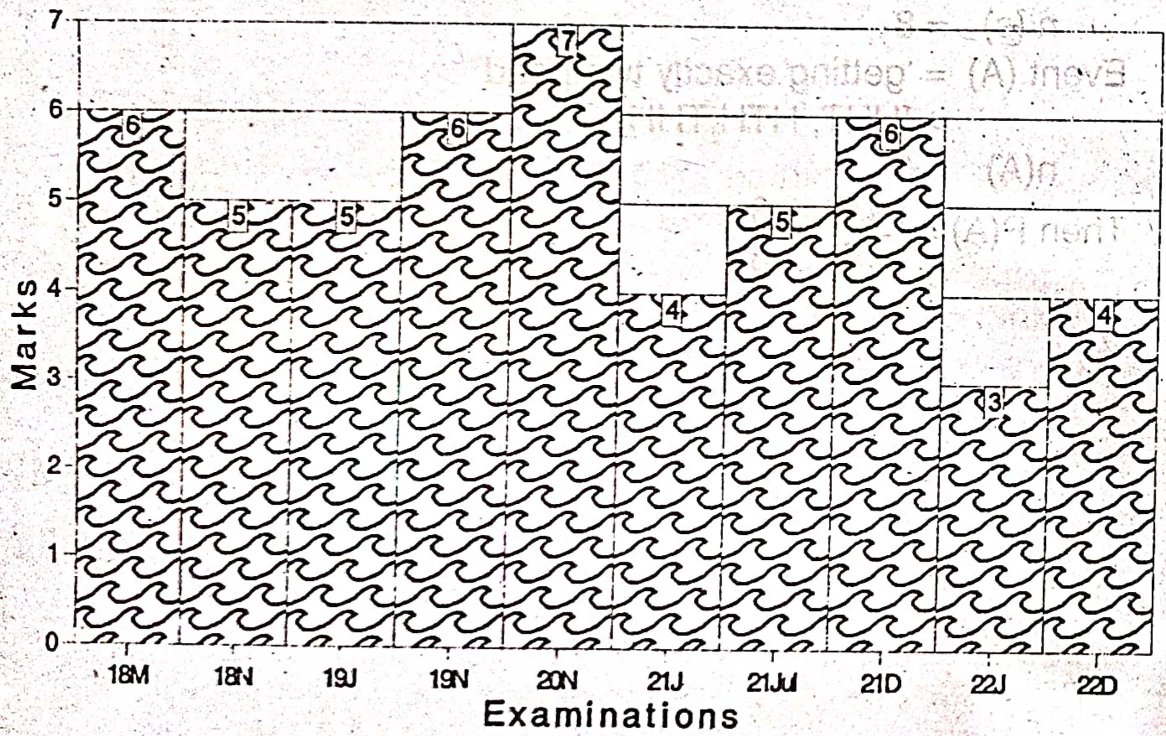
CHAPTER 16

THEORETICAL DISTRIBUTIONS

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective   Short Notes   Distinguish   Descriptive   Practical



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PAST YEAR QUESTIONS AND ANSWERS

2009 - JUNE

[1] In a poisson distribution P (x = 0) = P (X = 2). Find E (x).

(a) $\sqrt{2}$
(b) 2
(c) – 1
(d) 0

(1 mark)

Answer:

(a) E (x) stands for mean of the distribution.

Let x be a Poisson variate with parameter m.

The probability function of x is then given by :

$$f(x) = \frac{e^{-m}.m^x}{x!} \text{ for } x = 0, 1, 2 \text{ .... as}$$

now, P (x = o) = P (x = 2)

$$f(o) = f(2)$$

$$\frac{e^{-m}.m^{\circ}}{o!} = \frac{e^{-m}.m^2}{2!}$$

$$\frac{m^{\circ}}{1} = \frac{m^2}{2}$$

$$1 = \frac{m^2}{2}$$

$$m^2 = 2$$

$$m = \sqrt{2} \cong 1.414$$

Therefore, the mean of this distribution is $E(x) = m = \sqrt{2}$

2009 - DECEMBER

[2] Shape of Normal Distribution Curve:
(a) Depends on its parameters
(b) Does not depend on its parameters
(c) Either (a) or (b)
(d) Neither (a) nor (b) (1 mark)

Answer:

(a) Shape of the Normal Distribution curve depends on its parameters. [self- explanatory].

[3] For binomial distribution $E(x) = 2$, $V(x) = 4/3$. Find the value of n.
(a) 3
(b) 4
(c) 5
(d) 6 (1 mark)

Answer:

(d) $E(x) = np = 2$

$v(x) = npq = 4/3.$

$np = 2$ ....(1)

$$npq = \frac{4}{3}$$

substituting the value of np from (1):

$$2 \times q = \frac{4}{3}$$

$$2q = \frac{4}{3}$$

$$q = \frac{4}{3\times2} = \frac{2}{3}$$

$$\therefore \quad q = \frac{2}{3}$$

$$p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$$

$$np = 2$$

$$n \times \frac{1}{3} = 2$$

$$n = 6$$

$$\therefore \quad n = 6.$$

[4] What are the parameters of binomial distribution?

(a) n

(b) p

(c) Both n and p

(d) None of these (1 mark)

Answer:

(c) Binomial Distribution is a biparametric distribution, characterized by 'n' and 'p' [self-explanatory].

2010 - JUNE

[5] The Variance of standard normal distribution is

(a) 1

(b) μ

(c) σ^2

(d) 0 (1 mark)

Answer:

(a) In standard normal distribution

mean = 0

Variance = 9

[6] For a Poisson distribution P (x = 3) = 5 P(x = 5), then S.D. is

(a) 4 (b) 2

(c) 16 (d) $\sqrt{2}$ (1 mark)

Answer:

(d) Let x be a Poisson variate with parameter m. The probability function of x is then given by :

$f(x) = \frac{e^{-m}m^x}{x!}$ for x = 0, 1, 2 .... as now,

P(x = 3) = 5P (x = 5)

f(3) = 5 f(5)

$\frac{e^{-m}m^3}{3!} = \frac{5e^{-m}m^5}{5!}$

$20 = 5m^2$

$m^2 = 4$

Variance = m = 2

$\therefore$ SD = $\sqrt{\text{Variance}}$

SD = $\sqrt{2}$

[7] For a Binomial distribution B (6, p), P(x = 2) = 9p(x = 4), then P is

(a) 1/2 (b) 1/3

(c) 10/13 (d) 1/4 (1 mark)

Answer:

(d) We are given that n = 6. The probability mass function of x is given by

$f(x) = {}^nc_x\, p^x\, q^{n-x}$

$= {}^6c_x\, p^x\, q^{6-x}$, for x = 0, 1, 2 ...., 6

Thus, $P(x = 2) = f(2) = {}^6c_2\, p^2\, q^{6-2} = 15p^2\, q^4$

and $P(x = 4) = f(4) = {}^6c_4\, p^4\, q^{6-4} = 15p^4\, q^2$

Hence, P(x = 2) = 9P(x = 4)

$15p^2\, q^4 = 9.15p^4\, q^2$

$15p^2\, q^2\, (q^2 - 9p^2) = 0$

$q^2 - 9p^2 = 0$ (as $p \neq 0$ and $q \neq 0$)
$(1 - p)^2 - 9p^2 = 0$ (as $q = 1 - p$)
$(1 - p + 3p) = 0$ or $(1 - p - 3p) = 0$
$p = -½$ or $p = 1/4$
Thus, $p = 1/4$ (as $p \neq -½$)

[8] In Binomial distribution n = 9 and P = 1/3, what is the value of variance:
(a) 8 (b) 4
(c) 2 (d) 16 (1 mark)

Answer:

(c) In Binomial distribution,
Variance = npq
$n = 9$
$p = \frac{1}{3}$
$q = \frac{2}{3}$
$\therefore$ Variance = $9 \cdot \frac{1}{3} \cdot \frac{2}{3} = 2$

2010 - DECEMBER

[9] If standard deviation of a poisson distribution is 2, then its
(a) Mode is 2 (b) Mode is 4
(c) Modes are 3 and 4 (d) Modes are 4 and 5 (1 mark)

Answer:

(c) Given σ = S.D. = 2 $\Rightarrow$ Variance = $\sigma^2 = 4$
$\because$ In poisson distribution
Mean = Variance
$\because$ m = 4, which is an integer
$\because$ it is bi-modal
Modes are m and (m – 1)
hence, 4 and 3,

