100 Most Important MCQs in Mathematics

Question 1

If A : B = 3 : 5, B : C = 5 : 4, C : D = 2 : 3, and D is 50% more than E, find the ratio between A and E.

(a) 2 : 3 (b) 3 : 4 (c) 3 : 5 (d) 4 : 5

Solution

(b)

Let E be 1. Then, D will be 1.5. Therefore, D : E = 1.5 : 1, or, 3 : 2.

Thus, we have $\frac{A}{B} = \frac{3}{5}; \frac{B}{C} = \frac{5}{4}; \frac{C}{D} = \frac{2}{3}; \frac{D}{E} = \frac{3}{2}$

Therefore, $\frac{A}{E}=\frac{A}{B}\times\frac{B}{C}\times\frac{C}{D}\times\frac{D}{E}=\frac{3}{5}\times\frac{5}{4}\times\frac{2}{3}\times\frac{3}{2}=0.75$

Question 2

Find the value of $\sqrt{6561}+\sqrt[4]{6561}+\sqrt[8]{6561}$

(a) 81 (b) 93 (c) 121 (d) 243

Solution

(b)

$\sqrt{6561}+\sqrt[4]{6561}+\sqrt[8]{6561}=93$

Question 3

If $\frac{8^n \times 2^3 \times 16^{-1}}{2^n \times 4^2} = \frac{1}{4}$, then the value of n

(a) 1 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

Solution

(c)

$$\frac{8^n \times 2^3 \times 16^{-1}}{2^n \times 4^2} = \frac{1}{4}$$

$$\Rightarrow \frac{8^n \times 2^3}{2^n \times 4^2 \times 16} = \frac{1}{4}$$

$$\Rightarrow \frac{\left(2^3\right)^n \times 2^3}{2^n \times \left(2^2\right)^2 \times 2^4} = \frac{1}{4}$$

$$\Rightarrow \frac{2^{3n} \times 2^3}{2^n \times 2^4 \times 2^4} = \frac{1}{4}$$

$$\Rightarrow \frac{2^{3n+3}}{2^{n+4+4}} = \frac{1}{4}$$

$$\Rightarrow 2^{3n+3-n-4-4} = \frac{1}{4}$$

$$\Rightarrow 2^{2n-5} = \frac{1}{4}$$

We know that $\frac{1}{4}$ can be written as 2^{-2}.

Therefore, $2^{2n-5} = 2^{-2}$

Since the bases are same, powers can be equated.

Therefore, $2n - 5 = -2$

$$\Rightarrow 2n = 5 - 2$$

$$\Rightarrow 2n = 3$$

$$\Rightarrow n = \frac{3}{2}$$

Question 4

If $\log_{10} 5 + \log_{10}(5x+1) = \log_{10}(x+5)+1$, then x is equal to:

(a) 1 (b) 3 (c) 5 (d) 10

Solution

(b)

$\log_{10} 5 + \log_{10}(5x+1) = \log_{10}(x+5)+1$

$\Rightarrow \log_{10} 5 + \log_{10}(5x+1) = \log_{10}(x+5) + \log_{10} 10$

$\Rightarrow \log_{10}\{5\times(5x+1)\} = \log_{10}\{(x+5)\times 10\}$ (As $\log a + \log b = \log ab$)

Taking Anti-log on both sides, we'll get:

$$5(5x+1)=10(x+5)$$

$$\Rightarrow 5x+1=2(x+5)$$

$$\Rightarrow 5x+1=2x+10$$

$$\Rightarrow 5x-2x=10-1$$

$$\Rightarrow 3x=9$$

$$\Rightarrow x=\frac{9}{3}=3$$

Question 5

If $xy + yz + zx = -1$, then the value of $\left(\frac{x+y}{1+xy} + \frac{z+y}{1+zy} + \frac{x+z}{1+zx}\right)$ is:

(a) xyz (b) $-\frac{1}{yz}$ (c) $\frac{1}{xyz}$ (d) $\frac{1}{x+y+z}$

Solution

(c)

Given $xy + yz + zx = -1$

This means $1 + xy = -yz - zx \ldots$Eq. (1)

$$1+yz=-xy-zx \ldots \text{Eq. (2)}$$

$$1+zx=-xy-yz \ldots \text{Eq. (3)}$$

$$\frac{x+y}{1+xy}+\frac{z+y}{1+zy}+\frac{x+z}{1+zx}$$

Substituting the values of $1+xy$, $1+zy$, and $1+zx$ above from Eqs. (1), (2), and (3), we get:

$$\frac{x+y}{-yz-zx}+\frac{z+y}{-xy-zx}+\frac{x+z}{-xy-yz}$$

$$\Rightarrow \frac{x+y}{-z(y+x)}+\frac{z+y}{-x(y+z)}+\frac{x+z}{-y(x+z)}$$

$$\Rightarrow \frac{-1}{z} + \frac{-1}{x} + \frac{-1}{y}$$

$$\Rightarrow -\left(\frac{1}{z} + \frac{1}{x} + \frac{1}{y}\right)$$

$$\Rightarrow -\left(\frac{xy + yz + zx}{xyz}\right)$$

$$\Rightarrow -\left(\frac{-1}{xyz}\right)$$

$$\Rightarrow \frac{1}{xyz}$$

Question 6

The salaries of *A*, *B* and *C* are in the ratio 2 : 3 : 5. If increments of 15%, 10% and 20% are allowed respectively to their salary, then what will be the new ratio of their salaries?

(a) 23 : 33 : 60 (b) 33 : 23 : 60 (c) 23 : 60 : 33 (d) 33 : 60 : 23

Solution

(a)

Since the ratio of the salaries of *A*, *B* and *C* is 2:3:5, let the salaries be 200, 300, and 500 respectively.

A's new salary = 200 + (15% of 200) = 230

B's new salary = 300 + (10% of 300) = 330

C's new salary = 500 + (20% of 500) = 600

Therefore, clearly, the new ratio is 23:33:60.

Question 7

X and *Y* have their present ages in the ratio 6 : 7. 14 years ago, the ratio of the ages of the two was 4 : 5. What will be the ratio of their ages 21 years from now?

(a) 7 : 11 (b) 9 : 10 (c) 8 : 11 (d) 11 : 13

Solution

(b)

Let the ages of *X* and *Y* be $6x$ and $7x$ respectively.

14 years, ago, their ages would have been $(6x-14)$, and $(7x-14)$.

It is given that the ratio of their ages 14 years ago was 4 : 5.

Therefore, $\frac{(6x-14)}{(7x-14)}=\frac{4}{5}$

$\Rightarrow 5(6x-14)=4(7x-14)$

$\Rightarrow 30x-70=28x-56$

$\Rightarrow 30x-28x=70-56$

$\Rightarrow 2x=14$

$\Rightarrow x=\frac{14}{2}=7$

Therefore, the present ages are 6 × 7 = 42, and 7 × 7 = 49 respectively.

Their ages after 21 years will be 42 + 21 = 63, and 49 + 21 = 70 respectively.

Therefore, the ratio of their ages after 21 years will be 63 : 70 = 0.9.

Now, try the options.

Option (a) → 7 : 11 = 0.6363

Option (b) → 9 : 10 = 0.9

Therefore, option (b) is the answer.

