Ratio, proportion , Indices & logarithm

Ratio

Meaning of ratio

→ Ratio is a comparision between two quantities of the same kind.

→ Ratio is exist only between quantities of the same kind.

rules :

1) If a and b are two quantities of same kind then the ratio between a and b can be written $a:b$ or a to b or a/b → called term

2) If $a:b$ is ratio then
numerator a is called first term antecedent
denominator b is called second term. consequent

3) Compound ratio of $a:b$ and $c:d$ is $a/b \times c/d$
$= ac:bd$
example - $2:3$ $4:5$ $2\times4/3\times5 = 8/15$

4) Inverse ratio of $a:b = b:a$
Inverse ratio of $a:b:c = bc:ac:ab$
Inverse ratio of $a:b:c:d = bcd:acd:abd:abc$

5) If $a:b$ is ratio
duplicate ratio $= a^2:b^2$
Triplicate ratio $= a^3:b^3$
Subduplicate ratio $= \sqrt{a}:\sqrt{b}$
Subtriplicate ratio $= \sqrt[3]{a}:\sqrt[3]{b}$

→ A ratio $a:b$ is said to be of greater inequality if $a>b$ and of lesser inequality if $a<b$.

→ A ratio compounded of itself is called its duplicate Ratio

6) If a quantity Increase or decrease in the ratio $a:b$ then

new value = original value × b/a.

7) If $a/b > 1$ (Ratio of greater Inequality)

If $a/b < 1$ (Ratio of lesser Inequality)

$a/b = 1$ (Ratio of equality)

8) If $a/b = c/d = e/f =$

value of each ratio $= \frac{a+c+e}{b+d+f}$ ... (Addendo)

$= \frac{a-c-e}{b-d-f}$ ... (subtrandendo)

proportion

→ proportion is defined as the equality of two or more ratio. If $a/b = c/d$ in such a case of the quantities a, b, c, d are said to be proportional, here 'd' is called the fourth proportional.

→ If $a/b = c/d$ then a, b, c are said to be in continued proportion, where 'b' is called the mean proportional and 'c' is called third proportional.

→ If $a/b = b/c$ or $b^2 = ac$ ∴ $b = \sqrt{ac}$

fraction by which the orignial quantity is multiplied to get a new quantity is called the factor multiplying ratio.

If	Then	property
$\frac{a}{b} = \frac{c}{d}$	$ad = bc$	product of exterms = product of means.
	$b/a = d/c$	Invertendo
	$a/c = b/d$	Alternendo
	$a+b/b = c+d/d$	componendo
	$a-b/b = c-d/d$	dividendo
	$a+b/a-b = c+d/c-d$	componendo & dividendo
	$a-b/a+b = c-d/c+d$	dividendo & componendo

a = first proportional
b = second proportional
c = third proportional
d = fourth proportional

Ratio and proportion

Ratios

→ Ratio is a Comparison of two similar attributes in same units

Ratio
- Multiplier
- as a bridging element
- comparison

→ Types of ratio (a:b)

Duplicate	$a^2 : b^2$	
Sub dup.	$\sqrt{a} : \sqrt{b}$	
Triplicate	$a^3 : b^3$	
Sub tri	$\sqrt[3]{a} : \sqrt[3]{b}$	
compound	$(a:b, c:d)$	$\rightarrow a \times c : b \times d$
continued	$a : b : c$	$(a:b, b:c)$
Inverse	$b : a$	

proportion

→ If two ratios are equal they are said to be in proportion

each pair of ratio should have same units

proportion
- Mean proportion
- Third proportion
- Fourth proportion

$b^2 = \sqrt{ac}$ $\qquad \frac{a}{b} = \frac{b}{c}$

Product of means = product of extremes

properties of proportion {a:b}

Invertendo $\qquad \frac{b}{a} = \frac{d}{c}$

Alternendo $\qquad \frac{a}{c} = \frac{b}{d}$

componendo $\qquad \frac{a+b}{b} \quad \frac{c+d}{d}$

dividendo $\qquad \frac{a-b}{c} \quad \frac{c-d}{d}$

componendo & dividendo $\qquad \frac{a+b}{a-b} = \frac{c+d}{c-d}$

Indices

properties $\qquad (a \times b)^c = a^c \times b^c$

$\sqrt[b]{a} = a^{1/b}$ $\qquad (a+b)^c \neq a^c + b^c$

$a^b a^c = a^{b+c}$ $\qquad (a-b)^c \neq a^c - b^c$

$\frac{a^b}{a^c} = a^{b-c}$

$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$

$a^b = c \rightarrow a = c^{1/b}$

$a^b = a^c$ then Base same $b=c$ power equate

power - same base - equate
$a^b = c^b \rightarrow a = c$

$\frac{1}{a^{-b}} = a^b$, $\frac{1}{a^b} = a^{-b}$

$(a)^0 = 1$

$(a^m)^n = a^{mn}$; m is added n times

$(ab)^m = a^m \times b^m$

$a^m = a^n \Rightarrow m = n$; where $a \neq 0, 1, -1, \pm\infty$

For $a^m = b^m$ if $m \neq 0$ then
i) $a = b$ (when m is odd)
ii) $a = \pm b$ (when m is even)

$\sqrt[n]{a} = a^{1/n}$ $\qquad 1^a = 1$

$\sqrt{a} = a^{1/2}$ $\qquad a^1 = a$

0^0 = has No meaning

Basic
(a+b
(a-b
a^2 -
(a+b
(a+
(a-
a^3 +
a^3 -
+ a+
+ If
a =
si
i lo
ic
Al
th
ar
Th
t
c
c

Basic formula:

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a+b)(a-b)$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

$(a+b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b$

$= a^3 + b^3 + 3ab(a+b)$

$(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$= a^3 - b^3 - 3ab(a-b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

+ $a+b+c=0$ then $a^3+b^3+c^3 = 3abc$ (IMP)

+ If $a^3+b^3+c^3 = abc$, then either $a+b+c=0$ or $a=b=c$ but both the results are not true simultaneously.

Logarithms

$a^b = c$

$\log_a c = b$

Always assume base to be 10 — known as common logarithm

the base "a" of log can be any positive real number except 1.

The base of log can be clearly divided into two parts

$0 < a < 1$ (the proper fraction)

$a > 1$ (positive integer / mixed fraction)

The base is always taken to be "e", where "e" is a constant and this is known as "natural logarithm."

Common logarithm are used for numerical calculations and natural logarithms are used in calculus.

Basic rules

$\log_a mn = \log_a m + \log_a n$

$\log_a \frac{m}{n} = \log_a m - \log_a n$

$\log_a m^n = n \log_a m$ → logarithm of a number (n) to the base a is the power to which base must be raised to produce the number.

$\log_a a = 1$

$\log_a 1 = 0$

$\log_a 0 =$ undefined

$\log_2 8$ $2^8 = 3$

Base

$\log_a -ve =$ undefined

$\log_a m = \log_a n$

$m = n$

$\log_b a = \frac{\log_m a}{\log_m b}$ (m can be any common base)

$\rightarrow \log_b a = \frac{\log a}{\log b}$

$$\log_a b = \frac{1}{\log_b a}$$

$$a^{\log_a x} = x$$

$$\log_a b \times \log_b a = 1$$

$$\log_b a \times \log_c b \times \log_a c = 1$$

$$\log_b a \times \log_c b = \log_c a .$$

log = ? shortcut

Type No.

Type √ 13 times

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