1. Ratio & Proportion.

$a \rightarrow$ Antecedent
$b \rightarrow$ Consequent.

1) Inverse ratio $\rightarrow \frac{a}{b} \rightarrow \frac{b}{a}$.
2) duplicate Ratio $\rightarrow \frac{a}{b} \rightarrow \frac{a^2}{b^2}$.
3) Triplicate ratio $\rightarrow \frac{a}{b} \rightarrow \frac{a^3}{b^3}$
4) sub duplicate $\rightarrow \frac{a}{b} \rightarrow \frac{\sqrt{a}}{\sqrt{b}}$.
5) sub triplicate $\rightarrow \frac{a}{b} \rightarrow \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$.
6) Compound $\rightarrow \frac{a}{b} \times \frac{c}{d}$

operation on proportion.

1. Invertendo $\rightarrow \frac{a}{b} = \frac{c}{d} \rightarrow \frac{b}{a} = \frac{d}{c}$.
2. Alternendo $\rightarrow \frac{a}{b} = \frac{c}{d} = \frac{a}{c} = \frac{b}{d}$.
3. Componendo $\rightarrow \frac{a}{b} = \frac{c}{d} \rightarrow \frac{a+b}{b} = \frac{c+d}{d}$.
4. dividendo $\rightarrow \frac{a}{b} = \frac{c}{d} \rightarrow \frac{a-b}{b} = \frac{c-d}{d}$.
5. Comp – divid $\rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$.
6. Adendo $\rightarrow \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a-c-e}{b-d-f}$.

2. Logarithms.

* property.

1. $\log_a m = n \rightarrow a^n = m$.
2. $\log_a m + \log_a n \rightarrow \log_a mn$.
3. $\log_a m - \log_a n \rightarrow \log \frac{m}{n}$.
4. $\log_a (m^n) = n \cdot \log_a m$.
5. $\log_a 1 = 0$ 6. $\log_a a = 1$.
7. $\log_a b = \frac{\log_x b}{\log_x a} \rightarrow$ change of Base.

* Base is 10.

3. Indices

* Properties of Indices.

- $a^m \times a^n = a^{m+n}$.
- $\frac{a^m}{a^n} = a^{m-n}$.
- $(a^m)^n = a^{mn}$.
- $a^0 = 1$.
- $a^{-m} = \frac{1}{a^m}$.
- $a^m = \frac{1}{a^{-m}}$.
- $a^{\frac{m}{n}} = \sqrt[n]{a^m}$.

* Formula.

$\rightarrow (a+b)^3 = a^3 + b^3 + 3ab(a+b)$.
$\rightarrow (a-b)^3 = a^3 - b^3 - 3ab(a-b)$.
$\rightarrow (a^3) + (b^3) = (a+b)(a^2 - ab + b^2)$
$\rightarrow a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
$\rightarrow a^2 - b^2 = (a+b)(a-b)$.

* Roots

$2^1 = 2$	$3^1 = 3$	$5^1 = 5$
$2^2 = 4$	$3^2 = 9$	$5^2 = 25$
$2^3 = 8$	$3^3 = 27$	$5^3 = 125$
$2^4 = 16$	$3^4 = 81$	$5^4 = 625$
$2^5 = 32$	$3^5 = 243$	$5^5 = 3125$
$2^6 = 64$	$3^6 = 729$	$5^6 = 15625$
$2^7 = 128$	$3^7 = 2187$	$5^7 = 78125$
$2^8 = 256$	$3^8 = 6561$	$5^8 = 390625$
$2^9 = 512$	$3^9 = 19683$	$5^9 = 1953125$
$2^{10} = 1024$	$3^{10} = 59049$	$5^{10} = 9765625$

4. Equations

Quadratic Equations
$ax^2 + bx + c = 0$

formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

* properties of roots.

