 4 = 4x^2 + 2x - 1$

$\Rightarrow\ 2x = 3$

$\Rightarrow\ x = 3/2$

[13] If $A = \{p, q, r, s\}$

$B = \{q, s, t\}$

$C = \{m, q, n\}$

Find $C - (A \wedge B)$

(a) $\{m, n\}$ (b) $\{p, q\}$

(c) $\{r, s\}$ (d) $\{p, r\}$ (1 mark)

Answer:

(a) $A = \{p, q, r, s\}$

$B = \{q, s, t\}$

$A \cap B = \{q, s\}$

$C = \{m, q, n\}$

$C - (A \cap B) = \{m, n\}$

2009 - DECEMBER

[14] $X = \{x, y, w, z\}$, $y = \{1,2,3,4\}$

$H = \{(x, 1), (y, 2), (y, 3), (z, 4), (x, 4)\}$

(a) H is a function from X to Y

(b) H is not a function from X to Y

(c) H is a relation from Y to X

(d) None of the above (1 mark)

Answer:

(b) Any relation from X to Y in which no two different ordered pairs have the same first element is called a FUNCTION.
Therefore, in the given question, H is NOT a function from X to Y because the different ordered pairs of H have the same first element.

[15] Given the function $f(x) = (2x + 3)$, then the value of $f(2x) - 2f(x) + 3$ will be :

(a) 3 (b) 2
(c) 1 (d) 0 (1 mark)

(d) $f(x) = 2x + 3$
$f(2x) - 2f(x) + 3$
$= [2(2x) + 3] - [2(2x + 3)] + 3$
$= 4x + 3 - 4x - 6 + 3$
$= 4x - 4x + 6 - 6.$
$= 0.$

[16] If $f(x) = 2x + h$ then find $f(x + h) - 2f(x)$

(a) $h - 2x$ (b) $2x - h$
(c) $2x + h$ (d) None of these (1 mark)

Answer:

(a) $f(x) = 2x + h$
$f(x + h) - 2f(x)$
$= [2(x + h) + h] - [2(2x + h)]$
$= 2x + 2h + h - 4x - 2h$
$= -2x + h$
$= h - 2x.$

2010 - JUNE

[17] If $A = \{x : x^2 - 3x + 2 = 0\}$,
$B = \{x : x^2 + 4x - 12 = 0\}$, then
B – A is Equal to

(a) $\{-6\}$ (b) $\{1\}$
(c) $\{1,2\}$ (d) $\{2, -6\}$ (1 mark)

Answer:

(a) $A = \{x : x^2 - 3x + 2 = 0\}$

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = 1, 2$

$A = \{1, 2\}$

$B = \{x : x^2 + 4x - 12 = 0\}$

$x^2 + 4x - 12 = 0$

$x^2 + 6x - 2x - 12 = 0$

$(x - 2)(x + 6) = 0$

$x = 2, -6$

$B = \{2, -6\}$

$B - A$ = All elements present in B but not in $A = \{-6\}$

[18] If $F : A \rightarrow R$ is a real valued function defined by $f(x) = \frac{1}{x}$, then A = _____.

(a) R (b) R – {1}
(c) R – {0} (d) R – N (1 mark)

Answer:

(c) $f : A \rightarrow R$

$f(x) = \frac{1}{x}$

If $x = 0$

f(x) will be undefined

$A = R - \{0\}$

[19] In the set N of all natural numbers the relation R defined by a R b "if and only if, a divide b", then the relation R is :

(a) Partial order relation (b) Equivalence relation
(c) Symmetric relation (d) None of these (1 mark)

Answer:

(a) For a function to be a partial order Relation, it should be

(1) Reflexive

(2) Antisymmetric and

(3) Transitive

a divides b satisfies the above 3 relations as follows :

(1) a/a ∴ Reflexive

(2) a/b and b/a ∴ a = b ∴ Antisymmetric

(3) a/b, b/c ∴ a/c ∴ Transitive

a/b is not a symmetric function and hence, not an equivalence relation.

2010 - DECEMBER

[20] For any two sets A and B, A∩(A' ∪ B) =_____, where A' represent the compliment of the set A

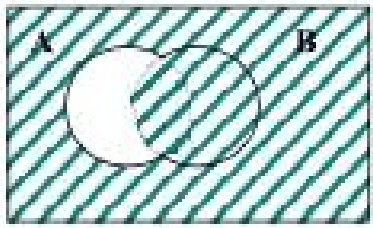
(a) A ∩ B (b) A∪B

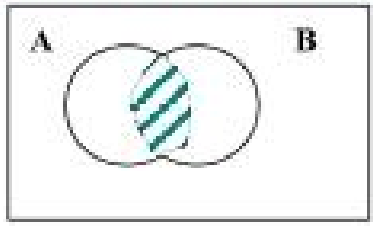
(c) A'∪B (d) None of these (1 mark)

Answer:

(a) A∩(A ∪B)

∵ A'∪B =





∴ A∩(A ∪B)

[21] If $f : R \to R$, $f(x) = x + 1$,
$g : R \to R$ $g(x) = x^2 + 1$
then $fog(-2)$ equals to

(a) 6 (b) 5
(c) -2 (d) None (1 mark)

Answer:

(a) $f(x) = x + 1$
$g(x) = x^2 + 1$
$fog(-2) = f[g(-2)] = f(5)$ [$\because g(-2) = 5$]
$f(5) = 5 + 1 = 6$

[22] If $A \subset B$, then which one of the following is true

(a) $A \cap B = B$ (b) $A \cup B = B$
(c) $A \cap B = A'$ (d) $A \cap B = \varphi$ (1 mark)

Answer:

(a) $A \subset B$

$A \cap B = B$ (as A is a subset of B)

[23] If $f(x - 1) = x^2 - 4x + 8$, then $f(x + 1) =$ _____

(a) x^2+8 (b) x^2+7
(c) x^2+4 (d) $x^2 - 4x$ (1 mark)

Answer:

(c) $f(x - 1) = x^2 - 4x + 8$
$= (x^2 - 2x + 1) - 2x + 7$
$= (x - 1)^2 - 2x + 2 + 7 - 2$
hence, $f(x - 1) = (x - 1)^2 - 2(x - 1) + 5$
$\therefore f(x + 1) = (x + 1)^2 - 2(x + 1) + 5$
$= x^2 + 2x + 1 - 2x - 2 + 5$
$= x^2 + 6 - 2$
$= x^2 + 4$

2011 - JUNE

[24] There are 40 students, 30 of them passed in English, 25 of them passed in Maths and 15 of them passed in both. Assuming that every Student has passed at least in one subject. How many student's passed in English only but not in Maths.

(a) 15 (b) 20

(c) 10 (d) 25 (1 mark)

Answer:

(a) Given :

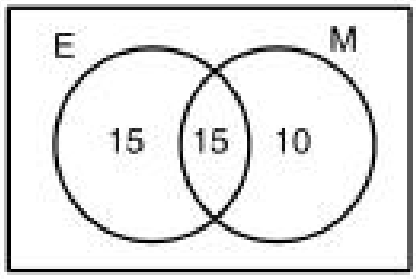
Total No. of Students $n(E \cup N) = 40$

No. of Students passed in Eng. $n(E) = 30$

No. of Students passed in Maths $n(M) = 25$

No. of Students passed in both $n(E \cap M) = 15$

Therefore, required to Find : n(only E) = ?



$\therefore$ $n(\text{only } E) = n(E) - n(E \cap M)$

$= 30 - 15$

$= 15$

[25] If $A = \{\pm 2, \pm 3\}$, $B = \{1,4,9\}$ and $F = \{(2, 4), (-2, 4), (3, 9), (-3, 4)\}$ then 'F' is defined as :

(a) One to one function from A into B.

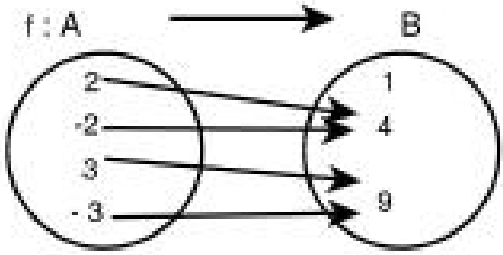
(b) One to one function from A onto B.

(c) Many to one function from A onto B.

(d) Many to one function from A into B. (1 mark)

Answer:

(c)



many one function from A onto B

[26] If $f(x) = \frac{x}{\sqrt{1+x^2}}$ and $g(x) = \frac{x}{\sqrt{1-x^2}}$ Find *fog* ?

(a) x (b) $\frac{1}{x}$

(c) $\frac{x}{\sqrt{1-x^2}}$ (d) $x\sqrt{1-x^2}$ (1 mark)

Answer:

(a) Given : $f(x) = \frac{x}{\sqrt{1+x^2}}$ and $g(x) = \frac{x}{\sqrt{1-x^2}}$

$\therefore fog(x) = f\{g(x)\}$

$= f\left\{\frac{x}{\sqrt{1-x^2}}\right\}$

$= \frac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{1+\left(\frac{x^2}{\sqrt{1+x^2}}\right)}}$

$= x$

2011 - DECEMBER

[27] f(x) = 3+x, for – 3< x <0 and 3 – 2x for 0< x <3, then Value of f(2) will be

(a) – 1 (b) 1

(c) 3 (d) 5 (1 mark)

Answer:

(a) f(x) = 3 + x if – 3 < x < 0

= 3 – 2x if 0 < x < 3

2 Lies 0 < x < 3

Then

f (x) = 3 – 2x

f (2) = 3 – 2 × 2 = 3 – 4 = – 1

[28] If A = (1, 2, 3, 4, 5), B = (2, 4) and C = (1, 3, 5) then (A – C) × B is

(a) {(2, 2), (2, 4), (4, 2), (4, 4), (5, 2), (5, 4)}

(b) {(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4)}

(c) {(2, 2), (4, 2), (4, 4), (4, 5)}

(d) {(2, 2), (2, 4), (4, 2), (4, 4)} (1 mark)

Answer :

(d) (A – C) = {1, 2, 3, 4, 5} – {1, 3, 5,} = {2, 4}

(A – C) × B = {2, 4,} × {2, 4} = {(2, 2,), (2, 4), (4, 2), (4, 4)

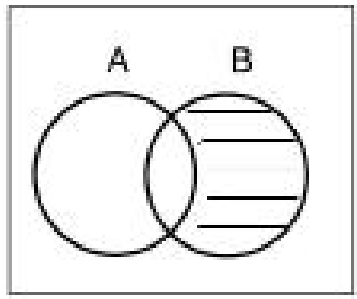
[29] For any two sets A and B the set (AUB')' is Equal to (where' denotes compliment of the set)

(a) B – A (b) A – B

(c) A' – B' (d) B' – A' (1 mark)

Answer:

(a) $(A \cup B')' = B - A$



2012 - JUNE

[30] The number of proper sub set of the set {3, 4, 5, 6, 7} is

(a) 32 (b) 31

(c) 30 (d) 25 (1 mark)

Answer:

(b) Given set A = {3, 4, 5, 6, 7}

Cardinal No n(A) = 5

No. of proper subset $= 2^n - 1$

$= 2^5 - 1$

$= 32 - 1$

$= 31$

[31] On the set of lines, being perpendicular is a _____ relation.

(a) Reflexive (b) Symmetric

(c) Transitive (d) None of these. (1 mark)

Answer:

(b) A set of lines, being perpendicular is a Symmetric Relation

[32] The range of the function $f : N \rightarrow N$; $f(x) = (-1)^{x-1}$, is

(a) {0, - 1} (b) {1, - 1}

(c) {1, 0} (d) {1, 0, - 1} (1 mark)

Answer:

(b) Given $f(x) = (-1)^{x-1}$

$x = 1 \quad f(1) = (-1)^{1-1} = 1 \; f = N \Rightarrow N$

$x = 2 \quad f(2) = (-1)^{2-1} = -1$

$x = 3 \quad f(3) = (-1)^{3-1} = 1$

$x = 4 \quad f(4) = (-1)^{4-1} = -1$

Range of function = $\{1, -1\}$

[33] The minimum value of the function $x^2 - 6x + 10$ is _____.

(a) 1 (b) 2

(c) 3 (d) 10 (1 mark)

Answer:

(a) Let $x^2 - 6x + 10 = y$

$x^2 - 6x + 10 - y = 0$

$x^2 - 6x + (10 - y) = 0$

$ax^2 + bx + c = 0$

we get

$a = 1, b = -6, c = (10 - y)$

For Real

$D \geq 0$

$b^2 - 4ac \geq 0$

$(-6)^2 - 4 \times 1 \times (10 - y) \geq 0$

$36 - 40 + 4y \geq 0$

$4y \geq 4$

$y \geq 1$

$y = \{1, 2, 3, \ldots\ldots\ldots\ldots\ldots \infty)$

Minimum value of function = 1

2012 - DECEMBER

[34] For a group of 200 persons, 100 are interested in music, 70 in photography and 40 in swimming, Further more 40 are interested in both music and photography, 30 in both music and swimming, 20 in photography and swimming and 10 in all the three. How many are interested in photography but not in music and swimming?

(a) 30 (b) 15

(c) 25 (d) 20 (1 mark)

Answer:

(d) Let Photography → P
Music → M
Swimming → S

$n(P \cup M \cup S) = 200, n(m) = 100, n(p) = 70$

$n(S) = 40, n(M \cap P) = 40, n(M \cap S) = 30, n(P \cap S) = 20$

$n(P \cap M \cap S) = 10$

$n(P \cap \bar{M} \cap \bar{S}) = n(P) - n(P \cap M) - n(P \cap S) + n(P \cap M \cap S)$

$= 70 - 40 - 20 + 10$

$= 80 - 60$

$= 20$

[35] If f: R → R is a function, defined by $f(x) = 10x - 7$, if $g(x) = f^{-1}(x)$, then g(x) is equal to

(a) $\frac{1}{10x - 7}$ (b) $\frac{1}{10x + 7}$

(c) $\frac{x + 7}{10}$ (d) $\frac{x - 7}{10}$ (1 mark)

Answer:

(c) If f : R → R is a function defined by

$f(x) = 10x - 7$ Let $y = f(x)$

$y = 10x - 7$ $x = f^{-1}(y)$ _____(1)

$$x = \frac{y+7}{10}$$

$$f^{-1}(y) = \frac{y+7}{10} \quad \text{[from eq (1)]}$$

$$f^{-1}(x) = \frac{x+7}{10}$$

The value of $g(x) = f^{-1}(x)$

$$= \frac{x+7}{10}$$

[36] The number of elements in range of constant function is

(a) One (b) Zero

(c) Infinite (d) Indetermined (1 mark)

Answer:

(c) The number of elements in range of constant function is infinite.

2013 - JUNE

[37] Let A = {1, 2, 3}, then the relation R = {1, 1), (2, 3), (2, 2), (3, 3), (1, 2)} is:

(a) Symmetric (b) Transitive

(c) Reflexive (d) Equivalence (1 mark)

Answer:

(c) If A = {1, 2, 3} then

R = {(1, 1) (2, 3) (2, 2) (3, 3) (1, 2)}

Here, R = {(1, 1) (2, 2) (3, 3)} shows reflexive

[38] If $f(x) = x + 2$, $g(x) = 7^x$, than $g \circ f(x) =$ _____

(a) $7^x \cdot x + 2 \cdot 7^x$ (b) $7^x + 2$

(c) $49\ (7^x)$ (d) None of these (1 mark)

Answer:

(c) If $f(x) = x + 2$, $g(x) = 7^x$ than

$$\begin{aligned} g\,of(x) &= g\{f(x)\} \\ &= g\{x+2\} \\ &= 7^{x+2} \\ &= 7^x \cdot 7^2 \\ &= 7^x \cdot (49) \\ &= 49 \cdot (7^x) \end{aligned}$$

[39] If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to:

(a) $f(x)$ (b) $2f(x)$

(c) $3f(x)$ (d) $-f(x)$ (1 mark)

Answer:

(b) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then

$$f\left(\frac{2x}{1+x^2}\right) = \log\left[\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right]$$

$$= \log\left[\frac{\frac{1+x^2+2x}{\cancel{(1+x^2)}}}{\frac{1+x^2-2x}{\cancel{(1-x)^2}}}\right]$$

$$= \log\left[\frac{(1+x)^2}{(1-x)^2}\right]$$

$$= \log\left[\frac{1+x}{1-x}\right]^2$$

$$= 2\log\left[\frac{1+x}{1-x}\right]$$

$$= 2f(x)$$

2013 - DECEMBER

[40] If $f(x) = (a - x^n)^{1/n}$, $a > 0$ and 'n' is a positive integer, then $f(f(x)) =$ _____.

(a) x (b) a
(c) $x^{1/n}$ (d) $a^{1/n}$ (1 mark)

Answer:

(a) If $f(x) = (a - x^n)^{1/n}$, $a > 0$

$$f\{f(x)\} = f\{(a - x^n)^{1/n}\}$$
$$= \{a - (a - x^n)^{1/n.n}\}^{1/n}$$
$$= \{a - a + x^n\}^{1/n}$$
$$= x^{n.1/n}$$
$$= x$$

[41] Of the 200 candidates who were interviewed for a position at call centre, 100 had a two-wheeler, 70 had a credit card and 140 had a mobile phone, 40 of them had both a two-wheeler and a credit card, 30 had both a credit card and a mobile phone, 60 had both a two-wheeler and a mobile phone, and 10 had all three. How many candidates had none of the three?

(a) 0 (b) 20
(c) 10 (d) 18 (1 mark)

Answer:

(c) A ⇒ Two wheeler candidate
B ⇒ Credit card candidate
C ⇒ Mobile phone candidate

Given $n(A) = 100$, $n(B) = 70$, $n(C) = 140$
$n(A \cap B) = 40$, $n(B \cap C) = 30$, $n(C \cap A) = 60$
$n(A \cap B \cap C) = 10$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 100 + 70 + 140 - 40 - 30 - 60 + 10$

$= 320 - 130$

$= 190$

No. of candidate who had none of the three

$= 200 - 190$

$= 10$

[42] If $f(x) = \frac{x^2 - 25}{x - 5}$, then $f(5)$ is

(a) 0 (b) 1

(c) 10 (d) not defined (1 mark)

Answer:

(d) If $f(x) = \frac{x^2 - 25}{x - 5}$

$f(5) = \frac{(5)^2 - 25}{5 - 5} = \frac{0}{0}$ = *does not exist* = *not defined*

2014 - JUNE

[43] Let A={1,2,3} and B={6,4,7}. Then, the relation R={(2,4), (3,6)} will be:

(a) Function from A to B (b) Function from B to A

(c) Both A and B (d) Not a function (1 mark)

Answer:

(a) A = {1, 2, 3} & B = {6, 4, 7}

Relation R = {(2, 4), (3, 6)} will be function from A to B.

[44] In a class of 50 students, 35 opted for Mathematics and 37 opted for Commerce. The number of such students who opted for both Mathematics and Commerce are:

(a) 13 (b) 15
(c) 22 (d) 28 (1mark)

Answer:

(c) Given $n(m \cup c) = 50$

$n(m) = 35$

$n(c) = 37$

$n(m \cup c) = n(m) + n(c) - n(m \cap c)$

$50 = 35 + 37 - n(m \cap c)$

$n(m \cap c) = 35 + 37 - 50$

$= 72 - 50$

$n(m \cap c) = 22$

[45] The range of {(1,0), (2,0), (3,0), (4,0), (0,0)} is:

(a) {1,2,3,4,0} (b) {0}
(c) {1,2,3,4} (d) None of these (1 mark)

Answer:

(b) The Range of {(1, 0), (2, 0), (3, 0), (4, 0), (5, 0)}
= {0}

2014 - DECEMBER

[46] Let N be the set of all Natural numbers; E be the set of all even natural numbers then the function

$f: N \to E$ defined as $f(x) = 2x + x \in N$ is:

(a) One-one into (b) One-one onto
(c) Many-one into (d) Many-one onto (1 mark)

Answer:

(b) N = Set of all Natural No.
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10 .....}

$= \{2, 4, 6, 8, 10, ...\}$

$f: N \rightarrow E$

$f(x) = 2x$

If $f(x_1) = f(x_2)$

$2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$ So $f(x)$ is one-one

at $f(x) = 2x$

$y = 2x$

$x = \frac{y}{2}$

Then Range of f = Even No. (E)

So $f(x)$ is onto

Hence, $f(x)$ is one-one onto.

[47] If A = {2, 3}, B = {4, 5}, C = {5, 6}, then A x (B ∩ C) = _____

(a) {(5, 2), (5, 3)} (b) {(2, 5), (3, 5)}

(c) {(2, 4), (3, 5)} (d) {(3, 5), (2, 6)} (1 mark)

Answer:

(b) A = {2, 3}, B = {4, 5}, C = {5, 6}

$B \cap C = \{5\}$

$A \times (B \cap C) = \{2, 3\} \times \{5\}$

$= \{(2, 5), (3, 5)\}$

[48] If S = {1, 2, 3} then the relation {(1, 1), (2, 2), (1, 2), (2, 1)} is symmetric and

(a) Reflexive but not transitive

(b) Reflexive as well as transitive

(c) Transitive but not reflexive

(d) Neither transitive nor reflexive (1 mark)

Answer:

(c) If S = {1, 2, 3} then

The Relation {(1, 1), (2, 2), (1, 2), (2, 1)} is symmetric and **transitive but not reflexive.**

[49] If $f(x) = \frac{x}{x-1}$, then $\frac{f(x/y)}{f(y/x)} =$ _____

(a) x/y (b) y/x
(c) -x/y (d) -y/x (1 mark)

Answer:

(c) If $f(x) = \frac{x}{x-1}$

$$f(x/y) = \frac{x/y}{\frac{x}{y}-1} = \frac{\frac{x}{y}}{\frac{x-y}{y}} = \frac{x}{x-y}$$

$$f(y/x) = \frac{y/x}{\frac{y}{x}-1} = \frac{\frac{y}{x}}{\frac{y-x}{x}} = \frac{y}{y-x}$$

$$\frac{f(x/y)}{f(y/x)} = \frac{x/(x-y)}{y/(y-x)} = \frac{x}{(x-y)} \cdot \frac{(y-x)}{y}$$

$$= \frac{-x(x-y)}{y(x-y)}$$

$$= \frac{-x}{y}$$

2015 - JUNE

[50] If N be the set of all natural numbers and E be the set of all even natural numbers then the function f: N → E, such that $f(x) = 2x$ for all $X \in N$ is

(a) one-one onto (b) one-one into
(c) many-one onto (d) constant (1 mark)

Answer:

(a) $N = \{1, 2, 3, 4, \ldots\ldots\ldots\ldots \infty\}$

$E = \{2, 4, 6, 8, \ldots\ldots\ldots\ldots \infty\}$

$f: N \rightarrow E$

$f(x) = 2x$

$f(1) = 2 \times 1 = 2$

$f(2) = 2 \times 2 = 4$

$f(3) = 2 \times 3 = 6$

(i) Range of function (R) = E

(ii) $f(x_1) = f(x_2)$ then function is one-one onto

2015 - DECEMBER

[51] If A = {x, y, z}, B = {a, b, c, d}, then which of the following relation from the set A to set B is a function?

(a) {(x, a), (x, b), (y, c), (z, d)}

(b) {(x, a), (y, b), (z, d)}

(c) {(x, c), (z, b), (z, c)}

(d) {a, z), (b, y), (c, z), (d, x)} (1 mark)

Answer:

(b) if A = {x, y, z}

B = {a, b, c, d}

A × B = {x, y, z} × {a, b, c, d}

= {(x, a) (x, b) (x, c) (x, d)
(y, a) (y, b) (y, c) (y, d)
(z, a) (z, b) (z, c) (z, d)}

Then {(x, a), (y, b), (z, d)} is a functions.

[52] In a class of 80 students, 35% students can play only cricket, 45% students can play only table tennis and the remaining students can play both the games. In all how many students can play cricket?

(a) 55 (b) 44
(c) 36 (d) 28 (1 mark)

Answer:

(b) Total students in the class = 80

$n(A \cup B) = 80$

Let no. of students who play both Table Tennis and Cricket = x

i.e. $n(A \cap B) = x$

No. of person who play only Cricket

$$n(A \cap \bar{B}) = 80 \times \frac{35}{100} = 28$$

$$n(A \cap \bar{B}) = 28$$

$$n(A) - n(A \cap B) = 28$$

$$n(A) - x = 28$$

$$n(A) = 28 + x$$

No. of students who play only Table Tennis

$$n(B \cap \bar{A}) = 45\% \text{ of } 80$$

$$= \frac{45}{100} \times 80$$

$$n(B \cap \bar{A}) = 36$$

$$n(B) - n(A \cap B) = 36$$

$$n(B) - x = 36$$

$$(B) = (36 + x)$$

We know that,

$$n(A \cup B) = n(A) + n(B) + n(B) - n(A \cap B)$$

$$80 = 28 + x + 36 + x - x$$

$$80 = 64 + x$$

$$n = 80 - 64$$

$$\boxed{n = 16}$$

i.e. $n(A \cap B) = 16$

No. of students who play cricket

$n(A) = 28 + x$

$= 28 + 16 = 44$

[53] If $f(x) = 2x + 2$ and $g(x) = x^2$, then the value of fog (4) is:

(a) 18 (b) 22

(c) 34 (d) 128 (1 mark)

Answer:

(c) $f(x) = 2x + 2$ and $g(x) = x^2$

$fog(x) = f\{g(x)\}$

$= f\{x^2\}$

$= 2x^2 + 2$

then

$fog(4) = 2(4)^2 + 2$

$= 2 \times 16 + 2$

$= 32 + 2$

$= 34$

2016 - JUNE

[54] If set $A = \left\{x: \frac{x}{2} \in z, \quad 0 \le x \le 10\right\}$,

$B = \{x : x \text{ is one digit prime number}\}$

and $C = \left\{x: \frac{x}{3} \in N, \quad x \le 12\right\}$

then $A \cap (B \cap C)$ is equal to –

(a) φ (b) Set A

(c) Set B (d) Set C (1 mark)

Answer:

(a) If $A = \left\{x: \frac{x}{2} \in z, \quad 0 \le x \le 10\right\}$

$A = \left\{\frac{0}{2}, \frac{2}{2}, \frac{4}{2}, \frac{6}{2}, \frac{8}{2}, \frac{10}{2}\right\} = \{0, 1, 2, 3, 4, 5\}$

$$B = \{x : x \text{ is one digit prime number}\}$$
$$= \{2, 3, 5, 7\}$$
$$\text{and } C = \left\{x: \frac{x}{3} \in N, \quad x \le 12\right\}$$
$$= \left\{\frac{3}{3}, \frac{6}{3}, \frac{9}{3}, \frac{12}{3}\right\}$$
$$= \{1, 2, 3, 4\}$$
$$B \cap C = \{2, 3\}$$
$$A \cap (B \cap C) = \{0, 1, 2, 3, 4, 5\} \cap \{2, 3\}$$
$$= \{2, 3\}$$

[55] Let A be the set of squares of natural numbers and let $x \in A$, $y \in A$ then

(a) $X + Y \in A$ (b) $X - Y \in A$

(c) $\frac{x}{Y} \in A$ (d) $xy \in A$ (1 mark)

Answer:

(d) Let A be the set of square of Natural No.

$A = \{1^2, 2^2, 3^2, 4^2, \ldots\ldots\ldots\ldots \infty\}$

$A = \{1, 4, 9, 16, \ldots\ldots\ldots\ldots\ldots\}$

If $x \in A$, $y \in A$ then $xy \in A$

[56] The domain (D) and range (R) of the function $f(x) = 2 - |x+1|$ is

(a) D = Real numbers, R = $(2, \infty)$

(b) D = Integers, R = (0, 2)

(c) D = Integers, R = $(-\infty, \infty)$

(d) D = Real numbers, R = $(-\infty, 2)$ (1 mark)

Answer:

(d) Given function

$f(x) = 2 - [(x + 1)]$

Domain = Real Number

and $f(x) = 2 - [x + 1]$

$y = 2 - [x + 1]$

$[x + 1] = 2 - y$

$\pm (x + 1) = 2 - y$

+ ve sign taking

$x + 1 = 2 - y$

$x = 2 - y - 1$

$x = 1 - y$

- ve sign

$-(x + 1) = 2 - y$

$x + 1 = -2 + y$

$x = y - 2 - 1$

$x = y - 3$

So Range = $[-\infty, 2]$

Domain = Real No, Range = $(-\infty, 2)$

2016 - DECEMBER

[57] If R is the set of all real numbers, then the function f: R→R defined by $f(x) = 2^x$

(a) one-one onto (b) one-one into

(c) many-one into (d) many-one onto (1 mark)

Answer:

(b) $f(x) = 2^x$

$f(x_1) = 2^{x_1}$ and $f(x_2) = 2^{x_2}$

Now, $f(x_1) = f(x_2)$

$2^{x_1} = 2^{x_2} \Rightarrow x_1 = x_2$

so, $f(x) = 2^x$ is one-one

and

$f(x) = 2^x$

$y = 2^x$

$\log y = \log 2^x$

$\log y = x \log 2$

$x = \log_2 y$ [log is not valid value if y is negative]

So, range of function ≠ B so it is into function.

[58] The inverse function f^{-1} of $f(x) = 100x$ is:

(a) $\frac{x}{100}$ (b) $\frac{1}{100x}$

(c) $\frac{1}{}$ (d) None of these (1 mark)

Answer:

(a) Given $f(x) = 100\,x$

$y = 100\,x$

$x = \frac{y}{100}$

$f^{-1}(y) = \frac{y}{100}$

$f^{-1}(x) = \frac{x}{100}$

[59] The number of subsets of the set formed by the word Allahabad is:

(a) 128 (b) 16

(c) 32 (d) 64 (1 mark)

Answer:

(c) A = Set of the letter of the word 'ALLAHABAD'

= {A,L,H,B,D}

n(A) = 5

No. of subset $= 2^n$

$= 2^5$

$= 32$

2017 - JUNE

[60] The range of function f defined by $f(x) = \frac{x}{x^2+1}$ is:

(a) $\{x: \frac{-1}{2} < x < \frac{1}{2}\}$ (b) $\{x: \frac{-1}{2} \leq x < \frac{1}{2}\}$

(c) $\{x: \frac{-1}{2} \leq x \leq \frac{1}{2}\}$ (d) $\{x: x > \frac{1}{2} \text{ or } x < \frac{-1}{2}\}$ (1 mark)

Answer:

(c) $f(x) = \frac{x}{x^2+1}$

$y = \frac{x}{x^2+1}$

$yx^2 + y = x$

$yx^2 - x + y = 0$

$a = y, b = -1, c = y$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times y \times y}}{2.y}$$

$$x = \frac{1 \pm \sqrt{1 - 4y^2}}{2y}$$

$1 - 4y^2 \geq 0$

$1 \geq 4y^2$

$\frac{1}{4} \geq y^2$

$\pm \frac{1}{2} \geq y$

Range → $\left\{x: -\frac{1}{2} \leq x \leq \frac{1}{2}\right\}$

[61] In a group of students 80 can speak Hindi, 60 can speak English and 40 can speak English and Hindi both, then number of students is:

(a) 100 (b) 140

(c) 180 (d) 60 (1 mark)

Answer:

(a) A = Hindi, B = English

$n(A) = 80, n(B) = 60, n(A \cap B) = 40$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 80 + 60 - 40$

$= 140 - 40$

$= 100$

[62] If $f(x) = \frac{x-1}{x}$ and $g(x) = \frac{1}{1-x}$ then (fog) (x) is equal to:

(a) $x - 1$ (b) x

(c) $1 - x$ (d) $-x$ (1 mark)

Answer:

(b) Given $f(x) = \frac{x-1}{x}$ and $g(x) = \frac{1}{1-x}$

$\log(x) = f\{g(x)\}$

$= f\left\{\frac{1}{1-x}\right\}$

$= \frac{\frac{1}{(1-x)} - \frac{1}{1}}{\frac{1}{(1-x)}} = \frac{\frac{x-x+x}{(1-x)}}{\frac{1}{(1-x)}} = \frac{x}{1} = x$

2017 - DECEMBER

[63] If $f(x) = \frac{x+1}{x+2}$, then $f\left\{f\left(\frac{1}{x}\right)\right\} =$ _____.

(a) $\frac{2x+3}{3x+5}$ (b) $\frac{2x+5}{3x+2}$

(c) $\frac{3x+2}{5x+3}$ (d) $\frac{5x+2}{2x+3}$ (1 mark)

Answer:

(c) Given $f(x) = \frac{x+1}{x+2}$

$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}+1}{\frac{1}{x}+2} = \frac{1+x}{1+2x}$

$f\left\{f\left(\frac{1}{x}\right)\right\} = f\left\{\frac{1+x}{1+2x}\right\}$

$= \frac{\frac{1+x}{1+2x}+1}{\frac{1+x}{1+2x}+2}$

$= \frac{\frac{1+x+1+2x}{(1+2x)}}{\frac{1+x+2+4x}{(1+2x)}} = \frac{(3x+2)}{(5x+3)}$

[64] In a class of 35 students, 24 like to play cricket and 16 like to play football. Also each student likes to play at least one of the two games. How many students like to play both cricket and football?

(a) 5 (b) 11
(c) 19 (d) 8 (1 mark)

Answer:

(a) Let A → Cricket
B → Football

$n(A) = 24, n(B) = 16, n(A \cup B) = 35$

$n(A \cap B) = ?$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$35 = 24 + 16 - n(A \cap B)$

$n(A \cap B) = 24 + 16 - 35$

$= 5$

2018 - MAY

[65] Let N be the set of all natural numbers; E be the set of all even natural numbers then the function;

$F:N \Rightarrow E$ defined as $f(x) = 2x - V x \in N$ is =

(a) One-one-into (b) Many-one-into
(c) One-one onto (d) Many-one-onto (1 mark)

Answer:

(c) Given

$N = \{1, 2, 3, 4, 5, 6 \ldots\ldots\ldots\ldots \infty\}$

$E = \{2, 4, 6, 8, \ldots\ldots\ldots\ldots \infty\}$

$f: N \to E$

$f(x) = 2x \qquad V\ X \in N$

$f(1) = 2 \times 1 = 2$

$f(2) = 2 \times 2 = 4$

$f(3) = 2 \times 3 = 6$

Range of function = {2,4,6,.....}
= E
and $f(x_1) = f(x_2)$
$2x_1 = 2x_2 = x_2$
So $f(x)$ function is one-one and one to.

[66] In a town of 20,000 families it was found that 40% families buy newspaper. A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C if 2% families buy all the three newspapers, then the number of families which buy A only is:

(a) 6600 (b) 6300
(c) 5600 (d) 600 (1 mark)

Answer:

(a) Total Families n(u) = 20000

No. of families who buy Newspapers 'A'
n(A) = 40% of 20000 = 8000

No. of families who buy Newspaper 'B'
n(B) = 20% of 20000 = 4000

No. of families who buy Newspaper 'C'
n(C) = 10% of 20000 = 2000

No. of families who buy Newspapers A & B
$n(A \cap B)$ = 5% of 20000 = 1000

No. of families who buy Newspapers B & C
$n(B \cap C)$ = 3% of 20000 = 600

No. of families who buy Newspapers C & A
$n(C \cap A)$ = 4% of 20000 = 800

No. of families who buy all newspapers
$n(A \cap B \cap C)$ = 2% of 20000 = 400

No. of families which buy Newspapers 'A' only

$= n\left(A \cap \bar{B} \cap \bar{C}\right)$

$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
$= 8000 - 1000 - 800 + 400$
$= 6600$

[67] The numbers of proper sub set of the set {3,4,5,6,7} is:
(a) 32 (b) 31
(c) 30 (d) 25 (1 mark)

Answer:

(b) Given

$A = \{3, 4, 5, 6, 7\}$
$n(A) = 5$
No. of proper subset $= 2^n - 1$
$= 2^5 - 1$
$= 32 - 1$
$= 31$

2018 - NOVEMBER

[68] A is {1,2,3,4} and B is {1,4,9,16,25} if a function f is defined from set A to B where $f(x) = x^2$ then the range of f is:
(a) {1,2,3,4} (b) {1,4,9,16}
(c) {1,4,9,16,25} (d) None of these (1 mark)

Answer:

(b) Given $A = \{1,2,3,4\}$
$B = \{1,4,9,16,25\}$
If $f : A \rightarrow B$ and $f(x) = x^2$
$f(1) = (1)^2 = 1$
$f(2) = (2)^2 = 4$
$f(3) = (3)^2 = 9$
$f(4) = (4)^2 = 16$
Range of f = {1,4,9,16}

[69] If A = {1,2} and B = {3,4}. Determine the number of relations from A and B:

(a) 3 (b) 16

(c) 5 (d) 6 (1 mark)

Answer:

(b) Given $A = \{1,2\}$

$B = \{3,4\}$

$A \times B = \{1,2\} \times \{3,4\}$

$= \{(1,3)\ (1,4)\ (2,3)\ (2,4)\}$

$n(A \times B) = 4$

No. of relation from A and B $= 2^n$

$= 2^4$

$= 16$

A liter or

Shortcut $A = \{1,2\}$, $n(A) = 2$

$B = \{3,4\}$, $n(B) = 2$

No. of Relation from A and B $= 2^{m \times n}$

$= 2^{2 \times 2}$

$= 2^4 = 16$

[70] If A = {1,2,3,4,5,6,7} and B = {2,4,6,8}. Cardinal number of A - B is:

(a) 4 (b) 3

(c) 9 (d) 7 (1 mark)

Answer:

(a) Given $A = \{1,2,3,4,5,6,7\}$

$B = \{2,4,6,8\}$

$A - B = \{1,2,3,4,5,6,7\} - \{2,4,6,8\}$

$= \{1,3,5,7\}$

$n(A - B) = 4$

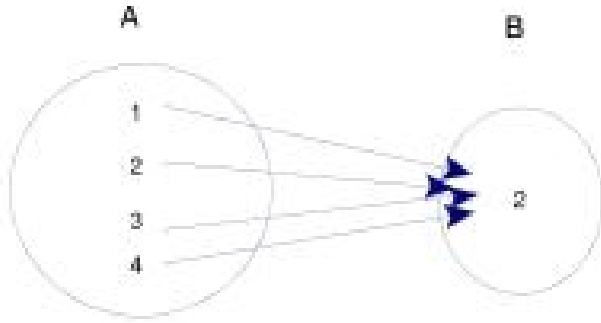
[71] Identity the function from the following:

(a) {(1,1), (1,2), (1,3)} (b) {(1,1), (2,1), (2,3)}

(c) {(1,2), (2,2), (3,2),(4,2)} (d) None of these (1 mark)

Answer:

(c) {(1,2) (2,2) (3,2) (4,2)} is the function.



Many one function

2019 - JUNE

[72] If A = { 1, 2, 3, 4, 5, 6, 7, 8, 9}

B = {1, 3, 4, 5, 7, 8}; C = {2, 6, 8} then find (A – B) ∪ C

(a) {2, 6}

(b) {2, 6, 8}

(c) {2, 6, 8, 9}

(d) None (1 mark)

Answer:

(c) A = {1, 2, 3, 4, 5, 6, 7, 8, 9}

B = {1, 3, 4, 5, 7, 8}, C = {2, 6, 8}

Then

$A - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 3, 4, 5, 7, 8\}$

$= \{2, 6, 9\}$

$(A - B) \cup C = \{2, 6, 9\} \cup \{2, 6, 8\}$

$= \{2, 6, 8, 9\}$

[73] A = {1 2 3 4 .....10} a relation on A, R = {(x, y)/ x + y = 10, x ∈A, Y ∈ A, x ≥ Y} then domain of R^{-1} is

(a) {1, 2, 3, 4, 5}
(b) {0, 3, 5, 7, 9}
(c) {1, 2, 4, 5, 6, 7}
(d) None (1 mark)

Answer:

(a) Given, A= {1,2,3,4,.....10}

$R = \{x, y): x + y = 10,\ x \in A, y \in A, x \geq y\}$

$\Rightarrow R = \{(5, 5), (6, 4), (7, 3), (8, 2) (9, 1)\}$

$R^{-1} = \{5, 5) (4, 6) (3, 7) (2, 8) (1, 9)\}$

Domain of $R^{-1} = \{5, 4, 3, 2, 1\}$

[74] The no. of subsets of the set {3, 4, 5} is :

(a) 4
(b) 8
(c) 16
(d) 32 (1 mark)

Answer:

(b) Here, A = {3, 4, 5}

n(A) = 3

No of Subset $= 2^n$

$= 2^3$

$= 8$

[75] If $f(x) = x^2$ and $g(x) = \sqrt{x}$ then

(a) go f(3) = 3
(b) go f(-3) = 9
(c) go f(9) = 3
(d) go f(-9) = 3 (1 mark)

Answer:

(a) Given, $f(x) = x^2$ and $g(x) = \sqrt{x}$

fog (x) $= f\{g(x)\}$

$= f\{\sqrt{x}\}$

$= (\sqrt{x})^2$

$fog(x) = x$

and $gof(x) = g\{f(x)\}$

$= g\{x^2\}$

$= . \sqrt{x^2}$

$gof(x) = x$

$gof(3) = 3$

[76] If A = {a, b, c, d}; B = {p, q, r, s} which of the following relation is a function from A to B

(a) $R_1 = \{(a, p), (b, q), (c, s)\}$

(b) $R_2 = \{(p, a), (b, r), (d, s)\}$

(c) $R_3 = \{(b, p), (c, s), (b, r)\}$

(d) $R_4 = \{(a, p), (b, r), (c, q), (d, s)\}$ (1 mark)

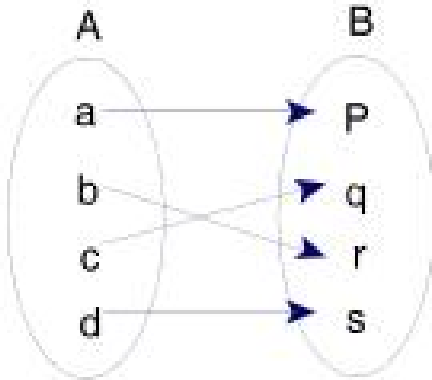
Answer:

(d) If A = {a, b, c, d}

B = {p, q, r, s}

$R_4 = \{(a, p), (b, r), (c, q), (d, s)\}$

is a function from A to B



2019 - NOVEMBER

[77] $(A^T)^T = ?$

(a) A

(b) A^T

(c) $A^T . A^T$

Answer:

(a) $(A^T)^T = A$

Example $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$(A^T)^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = A$$

So, $(A^T)^T = A$

[78] $f(n) = f(n-1) + f(n-2)$ when $n = 2, 3, 4 \ldots\ldots\ldots\ldots\ldots$ $f(0) = 0$, $f(1) = 1$ then $f(7) = ?$

(a) 3
(b) 5
(c) 8
(d) 13 (1 mark)

Answer:

(d) $f(n) = f(n-1) + f(n-2)$

$f(2) = f(1) + f(0) = 1 + 0 = 1 = f(2)$

$f(3) = f(2) + f(1) = 1 + 1 = 2 = f(3)$

$f(4) = f(3) + f(2) = 2 + 1 = 3$

Similarly,

$f(7) = f(6) + f(5)$

$f(7) = [f(5) + f(4)] + [f(4) + f(3)]$

$f(7) = [f(4) + f(3) + f(4)] + (f(4) + f(3))$

$f(7) = [3 + 2 + 3] + [3 + 2]$

$f(7) = 13$

[79] $f(x) = x + \frac{1}{x}$ find $f^{-1}(x)$

(a) $1/(x-1)$
(b) $1/(y-1)$
(c) $\frac{1}{y} - 1$
(d) x (1 mark)

Answer:

(a) $f(x) = \frac{x+1}{x}$ — Equation (1)

Let $f(x) = y$

$x = f^{-1}(y)$

Further Solving — Equation (1)

$$y = \frac{x+1}{x}$$

$$xy = x + 1$$

$$xy - x = 1$$

$$x(y-1) = 1$$

$$x = \frac{1}{(y-1)}$$

$$f^{-1}(y) = \frac{1}{(y-1)}$$

$$f^{-1}(x) = \frac{1}{(x-1)}$$

CHAPTER

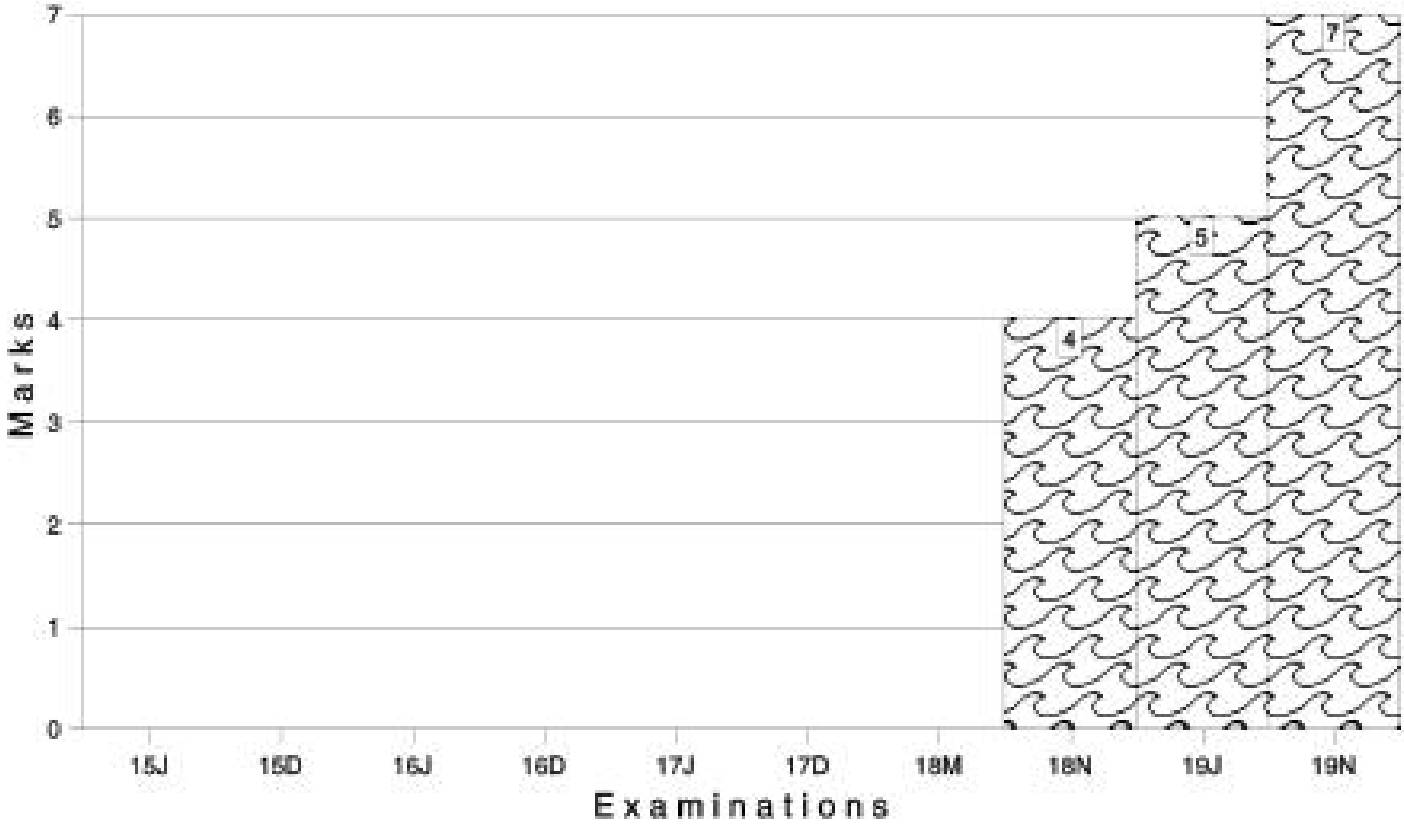
8

BASIC APPLICATIONS OF DIFFERENTIAL AND INTEGRAL CALCULUS

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



For detailed analysis Login at *www.scannerclasses.com*
for registration and password see first page of this book.

PAST YEAR QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] The slope of the tangent at the point (2, – 2) to the curve $x^2 + xy + y^2 - 4 = 0$ is given by :

(a) 0 (b) 1

(c) – 1 (d) None (1 mark)

Answer:

(b) $x^2 + xy + y^2 - 4 = 0$

Differentiating both sides w.r.t.x.

$$2x + x\frac{dy}{dx} + y.1 + 2y\frac{dy}{dx} - 0 = 0$$

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y + x) = -y - 2x.$$

$$\frac{dy}{dx} = -\frac{(2x + y)}{2y + x}$$

$$\therefore \left(\frac{dy}{dx}\right)_{at(2,2)} = -\frac{[(2\times 2)-2]}{2\times(-2)+2} = -\frac{(4-2)}{-4+2} = -\frac{2}{-2} = 1$$

[2] The derivative of $x^2 \log x$ is :

(a) 1 + 2 log x (b) 2 log x

(c) x (1 + 2 log x) (d) None of these (1 mark)

Answer:

(c) $x^2 \log x$

Differentiating both sides w.r.t. x

$$x^2 \frac{d}{dx}(\text{Log } x) + \log x \frac{d}{dx}(x^2)$$

$x^2 \frac{1}{x} + \text{Log}x.\ 2x.$

$x + 2x \log x.$

$x\,(1 + 2 \log x)$

[3] $\int_0^1 (e^x + e^{-x})dx$ is :

(a) $e - e^{-1}$ (b) $e^{-1} - e$

(c) $e + e^{-1}$ (d) None (1 mark)

Answer:

(a) $\int_0^1 (e^x + e^{-x})dx$

$= \left[\frac{e^x}{1} + \frac{e^{-x}}{-1}\right]_0^1$

$= \left[e^x - e^{-x}\right]_{-0}^{1}$

$= (e^1 - e^{-1}) - (e^0 - e^{-0})$

$= (e - e^{-1}) - (1 - 1)$

$= e - e^{-1}$

[4] $\int \frac{8x^2}{(x^3 + 2)^3}$ dx is equal to :

(a) $-\frac{4}{3}(x^3 + 2)^2 + C$ (b) $-\frac{4}{3}(x^3 + 2)^{-2} + C$

(c) $\frac{4}{3}(x^3 + 2)^2 + C$ (d) None of these (1 mark)

Answer:

(b) $\int \frac{8x^2}{(x^3 + 2)^3}$

$x^3 + 2 = t$

Differentiating both sides w.r.t.x.

$$3x^2 = \frac{dt}{dx}$$

$$3x^2\,dx = dt$$

$$x^2\,dx = \frac{dt}{3}$$

$$\therefore \int \frac{8x^2}{(x^3+2)^3}dx$$

$$\frac{8}{3}\int \frac{1}{t^3}\,dt = \frac{8}{3}\int t^{-3}\,dt$$

$$\frac{8}{3} \times \frac{t^{-3+1}}{-3+1} + C$$

$$\frac{8}{3} \times \frac{t^{-2}}{-2} + C$$

$$= -\frac{4,t^2}{3} + C$$

$$= -\frac{4}{3}(x^3+2)^{-2} + C$$

2007 - FEBRUARY

[5] If $x = y \log (xy)$, then $\frac{dy}{dx}$ is equal to:

(a) $\frac{x+y}{x(1+\log xy)}$ (b) $\frac{x-y}{x(1+\log xy)}$

(c) $\frac{x+y}{x(\log x+\log y)}$ (d) $\frac{x-y}{x(\log x+\log y)}$ (1 mark)

Answer:

(b) $x = y \text{ Log } (xy)$

Differentiating both sides w.r.t x

$$\frac{d}{dx}(x) = \frac{d}{dx}[y \log (xy)]$$

$$1 = y \frac{d}{dx} \log (xy) + \text{Log } (xy) : \frac{dy}{dx}$$

$$1 = y \frac{1}{xy} \frac{d}{dx} (xy) + \text{Log } (xy) \frac{dy}{dx}$$

$$1 = \frac{1}{x}\left(\frac{x dy}{dx} + y.1\right) + \text{Log } (xy) \frac{dy}{dx}$$

$$1 = \frac{1}{x}\left(\frac{x dy}{dx} + y\right) + \text{Log } (xy) \frac{dy}{dx}$$

$$1 = \frac{dy}{dx} + \frac{y}{x} + \text{Log } (xy) \frac{dy}{dx}$$

$$1 = [1 + \text{Log } (xy)] \frac{dy}{dx} + \frac{y}{x}$$

$$[1 + \text{Log } (xy)] \frac{dy}{dx} = 1 - \frac{y}{x}$$

$$\frac{dy}{dx} (1 + \text{Log } xy) = \frac{x - y}{x}$$

$$\frac{dy}{dx} = \frac{x - y}{x(1 + \text{Log } xy)}$$

[6] If $y = 2x + \frac{4}{x}$, then $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y$ yields

(a) 3 (b) 1

(c) 0 (d) 4 (1 mark)

Answer:

(c) $y = 2x + \frac{4}{x}$

Differentiating both sides w.r.t. x

$$\frac{dy}{dx} = 2 \times 1 + 4 (-1.x^{-1-1})$$

$$\frac{dy}{dx} = 2 - \frac{4}{x^2}$$

Differentiating again both sides w.r.t. x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(2 - \frac{4}{x^2}\right)$$

$$\frac{d^2y}{dx^2} = 0 - 4(-2.x^{-2-1})$$

$$\frac{d^2y}{dx^2} = \frac{8}{x^3}$$

Now. $\frac{x^2d^2y}{dx^2} + x\frac{dy}{dx} - y$

$$x^2\left(\frac{8}{x^3}\right) + x\left(2 - \frac{4}{x^2}\right) - \left(2x + \frac{4}{x}\right)$$

$$\frac{8}{x} + 2x - \frac{4}{x} - 2x - \frac{4}{x}$$

$$\frac{8}{x} - \frac{8}{x} = 0$$

[7] Evaluate : $\int \frac{dx}{\sqrt{x^2 + a^2}}$:

(a) $\frac{1}{2}\log(x + \sqrt{x^2 + a^2}) + C$ (b) $\log(x + \sqrt{x^2 + a^2}) + C$

(c) $\log(x\sqrt{x^2 + a^2}) + C$ (d) $\frac{1}{2}\log(x\sqrt{x^2 + a^2}) + C$ (1 mark)

Answer:

(b) $\int \frac{dx}{\sqrt{x^2 + a^2}}$

Let $\sqrt{x^2 + a^2} = t \quad x$

$t = x + \sqrt{x^2 + a^2}$

Differentiating both sides w.r.t. x

$$\frac{dt}{dx} = 1 + \frac{1}{2\sqrt{x^2 + a^2}} \frac{d}{dx}(x^2 + a^2)$$

$$\frac{dt}{dx} = 1 + \frac{2x}{2\sqrt{x^2 + a^2}}$$

$$\frac{dt}{dx} = 1 + \frac{x}{\sqrt{x^2 + a^2}}$$

$$\frac{dt}{dx} = \frac{\sqrt{x^2 + a^2} + x}{\sqrt{x^2 + a^2}}$$

$$\frac{dy}{dx} = \frac{t}{\sqrt{x^2 + a^2}}$$

$$\frac{dx}{\sqrt{x^2 + a^2}} = \frac{dt}{t}$$

$$\therefore \int \frac{dx}{\sqrt{x^2 + a^2}} = \int \frac{dt}{t}$$

$$= \log t + C$$

$$= \text{Log}\left(x + \sqrt{x^2 + a^2}\right) + c$$

[8] The value of $\int_0^2 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{2 - x}} dx$ is :

(a) 0 (b) 3
(c) 2 (d) 1 (1 mark)

Answer:

(d) $I = \int_0^2 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{2 - x}} dx$

$$I = \int_0^2 \frac{\sqrt{2 - x}}{\sqrt{2 - x} + \sqrt{2 - (2 - x)}} dx \quad \left[\text{since} \int_0^a f(x)dx = \int_0^a f(a - x)dx\right]$$

$$I = \int_0^2 \frac{\sqrt{2 - x}}{\sqrt{2 - x} + \sqrt{x}}$$

$$I = \int_0^2 \frac{\sqrt{2-x}}{\sqrt{x}+\sqrt{2-x}}\,dx$$

$$\therefore\ 2I = I + I$$

$$= \int_0^2 \frac{\quad}{\quad}$$

[10] Given $x = 2t + 5$; $y = t^2 - 2$, then $\frac{dy}{dx}$ is calculated as :

(a) t (b) 1/t

(c) − 1/t (d) None (1 mark)

Answer:

(a) $x = 2t + 5$

differentiating both sides w.r.t t.

$\frac{dx}{dt} = 2$ ..... (1)

$y = t^2 - 2$

differentiating both sides w.r.t t.

$\frac{dy}{dt} = 2t$ .....(2)

Dividing (2) by (1), we get

$\frac{dy}{dx} = \frac{2t}{2}$

$\frac{dy}{dx} = t$

[11] The integral of $(e^{3x} + e^{-3x}) / e^x$ is :

(a) $\frac{e^{2x}}{2} + \frac{e^{-4x}}{4} + C$ (b) $\frac{e^{2x}}{2} - \frac{e^{-4x}}{4} + C$

(c) $e^{2x} - e^{-4x} + C$ (d) None of these (1 mark)

Answer:

(b) $= \int \frac{e^{3x} + e^{-3x}}{e^x} dx$

$= \int \left(\frac{e^{3x}}{e^x} + \frac{e^{-3x}}{e^x} \right) dx$

$= \int (e^{2x} + e^{-4x}) dx$

$= \int e^{2x} dx + \int e^{-4x} dx$

$= \frac{e^{2x}}{2} + \frac{e^{-4x}}{(-4)} + C$

$= \frac{e^{2x}}{2} - \frac{e^{-4x}}{4} + C$

[12] $\int x^2 e^{3x} dx$ is :

(a) $x^2.e^{3x} - 2xe^{3x} + 2e^{3x} + C$

(b) $\frac{e^{3x}}{3} - \frac{x.e^{3x}}{9} + 2e^{3x} + C$

(c) $\frac{x^2.e^{3x}}{3} - \frac{2x.e^{3x}}{9} + \frac{2}{27}e^{3x} + C$

(d) None of these (1 mark)

Answer:

(c) $\int x^2 e^{3x} dx$

By the rule of ILATE. & uv rule, we get :

$x^2 \int e^{3x} dx - \int \left(\frac{d}{dx}(x^2) \int e^{3x} dx \right) dx$

$x^2 - \int 2x \frac{e^{3x}}{3} dx$

$\frac{1}{3}x^2e^{3x} - \frac{2}{3} \int x e^{3x} dx$

Again applying uv rule :

$\frac{1}{3}x^2e^{3x} - \frac{2}{3}\left[x \int e^{3x} dx - \int \left(\frac{d}{dx}(x) \int e^{3x} dx \right) dx \right]$

$\frac{1}{3} + x^2e^{3x} - \frac{2}{3}\left[\frac{xe^{3x}}{3} - \int \left(\frac{e1 \times e^{3x}}{3} \right) dx \right]$

$\frac{1}{3}x^2e^{3x} - \frac{2}{3}\left[\frac{1}{3}xe^{3x} - \frac{1}{3} \int e^{3x} dx \right]$

$\frac{1}{3}x^2e^{3x} - \frac{2}{3}\left[\frac{1}{3}xe^{3x} - \frac{1}{3} \times \frac{e^{3x}}{3} \right] + c$

$\frac{x^2e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{27} + c$

[13] $\int_1^2 \frac{2x}{1+x^2}\,dx$:

(a) $\log_e \frac{5}{2}$ (b) $\log_e 5 - \log_e 2 + 1$

(c) $\log_e \frac{2}{5}$ (d) None of these (1 mark)

Answer:

(a) $\int_1^2 \frac{2x}{1+x^2}\,dx$

Let $1 + x^2 = t$

differentiating both sides w.r.t x

$2x = \frac{dt}{dx}$

$2xdx = dt.$

When $x = 0$ then $t = 1$

When $x = 2$ then $t = 5$

So, $\int_1^5 \frac{1}{t}\,dt.$

$= [\log_e t]_1^5$

$= \text{Log}_e 5 - \text{Log}_e 2$

$= \text{Log}_e\left(\frac{5}{2}\right)$

2007 - AUGUST

[14] If $x^y = y^x$, then $\frac{dy}{dx}$ gives :

(a) $\frac{x\,(x\log y - y)}{y\,(y\log x - x)}$ (b) $\frac{x\,(y\log x - x)}{y\,(x\log y - y)}$

(c) $\frac{y\,(x\log y - y)}{x\,(y\log x - x)}$ (d) None of these (1 mark)

Answer:

(c) $x^y = y^x$

Taking log on both the sides :

$y \log x = x \log y$

Differentiating both sides w. r. t. x

$$y \frac{d}{dx}(\log x) + \log x \times \frac{dy}{dx} = x \frac{d}{dx}(\log y) + (\log y) \frac{d}{dx}(x)$$

$$y \times \frac{1}{x} + (\log x) \frac{dy}{dx} = x \times \frac{1}{y} \frac{dy}{dx} + (\log y) \times 1$$

$$\frac{y}{x} + \frac{dy}{dx}(\log x) = \frac{x}{y} \frac{dy}{dx} + \log y$$

$$\frac{dy}{dx}(\log x) - \frac{x}{y} \frac{dy}{dx} = \log y - \frac{y}{x}$$

$$\frac{dy}{dx}\left(\log x - \frac{x}{y}\right) = \frac{x \log y - y}{x}$$

$$\frac{dy}{dx}\left(\frac{y \log x - x}{y}\right) = \frac{x \log y - y}{x}$$

$$\frac{dy}{dx} = \frac{y\,(x \log y - y)}{x\,(y \log x - x)}$$

[15] If $x^3 - 2x^2y^2 + 5x + y = 5$, then $\frac{dy}{dx}$ at x = 1 and y = 1 is :

(a) 4/3 (b) – 5/4

(c) 4/5 (d) – 4/3 (1 mark)

Answer:

(a) $x^3 - 2x^2y^2 + 5x + y = 5$

Differentiating each term w.r.t x

$$3x^2 - 2\left(x^2 \frac{d}{dx}(y^2) + y^2 \frac{d}{dx}(x^2)\right) + 5 + \frac{dy}{dx} = 0$$

$$3x^2 - 2\left(x^2 2y \frac{dy}{dx} + y^2 . 2x\right) + 5 + \frac{dy}{dx} = 0$$

$$3x^2 - 4x^2 y \frac{dy}{dx} - 4xy^2 + 5 + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(1 - 4x^2y) = 4xy^2 - 3x^2 - 5$$

$$\frac{dy}{dx} = \frac{4xy^2 - 3x^2 - 5}{1 - 4x^2y}$$

$$\left(\frac{dy}{dx}\right)_{(1,1)} = \frac{4 \times 1 \times 1^2 - 3 \times 1^2 - 5}{1 - 4 \times 1^2 \times 1}$$

$$= \frac{4 - 3 - 5}{1 - 4} = \frac{-4}{-3} = \frac{4}{3}$$

[16] The value of $\int_1^e \frac{(1 + \log x)}{x} dx$ is : [Given Loge =1]

(a) 1/2 (b) 3/2
(c) 1 (d) 5/2 (1 mark)

Answer:

(b) $\int_1^e \frac{(1 + \log x)}{x} dx$

$$= \int_1^e \left(\frac{1}{x} + \frac{\log x}{x}\right) dx$$

$$= \int_1^e \frac{dx}{x} + \int_1^e \frac{\log x}{x} dx$$

$$[\log x]_1^e + \int_0^1 t \, dt$$

Where $\log x = t$

$$\frac{1}{x} = \frac{dt}{dx}$$

$$\frac{1}{x} dx = dt$$

So, when $x = 1$

$t = 0$

When $x = e$

$t = 1$

$$(\log e - \log 1) + \left[\frac{t^2}{2}\right]_0^1$$

$$(1 - 0) + \left(\frac{1}{2} - 0\right)$$

$$1 + \frac{1}{2} = \frac{3}{2}$$

[17] Find $\int \frac{x^3}{(x^2 + 1)^3}\, dx$:

(a) $\frac{1}{4}\left[\frac{2x^2 + 1}{(x^2 + 1)^2}\right]$ (b) $-\frac{1}{4}\left[\frac{2x^2 + 1}{(x^2 + 1)^2}\right]$

(c) $\frac{1}{2}\left[\frac{2x^2 + 1}{(x^2 + 1)^2}\right]$ (d) $-\frac{1}{2}\left[\frac{2x^2 + 1}{(x^2 + 1)^2}\right]$ (1 mark)

Answer:

(b) $\int \frac{x^3}{(x^2 + 1)^3}\, dx$

Let $x^2 + 1 = t$

Then $2x\, dx = dt$ & $x dx = \frac{dt}{2}$

$$\therefore \quad \int \frac{x^3}{(x^2 + 1)^3}\, dx$$

$$= \int \frac{x^2 \cdot x}{(x^2 + 1)^3}\, dx$$

$$= \frac{1}{2}\int \frac{t - 1}{t^3} dt$$

$$= \frac{1}{2}\int \left(\frac{t - 1}{t^3 \; t^3}\right)^{dt}$$

$$= \frac{1}{2}\int \left(\frac{1}{t^2} - \frac{1}{t^3}\right) dt = \frac{1}{2}\int (t^{-2} - t^{-3})\, dt$$

$$= \frac{1}{2}\left[\frac{t^{-2+1}}{-2+1} - \frac{t^{-3+1}}{-3+1}\right] + c$$

$$= \frac{1}{2}\left(\frac{t^{-1}}{-1} - \frac{t^{-2}}{-2}\right) + c$$

$$= \frac{-1}{2t} + \frac{1}{4t^2}$$

$$= \frac{-2t+1}{4t^2} = \frac{1-2t}{4t^2}$$

$$= \frac{1}{4}\left[\frac{1-2t}{t^2}\right]$$

$$= \frac{1}{4}\left[\frac{1-2\,(x^2+1)}{(x^2+1)^2}\right]$$

$$= \frac{1}{4}\left[\frac{1-2\,x^2-2}{(x^2+1)^2}\right] = \frac{1}{4}\left[\frac{-2\,x^2-1}{(x^2+1)^2}\right]$$

$$= \frac{-1}{4}\left[\frac{2x^2+1}{(x^2+1)^2}\right]$$

2007 - NOVEMBER

[18] If $y = (x + \sqrt{x^2 + m^2})^n$ then $\frac{dy}{dx} =$:

(a) $\frac{ny}{\sqrt{x^2 + m^2}}$ (b) ny

(c) $-\frac{ny}{\sqrt{x^2 + m^2}}$ (d) None (1 mark)

Answer:

(a) $y = (x + \sqrt{x^2 + m^2})^n$

Differentiating both sides w.r.t

$$\frac{dy}{dx} = n\,(x + \sqrt{x^2 + m^2})^{n-1}\,\frac{d}{dx}\,(x + \sqrt{x^2 + m^2})$$

$$\frac{dy}{dx} = n\,(x + \sqrt{x^2 + m^2})^{n-1}\left[1 + \frac{1}{2}(x^2+m^2)^{-\frac{1}{2}}\frac{1}{dx}(x^2+m^2)\right]$$

$$\frac{dy}{dx} = n\,(x + \sqrt{x^2 + m^2})^{n-1}\left[1 + \frac{1}{2\sqrt{x^2+m^2}}(2x)\right]$$

$$\frac{dy}{dx} = n\,(x + \sqrt{x^2 + m^2})^{n-1}\left[1 + \frac{x}{\sqrt{x^2+m^2}}\right]$$

$$\frac{dy}{dx} = n\,(x + \sqrt{x^2 + m^2})^{n-1}\left[\frac{\sqrt{x^2+m^2}+x}{\sqrt{x^2+m^2}}\right]$$

$$\frac{dy}{dx} = \frac{n\,(x+\sqrt{x^2+m^2})^{n-1+1}}{\sqrt{x^2+m^2}}$$

$$\frac{dy}{dx} = \frac{n\,y}{\sqrt{x^2+m^2}} \quad [\because\ y = (x + \sqrt{x^2 + m^2})^n]$$

[19] If $xy\,(x - y) = 0$, find $\frac{dy}{dx}$:

(a) $\frac{y\,(2x - y)}{x\,(2y - x)}$ (b) $\frac{x\,(2x - y)}{y\,(2y - x)}$

(c) $\frac{y\,(2y - x)}{x\,(2x - y)}$ (d) None of these (1 mark)

Answer:

(a) $xy\,(x - y) = 0$

$x^2y - xy^2 = 0$

Differentiating both sides wrt x

$$(x^2\frac{dy}{dx} + y\,2x) - (x\,2y\frac{dy}{dx} + y^2 \times 1) = 0$$

$$x^2\frac{dy}{dx} + 2xy - 2xy\frac{dy}{dx} - y^2 = 0$$

$$\frac{dy}{dx}(x^2 - 2xy) = y^2 - 2xy$$

$$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$$

$$\frac{dy}{dx} = \frac{-y\,(2x - y)}{-x\,(2y - x)}$$

$$\frac{dy}{dx} = \frac{y\,(2x - y)}{x\,(2y - x)}$$

[20] If $y = \sqrt{x}^{\sqrt{x}^{\cdots}}$ then $\frac{dy}{dx}$ is equal to :

(a) $\frac{y^2}{\log x}$ (b) $\frac{y^2}{2 - y\log x}$

(c) $\frac{y^2}{x(2 - y\log x)}$ (d) None (1 mark)

Answer:

(c) $y = \sqrt{x}^{\sqrt{x}^{\cdots}}$

$y = (\sqrt{x})^y$ ($\because \sqrt{x}^{\sqrt{x}^{\cdots}} = y$)

Taking log on both sides

$\text{Log } y = y \log \sqrt{x}$

$\text{Log } y = \frac{y}{2} \log x.$

Differentiating both sides w.r.t. x

$$\frac{1}{y}\frac{dy}{dx} = \frac{y}{2}\frac{d}{dx}\log x + \frac{1}{2}\log x.\ \frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{y}{2} \times \frac{1}{x} + \frac{1}{2}\log x\ \frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\log x\ \frac{dy}{dx} = \frac{y}{2x}$$

$$\frac{dy}{dx}\left(\qquad\right) =$$

[21] $\int \frac{1}{x^2 - a^2}$ dx is :

(a) log (x – a) – log (x + a) + C

(b) $\log x - \frac{a}{x + a} + C$

(c) $\frac{1}{2a}\log\left(\frac{x-a}{x+a}\right) + C$

(d) None of these (1 mark)

Answer:

(c) $\int \frac{1}{x^2 - a^2}$ dx

$\int \frac{1}{(x - a)(x - a)} dx$

Now let us use the method of partial fractions

Let $\frac{1}{(x + a)(x - a)}$

$= \frac{A}{(x + a)} + \frac{B}{(x - a)}$

1 = A (x – a) + B (x + a)

we put x = a and get

1 = A (a – a) + B(a + a) => B = ½a

we put x = – a and get

1 = A (– a – a) + B (– a + a) => A = ½ b

$\int \frac{1}{x^2 - a^2}$ dx $= \frac{-1}{2a}\int \frac{dx}{(x + a)} + \frac{1}{2a}\int \frac{dx}{(x + a)}$

$= \frac{1}{2a}$ [– log(x + a) + log(x – a)]

or $\frac{1}{2a}\log\left(\frac{x - a}{x + a}\right) + c$

[22] The value of $\int_0^1 \frac{dx}{(1+x)(2+x)}$ is :

(a) $\log \frac{3}{4}$ (b) $\log \frac{4}{3}$

(c) log 12 (d) None (1 mark)

Answer:

(b) $\int_0^1 \frac{dx}{(1+x)(2+x)}$

Let $\frac{1}{(1+x)(2+x)} = \frac{A}{(1+x)} + \frac{B}{(2+x)}$

$1 = A(2 + x) + B(1 + x)$

we put $x = -2$ and get

$1 = A(2 + 2) + B(1 - 2) = B = -1$

we put $x = -1$ and get

$1 = A(2 - 1) + B(1 - 1) = A = 1$

$\int \frac{dx}{(1+x)(2+x)} = \int \frac{dx}{(1-x)} - \int \frac{dx}{(2-x)}$

$= \log(1 + x) - \log(2 + x)$

or $\log \frac{(1+x)}{(2+x)}$

Now $\int_0^1 \frac{dx}{(1+x)(2+x)}$

$= \left[\frac{\log(1+x)}{(2+x)}\right]_0^1$

$= \log \frac{2}{3} - \log \frac{1}{2}$

$= \log \frac{4}{3}$ $\left(\because \log m - \log n = \log \frac{m}{n}\right)$

[23] If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots + \frac{x^n}{n} + \ldots\ldots \infty$

then $\frac{dy}{dx} - y$ is equal to :

(a) 1 (b) -1

(c) 0 (d) None (1 mark)

Answer:

(c) $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots + \frac{x^n}{n} + \ldots\ldots \infty$

Differentiating w.r.t x

$$\frac{dy}{dx} = 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \ldots\ldots + \frac{n x^{n-1}}{n!} + \ldots\ldots \infty$$

$\frac{dy}{dx} = y$ (Since R.H.S. is same as y)

$\therefore \frac{dy}{dx} - y = 0$

2008 - FEBRUARY

[24] The slope of the tangent to the curve $y = \sqrt{4 - x^2}$ at the point, where the ordinate and the abscissa are equal, is :

(a) -1 (b) 1

(c) 0 (d) None (1 mark)

Answer:

(a) Since the ordinate and abscissa are equal.

$\therefore x = y$

$x = \sqrt{4 - x^2}$ (Squaring both sides)

$\rightarrow x^2 = 4 - x^2$

$\rightarrow x = \sqrt{2}$

Differentiating the given equation w.r.t x

$$\frac{dy}{dx} = \frac{1(-2x)}{2\sqrt{4-x^2}}$$

$$\frac{dy}{dx}\ (\text{at } x = \sqrt{2}) = \frac{-2\sqrt{2}}{2\sqrt{2}} = -1$$

[25] The value of $\int_2^3 f(5-x)dx - \int_2^3 f(x)dx$ is:

(a) 1 (b) 0
(c) – 1 (d) None (1 mark)

Answer:

(b) $\int_2^3 f(5-x)dx - \int_2^3 f(x)dx$

$$= \int_2^3 f(5-x)dx - \int_2^3 f(2+3-x)dx$$

$$\left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx\right)$$

$$= \int_2^3 f(5-x)dx - \int_2^3 f(5-x)\,dx$$

$$= 0$$

[26] $\int \frac{e^{\log e^x}}{x} dx$ is :

(a) $x^{-1} + C$ (b) $x + C$
(c) $x^2 + C$ (d) None (1 mark)

Answer:

(b) $\int \frac{e^{\log e^x}}{x} dx$

Let $\log_e^x = t$

Differentiate w.r.t x

$\frac{1}{x} dx = dt$

Substituting $\frac{dx}{x}$ is the given integral

$\int e^t dt$

$= e^t$

$= e \log_e^x$

$= x + c$ [$\because \infty e \log_e^x - = x$]

2008 - JUNE

[27] Differentiate $e^{(x^x)}$:

(a) $(1 + \log x)$ (b) $x^x (1 + \log x)$

(c) $e^{x^x} (1 + \log x)x^x$ (d) $e^{x^x} (1 + \log x)$ (1 mark)

Answer:

(c) $y = e^{(x^x)}$

Taking log on both sides.

$\log y = x^x . \log e$

$\log y = (\because \log e = 1)$

Differentiating w.r.t on both sides.

$\frac{1}{y} \frac{dy}{dx} = \frac{d}{dx} (x^x)$ ..... (1)

Differentiating x^x Separately

Let $Z = x^x$

$\log Z = x \log x$

$\frac{1}{Z} \frac{dz}{dx} = x \frac{1}{x} + \log x.1$

$\frac{dz}{dx} = z(1 + \log x)$

$= x^x (1 + \log x)$ .....(2)

Substituting (2) in (1)

$\therefore \frac{1}{y}\frac{dy}{dx} = x^x (1 + \log^x)$

$\frac{dy}{dx} = e^{(xx)}\, x^x (1 + \log x)$

[28] If $x^m y^n = (x + y)^{m+n}$, then find $\frac{dy}{dx}$:

(a) $\frac{x}{y}$ (b) $\frac{y}{x}$

(c) xy (d) None (1 mark)

Answer:

(b) $x^m y^n = (n + y)^{m+n}$

Taking log on both the sides

$m \log x + n \log y = (m + n)\log(x + y)$

Differentiating w.r.t. x

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{(n+y)}\left[1 + \frac{dy}{dx}\right]$$

$$\frac{dy}{dx}\left(\frac{n}{y} - \frac{m+n}{x+y}\right) = \frac{m+n}{n+y} - \frac{m}{x}$$

$$\frac{dy}{dx}\left(\frac{nx + ny - my - ny}{y(x+y)}\right) = \frac{mx + nx - mx - my}{x(x+y)}$$

$$\frac{dy}{dx} = \frac{y\,(nx - my)\,(x+y)}{x\,(nx - my)\,(x+y)}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

[29] Evaluate $\int \frac{1}{(x-1)(x-2)}\, dx$:

(a) $\log\left(\frac{x-2}{x-1}\right) + C$ (b) $\log[(x-2)(x-1)] + C$

(c) $\log\left(\frac{x-1}{x-2}\right) + C$ (d) None (1 mark)

Answer:

(a) $\int \frac{1}{(x-1)(x-2)}\,dx$

$$\frac{1}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$$

$1 = A(x-2) + B(x-1)$

$x = 2,\ 1 = B$

$x = 1,\ -1 = A$

$$\int \frac{1}{(x-1)(x-2)}\,dx = \frac{-1}{(x-1)} + \frac{1}{(x-2)}$$

$= -\log(x-1) + \log(x-2)$

$= \log\left(\frac{x-2}{x-1}\right) + c$

[30] $\int_1^4 (2x+5)dx$ and the value is :

(a) 10 (b) 3

(c) 30 (d) None (1 mark)

Answer:

(c) $\int_1^4 (2x+5)dx$

$= \left[\frac{2x^2}{2} + 5x\right]_1^4$

$= \left[x^2 + 5x\right]_1^4$

$= 36 - 6$

$= 30$

2008 - DECEMBER

[31] If $f(x) = a^x x^a$ then find $f'(x)$.

(a) $f(x)[a + \log a]$ (b) $f(x)\left[\frac{a}{x} - \log a\right]$

(c) $f(x)\left[\frac{a}{x} - \log a\right]$ (d) $f(x)[a + x\log a]$ (1 mark)

Answer:

(c) $f(x) = a^x, x^a$

Differentiating w.r.t. x

$$f'(x) = \frac{d}{dx}(a^x . x^a)$$

$$= a^x . \frac{d}{dx}(x^a) + x^a . \frac{d}{dx}(a^x)$$

$$= a^x . a\, x^{a-1} + x^a a^x \log a$$

$$= x^a . a^x \left[\frac{a}{x} + \log a\right]$$

$$= f(x) \left[\frac{a}{x} + \log a\right] \quad (\because f(x) = x^a a^x)$$

[32] $\int \frac{1}{x(x^5+1)} dx$

(a) $\log\left(\frac{x^5}{x^5-1}\right) + C$ (b) $\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right) + C$

(c) $\frac{1}{3}\log\left(\frac{x^5}{x^5+1}\right) + C$ (d) $\frac{1}{3}\log\left(\quad + C$ (1 mark)

Now, we have to use partial fractions.

$$\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{(t-1)}$$

$1 = A(t-1) + Bt$

When, $t = 0$, $1 = -A$, $\therefore A = -1$

$t = 1$, $1 = B$, $\therefore B = 1$

$$\therefore \int \frac{dt}{4t(t-1.)} = \frac{1}{5}\int\left[\frac{-1}{t} + \frac{1}{t-1}\right]dt$$

$$= \frac{1}{5}[-\log t + \log(t-1)]$$

$$= \frac{1}{5}\left[\log\left(\frac{t-1}{t}\right)\right] \ldots\ldots\ldots\ldots (2)$$

Substituting the value of t in (2)

$$= \frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right) + C$$

2009 - JUNE

[33] Find the value of $\int_{-3}^{3} x\sqrt{8-x^2}\, d_x$

(a) 1 (b) – 1

(c) 0 (d) None of these (1 mark)

Answer:

(c) $\int_{-3}^{3} x\sqrt{8-x^2}\, dx$

Let $8 - x^2 = t$ ..... (1)

Differentiating both sides w.r.t. x

$$0 - 2x = \frac{dt}{dx}$$

$$-2x\, dx = d\, t$$

$$x\, d\, x = \frac{-1}{2} dt$$

So, $\int_{-3}^{3} x\sqrt{8 - x^2}\, dx$

$= \frac{-1}{2} \int_{-3}^{3} \sqrt{t}\, dt$

$= \frac{-1}{2} \int_{-3}^{3} t^{1/2} dt$

$= \frac{-1}{2} \left[\frac{t^{1/2} + 1}{\frac{1}{2} + 1}\right]_{-3}^{3}$

$= \frac{-1}{2} \left[\frac{t^{3/2}}{3/2}\right]_{-3}^{3}$

$= \frac{-1}{2} \left[\frac{2t^{3/2}}{3}\right]_{-3}^{3}$

$= \frac{-1}{3} \left[t^{3/2}\right]_{-3}^{3}$

$= \frac{-1}{3} \left[(8 - x^2)^{3/2}\right]_{-3}^{3}$ [Using (1)]

$= \frac{-1}{3} [(8 - (+3)^2)^{3/2} - (8 (-3)^2)^{3/2}]$

$= \frac{-1}{3}[(8 - 9)^{3/2} - (8 - 9)^{3/2}]$

$= \frac{-1}{3} \times 0$

$= 0$

$\therefore \int_{-3}^{3} x\sqrt{8 - x^2}\, dx = 0$

[34] If $x^3 y^2 = (x - y)^5$. Find $\frac{dy}{dx}$ at (1, 2).

(a) – 7/9 (b) 7/9
(c) 9/7 (d) – 9/7 (1 mark)

Answer:

(a) $x^3 y^2 = (x - y)^5$

Differentiating both sides w.r.t. x.

$$x^3 \frac{d}{dx}(y^2) + y^2 \frac{d}{dx}(x^3) = \frac{d}{dx}(x - y)^5$$

$$x^3 . \frac{2y\,dy}{dx} + y^2 . 3x^2 = 5(x - y)^4 \frac{d}{x}(x - y)$$

$$2x^3 y \frac{dy}{dx} + 3x^2 y^2 = 5(x - y)^4 \left(1 - \frac{dy}{dx}\right)$$

$$2x^3 y \frac{dy}{dx} + 3x^2 y^2 = 5(x - y)^4 - 5(x - y)^4 \frac{dy}{dx}$$

$$2x^3 y \frac{dy}{dx} + 5(x - y)^4 \frac{dy}{dx} = 5(x - y)^4 - 3x^2 y^2$$

$$\frac{dy}{dx}(2x^3 y + 5(x - y)^4) = 5(x - y)^4 - 3x^2 y^2$$

$$\frac{dy}{dx} = \frac{5(x - y)^4 - 3x^2 y^2}{2x^3 y + 5(x - y)^4}$$

$$\therefore \left(\frac{dy}{dx}\right)_{(1,2)} = \frac{5(x - y)^4 - 3x^2 y^2}{2x^3 y + 5(x - y)^4}$$

$$= \frac{5 - 12}{4 + 5}$$

$$\left(\frac{dy}{dx}\right)_{(1,2)} = \frac{-7}{9}$$

[35] Evaluate $\int x c^x \, dx$

(a) $e^x(x + 1) + c$ (b) $e^x(x - 1) + c$
(c) $e^x + c$ (d) $x - e^x + c$ (1 mark)

Answer:

(b) $\int x.\, e^x\, dx$

Integrating by parts we have

$$\int x.\, e^x\, dx = x \int e^x\, dx - \int \left\{ \frac{d}{dx}(x) \int e^x\, dx \right\} dx$$

$$= xe^x - \int 1.e^x\, dx$$

$$= xe^x - e^x + C$$

$$= e^x (x - 1) + C$$

Therefore, $\int xe^x\, dx = e^x (x - 1) + C$

[36] Find $\int \frac{x^3}{(x^2+1)^3} dx$

(a) $1/4\ (x^2 + 1)^{-2} + 1/2\ (x^2 + 1)^{-1} + C$

(b) $1/4\ (x^2 + 1)^{-1} - 1/2\ (x^2 + 1) + c$

(c) $1/4\ (x^2 + 1)^{-2} - 1/2\ (x^2 + 1)^{-1} + c$

(d) None of these (1 mark)

Answer:

(c) $\int \frac{x^3}{(x^2+1)^3} dx$

Let $x^2 + 1 = t$ .....(1)

differentiating both sides w.r.t. x

$$2x + 0 = \frac{dt}{dx}$$

$$xdx = \frac{dt}{2}$$

$$= \int \frac{x^3}{(x^2+1)^3} dx$$

$$= \int \frac{x^2 x}{(x^2+1)^3} dx$$

$$= \frac{1}{2}\int \frac{t-1}{t^3} dt \qquad \text{[Using (1)]}$$

$$= \frac{1}{2}\int \left(\frac{t}{t^3} - \frac{1}{t^3}\right) dt$$

$$= \frac{1}{2}\int \left(\frac{t}{t^2} - \frac{1}{t^3}\right)dt$$

$$= \frac{1}{2}\left[\frac{t^{-2+1}}{-2+1} - \frac{t^{-3+1}}{-3+1}\right] + c$$

$$= \frac{1}{2}\left[\frac{t^{-1}}{-1} - \frac{t^{-2}}{-2}\right] + c$$

$$= \frac{1}{2}\left[\frac{-1}{t} + \frac{1}{2t^2}\right] + c$$

$$= \frac{1}{4t^2} - \frac{1}{2t} + C = \frac{1}{4(x^2+1)^2} - \frac{1}{2(x^2+1)} + C$$

$$= 1/4(x^2+1)^{-2} - 1/2\,(x^2+1)^{-1} + C$$

2009 - DECEMBER

[37] $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

(a) $2x^{1/2}\left(\frac{1}{3}x - 1\right)$ (b) $2x^{1/2}\left(\frac{1}{3}x + 1\right)$

(c) $2\left(\frac{1}{3}x + x^{1/2}\right)$ (d) None of these. (1 mark)

Answer:

(b) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

$$= \int \sqrt{x}\,dx + \int \frac{1}{\sqrt{x}}dx$$

$$= x^{1/2}\,dx + x^{-1/2}\,dx$$

$$= \frac{x^{1/2+1}}{1/2+1} + \frac{x^{-1/2+1}}{-1/2+1} + C$$

$$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C$$

$$= \frac{2}{3}x^{3/2} + 2x^{1/2} + C$$

$$= 2x^{1/2}\left(\frac{1}{3}x + 1\right)$$

[38] $\int_0^1 \left(\frac{1-x}{1+x}\right) dx$

(a) $2 \log 2 - 1$ (b) $4 \log 2 - 1$

(c) $2 \log 2$ (d) None of these (1 mark)

Answer:

(a) $= \int_0^1 \frac{1-x}{1+x} dx$

$$= \int_0^1 \frac{2-1-x}{1+x} dx$$

$$= \int_0^1 \frac{2-(1+x)}{1+x} dx$$

$$= \int_0^1 \left(\frac{2}{1+x} - 1\right) dx$$

$$= [2 \log 1 + x - x]_0^1$$

$$= (2 \log 2 - 1) - (2 \log 1 - 0)$$

$$= (2 \log 2 - 1) - (0 - 0)$$

$$= 2 \log 2 - 1$$

[39] $x = 2t + 5$ and $y = t^2 - 5$, then $\frac{dy}{dx} = ?$

(a) t (b) $-1/t$

(c) $1/t$ (d) 0 (1 mark)

Answer:

(a) $x = 2t + 5$

differentiating both sides w.r.t. t.

$\frac{dx}{dt} = 2$.....(1)

$y = t^2 - 5$

differentiating both sides w.r.t. t.

$\frac{dy}{dt} = 2t$ .....(2)

Dividing (2) by (1) :

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{2}$$

$\therefore \frac{dy}{dx} = t.$

[40] $x = at^2$ $y = 2$ at, $\frac{dy}{dx} = ?$

(a) 1/t (b) – 1/t

(c) t (d) None of the above (1 mark)

Answer:

(a) $x = at^2$

differentiating both sides w.r.t. t.

$\frac{dx}{dt} = 2at$.....(1)

$y = 2at$

differentiating both sides w.r.t.t.

$\frac{dy}{dt} = 2a$.....(2)

Dividing (2) by (1)

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at}$$

$$\therefore \frac{dy}{dx} = \frac{1}{t}$$

[41] Find the second derivative of $y = \sqrt{x+1}$

(a) $\frac{1}{2}(x+1)^{-\frac{1}{2}}$ (b) $-1/4\,(x+1)^{-3/2}$

(c) $1/4\,(x+1)^{-\frac{1}{2}}$ (d) None of these. (1 mark)

Answer:

(b) $y = \sqrt{x+1}$

differentiating both sides w.r.t.x.

$$\frac{dy}{dx} = \frac{1}{2}(x+1)^{\frac{1}{2}-1} \times 1 \quad \left[\because \frac{dx^n}{dx} = nx^{n-1}\right]$$

$$\frac{dy}{dx} = \frac{1}{2}(x+1)^{-\frac{1}{2}}$$

Again, differentiating both sides w.r.t. x

$$\frac{d^2y}{dx^2} = \frac{1}{2}\left[-\frac{1}{2}(x+1)^{-\frac{1}{2}-1}\right]$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}(x+1)^{-3/2}$$

2010 - JUNE

[42] Equal to

(a) $\int \frac{dx}{\sqrt{3x+4}-\sqrt{3x+1}}$ $\frac{2}{27}\left[(3x+4)^{3/2}-(3x+1)^{3/2}\right]+c$

(b) $\frac{2}{27}\left[(3x+4)^{3/2}+(3x+1)^{3/2}\right]+c$

(c) $\frac{2}{3}\left[(3x+4)^{3/2}-(3x+1)^{3/2}\right]+c$

(d) None of these. (1 mark)

Answer:

(b) $\int \frac{dx}{\sqrt{3x+4}-\sqrt{3x+1}}$

$= \int \frac{dx}{\sqrt{3x+4}-\sqrt{3x+1}} \times \frac{\sqrt{3x+4}+\sqrt{3x+1}}{\sqrt{3x+4}+\sqrt{3x+1}}$

$= \int \frac{\sqrt{3x+4}+\sqrt{3x+1}\,dx}{3x+4-3x-1}$

$= \frac{1}{3}\int \sqrt{3x+4}\,dx + \frac{1}{3}\int \sqrt{3x+1}\,dx$

$= \frac{1}{3}\cdot\frac{(3x+4)^{3/2}}{3/2}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{(3x+1)^{3/2}}{3/2}\cdot\frac{1}{3}$

$= \frac{2(3x+4)^{3/2}}{27} + \frac{2(3x+1)^{3/2}}{27}$

$= \frac{2}{27}\left[(3x+4)^{3/2}+(3x+1)^{3/2}\right]$

[43] $\int_1^2 \frac{x\,dx}{x^2+2} =$ _____

(a) $\log \sqrt{2}$ (b) $\log \sqrt{3}$

(c) $\log \frac{1}{\sqrt{2}}$ (d) $\log \frac{1}{\sqrt{3}}$ (1 mark)

Answer:

(a) $\int_1^2 \frac{x\,dx}{x^2+2}$

Let $x^2 + 2 = t$

$\Rightarrow$ Differentiating both sides w.r.t. x

$2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$\frac{1}{2}\int_3^6 \frac{dt}{t}$ when, x = 1, t = 3

x = 2, t = 6

$= \left[\frac{\log|t|}{2}\right]_3^6$

$= \frac{1}{2}[\log 6 - \log 3]$

$= \frac{1}{2}\log\frac{6}{3}$

$= \frac{1}{2}\log 2$

$= \log\sqrt{2}$

[44] If $x^2 + y^2 = 4$ then

(a) $y\frac{d^2y}{dx^2} - \left(2\frac{dy}{dx}\right)^2 + 1 = 0$ (b) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0$

(c) $y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 - 1 = 0$ (d) $y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 1 = 0$ (1 mark)

Answer:

(b) $x^2 + y^2 = 4$

→ Differentiating both sides w. r. t. x

$2x + 2y\frac{dy}{dx} = 0$

$x + y\frac{dy}{dx} = 0$

$1 + y\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{dy}{dx} = 0$

$\therefore y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0$

[45] If a_1, a_2, a_3 represents first, second and third terms of an AP respectively, the first term is 2 and $(a_1 + a_2)a_3$ is minimum, then the common difference is equal to

(a) 5/2 (b) – 5/2
(c) 2/5 (d) – 2/5 (1 mark)

Answer:

(b) Given, $a_1 = 2$

$\because$ $a_2 = 2 + d$

$a_3 = 2 + 2d$

Consider a function, $f(d) = (a_1 + a_2)\, a_3$

$f(d) = [2 + (2 + d)]\,(2 + 2d)$

Solving, we get

$f(d) = 2d^2 + 10d + 8$

Differentiating the above equation

$f(d) = 4d + 10$ $(\therefore (a_1 + a_2)\, a_3$ is minimum)

$\therefore$ $d = -5/2$

Hence, the common difference $d = -5/2$

2010 - DECEMBER

[46] The cost function for the production of x units of a commodity is given by

$C(x) = 2x^3 - 15x^2 + 36x + 15$

The cost will be minimum when 'x' is equal to

(a) 3 (b) 2
(c) 1 (d) 4 (1 mark)

Answer:

(a) $C(x) = 2x^3 - 15x^2 + 36x + 15$

$C'(x) = 6x^2 - 30x + 36$

$= x^2 - 5x + 6$

$= x^2 - 3x - 2x + 6$

$= x(x - 3) - 2(x - 3)$

$= (x - 2)(x - 3)$

$x = 2, 3$

$C'(x) = 2x - 5$

Put $x = 2$

$= 2 \times 2 - 5 = -1$ (–)ve

⇒ Maxima Put $x = 3$

$6 - 5 = 1$ (+)ve

⇒ Minima

The cost will be minimum when $x = 3$

[47] $\int \frac{6x + 4}{(x - 2)(x - 3)} dx$ is equal to

(a) 22 log (x-3) -16 (x-2) (b) 11 log (x-3) – 8 (x-2)

(c) 22 log (x-3) -16 log (x-2) (d) 22 log (x-3) + 16 log (x-2)

(1 mark)

Answer:

(c) $\int \frac{6x + 4}{(x - 2)(x - 3)} dx$

or, $\frac{6x + 4}{(x - 2)(x - 3)} = \frac{A}{(x - 2)} + \frac{B}{(x - 3)}$ (Partial fractions)

$6x + 4 = A(x - 3) + B(x - 2)$

Put $x = 3$

We get $B = 22$ Put $x = 2$

We get $A = -16$

$\int \frac{-16}{x - 2} + \int \frac{22}{x - 3}$

$= -16 \log(x - 2) + 22 \log(x - 3)$

$= 22 \log(x - 3) - 16 \log(x - 2)$

[48] $\int \frac{1}{x(1 + \log x)^2} dx$ is equal to

(a) $-\frac{1}{2(1 + \log x)^2} + c$ (b) $\frac{1}{(1 + \log x)} + c$

(c) $-\frac{1}{(1 + \log x)} + c$ (d) None of these (1 mark)

Answer:

(c) $\int \frac{1}{x(1+\log x)^2} dx$

Let $1 + \log x = t$

$\frac{1}{x} dx = dt$

$\int \frac{dt}{t^2} = -\frac{1}{t} + c$

$= -\frac{-1}{(1+\log x)} + c$

2011 - JUN E

[49] Solve : $\int_{-1}^{1} (e^x - e^{-x}) dx$

(a) 0 (b) 1

(c) 12 (d) None of the above. (1 mark)

Answer:

(a) $\int_{-1}^{1} (e^x - e^{-x}) dx = 0$

because, given function $(e^x - e^{-x})$ is a ODD function

Say, $f(x) = e^x - e^{-x}$

then $f(-x) = -(e^x - e^{-x}) = -f(x)$

So, the given function is "ODD"

[50] Solve : $\int \frac{(\log x^n)^2}{x^3} .dx$

(a) $\frac{3}{2}(\log x)^3 + C$ (b) $\frac{1}{3}(\log x)^3 + C$

(c) $\frac{1}{6}(\log x)^3 + C$ (d) $\frac{3}{7}(\log x)^3 + C$ (1 mark)

Answer:

(b) $\int \frac{(\log x^2)^2}{x^3}\,dx = I$ (say)

$\therefore I = \int \frac{(x \log x)^2}{x^3}\,dx$

$= \int \frac{(\log x)^2}{x}\,dx$

Let $\log x = t$

$\therefore \frac{1}{x}\,dx = dt$

$\therefore I = \int t^2\,dt$

$= \frac{t^3}{3} + C = \frac{(\log x)^3}{3} + C$

[51] If $f(x) = {}^xC_3$; then $f'(1) = ?$

(a) $\frac{1}{6}$ (b) $\frac{-1}{6}$

(c) $\frac{5}{6}$ (d) $\frac{-5}{6}$ (1 mark)

Answer:

(b) Given $f(x) = {}^xC_3$

$= \frac{\lfloor x}{\lfloor 3 \lfloor x-3}$

$\therefore f(x) = \frac{(x^3 - 3x^2 + 2x)}{6}$

Differentiating w.r.t (x) both the sides, we get

$f'(x) = \frac{1}{6}(3x^2 - 6x + 2)$

$\therefore f'(1) = \frac{1}{6}(3 \times 1^2 - 6 \times 1 + 2)$

$\therefore f'(1) = \frac{-1}{6}$

[52] Given, $y = \int (e^{a \log x} + e^{x \log a}) dx$; then $\frac{dy}{dx}$

(a) $x^a a^x$ (b) $x^a + a^x$

(c) $ax^{x-1} + a^x \log a$ (d) None of the above. (1 mark)

Answer:

(b) Since, $e^{\log_e x^a} + e^{\log_a a^x} = x^a + a^x$ (1)

$\therefore Y = \int (e^{a \log x} + e^{x \log a})\, dx$

or $Y = \int (x^a + a^x)\, dx$ (using (1)

$\therefore Y = \frac{x^{a+1}}{(a+1)} + \frac{a^x}{\log a} + C$

On differentiating w.r.t (x) both the sides, we get

$\frac{dy}{dx} = x^a + a^x$

[53] If $f'(x) = 3x^2 - \frac{2}{x^3}$, $f(1) = 0$ and $f(x) =$ _____.

