

Ratio, proportion, Indices & logarithm

Ratio Meaning of ratio

→ Ratio is a comparison between two quantities of the same kind

→ Ratio is exist only between quantities of the same kind.

rules :

1) If a and b are two quantities of same kind then the ratio between a and b can be written $a:b$ or a to b or a/b

→ called term

2) If $a:b$ is ratio then

a is called

first term / antecedent

denominator b is called

consequent

second term.

3) Compound ratio of $a:b$ and $c:d$ is $a/b \times c/d$
 $= ac:bd$

example - $2:3$ $4:5$ $2 \times 4 / 3 \times 5 = 8/15$

4) Inverse ratio of $a:b = b:a$

Inverse ratio of $a:b:c = bc:ac:ab$

Inverse ratio of $a:b:c:d = bcd:acd:abd:abc$

5) If $a:b$ is ratio

duplicate ratio = $a^2:b^2$

triplicate ratio = $a^3:b^3$

subduplicate ratio = $\sqrt{a}:\sqrt{b}$

subtriplicate ratio = $\sqrt[3]{a}:\sqrt[3]{b}$

→ A ratio $a:b$ is said to be of **greater inequality** if $a > b$ and of **lesser inequality** if $a < b$.

→ A ratio compounded of itself is called its **duplicate ratio**.

6) IF a quantity **Increase** or **decrease** in the ratio $a:b$ then
new value = original value \times $\frac{b}{a}$.

7) IF $a/b > 1$ (Ratio of **greater inequality**)
IF $a/b < 1$ (Ratio of **lesser inequality**)
 $a/b = 1$ (Ratio of **equality**)

8) IF $a/b = c/d = e/f = \dots$
value of each = $\frac{a+c+e+\dots}{b+d+f+\dots}$ (Addendo)
 $b = \frac{a-c-e-\dots}{d-f-\dots}$ (Subtrahendo)

Proposition

→ **Proposition** is defined as the **equality of two or more ratio**. IF $a/b = c/d$ in such a case of the quantities a, b, c, d are said to be **proportional**, here ' d ' is called the **fourth proportional**.

→ IF $a/b = c/d$ then a, b, c are said to be in **continued proportion**, where ' b ' is called the **mean proportional** and ' c ' is called the **third proportional**.

→ IF $a/b = b/c$ or $b^2 = ac \therefore b = \sqrt{ac}$

fraction by which the original quantity is multiplied to get a new quantity is called the factor multiplying ratio.

IF	Then	property
$\frac{a}{b} = \frac{c}{d}$	$ad = bc$	product of extremes = product of means.
	$b/a = d/c$	Invertendo
	$a/c = b/d$	Alternendo
	$\frac{a+b}{b} = \frac{c+d}{d}$	componendo
	$\frac{a-b}{b} = \frac{c-d}{d}$	dividendo
	$\frac{a+b}{a-b} = \frac{c+d}{c-d}$	componendo & dividendo
	$\frac{a-b}{a+b} = \frac{c-d}{c+d}$	dividendo & componendo

a = first proportional

b = second proportional

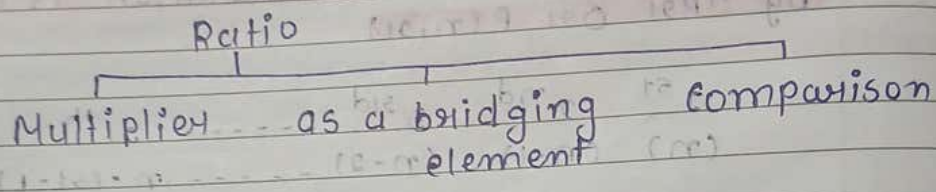
c = third proportional

d = fourth proportional

Ratio and proportion

Ratios

→ Ratio is a comparison of two similar attributes in same units



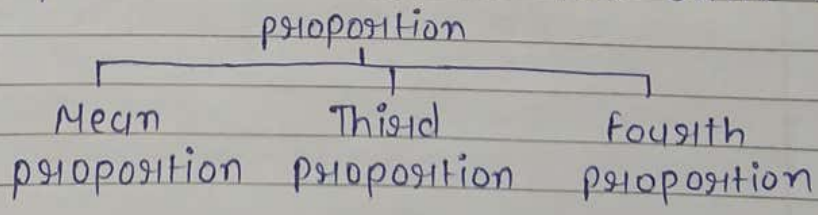
→ Types of ratio - (a:b)