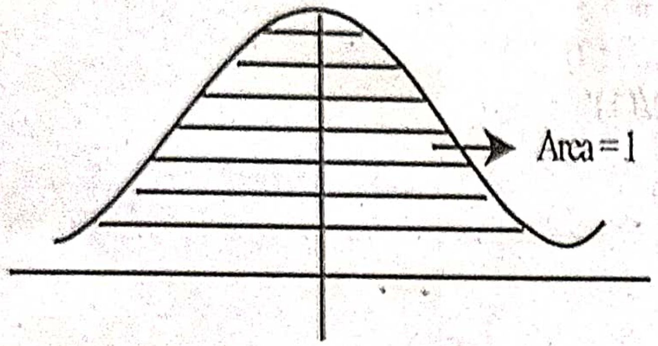
[10] The area under the Normal curve is

(a) 1 (b) 0

(c) 0.5 (d) – 1 (1 mark)

Answer:

(a) Area under the Normal curve = 1



[11] For a normal distribution $N(\mu,\sigma^2)$,
$P(\mu-3\sigma < x < \mu+3\sigma)$ is equal to

(a) 0.9973 (b) 0.9546

(c) 0.9899 (d) 0.9788 (1 mark)

Answer:

(a) We Know that

$P(\mu - 3\sigma < x < \mu + 3\sigma)$

$= 0.9973$

[12] If for a Binomial distribution B (n,p,) the mean = 6 and Variance = 2 then 'p' is

(a) 2/3 (b) 1/3

(c) 3/5 (d) 1/4 (1 mark)

Answer:

(a) Mean = 6 = np

Variance = 2 = npq $\left\{\begin{matrix}\text{For Binomial} \\ \text{Distribution}\end{matrix}\right\}$

$$\frac{npq}{np} = \frac{2}{6} \Rightarrow q = \frac{1}{3}$$

$$p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$$

$$p = \frac{2}{3}$$

2011 - JUNE

[13] If the inflexion points of a Normal Distribution are 6 and 14. Find its Standard Deviation ?

(a) 4 (b) 6

(c) 10 (d) 12. (1 mark)

Answer:

(a) ∵ The inflexion points of a Normal Distribution are given as

$(\mu + \sigma)$ and $(\mu - \sigma)$

here, we are given :

$\mu + \sigma = 14$ (1)

and, $\mu - \sigma = 6$ (2)

Solving (1) and (2) we get

$\mu = 10$ and $\sigma = 4$

Hence S.D $(\sigma) = 4$

[14] In a Binomial Distribution, if mean is k-times the variance, then the value of 'k' will be______.

(a) p (b) $\frac{1}{p}$

(c) $1 - p$ (d) $\frac{1}{1-p}$ (1 mark)

Answer:

(d) In Binomial Distribution :

Mean = np & Variance = npq

by question, here

Mean = K.

Variance np = K. npq

$\therefore \quad K = 1/q$

$\therefore \quad K = \frac{1}{1-p} \quad [\because p + q = 1]$

[15] If $x \sim N(3,36)$ and $y \sim N(5,64)$ are two independent Normal variate with their standard parameters of distribution, then if $(x+y) \sim N(8,A)$ also follows normal distribution. The value of A will be ________.

(a) 100 (b) 10

(c) 64 (d) 36 (1 mark)

Answer:

(b) We Know,

If $x \sim N(\mu_1, \sigma_1^2)$ and $y \sim N(\mu_2, \sigma_2^2)$

Then

$x + y \sim N\left(\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2}\right)$

Where $\sim N(\mu, \sigma)$

(Say)

$\mu = (\mu_1 + \mu_2)$ and $\sigma = \left(\sqrt{\sigma_1^2 + \sigma_2^2}\right)$

Here, $x \sim N(3,36)$ and $y \sim N(5,64)$

$\therefore \quad x + y \sim N(3 + 5, \sqrt{36+64} = N(8,A)$

$\Rightarrow A = \sqrt{36+64}$

$\therefore A = 10$

2011 - December

[16] The mean of Binomial distribution is 20 and Standard deviation is 4 then;

(a) $n = 100, p = 1/5, q = 4/5$

(b) $n = 50, p = 2/5, q = 2/5$

(c) $n = 100, p = 2/5, q = 4/5$

(d) $n = 100, p = 1/5, q = 3/5$ (1 mark)

Answer:

(a) Here,

Mean = 20 S.D = 4

$np = 20$ .....(1) Variance = $(4)^2$

Variance = 16

$npq = 16$.....(2)

Divide (2)/(1)

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

$$p = 1 - q$$

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Putting the value of p in eq (1)

$$n \times \frac{1}{5} = 20$$

$$n = 20 \times 5 = 100$$

[17] A Company has two cars which it hires out during the day. The number of Cars demanded in a day has poison distribution with mean 1.5. Then percentage of days on which only one car was in demand is equal to

(a) 23.26 (b) 33.47

(c) 44.62 (d) 46.40 (1 mark)

[Given Exp (– 1.5) = 0.2231]

Answer:

(b) Given the mean of Poisson distribution (m) = 1.5

Then

Poisson parameter (μ) = m = 1.5

We know by Poisson distribution

$$P(x) = \frac{e^{-m} . m^x}{x!}$$

Here

$m = 1.5, x = 1$

$$P(1) = \frac{e^{-1.5}.(1.5)}{1.!}$$

$$= \frac{0.2231 \times 1.5}{1}$$

$= 0.33465$

$= 0.3347$

% of P(1) = 0.3347 × 100 % = 33.47%

[18] The binomial distribution with mean 3 & variance 2 is:

(a) $\left(\frac{2}{4}+\frac{1}{4}\right)^{2\Rightarrow 9}$

(b) $\left(\frac{2}{6}+\frac{1}{6}\right)^{2\Rightarrow 9}$

(c) $\left(\frac{2}{3}+\frac{1}{3}\right)^{2\Rightarrow 9}$

(d) $\left(\frac{2}{5}+\frac{1}{5}\right)^{2\Rightarrow 9}$ (1 mark)

Answer:

(c) Given mean = 3

np = 3 ....(1)

Variance = 2

npq = 2 ....(2)

Divide (2)/(1) we get

$$\frac{npq}{np} = \frac{2}{3} \Rightarrow q = \frac{2}{3}$$

$p = 1 - q$

$$p = 1 - \frac{2}{3} = \frac{1}{3}$$

Putting the value of p in Equation (1)

$$n \times \frac{1}{3} = 3$$

$n = 9$

The Binomial distribution is

$$(q+p)^n = \left[\frac{2}{3}+\frac{1}{3}\right]^9$$

2012 - JUNE

[19] For binomial distribution

(a) Variance < Mean
(b) Variance = Mean
(c) Variance > Mean
(d) None of the above. (1 mark)

Answer:

(a) For Binomial distribution
npq < np
Variance < Mean

[20] If x is a Poisson variate and E(x) = 1, then P(x > 1) is

(a) $1 - \frac{e^{-1}}{2}$
(b) $1 - e^{-1}$
(c) $1 - 2e^{-1}$
(d) $1 - \frac{5}{2}e$ (1 mark)

Answer:

(c) E(x) = 1, we known P (x) $= \frac{e^{-m} m^x}{\underline{|x}}$; E(x) = m

$$\therefore P(x > 1) = 1 - P(x < 1)$$
$$= 1 - [P(x = 0) + P(x = 1)]$$
$$= 1 - \left[\frac{e^{-1} . 1^0}{\underline{|0}} + \frac{e^{-1} . 1^1}{\underline{|1}}\right]$$
$$= 1 - [e^{-1} + e^{-1}]$$
$$= 1 - 2e^{-1}$$

[21] The mean and the variance of a random variable X having the probability density function $P(X = x) = \exp\{-(x - 4)^2\}/\sqrt{\pi}, -\infty < x < \infty$ is.

(a) $4, \frac{1}{2}$
(b) $4, \frac{1}{\sqrt{2}}$
(c) 2, 2
(d) $2, \frac{1}{2}$ (1 mark)

Answer:

(a) We know, the probability distribution function for normal distribution is :

$$P(X = x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty$$

Given in equation :

$$P(X = x) = \frac{1}{\sqrt{\pi}} e^{-(x-4)2}$$

Comparing given function with the standard form, we get

Mean (u) = 4

$$S.D\ (\sigma) = \frac{1}{\sqrt{2}}$$

$$\therefore \text{Variance } (\sigma^2) = \frac{1}{2}$$

2012 - DECEMBER

[22] In a Normal Distribution

(a) The first and second quartile are equidistant from median

(b) The second and third quartiles are equidistant from the median

(c) The first and third quartiles are equidistant from the mean

(d) None of the above. (1 mark)

Answer:

(c) In a Normal Distribution :

"The first and third quartiles are equidistant from the mean".