(a) $\frac{x^3}{3} - x^{-2} - 2$ (b) $x^3 + x^2 + 2$

(c) $x^3 + x^{-2} - 2$ (d) None of these. (1 mark)

Answer:

(c) Given : $f'(x) = 3x^2 - \frac{2}{x^3}$

We know, $f(x) = \int f(x)\, dx = \int (3x^2 - \frac{2}{x^3}) dx$

$\therefore f(x) = x^3 + x^{-2} + c \quad - (i)$

given $f(1) = 0$

$\Rightarrow f(1) = (1)^3 + (1)^{-2} + c$

$0 = 1 + 1 + c$

$\Rightarrow c = -2$

$\therefore$ by (1)

$f(x) = x^3 + x^{-2} - 2$

2011 - DECEMBER

[54] $\int_{-1}^{1} \frac{|x|}{x} dx =$ _____

(a) -1 (b) 0
(c) 1 (d) 2 (1 mark)

Answer:

(b) $\int_{-1}^{1} \frac{|x|}{x} dx$

$f(x) = \frac{|x|}{-x}$

$f(-x) = \frac{|-x|}{-x} = \frac{|x|}{x} = \frac{-|x|}{x}$

$f(-x) = -f(x)$

So it is odd function

$\int_{-1}^{1} \frac{|x|}{x} dx = 0$

[55] $\frac{d}{dx}\left[2^{\log_2 x}\right] =$ _____

(a) 1 (b) 0
(c) 1/2 (d) $2^x.\log_2 x$ (1 mark)

Answer:

(a) $\frac{d}{dx} 2^{\log_2 x} = \frac{d}{dx}[x] \left[\because e^{\log_e x} = x\right]$

[56] $\int \frac{e^x}{(1+x)^3} dx - \int \frac{e^x}{2(1+x)^2} dx =$ _____

(a) 0 (b) $\frac{e^x}{2(1+x)^2} + C$

(c) $-\frac{e^x}{2(1+x)^2} + C$ (d) $\frac{e^x}{(1+x)^2} + C$ (1 mark)

Answer:

(c) $\int \frac{e^x}{(1+x)^3}dx - \int \frac{e^x}{2(1+x)^2}dx$

$= \int \left\{\frac{e^x}{(1+x)^3} - \frac{e^x}{2(1+x)^2}\right\}dx$

$= \int e^x \left\{\frac{1}{(1+x)^3} - \frac{1}{2(1+x)^2}\right\}dx$

$= \int e^x \left(-\frac{1}{2}\right)\left\{\frac{1}{(1+x)^2} - \frac{2}{(1+x)^3}\right\}dx$

$= \frac{-1}{2}\int e^x \left\{\frac{1}{(1+x)^2} - \frac{2}{(1+x)^3}\right\}dx$

$= \frac{-1}{2} e^x \frac{1}{(1+x)^2} + c \left[\because \left[\int e^x \{f(x) + f'(x)\} = e^x f(x) + e\right]\right.$

[57] If $Y = X^X$ then $\frac{d^2Y}{dx^2} =$ _____

(a) $\frac{dY}{dx}(1+\log x) + Y\frac{d}{dx}(1+\log x)$

(b) $\frac{dY}{dx}(1+\log x) + \frac{d}{dx}(1+\log x)$

(c) $\frac{dY}{dx}(1+\log x) - Y\frac{d}{dx}(1+\log x)$

(d) $\frac{dY}{dx}(1+\log x) - \frac{d}{dx}(1+\log x)$ (1 mark)

Answer:

(a) If $y = x^x$

taking log on both side

$\log y = \log x^x$

$\log y = x \log x$

Diff w.r.t (x)

$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{x} + \log x.1$

$\frac{1}{}\frac{dy}{} = 1 + \log x$

$$g^1(x) = \frac{x}{\sqrt{25 - x^2}}$$

$$g^1(1) = \frac{1}{\sqrt{25 - 1}}$$

$$= \frac{1}{\sqrt{24}}$$

[59] If $x = c\,t$, $y = c/t$, then $\frac{dy}{dx}$ is equal to:

(a) 1/t
(b) $t.e^t$
(c) -1/t2
(d) None of these. (1 mark)

Answer:

(c) Given $x = ct$ and $y = \frac{c}{t}$

$$\frac{dx}{dt} = \frac{d}{dt}(t) \qquad \frac{dy}{dt} = \frac{d}{dt} ct^{-1}$$

$$= c\,\frac{d}{dt}(t) \qquad = c.\,(-1)\,t^{-2}$$

$$= c \qquad = \frac{-c}{t^2}$$

$$\frac{dy}{dt} = \frac{dy/dt}{dx/dt} \qquad = \frac{-c/t^2}{c}$$

$$= \frac{-c}{t^2.c}$$

$$= -\frac{1}{t^2}$$

[60] $\int_0^1 \frac{dx}{[ax + b(1 - x)]^2} =$ _____

(a) a/b
(b) b/a
(c) ab
(d) 1/ab (1 mark)

Answer:

(d) $\int^1$

$$dt = [a + b(-1)]\,dx$$
$$dt = (a - b)\,dx$$
$$\text{or } dx = \frac{dt}{(a-b)}$$
$$\therefore \text{Say } I = \int_0^1 \frac{dx}{[ax + b(1-x)]^2}$$
$$= \int_b^a \frac{dt}{(a-b) \times t^2}$$
$$= \frac{1}{(a-b)} \int_b^a t^{-2} dt$$
$$= \frac{1}{(a-b)} \left[\frac{t^{-2+1}}{-2+1}\right]_b^a$$
$$= -\frac{1}{(a-b)} [t^{-1}]_b^a$$
$$= -\frac{1}{(a-b)} \left[\frac{1}{a} - \frac{1}{b}\right]$$
$$= \frac{1}{ab}$$

[61] If $y = e^{a \log x} + e^{x \log a}$, then $\frac{dy}{dx} =$

(a) $x^a + a^x$ (b) $a\,x^{a-1} + a^x \log a$
(c) $a\,x^{a-1} + x\,a^{x-1}$ (d) $x^x + a^a$ (1 mark)

Answer:

(b) $y = e^{a \log x} + e^{x \log a}$

$\Rightarrow y = e^{\log x^a} + e^{\log a^x}$

$\Rightarrow y = x^a + a^x$

$[\because e^{\log m} = m]$

Diff w.r.t (x) both the sides we get

$$\frac{dy}{dx} = a\,x^{a-1} + a^x \log a$$

2012 - DECEMBER

[62] $\int 2^{3x}.\ 3^{2x}.\ 5^{x}.\ dx =$ _____

(a) $\frac{2^{3x}.\ 3^{2x}.\ 5^{x}}{\log (720)} + c$ (b) $\frac{2^{3x}.\ 3^{2x}.\ 5^{x}}{\log (360)} + c$

(c) $\frac{2^{3x}.\ 3^{2x}.\ 5^{x}}{\log (180)} + c$ (d) $\frac{2^{3x}.\ 3^{2x}.\ 5^{x}}{\log (90)} + c$ (1 mark)

Answer:

(b) $\int 2^{3x}.\ 3^{2x}.\ 5^{x}\ dx$

$= \int 8^{x}.\ 9^{x}.\ 5^{x}\ dx$

$= \int (8.\ 9.\ 5)^{x}\ dx$

$= \int (360)^{x}\ dx$

$= \frac{(360)^{x}}{\log 360} + c$

$= \frac{2^{3x}.\ 3^{2x}.\ 5^{x}}{\log 360} + c$

[63] For the functions $y = x^3 - 3x$, the value of $\frac{d^2y}{dx^2}$ at which $\frac{dy}{dx}$ is zero, is

(a) ± 1 (b) ± 3

(c) ± 6 (d) None of these. (1 mark)

Answer:

(c) Given

$y = x^3 - 3x$

Diff w.r.t. 'x'

$\frac{dy}{dx} = 3x^2 - 3$ _____ (1)

$0 = 3(x^2 - 1)$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

Diff (1) w.r.t. 'x'

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2 - 3)$$

$$\frac{d^2y}{dx^2} = 6x$$

$$\left(\frac{d^2y}{dx^2}\right)_{(x = \pm 1)} = 6(\pm 1) = \pm 6$$

[64] The equation of the tangent to the curve, $f = x^3 - 2x + 3$, at the point (2, 7) is –

(a) $y = 2x - 13$ (b) $y = 10x$

(c) $y = 10x - 13$ (d) $y = 10$ (1 mark)

Answer:

(c) Given curve

$f(x) = x^3 - 2x + 3$

i.e. $y = x^3 - 2x + 3$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 2x + 3)$$

$$\frac{dy}{dx} = 3x^2 - 2$$

$$\left(\frac{dy}{dx}\right)_{(2,7)} = 3(2)^2 - 2$$

$$= 12 - 2$$

$$\left(\frac{dy}{dx}\right)_{(2,7)} = 10$$

Slope of tangent $m = \left(\frac{dy}{dx}\right)_{(2,7)} = 10$

The equation of tangent at (2,7)

$y - y_1 = m(x - x_1)$

$y - 7 = 10(x - 2)$

$y - 7 = 10x - 20$

$y = 10x - 20 + 7$

$y = 10x - 13$

[65] If $y = \log\left(\frac{5 - 4x^2}{3 + 5x^2}\right)$, then $\frac{dy}{dx} =$ ___

(a) $\frac{8}{4x - 5} - \frac{10}{3 + 5x}$ (b) $(4x^2 - 5) - (3 + 5x^2)$

(c) $\frac{8x}{4x^2 - 5} - \frac{10x}{3 + 5x^2}$ (d) $8x - 10$

Answer:

(c) Given,

$$y = \log\left(\frac{5 - 4x^2}{3 + 5x^2}\right)$$

$$y = \log(5 - 4x^2) - \log(3 + 5x^2)$$

Diff w.r.t. 'x',

$$\frac{dy}{dx} = \frac{d}{dx}\log(5 - 4x^2) - \frac{d}{dx}\log(3 + 5x^2)$$

$$= \frac{1}{(5 - 4x^2)}\frac{d}{dx}(5 - 4x^2) - \frac{1}{(3 + 5x^2)}\frac{d}{dx}(3 \times 5x^2)$$

$$= \frac{-8x}{5 - 4x^2}\frac{1}{(5 - 4x^2)}(0 - 8x) - \frac{1}{(3 + 5x^2)}(0 + 10x)$$

$$= -\frac{10x}{(3 + 5x^2)}$$

$$= \frac{8x}{4x^2 - 5} - \frac{10x}{3 + 5x^2}$$

2011 - JUNE

[66] If $y = \log_y x$, then $\frac{dy}{dx}$ is equal to:

(a) $\frac{1}{x + \log y}$ (b) $\frac{1}{x + x\log y}$

(c) $\frac{1}{1 + x\log y}$ (d) $\frac{1}{y + \log x}$ (1 mark)

Answer:

(b) If $y = \log_y x$

$$y = \frac{\log x}{\log y}$$

$$y \log y = \log x$$

Diff w.r.t. x

$$\frac{1}{y}\frac{dy}{dx} + \log y \cdot \frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx}(1 + \log y) = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x(1 + \log y)} = \frac{1}{(x + x \log y)}$$

[67] $\int_1^2 \frac{(\log_e (ex))^n}{x} dx\ (n + -1)$ is equal to:

(a) $\left[\frac{(\log_e (2e))^{n+1} - 1}{n+1}\right]$ (b) $[(\log_e (2e))^{(n+1)} + 1]$

(c) $\frac{(\log_e (2e))^{n+1}}{n+1} - \frac{(\log_e 2)^{n+1}}{n+1}$ (d) None of these (1 mark)

Answer:

(a) $\int_1^2 \frac{(\log_e ex)^n}{x} dx$

$= \int_1^2 \frac{(\log_e e + \log_e x)^n}{x} dx$

$= \int_1^2 \frac{(1 + \log_e x)^n}{x}$

let $1 + \log_e x = t$

$dx \quad \frac{1}{x} dx = dt$

x	1	2
t	1	1 + log e

$= \int_1^{1+\log_e 2} t^n dt$

$= \left[\frac{t^{n+1}}{n+1}\right]_1^{(1+\log_e 2)}$

$= \frac{1}{[n+1]}$ $^{(1+\log_e 2)}$

$$= \frac{1}{(n+1)}\left[(1+\log_e 2)^{n+1} - (1)^{n+1}\right]$$

$$= \frac{1}{(n+1)}\left[(\log_e e + \log_e 2)^{n+1} - 1\right]$$

$$= \frac{(\log_e 2e)^{n+1} - 1}{(n+1)}$$

[68] If $x = \log t$, $y = e^t$, then $\frac{dy}{dx} =$

(a) 1/t (b) $t.e^t$

(c) – 1/t2 (d) None of these (1 mark)

Answer:

(b) If $x = \log t$, diff w.r.t 't'

$$\frac{dx}{dt} = \frac{d}{dt}\log t = \frac{1}{t}$$

and $y = e^t$

$$\frac{dy}{dt} = \frac{d}{dt}(e^t) = e^t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^t}{1/t} = te^t$$

[69] $\int 2^{3x}.3^{2x}.5^x\,dx =$ _____

(a) $\frac{2^{3x}.3^{2x}.5^x}{\log(270)} + C$ (b) $\frac{2^{3x}.3^{2x}.5^x}{\log(360)} + C$

(c) $\frac{2^{3x}.3^{2x}.5^x}{\log(180)} + C$ (d) $\frac{2^{3x}.3^{2x}.5^x}{\log(90)} + C$ (1 mark)

Answer:

(b) $\int 2^{3x}.3^{2x}.5^x\,dx$

$$= \int (2^3)^x.(3^2)^x.5^x\,dx$$

$$= \int (8^x.9^x.5^x)\,dx$$

$$= \int (8.9.5)^x\,dx$$

$$= \int (360)^x\,dx$$

$$= \frac{(360)^x}{\log 360} + c$$

$$= \frac{2^{3x} . 3^{2x} . 5^x}{\log 360} + c$$

2013 - DECEMBER

[70] The points on the curve $y = x^3 - x^2 - x + 1$, where the tangent is parallel to x - axis are

(a) ( ) and (1, 0) (b) (0, 0) and (1, 0)

$$= \frac{-1}{27} - \frac{1}{9} + \frac{1}{3} + 1$$

$$= \frac{-1-3+9+27}{27}$$

$$y = \frac{32}{27}$$

Points are (-1/3, 32/27) and (1,0)

[71] $\int (a)^{2x}\,dx$ _____

(a) $\frac{a^{2x}}{2\log a}$ (b) $\frac{2.a^{2x}}{\log a}$

(c) $\frac{a^{2x}.\log a}{2}$ (d) None of these (1 mark)

Answer:

(a) $\int a^{2x}\,dx = \frac{a^{2x}}{2\log a}$ $\left[\because \int a^{kx}\,dx = \frac{a^{kx}}{k\log a}\right]$

2014 - JUNE

[72] $\int_0^5 \frac{x^2\,dx}{x^2+(5-x)^2}$ is equal to _____ .

(a) 5 (b) $\frac{5}{2}$

(c) 1 (d) None of these (1 mark)

Answer:

(b) $I = \int_0^5 \frac{x^2}{x^2+(5-x)^2}\,dx$ ..... (1)

$$I = \int_0^5 \frac{(5+0-x)^2}{(5+0-x)^2+(5-5-0+x)^2}\,dx$$

$\left[\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$

$$I = \int_0^5 \frac{(5-x)^2}{(5-x)^2 + x^2} dx \quad \text{..... (2)}$$

Adding (1) & (2)

$$2\ell = \int_0^5 \frac{x^2 + (5-x)^2}{x^2 + (5-x)^2} dx$$

$$= \int_0^5 dx$$

$$2 =$$

2014 - DECEMBER

[74] The value of definite integral $\int_0^2 |1 - x|\, dx =$ _____

(a) 0 (b) 1/2
(c) 3/2 (d) 1 (1 mark)

Answer:

(d) $\int_0^2 |1-x|dx = \int_0^1 |1-x|\, dx + \int_1^2 |1-x|\, dx$

$= \int_0^1 + (1-x)\, dx + \int_1^2 - (1-x)\, dx$

$= + \int_0^1 (1-x)\, dx - \int_1^2 (1-x)\, dx$

$= + \left[x - \frac{x^2}{2}\right]_0^1 - \left[x - \frac{x^2}{2}\right]_1^2$

$= + \left[\left\{1 - \frac{(1)^2}{2}\right\} - \left\{0 - \frac{0^2}{2}\right\}\right] - \left[\left\{2 - \frac{2^2}{2}\right\} - \left\{1 - \frac{1^2}{2}\right\}\right]$

$= + \left[\left(1 - \frac{1}{2}\right) - 0\right] - \left[(2-2) - \left(1 - \frac{1}{2}\right)\right]$

$= + \left[\frac{1}{2}\right] - \left[0 - 1 + \frac{1}{2}\right]$

$= + \frac{1}{2} + 1 - \frac{1}{2}$

$= 1$

[75] If $y = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \ldots\ldots\ldots\ldots + \frac{x^n}{\lfloor n} + \ldots\ldots\ldots\ldots$, then the value of $\frac{dy}{dx} - y =$ _____

(a) 1 (b) 0
(c) 2 (d) – 1 (1 mark)

Answer:

(b) $y = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \ldots\ldots\ldots\ldots + \frac{x^n}{\lfloor n} + \ldots\ldots\ldots\ldots \infty$

$\frac{dy}{dx} = 0 + \frac{1}{\lfloor 1} + \frac{2x}{\lfloor 2} + \ldots\ldots\ldots\ldots + \frac{nx^{n-1}}{\lfloor n} + \ldots\ldots\ldots\ldots \infty$

$\frac{dy}{dx} = 1 + \frac{2x}{2\lfloor 1} + \ldots\ldots\ldots\ldots \frac{nx^{n-1}}{n\lfloor n-1} + \ldots\ldots\ldots\ldots \infty$

$\frac{dy}{dx} = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \ldots\ldots\ldots\ldots \infty$

$\frac{dy}{dx} = y$

$\boxed{\frac{dy}{dx} - y = 0}$

2015 - JUNE

[76] The value of $\int_0^{1/2} \frac{dx}{\sqrt{3-2x}}$ is

(a) 1 (b) $1 - \sqrt{3/2}$

(c) $\sqrt{3} - \sqrt{2}$ (d) $\sqrt{2} - \sqrt{3}$ (1 mark)

Answer:

(c) $\int_0^{1/2} \frac{dx}{\sqrt{3-2x}}$

$= \int_0^{1/2} (3 - 2x)^{-1/2}\, dx$

$= \left[\frac{(3-2x)^{\frac{-1}{2}+1}}{\left(-\frac{1}{2}+1\right)} \left(\frac{1}{-2}\right) \right]_0^{1/2}$

$= \left[\frac{(3-2x)^{1/2}}{\frac{1}{2}} \left(\frac{1}{-2}\right) \right]_0^{1/2}$

$$= -\left[\sqrt{3-2x}\right]_0^{1/2}$$

$$= -\left[\sqrt{3-\cancel{2}\times\frac{1}{\cancel{2}}} - \sqrt{3-2\times 0}\right]$$

$$= -\left[\sqrt{2} - \sqrt{3}\right]$$

$$= (\sqrt{3} - \sqrt{2})$$

[77] The value of $\int_0^2 x e^{x^2} dx$ is

(a) 1 (b) e - 1

(c) (e/2) - 1 (d) $\frac{1}{2}(e^4 - 1)$ (1 mark)

Answer:

(d) $\int_0^2 x e^{x^2} dx$

Let $x^2 = t$

$2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$$= \int_0^2 e^{x^2} \times x\, dx$$

$$= \int_0^4 e^t \times \frac{dt}{2} \qquad (t = x^2 = 2^2 = 4)$$

$$= \frac{1}{2}\int_0^4 e^t dt$$

$$= \frac{1}{2}\left[e^t\right]_0^4 = \frac{1}{2}\left[e^4 - e^0\right] = \frac{1}{2}\left[e^4 - 1\right]$$

[78] If $x^p y^q = (x + y)^{p+q}$, then $\frac{dy}{dx}$ is equal to _____

(a) $\frac{q}{p}$ (b) $\frac{x}{y}$

(c) $\frac{y}{x}$ (d) $\frac{p}{q}$ (1 mark)

Answer:

(c) Given:

$$x^p y^q = (x + y)^{p+q}$$

$$\log (x^p y^q) = \log (x + y)^{p+q}$$

$$\log x^p + \log y^q = (p + q) \log (x + y)$$

$$p \log x + q \log y = (p + q) \log (x + y)$$

Diff. w.r.t 'x'

$$P \times \frac{1}{x} + q \times \frac{1}{y} \frac{dy}{dx} = (p + q) \frac{1}{(x + y)} \frac{d}{dx}(x + y)$$

$$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{(p + q)}{(x + y)}\left[1 + \frac{dy}{dx}\right]$$

$$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p + q}{x + y} + \left[\frac{p + q}{x + y}\right]\frac{dy}{dx}$$

$$\frac{q}{y}\frac{dy}{dx} - \left(\frac{p + q}{x + y}\right)\frac{dy}{dx} = \frac{p + q}{x + y} - \frac{p}{x}$$

$$\frac{dy}{dx}\left[\frac{q}{y} - \frac{(p + q)}{(x + y)}\right] = \frac{(p + q)x - p(x + y)}{(x + y)x}$$

$$\frac{dy}{dx}\left[\frac{q(x + y) - y(p + q)}{y(x + y)}\right] = \frac{\cancel{px} + qx - \cancel{px} - py}{(x + y)x}$$

$$\frac{dy}{dx}\left[\frac{qx + \cancel{qy} - py - \cancel{qy}}{y(x + y)}\right] = \frac{(qx - py)}{(x + y)x}$$

$$\frac{dy}{dx}\frac{(qx - py)}{y} = \frac{(qx - py)}{x}$$

$$\frac{dy}{dx} = \frac{y}{x}\frac{(qx - \cancel{py})}{\cancel{(qx - py)}}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

[79] If $e^{xy} - 4xy = 4$ then $\frac{dy}{dx} =$ _____

(a) $\frac{y}{x}$ (b) $\frac{-y}{x}$

(c) $\frac{x}{y}$ (d) $\frac{-x}{y}$ (1 mark)

Answer:

(b) $e^{xy} - 4xy = 4$

Diff. w.r.t 'x' on both side

$$\frac{d}{dx}(e^{xy} - 4xy) = \frac{d}{dx}(4)$$

$$\frac{d}{dx}e^{xy} - \frac{d}{dx}(4xy) = 0$$

$$e^{xy}\frac{d}{dx}(xy) - 4\frac{d}{dx}(xy) = 0$$

$$\frac{d}{dx}(xy)[e^{xy} - 4] = 0$$

$$e^{xy} - 4 = 0$$

$$e^{xy} = 4$$

Diff. w.r.t 'x'

$$\frac{d}{dx}e^{xy} = \frac{d}{dx}(4)$$

$$e^{xy} \times \frac{d}{dx}(xy) = 0$$

$$e^{xy} \times [x \times \frac{dy}{dx} + y.1] = 0$$

$$x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

2015 - DECEMBER

[80] If $u = 3t^4 + 5t^3 + 2t^2 + t + 4$, then the value of $\frac{du}{dt}$ at $t = -1$ is:

(a) 0 (b) 1
(c) 2 (d) 5 (1 mark)

Answer:

(a) if $u = 3t^4 + 5t^3 + 2t^2 + t + 4$

diff. w.r.t (t)

$$\frac{du}{} = \frac{d}{}(3t^4 + 5t^3 + 2t^2 + t + 4)$$

$$= \frac{d}{dt}(3t^4) + \frac{d}{dt}(5t^3) + \frac{d}{dt}(2t^2) + \frac{d}{dt}(t) + \frac{d}{dt}(4)$$

$$= 3.4t^3 + 5.3t^2 + 2.\ 2t + 1 + 0$$

$$= 12t^3 + 15t^2 + 4t + 1$$

$$\left(\frac{du}{dt}\right)_{t=-1} = 12(-1)^3 + 15(-1)^2 + 4(-1) + 1$$

$$= 12(-1) + 15(1) - 4 + 1$$

$$= -12 + 15 - 4 + 1$$

$$= -16 + 16$$

$$= 0$$

[81] The value of $\int_1^2 \frac{1-x}{1+x}\,dx$ is equal to:

(a) $\log \frac{3}{2} - 1$ (b) $2 \log \frac{3}{2} - 1$

(c) $\frac{1}{2} \log \frac{3}{2} - 1$ (d) $\frac{1}{2} \log \frac{2}{3} - 1$ (1 mark)

Answer:

(b) $\int_1^2 \left(\frac{1-x}{1+x}\right) dx$

$$= -\int_1^2 \left(\frac{x-1}{x+1}\right) dx$$

$$= -\int_1^2 \left(\frac{x+1-2}{x+1}\right) dx$$

$$= -\int_1^2 \left[\left(\frac{x+1}{x+1}\right) - \frac{2}{x+1}\right] dx$$

$$= -\int_1^2 1\,dx + \int_1^2 \frac{2}{x+1}\,dx$$

$$= -[x]_1^2 + [2 \log (x+1)]_1^2$$

$$= -(2-1) + 2[\log (2+1) - \log (1+1)]$$

$$= -1 + 2(\log 3 - \log 2)$$

$$= -1 + 2 \log \left(\frac{3}{2}\right)$$

$$= 2 \log \left(\frac{3}{2}\right) - 1$$

[82] The slope of the tangent to the curve $y = \frac{x-1}{x+2}$ at $x = 2$ is:

(a) $\frac{3}{16}$ (b) $-\frac{3}{16}$

(c) $\frac{1}{4}$ (d) $-\frac{1}{4}$ (1 mark)

Answer:

(a) Given curve

$$y = \frac{x-1}{x+2}$$

$$\frac{dy}{dx} = \frac{(x+2)\frac{d}{dx}(x-1) - (x-1)\frac{d}{dx}(x+2)}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{(x+2)(1-0) - (x-1).(1+0)}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{x+2-x+1}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{3}{(x+2)^2}$$

$$\left(\frac{dy}{dx}\right)_{(x=2)} = \frac{3}{(2+2)^2} = \frac{3}{(4)^2} = \frac{3}{16}$$

slope of tangent $= \left(\frac{3}{16}\right)$

2016 - JUNE

[83] $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}} dx$ is equal to _____

(a) $\frac{2\sqrt{2}}{\log_e 3}$ (b) 0

(c) $\frac{2}{\log_e 3}(3^{\sqrt{2}} - 1)$ (d) $\frac{3^{\sqrt{2}}}{\sqrt{2}}$ (1 mark)

Answer:

(c) $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}dx$

let: $\sqrt{x} = t$

$\frac{1}{2\sqrt{x}}dx = dt$

$\frac{1}{\sqrt{x}}dx = 2dt$

x	0	2
t	0	$\sqrt{2}$

$= \int_0^{\sqrt{2}} 3^t \cdot 2dt$

$= 2\int_0^{\sqrt{2}} 3^t dt$

$= 2\left[\frac{3^t}{\log_e^3}\right]_0^{\sqrt{2}}$

$= \frac{2}{\log_e^3}\left[3^t\right]_0^{\sqrt{2}} = \frac{2}{\log_e^3}\left[3^{\sqrt{2}} - 3^0\right]$

$= \frac{2}{\log_e^3}\left[3^{\sqrt{2}} - 1\right]$

[84] $\int \frac{x}{(x^2+1)(x^2+2)}dx$ is equal to_____

(a) $\log\left(\frac{x^2+1}{x^2+2}\right)+c$

(b) $\frac{1}{2}\log\left(\frac{x^2+1}{x^2+2}\right)+c$

(c) $\frac{1}{2}\log\left(\frac{x^2+2}{x^2+1}\right)+c$

(d) $-\log\left(\frac{x^2+1}{x^2+2}\right)+c$ (1 mark)

Answer:

(b) $\int \frac{x}{(x^2+1)(x^2+2)}dx$

Let:$x^2 = t$

$2xdx = dt$

$xdx = \frac{dt}{2}$

$$= \int \frac{1}{(t+1)(t+2)} \frac{dt}{2}$$

$$= \frac{1}{2}\int \frac{1}{(t+1)(t+2)} dt$$

$$= \frac{1}{2}\int \left[\frac{1}{(t+1)} - \frac{1}{(t+2)}\right]^{dt}$$

$$= \frac{1}{2}\left[\int \frac{1}{(t+1)} dt - \int \frac{1}{(t+2)} dt\right]$$

$$= \frac{1}{2}[\log(t+1) - \log(t+2)] + c$$

$$= \frac{1}{2}\left[\log \frac{(t+1)}{(t+2)}\right] + c$$

$$= \frac{1}{2}\left[\log\left(\frac{x^2+1}{x^2+2}\right)\right] + c$$

[85] If $y = \sqrt{\frac{1-x}{1+x}}$, then $\frac{dy}{dx}$ is equal to –

(a) $\frac{y}{X^2-1}$ (b) $\frac{y}{1-X^2}$

(c) $\frac{y}{1+X^2}$ (d) $\frac{y}{y^2-1}$ (1 mark)

Answer:

(a) If $y = \sqrt{\frac{1-x}{1+x}}$

$$y = \frac{\sqrt{1-x}}{\sqrt{1+x}}$$

diff w r t. 'x'

$$\frac{dy}{dx} = \frac{\sqrt{1+x}\cdot\frac{d}{dx}\sqrt{1-x} - \sqrt{1-x}\frac{d}{dx}\sqrt{1+x}}{(\sqrt{1+x})^2}$$

$$\frac{dy}{dx} = \frac{\sqrt{1+x}\cdot\frac{1}{2\sqrt{1-x}}\cdot(-1) - \sqrt{1-x}\cdot\frac{1}{2\sqrt{1+x}}\cdot(1)}{(1+x)}$$

$$\frac{dy}{dx} = \frac{(1+x)(-1) - (1-x)\cdot 1}{2\sqrt{1-x}\sqrt{1+x}(1+x)}$$

$$\frac{dy}{dx} = \frac{-1 \cancel{-x} - 1 \cancel{+x}}{2\sqrt{1-x^2}(1+x)}$$

$$\frac{dy}{dx} = \frac{\cancel{-2}}{\cancel{2}\sqrt{1-x^2}(1+x)}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2(1+x)}}\cdot\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{-1\ \sqrt{(1+x)(1-x)}}{(1-x^2)(1+x)}$$

$$\frac{dy}{dx} = \frac{-\cancel{\sqrt{1+x}}\sqrt{1-x}}{(1-x^2)\sqrt{1+x}\cancel{\sqrt{1+x}}}$$

$$\frac{dy}{dx} = -\sqrt{\frac{1-x}{1+x}}\cdot\frac{1}{(1-x^2)}$$

$$\frac{dy}{dx} = \frac{-y}{(1-x^2)}$$

$$\frac{dy}{dx} = \frac{y}{(x^2-1)}$$

2016 - DECEMBER

[86] Differential Co-efficient of $\log_e\left(\sqrt{x-1}+\sqrt{x+1}\right)$ with respect to x is:

(a) $\frac{1}{2\sqrt{x^2-1}}$ (b) $\frac{1}{2\sqrt{x^2+1}}$

(c) $\frac{1}{2(x^2-1)}$ (d) $\frac{1}{\sqrt{x-1}+\sqrt{x+1}}$ (1 mark)

Answer:

(a) If $y = \log_e (\sqrt{x-1} + \sqrt{x+1})$

Diff w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx} \log_e (\sqrt{x-1} + \sqrt{x+1})$$

$$= \frac{1}{(\sqrt{x-1} + \sqrt{x+1})} \cdot \frac{d}{dx} (\sqrt{x-1} + \sqrt{x+1})$$

$$= \frac{1}{(\sqrt{x-1} + \sqrt{x+1})} \cdot \left[\frac{1}{2\sqrt{x-1}} + \frac{1}{2\sqrt{x+1}}\right]$$

$$= \frac{1}{(\sqrt{x-1} + \sqrt{x+1})} \left[\right.$$

$$\frac{1}{1} = \frac{2}{x^2 - 1}$$

$$\Rightarrow x^2 - 1 = 2$$

$$x^2 = 2 + 1$$

$$x^2 = 3$$

$$x = \pm \sqrt{3}$$

[88] $\int_1^e \frac{e^x (x \log_e x + 1)}{x} dx$ is equal to:

(a) $e + 1$ (b) e^e

(c) $e - 1$ (d) $e^x + 1$

Answer:

(b) $\int_1^e \frac{e^x (x \log_e x + 1)}{x} dx$

$$= \int_1^e e^x \left(\frac{x \log_e x + 1}{x} \right) dx$$

$$= \int_1^e e^x \left(\frac{x \log_e x}{x} + \frac{1}{x} \right) dx$$

$$= \int_1^e e^x \left(\log_e x + \frac{1}{x} \right) dx$$

$$= \left[e^x . \log_e x \right]_1^e \qquad \left[\because \int e^x (f(x) + f^1(x)) = e^x f(x) \right]$$

$$= [e^e . \log_e x - e^1 . \log_e 1]$$

$$= e^e . 1 - e^1 . 0$$

$$= e^e - 0 = e^e$$

2017 - JUNE

[89] The equation of the curve which passes through the point (1, 2) and has the slope $3x - 4$ at any point (x, y) is:

(a) $2y = 3x^2 - 8x + 9$ (b) $y = 6x^2 - 8x + 9$

(c) $y = x^2 - 8x + 9$ (d) $2y = 3x^2 - 8x + c$ (1 mark)

Answer:

(a) Given, Slope $= 3x - 4$

$$\frac{dy}{dx} = 3x - 4$$

$$dy = (3x - 4)\,dx$$

On Integration

$$\int dy = \int (3x - 4)\,dx$$

$$y = \frac{3x^2}{2} - 4x + c \quad \text{_____(1)}$$

It Passes through (1, 2) then

$$2 = \frac{3\cdot(1)^2}{2} - 4 \times 1 + c$$

$$2 = \frac{3}{2} - 4 + c$$

$$c = 2 + 4 - \frac{3}{2}$$

$$c = \frac{9}{2} \text{ in equation (1)}$$

$$y = \frac{3x^2}{2} - 4x + \frac{9}{2}$$

$$2y = 3x^2 - 8x + 9$$

[90] The value of $\int_1^2 \frac{x}{x^2+1} dx$ is equal to:

(a) $\log_e\left(\frac{5}{2}\right)$ (b) $\frac{1}{2}\log_e\left(\frac{5}{2}\right)$

(c) $\log_e(5) - \log_e 2 + c$ (d) None of these. (1 mark)

Answer:

(b) $\int_1^2 \frac{x}{x^2+1} dx = \frac{1}{2}\int_1^2 \frac{2x}{(x^2+1)} dx$

$$= \frac{1}{2}\left[\log (x^2 + 1)\right]_1^2$$

$$= \frac{1}{2}[\log(2^2 + 1) - \log(1^2 + 1)]$$

$$= \frac{1}{2}[\log 5 - \log 2]$$

$$= \frac{1}{2}\log\left(\frac{5}{2}\right)$$

[91] If $x = at^3 + bt^2 - t$ and $y = at^2 - 2bt$, then the value of $\frac{dy}{dx}$ at $t = 0$ is :

(a) 2b (b) -2b

(c) $\frac{1}{2b}$ (d) $-\frac{1}{2b}$ (1 mark)

Answer:

(a) Given, $x = at^3 + bt^2 - t$ and $y = at^2 - 2bt$

$\frac{dx}{dt} = a \quad 3t^2 + b \quad 2t - 1$ and $\frac{dy}{dt} = 2at - 2b$

$= 3at^2 + 2bt - 1$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2at - 2b}{3at^2 + 2bt - 1}$

( ) = =

Answer:

(a) $\int e^x [f(x) + f^1 x)]\, dx$

$= \int e^x f(x)\, dx + \int e^x . f^1(x)\, dx$

$= \int f(x) . e^x\, dx + \int e^x . f^1(x)\, dx$

$= f(x) \int e^x\, dx - \int \left(\frac{d}{dx} f(x) \int e^x\, dx \right) dx + \int e^x . f^1(x)\, dx$

$= f(x) . e^x - \int f^1(x) . e^x\, dx + \int e^x . f^1(x)\, dx$

$= e^x . f(x) + c$

[93] If $x^y = e^{x-y}$ then $\frac{dy}{dx}$ is equal to:

(a) $\frac{2 \log x}{(1 + \log x)^2}$ (b) $\frac{\log x}{(1 + \log x)}$

(c) $\frac{\log x}{(1 + \log x)^2}$ (d) None of the above (1 mark)

Answer:

(c) If $x^y = e^{x-y}$

taking log on both side

$\log x^y = \log e^{x-y}$

$y \log x = (x - y) \log_e$

$y \log x = x - y \ (\log_e = 1)$

$y \log x + y = x$

$y (\log x + 1) = x$

$y = \frac{x}{(\log x + 1)}$

Diff w.r.f (x)

$$\frac{dy}{dx} = \frac{(\log x + 1) \frac{d}{dx}(x) - x . \frac{d}{dx}(\log x + 1)}{(\log x + 1)^2}$$

$$= \frac{(\log x + 1).1 - x . \left(\frac{1}{x} + 0 \right)}{(\log x + 1)^2}$$

$$= \frac{\log x + 1 - 1}{(\log x + 1)^2}$$

$$= \frac{\log x}{(\log x + 1)^2}$$

[94] If $y = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \ldots\ldots\ldots\ldots\ldots \infty$, then the value of $\frac{dy}{dx}$ is equal to:

(a) x (b) y

(c) 1 (d) 0 (1 mark)

Answer:

(b) If $y = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \ldots\ldots\ldots\ldots\ldots \infty$,

Diff w.r.t. 'r'

$$\frac{dy}{dx} = \frac{d}{dx}\left[1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \ldots\ldots\ldots\ldots\ldots \infty\right]$$

$$= \frac{d}{dx}(1) + \frac{d}{dx}\frac{x}{\lfloor 1} + \frac{d}{dx}\frac{x^2}{\lfloor 2} + \frac{d}{dx}\frac{x^3}{\lfloor 3} + \ldots\ldots\ldots\ldots\ldots \infty$$

$$= 0 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \ldots\ldots\ldots\ldots \infty$$

$$= 0 + 1 + \frac{\cancel{2}x}{\cancel{2}\lfloor 1} + \frac{\cancel{3}x^2}{\cancel{3}\lfloor 2} + \ldots\ldots\ldots\ldots \infty$$

$$\frac{dy}{dx} = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \ldots\ldots\ldots\ldots\ldots \infty$$

$$\frac{dy}{dx} = y$$

[95] $\int x.e^{x^2} dx$ is equal to:

(a) $2e^{x^2} + c$ (b) $e^{x^2} + c$

(c) $\frac{1}{2}.e^{x^2} + c$ (d) $xe^{x^2} + c$ (1 mark)

Answer:

(c) $I = x.e^{x2}\,dx$ Let $x^2 = t$

$I = e^t \frac{dt}{2}$ $2xdx = dt$

$I = \frac{1}{2} e^t dt$ $xdx = \frac{dt}{2}$

$= \frac{1}{2} e^t + c = \frac{1}{2} ex^2 + c$

[96] If $x = at^2$, $y = 2at$ then the value of $\frac{dy}{dx}$ at $t = 2$ is:

(a) 2 (b) 4

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$ (1 mark)

Answer:

(c) If $x = at^2$ and $y = 2at$ then $\left(\frac{dy}{dx}\right)_{t=2} = ?$

Now Given $x = at^2$

Diff. w.r.t 't'

$\frac{dx}{dt} = \frac{d}{dt}(at^2) = a\frac{d}{dt}(t^2) = a.2t = 2at$

and $y = 2at$

Diff. w.r.t 't'

$\frac{dy}{dt} = \frac{d}{dt}(2at) = 2a\frac{d}{dt}(t) = 2a.1 = 2a$

$\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{2a}{2at} = \frac{1}{t}$

$\left(\frac{dy}{dx}\right)_{t=2} = \frac{1}{2}$

[97] If $y = \log x^x$ then $\frac{dy}{dx}$ is equal to:

(a) log ex (b) $\log \frac{e}{x}$

(c) $\log \frac{x}{e}$ (d) 1 (1 mark)

Answer:

(a) Given $y = \log x^x$

$y = x \log x$

Diff. w.r.t (x)

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(x \log x)\\ &= x \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(x)\\ &= x \cdot \frac{1}{x} + \log x \cdot 1\\ &= 1 + \log x\\ &= \log^{e} + \log x\\ &= \log ex\end{aligned}$$

2018 - MAY

[98] The value of $\int_1^2 \frac{1-x}{1+x}\,dx$ is equal to:

(a) $\log \frac{3}{2} - 1$ (b) $2 \log \frac{3}{2} - 1$

(c) $\frac{1}{2} \log \frac{3}{2} - x$ (d) $\frac{1}{2} \log \frac{2}{3} - 1$ (1 mark)

Answer:

(b) $= \int_1^2 \left(\frac{1-x}{1+x}\right) dx = \int_1^2 \left(\frac{1}{1+x} - \frac{x}{1+x}\right) dx$

$= \int_1^2 \frac{1}{1+x}\,dx - \int_1^2 \frac{x}{1+x}\,dx$

$= \int_1^2 \frac{1}{1+x}\,dx - \int_1^2 \left(\frac{1+x-1}{1+x}\right) dx$

$= \int_1^2 \frac{1}{(1+x)}\,dx - \int_1^2 \left(1 - \frac{1}{1+x}\right) dx$

$= \int_1^2 \frac{1}{1+x}\,dx - \int_1^2 1 \times dx + \int_1^2 \frac{1}{1+x}\,dx$

$= 2\int_1^2 \frac{1}{1+x} - \int_1^2 1\,dx$

$= 2\,[\log(1+x)]_1^2 - [x]_1^2$

$= 2\,[\log(2+1) - \log(1+1)] - [2-1]$

$= 2\,[\log 3 - \log 2] - 1$

$= 2 \log \frac{3}{2} - 1$

[99] $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to

(a) $\frac{2\sqrt{2}}{\log_e 3}$ (b) 0

(c) $\frac{2(3\sqrt{2}-1)}{\log_e 3}$ (d) $\frac{3\sqrt{2}}{\sqrt{2}}$ (1 mark)

Answer:

(c) $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}\,dx$ let $\sqrt{x} = t$

$= \int_0^2 3^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}}\,dx$ $\frac{1}{2\sqrt{x}}dx = dt$

$\frac{1}{\sqrt{x}}dx = 2\,dt$

x	0	2
t	0	$\sqrt{2}$

$= \int_0^{\sqrt{2}} 3^t \cdot 2dt$

$= 2\int_0^{\sqrt{2}} 3^t\,dt$

$= 2\left[\frac{3^t}{\log 3}\right]_0^{\sqrt{2}}$

$= 2\left[\frac{3^{\sqrt{2}}}{\log 3} - \frac{3^0}{\log 3}\right]$

$= 2\left[\frac{3^{\sqrt{2}}-3^0}{\log 3}\right]$

$= \frac{2(3^{\sqrt{2}}-1)}{\log_e 3}$

[100] The value of $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}\,dx$ is:

(a) 0 (b) 3

(c) 2 (d) 1 (1 mark)

Answer:

(d) $I = \int^2 \frac{\sqrt{x}}{\quad}\,dx$.....(1)

$$I = \int_0^2 \frac{\sqrt{0+2-x}}{\sqrt{0+2-x} + \sqrt{2-(0+2-x)}}\,dx$$

$$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx\right]$$

$$I = \int_0^2 \frac{\sqrt{2-x}}{\sqrt{2-x} + \sqrt{x}}\,dx \quad \ldots\ldots(2)$$

Addly (1) and (2) we get

$$2I = \int_0^2 \left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{2-x}}} + \frac{\sqrt{2-x}}{\sqrt{2-x+\sqrt{x}}}\right) dx$$

$$2I = \int_0^2 \frac{(\sqrt{x} + \sqrt{2-x})}{(\sqrt{x} + \sqrt{2-x})}\,dx$$

$$2I = \int_0^2 1\,dx$$

$$2I = [x]_0^2$$

$$2I = [2-0]$$

$$2I = 2 \Rightarrow I = 1$$

[101] $\lim_{x\to 1}, \frac{x + x^2 + x^3 \ldots\ldots + x^n - n}{x-1}$

(a) (n) (b) $\frac{n(n+1)}{2}$

(c) (n+1) (d) n (n+1) (1 mark)

Answer:

(b) $\lim_{x\to 1} \frac{x + x^2 + x^3 + \cdots + x^n - n}{x-1}$ (:)

By L.H. Rule

$$\lim_{x\to 1} \frac{d/dx\,(x + x^2 + x^3 + \cdots + x^n) - n}{d/dx\,(x-1)}$$

$$= \lim_{x\to 1} \frac{1 + 2x + 3x^2 + \cdots + nx^{n-1}) - 0}{1-0}$$

$$= 1 + 2\times 1 + 3(i)^2 + \cdots + n\,(1)^{n-1}$$

$= 1 + 2 + 3 + \text{------n}$

$= \sum n = \frac{n(n+1)}{2}$

[102] The cost function for the production of x units of a commodity is given by $C(x) = 2x^3 + 15x^2 + 36x + 15$

The cost will be minimum. When 'x' is equal to:

(a) 3 (b) 2

(c) 1 (d) 4 (1 mark)

Answer:

(a) The cost function is given by

$C(x) = 2x^3 - 15x^2 + 36x + 15$

$\frac{d}{dx}C(x) = 6x^2 - 30x + 36$.....(1)

$C'(x) = 6x^2 - 30x + 36$.....(2)

For max/min

$C'(x) = 0$

$6x^2 - 30x + 36 = 0$

$6(x^2 - 5x + 6) = 0$

$= x^2 - 5x + 6 = 0$

$= x^2 - 3x - 2x + 6 = 0$

$= x(x-3) - 2(x-3) = 0$

$(x-3)(x-2) = 0$

$x = 3, 2$

Differentiating equation (2) again w.r.f. 'x'

$C'(x) = 12x - 30$ Eq.....(3)

Putting (x = 2) in Eq.....(3)

$C'(x) = 12 \times 2 - 30 = -6$

Putting (x= 3) in . .....(3)

$C'(x) = 12 \times 3 - 30 = 6$ (+ve) so function is minimum at x = 3

[103] $\lim_{x \to 0} \frac{2e^{1/x - 3x}}{e^{1/x + x}} = ?$

(a) – 3 (b) 0

(c) 2 (d) 9 (1 mark)

Answer:

(c) $\lim_{x\to\infty} \frac{2e^{1/x} - 3x}{e^{1/x} + x}$

let $\frac{1}{x} = y$ if $x \to 0, y \to \infty$

$$\lim_{x\to\infty} \frac{2e^{y} - 3\frac{1}{y}}{e^{y} + \frac{1}{y}}$$

$$= \lim_{x\to\infty} \frac{e^{y}\left[2 - 3\frac{1}{y.e^{y}}\right]}{e^{y}\left[1 + \frac{1}{y.e^{y}}\right]}$$

$$= \frac{2 - 3\frac{1}{\infty.e^{\infty}}}{1 + \frac{1}{\infty.e^{\infty}}}$$

$$= \frac{2 - 0}{1 + 0} = 2$$

2018 - NOVEMBER

[104] Let $x = at^3$, $y = \frac{a}{t^2}$. Then $\frac{dy}{dx} =$

(a) $\frac{-1}{t^6}$ (b) $\frac{-3a}{t^6}$

(c) $\frac{1}{3at^6}$ (d) None of the above (1 mark)

Answer:

(d) If $x = at^3$, and $y = \frac{a}{t^2} = at^{-2}$

Given

$x = at^3$

Different w.r.t. (t)

$$\frac{dx}{dt} = \frac{d}{dt} at^3 = a.3t^2 = 3at^2$$

and $y = at^{-2}$

different w.r.t (t)

$$\frac{dy}{dt} = \frac{d}{dt} at^{-2} = a \,.\, (-2t^{-3})$$

$$= -2at^{-3}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2at^{-3}}{3at^2} = \frac{-2}{3t^5}$$

[105] $\int x(x^2+4)^5 dx$ is equal to

(a) $(x^2+4)^6 + c$ (b) $\frac{1}{12}(x^2+4)^6 + c$

(c) $\frac{1}{6}(x^2+4)^6 + c$ (d) None of the above (1 mark)

Answer:

(b) $\int x(x^2+4)^5 dx$

Let $x^2 + 4 = t$

$2x\,dx = dt$

$$xdx = \frac{dt}{2}$$

$$\int (x^2+4)^5 . x dx$$

$$\int t^5 = \frac{dt}{2}$$

$$= \frac{1}{2}\int t^5\, dt$$

$$= \frac{1}{2}\,\frac{t^6}{6} + c = \frac{1}{12}(x^2+4)^6 + c$$

[106] $\int_{-1}^{3}(1 + 3x - x^3)\, dx$ is equal to

(a) -4 (b) 4

(c) 3 (d) -3 (1 mark)

Answer:

(a) $\int_{-1}^{3}(1+3x-x^3)dx$

$= \int_{-1}^{3}1dx + \int_{-1}^{3}3xdx - \int_{-1}^{3}x^3dx$

$= [x]_{-1}^{3} + 3\left[\frac{x^2}{2}\right]_{-1}^{3} - \left[\frac{x^4}{4}\right]_{-1}^{3}$

$= [3-(-1)] + \frac{3}{2}[(3)^2-(-1)^2] - \frac{1}{4}[(3)^4-(-1)^4]$

$= (3+1) + \frac{3}{2}[9-1] - \frac{1}{4}[81-1]$

$= 4 + \frac{3}{2}\times 8 - \frac{1}{4}\times 80$

$= 4 + 12 - 20$

$= -4$

[107] $xy = 1$ then $y^2 + \frac{dy}{dx} = ?$

(a) 1 (b) 0

(c) 2 (d) None of the above (1 mark)

Answer:

(b) $xy = 1$ then $y^2 + \frac{dy}{dx} = ?$

Given $xy = 1$

$y = \frac{1}{x}$ _____ (1)

$y = x^{-1}$

$\frac{dy}{dx} = (-1)\,x^{-2}$

$\frac{dy}{dx} = \frac{-1}{x^2}$

Now $y^2 + \frac{dy}{dx} = \left(\frac{1}{x}\right)^2 + \left(\frac{-1}{x^2}\right)$

$= \frac{1}{x^2} - \frac{1}{x^2}$

$= 0$

2019 - JUNE

[108] If the given cost function of commodity is given by $C = 150x - 5x^2 + \frac{x_3}{6}$, where C stands for cost and x stands for output, if the average cost is equal to the marginal cost then the output x = _____.

(a) 5
(b) 10
(c) 15
(d) 20 (1 mark)

Answer:

(c) The cost function of a commodity

$$C = 150x - 5x^2 + \frac{x^3}{6} \quad \text{_____ (i)}$$

$$\text{Average cost (AVC)} = \frac{C}{x}$$

$$= \frac{150x - 5x^2 + \frac{x^3}{6}}{x}$$

$$= 150 - 5x + \frac{x^2}{6}$$

Diff. eq (i) w.r.f. 'x'

$$\frac{dc}{dx} = \frac{d}{dx}($$

Average Cost = Marginal Cost

$$150 - 5x + \frac{x^2}{6} = 150 - 10x + \frac{x^2}{2}$$

$$\cancel{150} - 5x + \frac{x^2}{6} - \cancel{150} + 10x - \frac{x^2}{2} = 0$$

$$5x + \frac{x^2}{6} - \frac{x^2}{2} = 0$$

$$\frac{30x + x^2 - 3x^2}{6} = 0$$

$$30x - 2x^2 = 0$$

$$2x(15 - x) = 0$$

If $2x = 0$ If $15 - x = 0$

$\boxed{x = 0}$ $\boxed{x = 15}$

[109] If $2^x - 2^y = 2^{x-y}$ then $\frac{dy}{dx}$ at $x = y = 2$

(a) 1
(b) 2
(c) 4
(d) 5 (1 mark)

Answer:

(a) If $2^x - 2^y = 2^{x-y}$

Diff w.r.t. 'x'

$$\frac{d}{dx}(2^x - 2^y) = \frac{d}{dx}2^{x-y}$$

$$2^x \log 2 - 2^y \log 2 \cdot \frac{dy}{dx} = 2^{x-y} \log 2 \frac{d}{dx}(x - y)$$

$$2^x \log 2 - 2^y \log 2 \cdot \frac{dy}{dx} = 2^{x-y} \log 2 \left[1 - \frac{dy}{dx}\right]$$

$$2^x \log 2 - 2^y \log 2 \cdot \frac{dy}{dx} = 2^{x-y} \log 2 - 2^{x-y} \log 2 \frac{dy}{dx}$$

$$2^{x-y} \log 2 \frac{dy}{dx} - 2^y \log 2 \frac{dy}{dx} = 2^{x-y} \log 2 - 2^x \log 2$$

$$\frac{dy}{dx}(2^{x-y} \log 2 - 2^y \log 2) = 2^{x-y} \log 2 - 2^x \log 2$$

$$\frac{dy}{dx} = \frac{2^{x-y}\log 2 - 2^x \log 2}{2^{x-y}\log 2 - 2^y \log 2}$$

$$\left(\frac{dy}{dx}\right)_{x-y=2} = \frac{2^{2-2}\log 2 - 2^2 \log 2}{2^{2-2}\log 2 - 2^2 \log 2} = 1$$

[110] $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\, dx =$

(a) 1
(b) 1/2
(c) 2
(d) 3/2 (1 mark)

Answer:

(b) $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\, dx = \frac{3-2}{2} = \frac{1}{2}$

[111] $\int e^x (x^2 + 2x)\, dx =$

(a) $x^x \cdot e^2 + c$
(b) $e^x \cdot x + c$
(c) $-e^x x^2 + c$
(d) $-e^x \cdot x + c$ (1 mark)

Answer:

(a) $I = \int e^x (x^2 + 2x)\, dx$

$= \int e^x x^2\, dx + \int e^x \cdot 2x\, dx$

$= \int x^2 \cdot e^x\, dx + \int e^x \cdot 2^x\, dx$

$= \left[x^2 \int e^x dx - \int \left(\frac{d}{dx} x^2 \cdot \int e^x dx\right) dx\right] + \int e^x \cdot 2x\, dx$

$= \left[x^2 \cdot e^x - \int 2x \cdot e^x dx\right] + \int e^x \cdot 2^x\, dx$

$= x^2 \cdot e^x - \int 2x \cdot e^x\, dx + \int e^x \cdot 2^x\, dx$

$= x^2 e^x + c$

[112] $\int \log (a^x)\, dx =$

(a) $\log a \left(\frac{x^2}{2}\right) + c$

(b) $\log a \left(\frac{x}{2}\right) + c$

(c) $x \log a^x - x + c$

(d) $x \log a^x + c$ (1 mark)

Answer:

(a) $\int \log a^x\, dx$

$= \int x \log a\, dx$

$= \log a . \int x\, dx$

$= \log a . \left(\frac{x^2}{2}\right) + c$

2019 - NOVEMBER

[113] $\int a^x\, dx.$

(a) $x^x (1 + \log x)$

(b) $1 + \log x$

(c) $x . \log x$

(d) $\frac{a^x}{\log a} + c$ (1 mark)

Answer:

(d) Since, we know that

$$\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x$$

$$\int a^x\, dx = \frac{a^x}{\log a} + c$$

[114] $\int x.e^x\,dx.$

(a) $e^x\,(x-1)+c$

(b) $e^x\,.\,x+e^x+c$

(c) $\log x+e^x+c$

(d) $\frac{x^2}{e^x}+c$ (1 mark)

Answer:

(a) $\int x\cdot e^x\,dx$

Following I – Inverse
L – Logarithmic
A – Algebraic
T – Trigonometric
E – Exponential

So, x ⇒ I[st] function

e^x ⇒ II[nd] function

$\int \underset{\text{I}}{x}\ \underset{\text{II}}{e^x}\,dx$ or $\begin{matrix} x \to u \\ e^x \to v \end{matrix}$

Property

Since, $\int u\cdot v\,dx = u\cdot\int v\,dx - \int\left[\frac{d}{dx}(u)\cdot\int v\,dx\right]dx$

$x\cdot\int e^x\,dx - \int\left[\frac{d}{dx}(x)\cdot\int e^x\,dx\right]dx$

$x\ \ e^x - \int[1\times e^x]\,dx$

$x\ \ e^x - e^x + c$

$e^x\,(x-1)+c$

[115] $\int(4x+3)^6\,dx.$

(a) $\frac{1}{28}(4x+3)^7+c$

(b) $\frac{1}{7}(4x+3)^7+c$

(c) $\frac{1}{6}(4x+3)^6 + c$

(d) $\frac{4x}{5} + \frac{3}{5} + c$ (1 mark)

Answer:

(a) $\int (4x+3)^6 \, dx$

Since, $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{(n+1)} \times \frac{1}{a} + c$

So,

$\int (4x+3)^6 \, dx$

$= \frac{(4x+3)^{6+1}}{(6+1)} \times \frac{1}{4} + c$

$= \frac{1}{28}(4x+3)^7 + c$

[116] $\int_{-1}^{1} (2x^2 - x^3) \, dx.$

(a) 4/3
(b) 1
(c) 2
(d) 2/3 (1 mark)

Answer:

(a) $\int_{-1}^{1} (2x^2 - x^3) \, dx$ Since $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

$= \left[2 \times \frac{x^3}{3} \; \frac{-x^4}{4}\right]_{-1}^{1}$

$= \left[\left(\frac{2}{3} \times 1^3 - \frac{1^4}{4}\right) - \left\{\frac{2}{3} \times (-1)^3 - \frac{(-1)^4}{4}\right\}\right]$

$$= \left[\left(\frac{2}{3} - \frac{1}{4}\right) - \left(\frac{-2}{3} - \frac{1}{4}\right)\right]$$

$$= \frac{2}{3} - \frac{1}{4} + \frac{2}{3} + \frac{1}{4}$$

$$= \frac{4}{3}$$

[117] $\frac{d}{dx}(x \cdot \log x)$

(a) $x(1 + \log x)$
(b) $1 + \log x$
(c) $e^x x. \log x$
(d) $x^2 (\log x)$ (1 mark)

Answer:

(b) $\frac{d}{dx}(x. \log x)$

Since $\frac{d}{dx}(u.v) = u \frac{d}{dx}(v) + v \frac{d}{dx}(u)$

So here $u \Rightarrow x$

$v \Rightarrow \log x$

$\therefore \frac{d}{dx}(x \cdot \log x)$

$= x. \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(x)$

$= x \times \frac{1}{x} + \log x \times 1$ $\left[\because \frac{d}{dx}(\log x) = \frac{1}{x}\right]$

$= 1 + \log x$

[118] Differentiate x^x w.r.t x.

(a) $x^x (1 + \log x)$
(b) y/x
(c) $-y/x$
(d) $y + x^x \log x$ (1 mark)

Answer:

(a) $\frac{d}{dx}(x^x) = ?$

net $y = x^x$

Using log both sides

$\log y = x \log x$

On differentiating both sides w.r.t x

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \times \frac{d}{dx}(\log x) + \log x \times \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = y\left[x \times \frac{1}{x} + \log x \times 1\right]$$

$$\frac{d}{dx}(x^x) = x^x(1 + \log x)$$

[119] $\int x^2 . e^x dx.$

(a) $2x . e^x$

(b) $e^x (x^2 - 2x)$

(c) $x^2 . e^x - e^x . (2x) + 2$

(d) $e^x (x - 1)$ (1 mark)

Answer:

(c) $\int x^2 \ e^x dx$

Using I LATE

$a^2 \Rightarrow 1^{st}$ Function (u)

$e^x \Rightarrow 2^{nd}$ Function (v)

$$\int u . v \, dx = u . \int v dx - \int \left[\frac{d}{dx}(u) . \int v dx\right] dx$$

So $\int x^2 \ e^x dx$

$$x^2 \int e^x dx - \int \left[\frac{d}{dx}(x^2) . \int e^x\right] dx$$

$$x^2 \int e^x dx - \int [2x . e^x] dx$$

$x^2 . e^x - 2x \int x . e^x dx$ — Equation (1)

$* \int x \cdot e^x dx$

$= x \cdot \int e^x - \int \left[\frac{d}{dx}(x) \int e^x dx\right] dx$

$= x \cdot e^x - \int [1 \times e^x] dx$

$= x \cdot e^x - e^x$

$= e^x (x-1)$ — Equation (2)

Put Equation (2) in Equation (1)

$x^2 \cdot e^x - 2 e^x (x-1)$

$x^2 \cdot e^x - 2 e^x \cdot x + 2$

✿

Part - B

Logical Reasoning

CHAPTER

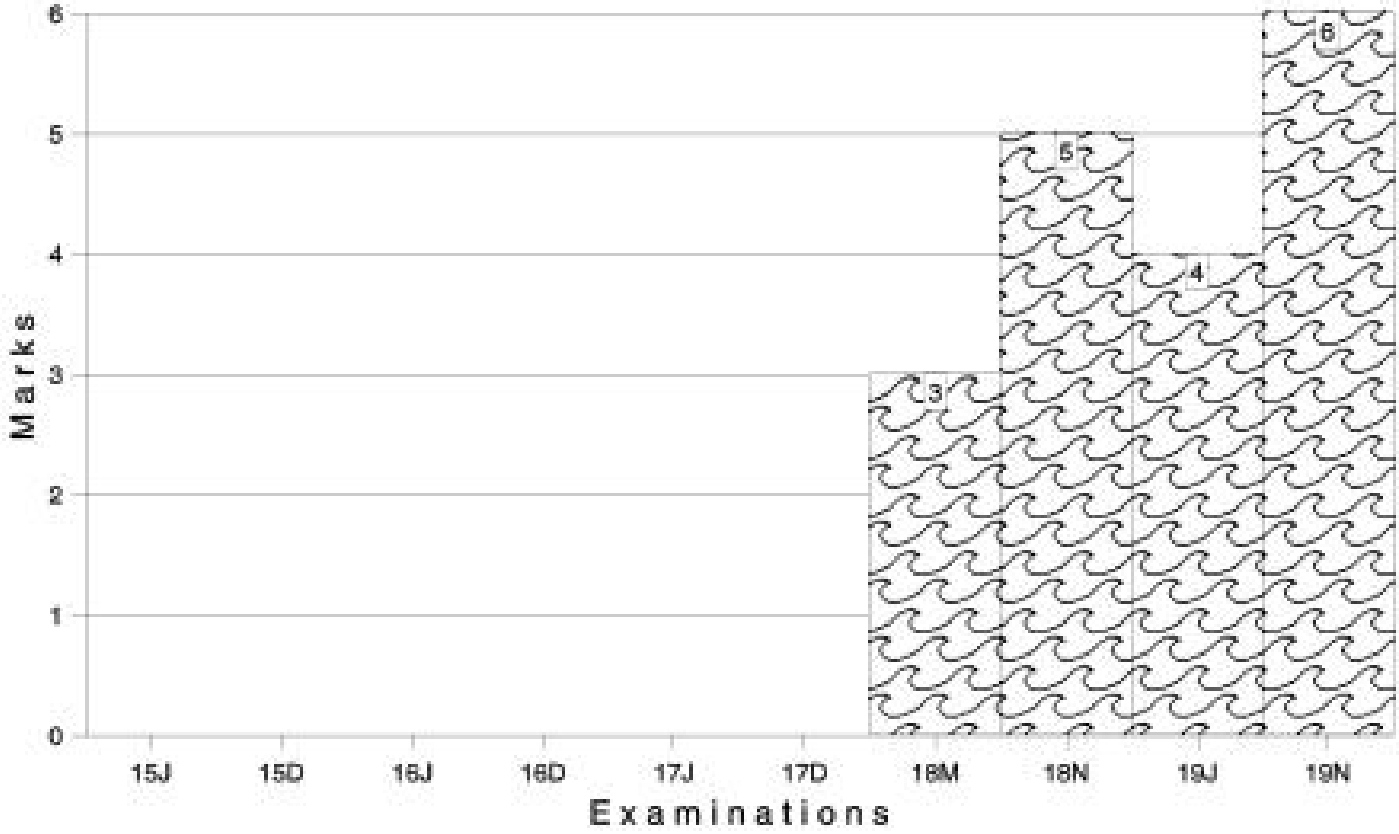
9

NUMBER SERIES, CODING AND DECODING AND ODD MAN OUT

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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for registration and password see first page of this book.

MULTIPLE CHOICE QUESTIONS AND ANSWERS

Q1. Series: A series is a sequence of numbers, where the sequence of numbers is obtained by some particular predefined rule and applying that predefined rule it is possible to find out the next term of the series.

Series can be classified into three types:

1. Number Series
2. Alphabet Series
3. Letter Series

1. **Number Series:**

There are many types of no Series

(i) **Arithmetic Series:** An Arithmetic Series is one in which successive numbers are obtained by adding (or Subtracting) a fixed number to the previous number.

e.g. (i) 5, 7, 9, 11, 13, 15 (adding 2 to the previous number)

(ii) 3 6 9 12 15 ..... (adding 3 to the previous number).

(ii) **Geometric Series:** Series in which each successive number is obtained by multiplying or dividing a fixed number by the previous number.

e.g. (i) 2, 4, 8, 16, 32 (multiply 2 to the previous number).

(ii) 15, – 30, 60, – 120, 240 ..... (multiply by – 2 to the previous number).

(iii) **Series of Squares, Cubes etc.:**

The series can be formed by squaring or cubing every successive number.

e.g. (i) 1, 2, 4, 16, 256 .....

(ii) 2, 8, 5 12 .....

(iv) **Two-tier Arithmetic Series:** In this series the difference of successive numbers themselves form an arithmetic series.

e.g. (i) 1, 2, 5, 10, 17, 26 .....

(The difference of successive numbers is 1, 3, 5, 7, 9, 11 ..... which is an arithmetic series).

Note: Two-tier arithmetic series can be denoted as a quadratic function 1, 2, 5, 10, 17, 26 can be denoted as

$0^2 + 1, 1^2 + 1, 2^2 + 1, 3^2 + 1, 4^2 + 1, 5^2 + 1$

$f(x) = x^2 + 1$

$x = 0, 1, 2, 3$ .....

(v) **Three-tier Arithmetic Series:** In this series find successive term differences are then again find successive term differences which give as Arithmetic Series.

e.g. (i) 336, 210, 120, 60, 24, 6, ... the difference of successive terms are 126, 90, 60, 36, 18, 6 .....

Again find successive differences of this new series

36, 30, 24, 18, 12 ..... which is an arithmetic series.

Note: Three-tier arithmetic series can be denoted as a cubic function.

(vi) **Arithmetic-Geometric Series:** In this series each successive term should be found by first adding a fixed no to the previous term and then multiplying if by another fixed number.

e.g. (i) 1, 9, 33, 105, 321, 969 ..... first add 2 to the previous term and then multiply it by 3.

(vii) **Geometric-Arithmetic Series:** In this series each successive term is found by first multiplying or dividing the previous term by a fixed number and then adding or subtracting another fixed number.

e.g. (i) 3, 9, 21, 45, 93, 189 ..... (multiply the previous number by 2 and then adding another fixed number that is 3)

2. **Alphabet Series:**

The English alphabet contains 26 letters as given below:

A B C D E F G H I J K L M N O P Q

R S T U V W X Y Z

First Alphabetical half A to M ⇒ 1 to 13

Second Alphabetical half N to Z ⇒ 14 to 26

The series (Alphabet order)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

3. **Letter Series:**

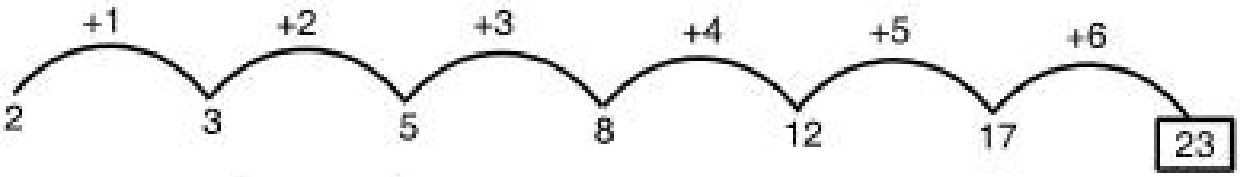
The letter of series will be such that each one follows its predecessor in a certain way, (according to a definite pattern). Students are required to find out the missing letters to complete the series.

This type of question usually follows a series of small letters.

PRACTICE QUESTIONS OF MCQ

Q1. 2, 3, 5, 8, 12, 17 _____.

Answer:



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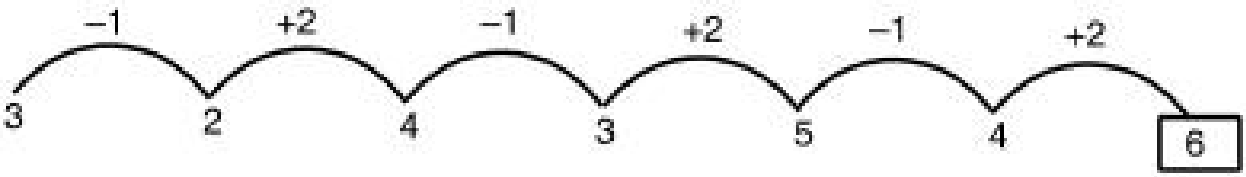
Q2. 3, 2, 4, 3, 5, 4 _____.

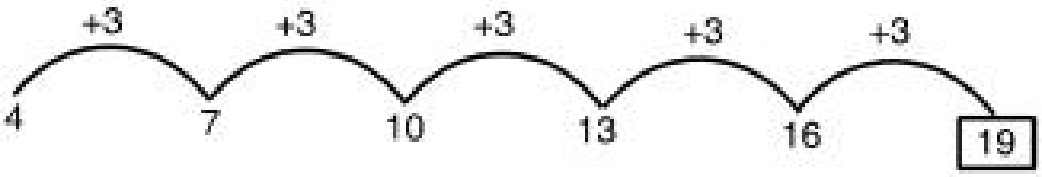
Answer:



6

Q3. 4, 7, 10, 13, 16 _____.

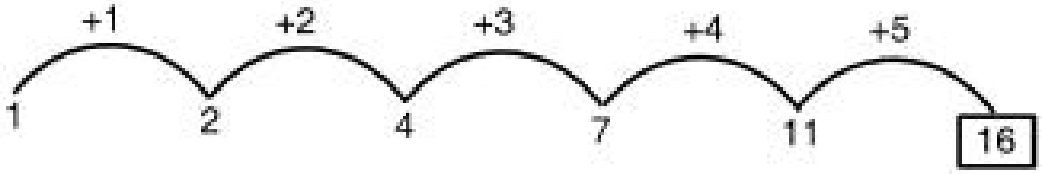
Answer:



19

Q4. 1, 2, 4, 7, 11 _____.

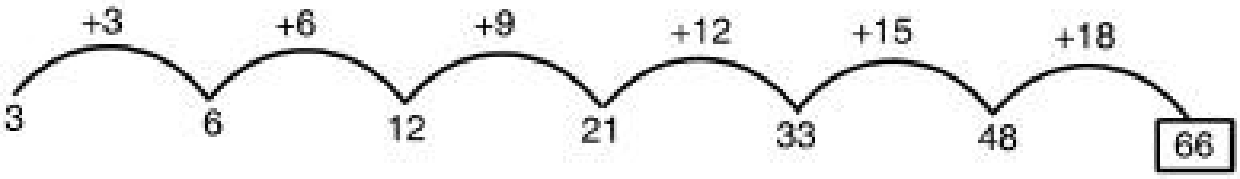
Answer:



16

Q5. 3, 6, 12, 21, 33, 48 _____.

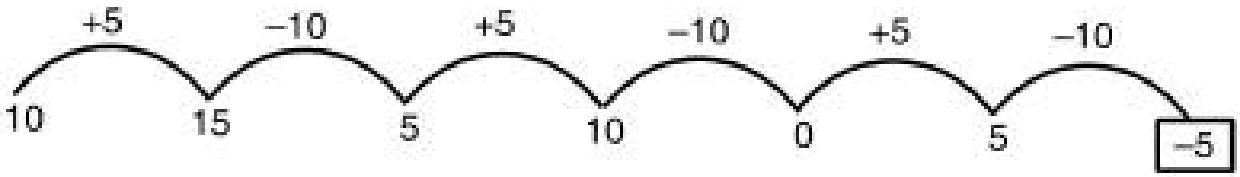
Answer:



66

Q6. 10, 15, 5, 10, 0, 5 _____.

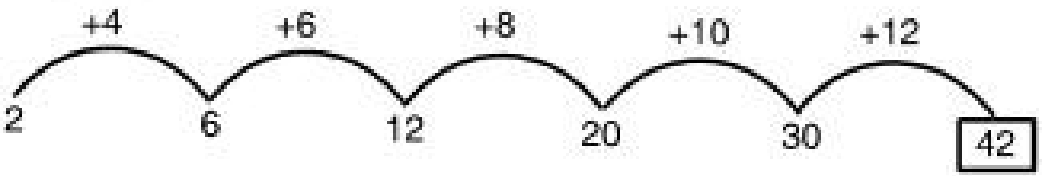
Answer:



−5

Q7. 2, 6, 12, 20, 30 _____.

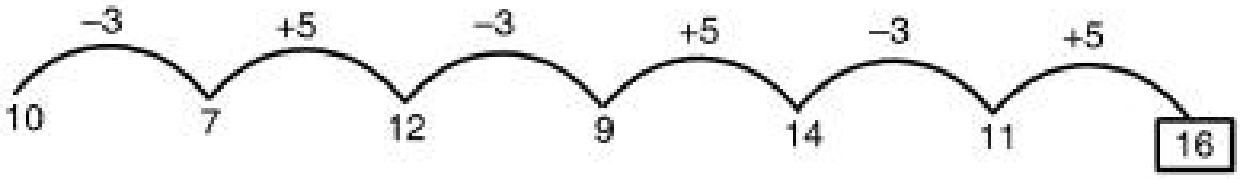
Answer:



42

Q8. 10, 7, 12, 9, 14, 11 _____.

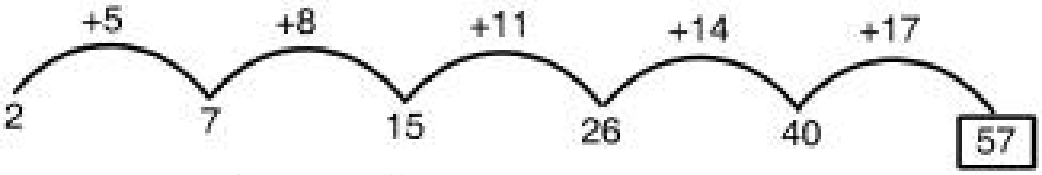
Answer:



16

Q9. 2, 7, 15, 26, 40 _____.

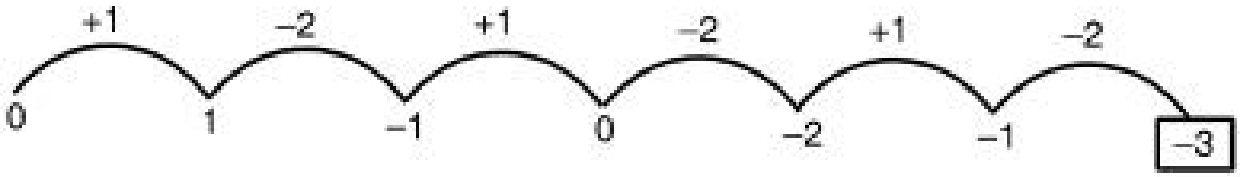
Answer:



57

Q10. 0, 1, −1, 0, −2, −1 _____.

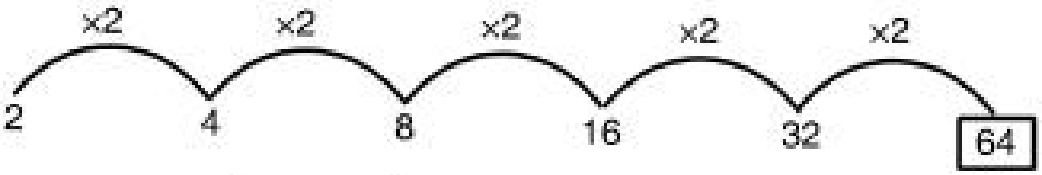
Answer:



−3

Q11. 2, 4, 8, 16, 32 _____.

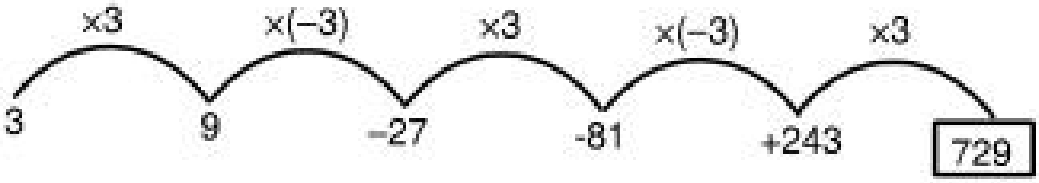
Answer:



64

Q12. 3, 9, −27, − 81, +243 _____.

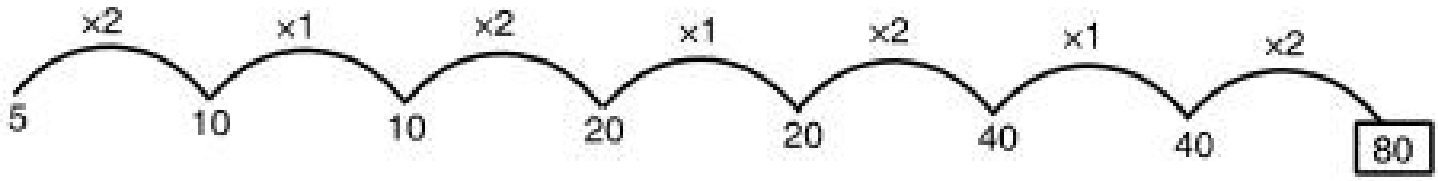
Answer:



729

Q13. 5, 10, 10, 20, 20, 40, 40 _____.

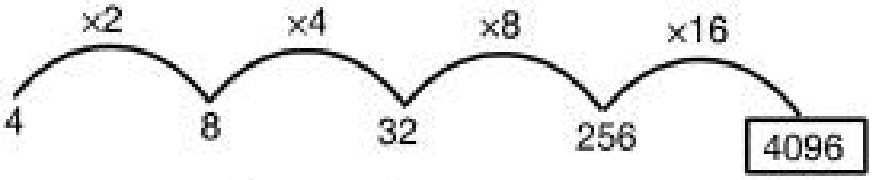
Answer:



80

Q14. 4, 8, 32, 256 _____.

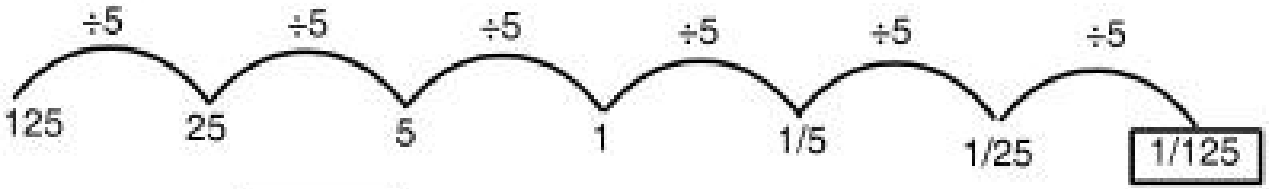
Answer:



4096

Q15. 125, 25, 5, 1, 1/5, 1/25 _____.

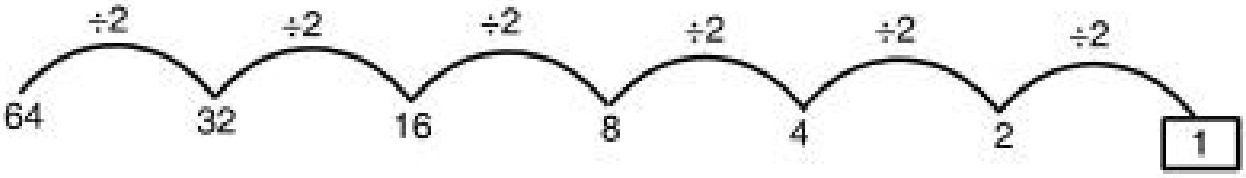
Answer:



1/125

Q16. 64, 32, 16, 8, 4, 2 _____.

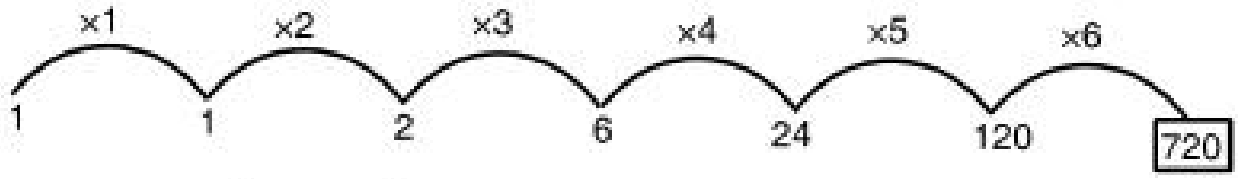
Answer:



1

Q17. 1, 1, 2, 6, 24, 120 _____.

Answer:



720

Q18. 1, 4, 9, 16, 25, 36 _____.

Answer:

Square of natural nos.

49

Q19. 0, 1, 8, 27, 64 _____.

Answer:

Cube of whole nos.

125

Q20. 4, 16, 36, 64, 100 _____.

Answer:

Square of even natural no.

144

Q21. 1, 9, 25, 49, 81 _____.

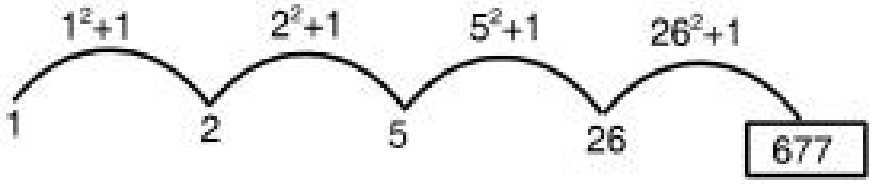
Answer:

Square of odd natural no.

121

Q22. 1, 2, 5, 26 _____.

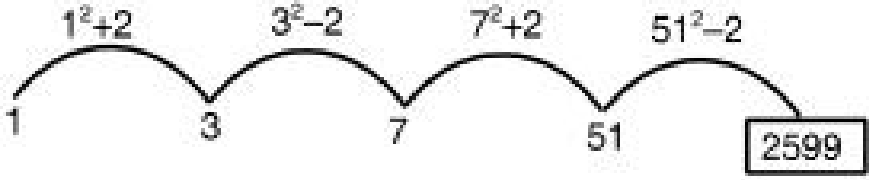
Answer:



677

Q23. 1, 3, 7, 51 _____.

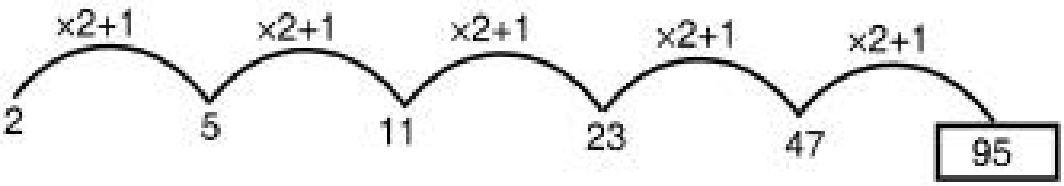
Answer:



2599

Q24. 2, 5, 11, 23, 47 _____.

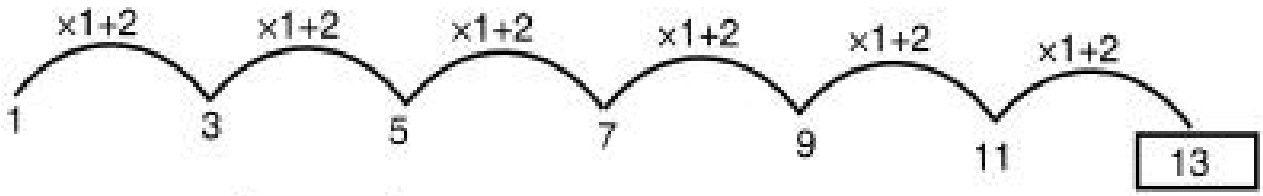
Answer:



95

Q25. 1, 3, 5, 7, 9, 11 _____.

Answer:



13

Q26. 1, 6, 21, 66, 201 _____.

Answer:

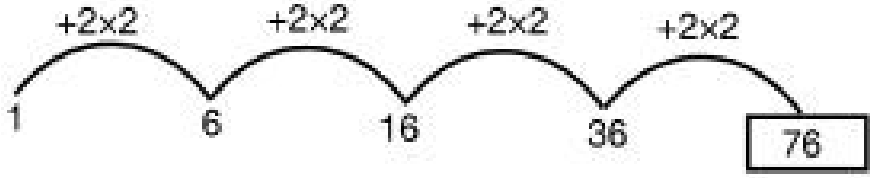
Adding 1 to the previous term and multiply by 3

606

Q27. 1, 6, 16, 36 _____.

Answer:

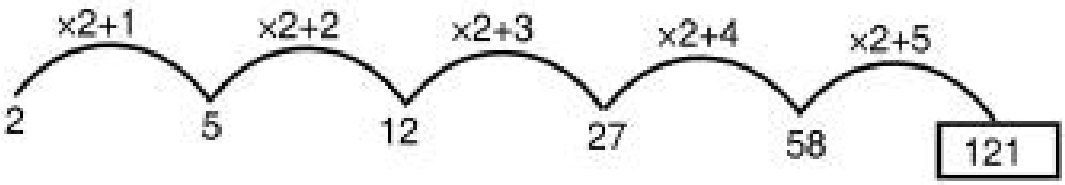
Adding 2 to the previous term and multiply by 2



76

Q28. 2, 5, 12, 27, 58 _____.

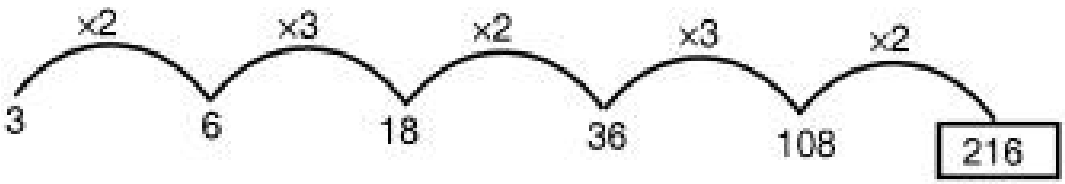
Answer:



121

Q29. 3, 6, 18, 36, 108 _____.

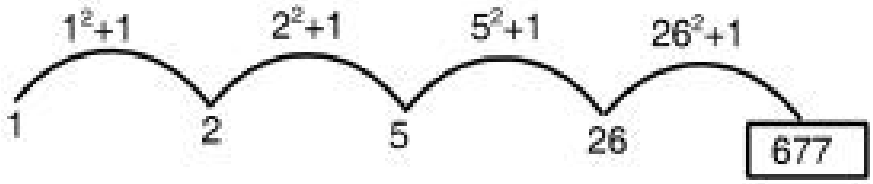
Answer:



216

Q30. 1, 2, 5, 26 _____.

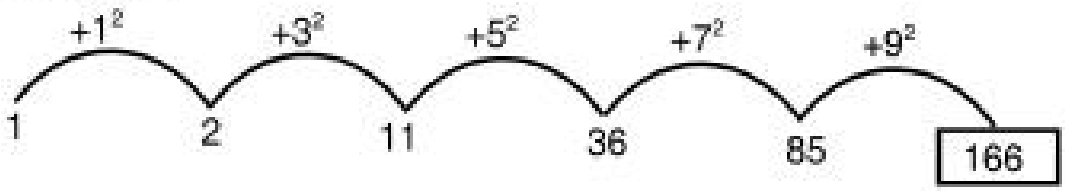
Answer:



677

Q31. 1, 2, 11, 36, 85 _____.

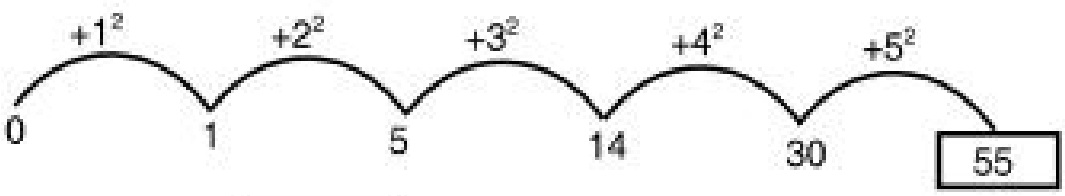
Answer:



166

Q32. 0, 1, 5, 14, 30 _____.

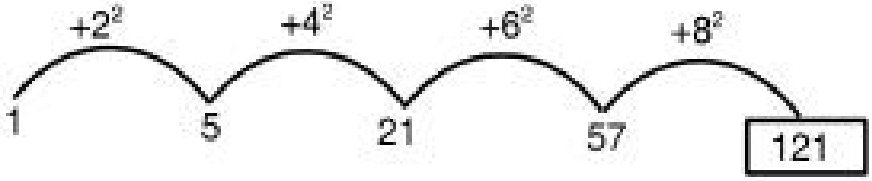
Answer:



55

Q33. 1, 5, 21, 57 _____.

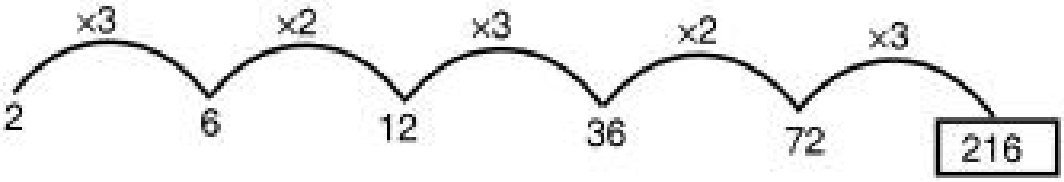
Answer:



121

Q34. 2, 6, 12, 36, 72 _____.

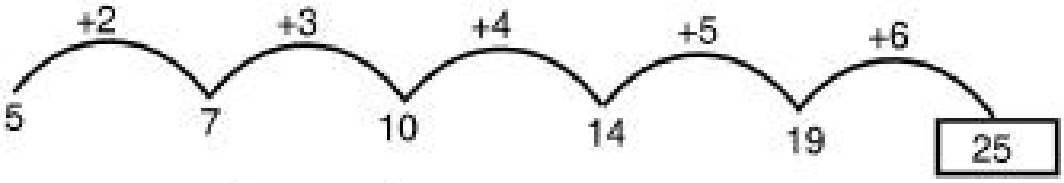
Answer:



216

Q35. 5, 7, 10, 14, 19 _____.

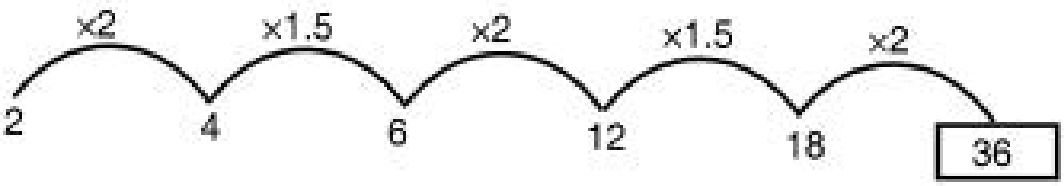
Answer:



25

Q36. 2, 4, 6, 12, 18 _____.

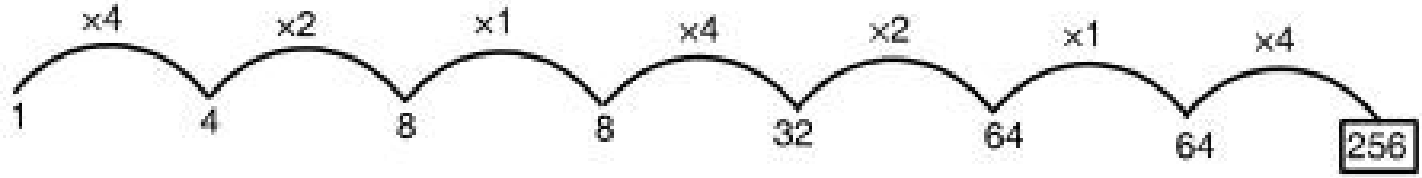
Answer:



36

Q37. 1, 4, 8, 8, 32, 64, 64 _____.

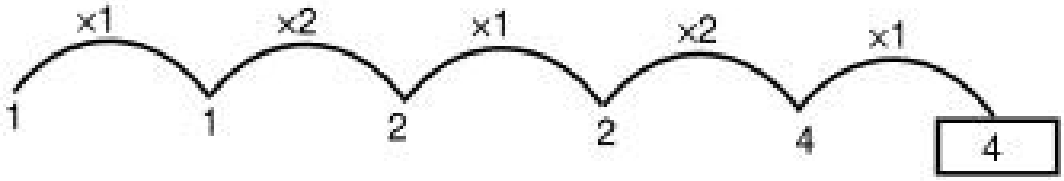
Answer:



256

Q38. 1, 1, 2, 2, 4 _____.

Answer:



4

Q39. 3, 4, 7, 16, 43 _____.

Answer:

(First multiply by 3 (previous term) and then subtract 5)

124

Q40. 3, 8, 18, 38 _____.

Answer:

First previous term multiply by 2 and then add 2

78

Q41. 1, 2, 5, 10, 17, 26 _____.

Answer:

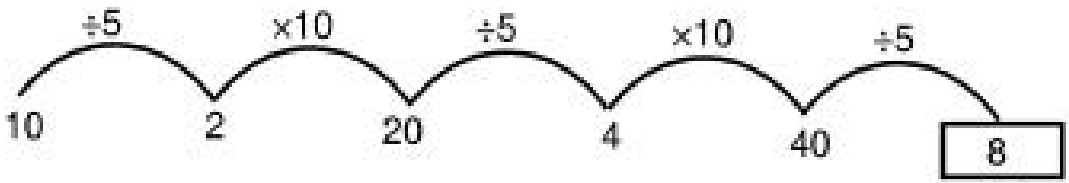
Follow the previous

0^2+1 1^2+1, 2^2+1, 3^2+1 _____.

37

Q42. 10, 2, 20, 4, 40 _____.

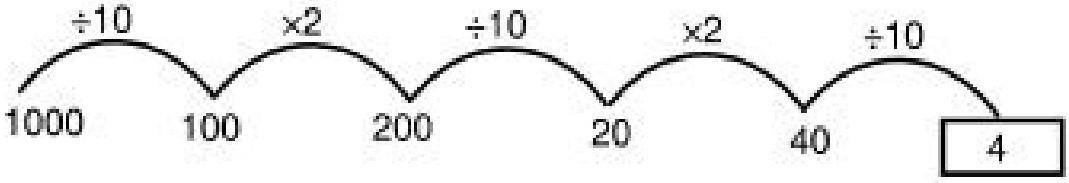
Answer:



8

Q43. 1000, 100, 200, 20, 40 _____.

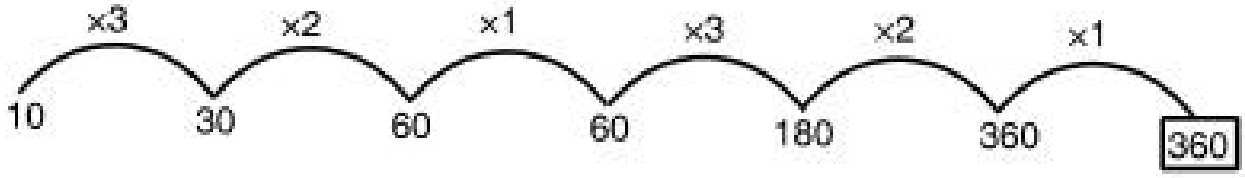
Answer:



4

Q44. 10, 30, 60, 60, 180, 360 _____.

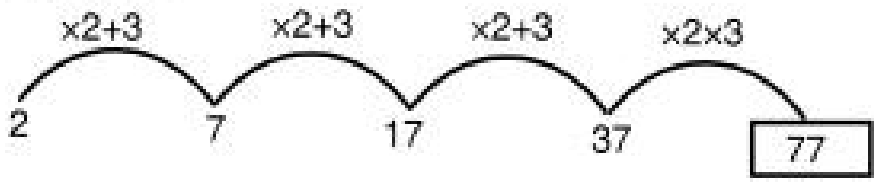
Answer:



360

Q45. 2, 7, 17, 37 _____.

Answer:



77

Q46. 1, 2, 6, 15 _____.

Answer:

$1+0^2$, 1^2+1^2, $1+1^2+2^2$, $1+1^2+2^2+3^2$

$1+1^2+2^2+3^2+4^2$

31

Q47. SCD, TEF, UGH _____ WKL.

(a) CNM (b) VJI

(c) VIJ (d) IJT

Answer:

There are two identical series here. The first series is with the first letter only. The second series involves the remaining letters CD, EF, GH, IJ, KL.

VIJ

Q48. C_3DE, CD_4E, CDE_5 _____ CD_7E.

(a) CDE (b) CDE_5

(c) C_6DE (d) CDE_4

Answer:

Letters are same change number only adding 1 in the previous one.

C_6DE

Q49. CMM, EOO, GQQ _____ KUU.

(a) GRR (b) GSS

(c) ISS (d) ITT

Answer:

The first letter will be in order CEGIK, and 2nd and 3rd letter is repeated.

ISS

Q50. ZA_5, Y_4B, XC_6, W_3D _____.

(a) E_6V (b) U_2E

(c) VE_5 (d) VE_7

Answer:

The 1st letter are alphabetically reverse, 2nd letter are in alphabetic order and numbers are 5, 4, 6, 3, 7.

VE_7

Q51. QPO, NML, KJI, HGF _____.

(a) EDC (b) HGE
(c) CAB (d) GHI

Answer:

This series consists of letters in a reverse alphabetic order.

EDC

Q52. AOU, BPV, CQW, DRX _____.

(a) ESY (b) JAK
(c) PFQ (d) MEN

Answer:

Follow an alphabetic order.

ESY

Q53. 1AZ, 2BY, 3CX, 4DW _____.

(a) 5EV (b) 6EU
(c) 7AE (d) 5FO

5EV

Q54. BCB, DED, FGF, HIH _____.

(a) HJH (b) JKJ
(c) KJK (d) HKH

Answer:

Follow an simple alphabetic order.

JKJ

Q55. A5A, C10C, E15E, G20G _____.

(a) I25I (b) I20I

(c) J25J (d) K20K

Answer:

Follow an simple alphabetic order.

I25I

Q56. QAR, RAS, SAT, TAU _____.

(a) UAV (b) UAT

(c) TAS (d) TAT

Answer:

In this 3rd letter will be the first letter in next and 3rd letter is in order.

UAV

Q57. AZ, BY, CX, DW _____.

(a) EV (b) FV

(c) VF (d) DV

Answer:

Simple forward and Backward steps.

EV

Q58. AZ, CX, FU _____.

(a) BC (b) JQ

(c) KP (d) PW

Answer:

First letter move forward by first, second, third _____ terms. Second letter move backwards.

JQ

Q59. AZ, GT, MN _____ YB.

(a) KF (b) XR
(c) HS (d) SH

Answer:

Moved forward and backward by six steps.

SH

Q60. CE, GI, KM, OQ _____.

(a) TW (b) TV
(c) TU (d) SU

Answer:

Letters of each term is alternate.

SU

Q61. D2E, H4J, L6O, P8T _____.

(a) T10Y (b) U5V
(c) L7O (d) X10Y

Answer:

First letter place is multiple of 4 while 3rd letter is multiple of 5.

T10Y

Q62. CAT, FDW, IGZ _____.

(a) KJA (b) TUV
(c) HDC (d) LJC

Answer:

All the letters of each terms moved three steps forward to obtain the corresponding letters.

LJC

Q63. BEH, KNO, TWZ _____.

(a) IJL (b) IFC
(c) CFI (d) RBI

Answer:

Each letter moved a steps forward.

CFI

Q64. FLP, INS, LPV _____.

(a) ORY (b) UXZ
(c) VXY (d) SVW

Answer:

1st and 3rd letter moved 3 steps which 2nd letter moved 2 steps.

ORY

Q65. LXF, MTJ, NPN, OLR _____.

(a) HAV (b) PHV
(c) PIU (d) PKX

Answer:

1st letter moved one step forward 2nd letter moved 4 step backward 3rd letter moved 4 step forward.

PHV

Q66. AB, BA, ABC, CBA, ABCD _____.

(a) DCBA (b) DCAB
(c) ABDC (d) BACD

Answer:

Inverse in next step.

DCBA

Q67. Coding and De-coding

Some words are stand for some another words which is known code of the word. Process of replacing some words by code word is known as coding. Decoding is reverse of coding.

The coding and Decoding is classified mainly into two types.

1. Letter coding
2. Numeric coding.

Letter Coding:

In this case Alphabets replaced by certain other Alphabets According to specific rule.

Numeric Coding:

In this case numeric values or letters can be changed according to specific rule.

Q68. MADRAS is coded as NBESBT, how is BOMBAY coded in that code?

Answer:

Each letter moved one step forward.

CPNCBZ

Q69. In a certain code, TRIPPLE is written as SQHOOKD. How is DISPOSE written in that code?

Answer:

Moved one step backward

CHRONRD

Q70. In a certain code, MONKEY is written as XDJMNL. How TIGER can be coded?

Answer:

First write the letter of the word in reverse order and then moved one step backward.

QDFHS

Q71. If DELHI is coded as 73541 and CALCUTTA as 82589662, then CALICUT be coded?

Answer:

∴ Coding as follows	D	E	L	H	I	C	A	U	T
	7	3	5	4	1	8	2	9	6

8251896

Q72. TWENTY is written as 863985 and ELEVEN is written as 323039, then TWELVE can be coded.

Answer:

∴ Coding as follows

T	W	E	N	Y	L	V
8	6	3	9	5	2	0

863203

Q73. In a system 15789 is coded as EGKPT and 2346 is coded as ALUR. How 23549 can be coded?

Answer:

Coding pattern is

1	5	7	8	9	2	3	4	6
E	G	K	P	T	A	L	U	R

ALGUT

Q74. How 184632 can be coded.

7	2	1	5	3	9	8	6	4
W	L	M	S	E	N	D	J	B

Answer:

MDBJEL

Q75. How 879341 can be coded.

7	2	1	5	3	9	8	6	4
W	L	M	S	E	N	D	J	B

Answer:

DWNEBM

Q76. If "isb ito inm" stands for "neat and tidy" "qpr inm sen" stands for "small but neat" "hsm sen rso" stands for "good but erratic" then "but" stands for.

Sen

Q77. In a certain code
"Pit dar na" means "you are good"
"dar tok pa" means "good and bad"
"Tim na tok" means "They are bad"
then "they" stands for

Answer:

Tim

Q78. In a coding
"37" means "which class"
"583" means "caste and class"
what is the code for caste

Answer:

either 5 or 8

Q79. In a coding
"743" means "mangoes are good"
"657" means "eat good food"
"934" means "mangoes are ripe"
then "ripe" can be coded as

Answer:

9

Q80. In a coding
"256" means "you are good"
"637" means "we are bad"
"358" means "good and bad"
then "and" is coded as
Answer:

8

Q81. What will be the coding for 649281

7	2	1	5	3	9	8	6	4
W	L	M	S	I	N	D	J	B

Answer:

JBNLDM

Q82. What will be the coding for MSINDWB

7	2	1	5	3	9	8	6	4
W	L	M	S	I	N	D	J	B

Answer:

1539874

Q83. Odd man out
Some thing or some one who differs markedly from the others in a group. These questions are based on words letters and numerals. In these problems we consider the defining quality of particular objects or things.

Q84. March May September December
Answer:
All are months having 31 days.

September

Q85. 25, 81, 144, 100, 90

Answer:

All are perfect square _____.

90

Q86. 1, 8, 27, 625, 124

Answer:

All are cubes

124

Q87. 5, 10, 25, 30, 42

Answer:

All are multiples of 5

42

Q88. 2, 3, 7, 9, 11

Answer:

All are prime no.

9

Q89. 2, 6, 88, 90, 58, 63

Answer:

All are even

63

Q90. 10, 19, 18, 16, 24

Answer:

All are even

19

Q91. Mithun, D, Sachin, T, Amitabh, B, Jitendra

Answer:

all are actors

Sachin T

Q92. P.T. Usha, Sania Mirza, Ganguli, Dharmendra

Answer:

all are Sports Star.

Dharmendra

Q93. Fish, Starfish, Crocodile, Hen

Answer:

all are aquatic

Hen

Q94. 2, 3, $\frac{4}{3}$, $\frac{1}{2}$, $\sqrt{3}$

Answer:

all are rational nos.

$\sqrt{3}$

Q95. 1, 2, 6, 15, 31, 56, 91

Answer:

Patter is 1, $1+1^2 = 2$

$2 + 2^2 = 6$ | $15 + 4^2 = 31$ | $56 + 6^2 = 92$

$6 + 3^2 = 15$ | $31 + 5^2 = 756$ |

91

Q96. 8, 13, 21, 32, 47, 63, 83

Answer:

go on adding 5, 8, 11, 14, 17, 20

47

Q97. 22, 33, 66, 77, 121, 279, 594

Answer:

rest are multiple of 11

279

Q98. 835, 734, 642, 751, 853, 981

Answer:

Pattern: The difference of third and first dight is the middle one.

751

PAST YEAR QUESTIONS AND ANSWERS

2018 - MAY

[1] In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code?

(a) 318826 (b) 318286

(c) 618826 (d) 338816 (1 mark)

Answer:

(a)

R	I	P	P	L	E
↓	↓	↓	↓	↓	↓
6	1	3	3	8	2

L	I	F	E
↓	↓	↓	↓
8	1	9	2

then

P	I	L	L	E	R
↓	↓	↓	↓	↓	↓
3	1	8	8	2	6

PILLER is written in that code is 318826

[2] In a certain code '256' means 'you are good', '637' means 'we are bad' and '358' means 'good and bad'. Which of the following represents 'and' in that code?

(a) 2 (b) 5
(c) 8 (d) 3 (1 mark)

Answer:

(c) 256 means 'You are good'
637 means 'We are Bad'
358 means 'Good and Bad'

Here Code of 'are' is 3
Code of good is 5
Code of and is 8

[3] If LOSE is coded as 1357 and GAIN is coded as 2468, what do figure 82146 for ?

(a) NGLAI (b) NGLIA
(c) GNLIA (d) GNLA (1 mark)

Answer:

(a)

L	O	S	E		G	A	I	N		8	2	1	4	6
↓	↓	↓	↓		↓	↓	↓	↓		↓	↓	↓	↓	↓
1	3	5	7		2	4	6	8		N	G	L	A	I

82146 is stands for NGLAI

2018 - NOVEMBER

[4] If PLAY is coded as 8123 and RHYME is coded as 49367. What will be code of MALE?

(a) 6217 (b) 6198
(c) 6395 (d) 6285 (1 mark)

Answer:

(a) If

P L A Y and R H Y M E

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

8 1 2 3 4 9 3 6 7

then Code for

M A L E

↓ ↓ ↓ ↓

6 2 1 7

[5] Find out the next number in the following series 7, 11, 13, 17, 19, 23, 25, 29?

(a) 30 (b) 31

(c) 32 (d) 33 (1 mark)

Answer:

(b) Given series

7, 11, 13, 17, 19, 23, 25, 29, 31

[6] If HONEY is coded as JQPGA, which word is code as VCTIGVU?

(a) CARPETS (b) TRAPETS

(c) TARGETS (d) UMBRELU (1 mark)

Answer:

(c)

H O N E Y then V C T I G V U

+2 +2 +2 +2 +2 -2 -2 -2 -2 -2 -2 -2

J Q P G A T A R G E T S

[7] Find odd man out of the following series 15, 21, 63, 81, 69

(a) 15 (b) 21

(c) 63 (d) 81 (1 mark)

Answer:

(d) 15, 21, 63, 81, 69

∵ Only 81 is a Perfect square.

[8] Find odd man out of the following series 7,9, 13, 17, 19

(a) 7 (b) 9

(c) 19 (d) 13 (1mark)

Answer:

(b) 7,9,13,17,19

9 is the odd man out

Since 9-7= 2 x 11 -7=4

13-9 = 4 13 -11 = 2

17-13 = 4 17-13 = 4

19 - 17 = 2 19 - 17 = 2

2019 - JUNE

[9] 7,23,47,119,167

(a) 211

(b) 223

(c) 287

(d) 319 (1 mark)

Answer:

(c) 7, 23, 47, 119, 167, 287

$3^2 - 2 = 7$

$5^2 - 2 = 23$

$7^2 - 2 = 47$

$11^2 - 2 = 119$

$13^2 - 2 = 167$

$17^2 - 2 = 287$

[10] Which of the following is odd one 4,12,44,176, 890 .....

(a) 4

(b) 12

(c) 44

(d) 176 (1 mark)

Answer:

(c) 4, 12, 44, 176, 890,

$4 \times 2 + 4 = 12$

$12 \times 3 + 6 = 42$

$42 \times 4 + 8 = 176$

$176 \times 5 + 10 = 890$

[11] If in a Certain language, MADRAS is code as NBESBT, How is BOMBAY coded in that language?

(a) CPNCBX

(b) CPNCBZ

(c) CPOCBZ

(d) CQOCBZ (1 mark)

Answer:

(b)

	M	A	D	R	A	S		B	O	M	B	A	Y
+1	↓	↓	↓	↓	↓	↓	+1	↓	↓	↓	↓	↓	↓
	N	B	E	S	B	T		C	P	N	C	B	Z

[12] Which of the following is odd one

(a) CEHL

(b) KMPT

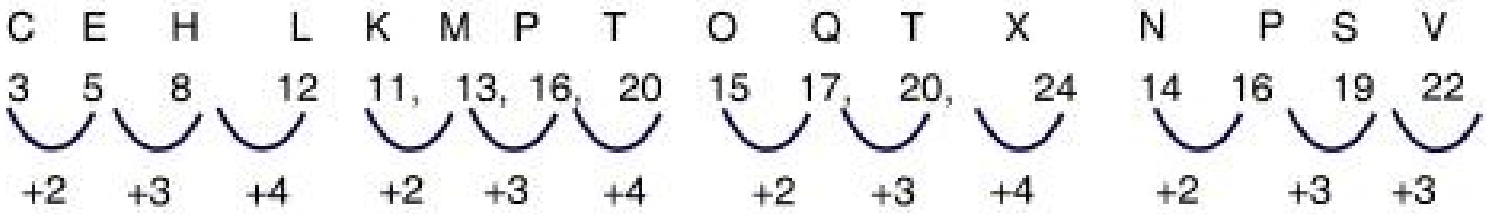
(c) OQTX

(d) NPSV (1 mark)

Answer:

(d)

C	E	H	L	→	3,	5,	8,	12
K	M	P	T	→	11,	13,	16,	20
O	Q	T	X	→	15,	17,	20,	24,
N	P	S	V	→	14,	16,	19,	22,



So NPSV is odd one.