1. $\alpha + \beta = \frac{-b}{a}$.
2. $\alpha \cdot \beta = \frac{c}{a}$.
3. $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$.
4. $(\alpha^2 - \beta^2) = \alpha^2 + \beta^2 - 2\alpha\beta$.
5. $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$
6. $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta)$

* In Case of reciprocal

If $\alpha = 5$ So, $\beta = \frac{1}{5}$.

Factorials

$0! = 1$
$1! = 1$
$2! = 2$
$3! = 6$
$4! = 24$
$5! = 120$
$6! = 720$
$7! = 5040$
$8! = 40320$
$9! = 362880$

5. Time Value of Money

1. Simple Interest.

$I = \frac{P \times n \times r}{100}$
$A = P + I$
$A = P\left[1 + \frac{nr}{100}\right]$

2. Compound Interest.

$A = P(1+i)^n$ i = rate of interest / $1/2/4/12$
$I = A - P$

yearly = $n = \times 1$.
Half yearly = $n = \times 2$.
Quaterly = $n = \times 4$.
Montly = $n = \times 12$.

3. Application of Compound int.

* In the problems of population

Increase
$A = P(1+i)^n$

Decrease
$A = P(1-i)^n$

P = Initial population
A = final population
n = no. of years.
i = rate of growth = Birth Rate – death rate.

4. In problem of depreciation.

$S.V = C.P(1-i)^n$

S.V = Scrap Value.
C.P = Cost price.
n = It is always 1 year.

5. Effective rate of interest

$ie = (1+i)^n - 1$

$ie \rightarrow$ Effective rate.
$n \rightarrow$ It is always 1 year.
$i \rightarrow$ Nominal value.

6. Future Value of Annuity.

a) regular annuity.

FV → [payment at end]

$$F.V = \frac{A[(1+i)^n - 1]}{i}$$

A = annuity.

b) Annuity due.

FV = [payment at start].

$$F.V = \frac{A[(1+i)^n - 1](1+i)}{i}$$

* If nothing is mention then, consider regular always.

7. Present Value.

V = present value

$$V = \frac{A[(1+i)^n - 1]}{i[(i+i)^n]}$$

Short formula.

A × PVF

$PVF \rightarrow \frac{1}{1+i} \rightarrow$ MR + (calculation)
MR +
MR +

6. Derivative & Integration.

	$f(x)$		$f'(x)$
1)	x^n	→	$n.x^{n-1}$.
2)	a^x	→	$a^x . \log_e a$.
3)	x^3	→	$2x$.
4)	x	→	1.
5)	e^x	→	e^x.
6)	$\log x$	→	$\frac{1}{x}$.
7)	$\frac{1}{x}$	→	$\frac{-1}{x^2}$.
8)	$\sqrt{x}$	→	$\frac{1}{2\sqrt{x}}$.
9)	K	→	0
10)	y	→	$u.v$

11) $\frac{dy}{dx} = \frac{d}{dx}(u).v + u\frac{d}{dx}(v)$

11) $y \rightarrow \frac{u}{v}$.

$$\frac{dy}{dx} = \frac{\frac{d}{dx}u.(x)(-)u\frac{d}{dx}(v)}{v^2}$$

Note: If variable ke upr variable has to both the side log lenka.

e.g.: $x^x = \log x^x = x.\log x$.

10B. Dispersion

1. Range

R = L-S

Co-efficient of range $= \frac{L-S}{L+S} \times 100$

L = Longest value.
S = Smallest value.

2. Quartile Deviation (Q.D).

$Q_1 \quad Q_2 \quad Q_3$

Inter Quartile range $= Q_3 - Q_1$

$$Q.D = \frac{Q_3 - Q_1}{2}.$$

Co-effint of $Q.D = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

3. Mean Deviation (M.D).

It is arithmatic mean of absolute deviation of all observation about central value.