Question 8

If $x=\sqrt{3}+\frac{1}{\sqrt{3}}$, then $\left(x-\frac{\sqrt{126}}{\sqrt{42}}\right)\left(x-\frac{1}{x-\frac{2\sqrt{3}}{3}}\right)=?$

(a) 5/6 (b) 6/5 (c) 2/3 (d) –3/5

Solution

(a)

$$x=\sqrt{3}+\frac{1}{\sqrt{3}}=2.3094$$

$$x-\frac{\sqrt{126}}{\sqrt{42}}=2.3094-1.7321=0.5773$$

$$x-\frac{1}{x-\frac{2\sqrt{3}}{3}}=2.3094-\frac{1}{2.3094-1.1547}=1.4434$$

$$\left(x-\frac{\sqrt{126}}{\sqrt{42}}\right)\left(x-\frac{1}{x-\frac{2\sqrt{3}}{3}}\right)=0.5773\times1.4434=0.8333$$

Now, try the options.

Option (a) → 5/6 = 0.8333

Therefore, option (a) is the answer.

Question 9

If $\log_a(ab) = x$, then $\log_b(ab) = ?$

(a) $1/x$ (b) $\frac{x}{1+x}$ (c) $\frac{x}{x-1}$ (d) None

Solution

(c)

$\log_a(ab) = x$

$\log_a a + \log_a b = x \quad [\text{As } \log m + \log n = \log mn]$

$1 + \log_a b = x$

$\log_a b = x - 1 \ldots$Eq. (1)

We know that $\log_a b \times \log_b a = 1$

Putting the value of $\log_a b$ from eq. (1), we get:

$$(x-1)\times \log_b a = 1$$

$$\log_b a = \frac{1}{x-1}$$

$$\log_a (ab) = \frac{\log_b (ab)}{\log_b a}$$ [As per Base Change Formula]

$$\log_b (ab) = \log_a (ab) \times \log_b a$$

$$\log_b(ab) = x \times \left(\frac{1}{x-1}\right) \quad \left[\text{As } \log_b(ab) = x \text{ and } \log_b a = \frac{1}{x-1}\right]$$

$$\log_a(ab) = \frac{x}{x-1}$$

Question 10

A vessel contained a solution of acid and water in which water was 64%. Four litres of the solution were taken out of the vessel and the same quantity of water was added. If the resulting solution contains 30% acid, the quantity (in litres) of the solution, in the beginning in the vessel, was:

(a) 12 (b) 36 (c) 24 (d) 27

Solution

(c)

Let the initial total volume be V.

Water = $0.64V$; Acid = $0.36V$

Now, 4 litres were taken out.

Remaining Water = $0.64V - (0.64 \times 4) = 0.64V - 2.56$

Remaining Acid = $0.36V - (0.36 \times 4) = 0.36V - 1.44$

To the above, 4 litres of water was added. Therefore, the total volume of the vessel would be V – 4 litres + 4 litres = V.

Now, it is given that this resulting solution contains 30% of acid.

Therefore, $\frac{0.36V - 1.44}{V} = 0.30$

$\Rightarrow 0.36V - 1.44 = 0.30V$

$\Rightarrow 0.36V - 0.30V = 1.44$

$\Rightarrow 0.06V = 1.44$

$$\Rightarrow V = \frac{1.44}{0.06} = 24$$

Question 11

If $\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$, then the value of x is:

(a) 64 (b) 4 (c) 16 (d) 2

Solution

(c)

$$\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$$

$$\Rightarrow \log_{2^2} x + \log_{2^4} x + \log_{2^6} x + \log_{2^8} x = \frac{25}{6}$$

$$\Rightarrow \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x + \frac{1}{6}\log_2 x + \frac{1}{8}\log_2 x = \frac{25}{6}$$

$$\Rightarrow \log_2 x\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right) = \frac{25}{6}$$

$$\Rightarrow \log_2 x\left(\frac{12+6+4+3}{24}\right) = \frac{25}{6}$$

$$\Rightarrow \log_2 x\left(\frac{25}{24}\right) = \frac{25}{6}$$

$$\Rightarrow \log_2 x = \frac{25}{6} \times \frac{24}{25}$$

$$\Rightarrow \log_2 x = 4$$

$$\Rightarrow x = 2^4 = 16$$

Question 12

If $x^2 + y^2 = 7xy$, then $\log \frac{1}{3}(x+y) = ?$

(a) $(\log x + \log y)$ (b) $\frac{1}{2}(\log x + \log y)$ (c) $\frac{1}{3}(\log x + \log y)$ (d) $3(\log / \log y)$

Solution

(b)

$x^2 + y^2 = 7xy$

$\Rightarrow x^2 + y^2 + 2xy - 2xy = 7xy$

$$\Rightarrow (x+y)^2 - 2xy = 7xy$$

$$\Rightarrow (x+y)^2 = 7xy + 2xy$$

$$\Rightarrow (x+y)^2 = 9xy$$

$$\Rightarrow x+y = \sqrt{9xy}$$

$$\Rightarrow x+y = 3\sqrt{xy}$$

$$\Rightarrow x+y = 3(xy)^{\frac{1}{2}}$$

We have to find the value of $\log_{\frac{1}{3}}(x+y)$

$$\Rightarrow \log \frac{1}{3} \times 3(xy)^{\frac{1}{2}}$$

$$\Rightarrow \log (xy)^{\frac{1}{2}}$$

$$\Rightarrow \frac{1}{2}(\log xy)$$

$$\Rightarrow \frac{1}{2}(\log x + \log y)$$

Question 13

The value of $\dfrac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$ is:

(a) ½ (b) 3/2 (c) 2/3 (d) 2

Solution

(b)

Since none of the options contains *n*, we can safely assume that *n* is going to get cancelled out.

For the sake of simplicity, let's assume the value of *n* to be 1.

$$\frac{2^1+2^{1-1}}{2^{1+1}-2^1}=\frac{2+2^0}{2^2-2}=\frac{2+1}{4-2}=\frac{3}{2}$$

Question 14

If $3^x = 5^y = 75^z$, then:

(a) $x + y - z = 0$ (b) $\frac{2}{x} + \frac{1}{y} = \frac{1}{z}$ (c) $\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$ (d) $\frac{2}{x} + \frac{1}{z} = \frac{1}{y}$

Solution

(c)

Question 15

A bag contains ₹187 in the form 1 rupee, 50 paise and 10 paise coins in the ratio 3:4:5. Find the number of each type of coins.

(a) 102, 136, 170 (b) 136, 102, 170 (c) 170, 102, 136 (d) None

(a)

Let the number of 1 rupee, 50 paise, and 10 paise coins be $3x$, $4x$ and $5x$ respectively.

Value of 1 rupee coins = $3x$ × ₹1 = ₹$3x$

Value of 50 paise coins = $4x$ × ₹0.50 = ₹$2x$

Value of 10 paise coins = $5x$ × ₹0.10 = ₹$0.50x$

Total value = $3x+2x+0.5x=5.5x$

We know that the total value is ₹187.

Therefore, $5.5x = 187$

$$\Rightarrow x = \frac{187}{5.5} = 34$$

Therefore, number of ₹1 coins = 3 × 34 = 102

Number of 50 paise coins = 4 × 34 = 136

Number of 10 paise coins = 5 × 34 = 170

Question 16

$\log_e x + \log(1+x) = 0$ is equivalent to:

(a) $x^2 + x + e = 0$ (b) $x^2 + x - e = 0$ (c) $x^2 + x + 1 = 0$ (d) $x^2 + x - 1 = 0$

(d)

In this question, the base of $\log(1+x)$ will be taken as *e*, as the base of the term log *x* is also *e*.