2019 - NOVEMBER

[13] Complete the series.
4, 16, 36, 64, 100 _____.
(a) 144
(b) 121
(c) 49
(d) 120 (1 mark)

Answer:

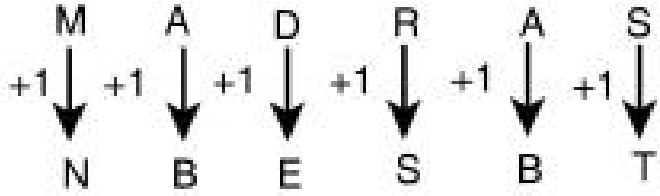
(a) Given series
4, 16, 36, 64, 100, _____
$2^2 = 4$, $4^2 = 16$, $6^2 = 36$, $8^2 = 64$, $10^2 = 100$
The series is of squares of even no.'s so after 100 it will be
$12^2 = 144$

[14] In a certain code MADRAS is NBESBT now DELHI is coded as:
(a) EMMJI
(b) JIFEM
(c) EFMIJ
(d) CDKGH (1 mark)

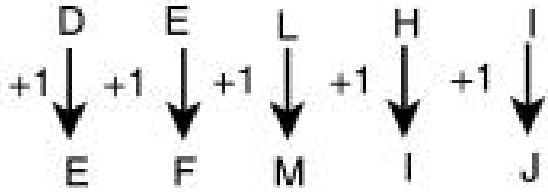
Answer:

(c) $5^2 = 25$
$7^2 = 49$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$

All the other no.'s are square of some natural no.'s except 143
So 143 is odd one out



Similarly,



[15] Find the odd man out 5, 10, 17, 27, 37:
(a) 5
(b) 17
(c) 27
(d) 10 (1 mark)

Answer:
(c) $(2 \times 2) + 1 = 5$
$(3 \times 3) + 1 = 10$
$(4 \times 4) + 1 = 17$
But $(5 \times 5) + 1 \neq 27$
$(6 \times 6) + 1 = 37$
So 27 is odd man out.

[16] Complete the series
4, 16 _____ 256, 1024
(a) 32
(b) 48
(c) 64
(d) 46 (1 mark)

Answer:
(c) $4^1 = 4$
$4^2 = 4 \times 4 = 16$
$*4^3 = 4 \times 4 \times 4 = 64*$

$4^4 = 4 \times 4 \times 4 \times 4 = 256$

$4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$

So the third term of the series is 64.

[17] SYSTEM is coded as 131625 then TERMS will be coded as?

(a) 62251
(b) 62451
(c) 64251
(d) 62415 (1mark)

Answer:

(b) Since in

S	Y	S	T	E	M
1	3	1	6	2	5

{given}

∴

T	E	R	M	S
6	2	4	5	1

As R cannot be 2 as E has already been assigned the value as 2

∴ R' S value left will be 4 as per given option.

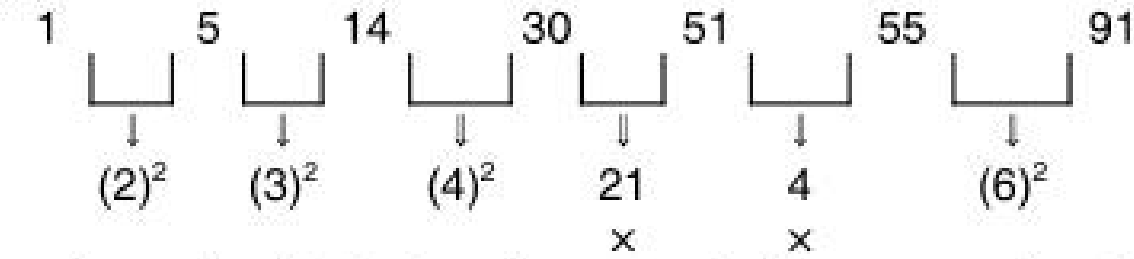
[18] Find the odd man out,

1, 5, 14, 30, 51, 55, 91

(a) 5
(b) 55
(c) 51
(d) 91 (1 mark)

Answer:

(c)



as the series is having the sum of all squares of natural number therefore 51 is the odd number.

CHAPTER

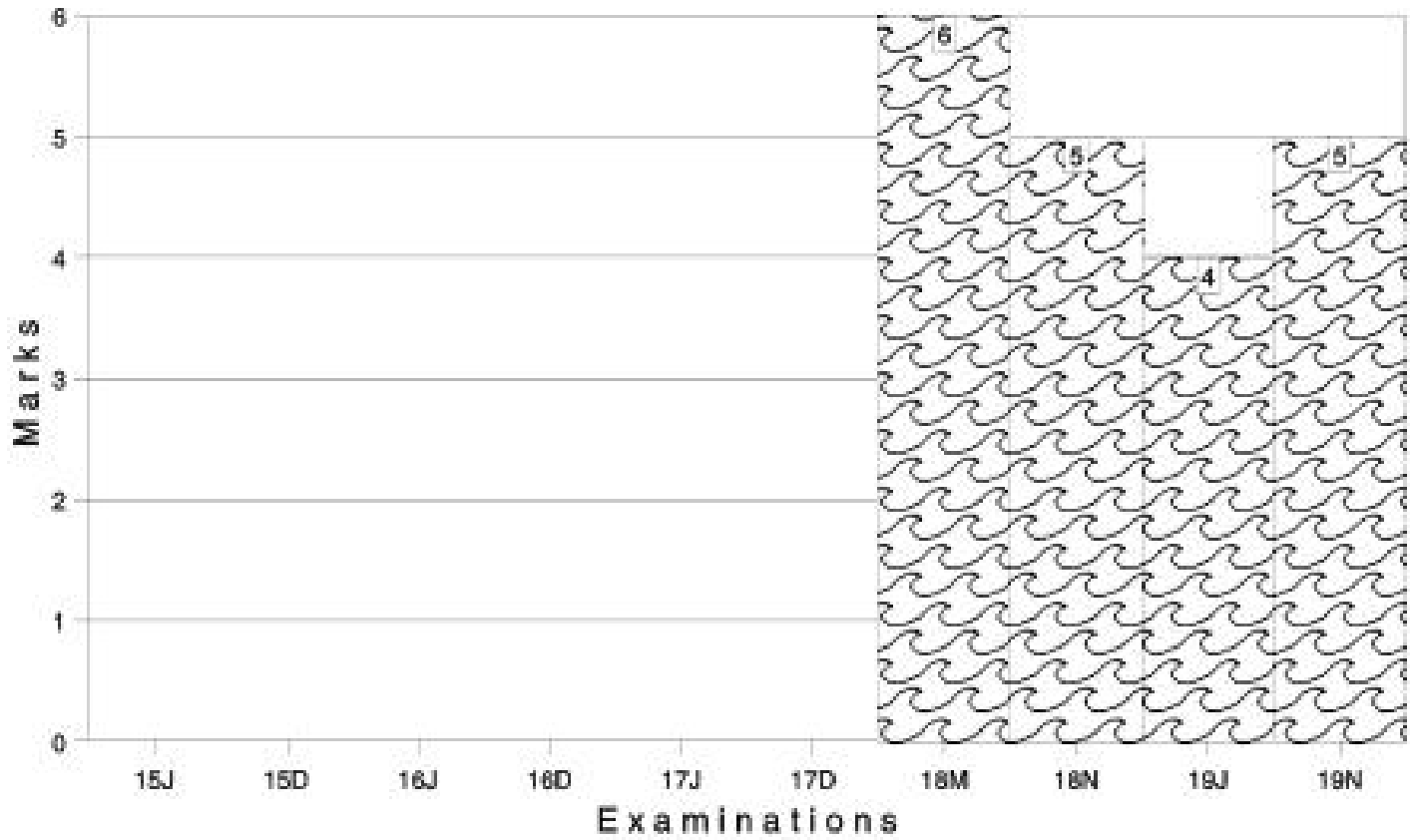
10

DIRECTION TESTS

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical

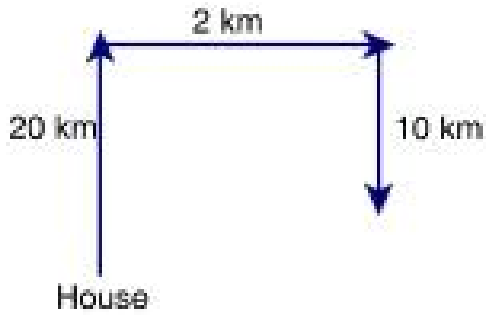


PRACTICE QUESTIONS OF MCQ

Q.1: Jairaj started from her house walked 20 km towards North. Now, he took a right turn and moved 2 km. Again he took right turn and walked for 10 km. In which direction he is going?

(a) North
(b) South
(c) East
(d) West

Answer:

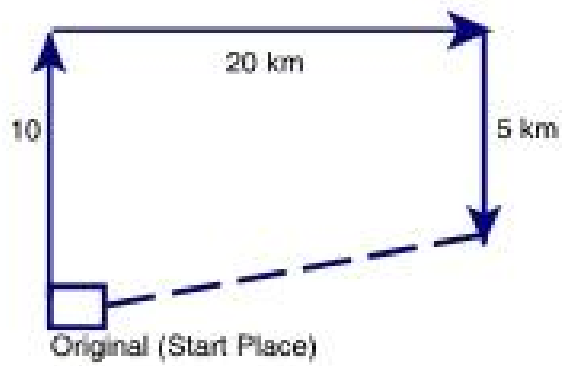


Moving towards | South |

Q.2: Raj moved 10 km towards North, now he took a right turn and moved 20 km. again took turn towards South and moved upto 5 km. From the original position he is in which direction.

(a) North
(b) South
(c) N-E
(d) S-E

Answer:

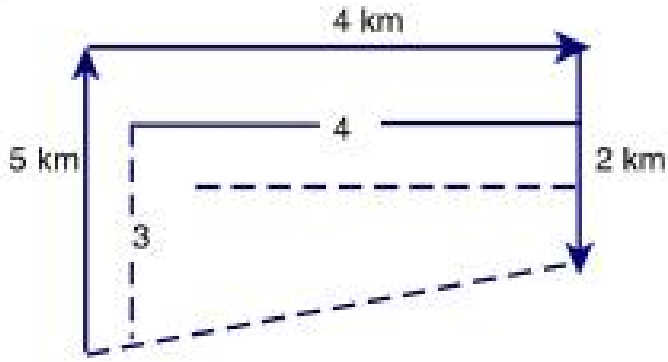


Now he is in N-E direction.

Q.3: Jai is going towards North upto 5 km he turned clockwise and moved upto 4 km and he took right turn moved upto 2 km. Find his distance from the start place.

(a) 2 km
(b) 3 km
(c) 4 km
(d) 5 km

Answer:



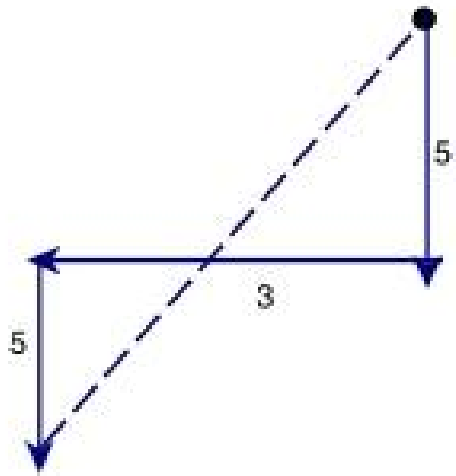
5 km from the start point.

Q.4: A man walks 5 km toward South and then turns to the right. After walking 3 km he turns to the left and walks five km. Now his direction from starting point is?

(a) West
(b) North
(c) N-E
(d) S-W

Answer:

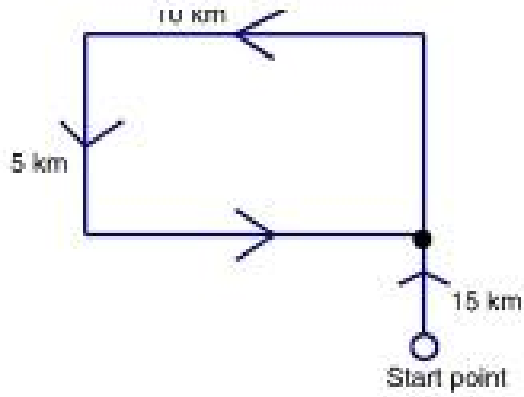
S - W



Q.5: From his house, Aditya went 15 km to the North. Then he turned west and covered 10 km. Then he turned South and covered 5 km. Finally turning to the East, he covered 10 km. In which direction is he from his house?

(a) East
(b) South
(c) North
(d) West

Answer:

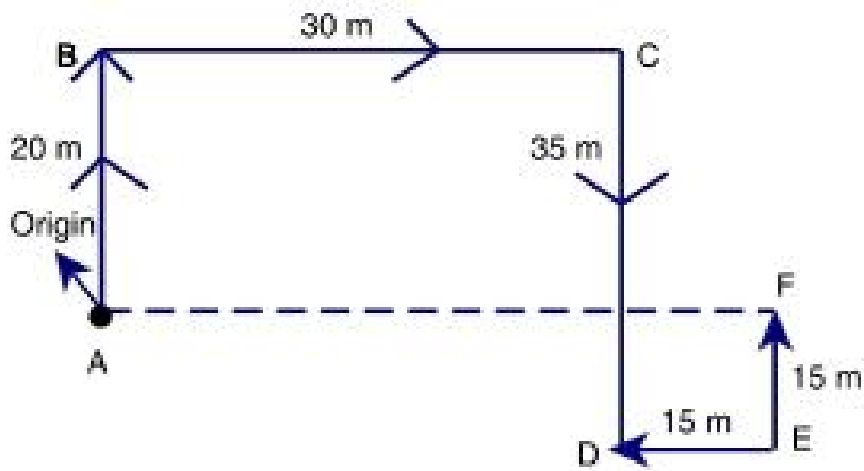


He is in North.

Q.6: Aditya walked 20 m towards North. Then he turned right and walks 30 m. Then he turns right and walk 35 m. Then he turns left and walk 15 m. Finally he turns left and walk 15 m. In which direction and how many meters is he from the starting point?

(a) 15 m West
(b) 30 m East
(c) 30 m West
(d) 45 m East

Answer:

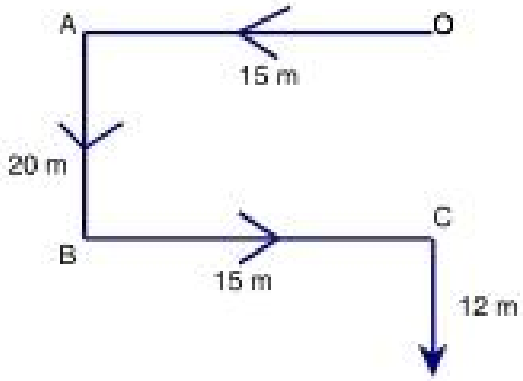


He is in 15 m in East

Q.7: Starting from the points O, Vaibhav walked 15 m towards West. He turned left and walked 20 m. Again turned left and walked 15 m. Now he turned to his right and walked 12 m. How far he is now from the point O.

(a) 32 m (b) 47 m
(c) 42 m (d) 27 m

Answer:

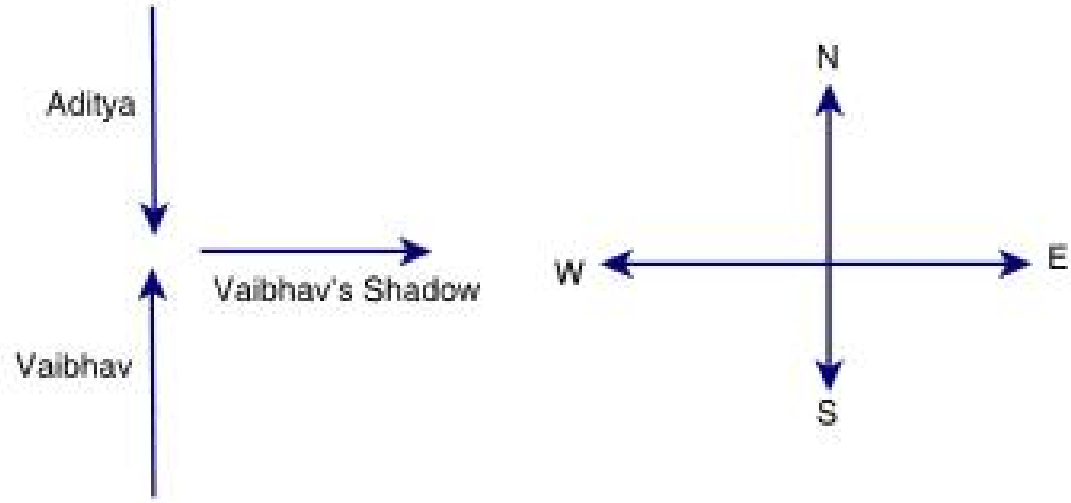


Now he is 32 m away from the points O.

Q.8: One evening before sunset Aditya and Vaibhav were talking, to each other face to face. If Vaibhav's shadow was exactly to the right of Vaibhav. Which direction was Aditya facing?

(a) North (b) South
(c) East (d) West

Answer:

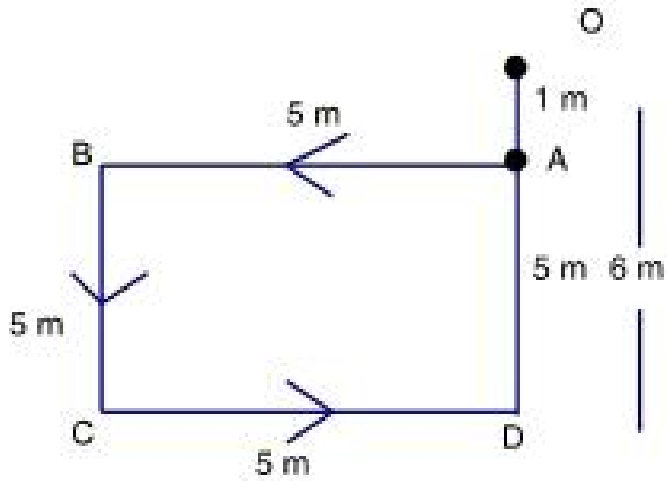


Aditya was facing towards South.

Q.9: Jay(Starts from A) walked 5 m towards West, then turned left and walked 5 m. again turned left and walked 5 m. again turned left and walked 6 m. How far he is from A.

(a) 1 m
(b) 2 m
(c) 3 m
(d) 4 m

Answer:

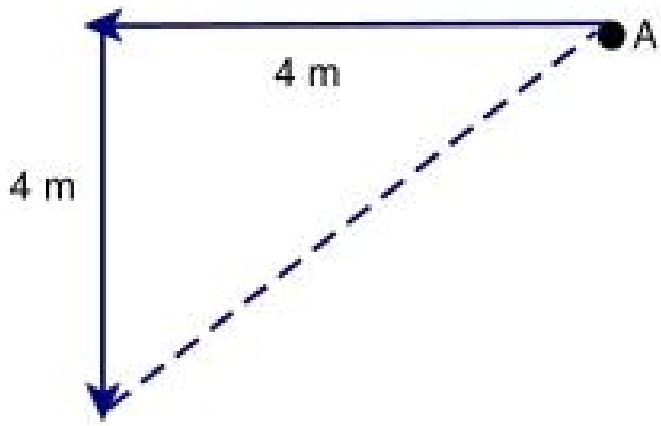


He is 1 m away.

Q.10: Raj started from point A, and walked towards West 4 m, then he turned towards South upto 4 m. In which direction he is from start point.

(a) W-S
(b) N-E
(c) South
(d) West

Answer:



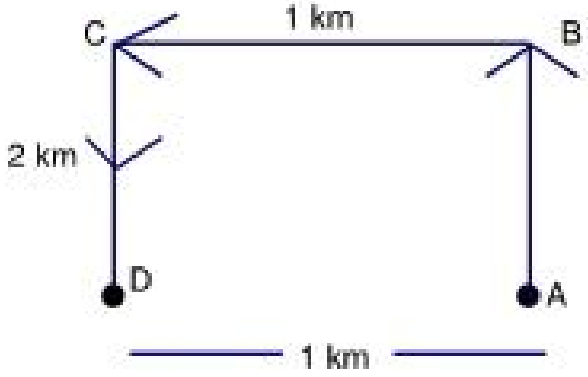
He is in South - West direction.

Q.11: A boy rode his bicycle Northward, then turned left and rode 1 km and again turned left and rode 2 km. He found himself 1 km West of his starting point. How far did he ride Northward initially?

(a) 1 km (b) 2 km

(c) 3 km (d) 4 km

Answer:



The boy rode 2 km Northward.

Q.12: A man starts from a point, walk 3 km towards North, turns towards his right and walks 3 km, turns right again and walk. What is the direction now he is facing?

(a) South (b) North

(c) West (d) East

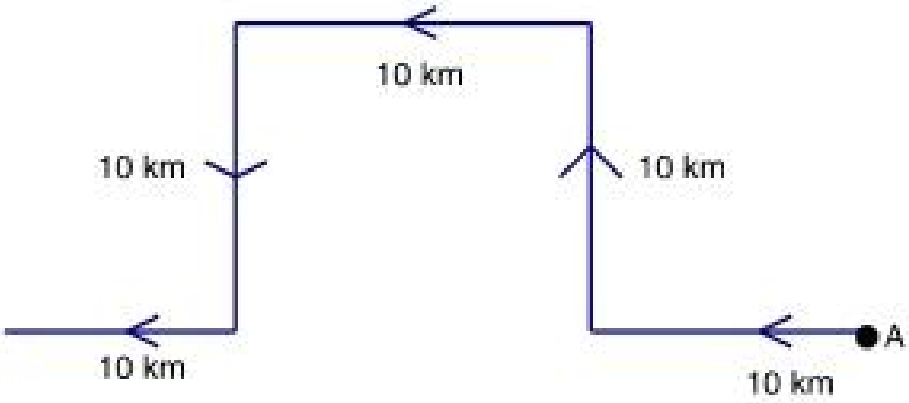
Answer:

Facing towards South.

Q.13: Sona walks 10 km towards West, then turns right and walks 10 km. Again she take left turn and walk 10 km. She turned again towards left and moved upto 10 km. Finally she turns right and walk 10 km. How far she is now from start.

(a) 10 m
(b) 20 m
(c) 30 m
(d) 40 m

Answer:

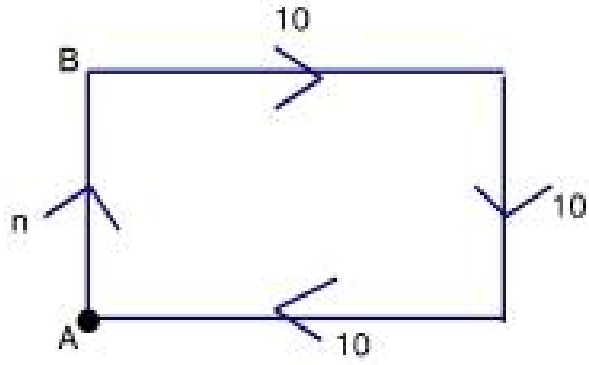


She is 30 m away from start point (A).

Q.14: A person moves towards North upto n meters, then walked towards East upto 10 km, then take right turn towards South and walked upto 10 km, again he moved towards West and reached at start point. What is the value of n.

(a) 10 km
(b) 20 km
(c) 1 km
(d) 2 km

Answer:

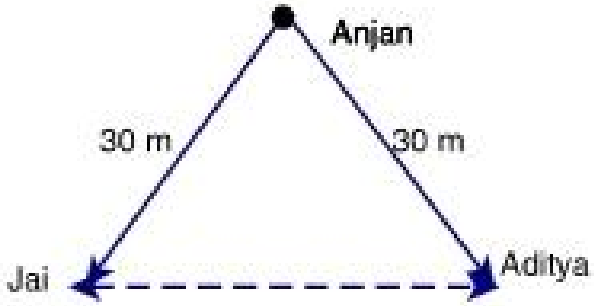


n = 10

Q.15: Jai is in which direction of Aditya. If Jai is 30 m S-W of Anjan. Aditya is 30 m S-E of Anjan.

(a) West
(b) East
(c) South
(d) North

Answer:

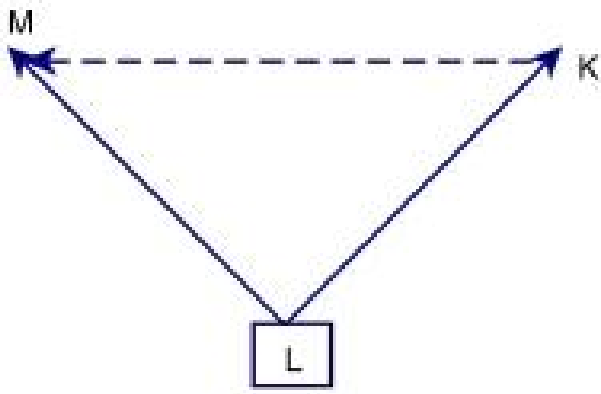


Jai is in West direction of Aditya.

Q.16: K is 40 m N-E of L, M is 40 m N-W of L. Then M is in which direction of k.

(a) East
(b) West
(c) South
(d) North

Answer:

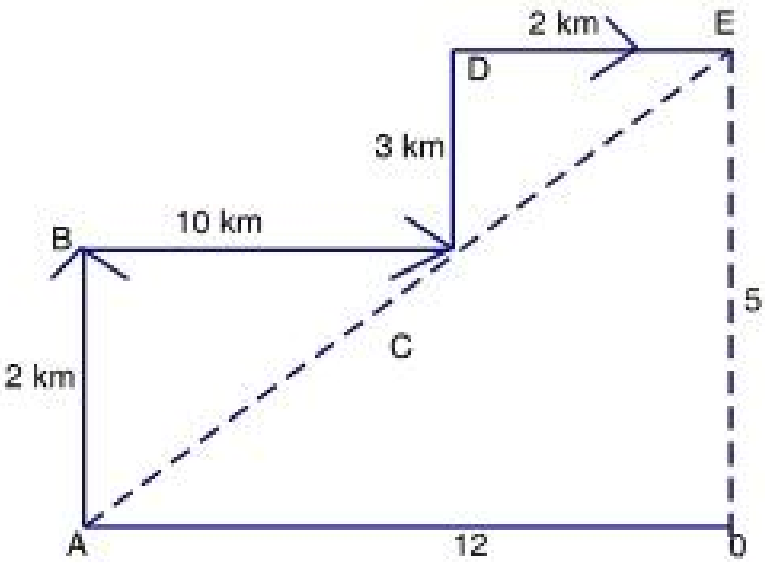


M is in West of K.

Q.17: A man walks 2 km towards North. Then he turns to East and walks 10 km. After this he turns to North and walks 3 km. Again he turns towards East and walks 2 km. How far is he from the starting point?

(a) 10 km
(b) 13 km
(c) 15 km
(d) 17 km

Answer:

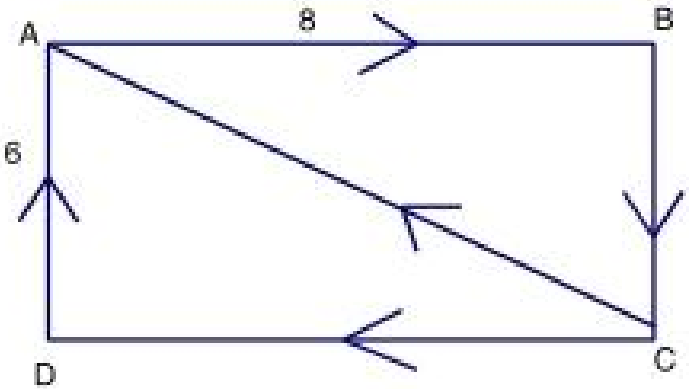


$AE^2 = A0^2 + 0E^2$
$AE^2 = 12^2 + 5^2$
$AE = 13$

Q.18: The length and breath of a room are 8 m and 6 m respectively. A cats runs along all the four walls and finally along a diagonal order to catch a rat. How much total distance is covered by the cat?

(a) 10 (b) 14
(c) 38 (d) 48

Answer:



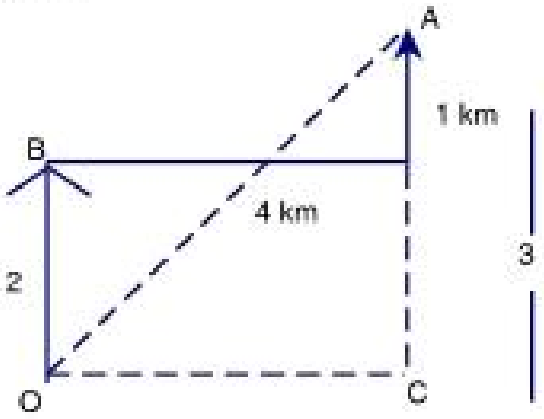
= Perimeter + AC
= $28 + \sqrt{8^2 + 6^2}$
= 28 + 10

Q.19: A man starts from O and moved towards North 2 km, then take right turn and moved 4 km. again turned towards North and walked utpo 1km reached at A. Find distance between OA.

(a) 6 (b) 7

(c) 4 (d) 5

Answer:



$OA^2 = OC^2 + AC^2$

$OA = \sqrt{16+9} = \sqrt{25} = 5$

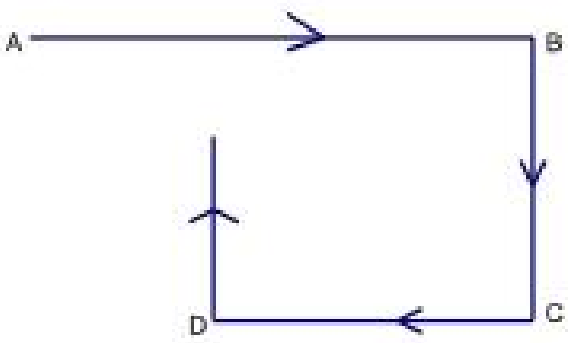
OA = 5 km.

Q.20: One morning Sujata started to walk towards the Sun. After covering some distance she turned to right then again to the right and after covering some distance she again turns to the right. Now in which direction is she facing?

(a) North (b) South

(c) N-E (d) S-W

Answer:



Sujata will face towards North.

Q.21: One day morning after sunrise, Vimal started to walk. During this he met Sheru who was coming from opposite direction. Vimal watch the shadow of Sheru to the right of him (Vimal) to which direction Vimal was facing?

(a) South (b) North
(c) East (d) West

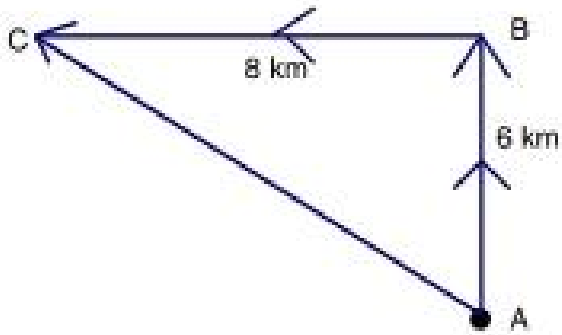
Answer:

(a) Sun rises in the East. So the shadow of a man will always fall towards the West. Since the shadow of Sheru is to the right of Vimal. Hence Vimal is facing towards South.

Q.22: Ramu is going towards North from his house. After covering a distance of 6 km he turned towards left and covered a distance of 8 km. What is the shortest distance from his house?

(a) 15 km (b) 10 km
(c) 25 km (d) 20 km

Answer:

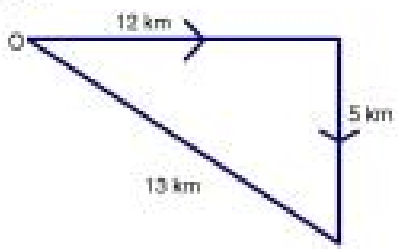


$CA = \sqrt{8^2 + 6^2} = 10$

Q.23: Ramji is going towards East, after 12 km he takes a right. Right turn and travel upto 5 km. Find his shortest distance now from his original place.

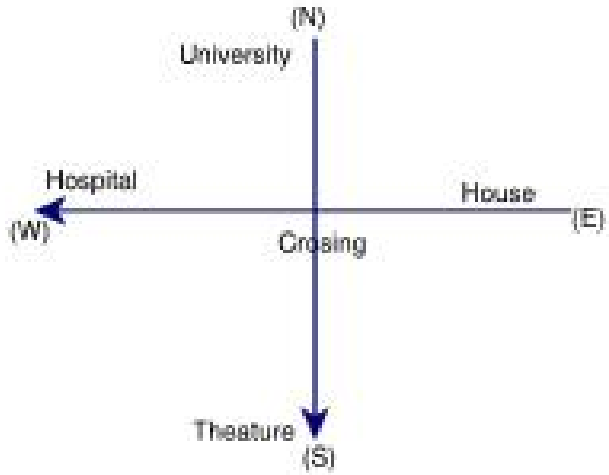
(a) 13 km (b) 12 km
(c) 5 km (d) 17 km

Answer:



Q.24: Shivam started from his house towards West. After walking a distance of 15 km he turned to the right and walked 10 km. He again turned to the right and walked 5 km. After this he is to turn right at 135 and covered 10

Answer:
(a) North



Q.27: After walking 6 km, I turned to the right and then walked 2 km. After that I turned to the left and walked 10 km. In the end, I was moving towards the North. From which direction did I start my Journey?

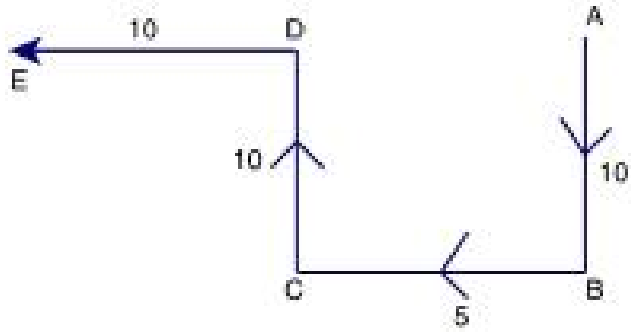
(a) North
(b) South
(c) East
(d) West

Answer:
(b) South

Q.28: Ravi left home and cycled 10 km towards South, then he turned right and cycled 5 km and then again turned right and cycled 10 km. After this he turned left and cycled 10 km. How many km. will he have to cycle to reach his home straight?

(a) 10 km
(b) 15 km
(c) 12 km
(d) 17 km

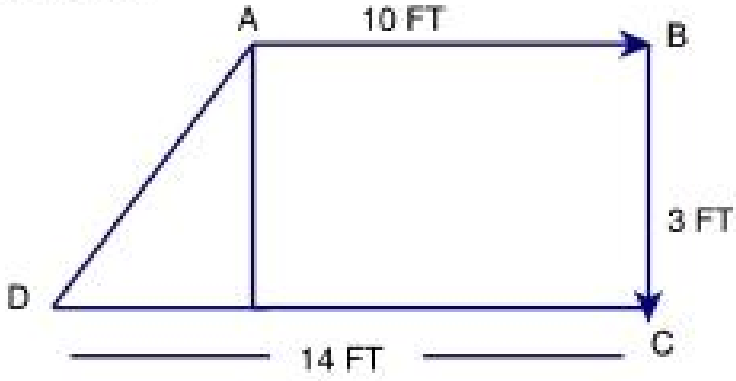
Answer:



Q.29: Reena walked from A to B in the East 10 Feet. The she turned to the right and walked 3 Feet. Again she turned to the right and walked 14 Feet. How far is she from A?

(a) 4 FT
(b) 5 FT
(c) 12 FT
(d) 13 FT

Answer:



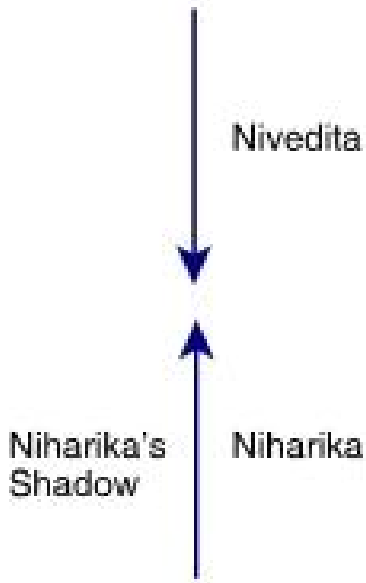
AD = 5 FT

Q.30: One morning after Sunrise Nivedita and Niharika were talking to each other face to face at Bara Chauraha. If Niharika's shadow was exactly to the right of Nivedita, which direction Niharika was facing?

(a) North
(b) South
(c) East
(d) West

Answer:

(a) North



Q.31: If A × B means A is to the South of B. A + B means A is to the North of B, A% B means A is to East of B, A – B means A is to the West of B, then in P% Q + R – S, S is in which direction with respect to Q.

(a) S-W
(b) S-E
(c) N-E
(d) N-W

Answer:

(b) S-E

Q.32: One morning after Sunrise, Suresh was standing facing a bole. The shadow of the bole fell exactly to his right. To which direction was he facing?

(a) East
(b) West
(c) North
(d) South

Answer:

(d) South

Q.33: Four friends A, B, C, D live in a same locality. The house of B is in the East of A's house but in the North of C's house. The house of C is in the West of D's house. D's house is in which direction of A's house?

(a) S-E
(b) N-E
(c) East
(d) North

Answer:

(a) S-E

Q.34: Shyam walks 5 km towards East and then turns left and walks 6 km. Again he turns right and walks 9 km. Finally he turns to his right and walks 6 km. How far is he from the starting point?

(a) 26 km
(b) 21 km
(c) 14 km
(d) 9 km

Answer:

(c) 14 km

Q.35: Amit started walking positioning his back towards the sun. After some time, he turned left, then turned right and towards the left again. In which direction is he going now?

(a) North or South
(b) East or West
(c) North or West
(d) South or West

Answer:

Q.36: Rohit walked 25 m towards South. Then he turned to his left and walked 20 m. He then turned to his left and walked 25 m. He again turned to his right and walked 15 m. At what distance is he from the starting point and in which direction?

(a) 35 m East
(b) 35 m North
(c) 30 m West
(d) 45 m East

Answer:

(a) 35 m East

Q.37: Village Q is to the North of the village P. The village R is in the East of village Q. The village S is to the left of the village P. In which direction is the village S with respect to village R?

(a) West
(b) S-W
(c) South
(d) N-W

Answer:

(b) S-W

Q.38: Sachin walk 20 km towards North. He turns left and walks 40 km. He again turns left and walks 20 km. Finally he moves 20 km after turning to the left. How far is he from his starting point.

(a) 20 km
(b) 40 km
(c) 60 km
(d) 10 km

Answer:

(a) 20 km

Q.39: Mohan starts from point A and walk 1 km towards South, turns left and walk 1 km. Then he turns left again and walk 1 km. Now he is facing.

(a) East
(b) West
(c) North
(d) South

Answer:

(c) North

Q.40: A starts from a point and walks 5 km North, turns left and walk 3 km. Again turns left and walk 5 km. In which direction he is going how.

(a) North
(b) South
(c) East
(d) West

Answer:

Q.41: A man started walking West. He turned right, then right again and finally turned left. In which direction he is going now?

(a) North (b) South
(c) West (d) East

Answer:

(a) North

Q.42: If a man on moped starts from a point and rides 4 km South then turns left and rides 2 km and turn again to the right to ride in which direction is he moving?

(a) North (b) West
(c) East (d) South

Answer:

(d) South

Q.43: If X stands on his head with his face towards South, to which direction will his left hand point?

(a) East (b) West
(c) North (d) South

Answer:

(b) West

Q.44: If A stands on his head with his face towards North. In which direction will his left hand point?

(a) N-E (b) North
(c) East (d) N-W

Answer:

(c) East

Q.45: A car traveling from South covers a distance of 4 km, then turns right and runs another 5 km and again turns to the right and was stopped. Which direction does it face now?

(a) South (b) North
(c) West (d) East

Answer:

(a) South

Q.46: Town D is 12 km towards the North of town A. Town C is 15 km towards the West of town D. Town B is 15 km towards the West of town A. How far and in which direction is town B from town C?

(a) 15 km towards North
(b) 12 km towards North
(c) 3 km towards South
(d) 12 km towards South

Answer:

(d) 12 km towards South

Q.47: Town D is to the West of town M. Town R is to the South of town D. Town K is to the East of town R. Town K is towards which direction of town D.

(a) South
(b) East
(c) N-E
(d) S-E

Answer:

(d) S-E

Q.48-49: A vehicle starts from point P and runs 10 km towards North. It takes a right turn and runs 15 km. It now runs 6 km after taking a left turn. It finally takes a left turn, runs 15 km and stops at point Q.

Q.48: How far is point Q with respect to point P?

(a) 16 km
(b) 25 km
(c) 4 km
(d) 0 km

Answer:

(a) 16 km

Q.49: Towards which direction was the vehicle moving before it stopped at point Q?

(a) North
(b) East
(c) South
(d) West

Answer:

(d) West

Q.50: Dev walks 20 m towards North. He then turns left and walks 40 m. He again turns left and walks 20 m. Further, he moved 20 m after turning to the right. How far is he from his original position?

(a) 20 m
(b) 30 m
(c) 50 m
(d) 60 m

Answer:

Q.51-52: A man went to meet his uncle in another village situated 5 km away in the N-E direction of his own village from there he came to meet his father-in-law living in a village situated 4 km in the South of his uncle's village.

Q.51: How far away he is? From his house.

(a) 3 km
(b) 4 km
(c) 5 km
(d) 6 km

Answer:

(a) 3 km

Q.52: Which direction he is now from his house?

(a) North
(b) East
(c) South
(d) West

Answer:

(b) East

Q.53: P,Q,R,S and T are sitting in a straight line facing North. P sits next to S. Q is sitting next to R who sits on the extreme left corner. Who sits to the left of S if T does not sit next to Q?

(a) P
(b) Q
(c) R
(d) T

Answer:

(b) Q

Q.54: Roza walked 25 m towards South, took a right turn and walked 15 m. She then took a left turn and walked 25 m. She is in which direction from starting point:

(a) S-E
(b) South
(c) S-W
(d) N-W

Answer:

(c) S-W

Q.55: Swati starts from a point walks 2 km towards North, turns towards her right and walks 2 km, turns right again and walks. In which direction is she going now?

(a) North
(b) S-E
(c) South
(d) West

Answer:

Q.56: After walking 6 km, I turned right and covered a distance of 2 km, then turned left and covered a distance of 10 km. In the end, I was moving towards the North. From which direction did I start my Journey?

(a) North (b) South
(c) East (d) West

Answer:

(b) South

Q.57: Rashmi leaves from her home. She first walks 30 m in N-W direction and then 30 m in S-W direction. Next, she walks 30 m in S-E direction. Finally, she turns towards her house. In which direction is she moving?

(a) N-E (b) N-W
(c) S-E (d) S-W

Answer:

(a) N-E

Q.58: I am facing South. I turn right and walk 20 m. Then I turn right again and walk 10 m. Then I turn left and walk 10 m and then turning right walk 20 m. Then I turn right again and walk 60 m. In which direction I am from the starting point?

(a) North (b) E-N
(c) East (d) N-E

Answer:

(d) N-E

Q.59: A Walks 3 km North and then he turns left and goes 2 km. He again turns left and goes 3 km. He turns right and walks straight. In which direction is he walking now?

(a) East (b) West
(c) North (d) South

Answer:

(b) West

Q.60: A walks Southwards, then turn right, then left and then right. In which direction is he from the starting point?

(a) South (b) East
(c) West (d) North

Answer:

Q.61: A boy start walking towards West he turns right and again he turns right and then turn left at last. He is in which direction?

(a) West (b) North
(c) South (d) East

Answer:

(b) North

Q.62: A person moves 4 km towards West, then turns right and moves 3 km and then turn right and moves 6 km, which direction he is moving?

(a) East (b) West
(c) North (d) South

Answer:

(a) East

Q.63: I started walking down a road in the morning facing the Sun. After walking for some time I turned to my left. Then I turned to my right. In which direction was I going then?

(a) East (b) West
(c) North (d) South

Answer:

(a) East

Q.64: You go North, turn right, then right again and then go to left. In which direction are you now?

(a) South (b) East
(c) West (d) North

Answer:

(b) East

Q.65: Suman starts from a point, walks 2 km towards North, turns towards her right and walks 2 km, turns right again and walks. What is the direction she is facing now?

(a) East (b) West
(c) South (d) North

Answer:

(c) South

Q.66: Ramu starts walking in the morning facing the sun. After sometime, he turned to the left later again he turned to his left. Ramu is in which direction?

(a) East (b) West
(c) South (d) North

Answer:

(b) West

Q.67: Mr. A walks Southwards and then turn right, then left and then right. He is in which direction?

(a) South (b) North
(c) West (d) S-N

Answer:

(c) West

Q.68: One morning, Rita started to walk toward the sun. After walking a while, she turned to her left and again to her left. She again turned right. Which direction is she facing now?

(a) East (b) West
(c) North (d) South

Answer:

(c) North

Q.69: One day Nita left home and cycled 10 km Southwards. Turned right and cycled 5 km and turned right and cycled 10 km and turned left cycled 10 km. How many km will she have to cycle to reach her home straight?

(a) 10 km (b) 15 km
(c) 20 km (d) 25 km

Answer:

(b) 15 km

Q.70: A located to the West of B. C is located at North in between A and B. D is exactly to the South of B and also in line with B. In which direction of C is D located?

(a) South (b) S-E
(c) West (d) S-W

Answer:

(b) S-E

Q.71: Deepak walks 20 m towards North. He turns left and walks 40 m, again turns left and walks 20 m. Further he moves 20 m after turning to the right. How far is he from his original position?

(a) 20 m
(b) 60 m
(c) 50 m
(d) 30 m

Answer:

(b) 60 m

Q.72: Asha walks 3 km Southward and then turns right and walks 2 km. She again turns right and walks 3 km and turns towards her left and start walking straight. In which direction is she walking now?

(a) East
(b) North
(c) South
(d) West

Answer:

(d) West

Q.73: Madhuri travels 14 km Westwards and then turns left and travels 6 km and further turns left and travels 26 km. How far is Madhuri now from the starting point?

(a) $\sqrt{180}$ km
(b) $\sqrt{80}$ km
(c) $\sqrt{110}$ km
(d) $\sqrt{50}$ km

Answer:

(a) $\sqrt{180}$ km

Q.74: Sohan drove 15 km to the West from his house, then turned left and drove 20 km. Again turned East and drove 25 km and finally turning left covered 20 km. How far he is from his house?

(a) 40 km
(b) 80 km
(c) 5 km
(d) 10 km

Answer:

(d) 10 km

Q.75: Sandesh starts driving from point A and drives 12 km towards North. He takes a right turn and drive 20 km. He now drives 8 km after taking a left turn. Finally he takes a left turn and drives 20 km and stops at point B. How far is point A with respect to point B?

(a) 20 km
(b) 10 km
(c) 5 km
(d) 9 km

Answer:

(a) 20 km

PAST YEAR QUESTIONS AND ANSWERS

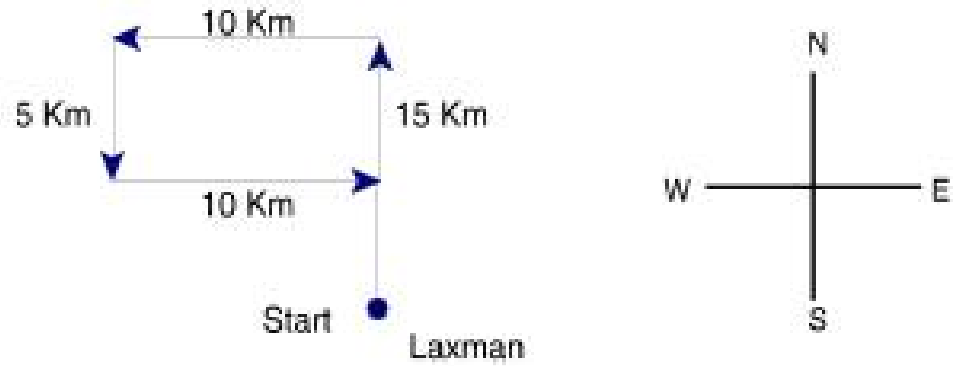
2018 - MAY

[1] Laxman went 15 Kms to North then he turned West and covered 10 Kms. Then he turned South and covered 5 Kms, finally turning to East he covered 10 Kms. In which direction in which he is from his house.

(a) East
(b) West
(c) North
(d) South (1 mark)

Answer:

(c)



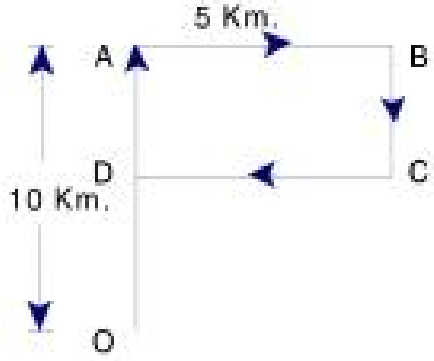
Laxman is moving North direction now.

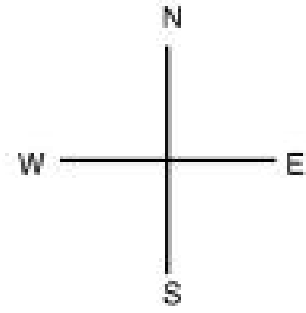
[2] A man is facing East, then he turns left and goes to 10 meter then turns right and goes 5 meter then goes 5 meter to the south and from their 5 meter to West. In which direction is he from his original place?

(a) East
(b) West
(c) North
(d) South (1 mark)

Answer:

(c)





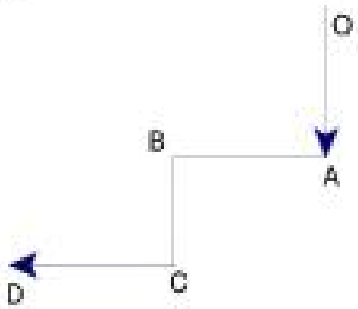
North direction is he form his original place.

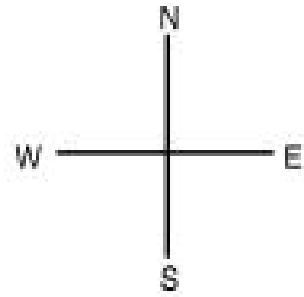
[3] X walks southwords and then turns right the left and the right. In which direction is he moving now?

(a) South (b) North
(c) West (d) South-west (1 mark)

Answer:

(c)



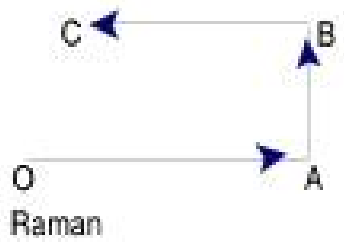


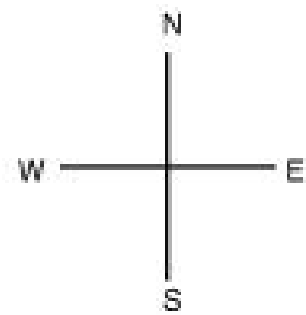
he is moving in west direction.

[4] Raman starts walking in the morning facing the sun. After sometimes, he turned to the left later again he turned to his left. At what direction is Raman moving how?

(a) East (b) West
(c) South (d) North (1 mark)

Answer: (b)





[5] I stand with my right hand extended side-ways towards south. Towards which direction will my back be?

(a) North (b) West
(c) East (d) South (1 mark)

Answer:

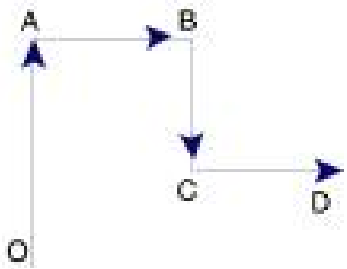
(b) I stand with my right hand extended side - ways to wards south. Towards west direction will my back.

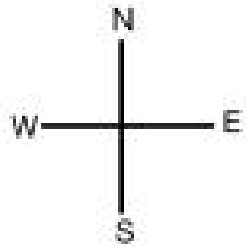
[6] You go North, turn right, then right then go to the left. In which direction are you now?

(a) South (b) East
(c) West (d) North (1 mark)

Answer:

(b)





You are in East direction.

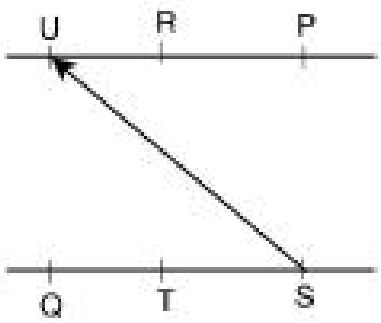
2018 - NOVEMBER

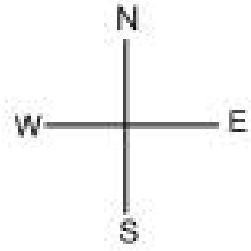
[7] Six flats on a floor in two rows facing North and South are allotted to P, Q, R, S, T and U. If Q gets a North facing flat and is not next to S. S and U get diagonally opposite flat. R next to U gets a South facing flat and T gets a North facing flat. Whose flat is between Q and S?

(a) T (b) U
(c) R (d) P (1 mark)

Answer:

(a)





Flat 'T' is b/w Q & S.

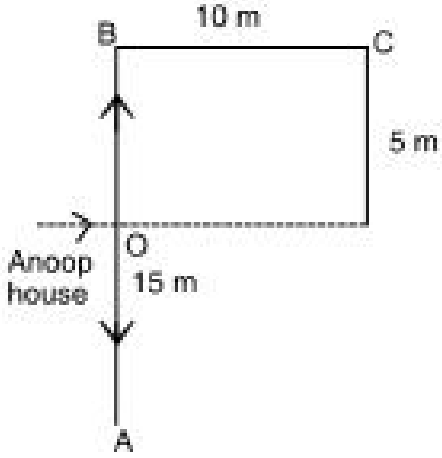
[8] Anoop Starts walking towards South after walking 15 metres he turns towards North. After walking 20 metres he turns towards East and walks 10 metres. He then turns towards south and walks 5 metres. In which direction is he from the original position.

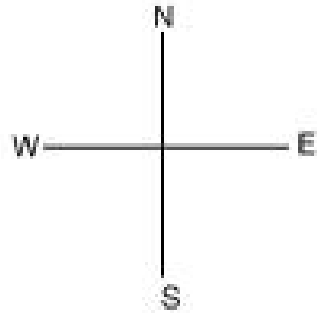
(a) North (b) South

(c) East (d) West (1 mark)

Answer:

(c)





'East direction is he from the original Position'.

[9] Rahim started from point X and walked straight 5 km. West, then turned left and walked straight 2 km. and again turned left and walked straight 7 km. In which direction is he from the point X?

(a) North-East (b) South-West
(c) South-East (d) North-West (1 mark)

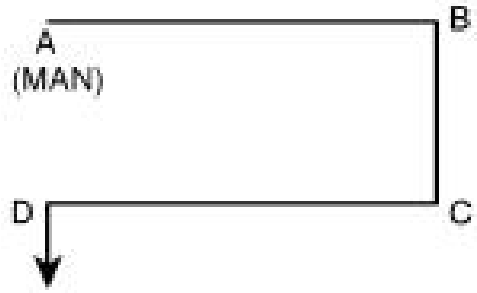
Answer:

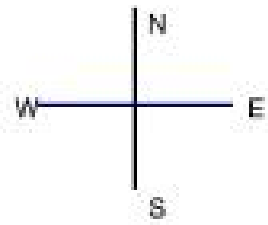
(c)

[10] A man started to walk East, After moving a certain distance, he turns to his right. After moving some distance, he turns to his right again. After moving a little he turns now to his left currently, he is going in _____ direction.

(a) East (b) West
(c) North (d) South (1 mark)

Answer:





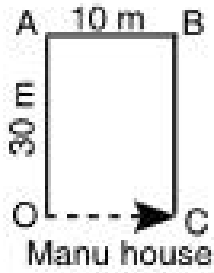
(d) He is going in south direction.

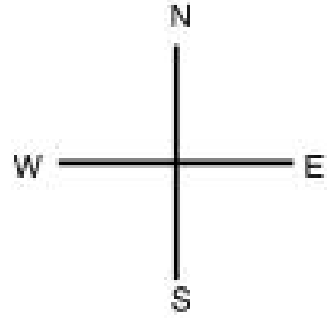
[11] Manu wants to go to the market. He starts from his house towards North reaches at a crossing after 30m. He turns towards East, goes 10m till the second crossing and turns again, moves towards South straight for 30m where marketing complex exits. In which direction is the market from his house?

(a) North (b) South
(c) East (d) West (1 mark)

Answer:

(c)





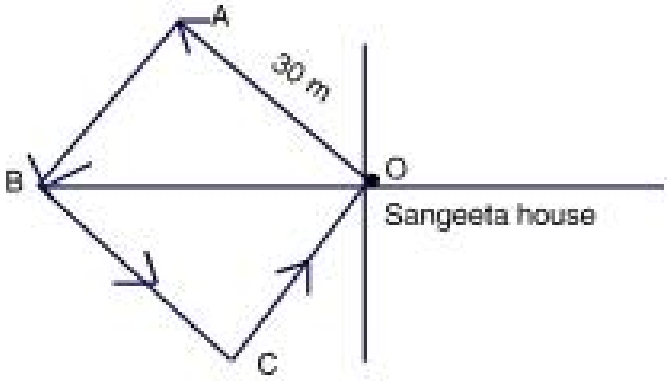
Direction is the market from his house is east.

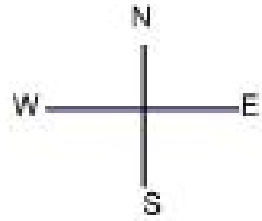
2019 - JUNE

[12] Sangeeta leaves from her home. She first walk 30 meters in north-west direction, and then 30m in south west direction, next she walks 30 metres in south-east direction. Finally she turns towards her house. In which direction is she moving

(a) North - West
(b) North - East
(c) South - East
(d) South - West (1 mark)

Answer: (b)





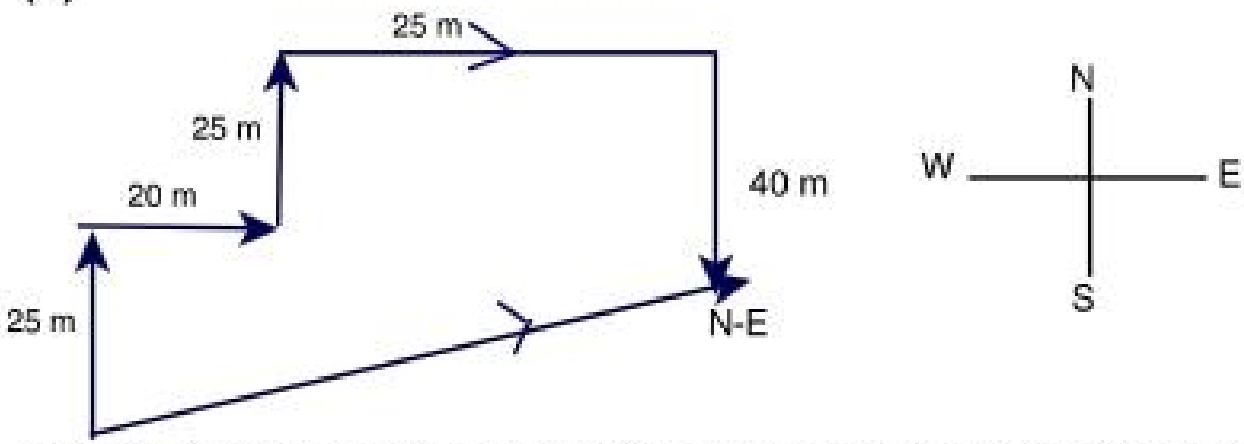
Sangeeta is moving North – East direction.

[13] When a person faces north and walk 25m right and she turn left and walk 20m and again turns right and walk 25m, and turns right 25 m and turns right and walks 40m in which direction is he now from his starting

(a) North – West
(b) North – East
(c) South – East
(d) South – West (1 mark)

Answer:

(b)

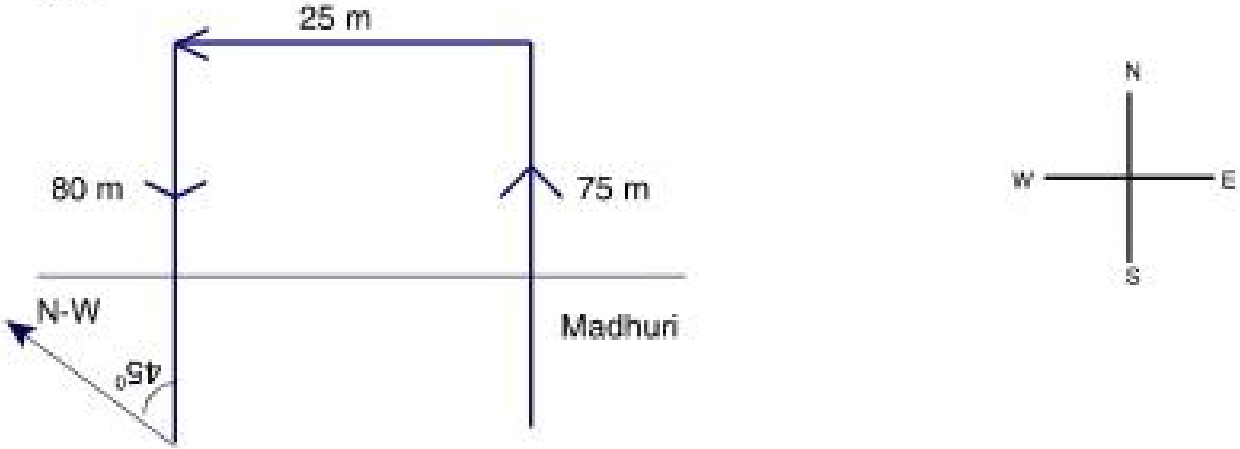


[14] Madhuri moved a distance of 75 meters toward north. She then turned to the left and walking for about 25m, turned left again and walks 80m. Finally she turned to the right at an angle of 45^0 . In which direction was she moving finally?

(a) South – East
(b) South – West
(c) North – West
(d) North – East (1 mark)

Answer:

(c)

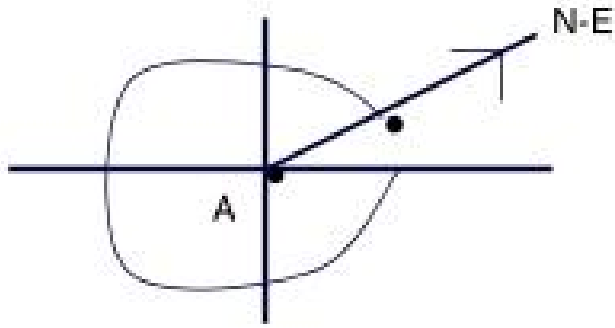


[15] A person facing North 7'o clock wise direction moving in clockwise direction and 300^0 clock wise direction. Now, in which direction he

(a) North – West (b) South – East
(c) North – East (d) South – West (1 mark)

Answer:

(c)



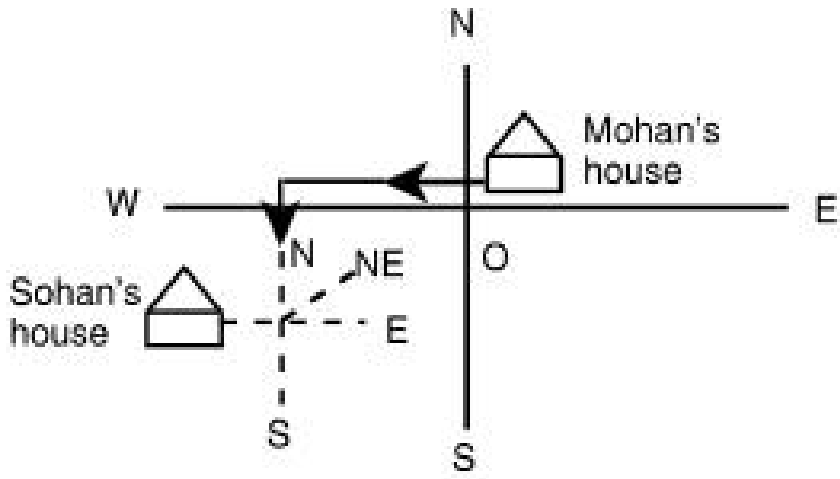
2019 - NOVEMBER

[16] Mohan started from a point and walked towards west. He took left to reach Sohan's house. In which direction should he move to reach his house:

(a) North east (b) South east
(c) South west (d) North west (1 mark)

Answer:

(a)



So, Mohan should move in north east direction from Sohan's house to reach his house.

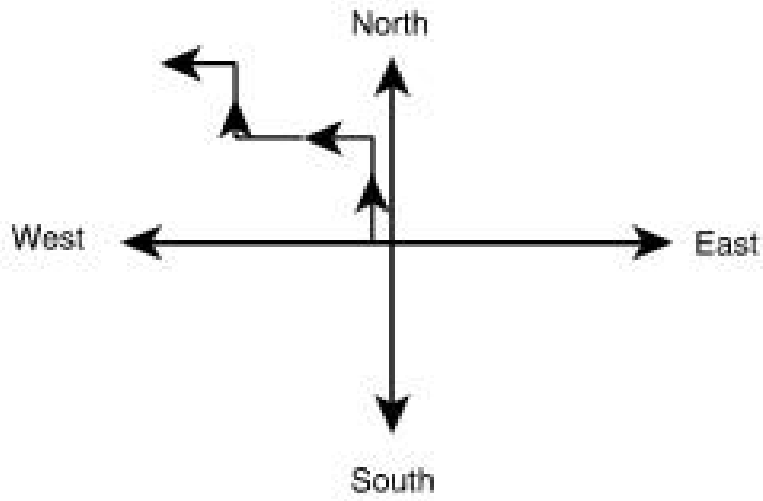
[17] A man stands on a point and starts walking towards north then turns. Left then turns right and then left in which direction he is moving.

(a) West (b) North

(c) East (d) South (1 mark)

Answer:

(a)



So, the man is moving in the west direction.

[18] A man started from a point facing north then turn left and then left then right. In which direction he is facing now?

(a) East

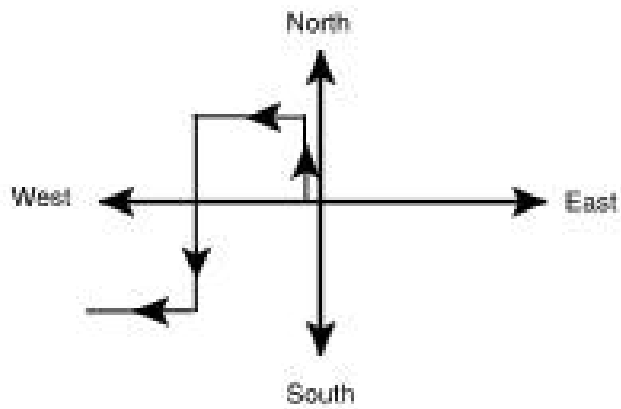
(b) West

(c) North

(d) South (1 mark)

Answer:

(b)



He is facing west.

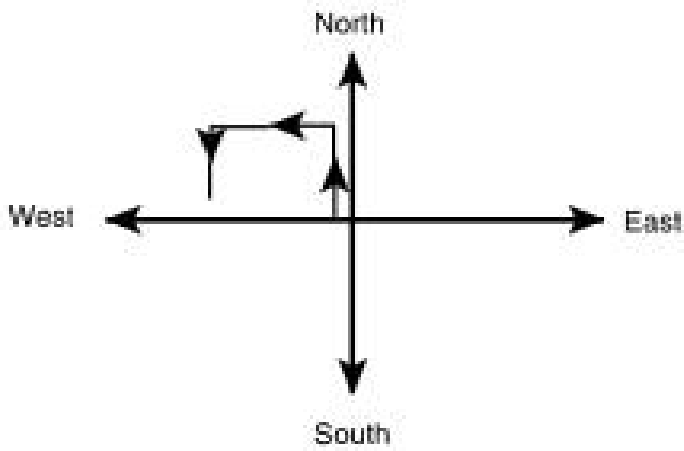
[19] Rohan driving cycle from house towards north, turn left and left again, which direction he is facing now?

(a) East (b) West

(c) North (d) South (1 mark)

Answer:

(d)



So Rohan is facing towards south.

[20] Sun rises behind the tower and sets behind the railway station. In which direction is the tower from railway station?

(a) North

(b) South

(c) East

(d) West (1 mark)

Answer:

(c) Since the sun rises behind the tower. So tower is in east direction, railway station is in west direction since sun sets behind it. So tower is in east direction from the railway station.

CHAPTER

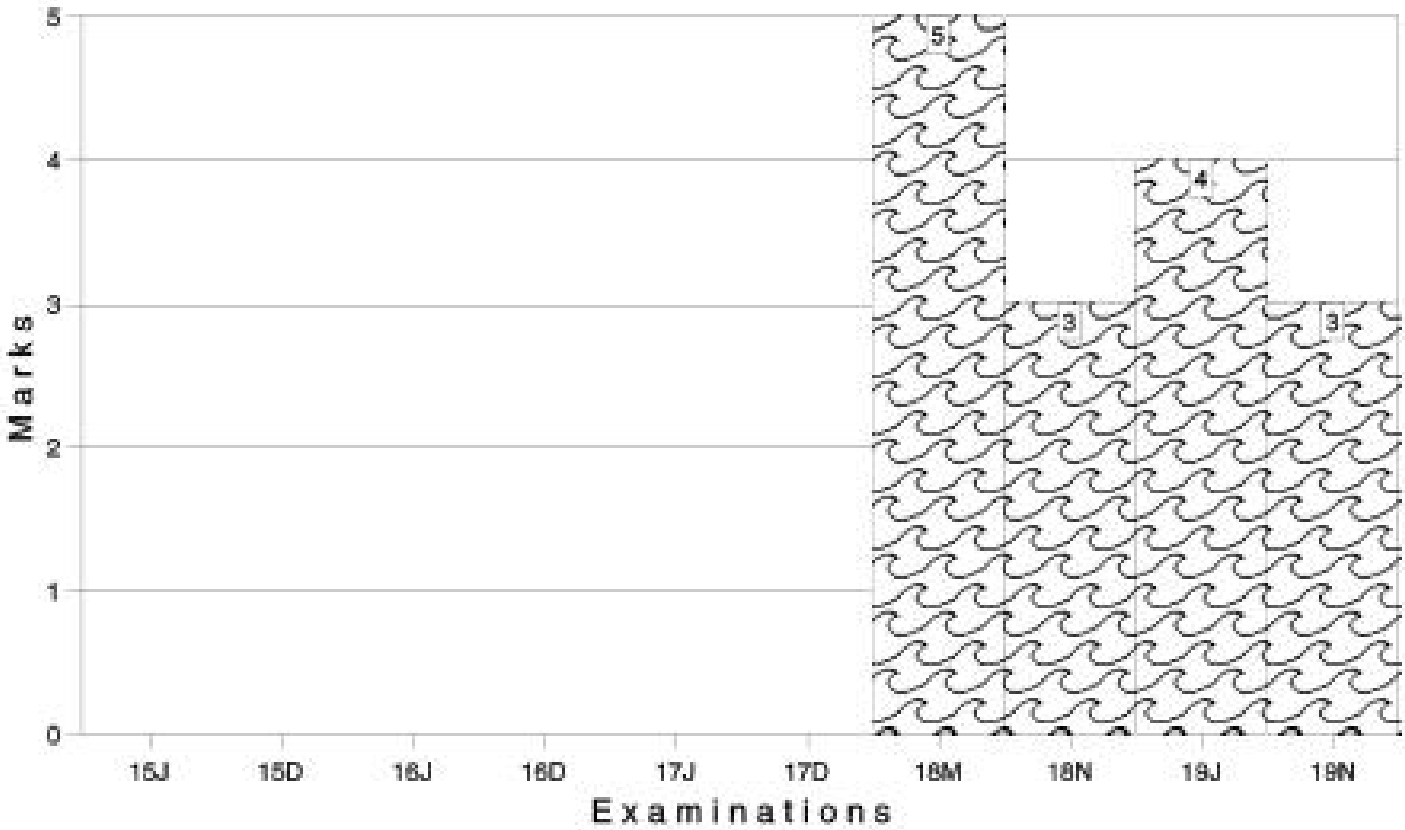
11

SEATING ARRANGEMENTS

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



PRACTICE QUESTIONS OF MCQ

Q.1:

(a) Eleven students A, B, C, D, E, F, G, H, I, J and K are sitting in first line facing to the teacher.

(b) D who is just to the left of F, is to the right of C at second place.

(c) A is second to the right of E who is at one end.

(d) J is the nearest neighbour of A and B and is to the left of G at third place.

(e) H is next to D to the left and is at the third place to the right of I who is just in the middle?

(a) A (b) B

(c) H (d) I

Answer:

I is in the middle.

(d) I

Q.2: Siva, Satish, Amar and Praveen are playing cards. Amar is to the right of Satish who is to the right of Siva. Who is to the right of Amar ?

(a) Satish (b) Amar

(c) Praveen (d) Shiva

Answer:

(c) Praveen

Q.3: A, R, P, X, S and Z are sitting in a row S and Z are in the Centre. A and P are at the ends. R is sitting to the left of A. Who is to the right of P ?

(a) A
(b) X
(c) S
(d) Z

Answer:

Right of P is X.

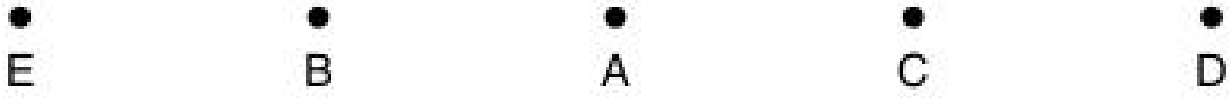
(b) X

Q.4: A, B, C, D and E are sitting on a bench. A is sitting next to B, C is sitting next to D, D is not sitting with E who is on the left end of the bench. C is on the second position from the right. A is to the right of B and E. A and C are sitting together. In which position A is sitting?

(a) Between B and D
(b) Between B and C
(c) Between E and D
(d) Between C and E

Answer:

A is sitting in between B and C



(b) Between B and C

Q.5-8: Directions to solve

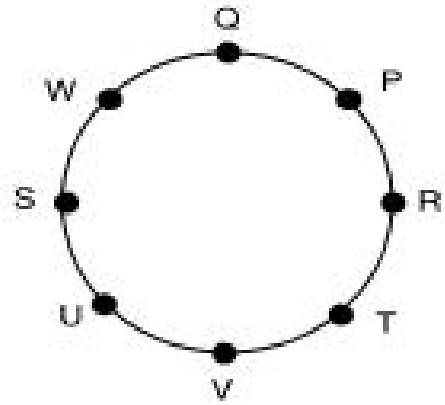
(a) P, Q, R, S, T, U, V and W are sitting round the circle and are facing the centre.
(b) P is second to the right of T who is the neighbour of R and V.
(c) S is not the neighbour of P
(d) V is the neighbour of U
(e) Q is not between S and W. W is not between U and S.

Q.5: Which two of the following are not neighbour
(a) RV (b) UV
(c) RP (d) QW
Answer:
(a) RV

Q.6: Who is immediate right to the V ?
(a) P (b) U
(c) R (d) T
Answer:
(d) T

Q.7: Which of the following is correct ?
(a) P is to the immediate right of Q. (b) R is between U and V
(c) Q is to the immediate left of W (d) U is between W and S
Answer:
(c) Q is to the immediate left of W

Q.8: What is the position of S ?
(a) Between U and V (b) Second to right of P
(c) To the immediate right of W (d) Data inadequate
Answer:
(c) To the immediate right of W



Q.9-12: Five girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary?

Q.9: Who is sitting immediate right to Reeta ?

(a) Bindu (b) Rani
(c) Mary (d) Seema

Answer:

(c) Mary

Q.10: Who is in the middle of the photograph ?

(a) Bindu (b) Rani
(c) Reeta (d) Seema

Answer:

(b) Rani

Q.11: Who is second from the right ?

(a) Mary (b) Rani
(c) Reeta (d) Bindu

Answer:

(c) Reeta

Q.12: Who is second from the left in photograph ?

(a) Reeta (b) Mary
(c) Bindu (d) Seema

Answer:

(d) Seema

Q.13-16: Six Friends are sitting in a circle and are facing the centre of the circle. Deepa is between Prakash and Pankaj. Priya is between Mukesh and Lalit. Prakash and Mukesh are opposite to each other.

Q. 13: Who is sitting right to Prakash?

(a) Mukesh (b) Deepa
(c) Pankaj (d) Lalit

Answer:

(d) Lalit

Q. 14: Who is just right to Pankaj?

(a) Deepa (b) Lalit
(c) Prakash (d) Priya

Answer:

(a) Deepa

Q. 15: Who are the neighbours of Mukesh?

(a) Prakash and Deepa (b) Deepa and Priya
(c) Priya and Pankaj (d) Lalit and Priya

Answer:

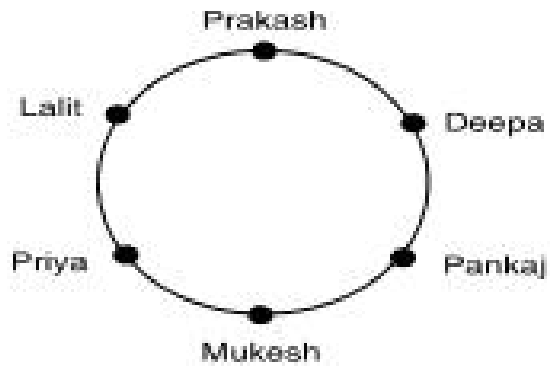
(c) Priya and Pankaj

Q.16: Who is sitting opposite to Priya?

(a) Prakash (b) Deepa
(c) Pankaj (d) Lalit

Answer:

(b) Deepa



Q.17-20: Six Friends P, Q, R, S, T and U are sitting around the hexagonal table each at one corner and are facing the centre of the hexagonal. P is second to the left of U. Q is neighbour of R and S. T is second to the left of S.

Q.17: Which one is sitting opposite to P?

(a) R (b) Q

(c) T (d) S

Answer:

(d) S

Q.18: Who is the fourth person to the left of Q ?

(a) P (b) U

(c) R (d) Data inadequate

Answer:

(a) P

Q.19: Who are neighbours of P ?

(a) U & P (b) T & R

(c) U & R (d) Data inadequate

Answer:

(b) T & R

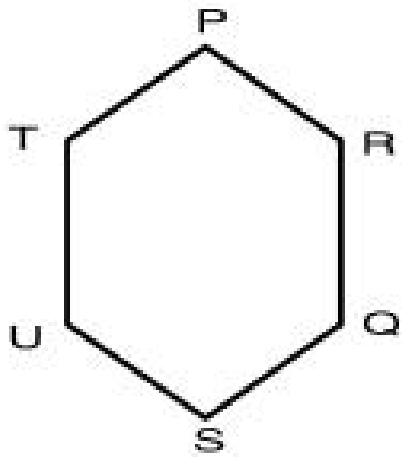
Q.20: Which one is sitting opposite to T ?

(a) R (b) Q

(c) Can't determine (d) S

Answer:

(b) Q



Q.21-25: Directions to solve

(a) A, B, C, D and E are five men sitting in a line facing to South-while M, N, O, P and Q are five ladies sitting in a second line parallel to the first line and are facing to North.

(b) B who is just next to the right of D, is opposite to Q

(c) C and N are diagonally opposite to each other.

(d) E is opposite to O who is just next right of M.

(e) P who is just to the left of Q, is opposite to D

(f) M is at one end of the line.

Q.21: Who is sitting third to the right of O ?

(a) Q (b) N

(c) M (d) Data Inadequate

Answer:

(b) N

Q.22: If B shifts to the place of E, E shifts to the place of Q, and Q shifts to the place of B, then who will be the second to the left of the person opposite to O ?

(a) Q (b) P

(c) E (d) D

Answer:

(a) Q

Q.23: Which of the following pair is diagonally opposite to each other ?

(a) EQ (b) BO

(c) AN (d) AM

Answer:

(d) AM

Q.24: If O and P, A and E, and B and Q interchange their positions, then who will be the second person to the right of the person who is opposite to the person second of the right of P ?

(a) D
(b) A
(c) E
(d) O

Answer:

(b) A

Q.25: In the original arrangement who is sitting just opposite to N ?

(a) B
(b) A
(c) C
(d) D

Answer:

C	E	D	B	A
↓	↓	↓	↓	↓
↑	↑	↑	↑	↑
M	O	P	Q	N

(b) A

Q.26-29: Directions to solve

(a) A, B, C, D, E, F and G are sitting in a row facing North.
(b) F is to the immediate right to E.
(c) E is 4th to the right to G.
(d) C is the neighbour of B and D.
(e) Person who is the third to the left at D is at one of ends.

Q.26: Who are to the left of C ?

(a) Only B
(b) G, B and D
(c) G and B
(d) D, E, F and A

Answer:

(c) G and B

Q.27: Which of the following statement not true ?

(a) E is to the immediate left of D.
(b) A is at one of the ends.
(c) G is to the immediate left of B
(d) F is second to the right of D.

Answer:

(a) E is to the immediate left of D.

Q.28: Who are the neighbours of B ?

(a) C and D
(b) C and G
(c) G and F
(d) C and E

Answer:

(b) C and G

Q.29: What is the position of A ?

(a) Between E and D
(b) Extreme left
(c) Centre
(d) Extreme right

Answer:

↑	↑	↑	↑	↑	↑	↑
G	B	C	D	E	F	A

(d) Extreme right

Q.30-34: Direction to solve

(1) 8 persons E, F, G, H, I, J, K and L are seated around a square table two on each side.
(2) There are 3 ladies who are not seated next to each other.
(3) J is between L and F.
(4) G is between I and F.
(5) H, a lady member is second to the left of J.
(6) F, a male member is seated opposite to E, a lady member.
(7) There is a lady member between F and I.

Q.30: Who among following is to the immediate left of F ?

(a) G
(b) I
(c) J
(d) H

Answer:

(c) J

Q.31: What is true about J and K?

(a) J is male, K is female.
(b) J is female, K is male.
(c) Both are female
(d) Both are male.

Answer:

(d) Both are male.

Q.32: How many persons are seated between K and F ?

(a) 1 (b) 2

(c) 3 (d) 4

Answer:

(c) 3

Q.33: Who among the following are three lady members ?

(a) EHJ (b) EFG

(c) EHG (d) CHJ

Answer:

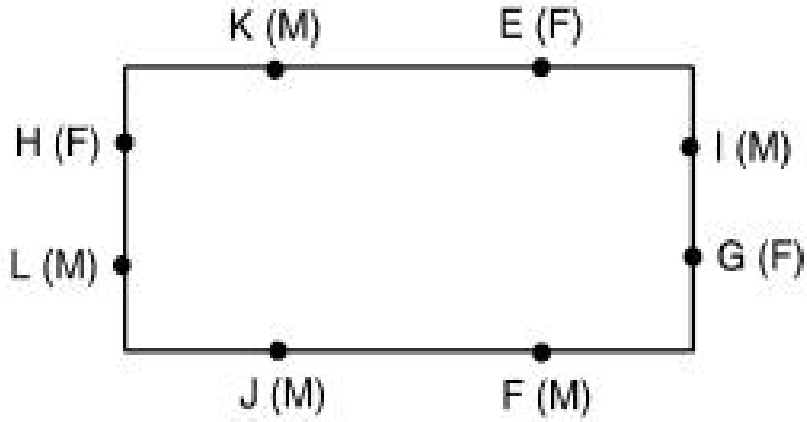
(c) EHG

Q.34: Who among the following is seated between E and H ?

(a) F (b) I

(c) K (d) Can't Say

Answer:



(c) K

Q.35-38: Directions to solve

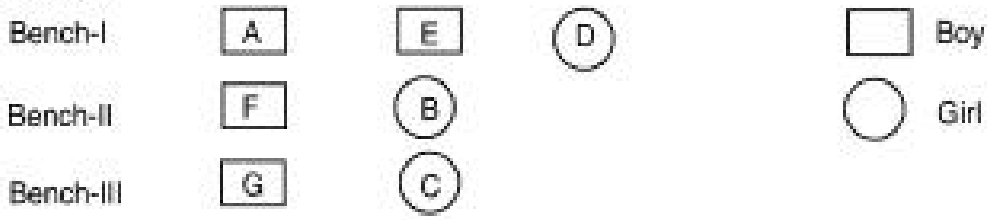
In a class there are seven students (including boys and girls) A, B, C, D, E, F and G. They sit on three benches I, II and III. Such that at least two students on each bench and at least one girl on each bench. C who is a girl student does not sit with A, E and D. F the boy student sits with only B. A sits on the bench I with his friends. G sits on the bench III. E is the brother of C.

Q.35: How many girls are there out of these 7 students ?
(a) 3 (b) 3 or 4
(c) 4 (d) Data inadequate
Answer:
(b) 3 or 4

Q.36: Which of the following is the group of girls ?
(a) BAC (b) BFC
(c) BCD (d) CDF
Answer:
(c) BCD

Q.37: Who sits with C ?
(a) B (b) D
(c) G (d) E
Answer:
(c) G

Q.38: On which bench there are three students ?
(a) Bench-I (b) Bench-II
(c) Bench-III (d) Bench-I or II
Answer:



(a) Bench-I

Q.39-42: Six girls are sitting in a circle facing to the centre of the circle. They are P, Q, R, S, T and V. T is not between Q and S, but some other one. P is next to the lift of V. R is 4^{th} to the right of P.
Q.39: Which of the following statement is not true ?
(a) S is Just next to the right of R. (b) T is just next to the right of V.
(c) R is second to the left of T. (d) P is second to the right of R.
Answer:

Q.40: If P and R interchange their positions then which of the following pair will sit together ?

(a) RT
(b) PV
(c) VT
(d) QV

Answer:

(c) VT

Q.41: What is the position of T ?

(a) Just next to the right of Q.
(b) Second to the left of P.
(c) Between Q and R.
(d) To the immediate right of V.

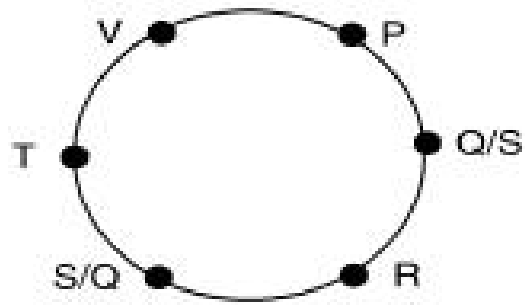
Answer:

(d) To the immediate right of V.

Q.42: Which one is sitting just right to V ?

(a) P
(b) T
(c) R
(d) S/Q

Answer:



(b) T

Q.43-47: Eight friends H, J, K, L, M, N, O and P are sitting around a circular table facing the centre but not necessarily in the same order. There are five females in the group of friends. No two male persons are immediate neighbours of each other. N sits third to the right of P, who sits second to the right of his wife. J sits second to the right of her husband H, who is not an immediate neighbour of P's wife. K is not an immediate neighbour of P, M sits second to the right of her husband. O is not an immediate neighbour of J. L sits second to the right of N, who is not a male.

Q.43: Who among the following sits exactly between two males ?
(a) M
(b) O
(c) J
(d) N
(e) K
Answer:
(b) O

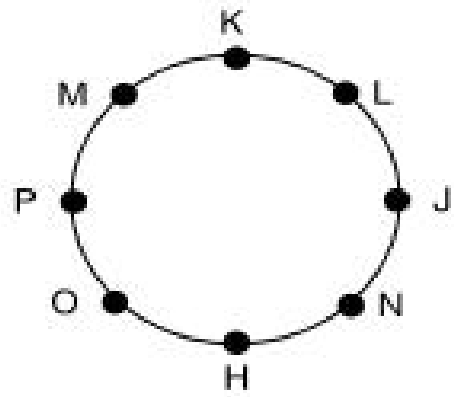
Q.44: Who is wife of P ?
(a) K
(b) O
(c) L
(d) N
(e) None of these
Answer:
(a) K

Q.45: In which of the following pairs is the second person second to the right of the first person ?
(a) M, O
(b) N, P
(c) K, J
(d) N, P
(e) None of these
Answer:
(a) M, O

Q.46: How many females are there exactly between J and N ?
(a) 1
(b) 2
(c) 3
(d) None
Answer:
(d) None

Q.47: Who among the following is third to the right of J ?
(a) H
(b) O
(c) Wife of L
(d) Wife of P

Answer:



(c) Wife of L

Q.48-52: A, B, C, D, E, F and G are sitting in a straight line facing North. There is only one person between F and C. E sits between A and D. There are only two persons between E and G. F sits on the immediate left of A, who sits in the middle of the row.

Q.48: How many persons are there between E & F ?

(a) 1 (b) 2
(c) 3 (d) 4

Answer:

(a) 1

Q.49: Who among the following sit at the extreme ends of the row ?

(a) D, F (b) G, C
(c) B, C (d) None of these

Answer:

(c) B, C

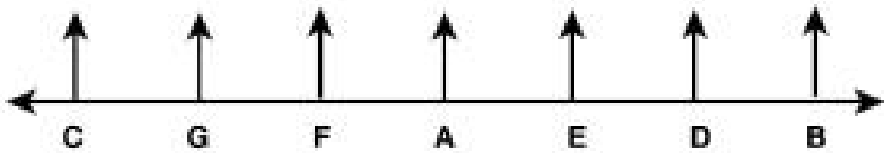
Q.50: Who among the following sits on the immediate right of D ?

(a) G (b) E
(c) F (d) B

Answer:

(d) B

Q.51: Who among the following sits third to the right of A ?
(a) C (b) G
(c) B (d) E
Answer:
(c) B
Q.52: Which is true with regard to B ?
(a) B is second to the right of A. (b) B is fourth to the left of G.
(c) B sits at the extreme right. (d) B sits at the extreme left.
Answer:



(c) B sits at the extreme right.

Q.53-57: Instructions to Solve
(1) P, Q, R, S, T, U and V are sitting on a wall and all of them are facing West.
(2) S is on the immediate left of R.
(3) T is at an extreme end and has Q as his neighbour.
(4) V is between Q and U.
(5) S is sitting third from the north end.

Q.53: Who is sitting to the left of S ?
(a) Q (b) U
(c) T (d) R
(e) P
Answer:
(b) U
Q.54: Which of the following pairs of people are sitting at the extreme ends?
(a) QV (b) PR
(c) TP (d) ST
(e) VP
Answer:
(c) TP

Q.55: Name the person who should change places with R such that he gets the fourth place from the South end ?

(a) P
(b) S
(c) Q
(d) T
(e) U

Answer:

(e) U

Q.56: Immediately between which of the following pairs of people S is sitting?

(a) UR
(b) PQ
(c) VP
(d) TU
(e) RV

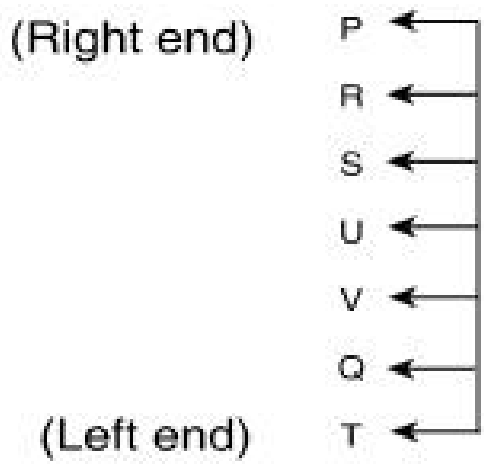
Answer:

(a) UR

Q.57: Which of the conditions given above are not required to find out the place in which P is sitting ?

(a) I
(b) II
(c) IV
(d) III
(e) All required

Answer:



(e) All required

Q.58-62: K, P, T, C, N, D, L and J are standing in a row facing north, but not necessarily in the same order. Only N is between C and D whereas only L is between D and J. T and K are P's neighbours. Neither T nor C is at the extreme end of the row. J is to the right of T but not necessarily on the immediate right.

Q.58: Who among the following are neighbour ?
(a) T, K
(b) T, C
(c) N, L
(d) K, J
(e) L, T
Answer:
(b) T, C

Q.59: Which of the following is true ?
(a) N is on the immediate right of D.
(b) K is not at either of the extreme end.
(c) P is between K and T.
(d) T is at one of the extreme end.
(e) None of these.
Answer:
(c) P is between K and T

Q.60: Which of the following defines the position of D ?
(a) D is second to the left of J.
(b) D is second to the right of L.
(c) D is at one of the extreme ends
(d) D is the neighbour of C.
(e) None of these.
Answer:
(a) D is second to the left of J.

Q.61: Which of the following pairs is the neighbour of N ?
(a) D, L
(b) C, T
(c) T, L
(d) C, D
(e) None of these
Answer:
(d) C, D

Q.62: Which of the following pairs is at the extreme ends of the row ?
(a) J, P
(b) K, P
(c) K, J
(d) Can't be determined
(e) None of these
Answer:
(c) K, J

Q.63-67: Directions: Twelve girl students L, M, N, O, P, Q, R, S, T, U, V, and W are sitting in a row facing east. Q is third to the right at M and T is sitting near L. Six girl students are sitting between W and O. R is fourth to the right of O and S is fifth to the left of P, who is eight to the right of W. L is sitting on sixth place from the right end. U is not near to R and S. V is sitting on the fourth place to the left of L.

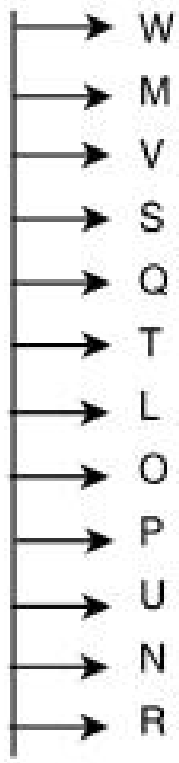
Q.63: Who is the two girl students sitting at the two ends ?
(a) W and N
(b) W and R
(c) V and O
(d) O and Q
Answer:
(b) W and R

Q.64: Find out the names of two girl students sitting near to Q ?
(a) T and V
(b) N and S
(c) V and U
(d) S and T
Answer:
(d) S and T

Q.65: Who is sitting at fifth to the left of U ?
(a) Q
(b) S
(c) T
(d) R
Answer:
(a) Q

Q.66: Who is second to right of L ?
(a) S
(b) P
(c) U
(d) V
Answer:
(b) P

Q.67: Which of the following statements is/are true ?
(a) More the one statements is true.
(b) There are six girl students sitting between N and Q.
(c) U is second to the right of O.
(d) M is sitting at fourth place to the right of T.
Answer:



(c) U is second to the right of O.

Q.68-72: Directions: A, B, C, D, E, F and H are sitting in a straight line but not necessarily in the same order. Two of them are facing south. C sits second to the right of E. A sits second to the left of F, who sits third to the left of H. H is facing south. B sits third to the right of C, who is third from left. E and B face opposite directions.

Q.68: Who among the following sits between F and B ?

(a) D
(b) E
(c) H
(d) C
(e) None of these

Answer:

(b) E

Q.69: Who among the following sits third to the right of F ?

(a) E
(b) C
(c) D
(d) A
(e) None of these

Answer:

(c) D

Q.70: Which of the following pairs sits on the extreme ends of the row ?

(a) F, D
(b) A, E
(c) H, B
(d) D, H
(e) None of these

Answer:

(d) D, H

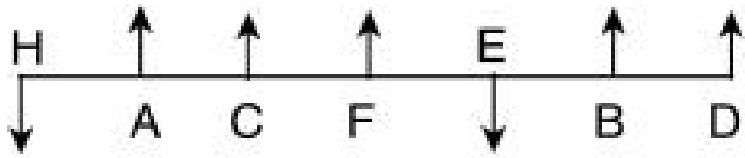
Q.71: Four of the following five are alike in a certain way and so form a group. Which is the one that does not belong to the group ?

(a) A, D
(b) B, D
(c) F, D
(d) C, D
(e) H, E

Answer:

(e) H, E

Q.72: Who among the following sits second to the left of E ?
(a) D
(b) C
(c) A
(d) B
(e) None of these
Answer:



(a) D

Q.73-77: Directions: Seven friends P, Q, R, S, T, U and V are standing in a straight line facing north but not necessarily in the same order.
- U stands second from the left end of the line.
- Only two persons stands between U and T.
- Only one person stands between P and R.
- V stands third to the left of R.
- Neither R nor U is an immediate neighbour of S.

Q.73: Which of the following is TRUF regarding Q ?
(a) P stands second to the left of Q.
(b) S is one of the immediate neighbour of Q.
(c) Only one person stands between Q and T.
(d) Q stands at one of the extreme ends of the line.
(e) V stands second to the right of Q.
Answer:
(c) Only one person stands between Q and T.

Q.74: Four of the following five form a group as per the given arrangement. Which of the following does not belong to that group ?
(a) R Q
(b) T P
(c) V T
(d) U V
(e) P S
Answer:
(c) V T

Q.75: What is the position of V with respect to U ?

(a) Immediate left
(b) Second to the right
(c) Third to the right
(d) Immediate right
(e) Second to the left

Answer:

(a) Immediate left

Q.76: Which of the following pairs stand at the extreme ends of the line ?

(a) Q, S
(b) Q, P
(c) Q, T
(d) V, S
(e) V, P

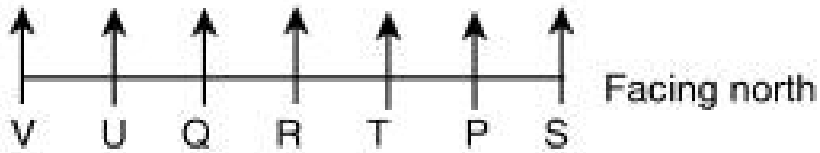
Answer:

(d) V, S

Q.77: Which of the following represents the position of R from the left end of the line ?

(a) Fifth
(b) First
(c) Sixth
(d) Fourth
(e) Second

Answer:



(d) Fourth

Q.78-82: Direction:

I. A, B, C, D, E, F, G, and H are sitting in a row facing North.
II. A is fourth to the right of E.
III. H is fourth to the left of D.
IV. C and F, who are not at the ends are neighbours of B and E, respectively.
V. H is next to the left of A and A is the neighbour of B.

Q.78: What is the position of F ?
(a) Next to the right of E.
(b) Next to the right of G.
(c) Sixth to the right of D.
(d) Between G and H.
(e) None of these.
Answer:
(a) Next to the right of E.

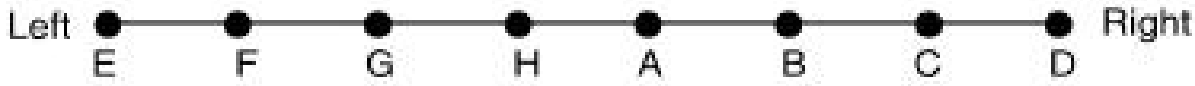
Q.79: Which of the following statements is not true ?
(a) G is the neighbour of H and F.
(b) B is next to the right of A.
(c) E is at left end.
(d) D is next to the right of B.
(e) None of these.
Answer:
(d) D is next to the right of B.

Q.80: Who is/are the neighbour/(s) of D ?
(a) F alone.
(b) C alone.
(c) B and C.
(d) Cannot be determined.
(e) None of these.
Answer:
(b) C alone.

Q.81: Which of the following statements is not true ?
(a) H is second to the right of F.
(b) E is fourth to the left of A.
(c) D is fourth to the right of H.
(d) None of these.
(e) Cannot be determined.
Answer:
(d) None of these.

Q.82: Who are sitting at the ends ?
(a) E and C.
(b) F and D.
(c) G and D.
(d) None of these.
(e) Cannot be determined.

Answer:



Seating Arrangements

(d) None of these.

PAST YEAR QUESTIONS

2018 - MAY

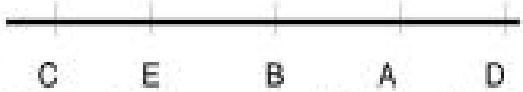
[1] Five boys A,B,C,D and E are sitting in a row A is to the right of B, and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end?

(a) D (b) A

(c) E (d) B (1 mark)

Answer:

(c)



E is the second from the left end.

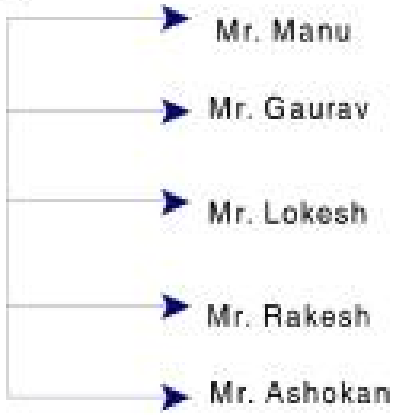
[2] Five senior citizens are living in a multi-storeyed building. Mr. Manu lives a flat above Mr. Ashokan, Mr. Lokesh in a flat below Mr. Gaurav, Mr. Ashokan lives in a flat below Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who lives in the top most flat?

(a) Mr. Lokesh (b) Mr. Gaurav

(c) Mr. Muan (d) Mr. Rakesh (1 mark)

Answer:

(c)



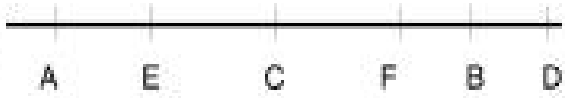
Mr. Manu lives in the top most flat.

[3] Six children A, B, C, D, E and F are standing in a row. B is between F and D. E is between A and C. A does not stand next to F or D. C does not stand next to D. F is between which of the following pairs of children?

(a) B and E (b) B and C
(c) B and D (d) B and A (1 mark)

Answer:

(b)



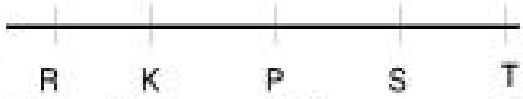
F is between B & C.

[4] Five children are sitting in a row. S is sitting next to P but not T. K is sitting next to R, who is sitting on the extreme left and T is not sitting next to K. Who is/are adjacent to S.

(a) K+P (b) R+P
(c) Only P (d) P and T. (1 mark)

Answer:

(d)



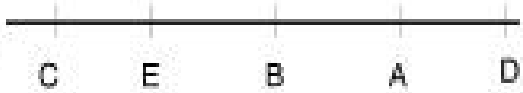
P and T are Adjacent to S

[5] Five boys A, B, C, D and E are sitting in a row. A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end?

(a) D (b) A
(c) E (d) B (1 mark)

Answer:

(c)



E is second from the left end.

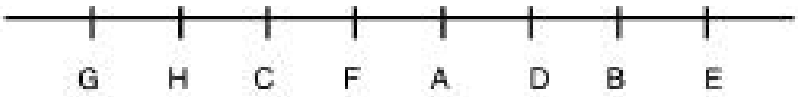
2018 - NOVEMBER

[6] Eight persons A, B, C, D, E, F, G and H are sitting in a line. E sits second right to D. H sits fourth left to D. C and F are immediate neighbours, but C is not immediate neighbour of A. G is not neighbour of E. Only two persons sit between A and E. The persons on left end and right end respectively are

(a) G and E (b) B and E
(c) H and E (d) G and B (1 mark)

Answer:

(a)



The Person on the left end = G
The Person on the right end = E

[7] Six children A, B, C, D, E and F are sitting in a row. B is between F and D. E is between A and C. However, A does not sit next to F or D. C does not sit next to D. Then, F is sitting between

(a) B and C (b) E and C
(c) B and D (d) None of the above (1 mark)

Answer:

(a)

F is siting b/w B and C

[8] Five students A, B, C, D and E are standing in a row. D is on the right of E, B is on the left of E but on the right of A. D is next to C on his left. The student in middle is

(a) B (b) E
(c) C (d) A (1 mark)

Answer:

(b)

The student in middle is 'E'.