$M.D = \frac{\Sigma |d|}{N}$ (without frequency)

$M.D = \frac{\Sigma F|d|}{\Sigma F}$ (with frequency)

$M.D = \frac{\Sigma F|d|}{\Sigma F}$ (continous)

∴ d = X-A (A = AM / Median / Mode)

∴ Co-eff. of $M.D = \frac{MD}{A} \times 100$

4. Standard deviation (S.D)

It is root of mean of squares of deviation of all observ. about AM.

$S.D \rightarrow \sigma$

$S.D = \sigma = \sqrt{\frac{\Sigma d^2}{N}}$ - without frequency

$d = X - \bar{X}$

Variance $= \sigma^2$

C.V = Co-eff of variation

$$\frac{\sigma}{\bar{x}} \times 100.$$

$S.D = \sigma = \sqrt{\frac{\Sigma Fd^2}{\Sigma F}}$ - with frequency

$\sigma = \sqrt{\frac{\Sigma Fd^2}{\Sigma F}}$ - continous

* Properties of dispersion

1. if all observation are same then Range = QD = MD = SD are also same.

[$\bar{x}$ = med = mode = GM = HM
R = MD = S.D = Q.D = 0]

2. for 2 numbers a and b

Range = b-a

$\sigma = \frac{b-a}{2} = \frac{R}{2}$.

3. Measure of dispersion.

- Absolute Measure → has same unit → R, MD, SD, QD
- Relative Measure. → No unit → Co-eff of range, Co-eff of MD, SD, QD

4. Application of C.V

Less C.V → more consistent, more stable.

5. Combined S.D

Steps:

1. $\bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$

2. $d_1 = \bar{X}_{12} - \bar{X}_1$.
$d_2 = \bar{X}_{12} - \bar{X}_2$.

3. $\sqrt{\dfrac{N_1(\sigma_1^2+d_1^2)+N_2(\sigma_2^2+\sigma_2^3)}{N_1+N_2}}$

6. change of scale and change of Origin:

Central tendency

↓

AM, Median, Mode

↓

$(\times, \div)$ Scale

$(+,-)$ Origin

$y = a+bx$

$\bar{y} = a+b\bar{x}$

$me_y = a+b\, me_x$

$mo_y = a+b\, mo_x$

Dispersion

↓

Range, S.D, M.D, Q.D

↓

Scale → ✓

Origin → ✗

↓

$y = n+bx$

$R_y = |b| \cdot R_x$

$\sigma_y = |b| \cdot \sigma_x$

$M.D_y = |b| \cdot MD_x$

$S.D_y = |b| \cdot S.D_x$

$Q.D_y = |b| \cdot Q.D_x$

Central Tendency

1. If all obser. are same then AM, GM & HM are also same.

$AM = GM = HM$

2. Relationship between AM, GM & HM

Case A: for 2 numbers

$AM = \dfrac{a+b}{2}$

$GM = \sqrt{ab}$

$HM = \dfrac{2ab}{a+b}$. $GM^2 = AM \times HM$

Case B: for any number

$AM \geq GM \geq HM$

3. Appliation of GM & HM

GM & HM are used for finding average rate & average interest.

4. problems of average speed.

$\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

[Case I] Distance is same → use HM

[Case II] Time is same → use AM

[Case III] Distance & time both are different

$\text{Average speed} = \dfrac{\text{Total distance}}{\text{Total Time}}$.

5. Combine Harmonic mean

$H_{12} = \dfrac{N_1+N_2}{\dfrac{N_1}{H_1}+\dfrac{N_2}{H_2}}$.

1. Arithmatic Mean. - Average.

$\bar{X} = \dfrac{\Sigma x}{N}$ .... discrete without frequency.

$\bar{X} = \dfrac{\Sigma fx}{\Sigma f}$ .... discrete with frequency

$\bar{X} = \dfrac{\Sigma fx_2}{\Sigma f}$ .... Continous data x = mid point.

2. Median - (Middle most Value)

a) discreate without frequency

→ first arrange data in A.D

→ Median $\left(\frac{n+1}{2}\right)^{th}$ Value.