[23] If parameters of a binomial distribution are n and p then, this distribution tends to a Poisson distribution when

(a) $n \to \infty, p \to 0$ (b) $p \to 0, np = \lambda$

(c) $n \to \infty, np = \lambda$ (d) $n \to \infty, p \to 0, np = \lambda$ (1 mark)

where 'λ' is a finite constant

Answer:

(d) If parameters of a binomial distribution are n and p then this distribution tends to a Poisson distribution when

$n \to \infty$, $p \to 0$, $np = \lambda$

Where 'λ' is a finite constant

[24] If a random variable x follows Poisson distribution such that E(x) = 30, then the variance of the distribution is

(a) 7

(b) 5

(c) 30

(d) 20 (1 mark)

Answer:

(c) In Poisson distribution

Mean = Variance

$\because$ E(x) = 30

Mean = E(x) = 30

So, Variance = 30

[25] In a normal distribution quartile deviation is 6, the standard deviation will be

(a) 4

(b) 9

(c) 7.5

(d) 6 (1 mark)

Answer:

(b) In normal distribution

4 S.D. = 6. Q.D.

$$S.D. = \frac{6}{4}\ Q.D.$$

$$= \frac{6}{4} \times 6$$

$$= 9$$

2013 - JUNE

[26] The mode of the Binomial Distribution for which the mean is 4 and variance 3 is equal to?

(a) 4 (b) 4.25

(c) 4.5 (d) 4.1 (1 mark)

Answer:

(a) In Binomial Distribution

Mean = np = 4 ....(1)

Variance = npq = 3 ....(2)

Sum (1) & (2)

$4q = 3$

$q = 3/4$

$p = 1-q$

$= 1-3/4$

$p = 1/4$ in equation (1)

$n \times \frac{1}{4} = 4$

$\boxed{n = 16}$

$(n+1)p = (16+1) \times \frac{1}{4}$

$= 4.25$ which is non Integer

So Mode $= 4$

[27] For Poisson Distribution:

(a) Mean and Standard Deviations are equal

(b) Mean and variance are equal

(c) Standard Deviation and variance are equal

(d) Both (a) and (b) are correct (1 mark)

Answer:

(b) In Poisson Distribution mean and variance are equal.

[28] Which of the following is not a characteristic of a normal probability distribution?

(a) Mean of the normally distributed population lies at the centre of its normal curve.

(b) It is multi-modal

(c) The mean, median and mode are equal

(d) It is a symmetric curve (1 mark)

Answer:

(b) It is multi-modal (False)

[29] An approximate relation between quartile deviation (QD) and standard deviation (S.D) of normal distribution is:

(a) 5 QD = 4 SD (b) 4 QD = 5 SD

(c) 2 QD = 3 SD (d) 3 QD = 2 SD (1 mark)

Answer:

(d) We know that

In normal distribution

4 S.D = 5 M. D = 6 Q.D

So 4 S.D = 6 Q.D

2 S.D = 3 Q.D

or 3 Q.D = 2 S.D

[30] In a binomial Distribution with 5 independent trials, probability of 2 and 3 successes are 0.4362 and 0.2181 respectively. Parameter 'p' of the binomial distribution is:

(a) 3/4 (b) 1/3

(c) 2/3 (d) 1/4 (1 mark)

Answer:

(b) Given

$n = 5, P(x = 2) = 0.4362$

$P(x = 3) = 0.2181$

$P(x = 3) = {}^5C_3 \cdot P^3 \cdot q^{5-3} = 10 \cdot P^3q^2$

$0.2181 = 10\, p^3q^2$ ——— (1)

and $P(x = 2) = {}^5C_2 \cdot P^2 \cdot q^{5-2} = 10\, p^2q^3$

$0.4362 = 10\, p^2q^3$ ——— (2)

equal (1)/ eq (2)

$$\frac{0.2181}{0.4362} = \frac{10p^3q^2}{10p^2q^3}$$

$$\frac{1}{2} = \frac{p}{q} \Rightarrow q = 2p$$

$$1 - p = 2p$$

$$2p + p = 1$$

$$3p = 1 \Rightarrow p = 1/3$$

2013 - DECEMBER

[31] In a certain Poisson frequency distribution, the probability corresponding to two successes is half the probability corresponding to three successes. The mean of the distribution is

(a) 6 (b) 12

(c) 3 (d) 2.45 (1 mark)

Answer:

(a) Given

$$P(x = 2) = \frac{1}{2} P(x = 3)$$

$$2 \quad P(x = 2) = P(x = 3)$$

$$2. \quad \frac{e^{-m}.m^2}{2!} = \frac{e^{-m}.m^3}{3!}$$

$$\frac{2}{2} = \frac{m}{6}$$

$$m = 6 \times \frac{2}{2} = 6$$

2014 - JUNE

[32] Mean and Variance of a binomial variance are 4 and $\frac{4}{3}$ respectively then $P(x \geq 1)$ will be______.

(a) $\frac{728}{729}$ (b) $\frac{1}{729}$

(c) $\frac{723}{729}$ (d) None of the above. (1 mark)

Answer:

(a) For Binomial Variable

$$\text{Mean} = np = 4 \quad \text{.........................(1)}$$

$$\text{Variance} = npq = \frac{4}{3} \quad \text{.........................(2)}$$

From (1) & (2)

$$4 \times q = \frac{4}{3}$$

$$q = \frac{1}{3}$$

$$p = 1 - \frac{1}{3} = \frac{2}{3}$$

$$np = 4$$

$$n \times \frac{2}{3} = 4$$

$$n = \frac{12}{2} = 6$$

$$p(x \geq 1) = 1 - p(x < 1)$$

$$= 1 - p(x = 0)$$

$$= 1 - {}^6C_0 . \left(\frac{2}{3}\right)^0 . \left(\frac{1}{3}\right)^6$$

$$= 1 - |x| \times \frac{1}{729} = 1 - \frac{1}{729} = \frac{728}{729}$$

[33] 5,000 students were appeared in an examination. The mean of marks was 39.5 with a Standard Deviation 12.5 marks. Assuming the distribution to be normal, find the number of students recorded more than 60% marks.

Given: When Z = 1.64, Area of normal curve = 0.4495

(a) 1,000

(b) 505

(c) 252

(d) 2,227 (1 mark)

Answer:

(c) Probability that students recorded more than 60% marks = P (x > 60)

$$= 1 - P(x \leq 60)$$

$$= 1 - P\left(\frac{x - \bar{x}}{\sigma} \leq \frac{60 - 39.5}{12.5}\right)$$

$$= 1 - P(Z \leq 1.64)$$

$$= 1 - \phi(1.64)$$

$$= 1 - (0.4495 + 0.5)$$

$$= 1 - 0.9495$$

$$= 0.0505$$

Thus, the Number of students having marks more than 60%

$$= 5000 \times 0.0505$$

$$= 252.5$$

[34] If a variate X has, mean > variance, then its distribution will be ______.

(a) Binomial distribution

(b) Poisson distribution

(c) Normal distribution

(d) T-distribution (1 mark)

Answer:

(a) In Binomial distribution

Mean > Variance

2014 - DECEMBER

[35] If six coins are tossed simultaneously. The probability of obtaining exactly two heads are:

(a) 1/64 (b) 63/64
(c) 15/64 (d) None of these (1 mark)

Answer:

(c) Here Total trial (n) = 6

For coin p = ½, q = 1 – ½ = ½

$$P(X = x) = {}^{n}C_{x}\, p^{x}.q^{n-x}$$

$$P(X = 2) = {}^{6}C_{2} \left(\frac{1}{2}\right)^{2} \times \left(\frac{1}{2}\right)^{6-2}$$

$$= \frac{6 \times 5}{2 \times 1} \times \left(\frac{1}{2}\right)^{2} \times \left(\frac{1}{2}\right)^{4}$$

$$= 15 \times \left(\frac{1}{2}\right)^{2+4}$$

$$= 15 \times \left(\frac{1}{2}\right)^{6}$$

$$= \left(\frac{15}{64}\right)$$

[36] If x and y are two independent normal random variables, then the distribution of x + y is:

(a) Normal (b) T-distribution
(c) Chi-square (d) F-distribution (1 mark)

Answer:

(a) If x and y are two independent Normal random variables, then the distribution of x + y is Normal.