2019 - JUNE

[9] Four girls are seated for a photograph Shikha is left of Reena. Manju is to the right of Reena. Reeta is between Reena and Manju. Who is the second left in photograph.

(a) Reena
(b) Manju
(c) Reeta
(d) Shikha (1 mark)

Answer:

(c)

[10] In a straight line there are six person sitting in a row? B is between F and D.E is between A and C. A does not stand next to F or D, C does not stand next to D. F is between which of the following person?

(a) B and E
(b) B and C
(c) B and D
(d) B and A (1 mark)

Answer:

(d)

[11] 5 children are sitting in a raw. S is sitting next to P but not T. K is sitting next to R. K is sitting on extreme end. T is not sitting next to K. Who are sitting adjacent S.

(a) K & P
(b) R & P
(c) Only P
(d) P & T (1 mark)

Answer:

(a)

[12] Five boys A, B, C, D, E are sitting in a row A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end?

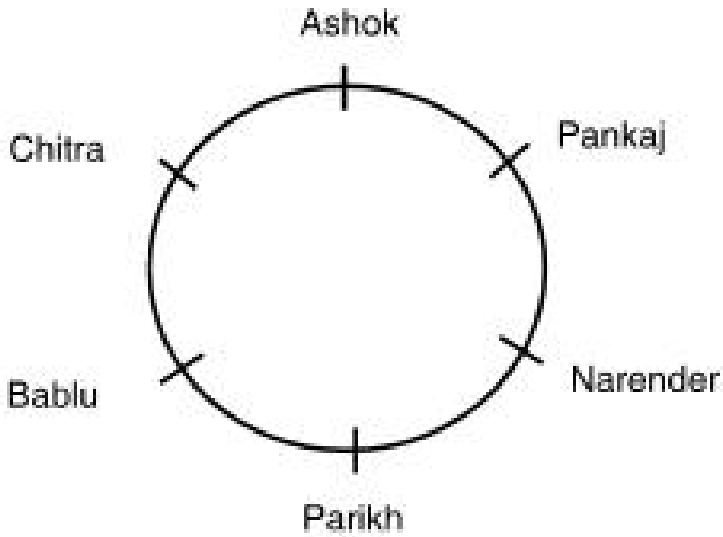
(a) D
(b) A
(c) E

Answer:
(c)

2019 - NOVEMBER

[13] Six person are sitting in a circle facing the center Parikh is between Bablu and Narender Ashok is between Chitra and Pankaj. Chitra is immediate left of Bablu. Who is immediate right of Bablu?
(a) Parikh
(b) Pankaj
(c) Narender
(d) Chitra (1 mark)

Answer:
(a) According to question;



So Parikh is immediate right of Bablu.

[14] C is between A and B, E is at the extreme right D is on the left of E. Who is at the middle?

(a) A
(b) B
(c) D
(d) E (1 mark)

Answer:

(b) According to question,

A C B D E

So B is in the middle.

[15] 5 person are standing in a line one of the 2 persons at the extreme ends is a professor and other a business max. An advocate is standing to the right of student. An author is to the left of the business man. The student is standing between the professor and advocate. Counting from left. The author is at which place?

(a) 2^{nd}
(b) 3^{rd}
(c) 4^{th}
(d) none (1 mark)

Answer:

(c) According to given ques;

Professor	Student	Advocate	Author	Businessman

Author is 4^{th} from the left

CHAPTER

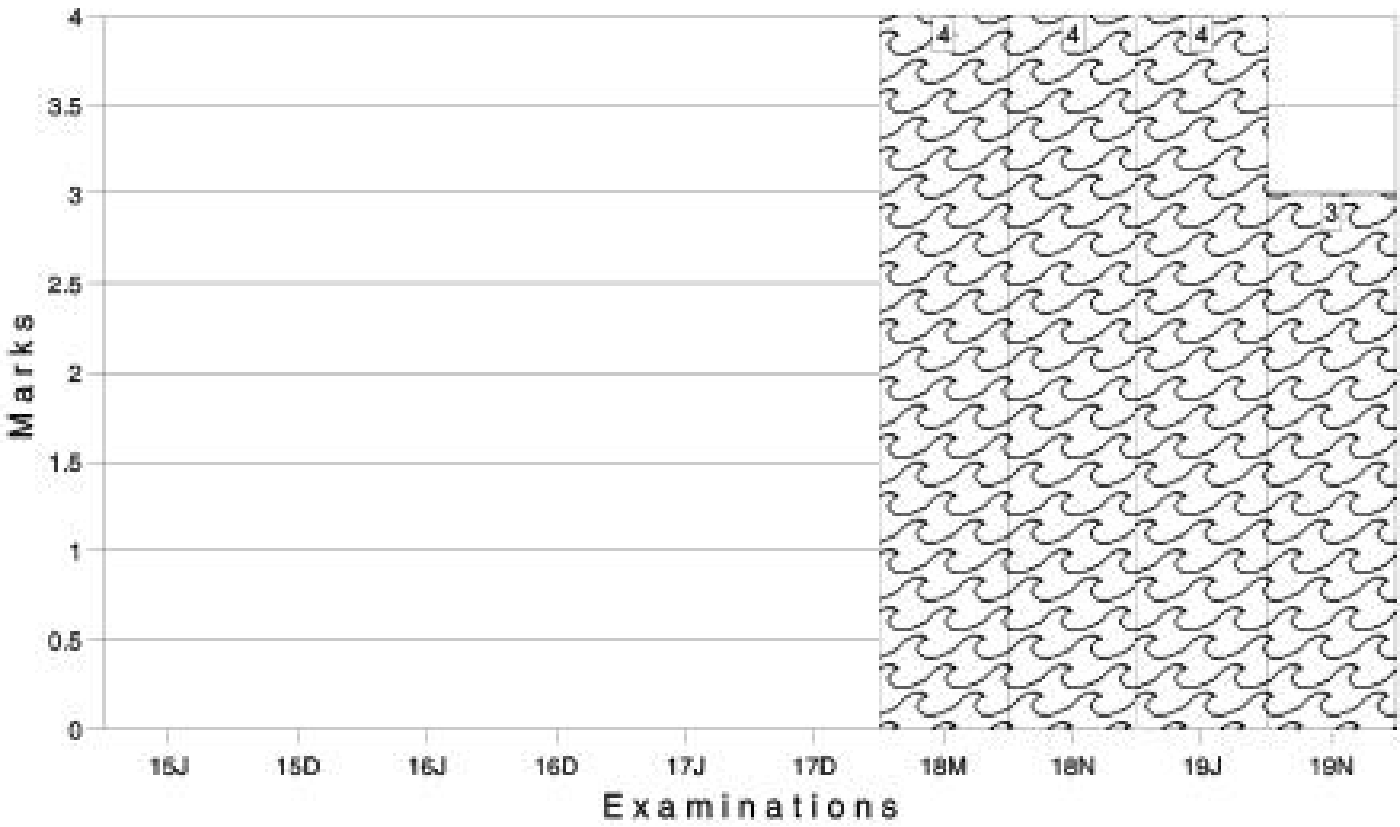
12

BLOOD RELATIONS

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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PRACTICE QUESTIONS OF MCQ

Q.1: Pointing to a lady in the photograph, Monika said, "Her son's father is the son-in-law of my mother" How is Monika related to the lady?

(a) Aunt (b) Sister
(c) Mother (d) Cousin

Answer:

Lady's son's father is lady's husband. Thus Monika is a lady's sister.

(b) Sister

Q.2: Amit introduced Akash of the son of the only brother of his father's wife. How is Akash related to Amit ?

(a) Cousin (b) Son
(c) Uncle (d) Son-in-law

Answer:

(a) Cousin

Q.3: Pointing to a man in a photograph, Sania said, "His mother's only daughter is my mother", How is Sania related to that man ?

(a) Nephew (b) Sister
(c) Wife (d) Niece

Answer:

Sania mother's mother is man's mother i.e. Sania mother is man's sister or Sania is man's niece.

(d) Niece

Q.4: Pointing to a woman, Rajesh said, "She is the daughter of the only child of my grandmother". How is the woman related to Rajesh ?

(a) Sister (b) Niece
(c) Cousin (d) Brother

Answer:

Only child of Rajesh's grandmother is Rajesh's mother/father. Daughter of Rajesh's father/mother - Rajesh's sister.

Q.5: K and L are brothers. M and N are sisters K's son is N's brother. How is L related to M ?

(a) Father
(b) Brother
(c) Grandfather
(d) Uncle

Answer:

(d) Uncle

Q.6: Pointing to a photograph of a boy Suresh said, "He is the son of the only son of my mother". How is Suresh related to that boy ?

(a) Brother
(b) Uncle
(c) Cousin
(d) Father

Answer:

The boy in the photograph is the only son of the son of Suresh's mother, the son of Suresh. Hence Suresh is the father of boy.

(d) Father

Q.7: If A is the brother of B, B is the sister of C, and C is the father of D, how D is related to A ?

(a) Brother
(b) Sister
(c) Nephew
(d) Can't say

Answer:

(c) Nephew

Q.8: If A + B means A is the brother of B, A - B means A is the sister of B, and A x B means A is the father of B. Which of the following means that C is the son of M ?

(a) M - N x C + F
(b) F - C + N x M
(c) N + M - F x C
(d) M x N - C + F

Answer:

M x N → M is the father of N
N - C → N is the sister of C
C + F → C is the brother of F

So M is the father of C or C is the son of M.

(d) M x N - C + F

Q.9: Introducing a boy, a girl said, "He is the son of the daughter of the father of my uncle." How is the boy related to the girl ?

(a) Brother
(b) Nephew
(c) Uncle
(d) Son-in-law

Answer:

The father of the boy's uncle → the grandfather of the boy and daughter of the grandfather → sister of father.

(a) Brother

Q.10: Pointing to a photograph Lata says, "He is the son of the only son of my grandfather". How is the man in the photograph related to Lata ?

(a) Brother
(b) Uncle
(c) Cousin
(d) Data is inadequate

Answer:

The man in the photograph is the son of the only son of Lata's grandfather i.e., the man is the son of Lata's father. Hence, the man is the brother of Lata.

(a) Brother

Q.11: If A + B means A is the brother of B. A x B means A is the son of B and A % B means B is the daughter of A then which of the following means M is the maternal uncle of N ?

(a) M + O x N
(b) M % O x N + P
(c) M + O % N
(d) None of these

Answer:

Because the sex of O is not known.

(d) None of these

Q.12: If D is the brother of B how B is related to C. To answer this question which of the statements is/are necessary ?

1. The son of D is the grandson of C.
2. B is the sister of D.

(a) Only 1
(b) Only 2
(c) Either 1 or 2
(d) 1 and 2 both are required

Answer:
Given D is the brother of B.
From statement 1, we can detect that D is son of C (son of D is the grandson of C).
From statement 2, we can detect that B is 'Female' (sister of D). Therefore, B is daughter of C.
(d) 1 and 2 both are required

Q.13: If A + B means A is the father of B. A - B means A is the brother B. A % B means A is the wife of B and A x B means A is the mother of B, which of the following shows that M is the maternal grandmother of T ?
(a) M x N % S + T
(b) M x N - S % T
(c) M x S - N % T
(d) M x N x S % T

Answer:
M x N → M is the mother of N.
N % S → N is the wife of S and
S + T → S is the father of T.
Hence M is the maternal grandmother of T.
(a) M x N % S + T

Q.14: Pointing to a photograph. Bajpai said, "He is the son of the only daughter of the father of my brother" How Bajpai is related to the man in the photograph ?
(a) Nephew
(b) Brother
(c) Father
(d) Maternal uncle

Answer:
The man in the photo is the son of the sister of Bajpai. Hence, Bajpai is the maternal uncle of the man in the photograph.
(d) Maternal uncle

Q.15: Deepak said to Nitin, "That boy playing with the football is the younger of the two brothers of the daughter of my father's wife", How is the boy playing football related to Deepak ?
(a) Son
(b) Brother
(c) Cousin
(d) Brother-in-law

Answer:
Father's wife – mother. Hence, the daughter of the mother means sister's younger brother means brother, Therefore, the boy is the brother of Deepak.
(b) Brother

Q.16: Pointing a photograph X said to his friend Y, "she is the only daughter of the father of my mother", How X is related to the person of photograph ?
(a) Daughter
(b) Son
(c) Nephew
(d) Cannot be decided

Answer:
(b) Son

Q.17: Veena who is the sister-in-law of Ashok is the daughter-in-law of Kalyani. Dheeraj is the father of Sudeep who is the only brother of Ashok. How Kalyani is related to Ashok ?
(a) Mother-in-law
(b) Aunt
(c) Wife
(d) None of these

Answer:
Ashok is the only brother of Sudeep and Veena is the sister-in-law of Ashok. Hence Veena is wife of Sudeep, Kalyani is the mother-in-law of Veena, Kalyani is the mother of Ashok.
(d) None of these

Q.18: If A + B means A is the sister of B, A x B means A is the wife of B, A % B means A is the father of B and A - B means A is the brother of B. Which of the following means T is the daughter of P ?
(a) P x Q % R + S - T
(b) P x Q % R - T + S
(c) P x Q % R + T - S
(d) P x Q % R + S + T

Answer:
P x Q → P is the wife of Q
Q % R → Q is the father of R.
R - T → R is the brother of T.
T + S → T is the sister of S.
Therefore, T is the daughter of P.
(b) P x Q % R - T + S

Q.19: Pointing to a woman, Abhijit said, "Her grand daughter is the only daughter of my brother", How is the woman related to Abhijit ?

(a) Sister
(b) Grandmother
(c) Mother-in-law
(d) Mother

Answer:

Daughter of Abhijit's brother → niece of Abhijit. Thus the grand daughter of the woman is Abhijit's niece. Hence the woman is the mother of Abhijit.

(d) Mother

Q.20: Amit said "This girl is the wife of the grandson of my mother", How is Amit related to the girl ?

(a) Brother
(b) Grandfather
(c) Husband
(d) Father-in-law

Answer:

The girl is wife of grandson of Amit's mother i.e. the girl is the wife of son of Amit. Hence, Amit is the father-in-law of the girl.

(d) Father-in-law

Q.21: Pointing toward a man, a woman said "His mother is the only daughter of my mother". How is the woman related to the man ?

(a) Mother
(b) Grandmother
(c) Sister
(d) Daughter

Answer:

Only daughter of my mother – myself. Hence, the woman is the mother of the man.

(a) Mother

Q.22: If P $ Q means P is the brother of Q. P # Q means P is the mother of Q, P * Q means P is the daughter of Q in A # B $ C * D, who is the father ?

(a) D
(b) C
(c) B
(d) Data is inadequate

Answer:

A is the mother of B, B is the brother of C and C is the daughter of D. Hence, D is the father.

(a) D

Q.23: Introducing Sonia, Aamir says, "She is the wife of only nephew of only brother of my mother". How Sonia is related to Aamir ?

(a) Wife
(b) Sister
(c) Sister-in-law
(d) Data is inadequate

Answer:

Brother of mother means maternal uncle. Hence only nephew of Aamir's maternal uncle means Aamir himself. Therefore Sonia is the wife of Aamir.

(a) Wife

Q.24: If A + B means A is the brother of B. A % B means A is the father of B and A x B means A is the sister of B. Which of the following means M is the uncle of P ?

(a) M % N x P
(b) N x P % M
(c) M + S % R % P
(d) M + K % T x P

Answer:

M + K → M is the brother of K.
K % T → K is the father of T.
T x P → T is the sister of P.

Therefore, K is the father of P and M is the uncle of P.

(d) M + K % T x P

Q.25: Pointing of Vaman, Madhav said, "I am the only son of one of the sons of his father", How is Vaman related to Madhav ?

(a) Nephew
(b) Uncle
(c) Father or uncle
(d) Father

Answer:

Madhav is only son of one of sons of Vaman's father – Either Vaman is the father or uncle of Madhav.

(c) Father or uncle

Q.26: Introducing a woman. Shashank said "she is the mother of the only daughter of my son", How that woman is related to Shashank ?

(a) Daughter
(b) Sister-in-law
(c) Wife
(d) Daughter-in-Law

Answer:
The woman is the mother of Shashank's grand daughter. Hence the woman is the daughter-in-law of Shashank.
(d) Daughter-in-Law

Q.27: If A + B means B is the brother of A, A x B means B is the husband of A, A - B means A is the mother of B and A % B means A is the father of B. Which of the following shows that Q is the grand mother of T ?
(a) Q - P + R % T
(b) P x Q % R - T
(c) P x Q % R + T
(d) P + Q % R - T

Answer:
Q - P → Q is the mother of P.
P + R → R is the brother of R .
Hence → Q is the mother of R.
R % T → R is the father of T.
Hence, Q is the grand mother of T.
(a) Q - P + R % T

Q.28: Pointing to a photograph. Anjali said, "He is the son of the only son of my grand father", How is the man in photograph related to Anjali ?
(a) Brother
(b) Uncle
(c) Son
(d) Data is inadequate

Answer:
The man in the photograph is son of Anjali's grandfather's son i.e. the son of Anjali's father. Hence, the boy is the brother of Ajanli.
(a) Brother.

Q.29: Pointing to a person, Deepak said, "His only brother is the father of my daughter's father", How is the person related to Deepak ?
(a) Father
(b) Grandfather
(c) Uncle
(d) Brother-in-law

Answer:
Father of Deepak's daughter's Father – Deepak's father. Hence the person in the brother of Deepak's father.
Therefore, the person is the uncle of Deepak.

Q.30: P is the mother of K. K is the sister of D. D is the father of J. How is P related to J ?

(a) Mother
(b) Grandmother
(c) Aunt
(d) Data is inadequate

Answer:

P is the mother of K.
K is the sister of D.
D is the father of J.
Therefore, J is the nephew or niece of K and P is the grandmother of J.

(b) Grandmother

Q.31: If P $ Q means P is the father of Q. P # Q means P is the mother of Q and P * Q means of P is the sister of Q then N # L $ P * Q shows which of the relation of Q to N ?

(a) Grandson
(b) Nephew
(c) Grand daughter
(d) Data is inadequate

Answer:

(d) Data is inadequate

Q.32: If A $ B means A is the brother of B. A @ B means A is the wife of B, A # B means A is the daughter of B and A * B means A is the father of B. Which of the following indicates that U is the father-in-law of P ?

(a) P @ Q $ T # U * W
(b) P @ W $ Q * T # U
(c) P @ Q $ W * T # U
(d) P @ Q $ T # W * U

Answer:

(a) P @ Q $ T # U * W

Q.33: Introducing a man, a woman said "He is the only son of the mother of my mother". How is the woman related to the man ?

(a) Mother
(b) Sister
(c) Niece
(d) Maternal aunt

Answer:

(c) Niece

Q.34: Pointing to Gopi, Naini says "I am the daughter of the only son of his grandfather". How Naini is related to Gopi ?

(a) Niece
(b) Daughter
(c) Sister
(d) Cannot be determined

Answer:

(c) Sister

Q.35: A's son B is married with C whose sister D is married to E the brother of B. How D is related to A ?

(a) Sister
(b) Daughter-in-law
(c) Sister-in-law
(d) Cousin

Answer:

(b) Daughter-in-law

Q.36: Pointing to a lady a person said. "The son of her only brother is the brother of my wife". How is the lady related to the person ?

(a) Maternal aunt
(b) Grand mother
(c) Sister of Father-in-law
(d) None of these

Answer:

(c) Sister of Father-in-law

Q.37:
1. B 5 D means B is the father of D.
2. B 9 D means B is the sister of D.
3. B 4 D means B is the brother of D.
4. B 3 D means B is the wife of D.

Which of the following means F is the mother of K ?

(a) F 3 M 5 K
(b) F 5 M 3 K
(c) F 9 M 4 N 3 K
(d) F 3 M 5 N 3 K

Answer:

(a) F 3 M 5 K

Q.38: A $ B means A is the father of B. A # B means A sis the sister of B, A * B means A is the daughter of B and A @ B means A is the brother of B. Which of the following indicates that M is the wife of Q ?

(a) Q $ R # T @ M
(b) Q $ R @ T # M
(c) Q $ R * T # M
(d) Q $ R @ T * M

Answer:
(d) Q $ R @ T * M

Q.39: If A $ B means A is the brother of B, B * C means B is the son of C. C @ D means C is the wife of D and A # D means A is the son of D, How C is related to A ?
(a) Maternal grandmother
(b) Aunt
(c) Maternal aunt
(d) Mother

Answer:
(d) Mother

Q.40: Pointing to a girl Sandeep said, "she is the daughter of the only sister of my father." How is Sandeep related to the girl ?
(a) Uncle
(b) Cousin
(c) Father
(d) Grand father

Answer:
(b) Cousin

Q.41: Pointing to a boy in the photograph Reena said, "He is the only son of the only child of my grandfather". How Reena related to that boy ?
(a) Mother
(b) Sister
(c) Aunt
(d) Cannot be determined

Answer:
(b) Sister

Q.42:
1. A * B means A is the sister of B.
2. A $ B means B is the mother of A.
3. A + B means A is the brother of B
4. A = B means B is the father of A

Which of the following means M is the maternal uncle of N ?

(a) M = P + Q * N
(b) N + P = Q * M
(c) N * P $ Q * M
(d) None of these

Answer:
(d) None of these

Q.43: If M x N means M is the daughter of N, M + N means M is the father of N. M % N means M is the mother of N and M - N means M is the brother of N then P % Q + R - T x K indicates which relation of P to K ?
(a) Daughter-in-law
(b) Sister-in-law
(c) Aunt
(d) None of these

Answer:
(a) Daughter-in-law

Q.44: If P + Q means P is the brother of Q, P x Q means P is the father of Q and P - Q means P is the sister of Q, which of the following relations shows that I is the niece of K ?
(a) K + Y + Z - I
(b) K + Y x I - Z
(c) Z - I x Y + K
(d) K x Y + I - Z

Answer:
(b) K + Y x I - Z

Q.45: Pointing towards a girl, Abhisek says "This girl is the daughter of only a child of my father". What is the relation of Abhisek's wife to that girl ?
(a) Daughter
(b) Mother
(c) Ant
(d) Sister-in-law

Answer:
(d) Sister-in-law

Q.46: P & Q are brothers, R and S are sister. P's son is S's brother. How is Q related to R ?
(a) Uncle
(b) Brother
(c) Father
(d) Grand father

Answer:
(a) Uncle

Q.47: A is B's brother. C is A's mother. D is C's father. F is A's son How is B related to F's child ?

(a) Aunt
(b) Cousin
(c) Nephew
(d) Grand father

Answer:

(d) Grand father

Q.48: A and B are brothers. E is the daughter of F. F is the wife of B. What is the relation of E to A ?

(a) Sister
(b) Daughter
(c) Niece
(d) Daughter in law

Answer:

(c) Niece

Q.49: A is B's wife's husband's brother. C and D are sisters of B. How A is related to C ?

(a) Brother
(b) Sister-in-law
(c) Wife
(d) Sister

Answer:

(a) Brother

Q.50: Vinod introduces Vishal as the son of the only brother of his father's wife. How is Vinod related to Vishal ?

(a) Cousin
(b) Brother
(c) Son
(d) Uncle

Answer:

(a) Cousin

Q.51: Pointing out a man receiving the prize, menu said, "He is the brother of my uncle's daughter". Who is the man to menu ?

(a) Son
(b) Brother-in-law
(c) Nephew
(d) Cousin

Answer:

(d) Cousin

Q.52: Pointing to a picture, Sumit said, she is the mother of my son's wife's daughter. How is lady related to the Sumit ?

(a) Uncle
(b) Cousin
(c) Daughter-in-law
(d) None of these

Answer:

(c) Daughter-in-law

Q.53: Introducing a boy, a girl said, "He is the only son of my mother's mother". How is the girl related to that boy ?

(a) Aunt
(b) Niece
(c) Sister
(d) Mother

Answer:

(b) Niece

Q.54: There are two couples in a family. K has two children. M is wife of O who is brother of B. F is daughter of K. U is sister of S, who is son of O. T is son of B, who is a male. How M is related to K ?

(a) Sister
(b) Sister-in-law
(c) Brother
(d) None of these

Answer:

(b) Sister-in-law

Q.55: There are two couple in a family. K has two children. M is wife of O, who is brother of B. F is daughter of K. U is sister of S, who is son of O. T is son of B, who is a male. How is U related to T ?

(a) Mother
(b) Brother
(c) Sister
(d) Cousin

Answer:

(d) Cousin

Q.56-58: Directions: R is the father of P, who is a son-in-law of M and S is the mother of G. S is a sister of K, who is a brother-in-law of P and H is the daughter of T, who is a grand mother of G ?

Q.56: How is G related to P ?

(a) Son
(b) Daughter
(c) Grandson
(d) Grand daughter
(e) Cannot be determined

Answer:

(e) Cannot be determined

Q.57: If M is a female, then how is H related to S ?

(a) Sister
(b) Sister-in-law
(c) Niece
(d) Cannot be determined
(e) None of these

Answer:

(b) Sister-in-law

Q.58: If K married to N, then how is N related to M ?

(a) Son-in-law
(b) Daughter-in-law
(c) Mother-in-law
(d) Father-in-law
(e) None of these

Answer:

(b) Daughter-in-law

Q.59: Pointing a photograph, Sheela said, he is the only son of my grandfather's only son. How is the boy in the photograph related to Sheela ?

(a) Brother
(b) Cousin
(c) Son
(d) Cannot be determined
(e) None of these

Answer:

(a) Brother

Q.60: Pointing a girl, Prasan said, she is the only granddaughter of my wife's grandfather's only child. How is the girl relate to Prasan ?

(a) Sister
(b) Niece
(c) Daughter
(d) Cannot be determined
(e) None of these

Answer:

(c) Daughter

Q.61: P is the sister of Q. R is the father of S, who is the brother of Q. R married to T. How is Q related to T ?

(a) Son
(b) Daughter
(c) Either son or daughter
(d) Data inadequate
(e) None of these

Answer:

(c) Either son or daughter

Q.62: G is the father of K, who is the brother of B. K married to U. B is the daughter of C. D is the father of U and M is the only son of D. How is U related to C ?

(a) Daughter
(b) Daughter-in-law
(c) Grand daughter
(d) Son-in-law
(e) None of these

Answer:

(b) Daughter-in-law

Q.63: K married to S. P is a brother of B. S is a mother of N, who is a brother of B. How is B related to K ?

(a) Son
(b) Daughter
(c) Either son or daughter
(d) None of these
(e) Data inadequate

Answer:

(c) Either son or daughter

Q.64: Pointing to a girl, Mala said, 'she is the grand daughter of my grand father's only son'. How is the girl related to Mala ?

(a) Niece
(b) Sister
(c) Daughter
(d) Cannot be determined
(e) None of these

Answer:

(c) Daughter

Q.65: Pointing a boy, Ram said, 'He is the only son of my grand father's son'. How is the boy related to Ram ?

(a) Brother
(b) Son
(c) Cousin
(d) Data inadequate
(e) None of these

Answer:

(a) Brother

Q.66-67: Directions: There are seven members A, C, D, E, F, G and H in a family. There are two fathers, one mother two sisters and four brothers. E is the sister-in-law of D. G is a daughter of C. F is the brother of E. A is a grandfather of G. E is a mother of H ?

Q.66: How is H related to A ?

(a) Grandson
(b) Grand daughter
(c) Son
(d) Cannot be determined
(e) None of these

Answer:

(a) Grandson

Q.67: How many male members in the family ?

(a) 4
(b) 5
(c) 3
(d) Data inadequate
(e) None of these

Answer:

(b) 5

Q.68-70: Directions: M is the grandson of G. D is the husband of G. K. is married to the son of S. T have two children of different gender. E is the daughter of K's brother N is brother-in-law of the son of S. G has one only one child. D is the father of N.

Q.68: How N is related to G ?

(a) Son
(b) Son-in-law
(c) Grandson
(d) Brother
(e) Father

Answer:

Q.69: If Q is married to N, then how is Q related to M ?
(a) Father
(b) Mother
(c) Sister
(d) Grand mother
(e) Cannot be determined
Answer:
(b) Mother

Q.70: If U is the son of T, then how is U related to N ?
(a) Brother
(b) Uncle
(c) Cousin
(d) Brother-in-law
(e) Data inadequate
Answer:
(d) Brother-in-law

Q.71-73: Directions: A family consists of six members P, Q, R, X, Y and Z. Q is the son of R but R is not the mother of Q. P and R are married couple. Y is the brother of R. X is the daughter of P and Z is the brother of P.
Q.71: Who is the brother-in-law of R ?
(a) P
(b) Z
(c) Y
(d) X
(e) Q
Answer:
(b) Z
Q.72: How many female members are there in the family ?
(a) One
(b) Two
(c) Three
(d) Four
(e) Five
Answer:
(b) Two
Q.73: Which of these is a pair of brothers ?
(a) P and X
(b) P and Z
(c) Q and X
(d) R and Y
(e) None of these
Answer:
(d) R and Y

Q.74-75: Directions: There are six children taking part in an essay competition, namely A, B, C, D, E, and F. A and E are brothers. F and D are the sister of E. C is the only son of A's uncle. B and D are the brother of C's father.

Q.74: How is D related to A ?

(a) Uncle
(b) Sister
(c) Niece
(d) Cousin
(e) None of these

Answer:

(b) Sister

Q.75: How many male competitors are there ?

(a) 6
(b) 5
(c) 4
(d) 3
(e) 2

Answer:

(d) 3

Q.76-78: Directions: PXQ means P is brother of Q.
P ÷ Q means Q is mother of P.
P - Q means P is father of Q.
P + Q means Q is sister of P.

Q.76: Which of the following means M is the daughter of T ?

(a) M + N ÷ J - T
(b) T - J x R + M
(c) M - J x T ÷ K
(d) M + W x R ÷ T
(e) None of these

Answer:

(b) T - J x R + M

Q.77: How is K related to R in the expression R ÷ T + K ?

(a) Daughter
(b) Sister
(c) Niece
(d) Cannot be determined

Answer:

(d) Cannot be determined

Q.78: Which of the following means D us grandfather of W ?
(a) D - K x T - W
(b) D ÷ K x T ÷ W
(c) D - K x T ÷ W
(d) D ÷ K x T - W
(e) None of these
Answer:
(a) D - K x T - W

Q.79-80: Directions: A is the mother of B. B is sister of C. D is son of C. E is brother of D. F is mother of E. G is grand daughter of A. H has only two children B and C.

Q.79: How is F related to H ?
(a) Son-in-law
(b) Daughter-in-law
(c) Father-in-law
(d) Grand daughter
(e) Niece
Answer:
(b) Daughter-in-law

Q.80: How is C related to E ?
(a) Father
(b) Son
(c) Mother
(d) Cousin
(e) Grand father
Answer:
(a) Father

PAST YEAR QUESTIONS AND ANSWERS

2018 - MAY

[1] Vinod introduce Vishal as the son of the only brother of his father's wife. How is Vinod related to Vishal?
(a) Cousin
(b) Brother

Answer:

(a) Vinod Introduces Vishal as the Son of the only brother of his father's wife then Vinod is cousin of Vishal.

[2] Suresh introduces a man as "he is the son of the woman who is the mother of the husband of my mother" . How is Suresh is related to the man?

(a) Brother-in-law (b) Son
(c) Brother (d) Nephew (1 mark)

Answer:

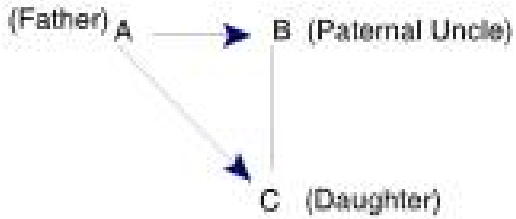
(b) Suresh introduces a man as "he is the son of the woman who is the mother of the husband of my mother". Then Suresh is the **'Son'** of the man.

[3] A reads a book and find the name of the author familiar. The author 'B' is the paternal uncle of C. C is the daughter of A. How is B related to A?

(a) Brother (b) Sister
(c) Father (d) Uncle (1 mark)

Answer:

(a)



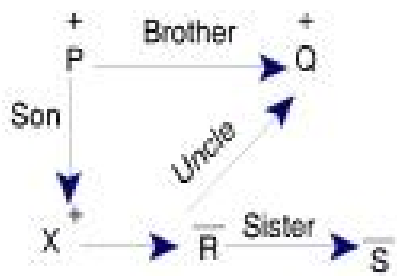
B is the Brother of 'A'.

[4] P and Q are brothers R and S are sisters. P's son is R's brother. How is Q related to R?

(a) Uncle (b) Brother
(c) Father (d) Grandfather (1 mark)

Answer:

(a)



2018 - NOVEMBER

[5] Pointing to a man in a photograph, a woman said, "the father of his brother is the only son of my grandfather", how is the woman related to the man in the photograph?

(a) Mother (b) Aunty
(c) Daughter (d) Sister (1 mark)

Answer:

(d) The father of his brother is the only son of my grand father. So the women is mans sister.

Explanation: Only son of women's grandfather = woman's father
Man's brother's father- man's father so. The women is man's sister.

[6] Six persons are seen together in a group. They are A, B, C, D, E and F, B is brother of D, but D is not brother of B. F is brother of B, C and A are married together . F is son of C, but C is not mother of F. E is brother of A. The number of female member in the group is

(a) 1 (b) 2
(c) 3 (d) 4 (1 mark)

Answer:

(b)

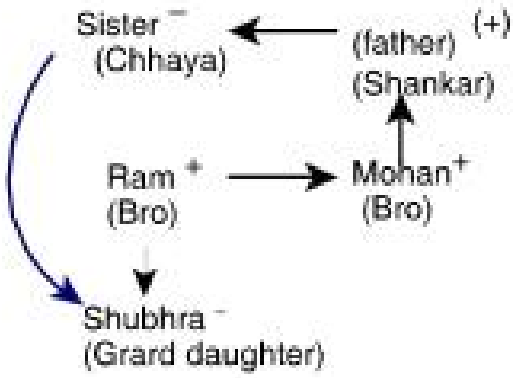
No. of female members = 2

[7] Ram and Mohan are brothers, Shankar is Mohan's father. Chhaya is Shankar's sister. Priya is Shankar's niece. Shubhra is Chhaya's grand daughter. Then Ram is Shubhra's

(a) Brother (b) Uncle
(c) Cousine (d) Nephew (1 mark)

Answer:

(b)



Ram is the uncle of Shubhra.

[8] If P + Q means P is the mother of Q. P ÷ Q means P is the father of Q. P–Q means P is the sister of Q. Then which of the following relationship shows that M is the daughter of R?

(a) R ÷ M + N (b) R + N ÷ M

(c) R – M ÷ N (d) None of the above (1 mark)

Answer:

(a) M+N i.e., R is the father of M.
Hence, R: M+N clearly means
That, M is the daughter of R.

2019 - JUNE

[9] Pointing to a picture, Sumit said, she is the mother of my son's wife's daughter. How is lady related to the Sumit?

(a) Uncle
(b) Cousin
(c) Daughter
(d) None (1 mark)

Answer:

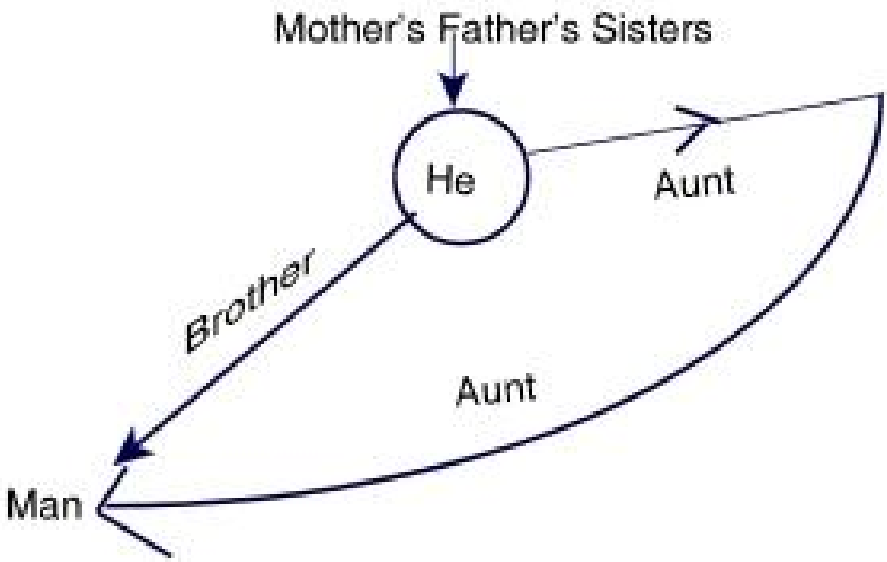
(d) Sumit's son's wife's daughter –
Sumit's son's daughter; mother of daughter of sumit's son – wife of Sumit son so Sumit is the father in law of the lady.
So, lady is daughter in law to the Sumit.

[10] Pointing to a photograph, a man said "His mother husband's sister is my aunt". Then what is relation between a man and he?

(a) Son
(b) Uncle
(c) Nephew
(d) Brother (1 mark)

Answer:

(d)

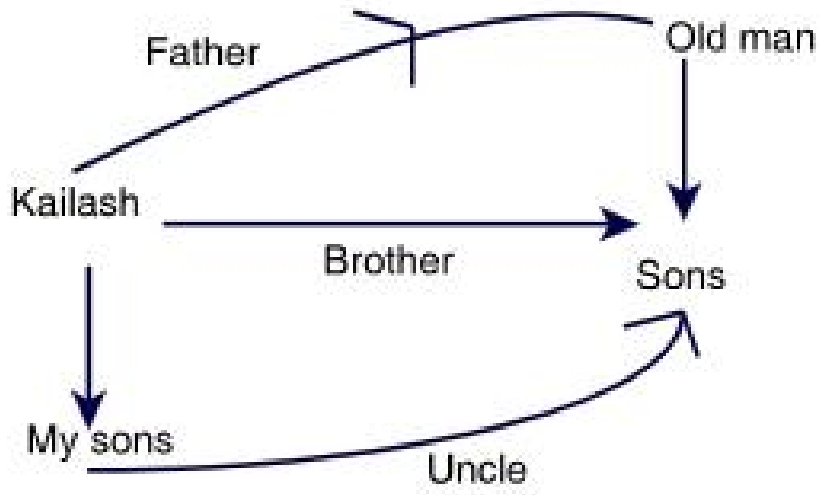


[11] Pointing the old man Kailash said "his son is my son's uncle". How is Kailash is related to old man.

(a) Brother
(b) Either son or son in law
(c) Father
(d) Grand Father (1 mark)

Answer:

(c)

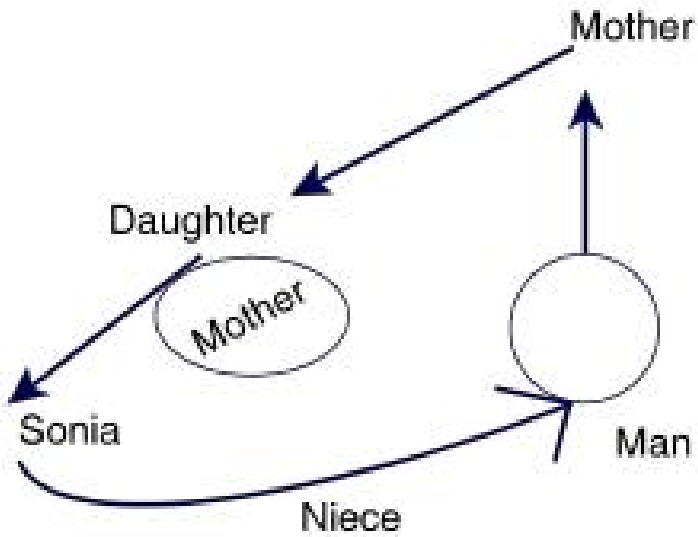


[12] Pointing in a photograph, Sonia said, "His mother's only daughter is my mother ". How is Sonia relates to that man?

(a) Nephew
(b) Sister
(c) Wife
(d) Niece (1 mark)

Answer:

(d)



2019 - NOVEMBER

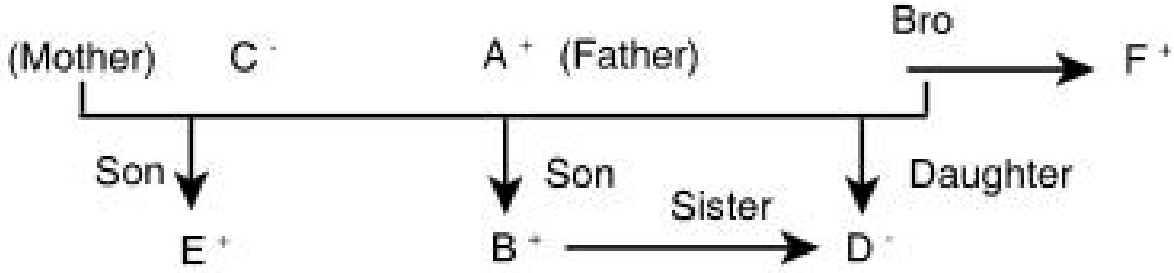
[For questions 16-18]
A, B, C, D, E and F are members of a family. B is the son of A but A is not the mother of B, A and C are married couple. F is the brother of A. D is the sister of B, E is the son of C.

[13] How many male members are there in the family.
(a) 1
(b) 2
(c) 3
(d) 4 (1 mark)

[14] How is F related to B?
(a) Uncle
(b) Daughter
(c) Son
(d) Niece (1mark)

[15] How many children A has?
(a) 3
(b) 2
(c) 4
(d) 1 (1 mark)

Answer:
For question [16-18]
(+) ⇒ male
(-) → female



[13] **(d)** There are four male members in the family i.e.
A (father or husband)
F (A's brother)
B (son of A and C)
E (son of A and C)

[14] **(a)** Since F is A's brother
and B is A's son.
father's brother = uncle
So, F is the uncle of B

[15] **(a)** A has 3 children
E, B and D

CHAPTER

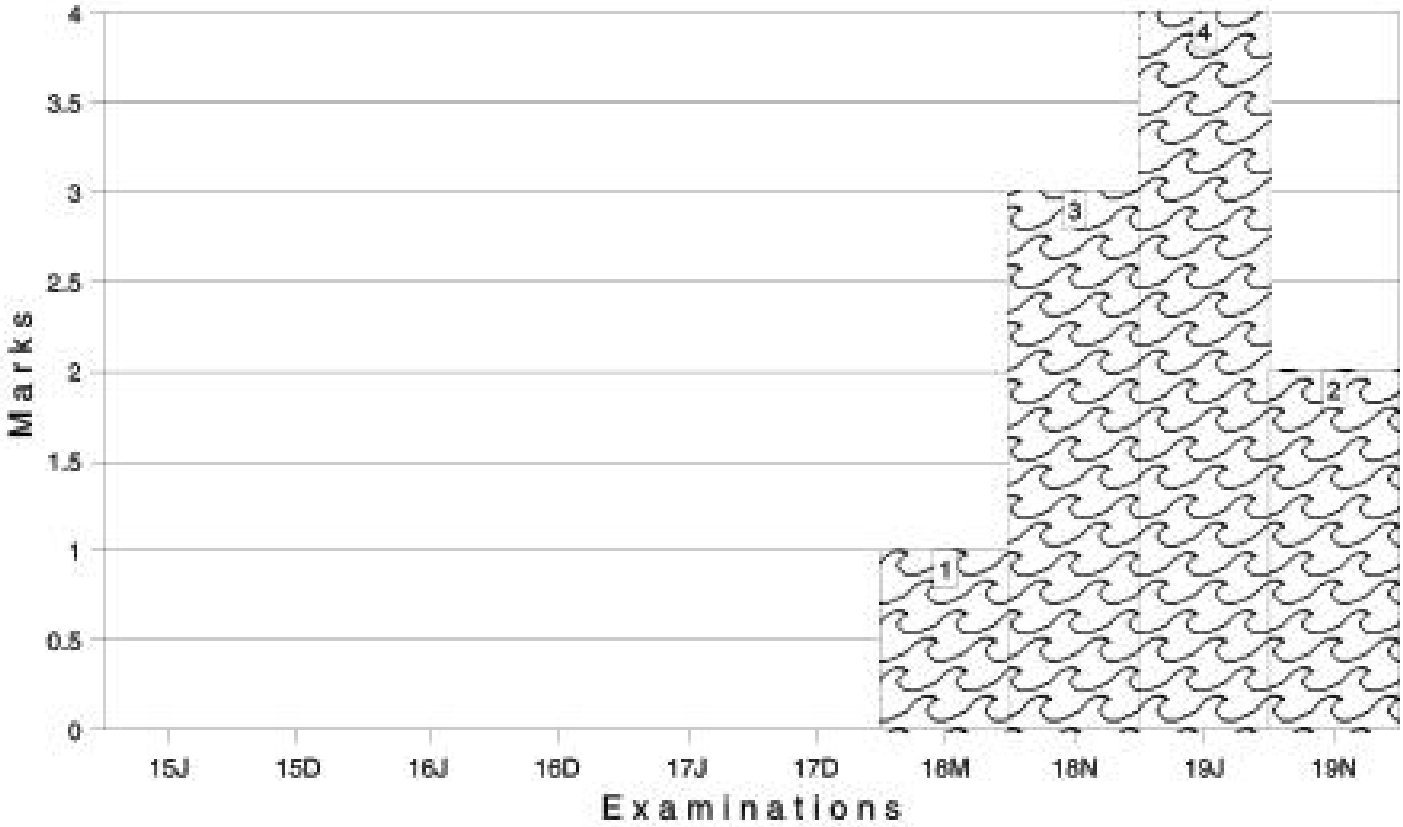
13

SYLLOGISM

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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for registration and password see first page of this book.

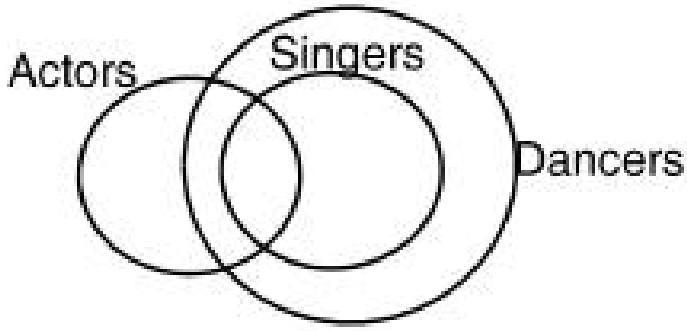
PRACTICE QUESTIONS OF MCQ

Direction to Solve :

A. If only (1) Conclusion follows.
B. If only (2) Conclusion follows.
C. If either (1) or (2) follows.
D. If neither (1) nor (2) follows.
E. If both (1) and (2) follows.

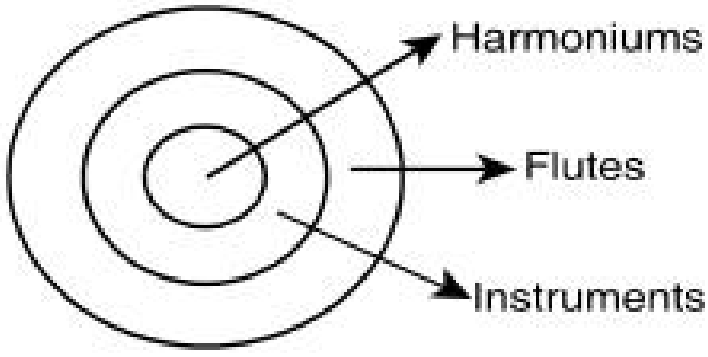
Q1. Statements: Some actors are Singers. All the Singers are dancers.
Conclusion: (1) Some actors are dancers.
(2) No Singer is actor.

Answer: A

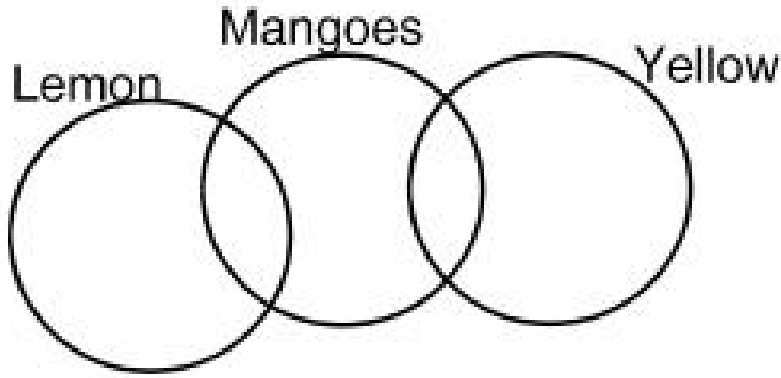


Q2. Statements: All the harmoniums are instruments. All the instruments are flutes.
Conclusions: (1) All flutes are instruments
(2) All the harmoniums are flutes

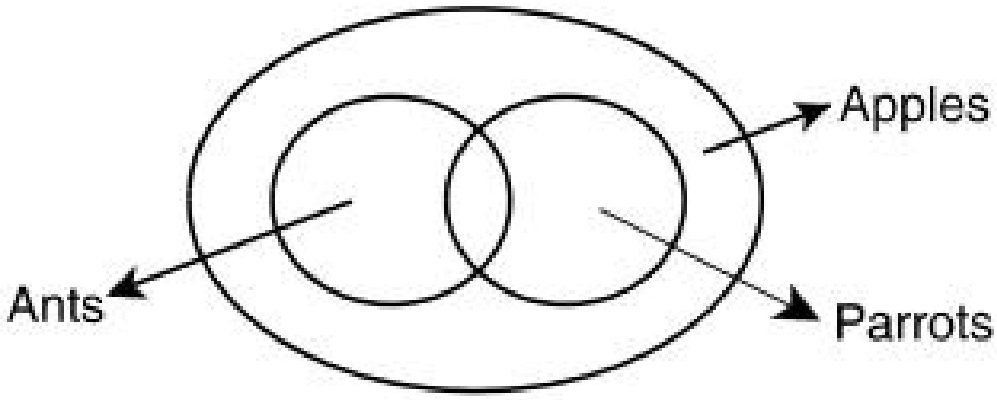
Answer: B



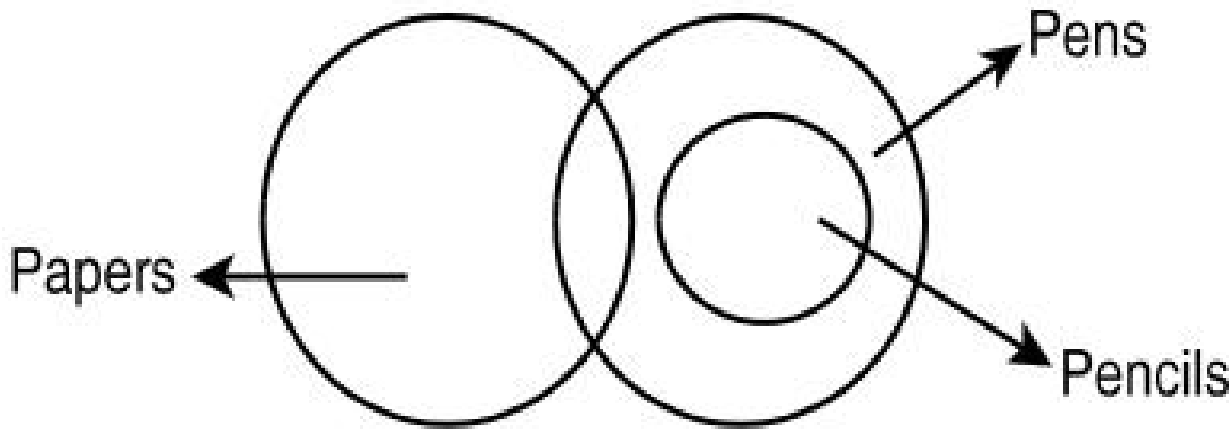
Q3. Statements: Some mangoes are yellow. Some lemons are mangoes.
Conclusions: (1) Some Mangoes are green
(2) Lemon is a yellow.
Answer: D



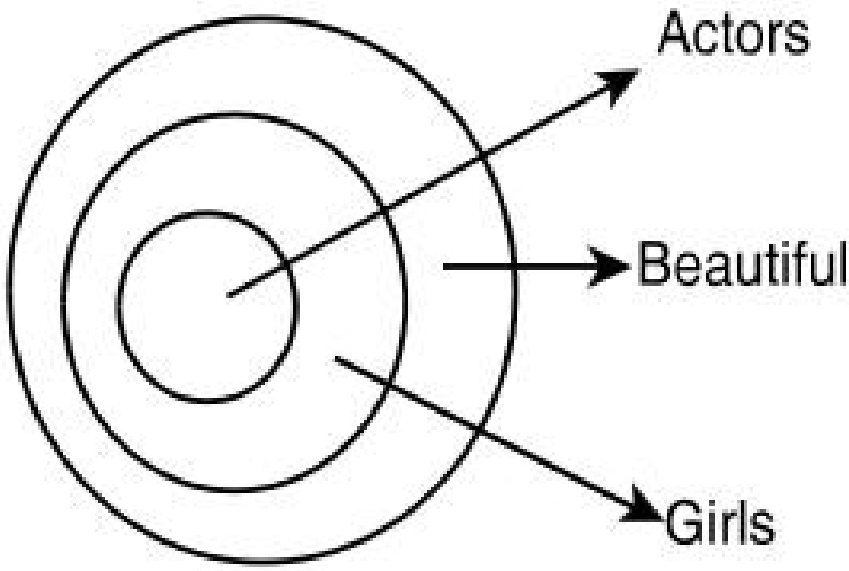
Q4. Statements: Some ants are parrots. All the parrots are apples.
Conclusions: (1) All the apples are parrots.
(2) Some ants are apples.
Answer: B



Q5. Statements: Some papers are pen. All the pencils are pens.
Conclusions: (1) Some pens are pencils.
(2) Some pens are papers.
Answer: E

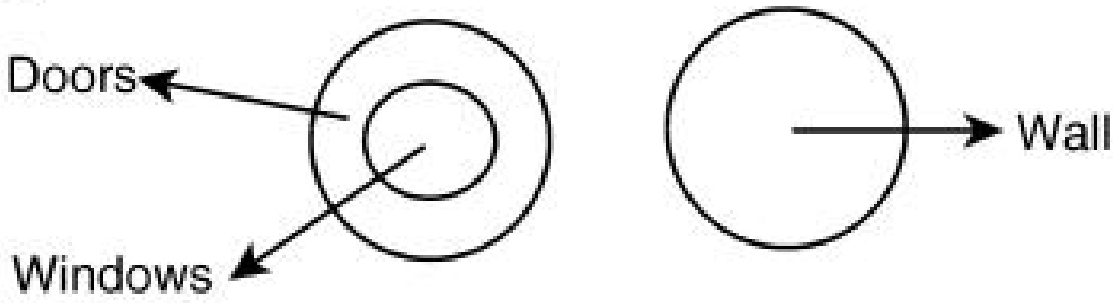


Q6. Statements: All the actors are girls. All the girls are beautiful.
Conclusions: (1) All the actors are beautiful.
(2) Some girls are actors.
Answer: E



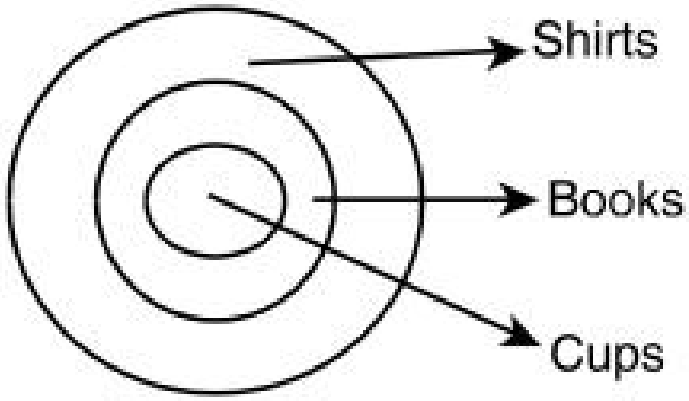
Q7. Statements: All the windows are doors. No door is a wall.
Conclusions: (1) Some windows are walls.
(2) No wall is a door.

Answer: B



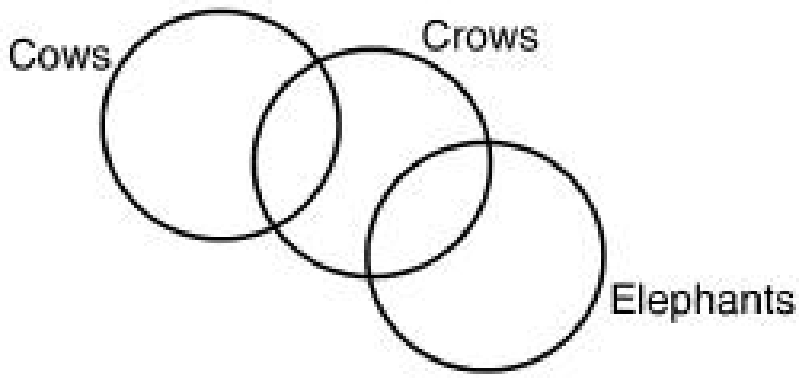
Q8. Statements: All cups are books. All books are shirts.
Conclusions: (1) Some cups are not shirts.
(2) Some shirts are cups.

Answer: B

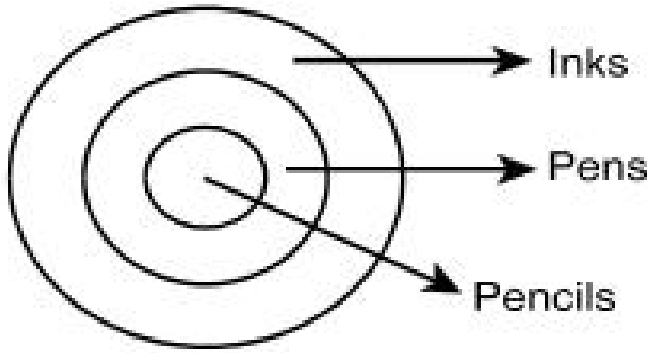


Q9. Statements: Some Cows are Crows. Some Crows are elephants.
Conclusions: (1) Some Cows are elephants.
(2) All Crows are elephants.

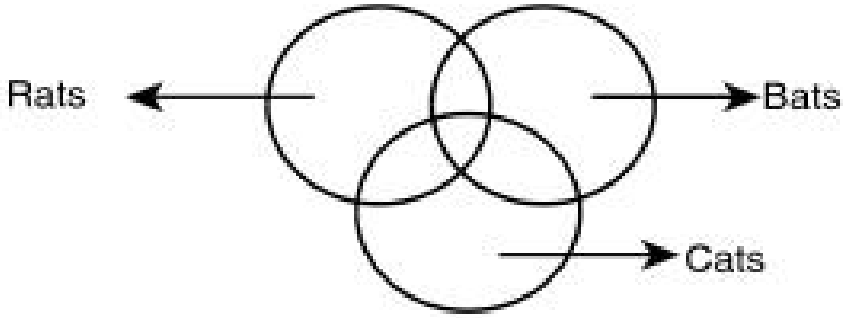
Answer: D



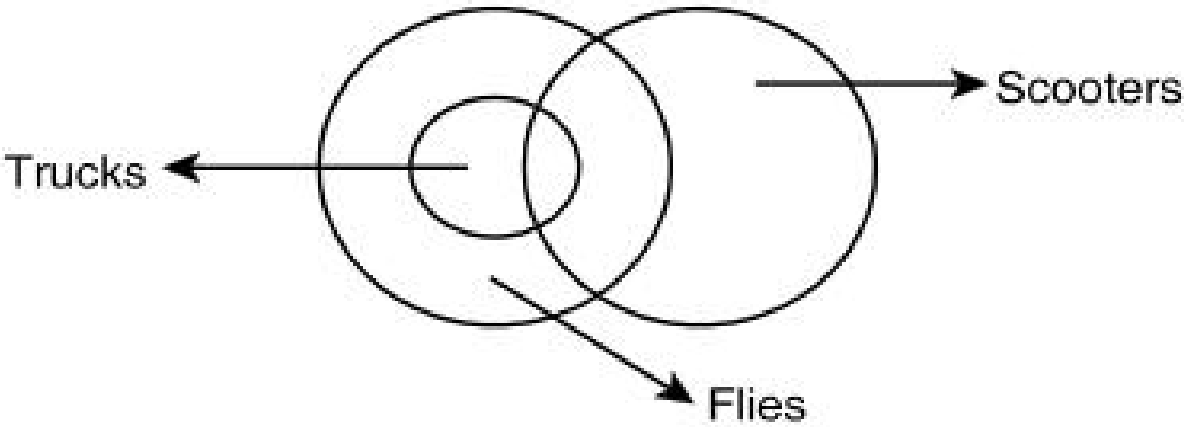
Q10. Statements: All the Pencils are Pens. All the pens are inks.
Conclusions: (1) All the pencils are inks.
(2) Some inks are pencils
Answer: E



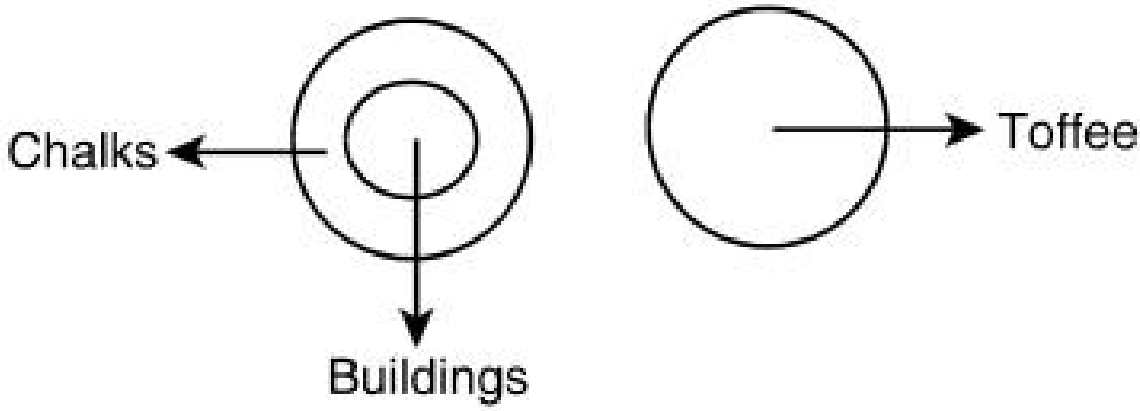
Q11. Statements: Some dogs are bats. Some bats are cats.
Conclusions: (1) Some dogs are Cats.
(2) Some Cats are dogs.
Answer: D



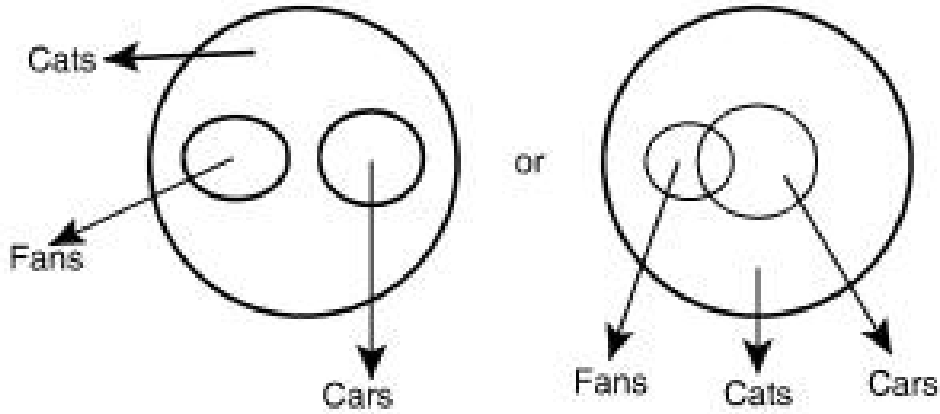
Q12. Statements: All the trucks are flies. Some Scooters are flies.
Conclusions: (1) All the trucks are Scooters.
(2) Some Scooters are trucks.
Answer: D



Q13. Statements: All buildings are Chalks. No chalk is toffee.
Conclusions: (1) No building is toffee
(2) All chalks are buildings.
Answer: A



Q14. Statements: All cars are cats. All fans are cats.
Conclusions: (1) All cars are fans.
(2) Some fans are cars.
Answer: D



Q15. Statements: All lions are dogs. All dogs are rat. No lions are tiger.
Conclusions: (1) No rat is a tiger
(2) Some dogs being tiger is a possibility.
Answer: B

Q16. Statements: Some red are blue. Ally yellow are blue. No blue are orange.
Conclusions: (1) All blue being red is a possibility.
(2) Some yellow are yellow is a possibility.
Answer: A

Q17. Statements: All orange are green. No blue is green. All green are red.
Conclusions: (1) No blue is orange
(2) All red being orange is a possibility.
Answer: E

Q18. Statements: All Monkey are rat. Some rat are lion. No lion is a dog.
Conclusions: (1) All Monkey being lion is a possibility.
(2) No dog is a Rat
Answer: A

Q19. Statements: No dollar is yen. Some yen are franc. All rupee are dollar.
Conclusions: (1) No dollar is franc.
(2) No yen is rupee.
Answer: B

Q20. Statements: All pen are eraser. All eraser are pencil. Some eraser are scale.
Conclusions: (1) All scale being pencil is a possibility.
(2) No eraser is a scale.
Answer: A

Q21. Statements: No table is bench. All bench are chair. Some chair are huts.
Conclusions: (1) No table is chair.
(2) Some bench are huts.
Answer: D

Q22. Statements: Some image are picture. Some picture are photo. Some photo are clip.
Conclusions: (1) All image are clip
(2) Some clip are picture.
Answer: D

Q23. Statements: All apple are grapes. Some apple are Mango. No mango is banana.
Conclusions: (1) All banana being apple is a possibility.
(2) All grapes being banana is a possibility.

Q24. Statements: All Png are bmp. Some bmp are jpg. All gif are jpg.
Conclusions: (1) Some bmp are gif.
(2) Some jpg are png.
Answer: D

Q25. Statements: Some chairs are bags. All trees are chairs.
Conclusions: (1) Some trees are bags.
(2) Some bags are trees.
Answer: D

Q26. Statements: No man is a lion. Ram is a man.
Conclusions: (1) Ram is not a lion.
(2) All men are not ram.
Answer: A

Q27. Statements: No tiger is a lion. Shiva is a tiger.
Conclusions: (1) Shiva is not a lion.
(2) All tigers are not Shiva.
Answer: A

Q28. Statements: All glasses are mirrors. Some mirrors are black.
Conclusions: I: All mirrors are glasses
II: Some glasses are black.
Answer: D

Q29. Statement: Some dogs are monkeys. No monkey is black.
Conclusions: I : Some dogs are black
II: Some monkeys are dogs
Answer: B

Q30. Statement: Many actors are directors. All directors are dancers.
Conclusions: I : Some actors are dancers
II: No director is an actor.
Answer: A

Q31. Statement: All roads are poles. No poles are Bungalows.
Conclusion: Some roads are Bungalows
Some Bungalows are poles
Answer: D

Q32. Statement: Only dogs are animals. No historian is an animals.
Conclusion: I: Some dogs are not historian
II: Some historians are not dogs.
Answer: A

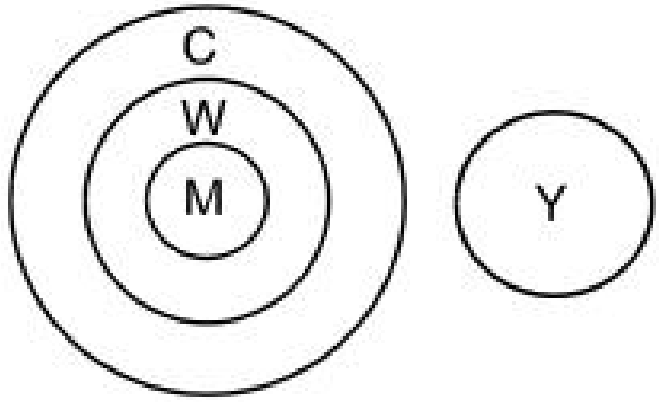
Q33. Statement: Some chairs are caps. No cap is red.
Conclusion: I: Some caps are chairs
II: No chair is red.
Answer: A

Q34. Statement: Some cups are belt. No belt is black.
Conclusion: I: Some cups are blacks.
II: Some cups are not black.
Answer: B

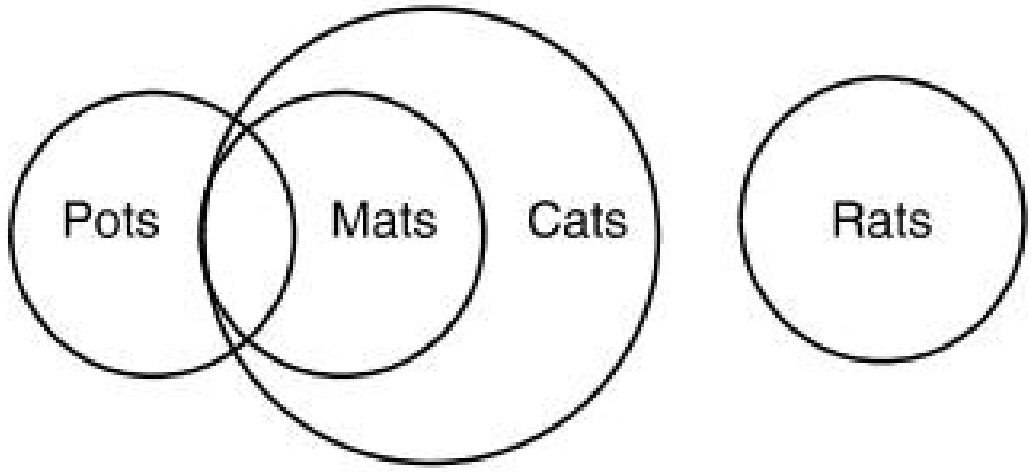
Q35. Statement: Some girls are flowers. Some flowers are books.
Conclusions: I: Some girls are books
II: No books are girls
Answer: C

Q36. Statement: Some files are rats. All animals are rats.
Conclusion: I: All files are rats.
II: Some rats are animals.
Answer: B
Q37. Statements: All Men are women. All women are children is young.
Conclusion: I: Some children are men.
II: No men is young.

Answer: E



Q38. Statement: Some Pots are Mats. All Mats are cats. No cat is Rat.
Conclusion: I: No rat is pot
II: Some rats are not mats.
Answer: B



Q39. Statement: Some pens are slots. Some slats are not pencils. All pencils are Erasers.
Conclusion: I: Some pencil are not slats
II: Some erasers are pens.
Answer: D

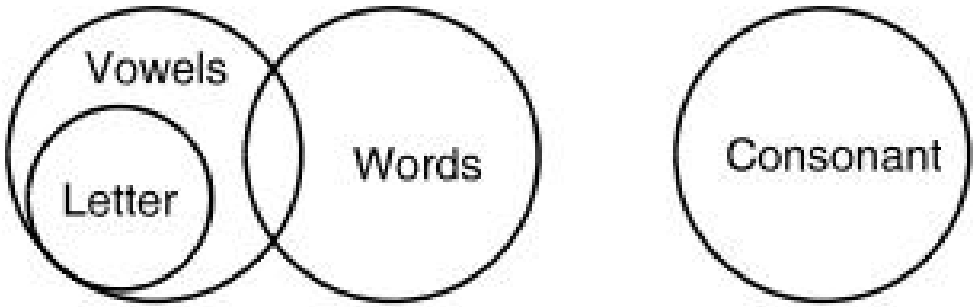
Q40. Statement: Some Bottles are lids. Some lids are flat. Some flat are round.
Conclusion: I : Some flat are lid.
II: Some round are not bottles
Answer: A

Q41. Statement: All letters are vowels. Some vowels are words. No word is consonant.

Conclusion: I: All consonant being vowels is a possibility.

II: No letter is consonants

Answer: A

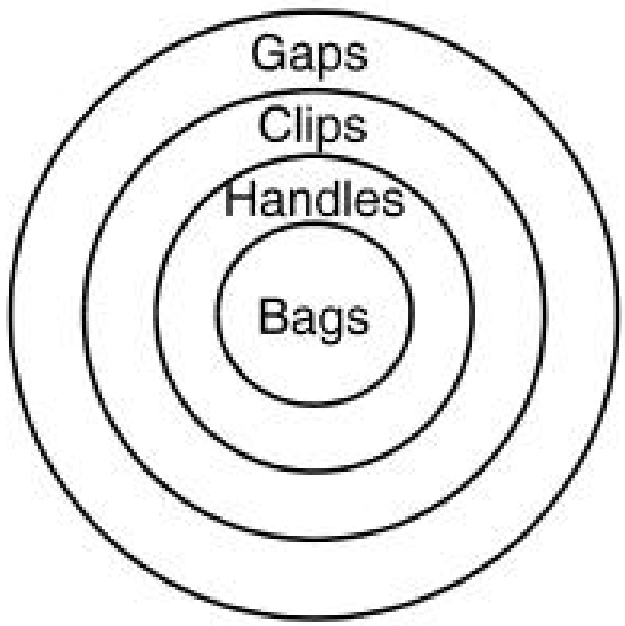


Q42. Statement: All the bags are handles. All the handles are clips. All the clips are gaps.

Conclusion: I: All the clips are bags.

II: All the handles are gaps.

Answer: B

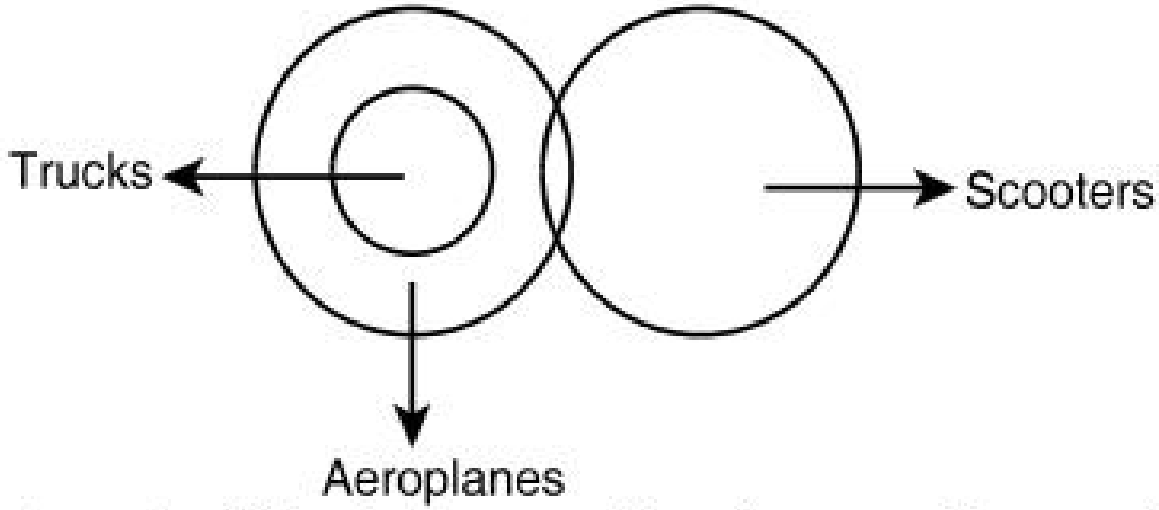


Q43. Statements: All trucks are aeroplanes. Some scooters are aeroplanes.

Conclusions: (1) All trucks are Scooters.

(2) Some aeroplanes are trucks.

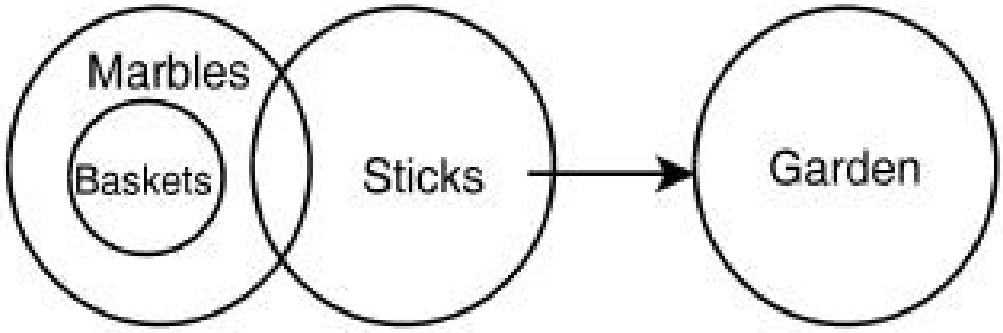
Answer: B



Q44. Statements: All baskets are marbles. Some marbles are sticks. No stick is garden.

Conclusions: (i) Some Gardens are basket.
(ii) No Garden is basket.

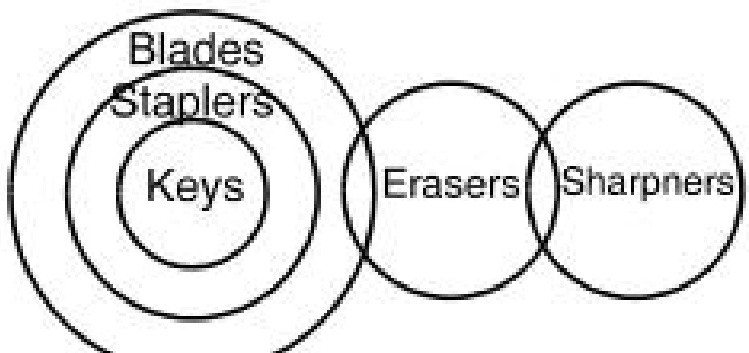
Answer: B



Q45. Statements: All keys are staplers. All staplers are blades. Some blades are eraser. Erasers are Sharpeners.

Conclusions: (i) Some sharpeners are keys.
(ii) Al keys are blades.

Answer: B



Q46. Statements: Some cats are dogs. All rats are dogs.
Conclusion: (1) Some cats are Rats
(2) Some rats are Cats.
Answer: D

Q47. Statement: Some chairs are hats. No hat is red.
Conclusions: (1) Some hats are chairs.
(2) No chair is red.
Answer: A

Q48. Statement: Some cups are laptop. No laptop is black.
Conclusions: (1) Some cups are black.
(2) Some cups are not black.
Answer: B

Q49. Statements: Some dogs are cows. No cow is black.
Conclusions: (1) Some dogs are black.
(2) Some cows are dogs
Answer: B

Q50. Statements: All papers are copies. All copies are erasers.
Conclusions: (i) Some erasers are papers.
(ii) Some copies are no papers.
Answer: A

Q51. Statement: Some ships are Monkeys. Some boats are ships.
Conclusions: (1) Some monkeys are boats.
(2) Some ships are neither boat nor monkey.
Answer: D

Q52. Statements: All sportsman are hardworking. No sportsman are superstitions.
Conclusions: (1) No sportsman are superstitions
(2) All superstitions are not sportsman
Answer: E

Q53. Statement: All cricketer are hardworking. No cricketer are superstitions.
Conclusions: (1) No cricketer are superstitions.
(2) All superstitions are not cricketer.
Answer: E

Q54. Statement: Some mirrors are glasses. All trees are mirrors.
Conclusions: (1) Some trees are glasses
(2) Some glasses are trees.
Answer: D

Q55. Statement: All player are hardworking. No player are superstitions.
Conclusion: (1) No players are superstitions.
(2) All superstitions are not players
Answer: E

Q56. Statements: All roads are villages. No villages are Bungalows.
Conclusions: (1) Some roads are Bungalows.
(2) Some Bungalows are villages.
Answer: D

Q57. Statements: Many actors are producers. All producers are dancers.
Conclusions: (1) Some actors are dancers.
(2) No producer is an actor. Full download
Answer: A

Q58. Statements: Many actors are Comedians. All comedians are dancers.
Conclusions: (1) Some actors are dancers
(2) No comedian is an actor.
Answer: A

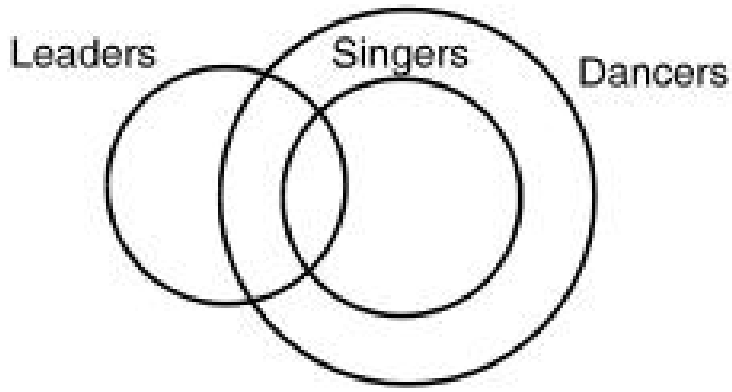
Q59. Statement: All cricketers are fall. Rajeshwar is falls.
Conclusions: (1) Rajeshwar is a cricketer
(2) Rajeshwar is not cricketer.
Answer: C

Q60. Statement: All players are fall. Sachin is fall.
Conclusions: (i) Sachin is a player.
(ii) Sachin is not player.
Answer: C

Q61. Statement: No man is a zebra. Ram is a man.
Conclusions: (1) Ram is not a zebra
(2) All men are not Ram
Answer: A

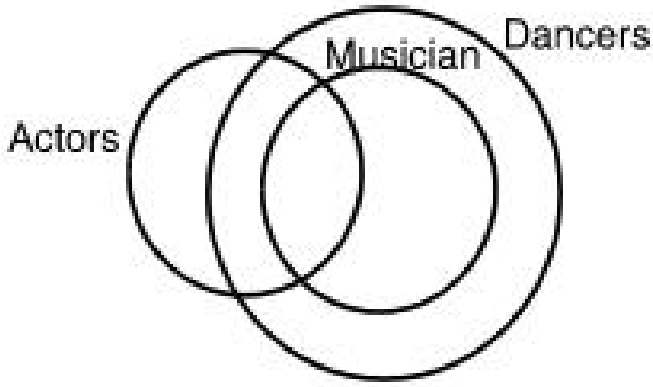
Q62. Statement: All Sportsman are strong. Rajesh is strong.
Conclusions: (i) Rajesh is a sportsman
(ii) Rajesh is not sportsman
Answer: C

Q63. Statements: Some leaders are singers. All the singers are dancers.
Conclusions: (1) Some leaders are dancers
(2) No singer is leader.
Answer: A



Q64. Statement: Some actors are musician. All the Musicians are dancers.
Conclusions: (1) Some actors are dancers.
(2) No Musician is actor.

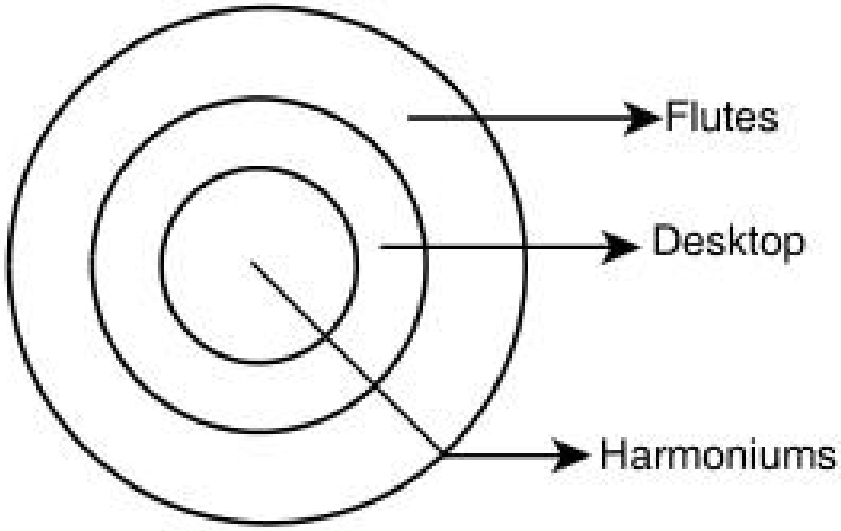
Answer: A



Q65. Statement: All the harmoniums are Desktop. All the Desktops are flutes.

Conclusion: (1) All the flutes are Desktop.
(2) All the harmoniums are flutes.

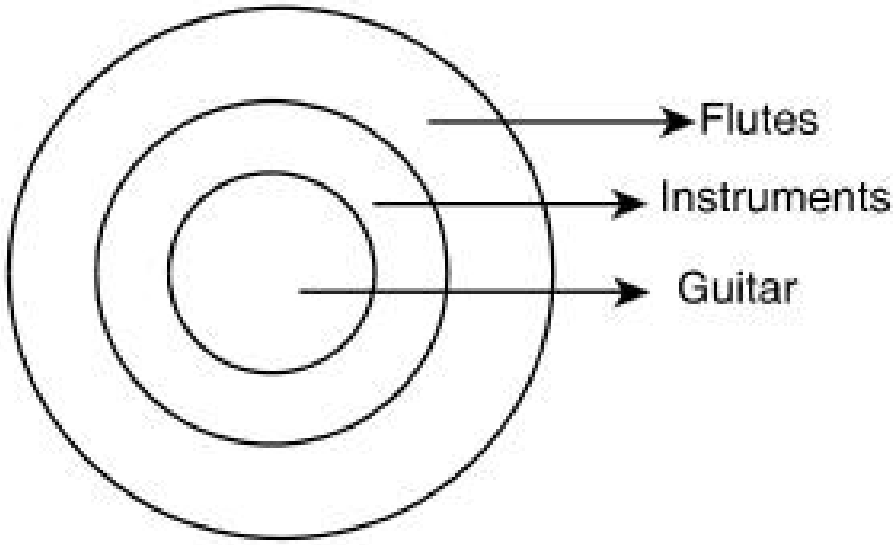
Answer: B



Q66. Statement: All the guitars are instruments. All the instruments are flutes.

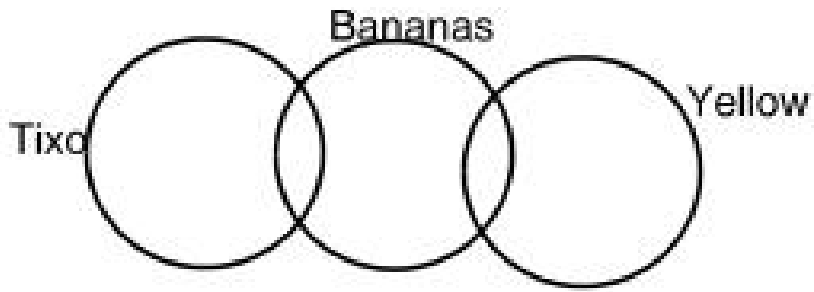
Conclusions: (1) All the flutes are instruments

Answer: B



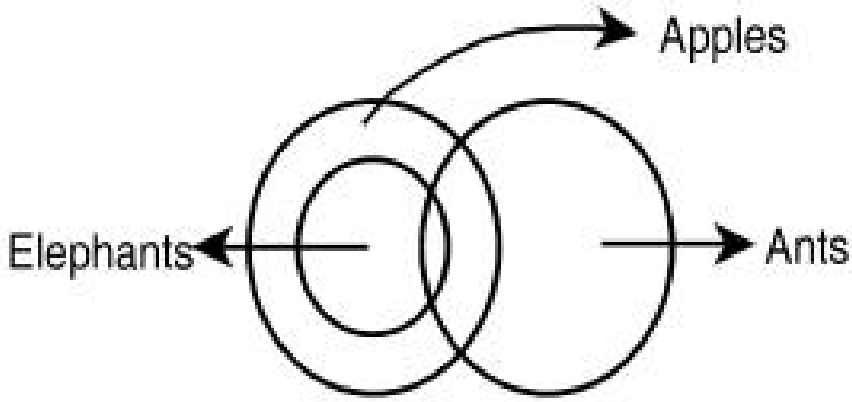
Q67. Statement: Some Bananas are yellow. Some tixo are Bananas.
Conclusions: (1) Some Bananas are green
(2) Tixo is a yellow

Answer: D



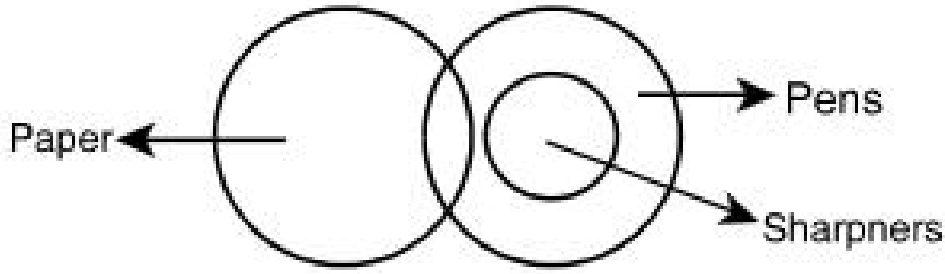
Q68. Statements: Some ants are elephants. All the elephants are apples.
Conclusions: (1) All the apples are elephants
(2) Some ants are apples.

Answer: B



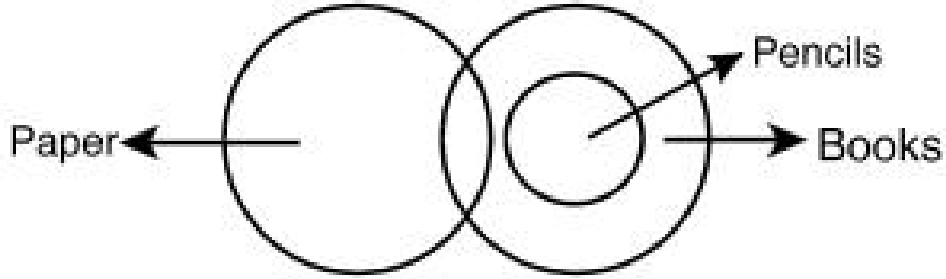
Q69. Statement: Some papers are pens. All sharpeners are pen.
Conclusion: (1) Some pens are sharpeners.
(2) Some pens are papers.

Answer: E



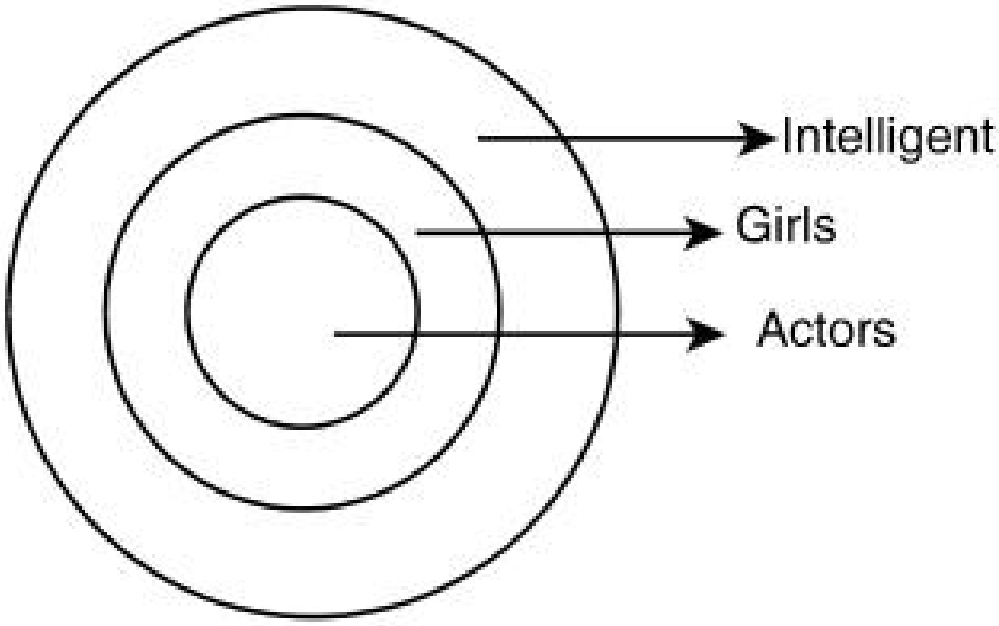
Q70. Statement: Some papers are books. All the pencils are books.
Conclusions: (1) Some books are pencils.
(2) Some books are papers.

Answer: E



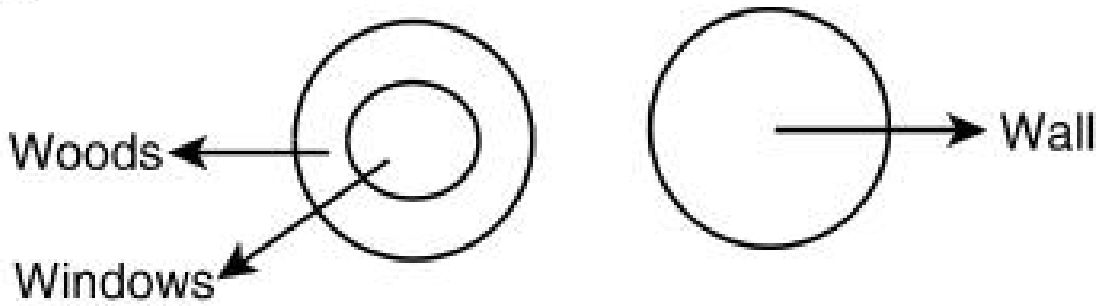
Q71. Statement: All the actors are girls. All the girls are intelligent.
Conclusion: (1) All the actors are intelligent.
(2) Some girls are actors.

Answer: E



Q72. Statement: All the windows are woods. No wood is a wall.
Conclusion: (1) Some windows are walls.
(2) No wall is a wood.

Answer: B



Q73. Statement: All cups are bats. All bats are shirts.
Conclusion: (1) Some cups are not shirts.
(2) Some shirts are cups.

Answer: B

Q74. Statement: Some cows are silvers. Some silvers are elephants.
Conclusion: (1) Some cows are elephants
(2) All silvers are elephants

Answer: D

Q75. Statements: All the pencils are boxes. All the boxes are inks.
Conclusions: (1) All the pencils are inks.
(2) Some inks are pencils.
Answer: E

Q76. Statements: Some dogs are rats. Some rats are cats.
Conclusions: (1) Some dogs are cats.
(2) Some cats are dogs.
Answer: D

Q77. Statements: Some tigers are bats. Some bats are cats.
Conclusions: (1) Some tigers are cats.
(2) Some cats are tigers.
Answer: D

Q78. Statements: All the cars are flies. Some scooters are flies.
Conclusions: (1) All the cars are scooters.
(2) Some scooters are cars.
Answer: D

Q79. Statements: All bags are chalks. No chalks is toffee.
Conclusions: (1) No bag is toffee.
(2) All chalks are bags.
Answer: A

Q80. Statements: All cars are Rockets. All fans are Cats.
Conclusions: (1) All cars are fans.
(2) Some fans are cars.
Answer: D

PAST YEAR QUESTIONS AND ANSWERS

2018 - MAY

[1] A prisoner introduced a boy who came to visit him to the jailor as "Brothers and sisters I have none, he is my father's son's son". Who is the boy?

(a) Nephew (b) Son
(c) Cousin (d) Uncle (1 mark)

Answer:

(b) 'Son'

2018 - NOVEMBER

Directions (2 - 4) : Each of the following questions contains two statements followed by two conclusions number I and II. You have to decide which of the given conclusions definitely follows from the given statements.

[2] Statements :

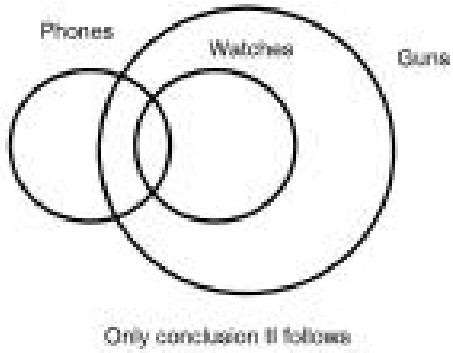
1. Some phones are watches
2. All watches are guns

Conclusions : I. All guns are watches
II. Some guns are phones.

(a) Only conclusion I follows (b) Only conclusion II follows
(c) Either I or II follows (d) Neither I nor II follows (1 mark)

Answer:

(b)



[3] Statements :

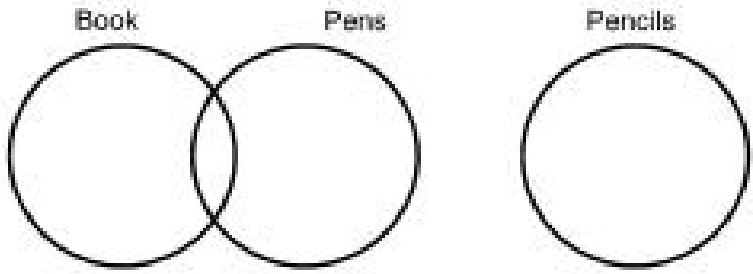
1. Some books are pens
2. No pen is pencil

Conclusions : I. Some books are pencil
II. No book is pencil

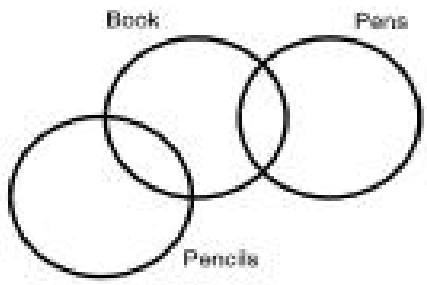
(a) Only conclusion I follows (b) Only conclusion II follows
(c) Either I or II follows (d) Neither I nor II follows (1 mark)

Answer:

(c)



Or



Either 'I' Or 'II' follows

[4] Statements :

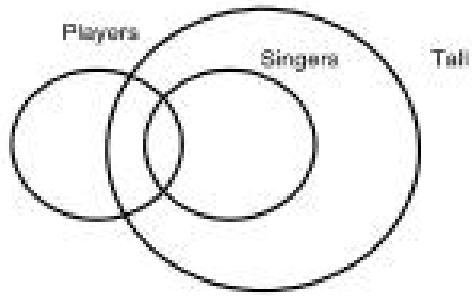
1. Some players are singers
2. All singers are tall

Conclusions : I. Some players are tall
II. All players are tall

(a) Only conclusion I follows (b) Only conclusion II follows
(c) Either I or II follows (d) Neither I nor II follows (1 mark)

Answer:

(a)



Only conclusion 'I' follows

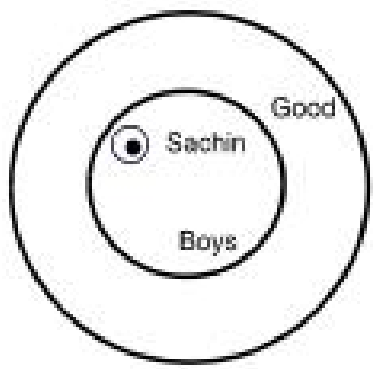
2019 - JUNE

[5] Statement I Sachin is a boy
II All boys are good.

Conclusion I : All boys are Sachin
II : Sachin is not a good boy

(a) If only I follow
(b) If only II follow
(c) If both I & II follow
(d) If neither I nor II follow (1 mark)

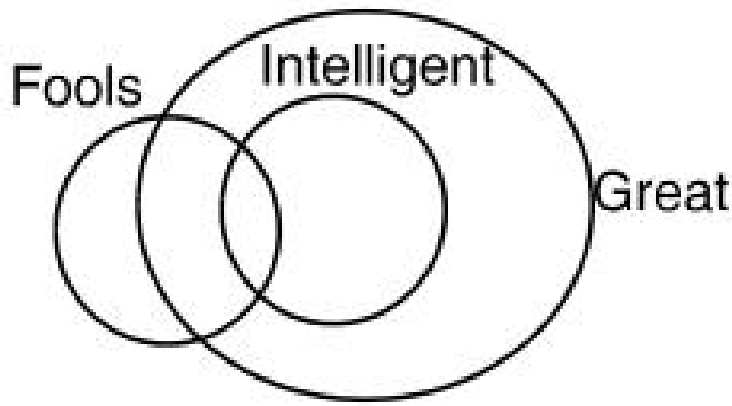
Answer: (d)



[6] Statement I: Some fools are intelligent
Statement II: All intelligent are great
Conclusion I: Some fools are great
Conclusion II: All greats are intelligent
(a) Conclusion I follows
(b) Conclusion II follows
(c) Neither I nor II follows
(d) either I or II follows (1 mark)

Answer:

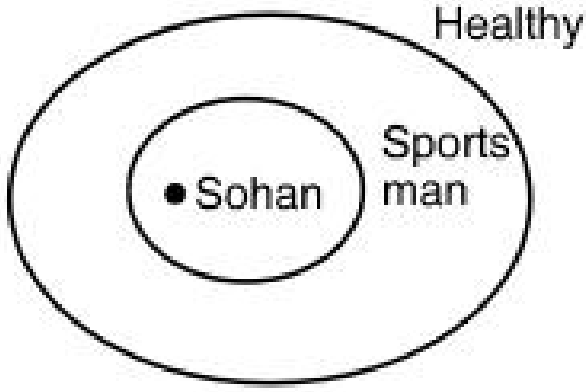
(a)



[7] Statement I: Sohan is good Sports man.
Statement II: Sports man is healthy.
Conclusion I: Sohan is healthy.
Conclusion II: All Sports mens are good.
(a) Conclusion I follows
(b) Conclusion II follows
(c) Neither I nor II follows
(d) Either I or II follows (1 mark)

Answer:

(a)



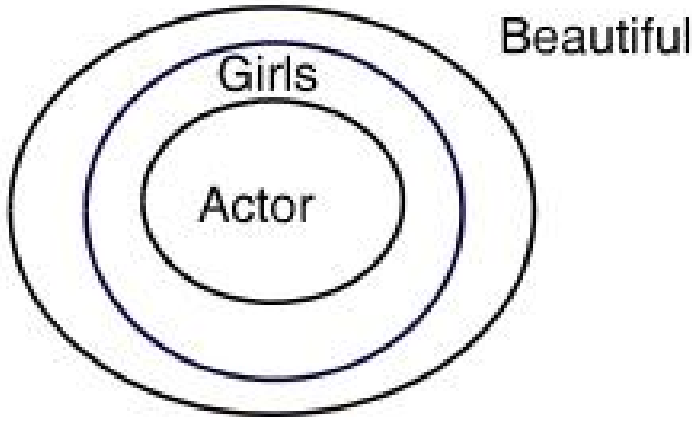
So 1st conclusion is follows.

[8] Statement: I All the actors are girls.
II All the girls are beautiful.
Conclusion : I All the actors are beautiful
II Some girls are actors

(a) If only I follow
(b) If only II follow
(c) If both I & II follow
(d) If neither I nor II follow (1 mark)

Answer:

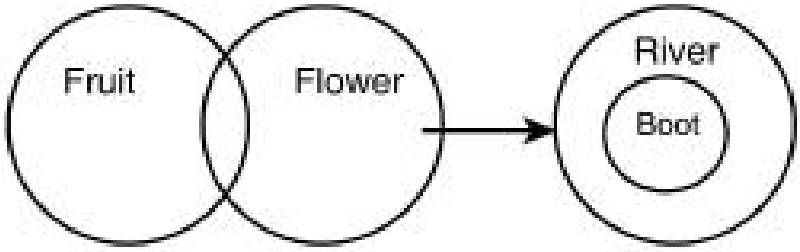
(c)



2019 - NOVEMBER

[9] Statements:
Some fruits are flowers
No flower is a boat
All boat are rivers
Conclusion:
I. Some fruits are rivers
II. Some rivers are boats
III. Some rivers are fruits
IV. Some flowers are fruits
(a) Only I & III follows
(b) Only II & III follows
(c) Only II & IV follows
(d) All follows (1 mark)

Answer:
(c)

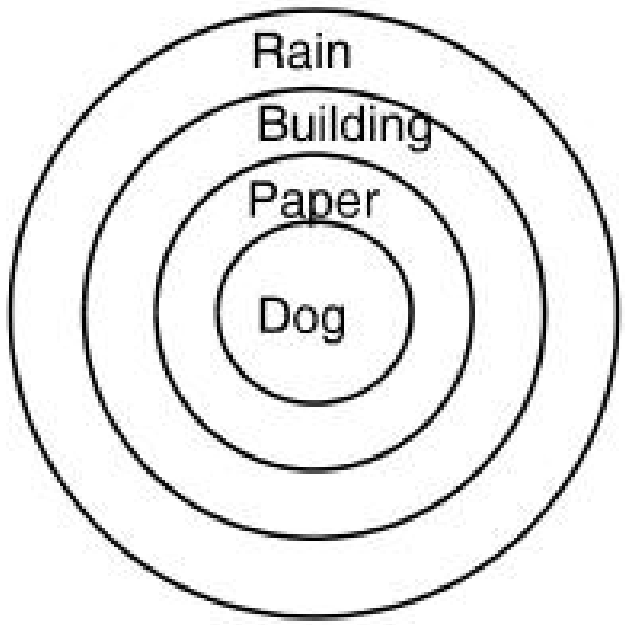


Conclusions: Some flowers are fruits some rivers are boots
So conclusion II & IV hold good.