N = no. of Observation.

b) discrete with frequency.

→ find less than C.F

→ find $\frac{N+1}{2}$ $(N = \Sigma F)$

→ check frequency ∵ $CF \geq \frac{N+1}{2}$

→ C.F → x → median

c) Median for Continous data.

→ find less than C.F

→ find $\frac{N}{2}$ $(N = \Sigma F)$

→ check $C.F \geq \frac{N}{2}$

→ C.F → class → median class

→ Median $= L + \left(\frac{N}{2} - C.F\right) \times \frac{h}{i}$.

3. Mode

Value having maximum frequency repetition.

→ Continuous Data.

$$\text{Mode} = L + \frac{(F_1 - F_0) \times h}{2F_1 - F_0 - F_2}$$

L = lower class frequency.

h = UCB - LCB

F_0 = previous class frequency

F_1 = maximum frequency.

F_2 = next class frequency.

4. Harmonic Mean.

$$HM = \frac{N}{\Sigma \frac{1}{x}}$$ - [discreate without frequency]

$$HM = \frac{\Sigma F}{\Sigma \frac{F}{x}}$$ → [discreate with frequency].

$$HM = \frac{\Sigma F}{\Sigma \frac{F}{x}}$$ - [Continuous].

5. Geometric Mean.

$$GM = (x_1 \cdot x_2 \cdot x_3 \ldots \ldots x_n)^{\frac{1}{n}}$$ - (Without frequency)

$$GM = (x_1^{F_1} \cdot x_2^{F_2} \cdot x_3^{F_3} \ldots)^{\frac{1}{\Sigma F}}$$ - (with frequency)

$$GM = (x_1^{F_1} \cdot x_2^{F_2} \cdot x_3^{F_3} \ldots)^{\frac{1}{\Sigma F}}$$ - (Continuous)

* Properties of mean, median & mode.

1. If all observation are same than mean, median, mode are also same. $[\bar{x} = \text{median} = \text{mode}]$

2. Relation between mean, median and mode

for symmetric, $\bar{x}$ = median = mode

for asymmetric, $\bar{x}$ = mode = 3 $(\bar{x} - \text{median})$

3. Combine arithmetic mean.

$$\bar{x}_{12} = \frac{N_1\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2}$$

4. change of scale & change of Origin.

Mean, median, mode are affected by both

change of scale $(\times, \div)$

Change of Origin $(+, -)$

$y = a + bx$

$\bar{y} = a + b\bar{x}$

$M_{ey} = a + b\,M_{ex}$

5. Sum of deviation of all Observation about Arithmatic mean is zero.

i.e. $\Sigma(x-\bar{x})=0$

6. Sum of absoulate deviation of all Observation is minimum when taken above median.

Absoulate = positive

Median = $Q_2 = D_5 = P_{50}$ are always equal

Median

Me

a) discrete without frequency
- first arrange in A.O
- Median $\left(\frac{n+1}{2}\right)^{th}$ Value

b) discrete with frequency
- find less than C.F
- find $\frac{N+1}{2}$
- check that $CF \geq \frac{N+1}{2}$
- Check → x → median

c) Continous data
- find less than CF.
- find $\frac{N}{2}$.
- $C.F \geq \frac{N}{2}$.
- C.F → Median class

Median $= L+\left(\frac{N}{2}-C.F\right)\times\frac{h}{F}$.

Quartile

Q_1 Q_2 Q_3

a) discrete without freq.
- first arrange in A.O
- $Q_P = \left[\frac{(N+1)}{4}\right]$ Value

b) discrete with freq.
- find less than C.F
- find → $\frac{(N+1)P}{4}$
- check that $C.F \geq \left(\frac{(N+1)P}{4}\right)$
- C.F → x → quartile

c) Continous data
- find less than C.F.
- find $\frac{NP}{4}$.
- $C.F \geq \frac{NP}{4}$
- C.F → class → quartile class

$Q_P = L+\left(\frac{NP}{4}-C.F\right)\times\frac{h}{F}$.