Therefore, the given equation can be written as:

$$\log_e x + \log_e(1+x) = 0$$

$$\Rightarrow \log_e x + \log_e(1+x) = \log_e 1$$

Also, we know that $\log a + \log b = \log ab$

Therefore, $\log_e x + \log_e (1+x) = \log_e 1$ can be written as:

$$\log_e \{x(1+x)\} = \log_e 1$$

$$\Rightarrow \log_e \{x + x^2\} = \log_e 1$$

$$\Rightarrow x + x^2 = 1$$

$$\Rightarrow x^2 + x - 1 = 0$$

Question 17

If $x = 3^{1/4} + 3^{-1/4}$, and $y = 3^{1/4} - 3^{-1/4}$, then the value of $3(x^2 + y^2)^2$ will be:

(a) 12 (b) 18 (c) 46 (d) 64

(d)

On calculator, press 3 → √ → √ → M+ → 1 → ÷ → MRC MRC = M+

Press 3 → √ → √ → + → MRC MRC = → × → =. This gives you the value of $x^2 = 4.3094$.

Press 3 → √ → √ → M+ → 1 → ÷ → MRC MRC = M+

Press 3 $\rightarrow \sqrt{} \rightarrow \sqrt{} \rightarrow - \rightarrow$ MRC MRC $= \rightarrow \times \rightarrow =$. This gives you the value of $y^2 = 0.3094$.

Press $4.3094 + 0.3094 = \times = \times 3 = 63.9999 \approx 64$

Question 18

Find the value of $(x+y)$, if $\left(x+\frac{y^3}{x^2}\right)^{-1}-\left(\frac{x^2}{y}+\frac{y^2}{x}\right)^{-1}+\left(\frac{x^3}{y^2}+y\right)^{-1}=\frac{1}{3}$.

(a) 1/3 (b) 3 (c) ½ (d) 2

(b)

$$\left(x+\frac{y^3}{x^2}\right)^{-1}-\left(\frac{x^2}{y}+\frac{y^2}{x}\right)^{-1}+\left(\frac{x^3}{y^2}+y\right)^{-1}=\frac{1}{3}$$

$$\left(\frac{x^3+y^3}{x^2}\right)^{-1}-\left(\frac{x^3+y^3}{xy}\right)^{-1}+\left(\frac{x^3+y^3}{y^2}\right)^{-1}=\frac{1}{3}$$

$$\left(\frac{x^2}{x^3+y^3}\right)-\left(\frac{xy}{x^3+y^3}\right)+\left(\frac{y^2}{x^3+y^3}\right)=\frac{1}{3}$$

$$\frac{x^2-xy+y^2}{x^3+y^3}=\frac{1}{3}$$

$$\frac{x^2-xy+y^2}{(x+y)\left(x^2-xy+y^2\right)}=\frac{1}{3}$$

$$\frac{1}{x+y}=\frac{1}{3}$$

$$3=x+y$$

$$x+y=3$$

Question 19 – Ambiguous

If $pqr = a^x$, $qrs = a^y$, $rsp = a^z$, then find the value of $(pqrs)^{1/2}$.

(a) a^{x+y+z} (b) $a^{\sqrt{x+y+z}}$ (c) $a^{\sqrt[4]{x+y+z}}$ (d) $\left(a^{x+y+z}\right)^{1/4}$

(d)

$pqr = a^x$

$qrs = a^y$

$rsp = a^z$

Multiplying these equations, we have

$$(pqr)\times(qrs)\times(rsp)=a^x\times a^y\times a^z$$

$$p^2q^2r^3s^2=a^{x+y+z}$$

$$p^2q^2r.r^2s^2=a^{x+y+z}$$

$$r\left(p^2q^2r^2s^2\right)=a^{x+y+z}$$

$$r(pqrs)^2=a^{x+y+z}$$

Now, ICAI has simply ignored this additional *r* outside the bracket on the left-hand side. So, we'll also do the same. Therefore, we'll have:

$$(pqrs)^2=a^{x+y+z}$$

Taking fourth root on both sides of the equation, we have:

$$\left\{\left(pqrs\right)^{2}\right\}^{1/4}=\left(a^{x+y+z}\right)^{1/4}$$

$$\left(pqrs\right)^{2\times\frac{1}{4}}=\left(a^{x+y+z}\right)^{1/4}$$

$$\left(pqrs\right)^{1/2}=\left(a^{x+y+z}\right)^{1/4}$$

Question 20

If $x = 5^{1/3} + 5^{-1/3}$, then $5x^3 - 15x$ is given by:

(a) 25 (b) 26 (c) 27 (d) 30

Solution

(b)

Question 21

One student is asked to divide a half of a number by 6 and other half by 4 and then to add the two quantities. Instead of doing so, the student divides the given number by 5. If the answer is 4 short of the correct answer, then the number was:

(a) 320 (b) 400 (c) 480 (d) None

Solution

(c)

Question 22

The cab bill is partly fixed and partly varies on the distance covered. For 456 km, the bill is ₹8252, for 484 km the bill is ₹8728. What will the bill be for 500 km?

(a) ₹8876 (b) ₹9156 (c) ₹9472 (d) ₹9000

Solution

(d)

Variable Cost per unit = $\frac{8728-8252}{484-456}=17$

Therefore, Fixed Cost = 8252 – (17 × 456) = 8252 – 7752 = ₹500

The bill for 500 km will be ₹500 + (500 × ₹17 p.u.) = ₹500 + ₹8,500 = ₹9,000

Alternatively,

Let the fixed cost be x and variable cost per unit be y.

For 456 km, we have $x + 456y = 8252$…Eq. (1)

For 484 km, we have $x + 484y = 8728$…Eq. (2)

Subtracting Eq. (1) from Eq. (2), we have:

$x - x + 484y - 456y = 8728 - 8252$

$\Rightarrow y(484 - 456) = 476$

$\Rightarrow y \times 28 = 476$

$\Rightarrow y = \frac{476}{28} = 17$

Putting this value in Eq. (1), we have:

$$x+(456\times 17)=8252$$

$$\Rightarrow x=8252-(456\times 17)=8252-7752=500$$

Therefore, for 500 km, the bill will be $x+500y=500+(500\times 17)=500+8500=9000$

Question 23

The value of k for the system of equations $kx+2y=5$ and $3x+y=1$ has no solution is:

(a) 5 (b) 2/3 (c) 6 (d) 3/2

Solution

(c)

$kx+2y=5$...Eq. (1)

$3x+y=1$...Eq. (2)

Multiplying Eq. (2) with 2, we'll get:

$6x+2y=2$...Eq. (3)

Subtracting Eq. (3) from Eq. (1), we'll get:

$$kx - 6x + 2y - 2y = 5 - 2$$

$$\Rightarrow kx - 6x = 3$$

$$\Rightarrow x(k - 6) = 3$$

$$\Rightarrow x = \frac{3}{k - 6}$$

Now, clearly, if k takes the value 6, then denominator becomes zero, and x becomes not defined, and hence the system of equations won't have any solution.

Question 24

Examine the nature of roots of the following equation: $5x^2 - 4x + 2 = 0$.

(a) Real and Equal
(b) Real and Unequal
(c) Imaginary and Unequal
(d) Real, Rational, Unequal

Solution

(c)

$$5x^2 - 4x + 2 = 0$$

$$a = 5;\ b = -4;\ c = 2$$

$$b^2 - 4ac = (-4)^2 - (4)(5)(2) = -24$$

Since $D < 0$, the roots are imaginary and unequal.

Question 25

If α and β be the roots of $x^2 + 7x + 12 = 0$, find the equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$.