[10] Statements: All dogs are papers
All building are rains
All Papers are building
I some rains are buildings.
Conclusion II some rains are papers.
III All dogs are rains.
IV some papers are rains.
(a) only I follows
(b) only I, II, III follows
(c) either I or II follows

Answer:

(b) According to Question



Conclusions

- Some rains are buildings.
- Some buildings are papers.
- Some papers are dogs.

So,

Conclusion I, II, III follow

✿

Part - C

Statistics

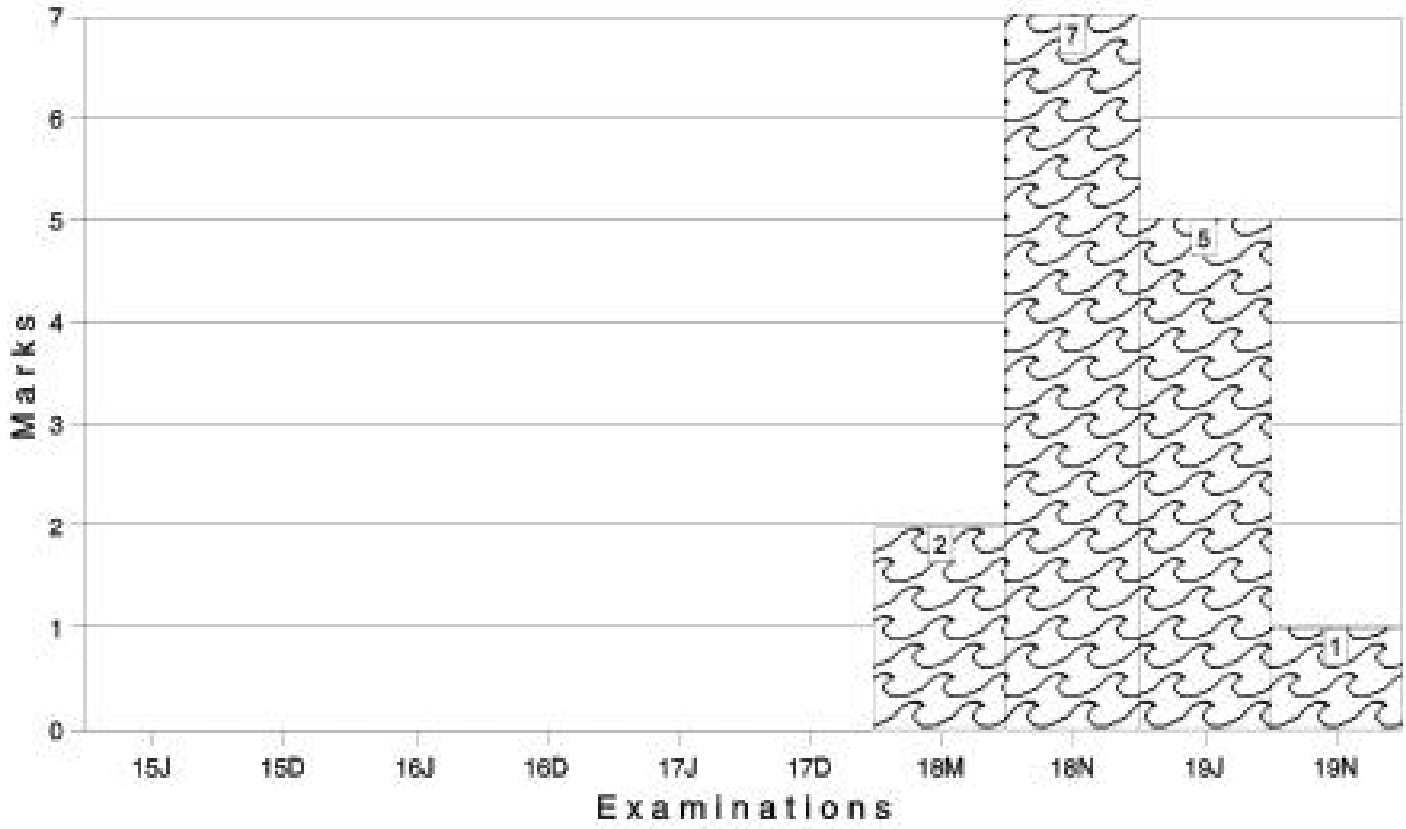
CHAPTER

14 STATISTICAL DESCRIPTION OF DATA

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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PAST YEAR QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] The quickest method to collect primary data is :
(a) Personal Interview
(b) Indirect Interview
(c) Mailed Questionnaire Method
(d) Telephonic Interview (1 mark)

Answer:

(d) Telephonic interview method is considered as the quickest method to collect primary data as the relevant information can be gathered by the researcher himself by contacting the interviewer over the phone without any time log.

[2] Which of the following statement is true?
(a) Statistics is derived from the French word 'Statistik'
(b) Statistics is derived from the Italian word 'Statista'.
(c) Statistics is derived from the Latin word 'Statistique'.
(d) None of these (1 mark)

Answer:

(b) According to the History of Statistics we can see that one school of thought is of the view that statistics is derived from the Italian word 'Statista'

[3] The following data relates to the incomes of 90 persons :

Income in ₹ :	1500-1999	2000-2499	2500-2999	3000-3499
No. of Persons :	13	32	20	25

What is the percentage of persons earning more than ₹ 2,500 ?
(a) 45 (b) 50
(c) 52 (d) 55 (1 mark)

Answer:

(b) No. of persons earning more than ₹ 2500 = 20 + 25 = 45

∴ The percentage of persons earning more than

₹ 2,500 = $\frac{45}{90} \times 100 = 50\ \%$

2007 - FEBRUARY

[4] In tabulation, source of data, if any, is shown in the :

(a) Stub (b) Body

(c) Caption (d) Footnote (1 mark)

Answer:

(d) The source of data, if any, in any kind of tabulation is shown in the footnote.

[5] Divided bar chart is good for :

(a) Comparing various components of a variable

(b) Relating the different components to the whole.

(c) (a) and (b)

(d) (a) or (b) (1 mark)

Answer:

(c) Divided Bar Chart, also known as percentage Bar Diagrams, is good for both the things i.e. for comparing different components of a variable as well as the relating of the different components to the whole.

2007 - MAY

[6] Relative frequency for a particular class lies between :

(a) 0 and 1 (b) 0 and 1, both inclusive

(c) – 1 and 0 (d) – 1 and 1 (1 mark)

Answer:

(a) Relative frequency of a class interval is defined as the ratio of the class frequency to the total frequency. Therefore, Relative frequency for a particular class lies between 0 and 1.

[7] Find the number of observations between 350 and 400 from the following data :

Value :	More than 200	More than 350	More than 400	More than 450
No. of observations :	48	25	12	0

(a) 13 (b) 15
(c) 17 (d) 19 (1 mark)

Answer:

(a) The number of observation which are more than 350 is inclusive of those observations which are more than 400 and 450.

∴ Deducting those number of observations which are more than 400 and 450 from the number of observations which are 350, we will get the number of observations lying between 350 and 400.

So, the number of observations lying between 350 and 400 = 25 – 12 – 0 =13

[8] When the width of all classes is same, frequency polygon has not the same area as the Histogram :

(a) False (b) True
(c) Both (d) None (1 mark)

Answer:

(a) When the width of all classes is same frequency, polygon has the same area as the histogram.

[9] The graphical representation of a cumulative frequency distribution is called :

(a) Histogram (b) Ogive
(c) Both (d) None (1 mark)

Answer:

(b) The graphical representation of a cumulative frequency distribution is called Ogive i.e. by plotting the cumulative frequency against the respective class boundary, we get olives which can be less than type ogive are these than type olives depending upon the type of

2007 - AUGUST

[10] A table has _____ parts.

(a) Four (b) Two

(c) Five (d) None (1 mark)

Answer:

(c) A table has five parts namely.

(i) Stub
(ii) Caption
(iii) Body
(iv) Box head
(v) Footnote.

[11] Cost of sugar in a month under the heads raw materials, labour, direct production and others were 12, 20, 35 and 23 units respectively. What is the difference between the central angles for the largest and smallest components of the cost of sugar ?

(a) 72^0 (b) 48^0

(c) 56^0 (d) 92^0 (1 mark)

Answer:

(d) Total components of the cost of sugar

= (12 + 20 + 35 + 23) units

= 90 units

Largest component of cost of sugar

= 35 units

i.e. $\frac{35}{90} \times 360^0 = 140$

Smallest component of cost of sugar

= 12 units

i.e. $\frac{12}{90} \times 360^0 = 48^0$

∴ Difference between the central angles for the largest and smallest components of the cost of sugar

$= 140^0 - 48^0 = 92^0$

[12] Frequency density corresponding to a class interval is the ratio of :
(a) Class Frequency to the Total Frequency
(b) Class Frequency to the Class Length
(c) Class Length to the Class Frequency
(d) Class Frequency to the Cumulative Frequency (1 mark)

Answer:

(b) Frequency density of a class interval is defined as the ratio of the frequency of that class interval to the corresponding class length.

2007 - NOVEMBER

[13] In order to compare two or more related series, we consider :
(a) Multiple Bar Chart (b) Grouped Bar Chart
(c) (a) or (b) (d) (a) and (b) (1 mark)

Answer:

(c) Multiple Bar Chart also known as Grouped Bar Chart is one dimensional diagram in which two or more bars adjoining each other are constructed to represent the values of different variables or the values of various components of the same variable.
Multiple Bar Chart or Grouped Bar Chart is considered to compare two or more related series.

[14] An area diagram is :
(a) Histogram (b) Ogive
(c) Frequency Polygon (d) None of these (1 mark)

Answer:

(a) Histogram is a graph that represents the class frequencies in a frequency distribution by vertical adjacent rectangles. A Histogram is two - dimensional, ie. a histogram comprises of both length as well as the width. As the Product of length and width indicates the area. Therefore Area Histogram is referred to as an Area Diagram. Its area represents the total frequency as distributed throughout the classes.

[15] Most extreme values which would ever be included in a class interval are called:

(a) Class Interval (b) Class Limits

(c) Class Boundaries (d) None of these (1 mark)

Answer:

(c) Most extreme values which would be ever included in a class-interval are called as class boundaries, also referred to as actual class limit, are defined as the limits up to which the two limits, (actual) of each class may be extended to fill up the gap that exist between the classes.

[16] In 2000, out of total of 1,750 workers of a factory, 1,200 were members of a trade union. The number of women employed was 200 of which 175 did not belong to a trade union. In 2004, there were 1,800 employees who belong to a trade union and 50 who did not belong to trade union. Of all the employees in 2004, 300 were women of whom only 8 did not belong to the trade union. On the basis of this information, the ratio of female members of the trade union in 2000 and 2004 is :

(a) 292 : 25 (b) 8 : 175

(c) 175 : 8 (d) 25 : 292 (1 mark)

Answer:

(d) TITLE : Sex distribution of Trade Union and Non- union members.

Year	2000			2004		
Category	Male	Female	Total	Male	Female	Total
Member	1175	25	1200	1508	292	1800
Non-member	375	175	550	42	8	50
Total	1550	200	1750	1550	300	185

Required ratio of female members of the trade union is 2000 : 2004 = 25 : 292

2008 - FEBRUARY

[17] The lower class boundary is :

(a) An upper limit to Lower Class Limit

(b) A lower limit to Lower Class Limit.

(c) Both (a) & (b)

(d) None of these (1 mark)

Answer:

(b) Lower class Boundary

Lower class limit $-\frac{1}{2}$(upper class limit of the class – lower class limit of the succeeding class). Therefore, lower class boundary is a lower limit to lower class limit.

[18] The distribution of profits of a company follows :

(a) J - shaped frequency curve

(b) U - shaped frequency curve

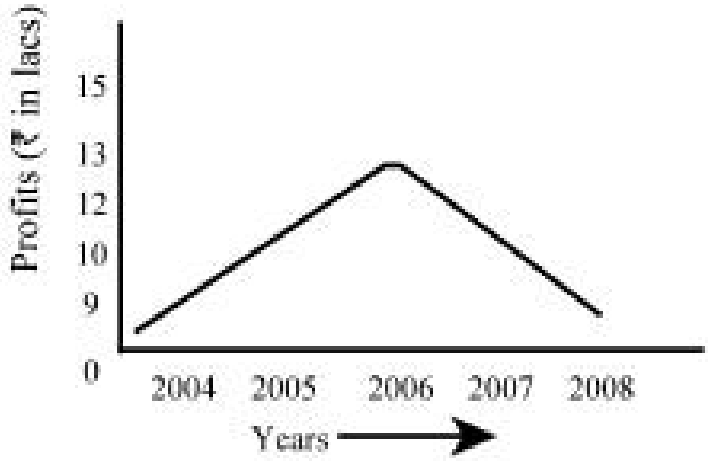
(c) Bell - shaped frequency curve

(d) Any of these (1 mark)

Answer:

(c) The bell- shaped curve looks like a bell. On a bell- shape curve, the frequency, starting from a rather low value, gradually reaches the maximum value, somewhere near the central part and then gradually decreases to reach its lowest value at the other extremity. Similar is the case of profits of a company. It rises till the resources are fully utilized and if the resources are still utilized then due to over-utilization of resources, the profits start declining. This can be clearly depicted through the data given below:

Year	Profits (₹ in lacs)
2004	10
2005	12
2006	15
2007	13



[19] Out of 1000 persons, 25 per cent were industrial workers and the rest were agricultural workers. 300 persons enjoyed world cup matches on T.V. 30 per cent of the people who had not watched world cup matches were industrial workers. What is the number of agricultural workers who had enjoyed world cup matches on TV?

(a) 230 (b) 250

(c) 240 (d) 260 (1 mark)

Answer:

(d)

Category	T.V.	NTV	TOTAL
Agricultural workers	260	490	750
Industrial workers	40	210	250
Total	300	700	1000

Therefore, number of agricultural workers who had enjoyed world Cup matches on T.V. = 260

[20] Median of a distribution can be obtained from ;

(a) Histogram (b) Frequency Polygon

(c) Less than type Ogives (d) None of these (1 mark)

Answer:

(c) Olives are considered for obtaining quartiles graphically. If a perpendicular is drawn from the point of intersection of the two o-gives, i.e. less than type ogive and more than type give, on the horizontal axis, then x- value of this point gives us the value of median, the second or middle quartile.

2008 - JUNE

[21] In indirect oral investigation :

(a) Data is not capable of numerical expression

(b) Not possible or desirable to approach informant directly.

(c) Data is collected from the books.

(d) None of these (1 mark)

Answer:

(b) Indirect oral investigation is a method in which a third person is contacted who is expected to know the necessary details about the persons for whom the enquiry is meant. This method is suitable when it is not possible or deliverable to approach informant directly.

[22] Circular diagrams are always :

(a) One - dimensional (b) Two - dimensional

(c) Three - dimensional (d) Cartograms (1 mark)

Answer:

(b) Circular diagram is a Two- dimension diagram in which a circle is prepared and the radius of circle is determined on the basis of minimum square root value of the variable. Two- dimensional diagram is a diagram which is prepared on the basis of two dimension i.e. length and width.

[23] The column headings of a table are known as :

(a) Body (b) Stub

(c) Box - head (d) Caption (1 mark)

Answer:

(d) Each column is given a heading to explain what the figures in the columns represent. These column headings of a table are known as caption.

[24] Some important sources of secondary data are _____ :

(a) International and Government sources

(b) International and primary sources

(c) Private and primary sources

(d) Government sources (1 mark)

Answer:

(a) The Government source like Indian trade journal - weekly, reserve Bank of India Bulletin - monthly, etc and International sources like WHO, World Bank, IMF, etc are some of the important sources of secondary data.

2008 - DECEMBER

[25] From the following data find the number class intervals if class length is given as 5.

73, 72, 65, 41, 54, 80, 50, 46, 49, 53.

(a) 6 (b) 5

(c) 7 (d) 8 (1 mark)

Answer:

(d) We have, Range = Maximum value – Minimum value = 80 – 41 = 39

Class length = 5

No. of class Intervals × class lengths ≅ Range

$\Rightarrow$ No. of class Intervals × 5 ≅ 39

$\Rightarrow$ No. of class Intervals = $\frac{39}{5}$ ≅ 8

(We always take the next integer as the no. of class intervals so as to include both the minimum and maximum values).

[26] The most appropriate diagram to represent the data relating to the monthly expenditure on different items by a family is

(a) Histogram (b) Pie-diagram.

(c) Frequency polygon (d) Line graph. (1 mark)

Answer:

(b) Pie diagram

[27] Which of the following is a statistical data ?

(a) Ram is 50 years old.

(b) Height of Ram is 5'6" and of Shyam and Hari is 5'3" and 5'4" respectively.

(c) Height of Ram is 5'6" and weight is 90kg

(d) Sale of A was more than B and C. (1 mark)

Answer:

(b) Option (b) represents statistical data which can be understood by referring the definition of statistics keeping note of the following points.

(1) Statistics are aggregate of facts. A single figure cannot be called as statistics because it cannot be compared to draw any conclusion out of it.

(2) All statistical facts are expressed in numbers. Qualitative expressions like young, old, etc do not constitute statistics.

(3) Statistics should be placed in relation to each other so as to facilitate comparison. For this purpose, the data must be homogenous and not heterogenous. e.g. height and weight are heterogenous in character.

[28] Sales of XYZ Ltd. for 4 months is :

Months	Sales
Jan.	10,000
Feb.	15,000
May	18,000
Apr.	9,000

The above data represents :

(a) Discrete (b) Continuous

(c) Individual (d) None of these (1 mark)

Answer:

(c) Given data represents unclassified and ungrouped data. Therefore, the given series is an **individual series**.

2009 - JUNE

[29] Mid values are also called _____

(a) Lower limit (b) Upper limit

(c) Class mark (d) None (1 mark)

Answer:

(c) Mid-values are also called class mark.

$$\text{Class Mark} = \frac{\text{Lower class limt} + \text{Upper class limit}}{2}$$

[30] Which of the following is not a two-dimensional figure ?

(a) Line Diagram (b) Pie Diagram

(c) Square Diagram (d) Rectangle Diagram (1 mark)

Answer:

(a) Line Diagram.

[31] Less than type and more than type Ogives meet at a point known as:

(a) Mean (b) Median

(c) Mode (d) None (1 mark)

Answer:

(b) By plotting cumulative frequency against the respective class boundary, we get olives. There are two type of olives :

(i) Less than type ogive.

(ii) More than type ogive.

Olives may be considered for obtaining quartiles graphically. If a perpendicular is drawn from the point of intersection of two olives on the horizontal axis, then the x-value of this point gives us the value of median, the second or middle quartile.

Therefore, the meeting point of less than type ogive and more than

2009 - DECEMBER

[32] Arrange the dimensions of Bar diagram, Cube diagram, Pie diagram in sequence.

(a) 1, 3, 2 (b) 2, 1, 3
(c) 2, 3, 1 (d) 3, 2, 1 (1 mark)

Answer:

(a) Bar diagram is one dimensional.
Cube diagram has 3 dimensions viz. length breadth and height and hence is three-dimensional.
Pie-diagram is two-dimensional.
Therefore, if we arrange it in sequence, we get:
Bar diagram, cube-diagram and Pie diagram i.e. 1, 3, 2.

[33] With the help of histogram one can find.

(a) Mean (b) Median
(c) Mode (d) First Quartile (1 mark)

Answer:

(c) Histogram is used to find Mode. [Self Explanatory]

[34] Nationality of a person is :

(a) Discrete variable (b) An attribute
(c) Continuous variable (d) None (1 mark)

Answer:

(b) A qualitative characteristic is known as an attribute.
So the nationality of a person is an attribute as it is a qualitative characteristic.

[35] If we plot less than and more than type frequency distribution, then the graph plotted is _____.

(a) Histogram (b) Frequency Curve
(c) Ogive (d) None of these (1 mark)

Answer:

(c) If we plot less than and more than type frequency distribution, then the graph plotted is Ogive.
Ogive are of two types - Less than type ogive and more than type ogive. [self-explanatory]

2010 - JUNE

[36] The primary rules that should be observed in classification

(i) As far as possible, the class should be of equal width

(ii) The classes should be exhaustive

(iii) The classes should be unambiguously defined.

Then which of the following is correct

(a) only (i) and (ii) (b) only (ii) and (iii)

(c) only (i) and (iii) (d) all (i), (ii) and (iii) (1 mark)

Answer:

(b) Requisites of a good classification are:

(1) It should be exhaustive

(2) It should be mutually exclusive

(3) It should be unambiguous

(4) It should be stable and flexible

(5) It should be homogeneous

(6) It should be a revealing classification

[37] Using Ogive Curve, we can determine

(a) Median (b) Quartile

(c) Both (a) and (b) (d) None. (1 mark)

Answer:

(c) Olives are considered for obtaining quartiles graphically. If a perpendicular is drawn from point of intersection of two Olives on horizontal axis, then x-value of this point gives us the value of median (2^{nd} or middle quartile).

2010 - DECEMBER

[38] Mode can be obtained from

(a) Frequency polygon. (b) Histogram.

(c) Ogive (d) All of the above (1 mark)

Answer:

[39] The data obtained by the internet are

(a) Primary data (b) Secondary data

(c) Both (a) and (b) (d) None of these. (1 mark)

Answer:

(b) Secondary data

[40] The statistical measure computed from the sample observations alone have been termed as

(a) estimate (b) parameter.

(c) statistic (d) attribute. (1 mark)

Answer:

(c) Statistic

2011 - JUNE

[41] When the two curves of ogive intersect, the point of intersection provides :

(a) First Quartile (b) Second Quartile

(c) Third Quartile (d) Mode. (1 mark)

Answer:

(b) We know, that the two curves viz. Less than Ogive & More than Ogive intersect at a point called MEDIAN or we can say Second Quartile.

[42] Frequency Density can be termed as:

(a) Class frequency to the cumulative frequency

(b) Class frequency to the total frequency

(c) Class frequency to the class length

(d) Class length to the class frequency. (1 mark)

Answer:

(c) Class frequency to the class length.

[43] The Choronological classification of data are classified on the basis of :

(a) Attributes (b) Area

(c) Time (d) Class Interval (1 mark)

Answer:

(c) Chronological Classification data are classified on the basis of "TIME".

[44] Arrange the following dimension wise : pie-diagram, bar-diagram and cubic diagram.

(a) 1,2,3 (b) 3,1,2

(c) 3,2,1 (d) 2,1,3 (1 mark)

Answer:

(d) Pie-Diagram :Two Dimensional Diagram (2)

These Diagrams are also called as "Area-Diagrams".

Used when different segments or components of values are also to be presented.

Bar-Diagram : One Dimensional Diagram (1) means such diagrams where only one dimensional measurement i.e. height is used. There is no importance of width or thickness in these diagrams. The heights of bars are taken on the basis of values.

Cubic-Diagram :Three Dimensional Diagram (3) are those in which three dimensions viz length, breadth & height are taken into account. used when these is wide range of data and three different but inter-related features of data are to be represented simultaneously.

2011 - DECEMBER

[45] The frequency of class 20-30 in the following data is

Class	0-10	10-20	20-30	30-40	40-50
Cumulative Frequency	5	13	28	34	38

(a) 5 (b) 28

(c) 15 (d) 13 (1 mark)

Answer:

(c)

Class	Cumulative freq.	Frequency
0-10	5	5
10-20	13	13 – 5 = 8
20-30	28	28 – 13 = 15
30-40	34	34 – 28 = 6
40-50	38	38 – 34 = 4

[46] The Graphical representation by which median is calculated is called

(a) Ogive Curve (b) Frequency Curve

(c) Line diagram (d) Histogram (1 mark)

Answer:

(a) The median is calculated by Ogive Curve

[47] Which of the following is not a two dimensional diagram?

(a) Square diagram (b) Line diagram

(c) Rectangular diagram (d) Pie-chart (1 mark)

Answer:

(b) Line diagram is not two dimensional diagram

2012 - JUNE

[48] From which graphical representation, we can calculate partition values ?

(a) Lorenz curve (b) Ogive curve

(c) Histogram (d) None of the above. (1 mark)

Answer:

(b) We can calculate partition values with the help of **O'Give Curve** for graphical representation.

[49] The data given below refers to the marks gained by a group of students:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
No. of Students	15	38	65	84	100

Then the no. of students getting marks more than 30 would be _____.

(a) 50 (b) 53
(c) 35 (d) 62 (1 mark)

Answer:

(c) Converting the given "Less than" type frequency distribution to Normal frequency distribution :

Class interval	**(f) frequency**
0 - 10	15
10 - 20	23
20 - 30	27
30 - 40	19
40 - 50	16

Hence,

The no. of students getting marks more than 30 is 19 + 16 = 35

[50] Cost of Sugar in a month under the heads raw materials, labour, direct production and others were 12,20,35 & 23 units respectively. The difference between their central angles for the largest & smallest components of the cost of Sugar is

(a) 92 (b) 72
(c) 48 (d) 56 (1 mark)

Answer:

(a)

Cost of SUGAR :

HEAD	**Units**	**Angular-Value**
Raw-Material	12	$\frac{12}{} \times 360 = 48$ (Smallest)

Labour	20	$\frac{20}{90} \times 360 = 80$
Direct Production	35	$\frac{35}{90} \times 360 = 140$ (Largest)
Others	23	$\frac{23}{90} \times 360 = 92$
	90 Units	

∴ Difference between their central angles of largest components = 140 - 48
= 92

2012 - DECEMBER

[51] What is a exclusive series?
(a) In which both upper and lower limit are not included in class frequency.
(b) In which lower limit is not included in class frequency.
(c) In which upper limit is not included in class frequency.
(d) None of the above. (1 mark)

Answer:
(c) In exclusive series, upper limit is not included in class frequency.

2013 - JUNE

[52] A pie diagram is used to represent the following data:

Source:	Customs	Excise	Income tax	Wealth tax
Revenue in million rupees:	120	180	240	180

The central angles in the pie diagram corresponding to income tax and wealth tax respectively:

(a) (120 , 90) (b) (90 , 120)
(c) (60 , 120) (d) (90 , 60) (1 mark)

Answer:

(a) Central Angle $= \frac{\text{Revenue of income tax}}{\text{Total Revenue}} \times 360$

$= \frac{240}{120 + 180 + 240 + 180} \times 360$

$= \frac{240}{720} \times 360 = 120$

Central Angle of wealth tax $= \frac{\text{Revenue of wealth tax}}{\text{Total Revenue}} \times 360$

$= \frac{180}{720} \times 360$

$= 90$

2013 - DECEMBER

[53] Difference between the maximum and minimum value of a given data is called

(a) Width (b) Size
(c) Range (d) Class (1 mark)

Answer:

(c) Difference between the maximum and minimum value of given data is called **Range**.

[54] If class interval is 10 - 14, 15 - 19, 20 - 24, then the first class is

(a) 10 - 15 (b) 9.5 - 14.5
(c) 10.5 - 15.5 (d) 9 - 15 (1 mark)

Answer:

(b) Class intervals is 10 - 14, 15 - 19, 20 - 24,

$D = 15 - 14 = 1$

$\frac{D}{2} = \frac{1}{2} = 0.5$

First class is (10 - 0.5) - (14 + 0.5)

[55] The difference between the upper and lower limit of a class is called _____.

(a) Class Interval (b) Mid Value
(c) Class boundary (d) Frequency (1 mark)

Answer:

(a) The difference between the upper and lower limit of class is called class interval (class width).

2014 - JUNE

[56] There were 200 employees in an office in which 150 were married. Total male employees were 160 out of which 120 were married. What was the number of female unmarried employees?

(a) 30 (b) 10
(c) 40 (d) 50 (1 mark)

Answer:

(b) Total Employees in the office = 200
No. of Employees who are married =150
No. of Employees who are unmarried = 200 - 150 = 50
No. of Total male Employees = 160
No. of Married male Employees = 120
No. of unmarried male Employees = 160 - 120 = 40
No. of females who are unmarried = 50 - 40 = 10

[57] "The less than Ogive" is a:

(a) U-Shaped Curve (b) J-Shaped Curve
(c) S-Shaped (d) Bell Shaped Curve (1 mark)

Answer:

(c) "The less than Ogive" is a s - shaped.

[58] The following data relates to the marks of a group of students.

Marks	No. of Students
More than 70%	07
More than 60%	18
More than 50%	40
More than 40%	60
More than 30%	75
More than 20%	100

How many students have got marks less than 50%?

(a) 60 (b) 82

(c) 40 (d) 53 (1 mark)

Answer:

(a)

Marks (in%) C.I.	Frequency
20 - 30	100 - 75 =25
30 - 40	75 - 60 =15
40 - 50	60 - 40 =20
50 - 60	40 - 18 =22
60 - 70	18 - 07 =11
70 - 80	=07
No. of students who got marks less than 50% = 25 + 15 + 20 = 60	

[59] To draw Histogram, the frequency distribution should be:

(a) Inclusive type

(b) Exclusive type

(c) Inclusive and Exclusive type

(d) None of these. (1 mark)

Answer:

(b) To Draw Histogram, the frequency distribution should be exclusive type.

2014 - DECEMBER

[60] The most appropriate diagram to represent the five - year plan outlay of India in different economic sectors is:

(a) Pie diagram (b) Histogram
(c) Line-Graph (d) Frequency Polygon (1 mark)

Answer:

(a) Pie diagram

[61] If the fluctuations in the observed value are very small as compared to the size of the item, it is presented by:

(a) Z chart (b) Ogive curve
(c) False base line (d) Control chart (1 mark)

Answer:

(c) If the fluctuations in the observed value are very small as compared to the size of the item, it is present by **false base line**.

[62] For constructing a histogram, the class-intervals of a frequency distribution must be

(a) equal (b) unequal
(c) equal or unequal (d) none of these (1 mark)

Answer:

(a) For constructing a histogram, the class-intervals of a frequency distribution must be **equal**.

[63] 100 persons are classified into male/female and graduate/non-graduate classes. This data classification is:

(a) Cardinal data (b) Ordinal data
(c) Spatial Series data (d) Temporal data (1 mark)

Answer:

(b) ordinal data

2015 - JUNE

[64] If we draw a perpendicular on x-axis from the point of inter-section of both 'less than' and 'more than' frequency curves we will get the value of _____

(a) mode (b) median
(c) arithmetic mean (d) third quartile (1 mark)

Answer:

(b) If we draw a perpendicular on x-axis from the point of intersection of both 'less than' and 'more than' frequency curve. We will get the value of 'Median'.

[65] Histogram is used for the presentation of the following type of series

(a) Time series
(b) Continuous frequency distribution
(c) Discrete frequency distribution
(d) Individual observation (1 mark)

Answer:

(b) Histogram is used for the presentation of the **continuous frequency distribution** of the series.

[66] Curve obtained by joining the points whose x coordinates are the upper limits of the class intervals and y coordinates are the corresponding cumulative frequencies is called.

(a) Frequency Polygon (b) Frequency curve
(c) Histogram (d) Ogive. (1 mark)

Answer:

(d) Curve obtained by joining the points whose x co-ordinate are the upper limits of the class intervals and y co-ordinates are the corresponding cumulative frequencies is called **'o' give**.

[67] The number of observations between 150 and 200 based on the following data is

Value:	More than 100	More than 150	More than 200	More than 250
No. of observations:	76	63	28	05

(a) 46 (b) 35
(c) 28 (d) 23 (1 mark)

Answer:

(b)

C.I.	Frequency
100-150	76-63 = 13
150-200	63-28 = 35
200-250	28-05 = 23
250-300	05

The No. of observation b/w 150 and 200 is 35

[68] The number of car accidents in seven days in a locality are given below:

No. of accidents:	0	1	2	3	4	5	6	7
Frequency:	12	9	11	13	8	9	6	3

What will be the number of cases when 4 or more accidents occurred?

(a) 32 (b) 41
(c) 26 (d) 18 (1 mark)

Answer:

(c)

No. of Accident:	0	1	2	3	4	5	6	7
Frequency:	12	9	11	13	8	9	6	3

No. of Cases when 4 or more Accidents occurred
= 8 + 9 + 6 + 3 = 26

2015 - DECEMBER

[69] The most common form of diagrammatic representation of a grouped frequency distribution is:

(a) Histogram (b) Ogive
(c) Both (d) None (1 mark)

Answer:

(a) The most common form of diagrammatic representation of a group frequency distribution is **Histogram.**

[70] Classification is of _____ kinds.

(a) Two (b) Three
(c) One (d) Four (1 mark)

Answer:

(d) Classification is of four kind.

[71] The chart that uses logarithm of variable is known as:

(a) Ratio chart (b) Line chart
(c) Multiple line chart (d) Component line chart (1 mark)

Answer:

(a) The chart that uses logarithm of variable is known as Ratio Chart.

[72] Find the number of observation between 250 and 300 from the following data:

Value more than:	200	250	300	500
No. of observation:	56	38	15	0

(a) 38 (b) 23
(c) 15 (d) None of the above (1 mark)

Answer:

(b)

C.I	Frequency
200-250	56-38 = 18
250-300	38-15 = 23

300-350	15-0 = 15
350-400	0 - 0 = 0

No. of observation b/w 250 and 350 = 23

2016 - JUNE

[73] Data collected on religion from the census reports are:

(a) Primary data (b) Secondary data
(c) Sample data (d) (a) or (b) (1 mark)

Answer:

(b) DATA collected on religion from the census reports are **secondary data**.

2016 - DECEMBER

[74] In collection of data which of the following interview methods:

(a) Personal interview method
(b) Telephone interview method
(c) Published data
(d) (a) and (b) (1 mark)

Answer:

(d) Personal Interview Method and Telephone Interview Method are the Interview Method.

[75] For constructing a histogram the class intervals of a frequency distribution must be of the following type:

(a) Equal (b) Unequal
(c) Equal or Unequal (d) None of these (1 mark)

Answer:

(a) For constructing a histogram the class intervals of a frequency distribution must be equal.

[76] Profits made by XYZ Bank which is a blue chip company in different years refer to:

(a) An attribute (b) A discrete variable
(c) A continuous variable (d) None of these. (1 mark)

Answer:

(c) Profit made by XYZ Bank in different years refer to a continuous variable because Blue chips company's profit always increased.

[77] Mode of presentation data

(a) Textual presentation (b) Tabulation
(c) Oral presentation (d) (a) and (b) (1 mark)

Answer:

(d) Mode of presentation data are textual presentation and tabulation.

2017 - JUNE

[78] If the data represent costs spent on conducting an examination under various needs, then the most suitable diagram will be:

(a) Pie diagram (b) Frequency diagram
(c) Bar diagram (d) Multiple bar diagram (1 mark)

Answer:

(a) If the data represent cost spent on conducting an examination under various heads then the most suitable diagram will be **Pie diagram**.

[79] Frequency density corresponding to class interval is the ratio of :

(a) Class frequency to the total frequency
(b) Class frequency to the class length
(c) Class length to the class frequency
(d) Class frequency to the cumulative frequency (1 mark)

Answer:

(b) Frequency density corresponding to a class Interval is the Ratio of class Frequency to the class Length.

$$\text{Frequency density} = \frac{\text{Frequency of the class}}{}$$

[80] The point of intersection of less than ogive and greater than ogive curve gives us:

(a) Mean (b) Mode

(c) Median (d) None of the above. (1 mark)

Answer:

(c) The point of intersection of less than '0' give and greater than '0' give curve gives us Median.

2017 - DECEMBER

[81] 'Stub' of a table is the

(a) Left part of the table describing the columns

(b) Right part of the table describing the columns

(c) Right part of the table describing the rows

(d) Left part of the table describing the rows. (1 mark)

Answer:

(d) 'Stub' of a table is the left part of the table describing the rows.

[82] Frequency density corresponding to a class interval is the ratio of

(a) Class frequency to total frequency

(b) Class frequency to the class length

(c) Class length to class frequency

(d) Class frequency to the cumulative frequency. (1 mark)

Answer:

Please refer question no. [79] on page no. 695.

2018 - MAY

[83] Frequency density is used in the construction of
(a) Histogram
(b) Ogive
(c) Frequency polygon
(d) None when the classes are of unequal width (1 mark)

Answer:

(a) Frequency density is used in the construction of Histogram.

[84] Divided bar chart is considered for
(a) Comparing different components of a variable
(b) The relation of different components to the table
(c) (a) or (b)
(d) (a) and (b) (1 mark)

Answer:

(d) Divided Bar Chart is considered for comparing different components of a variable and the relation of different components to the table.

2018 - NOVEMBER

[85] The following frequency distribution

X :	12	17	24	36	45
F :	2	5	3	8	9

is classified as
(a) Continuous distribution
(b) Discrete distribution
(c) Cumulative frequency distribution
(d) None of the above (1 mark)

Answer:

(b)

X:	12	17	24	36	45
F:	2	5	3	8	9

is classified as Discrete distribution.

[86] Histogram is useful to determine graphically the value of

(a) Arithmetic mean (b) Median

(c) Mode (d) None of the above (1 mark)

Answer:

(c) Histogram is useful to determine graphically the value of "mode".

[87] Data are said to be _____ if the investigator himself is responsible for the collection of the data.

(a) Primary data

(b) Secondary data

(c) Mixed of primary and secondary data

(d) None of the above (1 mark)

Answer:

(a) Data are said to be Primary data if the Investigator himself is responsible for the collection of the data.

[88] A suitable graph for representing the portioning of total into sub parts in statistics is:

(a) A Pie chart (b) A pictograph

(c) An ogive (d) Histogram (1 mark)

Answer:

(a) A suitable graph for representating the portioning of total into sub parts in statistics is A Pie chart.

[89] The number of times a particular items occurs in a class interval is called its:

(a) Mean (b) Frequency

(c) Cumulative frequency (d) None of the above (1 mark)

Answer:

(b) The number of times a particular items occurs in a Class Interval

[90] An ogive is a graphical representation of
(a) Cumulative frequency distribution
(b) A frequency distribution
(c) Ungrouped data
(d) None of the above (1 mark)

Answer:

(a) An 'O' give is a graphical representation of cumulative frequency distribution.

[91]

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	4	6	20	8	3

For the class 20-30. Cumulative frequency is:
(a) 10 (b) 26
(c) 30 (d) 41 (1 mark)

Answer:

(c)

C.I	F	C. F
0-10	4	4
10-20	6	10
20-30	20	30
30-40	8	38
40-50	3	

Cumulative frequency of Class Interval '20-30' is 30.

2019 - JUNE

[92] Which of the following graph is suitable for cumulative frequency distribution?
(a) 'O'give
(b) Histogram

(c) G.M
(c) A.M (1 mark)

Answer:

(a) 'O'give is graph suitable for cumulative frequency distribution.

[93] Histogram can be shown as
(a) Ellipse
(b) Rectangle
(c) Hyperbola
(d) Circle (1 mark)

Answer:

(b) Histogram can be shown as Rectangle.

[94] _____ Series is continuous.
(a) Open ended
(b) Exclusive
(c) Close ended
(d) Unequal call intervals (1 mark)

Answer:

(b) Exclusive Series is continuous.

[95] Ogive graph is used for finding
(a) Mean
(b) Mode
(c) Median
(d) None (1 mark)

Answer:

(c) 'O'give graph is used for finding 'Median'.

[96] Histogram is used for finding
(a) Mode
(b) Mean
(c) First quartile
(d) None (1 mark)

Answer:

(a) Histogram is used for finding 'Mode'.

2019 - NOVEMBER

[97] The graphical representation of cumulative frequency distribution is called.

(a) Histogram
(b) Historiagram
(c) Ogive
(d) None (1 mark)

Answer:

(c) A curve that represents the cumulative frequency distribution of a grouped data on a graph is called ogive.
Cumulative frequency on y-axis
Class interval on x-axis

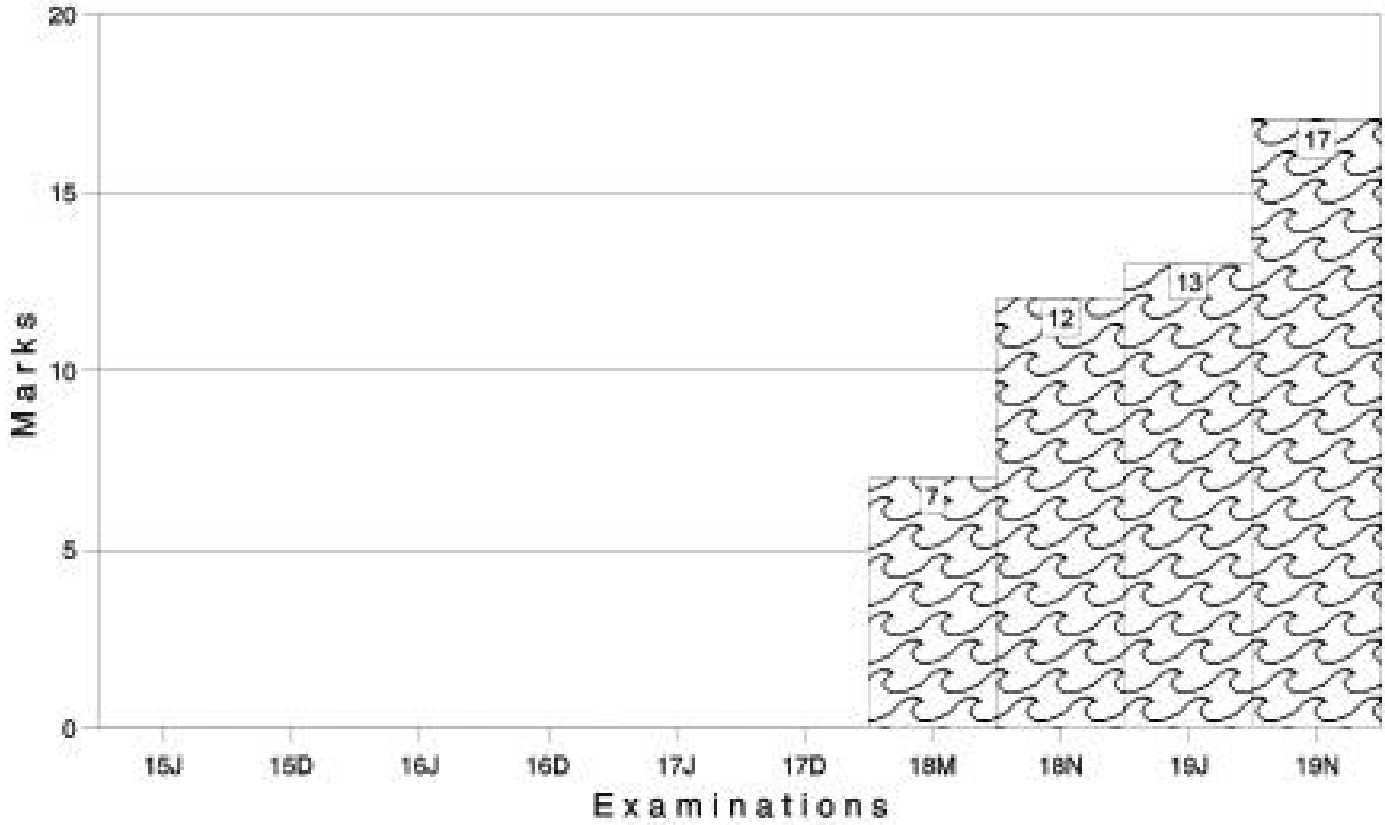
CHAPTER

15 MEASURES OF CENTRAL TENDENCY AND DISPERSION

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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PAST YEAR QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] If x and y are related by x - y - 10 = 0 and mode of x is known to be 23, then the mode of y is :

(a) 20 (b) 13

(c) 3 (d) 23 (1 mark)

Answer:

(b) Mode of x = 23

$x - y - 10 = 0$

$y = x - 10$

Mode of y = Mode of x – 10

= 23 – 10

= 13

[2] A man travels at a speed of 20 km/hr and then returns at a speed of 30 km/ hr. His average speed of the whole journey is :

(a) 25 km/ hr (b) 24.5 km/hr

(c) 24 km/hr (d) None (1 mark)

Answer:

(c) Harmonic Mean is the method which is preferred for the computation of average speed.

$$H.M = \frac{2ab}{a+b} = \frac{2 \times 20 \times 30}{20 + 30} = 24 \text{ km/hr}$$

[3] A student obtained the mean and standard deviation of 100 observations as 40 and 5.1 respectively. It was later discovered that he had wrongly copied down an observation as 50 instead of 40. The correct standard deviation is :

(a) 5 (b) 6

(c) 3 (d) 7 (1 mark)

Answer:

(a) $n = 100$

wrong $\bar{x} = 40$

wrong $\sigma = 5.1$

$$\bar{x} = \frac{\sum x}{n}$$

$$40 = \frac{\sum x}{100}$$

wrong $x = 40 \times 100 = 4000$

correct $x = 4000 - 50 + 40 = 3990$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$$

$$(\sigma)^2 = \frac{\sum x^2}{100} - (40)^2$$

$$(5.1)^2 = \frac{\sum x^2}{100} - 1600$$

$$26.01 = \frac{\sum x^2}{100} - 1600$$

$$\frac{\sum x^2}{100} = 26.01 + 1600$$

Wrong $x^2 = 162601$

Correct $x^2 = 162601 - (50)^2 + (40)^2 = 161701$

$\therefore$ correct $\bar{x} = \frac{\text{Correct } \sum x}{n} = \frac{3990}{100} = 39.90$

correct $\sigma = \sqrt{\frac{\text{Correct } \sum x^2}{n} - (\bar{x})^2}$

$$\sigma = \sqrt{\frac{161701}{100} - (39.90)^2}$$

$$= (\sqrt{1617.01 - 1592.01}) = \sqrt{25} = 5$$

[4] For a moderately skewed distribution, quartile deviation and the standard deviation are related by:

(a) S. D. = $\frac{2}{3}$ Q.D (b) S. D.= $\frac{3}{4}$ Q.D

(c) S. D.= $\frac{4}{3}$ Q.D (d) S. D.= $\frac{3}{2}$ Q.D (1 mark)

Answer:

(d) As we know in a normal distribution

$Q_1 = u\ 0.675\ \sigma$ .....(1)

and $Q_3 = u + 0.675\ \sigma$.....(2)

Subtracting (1) from (2) we get

$$\frac{Q_3 - Q_1}{2} = 0.675\sigma$$

$$\frac{Q.D.}{0.675} = \sigma \left[\because Q.D. = \frac{(Q_3 - Q_1)}{2} \right]$$

$1.5\ Q.D. = \sigma$

$S.D. = \frac{3}{2} \times Q.D.$ $(S.D. = \sigma)$

2007 - FEBRUARY

[5] The median of the data 13, 8, 11, 6, 4, 15, 2, 18, is :

(a) 5 (b) 8

(c) 11 (d) 9.5 (1 mark)

Answer:

(d) Arranging data in ascending order, we get :

2, 4, 6, 8, 11, 13, 15, 18

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{items}$$

$$= \left(\frac{8+1}{2}\right)^{th} \text{items}$$

$= 4.5^{th}$ items

$= 4^{th}$ item + 0.5 (5^{th} term – 4^{th} term)
$= 8 + 0.5\ (11 - 8)$
$= 8 + 1.5$
$= 9.5$

[6] The sum of the squares of deviations of a set of observations has the smallest value, when the deviations are taken from their :

(a) A . M. (b) H. M.
(c) G. M. (d) None (1 mark)

Answer:

(a) The sum of the squares of deviations of a set of observations has the smallest value, when the deviations are taken from their Arithmetic mean or the algebraic sum of the squares of deviations or In other words, this sum will always be less than the sum of squared deviations of the items from any other value of a set of observations from their. A.M. is minimum or the least i.e. for unclassified data $\Sigma(x_1 - \bar{x})^2 = 0$

[7] If two samples of sizes 30 and 20 have means as 55 and 60 and variances as 16 and 25 respectively, then what would be the S. D. of the combined sample size 50 ?

(a) 5.33 (b) 5.17
(c) 5.06 (d) 5 (1 mark)

Answer:

(c) As we know, combined mean is given by

$$\bar{x}_{12} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_{2_1}}{n_1 + n_2}$$

$$\bar{x}_{12} = \frac{30 \times 55 + 20 \times 60}{30 + 20} = 57$$

Also $d^2_1 = (\bar{x}_1 - \bar{x}_{12})^2 = (55 - 57)^2 = 4$

and $d^2_2 = (\bar{x}_2 - \bar{x}_{12})^2 = (60 - 57)^2 = 9$

Combined Standard Deviation, i.e.

$$\sigma_{12} = \sqrt{\frac{\sigma_1^2 n_1 + \sigma_2^2 n_2 + d_1^2 n_1 + d_2^2 n_2}{n_1 + n_2}}$$

$$\sigma_{12} = \sqrt{\frac{16\times30 + 25\times20 + 4\times30 + 9\times20}{30+20}}$$

$$= \frac{1280}{50} = 25.6$$

S.D. $\sqrt{25.6} = 5.06$

[8] If two variables x and y are related by $2x + 3y - 7 = 0$ and the mean and mean deviation about mean of x are 1 and 0.3 respectively, then the co-efficient of mean deviation of y about mean is :

(a) -5 (b) 4

(c) 12 (d) 50 (1 mark)

Answer:

(c) If $y = a + bx$, a and b being contact, then

M.D of y = b (M.D. of x)

(Single M.D. changes due to change in scale)

$$\therefore y = -\frac{2}{3}x + \frac{7}{3}$$

M.D. of $y = \left|\frac{2}{3}\right|$ (M.D. of x)

$$\left(\frac{2}{3}\right) \times 0.3 = 0.2$$

Also $2x + 3y - 7 = 0$

$2\bar{x} + 3\bar{y} - 7 = 0$ (Since, the A.M. is affected by change of origin as well as change of scale)

$$\bar{y} = -\frac{2}{3}\bar{x} + \frac{7}{3}$$

$$\bar{y} = -\frac{2}{3}\times1 + \frac{7}{3} = \frac{5}{3} \text{ (Given } \bar{x} = 1)$$

(Coefficient of mean deviation of y about mean)

$$= \frac{\text{M.D of } y}{\bar{y}} \times 100 = \frac{0.2}{5/3} \times 100 = 12$$

2007 - MAY

[9] Which of the following result hold for a set of distinct positive observations?

(a) A . M. ≥ G. M. ≥ H. M. (b) G. M. > A. M. > H. M.

(c) G. M. ≥ A. M. ≥ H. M. (d) A . M. > G. M. > H. M. (1 mark)

Answer:

(d) For a distinct set of positive observation AM>GM>H.M. while the equality holds when all the observations are equal.

[10] Measures of dispersion are called averages of the ____ order.

(a) 1^{st} (b) 2^{nd}

(c) 3^{rd} (d) None (1 mark)

Answer:

(b) The measures of dispersion are also called averages of the second order because these measures give an average of the differences of various items from an average.

[11] For a set of 100 observations, taking assumed mean as 4, the sum of the deviations is – 11 cm, and the sum of the squares of these deviations is 257 cm^2. The coefficient of variation is :

(a) 41.13% (b) 42.13%

(c) 40.13% (d) None (1 mark)

Answer:

(a) According to short-cut method, we have

$$\bar{x} = A + \frac{\sum fd}{n} = 4 + \frac{-11}{100} = 3.89$$

Standard Deviation

$$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} = \sqrt{\frac{257}{100} - \left(\frac{-11}{100}\right)^2} = 1.6$$

Coefficient of variation C.V. = $\frac{\sigma}{x} \times 100$

$$= \frac{1.6}{3.89} \times 100 = 41.13\%$$

2007 - AUGUST

[12] If the A. M. and H.M. for two numbers are 5 and 3.2 respectively then the G.M. will be :

(a) 4.05 (b) 16

(c) 4 (d) 4.10 (1 mark)

Answer:

(c) Let the two number be a and b, their

A.M. $\frac{a+b}{2} = 5 \Rightarrow a + b = 10$.....(1)

H.M $\frac{2ab}{a+b} = 3.2 \Rightarrow ab = 16$

$\therefore a - b = \sqrt{(a+b)^2 - 4ab}$

$= \sqrt{100 - 64} = 6$..... (2)

Now (1) and (2)

$\Rightarrow a = 8, b = 2$

G.M. $= \sqrt{ab} = \sqrt{16} = 4$

[13] ___ are used for measuring central tendency, dispersion and skewness:

(a) Median (b) Deciles

(c) Percentiles (d) Quartiles (1 mark)

Answer:

(d) Qualitites are used for measuring Central Tendency, Dispersion and Skewness.

[14] Which of the following companies A or B is more consistent so far as the payment of dividend is concerned ?

Dividend paid by A :	5	9	6	12	15	10	8	10
Dividend paid by B :	4	8	7	15	18	9	6	6

(a) A (b) B

(c) Both A & B (d) Neither A nor B (1 mark)

Answer:

(a) There, $\Sigma x_A = 75$

$\therefore \bar{x}_A = \frac{75}{8} = 9.375$

Also $\Sigma x^2_A = 775$

$\therefore \sigma^2_A = \frac{\Sigma x_A^2}{N} = \left(\frac{\Sigma x_A}{N}\right)^2$

$= \frac{775}{8} - \left(\frac{75}{8}\right)^2$

$\sigma_A = 3$

$\therefore C.V_A = \frac{\sigma_A}{\bar{x}_A} \times 100 = \frac{3}{9.375} \times 100 = 32$

Similarly,

$\Sigma x_B = 73$, $\therefore \bar{x}_B = \frac{73}{8} = 9.125$

$\Sigma x^2_B = 831$,

$\therefore \sigma^2_B = \frac{831}{8} - \left(\frac{73}{8}\right)^2$

$= 20.61$

$\therefore \sigma_B = 4.54$

$C.V_B = \frac{4.54}{9.125} \times 100 = 49.75$

Since, $C.V_A < C.V_B$

Company A is more consistent than B.

[15] What is the coefficient of range for the following distribution?

Class Interval:	10-19	20-29	30-39	40-49	50-59
Frequency:	11	25	16	7	3

(a) 22 (b) 50

(c) 75.82 (d) 72.46 (1 mark)

Answer:

(d) The lowest class boundary is 9.50 and the highest class boundary is 59.50. We get this by converting the given data into a continuous series.

$\therefore$ Range = L – S = 59.50 – 9.50 = 50

$$\text{Coefficient of Range} = \frac{L-S}{L+S} \times 100$$

$$= \frac{59.50-9.50}{59.50+9.50} \times 100$$

$$= \frac{50}{69} \times 100$$

$$= 72.46$$

2007 - NOVEMBER

[16] An aeroplane flies from A to B at the rate of 500 km / hr and comes back from B to A at the rate of 700 km / hr. The average speed of the aeroplane is :

(a) 600 km / hr (b) 583.33 km / hr

(c) $100\sqrt{35}$ km / hr (d) 620 km / hr. (1 mark)

Answer:

(b) Required average speed of the aeroplane = the harmonic mean between the two speeds.

$$= \frac{2}{\frac{1}{500}+\frac{1}{700}} = \frac{2\times 3500}{7+5} = 583.33 \text{ km/hr.}$$

[17] For a moderately skewed distribution, which of the following relationship holds ?

(a) Mean – Median = 3 (Median – Mode)

(b) Median – Mode = 3 (Mean – Median)

(c) Mean – Mode = 3 (Mean – Median)

(d) Mean – Median = 2 (Mean – Mode) (1 mark)

Answer:

(c) For a moderately skewed distribution the empirical relationship between mean, median and mode is

Mean – Mode = 3 (Mean – Median)

[18] _____ & _____ are called ratio averages:

(a) H. M. & G. M. (b) H. M. & A. M.

(c) A. M. & G. M. (d) None (1 mark)

Answer:

(c) A. M. and G.M. are used in averaging ratios and percentages and in computing average rates of increase or decrease.

[19] A sample of 35 observations has the mean 80 and S.D. as 4. A second sample of 65 observations from the same population has mean 70 and S.D. 3. The S.D. of the combined sample is :

(a) 5.85 (b) 5.58

(c) 10.23 (d) None of these (1 mark)

Answer:

(a) Here, $n_1 = 35,\ \bar{x}_1 = 80,\ \sigma_1 = 4$

and , $n_2 = 65,\ \bar{x}_2 = 70,\ \sigma_2 = 3$

$$\therefore \bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{35\times 80 + 65\times 70}{35 + 65} = 73.5$$

$$\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

$$= \sqrt{\frac{35\times 16 + 65\times 9 + 35\times(-6.5)^2 + 65\times(3.5)^2}{35 + 65}}$$

$$= \sqrt{34.21} = 5.85$$

Note : $d_1 = \bar{x}_{12} - \bar{x}_1 = 73.5 - 80 = -6.5$

$d_2 = \bar{x}_{12} - \bar{x}_2 = 73.5 - 70 = 3.5$

[20] If x and y are related as $3x - 4y = 20$ and the quartile deviation of x is 12, then the quartile deviation of y is :

(a) 14 (b) 15

(c) 16 (d) 9 (1 mark)

Answer:

(d) If y = a+bx, a and b being constant, then Q.D. y = |b| Q. D.x.

$3x - 4y = 20$ or $y = \frac{3}{4}x + 5$

$Q.Dy = \left(\frac{3}{4}\right) \times 12 = 9$ [Q.Dx = 12]

2008 - FEBRUARY

[21] Extreme values have _____ effect on mode.

(a) High (b) low

(c) No (d) None of these (1 mark)

Answer:

(c) **Mode** is defined as the value that occurs the maximum number of times i.e. the value which has the maximum concentration of the observations around it. Therefore, extreme values have no effect **on mode.**

[22] The mean salary for a group of 40 female workers is ₹ 5,200 per month and that for a group of 60 male workers is ₹ 6,800 per month. What is the combined salary ?

(a) ₹ 6,160 (b) ₹ 6,280

(c) ₹ 6,890 (d) ₹ 6,920 (1 mark)

Answer:

(a) We have $n_1 = 40$, $n_2 = 60$

$\bar{x}_1 =$ ₹ 5200 and $\bar{x}_2 = 6800$

Hence, the combined mean salary per month is :

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_{2_1}}{n_1 + n_2} = \frac{40 \times 5200 + 60 \times 6800}{40 + 60} = ₹\ 6,160$$

[23] The best measure of dispersion is:

(a) Q. D. (b) M. D.

(c) Range (d) S. D. (1 mark)

Answer:

(d) Standard Deviation (S.D) in the best measure of dispersion because of the following reasons.

(i) It is rigidly defined, based on all the observations.

(ii) It is not much affected by sampling fluctuations.

(iii) It has some desirable mathematical properties.

[24] If the mean and S.D. of x are a and b respectively, then the S.D. of $\frac{x - a}{b}$ is:

(a) a / b (b) – 1

(c) 1 (d) ab (1 mark)

Answer:

(c) Let $y = \frac{x - a}{b}$

$$y = \frac{1}{b}x - \frac{a}{b}$$

$$\sigma_y = \left|\frac{1}{b}\right| \sigma_x$$

$$= \frac{1}{b} \times b = 1$$

[25] Suppose a population A has 100 observations 101, 102, 103, ... 200 and another population B has 100 observations 151, 152, 153, ... 250. If V_A and V_B represents the variance of the two populations respectively, then V_A / V_B = :

(a) 9 / 4 (b) 1

(c) 4 / 9 (d) 2 / 3 (1 mark)

Answer:

(b) **Population** A : S.D. of 101, 102, 103, ...20 is equal to the S.D. of 1,23,....., 100.

Population B: S.D. of 151,152,153, ...25 is equal to the S.D. of 1,2,3....., 100.

[Since S.D. (and variance) remains unchanged when each observation is increased or decreased) by the same constant in this case, observations are increased by 100 and 150, respectively.

As variance of 1st n natural numbers is given by

$$\sigma^2 = n^2b\ n\ verb\ \frac{n^2-1}{12}$$

$$\therefore V_A = \frac{100^2-1}{12} \text{ (Variance of A)}$$

$$V_B = \frac{100^2-1}{12} \text{ (Variance of B)}$$

$$\therefore \frac{V_A}{V_B} = 1$$

2008 - JUNE

[26] If there are two groups with 75 and 65 as harmonic means and containing 15 and 13 observations. then the combined H.M. is given by:

(a) 70 (b) 80

(c) 70.35 (d) 69.48 (1 mark)

Answer:

(a) Combined H.M. is given by

$$= \frac{n_1+n_2}{\left(\frac{n_1}{H_1}+\frac{n_2}{H_2}\right)} = \frac{15+13}{\left(\frac{15}{75}+\frac{13}{65}\right)} = \frac{5\times 28}{2} = 70$$

[27] The G.M. of 4, 6 and 8 is :

(a) 4.77 (b) 5.32

(c) 6.14 (d) 5.77 (1 mark)

Answer:

(d) When a variable x assumes n values $x_1, x_2, x_3, \ldots\ldots x_n$, all being positive, then G.M. of x is given by

$G = (x_1 \times x_2 \times x_3 \ldots\ldots x_n)^{1/n}$

$G = (4 \times 6 \times 8)^{1/3} = (192)^{1/3} = 5.77$

[28] The Mean and S.D. for group of 100 observations are 65 and 7.03 respectively If 60 of these observations have mean and S.D. as 70 and 3 respectively, what is the S.D. for the group comprising 40 observations?

(a) 2.03 (b) 4.03

(c) 8.03 (d) 9.33 (1 mark)

Answer:

(b) Given : $n_1 = 60, \bar{x}_1 = 70, \sigma_1 = 3$

$n_2 = 40, \bar{x}_2 = ?, \sigma_2 = ?$

and $n = 100, \bar{x}_{12} = 65, \sigma_{12} = 7.03$

$$\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{60 \times 70 + 40\bar{x}_2}{60 + 40} = 65$$

$\Rightarrow \bar{x}_2 = 57.5$

Now $d_1 = \bar{x}_{12} - \bar{x}_1 = 65 - 70 = 5$

$d_2 = \bar{x}_{12} - \bar{x}_2 = 65 - 57.5 = 7.5$

$$\sigma^2_{12} = \frac{n_1c_1^2 + n_2c^2 + n_1d_1^2 + n_2d_2^2}{n_1 + n_2}$$

$$(703)^2 = \frac{60 \times 9 + 40 \times c_2^2 - 60 \times (-5)^2 + 40 \times (7.5)^2}{60 + 40}$$

$\sigma^2_2 = 16.3$

$\sigma_2 = \sqrt{16.3}$

$\sigma_2 = 4.03$

[29] The quartile deviation for the data is:

x :	2	3	4	5	6
f :	3	4	8	4	1

(a) ¼ (b) ½

(c) 1 (d) 0 (1 mark)

Answer:

(c)

x :	2	3	4	5	6
f :	3	4	8	4	1
c.f :	3	7	15	19	20

$Q1 = \text{size of } \frac{N+1^{th}}{4} \text{ item}$

$= \frac{20+1}{4} = \frac{21^{th}}{4} = 5.25^{th} \text{ item}$

$\therefore Q_1 = 3$

and $Q_2 = \text{size of } \frac{3(N+1)^{th}}{4} \text{ item}$

$= \text{size of } 15.75^{th} \text{ item}$

$\therefore Q_3 = 5$

We know that

$Q.D. = \frac{1}{2}(Q_3 - Q_1) = \frac{1}{2}(5-3) = 1$

2008 - DECEMBER

[30] If X and Y are two random variables then v (x + y) is :

(a) v (x) + v (y) (b) v (x) + v (y) – 2 v (x, y)

(c) v (x) + v (y) + 2 v (x, y) (d) v (x) – v (y) (1 mark)

Answer:

(a) Variance of sum of two random variables is the sum of their variances. i.e. V(x + y) = V (x) + V(y) for any two random variables x and y.

[31] G.M is a better measure than others when,

(a) ratios and percentages are given

(b) interval of scale is given

(c) Both (a) and (b)

(d) Either (a) or (b) (1 mark)

Answer:

(a) G.M. is not as widely used as the A.M. but it is sometimes more significant than the A.M. and useful in averaging rates of changes when ratio and percentage changes are more important than the absolute changes.

[32] Mean and S. D. of x is 50 and 5 respectively. Find mean and S.D. of $\frac{x-50}{5}$.

(a) (1,0) (b) (0, 1)

(c) (1, :) (d) (0,– 1) (1 mark)

Answer:

(b) $\bar{x} = 50$, and $\sigma_x = 5$

Let $y = \frac{x - 50}{5}$ ..... (1)

As we know, AM is affected due to a change of origin and/or scale, therefore

$$\bar{y} = \frac{\bar{x} - 50}{5} = \frac{50 - 50}{5} = 0$$

Regarding S.D., it remains unaffected due to a change of origin but it is affected is the same ratio due to a change of scale.

$\sigma_y = |b| \sigma_x$

Comparing (1) with $y = a + b x$

$\therefore |b| = 1/5$

$\sigma_y = \frac{1}{5} \times 5 = 1$

[33] Mean and S. D. of a given set of observations is 1,500 and 400 respectively. If there is an increment of 100 in the first year and each observation is hiked by 20% in 2nd years, then find new mean and S.D.

(a) 1920, 480 (b) 1920, 580

(c) 1600, 480 (d) 1600, 400 (1 mark)

Answer:

(a) $\bar{x} = 1500, \sigma = 400$

After 1st year,

Mean	**S.D.**
1500 + 100 = 1600	No Change

After 2nd year,

Mean	**S.D.**
$1600 \times \frac{20}{100} + 1600$	$400 \times \frac{20}{100} + 400$
= 1,920	= 480

[34] If 5 is subtracted from each observation of some certain item then its co-efficient of variation is 10% and if 5 is added to each item then its coefficient of variation is 6%. Find original coefficient of variation.

(a) 8% (b) 7.5%

(c) 4% (d) None of these (1 mark)

Answer:

(b) Coefficient of variation is given by $\frac{\sigma}{\text{Mean}} \times 100$

We know, S.D. remains invariant due to a change of origin.

Let original S.D. be σ and mean be $\bar{x}$

$\therefore$ In first case, $\frac{\sigma}{\bar{x}-5} \times 100 = 10$..... (1)

In second case $\frac{\sigma}{\bar{x}+5} \times 100 = 6$ ..... (2)

Divide (2) by (1), we get

$\bar{x} = 20$ and consequently $\sigma = 1.5$

Now, substitute the values of σ and $\bar{x}$ in any of the above equations, we get C.V = $\frac{1.5}{}$ x 100 = 7.5%

2009 - JUNE

[35] The median of x, $\frac{x}{2}, \frac{x}{3}, \frac{x}{5}$ is 10.

Find x where x > 0

(a) 24 (b) 32

(c) 8 (d) 16 (1 mark)

Answer:

(a) Step - 1 :Arrange the data in ascending order.

$$\frac{x}{5}, \frac{x}{3}, \frac{x}{2}, x$$

Step - 2: Median $= \left(\frac{n+1}{2}\right)^{th}$ term

$$= \left(\frac{4+1}{2}\right)^{th} \text{term}$$

$$= (2.5)^{th} \text{ term}$$

So, Median = 2^{nd} term + 0.5 (3^{rd} term 2^{nd} term)

$$10 = \frac{x}{3} + 0.5\left(\frac{x}{2} - \frac{x}{3}\right)$$

$$10 = \frac{x}{3} + 0.5\left(\frac{3x - 2x}{6}\right)$$

$$10 = \frac{x}{3} + \frac{x}{12}$$

$$10 = \frac{4x - x}{12}$$

$$10 = \frac{5x}{12}$$

$$x = \frac{10 \times 12}{5}$$

$$x = 24$$

∴The value of x is 24.

[36] The average salary of 50 men was ₹ 80 but it was found that salary of 2 of them were ₹ 46 and ₹ 28 which was wrongly taken as ₹ 64 and ₹ 82. The revised average salary is :

(a) ₹ 80 (b) ₹ 78.56

(c) ₹ 85.26 (d) ₹ 82.92 (1 mark)

Answer:

(b) Mean = $\frac{\Sigma x}{N}$

$80 = \frac{\Sigma x}{50}$

$\therefore$ Wrong $\Sigma x = 80 \times 50 = 4000$

So, corrected $\Sigma x = 4000 - 64 - 82 + 46 + 28 = 3928$

Therefore, revised average salary = $\frac{\Sigma x}{N} = \frac{3,928}{50}$

Revised Average Salary = ₹ 78.56

[37] Inter Quartile Range is_____of Quartile Deviation.

(a) Half (b) Double

(c) Triple (d) Equal (1 mark)

Answer:

(b) Quartile Deviation or Semi - inter quartile Range = $\frac{Q_3 - Q_1}{2}$

Inter-quartile Range = $Q_3 - Q_1$

Therefore, inter-quartile range is double of quartile deviation. In other words, quartile deviation is half of inter-quartile range.

[38] The sum of squares of deviation from mean of 10 observations is 250. Mean of the data is 10. Find the co-efficient of variation.

(a) 10 % (b) 25 %

(c) 50 % (d) 0 % (1 mark)

Answer:

(c) S.D. = $\sqrt{\frac{\Sigma(x - \bar{x})^2}{N}}$

In the given data,

$\Sigma(x - \bar{x})^2 = 250$

N = 10

Mean = 10

Therefore, S.D. = $\sqrt{\frac{250}{10}}$

S.D. = 5

So, coefficient of variation = $\frac{S.D.}{Mean} \times 100 = \frac{5}{10} \times 100 = 50\%$

[39] If A be the A.M. of two positive unequal quantities X and Y and G be their G.M., then ;

(a) $A < G$ (b) $A > G$

(c) $A \leq G$ (d) $A \geq G$ (1 mark)

Answer:

(b) For any set of positive observations, we have the following inequality :

$A.M. \geq G.M \geq H.M$

The equality sign occurs, when all the observations are equal.

If all the observations are positive and unequal then the inequality is:

AM > G.M > H M

Therefore, we can conclude that AM > G.M for positive unequal quantities.

2009 - DECEMBER

[40] When mean is 3.57 and mode is 2.13 then the value of median is _____.

(a) 3.09 (b) 5.01

(c) 4.01 (d) None of these (1 mark)

Answer:

(a) Mean = 3.57

Mode = 2.13

As per the empirical formula,

Mode = 3 Median – 2 Mean

2.13 = 3 Me – 2 × 3.57

2.13 = 3 Me – 7.14

3 Me = 2.13 + 7.14

3 Me = 9.27

$Me = \frac{9.27}{3} = 3.09.$

∴ Median = 3.09

[41] If L_1 = highest observation and L_2 = smallest observation, then Coefficient of Range =

(a) $\frac{L_1 \times L_2}{L_1/L_2} \times 100$ (b) $\frac{L_1 - L_2}{L_1 + L_2} \times 100$

(c) $\frac{L_1 + L_2}{L_1 - L_2} \times 100$ (d) $\frac{L_1/L_2}{L_1 \times L_2} \times 100$ (1 mark)

Answer:

(b) Coefficient of Range $= \frac{\text{highest observation} - \text{smallest observation}}{\text{highest observation} + \text{smallest observation}} \times 100$

$= \frac{L_1 - L_2}{L_1 + L_2} \times 100$

[42] The equation of a line is $5x + 2y = 17$. Mean deviation of y about mean is 5. Calculate mean deviation of x about mean.

(a) – 2 (b) 2

(c) – 4 (d) None (1 mark)

Answer:

(b) $5x + 2y = 17$

$x = -\frac{2y}{5} + \frac{17}{5}$

M.D. of x = b × M.D. of y.

$$= \left|-\frac{2}{5}\right| \times 5$$

$$= \frac{2}{5} \times 5$$

$$= 2$$

[43] If variance of x is 5, then find the variance of (2 – 3x)

(a) 10 (b) 15

(c) 5 (d) –13 (1 mark)

Answer:

(b) y = 2 – 3x

Variance of y = b × Variance of x

= –3 × 5 = 3 × 5 = 15

2010 - JUNE

[44] The harmonic mean of 1, 1/2, 1/3 ..... 1/n is

(a) 1/(n + 1) (b) 2/(n + 1)

(c) (n + 1)/2 (d) 1/(n – 1) (1 mark)

Answer:

(b) For a given set of non-zero observations, harmonic mean is defined as the reciprocal of the A.M. of the reciprocals of the observations. Therefore, H.M. for a variable x is given by

$$H = \frac{n}{\sum(1/x_i)}$$

$$= \frac{n}{1 + 2 + 3 + \ldots\ldots\ldots\ldots + n}$$

$$= \frac{n}{\frac{n}{2}(n + 1)}$$

$$= \frac{2}{(n + 1)}$$

[45] The mean weight of 15 students is 110 kg. The mean weight of 5 of them is 100 kg. and of another five students is 125 kg. then the mean weight of the remaining students is :

(a) 120 (b) 105

(c) 115 (d) None of these (1 mark)

Answer:

(b) Total weight of 1st five students = 5 × 100 = 500

Total weight of another five students = 5 × 125 = 625

Total weight of 10 students = 500 + 625 = 1125

Total weight of 15 students = 15 × 110 = 1650

∴ Total weight of remaining 5 students = 1650 – 1125 = 525

∴ Mean weight of remaining 5 students = $\frac{525}{5}$ = 105

[46] In a class of 11 students, 3 students were failed in a test. 8 students who passed secured 10,11, 20, 15, 12, 14, 26 and 24 marks respectively. What will be the median marks of the students :

(a) 12 (b) 15

(c) 13 (d) 13.5 (1 mark)

Answer:

(a) Let x_1, x_2, x_3 be the 3 students failing in test Marks of 11 students in ascending order are –

x_1, x_2, x_3, 10, 11, 12, 14, 15, 20, 24, 26

Median of discrete series = $\frac{n+1^{th}}{2}$ term

$= \frac{11+1^{th}}{2}$ term

$= 6^{th}$ term

= 12

2010 - DECEMBER

[47] The variance of data : 3,4,5,8 is

(a) 4.5 (b) 3.5

(c) 5.5 (d) 6.5 (1 mark)

Answer:

(b) 3, 4, 5, 8

$$\text{Variance} = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

$$= \frac{114}{4} - 25$$

$$= \frac{114 - 100}{4} = \frac{14}{4}$$

$$= \frac{7}{2}$$

$$= 3.5$$

[48] A lady travel at a speed of 20km/h and returned at quicker speed. If her average speed of the whole journey is 24km/h, find the speed of return journey (in km/h)

(a) 25 (b) 30

(c) 35 (d) 38 (1 mark)

Answer:

(b) In this question we will apply formula for harmonic mean as equal Distance(s) covered with variable speed.

Since, H.M. $= \frac{N}{\sum 1/x}$

Let x km/hr be speed of return journey

$$24 = \frac{2}{\frac{1}{20} + \frac{1}{x}}$$

$$24 = \frac{2 \times 20x}{x + 20}$$

$24x + 480 = 40\ x$

$16x = 480$

$x = 30$ km/hr.

[49] Let the mean of the variable 'x' be 50, then the mean of u=10+5x will be :

(a) 250 (b) 260

(c) 265 (d) 273 (1 mark)

Answer:

(b) $u = 10 + 5x$

Since Mean is dependent of change of Origin & Scale

$\therefore$ New Mu = 10 + 5 Mx

$Mu = 10 + 5 \times 50 = 260$

[50] Given the observations : 4,9,11,14,37. The Mean deviation about the Median is

(a) 11 (b) 8.5

(c) 7.6 (d) 7.45 (1 mark)

Answer:

(c) 4, 9, 11, 14, 37

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{term}$$

$$= \left(\frac{5+1}{2}\right)^{th} \text{term}$$

$= 3^{rd}$ term

$= 11$

x	$\lvert d\rvert = \lvert x - 11\rvert$
4	7
9	2
11	0
14	3
37	26
	38

Mean deviation about median

$$M.D. = \frac{\sum |d|}{n} = \frac{38}{5} = 7.6$$

2011 - JUNE

[51] If the difference between mean and Mode is 63, then the difference between Mean and Median will be _____.

(a) 63 (b) 31.5
(c) 21 (d) None of the above. (1 mark)

Answer:

(c) Given : Mode – Mean = 63

We know, the Empirical Relationship between Mean, Median & Mode i.e.

(Mode – Mean) = 3(Median – Mean)

$\therefore$ Median – Mean = $\frac{63}{3} = 21$

[52] If the Arithmetic mean between two numbers is 64 and the Geometric mean between them is 16. The Harmonic Mean between them is _____.

(a) 64 (b) 4
(c) 16 (d) 40 (1 mark)

Answer:

(b) Given : A.M = 64

G.M = 16

H.M = ?

We know, $(G.M)^2 = A.M \times H.M$

$(16)^2 = 64 \times H.M$

$\therefore HM = \frac{256}{64}$

$\therefore H.M = 4$

[53] If all observations in a distribution are increased by 6, then the variance of the series will be _____.

(a) Increased (b) Decreased
(c) Unchanged (d) None of these. (1 mark)

Answer:

(c) Since, Standard Deviation is independent of change of origin, therefore VARIANCE is also independent of change of origin.

[54] The average of 5 quantities is 6 and the average of 3 is 8. what is the average of the remaining two.

(a) 4 (b) 5

(c) 3 (d) 3.5 (1 mark)

Answer:

(c) The avg. of 5 quantities = 6

$\therefore$ The sum of 5 quantities = $6 \times 5 = 30$

$\because$ The avg. of 3 quantities = 8

$\therefore$ The sum of 3 quantities = $8 \times 3 = 24$

$\therefore$ Sum of Remaining Two Nos. = $30 - 24 = 6$

$\therefore$ Avg. of Remaining two = $\frac{6}{2} = 3$

2011 - DECEMBER

[55] The standard deviation of the weights (in kg) of the students of a class of 50 students was calculated to be 4.5 kg. Later on it was found that due to some fault in weighing machine, the weight of each student was under measured by 0.5 kg. The Correct standard deviation of the weight will be:

(a) Less than 4.5 (b) Greater than 4.5

(c) Equal to 4.5 (d) Can not be determined (1 mark)

Answer:

(c) S.D. remains unaffected due to a change of origin so correct S.D. of 50 students is 4.5

[56] For Normal distribution the relation between quartile deviation (Q.D) and standard deviation (S.D) is

(a) Q.D > S.D (b) Q.D < S.D

(c) Q.D = S.D (d) None of the above (1 mark)

Answer:

(b) We know $Q.D = \frac{2}{3}$ S.D

$\Rightarrow$ Q. D < S.D

[57] The median of following numbers, which are given is ascending order is 25. Find the Value of X.

11 13 15 19 (x + 2) (x + 4) 30 35 39 46

(a) 22 (b) 20

(c) 15 (d) 30 (1 mark)

Answer:

(a) Numbers in Ascending Order are

11, 13, 15, 19, (x + 2), (x + 4), 30, 35, 39, 46

Here

No. of terms (N) = 10

$$\text{Median} = \frac{1}{2}\left[\frac{N^{th}}{2}\text{ term} + \left(\frac{N}{2}+1\right)^{th}\text{term}\right]$$

$$25 = \frac{1}{2}\left[\frac{10^{th}}{2}\text{ term} + \left(\frac{10}{2}+1\right)^{th}\text{term}\right]$$

$$25 = \frac{1}{2}\left[5^{th}\text{ term} + 6^{th}\text{ term}\right]$$

$$25 = \frac{1}{2}\left[(x+2)+(x+4)\right]$$

$$50 = 2x + 6$$

$$2x = 50 - 6$$

$$2x = 44$$

$$x = 22$$

[58] The average age of a group of 10 students was 20 years. The average age increased by two years when two new students joined the group. What is the average age of two new students who joined the group?

(a) 22 years (b) 30 years
(c) 44 years (d) 32 years (1 mark)

Answer:

(d) ∴ Average age of 10 students = 20 yrs

∴ the sum of age of 10 students = 20 × 10 = 200 yrs
if two boys are Increased
the total no. of students = 10 + 2 = 12
and Average Increased by 2 yrs
Then New Average = 20 + 2 = 22

∴ The Average age of 12 students = 22
The sum of age of 12 students = 22 × 12 = 264
The sum of age of two boys = 264 − 200 = 64

Average Age of two boys = $\frac{64}{2}$ = 32

2012 - JUNE

[59] If standard deviation of first 'n' natural numbers is 2 then value of 'n' is

(a) 10 (b) 7
(c) 6 (d) 5 (1 mark)

Answer:

(b) S.D. of First 'n' natural Numbers= $\sqrt{\frac{n^2-1}{12}}$

$$\frac{2}{1} = \sqrt{\frac{n^2-1}{12}}$$

$$4 = \frac{n^2-1}{12}$$

$$n^2 - 1 = 48$$

[60] Geometric Mean of three observations 40, 50 and X is 10. The value of X is

(a) 2 (b) 4

(c) 1/2 (d) None of the above. (1 mark)

Answer:

(c) Given G.M of three observation = 10

Given No. of observation (n) = 3

$x_1 = 40, x_2 = 50, x_3 = x$

Geometrical Mean

$$G.M = (x_1 \cdot x_2 \cdot x_3)^{1/3}$$

$$10 = (40 \cdot 50 \cdot x)^{1/3}$$

$$(10)^3 = 40 \cdot 50 \cdot x$$

$$1{,}000 = 40 \cdot 50 \cdot x$$

$$x = \frac{1{,}000}{40.50}$$

$$x = \frac{10}{20}$$

$$x = \frac{1}{2}$$

[61] The mean of first three term is 14 and mean of next two terms is 18. The mean of all five term is :

(a) 14.5 (b) 15

(c) 14 (d) 15.6 (1 mark)

Answer:

(d) Given x_1, x_2, x_3, x_4, x_5 (Say)

$\therefore$ For first three terms (x_1, x_2, x_3)

we know $\therefore$ Mean $= \frac{\sum x}{n}$

$$\therefore \quad 14 = \frac{(\sum x)_{(x_1, x_2, x_3)}}{3}$$

$$(\sum x)_{(x_1, x_2, x_3)} = 42$$

$$\text{\& also } 18 = \frac{(\sum x)_{(x_4, x_5)}}{2}$$

$$\therefore \quad (\sum x)_{(x_4, x_5)} = 36$$

$$\therefore \quad (\sum x)_{(x_1, x_2, x_3, x_4, x_5)} = (\sum x)_{(x_1, x_2, x_3)} + (\sum x)_{(x_4, x_5)}$$

$$= 42 + 36$$

$$= 78$$

$$\text{Mean of all 5 terms} = \frac{(\sum x)_{(x_1, x_2, x_3, x_4, x_5)}}{5}$$

$$= \frac{78}{5}$$

$$= 15.6$$

[62] The standard deviation is independent of change of

(a) Scale (b) Origin

(c) Both origin and scale (d) None of these. (1 mark)

Answer:

(b) We know,

$$\text{S.D} = = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

Where d = x - A

A = Assumed mean

Comparing above both the formula's, we immediately conclude that S.D. is independent of change of origin.

[63] In a normal distribution, the relationship between the three most commonly used measures of dispersion are:

(a) Standard Deviation > Mean Deviation > Quartile Deviation

(b) Mean Deviation > Standard Deviation > Quartile Deviation

(c) Standard Deviation > Quartile Deviation > Mean Deviation

(d) Quartile Deviation > Mean Deviation > Standard Deviation

(1 mark)

Answer:

(a) In a normal distribution,

S.D. > M. D > Q. D.

$\left\{\begin{array}{l}\text{Since M.D.} = 0.8 \times \text{S.D.} \\ \text{and Q.D.} = 0.675 \times \text{S.D.}\end{array}\right\}$

2012 - DECEMBER

[64] If Standard deviation of x is σ, then Standard deviation of $\frac{ax + b}{c}$, where a, b and c (c ± 0) are arbitrary constants, will be

(a) σ (b) $\frac{a\sigma + b}{c}$

(c) $\frac{a}{c}.\sigma$ (d) $\left|\frac{a}{c}\right|\sigma$ (1 mark)

Answer:

(d) ∵ S.D. of x = σ

Let $y = \frac{ax + b}{c}$

$y = \frac{ax}{c} + \frac{b}{c}$

$y = \frac{b}{c} + \frac{a}{c}x$

S.D. of $y = \left|\frac{a}{c}\right|$ S.D. of x

$= \left|\frac{a}{c}\right| \sigma$

[65] The mean salary of a group of 50 persons is ₹ 5,850. Later on it is discovered that the salary of one employee has been wrongly taken as ₹ 8,000 instead of ₹ 7,800. The corrected mean salary is

(a) ₹ 5,854 (b) ₹ 5,846

(c) ₹ 5,650 (d) None of the above (1 mark)

Answer:

(b) Mean $\bar{x} = \frac{\sum x}{N}$

In correct $\sum x = N \bar{x}$

$= 50 \times 5,850$

$= 2,92,500$

Correct $\sum x$ = In correct $\sum x$ + Right value – wrong value

$= 2,92,500 + 7,800 - 8,000$

$= 2,92,500 - 200$

$= 2,92,300$

Correct mean $= \frac{\text{Correct } \sum x}{N}$

$= \frac{2,92,300}{50}$

$= 5,846$

[66] Which of the following measures of dispersion is used for calculating the consistency between two series?

(a) Quartile deviation (b) Standard deviation

(c) Coefficient of variation (d) None of the above. (1 mark)

Answer:

(c) Coefficient of variation is used for calculating the consistency between two series.

[67] If the mode of a data is 18 and mean is 24, then median is —

(a) 18 (b) 24

(c) 22 (d) 21 (1 mark)

Answer:

(c) Mode = 18, Mean = 24

Mode = 3 Median – 2 Mean

$18 = 3 \text{ Median} - 2 \times 24$

$18 = 3 \text{ Median} - 48$

$18 + 48 = 3 \text{ Median}$

$66 = 3 \text{ Median}$

$\text{Median} = \frac{66}{3} = 22$

[68] For data on frequency distribution of weights:
70, 73, 49, 57, 56, 44, 56, 71, 65, 62, 60, 50, 55, 49, 63 and 45
If we assume class length as 5, the number of class intervals would be
(a) 5 (b) 6
(c) 7 (d) 8 (1 mark)

Answer:

(c)

C.I.	40 45	45-50	50-55	55-60	60-65	65-70	70-75
No. of class intervals = 7							

[69] The point of intersection of the "less than" and "more than" ogives correspond to
(a) Mean (b) Mode
(c) Median (d) 10^{th} Percentile (1 mark)

Answer:

(c) Median

[70] A man travels from Agra to Gwalior at an average speed of 30 km per hour and back at an average speed of 60 km per hour. What is his average speed?
(a) 38 km per hour (b) 40 km per hour
(c) 45 km per hour (d) 35 km per hour (1 mark)

Answer:

(b) Average speed = $\frac{2xy}{x + y}$

Given x = 30 km/h & y = 60 km/h

Average speed = $\frac{2 \times 30 \times 60}{30 + 60}$

$= \frac{2 \times \cancel{30} \times \cancel{60}^{20}}{\cancel{90}_{3}}$

= 40 km per hour

2013 - JUNE

[71] If sum of squares of the values = 3390, N = 30 and standard deviation = 7, find out the mean.

(a) 113 (b) 210

(c) 8 (d) None of these (1 mark)

Answer:

(c) S.D. = 7, $\Sigma d^2 = 3390, N = 30$

we know that find $\bar{x}$ = ?

$$S.D. = \sqrt{\frac{\Sigma d^2}{N} - (\bar{x})^2}$$

$$7 = \sqrt{\frac{3390}{30} - (\bar{x})^2}$$

on squaring

$$49 = \frac{3390}{30} - (\bar{x})^2$$

$$49 = 113 - (\bar{x})^2$$

$$(\bar{x})^2 = 113 - 49$$

$$(\bar{x})^2 = 64$$

$$\bar{x} = \sqrt{64}$$

$$= 8$$

[72] If the mean of a frequency distribution is 100 and coefficient of variation is 45% then standard deviation is:

(a) 45 (b) 0.45

(c) 4.5 (d) 450 (1 mark)

Answer:

(a) Given Mean $\bar{x} = 100$

Coeff. of variation (C.V.) = 45%

$$C.V. = \frac{S.D.}{A.M.} \times 100$$

$$45 = \frac{S.D.}{100} \times 100$$

[73] Which of the following measures of central tendency cannot be calculated by graphical method?

(a) Mean (b) Mode
(c) Median (d) Quartile (1 mark)

Answer:

(a) Mean can not be calculate by graphical method.

[74] Geometric mean of 8, 4, 2 is

(a) 4 (b) 2
(c) 8 (d) None of these (1 mark)

Answer:

(a) G.M. $= (x_1 . x_2 . x_3)^{1/3}$

$= (8 . 4 . 2)^{1/3}$

$= (64)^{1/3}$

$= 4^3 \times \frac{1}{3}$

$= 4$

[75] The average age of 15 students of a class is 15 years. Out of them, the average age of 5 students is 14 years and that of the other 9 students is 16 years. The age of the 15th student is:

(a) 11 years (b) 14 years
(c) 15 years (d) None of these (1 mark)

Answer:

(a) The age of 15th student

$= (15 \times 15) - [(5 \times 14) + (9 \times 16)]$

$= 225 - [70 + 144]$

$= 225 - 214$

$= 11$

2013 - DECEMBER

[76] Find at the variance given that the Arithmetic Mean = (8 + 4)/2

(a) 2 (b) 6
(c) 1 (d) 4 (1 mark)

Answer:

(d) Given Arithmetic Mean = $\frac{(8+4)}{2}$

Here Largest value (L) = 8

Smallest value (S) = 4

Range = L - S

= 8 - 4

= 4

we know that

$S.D = \frac{\text{Range}}{2}$

$S.D = \frac{4}{2} = 2$

Variance = $(S.D)^2 = (2)^2 = 4$

[77] In normal distribution mean, median and mode are

(a) Equal (b) Not Equal

(c) Zero (d) None of above (1 mark)

Answer:

(a) In Normal distribution Mean, Median and Mode are **equal**.

[78] Coefficient of mean deviation about mean for the first 9 natural numbers is

(a) 200/9 (b) 80

(c) 400/9 (d) 50 (1 mark)

Answer:

(c) The First 9 natural Number are 1, 2, 3, 4, 5, 6, 7, 8, 9

$$\text{Mean } \bar{x} = \frac{\sum x}{N} = \left(\frac{1+2+3+4+5+6+7+8+9}{9}\right) = \frac{45}{9} = 5$$

x	$\bar{x}$	$\lvert d \rvert = \lvert x - \bar{x} \rvert$
1	5	$\lvert 1 - 5 \rvert = 4$
2	5	$\lvert 2 - 5 \rvert = 3$
3	5	$\lvert 3 - 5 \rvert = 2$
4	5	$\lvert 4 - 5 \rvert = 1$

5	5	$\|5 - 5\| = 0$
6	5	$\|6 - 5\| = 1$
7	5	$\|7 - 5\| = 2$
8	5	$\|8 - 5\| = 3$
9	5	$\|9 - 5\| = 4$
N = 9		$\Sigma \|d\| = 20$

Mean Deviation

$$M.D. = \frac{\Sigma |d|}{N} = \frac{20}{9}$$

$$\text{Coeff of M.D} = \frac{M.D}{Mean} \times 100$$

$$= \frac{\frac{20}{9} \times 100}{5}$$

$$= \frac{20 \times 100}{9 \times 5} = \frac{400}{9}$$

[79] The pair of averages whose value can be determined graphically?

(a) Mode, Median (b) Mean, Mode

(c) Mean, Median (d) None of the above (1 mark)

Answer:

(a) The pair of averages whose value can be determined graphically are mode and median

[80] If mean = 5, Standard deviation = 2.6, median = 5 and quartile deviation = 1.5, then the coefficient of quartile deviation equals

(a) 35 (b) 39

(c) 30 (d) 32 (1 mark)

Answer:

(c) Given

Mean $\bar{x} = 5$, S.D $(\sigma) = 2.6$, Median = 5 and Q.D. = 1.5

$$\text{Coeff of Q.D} = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

$$= \frac{\left(\frac{Q_3 - Q_1}{2}\right)}{\left(\frac{Q_3 + Q_1}{2}\right)} \times 100$$

$$= \frac{\text{Q.D.}}{\text{Median}} \times 100$$

$$= \frac{1.5}{5} \times 100$$

$$= \frac{150}{5}$$

$$= 30$$

2014 - JUNE

[81] What will be the probable value of mean deviation? When $Q_3 = 40$ and $Q_1 = 15$

(a) 17.50 (b) 18.75

(c) 15.00 (d) None of the above (1 mark)

Answer:

(c) $Q_3 = 40$ and $Q_1 = 15$

$$\text{Q.D.} = \frac{Q_3 - Q_1}{2} = \frac{40 - 15}{2} = \frac{25}{2} = 12.50$$

$$5 \text{ M.D.} = 6 \text{ Q.D.} \Rightarrow \text{M.D.} = \frac{6}{5}\text{Q.D.}$$

$$= \frac{6}{5} \times 12.50 = 15$$

[82] Which of the following statements is true?

(a) Median is based on all the observations

(b) The mode is the mid value

(c) The median is the second quartile

(d) The mode is the fifth decile. (1 mark)

Answer:

[83] The mean of the following data is 6. Find the value of 'P'.

x :	2	4	6	10	P+5
f :	3	2	3	1	2

(a) 4 (b) 6
(c) 8 (d) 7 (1 mark)

Answer:

(d)

x	f	fx
2	3	6
4	2	8
6	3	18
10	1	10
P + 5	2	2P + 10
	N = 11	$\sum fx = 2P + 52$

$$\bar{x} = \frac{\sum fx}{N}$$

$$6 = \frac{2P + 52}{11}$$

$$6 \times 11 = 2P + 52$$

$$66 = 2P + 52$$

$$2P = 14$$

$$P = 7$$

[84] The formula for range of middle 50% items of a series is :

(a) $Q_3 - Q_1$ (b) $Q_3 - Q_2$
(c) $Q_2 - Q_1$ (d) $\frac{Q_3 - Q_1}{2}$ (1 mark)

Answer:

(d) The formula for Range of middle 50% items of a series is (Q.D.).

$$Q.D. = \frac{Q_3 - Q_1}{2}$$

2014 - DECEMBER

[85] The third decile for the numbers 15, 10, 20, 25, 18, 11, 9, 12, is:

(a) 13 (b) 10.70

(c) 11 (d) 11.50 (1 mark)

Answer:

(b) Write the terms in Ascending order 9, 10, 11, 12, 15, 18, 20, 25

No. of terms (N) = 8

$$\text{Third Decile } D_3 = \frac{3(N+1)^{th}}{10}\text{ term}$$

$$= \frac{3(8+1)^{th}}{10}\text{ term}$$

$$= 2.7^{th}\text{ term}$$

$$= 2^{nd}\text{ term} + 0.7\,(3^{rd}\text{ term} - 2^{nd}\text{ term})$$

$$= 10 + 0.7\,(11 - 10)$$

$$= 10 + 0.7$$

$$= 10.70$$

[86] A random variable X has uniform distribution on the interval (-3, 7). The mean of the distribution is:

(a) 2 (b) 4

(c) 5 (d) 6 (1 mark)

Answer:

(a) A random variable x has uniform distribution. Given Interval (– 3, 7).

$$\text{Mean} = \frac{7+(-3)}{2} = \frac{7-3}{2} = \frac{4}{2} = 2$$

[87] If the first quartile is 142 and semi-inter quartile range is 18, then the value of median is:

(a) 151 (b) 160

(c) 178 (d) None of these (1 mark)

Answer:

(b) First Quartile $Q_1 = 142$

Semi Inter quartile Range (Q.D.) $= 18$

$$\frac{Q_3 - Q_1}{2} = 18$$

$$Q_3 - Q_1 = 36$$

$$Q_3 - 142 = 36$$

$$Q_3 = 36 + 142$$

Third Quartile $Q_3 = 178$

$$\text{Median} = \frac{Q_1 + Q_3}{2}$$

$$= \frac{142 + 178}{2}$$

$$= \frac{320}{2} = 160$$

[88] The quartile deviation is:

(a) 2/3 of S.D. (b) 4/5 of S.D.

(c) 5/6 of S.D. (d) None of these (1 mark)

Answer:

(a) We know that

4 S.D. = 6 Q.D.

then Q.D. $= \frac{4}{6}$S.D.

$$\boxed{\text{Q.D.} = \frac{2}{3}\text{S.D.}}$$

[89] If the arithmetic mean of two numbers is 10 and the geometric mean of these numbers is 8, then the harmonic mean is:

(a) 9 (b) 8.9

(c) 6.4 (d) None of these (1 mark)

Answer:

(c) Given Arithmetic Mean (A.M.) = 10

(G.M.) = 8

We know that $(G.M.)^2 = A.M. \times H.M.$

$(8)^2 = 10 \times H.M.$

$H.M. = \frac{(8)^2}{10} = \frac{64}{10} = 6.4$

2015 - JUNE

[90] The standard deviation of a variable x is known to be 10. The standard deviation of 50 + 5x is

(a) 50 (b) 100

(c) 10 (d) 500 (1 mark)

Answer:

(a) S.D. of x= 10

Given y = 50 + 5x

5x- y + 50 = 0

$b = -\frac{\text{coefficient of x}}{\text{coefficient of y}} = \frac{-5}{-1} = 5$

S.D. of y= |3| S.D. of x

= |5| × 10

= 5 × 10

= 50

[91] The harmonic mean H of two numbers is 4 and their arithmetic mean A and the geometric mean G satisfy the equation $2A + G^2 = 27$, then the numbers are

(a) (1, 3) (b) (9, 5)

(c) (6, 3) (d) (12, 7) (1 mark)

Answer:

(c) Let two Nos. are a & b

Given Harmonic mean of two Nos. (H) = 4

$\frac{2ab}{a+b} = 4$

$2ab = 4(a + b)$

Given $2A + G^2 = 27$

$$2\frac{(a+b)}{2} + ab = 27$$

$$a + b + 2(a + b) = 27$$

$$a + b + 2a + 2b = 27$$

$$3a + 3b = 27$$

$$3(a + b) = 27$$

$$a + b = 9 \quad \text{_____ (2)}$$

Solving equation (1) & (2) we get

$a = 6, b = 3$

[92] Coefficient of quartile deviation is equal to

(a) Quartile deviation × 100/median

(b) Quartile deviation × 100 / mean

(c) Quartile deviation × 100 / mode

(d) None (1 mark)

Answer:

(a) Coeff. of Q.D.$= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

$$= \frac{\frac{Q_3 - Q_1}{2}}{\frac{Q_3 + Q_1}{2}} \times 100$$

$$= \frac{\text{Quartile Deviation} \times 100}{\text{Median}}$$

[93] If all the observations are increased by 5, then

(a) S.D. would be increased by 5

(b) Mean deviation would be increased by 5

(c) Quartile deviation would be increased by 5

(d) All the three would not be increased by 5 (1 mark)

Answer:

(d) If all observations are Increased by 5 then S.D, M.D, Q.D and Range remains unchanged.

2015 - DECEMBER

[94] What is value of mean deviation about mean from the number 5, 8 , 6, 3 and 4 ?

(a) 5.20 (b) 7.20

(c) 1.44 (d) 2.23 (1 mark)

Answer:

(c) Given data 3, 4, 5, 6, 8

Mean $\bar{x} = \frac{\sum x}{N} = \frac{3+4+5+6+8}{5} = \frac{26}{5} = 5.2$

x	$\bar{x}$	$\lvert d \rvert = \lvert x - \bar{x} \rvert$
3	5.2	$\lvert 3 - 5.2 \rvert = 2.2$
4	5.2	$\lvert 4 - 5.2 \rvert = 1.2$
5	5.2	$\lvert 5 - 5.2 \rvert = 0.2$
6	5.2	$\lvert 6 - 5.2 \rvert = 0.8$
8	5.2	$\lvert 8 - 5.2 \rvert = 2.8$
N = 5		$\sum \lvert d \rvert = 7.2$

M.D. $= \frac{\sum \lvert d \rvert}{N} = \frac{7.2}{5} = 1.44$

[95] For the observation of 6, 4, 1, 6, 5, 10, 4, 8 the range is:

(a) 10 (b) 9

(c) 8 (d) None (1 mark)

Answer:

(b) Given data in Ascending Order

1, 4, 4, 5, 6, 6, 8, 10

Largest value (L) = 10

Smallest value (S) = 1

Range (R) = L - S

=10 -1

= 9

[96] If a variance of a random variable 'x' is 23, then what is variance of $2x + 10$?

(a) 56 (b) 33
(c) 46 (d) 92 (1 mark)

Answer:

(d) Given Variance of x $= 23$

$V(x) = 23$

S.D. of x $= \sqrt{23}$

Given $y = 2x + 10$

$2x - y + 10 = 0$

$b = -\frac{\text{coefficient of x}}{\text{coefficient of y}} = \frac{-2}{-1} = 2$

S.D. of Y $= |b|$ S.D. of x

$= |2|.\sqrt{23}$

$= 2\sqrt{23}$

Variance of y $= (\text{S.D. of y})^2 = (2\sqrt{23})^2$

$= 2^2 \times 23$

$= 4 \times 23$

$= 92$

[97] If variance = 148.6 and $\bar{x} = 40$, then the coefficient of variation is:

(a) 37.15 (b) 30.48
(c) 33.75 (d) None of the above (1 mark)

Answer:

(b) Variance $= 148.6$

S.D. $= \sqrt{\text{Variance}}$

$= \sqrt{148.6}$

$= 12.19$

And A.M. $(\bar{x}) = 40$

Coefficient of Variation C.V. $= \frac{\text{S.D.}}{\text{A.M.}} \times 100$

$= \frac{12.19}{40} \times 100$

$= 30.48$ (Approx.)

[98] Quartiles can be determined graphically using:

(a) Histogram (b) Frequency polygon

(c) Ogive curve (d) Pie chart (1 mark)

Answer:

(c) Quartile can be determined graphically using **'o' give curve**.

[99] In a class of 50 students, 10 have failed and their average marks in 2.5. The total marks secured by the entire class were 281. The average marks who have passed is:

(a) 5.32 (b) 7.25

(c) 6.40 (d) None of the above. (1 mark)

Answer:

(c) Total No. of students = 50

No. of Failed students $(n_1) = 10$

No. of Passed students $(n_2) = 40$

Average marks of Failed students $\bar{x}_1 = 2.5$

Let Average marks of Passed students $\bar{x}_2 = x$

Combined Mean $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

Total sum of all observation $= (n_1 + n_2)\,\bar{x}$

$$n_1\bar{x}_1 + n_2\bar{x}_2 = (n_1 + n_2)\,\bar{x}$$

$$10 \times 2.5 + 40 \times x = 281$$

$$25 + 40 \times x = 281$$

$$40x = 281 - 25$$

$$40x = 256$$

$$x = \frac{256}{40}$$

$$x = 6.40$$

2016 - JUNE

[100] The SD of first n natural number is _____

(a) $\sqrt{\frac{n^2-1}{12}}$ (b) $\sqrt{\frac{n(n+1)}{12}}$

(c) $\sqrt{\frac{n(n-1)}{6}}$ (d) None of these. (1 mark)

Answer:

(a) The S.D. of First n natural Number is

$$S.D. = \sqrt{\frac{n^2-1}{12}}$$

[101] If mean and coefficient of variation of the marks of 10 students is 20 and 80 respectively. What will be variance of them?

(a) 256 (b) 16

(c) 25 (d) None of these. (1 mark)

Answer:

(a) Given No. of observation N = 10

Mean $(\bar{x}) = 20$

$$c.v. = 80$$

$$c.v. = \frac{S.D.}{A.M.} \times 100$$

$$80 = \frac{S.D.}{20} \times 100$$

$$S.D. = \frac{80 \times 20}{100}$$

$$S.D. = 16$$

$$\text{Variance} = (S.D.)^2$$

$$= (16)^2$$

$$= 256$$

[102] If same amount is added to or subtracted from all the values of an individual series then the standard deviation and variance both shall be _____

(a) changed (b) unchanged
(c) same (d) none of these (1 mark)

Answer:

(b) If same amount is added to or subtracted from all the values of an individual series then S.D and variance both shall be unchanged.

[103] If the mean of two numbers is 30 and geometric mean is 24 then what will be these two numbers?

(a) 36 and 24 (b) 30 and 30
(c) 48 and 12 (d) None of these (1 mark)

Answer:

(c) Let two number be a & b

$$A.M = \frac{a+b}{2}$$

$$30 = \frac{a+b}{2}$$

$$a + b = 60 \text{ ——— (1)}$$

$$G.M = \sqrt{ab}$$

$$24 = \sqrt{ab}$$

$$ab = 576 \text{ ——— (2)}$$

Solving (1) & (2) we get

$a = 48$ and $b = 12$

2016 - JUNE

[104] For moderately skewed distribution of marks in commerce for a group of 200 students the mean marks and mode marks were found to be 55.60 and 46. What is the median marks?