[37] For a normal distribution having mean = 2 and variance = 4, the fourth central moment μ_4 is:

(a) 16 (b) 32
(c) 48 (d) 64 (1 mark)

Answer:

(c) For Normal Distribution Mean = 2, Variance = 4

Fourth central moments μ_4 =?

We know that Normal curve is always

Meso kuritic then $\beta_2 = 3$

moment coefficient of kurtosis

$$(\beta_2) = \frac{\mu_4}{\mu_2^2}$$

Here, μ_2 = Variance = 4, $\beta = 3$

$$3 = \frac{\mu_4}{4^2}$$

$$\mu_4 = 3 \times 4^2 = 3 \times 16 = 48$$

Shortcut: Fourth moments $\mu_4 = 3\sigma^4 = 3(4)^2 = 48$

[38] T-test can be used only when the sample has been taken from

(a) Binomial Population
(b) Poisson Population
(c) Normal Population
(d) Exponential Population (1 mark)

Answer:

(c) t-test can be used only when the sample has been taken from **Normal Population.**

[39] For a binomial distribution with mean = 4 and variance = 3, the third central moment μ_3 is:

(a) 5/2
(b) 7/4
(c) 3/2
(d) 1/3 (1 mark)

Answer:

(c) For Binomial distribution

Mean = 4 Variance = 3

np = 4 ____ (1) npq = 3 ____ (2)

4q = 3

q = 3/4

then p = 1 – q = 1 $\frac{3}{4}$ = 1/4

Putting p = 1/4 in equation (1)

$$n\frac{1}{4} = 4 \Rightarrow n = 16$$

The third central moment

$$\mu_3 = npq\,(q - p)$$

$$= 16 \times \frac{1}{4} \times \frac{3}{4}\left(\frac{3}{4} - \frac{1}{4}\right)$$

$$= 3\left(\frac{2}{4}\right) = \frac{3}{2}$$

2015 - JUNE

[40] If x is a binomial variable with parameters n and p, then x can assume
(a) any value between 0 and n
(b) any value between 0 and n, both inclusive
(c) any whole number between 0 and n, both inclusive
(d) any number between 0 and infinity (1 mark)

Answer:

(c) If x is a binomial variable with parameters n & p, then x can assume any whole number between 0 and n, both inclusive.

[41] In ________ distribution, mean = variance
(a) Normal (b) Binomial
(c) Poisson (d) None (1 mark)

Answer:

(c) In Poisson Distribution, Mean = Variance

[42] Under a normal curve $\bar{x} \pm 3\sigma$ covers ________
(a) 100% of the area (item values)
(b) 99%
(c) 99.73%
(d) 99.37% (1 mark)

Answer:

(c) Under a normal curve $(\bar{x} \pm 3\sigma)$ covers **99.73%** of the Area

2015 - DECEMBER

[43] If 'x' is a binomial variable with parameter 15 and $\frac{1}{3}$, then the value of the mode of the distribution:

(a) 5
(b) 5 and 6
(c) 5.50
(d) 6
(1 mark)

Answer:

(a) In Binomial Variable (Distribution)

$$x \sim B(n, p)$$

$$x \sim B(15, \frac{1}{3})$$

$$n = 15, P = \frac{1}{3}$$

$$\text{Mode} = (n + 1)P$$

$$= (15 + 1).\frac{1}{3}$$

$$= 16 \times \frac{1}{3} = 5.33 \text{ (which is non Integer)}$$

$$= 5$$

[44] Standard deviation of binomial distribution is:

(a) $\sqrt{np}$
(b) $(np)^2$
(c) $\sqrt{npq}$
(d) $(npq)^2$
(1 mark)

Answer:

(c) Standard Deviation of binomial distribution is $\sqrt{npq}$

[45] The wages of workers of factory follows:

(a) Binomial distribution
(b) Poisson distribution
(c) Normal distribution
(d) Chi-square distribution
(1 mark)

Answer:

(c) The wages of workers of factory follow **Normal Distribution.**

2016 - June

[46] The normal curve is:

(a) Positively skewed (b) Negatively skewed

(c) Symmetrical (d) All these (1 mark)

Answer:

(c) Normal curve is symmetrical.

[47] For a Poisson variate X, P (X = 1) = P (X = 2), what is the mean of X ?

(a) 1 (b) $\frac{3}{2}$

(c) 2 (d) $\frac{5}{2}$ (1 mark)

Answer:

(c) For x ~ P(m)

$P(x = 1) = P(x = 2)$

$$\frac{e^{-m}.m^1}{1!} = \frac{e^{-m}.m^2}{2!}$$

$$\frac{m}{1} = \frac{m^2}{2}$$

$$\boxed{m = 2}$$

[48] In a discrete random variable X follows uniform distribution and assumes only the values 8,9,11,15,18,20. Then P (X≤15) is____

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{2}{5}$ (1 mark)

Answer:

(c) Given data

8, 9, 11, 15, 18, 20

Total No. of data n(s) = 6

$$P(x \leq 15) = \frac{n(A)}{n(s)} = \frac{4}{6} = \frac{2}{3}$$

2016 - DECEMBER

[49] If x and y are independent normal variates with Mean and Standard Deviation as μ_1 and μ_2 and σ_1 and σ_2 respectively, then $z = x+y$ also follows normal distribution with

(a) Mean = $\mu_1 + \mu_2$ and S.D. = 0 respectively

(b) Mean = 0 and S.D. = $\sigma_1^2 + \sigma_2^2$

(c) Mean = $\mu_1 + \mu_2$ and S.D. = $\sqrt{\sigma_1^2 + \sigma_2^2}$

(d) None of these. (1 mark)

Answer:

(c) If x and y are two Independent variables of Normal Distribution

if $x \sim N(\mu_1, \sigma_1^2)$

and $y \sim N(\mu_2, \sigma_2^2)$

then $z = x + y$

$z = N(\mu_1, \sigma_1^2) + N(\mu_2, \sigma_2^2)$

$z = N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

Mean = $\mu_1 + \mu_2$, Variance = $\sigma_1^2 + \sigma_2^2$

S.D = $= \sqrt{\sigma_1^2 + \sigma_2^2}$

[50] A Poisson random variable has $\mu_4 = 2$, its variance i.e. μ_2 is

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{3}{2}$ (1 mark)

Answer:

(a) If X ~ P (m) Fourth moment of Poisson distribution

$\mu_2 = 3\,\mu_2$

$2 = 3\sigma^2$ [$\therefore \mu_2 \rightarrow \sigma^2$]

$\sigma^2 = \frac{2}{3}$

Variance $= \frac{2}{3}$

[51] Name the distribution which has Mean = Variance

(a) Binomial (b) Poisson

(c) Normal (d) Chi-square (1 mark)

Answer:

(b) In Poisson distribution, mean and variance are equal.

[52] An example of a bi-parametric continuos probability distribution:

(a) Binomial (b) Poisson

(c) Normal (d) (a) and (b) (1 mark)

Answer:

(c) Normal distribution is the example of bi-parametric probability distribution.

If x- is a continuous variable.

2017 - JUNE

[53] If X ~ N (50, 16), then which of the following is not possible:

(a) P(X > 60) = 0.30

(b) P(X < 50) = 0.50

(c) P(X < 60) = 0.40

(d) P(X > 50) = 0.50 (1 mark)

Answer:

(c) If X ~ N (50, 16) then P(X<60) = 0.40 is not possible.

[54] If for a distribution mean = variance, then the distribution is said to be:

(a) Normal (b) Binomial
(c) Poisson (d) None of the above. (1 mark)

Answer:

(c) In Poisson distribution mean and variance are equal.

[55] For a Binomial distribution if variance = $(\text{Mean})^2$, then the values of n and p will be:

(a) 1 and $\frac{1}{2}$ (b) 2 and $\frac{1}{2}$
(c) 3 and $\frac{1}{2}$ (d) 1 and 1 (1 mark)

Answer:

(a) Given, In Binomial Distribution
we know that mean = np
Variance = npq
Given, Variance = $(\text{Mean})^2$
Here option (A) we have n = 1, $p = \frac{1}{2}$, $q = 1 - \frac{1}{2} = \frac{1}{2}$

$$(\text{Mean})^2 = (np)^2 = \left(1 \times \frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\text{Variance} = npq = 1 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

So that Variance = $(\text{Mean})^2$

∴ Option a is correct.

2017 - DECEMBER

[56] In a normal distribution about 95 per cent of the observations lie between ________ and ________.