- Duplicate $a^2 : b^2$
- Sub dup. $\sqrt{a} : \sqrt{b}$
- Triplicate $a^3 : b^3$
- Sub tri $\sqrt[3]{a} : \sqrt[3]{b}$
- compound (a:b, c:d) → a x c : b x d
- continued a : b : c - (a:b, b:c)
- Inverse b : a

proportion

→ If two ratios are equal they are said to be in proportion

each pair of ratio should have same units



$$b^2 = \sqrt{ac} \quad \frac{a}{b} = \frac{b}{c}$$

Product of means = product of extremes

properties of proportion $a:b::c:d$

Invertendo $\frac{b}{a} = \frac{d}{c}$

Alternendo $\frac{a}{c} = \frac{b}{d}$

componendo $\frac{a+b}{b} = \frac{c+d}{d}$

dividendo $\frac{a-b}{c} = \frac{d-b}{d}$

componendo & dividendo $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Indices

properties $(a \times b)^c = a^c \times b^c$

$b \sqrt[a]{a} = a^{1/b}$ $(a+b)^c \neq a^c + b^c$

$ab a^c = a^{b+c}$ $(a-b)^c \neq a^c - b^c$

$\frac{a^b}{a^c} = a^{b-c}$

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$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

$$a^b = c \rightarrow a = c^{1/b}$$

$a^b = a^c$ then Base same $b=c$ power equate

power - same base - equate

$$a^b = c^b \rightarrow a = c$$

$$\frac{1}{a^{-b}} = a^b, \quad \frac{1}{a^b} = a^{-b}$$

$$(a)^0 = 1$$

$(a^m)^n = a^{mn}$; m is added n times

$$(ab)^m = a^m \times b^m$$

$$a^m = a^n \Rightarrow m = n, \text{ where } a \neq 0, 1, -1 \neq m$$

For $a^m = b^m$ if $m \neq 0$ then

- i) $a = b$ (when m is odd)
- ii) $a = \pm b$ (when m is even)

$$\sqrt[n]{a} = a^{1/n}, \quad 1^a = 1$$

$$\sqrt{a} = a^{1/2}, \quad a^1 = a$$

$0^0 = \text{has NO meaning}$

Basic formulae:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$

$$(a+b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b$$

$$= a^3 + b^3 + 3ab(a+b)$$

$$(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$$

$$= a^3 - b^3 - 3ab(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

+ $a+b+c=0$ then $a^3 + b^3 + c^3 = 3abc$ (IMP)

+ If $a^3 + b^3 + c^3 = 3abc$, then either $a+b+c=0$ or $a=b=c$ but both the results are not true simultaneously.

Logarithms

$$a^b = c$$

$$\log_a c = b$$

Always assume base to be 10 ← known as common logarithm

the base "a" of log can be any positive real number except 1.

The base of log can be clearly divided into two parts

$0 < a < 1$ (the proper fraction)

$a > 1$ (positive integer / mixed fraction)

The base is always taken to be "e", where "e" is a constant and this is known as "natural logarithm".

Common logarithm are used for numerical calculations and natural logarithms are used in calculus.

Basic rules

$$\log_a mn = \log_a m + \log_a n$$

$$\log_a \frac{m}{n} = \log_a m - \log_a n$$

$$\log_a m^n = n \log_a m$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

$$\log_a 0 = \text{undefined}$$

$$\log_a \text{-ve} = \text{undefined}$$

$$\log_a m = \log_a n \quad m=n$$

$$\log_b a = \frac{\log m a}{\log m b} \quad (m \text{ can be any common base})$$

$$\log_b a = \frac{\log a}{\log b}$$

-) logarithm of a number (n) to the base a is the power to which base must be raised to produce the number.

$$\log_2 8 \quad 2^3 = 8$$

↑
Base

$$\log_a b = \frac{1}{\log_b a}$$

$$a^{\log_a x} = x$$

$$\log_a b \times \log_b a = 1$$

$$\log_b^a \times \log_c^b \times \log_a^c = 1$$

$$\log_b^a \times \log_c^b = \log_c^a$$

log = ? shortcut

Type no.

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