(a) 55.5 (b) 60.5
(c) 52.4 (d) None of these (1 mark)

Answer:

(c) Here Mean $(\bar{x}) = 55.60$

Mode $(M_0) = 46$.

For moderately skewed distribution of marks

Mode = 3 Median – 2 Mean

46 = 3 Median – 2 × 55.60

46 = 3 Median – 111.20

3 Median = 46 + 111.20

3 Median = 157.20

$$\text{Median} = \frac{157.20}{3} = 52.40$$

[105] Mean for the data 6, 4, 1, 6, 5, 10, 3 is 5 when each observation added by 2, what is mean of the data

(a) 5 (b) 6

(c) 7 (d) 10 (1 mark)

Answer:

(c) Original Mean = 5

K = 2

By shifting the origin, Mean is changed

New Mean $\bar{X}$ = Original Mean + k

= 5 + 2

New Mean $\bar{X}$ = 7

[106] The average of 10 observations is 14.4. If the average of first 4 observations is 16.5. The average of remaining 6 observations is:

(a) 13.6 (b) 13.0

(c) 13.2 (d) 12.5 (1 mark)

Answer:

(b) Given $n_1 = 4$ $n_2 = 6$

$\overline{X_1} = 16.5$ $\overline{X_2} = x$ (let)

Combined Average $\bar{X} = 14.4$

$$\text{Combined Average } (\bar{x}) = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

$$14.4 = \frac{4 \times 16.5 + 6 \times x}{4 + 6}$$

$$\frac{14.4}{1} = \frac{66.0 + 6x}{10}$$

$$14.4 \times 10 = 66 + 6x$$

$$144 = 66 + 6x$$

$$6x = 144 - 66$$

$$6x = 78$$

$$x = \frac{78}{6} = 13$$

[107] The ordering of a particular design of a cloth show room, a _____ size be more appropriate.

(a) median (b) mean

(c) mode (d) all of these (1 mark)

Answer:

(c) The ordering of the particular design of a cloth showroom, a mode size be more appropriate.

[108] The second and third moments of a sample of seven observation (-6, -4, -2, 0, 2, 4, 6) are

(a) (12, 0) (b) (0, 12)

(c) (0, 16) (d) (16, 0) (1 mark)

Answer:

(d) Calculation for moment:

X	$(X - \bar{x})$	$(X - \bar{x})^2$	$(X - \bar{x})^3$
– 6	(– 6 – 0) = – 6	36	– 216
– 4	(– 4 – 0) = – 4	16	– 64
– 2	(– 2 – 0) = – 2	4	– 8
0	(0 – 0) = 0	0	0

4	(4 – 0) = 4	16	64
6	(– 6 – 0) = 6	36	216
		$\Sigma (X - \bar{x})^2 = 112$	$\Sigma (X - \bar{x})^3 = 0$

Here $\bar{X} = \frac{\Sigma x}{N} = \frac{-6-4-2+0+2+4+6}{7} = \frac{0}{7} = 0$

Second Moment

$\mu_2 = \frac{\Sigma (x - \bar{X})^2}{N}$ (Here N = 7)

$= \frac{112}{7}$

$= 16$

Third moment

$\mu_3 = \frac{\Sigma (X - \bar{X})^3}{N}$

$= \frac{0}{7}$

$= 0$

(16 , 0)

[109] The geometric mean of three numbers 40, 50 and x is 10, the value of x is

(a) 5 (b) 4

(c) 2 (d) $\frac{1}{2}$ (1 mark)

Answer:

(d) G.M = $(x_1 . x_2 . x_3)^{\frac{1}{3}}$

$10 = (40 \times 50 \times x)^{\frac{1}{3}}$

Cube on both side

$10^3 = (40 \times 50 \times x)$

$1,000 = 2,000x \Rightarrow x = \frac{1,000}{2,000} = \frac{1}{2}$

2017 - JUNE

[110] The rates of returns from three different shares are 100%, 200% and 400% respectively. The average rate of return will be:

(a) 350% (b) 233.33%

(c) 200% (d) 300% (1 mark)

Answer:

(c) If given data are in the form of % then

We use G.M for average

$$G.M. = (x_1 . x_2 . x_3)^{1/3}$$
$$= (100 \times 200 \times 400)^{1/3}$$
$$= (80,00,000)^{1/3}$$
$$= (200)^{3 \times 1/3}$$
$$= 200\%$$

[111] If geometric mean is 6 and arithmetic mean is 6.5, then harmonic mean will be:

(a) $\frac{6^2}{6.5}$ (b) $\frac{6}{6.5^2}$

(c) $\frac{6}{6.5}$ (d) None of the above. (1 mark)

Answer:

(a) $\because$ G.M = 6, A.M = 6.5

$$H.M = \frac{(G.M)^2}{A.M} = \frac{6^2}{6.5}$$

[112] A company's past 10 years average earning is ₹ 40 crores. To have the same average earning for 11 years including these 10 years, how much earning must be made by the company in the eleventh year?

(a) ₹ 40 crores (b) ₹ $\frac{40 \times 10}{11}$ crores

(c) More than ₹ 40 crores (d) None of the above. (1 mark)

Answer:

(a) Given $n_1 = 10$ $n_2 = 1$ $\bar{x} = 40$

$\bar{x}_1 = 40$ $\bar{x}_2 = x$

Combined mean $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$$40 = \frac{10 \times 40 + 1 \times x}{10 + 1}$$

$$40 = \frac{400 + x}{11}$$

$$440 = 400 + x$$

$$x = 440 - 400$$

$$x = 40 \text{ crores}$$

[113] A person purchases 5 rupees worth of eggs from 10 different markets. You are to find the average number of eggs per rupee purchased from all the markets taken together. The suitable average in this case is:

(a) A.M. (b) G.M.

(c) H.M. (d) None of the above. (1 mark)

Answer:

(c) If rate are given so we use H.M. for suitable average.

[114] For a moderately skewed distribution, the relationship between mean, median and mode is :

(a) Mean – Mode = 2 (Mean – Median)

(b) Mean – Median = 3 (Mean – Mode)

(c) Mean – Median = 2 (Mean – Mode)

(d) Mean – Mode = 3 (Mean – Median). (1 mark)

Answer:

(d) For a moderately skewed distribution

Mean – Mode = 3 (Mean – Median)

[115] If arithmetic mean and coefficient of variation of x are 10 and 40, respectively then the variance of $-15 + \frac{3x}{2}$ will be:

(a) 64 (b) 81
(c) 49 (d) 36 (1 mark)

Answer:

(d) Given Mean of x= 10

C.V. (x) = 40

$$\text{C.V. of } x = \frac{\text{S.D of } x}{\text{A.M. of } x} \times 100$$

$$40 = \frac{\text{S.D of } x}{10} \times 100$$

$$400 = \text{S.D of } x \times 100$$

$$\text{S.D. of } x = \frac{400}{100} = 4$$

$$\text{Now } y = -15 + \frac{3x}{2}$$

$$2y = -30 + 3x$$

$$3x - 2y - 30 = 0$$

$$b = \frac{-\text{Coefficient of } x}{-\text{Coefficient of } y} = \frac{-3}{-2} = \frac{3}{2}$$

$$\text{S.D. of } y = |b| \text{ S.D of } x$$

$$= \left|\frac{3}{2}\right| \times 4$$

$$= \frac{3}{2} \times 4\ 2$$

$$= 6$$

$$\text{Variance of } y = (6)^2 = 36$$

2017 - DECEMBER

[116] _____ is the reciprocal of the AM of the reciprocal of observations.

(a) HM (b) GM

Answer:

(a) Harmonic Mean (H.M.) is the reciprocal of the A.M. of the reciprocal of observations.

[117] Mean deviation is the least when deviations are taken from

(a) Mean (b) Median

(c) Mode (d) Harmonic mean (1 mark)

Answer:

(b) Mean Deviation is the least when deviation are taken from **Median**.

[118] If the mean value of seven numbers 7, 9, 12, X, 4, 11 and 5 is 9, then the missing number X will be:

(a) 13 (b) 14

(c) 15 (d) 8 (1 mark)

Answer:

(c) Given observations are

7, 9, 12, x, 4, 11, 5

No. of observation (N) = 7

Sum of all observation $\Sigma x = 7 + 9 + 12 + x + 4 + 11 + 5$

$= 48 + x$

$$\text{Mean } \bar{x} = \frac{\Sigma x}{N}$$

$$9 = \frac{48 + x}{7}$$

$$63 = 48 + x$$

$$x = 15$$

[119] When all observations occur with equal frequency _____ does not exist.

(a) median (b) mode

(c) mean (d) none of the above. (1 mark)

Answer:

(b) When all observations occur with equal frequency, mode does not exist.

2018 - May

[120] If the variables x and z are so related that z = ax+b for each $x=x_1$, where a and b are constant, then $\bar{z} = a\bar{x} + b$

(a) True (b) false
(c) both (d) none (1 mark)

Answer:

(a) If the Variable 'X' and 'Z' are so related that Z = ax+b for each x $= x_1$, where and a and b are constant then $\bar{Z} = \overline{ax} + \bar{b}$ then it is True.

[121] Relation between mean, median and mode is

(a) mean-mode = 2 (mean-median)
(b) mean-median = 3(mean-mode)
(c) mean-median = 2 (mean-mode)
(d) mean-mode = 3(mean-median) (1 mark)

Answer:

(d) We know that
Mode = 3 Median – 2 Mean
Mode – Mean = 3 Median – 2 Mean – Mean
Mode – Mean = 3 Median – 3 Mean
Mode – Mean = 3 (Median – Mean)
Mean – Mode = 3 (Mean – Median) (1 mark)

[122] $\frac{(Q_3-Q_1)}{(Q_3+Q_1)}$ is known as

(a) Coefficient of Range (b) Coefficient of Q.D.
(c) Coefficient of S.D. (d) Coefficient of M.D. (1 mark)

Answer:

(b) Coefficient of Q.D. = $\frac{(Q_3-Q_1)}{(Q_3+Q_1)}$

[123] If each item is reduced by 15 A. M is

(a) reduced by 15 (b) increased by 15

(c) reduced by 10 (d) none (1 mark)

Answer:

(a) If each item is reduced by 15 then new A.M is reduced by 15. Because the shifting of origin, the A.M is changed.

[124] For 899, 999, 391, 384, 390, 480, 485, 760, 111, 240 Rank of median is

(a) 2.75 (b) 5.5

(c) 8.25 (d) none (1 mark)

Answer:

(b) Write the terms in Ascending Order 111, 240, 384, 391, 480, 485, 590, 760, 899, 999.

Here No of observation (N) = 10

$$\text{median } (m_e) = \left(\frac{n+1}{2}\right)^{th} \text{ term}$$

$$= \left(\frac{10+1}{2}\right)^{th} \text{ term}$$

$$= 5.5^{th} \text{ term}$$

rank of median $(m_e) = 5.5$

[125] The average of a series of overlapping averages, each of which is based on a certain number of item within a series is know as.

(a) Moving average (b) Weighted average

(c) Simple average (d) None (1 mark)

Answer:

(a) The Average of a series of overlapping averages, each of which is based on a certain number of item within a series is know as **Moving Average**.

[126] If the S.D. of the 1st n natural Nos. is $\sqrt{30}$, Then the value of n is

(a) 19 (b) 20

(c) 21 (d) None (1 mark)

Answer:

(a) S.D of First 'n' natural Numbers

$$= \sqrt{\frac{n^2-1}{12}}$$

$$\sqrt{30} = \sqrt{\frac{n^2-1}{12}}$$

On squaring both side

$$30 = \frac{n^2-1}{12}$$

$$360 = n^2 - 1$$

$$n^2 = 360 + 1$$

$$n^2 = 361$$

$$n = \sqrt{361}$$

$$n = 19$$

2018 - November

[127] The median of the data 5, 6, 7, 7, 8, 9, 10, 11, 11, 12, 15, 18, 18 and 19 is

(a) 10.5 (b) 10

(c) 11 (d) 11.5 (1 mark)

Answer:

(a) Write the term is Ascending order 5, 6, 7, 7, 8, 9, 10, 11, 11, 12, 15, 18, 18 and 19

Here, No. of terms (N) = 14

$$\text{Median} = \frac{1}{2}\left[\frac{N}{2}^{th} \text{ term} + \left(\frac{N}{2}+1\right)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[\frac{14}{2}^{th} \text{ term} + \left(\frac{14}{2}+1\right)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[7^{th} \text{ term} + 8^{th} \text{ term}\right]$$

$$= \frac{1}{2}[10 + 11]$$

$$= \frac{1}{2} \times [21]$$

$$= 10.5$$

[128] The mean of 20 items of a data is 5 and if each item is multiplied by 3, then the new mean will be

(a) 5 (b) 10

(c) 15 (d) 20 (1 mark)

Answer:

(c) By shifting the scale Mean is changed

New Mean = K x original Mean = 5

K = 3

New Mean = 3 × 5

=15

[129] The Geometric mean of 3, 6, 24 and 48 is

(a) 8 (b) 12

(c) 24 (d) 6 (1 mark)

Answer:

(b) G.M. $= (x_1 \cdot x_2 \cdot x_3 \cdot x_4)^{1/4}$ {Here, n = 4}

$= (3 \times 6 \times 24 \times 48)^{1/4}$

$= \sqrt[4]{3 \times 6 \times 24 \times 48}$

$= \sqrt[4]{3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3}$

$= \sqrt[4]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$

$= 2 \times 2 \times 3$

$= 12$

[130] The Algebraic sum of the deviation of a set of values from their arithmetic mean is

(a) >0 (b) = 0

(c) < 0 (d) None of the above (1 mark)

Answer:

(b) The Arithmetic sum of the deviation of a set of value from their A.M is always Zero.

[131] Which one of the following is not a central tendency?

(a) Mean Deviation (b) Arithmetic mean

(c) Median (d) Mode (1 mark)

Answer:

(a) M.D is not a Central tendency.

[132] If the range of a set of values is 65 and maximum value in the set is 83, then the minimum value in the set is

(a) 74 (b) 9

(c) 18 (d) None of the above (1 mark)

Answer:

(c) Given: Maximum Value (L) = 83

Range (R) = 65

Minimum Value (S) = ?

Range (R) = L – S

65 = 83 – S

S = 83 – 65

S = 18

[133] If total frequencies of three series are 50, 60 and 90 and their means are 12, 15 and 20 respectively, then the mean of their composite series is

(a) 16 (b) 15.5

(c) 16.5 (d) 14.5 (1 mark)

Answer:

(c)

$n_1 = 50$ and $\bar{x}_1 = 12$

$n_2 = 60$ $\bar{x}_2 = 15$

$n_3 = 90$ $\bar{x}_3 = 20$

$$\text{Compared Mean } \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3}$$

$$= \frac{50 \times 12 + 60 \times 15 + 90 \times 20}{50 + 60 + 90}$$

$$= \frac{600 + 900 + 1800}{200}$$

$$= \frac{3300}{200}$$

$$= 16.5$$

[134] If the variance of 5, 7, 9 and 11 is 4, then the coefficient of variation is:

(a) 15 (b) 25

(c) 17 (d) 19 (1 mark)

Answer:

(b) Variance of 5, 7, 9 and 11 is 4

i.e. Variance = 4

S.D (σ) = $\sqrt{4} = 2$

$$\text{Mean } (\bar{x}) = \frac{\Sigma x}{N} = \frac{5 + 7 + 9 + 11}{4}$$

$$= \frac{32}{4} = 8$$

$$\text{Coeff. of Variation (C.V.)} = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{2}{8} \times 100 = 25$$

[135] Standard Deviation for the marks obtained by a student in monthly test in mathematic (out of 50) as 30, 35, 25, 20, 15 is

(a) 25 (b) $\sqrt{50}$

(c) $\sqrt{30}$ (d) 50 (1 mark)

Answer:

(b) Given data's are

15, 20, 25, 30, 35

$$\text{Mean } (\bar{x}) = \frac{\Sigma x}{N} = \frac{15 + 20 + 25 + 30 + 35}{5} = \frac{125}{5} = 25$$

For S.D

x	$\bar{x}$	$d = x - (\bar{x})$	d^2
15	25	− 10	100

25	25	0	0
30	25	5	25
35	25	10	100
N = 5			$\Sigma d^2 = 250$

$$S.D = \sqrt{\frac{\Sigma d^2}{N}} = \sqrt{\frac{250}{5}} = \sqrt{50}$$

[136] If in a moderately skewed distribution the values of mode and mean are 32.1 and 35.4 respectively, then the value of the median is

(a) 34.3 (b) 33.3

(c) 34 (d) 33 (1 mark)

Answer:

(a) Given:

Mode = 32.1, Median = ?

Mean = 35.4

Mode = 3 Median – 2 Mean

32.1 = 3 Median – 2 × 35.4

32.1 = 3 Median – 70.8

3 Median = 32.1 + 70.8

3 Median = 102.9

$$\text{Median} = \frac{102.9}{3} = 34.3$$

[137] If the standard deviation for the marks obtained by a student in monthly test is 36, then the variance is

(a) 6 (b) 36

(c) 1296 (d) None of the above (1 mark)

Answer:

(c) If S.D. = 36

Variance (σ) = $(36)^2$

= 1,296

[138] If the mean of the following distribution is 6 then the value of P is

X:	2	4	6	10	P + 5
f:	3	2	3	1	2

(a) 7 (b) 5
(c) 8 (d) 11 (1 mark)

Answer: (a)

x	f	f.x
2	3	6
4	2	8
6	3	18
10	1	10
P + 5	2	2P + 10
	N = 11	$\Sigma fx = 2P + 52$

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{2P + 52}{11}$$

Given:

$$\bar{x} = 6$$

$$\frac{6}{1} = \frac{2P + 52}{11}$$

$$2P + 52 = 66$$

$$2P = 66 - 52$$

$$2P = 14$$

$$P = 7$$

2019 - June

[139] If $\sigma^2 = 100$ and coefficient of variation = 20% then $\bar{x}$ =

(a) 60
(b) 70
(c) 80
(d) 50 (1 mark)

Answer:

(d) If $\sigma^2 = 100$ and c.v. = 20%

$\sigma = \sqrt{100} = 10$ $\bar{x} = ?$

$$\text{c.v.} = \frac{\text{S.D}}{\text{A.M}} \times 100$$

$$20 = \frac{10}{x} \times 100$$

$$20x = 1000$$

$$x = \frac{1000}{20} = 50$$

[140] The AM of 15 Observation is 9 and the AM of first 9 Observation is 11 and then AM of remaining Observations is

(a) 11
(b) 6
(c) 5
(d) 9 (1 mark)

Answer:

(b)

A.M of 15 observations	= 9
Sum of 15 observations	= 9 × 15
	= 135
A.M of 9 observations	= 11
Sum of 9 observations	= 11 × 9
	= 99
Sum of remaining 6 observations	= 135 - 99
	= 36
Average of 6 observations	$= \frac{36}{6}$
	= 6

[141] In a moderately Skewed distribution the values of mean & median are 12 & 8 respectively. The value of mode is

(a) 0
(b) 12
(c) 15
(d) 30 (1 mark)

Answer:

(a) Given,

Mean = 12
Median = 8
Mode = 3 Median - 2 Mean
= 3 × 8 - 2 × 12
= 24 - 24
= 0

[142] If the points of inflexion of a normal curve are 40 and 60 respectively, then its mean deviation is

(a) 8
(b) 45
(c) 50
(d) 60 (1 mark)

Answer:

(a) If the point of Inflexion of a Normal Distribution are 40 and 60.

Then

$\mu - \sigma = 40$ ———— (1)

$\mu + \sigma = 60$ ———— (2)

Solving eq. (1) and (2) we get

$\mu = 50$, $\sigma = 10$

Then M.O $= \frac{4}{5}$ S.D

$= \frac{4}{5} \times 10$

$= 8$

[143] Standard deviation is _____ times of $\sqrt{MD \times QD}$

(a) 2/3
(b) 4/5
(c) $\sqrt{\frac{15}{8}}$
(d) $\sqrt{\frac{8}{}}$ (1 mark)

Answer:

(c) We know that,

$$4\ S.D = 5\ M.D = 6\ Q.D$$

$$4\ S.D = 5\ M.D$$

$$\frac{S.D}{M.D} = \frac{5}{4} \quad ...(1)$$

$$4\ S.D = 6\ Q.D$$

$$S.D = \frac{6}{4} Q.D$$

$$\frac{S.D}{Q.D} = \frac{6}{4} \quad ...(2)$$

Multiply by (1) and (2)

$$\frac{S.D}{M.D} \times \frac{S.D}{Q.D} = \frac{5}{4} \times \frac{6}{4}$$

$$(S.D)^2 = \frac{15}{8}\ M.D \times Q.D$$

$$(S.D) = \sqrt{\frac{15}{8} M.D \times Q.D}$$

$$S.D = \sqrt{\frac{15}{8}} \sqrt{M.D \times Q.D}$$

[144] The Q.D of 6 numbers 15, 8, 36, 40, 38, 41 is equal to

(a) 12.5

(b) 25

(c) 13.5

(d) 37 (1 mark)

Answer:

(c) Write the terms in Ascending order,

8, 15, 36, 38, 40, 41

Here, N = 4

$$Q_1 = \left(\frac{N+1}{4}\right)^{th} \text{term}$$

$$= \left(\frac{6+1}{4}\right)^{th} \text{term}$$

$= 1^{st}$ term + 0.75 (2^{nd} term - 1^{st} term)
$= 8 + 0.75 \times (15 - 8)$
$= 8 + 0.75 \times 7$
$= 8 + 5.25$
$= 13.25$

$$Q_3 = \frac{3(N+1)^{th}}{4} \text{ term}$$

$$= \frac{3(6+1)^{th}}{4} \text{ term}$$

$= 5.25^{th}$ term
$= 5^{th}$ term + 0.25 (6^{th} term - 5^{th} term)
$= 40 + 0.25 (41 - 40)$
$= 40 + 0.25 \times 1$
$= 40 + 0.25$
$= 40.25$

$$QD = \frac{Q3 - Q1}{2}$$

$$= \frac{40.25 - 13.25}{2}$$

$$= \frac{27}{2} = 13.5$$

[145] Which of the following is positional average ?
(a) Median
(b) GM
(c) HM
(d) AM (1 mark)

Answer:
(a) Median is the known as positional average.

[146] S.D of first five consecutive natural numbers is
(a) $\sqrt{10}$
(b) $\sqrt{8}$
(c) $\sqrt{3}$

Answer:

(d) S.D of Ist 'n' Natural No. $= \sqrt{\frac{n^2-1}{12}}$

$n = 5$

$S.D = \sqrt{\frac{5^2-1}{12}} = \sqrt{\frac{24}{12}}$

$= \sqrt{2}$

[147] If the profits of a company remain some for the last ten months then the S.D. of profits of the company would be:

(a) Positive
(b) Negative
(c) Zero
(d) (a) or (c) (1 mark)

Answer:

(c) If the profits of a company remain same for ten months.

then $\boxed{S.D = 0}$

(Since shifting of origin S.D is not changed)

[148] For a symmetric distribution

(a) Mean = Median = Mode
(b) Mode = 3 Median – 2 Mean
(c) Mode $= \frac{1}{3}$ Median $= \frac{1}{2}$ Mean
(d) None (1 mark)

Answer:

(a) For a symmetric distribution

Mean = Median = Mode

[149] For the distribution

X	1	2	3	4	5	6
F	6	9	10	14	12	8

The value of median is

(a) 3.5

(b) 3

(c) 4

(d) 5 (1 mark)

Answer:

(c)

x	1	2	3	4	5	6	
f	6	9	10	14	12	8	N = 59
C.F	6	15	25	39	51	59	

Median (Me) = $\left(\frac{N+1}{2}\right)^{th}$ term

= $\left(\frac{59+1}{2}\right)^{th}$ term

= 30^{th} term

= 4

[150] Coefficient of quartile deviation is 1/4 then Q_3/Q_1 is

(a) 5/3

(b) 4/3

(c) 3/4

(d) 3/5 (1 mark)

Answer:

(a) Coeff. of Q.D = $\frac{1}{4}$

$$\frac{Q_3-Q_1}{Q_3+Q_1} \times \frac{1}{4}$$

$4Q_3 - 4Q_1 = Q_3 + Q_1$

$4Q_3 - Q_3 = Q_1 + 4Q_1$

$3Q_3 = 5Q_1$

$\frac{Q_3}{Q_1} = \frac{5}{3}$

[151] The sum of mean and SD of a series is a + b, if we add 2 to each observation of the series then the sum of mean and SD is

(a) a + b + 2
(b) 6 – a + b
(c) 4 + a – b
(d) a + b + 4 (1 mark)

Answer:

(a) By shifting the origin, Mean is change but S.D is not changed.
The sum of Mean and S.D of a series.
= (a+b)
If we add '2' in each term then the new sum of mean and S.D
= (a+b+2)

2019 - November

[152] The approximate ratio of SD, MD, QD is:

(a) 3 : 4 : 5
(b) 2 : 3 : 4
(c) 15 : 12 : 10
(d) 5 : 6 : 7 (1 mark)

Answer:

(c) We know that,
$4SD = 5MD = 6QD$
net $4SD = 5MD = 6QD = K$
So,
$$SD = \frac{K}{4},\ MD = \frac{K}{5};\ QD = \frac{K}{6}$$
Now, SD : MD: QD
$$\frac{K}{4} : \frac{K}{5} : \frac{K}{6}$$
$$\frac{30K}{120} : \frac{24K}{120} : \frac{20K}{120}$$ [∵ LCM of 4,5,6 is 120]

30 : 24 : 20

15 : 12 : 10

So,

SD : MD : QD = 15 : 12 : 10

[153] The deviations are minimum when taken from:

(a) Mean

(b) Median

(c) Mode

(d) None (1 mark)

Answer:

(b) The sum of deviations are minimum when taken from median

x - Mean

x - Median {Minimum}

x- Mode

[154] If the AM & GM of two numbers are 30 and 24 respectively. Find the no.'s

(a) 12 and 24

(b) 48 and 12

(c) 30 and 30

(d) 40 and 20 (1 mark)

Answer:

(b) Let the two no.'s be a and b

AM = 30 GM = 24

$\frac{a+b}{2} = 30$ $\sqrt{ab} = 24$ (– 2)

a + b = 60

a = 60 - b (– 1)

Put Eq 1 in Eq 2

$\sqrt{(60-b)b} = 24$

On squaring both sides

(60 - b) b = 576

$60b - b^2 = 576$

$b^2 - 60b + 576 = 0$

$b^2 - 48b - 12b + 576 = 0$

$b(b - 48) - 12(b - 48) = 0$

$(b-12)(b - 48) = 0$

$b = 12$	or	$b = 48$
$a = 60 - b$		$a = 60 - 48$
$a = 48$		$a = 12$
(12, 48)	or	(48, 12)

So the two no.'s are 48 and 12

After Method [Do by hit and trial]

i.e. Try with the given options whether their AM is 30 and GM 24

[155] Origin is shifted by 5, what will happen

(a) SD will increase by 5

(b) QD will increase by 5

(c) MD will increase by 5

(d) There will be no change (1 mark)

Answer:

(d) SD is not affected or remains inchanged by shifting of origin. So here if the origin is shifted by 5 there will be no change in SD.

[156] Coefficient of variation is equal to:

(a) $\frac{SD}{Mean}$

(b) $\frac{SD}{Mean} \times 100$

(c) $\frac{Mean}{SD} \times 100$

(d) $\frac{Mean}{SD}$ (1 mark)

Answer:

(b) In probability theory and statistics the coefficient of variation also known as relative standard deviation is a standardized measure of dispersion of frequency distribution.

It is expressed as a percentage and defined as the ratio of SD and mean.

So, coefficient of variation = $\frac{SD}{Mean} \times 100$

[157] Find mode of the following date

3 – 6	6 – 9	9 – 12	12 – 15	15 – 18	18 – 21
2	5	10	23	21	12

(a) 25
(b) 4.6
(c) 14.6
(d) 13.5 (1 mark)

Answer:

(c)

CI	f	
3-6	2	
6-9	5	
9-12	10	
12-15	23	*Modal Class
15-18	21	
18-21	12	

Since 23 is the highest frequency, so 12-15 is the modal class.

So, $f_1 = 23$, $f_0 = 10$, $f_2 = 21$

$L_1 = 12$ $\quad$ $i = 3$

$$\text{Mode} = L_1 + \frac{f_1 - f_0}{^2f_1 - f_0 - f_2} \times i$$

$$= 12 + \frac{23 - 10}{2\,(23) - 10 - 21} \times 3$$

$$= 12 + \frac{13}{15} \times 3$$

$$= 12 + 2.599$$

$$= 14.59$$

$$= 14.6 \text{ (approx)}$$

[158] Find SD of the following
1, 2, 3, 4, 5, 6, 7, 8, 9.
(a) 2.58
(b) 60/9
(c) 60/3
(d) 3.20 (1 mark)

Answer:

(a) $SD = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$

Here N = 9

$x^2 = 1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots\ldots\ldots 9^2$

$= 285$

$\frac{\sum x}{N} = \frac{1+2+3+4+5+6+7+8+9}{9} = 5$

Put in above formula,

$SD = \sqrt{\frac{285}{9} - \frac{25}{1}}$

$SD = \sqrt{\frac{60}{9}}$

$SD = \sqrt{6.67}$

$SD = 2.58$

[159] If mean = 200 and variance = 80. Find coefficient of variation.
(a) 2.56
(b) 4.47
(c) 32
(d) 0.32 (1 mark)

Answer:

(b) We know

$CV = \frac{SD}{Mean} \times 100$

$CV = \frac{\sqrt{Variance}}{Mean} \times 100$ $SD = \sqrt{Variance}$

$$CV = \frac{\sqrt{80}}{200} \times 100$$

$$CV = \frac{\sqrt{80}}{2}$$

$$CV = 4.47 \text{ (approx)}$$

[160] Which of the following is affected by shifting of scale.

(a) SD
(b) MD
(c) QD
(d) None of these (1 mark)

Answer:

(d) Since SD, MD, QD are measures of absolute dispersion. So, a change in scale will neither affect SD nor MD and QD.

[161] Histogram is used to represent

(a) Mode
(b) Median
(c) Percentile
(d) Quartile (1 mark)

Answer:

(a) Histogram is a graphical representation of grouped frequency distribution.
It is used to locate mode.
x-axis class interval
y-axis Frequency.

[162] Coefficient of variation is 80. Mean is 20. Find variance:

(a) 640
(b) 256
(c) 16
(d) 250 (1 mark)

Answer:

(b) We know,

Coefficient of variation (CV) = $\frac{\text{S.D.}}{\text{Mean}} \times 100$

Here mean = 20 ; CV = 80

$80 = \frac{\text{S.D.}}{20} \times 100$

S.D. = 16

Variance = $(\text{S.D.})^2$

Variance = $(16)^2 = 256$

[163] Find the median of the following.

CI	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	2	3	4	5	6

(a) 35
(b) 32
(c) 36
(d) 37.5 (1 mark)

Answer:

(b)

CI	f	c.f
0-10	2	2
10-20	3	5
20-30	4	9
*30-40	5	14
40-50	6	20

$\Sigma f = 20$

N = 20 $\frac{N}{2} = 10$

So 30 - 40 is the median class

$L_1 = 30$ C ⇒ Pre. Cof. of median class

C ⇒ 9 F = 5

$$\text{Median} = 4 + \frac{(N/2 - C)}{f} \times i$$

$$= 30 + \frac{(10 - 9)}{5} \times 10$$

$$= 30 + 2$$

$$= 32$$

[164] Difference between upper limit and lower limit of a class is known as.

(a) Range
(b) Class mark
(c) Class size
(d) Class boundary (1 mark)

Answer:

(c) Difference between upper limit and lower limit of class is class size.

Range = largest value – Smallest value

Class mark = (Lower Limit + Upper Limit)/2

Class boundary = Class interval of exclusive date series.

[165] Find the made of the following:

0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
7	14	22	34	20	19

(a) 32
(b) 34.61
(c) 25.42
(d) 35 (1 mark)

Answer:

(b)

CI	f
0-10	7
10-20	14
20-30	22
*30-40	34
40-50	20

Since 34 is the highest frequency so, 30 - 40 is the modal class

$f_1 = 34\ f_0 = 22\ f_2 = 20$

$i = 10$

$$\text{Mode} = L_1 + \frac{f_1 - f_0 \times i}{2f_1 - f_0 - f_2}$$

$$= 30 + \frac{(34 - 22)}{2 \times 34 - 22 - 20} \times 10$$

$$= 30 + \frac{12}{26} \times 10$$

$$= 34.61$$

[166] Find the median of the following:

CI	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	5	15	28	10	2

(a) 10.57
(b) 23.57
(c) 25
(d) None (1 mark)

Answer:

(b)

CI	f	c.f	
0-10	5	5	
10-20	15	20	
*20-30	28	48	*
30-40	10	58	
40-50	2	60	
	$\sum f = 60$		

$$\frac{N}{2} = \frac{60}{2} = 30$$

So 20 -30 is the median class

$L_1 = 20\ L, = 30$

$c \Rightarrow 20\ f \Rightarrow 28$

$$\text{Median} = L_1 + \frac{(N/2 - C)}{f} \times i$$

$$= 20 + \frac{(30 - 20)}{28} \times 10$$

$$= 23.57$$

[167] $\sum_{i=1}^{n} (\bar{x} - x_i)$ is equal to

(a) $\bar{x} \sum_{i=1}^{n} x_i$

(b) $n (\bar{x} \sum_{i=1}^{n} x_i)$

(c) $\bar{x} - n\,\bar{x}$

(d) zero (1 mark)

Answer:

(d) $\sum_{i=1}^{n} (\bar{x} - x_i) = 0$

Since the sum of deviations about their AM is always zero.

[168] SD from numbers 1, 4, 5, 7, 8 is 2.45. If 10 is added to each then SD will be:

(a) 12.45

(b) 24.5

(c) 12

(d) will not change. (1 mark)

Answer:

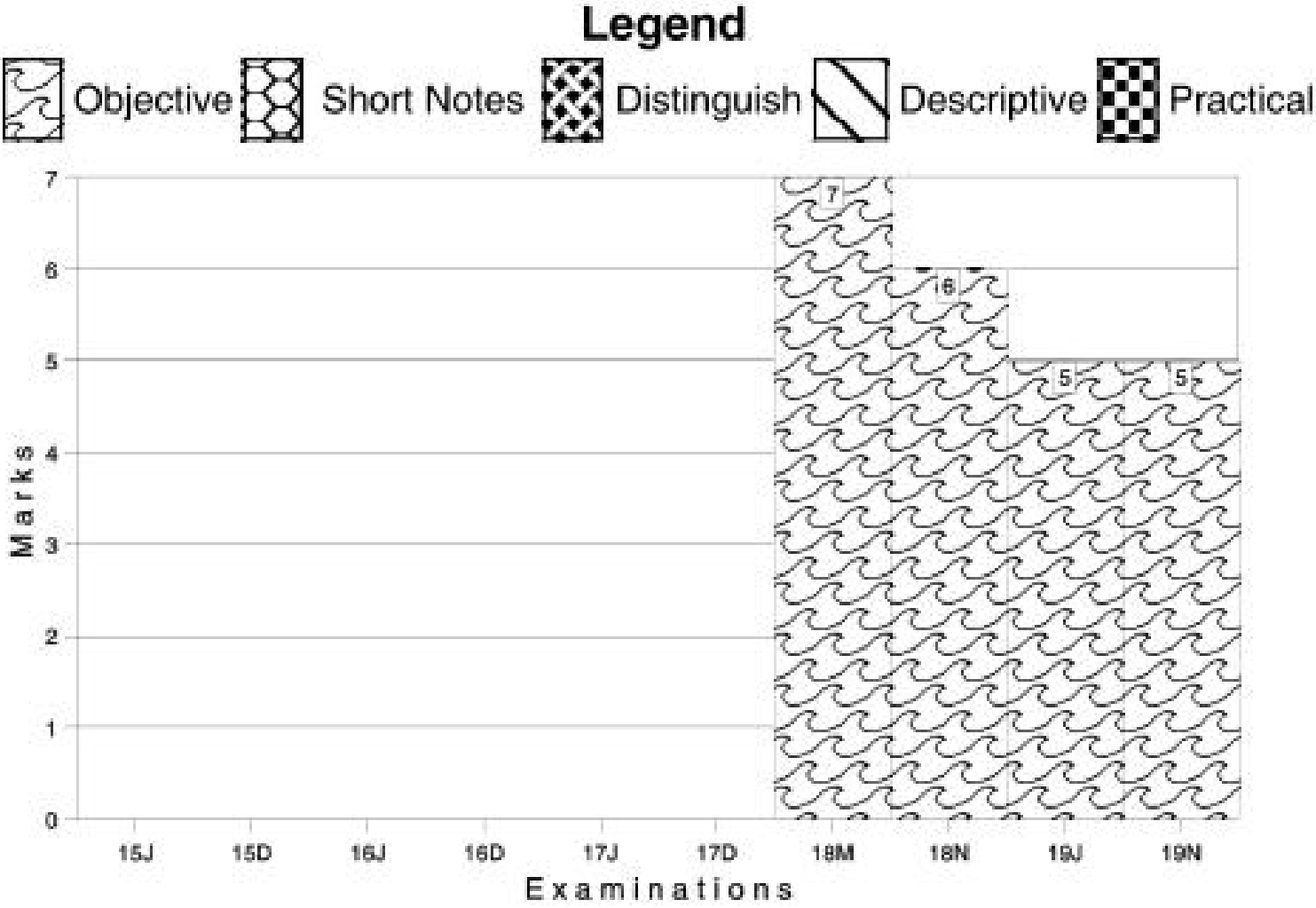
(d) We know a change in origin of SD causes no change in SD
So, New SD = Original SD when 10 will be added
So, SD will not change

CHAPTER

16

PROBABILITY

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions



PAST YEAR QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] There are six slips in a box and numbers 1, 1, 2, 2, 3, 3 are written on these slips. Two slips are taken at random from the box. The expected values of the sum of numbers on the to slips is :

(a) 5 (b) 3

(c) 4 (d) 7 (1 mark)

Answer:

(c) Number on the slips: (1, 1), (1, 2), (1, 3), (2, 1), (3, 1), (2, 2), (2, 3), (3, 2), (3, 3)

Sum of Numbers (x_1) : 2, 3, 4, 5, 6.

Probability distribution of sum of Numbers

x_1 : 2, 3, 4, 5, 6

$P(x_1)$: $\frac{1}{9}$ $\frac{2}{9}$ $\frac{3}{9}$ $\frac{2}{9}$ $\frac{1}{9}$

Thus, expected value of sum of numbers

$$E(X) = 2\times\frac{1}{9} + 3\times\frac{2}{9} + 4\times\frac{3}{9} + 5\times\frac{2}{9} + 6\times\frac{1}{9} = \frac{36}{9} = 4$$

[2] A letter is taken out at random from the word RANGE and another is taken out from the word PAGE. The probability that they are the same letters is :

(a) 1/20 (b) 3/20

(c) 3/5 (d) 3/4

Answer:

(b) Either the letter can be a, g or e.

Therefore , the probability that they are same

$$= \frac{1}{5}\times\frac{1}{4} + \frac{1}{5}\times\frac{1}{4} + \frac{1}{5}\times\frac{1}{4} = \frac{3}{20}$$

[3] An urn contains 9 balls two of which are red, three blue and four black. Three balls are drawn at random. The probability that they are of same colour is :

(a) $\frac{3}{27}$ (b) $\frac{20}{31}$

(c) $\frac{5}{84}$ (d) None (1 mark)

Answer:

(c) 3 balls out of 9 can be drawn in ${}^{9}C_{3} = 84$ ways

They can be of the same colour in

${}^{3}C_{3} + {}^{4}C_{3} = 1 + 4 = 5$ way

(Since, either all the balls can be blue or black but not red)

$\therefore$ Required Probability $= \frac{5}{84}$

[4] A card is drawn from a well shuffled pack of 52 cards. Let E_1 "a king or a queen is drawn" & E_2 : "a queen or a jack is drawn", then:

(a) E_1 and E_2 are not independent

(b) E_1 and E_2 are mutually exclusive

(c) E_1 and E_2 are independent

(d) None of these (1 mark)

Answer:

(a) E_1 = a king or a queen is drawn

E_2 = a queen or a jack is drawn

$P(E_1) = \frac{4+4}{52} = \frac{8}{52}$

$P(E_2) = \frac{4+4}{52} = \frac{8}{52}$

$P(E_1 \cap E_2)$ = P (drawing a queen) $= \frac{4}{52}$

Here $P(E_1 \cap E_2 \neq P(E_1).\ P(E_2)$

Hence E_1 and E_2 are not independent

2007 - FEBRUARY

[5] In a non - leap year, the probability of getting 53 sundays or 53 Tuesdays or 53 Thursdays is :

(a) $\frac{4}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{1}{7}$ (1 mark)

Answer:

(c) A non leap year certain 365 days , i.e. 52 complete weeks and an additional day which may be Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, or Saturday.

Out of these 7 possible days, the favorable are Sunday, Tuesday, Thursday.

$\therefore$ Required Probability $= \frac{3}{7}$

[6] If A and B are two events and $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{4}$, then the value of $P(A' \cup B')$ is :

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{5}{8}$ (d) $\frac{5}{4}$ (1 mark)

Answer:

(b) $P(A \cap B) = \frac{1}{4}$

According to De- Morgans Law

$(A \cap B)' = (A' \cup B')$

$\therefore P(A' \cup B') = P[(A \cap B)']$

$= 1 - P(A \cap B)$

$= 1 - \frac{1}{4}$

$\frac{3}{}$

[7] The probability that there is at least one error in an account statement prepared by A is 0.3 and for B and C, they are 0.4 and 0.45 respectively. A, B and C prepared 20, 10 and 40 statements respectively. The expected number of correct statements in all is:

(a) 32 (b) 45

(c) 42 (d) 25 (1 mark)

Answer:

(c) Let P_1, P_2, P_3 respectively denote the probability of the events that there is no error in the accounts prepared by A, B and C : Then

$P_1 = 1 -$ (probability of at least one error occurring in the account statement prepared by (A) $= 1 - 0.3 = 0.7$

Similarly, $P_2 = 1 - 0.4 = 0.6$ and,

$P_3 = 1 - 0.45 = .55$

Now $E(x) = \Sigma$ pi x_i

$E(x) = (0.7) \times 20 + (0.6) \times 10 + (0.55) \times 40$

$= 14 + 6 + 22$

$= 42$

2007 - MAY

[8] From a pack of cards, two are drawn, the first being replaced before the second is drawn. The chance that the first is a diamond and the second is king is:

(a) $\frac{1}{52}$ (b) $\frac{3}{2704}$

(c) $\frac{4}{13}$ (d) $\frac{3}{52}$ (1 mark)

Answer:

(a) A pack of cards has 52 cards

Required probability

= P (card drawn is a diamond) × P (card drawn is a king)

$= \frac{13}{52} \times \frac{4}{52}$ (since there are 13 diamonds and 4 Kings).

$= \frac{1}{}$

[9] The theory of compound probability states that for any two events A and B :

(a) $P(A \cap B) = P(A) \times P(B)$

(b) $P(A \cap B) = P(A) \times P(B/A)$

(c) $P(A \cup B) = P(A) \times P(B/A)$

(d) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (1 mark)

Answer:

(b) The probability of the simultaneous occurrence of the two events A and B is equal is the probability of the events multiplied by the conditional probability of other, given the occurrence of the first, i.e.

$P(A \cap B) = P(A) \times P(B/A)$

[10] The probability of getting qualified in IIT- JEE and AIEEE by a student are respectively $\frac{1}{5}$ and $\frac{3}{5}$. The probability that the student gets qualified for one of the these tests is:

(a) $\frac{17}{25}$ (b) $\frac{22}{25}$

(c) $\frac{8}{25}$ (d) $\frac{3}{25}$ (1 mark)

Answer:

(a) The probability that the student is selected for IIT JEE

$= P(E_1) = \frac{1}{5} \Rightarrow P(\bar{E}_1)\frac{4}{5}$

The probability that the student is selected for AIEEE

$= P(E_2) = \frac{3}{5} \Rightarrow P(E_2)\frac{2}{5}$

The Probability that he is selected for one of the examinations

$= 1 - p(\bar{E}_1)p(\bar{E}_2) = 1 - \frac{4}{5} \times \frac{2}{5} = \frac{17}{25}$

[11] Amitabh plays a game of tossing a dice. If the number less than 3 appears, he is getting ₹ a, otherwise he has to pay ₹ 10. If the game is fair, find a :

(a) 25 (b) 20

(c) 22 (d) 18 (1 mark)

Answer:

(b) Let x = gain of Amitabh

Sample space of tossing a dice

S = {1, 2, 3, 4, 5, 6}

This implies that

P_1 = (less than 3) = $\frac{2}{6}$

P_2 = (more than 3) = $\frac{4}{6}$

Also X_1 = ₹ a or X_2 = ₹ 10

E(x) = xp(x)

$\therefore E(x) = \frac{2a}{6} + \frac{4}{6} \times (-10)$

As the game is fair, E(x) = 0

i.e. $\frac{2}{6}a - \frac{40}{6} = 0$

or $\frac{2}{6}a = \frac{40}{6}$

a = 20

2007 - AUGUST

[12] Suppose E and F are two events of a random experiment. If the probability of occurrence of E is 1/5 and the probability of occurrence of F given E is 1/10, then the probability of non- occurrence of at least one of the events E and F is:

(a) $\frac{1}{50}$ (b) $\frac{1}{25}$

(c) $\frac{13}{50}$ (d) $\frac{49}{50}$ (1 mark)

Answer:

(d) Given $P(E) = \frac{1}{5}$, $P(F/E) = \frac{1}{10}$

$\therefore \quad P(E \cap F) = P(E).P(F/E)$

$= \frac{1}{5} \cdot \frac{1}{10} = \frac{1}{50}$

Probability of non-occurrence of at least one of the events of E and F

$= 1 - P(E \cap P)$

$= 1 - \frac{1}{50} = \frac{49}{50}$

[13] A bag contains 8 red and 5 white balls. Two successive draws of 3 balls are made without replacement. The probability that the first draw will produce 3 white balls and second 3 red balls is :

(a) $\frac{6}{255}$ (b) $\frac{5}{548}$

(c) $\frac{7}{429}$ (d) $\frac{3}{233}$ (1 mark)

Answer:

(c) Let A be the event that first 3 balls are drawn white Probability of drawing 3 white balls is the first draw.

$P(A) = \frac{5C_3}{13C_3}$

Let B be the event that second 3 balls, drawn are red.

If the 3 white balls drawn in the first draw are not replaced then the total number of balls remaining for second draw are 10.

$\therefore$ P(B/A) = Probability that 3 red balls are drawn in second draw

$$P(B/A) \frac{8C_3}{10C_3}$$

Hence, required probability is

$$P(A \cap B) = P(A) \times P(B/A)$$

$$= \frac{5}{143} \times \frac{7}{15}$$

$$= \frac{7}{429}$$

[14] A box contains 12 electric lamps of which 5 are defectives. A man selects three lamps at random. What is the expected number of defective lamps in his selection ?

(a) 1.25 (b) 2.50

(c) 1.05 (d) 2.03 (1 mark)

Answer:

(a) Let x denote the number of defective lamps and it can assume the values 0, 1, 2 and 3

P(x = 0) = Prob. of having O defective out of 5 defective and 3 non defective out of 7 non defective.

$$= \frac{5_0 \times 7_{c_3}}{12c_3} = \frac{35}{220}$$

Similarly

$$P(x = 1) = \frac{5_{c_1} \times 7c_2}{12_{c_3}} = \frac{105}{220}$$

$$P(C = 2) = \frac{5_{c_2} \times 7_{c_1}}{12_{c_3}} = \frac{70}{220}$$

$$\text{and } P(x = 3) = \frac{5_{c_3} \times 7_{c_0}}{12_{c_3}} = \frac{10}{220}$$

Probability distribution of No. of defective lamps:

X : 0 1 2 3

P : $\frac{35}{220}$ $\frac{105}{220}$ $\frac{70}{220}$ $\frac{10}{220}$

Thus the expected number of defective is given by

$$\frac{35}{220} \times 0 + \frac{105}{220} \times 1 + \frac{70}{220} \times 2 + \frac{10}{220} \times 3 = 1.25$$

2007 - NOVEMBER

[15] Three identical dice are rolled. The probability that the same number will appear on each of them is :

(a) 1/6 (b) 1/12

(c) 1/36 (d) 1 (1 mark)

Answer:

(c) The equiprobable sample space of the experiment consists of 6 × 6 × 6 sample points

Event A = same number appears on each of the three die.

= {(1, 1, 1)(2, 2, 2)(3, 3, 3)(4, 4, 4)(5, 5, 5)(6, 6, 6)}

i.e. n(A) = 6

Hence, the required probability

$$= \frac{6}{6 \times 6 \times 6} = \frac{1}{36}$$

[16] Among the examinees in an examination 30%, 35% and 45% failed in Statistics, in Mathematics and in at least one of the subjects respectively. An examinee is selected at random. Find the probability that he failed in Mathematics only :

(a) 0.15 (b) 0.25

(c) 0.254 (d) 0.55 (1 mark)

Answer:

(a) Event S : Students failed in statistics, and

Event M : Students failed in mathematics

$P(S) = 0.30, P(M) = 0.35$

and $P(S \cup M) = 0.45$

$\therefore P(S \cup M) = P(S) + P(M) - P(S \cap M)$

$\Rightarrow 0.45 = 0.30 + 0.35 - P(S \cap M)$

or $P(S \cap M) = 0.20$

Hence P (failed in mathematics only) = P $(M \cap S)$

$= P(M) - P(S \cap M)$
$= 0.35 - 0.20$
$= 0.15$

[17] An article consists of two parts A and B. The manufacturing process of each part is such that probability of defect in A is 0.08 and that B is 0.05. What is the probability that the assembled product will not have any defect?

(a) 0.934 (b) 0.864
(c) 0.85 (d) 0.874 (1 mark)

Answer:

(d) Event X : Part A is free from defect and Event Y: Part B is free from defect

$\therefore P(X) = 1 - 0.08 = 0.92$

$P(Y) = 1 - 0.05 = 0.95$

The two events X and Y are independent as part A having no defects or otherwise does not influence on part B's being defective or otherwise.

$\therefore P(X \cap Y) = P(X), P(Y) = 0.92 \times 0.95 = 0.874$

[18] Daily demand for calculators is having the following probability distribution :

Demand :	1	2	3	4	5	6
Probability	0.10	0.15	0.20	0.25	0.18	0.12

Determine the variance of the demand.

(a) 2.54 (b) 2.93

(c) 2.22 (d) 2.19 (1 mark)

Answer:

Demand :	1	2	3	4	5	6
(x)	0.10	0.15	0.20	0.25	0.18	0.12

$E(X) = P(x).X$

$= 1 \times 0.10 + 2 \times 0.15 + 3 \times 0.20 + 4 \times 0.25 + 5 \times 0.18 + 6 \times 0.12$

$= 3.62$

$E(X^2) = 1 \times 0.10 + 4 \times 0.15 + 9 \times 0.20 + 16 \times 0.25 + 25 \times 0.18 + 36 \times 0.12$

$= 15.32$

Variance $(X) = E(X^2) - [E(X)]^2 = 15.32 - (3.62)^2 = 2.22$

2008 - FEBRUARY

[19] If 10 men, among whom are A and B, stand in a row, what is the probability that there will be exactly 3 men between A and B?

(a) 11/15 (b) 4/15

(c) 1/15 (d) 2/15 (1 mark)

Answer:

(d) Total probability to arrange 10 men in a row without any restrictions = 10!

Let us fix A and B so as to incorporate in exactly 3 men in between them as shown

A × × × B

These 3 men can be selected and arranged in $8P_3$ ways while A and B can also interchange themselves in 2! ways

More over, the rest 5 people among the given 10 including the above 5 considering them as a single person make a row of 6 men which can be arranged in 6! ways

$\therefore$ No. of ways to arrange 10 men is a row following the given restriction

$= {}^{8}P_{3} \times 2! \times 6!$

Hence , the required probability

$$= \frac{{}^{8}P_{3} \times 2! \times 6!}{10!} = \frac{2}{15}$$

[20] The probability of an event can assume any value between :

(a) 0 and 1 (b) -1 and 0

(c) -1 and 1 (d) None of these (1 mark)

Answer:

(d) Since the Probability of an event lies between 0 and 1, both inclusive i.e. $o \leq P(A) \leq 1$ therefore the answer will be none of these.

[21] The odds are 9 : 5 against a person who is 50 years living till he is 70 and 8 : 6 against a person who is 60 living till he is 80. Find the probability that at least one of them will be alive after 20 years:

(a) $\frac{11}{14}$ (b) $\frac{22}{49}$

(c) $\frac{31}{49}$ (d) $\frac{35}{49}$ (1 mark)

Answer:

(c) Event A : Person aged 50 years will remain alive after 20 years

Event B : Person aged 60 years will remain alive after 20 years

$\therefore P(A) = \frac{5}{9+5} \quad \frac{5}{14}$ and $P(B) = \frac{6}{8+6} \quad \frac{6}{14}$

$$\therefore P(A \cup B) = \frac{5}{14} + \frac{6}{14} - \frac{5}{14} \times \frac{6}{14} = \frac{31}{49}$$

[22] An urn contains 6 white and 4 black balls. 3 balls are drawn without replacement. What is the expected number of black balls that will be obtained ?

(a) 6/5 (b) 1/5

(c) 7/5 (d) 4/5 (1 mark)

Answer:

(a) The random variable X can assume 0, 1, 2 and 3 with respective probabilities P_1, P_2, P_3 and P_4 of black balls

$$P(x = 0) = \frac{4_{C_0} \times 6_{C_3}}{10_{C_3}} = \frac{1}{6}$$

$$P(x = 1) = \frac{4_{C_1} \times 6_{C_2}}{10_{C_3}} = \frac{1}{2}$$

$$P(x = 3) = \frac{4_{C_2} \times 6_{C_1}}{10_{C_3}} = \frac{3}{10}$$

$$P(x = 4) = \frac{4_{C_3} \times 6_{C_0}}{10_{C_3}} = \frac{1}{30}$$

Thus,

$X^2 (= x_i)$:	0	1	2	3
P(i) :	1/6	1/2	3/10	1/30

$$E(x)\ 0 \times \frac{1}{6} + 1 \times \frac{1}{2} + 2 \times \frac{3}{10} + 3 \times \frac{1}{30} = \frac{6}{5}$$

2008 - JUNE

[23] If P(A) = p and P (B) = q, then :

(a) P (A / B) ≤ q / p (b) P (A / B) ≥ p / q

(c) P (A / B) ≤ p / q (d) P (A / B) ≥ q / p (1 mark)

Answer:

(c) Given P(A) = P and P(B) = q

We know that,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

and also

$P(A \cap B) < P(A)$

$\therefore P(A/B) \leq p/q$

[24] The probability that a trainee will remain with a company is 0.8. The probability that an employee earns more than ₹ 20,000 per month is 0.4. The probability that an employee, who was a trainee and remained with the company or who earns more than ₹ 20,000 per month is 0.9. What is the probability that an employee earns more than ₹ 20,000 per month given that he is a trainee, who stayed with the company?

(a) 5/8 (b) 3/8
(c) 1/8 (d) 7/8 (1 mark)

Answer:

(b) Event A : A trainee will remain with the company Event B: An Employee earns more than ₹ 20,000.

Then $P(A) = 0.8$, $P(B) = 0.4$ and $P(A \cup B) = 0.9$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.9 = 0.8 + 0.4 - P(A \cap B)$

$\Rightarrow P(A \cap B) = 0.3$

Hence $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.3}{0.8} = \frac{3}{8}$

[25] A random variable X has the following probability distribution :

X	:	−2	3	1
P (X= x):		1/3	1/2	1/6

Find $E(X^2)$ and $E(2X + 5)$.

(a) 6 and 7 respectively (b) 5 and 7 respectively
(c) 7 and 5 respectively (d) 7 and 6 respectively (1 mark)

Answer:

(a) $E(2x + 5) = Pi\,(2x + 5) = 2\ Pi\ xi + 5$

$$= 2\left[\frac{1}{3} \times -2 + \frac{1}{2} \times 3 + \frac{1}{6} \times 1\right] + 5 = 7$$

$$E(x^2) = \frac{1}{3} \times 4 + \frac{1}{2} \times 9 + \frac{1}{6} \times 1 = 6$$

2008 - DECEMBER

[26] The limiting relative frequency of probability is :

(a) Axiomatic (b) Classical

(c) Statistical (d) Mathematical (1 mark)

Answer:

(c) Owing to the limitations of the classical definition of probability, there are cases when we consider the statistical definition of probability based on the concept of relative frequency.

Assume that an event A occurs fA times. Then, the limiting value of the ratio of f_A to n as n tends to infinity is defined as the probability of A.

i.e. $P(A) = \lim_{n \to \infty} \frac{f_A}{n}$

Therefore, the limiting relative frequency of probability is statistical.

[27] If a probability density function is $f(x) = \begin{cases} 1 \text{ if } o < x < 1 \\ 0 \text{ otherwise} \end{cases}$ then find E (x)

(a) ∞ (b) 0

(c) 1 (d) $-\infty$ (1 mark)

Answer:

(c) Here, x is a continuous random variable defined over an interval [0, 1] where 1>0,

$\therefore$ its expected value is given by

$$\int_{-}^{\beta} x f(x)\, dx$$

$$= \int_{0}^{1} 1\, dx$$

$$= [x]_0^1 = 1 - 0 = 1$$

[28] If :

x :	− 2	3	1
P (x) :	1/3	1/2	1/6

then find E (2x + 5)

(a) 7 (b) 6

(c) 9 (d) 4 (1 mark)

Answer:

(a)

x :	− 2	3	1
P (x) :	1/3	1/2	1/6

The expected value of x is given by

$E(x) = \Sigma p_i x_i$

$= \frac{1}{3} \times -2 + \frac{1}{2} \times 3 + \frac{1}{6} \times 1$

$= \frac{-2}{3} + \frac{3}{2} + \frac{1}{6}$

$= \frac{-4+9+1}{6} = 1$

$E(2x + 5) = 2\,E(x) + E(5)$ ($\because E(kx) = k.E(x)$ for any constant k)

$= 2 \times 1 + 5$ [$\because E(k) = k$ for any constant K]

$= 2 + 5$

$= 7$

2006 - JUNE

[29] If A and B are two independent events and P(AUB) = 2/5; P(B) = 1/3. Find P(A).

(a) 2/9 (b) - 1/3

(c) 2/10 (d) 1/10 (1 mark)

Answer:

(d) Since A and B are two independent events, therefore -

$P(A \cap B) = P(A) \times P(B)$

Let P (A) be x

then, $P(A \cap B) = P(A) \times P(B)$

$= x \times \frac{1}{3} = \frac{x}{3}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{2}{5} = x + \frac{1}{3} - \frac{x}{3}$

$\frac{2}{5} = \frac{3x + 1 - x}{3}$

$\frac{2}{5} = \frac{2x}{3} + \frac{1}{3}$

$\frac{2x}{3} = \frac{2}{5} - \frac{1}{3}$

$\frac{2x}{3} = \frac{6-5}{15}$

$x = \frac{1}{15} \times \frac{3}{2}$

$x = \frac{1}{10}$

$\therefore P(A) = \frac{1}{10}$

[30] A bag contains 12 balls of which 3 are red 5 balls are drawn at random. Find the probability that in 5 balls 3 are red.

(a) 3/132 (b) 5/396

(c) 1/36 (d) 1/22 (1 mark)

Answer:

(d) Total number of cases of drawing 5 balls out of 12 balls = $^{12}C_5$

Cases when out of 5 balls drawn, 3 are red = $^{5}C_3$

If 3 are red, then the other 2 balls may be of any colour which may be drawn from remaining 9 balls. Therefore, the cases are ${}^{9}C_{2}$.

So, the probability that in 5 balls 3 are red

$$= \frac{{}^{3}C_{3} \times {}^{9}C_{2}}{{}^{12}C_{5}} = \frac{1 \times 36}{729} = \frac{1}{22}$$

[31] A random variable X has the following probability distribution.

X	0	1	2	3
P(x)	0	2K	3K	K

Then, P (x<3) would be :

(a) 1/6 (b) 1/3

(c) 2/3 (d) 5/6 (1 mark)

Answer:

(d) Since $\Sigma P(x) = 1$

therefore $0 + 2k + 3k + k = 1$

$6k = 1$

$k = \frac{1}{6}$

$P(x < 3) = P(x = o) + P(x = 1) + P(x = 2)$

$= 0 + 2k + 3k = 5k$

$= 5 \times \frac{1}{6} \left(\text{as } k = \frac{1}{6}\right) = \frac{5}{6}$

2009 - DECEMBER

[32] P(A) = 2/3; P (B) = 3/5; P(A∪B) = 5/6. Find P (B/A)

(a) 11/20 (b) 13/20

(c) 13/18 (d) None (1 mark)

Answer:

(b) P (A) = 2/3

P (B) = 3/5

P (A∪B) = 5/6

$$P(A) + P(B) - P(A \cap B) = P(A \cup B)$$

$$\frac{2}{3} + \frac{3}{5} - P(A \cap B) = \frac{5}{6}$$

$$\frac{10+9}{15} - P(A \cap B) = \frac{5}{6}$$

$$P(A \cap B) = \frac{19}{15} - \frac{5}{6}$$

$$P(A \cap B) = \frac{38-25}{30} = \frac{13}{30}$$

$$\text{Now, } P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{13}{30}}{\frac{2}{3}} = \frac{13}{30} \times \frac{3}{2} = \frac{13}{20}$$

$$\therefore \quad P(B/A) = \frac{13}{20}$$

[33] If $P(A \cap B) = P(A) \times P(B)$, then the events are:

(a) Independent events (b) Mutually exclusive events
(c) Exhaustive events (d) Mutually inclusive events

(1 mark)

Answer:

(a) If A and B are two independent events, then $P(A \cap B) = P(A) \times P(B)$. [self-explanatory].

[34] E (XY) is also known as:

(a) E (X) + E (Y) (b) E (X) E (Y)
(c) E (X) – E (Y) (d) E (X) ÷ E (Y) (1 mark)

Answer:

(b) Expectation of the product of two random variables is the product of the two random variables, provided the two variables are independent.
i.e. $E(xy) = E(x) \times E(y)$
whenever x and y are independent.

[35] In a bag, there were 5 white, 3 red, and 2 black balls. Three balls are drawn at a time what is the probability that the three balls drawn are white?

(a) 1/12 (b) 1/24
(c) 1/120 (d) None of these (1 mark)

Answer:

(a) Number of ways of drawing 3 balls at a time = $^{10}C_3 = 120$ ways.
Number of ways of drawing 3 white balls out of 5 white balls = $^5C_3 = 10$ ways.

$\therefore$ Total number of ways

$$= \frac{\text{Favourable cases}}{\text{Total number of cases}} = \frac{10}{120} = \frac{1}{12}$$

[36] In how many ways can the letters of 'REGULATION' be arranged so that the vowels come at odd places?

(a) 1/252 (b) 1/144
(c) 144/252 (d) None of these (1 mark)

Answer:

(a) Number of vowels = 5
Number of old places = 5
Number of ways of arranging vowels at odd places = $^5P_5 = 120$ ways
Remaining letters = 5
Number of ways of arranging these 5 letters in the remaining 5 places
$= {}^5P_5 = 120$ ways
Total ways = $^{10}P_{10} = 36,28,800$

$\therefore$ Number of ways of arranging the letters of the word 'REGULATION' so that the vowels come at odd places

$$= \frac{120 \times 120}{36,28,800} = \frac{1}{252}$$

2010 - JUNE

[37] In a pack of playing cards with two jokers probability of getting king of spade is

(a) 4/13 (b) 4/52

(c) 1/52 (d) 1/54 (1 mark)

Answer:

(d) Pack of playing cards contain 52 cards + 2 Jokers = Total cards are 54

Total no. of spade king = 1

$\therefore$ Probability of getting spade king $= \frac{1}{54}$

[38] Consider two events A and B not mutually exclusive, such that P(A) = 1/4, P(B) = 2/5, P(A∪B) =1/2, then P ($A\overline{B}$) is

(a) 3/7 (b) 2/10

(c) 1/10 (d) None of the above (1 mark)

Answer:

(d) Since the two events are not mutually exclusive, they are independent events. The events A and B are independent if

$P(AB) = P(A) \cdot P(B)$

$\therefore P(A\overline{B}) = P(A) \cdot P(\overline{B})$

$= P(A).\ [1 - P(B)]$

$= \frac{1}{4} \cdot \left(1 - \frac{2}{5}\right)$

$= \frac{1}{4} \cdot \frac{3}{5}$

$= \frac{3}{20}$

Moreover,

$$P(A\ 1B) = P(A) + P(B) - P(AUB)$$

$$= \frac{1}{4} + \frac{2}{5} - \frac{1}{2} = \frac{3}{20}$$

Note : In case of independent events, the multiplication theorem becomes

$P(A \cap B) = P(A) \times P(B)$ [$\because$ $P(A/B) = P(A)$ and $P(B/A) = P(B)$]

[39] If x be the sum of two numbers obtained when two die are thrown simultaneously then $P(x \geq 7)$ is

(a) 5/12 (b) 7/12

(c) 11/15 (d) 3/8 (1 mark)

Answer:

(b) While, throwing two dice

Total no. of outcomes = 36

Probability of sum = 7 is $\frac{6}{36}$

Probability of sum = 8 is $\frac{5}{36}$

Probability of sum = 9 is $\frac{4}{36}$

Probability of sum = 10 is $\frac{3}{36}$

Probability of sum = 11 is $\frac{2}{36}$

Probability of sum = 12 is $\frac{1}{36}$

$\therefore$ Required Probability $= \frac{21}{36} = \frac{7}{12}$

[40] E (13x + 9) = _____.

(a) 13x (b) 13E(x)

(c) 13E(x) + 9 (d) 9 (1 mark)

Answer:

(c) $E(x + y) = E(x) + E(y)$

$\therefore \quad E(13x + 9) = 13E(x) + E(9)$

$= 13E(x) + 9 \qquad [\because E(K) = K$ for any constant $K]$

2010 - DECEMBER

[41] A dice is thrown once. What is the mathematical expectation of the number on the dice ?

(a) 16/6 (b) 13/2

(c) 3.5 (d) 4.5 (1 mark)

Answer:

(c)

x	p(x)	x.p(x)
1	1/6	1/6
2	1/6	2/6
3	1/6	3/6
4	1/6	4/6
5	1/6	5/6
6	1/6	6/6
	1	21/6

Expected value $= \frac{\sum xp(x)}{\sum p(x)} = \frac{21}{6} = 3.5$

[42] If P(A/B) = P(A), then A and B are

(a) Mutually exclusive events (b) Dependent events

(c) Independent events (d) Composite events (1 mark)

Answer:

(c) $P(A/B) = P(A)$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\,P(B)}{P(B)} = P(A)$

Since, $P(A \cap B) = P(A)\,P(B)$

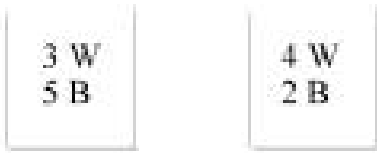
[43] A bag contains 3 white and 5 black balls and second bag contains 4 white and 2 black balls. If one ball is taken from each bag, the probability that both the balls are white is _____

(a) 1/3 (b) 1/4

(c) 1/2 (d) None of these (1 mark)

Answer:

(b)



Required Probability = White ball from Urn1 * White ball from Um 2

$$= \frac{{}^{3}C_1 \times {}^{4}C_1}{{}^{8}C_1 \times {}^{6}C_1} = \frac{3}{8} \times \frac{4}{6} = \frac{1}{4}$$

[44] The odds in favour of A solving a problem is 5:7and odds against B solving the same problem is 9:6. What is the probability that if both of them try, the problem will be solved?

(a) 117/180 (b) 181/200

(c) 147/180 (d) 119/180 (1 mark)

Answer:

(a) $P(A) = \frac{5}{12}$

$P(B) = \frac{6}{15}$

Probability that problem will be Solved

$= 1 - [\,1 - P(A)]\,[1 - P(B)]$

$= 1 - \left(1 - \frac{5}{12}\right)\left(1 - \frac{6}{15}\right)$

$= 1 - \frac{7}{12} \times \frac{9}{15}$

$= \frac{180 - 63}{180}$

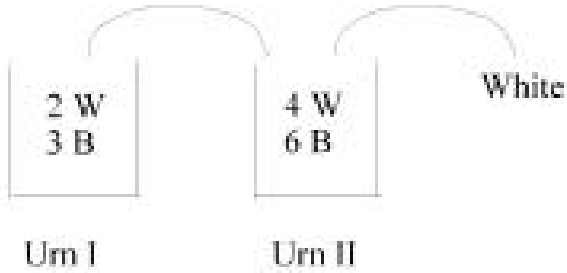
$= \frac{117}{}$

[45] Consider Urn I : 2 white balls, 3 black balls
Urn II : 4 white balls, 6 black balls
One ball is randomly transferred from first to second Urn, then one ball is drawn from II Urn. The probability that drawn ball is white is

(a) 22/65 (b) 22/46
(c) 22/55 (d) 21/45 (1 mark)

Answer:

(c)



Case I :

Black ball is transferred from Urn I to Urn II and then a white ball is picked from Urn II

$$\frac{3}{5} \times \frac{4}{11} = \frac{12}{55}$$

Case II :

White ball is transferred from Urn I to Urn II and then a white ball is picked from Urn II

$$\frac{2}{5} \times \frac{5}{11} = \frac{10}{55}$$

Case I + Case II

$$\frac{12}{55} + \frac{10}{55} = \frac{22}{55}$$

2011 - JUNE

[46] If $P(A \cup B) = P(A)$, Find $P(A \cap B)$.

(a) P(A).P(B) (b) P(A) + P(B)
(c) 0 (d) P(B) (1 mark)

Answer:

(d) Given : $P(A \cup B) = P(A)$ (1)

we know,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore$ we get $P(A \cap B) = P(B)$ using (1)

[47] In how many ways a team of 5 can be made out of 7 Boys and 8 Girls, if 2 Girls are compulsory to form a Team.

(a) 2,646 (b) 1,722

(c) 2,702 (d) 980 (1 mark)

Answer:

(c) By Question : 7 Boys (B); 8 Girls (G)

Term = 5 Members (2 Girls are Compulsory)

$\Rightarrow$ Term = (3B, 2G) + (2B, 3G) + (1B, 4G) + (5G)

$= {}^{7}C_3 \times {}^{8}C_2 + {}^{7}C_2 \times {}^{8}C_3 + {}^{7}C_1 \times {}^{8}C_4 + {}^{8}C_5$

$= 980 + 1,176 + 490 + 56$

$= 2,702$

[48] A bag contains 5 Red balls, 4 Blue Balls and 'm' Green Balls. If the random probability of picking two green balls is 1/7. What is the no. of green Balls (m).

(a) 5 (b) 7

(c) 6 (d) None of the above. (1 mark)

Answer:

(c) Given : BAG = [5Red (R) + 4 Blue (B) + 'm' Green (G)] Balls

$\therefore$ Probability of picking two green balls = $\frac{1}{7}$

$$\frac{M_{C2}}{9 + m_{C2}} = \frac{1}{7}$$

$$\frac{m(m-1)}{(m+9)(m+8)} = \frac{1}{7}$$

$\Rightarrow m^2 - 4m - 12 = 0$

$\Rightarrow (m - 6)(m + 2) = 0$

$\Rightarrow m = 6$

[49] The probability of Girl getting scholarship is 0.6 and the same probability for Boy is 0.8. Find the probability that at least one of the categories getting scholarship.

(a) 0.32 (b) 0.44

(c) 0.92 (d) None of the above. (1 mark)

Answer:

(c) Probability of Girl getting scholarship P(A) = 0.6

Probability of Boy getting scholarship P(B) = 0.8

Required to find :

Probability of at least one category getting scholarship :

$\Rightarrow$ $P(\bar{A} \cap B) + (A \cap \bar{B}) + P(A \cap B)$

$= P(\bar{A})\,P(B) + P(A).P(\bar{B}) + P(A).P(B)$

$= [1 - P(A)].P(B) + P(A).[1 - P(B)] + P(A).P(B)$

$= (1 - 0.6)(0.8) + 0.6\,(1 - 0.8) + 0.6 \times 0.8$

$= 0.32 + 0.12 + 0.48$

$= 0.92$

[50] If 15 persons are to be seated around 2 round tables, one occupying 8 persons and another 7 persons. Find the number of ways in which they can be seated.

(a) $\frac{15!}{18!}$ (b) $^{15}C_7 \frac{7!}{8!}$

(c) 7!.8! (d) $2.\ ^{15}C_7\ 6!\ 7!$ (1 mark)

Answer:

(d) No. of ways of selecting two Round Tables $= 2! = 2$

$\therefore$ No. of ways of selecting 7 persons = $^{15}C_7$

$\therefore$ No. of ways 7 persons to sit in a round table $= (7 - 1)! = 6!$

$\therefore$ Remaining Person $= 15 - 7 = 8$ person

$\therefore$ No. of ways 8 persons to sit on around table $= (8 - 1)! = 7!$

$\therefore$ Required total No. of ways $= 2!\ x^{15}c_7 \times 6!\ \times 7! = 2x^{15}C_7 \times 6! \times 7!$

[51] A coin is tossed 5 times, what is the probability that exactly 3 heads will occur.

(a) $\frac{5}{16}$ (b) $\frac{1}{32}$

(c) $\frac{5}{36}$ (d) $\frac{3}{32}$ (1 mark)

Answer:

(a) Total No. of Trails (n) = 5

r = 3

Probability of getting Head (p) = 1/2

Probability of getting Tail (q) = 1/2

By Binomial Distribution,

$P(x = r) = {}^{n}Cr\ p^{r}\ q^{n-r}$

$\therefore$ P [Exactly 3 Heads (r = 3)] = $P(r = 3)_3$

$= 5_{C3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{5-3} = \frac{5}{16}$

[52] Exactly 3 girls are to be selected from 5 Girls and 3 Boys. The probability of selecting 3 Girls will be_____.

(a) $\frac{5}{28}$ (b) $\frac{1}{56}$

(c) $\frac{15}{28}$ (d) None. (1 mark)

Answer:

(a) Given : 5 Girls (G) + 3 Boys (B)

Required : Exactly 3 Girls

$\therefore$ Probability for exactly 3 Girls = $\frac{5_{C_3}}{8_{C_3}} = \frac{10}{56} = \frac{5}{28}$

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[53] Two unbiased dice are thrown. The Expected value of the sum of numbers on the upper side is;

(a) 3.5 (b) 7

Answer:

(b) According to the formula of Addition Law of Expectation

$E(x + y) = E(x) + E(y)$

$\therefore$ Expectation of a number on a dice :

$E(x) = p_1 x_1 + p_2 x_2 + \ldots\ldots\ldots\ldots + p_6 x_6$

$= \left[\frac{1}{6}\right] \times 1 + \left[\frac{1}{6}\right] \times 2 + \left[\frac{1}{6}\right] \times 3 + \left[\frac{1}{6}\right] \times 4 + \left[\frac{1}{6}\right] \times 5 + \left[\frac{1}{6}\right] \times 6$

$= \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6) = \frac{7}{2}$

$\therefore$ Expectation of a number on a dice = $\frac{7}{2}$

$\therefore$ $E(x) = \frac{7}{2}$; $E(y) = \frac{7}{2}$ (since IInd dice will also give same result)

$\therefore$ $E(x + y) = E(x) + E(y) = \frac{7}{2} + \frac{7}{2} = 7$

[54] One Card is drawn from pack of 52, what is the probability that it is a king or a queen?

(a) 11/13 (b) 2/13

(c) 1/13 (d) None of these (1 mark)

Answer:

(b) One card is drawn from pack of 52, then

Sample Space $n(s) = {}^{52}C_1 = 52$

Event (A) = 'a king or a queen'

$n(A) = ({}^4C_1 + {}^4C_1) = (4 + 4) = 8$

Probability (a king or a Queen)

$P(A) = \frac{n(A)}{n(S)} = \frac{8}{52} = \frac{2}{13}$

[55] In a packet of 500 pens, 50 are found to be defective. A pen is selected at random. Find the probability that it is non defective.

(a) 8/9 (b) 7/8

(c) 9/10 (d) 2/3 (1 mark)

Answer:

(c) Total pen in the packet = 500

No. of defective pen = 50

if a pen is selected sample space $n(s) = {}^{500}C_1 = 500$
Event (A) = 'pen is non defective' $n(A) = {}^{450}C_1 = 450$

$$P(\text{non defective pens}) = \frac{n(A)}{n(S)} = \frac{450}{500} = \frac{9}{10}$$

[56] Four married couples have gathered in a room. Two persons are selected at random amongst them, find the probability that selected persons are a gentleman and a lady but not a couple.

(a) 1/7 (b) 3/7
(c) 1/8 (d) 3/8 (1 mark)

Answer:

(b) Total person = 4 married couples = $4 \times 2 = 8$
Two person are selected then sample space $n(s) = {}^{8}C_2$
Event (A) = Selected persons are a gentle man and a lady, but not couple
$n(A) = {}^{4}C_1 \times {}^{3}C_1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4 \times 3}{28} = \frac{3}{7}$$

[57] A team of 5 is to be selected from 8 boys and three girls. Find the probability that it includes two particular girls.

(a) 2/30 (b) 1/5
(c) 2/11 (d) 8/9 (1 mark)

Answer:

(c) Total student = 8 boys + 3 girls = 11
if 5 person can be selected from 11 persons then
Sample space $n(s) = {}^{11}C_5$
Event (A) = 5 person selected if it included two particular girl
$n(A) = {}^{3}C_3 \times {}^{8}C_2 + {}^{2}C_2 \times {}^{8}C_3 = 1 \times 28 + 1 \times 56 = 84$

$$P(A) = \frac{n(A)}{n(S)} = \frac{84}{11_{C_5}} = \frac{84}{11 \times 42} = \frac{2}{11}$$

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[58] Let A and B two events in a sample space S such that $P(A) = \frac{1}{2}$; $P(\overline{B}) = \frac{5}{8}$, $P(A \cup B) = \frac{3}{4}$; Find $P(\overline{A} \cap \overline{B})$

(a) 3/4 (b) 1/4
(c) 3/16 (d) None of these. (1 mark)

Answer:

(b) Given $P(A) = \frac{1}{2}$, $P(\overline{B}) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$ then we know that

$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - \frac{3}{4}$

$= \frac{1}{4}$

[59] A card is drawn out of a standard pack of 52 cards. What is the probability of drawing a king or red colour ?

(a) 1/4 (b) 4/13
(c) 7/13 (d) ½ (1 mark)

Answer:

(c) A card is drawn out of a standard pack of 52 cards.
Then
Sample space $n(s) = {}^{52}C_1 = 52$
Event (A) = 'King or Red Colour'
$n(A) = 4 + 24$
$= 28$

Probability P (King or Red Colour) $= \frac{n(A)}{n(S)}$

$= \frac{28}{52}$

$= \frac{7}{}$

[60] A player tosses two fair coins, he wins ₹ 5 if 2 heads appear, ₹ 2 if one head appears and ₹ 1 if no head occurs. Find his expected amount of winning.

(a) 2.5 (b) 3.5

(c) 4.5 (d) 5.5 (1 mark)

Answer:

(a) For tossed two coins, the prob. distribution of getting head

x_i :	0	1	2
$P(x_i)$:	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
Getting Amount (₹) m_i :	1	2	5

Expected Amount of winning

$$E(x) = \sum m_i P(x_i)$$
$$= m_1 P(x_1) + m_2 P(x_2) + m_3 P(x_3)$$
$$= 1 \times \frac{1}{4} + 2 \times \frac{1}{2} + 5 \times \frac{1}{4}$$
$$= 0.25 + 1 + 1.25$$
$$= ₹ 2.50$$

[61] Arun & Tarun appear for an interview for two vacancies. The probability of Arun's selection is 1/3 and that of Tarun's selection is 1/5 Find the probability that only one of them will be selected.

(a) 2/5 (b) 4/5

(c) 6/5 (d) 8/5 (1 mark)

Answer:

(a) Probability of Arun's Selection & $P(A) = \frac{1}{3}$

$\therefore$ Probability of Arun not selected $P(\bar{A}) = 1 - \frac{1}{3} = \frac{2}{3}$

and, Probability of Tarun's Selection $P(T) = \frac{1}{5}$

$\therefore$ Probability of Tarun not Selected $P(\bar{T}) = 1 - \frac{1}{5} = \frac{4}{5}$

∴ Probability that only one will be selected

$$= P(A) \cdot P(\bar{T}) + P(\bar{A}) \cdot P(T)$$
$$= \frac{1}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{5}$$
$$= \frac{4}{15} + \frac{2}{15}$$
$$= \frac{6}{15}$$
$$= \frac{2}{5}$$

[62] A company employed 7 CA's, 6 MBA's and 3 Engineer's. In how many ways the company can form a committee, if the committee has two members of each type.

(a) 900 (b) 1,000
(c) 787 (d) 945 (1 mark)

Answer:

(d) There are 7CA's, 6MBA's and 3 Engineer's. The number of ways in which a committee containing two members of each type can be formed is

$= {}^{7}C_2 \times {}^{6}C_2 \times {}^{3}C_2$
$= 21 \times 15 \times 3$
$= 945$

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[63] Two dice are thrown together. Find the probability of getting a multiple of 2 on one Ist dice and multiple of 3 on the other dice.

(a) 2/3 (b) 1/6
(c) 1/3 (d) None of the above. (1 mark)

Answer:

(b) Two dice are thrown together

Sample space n(S) = 36

Event 'E' = 'getting a multiply of 2 on the Ist die and multiple of 3 on the ^{lind}die'.

= {(2,3) (2,6) (4,3) (4,6) (6,3) (6,6)}

n(E) = 6

$p(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

[64] The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5.
What is the probability that the problem will be solved if they both try ?

(a) 15/21 (b) 16/21

(c) 17/21 (d) 13/21 (1 mark)

Answer:

(b) The odd against A solving a certain problem = 4 : 3

$P(A) = \text{Prob (solve the problem)} = \frac{3}{4+3} = \frac{3}{7}$

$P(\overline{A}) = \text{Prob (not solve the problem)} = \frac{4}{4+3} = \frac{4}{7}$

The odds in favour of B solving the same problem = 7 : 5

$P(B) = \text{Prob (solve the problem)} = \frac{7}{7+5} = \frac{7}{12}$

$P(\overline{B}) = \text{Prob (not solve the problem)} = \frac{5}{7+5} = \frac{5}{12}$

Probability (Both are not solved the problem)

$= P(\overline{A} \cap \overline{B})$

$= P(\overline{A}) \cdot P(\overline{B})$

$= \frac{4}{7} \times \frac{5}{12} = \frac{5}{21}$

Probability (problem is solved) $= 1 - \frac{5}{21}$

$= \frac{16}{21}$

[65] Find the expected value of the following probability distribution

x :	– 20	– 10	30	75	80
p(x) :	3/20	1/5	½	1/10	1/20

(a) 20.5 (b) 21.5
(c) 22.5 (d) 24.5 (1 mark)

Answer:

(b) Expected value

$$E(n) = \Sigma p_i x_i$$
$$= p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4 + p_5 x_5$$
$$= \frac{3}{20}(-20) + \frac{1}{5}(-10) + \frac{1}{2}(30) + \frac{1}{10}(75) + \frac{1}{20}(80)$$
$$= -3 - 2 + 15 + 7.5 + 4$$
$$= 21.5$$

[66] A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball form the bag is twice that of a red ball, find the number of blue balls in the bag

(a) 10 (b) 12
(c) 14 (d) 16 (1 mark)

Answer:

(b) Let No. of Blue ball $= x$

Red Ball $= 6$

Total Ball in a Bag $= (6 + x)$

Prob of a Red ball P(R) $= \frac{6}{6+x}$

and prob of a Blue Ball P(B) $= \frac{x}{6+x}$

Given,

$$P(B) = 2\,P(R)$$
$$\frac{x}{(6+x)} = \frac{2 \times 6}{(6+x)} \Rightarrow x = 12$$

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[67] The probability of selecting a sample of size 'n' out of a population of size N by simple random sampling with replacement is:

(a) 1/N (b) $1/N^n$

(c) $1/{}^{N}c_n$ (d) $\frac{1}{N_{C_n}\ n!}$ (1 mark)

Answer:

(b) Total sample space = N^n

Event = 1

Probability = $\frac{\text{Event}}{\text{Sample space}}$

$= \frac{1}{N^n}$

[68] A box contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

(a) 10/21 (b) 11/21

(c) 2/7 (d) 5/7 (1 mark)

Answer:

(a) Total Ball = 2R + 3G + 2B = 7

Two Balls are drawn

Sample space n(S) = ${}^{7}C_2 = 21$

Events E = 'Ball is not Blue'

n(E) = ${}^{5}C_2 = 10$

Prob p(E) = $\frac{n(E)}{n(S)} = \frac{10}{21}$

[69] The odds that a book will be favourably received by 3 independent reviewers are 5 to 2, 4 to 3 and 3 to 4 respectively. What is the probability that out of 3 reviewers a majority will be favourable?

(a) $\frac{209}{343}$ (b) $\frac{209}{434}$

(c) $\frac{209}{\ }$ (d) $\frac{209}{\ }$ (1 mark)

Answer:

(a) Given: Say A, B, C be three Reviews.

$\therefore$ The odds in favour of A is 5 : 2 $\Rightarrow$ P (A) $= \frac{5}{7}$ & P (A^1) $= \frac{2}{7}$

And for B $\Rightarrow$ P (B) $= \frac{4}{7}$; P (B^1) $\frac{3}{7}$

for C $\Rightarrow$ P (C) $= \frac{3}{7}$; P (C^1) $= \frac{4}{7}$

$\therefore$ Probability of out of 3 = P (A) . P (B) . P (C^1) + P (A) . P (B^1) . P (C) +
Majority to be favourable P (A^1) . P (B) . P (C) + P (A) . P (B) . P (C)

$$= \frac{5}{7} \times \frac{4}{7} \times \frac{4}{7} + \frac{5}{7} \times \frac{3}{7} \times \frac{3}{7} + \frac{2}{7} \times \frac{4}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7}$$

$$= \frac{80}{343} + \frac{45}{343} + \frac{24}{343} + \frac{60}{343}$$

$$= \frac{209}{343}$$

[70] A player tosses 3 fair coins. He wins ₹ 5 if three heads appear, ₹ 3 if two heads appear, ₹ 1 if one head occurs. On the other hand, he losses ₹ 15 if 3 tails occur. Find expected gain of the player:

(a) 0.15 (b) 0.25
(c) 0.35 (d) 0.45 (1 mark)

Answer:

(b) Prob Distribution of getting no. of heads

Xi	0	1	2	3
Pi	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Expected gain $= \frac{1}{8} \times 5 + \frac{3}{8} \times 3 + \frac{3}{8} \times 1 + \frac{1}{8} \times (-15)$

$$= \frac{5}{8} + \frac{9}{8} + \frac{3}{8} - \frac{15}{8}$$

$$= \frac{5 + 9 + 3 - 15}{8}$$

$$= \frac{2}{8} = 0.25$$

[71] Find the probability of drawing an ace on each of two consecutive draws from a well shuffled pack of cards, without replacement

(a) $\frac{2}{51}$ (b) $\frac{1}{221}$

(c) $\frac{4}{51}$ (d) $\frac{5}{51}$ (1 mark)

Answer:

(b) Prob of getting Ace

$$P(\text{Ace}) = \frac{4_{C_1}}{52_{C_1}} = \frac{4}{52}$$

$$P(A) = \frac{1}{13}$$

other card is drawn without Replacement

$$P(B/A) = \frac{3_{C_1}}{51_{C_1}} = \frac{3}{51}$$

$$P(\text{Require}) = P(A) \cdot P(B/A)$$

$$= \frac{4^1}{52_{13}} \times \frac{3^1}{51_{17}} = \frac{1}{13 \times 17} = \frac{1}{221}$$

2013 - DECEMBER

[72] If P(A) = 0.45, P(B) = 0.35 and P(A&B) = 0.25, then P(A/B) =?

(a) 1.4 (b) 1.8

(c) 0.714 (d) 0.556 (1 mark)

Answer:

(c) Given $P(A) = 0.45, P(A\&B) = 0.25$

$P(B) = 0.35 P(A \cap B) = 0.25$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.35} = 0.714$$

[73] The probability of a cricket team winning match at Kanpur is 2/5 and hosing losing match at Delhi is 1/7 what is the Probability of the team winning atleast one match?

(a) 3/35 (b) 32/35
(c) 18/35 (d) 17/35 (1 mark)

Answer:

(b) Here A & B are Independent Event.

$P(\overline{A}) = 2/5$ and $P(\overline{B}) = \frac{1}{7}$

$P(\overline{A}) = 1 - \frac{2}{5} = \frac{3}{5}$, $P(\overline{B}) = 1 - \frac{1}{7} = \frac{6}{7}$

$P(A \cap \overline{B}) = P(\overline{A}).P(\overline{B}) = \frac{3}{5} \times \frac{1}{7} = \frac{3}{35}$

P(winning at least are match) $= 1 - P(A \cap B)$

$= 1 - \frac{3}{35} = \frac{32}{35}$

[74] Find the expected value of the following probability distribution

X:	-20	-10	30	75	80
P(x):	3/20	1/5	1/2	1/10	1/20

(a) 20.5 (b) 21.5
(c) 22.5 (d) 24.5 (1 mark)

Answer:

(b)

x	– 20	– 10	30	75	80
P(x)	3/20	1/5	½	1/10	1/20

Expected value

$E(x) = \Sigma p_i x_i$

$= p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4 + p_5 x_5$

$= \frac{3}{20} \times (-20) + \frac{1}{5} \times (-10) + \frac{1}{2} \times 30 + \frac{1}{10} \times 75 + \frac{1}{20} \times 80$

$= -3 - 2 + 15 + 7.5 + 4$

[75] Two coins are tossed simultaneously. Find the probability of getting exactly are head.

(a) 3/4 (b) 2/3
(c) 1/4 (d) ½ (1 mark)

Answer:

(d) Two coins are tossed

Sample Space (S) = {HH, HT, TH, TT}

n(S) = 4

A = 'Exactly are head'

A = 'HT, TH'

n(A) = 2

$$P(\text{exactly are head}) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

2014 - JUNE

[76] If a pair of dice is thrown then the probability that the sum of the digit is neither 7 nor 11 is_____.

(a) $\frac{1}{6}$ (b) $\frac{1}{18}$
(c) $\frac{2}{9}$ (d) $\frac{7}{9}$ (1 mark)

Answer:

(d) A pair of dice is thrown

Sample space n(s) = 36

Event (A) = 'Sum is 7 or 11'

= {(6, 1) (1, 6) (4, 3) (3, 4) (5, 2) (2, 5) (6,5) (5, 6)}

n(A) = 8

$$P(A) = \frac{n(A)}{n(S)} = \frac{\cancel{8}}{\cancel{36}} = \frac{2}{9}$$

P (Sum is neither 7 nor 11) = $P(\overline{A})$

$= 1 - P(A)$

$= 1 - \frac{2}{9}$

$= \frac{7}{9}$

[77] An urn contains 2 red and 1 green balls. Another urn contains 2 red and 2 green balls. An urn was selected at random and then a ball was drawn from it. If it was found to be red then the probability that it has been drawn from urn one is___.

(a) $\frac{4}{7}$ (b) $\frac{3}{7}$

(c) $\frac{2}{3}$ (d) $\frac{7}{12}$ (1 mark)

Answer:

(c) 2 Red, 1Green — Urn-1; 2 Red, 2 Green — Urn-2

Probability of Red ball from the first urn = $\frac{2}{3}$

$\therefore$ [Sample space n(s) = 2R + 1G = 3]

[78] For any two events A_1, A_2 let $P(A_1) = \frac{2}{3}, P(A_2) = \frac{3}{8}$ and $P(A_1 \cap A_2) = \frac{1}{4}$

then A_1, A_2 are:

(a) Mutually exclusive but not independent events

(b) Mutually exclusive and independent events

(c) Independent but not mutually exclusive

(d) None of these (1 mark)

Answer:

(c) Given

$P(A_1) = \frac{2}{3}$

$P(A_2) = \frac{3}{8}$

$P(A_1 \cap A_2) = \frac{1}{4}$

$P(A_1) \cdot P(A_2) = \frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$

$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2)$

2014 - DECEMBER

[79] An unbiased die is thrown twice. The probability of the sum of numbers obtained on the two faces being divisible by 4 is:

(a) 7/36 (b) 1/3

(c) 11/36 (d) 1/4 (1 mark)

Answer:

(d) One unbiased die is thrown twice

Sample space n(s) = 36

Event (A) = 'sum is divisible by 4'

= {(1, 3), (3, 1), (2, 2), (5, 3), (3, 5), (6, 2), (2, 6), (4, 4), (6, 6)}

n(A) = 9

Probability p(A) = $\frac{n(A)}{n(S)}$

= $\frac{9}{36} = \frac{1}{4}$

[80] Let the distribution function of a random variable X be $F(X) = P(X \leq X)$. Then F(5) - F(2) is:

(a) $P(2 < X < 5)$ (b) $P(2 \leq X < 5)$

(c) $P(2 \leq X \leq 5)$ (d) $P(2 < X \leq 5)$ (1 mark)

Answer:

(d) If the distribution function of a random variable X be F(x)

$= P(X \leq x)$ then

$F(5) = P(X \leq 5)$

& $F(2) = P(X \leq 2)$

$F(5) - F(2) = P(X \leq 5) - P(X \leq 2)$

$= P[X = (0, 1, 2, 3, 4, 5)] - P[X = (0, 1, 2)]$

$= P(X = 3, 4, 5)$

$= P(2 < X \leq 5)$

[81] A discrete random variable X takes three values -1, 2 and 3 with probabilities

$p(-1) = \frac{1}{3}$, $p(2) = \frac{1}{3}$, $p(3) = \frac{1}{3}$, then E(|X|) is:

(a) 3/2 (b) 5/2
(c) 2 (d) 9/2 (1 mark)

Answer:

(c) Given,

X_i :	-1	2	3
P_i :	1/3	1/3	1/3

$$E(|x|) = \Sigma p_i |x_i|$$
$$= p_1 |x_1| + p_2 |x_2| + p_3 |x_3|$$
$$= \frac{1}{3}|-1| + \frac{1}{3}|2| + \frac{1}{3}|3|$$
$$= \frac{1}{3} \times 1 + \frac{1}{3} \times 2 + \frac{1}{3} \times 3$$
$$= \frac{1}{3} + \frac{2}{3} + \frac{3}{3}$$
$$= \frac{6}{3}$$
$$= 2$$

2015 - JUNE

[82] The sum of numbers obtained in throw of a dice twice is S. Probability of S will be maximum if S is

(a) 5 (b) 7
(c) 6 (d) 8 (1 mark)

Answer:

(b) If A die is thrown twice:

Sum is 2 → (1, 1) = 1

Sum is 4 → (1, 3) (3, 1) (2, 2) = 3
Sum is 5 → (1, 4) (4, 1) (2, 3) (3, 2) = 4
Sum is 6 → (1, 5) (5, 1) (4, 2) (2, 4) (3, 3) = 5
Sum is 7 → (6, 1) (1, 6) (5, 2) (2, 5) (4, 3) (3, 4) = 6
Sum is 8 → (6, 2) (2, 6) (5, 3) (3, 5) (4, 4) = 5
Sum is 9 → (6, 3) (3, 6) (4, 5) (5, 4) = 4
Sum is 10 → (6, 4) (4, 6) (5, 5) = 3
Sum is 11 → (6, 5) (5, 6) = 2
Sum is 12 → (6, 6) = 1

Here we see
Probability of s will be maximum if sum is 7.

[83] An unbiased coin is tossed three times. The expected value of the number of heads is

(a) 2.5 (b) 1.0
(c) 1.5 (d) 2.0 (1 mark)

Answer:

(c) An unbiased coins is tossed three times.
Then, Probability Distribution of No. of getting 'Head' is

$x_i =$	0	1	2	3
$p_i =$	1/8	3/8	3/8	1/8

Expected value $= \sum p_i x_i$

$= p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4$

$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$

$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$

$= \frac{12}{8}$

$= 1.5$

[84] For two events A_1 and A_2, let $P(A_1) = \frac{2}{3}$ and $P(A_2) = \frac{3}{8}$ and $P(A_1 \cap A_2) = \frac{1}{4}$, then

(a) mutually exclusive but not independent
(b) mutually exclusive and independent
(c) independent but not mutually exclusive
(d) none of these (1 mark)

Answer:

(c) $P(A_1) = \frac{2}{3}$, $P(A_2) = \frac{3}{8}$

$P(A_1 \cap A_2) = 1/4$

$P(A_1) \times P(A_2) = \frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$

$P(A_1).\ P(A_2) = P(A_1 \cap A_2)$

So, A_1 & A_2 are independent but not mutually exclusive.

[85] From 6 positive and 8 negative numbers, 4 numbers are chosen at random without replacement and are then multiplied. The probability that the product of the chosen numbers will be positive number is

(a) $\frac{409}{1001}$ (b) $\frac{70}{1001}$

(c) $\frac{505}{1001}$ (d) $\frac{420}{1001}$ (1 mark)

Answer:

(c) Positive No. = 6
Negative No. = 8
Total No. = 6 + 8 = 14

Require Probability = P(all negative) + P(all + ve) + P(two negative & two positive)

$$= \frac{{}^8C_4}{{}^{14}C_4} + \frac{{}^6C_4}{{}^{14}C_4} + \frac{{}^6C_2 \times {}^8C_2}{{}^{14}C_4}$$

$$= \frac{15 + 70 + (28 \times 15)}{} = \frac{505}{}$$

2015 - DECEMBER

[86] If an unbiased die is rolled once, the odds in favour of getting a point which is multiple of 3 is:

(a) 1:2 (b) 2:1

(c) 1:3 (d) 3:1 (1 mark)

Answer:

(a) One die is Rolled

Sample space n(S) = 6

Event(A) = 'getting no. which is multiple of 3'

$= \{3,6\}$

$n(A) = 2$

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Odd in favour of an events $= P(A) : P(\bar{A})$

$= \frac{1}{3} : (1 - \frac{1}{3})$

$= \frac{1}{3} : \frac{2}{3}$

$= 1 : 2$

[87] A bag contains 15 one rupee coins, 25 two rupees coins and 10 five rupees coins, if a coin is selected at random then probability for not selecting a one rupee coin is:

(a) 0.30 (b) 0.20

(c) 0.25 (d) 0.70 (1 mark)

Answer:

(d) Total No. of coins $= 15 + 25 + 10$

$= 50$

Sample Space n(S) $= 50$

Event 'A' = 'not getting one Rupee coins'

$n(A) = 25 + 10$

$= 35$

$P(A) = \frac{n(A)}{n(S)} = \frac{35}{50} = 0.7$

[88] Three coins are tossed together, the probability of getting exactly two head is:

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$

(c) $\frac{1}{8}$ (d) None (1 mark)

Answer:

(b) Three coins are tossed

then Sample Space S = {HHH, HHT, HTH, HTT, TTT, TTH, THT, THH}

n(S) = 8

Event (A) = 'getting Exactly two head' {HHT, HTH, THH}

n(A) = 3

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

[89] If two letters are taken at random from the word "HOME", what is the probability that none of the letters would be vowels?

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$ (1 mark)

Answer:

(a) Given word 'HOME'

If two letters are taken then

Sample Space $n(S) = {}^4c_2$

$= \frac{4\times3}{2\times1} = 6$

Event (A) 'none of the letters would be Vowels'

$n(A) = {}^2c_2 = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$

2016 - JUNE

[90] In a game, cards are thoroughly shuffled and distributed equally among four players. What is the probability that a specific player gets all the four kings?

(a) $\frac{{}^{13}C_4 \times {}^{48}C_{13}}{{}^{52}C_{13}}$ (b) $\frac{{}^{4}C_4 \times {}^{46}C_9}{{}^{52}C_{13}}$

(c) $\frac{{}^{13}C_4 \times {}^{52}C_4}{{}^{52}C_{13}}$ (d) $\frac{{}^{4}C_4 \times {}^{39}C_9}{{}^{52}C_{13}}$ (1 mark)

Answer:

(b) In a game, cards are thoroughly shuffled and distributed equally among four players.

Sample space n(s) = ${}^{52}C_{13}$

Event (A) = 'a specific player gets all four king'

n(A) = ${}^{4}C_4 \times {}^{48}C_9$

Probability P(A) = $\frac{n(A)}{n(S)}$

= $\frac{{}^{4}C_4 \times {}^{48}C_9}{{}^{52}C_{13}}$

[91] A bag contains 4 Red and 5 Black balls. Another bag contains 5 Red and 3 Black balls. If one ball is drawn at random from each bag. Then the probability that one Red and one Black drawn is –

(a) $\frac{12}{72}$ (b) $\frac{25}{72}$

(c) $\frac{37}{72}$ (d) $\frac{13}{72}$ (1 mark)

Answer:

(c)

Require Probability = P(one Red from the Ist bag and one Black ball from the IInd bag) + P(one Red ball from the IInd bag and one Black ball from the Ist bag)

$$= P(R_1 \cap B_2) + P(R_2 \cap B_1)$$
$$= P(R_1).\ P(B_2) + P(R_2).\ P(B_1)$$
$$= \frac{4}{9}.\frac{3}{8} + \frac{5}{8}.\frac{5}{9}$$
$$= \frac{12}{72} + \frac{25}{72}$$
$$= \frac{37}{72}$$

[92] If $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{5}$ and $P(A \cup B) = \frac{5}{6}$ then $P\left(\frac{A}{B^1}\right)$ is

(a) $\frac{7}{12}$ (b) $\frac{5}{12}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$ (1 mark)

Answer:

(a) If $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{5}$ and $P(A \cup B) = \frac{5}{6}$ then $P(A/B') = ?$

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\frac{5}{6} = \frac{2}{3} + \frac{3}{5} - P(A \cap B)$$
$$P(A \cap B) = \frac{2}{3} + \frac{3}{5} - \frac{5}{6} = \frac{20 + 18 - 25}{30} = \frac{13}{30}$$
$$P(A/\overline{B}) = \frac{P(A \cap \overline{B})}{P(\overline{B})} \qquad [P(A/B') = P(A/\overline{B})]$$
$$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$$
$$= \frac{\frac{2}{3} - \frac{13}{30}}{1 - \frac{3}{5}} = \frac{\frac{20 - 13}{30}}{\frac{5 - 3}{5}} = \frac{7}{30} \times \frac{5}{2} = \frac{7}{12}$$

[93] If two unbiased dice are rolled, what is the probability of getting points neither 3 nor 6?

(a) 0.25 (b) 0.50

(c) 0.75 (d) 0.80 (1 mark)

Answer:

(d) If two dice are rolled then

Sample space n(s) = 36

Event 'A' = 'getting sum is either 3 or 6'

n(A) = 36 – 7

= 29

$P(A) = \frac{n(A)}{n(s)} = \frac{29}{36} = 0.80$

[94] Two dice are tossed. What is the probability that the total is divisible by 3 or 4.