(a) $\mu - 2\sigma, \mu + 2\sigma$ (b) $\mu - 3\sigma, \mu + 3\sigma$
(c) $\mu - 1.96\sigma, \mu + 1.96\sigma$ (d) $\mu - 2.58\sigma, \mu + 2.58\sigma$ (1 mark)

Answer:

(c) In a Normal distribution about 95% of the observations lies b/w **$\mu - 1.96\sigma$** and **$\mu + 1.96\sigma$.**

[57] An example of a bi-parametric discrete probability distribution is
(a) Binomial distribution (b) Poisson distribution
(c) Normal distribution (d) Both (a) and (b) (1 mark)

Answer:

(a) An example of a bi-parametric discrete probability distribution is Binomial distribution.

[58] In ______ distribution, mean = variance
(a) Normal (b) Binomial
(c) Poisson (d) None of the above. (1 mark)

Answer:

(c) In **Poisson** Distribution, Mean = Variance

2018 - MAY

[59] The variance of a binomial distribution with parameters n and p is:
(a) $np^2 (1-p)$ (b) $\sqrt{np-(l-p)}$
(c) $nq(1-q)$ (d) $n^2p^2 (1-P)^2$ (1 mark)

Answer:

(c) The variance of a binomial distribution
= npq
= nqp
= nq(1-q)

[60] X is a poisson variate satisfying the following condition $9\,P(X = 4) + 90\,(X = 6) = P\,(X = 2)$. What is the value of $P(X \leq 1)$?
(a) 0.5655 (b) 0.6559
(c) 0.7358 (d) 0.8201 (1 mark)

Answer:

(c) Given $X \sim P(m)$

$P(x=2) = 9\ P(x=4) + 90\ P(x=6)$

$$\frac{e^{-m}.m^2}{2!} = +\frac{9.e^{-m}.m^4}{4!} + \frac{90.e^{-m}.m^6}{6!}$$

$$\frac{90.e^{-m}.m^6}{6!} + \frac{9.e^{-m}.m^4}{4!} - \frac{e^{-m}.m^2}{2!} = 0$$

$$e^{-m}.m^2\left[\frac{90.m^4}{6!} + \frac{9m^2}{4!} - \frac{1}{2!}\right] = 0$$

$$e^{-m}.m^2\left[\frac{90.m^4}{720} + \frac{9m^2}{24} - \frac{1}{2}\right] = 0$$

$$\frac{e^{-m}.m^2}{2}\left[\frac{90.m^4}{360} + \frac{9m^2}{12} - 1\right] = 0$$

$$\frac{e^{-m}.m^2}{2}\left[\frac{m^4}{4} + \frac{3m^2}{4} - 1\right] = 0$$

$$\frac{e^{-m}.m^2}{2}\left[\frac{m^4 + 3m^2 - 4}{4}\right] = 0$$

$$\frac{e^{-m}.m^2}{8}(m^4 + 3m^2 - 4) = 0$$

$m^4 + 3m^2 - 4 = 0$

$m^4 + 4m^2 - m^2 - 4 = 0$

$m^2(m^2 + 4) - 1(m^2 + 4) = 0$

$(m^2 + 4)(m^2 - 1) = 0$

if $m^2 + 4 = 0$ if $m^2 - 1 = 0$

$m^2 = -4$ if $m^2 = +1$

$m^2 = \pm\sqrt{1}$

$m^2 = \pm 1$

$m = (\because m > 0)$

$P(x \leq 1) = P(x = 0) + P(x = 1)$

$= \frac{e^{-1}.1^0}{0!} + \frac{e^{-1}.1^1}{1!} = \frac{1}{e} + \frac{1}{e} = \frac{2}{e}$

$\frac{2}{2.7182} = 0.7358$

[61] What is the first quartile of x having the following probability density function?

$f(x) = \frac{1}{\sqrt{72\pi}} e^{-(x-10)^2/72}$ for $-\infty < x < \infty$

(a) 4 (b) 5

(c) 5.95 (d) 6.75 (1 mark)

Answer:

(c) Given $f(x) = \frac{1}{\sqrt{72\pi}} \cdot e^{\frac{-(x-10)^2}{72}}$ for $-\infty < x < \infty$

$f(x) = \frac{1}{6\sqrt{2\pi}} \cdot e^{\frac{-1(x-\mu)^2}{72}}$

on company

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-1(x-\mu)^2}{2\sigma^{2}}}$

We get

$\sigma = 6, \mu = 10$

First quartile $Q_1 = \mu - 0.675\,\sigma$

$= 10 - 0.675 \times 6$

$= 10 - 4.05$

$= 5.95$

[62] An example of a bi-parametric discrete probability distribution is

(a) binomial distribution (b) Poisson distribution

(c) normal distribution (d) both (a) and (b) (1 mark)

Answer:

(a) Binomial distribution is example of a bi-parametric discrete probability distribution.

[63] Probability distribution may be

(a) discrete (b) continuous

(c) infinite (d) (a) or (b) (1 mark)

Answer:

(d) Probability distribution may be discrete or continuous .

[64] If the area of standard normal curve between z = 0 to z = 1 is 0.3412, then the value of ϕ (1) is.

(a) 0.5000 (b) 0.8413

(c) – 0.5000 (d) 1 (1 mark)

Answer:

(b) The area of standard normal curve between z = 0 to z = 1 is 0.3413 then

$$\phi(1) = 0.3413 + 0.5 = 0.8413$$

2018 - NOVEMBER

[65] For a Poisson variate X, P(X = 2) = 3P (X = 4), then the standard deviation of X is

(a) 2 (b) 4

(c) $\sqrt{2}$ (d) 3 (1 mark)

Answer:

(c) For a Poisson Variate X,

P(X = 2) = 3P (X = 4),

$$\frac{e^{-m}m^2}{2!} = \frac{3e^{-m}m^4}{4!}$$

$$\frac{m^2}{2} = \frac{3m^4}{24}$$

$$6\,m^4 = 24m^2$$

$$m^2 = \frac{24}{6}$$

$$m^2 = 4$$

$$m = 2$$

$$S.D = \sqrt{m} = \sqrt{2}$$

[66] The mean of the Binomial distribution $B\left(4,\frac{1}{3}\right)$ is equal to

(a) $\frac{3}{5}$ (b) $\frac{8}{3}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$ (1 mark)

Answer:

(d) $X_4\ B(n, P) = B(4, \frac{1}{3})$

We get n = 4, P = 1/3

Mean (μ) = np

= 4 × 1/3 = 4/3

[67] If for a normal distribution $Q_1 = 54.52$ and $Q_3 = 78.86$, then the median of the distribution is

(a) 12.17 (b) 39.43

(c) 66.69 (d) None of these (1 mark)

Answer:

(c) For a Normal Distribution

$Q_1 = 54.52$ and $Q_3 = 78.86$

We known that

$Q_1 = \mu - 0.675 = 54.52$ ______(1)

$Q_3 = \mu - 0.675 = 78.86$ ______(2)

On Adding ______________________

$2\mu = 133.38$

$\mu = \frac{133.38}{2}$

$\mu = 66.69$

In Normal Distribution Mean, Median and Mode are equal.

So, Median = Mean = 66.69

[68] What is the mean of X having the following density function?

$$f(x) = \frac{1}{4\sqrt{2\pi}} . e^{\frac{-(x-10)^2}{32}} \quad \text{for} - \infty < x < \infty$$

(a) 10 (b) 4

(c) 40 (d) None of the above (1 mark)

Answer:

(a) Given Normal distribution

$$f(x) = \frac{1}{4\sqrt{2\pi}} \, e^{\frac{-(x-10)^2}{32}} \text{ for } -\infty < x < \infty$$

On comparing from

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

We get:

Mean (μ) = 10

[69] The probability that a student is not a swimmer is $\frac{1}{5}$, then the probability that out of five students four are swimmer is

(a) $\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$

(b) ${}^5C_1\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)$

(c) ${}^5C_4\left(\frac{4}{5}\right)^1\left(\frac{1}{5}\right)^4$

(d) None of the above

(1 mark)

Answer:

(d) Given:

Probability that a student is not a swimmer (q) = $\frac{1}{5}$

Probability that a student is a swimmer (p) = $1 - q = 1 - \frac{1}{5} = \frac{4}{5}$

Total No. of students (n) = 5

P(Exactly 4 students are swimmer)

$= P(x = 4) = {}^5C_4 \cdot \left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)^1$

$\{\because P(x = n) = {}^nc_x \cdot p^x \cdot q^{n-x}\}$ So, ans. (d)

2019 - JUNE

[70] 4 coins were tossed 1600 times. What is the probability that all 4 coins do not turn head upward at a time?