(a) $x^2+50x+49=0$

(b) $x^2-24x+144=0$

(c) $x^2-50x+49=0$

(d) $x^2-19x+49=0$

Solution

(c)

$x^2+7x+12=0$

Here, $a = 1$; $b = 7$; $c = 12$

$$\alpha+\beta=-\frac{b}{a}=-\frac{7}{1}=-7$$

$$\alpha\beta=\frac{c}{a}=\frac{12}{1}=12$$

As per the fastest method,

$$\left(\frac{-7}{2}+x\right)\left(\frac{-7}{2}-x\right)=12$$

$$\left(\frac{-7}{2}\right)^2-x^2=12$$

$$x^2=\frac{49}{4}-12=\frac{49-48}{4}=\frac{1}{4}$$

$$x = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Therefore, $\alpha = \frac{-7}{2} + \frac{1}{2} = -\frac{6}{2} = -3$

$$\beta = \frac{-7}{2} - \frac{1}{2} = -\frac{8}{2} = -4$$

The roots of the new equation will be:

$(\alpha + \beta)^2 = (-3 - 4)^2 = 49$, and

$$(\alpha - \beta)^2 = \{-3 - (-4)\}^2 = 1$$

When roots of the equation are known, the equation is given by:

$$x^2 - (Sum\,of\ Roots)x + Product\,of\ Roots = 0$$

Therefore, the equation will be $x^2 - (49+1)x + (49 \times 1) = 0$

$$\Rightarrow x^2 - 50x + 49 = 0$$

Question 26

If α, β are the two roots of the equation $x^2 + px + q = 0$, form the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

(a) $qx^2 - \left(p^2 - 2q\right)x + q = 0$

(b) $px^2 - \left(p^2 - 2q\right)x + q = 0$

(c) $qx^2 - \left(p^2 - 2q\right)x + p = 0$

(d) $qx^2 + \left(p^2 - 2q\right)x + p = 0$

Solution

(a)

$$x^2 + px + q = 0$$

$$\Rightarrow \alpha + \beta = -\frac{b}{a} = -\frac{p}{1} = -p \text{, and}$$

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

We need an equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

Quadratic equation is given by: $x^2 - (Sum\ of\ Roots)x + Product\ of\ Roots = 0$

Therefore,

$$x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \left(\frac{\alpha}{\beta} \times \frac{\beta}{\alpha}\right) = 0$$

$$\Rightarrow x^2-\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)x+1=0$$

$$\Rightarrow x^2-\left(\frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha\beta}\right)x+1=0$$

$$\Rightarrow x^2-\left\{\frac{\left(\alpha^2+\beta^2+2\alpha\beta\right)-2\alpha\beta}{\alpha\beta}\right\}x+1=0$$

$$\Rightarrow x^2-\left\{\frac{\left(\alpha+\beta\right)^2-2\alpha\beta}{\alpha\beta}\right\}x+1=0$$

$$\Rightarrow x^2 - \left\{\frac{(-p)^2 - (2q)}{q}\right\}x + 1 = 0$$

$$\Rightarrow x^2 - \left\{\frac{p^2 - 2q}{q}\right\}x + 1 = 0$$

Multiplying the entire equation with q, we get:

$$qx^2 - q\left\{\frac{p^2 - 2q}{q}\right\}x + q = 0$$

$$\Rightarrow qx^2 - \left(p^2 - 2q\right)x + q = 0$$

Question 27

If α and β are the roots of $x^2 = x + 1$, then the value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ is:

(a) $2\sqrt{5}$ (b) $\sqrt{5}$
(c) $3\sqrt{5}$ (d) $-2\sqrt{5}$

Solution

(d)

As per the fastest method:

$$\left(\frac{1}{2} + x\right)\left(\frac{1}{2} - x\right) = -1$$

$$\left(\frac{1}{2}\right)^2 - x^2 = -1$$

$$\frac{1}{4} + 1 = x^2$$

$$x^2 = \frac{5}{4}$$

$$x = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

$$\alpha = \frac{1}{2} + \frac{\sqrt{5}}{2} = \frac{1+\sqrt{5}}{2}$$

$$\beta = \frac{1}{2} - \frac{\sqrt{5}}{2} = \frac{1-\sqrt{5}}{2}$$

$$\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha} = \left[\left\{\left(\frac{1+\sqrt{5}}{2}\right)^2\right\} \div \left(\frac{1-\sqrt{5}}{2}\right)\right] - \left[\left\{\left(\frac{1-\sqrt{5}}{2}\right)^2\right\} \div \left(\frac{1+\sqrt{5}}{2}\right)\right]$$

$$\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha} = -4.2361 - 0.2361 = -4.4722 = -2\sqrt{5}$$

Question 28

The value of $4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\ldots\infty}}}$ is:

(a) $1\pm\sqrt{2}$ (b) $2+\sqrt{5}$ (c) $2\pm\sqrt{5}$ (d) None

Solution

(b)

Let $4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\ldots\infty}}}=x$

$$x=4+\frac{1}{x}$$

$$x=\frac{4x+1}{x}$$

$$x^2=4x+1$$

$$x^2-4x-1=0$$

$$a=1;\ b=-4;\ c=-1$$

$$\alpha+\beta=-\frac{b}{a}=-\frac{-4}{1}=4$$

$$\alpha\beta=\frac{c}{a}=\frac{-1}{1}=-1$$

$$\left(\frac{4}{2}+x\right)\left(\frac{4}{2}-x\right)=-1$$

$$(2)^2-x^2=-1$$

$$x^2=4+1=5$$

$$x=\sqrt{5}$$

$$\alpha = \frac{4}{2} + \sqrt{5} = 2 + \sqrt{5} = 4.23$$

$$\beta = \frac{4}{2} - \sqrt{5} = 2 - \sqrt{5} = -0.24$$

Clearly, the answer cannot be negative. Therefore, option (b) is the answer.

Question 29

If p and q are the roots of the $x^2+2x+1=0$, then the values of p^3+q^3 becomes:

(a) 2 (b) –2 (c) 4 (d) –4

Solution

(b)

$x^2+2x+1=0$

$a=1;\ b=2;\ c=1$

$$p+q=-\frac{b}{a}=-\frac{2}{1}=-2$$

$$pq = \frac{c}{a} = \frac{1}{1} = 1$$

Therefore, obviously $p = -1$ and $q = -1$

$$p^3 + q^3 = (-1)^3 + (-1)^3 = -2$$

Alternatively,

We know that $(p+q)^3 = p^3 + q^3 + 3pq(p+q)$

$$\Rightarrow p^3 + q^3 = (p+q)^3 - 3pq(p+q)$$

$$p^3 + q^3 = (-2)^3 - (3)(1)(-2) = -2$$

Question 30

The harmonic mean of the roots of the equation $\left(5+\sqrt{2}\right)x^2-\left(4+\sqrt{5}\right)x+8+2\sqrt{5}=0$ is:

(a) 2 (b) 4 (c) 6 (d) 8

Solution

(b)

$$\left(5+\sqrt{2}\right)x^2-\left(4+\sqrt{5}\right)x+8+2\sqrt{5}=0$$

Here,

$$a=5+\sqrt{2};\ b=-\left(4+\sqrt{5}\right);\ c=8+2\sqrt{5}$$

Therefore, $\alpha + \beta = -\frac{b}{a} = -\frac{-\left(4+\sqrt{5}\right)}{5+\sqrt{2}} = 0.9722$

$$\alpha\beta = \frac{c}{a} = \frac{8+2\sqrt{5}}{5+\sqrt{2}} = 1.9444$$

$$HM = \frac{2\alpha\beta}{\alpha+\beta} = \frac{2\times 1.9444}{0.9722} = 4$$

Question 31

On the average, an experienced person does 7 units of work while a fresh one work 5 units of work daily but the employer has to maintain an output of at least 35 units of work per day. The situation can be expressed as:

(a) $7x + 5y < 35$ (b) $7x + 5y \leq 35$ (c) $7x + 5y > 35$ (d) $7x + 5y \geq 35$

Solution

(d)

Question 32

Mr. A plans to invest up to ₹30,000 in two stocks *X* and *Y*. Stock *X* (x) is priced at ₹175 and Stock *Y* (y) at ₹95 per share. This can be shown by:

(a) $175x + 95y < 30{,}000$ (b) $175x + 95y > 30{,}000$ (c) $175x + 95y = 30{,}000$ (d) None

Solution

(a)

Question 33

The rules and representations demand that employer should employ not more than 8 experienced leads to 1 fresh one and this fact can be expressed as:

(a) $y \geq x/8$ (b) $8y \leq x$ (c) $8y = x$ (d) $y = 8x$

Solution

(a)

Question 34

The common region in the graph of the inequalities $x+y\le 4,\ x-y\le 4,\ x\ge 2$ is

(a) Equilateral triangle
(b) Isosceles triangle
(c) Quadrilateral
(d) Square

Solution

(b)

Inequalities graph for

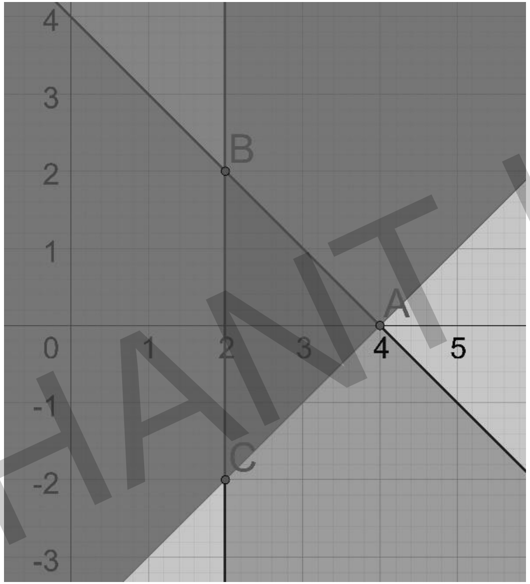
$x+y\le 4,$

x	0	4
y	4	0

$x-y\le 4$

x	0	4
y	–4	0

$x\ge 2$

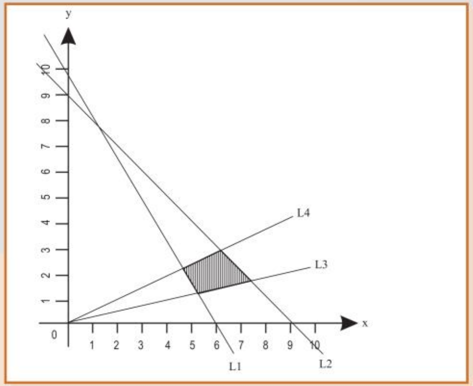


Common Area in the graph is Δ ABC

Clearly, it is an isosceles triangle.

Question 35

(viii)



L1 : 5x + 3y = 30 L2 : x+y = 9 L3 : y = x/3 L4 : y = x/2

The common region (shaded part) shown in the diagram refers to

(a)	(b)	(c)	(d)	(e)
$5x + 3y \le 30$	$5x + 3y \ge 30$	$5x + 3y \ge 30$	$5x + 3y > 30$	None of these
$x + y \le 9$	$x + y \le 9$	$x + y \ge 9$	$x + y < 9$	
$y \le 1/5\ x$	$y \ge x/3$	$y \le x/3$	$y \ge 9$	
$y \le x/2$	$y \le x/2$	$y \ge x/2$	$y \le x/2$	
	$x \ge 0, y \ge 0$	$x \ge 0, y \ge 0$	$x \ge 0, y \ge 0$	

Question 36

₹8,000 becomes ₹10,000 in 1 year 8 months at simple interest. The amount that will become ₹6,875 in 2 years 7 months at the same rate of interest is:

(a) ₹4,850 (b) ₹5,000 (c) ₹4,955 (d) ₹5,275

Solution

(c)

First, let's calculate rate of interest.

$$i=\frac{A-P}{Pt}=\frac{1000-8000}{8000\times\left(1+\frac{8}{12}\right)}=0.15$$

Now, let's calculate P.

$$P = \frac{A}{1+it} = \frac{6,875}{1+\left\{0.15\times\left(2+\frac{7}{12}\right)\right\}} = 4,955$$

Question 37

A sum of money gets doubled in 5 years at x% simple interest. If the interest was y%, the sum of money would have become ten-fold in thirty years. What is $y - x$ (in %)?

(a) 10 (b) 5 (c) 8 (d) None

Solution

(a)

Let's find out x first.

We have $P = 100$; $A = 200$; $t = 5$; $i = x/100$

$I = A - P = 200 - 100 = 10$

We know that $I = Pit$

Therefore, $100 = 100 \times \frac{x}{100} \times 5$

$$\Rightarrow \frac{100 \times 100}{100 \times 5} = x$$

$$\Rightarrow x = 20\%$$

Now, let's find out y.

$P = 100$; $A = 10 \times 100 = 1{,}000$; $t = 30$; $i = y/100$

$I = A - P = 1{,}000 - 100 = 900$

We know that $I = Pit$

Therefore, $900 = 100 \times \frac{y}{100} \times 30$

$$\Rightarrow \frac{900 \times 100}{100 \times 30} = y$$

$$\Rightarrow y = 30\%$$

Therefore, $y - x = 30\% - 20\% = 10\%$

Question 38

If a simple interest on a sum of money at 6% p.a. for 7 years is equal to twice of simple interest on another sum for 9 years at 5% p.a., the ratio will be:

(a) 2 : 15 (b) 7 : 15 (c) 15 : 7 (d) 1 : 7

Solution

(c)

$$P_1 \times 0.06 \times 7 = 2 \times P_2 \times 0.05 \times 9$$

$$\frac{P_1}{P_2} = \frac{2 \times 0.05 \times 9}{0.06 \times 7} = 2.1428$$

Question 39

A sum of money amounts to ₹20,800 in 5 years and ₹22,720 in 7 years. Find the principal and rate of interest.

(a) ₹5,000; 6% (b) ₹16,000; 6% (c) ₹80,000; 8% (d) ₹10,000; 10%

Solution

(b)

Since the question is silent about Simple Interest or Compound Interest, we'll try both. First, let's try Simple Interest.

$A = P + I$

Try the options:

Option (b) → 16,000; 6%

Amount after 5 years:

$A = 16{,}000 + (16{,}000 \times 0.06 \times 5) = ₹20{,}800$

Amount after 7 years:

$A = 16{,}000 + (16{,}000 \times 0.06 \times 7) = ₹22{,}720$

Therefore, option (b) is the answer.

Question 40

Two equal sums were lent out at 7% and 5% simple interest respectively. The interest earned on the two loans adds upto ₹960 for four years. Find the sum lent out.

(a) ₹4,000 (b) ₹3,000 (c) ₹5,000 (d) ₹6,000

Solution

(a)

Let the sum lent out be x each.