Decile

D Q_2 D_5 D_9.....

a) discrete without frequency.

→ first arrange in A.O

→ $D_P = \left[\frac{(N+1)P}{10}\right]^{th}$ Value

b) discrete with frequency

→ find less than C.F

→ find $\frac{(N+1)P}{10}$

→ find check that $C.F \geq \frac{(N+1)P}{10}$

→ C.F → x → decile

Percentile

P_1 P_2 P_3 ...... P_{99}

a) Discrete without frequency.

→ first arrange in A.O

→ $P_P = \left[\frac{(N+1)P}{100}\right]^{th}$ Value

b) discrete with frequency.

→ find less than C.F

→ find $\frac{(N+1)P}{100}$.

→ check that $C.F \geq \frac{(N+1)P}{100}$

→ C.F → x → percentile

Sets, function & Relation

- Cardinal number = $n(A) = xx$
- Types of Set

Smaleton set
$A = \{7\}$
$B = \{8\}$

Null set / empty set
$\phi \; \{\}$

- Subset = 2^n
  - Proper subset 2^{n-1}
  - Improper subset 1

- Subset
eg = $A = \{2,3,5\}$
$B = \{3,5\}$
$B \subset A$

No. of subset = 2^n

- **Operation of Set**

1. Union → $(A \cup B)$
2. Intersection → $(A \cap B)$
3. Subtraction → $A = \{2,3,7,8,9\}$
   $B = \{1,5,7,9,10\}$
   $A - B = \{2,3,8\}$

* Theorem of addition:

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
3. $n(A \cap B') = n(A) - n(A \cap B)$
4. $n(A' \cap B) = n(B) - n(A \cap B)$
5. Total = Atleast + Nothing.

* Formula for three sets:

1. $n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C)$
2. $n(A \cap B' \cap C) = n(A \cap C) - n(A \cap B \cap C)$
3. $n(A' \cap B \cap C) = n(B \cap C) - n(A \cap B \cap C)$
4. $n(A \cap B' \cap C') = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$.
5. $n(A' \cap B \cap C') = n(B) - n(B \cap A) - n(B \cap C) + n(A \cap B \cap C)$

* Domain and Co-domain:

eg. $\{(3,8), (5,2), (7,5), (6,1)\}$

domain = input = $\{3,5,7,6\}$

Co-domain = Output = $\{8,2,5,1\}$

Index Number

1. Price index no. = $P_{on} = \frac{P_n}{P_0} \times 100$
2. Quantity index no. = $Q_{on} = \frac{Q_n}{Q_0} \times 100$
3. Value index no. = $V_{on} = \frac{V_n}{V_0} \times 100$
4. Simple Aggregative method.
   $P_{on} = \frac{\Sigma P_o}{\Sigma P_o} \times 100$
5. Simple relative method.
   $P_{on} = \frac{\Sigma \frac{P_n}{P_o}}{N} \times 100$
6. Weighted relative method
   $P_{on} = \frac{\Sigma \frac{P_n}{P_o} \cdot W}{\Sigma W} \times 100 = \frac{\Sigma I \cdot W}{\Sigma W}$.
7. Weighted aggregative Metho.
   $P_{on} = \frac{\Sigma P_n \cdot W}{\Sigma P_o \cdot W} \times 100$.
8. Laspayre's method
   $= \frac{\Sigma P_A \cdot q_0}{\Sigma P_0 \cdot q_0} \times 100$
9. Paasche's method
   $\frac{\Sigma P_n \cdot q_n}{\Sigma P_n \cdot q_n} \times 100$
10. Fisher = $P_{on} = \sqrt{L \times P}$.
11. Bowley = $P_{on} = \frac{L+P}{2}$.
12. Marshall-edgeworth Metho.
    $P_{on} = \frac{\Sigma P_n \cdot \left[\frac{q_0 + q_n}{2}\right]}{\Sigma P_0 \cdot \left[\frac{q_0 + q_n}{2}\right]} \times 100$
13. $CLI = \frac{\Sigma I \cdot W}{\Sigma W}$.