(a) $1600\, e^{-100}$

(b) $1000\, e^{-100}$

(c) $100\, e^{-1600}$

(d) e^{-100}

(1 mark)

Answer:

(d) Probability of getting a head in a throw of a coin = $\frac{1}{2}$

Probability of getting 4 heads in a throw of four coins = $\frac{1}{2^4} = \frac{1}{16}$

Here, n $= 1600$

Mean $= m = np$

$= 1600 \times \frac{1}{16}$

$= 100$

P(No. Head) $= P\ (X\ 20)$

$= \frac{e^{-100}.(100)^0}{01}$

$= \frac{e^{-100}.1}{1}$

$= e^{-100}$

[71] If mean and variance are 5 and 3 respectively then relation between p and q is:

(a) $p > q$

(b) $p < q$

(c) $p = q$

(d) p is symmetric

(1 mark)

Answer:

(b) Mean = 5, Variance = 3

np = 5 ...(1), npq = 3 ...(2)

eq(2)/eq(1)

$$\frac{npq}{np} = \frac{3}{5}$$

q = 3/5

p = 1-q

= 1- 3/5 = 2/5

Here, p < q

[72] In a Poisson distribution if P(x = 4) = P(x = 5) then the parameter of Poisson distribution is:

(a) $\frac{4}{5}$

(b) $\frac{5}{4}$

(c) 4

(d) 5 (1 mark)

Answer:

(d) In Poisson distribution

P (x = 4) = P (x = 5)

$$\frac{e^{-m} \cdot m^4}{4!} = \frac{e^{-m} \cdot m^5}{5!}$$

$$\frac{1}{4!} = \frac{m}{5!}$$

$$\frac{1}{24} = \frac{m}{120}$$

24m = 120

m = 5

[73] Area between = 1.96 to + 1.96 in a normal distribution is:

(a) 95.45%

(b) 95%

(c) 96%

(d) 99% (1 mark)

Answer:

(b) Area between - 1.96 to +1.96 in a Normal distribution is 95%.

[74] If the points of inflexion of a normal curve are 40 and 60 respectively, then its mean deviation is:
(a) 8
(b) 45
(c) 50
(d) 60 (1 mark)

Answer:

(a) If the point of Inflexion of a Normal Distribution are 40 and 60. Then

$\mu - \sigma = 40$ ——— (1)

$\mu + \sigma = 60$ ——— (2)

Solving eq. (1) and (2) we get

$\mu = 50, \quad \sigma = 10$

Then $M.O = \frac{4}{5}$ S.D

$= \frac{4}{5} \times 10$

$= 8$

2019 - NOVEMBER

[75] Area under U ± 3σ
(a) 99.73%
(b) 99%
(c) 100%
(d) 99.37% (1 mark)

Answer:

(a) We know that 99.73 per cent of the values of a normal variable lies between (u - 3σ) and (u + 3σ)

Thus probability that a value of x lies. Outside the limit is as low as (100 - 99.73) = 0.27%.

[76] For a Poisson distribution:
(a) mean and SD are equal
(b) mean and variance are equal
(c) SD and Variance
(d) both a and b (1 mark)

Answer:

(b) Poisson distribution is theoretical discrete probability distribution which can describe many processes
Mean is given by m i.e, U = m
Variance is also given by m i.e. $\sigma^2 = m$
So in pass on distribution mean and variance are equal.

[77] Find mode when n = 15 and $p = \frac{1}{4}$ in binomial distribution?

(a) 4
(b) 4 and 3
(c) 4.2
(d) 3.75 (1 mark)

Answer:

(b) In binomial distribution,

$$m = (n + 1)\,p$$

$$m = (15 + 1) \times \frac{1}{4}$$

$$m = 4$$

Since 4 is a integer so there will 2 modes
4 and (4 - 1)
Mode = 4 and 3

[78] In Poisson distribution, if $P(x = 2) = \frac{1}{2} P(x = 3)$ find m?

(a) 3
(b) 1/6
(c) 6
(d) 1/3 (1 mark)

Answer:

(c) In Poisson distribution $P(x = x) = \dfrac{e^{-m}.m^2}{x!}$

Here $P(x = 2) = \dfrac{1}{2} P(x = 3)$

$$\frac{e^{-m}.m^2}{2!} = \frac{1}{2} \times \frac{e^{-m}.m^3}{3!}$$

$$\frac{e^{-m}.m^2}{2} = \frac{1}{2} \frac{xe^{-m}.m^3}{2\times6}$$

$m = 6$

[79] In a binomial distribution B(n, p)

n = 4 P(x = 2) = 3x P (x = 3) find P

(a) 1/3

(b) 2/3

(c) 6/4

(d) 4/3

(1 mark)

Answer:

(a) n = 4

we know $P(x = r) = {}^nC_r (p)^r (q)^{n-r}$

here $p(x = 2) = 3 \times P(x = 3)$

$${}^4c_2 (p)^2 (q)^{4-2} = 3 \times {}^4c_3 (p)^3 (q)^1$$

$$\frac{4!}{(4-2)\,1\times2!} (p)^2 (1-p)^2 = 3 \times \frac{4!}{(4-3)\,1\times3!} \times (p)^3 (1 - p)$$

Since ${}^nc_r = \dfrac{n!}{(n \div r)!\,1\times r!}$

$6 \times (1 - p) = 3 \times 4\,p$

$6 - 6p = 12\,p$

$18\,p = 6$

$p = \dfrac{1}{3}$

[80] What is the SD and mean

x if $f(x) = \frac{\sqrt{2}}{\sqrt{\pi}} e^{-2(x-3)^2}$, $-\infty < x < \infty$.

(a) 3, $\frac{1}{2}$

(b) 3, $\frac{1}{4}$

(c) 2, $\frac{1}{2}$

(d) 2, $\sqrt{2}$

(1 mark)

Answer:

(a) The standard form of probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}} \text{ -------------------(1)}$$

Here, $\sqrt{\frac{2}{\pi}} \cdot e^{-2(x-3)2}$

$= \sqrt{\frac{2}{\pi}} \cdot e^{-\left(\frac{1-3}{1/2}\right)^2}$

on comparing with Equation ---------------(1)

$2\sigma^2 = \frac{1}{2}$ u = 3

$\sigma^2 = \frac{1}{4}$

$\sigma = \frac{1}{2}$

So SD = $\frac{1}{2}$, mean = 3

2020 - November

[81] Which of the following is uni-parametric distribution?

(a) Poisson

(b) Normal

(c) Binomial

(d) Hyper geometric (1 mark)

Answer:

(a) Poisson distribution is uni parametric distribution.

[82] If the probability of success in a binomial distribution is less than one-half, then the binomial distribution ______.

(a) is skewed to left

(b) is skewed to right

(c) has two modes

(d) has median at a point > mean + 1/2 (1 mark)

Answer:

(a) is skewed to left.

[83] If we change the parameter(s) of a ______ distribution the sharpe of probability curve does not change.

(a) Normal

(b) Binomial

(c) Poisson

(d) Non-Gaussion (1 mark)

Answer:

(a) Normal Distribution.

[84] Which one of the following has Poisson distribution?

(a) The number of days to get a complete cure.

(b) The number of defects per meter on long roll of coated polythene sheet.

(c) The errors obtained in repeated measuring of the length of a rod.

(d) The number of claims rejected by an insurance agency. (1 mark)

Answer:

(b) The Number of defects per meter on long roil of coated polythene sheet is the example of Poisson distribution.

[85] Far a Poisson distributed variable X, we have P(X = 7) = 8 P(X = 9), the mean of the distribution is:

(a) 3
(b) 4
(c) 7
(d) 9

(1 mark)

Answer:

(a) If $p(x = 7) = 8\,p(x = 9)$

$$\frac{e^{-m}.m^7}{7!} = 8.\frac{e^{-m}.m^9}{9!}$$

$$\frac{9!}{8\times 7!} = \frac{m^9}{m^7}$$

$$\frac{9\times 8\times 7!}{8\times 7!} = m^2$$

$$m^2 = 9$$

$$m = 3$$

mean of Poisson distribution

$= m = 3$

[86] The quartile deviation of a normal distribution with mean 10 and standard deviation 4 is ________.