Interest from 7% = $x \times 0.07 \times 4 = 0.28x$

Interest from 5% = $x \times 0.05 \times 4 = 0.2x$

As per the question, $0.28x + 0.2x = 960 \Rightarrow 0.48x = 960 \Rightarrow x = 960 \div 0.48 =$ ₹2,000

Therefore, total sum lent out = ₹2,000 + ₹2,000 = ₹4,000

Question 41

₹100 will become after 20 years at 5% p.a. compound interest of:

(a) ₹250
(b) ₹205
(c) ₹165.33
(d) None of these

Solution

(c)

We have $P = 100; t = 20; i = 0.05; NOCPPY = 1$

$$A = P\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}$$

$$A = 100\left(1 + \frac{0.05}{1}\right)^{20\times1} = 265.33$$

As per the language of the question, this should be the answer.

However, ICAI has given the answer ₹165.33 (as on the date of writing this question).

This implies that ICAI wants us to calculate the interest.

You are advised to check the answer from ICAI Material right now. If the answer has changed, very good, you know how to solve it; if not, then memorize this question.

Question 42

On what sum will the compound interest at 5% p.a. for two years compounded annually be ₹1,640?

(a) ₹2,200 (b) ₹1,487.53 (c) ₹16,000 (d) None

Solution

(c)

$$CI = P\left[\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY} - 1\right]$$

$$P=\frac{CI}{\left[\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1\right]}=\frac{1,640}{\left[\left(1+\frac{0.05}{1}\right)^{2\times 1}-1\right]}=16,000$$

Question 43

What sum of money will amount to ₹11,035.50 in four years at compound interest for 1st, 2nd, 3rd, and 4th years being 4%, 3%, 2% and 1% respectively?

(a) ₹10,000 (b) ₹11,000 (c) ₹1,035 (d) ₹11,305

(a)

$$11,035.50 = P(1.04)(1.03)(1.02)(1.01)$$

$$P = \frac{11,035.50}{(1.04)(1.03)(1.02)(1.01)} = 9,999.997 \approx 10,000$$

Question 44

The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is:

(a) 7 years (b) 10 years (c) 17 years (approx.) (d) None

Solution

(c)

Question 45

The annual birth and death rates per 1,000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is:

(a) 35 years (b) 30 years (c) 25 years (d) None

Solution

(a)

Question 46

A = ₹5,200, R = 5% p.a., T = 6 years, P will be

(a) ₹2,000 (b) ₹3,880 (c) ₹3,000 (d) None

(b)

It is not mentioned in the question whether we have to use Simple Interest or Compound Interest. So, we'll try both.

First, let's try Simple Interest.

$$I = Pit$$

$$A = P + I$$

$$A = P + Pit$$

$$A = P(1 + it)$$

$$P = \frac{A}{1 + it}$$

$$P = \frac{A}{1 + it} = \frac{5,200}{1 + (0.05 \times 6)} = ₹4,000$$

Clearly, ₹4,000 is not present in any of the options. Now, don't just straightaway mark the option (d). Try with Compound Interest first.

$$A = P\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}$$

$$P = \frac{A}{\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}}$$

$$P = \frac{A}{\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}} = \frac{5,200}{\left(1 + \frac{0.05}{1}\right)^{6 \times 1}} = ₹3,880$$

Therefore, option (b) is the answer.

Question 47

A man borrows ₹4,000 from a bank at 10% compound interest. At the end of every year ₹1,500 as part of repayment of loan and interest. How much is still owed to the bank after three such instalments [Given: $(1.1)^3 = 1.331$]

(a) ₹359 (b) ₹820 (c) ₹724 (d) ₹720

(a)

Amount owed at the end of first year before payment of instalment = $4,000\left(1+\frac{0.10}{1}\right)^{1\times 1}=4,400$

From this, instalment of ₹1,500 is paid.

Therefore, amount owed at the end of first year after payment of instalment = ₹4,400 – ₹1,500 = ₹2,900

Now, amount owed at the end of the second year before payment of instalment = $2,900\left(1+\frac{0.10}{1}\right)^{1\times 1} = 3,190$

From this, instalment of ₹1,500 is paid.

Therefore, amount owed at the end of the second year after payment of instalment = ₹3,190 – ₹1,500 = ₹1,690

Now, amount owed at the end of the third year before payment of instalment = $1,690\left(1+\frac{0.10}{1}\right)^{1\times 1} = 1,859$

From this, instalment of ₹1,500 is paid.

Therefore, amount owed at the end of the third year after payment of instalment = ₹1,859 – ₹1,500 = ₹359

Therefore, amount owed after payment of the third instalment = ₹359.

Question 48

A Maruti Zen costs ₹3,60,000. Its price depreciates at the rate of 10% of a year during the first two years and at the rate of 20% in the third year. Find the total depreciation.

(a) ₹1,26,720 (b) ₹1,15,620 (c) ₹1,25,000 (d) ₹1,10,520

Solution

(a)

For the first two years, we have P = ₹3,60,000; $i = -0.10$; $t = 2$ years

$$\text{WDV at the end of 2 years} = 3,60,000\left(1+\frac{-0.10}{1}\right)^{2\times 1} = 2,91,600$$

For the third year, we have P = ₹2,91,600; $i = -0.20$; $t = 1$ year

WDV at the end of 3rd year = $2,91,600\left(1+\frac{-0.20}{1}\right)^{1\times1} = 2,33,280$

Therefore, total depreciation = ₹3,60,000 – ₹2,33,280 = ₹1,26,720

Question 49

The difference between the simple and compound interest on a certain sum for 3 years at 5% p.a. is ₹228.75. The compound interest on the sum for 2 years at 5% p.a. is:

(a) ₹3,175 (b) ₹3,075 (c) ₹3,275 (d) ₹2,975

Solution

(b)

Question 50

Which is a better investment 3% per year compounded monthly or 3.2% per year simple interest? Given that $(1 + 0.0025)^{12} = 1.0304$.

(a) Compound Interest (b) Simple Interest (c) Don't Know (d) None

Solution

(b)

Question 51

₹200 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10th payment? Given that $(1.005)^{10} = 1.0511$.

(a) ₹2,047 (b) ₹2,046 (c) ₹2,044 (d) ₹2,045

Solution

(c)

Question 52

A person invests ₹500 at the end of each year with a bank which pays interest at 10% p.a. C.I. annually. The amount standing to his credit one year after he has made his yearly investment for the 12^{th} time is:

(a) ₹11,761.36 (b) ₹10,000 (c) ₹12,000 (d) None

Solution

(a)

Question 53

How much money is to be invested every year so to accumulate ₹3,00,000 at the end of 10 years if interest is compounded annually at 10% [A(10, 0.1) = 15.9374]

(a) ₹18,823.65 (b) ₹18,833.64 (c) ₹18,223.60 (d) ₹16,823.65

Solution

(a)

Question 53 – Alternatively

How much amount is required to be invested every year so as to accumulate ₹3,00,000 at the end of 10 years, if interest is compounded annually at 10%?

{Given $(1.1)^{10} = 2.5937$}

(a) ₹18,823.65 (b) ₹18,828.65 (c) ₹18,832.65 (d) ₹18,182.65

Solution

(a)

Question 54

A machine costs ₹5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of ₹25,000. What amount should be set aside every year if the sinking fund investments accumulate at 3.5% compound interest p.a.?

(a) ₹16,000 (b) ₹16,500 (c) ₹16,050 (d) ₹16,005

Solution

(c)

Question 55

Raja aged 40 wishes his wife Rani to have ₹40 lakhs at his death. If his expectation of life is another 30 years and he starts making equal annual investments commencing now at 3% compound interest p.a. how much should he invest annually?