Probability

Farmula = $\frac{\text{Special Way}}{\text{Normal Way}}$.

52 Card

- Red 26
  - Heart 13
  - diamond 13
- Black 26
  - Spade 13
  - Club 13

$P(A) = 0.3$ | $P'(A) = 1 - 0.3 = 0.7$

$P(B) = 0.45$ | $P'(B) = 1 - 0.45 = 0.55$

$P(C) = 0.75$ | $P'(C) = 1 - 0.75 = 0.25$

* ODD's in favour of event $= \frac{\text{favourable}}{\text{unfavourable}}$.

* ODD's against $= \frac{\text{unfavourable}}{\text{favourable}}$

- $P(A) = \frac{F}{F+U} = \frac{3}{10}$.

* Types of Event.

1. Sure event $\rightarrow P(A) = 1$.
2. Impossible event $\rightarrow P(A) = 0$
3. Exclusive event $\rightarrow$
   $P(A \cap D) = 0$.
4. Exhaustive Event $\rightarrow$
   $P(A \cup B) = 1$.
5. Equally likely event $\rightarrow$
   $P(A) = P(B)$
6. dependent event

   $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

   $P(B|A) = \frac{P(A \cap B)}{P(B)}$.
7. Independent event.

   $P(A \cap B) = P(A) \cdot P(B)$

* Problems of expected value

Expected Value $\rightarrow$ [avg. Value]
[mean]

$E(x) = \sum x \cdot P$

$E(x^2) = \sum x^2 \cdot P$

Variance $= E(x^2) - [E(x)]^2$

Basic Concepts of differential & Integral calculus

A. Differential Calculus.

Functions	derivation	Functions	Derivation
$\frac{d}{dx}(\text{Constant})$	0	$\frac{d}{dx}(\log x)$	$\frac{1}{x}$.
$\frac{d}{dx}[cF(x)]$	$c\frac{d}{dx}[F(x)]$	$\frac{d}{dx}[F(x) \mp g(x)]$	$\frac{d}{dx}[F(x)] \mp \frac{d}{dx}[g(x)]$
$\frac{d}{dx}(x^n)$	nx^{n-1}	$\frac{d}{dx}[f(x)g(x)]$	$f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$
$\frac{d}{dx}(e^x)$	e^x	$\frac{d}{dx}\left[\frac{F(x)}{g(x)}\right]$	$\frac{g(x)\frac{d}{dx}[F(x)] - F(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$
$\frac{d}{dx}(a^x)$	$a^x \log_e a$	$\frac{d}{dx}\left[[F(x)]^{(g(x))}\right]$	$[F(x)]^{(g(x))}$ $\left[\frac{g(x)}{F(x)} \cdot \frac{d}{dx}[F(x)] + \log F(x)\frac{d}{dx}[g(x)]\right]$
$\frac{d}{dx}(e^{ax})$	ae^{ax}		**Gradient/Slope :** Derivation of $f(x)$ at a point x represent the slope of the tangent to the Curve $y = f(x)$ at the point x.

Note:

If $x = \phi(t)$ and $y = \psi(t)$, then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. If u and v are the functions of x, then $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$.

Some standards results:

1. $\frac{d}{dx}\frac{1}{x^2} = -\frac{2}{x^3}$.

2. $\frac{d}{dx}\frac{1}{x^7} = -\frac{7}{x^8}$

3. differentiation of $x^p y^q = (x+y)^{p+q}$, $\frac{dy}{dx} = \frac{y}{x}$.

4. $y = ae^{nx} + be^{-nx}$ then $\frac{d^2y}{dx^2} = n^2 y$.