(a) $\frac{20}{36}$ (b) $\frac{21}{36}$

(c) $\frac{14}{36}$ (d) None of these. (1 mark)

Answer:

(a) If two dice are rolled

Sample Space n(s) = 36

Event 'A' = 'The total sum is divisible by 3 or 4'

= {(1, 2) (2, 1) (5, 1) (1, 5) (3, 3) (4, 2) (2, 4)
(4, 5) (5, 4) (6, 3) (3, 6) (6, 6)
(1, 3) (3, 1) (2, 2) (6, 2) (2, 6)
(5, 3) (3, 5) (4, 4)}

n(A) = 20

$P(A) = \frac{n(A)}{n(B)} = \frac{20}{36}$

2016 - DECEMBER

[95] If two events A, B, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{2}{3}$ then $P(A \cap B)$ is:

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$ (1 mark)

Answer:

(b) $P(A) = \frac{1}{2}$, $P(3) = \frac{1}{3}$, $P(A\ B) = ?$

$P(A \cup B) = \frac{2}{3}$

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{2}{3} = \frac{1}{2} + \frac{1}{3} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{2}{3}$$

$$P(A \cap B) = \frac{3 + 2 - 4}{6}$$

$$P(A \cap B) = \frac{1}{6}$$

[96] A bag contains 6 white and 5 red balls. One ball is drawn. The probability that it is red is:

(a) $\frac{5}{11}$ (b) $\frac{6}{11}$

(c) $\frac{1}{11}$ (d) None of these (1 mark)

Answer:

(a) Total ball in the bag $= 6W + 5R$

$= 11$

If one ball is drawn from the bag then

sample space $n(S) = 11c_1 = 11$

Event (A)= 'getting ball is Red'

$= 5c_1$

$n(A) = 5$

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{11}$

[97] For two events A, B let $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{8}$ and $P(A \cap B) = \frac{1}{4}$ then A and B are:

(a) Mutually exclusive but not independent

(b) Independent but not mutually exclusive

(c) Mutually exclusive and independent

(d) None of these (1 mark)

Answer:

(b) Given $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{8}$, $P(A \cap B) = \frac{1}{4}$

$P(A) \times P(B) = \frac{2}{3} \times \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$

$P(A \cap B) = \frac{1}{4}$

so, $P(A \cap B) = P(A).\ P(B)$

so, A and B are Independent but not mutually exclusive.

2017 - JUNE

[98] Let A and B are two events with $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{12}$, then P(B/A) will be :

(a) 7/8 (b) 1/3

(c) 1/8 (d) 8/7 (1 mark)

Answer:

(c) Given $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{12}$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{12}}{\frac{2}{3}} = \frac{1}{12} \times \frac{3}{2} = \frac{1}{8}$$

[99] What is the probability of having at least one 'SIX' from 3 throws of an unbaised die?

(a) $\frac{5}{6}$ (b) $\left(\frac{5}{6}\right)^3$

(c) $1 - \left(\frac{1}{6}\right)^3$ (d) $1 - \left(\frac{5}{6}\right)^3$ (1 mark)

Answer:

(d) One die is Rolled

No. of sample space n(s) = 6

P(getting six) = $\frac{1}{6}$

P(getting not a six) = $1 - \frac{1}{6} = \frac{5}{6}$

Probability of not getting a six in first throw = $\frac{5}{6}$

Probability of not getting a six in second throw = $\frac{5}{6}$

Probability of not getting a six in third throw = $\frac{5}{6}$

Probability of not getting a six in all three throw $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

$= \left(\frac{5}{6}\right)^3$

Probability of getting at least one six $= 1 - \left(\frac{5}{6}\right)^3$

[100] For any two events A and B:

(a) $P(A - B) = P(A) - P(B)$

(b) $P(A - B) = P(A) - P(A \cap B)$

(c) $P(A - B) = P(B) - P(A \cap B)$

(d) $P(B - A) = P(B) + P(A \cap B)$ (1 mark)

Answer:

(b) If A & B two events

$P(A - B) = P(A \cap B)$

$= P(A) - P(A \cap B)$

2017 - DECEMBER

[101] If for two mutually exclusive events A and B $P(A \cup B) = \frac{2}{3}$ and $P(A) = \frac{2}{5}$ then what is the value of P (B)?

(a) $\frac{4}{15}$ (b) $\frac{4}{9}$

(c) $\frac{5}{9}$ (d) $\frac{7}{15}$ (1 mark)

Answer:

(a) Given $P(A \cup 3) = \frac{2}{3}$, $P(A) = \frac{2}{5}$

$\because$ A and B are two mutually exclusive events then $P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{2}{3} = \frac{2}{5} + P(B) - 0$$

$$P(B) = \frac{2}{3} - \frac{2}{5} = \frac{10-6}{15} = \frac{4}{15}$$

[102] The probability distribution of the demand for a commodity is given below:

Demand (x)	5	6	7	8	9	10
Probability [P(x)]	0.05	0.10	0.30	0.40	0.10	0.05

The expected value of demand will be

(a) 7.55 (b) 7.85

(c) 1.25 (d) 8.35 (1 mark)

Answer:

(a)

Given Demand (x)	x_1 5	x_2 6	x_3 7	x_4 8	x_5 9	x_6 10
Probability P(x)	0.05	0.10	0.30	0.40	0.10	0.05
	P_1	P_2	P_3	P_4	P_5	P_6

Expected Value

$E(x) = \sum P_i x_i$

$= P_1x_1 + P_2x_2 + P_3x_3 + P_4 x_4 + P_5 \times 5 + P_6 \times 6$

$= 0.05 \times 5 + 0.10 \times 6 + 0.30 \times 7 + 0.40 \times 8 + 0.10 \times 9 + 0.05 \times 10$

$= 0.25 + 0.60 + 2.10 + 3.20 + 0.90 + 0.50$

$= 7.55$

[103] Given $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$, the value of P(A/B) is

(a) 1/2 (b) 1/6

(c) 2/3 (d) 3/4 (1 mark)

Answer:

(d) Given

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$

[104] A brother and his sister appear in an interview for two vacancies for the same post. The probability of brother's selection is $\frac{1}{7}$ and the probability of sister's selection is $\frac{1}{5}$. The probability that (i) both are selected (ii) only one of them is selected and (iii) none of them is selected will be:

(a) $\frac{1}{35}, \frac{10}{35}, \frac{24}{35}$ (b) $\frac{24}{35}, \frac{1}{35}, \frac{10}{35}$

(c) $\frac{10}{35}, \frac{1}{35}, \frac{24}{35}$ (d) $\frac{24}{35}, \frac{10}{35}, \frac{1}{35}$ (1 mark)

Answer:

(a) Given

Probability of brother's selection $P(A) = \frac{1}{7}$

Probability of brother's 'not selection' $P(\overline{A}) = \frac{6}{7}$

Probability of brother's selection $P(B) = \frac{1}{5}$

Probability of sister's not selection $P(\overline{B}) = 1 - \frac{1}{5} = \frac{4}{5}$

(i) Probability of both selected $= P(A \cap B)$

$= P(A) \cdot P(B)$

$= \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$

(ii) P (only one is selected) $= P(A \cap \bar{B}) + P(B \cap \bar{A})$

$= P(A).\ P(\bar{B}) + P(B).\ P(\bar{A})$

$= \frac{1}{7} \times \frac{4}{5} + \frac{1}{5} \times \frac{6}{7} = \frac{4+6}{35} = \frac{10}{35}$

(iii) P(none of them is selected) $= P(\bar{A} \cap \bar{B})$

$= P(\bar{A}).\ P(\bar{B})$

$= \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$

2018 - MAY

[105] Two broad divisions of probability are:

(a) Subjective probability and objective probability

(b) Deductive probability and mathematical probability

(c) Statistical probability and mathematical probability

(d) None of these (1 mark)

Answer:

(a) Two broad and divisions of Probability are

(1) Subjective Probability

(2) Objective Probability

[106] The term "chance" and probability are synonyms:

(a) True (b) False

(c) Both (d) None (1 mark)

Answer:

(a) The terms "chance" and probability are synonyms is True.

[107] The theorem of compound probability states that for any two events A an B

(a) $P(A \cap B) = P(A) \times P(B/A)$

(b) $P(A \cup B) = P(A) \times P(B/A)$

(c) $P(A \cap B) = P(A) \times P(B)$

(d) $P(A \cup B) = P(A) \times P(B) - P(A \cap B)$ (1 mark)

Answer:

(a) The theorem of compound probability states that for only two events A an B given by

$$P(A \cap B) = P(A) \times P(B/A)$$

[108] Variance of a random variable x is given by

(a) $E(X-\mu)^2$ (b) $E[X- E(X)]^2$

(c) $E(X^2 - \mu)$ (d) (a) or (b) (1 mark)

Answer:

(d) Variance of a random variable x is given by

$$V(x) = E(X-\mu)^2$$

or

$$V(x) = [E(X-E(x)]^2$$

[109] If two random variables x and y are related by y=2-3x, then the SD of y is given by

(a) $-3 \times$ SD of x (b) $3 \times$ SD of x

(c) $9 \times$ SD of x (d) $2 \times$ SD of x (1 mark)

Answer:

(b) Given Equation

$$y = 2-3x$$

$$3x+y-2=0$$

$$b = \frac{-\text{Coefficient of } x}{\text{Coefficient of } y} = \frac{-3}{1} = -3$$

S.D of $y = |b|$ S.D of X

$= |-3|$. S.D of x

$= 3x$ S.D of x

[110] What is the probability of having at least one' six' in 3 throws of a project die?

(a) 5/6 (b) $(5/6)^3$

(c) $1-(1/6)^3$ (d) $1-(5/6)^3$ (1 mark)

Answer:

(d) For a die Probability of getting Six

$$P(A) = \frac{1}{6} \Rightarrow p$$

$$P(\bar{A}) = 1 - \frac{1}{6} = \frac{5}{6} \Rightarrow q$$

Here n = 3

P(getting at least '1' Six) = $P(X \geq 1)$

$= 1 - P((X < 1)$

$= 1 - P(X = 0)$

$= 1 - {}^{3}C_{0} \cdot \left(\frac{1}{6}\right)^{0} \cdot \left(\frac{1}{6}\right)^{3-0}$

$= 1 - 1 \times 1 \times \left(\frac{5}{6}\right)^{3}$

$= 1 - \left(\frac{5}{6}\right)^{3}$

[111] Sum of all probabilities mutually exclusive and exhaustive events is equal to

(a) 0 (b) 1/2

(c) 1/4 (d) 1 (1 mark)

Answer:

(d) Sum of all probabilities mutually exclusive and exhaustive events is equal to 1.

2018 - NOVEMBER

[112] If, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, and $P(A \cap B) = \frac{1}{4}$, then $P(A \cup B)$ is equal to

(a) $\frac{11}{12}$ (b) $\frac{10}{12}$

(c) $\frac{7}{12}$ (d) $\frac{1}{6}$ (1 mark)

Answer:

(c) Given:

$$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(A \cap B) = \frac{1}{4}$$

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$$

$$= \frac{6+4-3}{12}$$

$$= \frac{7}{12}$$

[113] The probability that a leap year has 53 Wednesday is

(a) $\frac{2}{7}$ (b) $\frac{3}{5}$

(c) $\frac{2}{3}$ (d) $\frac{1}{7}$ (1 mark)

Answer:

(a) In a leap years, there are 366 days.
366 days. = 52 weeks and 2 days.
2 odd days may be:

(a) Sunday and Monday
(b) Monday and Tuesday
(c) Tuesday and Wednesday
(d) Wednesday and Thursday
(e) Thursday and Friday
(f) Friday and Saturday
(g) Saturday and Sunday

No. of sample space
n(s) = 7
Event (A) = 'getting Wednesday'
n(A) = 2
P(A) = 2/7

[114] A coin is tossed six times, then the probability of obtaining heads and tails alternatively is

(a) $\frac{1}{2}$ (b) $\frac{1}{64}$

(c) $\frac{1}{32}$ (d) $\frac{1}{16}$ (1 mark)

Answer:

(c) If one coin is tossed '6' times
P(H) = 1/2, P(T) = 1/2

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{64} \times \frac{1}{64} = \frac{2}{64} = \frac{1}{32}$$

[115] Ram is known to hit a target in 2 out of 3 shots where as Shyam is known to hit the same target in 5 out of 11 shots. What is the probability that the target would be hit if they both try?

(a) $\frac{9}{11}$ (b) $\frac{3}{11}$

(c) $\frac{10}{33}$ (d) $\frac{6}{11}$ (1 mark)

Answer:

(a) Probability of hitting the target by Ram $P(A) = \frac{2}{3}$

Probability of hitting the target by Shyam $P(B) = \frac{5}{11}$

$P(\overline{A}) = 1 - \frac{2}{3} = \frac{1}{3}$

$P(\overline{B}) = 1 - \frac{5}{11} = \frac{6}{11}$

$P(\text{Target WOULT be HIT}) = 1 - P(\overline{A} \cap \overline{B})$

$= 1 - P(\overline{A}).\ P(\overline{B})$

$= 1 - \frac{1}{3} \times \frac{6}{11}$

$= 1 - \frac{2}{11}$

$= \frac{9}{11}$

[116] Two different dice are thrown simultaneously, then the probability, that the sum of two numbers appearing on the top of dice is 9 is

(a) $\frac{8}{9}$ (b) $\frac{1}{9}$

(c) $\frac{7}{}$ (d) None of the above (1 mark)

Answer:

(b) If two dice are Rolled then

Sample space $n(s) = 6^2 = 36$

Event (A) = Getting the sum is '9'

= {(6,3) (3,6) (4,5) (5,4)}

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

[117] If $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$, then $P(\bar{A}) + P(\bar{B})$ is equal to

(a) 0.3 (b) 0.5

(c) 0.7 (d) 0.9 (1 mark)

Answer:

(d) Given:

$P(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$

We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = [1 - P(\bar{A})] + [1 - P(\bar{B})] - 0.3$

$0.8 = 1 - P(\bar{A}) + 1 - P(\bar{B}) - 0.3$

$P(\bar{A}) + P(\bar{B}) = 2 - 0.3 - 0.8$

$P(\bar{A}) + P(\bar{B}) = 0.9$

2019 - JUNE

[118] If $Y \geq x$ then mathematical expectation is

(a) $E(X) > E(Y)$

(b) $E(X) \leq E(Y)$

(c) $E(X) = E(Y)$

(d) $E(X) \cdot E(Y) = 1$

Answer:

(b) If $y \geq x$

then $E(y) \geq E(x)$

[119] Two event A and B are such that they do not occurs simultaneously then they are called _____ events

(a) Mutually exhaustive
(b) Mutually exclusive
(c) Mutually independent
(d) Equally likely

Answer:

(b) Two events A and B are such that they do not occurs simultaneously then they are called **Mutually Exclusive** Events.

[120] According to bayee's theorem,

$$P(E_K / A) = \frac{P(E_K)\,P(A/E_K)}{\sum_{i=1}^{n} P(E_1)\,P(A/E_1)} \text{ here}$$

(a) E_1, E_2 ..... are mutually exclusive
(b) $P(E/A_1), P(E/A_2)$..... are equal to 1
(c) $P(A_1/E), P(A_2/E)$ ..... are equal to 1
(d) A & E_1's are disjoint sets.

Answer:

(a) According to Bayee' Theorem

$$P(E_k/A) = \frac{P(E_k)P(A/E_k)}{\sum_{i=1}^{h} P(E_i)\cdot P(A/E_i)}$$

Here, E_1, E_2, E_3 .... are Mutually Exclusive.

[121] If a coin is tossed 5 times then the probability of getting Tail and Head occurs alternatively is

(a) $\frac{1}{8}$
(b) $\frac{1}{16}$
(c) $\frac{1}{32}$
(d) $\frac{1}{64}$

Answer:

(b) P(getting tail and Head occurs Alternative)

$= \quad P(HTHTH) \text{ or } P(THTHT)$

$= \quad \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$

$= \quad \frac{1}{32} + \frac{1}{32}$

$= \quad \left(\frac{1+1}{32}\right)$

$= \quad \frac{2}{32}$

$= \quad \frac{1}{16}$

[122] When 2 - dice are thrown Simultaneously then the probability of getting at least one 5 is

(a) $\frac{11}{36}$ (b) $\frac{5}{36}$

(c) $\frac{8}{15}$ (d) $\frac{1}{7}$

Answer:

(a) If two dice are thrown then sample space n(S) = 36

Events 'A' = getting at least one '5'

$$\text{'A'} = \begin{bmatrix} (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (1,5) & (2,5) & (3,5) & (4,5) & (6,5) & \end{bmatrix}$$

$n(A) = 11$

$p(A) = \frac{n(A)}{n(S)}$

$= \frac{11}{36}$

In Binomial Distribution

2019 - NOVEMBER

[123] Two letters are choosen from the word HOME. What is the probability that the letters choosen are not vowels.

(a) ½
(b) 1/6
(c) 2/3
(d) 0 (1 mark)

Answer:

(b) HOME

Total letters = 4

Total vowels = 2 {O, E}

Total Consonants = 2 {H, M}

P (that 2 letters choosen are not vowels)

P (that 2 letters choosen are consonants)

$= \frac{^2C_2}{^4C_2} = \frac{1}{6}$ (Required probability)

[124] If A, B, C are three mutually exclusive and exhaustive events such that:

P(A) = 2(B) = 3P(C) what is P(B)?

(a) 6/11
(b) 3/11
(c) 1/6
(d) 1/3 (1 mark)

Answer:

(b) Since A, B, C are mutually exclusive events

$P(A \cap B) = 0$, $P(B \cap C) = 0$, $P(C \cap A) = 0$ and $P(A \cap B \cap C) = 0$

Since A, B, C are mutually exhaustive $P(A \cup B) = 1$

We know,

$P(A \cup B) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$$1 = P(A) + P(B) + P(C) - 0 - 0 - 0 + 0$$

$$P(A) + P(B) + P(C) = 1 \qquad \text{Eq} - 1$$

In given question; $P(A) = 2P(B) = 3P(C)$

$$P(A) = 2P(B) \qquad \text{Eq} - 2$$

$$\text{and } P(C) = \frac{2}{3}P(B) \qquad \text{Eq} - 3$$

Put Eq 2 and 3 in Eq 1

$$2P(B) + P(B) + \frac{2}{3}P(B) = 1$$

$$\frac{11}{3}P(B) = 1$$

$$P(B) \frac{3}{11}$$

[125] What is the probability of getting 7 or 11 when two dices are thrown?

(a) 2/9
(b) 6/36
(c) 10/36
(d) 2/36 (1 mark)

Answer:

(a) When two dices are thrown

$n(S) = 36$

A Event of getting sum 7

B Event of getting sum 11

A {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

$n(A) = 6$

B {(5,6), (6,5)}

$n(B) = 2$

$$P(\text{of getting sum 7 or 11}) = \frac{6+2}{36}$$

$$= \frac{8}{36} = \frac{2}{9}$$

[126] A log contains 15 one rupee Coins, 25 two rupees coins and 10 five rupee coins if a coin is selected at random than probability for not selecting a one rupee coin is:

(a) 0.30
(b) 0.20
(c) 0.25
(d) 0.70 (1 mark)

Answer:

(d) No. of one rupee coin = 15
No. of two rupee coin = 25
No. of five rupee coin = 10
Total coin = 15 + 25 + 10 = 50

$$P \text{ (of not selecting one rupee coin)} = \frac{25 + 10}{50}$$

$$= \frac{35}{50} = 0.7$$

0.7 is the required probability.

[127] What is the probability of occurring 4 or more than 4 accidents.

No. of acc.	1	2	3	4	5	6	7
Frequency	8	17	15	24	27	18	9

(a) 24
(b) 69
(c) 78
(d) 80 (1 mark)

Answer:

(c) (No. of 4 or more accidents) = 24 + 27 + 18 + 9
= 78

Total accidents = 8 + 17 + 15 + 24 + 27 + 18 + 9
= 118

CHAPTER

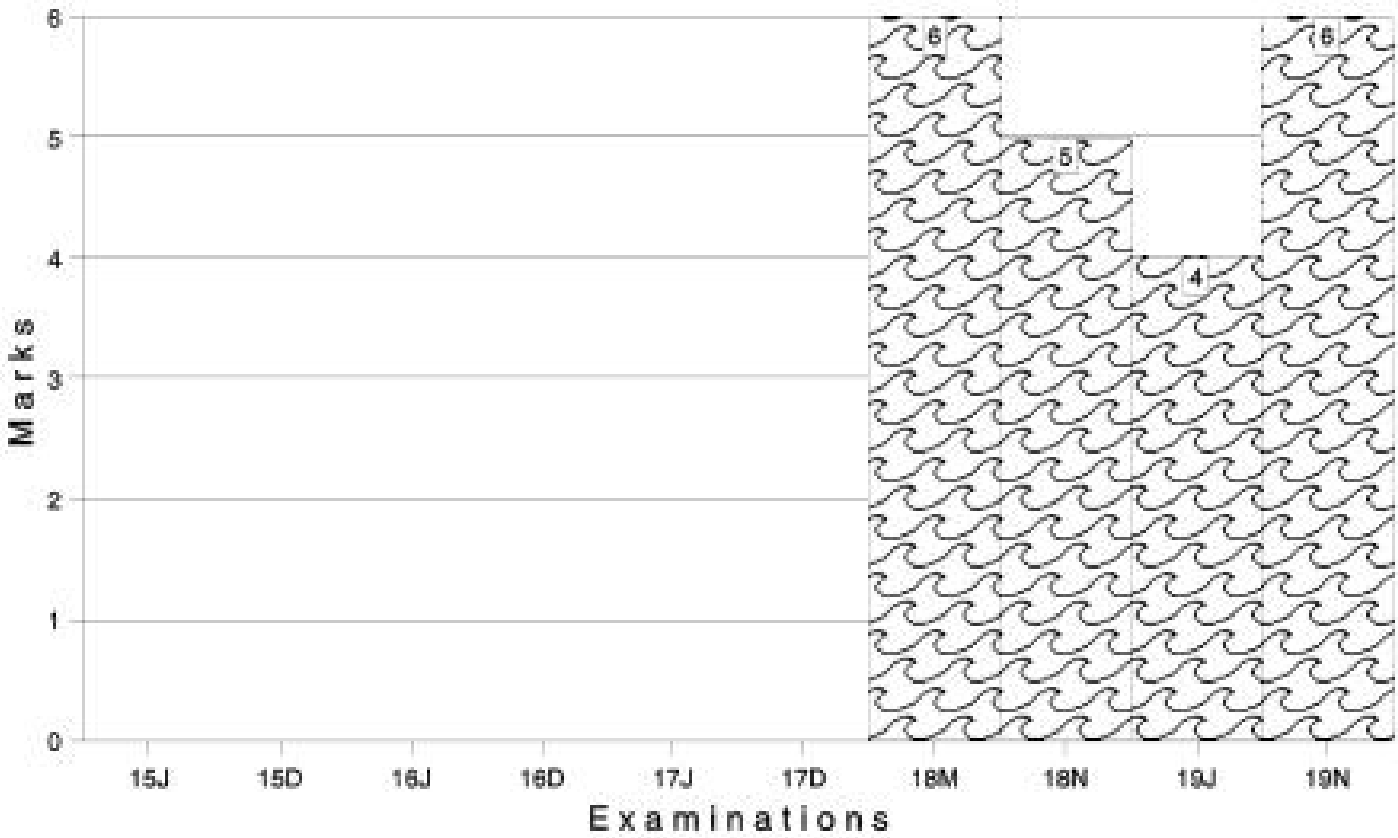
17

THEORETICAL DISTRIBUTIONS

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



PAST YEAR QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] Parameter is a characteristic of :

(a) Population (b) Sample

(c) Probability distribution (d) Both (a) & (b) (1 mark)

Answer:

(a) A parametre is defined as a characteristic of a population based on all the units of the population.

Therefore, parameter is a characteristic of population.

[2] What is the probability of making 3 correct guesses in 5 True - False answer type questions ?

(a) 0.4156 (b) 0.32

(c) 0.3125 (d) 0.5235 (1 mark)

Answer:

(c) Here, $p = 0.5$, $q = 0.5$

(Since the answer can either be True of false), $n = 5$, $r = 3$

$P(x = r) = {}^{n}C_{r}\, p^{r}\, q^{n-x}$

$P(x = 3) = {}^{5}C_{3}\, (0.5)^{3}\, (0.5)^{2} = 0.3125$

[3] The 1.Q.'s of army volunteers in a given year are normally distributed with Mean = 110 and Standard Deviation = 10. The army wants to give advance training to 20% of those recruits with the highest scores. What is the lowest 1.Q score acceptable for the advanced training? The value of Z for the area 0.3 = 0.84.

(a) 0.84 (b) 118.4

(c) 138.4 (d) 115.4 (1 mark)

Answer:

(b) Let X denote the scores acceptable for advanced training. We assume that X is normally distributed with mean 110 and standard deviation 10 respectively.

The value of X for the area (0.5 – 0.2) = 0.3 is 0.84 (As given)

Let the lowest score acceptable be X_0

$\therefore P(X<X_0) = 0.84$

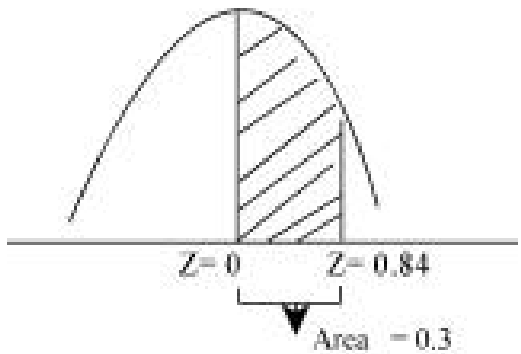
$$\Rightarrow P\frac{(x-110)}{10} < \frac{(x_0-110)}{10} = \phi(0.3)$$

$$\Rightarrow P\frac{X_0-110}{10} = \phi(0.3)$$

$$\frac{(X_0-110)}{10} = \phi(0.3)$$

$$\frac{X_0-110}{10} = \phi(0.3)$$

$X_0 = 118.4$



2007 - FEBRUARY

[4] The number of calls arriving at an internal switch board of an office is 96 per hour. Find the probability that there will be:

(i) not more than 3 calls on the board, per minute.

(ii) at least three calls in a minute on the board. [Given : $e^{-1.6} = 0.2019$]

(a) 0.08 and 0.92 respectively (b) 0.19 and 0.92 respectively

(c) 0.92 and 0.13 respectively (d) 0.92 & 0.08 respectively

(1 mark)

Answer:

(d) Let X be the no. of calls on board. Therefore, no. of calls per hour.

i.e. $m = \frac{96}{60} = 1.6$

(i) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$$= \frac{e^{-1.6}(1.6)^0}{0!} + \frac{e^{-1.6}(1.6)^1}{1!} + \frac{e^{-1.6}(1.6)^2}{21} + \frac{e^{-1.6}(1.6)^3}{31}$$

$$= e^{-1.6}\left[1 + 1.6 + \frac{(1.6)^2}{21} + \frac{(1.6)^3}{31}\right]$$

$= 0.2019 \times 4.56$

$= 0.920$

(ii) $P(X \geq 3)$

$= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$= 1 - 0.92$ (from (i))

$= 0.08$

[5] For a normal distribution with mean 150 and S.D. 45; find Q_1 and Q_3 :

(a) 119.35 and 190.65 respectively

(b) 119.65 and 180.35 respectively

(c) 180.35 and 119.65 respectively

(d) 123.45 and 183.65 respectively (1 mark)

Answer:

(b) Quartiles for Normal Distribution are given by;

$Q_1 = u - 0.6745\ \sigma$

$Q_3 = u + 0.6745\ \sigma$

$\therefore Q_1 = 150 - 0.6745 \times 45 = 119.65$

$Q_3 = 150 + 0.6745 \times 45 = 180.35$

[6] The probability density function of a normal variable x is given by :

(a) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ for

$0 < x < \infty$

(b) $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ for

$-\infty < x < \infty$

(c) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for

$-\infty < x < \infty$

(d) None of these (1 mark)

Answer:

(c) $f(x) = \frac{1}{\sigma}\sqrt{2\pi} \cdot C - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$ for $-< x < \infty$

Where m is the mean, σ is the standard deviation, $N = \frac{22}{7}$ or 3.1416 and
e = 2.71828 respectively.

2007 - MAY

[7] The Interval (μ – 3δ, μ + 3δ) covers:

(a) 95% area of normal distribution

(b) 96% area of normal distribution

(c) 99% area of normal distribution

(d) All but 0.27% area of a normal distribution (1 mark)

Answer:

(d) 99.73 percent of the values of a normal variable lies between (μ – 3σ) and (μ + 3σ). Thus, the probability that a value of x lies outside that limit is as low as, 0.0027. Hence, the interval (μ – 3σ, μ + 3σ) covers all but 0.27% area of a normal distribution.

[8] The overall percentage of failure in a certain examination is 0.30. What is the probability that out of a group of 6 candidates at least 4 passed the examination ?

(a) 0.74 (b) 0.71

(c) 0.59 (d) 0.67 (1 mark)

Answer:

(a) Let p = probability of success and q = probability of failure

$p = 0.70$, $q = 0.30$

Probability that atleast 4 candidates passed the examination

i.e. $p(x \geq 4)$

$= {}^6C_4 (0.7)^4 (0.3)^2 + {}^6C_5 (0.7)^5 (0.3)^1 + {}^6C_6 (0.7)^6 (0.3)^0$

$= 0.74$

[9] A manufacturer, who produces medicine bottles, finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contains at least two defectives :

[Given: $e^{-0.5} = 0.6065$]

(a) 7 (b) 13

(c) 9 (d) 11 (1 mark)

Answer:

(c) Given : $n = 500$ and $p = \frac{0.1}{100} = 0.001$

This implies $m = np = 500 \times 0.001 = 0.5$

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - [P(x = 0) + P(x = 1)]$

$= 1 - \left[\frac{e^{-0.5}(0.5)^0}{0!} + \frac{e^{-0.5}(0.5)^1}{1!}\right]$

$= 1 - [0.6065 + 0.5 \times 0.6065]$ ($\because$ Given $e^{-0.5} = 0.6065$)

$= 1 - 0.9097$

$= 0.0902$

Hence, the approx. number of boxes containing atleast 2 defective bottles.

$= 100 \times P(x \geq 2)$

$= 100 \times 0.0902$

$= 9$, approx.

2007 - AUGUST

[10] The number of methods of fitting the normal curve is :

(a) 4 (b) 3

(c) 2 (d) 1 (1 mark)

Answer:

(c) There are 2 methods of fitting the normal curve.

1. Ordinate method
2. Area method

[11] Suppose that weather records show that on an average 5 out of 31 days in October are rainy days. Assuming a binomial distribution with each day of October as an independent trial, then the probability that the next October will have at most three rainy days is :

(a) 0.4403 (b) 0.2403

(c) 0.3403 (d) None (1 mark)

Answer:

(b) p = Probability of a rainy day in October : $P = 5/31$

q = Probability of a non- rainy day in October

$\therefore q = 1 - P$

$\therefore q = 1 - \frac{5}{31} = \frac{26}{31}$

$n = 31$ (Number of days in October)

$$P(x) = {}^{31}C_r \left(\frac{5}{31}\right)^r \left(\frac{26}{31}\right)^{31-r}$$

Required Probability

$= P(0) + P(1) + P(2) + P(3)$

$= {}^{31}C_0 \left(\frac{5}{31}\right)^0 \left(\frac{26}{31}\right)^{31} + {}^{31}C_1 \times \left(\frac{5}{31}\right)^1 \left(\frac{26}{31}\right)^{30} + {}^{31}C_2 \left(\frac{5}{31}\right)^2 \left(\frac{26}{31}\right)^{29} + {}^{31}C_3 \left(\frac{5}{31}\right)^3 \left(\frac{26}{31}\right)^{28}$

$\left[1 \times \left(\frac{26}{31}\right)^3 - 31 \times \frac{5}{31} \times \left(\frac{26}{31}\right)^2 + {}^{31}C_2 \left(\frac{5}{31}\right)^2 \left(\frac{26}{31}\right) + {}^{31}C_3 \left(\frac{5}{31}\right)^3\right]$

$= 0.2403$

[12] If 5% of the families in Kolkata do not use gas as a fuel, what will be the probability of selecting 10 families in a random sample of 100 families who do not use gas as fuel?

[Given : $e^{-5} = 0.0067$]

(a) 0.038 (b) 0.028

(c) 0.048 (d) 0.018 (1 mark)

Answer:

(d) Given, P = 0.05 (families who do not use gas as fuel)

No. of families in the random sample n = 100

$\therefore \quad m = np = 100 \times 0.05 = 5$

Thus, Probability of selecting 10 families in the random sample.

$P(X=10) = \frac{e^{-5}(5)^{10}}{10!}$ (Given $e^{-5} = 0.0067$)

$= \frac{0.0067.(5)^{10}}{10!} = 0.018$

[13] If the 1st quartile and Mean Deviation about median of a normal distribution are 13.25 and 8 respectively, then the mode of the distribution is:

(a) 20 (b) 10

(c) 15 (d) 23 (1 mark)

Answer:

(a) 1st Quartile and M.D. a normal distribution are given as

$$Q_1 = \mu - 0.675\,\sigma$$

M.D. $= 0.8\,\sigma$

$\therefore\ \mu - 0.675\,\sigma = 13.25$

$0.8\,\sigma = 8,\ \sigma = 10$

$\therefore\ \mu = 20$

Mode $= \mu = 20$

2007 - NOVEMBER

[14] If 15 dates are selected at random, then the probability of getting two sundays is:

(a) 0.29 (b) 0.99

(c) 0.49 (d) 0.39 (1 mark)

Answer:

(a) Let p = probability of getting a Sunday in a week

$\therefore\ P = \frac{1}{7}$ and $q = 1 - P = \frac{6}{7}$

required probability

$$P(2) = {}^{15}C_2 \left(\frac{1}{7}\right)^2 \left(\frac{6}{7}\right)^{13} = 0.288 \approx 0.29$$

[15] If X is a Poisson variate with P (X = 0) = P (X = 1), then P (X = 2) = :

(a) $\frac{1}{6e}$ (b) $\frac{e}{6}$

(c) $\frac{1}{2e}$ (d) $\frac{e}{3}$ (1 mark)

Answer:

(c) We have,

$P(X = 0) = P(X = 1)$

$$\frac{e^{-m}.m^0}{0!} = \frac{e^{-m}.m^1}{1!} \quad m = 1$$

$$\therefore P(X = 2) = \frac{e^{-m}.m^2}{2!} = \frac{e^{-1}.(1)^2}{2!} = \frac{1}{2e}$$

[16] A sample of 100 dry battery cells tested to find the length of life produced the following results : $\bar{x}$ = 12 hours, σ = 3 hours. What percentage of battery cells are expected to have life less than 6 hours?

[Area under the normal curve from z = 0 to z = 2 is 0.4772]

(a) 2.28% (b) 2.56%

(c) 4.56% (d) 1.93% (1 mark)

Answer:

(a) Let X denote the length of life in hours. We assume that X is normally distributed with mean of life as 12 hour and standard deviations as 3 hours respectively

Probability of battery cells expected to have life less than 6 hours.

$= P(X < 6)$

$$= P\left(\frac{X-12}{3}\right) < \left(\frac{6-12}{3}\right)$$

$= P(Z < -2)$

$= \phi(-2)$

$= 1 - \phi(2)$

$= 1 - 0.9772$

$= 0.0228$ [Given z = 0 to z = 2 is 0.4772 $\therefore \phi(2) = 0.5 + 0.4772 = 0.9772$]

Hence the percentage of cells expected to have life than 6 hours

$= 0.228 \times 100 = 2.28\ \%$

2008 - FEBRUARY

[17] The method usually applied for fitting a binomial distribution is known as :

(a) Method of probability distribution
(b) Method of deviations
(c) Method of moments
(d) Method of least squares. (1 mark)

Answer:

(c) As regards, fitting a binomial distribution to an observed frequency distribution, it is necessary to estimate the parameters of the distribution.

One of the most convenient methods is **method of moments** which comprise of equating p moments of binomial distribution to p moments of the observed frequency distribution. Where p is the number of parameters to be estimated.

[18] If X follows a normal distribution with $\mu = 50$ and $\sigma = 10$. What is the value of $P\left(\frac{x \le 60}{x > 50}\right)$: [Area under the normal curve from z = 0 to z =1 is 0.3413].

(a) 0.6826 (b) 0.7354
(c) 0.1983 (d) 0.5492 (1 mark)

Answer:

(a) $P\left(\frac{X \le 60}{X > 50}\right) = \frac{p(x \le 60) \cap p(x > 50)}{p(x > 50)}$ [Since $p(A/B) = \left[\frac{p(A \cap B)}{P(B)}\right]$

$$= \frac{P(50 < x \le 60)}{1 - P(x \le 50)}$$

$$= \phi\left(\frac{60-50}{10}\right) \quad \phi\left(\frac{50-50}{10}\right)$$

$$= 1 - P\left(\frac{x-50}{10}\right) \le \left(\frac{50-50}{10}\right)$$

$$= \frac{\phi.(1) - \phi(0)}{1 - P(Z \leq 0)}$$

$$= \frac{\phi.(1) - \phi(0)}{1 - \phi(0)}$$

$$= \frac{0.8413 - 0.5}{1 - 0.5}$$

$$= 0.6826$$

[19] In a certain manufacturing process, 5% of the tools produced turn out to be defective. Find the probability that in a sample of 40 tools, atmost 2 will be defective :

[Given : $e^{-2} = 0.135$]

(a) 0.555 (b) 0.932

(c) 0.785 (d) 0.675 (1 mark)

Answer:

(d) Given n = 40 tools and p = defective tools = 0.05

$\therefore$ m = n p = 40 × 0.05

m = 2

Thus, P (X ≤ 2)

= P(X = 0) + P(X = 1) + p(X = 2)

$$= \frac{e^{-2}.(2)^0}{0!} + \frac{e^{-2}.(2)^1}{1!} + \frac{e^{-2}.(2)^2}{2!}$$

$$= e^{-2}[1 + 2 + 2]$$

$$= 0.135 \times 5$$

$$= 0.675$$

[20] Examine the validity of the following:
Mean and standard Deviation of a binomial distribution are 10 and 4 respectively.

(a) Not valid (b) Valid

(c) Both (a) & (b) (d) Neither (a) nor (b) (1 mark)

Answer:

(a) Given : $\mu = 10, \sigma = 4$

As we know, mean and standard deviation of a binomial distribution are given as $\mu = n\,p$, $\sigma = \sqrt{npq}$ respectively.

$\therefore n\,p = 10$, $n\,p\,q = 16$

$q = 1.6$ which is not possible

since $p(x) + p(x^1) = 1$

or $p + q = 1$

2008 - JUNE

[21] An experiment succeeds twice as often as it fails. What is the probability that in next five trials there will be at least three successes?

(a) $\frac{33}{81}$ (b) $\frac{46}{81}$

(c) $\frac{64}{81}$ (d) $\frac{25}{81}$ (1 mark)

Answer:

(c) According to the given statement

$p = 2q$

We know, $p + q = 1$

$\therefore p = \frac{2}{3}$, $q = \frac{1}{3}$

Required probability :

$P(x \geq 3) = p(3) + p(4) + p(5)$

$$= {}^5C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2 + {}^5C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^1 + {}^5C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^0$$

$$= \left(\frac{2}{3}\right)^3\left[\frac{10}{9} + \frac{10}{9} + \frac{4}{9}\right]$$

$$= \frac{64}{81}$$

[22] The probability than a man aged 45 years will die within a year is 0.012. What is the probability that of 10 men, at least 9 will reach their 46th birthday?

[Given : $e^{-0.12} = 0.88692$]

(a) 0.0935 (b) 0.9934

(c) 0.9335 (d) 0.9555 (1 mark)

Answer:

(b) Given : $p = 0.012$ and $n = 10$

$\therefore m = np = 10 \times 0.012 = 0.12$

Hence P(at least 9 will reach their 46th birthday)

$= p(X = 0) + p(X = 1)$

($\therefore$ if 9 men survive , 1 will die and if all 10 survive, 0 will die)

$$= \frac{e^{-12}.(.12)^0}{0!} + \frac{e^{-12}.(.12)^1}{1!}$$

$= e^{-0.12}[1 + 0.12]$

$= 0.88692 \times 1.12$

$= 0.9934$

[23] For a certain normal variate X, the mean is 12 and S.D. is 4. Find P ($X \geq 20$) : [Area under the normal curve from $z = 0$ to $z = 2$ is 0.4772]

(a) 0.5238 (b) 0.0472

(c) 0.7272 (d) 0.0228 (1 mark)

Answer:

(d) $p(x \geq 20) = 1 - p(x<20)$

$$= 1 - p\left(\frac{x-12}{4}\right) < \left(\frac{20-12}{4}\right)$$

$= 1 - p(z < 2)$

$= 1 - 0.9772$

$= 0.0228$

[24] In Poisson Distribution, probability of success is very close to :

(a) – 1 (b) 0

(c) 1 (d) None (1 mark)

Answer:

(b) n ⟶ number of trials, is indefinitely large , ie. n → ∞ while p ie. the probability of success for each trial is indefinitely small i.e p → 0. Therefore, the probability of success is very close in **zero.**

2008 - DECEMBER

[25] If x and y are two independent standard normal variables, then the distribution of $\frac{x}{y}$ is :

(a) Normal Distribution (b) Exponential Distribution

(c) Couchy's Distribution (d) Binomial Distribution (1 mark)

Answer:

(a) If x and y are two independent standard normal variables, then z = x/y also follows **normal distribution**.

[26] If the mean of a poisson variable X is 1, what is P (x = at least one)?

(a) 0.456 (b) 0.821

(c) 0.632 (d) 0.254 (1 mark)

Answer:

(c) Mean of a Poisson variable is given by,

$\mu = m$

i.e. $\mu = m = 1$

We have to find,

$P(x \geq 1)$

$= 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - \frac{e^{-1} \cdot (1)^0}{0!}$

$= 1 - \frac{1}{e}$

$= 1 - 0.3679$ (as e = 2.71828)

$= 0.632$

[27] What is the probability of getting 3 heads if 6 unbiased coins are tossed simultaneously?

(a) 0.3125 (b) 0.25

(c) 0.6875 (d) 0.50 (1 mark)

Answer:

(a) If x denotes the number of heads, then x follows binomial distribution with parameters n = 6 and

p = prob of success = 1/2

q = prob of failure = 1 – 1/2 = 1/2, being given the coins are unbiased.

The probability mass function of x is given by :

$f(x) = {}^6C_3 (1/2)^3 (1/2)^3$

$= 20 \times \left(\frac{1}{2}\right)^6$

$= 0.3125$

2009 - JUNE

[28] In a poisson distribution P (x = 0) = P (X = 2). Find E (x).

(a) $\sqrt{2}$ (b) 2

(c) – 1 (d) 0 (1 mark)

Answer:

(a) E (x) stands for mean of the distribution.

Let x be a Poisson variate with parameter m.

The probability function of x is then given by :

$f(x) = \frac{e^{-m} \cdot m^x}{x!}$ for x = 0, 1, 2 ..... as

$$f(o) = f(2)$$

$$\frac{e^{-m}.m^{o}}{o!} = \frac{e^{-m}.m^{2}}{2!}$$

$$\frac{m^{o}}{1} = \frac{m^{2}}{2}$$

$$1 = \frac{m^{2}}{2}$$

$$m^2 = 2$$

$$m = \sqrt{2} \cong 1.414$$

Therefore, the mean of this distribution is $E(x) = m = \sqrt{2}$

2009 - DECEMBER

[29] Shape of Normal Distribution Curve:

(a) Depends on its parameters

(b) Does not depend on its parameters

(c) Either (a) or (b)

(d) Neither (a) nor (b) (1 mark)

Answer:

(a) Shape of the Normal Distribution curve depends on its parameters. [self- explanatory].

[30] For binomial distribution $E(x) = 2$, $V(x) = 4/3$. Find the value of n.

(a) 3 (b) 4

(c) 5 (d) 6 (1 mark)

Answer:

(d) $E(x) = np = 2$

$v(x) = npq = 4/3.$

$np = 2$ .....(1)

$$npq = \frac{4}{3}$$

substituting the value of np from (1):

$$2 \times q = \frac{4}{3}$$

$$2q = \frac{4}{3}$$

$$q = \frac{4}{3 \times 2} = \frac{2}{3}$$

$$\therefore \quad q = \frac{2}{3}$$

$$p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$$

$$np = 2$$

$$n \times \frac{1}{3} = 2$$

$$n = 6$$

$$\therefore \quad n = 6.$$

[31] What are the parameters of binomial distribution?

(a) n (b) p

(c) Both n and p (d) None of these (1 mark)

Answer:

(c) Binomial Distribution is a biparametric distribution, characterized by 'n' and 'p' [self-explanatory].

2010 - JUNE

[32] The Variance of standard normal distribution is

(a) 1 (b) μ

(c) σ^2 (d) 0

Answer:

(a) In standard normal distribution

mean = 0

Variance = 9

[33] For a Poisson distribution P (x = 3) = 5 P(x = 5), then S.D. is

(a) 4 (b) 2

(c) 16 (d) $\sqrt{2}$ (1 mark)

Answer:

(d) Let x be a Poisson variate with parameter m. The probability function of x is then given by :

$f(x) = \frac{e^{-m} m^x}{x!}$ for x = 0, 1, 2 ..... as now,

$P(x = 3) = 5P(x = 5)$

$f(3) = 5 f(5)$

$\frac{e^{-m} m^3}{3!} = \frac{5e^{-m} m^5}{5!}$

$20 = 5m^2$

$m^2 = 4$

Variance = m = 2

$\therefore$ SD = $\sqrt{\text{Variance}}$

SD = $\sqrt{2}$

[34] For a Binomial distribution B (6, p), P(x = 2) = 9p(x = 4), then P is

(a) 1/2 (b) 1/3

(c) 10/13 (d) 1/4 (1 mark)

Answer:

(d) We are given that n = 6. The probability mass function of x is given by

$f(x) = {}^nc_x p^x q^{n-x}$

$= {}^6c_x p^x q^{6-x}$, for x = 0, 1, 2 ....., 6

Thus, $P(x = 2) = f(2) = {}^6c_2 p^2 q^{6-2} = 15p^2 q^4$

and $P(x = 4) = f(4) = {}^6c_4 p^4 q^{6-4} = 15p^4 q^2$

Hence, $P(x = 2) = 9P(x = 4)$

$15p^2 q^4 = 9.15p^4 q^2$

$15p^2 q^2 (q^2 - 9p^2) = 0$

$q^2 - 9p^2 = 0$ (as $p \neq 0$ and $q \neq 0$)
$(1-p)^2 - 9p^2 = 0$ (as $q = 1-p$)
$(1 - p + 3p) = 0$ or $(1 - p - 3p) = 0$
$p = -\frac{1}{2}$ or $p = 1/4$

Thus, $p = 1/4$ (as $p \neq -\frac{1}{2}$)

[35] In Binomial distribution n = 9 and P = 1/3, what is the value of variance:
(a) 8 (b) 4
(c) 2 (d) 16 (1 mark)

Answer:

(c) In Binomial distribution,
Variance = npq
n = 9
$p = \frac{1}{3}$
$q = \frac{2}{3}$
$\therefore$ Variance = $9 . \frac{1}{3} . \frac{2}{3} = 2$

2010 - DECEMBER

[36] If standard deviation of a poisson distribution is 2, then its
(a) Mode is 2 (b) Mode is 4
(c) Modes are 3 and 4 (d) Modes are 4 and 5 (1 mark)

Answer:

(c) Given σ = S.D. = 2 $\Rightarrow$ Variance = $\sigma^2 = 4$

$\because$ In poisson distribution
Mean = Variance

$\because$ m = 4, which is an integer

$\because$ it is bi-modal
Modes are m and (m – 1)

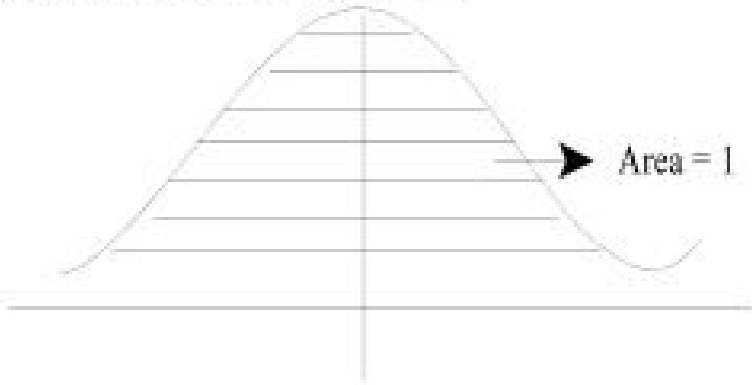
[37] The area under the Normal curve is

(a) 1 (b) 0

(c) 0.5 (d) – 1 (1 mark)

Answer:

(a) Area under the Normal curve = 1



[38] For a normal distribution $N(\mu,\sigma^2)$,
$P(\mu-3\sigma < x < \mu+3\sigma)$ is equal to

(a) 0.9973 (b) 0.9546

(c) 0.9899 (d) 0.9788 (1 mark)

Answer:

(a) We Know that
$P(\mu - 3\sigma < x < \mu + 3\sigma)$
$= 0.9973$

[39] If for a Binomial distribution B (n,p,) the mean = 6 and Variance = 2 then 'p' is

(a) 2/3 (b) 1/3

(c) 3/5 (d) 1/4 (1 mark)

Answer:

(a) Mean = 6 = np

Variance = 2 = npq {For Binomial Distribution}

$$\frac{npq}{np} = \frac{2}{6} \Rightarrow q = \frac{1}{3}$$

$$p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$$

$$n = \frac{2}{}$$

2011 - JUNE

[40] If the inflexion points of a Normal Distribution are 6 and 14. Find its Standard Deviation ?

(a) 4 (b) 6
(c) 10 (d) 12. (1 mark)

Answer:

(a) ∵ The inflexion points of a Normal Distribution are given as
$(\mu + \sigma)$ and $(\mu - \sigma)$
here, we are given :
$\mu + \sigma = 14$ (1)
and, $\mu - \sigma = 6$ (2)
Solving (1) and (2) we get
$\mu = 10$ and $\sigma = 4$
Hence S.D $(\sigma) = 4$

[41] In a Binomial Distribution, if mean is k-times the variance, then the value of 'k' will be_____.

(a) p (b) $\frac{1}{p}$
(c) $1 - p$ (d) $\frac{1}{1-p}$ (1 mark)

Answer:

(d) In Binomial Distribution :
Mean = np & Variance = npq
by question, here
Mean = K.
Variance np = K. npq

$\therefore \quad K = 1/q$

$\therefore \quad K = \frac{1}{1-p} \quad [\because p + q = 1]$

[42] If $x \sim N(3,36)$ and $y \sim N(5,64)$ are two independent Normal variate with their standard parameters of distribution, then if $(x + y) \sim N(8,A)$ also follows normal distribution. The value of A will be _____.

(a) 100 (b) 10

(c) 64 (d) 36 (1 mark)

Answer:

(b) We Know,

If $x \sim N(\mu_1, \sigma_1^2)$ and $y \sim N(\mu_2, \sigma_2^2)$

Then

$x + y \sim N\left(\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2}\right)$

Where $\sim N(\mu,\sigma)$

(Say)

$\mu = (\mu_1 + \mu_2)$ and $\sigma = \left(\sqrt{\sigma_1^2 + \sigma_2^2}\right)$

Here, $x \sim N(3,36)$ and $y \sim N(5,64)$

$\therefore \quad x + y \sim N(3 + 5, \sqrt{36 + 64} = N(8,A)$

$\Rightarrow A = \sqrt{36 + 64}$

$\therefore A = 10$

2011 - DECEMBER

[43] The mean of Binomial distribution is 20 and Standard deviation is 4 then;

(a) $n = 100, p = 1/5, q = 4/5$ (b) $n = 50, p = 2/5, q = 2/5$

(c) $n = 100, p = 2/5, q = 4/5$ (d) $n = 100, p = 1/5, q = 3/5$

(1 mark)

Answer:

(a) Here,

Mean = 20 S.D = 4

np = 20 .....(1) Variance = $(4)^2$

Variance = 16

npq = 16.....(2)

Divide (2)/(1)

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

$$p = 1 - q$$

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Putting the value of p in eq (1)

$$n \times \frac{1}{5} = 20$$

$$n = 20 \times 5 = 100$$

[44] A Company has two cars which it hires out during the day. The number of Cars demanded in a day has poison distribution with mean 1.5. Then percentage of days on which only one car was in demand is equal to

(a) 23.26 (b) 33.47

(c) 44.62 (d) 46.40 (1 mark)

[Given Exp (– 1.5) = 0.2231]

Answer:

(b) Given the mean of Poisson distribution (m) = 1.5

Then

Poisson parameter (μ) = m = 1.5

We know by Poisson distribution

$$P(x) = \frac{e^{-m} \cdot m^x}{x!}$$

Here

$m = 1.5, x = 1$

$$P(1) = \frac{e^{-1.5} \cdot (1.5)}{1!}$$

$$= \frac{0.2231 \times 1.5}{1}$$

$$= 0.33465$$

$$= 0.3347$$

% of $P(1) = 0.3347 \times 100\ \% = 33.47\%$

[45] The binomial distribution with mean 3 & variance 2 is:

(a) $\left(\frac{2}{4} + \frac{1}{4}\right)^{2-9}$ (b) $\left(\frac{2}{6} + \frac{1}{6}\right)^{2-9}$

(c) $\left(\frac{2}{3} + \frac{1}{3}\right)^{2-9}$ (d) $\left(\frac{2}{5} + \frac{1}{5}\right)^{2-9}$ (1 mark)

Answer:

(c) Given mean = 3

$np = 3$ .....(1)

Variance = 2

$npq = 2$ .....(2)

Divide (2)/(1) we get

$$\frac{npq}{np} = \frac{2}{3} \Rightarrow q = \frac{2}{3}$$

$$p = 1 - q$$

$$p = 1 - \frac{2}{3} = \frac{1}{3}$$

Putting the value of p in Equation (1)

$$n \times \frac{1}{3} = 3$$

$n = 9$

The Binomial distribution is

$$(q + p)^n = \left[\frac{2}{3} + \frac{1}{3}\right]^9$$

2012 - JUNE

[46] For binomial distribution

(a) Variance < Mean (b) Variance = Mean

(c) Variance > Mean (d) None of the above. (1 mark)

Answer:

(a) For Binomial distribution

$npq < np$

Variance < Mean

[47] If x is a Poisson variate and E(x) = 1, then P(x > 1) is

(a) $1 - \frac{e^{-1}}{2}$ (b) $1 - e^{-1}$

(c) $1 - 2e^{-1}$ (d) $1 - \frac{5}{2}e$ (1 mark)

Answer:

(c) E(x) = 1, we known P (x) $= \frac{e^{-m} m^x}{L_x}$; E(x) = m

$\therefore \quad P(x > 1) = 1 - P(x < 1)$

$= 1 - [P(x = 0) + P(x = 1)]$

$= 1 - \left[\frac{e^{-1} \cdot 1^0}{L^0} + \frac{e^{-1} \cdot 1^1}{L^1}\right]$

$= 1 - [e^{-1} + e^{-1}]$

$= 1 - 2e^{-1}$

[48] The mean and the variance of a random variable X having the probability density function $P(X = x) = \exp\{-(x - 4)^2\}/\sqrt{\pi}, -\infty < x < \infty$ is.

(a) 4, $\frac{1}{2}$ (b) 4, $\frac{1}{\sqrt{2}}$

(c) 2, 2 (d) 2, $\frac{1}{2}$ (1 mark)

Answer:

(a) We know, the probability distribution function for normal distribution is :

$$P(X = x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty$$

Given in equation :

$$P(X = x) = \frac{1}{\sqrt{\pi}} e^{-(x-4)^2}$$

Comparing given function with the standard form, we get

Mean (u) $= 4$

S.D (σ) $= \frac{1}{\sqrt{2}}$

$\therefore$ Variance (σ^2) $= \frac{1}{2}$

2012 - DECEMBER

[49] In a Normal Distribution

(a) The first and second quartile are equidistant from median

(b) The second and third quartiles are equidistant from the median

(c) The first and third quartiles are equidistant from the mean

(d) None of the above. (1 mark)

Answer:

(c) In a Normal Distribution :
"The first and third quartiles are equidistant from the mean".

[50] If parameters of a binomial distribution are n and p then, this distribution tends to a Poisson distribution when

(a) $n \to \infty, p \to 0$ (b) $p \to 0, np = \lambda$

(c) $n \to \infty, np = \lambda$ (d) $n \to \infty, p \to 0, np = \lambda$ (1 mark)

where 'λ' is a finite constant

Answer:

(d) If parameters of a binomial distribution are n and p then this distribution tends to a Poisson distribution when

$n \to \infty$, $p \to 0$, $np = \lambda$

Where 'λ' is a finite constant

[51] If a random variable x follows Poisson distribution such that E(x) = 30, then the variance of the distribution is

(a) 7 (b) 5

(c) 30 (d) 20 (1 mark)

Answer:

(c) In Poisson distribution

Mean = Variance

$\because$ E(x) = 30

Mean = E(x) = 30

So, Variance = 30

[52] In a normal distribution quartile deviation is 6, the standard deviation will be

(a) 4 (b) 9

(c) 7.5 (d) 6 (1 mark)

Answer:

(b) In normal distribution

4 S.D. = 6. Q.D.

$$\text{S.D.} = \frac{6}{4}\ \text{Q.D.}$$

$$= \frac{6}{4} \times 6$$

$$= 9$$

2013 - JUNE

[53] The mode of the Binomial Distribution for which the mean is 4 and variance 3 is equal to?

(a) 4 (b) 4.25

(c) 4.5 (d) 4.1 (1 mark)

Answer:

(a) In Binomial Distribution

Mean = np = 4 .....(1)

Variance = npq = 3 .....(2)

Sum (1) & (2)

$4q = 3$

$q = 3/4$

$p = 1 - q$

$= 1 - 3/4$

$p = 1/4$ in equation (1)

$n \times \frac{1}{4} = 4$

$\boxed{n = 16}$

$(n + 1)\, p = (16 + 1) \times \frac{1}{4}$

$= 4.25$ which is non Integer

So Mode $= 4$

[54] For Poisson Distribution:

(a) Mean and Standard Deviations are equal

(b) Mean and variance are equal

(c) Standard Deviation and variance are equal

(d) Both (a) and (b) are correct (1 mark)

Answer:

(b) In Poisson Distribution mean and variance are equal.

[55] Which of the following is not a characteristic of a normal probability distribution?

(a) Mean of the normally distributed population lies at the centre of its normal curve.
(b) It is multi-modal
(c) The mean, median and mode are equal
(d) It is a symmetric curve (1 mark)

Answer:

(b) It is multi-modal (False)

[56] An approximate relation between quartile deviation (QD) and standard deviation (S.D) of normal distribution is:

(a) 5 QD = 4 SD (b) 4 QD = 5 SD
(c) 2 QD = 3 SD (d) 3 QD = 2 SD (1 mark)

Answer:

(d) We know that
In normal distribution
4 S.D = 5 M. D = 6 Q.D
So 4 S.D = 6 Q.D
2 S.D = 3 Q.D
or 3 Q.D = 2 S.D

[57] In a binomial Distribution with 5 independent trials, probability of 2 and 3 successes are 0.4362 and 0.2181 respectively. Parameter 'p' of the binomial distribution is:

(a) 3/4 (b) 1/3
(c) 2/3 (d) 1/4 (1 mark)

Answer:

(b) Given
$n = 5$, $P(x = 2) = 0.4362$
$P(x = 3) = 0.2181$
$P(x = 3) = {}^5C_3 \cdot P^3 \cdot q^{5-3} = 10 \cdot P^3q^2$
$0.2181 = 10\ p^3q^2$ _____ (1)
and $P(x = 2) = {}^5C_2 \cdot P^2 \cdot q^{5-2} = 10\ p^2q^3$

equal (1)/ eq (2)

$$\frac{0.2181}{0.4362} = \frac{10\,p^2q^2}{10\,p^2q^3}$$

$$\frac{1}{2} = \frac{p}{q} \Rightarrow q = 2p$$

$$1 - p = 2p$$

$$2p + p = 1$$

$$3p = 1 \Rightarrow p = 1/3$$

2013 - DECEMBER

[58] In a certain Poisson frequency distribution, the probability corresponding to two successes is half the probability corresponding to three successes. The mean of the distribution is

(a) 6 (b) 12

(c) 3 (d) 2.45 (1 mark)

Answer:

(a) Given

$$P(x = 2) = \frac{1}{2} P(x = 3)$$

$$2\ P(x = 2) = P(x = 3)$$

$$2.\ \frac{e^{-m}.m^2}{2!} = \frac{e^{-m}.m^3}{3!}$$

$$\frac{2}{2} = \frac{m}{6}$$

$$m = 6 \times \frac{2}{2} = 6$$

2014 - JUNE

[59] Mean and Variance of a binomial variance are 4 and $\frac{4}{3}$ respectively then $P(x \geq 1)$ will be_____.

(a) $\frac{728}{729}$ (b) $\frac{1}{729}$

(c) $\frac{723}{729}$ (d) None of the above. (1 mark)

Answer:

(a) For Binomial Variable

Mean = np = 4 .....(1)

Variance = npq = $\frac{4}{3}$ .....(2)

From (1) & (2)

$$4 \times q = \frac{4}{3}$$

$$q = \frac{1}{3}$$

$$p = 1 - \frac{1}{3} = \frac{2}{3}$$

$$np = 4$$

$$n \times \frac{2}{3} = 4$$

$$n = \frac{12}{2} = 6$$

$$p(x \geq 1) = 1 - p(x < 1)$$

$$= 1 - p(x = 0)$$

$$= 1 - {}^{6}C_{0} \cdot \left(\frac{2}{3}\right)^{0} \cdot \left(\frac{1}{3}\right)^{6}$$

$$= 1 - |x| \times \frac{1}{729} = 1 - \frac{1}{729} = \frac{728}{729}$$

[60] 5,000 students were appeared in an examination. The mean of marks was 39.5 with a Standard Deviation 12.5 marks. Assuming the distribution to be normal, find the number of students recorded more than 60% marks.

Given: When $Z = 1.64$, Area of normal curve = 0.4495

(a) 1,000 (b) 505

(c) 252 (d) 2,227 (1 mark)

Answer:

(c) Probability that students recorded more than 60% marks = $P(x > 60)$

$$= 1 - P(x \leq 60)$$

$$= 1 - P\left(\frac{x - \bar{x}}{\sigma} \leq \frac{60 - 39.5}{12.5}\right)$$

$$= 1 - P(Z \leq 1.64)$$

$$= 1 - \phi(1.64)$$

$$= 1 - (0.4495 + 0.5)$$

$$= 1 - 0.9495$$

$$= 0.0505$$

Thus, the Number of students having marks more than 60%

$$= 5000 \times 0.0505$$

$$= 252.5$$

[61] If a variate X has, mean > variance, then its distribution will be _____.

(a) Binomial distribution (b) Poisson distribution

(c) Normal distribution (d) T-distribution (1 mark)

Answer:

(a) In Binomial distribution

Mean > Variance

2014 - DECEMBER

[62] If six coins are tossed simultaneously. The probability of obtaining exactly two heads are:

(a) 1/64 (b) 63/64

(c) 15/64 (d) None of these (1 mark)

Answer:

(c) Here Total trial (n) = 6

For coin $p = \frac{1}{2}$, $q = 1 - \frac{1}{2} = \frac{1}{2}$

$$P(X = x) = {}^{n}C_x \, p^x . q^{n-x}$$

$$P(X = 2) = {}^{6}C_2 \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{6-2}$$

$$= \frac{6 \times 5}{2 \times 1} \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^4$$

$$= 15 \times \left(\frac{1}{2}\right)^{2+4}$$

$$= 15 \times \left(\frac{1}{2}\right)^6$$

$$= \left(\frac{15}{64}\right)$$

[63] If x and y are two independent normal random variables, then the distribution of x + y is:

(a) Normal (b) T-distribution

(c) Chi-square (d) F-distribution

Answer:

(a) If x and y are two independent Normal random variables, then the distribution of x + y is Normal.

[64] For a normal distribution having mean = 2 and variance = 4, the fourth central moment μ_4 is:

(a) 16 (b) 32

(c) 48 (d) 64 (1 mark)

Answer:

(c) For Normal Distribution Mean = 2, Variance = 4

Fourth central moments μ_4 =?

We know that Normal curve is always

Meso kuritic then $\beta_2 = 3$

moment coefficient of kurtosis

$$(\beta_2) = \frac{\mu_4}{\mu_2^2}$$

Here, μ_2 = Variance = 4, $\beta = 3$

$$3 = \frac{\mu_4}{4^2}$$

$$\mu_4 = 3 \times 4^2 = 3 \times 16 = 48$$

Shortcut: Fourth moments $\mu_4 = 3\sigma^4 = 3(4)^2 = 48$

[65] T-test can be used only when the sample has been taken from

(a) Binomial Population (b) Poisson Population

(c) Normal Population (d) Exponential Population (1 mark)

Answer:

(c) t-test can be used only when the sample has been taken from **Normal Population**.

[66] For a binomial distribution with mean = 4 and variance = 3, the third central moment μ_3 is:

(a) 5/2 (b) 7/4

(c) 3/2 (d) 1/3 (1 mark)

Answer:

(c) For Binomial distribution

Mean = 4 Variance = 3

$np = 4$ _____ (1) $npq = 3$ _____ (2)

$4q = 3$

$q = 3/4$

then $p = 1 - q = 1 - \frac{3}{4} = 1/4$

Putting p = 1/4 in equation (1)

$$n\frac{1}{4} = 4 \Rightarrow n = 16$$

The third central moment

$$\mu_3 = npq\,(q - p)$$

$$= 16 \times \frac{1}{4} \times \frac{3}{4}\left(\frac{3}{4} - \frac{1}{4}\right)$$

$$= 3\left(\frac{2}{4}\right) = \frac{3}{2}$$

2015 - JUNE

[67] If x is a binomial variable with parameters n and p, then x can assume
(a) any value between 0 and n
(b) any value between 0 and n, both inclusive
(c) any whole number between 0 and n, both inclusive
(d) any number between 0 and infinity (1 mark)

Answer:

(c) If x is a binomial variable with parameters n & p, then x can assume any whole number between 0 and n, both inclusive.

[68] In _____ distribution, mean = variance
(a) Normal (b) Binomial
(c) Poisson (d) None

Answer:

(c) In Poisson Distribution, Mean = Variance

[69] Under a normal curve $\bar{x} \pm 3\sigma$ covers _____
(a) 100% of the area (item values)
(b) 99%
(c) 99.73%
(d) 99.37% (1 mark)

Answer:

(c) Under a normal curve $(\bar{x} \pm 3\sigma)$

2015 - DECEMBER

[70] If 'x' is a binomial variable with parameter 15 and $\frac{1}{3}$, then the value of the mode of the distribution:

(a) 5 (b) 5 and 6
(c) 5.50 (d) 6 (1 mark)

Answer:

(a) In Binomial Variable (Distribution)

$$x \sim B(n, p)$$

$$x \sim B(15, \frac{1}{3})$$

$$n = 15, P = \frac{1}{3}$$

$$\text{Mode} = (n + 1)P$$

$$= (15 + 1).\frac{1}{3}$$

$$= 16 \times \frac{1}{3} = 5.33 \text{ (which is non Integer)}$$

$$= 5$$

[71] Standard deviation of binomial distribution is:

(a) $\sqrt{np}$ (b) $(np)^2$
(c) $\sqrt{npq}$ (d) $(npq)^2$ (1 mark)

Answer:

(c) Standard Deviation of binomial distribution is $\sqrt{npq}$

[72] The wages of workers of factory follows:

(a) Binomial distribution (b) Poisson distribution
(c) Normal distribution (d) Chi-square distribution

Answer:

(c) The wages of workers of factory follow **Normal Distribution.**

2016 - JUNE

[73] The normal curve is:

(a) Positively skewed (b) Negatively skewed

(c) Symmetrical (d) All these (1 mark)

Answer:

(c) Normal curve is symmetrical.

[74] For a Poisson variate X, P (X = 1) = P (X = 2), what is the mean of X ?

(a) 1 (b) $\frac{3}{2}$

(c) 2 (d) $\frac{5}{2}$ (1 mark)

Answer:

(c) For $x \sim P(m)$

$P(x = 1) = P(x = 2)$

$$\frac{e^{-m}.m^1}{1!} = \frac{e^{-m}.m^2}{2!}$$

$$\frac{m}{1} = \frac{m^2}{2}$$

$$\boxed{m = 2}$$

[75] In a discrete random variable X follows uniform distribution and assumes only the values 8,9,11,15,18,20. Then P ($X \leq 15$) is____

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{2}{5}$ (1 mark)

Answer:

(c) Given data

8, 9, 11, 15, 18, 20

Total No. of data n(s) = 6

$$P(x \leq 15) = \frac{n(A)}{n(s)} = \frac{4}{6} = \frac{2}{3}$$

2016 - DECEMBER

[76] If x and y are independent normal variates with Mean and Standard Deviation as μ_1 and μ_2 and σ_1 and σ_2 respectively, then z = x+y also follows normal distribution with

(a) Mean = $\mu_1 + \mu_2$ and S.D. = 0 respectively

(b) Mean = 0 and S.D. = $\sigma_1^2 + \sigma_2^2$

(c) Mean = $\mu_1 + \mu_2$ and S.D. = $\sqrt{\sigma_1^2 + \sigma_2^2}$

(d) None of these. (1 mark)

Answer:

(c) If x and y are two Independent variables of Normal Distribution

if $x \sim N(\mu_1, \sigma_1^2)$

and $y \sim N(\mu_2, \sigma_2^2)$

then z = x + y

$z = N(\mu_1, \sigma_1^2) + N(\mu_2, \sigma_2^2)$

$z = N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

Mean = $\mu_1 + \mu_2$, Variance = $\sigma_1^2 + \sigma_2^2$

S.D = $\sqrt{\sigma_1^2 + \sigma_2^2}$

[77] A Poisson random variable has $\mu_4 = 2$, its variance i.e. μ_2 is

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{3}{2}$ (1 mark)

Answer:

(a) If X ~ P (m) Fourth moment of Poisson distribution

$$\mu_2 = 3\mu_2$$

$$2 = 3\sigma^2 \ [\because \mu_2 \rightarrow \sigma^2]$$

$$\sigma^2 = \frac{2}{3}$$

$$\text{Variance} = \frac{2}{3}$$

[78] Name the distribution which has Mean = Variance

(a) Binomial (b) Poisson

(c) Normal (d) Chi-square (1 mark)

Answer:

(b) In Poisson distribution, mean and variance are equal.

[79] An example of a bi-parametric continuos probability distribution:

(a) Binomial (b) Poisson

(c) Normal (d) (a) and (b) (1 mark)

Answer:

(c) Normal distribution is the example of bi-parametric probability distribution.

If x- is a continuous variable.

2017 - JUNE

[80] If X ~ N (50, 16), then which of the following is not possible:

(a) P(X > 60) = 0.30 (b) P(X < 50) = 0.50

(c) P(X < 60) = 0.40 (d) P(X > 50) = 0.50 (1 mark)

Answer:

(c) If X ~ N (50, 16) then P(X<60) = 0.40 is not possible.

[81] If for a distribution mean = variance, then the distribution is said to be:

(a) Normal (b) Binomial

(c) Poisson (d) None of the above. (1 mark)

Answer:

(c) In Poisson distribution mean and variance are equal.

[82] For a Binomial distribution if variance = $(\text{Mean})^2$, then the values of n and p will be:

(a) 1 and $\frac{1}{2}$ (b) 2 and $\frac{1}{2}$

(c) 3 and $\frac{1}{2}$ (d) 1 and 1 (1 mark)

Answer:

(a) Given, In Binomial Distribution

we know that mean = np

Variance = npq

Given, Variance = $(\text{Mean})^2$

Here option (A) we have $n = 1, p = \frac{1}{2}, q = 1 - \frac{1}{2} = \frac{1}{2}$

$$(\text{Mean})^2 = (np)^2 = \left(1 \times \frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\text{Variance} = npq = 1 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

So that Variance = $(\text{Mean})^2$

∴ Option a is correct.

2017 - DECEMBER

[83] In a normal distribution about 95 per cent of the observations lie between _____ and _____.

(a) $\mu - 2\sigma, \mu + 2\sigma$ (b) $\mu - 3\sigma, \mu + 3\sigma$

(c) $\mu - 1.96\sigma, \mu + 1.96\sigma$ (d) $\mu - 2.58\sigma, \mu + 2.58\sigma$ (1 mark)

Answer:

(c) In a Normal distribution about 95% of the observations lies b/w $\mu - 1.96\sigma$ and $\mu + 1.96\sigma$.

[84] An example of a bi-parametric discrete probability distribution is

(a) Binomial distribution (b) Poisson distribution

(c) Normal distribution (d) Both (a) and (b) (1 mark)

Answer:

(a) An example of a bi-parametric discrete probability distribution is Binomial distribution.

[85] In _____ distribution, mean = variance

(a) Normal (b) Binomial

(c) Poisson (d) None of the above. (1 mark)

Answer:

(c) In **Poisson** Distribution, Mean = Variance

2018 - MAY

[86] The variance of a binomial distribution with parameters n and p is:

(a) $np^2(1-p)$ (b) $\sqrt{np-(1-p)}$

(c) $nq(1-q)$ (d) $n^2p^2(1-P)^2$ (1 mark)

Answer:

(c) The variance of a binomial distribution

$= npq$

$= nqp$

$= nq(1-q)$

[87] X is a poisson variate satisfying the following condition 9 P(X = 4) + 90 (X = 6) = P (X = 2).What is the value of $P(X \leq 1)$?

(a) 0.5655 (b) 0.6559

(c) 0.7358 (d) 0.8201 (1 mark)

Answer:

(c) Given $X \sim P(m)$

$$P(x = 2) = 9\ P(x = 4) + 90\ P(x = 6)$$

$$\frac{e^{-m}.m^2}{2!} = +\frac{9.e^{-m}.m^4}{4!} + \frac{90.e^{-m}.m^6}{6!}$$

$$\frac{90.e^{-m}.m^6}{6!} + \frac{9.e^{-m}.m^4}{4!} - \frac{e^{-m}.m^2}{2!} = 0$$

$$e^{-m}.m^2\left[\frac{90.m^4}{6!} + \frac{9m^2}{4!} - \frac{1}{2!}\right] = 0$$

$$e^{-m}.m^2\left[\frac{90.m^4}{720} + \frac{9m^2}{24} - \frac{1}{2}\right] = 0$$

$$\frac{e^{-m}.m^2}{2}\left[\frac{90.m^4}{360} + \frac{9m^2}{12} - 1\right] = 0$$

$$\frac{e^{-m}.m^2}{2}\left[\frac{m^4}{4} + \frac{3m^2}{4} - 1\right] = 0$$

$$\frac{e^{-m}.m^2}{2}\left[\frac{m^4 + 3m^2 - 4}{4}\right] = 0$$

$$\frac{e^{-m}.m^2}{8}(m^4 + 3m^2 - 4) = 0$$

$$m^4 + 3m^2 - 4 = 0$$

$$m^4 + 4m^2 - m^2 - 4 = 0$$

$$m^2(m^2 + 4) - 1(m^2 + 4) = 0$$

$$(m^2 + 4)(m^2 - 1) = 0$$

if $m^2 + 4 = 0$ if $m^2 - 1 = 0$

$m^2 = -4$ if $m^2 = +1$

$m^2 = \pm\sqrt{1}$

$m^2 = \pm 1$

$m = (\because m > 0)$

$$P(x \leq 1) = P(x = 0) + P(x = 1)$$

$$= \frac{e^{-1}.1^0}{0!} + \frac{e^{-1}.1^1}{1!} = \frac{1}{e} + \frac{1}{e} = \frac{2}{e}$$

$$\frac{2}{2.7182} = 0.7358$$

[88] What is the first quartile of x having the following probability density function?

$$f(x) = \frac{1}{\sqrt{72\pi}} e^{-(x-10)^2/72} \text{ for } -\infty < x < \infty$$

(a) 4 (b) 5
(c) 5.95 (d) 6.75 (1 mark)

Answer:

(c) Given $f(x) = \frac{1}{\sqrt{72\pi}} . e^{\frac{-(x-10)^2}{72}}$ for $-\infty < x < \infty$

$$f(x) = \frac{1}{6\sqrt{2\pi}} . e^{\frac{-1(x-\mu)^2}{72}}$$

on company

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} . e^{\frac{-1(x-\mu)^2}{2\sigma^2}}$$

We get

$\sigma = 6, \mu = 10$

First quartile $Q_1 = \mu - 0.675\ \sigma$

$= 10 - 0.675 \times 6$

$= 10 - 4.05$

$= 5.95$

[89] An example of a bi-parametric discrete probability distribution is

(a) binomial distribution (b) Poisson distribution
(c) normal distribution (d) both (a) and (b) (1 mark)

Answer:

(a) Binomial distribution is example of a bi-parametric discrete probability distribution.

[90] Probability distribution may be

(a) discrete (b) continuous

(c) infinite (d) (a) or (b) (1 mark)

Answer:

(d) Probability distribution may be discrete or continuous .

[91] If the area of standard normal curve between z = 0 to z = 1 is 0.3412, then the value of ϕ (1) is.

(a) 0.5000 (b) 0.8413

(c) – 0.5000 (d) 1 (1 mark)

Answer:

(b) The area of standard normal curve between z = 0 to z = 1 is 0.3413

then $\phi(1) = 0.3413 + 0.5$

$= 0.8413$

2018 - NOVEMBER

[92] For a Poisson variate X, P(X = 2) = 3P (X = 4), then the standard deviation of X is

(a) 2 (b) 4

(c) $\sqrt{2}$ (d) 3 (1 mark)

Answer:

(c) For a Poisson Variate X,

P(X = 2) = 3P (X = 4),

$$\frac{e^{-m} m^2}{2!} = \frac{3e^{-m} m^4}{4!}$$

$$\frac{m^2}{2} = \frac{3m^4}{24}$$

$$6m^4 = 24m^2$$

$$m^2 = \frac{24}{6}$$

$$m^2 = 4$$

$$m = 2$$

S.D $= \sqrt{m} = \sqrt{2}$

[93] The mean of the Binomial distribution $B\left(4, \frac{1}{3}\right)$ is equal to

(a) $\frac{3}{5}$ (b) $\frac{8}{3}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$ (1 mark)

Answer:

(d) $X_4\ B(n, P) = B(4, \frac{1}{3})$

We get $n = 4, P = 1/3$

Mean (μ) $= np$

$= 4 \times 1/3 = 4/3$

[94] If for a normal distribution $Q_1 = 54.52$ and $Q_3 = 78.86$, then the median of the distribution is

(a) 12.17 (b) 39.43

(c) 66.69 (d) None of these (1 mark)

Answer:

(c) For a Normal Distribution

$Q_1 = 54.52$ and $Q_3 = 78.86$

We known that

$Q_1 = \mu - 0.675 = 54.52$ _____(1)

$Q_3 = \mu - 0.675 = 78.86$ _____(2)

On Adding _____

$2\mu = 133.38$

$\mu = \frac{133.38}{2}$

$\mu = 66.69$

In Normal Distribution Mean, Median and Mode are equal.

So, Median = Mean = 66.69

[95] What is the mean of X having the following density function?

$$f(x) = \frac{1}{4\sqrt{2\pi}} . e^{\frac{-(x-10)^2}{32}} \quad \text{for } -\infty < x < \infty$$

(a) 10 (b) 4
(c) 40 (d) None of the above (1 mark)

Answer:

(a) Given Normal distribution

$$f(x) = \frac{1}{4\sqrt{2\pi}} \; e^{\frac{-(x-10)^2}{32}} \quad \text{for } -\infty < x < \infty$$

On comparing from

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} . e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

We get:

Mean (μ) = 10

[96] The probability that a student is not a swimmer is $\frac{1}{5}$, then the probability that out of five students four are swimmer is

(a) $\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$ (b) ${}^5C_1 \left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)$

(c) ${}^5C_4 \left(\frac{4}{5}\right)^1\left(\frac{1}{5}\right)^4$ (d) None of the above (1 mark)

Answer:

(d) Given:

Probability that a student is not a swimmer (q) = $\frac{1}{5}$

Probability that a student is a swimmer (p) = $1 - q = 1 - \frac{1}{5} = \frac{4}{5}$

Total No. of students (n) = 5

P(Exactly 4 students are swimmer)

$$= P(x = 4) = {}^5C_4 . \left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)^1$$

[$\because P(x = r) = {}^nC_r \; p^x \; q^{n-x}$] So, ans. (d)

2019 - JUNE

[97] 4 coins were tossed 1600 times. What is the probability that all 4 coins do not turn head upward at a time?

(a) $1600\ e^{-100}$
(b) $1000\ e^{-100}$
(c) $100\ e^{-1600}$
(d) e^{-100} (1 mark)

Answer:

(d) Probability of getting a head in a throw of a coin = $\frac{1}{2}$

Probability of getting 4 heads in a throw of four coins = $\frac{1}{2^4} = \frac{1}{16}$

Here, n = 1600

Mean = m = np

$= 1600 \times \frac{1}{16}$

$= 100$

P(No. Head) = P (X 20)

$= \frac{e^{-100}.(100)^0}{01}$

$= \frac{e^{-100}.1}{1}$

$= e^{-100}$

[98] If mean and variance are 5 and 3 respectively then relation between p and q is:

(a) $p > q$
(b) $p < q$
(c) $p = q$
(d) p is symmetric (1 mark)

Answer:

(b) Mean = 5, Variance = 3

$np = 5$ ...(1), $npq = 3$ ...(2)

eq(2)/eq(1)

$$\frac{npq}{np} = \frac{3}{5}$$

$q = 3/5$

$p = 1-q$

$= 1- 3/5 = 2/5$

Here, $p < q$

[99] In a Poisson distribution if $P(x = 4) = P(x = 5)$ then the parameter of Poisson distribution is:

(a) $\frac{4}{5}$

(b) $\frac{5}{4}$

(c) 4

(d) 5 (1 mark)

Answer:

(d) In Poisson distribution

$$P(x = 4) = P(x = 5)$$

$$\frac{e^{-m} \cdot m^4}{4!} = \frac{e^{-m} \cdot m^5}{5!}$$

$$\frac{1}{4!} = \frac{m}{5!}$$

$$\frac{1}{24} = \frac{m}{120}$$

$$24m = 120$$

$$m = 5$$

[100] Area between = 1.96 to + 1.96 in a normal distribution is:

(a) 95.45%

(b) 95%

(c) 96%

(d) 99% (1 mark)

Answer:

2019 - NOVEMBER

[101] Area under $U \pm 3\sigma$

(a) 99.73%
(b) 99%
(c) 100%
(d) 99.37% (1 mark)

Answer:

(a) We know that 99.73 per cent of the values of a normal variable lies between $(u - 3\sigma)$ and $(u + 3\sigma)$
Thus probability that a value of x lies. Outside the limit is as low as $(100 - 99.73) = 0.27\%$.

[102] For a Poisson distribution:

(a) mean and SD are equal
(b) mean and variance are equal
(c) SD and Variance
(d) both a and b (1 mark)

Answer:

(b) Poisson distribution is theoretical discrete probability distribution which can describe many processes
Mean is given by m i.e, $U = m$
Variance is also given by m i.e. $\sigma^2 = m$
So in pass on distribution mean and variance are equal.

[103] Find mode when $n = 15$ and $p = \frac{1}{4}$ in binomial distribution?

(a) 4
(b) 4 and 3
(c) 4.2
(d) 3.75 (1 mark)

Answer:

(b) In binomial distribution,

$$m = (n + 1)\, p$$

$$m = (15 + 1) \times \frac{1}{4}$$

$$m = 4$$

Since 4 is a integer so there will 2 modes

4 and (4 - 1)

Mode = 4 and 3

[104] In Poisson distribution, if $P(x = 2) = \frac{1}{2} P(x = 3)$ find m?

(a) 3
(b) 1/6
(c) 6
(d) 1/3 (1 mark)

Answer:

(c) In Poisson distribution $P(x = x) = \frac{e^{-m} \cdot m^2}{x!}$

Here $P(x = 2) = \frac{1}{2} P(x = 3)$

$$\frac{e^{-m} \cdot m^2}{2!} = \frac{1}{2} \times \frac{e^{-m} \cdot m^3}{3!}$$

$$\frac{e^{-m} \cdot m^2}{2} = \frac{1}{2} \frac{x e^{-m} \cdot m^3}{2 \times 6}$$

$$m = 6$$

[105] In a binomial distribution B(n, p)

$n = 4$ $P(x = 2) = 3x\ P(x = 3)$ find P

(a) 1/3
(b) 2/3
(c) 6/4
(d) 4/3 (1 mark)

Answer:

(a) $n = 4$

we know $P(x = r) = {}^nC_r (p)^r (q)^{n-r}$

here $p(x = 2) = 3 \times P(x = 3)$

$${}^4c_2 (p)^2 (q)^{4-2} = 3 \times {}^4c_3 (p)^3 (q)^1$$

$$\frac{4!}{(4-2)\,1 \times 2!} (p)^2 (1-p)^2 = 3 \times \frac{4!}{(4-3)\,1 \times 3!} \times (p)^3 (1-p)$$

Since ${}^nc_r = \frac{n!}{(n-r)!\,1 \times r!}$

$$6 \times (1-p) = 3 \times 4p$$

$$6 - 6p = 12p$$

$$18p = 6$$

$$p = \frac{1}{3}$$

[106] What is the SD and mean

x if $f(x) = \frac{\sqrt{2}}{\sqrt{\pi}} e^{-2(x-3)^2}, -\infty < x < \infty.$

(a) $3, \frac{1}{2}$

(b) $3, \frac{1}{4}$

(c) $2, \frac{1}{2}$

(d) $2, \sqrt{2}$ (1 mark)

Answer:

(a) The standard form of probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}} \text{ ----------(1)}$$

Here, $\sqrt{\frac{2}{\pi}} \cdot e^{-2(x-3)^2}$

$$= \sqrt{\frac{2}{\pi}} \cdot e^{-\left(\frac{1-3}{1/2}\right)^2}$$

on comparing with Equation ----------(1)

$2\sigma^2 = \frac{1}{2}$ u = 3

$\sigma^2 = \frac{1}{4}$

$\sigma = \frac{1}{2}$

So SD = $\frac{1}{2}$, mean = 3

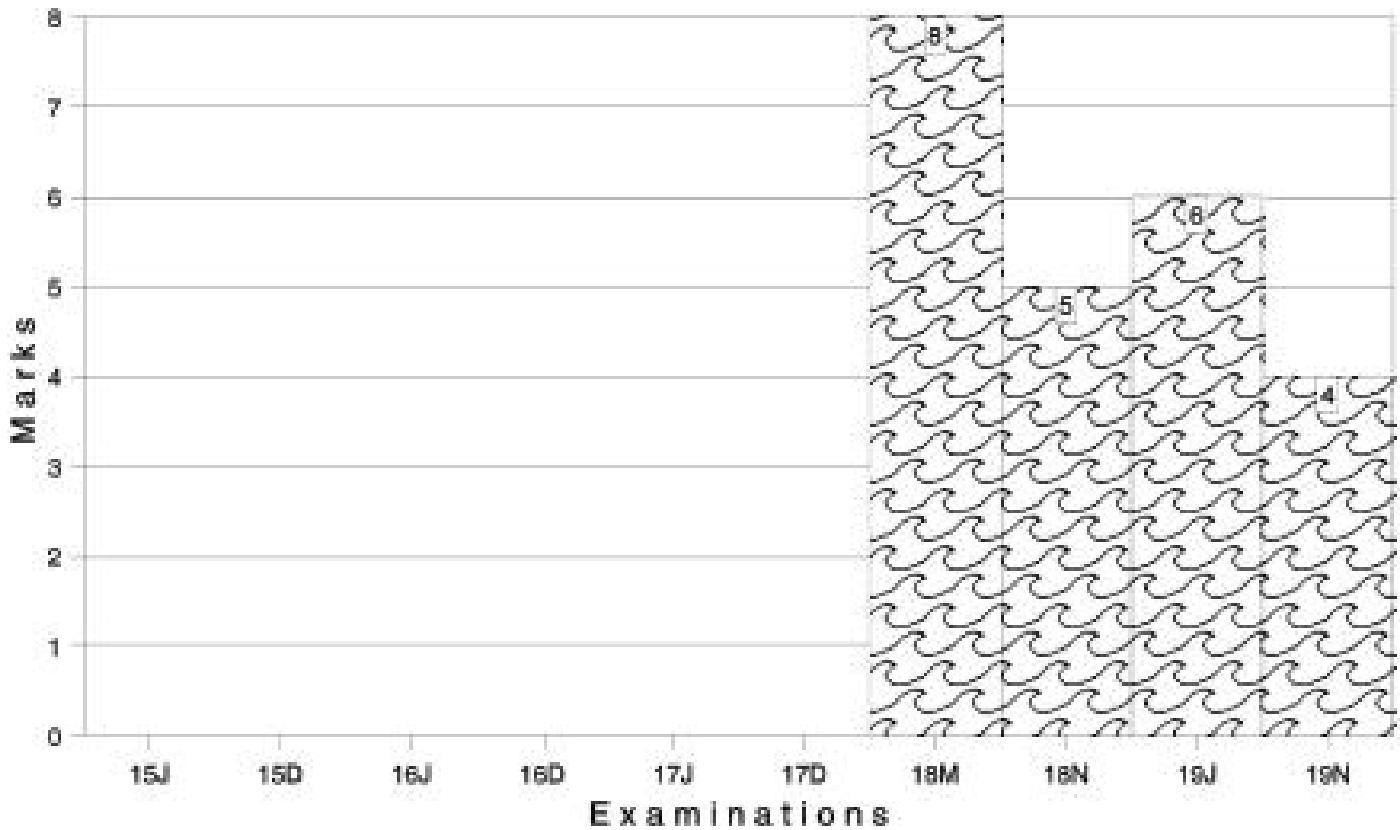
CHAPTER

18 CORRELATION AND REGRESSION

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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PAST YEAR QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] The coefficient of correlation r between x and y when :
Cov (x, y) = – 16.5, Var (x) = 2.89, Var (y) = 100 is :
(a) – 0.97 (b) 0.97
(c) 0.89 (d) – 0.89 (1 mark)

Answer:

(a) Coefficient of correlation

$$r = \frac{cov(x,y)}{\sigma_x . \sigma_y}$$

$$\text{or } r = \frac{Cov(x,y)}{\sqrt{var(x) - var(y)}}$$

$$= \frac{-16.5}{\sqrt{2.89 \times 100}}$$

$$= -0.97$$

[2] Take 200 and 150 respectively as the assumed mean for X and Y series of 11 values, then dx = X – 200, dy = Y – 150, $\Sigma dx = 13$, $\Sigma dx^2 = 2667$, $\Sigma dy = 42$, $\Sigma dy^2 = 6964$, $\Sigma dx\, dy = 3943$. The value of r is:
(a) 0.77 (b) 0.98
(c) 0.92 (d) 0.82

Answer:

(c) As we know

$$r = \frac{n\Sigma dxdy - \Sigma dx . \Sigma dy}{\sqrt{n\Sigma dx^2 - (\Sigma dx)^2}\sqrt{n\Sigma dy^2 - (\Sigma dy)^2}}$$

$$r = \frac{11 \times 3943 - 13 \times 42}{\sqrt{11 \times 2667 - (13)^2}.\sqrt{11 \times 6964 - (42)^2}}$$

$$= 0.92$$

[3] For some bivariate data, the following results were obtained for the two variables x and y :

$\bar{x} = 53.2$, $\bar{y} = 27.9$, $b_{yx} = -1.5$, $b_{xy} = -0.2$

The most probable value of y when x = 60 is :

(a) 15.6 (b) 13.4

(c) 19.7 (d) 17.7 (1 mark)

Answer:

(d) The regression equation of y of x is:

$y - \bar{y} = by\ x\ (x - \bar{x})$

$\Leftarrow y - 27.9 = (-1.5)(x - 53.2)$

or $y = 107.7 - 1.5x$

when x = 60 then

$y = 107.7 - 1.5 \times 60 = 17.7$

2007 - FEBRUARY

[4] If the sum of squares of the rank difference in mathematics and physics marks of 10 students is 22, then the coefficient of rank correlation is:

(a) 0.267 (b) 0.867

(c) 0.92 (d) None (1 mark)

Answer:

(b) Co. efficient of rank correlation

$$= 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 22}{10(10^2-1)}$$

$$= 1 - \frac{6 \times 2}{10 \times 9}$$

$$= \frac{13}{15} = 0.867 \text{(approx)}$$

[5] Two random variables have the regression lines $3x + 2y = 26$ and $6x + y = 31$. The coefficient of correlation between x and y is :

(a) -0.25 (b) 0.5

(c) -0.5 (d) 0.25 (1 mark)

Answer:

(c) The regression lines $3x + 2y = 26$ and $6x + y = 31$ are given

Let first equation be y on x and second be x on y respectively

Therefore, $3x + 2y = 26$

$$\Rightarrow y = \left(\frac{-3}{2}\right)x + \left(\frac{26}{2}\right)$$

$\therefore byx = -3/2$

and $6x + y = 31$

$$\Rightarrow x = \left(\frac{-1}{6}\right)y + \left(\frac{31}{6}\right)$$

$\therefore bxy = -1/6$

Now $r^2 = byx.bxy$

$$= \left(\frac{-3}{2}\right)\left(\frac{-1}{6}\right)$$

$= -0.25$

$\Rightarrow r = -0.5$

Hence, our assumption holds true and $r = -0.5$ ($\because -1 \leq r \leq 1$)

Note : r is negative because byx and bxy < 0

$\therefore x < 0$

2007 - MAY

[6] The coefficient of correlation between X and Y is 0.6. U and V are two variables defined as $U = \frac{x - 3}{2}$, $V = \frac{y - 2}{3}$, then the coefficient of correlation between U and V is :

(a) 0.6 (b) 0.4
(c) 0.8 (d) 1 (1 mark)

Answer:

(a) Since correlation coefficient (Karl Pearson's) is independent of both scale and origin, therefore,

$p(u, v) = p(x, y) = 0.6$

It may be noted that if

$\mu_1 = a x_1 + b$ and $V_i = cy_i + d$ then

$r(u, v) = P(x, y)$ if a and c are of same signs

$-p(x, y)$ if a and c are of opposite signs

[7] For the following data, the coefficient of rank correlation is :

Rank in Botany: 1 2 3 4 5

Rank in Chemistry 2 3 1 5 4

(a) 0.93 (b) 0.4
(c) 0.6 (d) None (1 mark)

Answer:

(c)

S. No.	Rank in Botany (x_i)	Rank in Chem (y_i)	$d = x_i\ y_i$	d^2
1	1	2	– 1	1
2	2	3	– 1	1
3	3	1	2	4
4	4	5	– 1	1
5	5	4	1	1
Total	–	–	0	8

Hence, coefficient of rank correlation

$$= 1 - \frac{6 \times 8}{5(5^2 - 1)}$$

$$S = 1 - \frac{2}{5} = 0.6$$

[8] The following data is given, based on 450 students for marks in Statistics and Economics at a certain examination :

Mean marks in Statistics = 40

S.D. of marks (Statistics) = 12
Variance of marks (Economics) = 256
Sum of the products of deviations of marks from their respective mean = 42075
The average marks in Economics of candidates who obtained 50 marks in Statistics is :

(a) 45 (b) 54.5
(c) 54 (d) 47.5 (1 mark)

Answer:

(b) Let x = Marks in statistics
and y = Marks in Economics
We know that

$$r_{xy} = \Sigma \frac{(\Sigma dx \times dy)}{n \times \sigma_x \times \sigma_y}$$

Where, $dx = x_1 - \bar{x}$ and $dy = yi - \bar{y}$

$$\therefore r_{xy} = \frac{(42075)}{450 \times 12 \times 16} = 0.49$$

Now regression equation of Y on X

$$y - \bar{y} = \frac{r\sigma_y(x - \bar{x})}{\sigma_x}$$

$$\Rightarrow y - 48 = \times \frac{0.49 \times 16}{12}(x - 40)$$

$$\Rightarrow y = 0.65x + 22$$

when x = 50, then
Y = 0.65 × 50 + 22 = 54.5

2007 - AUGUST

[9] For 10 pairs of observations, number of concurrent deviations was found to be 4. What is the value of the coefficient of concurrent deviation ?

(a) $\sqrt{0.2}$ (b) 1/3

(c) – 1/3 (d) $-\sqrt{0.2}$ (1 mark)

Answer:

(c) Here C = 4, N = 10, So

$n = N - 1 = 10 - 1 = 9$

$$r_c = \pm\sqrt{\frac{\pm(2c-n)}{n}}$$

$$= \pm\sqrt{\frac{\pm(2\times4-9)}{9}}$$

Here (2c – n) is negative, so negative sign is taken at both the places so,

$$r_c = \frac{(-1)}{3}$$

[10] If the covariance between two variables is 20 and the variance of one of the variables is 16, what would be the variance of the other variable?

(a) More than 10 (b) More than 100

(c) More than 1.25 (d) Less than 10 (1 mark)

Answer:

(b) We know that,

$$r = \frac{(Cov.y)}{\sigma_x \times \sigma_y}$$

$$\therefore r = \frac{20}{4\sigma_y} \quad (\sigma_x^2 = 16 \ \therefore \sigma_x = 4)$$

$$\Rightarrow r = \frac{5}{\sigma_y}$$

Now , by definition $-1 \le x \le 1$,

$\therefore +5 \le \sigma_y \le -5$

$(\therefore r \times \frac{1}{\sigma_y})$

$\Rightarrow 25 \le \sigma^2_y$

i.e. variance (σ^2_y) must be more than at least 25.

In other words , variance of the other variable, should be more than 100 in this case.

2007 - NOVEMBER

[11] Assume 69 and 112 as the mean values for X and Y respectively. $\Sigma dx = 47$, $\Sigma dx^2 = 1475$, $\Sigma dy = 108$, $\Sigma dy^2 = 3468$, $\Sigma dx\, dy = 2116$ and $N = 8$.

Where $dx = X - 69$, $dy = Y - 112$. Then the value of r is :

(a) 0.95 (b) 0.65

(c) 0.75 (d) 0.85 (1 mark)

Answer:

(a) $$r = \frac{N\sum dxdy - (\sum dx)(\sum dy)}{\sqrt{N\sum dx^2 - (\sum dx)^2}\sqrt{N\sum dy^2 - (\sum dy)^2}}$$

$$= \frac{8\times 2116 - (47)(108)}{\sqrt{18\times 1475 - (47)^2}\sqrt{8\times 3468 - (108)^2}} = 0.95$$

[12] In rank correlation, the association need not be linear :

(a) True (b) False

(c) Partly True (d) Partly False (1 mark)

Answer:

(a) In spearman's rank correlation, the association need not be linear since it is based on ranks rather than on actual observations.

[13] The lines of regression are as follows :
$5x - 145 = -10y$; $14y - 208 = -8x$. The mean values $(\bar{x}, \bar{y})$ is :

(a) (12, 5) (b) (5, 7)
(c) (7, 12) (d) (5, 12)

Answer:

(d) As we know, that the mean values of 2 regression equations are their points of intersection, therefore solving the equations simultaneously.

Given :

$5x - 145 = -10\bar{y}$ and $14\bar{y} - 20 = -8\bar{x}$

$\Rightarrow 5\bar{x} + 10\bar{y} = 145$ .....(1)

$8\bar{x} + 14\bar{y} = 208$ .....(2)

$\therefore 40\bar{x} + 80\bar{y} = 1160$ .....(3)

$40\bar{x} + 70\bar{y} = 1040$ .....(4)

Subtracting (4) from (3), we get:

$\bar{y} = 12$ Putting $\bar{y} = 12$ in (1), we get

$\bar{x} = 5$

$\therefore (\bar{x}, \bar{y}) = (5, 12)$

2008 - FEBRUARY

[14] The coefficient of rank correlation of marks obtained by 10 students, in English and Economics was found to be 0.5. It was later discovered that the difference in ranks in the two subjects obtained by one student was wrongly taken as 3 instead of 7. The correct coefficient of rank correlation is :

(a) 0.32 (b) 0.26
(c) 0.49 (d) 0.93 (1 mark)

Answer:

(b) Coefficient of rank correlation:

$$R = \frac{6\sum d^2}{n(n^2-1)}$$

$$\Rightarrow 0.5 = 1 - \frac{6\sum d^2}{10(10^2-1)}$$

$$\Rightarrow \frac{6\sum d^2}{990} = 0.5$$

$$\Rightarrow \sum d^2 = 82.5$$

Corrected values of $\sum d^2 = 82.5 - 3^2 + 7^2 = 122.5$

Corrected $R = 1 - \frac{6 \times 122.5}{990} = 1 - \frac{735}{990} = 1 - 0.74 = 0.26$

[15] Given the following data :

$b_{xy} = 0.4$ & $b_{yx} = 1.6$. The coefficient of determination is :

(a) 0.74 (b) 0.42

(c) 0.58 (d) 0.64 (1 mark)

Answer:

(d) We know that , Coefficient of determination

$r^2 = byx \times bxy$

$\therefore \quad r^2 = 16 \times 0.4$

$r^2 = 0.6\ y$

[16] The method applied for deriving regression equations is known as :

(a) Concurrent deviation (b) Product moment

(c) Least squares (d) Normal equation (1 mark)

Answer:

(c) When the relationship between two variables is linear, **the method of least** squares is used for obtaining the best values of the constants in an appropriate equation. Therefore, the above method is applied for **deriving regression** equation.

2008 - JUNE

[17] The coefficient of correlation between x and y series from the following data :

	X series	Y series
Number of pairs of observations	15	15
Arithmetic Mean	25	18
Standard Deviation	3.01	3.03
Sum of the squares of deviation from mean	136	138

Sum of the product of the deviations of x and y series from their respective means = 122, is :

(a) 0.89 (b) 0.99
(c) 0.69 (d) 0.91 (1 mark)

Answer:

(a) Here $\sigma_x = 301$, $\sigma_y = 3.03$, $\Sigma dx = 136$
$\Sigma dy^2 = 138$, $\Sigma dx\, dy = 122$
We know that,

$$X = \frac{\sum dx \times dy}{\sqrt{\sum dx^2 \times \sum dy^2}} = \frac{122}{\sqrt{136 \times 138}} = 0.89$$

[18] If the lines of regression in a bivariate distribution are given by $x + 2y = 5$ and $2x + 3y = 8$, then the coefficient of correlation is :

(a) 0.866 (b) – 0.666
(c) 0.667 (d) – 0.866 (1 mark)

Answer:

(d) Let the lines of regression of Y on X and X on Y be $x + 2y = 5$ and $2x + 3y = 8$ respectively,
then

$$byx = \left(\frac{-1}{2}\right) \text{ and } bxy = \left(\frac{-3}{2}\right)$$

$\therefore r^2 = byx \times bxy$

$= \left(\frac{-1}{2}\right) \times \left(\frac{-3}{2}\right)$

$\Rightarrow r^2 = \left(\frac{3}{4}\right)$

$\Rightarrow r = \left(-\frac{\sqrt{3}}{2}\right) = -0.866$

$(\therefore byx, bxy < 0, \ \therefore r < 0)$

[19] If the correlation coefficient between two variables is 1, then the two lines of regressions are :

(a) Parallel (b) At right angles

(c) Coincident (d) None of these (1 mark)

Answer:

(c) As the value of r increases numerically from O to 1, the angle between regression equations decreases from 90^0 to 0^o. In other words, the farther the two regression lines are from each other, the lesser is the degree of correlation (i.e. approaching 1) and nearer the two regression lines are to each other , the higher is the degree of correlation (i.e. approaching +1).

The above explanation clarifies if r = 1, The two lines of regressions are coincident.

2008 - DECEMBER

[20] If the sum of square of differences of rank is 50 and number of items is 8 then what is the value of rank correlation coefficient.

(a) 0.59 (b) 0.40

(c) 0.36 (d) 0.63 (1 mark)

Answer:

(b) Given $\Sigma d^2 = 50$, $n = 8$

$$'R = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6\times50}{8(8^2-1)} = 1 - \frac{6\times50}{8\times63} = 0.40$$

[21] If coefficient of correlation between x and y is 0.46. Find coefficient of correlation between x and $\frac{y}{2}$

(a) 0.46 (b) 0.92
(c) – 0.46 (d) – 0.92 (1 mark)

Answer:

(a) $r_{xy} = 0.46$

We know that , $r_{xy} = \frac{bd}{|b|.|d|}\ 'uv$

Let $u = x$ and $v = \frac{y}{2}$

$$0.46 = \frac{1\times1/2}{1\times1/2} \times r_{uv} = 1\frac{1}{2}\times ruv$$

$\Rightarrow\ r_{uv} = 0.46$

Note : The two correlation coefficients remain equal when b and d, the two scales have similar signs.

[22] Given the regression equations as $3x + y = 13$ and $2x + 5y = 20$. Find regression equation of y on x.