(a) ₹84,448 (b) ₹84,450 (c) ₹84,449 (d) ₹84,080

Solution

(d)

Question 56

Appu retires at 60 years receiving a pension of ₹14,400 a year paid in half-yearly installments for rest of his life after reckoning his life expectation to be 13 years and that interest at 4% p.a. is payable half-yearly. What single sum is equivalent to his pension?

(a) ₹1,45,000 (b) ₹1,44,900 (c) ₹1,44,800 (d) ₹1,44,700

Solution

(b)

Question 57

A took a loan from B. The loan is to be repaid in annual installments of ₹2,000 each. The first instalment is to be paid three years from today and the last one is to be paid 8 years from today? What is the value of loan today, using a discount rate of eight percent?

(a) ₹9,246 (b) ₹7,927 (c) ₹8,567 (d) None

Solution

(b)

The first instalment is to be paid at the end of 3^{rd} year, and the last instalment is to be paid at the end of 8^{th} year. Therefore, total number of instalments = 6.

If we calculate the present value of this annuity regular, we'll get the value at the end of 2nd year.

$$PVAR = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}\times\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]$$

$$\Rightarrow PVAR = 2,000\left[\frac{\left(1+\frac{0.08}{1}\right)^{6\times1}-1}{\left(\frac{0.08}{1}\right)\times\left(1+\frac{0.08}{1}\right)^{6\times1}}\right]=9,246$$

Now, this amount is standing at the end of 2nd year.

Let’s calculate the Present Value of this amount now:

$$P = \frac{A}{\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}}$$

$$\Rightarrow P = \frac{9,246}{\left(1 + \frac{0.08}{1}\right)^{2 \times 1}} = 7,927$$

Question 58

A man purchased a house valued at ₹3,00,000. He paid ₹2,00,000 at the time of purchase and agreed to pay the balance with interest at 12% per annum compounded half yearly in 20 equal half-yearly instalments. If the first instalment is paid after six months from the date of purchase then the amount of each instalment is:

(a) ₹8,718.45 (b) ₹8,769.21 (c) ₹7,893.13 (d) None

(a)

The value of the house at the time of purchase is ₹3,00,000. The man has paid ₹2,00,000 upfront, and ₹1,00,000 is pending. This is the present value of all the instalments that he is going to pay. We need to find out the amount of each instalment. Therefore, we have

PV = ₹1,00,000; i = 0.12; $NOCPPY$ = 2; t = 10 years (since there are 20 half yearly instalments); A = ?

$$PV = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\left(\frac{i}{NOCPPY}\right)\times\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]$$

$$A=\frac{PV}{\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\left(\frac{i}{NOCPPY}\right)\times\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]}$$

$$A = \frac{1,00,000}{\left[\frac{\left(1+\frac{0.12}{2}\right)^{10\times 2}-1}{\left(\frac{0.12}{2}\right)\times\left(1+\frac{0.12}{2}\right)^{10\times 2}}\right]}$$

$$A = \frac{1,00,000}{\frac{(1.06)^{20}-1}{0.06\times(1.06)^{20}}} = 8718.45$$

Question 59

Arun purchased a vacuum cleaner by giving ₹1700 as cash down payment, which will be followed by five EMIs of ₹480 each. The vacuum cleaner can also be bought by paying ₹3900 cash. What is the approx. rate of interest p.a. (at simple interest) under this instalment plan?

(a) 18% (b) 19% (c) 22% (d) 20%

Solution

(c)

Cash Down Price = ₹3,900

Down Payment = ₹1,700

Loan Amount = ₹3,900 – ₹1,700 = ₹2,200

Total amount paid in instalments = ₹480 × 5 = ₹2,400

Therefore, interest paid = ₹2,400 – ₹2,200 = ₹200

Now, P = ₹2,200; t = 5/12 years; A = ₹2,400; i = ?

$$i = \frac{A - P}{Pt} = \frac{2400 - 2200}{2200 \times \frac{5}{12}} = 0.21818 = 21.82\% \approx 22\%$$

Question 60

If the cost of capital be 12% per annum, then the Net Present Value (in nearest ₹) from the given cash flow is given as:

Year	0	1	2	3
Operating Profit (in thousand ₹)	(100)	60	40	50

(a) ₹34,048 (b) ₹34,185 (c) ₹51,048 (d) ₹21,048

Solution

(d)

$$\text{Present Value of Inflows} = \frac{60,000}{\left(1+\frac{0.12}{1}\right)^{1\times1}} + \frac{40,000}{\left(1+\frac{0.12}{1}\right)^{2\times1}} + \frac{50,000}{\left(1+\frac{0.12}{1}\right)^{3\times1}} = 1,21,048$$

Net Present Value = PV of Inflows – PV of Outflows

Net Present Value = ₹1,21,048 – ₹1,00,000 = ₹21,048

Question 61

An investor intends purchasing a three-year ₹1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

(a) ₹907.125 (b) ₹800.125 (c) ₹729.12 (d) None

Solution

(a)

Question 62

A stock pays annually an amount of ₹10 from 6th year onwards. What is the present value of the perpetuity, if the rate of return is 20%?

(a) 20.1 (b) 19.1 (c) 21.1 (d) 22.1

Solution

(a)

Since the stock starts paying annually from 6th year onwards, if we use the present value of perpetuity formula to find out the present value, it'll give us the value at the 5th year. Think about it logically. In all the questions on perpetuity that we've done so far, the amount was supposed to be received from the end of the first year, and then, when we calculated the present value, it gave us the value at the beginning of the first year. In similar lines, if the stock will start paying the interest from the end of the 6th year, and we use the same formula to calculate the present value, it'll give the present value of only one year before, i.e., at the end of the fifth year.

Let's first calculate that:

$$PV = \frac{A}{i / NOCPPY} = \frac{10}{0.20 / 1} = 50$$

Now, this ₹50 is the amount standing at the end of the 5th year. Since we are required to find out the present value, we need to discount it to the present. Again, think about it logically. This is the amount that is standing at the end of the 5th year. We need to find out the sum that we could invest right now so as to get this 50 at the end of the 5th year. Therefore, this 50 is the amount, and we need to find out the principal.

$$P = \frac{A}{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}$$

$$P = \frac{50}{\left(1+\frac{0.20}{1}\right)^{5\times 1}} = 20.09$$

Question 63

Assuming that the discount rate is 7% per annum, how much would you pay to receive ₹50, growing at 5%, annually, forever?

(a) ₹4,300 (b) ₹2,500 (c) ₹4,200 (d) None

Solution

(b)

Question 64

The nominal rate of growth is 17% and inflation is 9% for the five years. Let *P* be the Gross Domestic Product (GDP) amount at the present year, then the projected real GDP after 6 years is:

(a) 1.587*P* (b) 1.921*P* (c) 1.403*P* (d) 2.51*P*

Solution

(a)

Nominal Rate = Real Rate + Inflation Rate

17% = Real Rate + 9%

Real Rate = 17% – 9% = 8%

Present GDP = P

GDP after 6 years = $P(1.08)^6 = 1.5869P \approx 1.587P$

Question 65

Let the operating profit of a manufacturer for five years is given as:

Year	1	2	3	4	5	6
Operating Profit (in lakh ₹)	90	100	106.4	107.14	120.24	157.35

(a) 9% (b) 12% (c) 11% (d) 13%

Solution

(b)

Question me kuch diya toh hai nahi ki kya nikaalna hai…CAGR jaisa hi lag raha hai…wohi nikaal lete hain.