Some short trick of integrations:

$$\int e^x (f(x) + f'(x))\, dx = e^x f(x)$$

standard formula. $\int e^x (x' + 1)\, dx = e^x x^1$

$$\int e^x \left(\log x + \frac{1}{x}\right) dx = e^x \log x.$$

$$\int e^x (x^2 + 2x)\, dx = e^x x^2.$$

$\int_{-a}^{a} f(x) = 0$, if $f(x)$ is th odd function, $f(-x) = -f(x)$

$\int_{-1}^{1} \frac{|x|}{x} = 0$, if $f(x)$ is the odd function

- $\int_{-1}^{1} e^x - e^{-x} = 0$, if $f(x)$ is the odd function.

$$\int_{LL}^{UL} \frac{x^n}{x^n + (UL + LL - x)^n} = \frac{UL - LL}{2}.$$

* Integral Calculus

1. $\int K F(x)\,dx \longrightarrow K\int F(x)\,dx$.

2. $\int F(x) \pm g(x)\,dx \longrightarrow \int F(x)\,dx \pm \int F(x)\,dx$.

3. $\int x^n dx$ except for $n \neq -1 \longrightarrow \dfrac{x^{n+1}}{n+1} + c$.

4. $\int uv\,dx = u\int v\,dx - \int\left[\dfrac{du}{dx}\int v\,dx\right]dx \longrightarrow \log_e x + c$.

5. $\int \frac{1}{x}\,dx \longrightarrow \log_e x + c$.

6. $\int a^x dx \longrightarrow \dfrac{a^x}{\log_e a} + c,\ a \neq 1,\ a > 0$.

7. $\int e^x dx \longrightarrow e^x + c$.

8. $\int x^x dx \longrightarrow x^x(1 + \log x) + c;\ x^x \log e x;\ x^x(\log 1 + \log x)$.

9. $\int \dfrac{dx}{x^2 - a^2} \longrightarrow \dfrac{1}{2a}\log\dfrac{x-a}{x+a} + c$.

10. $\int \dfrac{dx}{a^2 - x^2} \longrightarrow \dfrac{1}{2a}\log\dfrac{x+a}{a-x} + c$.

11. $\int \dfrac{dx}{\sqrt{a^2 + x^2}} \longrightarrow \log\{x + \sqrt{(x^2 + a^2)}\} + c$.

12. $\int \sqrt{(x^2 + a^2)}\,dx \longrightarrow \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log(x + \sqrt{(x^2 + a^2)}) + c$.

13. $\int \sqrt{(x^2 - a^2)}\,dx \longrightarrow \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log(x + \sqrt{(x^2 - a^2)}) + c$.

14. $\int \dfrac{F(x)\,dx}{f'(x)} \longrightarrow \log F(x) + c$.

15. $\int F(x)^n F'(x)\,dx \longrightarrow \dfrac{[F(x)]^{n+2}}{n+1} + c$.

Correlation & Regression

Correlation : It analysis is a measure of closeness of the relationship between two random variables and is bounded by the values +1 and -1.

A. Karl pearson's product Moment Correlation Coefficient.

* Alternative 1 : with Covariance

$$P_{xy} = \frac{Cov_{xy}}{\sigma_x \sigma_y}$$ where

$$Cov_{xy} = \Sigma[D_x \times D_y] \div n$$

$$\sigma_x = \sqrt{\frac{\Sigma x_i^2}{n} - (\bar{x})^2}$$

$$\sigma_y = \sqrt{\frac{\Sigma y_i^2}{n} - (\bar{y})^2}$$

Alt 2 : with probability

$$\frac{\Sigma|x_i - E(x_i)| \times |y_i - E(y_i)|}{\sigma_x \sigma_y}.$$

Alt 3 : With bivariate data

$$\frac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{\sqrt{[n\Sigma x_i^2 - \Sigma(x_i)^2][n\Sigma y_i^2 - \Sigma(y$$

* Spearman's rank Correlation Coefficient

$$\gamma_R = 1 - \frac{6\Sigma D^2}{n(n^2-1)}.$$