(a) 54.24
(b) 23.20
(c) 0.275
(d) 2.70

(1 mark)

Answer:

(d) For Normal Distribution

Given Mean (μ) = 10

S.D. (σ) = 4

Quartile Deviation

$$Q.D. = 0.675\,\sigma$$

$$= 0.675 \times 4$$

$$= 2.70$$

[87] If the parameter of Poisson distribution is m and (Mean + S. D.) = 6/25 then find m:

(a) 3/25
(b) 1/25
(c) 4/25
(d) 3/5 (1 mark)

Answer:

(b) In Poisson distribution

Mean = m

S.D. $= \sqrt{m}$

Given Mean + S.D. $= \frac{6}{25}$

$m + \sqrt{m} = \frac{6}{25}$ ________ (1)

By Hits and Trial

option (b) satisfied the eq. (1)

Here, $m = \frac{1}{25}$ putting in eq. (1)

L.H.S. $= \frac{1}{25} + \sqrt{\frac{1}{25}} = \frac{1}{25} + \frac{1}{5} = \frac{6}{25}$ = R.H.S.

So, option **(b)** is correct. (1 mark)

2021 - JANUARY

[88] A coin with probability for head as $\frac{1}{5}$ is tossed 100 times. The standard deviation of the number of head 5 turned up is.

(a) 3
(b) 2
(c) 4
(d) 6 (1 mark)

Answer:

(c) Here n = 100

Probability of success (p) $= \frac{1}{5}$

Probability of failure (q) $= 1 - p$

$= 1 - \frac{1}{5}$

$= \frac{4}{5}$

S.D. $= \sqrt{npq}$

$= \sqrt{100 \times \frac{1}{5} \times \frac{4}{5}}$

$= \sqrt{4 \times 4}$

$= 4$

[89] If x is a Poisson variable and P (x = 1) = P (x = 2), then P (x = 4) is

(a) $\frac{2}{3}e^{-2}$

(b) $\frac{2}{3}e^{4}$

(c) $\frac{3}{2}e^{-2}$

(d) $\frac{3}{2}e^{4}$ (1 mark)

Answer:

(a) If $X \sim P(m)$

and $P(x=1) = P(x=2)$

$\frac{e^{-m}.m^1}{1!} = \frac{e^{-m}.m^2}{2!}$

$\frac{m}{1} = \frac{m^2}{2}$

$2m = m^2$

$\boxed{2 = m}$

$m = 2$

$$P(x = x) = \frac{e^{-m}.m^x}{x!}$$

$$P(x = 4) = \frac{e^{-2}.2^4}{4!}$$

$$= \frac{e^{-2}.16}{24}$$

$$= \frac{2e^{-2}}{3}$$

[90] Which one of the following is an uniparametric distribution?
(a) Poisson
(b) Normal
(c) Binomial
(d) Hyper geometric (1 mark)

Answer:

(a) Poisson distribution is an uniparametric distribution.

[91] For a normal distribution, the value of third moment about mean is.
(a) 0
(b) 1
(c) 2
(d) 3 (1 mark)

Answer:

(a) For a Normal Distribution the value of third moment about mean is Zero.

2021 - JULY

[92] In normal distribution, Mean, Median and Mode are:
(a) Zero
(b) Not Equal
(c) Equal
(d) Null (1 mark)

Answer:

(c) In Normal Distribution, Mean, Median and Mode are equal.

[93] It is Poisson variate such that P(x = 1) = 0.7, P (x=2) = 0.3, then P(x = 0) =

(a) $e^{6/7}$

(b) $e^{-6/7}$

(c) $e^{-2/3}$

(d) $e^{-1/3}$

(1 mark)

Answer:

(b) In a poisson variate

$p(x = 1) = 0.7$ and $p(x = 2) = 0.3$

$\frac{e^{-m}.m^1}{1!} = 0.7$ $\qquad$ $\frac{e^{-m}.m^2}{2!} = 0.3$

$e^{-m}.m = 0.7$ ——— (1) $\qquad$ $e^{-m}.m^2 = 0.3 \times 2!$

$e^{-m}.m^2 = 0.3 \times 2$

$e^{-m}.m^2 = 0.6$ ——— (2)

eq (1)/ eq (2)

$\frac{e^{-m}.m}{e^{-m}.m^2} = \frac{0.7}{0.6}$

$\frac{1}{m} = \frac{7}{6}$

$m = 6/7$

Now, $p(x = 0) = \frac{e^{-m}.m^0}{0!}$

$= \frac{e^{-6/7}.1}{1}$

$= e^{-6/7}$

[94] Which of the following diagram is the most appropriate to represents various heads in total cost?

(a) Pie chart

(b) Bar graph

(c) Multiple Line chart

(d) Scatter Plot

(1 mark)

Answer:

(a) Pie chart is the most appropriate to represents various heads in total cost.

[95] If x is a binomial variate with P=1/3, for the experiment of 90 trials, then the standard deviation is equal to:

(a) $-\sqrt{5}$

(b) $\sqrt{5}$

(c) $2\sqrt{5}$

(d) $\sqrt{15}$ (1 mark)

Answer:

(c) P if x ~ B (n, p)

Here n = 90, p = 1/3, q = 1 – p

$$= 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

$$S.D = \sqrt{npq}$$

$$= \sqrt{90 \times \frac{1}{3} \times \frac{2}{3}}$$

$$= \sqrt{20}$$

$$S.D = 2\sqrt{5}$$

[96] For a certain type of mobile, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. A person owns one of these mobiles and want to know the probability that the length of time will be between 50 and 70 hours is (given φ (1.33)) = 0.9082 , φ (0) = 0.5)?

(a) - 0.4082

(b) 0.5

(c) 0.4082

(d) -0.5 (1 mark)

Answer:

(c) Here mean (μ) = 50 hours

S- D (σ) = 15 hours

$$P(50 < x < 70) = P\left(\frac{50-50}{15} < \frac{x-\mu}{\sigma} < \frac{70-50}{15}\right)$$
$$= P(0 < z < 1.33)$$
$$= \varphi(1.33) - \varphi(0)$$
$$= 0.9082 - 0.5000$$
$$= 0.4082$$

2021 - DECEMBER

[97] The average number of advertisements per page appearing in a newspaper is 3. What is the probability that in a particular page zero number of advertisements are there?

(a) e^{-3}
(b) e^{0}
(c) e^{+3}
(d) e^{-1}

(1 mark)

Answer:

(a) Given m = 3; x = 0

As per Poisson Distribution, $P(x) = \frac{e^{-m}m^{x}}{x!}$

$P(x = 0) = \frac{e^{-3}3^{0}}{0!} = e^{-3}$

[98] Four unbiased coins are tossed simultaneously. The expected number of heads is:

(a) 1
(b) 2
(c) 3
(d) 4

(1 mark)

Answer:

(b) Since four coins are being tossed, we have n = 4.
Probability of getting a "heads" in each trial (p) = ½
Expected number of Heads = np = 4 × ½ = 2.

[99] If, for a Poisson distributed random variable X, the probability for X taking value 2 is 3 times the probability for X taking value 4, then the variance of X is

(a) 4
(b) 3
(c) 2
(d) 5 (1 mark)

Answer:

(c) In Poisson Distribution, $P(x) = \frac{e^{-m}m^x}{x!}$

$$P(x = 2) = 3P(x = 4)$$

$$\frac{e^{-m}m^2}{2!} = 3 \times \frac{e^{-m}m^4}{4!}$$

$$\frac{1}{2} = \frac{3m^2}{24}$$

$$\frac{6m^2}{24} = 1$$

$$m^2 = \frac{24}{6} = 4$$

$$m = \sqrt{4} = 2$$

[100] Let X be normal distribution with mean 2.5 and variance 1. If P[a < X < 2.5] = 0.4772 and that the cumulative normal probability value at 2 is 0.9772, then a =?