(a) $3x + y = 13$ (b) $2x + y = 20$
(c) $3x + 5y = 13$ (d) $2x + 5y = 20$ (1 mark)

Answer:

(d) Let us assume that $2x + 5y - 20 = 0$ represents the regression line of y on x and $3x + y - 13 = 0$ represent the line of x on y.

Now, $2x + 5y - 20 = 0$

$$\Rightarrow\ y = (4) + \left(\frac{-2}{5}\right)x$$

$\therefore b_{yx} = \frac{-2}{5}$

Again, $3x + y - 13 = 0$

$\Rightarrow x = \frac{13}{3} + \left(\frac{-1}{3}\right) y$

$\therefore b_{xy} = \frac{-1}{3}$

Thus, $r^2 = b_{yx} \times b_{xy}$

$= \frac{-2}{5} \times \frac{-1}{3}$

$= \frac{2}{15} < 1$

Since $|r| \leq 1 \Rightarrow r^2 \leq 1$, our assumptions are correct. Thus, $2x + 5y - 20 = 0$ truly represents the regression line of y on x.

[23] The coefficient of correlation is significant if :

(a) $r > 5$ P. E (b) $r < 6$ P. E

(c) $r \geq 6$ P. E (d) $r = 6$ P. E (1 mark)

Answer:

(c) The probable error of the coefficient of correlation helps in interpreting its value. Since the coefficients of correlation are, generally, computed from samples, they, like other statistical quantities, are subject to errors of sampling. So from interpretation point of view **probable error of the coefficient of correlation is very useful.** When $r \geq 6 \times$ P.E., then coefficient of correlation is significant and the correlation exists.

2009 - JUNE

[24] The two regression equations are :

$2x + 3y + 18 = 0$

$x + 2y - 25 = 0$

find the value of y if $x = 9$

(a) – 8 (b) 8
(c) – 12 (d) 0 (1 mark)

Answer:

(b) To find the value of y when x's value is given, regression equation of Y on X should be known.

Let us assume that $2x + 3y + 18 = 0$ represents the regression line of Y on X and $x + 2y - 25 = 0$ represents the regression line of X on Y.

Now, $2x + 3y + 18 = 0$

$$Y = -18 + \frac{(-2)x}{3}$$

$$\therefore b_{yx} = \frac{-2}{3}$$

Again $x + 2y - 25 = 0$

$x = 25 - 2y$

$\therefore b_{xy} = -2$

Thus, $r^2 = b_{yx} \times b_{xy}$

$$= \frac{-2}{3} \times -2$$

$$= \frac{4}{3} > 1$$

Since $|r| \leq 1 = r^2 \leq 1$, our assumption is wrong. Thus, $2x + 3y + 18 = 0$ truly represents the regression line of X on Y and $x + 2y - 25 = 0$ truly represents the regression line of Y on X.

$\therefore \quad x + 2y - 25 = 0$ .....(1)

Substituting $x = 9$ in (1)

$9 + 2y - 25 = 0$

$2y = 25 - 9$

$$y = \frac{16}{2}$$

$y = 8$

$\therefore$ When $x = 9$ then $y = 8$

[25] The correlation coefficient between x and y is – 1/2. The value of $^{b}xy = -1/8$. Find by x.

(a) – 2 (b) – 4

(c) 0 (d) 2 (1 mark)

Answer:

(a) Since $r^2 = b_{xy} \times b_{yx}$

$$\left(\frac{-1}{2}\right)^2 = \frac{-1}{8} \times b_{yx}$$

$$\frac{1}{4} = \frac{-1}{8} \times b_{yx}$$

$$b_{yx} = \frac{1}{4} \times (-8)$$

$$b_{yx} = -2$$

[26] Ranks of two_____.characteristics by two judges are in reverse order then find the value of Spearman rank correlation co-efficient.

(a) – 1 (b) 0

(c) 1 (d) 0.75 (1 mark)

Answer:

(a) Lets solve this question by taking an example. Suppose the hypothetical value of n be 5.

Then,

Rank by I Judge (R_x)	**Rank by II Judge (R_y)**	**$d = R_x \; R_y$**	**d^2**
1	5	– 4	16
2	4	– 2	4
3	3	0	0
4	2	+ 2	4
5	1	+ 4	16
			40

Spearman's Rank Correlation Coefficient

$$= 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 40}{5(5^2-1)}$$

$$= 1 - \frac{240}{5 \times 24}$$

$$= 1 - 2$$

$$= -1$$

$$\therefore r_{xy} = -1$$

NOTE :

Students may, however, solve this question by taking any value of n. But, they should remember that the ranks given by two judges are in reverse order.

2009 - DECEMBER

[27] Which of the following regression equations represent regression line of Y on X :

$7x + 2y + 15 = 0$, $2x + 5y + 10 = 0$

(a) $7x + 2y + 15 = 0$ (b) $2x + 5y + 10 = 0$

(c) Both (a) and (b) (d) None of these (1 mark)

Answer:

(b) $7x + 2y + 15 = 0$ .....(1)

$2x + 5y + 10 = 0$ .....(2)

Assume that $7x + 2y + 15 = 0$ is the regression line of X on Y and $2x + 5y + 10 = 0$ is the regression line of Y on X.

$7x + 2y + 15 = 0$

$$x = \frac{-2y}{7} \quad \frac{-15}{7}$$

$$b_{xy} = -\frac{2}{7}$$

$$y = -\frac{2x}{5} - \frac{10}{5}$$

$$b_{yx} = -\frac{2}{5}$$

$$r^2 = b_{xy} \times b_{yx}$$

$$= -\frac{2}{7} \times -\frac{2}{5}$$

$$r = \sqrt{\frac{4}{35}}$$

$$r = -0.33$$

Since $-1 \leq r \leq 1$ $\therefore$ our assumption is correct So, $2x + 5y + 10 = 0$ is the regression line of Y on X.

[28] If the rank correlation co-efficient between marks in Management and Mathematics for a group of students is 0.6 and the sum of the squares of the difference in ranks is 66. Then what is the number of students in the group?

(a) 9 (b) 10

(c) 11 (d) 12 (1 mark)

Answer:

(b) $r = 0.6$

$d^2 = 66$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$0.6 = 1 - \frac{6 \times 66}{n(n^2-1)}$$

$$1 - 0.6 = \frac{396}{n(n^2-1)}$$

$$0.4 = \frac{396}{n(n^2-1)}$$

$$n(n^2 - 1) = \frac{396}{0.4}$$

$$n(n^2 - 1) = 990$$

$$n = 10$$

Therefore, the number of students = 10

[29] Correlation coefficient between X and Y will be negative when:-

(a) X and Y are decreasing

(b) X is increasing, Y is decreasing

(c) X and Y are increasing

(d) None of these (1 mark)

Answer:

(b) When X and Y move in opposite direction, then correlation coefficient is negative. Therefore, if X is increasing, and Y is decreasing the correlation coefficient will be negative.

[30] The two regression lines are $7x - 3y - 18 = 0$ and $4x - y - 11 = 0$.Find the values of b_{yx} and b_{xy}

(a) 7/3, 1/4 (b) – 7/3, – 1/4

(c) – 3/7, – 1/4 (d) None of these. (1 mark)

Answer:

(a) Assume that $7x - 3y - 18 = 0$ is the line

$7x - 3y - 18 = 0$ of Y on X and $4x - y - 11 = 0$ is of X on Y.

$$3y = 7x - 18$$

$$y = \frac{7x}{3} - \frac{18}{3}$$

$$b_{yx} = \frac{7}{3}$$

$$4x - y - 11 = 0$$

$$4x = y + 11$$

$$x = \frac{y}{4} + \frac{11}{4}$$

$$b_{xy} = \frac{1}{4}$$

$$r^2 = b_{xy} \times b_{yx}$$

$$r = \sqrt{\frac{1}{4}} \times \frac{7}{3}$$

$$r = \sqrt{\frac{7}{12}} = 0.764$$

since value of r is lying between – 1 and 1 therefore our assumption was correct.

So, $b_{yx} = \frac{7}{3}$ and $b_{xy} = \frac{1}{4}$

2010 - JUNE

[31] If 'P' is the simple correlation coefficient, the quantity P^2 is known as:

(a) Coefficient of determination
(b) Coefficient of Non-determination
(c) Coefficient of alienation
(d) None of the above. (1 mark)

Answer:

(a) Better measure for measuring correlation is provided by the square of correlation coefficient, known as 'coefficient of determination' which is expressed as –

$$r^2 = \frac{\text{Explained Variance}}{\text{Total Variance}}$$

[32] _____ of the regression Coefficients is greater than the correlation coefficient

(a) Combined mean (b) Harmonic mean
(c) Geometric mean (d) Arithmetic mean (1 mark)

Answer:

(d) Correlation Coefficient (r) is the Geometric Mean (G.M.) between two co regression coefficients.

$$r = \pm\sqrt{b_{xy}.b_{yx}}$$

Since, AM > GM > HM

Therefore, AM of regression coefficients is greater than correlation coefficient

[33] If the correlation coefficient between x and y is r, then between $U = \frac{x-5}{10}$ and $V = \frac{y-7}{2}$ is

(a) r (b) – r
(c) (r-5)/2 (d) (r – 7)/10 (1 mark)

Answer:

(a) x – 10u = 5 → (1) eq.
y – 2v = 7 → (2) eq.
Since correlation coefficient (Karl Pearson's) is independent of both scale and origin, therefore,
P(u, v) = p(x, y) = r
It may be noted that if
$u_i = ax_i + b$ and $v_i = cy_i + d$, then
r(u, v) = p(x, y) if a and c are of same signs
r(u, v) = – p(x, y) if a and c are of opposite signs.

[34] If the two lines of regression are
x + 2y - 5 = 0 and 2x + 3y - 8 = 0
The regression line of y on x is

(a) x + 2y - 5 = 0 (b) 2x + 3y - 8 = 0
(c) Any of the two line (d) None of the two lines. (1 mark)

Answer:

(c) Let us take equation (1) as
x + 2y – 5 = 0

$$byx = \frac{\text{coeff. of x}}{\text{coeff. of y}} = \frac{-1}{2} = -0.5$$

Now, let us take equation (2) as
2x + 3y – 8 = 0

$$byx = -\frac{2}{3} = -0.66$$

In both the cases r < 1
Hence, any of the two lines can be regression line of y on x

2010 - DECEMBER

[35] If the sum of the product of deviations of x and y series from their means is zero, then the coefficient of correlation will be

(a) 1 (b) -1

(c) 0 (d) None of these (1 mark)

Answer:

(c) Coefficient of correlation = $\frac{Cov(x, y)}{Sx \times Sy} = \frac{\sum(x - \bar{x})(y - \bar{y})}{n \times \sigma_x \times \sigma_y}$

$Cov(x, y) = \frac{\sum xy}{x} - \bar{x}\,\bar{y} = 0$

It is given that the above value

$\Rightarrow \sum(x - \bar{x})(y - \bar{y}) = 0$ (Numerator)

Hence, Coefficient of correlation = $\frac{0}{Sx \times Sy} = 0$

[36] The ranks of five participants given by two judges are

		Participants				
		A	B	C	D	E
Judge	1	1	2	3	4	5
Judge	2	5	4	3	2	1

Rank correlation coefficient between ranks will be

(a) 1 (b) 0

(c) -1 (d) 1/2 (1 mark)

Answer:

(c)

	Judge 1 (r_1)	Judge 2 (r_2)	d	d^2
A	1	5	– 4	16
B	2	4	– 2	4
C	3	3	0	0

D	4	2	2	4
E	5	1	2	16
				40

$$\text{Rank correlation coefficient} = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 40}{5 \times 24}$$

$$= 1 - 2 = -1$$

[37] Regression coefficient are _____

(a) dependent of change of origin and of scale.
(b) independent of both change of origin and of scale.
(c) dependent of change of origin but not of scale.
(d) independent of change of origin but not of scale (1 mark)

Answer:

(d) Regression coefficient are independent of change of origin but not of scale (As per Fundamental Principle)

[38] Given : $\bar{x} = 16$, $\sigma x = 4.8$

$\bar{y} = 20$, $\sigma y = 9.6$

The coefficient of correlation between x and y is 0.6. What will be the regression coefficient of 'x' on 'y'?

(a) 0.03 (b) 0.3
(c) 0.2 (d) 0.05 (1 mark)

Answer:

(b) $bxy = r \times \frac{SDx}{SDy}$

$r = 0.6$
$SDx = 4.8$
$SDy = 9.6$

$bxy = 0.6 \times \frac{4.8}{9.6} = 0.3$

[39] If the two lines of regression are
$x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$
The regression line of y on x is
(a) $x+2y-5 = 0$ (b) $2x+3y-8 = 0$
(c) Any of the two line (d) None of the two lines. (1 mark)

Answer:

(c) Let us take equation (1) as
$x + 2y - 5 = 0$

$$byx = \frac{\text{coeff. of x}}{\text{coeff. of y}} = \frac{-1}{2} = -0.5$$

Now, let us take equation (2) as
$2x + 3y - 8 = 0$

$$byx = -\frac{2}{3} = -0.66$$

In both the cases $r < 1$
Hence, any of the two lines can be regression line of y on x

2011 - JUNE

[40] The covariance between two variables X and Y is 8.4 and their variances are 25 and 36 respectively. Calculate Karl Pearson's coefficient of correlation between them.
(a) 0.82 (b) 0.28
(c) 0.01 (d) 0.09 (1 mark)

Answer:

(b) Given : Cov (x, y) = 8.4
Variance of x = 25
$\therefore$ S.D of x $(\sigma_x) = \sqrt{25} = 5$
and
$\because$ Variance of y = 36
$\therefore$ S.D of y $= \sqrt{36} = 6$

$\therefore$ Karl Pearson's Coefficient of Correlation $= \frac{Cov(x,y)}{\sigma x, \sigma y}$

$= \frac{8.4}{5\times6}$

$= \frac{8.4}{30}$

$= 0.28$

[41] For a bivariate data, two times of regression are $40x - 18y = 214$ and $8x - 10y + 66 = 0$, then find the values of $\bar{x}$ and $\bar{y}$

(a) 17 and 13 (b) 13 and 17

(c) 13 and -17 (d) -13 and 17 (1 mark)

Answer:

(b) Given : $40x - 18y = 214$ _____(1)

$8x - 10y = -66$ _____(2)

On solving (1) and (2) we get

$x = 13$ and $y = 17$

$\therefore \bar{x} = 13$ and $\bar{y} = 17$

[42] Three competitors in a contest are ranked by two judges in the order 1,2,3 and 2,3,1 respectively. Calculate the Spearman's rank correlation coefficient.

(a) -0.5 (b) -0.8

(c) 0.5 (d) 0.8 (1 mark)

Answer:

(a)

Rank by IstJudge R_1	Rank by IInd Judge R_2	Diff $D = R_1 - R_2$	D^2
1	2	−1	1
2	3	−1	1
here $\frac{3}{n=3}$	1	+2	$\frac{4}{\sum D^2 = 6}$

$$\text{Spearman's Rank Correlation coefficient} = 1 - \frac{6\sum D^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 6}{3(3^2-1)}$$

$$= -0.5$$

2011 - DECEMBER

[43] Out of the following which one affects the regression co-efficient.

(a) Change of Origin Only
(b) Change of scale Only
(c) Change of scale & origin both
(d) Neither Change of origin nor change of scale (1 mark)

Answer:

(b) The regression coefficients remain unchange due to a shift of origin but change due to a shift of scale.

[44] For a bivariate data, the lines of regression of Y on X, and of X on Y are respectively $2.5Y - X = 35$ and $10X - Y = 70$, then the Correlation coefficient r is equal to:

(a) 0.2 (b) – 0.2
(c) 0.5 (d) – 0.5 (1 mark)

Answer:

(a) The equation of regression line y on x is given by

$2.5y - x = 35$

$2.5y = x + 35$

$$y = \frac{x + 35}{2.5}$$

$$y = \frac{x}{2.5} + \frac{350}{25}$$

$$y = 14 + \frac{2}{5}x$$

On comparing

$y = a + bx$

we get $b = \frac{2}{5} \Rightarrow b_{yx}$

Now the equation of Regression line x on y in given by

$10x - y = 70$

$10x = 70 + y$

$x = \frac{70}{10} + \frac{y}{10}$

$x = 7 + \frac{1}{10}y$

Comparing from $x = a + by$

we get $b = \frac{1}{10} \Rightarrow bxy$

coefficient of correlation $(r) = \sqrt{bxy \times byx}$

$= \sqrt{\frac{2}{5} \times \frac{1}{10}}$

$= \sqrt{\frac{1}{25}}$

$= \frac{1}{5}$

$= 0.2$

[45] If one of regression coefficient is _____ unity, the other must be _____ unity.

(a) more than, more then (b) Less than, Less then

(c) more than, less than (d) Positive, Negative (1 mark)

Answer:

(c) If one of regression Co-efficient is **more than** unity, the other must be **less than** unity.

[46] If Y is dependent variable and X is Independent variable and the S.D of X and Y are 5 and 8 respectively and Co-efficient of co-relation between X and Y is 0.8. Find the Regression coefficient of Y on X.

(a) 0.78 (b) 1.28

(c) 6.8 (d) 0.32 (1 mark)

Answer:

(b) Given

S. D of x (σ_x) = 5

S. D of y (σ_y) = 8

Co-eff. of Correlation (r) = 0.8

Regression Co-eff of y on x

$$b_{yx} = r.\frac{\sigma_y}{\sigma_x} = \frac{0.8 \times 8}{5} = \frac{6.4}{5} = 1.28$$

2012 - JUNE

[47] If the regression lines are 8x - 10y + 66 = 0 and 40x - 18y = 214, the correlation coefficient between 'x' and 'y' is :

(a) 1 (b) 0.6

(c) - 0.6 (d) - 1 (1 mark)

Answer:

(b) Given Ist Regression line

$8x - 10y + 66 = 0$

$10y = 66 + 8x$

$$y = \frac{66}{10} + \frac{8x}{10}$$

$y = 6.6 + 0.8x$

on comparing $y = a + bx$

we get $b = byx = 0.8$

and IInd Regression line

$40x - 18y = 214$

$40x = 214 + 18y$

$$x = \frac{214}{40} + \frac{18y}{40}$$

$x = 5.35 + 0.45y$

on comparing $x = a + by$

we get $b = bxy = 0.45$

coefficient of correlation between x & y

$r = \pm \sqrt{byx \times bxy}$

$= \pm \sqrt{0.8 \times 0.45}$

$= \pm \sqrt{.36}$

$= \pm 0.6$

[48] The coefficient of correlation between two variables x and y is the simple _____ of the two regression coefficients.

(a) Arithmetic Mean (b) Geometric Mean

(c) Harmonic Mean (d) None of the above. (1 mark)

Answer:

(b) The coefficient of correlation between two variables x and y is the simple **geometric mean** of the two regression coefficient.

[49] If 2 variables are uncorrelated, their regression lines are:

(a) Parallel (b) Perpendicular

(c) Coincident (d) Inclined at 45 degrees. (1 mark)

Answer:

(b) If two variables are uncorrelated (i.e. r = 0) then regression lines are **perpendicular**.

[50] If the covariance between variables X and Y is 25 and variance of X and Y are respectively 36 and 25, then the coefficient of correlation is

(a) 0.409 (b) 0.416

(c) 0.833 (d) 0.0277 (1 mark)

Answer:

(c) Given (Covariance) Cov (x, y) = 25

Variance of x = 36

S.D of x (σ_x) = $\sqrt{36} = 6$

Variance of y = 25

S.D of (σ_y) = $\sqrt{25} = 5$

Coefficient of correlation

$$r = \frac{Cov(x,y)}{\sigma_x . \sigma_y}$$

$$= \frac{25}{6 \times 5}$$

$$= \frac{5}{6}$$

$$= 0.833$$

[51] If $\bar{x}$, $\bar{y}$ denote the arithmetic means, σ_x, σ_y denotes the standard deviations. b_{xy}, b_{yx} denote the regression coefficients of the variables 'x' and 'y' respectively, then the point of intersection of regression lines x on y & y on x is _____.

(a) $(\bar{x}, \bar{y})$ (b) (σ_x, σ_y)

(c) (b_{xy}, b_{yx}) (d) (σ_x^2, σ_y^2) (1 mark)

Answer:

(a) Since the two lines of regression pass through the point $(\bar{x}, \bar{y})$, the mean values $(\bar{x}, \bar{y})$ can be obtained as the point of intersection of the two regression lines.

2012 - DECEMBER

[52] In Spearman's Correlation Coefficient, the sum of the differences of ranks between two variables shall be _____.

(a) 0 (b) 1

(c) – 1 (d) None of the above. (1 mark)

Answer:

(a) In spearman's correlation coefficient, the sum of the differences of ranks between two variable shall be **any number.**

[53] For certain x and y series which are correlated, the two lines of regression are

$5x - 6y + 9 = 0$

$15x - 8y - 130 = 0$

The correlation coefficient is

(a) 4/5 (b) 3/4
(c) 2/3 (d) 1/2 (1 mark)

Answer:

(c) The lines of regression are

$5x - 6y + 9 = 0$ and $15x - 8y - 130 = 0$

$6y = 5x + 9$ $15x = 8y + 130$

$y = \frac{5}{6}x + \frac{9}{6}$ $x = \frac{8}{15}y + \frac{130}{15}$

$y = \frac{9}{6} + \frac{5}{6}x$ $x = \frac{130}{15} + \frac{8}{15}y$

$y = a + bx$ $x = a + by$

We get We get

$b = \frac{5}{6} = b_{yx}$ $b = \frac{8}{15} = b_{xy}$

Coefficient of correlation $r = \pm \sqrt{b_{yx} \times b_{xy}}$

$= \pm \sqrt{\frac{5}{6} \times \frac{8}{15}}$

$= \pm \sqrt{\frac{4}{9}}$

$= \pm 2/3$

[54] The Coefficient of correlation between x and y series is – 0.38. The linear relation between x & u and y & v are $3x + 5v = 3$ and $-8x - 7v = 44$, what is the coefficient of correlation between u & v?

(a) 0.38 (b) – 0.38
(c) 0.40 (d) None of the above. (1 mark)

Answer:

(b) Given $r_{xy} = -0.38$

Given linear relation between x & u and y & v are

$3x + 5u = 3$ and $-8x - 7v = 44$

$5u = 3 - 3x$ $7v = -44 - 8x$

$u = \frac{3}{5} - \frac{3}{5}x$ $v = -\frac{44}{7} - \frac{8}{7}x$

$u = a + bx$ $v = c + dx$

We get $b = -\frac{3}{5}$

We get $d = -\frac{8}{7}$

$$r_{xy} = \frac{bd}{|b|\,|d|} r_{uv}$$

$$-0.38 = \frac{\left(\frac{-3}{5}\right)\left(\frac{-8}{7}\right)}{\left[\left(\frac{-3}{5}\right)\right]\left[\left(\frac{-8}{7}\right)\right]} \times r_{uv}$$

$$-0.38 = \frac{24}{35} \times \frac{35}{24} r_{uv}$$

$$r_{uv} = -0.38$$

[55] If $y = 18x + 5$ is the regression line of y on x value of b_{xy} is

(a) 5/18 (b) 18

(c) 5 (d) 1/18 (1 mark)

Answer:

(d) If $y = 18x + 5$

$$18x = -5 + y$$

$$x = \frac{-5 + y}{18}$$

$$x = \frac{-5}{18} + \frac{1}{18} y$$

$x = a + by$

We get $b = b_{xy} = 1/18$

2013 - JUNE

[56] If 'r' be the Karls Pearson's coefficient of correlation in a bivariate distribution then the two regression lines are at right angle if:

(a) $r = \pm 1$

(b) $r = 0$

(c) r = ± any finite value whose numerical value is less than 1
(d) None of these (1 mark)

Answer:

(b) If r = 0 the two regression lines are at right angle.

[57] If the regression equations are 8x – 3y + 50 = 0 and 14x – 7y – 60 = 0 and standard deviation of y is 1. The coefficient of correlation is = _____

(a) 2 (b) 1
(c) 0.87 (d) – 0.87 (1 mark)

Answer:

(c) Regression Equation are

$8x - 3y + 50 = 0$ and $14x - 7y - 60 = 0$

$8x = -50 + 3y$ $7y = -60 + 14x$

$x = \frac{-50}{8} + \frac{3}{8}y$ $y = \frac{-60}{7} + \frac{14}{7}x$

$x = a + 3y$ $y = a + bx$

We get We get

$b = \frac{3}{8} \rightarrow bxy$ $b = \frac{14}{7} = 2 \rightarrow byx$

$r = \pm \sqrt{bxy \times byx}$

$= \pm \sqrt{\frac{3}{8} \times 2}$

$= \pm \frac{\sqrt{3}}{2}$

$= + \frac{1.732}{2} = + 0.866$

$= + 0.87$

[58] The coefficient of correlation between two variables x and y is 0.28. Their covariance is 7.6. If the variance of x is 9, then the standard deviation of y is:

(a) 8.048 (b) 9.048
(c) 10.048 (d) 11.048 (1 mark)

Answer:

(b) Coeff of correlation (r) = 0.28

Cov (x, y)= 7.6

Var (x) = 9

S.D. (σx) = $\sqrt{9}$ = 3

S.D. of y (σy)= ?

We know that

$$r = \frac{Cov(x, y)}{\sigma x . \sigma y}$$

$$0.28 = \frac{7.6}{3 \times \sigma y}$$

$$\sigma y = \frac{760^{190}}{3 \times 0.28_{7}}$$

$$\sigma y = 9.048$$

[59] Two variables x and y are related according to 4x + 3y = 7. Then x and y are:

(a) Positively correlated. (b) Negatively correlated.

(c) Correlation is zero. (d) None of these. (1 mark)

Answer:

(b) Given Regression Equation

$4x + 3y = 7$ and $4x + 3y = 7$

$3y = 7 - 4x$ $4x = 7 - 3y$

$y = \frac{7}{3} \frac{- 4x}{3}$ $x = \frac{7}{4} \frac{- 3y}{4}$

$y = a + bx$ $x = a + by$

We get We get

$b = -4/3 = byx$ $b = -3/4 = bxy$

$r = \pm\sqrt{byx \times bxy}$

$= \pm \sqrt{(\frac{-4}{3})(\frac{-3}{4})}$

$= - \sqrt{1}$ [∵both bxy & byx are negative]

$r = -1$ (Negative correlated)

2013 - DECEMBER

[60] Determine the coefficient of correlation between x and y series:

	x Series	y Series
No. of items	15	15
Sum of sq. of deviation of mean	25	18
Arithmetic mean	136	138

Sum of product of deviation of x and y series from mean = 122

(a) - 0.89 (b) 0.89

(c) 0.69 (d) - 0.69 (1 mark)

Answer:

(b) Given N = 15, $\bar{x} = 25$, $\bar{y} = 18$, $\Sigma dx^2 = 136$, $\Sigma dy^2 = 138$ $\Sigma dx\, dy = 122$

Coeff of correlation

$$r = \frac{\sum dx\, dy}{\sqrt{\sum dx^2 \sum dy^2}}$$

$$= \frac{122}{\sqrt{136 \times 138}} = \frac{122}{136.99} = 0.89$$

[61] Price and Demand is the example for

(a) No correlation (b) Positive correlation

(c) Negative (d) None of the above (1 mark)

Answer:

(c) Price and Demand is the example for negative correlation.

[62] If mean of x and y variables is 20 and 40 respectively and the regression coefficient of y on x is 1.608, then the regression line of y on x is

(a) y = 1.608x + 7.84 (b) y = 1.5x + 4.84

(c) y = 1.608x + 4.84 (d) y = 1.56x + 7.84 (1 mark)

Answer:

(a) $\bar{x} = 20, \bar{y} = 40$, byx = 1.608

The Regression equation of line y on x

$y - \bar{y} = byx\,(x - \bar{x})$

$y - 40 = 1.608\,(x - 20)$

$y - 40 = 1.608x - 32.16$

$y = 1.608x - 32.16 + 40$

$y = 1.608x + 7.84$

[63] When the value of correlation coefficient is +1 or -1, then the two regression lines will _____.

(a) have 30° angle between them.

(b) have 45° angle between them.

(c) coincide.

(d) be perpendicular to each other (1 mark)

Answer:

(c) When the value of correlation coefficient is + 1 or - 1 then the two regression line will **coincide**

2014 - JUNE

[64] Two regression lines for a bivariate data are: 2x - 5y + 6 = 0 and 5x-4y + 3 = 0. Then the coefficient of correlation should be:

(a) $\frac{-2\sqrt{2}}{5}$ (b) $\frac{2}{5}$

(c) $\frac{+2\sqrt{2}}{5}$ (d) $\frac{\sqrt{2}}{5}$ (1 mark)

Answer:

(c) Two Regression lines

$2x - 5y + 6 = 0$ and $5x - 4y + 3 = 0$

$byx = \frac{-\text{Coeff. of x}}{\text{Coeff. of y}}$, $bxy = \frac{-\text{Coeff. of y}}{\text{Coeff. of x}}$

$$= \frac{-(2)}{-5} \qquad = \frac{-(-4)}{5}$$

$$= \frac{2}{5} \qquad = \frac{4}{5}$$

$$r = \pm \sqrt{byx \times bxy}$$

$$= \pm \sqrt{\frac{2}{5} \times \frac{4}{5}}$$

$$= + \sqrt{\frac{8}{25}}$$

$$= + \frac{2\sqrt{2}}{5}$$

[65] When each individual gets the exactly opposite rank by the two Judges, then the rank correlation will be _____.

(a) 0 (b) -1

(c) +1 (d) $\frac{1}{2}$ (1 mark)

Answer:

(b) When each individual gets the exactly opposite rank by the two Judges, then the rank correlation will be – 1.

[66] If the mean of the two variables 'x' and 'y' are 3 and 1 respectively. Then the equation of two regression lines are_____.

(a) 5x+7y-22=0, 6x+2y-20 = 0 (b) 5x+7y-22=0, 6x+2y+20=0

(c) 5x+7y+22=0, 6x+2y-20 = 0 (d) 5x+7y+22=0, 6x+2y+20=0

(1 mark)

Answer:

(a) The equation of two Regression lines are

$5x + 7y - 22 = 0, 6x + 2y - 20 = 0$

by solving these equations we get.

$x = 3$ & $y = 1$

So $\bar{x} = 3$, & $\bar{y} = 1$

(The Intersection of two regression lines are $\bar{x}$, $\bar{y}$).

[67] The equation of two lines of regression for 'x' and 'y' are $5x = 22 + y$ and $64x = 24 + 45y$ then the value of regression coefficient of 'y' on 'x' will be _____.

(a) 5 (b) $\frac{1}{5}$

(c) $\frac{64}{45}$ (d) $\frac{45}{64}$ (1 mark)

Answer:

(c) Given Regression Equation

$5x = 22 + y$ and $64x = 24 + 45y$

$5x - y - 22 = 0$ and $64x - 45y - 24 = 0$

$bxy = \frac{1}{5}$ and $byx = \frac{+64}{45}$

So, $bxy \times byx \leq 1$ then $byx = 64/45$

2014 - DECEMBER

[68] If the correlation coefficient between two variables is zero, then the lines of regression are:

(a) Parallel (b) Perpendicular

(c) Coincide (d) None of these (1 mark)

Answer:

(b) If the correlation coefficient b/w two variables is zero, then the lines of regression are **perpendicular.**

[69] If the value of correlation coefficient between x & y is 1, then the value of correlation coefficient between x - 2 and $\frac{-y}{2}+1$ is:

(a) 1 (b) -1

(c) -1/2 (d) 1/2 (1 mark)

Answer:

(b) Given $r_{xy} = 1$

Let $x - 2 = u$ and $\frac{-y}{2} + 1 = v$

$x = 2 + u$ $\frac{-y+2}{2} = v$

Comparing from

$x = a + bu$ $-y + 2 = 2v$

we get $b = 1$ $y = 2 - 2v$

on comparing

$y = c + dv$

we get

$d = -2$

$$r_{xy} = \frac{b.d}{|b|\,|d|} r_{uv}$$

$$1 = \frac{1 \times (-2)}{|1|\,|-2|} . r_{uv}$$

$$1 = \frac{-2}{2} r_{uv}$$

$$r_{uv} = -1$$

[70] The equations of two regression lines are $x + y = 6$ and $x + 2y = 10$, then the value of correlation coefficient between x and y is:

(a) -1/2 (b) +1/2

(c) $-1/\sqrt{2}$ (d) $+1/\sqrt{2}$ (1 mark)

Answer:

(c) Given two Regression lines:

$x + y = 6$ and $x + 2y = 10$

$x + y - 6 = 0$ $x + 2y - 10 = 0$

$b_{xy} = \frac{-\text{Coeff. of } y}{\text{Coeff. of } x}$ $b_{yx} = \frac{-\text{Coeff. of } x}{\text{Coeff. of } y}$

$= \frac{-1}{1} = -1$ $= \frac{-1}{2}$

$r = +\sqrt{b_{xy} \times b_{yx}} = +\sqrt{(-1)\left(\frac{-1}{2}\right)} = -\frac{1}{\sqrt{2}}$

2015 - JUNE

[71] Two regression lines are
$16x - 20y + 132 = 0$
$80x - 36y - 428 = 0$
The value of the correlation coefficient is
(a) 0.6 (b) - 0.6
(c) 0.54 (d) 0.45 (1 mark)

Answer:

(a) Given: Regression line

$$16x - 20y + 132 = 0$$

$$byx = -\frac{\text{coefficient of } x}{\text{coefficient of } y} = \frac{-16}{-20} = \frac{4}{5}$$

and other Regression line

$$80x - 36y - 428 = 0$$

$$bxy = -\frac{\text{coefficient of } y}{\text{coefficient of } x} = -\left(\frac{-36}{80}\right)$$

$$= \frac{36}{80} = \frac{9}{20}$$

$$byx \times bxy = \frac{4}{5} \times \frac{9}{20} = 0.36$$

$byx \times bxy \leq 1$ so it is satisfies

Then,

$$r = \pm \sqrt{byx \times bxy}$$

$$= \pm \sqrt{\frac{4}{5} \times \frac{9}{20}}$$

$$= + \sqrt{\frac{36}{100}}$$

$$= + 0.6$$

[72] When the correlation coefficient r is equal to + 1, all the points in a scatter diagram would be

(a) On a straight line directed from upper left to lower right
(b) On a straight line directed from lower left to upper right
(c) On a straight line
(d) Both (a) and (b) (1 mark)

Answer:

(b) When the correlation coefficient r is equal to '+1', all the points in a scatter diagram on a straight line directed from lower left to upper Right.

2015 - DECEMBER

[73] Out of following which is correct?

(a) $b_{yx} = r \frac{\sigma_x}{\sigma_y}$ (b) $b_{yx} = r \frac{\sigma_y}{\sigma_x}$

(c) $b_{yx} = \frac{\pi . \Sigma xy}{\sigma_x}$ (d) $b_{yx} = \frac{\pi . \Sigma xy}{\sigma_y}$ (1 mark)

Answer:

(b) $byx = \frac{r.\sigma_y}{\sigma_x}$

Where σ_y = S.D. of y, σ_x = S.D. of x
r = Coeff. of Correlation

[74] In case of "Insurance Companies" profits and the number of claims they have to pay there is _____ correlation.

(a) Positive (b) Negative
(c) No correlation (d) None of the above (1 mark)

Answer:

(b) In case of Insurance Companies Profits and the Number of claims they have to pay there is **Negative** Correlation.

2016 - JUNE

[75] Two regression equations are as follows:
Regression equation of x on y : $5x - y = 22$
Regression equation of y on x : $64x - 45y = 24$
What will be the mean of x and y?

(a) $\bar{x} = 8$, $\bar{y} = 6$ (b) $\bar{x} = 6$, $\bar{y} = 6$
(c) $\bar{x} = 6$, $\bar{y} = 8$ (d) $\bar{x} = 8$, $\bar{y} = 8$ (1 mark)

Answer:

(c) Given Regression Equations

$5x - y = 22$ _____ (1)
$64x - 45y = 24$ _____ (2)

Multiply by 45 in equation (1) we get

$225x - 45y = 990$ _____ (3)

equation (3) – equation (2)

$$\begin{array}{r} 225x - 45y = 990 \\ 64x - 45y = 24 \\ - \quad + \quad - \\ \hline 161x = 966 \end{array}$$

$\boxed{x = 6}$

Putting $x = 6$ in equation (1)

$5 \times 6 - y = 22$
$30 - y = 22$
$\boxed{y = 8}$

$\bar{x} = x = 6$
$\bar{y} = y = 8$

[76] If the coefficient of correlation between X and Y variables is +0.90 then what will be the coefficient of determination?

(a) 0.30 (b) 0.81
(c) 0.94 (d) None of these (1 mark)

Answer:

(b) If Coeff. of Correlation (r) = 0.90

Coeff. of Determination = r^2

= $(0.90)^2$

= 0.81

[77] The two lines of regression become identical when

(a) r = 1 (b) r = – 1

(c) r = 0 (d) (a) or (b) (1 mark)

Answer:

(d) If r = – 1 or + 1 then two lines of Regression become Identical.

[78] If r = 0.6, then the coefficient of determination is.

(a) 0.4 (b) – 0.6

(c) 0.36 (d) 0.64 (1 mark)

Answer:

(c) If r = 0.6

Then Coeff. of determination = r^2

= $(0.6)^2$

= 0.36

2016 - DECEMBER

[79] The two regression lines passing through

(a) Represent means (b) Represent S.Ds

(c) (a) and (b) (d) None of these. (1 mark)

Answer:

(a) The two Regression lines passing through or (Intersect) at their means.

[80] Out of the following the one which effects the regression coefficient is
(a) Change of origin only
(b) Change of scale only
(c) Change of scale and origin both
(d) Neither change in origin nor change of scale (1 mark)

Answer:

(b) By shifting the scale, coefficient of regression is changed.

[81] The regression equation x and y is $3x + 2y = 100$, the value of b_{xy}

(a) $-\frac{2}{3}$ (b) $\frac{100}{3}$

(c) $\frac{3}{2}$ (d) $\frac{2}{3}$ (1 mark)

Answer:

(a) The regression equation of x & y is

$3x + 2y = 100$

$3x + 2y - 100 = 0$

$$b_{xy} = -\frac{\text{Coefficient of y}}{\text{Coefficient of x}} = \frac{-2}{3}$$

[82] In a beauty contest there were 10 competitors. Rank of these candidates are assigned by two judges A and B. The sum of squares of differences of ranks is 44. The value of rank correlation is:

(a) 0.70 (b) 0.73
(c) 0.80 (d) 0.60 (1 mark)

Answer:

(b) Sum of squares of differences of ranks$(\Sigma d^2) = 44$

No. of candidates $n = 10$

$r_R = ?$

Formula:

Rank correlation

$$r_R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6 \times 44}{10(10^2 - 1)}$$

$$= 1 - \frac{6 \times 44}{10 \times 99}$$

$$= 1 - 0.267$$

$$= 0.733$$

$$= 0.73$$

2017 - JUNE

[83] The coefficient of correlation between the temperature of environment and power consumption is always:

(a) Positive (b) Negative

(c) Zero (d) Equal to 1 (1 mark)

Answer:

(a) The coefficient of correlation between the temperature of environment and power consumption is always positive.

[84] If two regression lines are $x + y = 1$ and $x - y = 1$ then mean values of x and y will be:

(a) 0 and 1 (b) 1 and 1

(c) 1 and 0 (d) -1 and -1 (1 mark)

Answer:

(c) Given Regression line

$x + y = 1$ _____ (1)

Adding $x - y = 1$ _____ (2)

$2x = 2$

$x = \frac{2}{2} = 1$

$x = 1$ in equation (1) we get

$1 + y = 1$

$y = 0$

Mean of $x = \bar{x} = 1$

Mean of $y = \bar{y} = 0$

[85] The coefficient of correlation between x and y is 0.6. If x and y values are multiplied by -1, then the coefficient of correlation will be:

(a) 0.6 (b) -0.6

(c) $\frac{1}{0.6}$ (d) $1 - 0.6$ (1 mark)

Answer:

(a) The coefficient of correlation between X and Y is 0.6. If X and Y values are multiplied by -1 then coefficient of correlation remains unchanged. Then are coefficient of correlation will be 0.6.

2017 - DECEMBER

[86] If two regression lines are $5y = 9x - 22$ and $20x = 9y + 350$, then the value of correlation coefficient (r) will be:

(a) 0.10 (b) -0.10

(c) -0.90 (d) 0.90 (1 mark)

Answer:

(d) Given two regression lines are

$5y = 9x - 22$ ——— (1)

$9x - 5y - 22 = 0$

$b_{yx} = \frac{-\text{coeff. of } x}{\text{coeff. of } y} = \frac{-9}{-5}$

$b_{yx} = \frac{9}{5}$

and $20x = 9y + 350$ ——— (2)

and $20x - 9y - 350 = 0$

and $b_{xy} = \frac{-\text{coeff. of } y}{\text{coeff. of } x}$

$b_{xy} = \frac{-(-9)}{20} = \frac{9}{20}$

$r = \pm \sqrt{b_{yx} \times b_{xy}}$

$$= \pm \sqrt{\frac{9}{5} \times \frac{9}{20}}$$

$$= \pm \sqrt{\frac{81}{100}}$$

$$= + \left(\frac{9}{10} \right) = + 0.90$$

[87] The regression coefficient is independent of the change of:

(a) Origin (b) Scale

(c) Both (a) and (b) (d) Neither (a) nor (b). (1 mark)

Answer:

(a) The regression coefficient is independent of the change of 'origin'.

or

By shifting the origin, coefficient of regression is not changed.

[88] If r = 0.6 then the coefficient of non-determination will be:

(a) 0.40 (b) -0.60

(c) 0.36 (d) 0.64 (1 mark)

Answer:

(d) Given r = 0.6

Coefficient of non determination $= 1 - r^2$

$= 1 - (0.6)^2$

$= 1 - 0.36$

$= 0.64$

[89] The correlation coefficient (r) is the _____ of the two regression coefficients (b_{yx} and b_{xy})

(a) AM (b) GM

(c) HM (d) Median (1 mark)

Answer:

(b) The coefficient of correlation (r) is the G.M. of the two regression coefficient (byx × bxy)

$r = \pm \sqrt{byx \times bxy}$

[90] If there is a constant increase in a series, then the corresponding graph will be
(a) Convex curve
(b) Concave curve
(c) Parabola
(d) Straight line from the left to the right (1 mark)

Answer:

(d) If there is a constant increase in a series, then the corresponding graph will be straight line from the left to the right.

2018 - MAY

[91] If the plotted points is a scatter diagram are evenly distributed, then the correlation is
(a) Zero (b) Negative
(c) Positive (d) (a) Or (b) (1 mark)

Answer:

(a) If the plotted points in a scatter diagram are evenly distributed, then the correlation is **Zero**

[92] The covariance between two variables is
(a) Strictly positive
(b) Strictly negative
(c) Always Zero
(d) Either positive or negative or zero (1 mark)

Answer:

(d) The Co-variance between two variables is either positive or negative or zero.

[93] The coefficient of determination is defined by the formula

(a) $r^2 = \frac{1 - \text{unexplained variance}}{\text{total variance}}$

(b) $r^2 = \frac{\text{explained variance}}{\text{total variance}}$

(c) both (a) and (b)

(d) none (1 mark)

Answer:

(c) The coefficient of determination $= 1 - \frac{\textit{unexplained variance}}{\textit{total variance}}$

$= \frac{\text{explained variance}}{\text{total variance}}$

[94] In the method of Concurrent Deviations, only the directions of change (Positive direction/Negative direction) in the variables are taken into account for calculation of

(a) Coefficient of SD.

(b) Coefficient of regression

(c) Coefficient of correlation

(d) none (1 mark)

Answer:

(c) The Method of concurrent Deviation, only the directions of change (Positive direction/Negative direction) in the variables are taken into account for calculation of Coefficient of correlation.

[95] Correlation coefficient is _____ of the units of measurement.

(a) dependent (b) independent

(c) both (d) none (1 mark)

Answer:

(b) Correlation coefficient is **Independent** of the units of measurement.

[96] In case speed of an automobile and the distance required to stop the car after applying brakes correlation is

(a) Positive (b) Negative
(c) Zero (d) None (1 mark)

Answer:

(a) In case 'speed of on automobile and the distance required to stop the car often applying brakes' correlation is positive

[97] A relationship $r^2 = 1 - \frac{500}{300}$ is not possible

(a) True (b) False
(c) Both (d) None (1 mark)

Answer:

(a) Given

$r^2 = 1 - \frac{500}{300}$ is not possible

$r^2 = -\frac{200}{300}$ is not possible

So, It is true.

[98] Rank correlation coefficient lies between

(a) 0 to 1
(b) – 1 to +1 inclusive of these value
(c) – 1 to 0
(d) both (1 mark)

Answer:

(b) Rank correlation coefficient lies between – 1 to +1 inclusive of both value

2018 - NOVEMBER

[99] The two line of regression intersect at the point

(a) Mean (b) Mode
(c) Median (d) None of these (1 mark)

Answer:

[100] If the two lines of regression are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$, then the regression line of y on x is:

(a) $x + 2y - 5 = 0$ (b) $2x + 3y - 8 = 0$

(c) $x + 2y = 0$ (d) $2x + 3y = 0$ (1 mark)

Answer:

(a) Given two Regression lines are

$x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$

$$byx = \frac{-\text{Coeff. of } x}{\text{Coeff. of } y} = \frac{-1}{2} \text{ and } bxy = \frac{-\text{Coeff. of } y}{\text{Coeff. of } x} = \frac{-3}{2}$$

Here, $byx \times bxy \leq 1$ which is satisfied.

So 1st equation $x + 2y - 5 = 0$ is the Regression Equation y on x.

[101] If the two regression lines are $3X = Y$ and $8Y = 6X$, then the value of correlation coefficient is

(a) 0.5 (b) – 0.5

(c) 0.75 (d) – 0.80 (1 mark)

Answer:

(a) Given:

Regression line

$3x = y$ and $8y = 6x$

$3x - y = 0$ $6x - 8y = 0$

$$bxy = \frac{-\text{Coeff. of } y}{\text{Coeff. of } x} \text{ and } byx = \frac{-\text{Coeff. of } x}{\text{Coeff. of } y}$$

$$= \frac{-(-1)}{3} = \frac{-6}{-8} = \frac{3}{4}$$

$$bxy = \frac{1}{3}$$

Coeff. of correlation is given by

$$r = \pm \sqrt{byx \times bxy}$$

$$= \pm \sqrt{\frac{3}{4} \times \frac{1}{3}}$$

$$= + \sqrt{\frac{1}{4}}$$

$$= + \frac{1}{2}$$

$$= 0.5$$

[102] The regression coefficient is independent of the change of

(a) Scale (b) Origin

(c) Scale and origin both (d) None of these (1 mark)

Answer:

(b) The Regression coefficient is independent of the change of 'Origin'.

[103] If the correlation coefficient between the variables X and Y is 0.5, then the correlation coefficient between the variables $2x - 4$ and $3 - 2y$ is

(a) 1 (b) 0.5

(c) –0.5 (d) 0 (1 mark)

Answer:

(c) If coefficient of correlation $r_{xy} = 0.5$

Given $u = 2x - 4$ and $v = 3 - 2y$

$2x - u - 4 = 0$ and $2y + v - 3 = 0$

$b = \frac{-\text{Coeff. of } u}{\text{Coeff. of } x}$ and $d = \frac{-\text{Coeff. of } v}{\text{Coeff. of } y}$

$= \frac{-(-1)}{2}$ $d = \frac{-1}{2}$

$b = \frac{1}{2}$ $d = \frac{-1}{2}$

Here, b and d both have different sign so $r_{uv} = - r_{xy}$

$= - 0.5$

2019 - JUNE

[104] A.M. of regression coefficients is

(a) Equal to r

(c) Half of r
(d) None (1 mark)

Answer:

(b) A.M of Regression Coefficient is greater than or equal to r.

[105] Given that

X	– 3	– 3/2	0	3/2	3
Y	9	9/4	0	9/4	9

Then Karipearson's coefficient of correlation is
(a) Positive
(b) Zero
(c) Negative
(d) None (1 mark)

Answer:

(b) Given that

x	-3	-3/2	0	3/2	3
y	9	9/4	0	9/4	9

then Karlpearson's Coefficient of Correlation is "Zero" because it is equally distribute.

[106] Find the probable error if $r = \frac{2}{\sqrt{10}}$ and n = 36

(a) 0.6745
(b) 0.067
(c) 0.5287
(d) None (1 mark)

Answer:

(b) $r = \frac{2}{\sqrt{10}}$, n = 36, P.E = ?

Probable Error $P.E = \frac{2}{3}$ S.E

$$= \frac{2}{3}\frac{1-r^2}{\sqrt{n}}$$

$$= \frac{2}{3}\left[\frac{1-\left(\frac{2}{\sqrt{10}}\right)^2}{\sqrt{36}}\right]$$

$$= \frac{2}{3}\,\frac{\left(1-\frac{4}{10}\right)}{6}$$

$$= \frac{2}{3} \times \frac{\cancel{6}}{10 \times \cancel{6}}$$

$$= \frac{1}{15}$$

$$= 0.067$$

[107] Given the following series:

X	10	13	12	15	8	15
Y	12	16	18	16	7	18

The rank correlation coefficient r =

(a) $1 - \dfrac{6\sum d^2 + \sum_{i-1}^{2} \dfrac{m_1(m_i^2 - 1)}{12}}{n(n^2 - 1)}$

(b) $1 - \dfrac{6\left[\sum d^2 + \sum_{1=i}^{2} \dfrac{m_1(m_i^2 - 1)}{12}\right]}{n(n^2 - 1)}$

(c) $1 - 6\sum d^2 + \sum_{i=1}^{2} \dfrac{\dfrac{m_1(m_i^2 - 1)}{12}}{n(n^2 - 1)}$

(d) $1 - 6\sum d^2 + \sum_{i=1}^{3} \dfrac{\dfrac{m_1(m_i^2 - 1)}{12}}{n(n^2 - 1)}$

Answer:

(b) Given the following series:

x	10	13	12	15	8	15
y	12	16	18	16	7	18

In this question we use the formula of the Rank Correlation Coefficient.

$$r = 1 - \frac{6\left[\sum d^2 + \sum_{i=1}^{2} \frac{m_i(m^2_i - 1)}{12}\right]}{n(n^2-1)}$$

[108] Determine Spearman's rank correlation coefficient from the given data $\sum d^2 = 30$, n = 10:

(a) r = 0.82

(b) r = 0.32

(c) r = 0.40

(d) None of the above (1 mark)

Answer:

(a) Here, $\sum d^2 = 30$, n = 10

Spearman's rank correlation

$$r_R = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 30}{10(10^2-1)} = 1 - \frac{180}{990} = 1 - \frac{2}{11} = \frac{9}{11}$$

$$= 0.82$$

[109] If the regression line of Y on X is given by Y = X + 2 and Karlpearson's coefficient of correlation is 0.5 then $\frac{QY^2}{QX^2}$ = _____.

(a) 3

(b) 2

(c) 4

(d) None (1 mark)

Answer:

(c) The regression line of y on x is given by $y = x + 2$

$x - y + 2 = 0$

$$\text{by x} = -\frac{\text{coefficient of x}}{\text{coefficient of y}}$$

$$= \frac{-1}{-1} = 1$$

$\text{by x} = 1$

coeff. of correlation (r) = 0.5

then Regression coefficient y or x

$$\text{by x} = r \frac{\sigma y}{\sigma x}$$

$$1 = 0.5 \frac{\sigma y}{\sigma x}$$

$$\frac{\sigma y}{\sigma x} = \frac{\cancel{10}^2}{\cancel{5}}$$

$$\left(\frac{\sigma y}{\sigma x}\right)^2 = (2)^2$$

$$\frac{\sigma y^2}{\sigma x^2} = 4$$

2019 - NOVEMBER

[110] If two line of regression are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$. So $x + 2y - 5 = 0$ is

(a) y on x

(b) x on y

(c) both

(d) None (1 mark)

Answer:

(a) $x + 2y - 5 = 0$ — Eq 1 $2x + 3y - 8 = 0$ — Eq 2

Let Eq 1 be y on x from Eq 2

$$b_{yx} = \frac{-\text{coeff of } x}{\text{coeff of } y} \quad b_{xy} = \frac{-\text{coeff of } y}{\text{coeff of } x}$$

$$b_{yx} = \frac{-1}{2} \quad b_{xy} = \frac{-3}{2}$$

$$b_{yx} \times b_{xy} = \left(\frac{-1}{2}\right) \times \left(\frac{-3}{2}\right) = \frac{3}{4}$$

So, $b_{yx} \times b_{xy} < 1$

So, $x + 2y - 5 = 0$ is y on x

and $2x + 3y - 8 = 0$ is x on y.

[111] Find the coefficient of regression.

$2x + 3y = 2$

$4x + 3y = 4$

(a) 0.5

(b) – 0.5

(c) 0.25

(d) – 0.25 (1 mark)

Answer:

(b) $2x + 3y = 2$ — Eq 1 $ax + 3y = 4$ — Eq 2

Let Eq1 be y on x

From Eq 1

$$b_{yx} = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-2}{3}$$

From Eq 2

$$b_{xy} = \frac{-\text{coeff of } y}{\text{coeff of } x} = \frac{-3}{4}$$

$$b_{yx} \times b_{xy} = \frac{-2}{3} \times \frac{(-3)}{4} = \frac{1}{2} < 1$$

So above assumption holds true.

$r = \pm \sqrt{b_{yx} \times b_{xy}}$

$r = \pm \sqrt{\left(\frac{-2}{3}\right) \times \left(\frac{-3}{4}\right)}$

$r = \frac{-1}{2}$ [Since b_{yx} and b_{xy} are negative]

$r = -0.5$

[112] What is the coefficient of correlation from the following data?

x :	1	2	3	4	5
y :	5	4	3	2	6

(a) 0
(b) – 0.75
(c) – 0.85
(d) 0.82 (1 mark)

Answer:

(a)

x	y	xy
1	5	5
2	4	8
3	3	9
4	2	8
5	6	30
$\Sigma x = 15$	$\Sigma y = 20$	$\Sigma xy = 60$

$\text{cov}(x, y) = \frac{\Sigma xy}{n} - \bar{x}, \bar{y}$

$= \frac{60}{5} - \left(\frac{15}{5}\right) \times \left(\frac{20}{5}\right)$

$= 12 - 12$

$\text{cov}(x, y) = 0$

$r = \frac{\text{Cov}(x, y)}{\sigma_x . \sigma_y} = \frac{0}{\sigma_x . \sigma_y}$

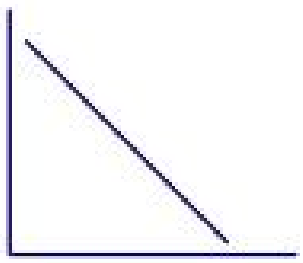
$r = 0$

[113] If the plotted points in a scatter diagram lie from upper left to lower right, then correlation is

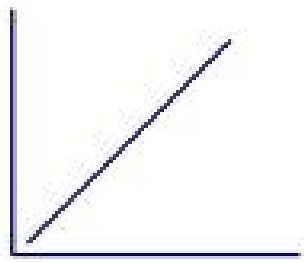
(a) Positive
(b) Negative
(c) Zero
(d) None of these (1 mark)

Answer:

(c) If the plotted points in a scatter diagram lie from upper left to lower right them correlation us negative.



Negative correlation



Positive correlation

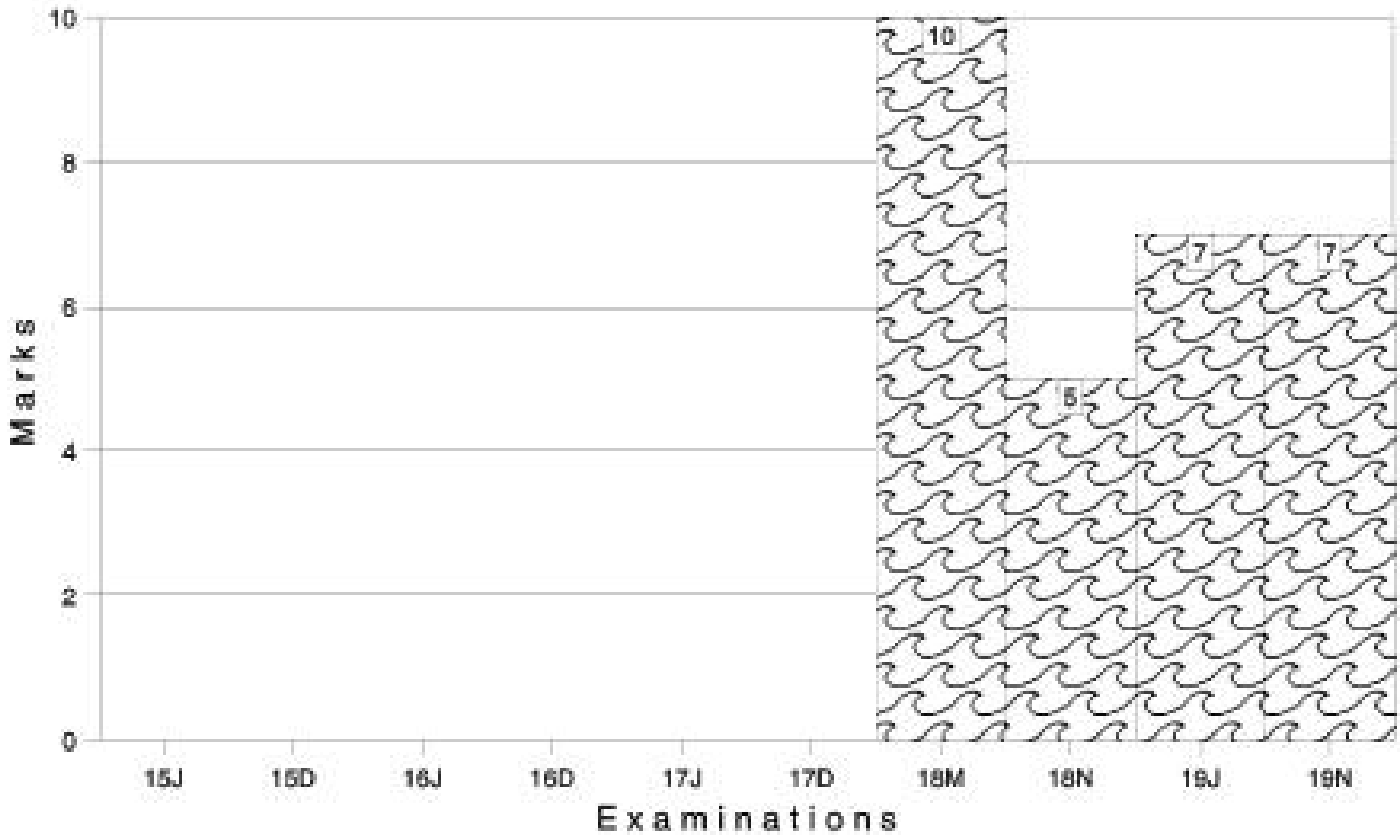
CHAPTER

19 INDEX NUMBERS AND TIME SERIES

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective | Short Notes | Distinguish | Descriptive | Practical



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PAST YEAR QUESTIONS AND ANSWERS

2006 - NOVEMBER

[1] The number of test of Adequacy is :

(a) 2 (b) 3

(c) 4 (d) 5 (1 mark)

Answer:

(c) Number of Test of Adequacy are 4 which are.

(a) Unit test

(b) Time reversal test

(c) Factor reversal test

(d) Circular test

[2] The consumer price index for 2006 on the basis of 2005 from the following data is:

Commodities	Quantities consumed in 2005	Price in 2005	Prices in 2006
A	6	5.75	6.00W
B	6	5.00	8.00
C	1	6.00	9.00
D	6	8.00	10.00
E	4	2.00	1.50
F	1	20.00	15.00

(a) 128.77 (b) 108.77

(c) 138.77 (d) 118.77 (1 mark)

Answer:

(d)

Commodities	Quantities consumed in 2005 (Q_0)	Price in 2005 (P_0)	Price in 2006 (A)	P_1q_0	P_0q_0
A	6	5.75	6.00	36	34.50
B	6	5.00	8.00	48	30.00
C	1	6.00	9.00	9	6.00
D	6	8.00	10.00	60	48.00
E	4	2.00	1.50	6	8.00
F	1	20.00	15.00	15	20.00
				$\Sigma P_1q_0 = 174$	$\Sigma P_1q_0 = 146.5$

$$\text{Consumer Price Index} = \frac{\sum P_1q_0}{\sum P_0q_0} \times 100 = \frac{174}{146.5} \times 100 = 118.77$$

[3] Suppose a business executive was earning ₹ 2,050 in the base period, what should be his salary in the current period if his standard of living is to remain the same? Given $\Sigma W = 25$ and $\Sigma IW = 3544$:

(a) ₹ 2096 (b) ₹ 2906

(c) ₹ 2106 (d) ₹ 2306 (1 mark)

Answer:

(b) Cost of Living Index $= \frac{\sum IW}{\sum W} = \frac{3544}{25} = 141.76 \rightarrow$ (i)

Since the Cost of living Index of the base period is always taken to be 100

$\therefore$ In order to maintain his standard of living, his salary in the current year should be :

Given his salary in the base period = ₹ 2,050

Cost of living Index from (i) is141.76

100	₹ 2,050
141.76	x

$$x = \frac{2,050 \times 141.76}{100} = ₹\ 2,906.08 = ₹\ 2,906 \text{ (approx)}$$

2007 - FEBRUARY

[4] Bowley's index number is expressed in terms of :

(a) $\frac{\text{Laspeyre's} + \text{Paasche's}}{2}$ (b) $\frac{\text{Laspeyre's} \times \text{Paasche's}}{2}$

(c) $\frac{\text{Laspeyre's} - \text{Paasche's}}{2}$ (d) None of these (1 mark)

Answer:

(a) Bowley's has suggested simple arithmetic mean of the two indices (Laspeyres and Paasche) mentioned above so as to take into account the influence of both the periods, i.e, current as well as base periods.

$$P_{01} = \frac{L + P}{2}$$

Where L = Laspeyer's Index, P = Paasche's Index

$$\frac{P_{01} = \frac{\sum P_1 q_0 + \sum P_1 q_1}{\sum P_0 q_0 + \sum P_0 q_1} \times 100}{2}$$

[5] Fisher's ideal formula for calculating index number satisfies the ___ :

(a) Unit Test (b) Factor Reversal Test

(c) Both (a) & (b) (d) None of these (1 mark)

Answer:

(c) Fisher's Ideal formula for calculating index number satisfies the unit test as unit test requires that the formula should be independent of the unit in which or for which prices and quantities are quoted and this is fulfilled by Fisher's Ideal Index.

Factor reversal test holds when the product of price Index and the quantity index should be equal to corresponding value index ie.

$$\frac{\sum P_1 Q_1}{\sum P_0 Q_0}$$

$$P_{01} \times Q_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_0}$$

Hence it is satisfied by Fisher's Ideal Index.

Proof :

$$P_{01} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum p_0 q_1}$$

Changing p to q and q to p.

$$Q_{01} = \sqrt{\frac{\sum q_1 P_0}{\sum q_0 p_0} \times \frac{\sum q_1 P_1}{\sum q_0 p_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1} \times \frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 P_1}{\sum q_0 p_1}}$$

$$= \sqrt{\frac{(\sum P_1 q_1)^2}{\sum P_0 q_0)^2}} = \frac{\sum P_1 q_1}{\sum P_0 q_0}$$

Hence, we can say that Fisher's ideal formula for calculating index number satisfies both Unit Test as well as Factor Reversal test.

[6] Calculate the Fisher ideal index from the following data :

	Price (₹)		Quantity ('000 kg.)	
Commodity	**2004**	**2005**	**2004**	**2005**
Rice	9.3	4.5	100	90
Wheat	6.4	3.7	11	10
Pulse	5.1	2.7	5	3

(a) 49.13 (b) 48.13

(c) 84.13 (d) 46.12 (1 mark)

Answer:

(a)

Commodity	P_0	P_1	q_0	q_1	P_0q_0	P_1q_0	P_0q_1	P_1q_1
Rice	9.3	4.5	100	90	930	450	837	405
Wheat	6.4	3.7	11	10	70.4	40.7	64	37
Pulse	5.1	2.7	5	3	25.5	13.5	15.3	8.1
					1025.9	504.2	916.3	450.1

$$\text{Fisher's Ideal Index : } P_{01} = \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100$$

$$= \sqrt{\frac{504.2}{1025.9} \times \frac{450.1}{916.3}} \times 100$$

$$= \sqrt{0.2414} \times 100$$

$$= 0.4913 \times 100$$

$$= 49.13$$

2007 - MAY

[7] Circular Test is satisfied by :

(a) Paasche's Index Number.

(b) The simple geometric mean of price relatives and the weighted aggregative with fixed weights

(c) Laspeyre's Index Number

(d) None of these (1 mark)

Answer:

(b) By weighted aggregative formula with fixed weights

$$Io_1 = \frac{\sum P_1 w}{\sum P_0 w},\ I_{12} = \frac{\sum P_2 w}{\sum P_1 w} \text{ and } I_{20} = \frac{\sum P_0 w}{\sum P_2 w}$$

$$\therefore Io_1 \times I_{12} \times I_{20}$$

$$= \frac{\sum P_1 w}{\sum P_0 w} \times \frac{\sum P_2 w}{\sum P_1 w} \times \frac{\sum P_0 w}{\sum P_2 w} = 1$$

Hence, the weighted aggregative formula with fixed weight satisfies circular test.

Again for the simple geometric mean of price relatives.

$$P_{01} = \frac{P_1}{P_0},\ P_{12} = \frac{P_2}{P_1},\ P_{20} = \frac{P_0}{P_2}$$

Simple Geometric Mean of the above price relative i.e P_{01}, P_{12}, & P_{20}

$$= \sqrt{P_{01}\ P_{12}\ P_{20}} = \sqrt{\frac{P_1}{P_0} \times \frac{P_2}{P_1} \times \frac{P_0}{P_2}} = 1$$

Hence, Geometric Mean of Price Relatives also satisfies Circular Test.

[8] From the following data :

Group	:	A	B	C	D	E	F
Group Index	:	120	132	98	115	108	95
Weight	:	6	3	4	2	1	4

The general index is given by :

(a) 113.54 (b) 115.30

(c) 117.92 (d) 111.30 (1 mark)

Answer:

(d)

Group	Group Index (I)	Weight (W)	I.W.
A	120	6	720
B	132	3	396
C	98	4	392
D	115	2	230
E	108	1	108
F	95	4	380
		= 20 = ΣW.	= 2226 = ΣIW

General Index = $\frac{\sum IW}{\sum W} = \frac{2226}{20} = 111.30$

2007 - AUGUST

[9] Cost of living index numbers are also used to find real wages by the process of :

(a) Base shifting (b) Splicing of index numbers

(c) Deflating of index numbers (d) None of these (1 mark)

Answer:

(c) Since deflation is a technique of adjusting the rupee value figure at a time period dividing it by the appropriate price index (usually Cost of Living Index) of the same time period after considering changes in price level.

When prices increases, money wages are deflated by the price index to find the real wages.

Thus real wage is determined after deflating the money by the Cost of Living Index by using the formula.

$$\text{Real wage} = \frac{\text{Money (or Actual) wage}}{\text{Cost of Living Index}} \times 100$$

[10] The prices of a commodity in the year 1975 and 1980 were 25 and 30 respectively. Taking 1980 as the base year the price relative is :

(a) 113.25 (b) 83.33

(c) 109.78 (d) None (1 mark)

Answer:

(b) $\text{Price relative} = \frac{\text{Current Year 's Price}}{\text{Base year 's Price}} \times 100 = \frac{25}{30} \times 100 = 83.33$

[11] From the following data :

Commodity	Base Year Price	Base Year Quantity	Current Year Price	Current Year Quantity
A	7	17	13	25
B	6	23	7	25
C	11	14	13	15
D	4	10	8	8

The Marshal Edgeworth index number is :

(a) 144.19 (b) 143.91

(c) 4900 (d) 140.31 (1 mark)

Answer:

(a)

CALCULATIONS FOR MARSHALL EDGE WORTH INDEX								
Commodity	Price P_o	Quantity q_o	Price P_1	Quantity q_1	P_oq_o	P_oq_1	P_1q_o	P_1q_1

B	6	23	7	25	138	150	161	175
C	11	14	13	15	154	165	182	195
D	4	10	8	8	40	32	80	64
Total	-	-	-	-	451	522	644	759
					$\sum P_0q_0$	$\sum P_0q_1$	$\sum P_1q_0$	$\sum P_1q_1$

Marshal-Edge worth Index

$$= \frac{\sum P_1(q_0+q_1)}{\sum P_0(q_0+q_1)} \times 100$$

$$= \frac{\sum P_1q_0+\sum P_1q_1}{\sum P_0q_0+\sum P_0q_1} \times 100$$

$$= \frac{644 + 759}{451 + 522}$$

$$= 144.19$$

2007 - NOVEMBER

[12] Net monthly salary of an employee was ₹ 3,000 in 1980. The consumer price index number in 1985 is 250 with 1980 as base year. If he has to be rightly compensated, then the Dearness Allowance to be paid to the employee is:

(a) ₹ 4,200 (b) ₹ 4,500

(c) ₹ 4,900 (d) ₹ 7,500 (1 mark)

Answer:

(b) Given :

Net monthly salary of the employee = ₹ 3,000 in 1980

Consumer price Index in 1985 with 1980 as the base year = 250

Since it is always assumed that the consumer price index is always taken to be as 100.

∴ When consumer price Index was 100 then monthly salary of the employee was ₹ 3,000.

And when the consumer Price Index. was 250 in 1985 then his monthly salary will be

$= \frac{250 \times 3,000}{100} = ₹ 7,500$

∴ The Dearness Allowance to be paid to the employee is
= Amount he should receive – Amount that he actually received.
= ₹ 7,500 – ₹ 3,000
= ₹ 4,500.

[13] P_{10} is the index for time :
(a) 0 on 1 (b) 1 on 0
(c) 1 on 1 (d) 0 on 0 (1 mark)

Answer:

(a) Index No. is defined as the ratio of the price of a commodity in a given period to its price in base or reference period.
Here, P_{10} is the ratio of price of the current year i.e. P_0 to the price of the base year i.e. P_1. Hence P_{10} signifies the index for time O on 1.

2008 - FEBRUARY

[14] An enquiry into the budgets of middle class families in a village gave the following information :

Expenses on :	Food	Rent	Clothing	Fuel	Others
	30%	15%	20%	10%	25%
Price in ₹ in 1987:	100	20	70	20	40
Price in ₹ in 2005:	90	20	60	10	55

$\Sigma PW = 10101.5$, $\Sigma W \log P = 199.494$. The price index number based on Weighted Arithmetic Mean of price relatives is :
(a) 111.015 (b) 101.015
(c) 0.0197 (d) None (1 mark)

Answer:

(b) Given $\Sigma PW = 1010.5$

$\Sigma W = 30\% + 15\% + 20\% + 10\% + 25\% = 100\%$

Price Index Number based on

Weighted Arithmetic Mean of price relatives =

$$P_{01}\text{ (A.M.)} = \frac{\Sigma PW}{\Sigma W} = \frac{1010.5}{100} = 101.015$$

[15] Shifted Price index

$$= \frac{\text{Original Price Index}}{\text{Price index of the year on which } \textit{it has to be shifted}} \times 100:$$

(a) True (b) False

(c) Partly True (d) Partly False (1 mark)

Answer:

(a) As we know, that base shifting is a technique of charging the old base period to new base period.

Therefore, we have

$$\text{Shifted price Index} = \frac{\text{Original Price Index}}{\text{Price Index of the year on which it has to be shifted}} \times 100$$

[16] Given the following information :

Commodity	2000		2003	
	Price	Quantity	price	Quantity
A	2	74	3	82
B	5	125	4	140
C	7	40	6	33

Which of the following is true :

(a) Marshall Edgeworth index for 2003 is 105.13

(b) Fisher's index for 2003 is 90.15.

(c) Marshall Edgeworth Index Number is good approximation to Fisher's Index Number

(d) None of these (1 mark)

Answer:

(c)

Commodity	Price P_0	Quantity q_0	Price P_1	Quantity q_1	P_0q_0	P_0q_1	P_1q_0	P_1q_1
A	2	74	3	82	148	164	222	246
B	5	125	4	140	625	700	500	560
C	7	40	6	33	280	321	240	198
Total					1053	1095	962	1004
	-	-	-	-	$=\Sigma P_0q_0$	$=\Sigma P_0q_1$	$=\Sigma P_1q_0$	$=\Sigma P_1q_1$

$$\text{Marshall - Edge Worth Index} = \frac{\Sigma P_1q_0+\Sigma P_1q_1}{\Sigma P_0q_0+\Sigma P_0q_1}\times 100$$

$$= \frac{962+1004}{1053+1095}\times 100 = \frac{196600}{2146} = 91.527$$

$$\text{Fisher's Ideal Index} = \sqrt{\frac{\Sigma P_1q_0+\Sigma P_1q_1}{\Sigma P_0q_0+\Sigma P_0q_1}\times 100}$$

$$= \sqrt{\frac{962}{1053}\times\frac{1004}{1095}\times 100} = 91.532$$

From above, we see that Marshall Edgeworth's Index is a good approximation to Fisher's Ideal Index.

2008 - JUNE

[17] Laspeyare's and Paasche's Method _____ Time Reversal Test :

(a) Do not satisfy (b) Satisfy

(c) Depends on the case (d) Can't say. (1 mark)

Answer:

(a) Laspeyre's formula (omitting the factor 100) is

$$I_{01} = \frac{\Sigma P_1q_0}{\Sigma P_0q_0} \text{.....(i)}$$

Interchanging the suffixes 0 and 1, we get

$$I_{10} = \frac{\sum P_0 q_1}{\sum P_1 q_1} \text{.....(ii)}$$

Multiplying (i) and (ii), we have

$$I_{01} \times I_{10} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_0 q_1}{\sum P_1 q_1} \neq 1$$

Therefore Laspeyre's formula do not satisfy Time Reversal Test which gives $I_{10} \times I_{01} = 1$

Again Paasche's formula (omitting the factor 100) is

$$I_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_1} \text{ ..... (iii)}$$

Interchanging the suffixes O and 1, we get

$$I_{10} = \frac{\sum P_0 q_0}{\sum P_1 q_0} \text{ ..... (iv)}$$

Multiplying (iii) and (iv), we have,

$$I_{01} \times I_{10} = \frac{\sum P_1 q_1}{\sum q_0 p_1} \times \frac{\sum P_0 q_0}{\sum P_1 q_0} \neq 1$$

Thus, Paasche's formula also do not satisfy Time Reversal test.

[18] Chain index is equal to :

(a) $\dfrac{\text{link relative of current year} \times \textit{chain index of the current year}}{100}$

(b) $\dfrac{\text{link relative of current year} \times \textit{chain index of the previous year}}{100}$

(c) $\dfrac{\text{link relative of previous year} \times \textit{chain index of the current year}}{100}$

(d) None of these (1 mark)

Answer:

(b) Chain Base Method of computing Index numbers, the price of each item in any year is compared with its price in the preceding year and expressed as a percentage. These are known as link relative.

These Link Indices for the successive years are then chained together by multiplication to obtain Chain Index Numbers, relative to a common base.

Thus chain Index for any year,

$$= \frac{\text{Link relative (Index) of current year} \times \text{Chain Index of the previous year.}}{100}$$

[19] In 2004 for working class people wheat was selling at an average price of ₹ 16 per 20 kg, cloth at ₹ 2 per meter, house rent ₹ 30 per house and other items at ₹ 10 per unit By 2005 cost of wheat rose by ₹ 4 per 20 Kg, house rent by ₹ 15 per house and other items doubled in price. The working class cost of living index for the year 2005 (with 2004 as base) was 160. By how much did cloth rose in price during the period :

(a) 1.28 (b) 0.99

(c) 1.73 (d) 1.30 (1 mark)

Answer:

(d) Let the Price of cloth be x in 2005

By applying simple average of price relative method.

$$P_{01} = \frac{\frac{\sum P_1}{\sum P_0} \times 100}{N} \Rightarrow 160 = \frac{475 + 50x}{4}$$

$\Rightarrow x = 3.30$

Hence increase in price of cloth

= ₹ 3.30 – ₹ 2 = ₹ 1.30

[20] The ratio of price of the single commodity in a given period to its price in another period is called :

(a) Price Ratio (b) Price Relative

(c) Base Period (d) None of these (1 mark)

Answer:

(b) The ratio of price of the single commodity is a given period to its price in another period is called price relative.

ie $P_{01} = \frac{P_1}{P_0}$

2008 - DECEMBER

[21] Consumer Price Index Number goes up from 100 to 200 and salary of a worker is also raised from 300 to 500

(a) 300 (b) 250

(c) 600 (d) 350 (1 mark)

Answer:

(b) Here, we are given the following data :

Year	Consumer Price	Index Salary
1990	100	300
1993	200	500

$$\text{Real wages of a worker in 1993} = \frac{\text{Salary in 93} \times \text{C.P.I. in 1990}}{\text{C.P.I. in 1993}}$$

$$= \frac{500 \times 100}{200} = ₹\ 250$$

[22] Using following data, find Paasche's Index Number

Commodities	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	5	25	6	30
B	3	8	4	10
C	2	10	3	8
D	10	4	3	5

(a) 109.21 (b) 105.28

(c) 110.32 (d) 120.21 (1 mark)

Answer:

(b)

Commodities	P_o	q_o	P_1	q_1	p_1q_1	p_0q_1
A	5	25	6	30	180	150
B	3	8	4	10	40	30
C	2	10	3	8	24	16
D	10	4	3	5	15	50
					$\Sigma p_1q_1 = 259$	$\Sigma p_0q_1 = 246$

Paasche's Index number = $\frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100 = \frac{259}{246} \times 100 = 105.28$

[23] The Circular Test is known as :

(a) $P_{01} \times P_{12} \times P_{20} = 1$ (b) $P_{12} \times P_{01}\ P_{20} = 1$

(c) $P_{20} \times P_{12}\ P_{01} = 1$ (d) $P_{02} \times P_{21}\ P_{12} = 1$ (1 mark)

Answer:

(a) Circular Test is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base.

The following equation,

$P_{01} \times P_{12} \times P_{20} =$ must be satisfied.

2009 - JUNE

[24] Fisher's Index is based on :-

(a) Arithmetic Mean of Laspeyre and Paasche

(b) Geometric Mean of Laspeyre and Paasche

(c) Harmonic Mean of Laspeyre and Paasche

(d) Median of Laspeyre and Paasche. (1 mark)

Answer:

(b) Fisher's ideal price index is the geometric mean of Laspeyres' Index and Paasche's Index.

Therefore,

Fisher Index = $\sqrt{\text{Laspeyre} \times \text{Paasche}}$

$$= \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1}} \times 100$$

[25] In Passche's index, weights are based on :

(a) Current year quantities (b) Base year quantities

(c) Weighted average prices (d) None of these (1 mark)

Answer:

(a) In Paasche's Index, the quantity weights of the current year are used. Therefore,

Paasche's Index = $\frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n}$

[26] Fisher's Ideal Index does not satisfy:

(a) Time Reversal Test (b) Factor Reversal Test

(c) Unit Test (d) Circular test (1 mark)

Answer:

(d) Time Reversal Test :

It is a test to determine whether a given method will work both ways in time, forward and backward.

Laspeyre's and Paasche's method do not satisfy this test, but Fisher Ideal Formula does.

Factor Reversal Test :

This holds when the product of price index and the quantity index should be equal to the corresponding value index i.e. $\frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0}$

Symbolically : $P_{01} \times Q_{01} = V_{01}$

Only Fisher's Ideal Index satisfies Factor Reversal Test.

Unit Test -

This test requires that the formula should be independent of the unit in which or for which prices and quantities are quoted.

Except for the simple (unweighted) aggregative index all other formulae satisfy this test. (This means that Fisher's Index also

Circular Test -

It is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base.

This test is **not met** by Laspeyres or Paasche's or the Fisher's Ideal Index.

The simple geometric mean of price relatives and the weighted aggregative with fixed weights **meet** this test.

Therefore, we can conclude that Fisher's Ideal Index satisfies all other tests except the Circular Test.

[27] _____ $P_{01}Q_{01} = \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0}$ which of following test satisfies the above?

(a) Time Reversal Test (b) Factor Reversal Test

(c) Circular Test (d) None of these. (1 mark)

Answer:

(b) Factor Reversal Test holds when the product of price index and the quantity index should be equal to the corresponding value index.

i.e. $\frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0}$

Symbolically : $P_{01} \times Q_{01} = V_{01}$

$$P_{01} \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1}} \quad Q_{01} \sqrt{\frac{\Sigma Q_1 P_0}{\Sigma Q_0 P_0} \times \frac{\Sigma P_1 Q_1}{\Sigma Q_0 P_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1} \times \frac{\Sigma Q_1 P_0}{\Sigma Q_0 P_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0}}$$

$$P_{01} \times Q_{01} = \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0}$$

This test is satisfied only by Fisher's Ideal Index Number.

2009 - DECEMBER

[28] Time reversal & factor reversal are:

(a) Quantity Index (b) Ideal Index

(c) Price Index (d) Test of Consistency (1 mark)

Answer:

(d) While selecting an appropriate index formula the Time Reversal Test and the Factor Reversal Test are considered necessary in testing the consistency and hence these tests are also known as test of consistency [self- explanatory].

[29] In Laspeyres Index Number _____are used as weights?

(a) Base year price (b) Current year price

(c) Base year quantities (d) Current year quantities (1 mark)

Answer:

(c) Laspeyre's Index = $\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0}$

Therefore from the above we can see that base year quantities are used as weights in Laspeyre's Index.

2010 - JUNE

[30] In the data group Bowley's and Laspeyre's index number is as follows. Bowley's index number = 150, Lspeyre's index number = 180 then Paesche's index number is

(a) 120 (b) 30

(c) 165 (d) None of these (1 mark)

Answer:

(a) Drobish and Bowley Index No. $= \frac{\text{Laspeyres} + \text{Paasche's}}{2}$

$150 = \frac{180 + Paasche's}{2}$

180 + Paasches = 300

∴ Paasche's Index No. = 120

[31] Consumer price index is commonly known as

(a) Chain Based index (b) Ideal index

(c) Wholesale price index (d) Cost of living index. (1 mark)

Answer:

(d) Consumer Price Index is also called Cost of Living Index [Self-Explanatory].

[32] Find the paasche's index number for prices from the following data taking 1970 as the base year.

Commodity	1970		1975	
	Price	Commodity	Price	Commodity
A	1	6	3	5
B	3	5	8	5
C	4	8	10	6

(a) 261.36 (b) 265.48

(c) 274.32 (d) 282 (1 mark)

Answer:

(a)

P_0	Q_0	P_1	Q_1	$P_1 Q_1$	$P_0 Q_1$
1	6	3	5	15	5
3	5	8	5	40	15
4	8	10	6	60	24
				$\Sigma = 115$	$\Sigma = 44$

Paasche's Index No. $= \frac{\sum P_1 Q_1}{\sum P_0 Q_0} \times 100 = \frac{115}{44} \times 100 = 261.36$

[33] The life expectancy, E of male is o linear function of time (year). It is given that in 1980 the life expectancy was 70 years and in 2000 it was

(a) 78 (b) 80
(c) 82 (d) 84 (1 mark)

Answer:

(a) Given (E = At) + B
(E being a linear function of time)
$70 = A \times 1980 + B$ (1) (At t = 1980)
$75 = A \times 2000 + B$ (2) (At t = 2000)
Solving (1) & (2), we get
A = 1/4 and B = – 425
Now, $E = A \times 2012 + B$ (3)
Substituting A and B in 3 above

$$E = \frac{2012}{4} + (-425) = 503 - 425$$

2010 - DECEMBER

[34] If Laspeyre's index number is 90 and Paasche's index number is 160, then Fisher's index number will _____.

(a) 144 (b) 120
(c) 125 (d) None of these (1 mark)

Answer:

(b) Fisher's index No. = $\sqrt{\text{Laspeyre} \times \text{Paasche}}$
Fisher's index No. = $\sqrt{90 \times 160} = 120$

2011 - JUNE

[35] Wholesale Price Index (WPI) is given by :

(a) Marshall-Edge worth Index (b) Laspeyre's Index
(c) Paasche's Index (d) None of the above. (1 mark)

Answer:

(b) WPI : In this method, the quantities of various commodities consumed by a particular class of people in the base year are taken as weights. This method is based on laspeyre's Price Index.

$$= \frac{\sum P_1 \sum q_0}{\sum P_0 \sum q_0} \times 100 = \frac{\text{Total Expenditure in CurrentYear}}{\text{Total Expenditure in BaseYear}} \times 100$$

[36] Fisher's Ideal index is obtained by :

(a) Arithmetic Mean of Laspeyre's & Paasche's index

(b) Geometric Mean of Laspeyre's & Paasche's index

(c) Sum of Laspeyre's & Paasche's index.

(d) None of the above. (1 mark)

Answer:

(b) Fishers Ideal Index is obtained by geometric mean of Laspeyre's & Paasche's Price Index.

Fisher Ideal Index = $\sqrt{\text{Laspeyre Price Index} \times \text{Paasche Price Index}}$

[37] The index number of prices at a place in the year 2008 is 225 with 2004 as the base year then there is:

(a) average 125% increase in prices.

(b) average 225% increase in prices.

(c) average 100% increase in prices.

(d) None of the above. (1 mark)

Answer:

(a) Say, the price of base year 2004 = 100

$\therefore$ the price of current year 2008 = 225

Increase in Price = 225 – 100

= 125

$\therefore$ % of increase in price = $\frac{\text{Increase in price}}{\text{Price of Base year}} \times 100 = \frac{125}{100} \times 100 = 125\%$

2011 - NOVEMBER

[38] The simple index number for the current year using simple aggregative method for the following data is _____.

Commodity Base	Year price (P_0)	Current year price (P_1)
Wheat	80	100
Rice	100	150
Gram	120	250
Pulses	200	300

(a) 200 (b) 150
(c) 240 (d) 160 (1 mark)

Answer:

(d)

Commodity	Base year Price P_0	Current yrs Price (P_1)
Wheat	80	100
Rice	100	150
Grain	120	250
Pulses	200	300
	$\Sigma P_0 = 500$	$\Sigma P_1 = 800$

Simple Index No. $\frac{\Sigma P_1}{\Sigma P_0} \times 100 = \frac{800}{500} \times 100 = 160$

[39] Fishers Ideal Index Number not satisfies _____.

(a) Unit Test
(b) Time Reversal Test
(c) Circular Test
(d) Factor Reversal Test (1 mark)

Answer:

(c) Fishers Ideal Index Number does not Satisfy Circular Test

[40] If the prices of all commodities in a place has increased 20% in comparison to the base period prices, then the index number of prices for the place is now _____.

(a) 100 (b) 120

(c) 20 (d) 150 (1 mark)

Answer:

(b) Price of Base yrs (P_0) = ₹ 100

Price of Current yrs (P_1) = ₹ 100 + 20% of ₹ 100

= ₹ 100 + ₹ 20 = ₹ 120

$$\text{Index No.} = \frac{P_1}{P_0} \times 100 = \frac{120}{100} \times 100 = 120$$

2012 - JUNE

[41] If $\Sigma P_0 Q_0 = 116$, $\Sigma P_0 Q_1 = 140$

$\Sigma P_1 Q_0 = 97$, $\Sigma P_1 Q_1 = 117$

then Fisher's ideal index number is _____.

(a) 184 (b) 83.59

(c) 119.66 (d) 120 (1 mark)

Answer:

(b) Given : $\Sigma P_0 Q_0 = 116$

$\Sigma P_0 Q_1 = 140$

$\Sigma P_1 Q_0 = 97$

$\Sigma P_1 Q_1 = 117$

$\because$ Fisher's index formula

$$= \sqrt{\frac{\Sigma P_1 Q_0 \cdot \Sigma P_1 Q_1}{\Sigma P_0 Q_0 \ \ \Sigma P_0 Q_1}} \times 100$$

$$= \sqrt{\frac{97 \times 117}{116 \times 140}} \times 100$$

= 83.59

[42] Find the Paasche's Index number for prices from the following data taking 1970 as the base year.

Commodity	1970		1975	
	Price	Commodity	price	Commodity
A	1	6	3	5
B	3	5	8	5
C	4	8	10	6

(a) 261.36 (b) 265.48
(c) 274.32 (d) 282 (1 mark)

Answer:

(a)

Given :

	1970		1975			
Commodity	Price (P_0)	Qty (Q_0)	Price (P_1)	Qty (Q_1)	$P_1 Q_1$	$P_0 Q_1$
A	1	6	3	5	15	5
B	3	5	8	5	40	15
C	4	8	10	6	60	24
					$\sum P_1 Q_1 = 115$	$\sum P_0 Q_1 = 44$

$\therefore$ Pasche's Index

$$= \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$$

$$= \frac{115}{44} \times 100$$

2012 - DECEMBER

[43] If Fisher's index = 150 and Paasche's Index = 144, then Laspeyre's index is _____.

(a) 147 (b) 156.25
(c) 104.17 (d) 138 (1 mark)

Answer:

(b) If fisher's index = 150 and Paasche's index = 144

Then, we know that

$$\text{Fisher's index} = \sqrt{\text{Laspeyre's} \times \text{Paasche's}}$$

$$150 = \sqrt{\text{Laspeyre's} \times 144}$$

on squaring both side

$$150 \times 150 = \text{Laspeyre's} \times 144$$

$$\text{Laspeyre's index} = \frac{150 \times 150}{144}$$

$$= 156.25$$

[44] Net monthly salary of an employee was ₹ 3,000. The consumer price index number in 1985 is 250 with 1980 as base year. If he has to be rightly compensated then the additional dearness allowance to be paid to the employee is:

(a) ₹ 4,000 (b) ₹ 4,800

(c) ₹ 5,500 (d) ₹ 4,500 (1 mark)

Answer:

(d) Given,

Net monthly salary of the employee = 3,000 in 1980 consumer price index in 1985 with 1980 as the base year = 250

Since, it is always assumed that the consumer price index is always taken to be as 100

∴ When the consumer price index, was 100 in 1985

Then his monthly salary was = 3,000

∴ When the consumer price index was 250 in 1985 then his monthly salary will be = $\frac{250 \times 3,000}{100}$

= ₹ 7,500

∴ The Dearness allowance to be paid to the employee

= ₹ (7,500 – 3,000)

= ₹ 4,500

2013 - JUNE

[45] Time reversal test is satisfied by:

(a) Fisher's formula (b) Laspeyre's formula

(c) Paasche's formula (d) Dorbish formula (1 mark)

Answer:

(a) Time reversal test is satisfied by Fisher's Index No.

[46] In year 2005 the wholesale price index number is 286 with 1995 as base year, then how much the prices have increased in 2005 in comparison to 1995?

(a) 286% (b) 386%

(c) 86% (d) 186% (1 mark)

Answer:

(d) Price Index of base year = 1995 = 100

Price Index of current year 2005 = 286

Price Increase in 2005 = (286 – 100)

= 186

Increased % = 186%

[47] Bowley's index = 150, Laspeyer's index = 180, then Paasche's index = _____

(a) 120 (b) 30

(c) 165 (d) None of these (1 mark)

Answer:

(a) Bowley's Index = 150Paasche's Index = ?

Laspeyer's Index = 180

Bowley's Index = $\left(\frac{L+P}{2}\right)$

$$\frac{150}{1} = \frac{(180+P)}{2}$$

$$300 = 180 + P$$

$$P = 300 - 180 = 120$$

2013 - DECEMBER

[48] An index time series is a list of _____ number of two or more period of time, where each index number employs the same base y can

(a) Index (b) Absolute
(c) Relative (d) Sample (1 mark)

Answer:

(a) An Index time series is a list of **Index** Number of two or more period of time, where each Index Number employs the same base year.

[49] The index number for the year 2012 taking 2011 as the base year from the data given below by using simple average of price relative method is.

Commodity	A	B	C	D	E
Price in 2011	115	108	95	80	90
Price in 2012	125	117	108	95	95

(a) 112 (b) 117
(c) 120 (d) 111 (1 mark)

Answer:

(d)

Commodity	Price in 2011 (P_0)	Price in 2012 (P_1)	$\frac{P_1}{P_0} \times 100$
A	115	125	$\frac{125}{115} \times 100 = 108.70$
B	108	117	$\frac{117}{108} \times 100 = 108.33$
C	95	108	$\frac{108}{95} \times 100 = 113.69$
D	80	95	$\frac{95}{80} \times 100 = 118.75$

E	90	95	$\frac{95}{90} \times 100 = 105.56$
N = 5			$\Sigma\left(\frac{P_1}{P_0} \times 100\right) = 555.03$

$$\text{INDEX No.} = \frac{\Sigma\left(\frac{P_1}{P_0} \times 100\right)}{N}$$

$$= \frac{555.03}{5}$$

$$= 111$$

[50] What is the formula for calculating the deflated value?

(a) Current value/Price index of current year

(b) (Current value/Price index of current year) × 100

(c) Price index of current year/Current value

(d) (Current value/Price index of last year) × 100 (1 mark)

Answer:

(a) Deflated value = $\frac{\text{Current value}}{\text{Price index of current year}}$

2014 - JUNE

[51] Circular test is satisfied by which index number?

(a) Laspeyre's (b) Paasche's

(c) Fisher's (d) None of the above. (1 mark)

Answer:

(d) Laspeyre's, Paasche's & Fisher's Index No. do not satisfied circular test so (D) is correct.

[52] Fisher's Index Number is _____of Laspeyre's and Paasche's Index Number

(a) A.M. (b) G.M.
(c) H.M. (d) None of the above. (1 mark)

Answer:

(b) Fisher's Index Number is G.M of Laspeyre's and Paasche's Index Number.

[53] Which of the following statements is true?

(a) Paasche's Index Number is based on base year quantity
(b) Fisher's Index Number satisfies the circular test
(c) Arithmetic Mean is the most appropriate average for constructing the Index Number
(d) Splicing means constructing one continuous series from two different indices on the basis of common base. (1 mark)

Answer:

(c) Arithmetic Mean is the most appropriate average for constructing the Index Number.

[54] Monthly salary of an employee was ₹ 10,000 in the year 2000 and it was increased to ₹ 20,000 in year 2013 while the consumer price Index No. is 240 in year 2013 with the base year 2000. What should be his salary in comparison of consumer price index in the year 2013?

(a) ₹ 20,000 (b) ₹ 16,000
(c) ₹ 24,000 (d) None of the above (1 mark)

Answer:

(c)

Consumer Price Index	Salary
100	10,000
240	x

∵ If C.P.I. is 100 then salary = 10,000

∴ C.P.I. is 1 then salary $= \frac{10,000}{100}$

$= 100$

∴ C.P.I. is 240 then salary $= 100 \times 240$

$= 24,000$

2014 - DECEMBER

[55] If $\Sigma P_1Q_0 = 1180$, $\Sigma P_0Q_0 = 1170$, $\Sigma P_1Q_1 = 1064$, $\Sigma P_0Q_1 = 1100$. The Fisher's Ideal Index is:

(a) 96.73 (b) 98.795

(c) 98.77 (d) 100.86 (1 mark)

Answer:

(c) Given $\Sigma P_1 Q_0 = 1,180$, $\Sigma P_0Q_0 = 1,170$

$\Sigma P_1Q_1 = 1,064$, $\Sigma P_0Q_1 = 1,100$

$$\text{Fisher Index No.} = \sqrt{\frac{\Sigma P_1Q_0}{\Sigma P_0Q_0} \times \frac{\Sigma P_1Q_1}{\Sigma P_0Q_1}} \times 100$$

$$= \sqrt{\frac{1,180}{1,170} \times \frac{1,064}{1,100}} \times 100$$

$$= \sqrt{\frac{12,55,520}{12,87,000}} \times 100$$

$$= 98.77$$

[56] If the price of a commodity in a place have decreased by 30% over the base period prices, then the index number of that place is:

(a) 30 (b) 60

(c) 70 (d) 80 (1 mark)

Answer:

(c) Base price of any commodity = 100 decreased price

= 30% of 100

= 30

Index No. of that Place Now = 100 – 30

= 70.

2015 - JUNE

[57] Factor reversal test is expressed in terms of

(a) $\frac{\sum P_1 Q_1}{\sum P_0 Q_0}$ (b) $\frac{\sum P_1 Q_1}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}$

(c) $\frac{\sum P_1 Q_1}{\sum Q_0 P_1}$ (d) $\frac{\sum Q_1 P_0}{\sum Q_0 P_0} \times \frac{\sum P_1 Q_1}{\sum Q_0 P_1}$ (1 mark)

Answer:

(a) Factor reversal test is expressed in terms of $\frac{\sum P_1 Q_1}{\sum P_0 Q_0}$

[58] If with an increase of 10% in prices, the rise in wages is 20% then the real wage has increased by

(a) 20% (b) 10%

(c) Less than 10% (d) More than 10%

Answer:

(a) Real wages $= \frac{\text{Real Wage of current year}}{\text{Real wage of base year}} \times 100$

$= \frac{120}{100} \times 100$

$= 120 - 100$

$= 20\%$

[59] _____ play a very important role in the construction of index numbers.

(a) Weights (b) Classes

(c) Estimations (d) None (1 mark)

Answer:

(a) Weights play a very important role in the construction of Index Numbers.

2015 - DECEMBER

[60] Consumer price index number for the year 1977, was 313, with 1960 as the base year, and was 100 for the year 1960. The average monthly wages in 1977 of the workers into factory be ₹ 160, their real wages is:

(a) ₹ 48.40 (b) ₹ 51.12

(c) ₹ 40.30 (d) None of the above (1 mark)

Answer:

(b) Let Real wage is x

	C.P.I.	Wages
1977	313	160
1960	100	X

$$\frac{313}{100} = \frac{160}{x}$$

$$313\,x = 160 \times 100$$

$$x = \frac{160 \times 100}{313}$$

$$= \frac{16000}{313}$$

$$= 51.12$$

2016 - JUNE

[61] Purchasing power of money is

(a) Reciprocal of price index number

(b) Equal to price index number

(c) Unequal to price index number

(d) None of these. (1 mark)

Answer:

(a) Purchasing power of money is the Reciprocal of Price Index

[62] If $\Sigma P_0Q_0 = 1360$, $\Sigma P_nQ_0 = 1900$, $\Sigma P_0Q_n = 1344$, $\Sigma P_nQ_n = 1880$, then the Laspeyre's Index Number is

(a) 0.71 (b) 1.39

(c) 1.76 (d) none. (1 mark)

Answer:

(b) $\Sigma P_0Q_0 = 1360$, $\Sigma P_nQ_0 = 1900$

$\Sigma P_0Q_n = 1344$, $\Sigma P_nQ_n = 1880$

$$\text{Laspeyre's Index Number} = \frac{\Sigma P_nQ_0}{\Sigma P_0Q_0}$$

$$= \frac{1900}{1360}$$

$$= 1.39$$

[63] In the year 2010 the monthly salary of a clerk was ₹ 24,000. The consumer price Index was 140 in the year 2010, which rises to 224 in the year 2016. If he has to be rightly compensated, what additional monthly salary to be paid to him?

(a) ₹ 14,400 (b) ₹ 38,400

(c) ₹ 7,200 (d) None of these (1 mark)

Answer:

(a)

Years	Consumer Price index	Salary
2010	140	24,000
2016	224	X

$$\frac{140}{224} = \frac{24,000}{x}$$

$$x = \frac{24,000 \times 224}{140}$$

$$x = 38,400$$

$$D.A = 38,400 - 24,000$$

$$= 14,400$$

[64] The suitable index number for the comparison of changes in price level of every year is _____

(a) Fixed Base Index Number

(b) Fisher's Ideal Index Number

(c) Chain Base Index Number
(d) Both (a) and (c) (1 mark)

Answer:

(c) The suitable Index No. for the comparison of changes in price level of every year is Chain Base Index Number.

2016 - DECEMBER

[65] Following is the data concerning to commodities A, B, C and D in the base period 1992 and current period 1993.

	Base Year 1992		Current Year 1993	
Commodities	**Price**	**Quantity**	**Price**	**Quantity**
A	3	18	4	15
B	5	6	5	9
C	4	20	6	26
D	1	14	3	15

The Paasche's price index number is:

(a) 148.25 (b) 146.41
(c) 144.25 (d) None of these (1 mark)

Answer:

(b)

	Base year-1992		Current year 1993			
Commodities	Price p_0	Qty q_0	Price p_1	Qty q_1	$p_0 q_1$	$p_1 q_1$
A	3	18	4	15	45	60
B	5	6	5	9	45	45
C	4	20	6	26	104	156
D	1	14	3	15	15	45
					$\Sigma p_0 q_1 = 209$	$\Sigma p_1 q_1 = 306$

Paasche's Index No. $= \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$

$= \frac{306}{209} \times 100$

$= 146.41$

[66] Which method satisfy time reversal test?

(a) Laspeyer's method (b) Paasche's method

(c) Fishers method (d) None of these. (1 mark)

Answer:

(c) Fishers Index No. satisfy 'Time Reversal Test'.

[67] Index number are the _____.

(a) Economic (b) Statistics

(c) (a) and (b) (d) None of these. (1 mark)

Answer:

(c) Index number are used in Economic and Statistics.

2017 - JUNE

[68] The monthly income of an employee was ₹ 8,000 in 2014. The consumer price index number was 160 in 2014, which rose to 200 in 2017. If he has to be rightly compensated, the additional dearness allowance to be paid to him in 2017 would be:

(a) ₹ 2,400 (b) ₹ 2,750

(c) ₹ 2,500 (d) None of these. (1 mark)

Answer:

(d)

C.P.I	Salary
160 ↓	8,000 ↓
200	x

$\frac{x}{8,000} = \frac{200}{160}$

$x = \frac{200 \times 8,000}{160} = 10,000$

Addition D.A to be paid = 10,000 – 8,000

[69] If Laspeyre's index number (L) and Paasche's index number (P) are known, then one can compute Fisher's index number (F) by:

(a) $F = LP$ (b) $\sqrt{F} = LP$

(c) $F = \frac{1}{LP}$ (d) $F^2 = LP$ (1 mark)

Answer:

(d) The relation between Laspeyre, Paasche & Fisher Index is given by

$F = \sqrt{L \times P}$

$F^2 = L \times P$

Where L → Laspeyre Index
P → Paasche Index
F → Fisher Index

[70] Fisher's index number does not satisfy:

(a) Unit Test (b) Circular Test

(c) Time reversal test (d) Factor reversal test. (1 mark)

Answer:

(b) Fisher Index no. does not satisfy **circular test.**

2017 - December

[71] Circular Test is an extension of _____ .

(a) Factor reversal test (b) Time reversal test

(c) Neither (a) nor (b) (d) Both (a) and (b). (1 mark)

Answer:

(b) Circular test is an extension of 'Time Reversal Test'.

[72] Fishers index number is based on:

(a) The arithmetic mean of Laspeyre's and Paasche's index numbers

(b) The median of Laspeyre's and Paasche's index numbers

(c) The mode of Laspeyre's and Paasche's index numbers

(d) None of the above. (1 mark)

Answer:

(d) Fishers Index Number is the G.M. of Laspeyre and Paasches Index Numbers.

[73] Price relative is equal to:

(a) $\frac{\text{Price in the given year}}{\text{Price in the base year}} \times 100$

(b) $\frac{\text{Price in the base year}}{\text{Price in the given year}} \times 100$

(c) Price in the given year × 100

(d) Price in the base year × 100. (1 mark)

Answer:

(a) Price Relative = $\frac{\text{Price of given (Current) year}}{\text{Price in the base year}} \times 100$

[74] For consumer price index, prices are collected from:

(a) Retail traders (b) Wholesale traders

(c) Fair price shops (d) Government Depots. (1 mark)

Answer:

(a) For consumer Price Index, Prices are collected from **Retail traders**.

2018 - May

[75] Time reversal & factor reversal are:

(a) Quantity Index (b) Ideal Index

(c) Price Index (d) Test of consistency (1 mark)

Answer:

(d) Time reversal and factor reversal test are test of consistency.

[76] A series of numerical figures which show the relative position is called.

(a) Index number (b) Relative number

(c) Absolute number (d) None (1 mark)

Answer:

(a) A series of numerical figures which show the relative position is called. Index Number