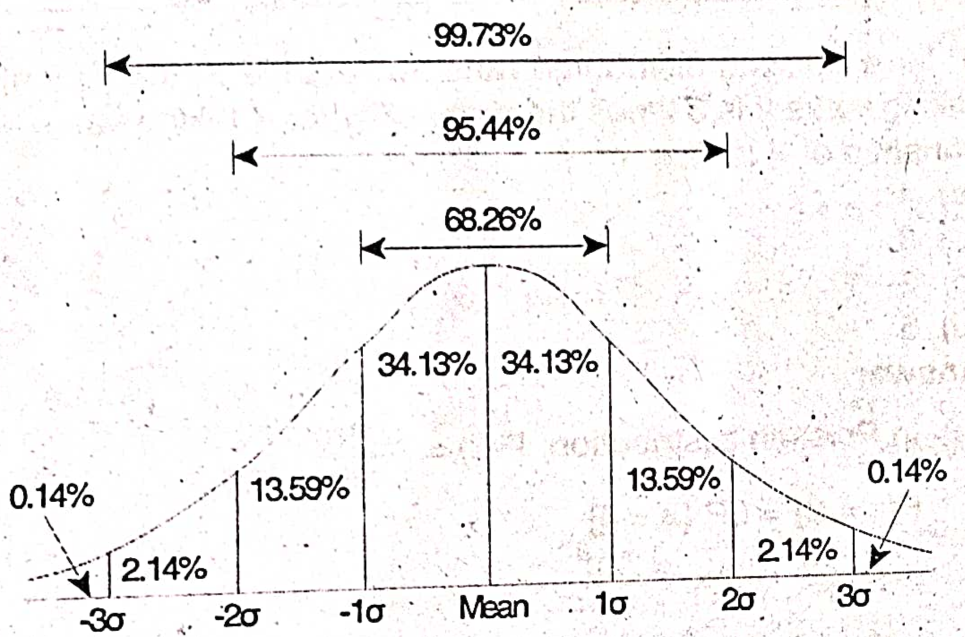
(a) 0.5
(b) 3
(c) -3.5
(d) -4.5 (1 mark)

Answer:

(a) We know that for a standard normal deviate, $z = \frac{x-\mu}{\sigma}$

Therefore, for x = 2.5, $z = \frac{2.5-2.5}{1} = 0$

Therefore, we need the area of 0.4772 from the mean till a certain point on the left-hand side.



From the graph above, we can see that the area from mean till -2σ is 47.72%, i.e., 0.4772.

Thus, the corresponding z for the value of x = a should be –2.

Therefore, $-2 = \frac{a - 2.5}{1}$

$$-2 = a - 2.5$$
$$2.5 - 2 = a$$
$$a = 0.5$$

[101] The manufacturer of a certain electronic component is certain that 2% of his product is defective. He sells the components in boxes of 120 and guarantees that not more than 2% in any box will be defective. Find the probability that a box, selected at random would fail to meet the guarantee? (Given that $e^{-2.4} = 0.0907$)

(a) 0.49
(b) 0.39
(c) 0.37
(d) 0.43

(1 mark)

Answer:

(d) Here, n = 120; p = $\frac{2}{100}$ = 0.02

m = np = 120 × 0.02 = 2.40

As per Poisson Distribution, $P(x) = \frac{e^{-m}.m^x}{x!}$

A box, selected at random would fail to meet the guarantee if more than 2.40 components turn out to be defective.

$P(x > 2.40) = 1 - P(x \leq 2.40)$

$P(x > 2.40) = 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$

$$P(x > 2.40) = 1 - \left[\frac{e^{-240}.(2.40)^0}{0!} + \frac{e^{-240}.(2.40)^1}{1!} + \frac{e^{-240}.(2.40)^2}{2!}\right]$$

$$P(x > 2.40) = 1 - \left[\frac{0.0907 \times 1}{1} + \frac{0.0907 \times 2.40}{1} + \frac{0.0907 \times (2.40)^2}{2}\right]$$

$P(x > 2.40) \approx 0.43$

[102] A renowned hospital usually admits 200 patients everyday. One percent patients, on an average, require special room facilities. On one particular morning, it was found that only one special room is available. What is the probability that more than 3 patients would require special room facilities?

(a) 0.1428
(b) 0.1732
(c) 0.2235
(d) 0.3450

(1 mark)

Answer:

(a) Here n = 200; p = $\frac{1}{100}$

Therefore, m = np = 200 × $\frac{1}{100}$ = 2

As per Poisson Distribution, $P(x) = \frac{e^{-m}.m^x}{x!}$

$P(x > 3) = 1 - P(x \leq 3)$

$P(x > 3) = 1 - [P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)]$

$$P(x > 3) = 1 - \left[\frac{e^{-2} \times 2^0}{0!} + \frac{e^{-2} \times 2^1}{1!} + \frac{e^{-2} \times 2^2}{2!} + \frac{e^{-2} \times 2^3}{3!}\right]$$

$$P(x>3)=1-\left[\frac{(271828)^{-2}\times 2^0}{0!}+\frac{(271828)^{-2}\times 2^1}{1!}+\frac{(271828)^{-2}\times 2^2}{2!}+\frac{(271828)^{-2}\times 2^3}{3!}\right]$$

$$P(x>3)=1-\left[\frac{1}{(2.71828)^2}+\frac{2}{(2.71828)^2}+\frac{4}{2\times(2.71828)^2}+\frac{8}{6\times(2.71828)^2}\right]$$

$$P(x>3)=1-\left[\frac{1}{(2.71828)^2}\left\{1+2+\frac{4}{2}+\frac{8}{6}\right\}\right]$$

$$P(x>3)=1-[0.8571]=0.1428$$

2022 - June

[103] If Standard Deviation is 1.732 then what is the value of poisson distribution. The P [- 2.48 < x < 3.54] is

(a) 0.73
(b) 0.65
(c) 0.86
(d) 0.81 (1 mark)

Answer:

(b) Given S.D = 1.732

$S.D = \sqrt{3}$

In Poisson distribution

$S.D = \sqrt{m}$

$\sqrt{3} = \sqrt{m}$

$\boxed{m=3}$

$P(-2.48 < n < 3.54)$

$= P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$$=\frac{e^{-3}.3^0}{0!}+\frac{e^{-3}.3^1}{1!}+\frac{e^{-3}.3^2}{2!}+\frac{e^{-3}.3^3}{3!}$$

$$=e^{-3}\left[\frac{1}{0!}+\frac{3}{1!}+\frac{9}{2!}+\frac{27}{3!}\right]$$

$$=e^{-3}\left[1+3+\frac{9}{2}+\frac{27}{6}\right]$$

$$= \frac{1}{e^3}[1+3+4.5+4.5]$$

$$= \frac{1}{(2.72)^3} \times 13$$

$$= \frac{13}{(2.72)^3} = \frac{13}{20.12} = 0.6461 = 0.65$$

[104] In a normal distribution, variance is 16 then the value of mean deviation is.

(a) 4.2 (b) 3.2

(c) 4.5 (d) 2.5 (1 mark)

Answer:

(b) Variance = 16 (In Normal Distribution)

$$S.D = \sqrt{16} = 4$$

$$M.D = 0.8\ S.D$$
$$= 0.8 \times 4$$
$$= 3.2$$

[105] For a binomial distribution, there may be –

(a) One mode (b) Two mode

(c) Multi mode (d) No mode (1 mark)

Answer:

(c) For a binomial distribution, there may be multimode

2022 - DECEMBER

[106] Skewness of Normal Distribution is:

(a) Negative (b) Positive

(c) Zero (d) Undefined (1 mark)

Answer:

(c) Skewness of Normal Distribution is zero because Normal Curve is Symmetrical.

[107] If a Poisson distribution is such that P(X = 2) = P(X = 3) then the variance of the distribution is:

(a) $\sqrt{3}$ (b) 3

(c) 6 (d) 9 (1 mark)

Answer:

(b) In Poisson distribution

$$P(x = 2) = P(x = 3)$$

$$\frac{e^{-m}.m^2}{2!} = \frac{e^{-m}.m^3}{3!}$$

$$\Rightarrow \frac{m^2}{2} = \frac{m^3}{6}$$

$$\Rightarrow 2m = 6$$

$$\Rightarrow m = 3$$

So Variance = m = 3

[108] The Standard Deviation of Binomial distribution is:

(a) npq (b) $\sqrt{npq}$

(c) np (d) $\sqrt{np}$ (1 mark)

Answer:

(b) S.D of Binomial Distribution = $\sqrt{npq}$

[109] The speeds of a number of bikes follow a normal distribution model with a mean of 83 km/hr and a standard deviation of 9.4 km. /hr. Find the probability that a bike picked at random is travelling at more than 95 km/hr.? Given [P (Z > 1.28) = 0.1003]

(a) 0.1003 (b) 0.38

(c) 0.49 (d) 0.278 (1 mark)

Answer:

(a) Mean (M) = 83 , S.D (σ) = 9.4

$$P(x > 95) = P\left(\frac{x-M}{\sigma} > \frac{95-83}{9.4}\right)$$

$$= P(Z > 1.28)$$

$$= 0.1003$$