$P = 90$; $A = 157.35$; $t = 5$

$$A = P\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}$$

$$\Rightarrow 157.35 = 90\left(1 + \frac{i}{1}\right)^{5 \times 1}$$

$$\Rightarrow 157.35 = 90(1+i)^{5}$$

Now, try the options.

Option (a) $\rightarrow$ 9%

RHS $= 90(1+0.09)^{5} = 138.48$

Option (b) $\rightarrow$ 12%

$$\text{RHS} = 90(1+0.12)^5 = 158.61$$

Option (c) $\rightarrow$ 11%

$$\text{RHS} = 90(1+0.11)^5 = 151.66$$

Option (d) $\rightarrow$ 13%

$$\text{RHS} = 90(1+0.13)^5 = 165.82$$

Closest is option (b). Therefore, option (b) is the answer.

Question 66

In an election the number of candidates is one more than the number of members to be elected. If a voter can vote in 254 different ways; find the number of candidates.

(a) 8 (b) 10 (c) 7 (d) None

Solution

(a)

In an election the number of candidates is one more than the number of members to be elected. This means that if, suppose the total number of candidates is 11, then only 10 are to be selected. In other words, if, suppose the total number of candidates is n, then the number of candidates to be voted for are $n - 1$.

Given that a voter can vote in 254 different ways, it is clear that the voter can vote for one or more members.

Now, number of ways of selecting one or more items from a set of n items = $2^n - 1$. However, this would also consider the one extra candidate which cannot be voted for. Therefore, we need to subtract that one extra candidate as well.

$\Rightarrow 2^n - 1 - 1$

$= 2^n - 2$

Given that a voter can vote in 254 different ways.

$\Rightarrow 2^n - 2 = 254$

$\Rightarrow 2^n = 254 + 2$

$\Rightarrow 2^n = 256$

$$\Rightarrow 2^n = 2^8$$

$$\Rightarrow n = 8$$

Question 67

There are 12 questions to be answered in Yes or No. How many ways can these be answered?

(a) 1024 (b) 2048 (c) 4096 (d) None

Solution

(c)

Every question can be answered in 2 ways, i.e., Yes, or No.

Therefore, all the 12 questions can be answered in $2^{12} = 4096$ ways.

Question 68

n locks and n corresponding keys are available but the actual combination is not known. The maximum number of trials that are needed to assigns the keys to the corresponding locks is:

(a) ${}^{(n-1)}C_2$ (b) ${}^{(n+1)}C_2$ (c) $\sum_{k=2}^{n}(k-1)$ (d) $\sum_{k=2}^{n}k$

Solution

(c)

Assume that there are 3 locks and 3 corresponding keys.

The maximum number of trials to assign key to the first lock will be 2. This is because if the first two keys are incorrect, then obviously, the third key is the key for the first lock.

Similarly, the number of trials to assign key to the second lock will be 1.

This would automatically assign the third key to the third lock, so the number of trials to assign a key to the third lock will be 0.

Therefore, the maximum number of trials to assign keys to all three locks = 2 + 1 = 3.

Now, try the options:

Option (a) → $^{(n+1)}C_2 = {}^{(3-1)}C_2 = {}^{2}C_2 = 1$

Therefore, option (a) cannot be the answer.

Option (b) → $^{(n+1)}C_2 = {}^{(3+1)}C_2 = {}^{4}C_2 = \frac{4\times 3}{1\times 2} = 6$

Therefore, option (b) cannot be the answer.

Option (c) → $\sum_{k=2}^{n}(k-1)=(2-1)+(3-1)=1+2=3$

Therefore, option (c) is the answer.

Question 69

If 15 persons are to be seated around 2 round tables, one occupying 8 persons and another 7 persons. Find the number of ways in which they can be seated.

(a) $\frac{15!}{18!}$

(b) ${}^{15}C_7 \frac{7!}{8!}$

(c) 7!.8!

(d) $2.{}^{15}C_7\ 6!\ 7!$

Solution

(d)

No. of ways of selecting two Round Tables = 2C1 = 2

∴ No. of ways of selecting 7 persons = ${}^{15}C_7$

∴ No. of ways 7 persons to sit in a round table = (7 – 1)! = 6!

∴ Remaining Person = 15 – 7 = 8 person

∴ No. of ways 8 persons to sit on around table = (8 – 1)! = 7!

∴ Required total No. of ways = $2!\, x\, {}^{15}c_7 \times 6! \times 7! = 2x\, {}^{15}C_7 \times 6! \times 7!$

Question 70

The number of permutations of 10 different things taken 4 at a time in which one particular thing never occurs is:

(a) 3,020 (b) 3,025 (c) 3,024 (d) None

Solution

(c)

Question 71

Eight guests have to be seated 4 on each side of a long rectangular table. 2 particular guests desire to sit on one side of the table and 3 on the other side. The number of ways in which the sitting arrangements can be made is:

(a) 1732 (b) 1728 (c) 1730 (d) 1278

Solution

(b)

Question 72

The number of even numbers greater than 300 can be formed with the digits 1, 2, 3, 4, 5 without repetition is:

(a) 110 (b) 112 (c) 111 (d) None

Solution

(c)

Case I – 3 Digit-Numbers are formed

___ ___ ___

Since we need even numbers, the last place can be filled either with 2, or with 4.

Also, since the number has to be greater than 300, the first place cannot be filled with either 1 or 2. Therefore, the first place can be filled only with 3, or 4, or 5. Therefore, there are 3 ways of filling the first place.

Now, again, two cases arise. If the first place is filled with 4, then the last place can be filled with only 1 number, i.e., 2. In such a case, the middle position can be filled in 3 ways. Therefore, the different numbers formed in this case are:

$$\underline{\ 1\ } \times \underline{\ 3\ } \times \underline{\ 1\ } = 3$$

If the first place is filled either with 3, or 5, then the last place can be filled either with 2, or 4. In this case also, the middle position can be filled in 3 ways. Therefore, the different numbers formed in this case are:

$$\underline{\ 2\ } \times \underline{\ 3\ } \times \underline{\ 2\ } = 12$$

Therefore, the total number of 3-digit even numbers that can be formed are 3 + 12 = 15.

Case II – 4-Digit Numbers are formed

In this case, there is not restriction for the first position. The last place can be filled in 2 ways, i.e., either with 2, or with 4. The number of ways each place can be filled is demonstrated below:

$$\underline{\;2\;} \times \underline{\;3\;} \times \underline{\;4\;} \times \underline{\;2\;} = 48$$

Case III – 5-Digit Numbers are formed

In this case, there is not restriction for the first position. The last place can be filled in 2 ways, i.e., either with 2, or with 4. The number of ways each place can be filled is demonstrated below:

$$\underline{\;1\;} \times \underline{\;2\;} \times \underline{\;3\;} \times \underline{\;4\;} \times \underline{\;2\;} = 48$$

Therefore, total number of ways = 15 + 48 + 48 = 111

Question 73

The number of words that can be made by rearranging the letters of the word APURNA so that vowels and consonants appear alternate is:

(a) 18 (b) 35 (c) 36 (d) None

Solution

(c)

Question 74

The Supreme Court has given a 6 to 3 decision upholding a lower court; the number of ways it can give a majority decision reversing the lower court is:

(a) 256 (b) 276 (c) 245 (d) 226

Solution

(a)

"6 to 3 decision upholding a lower court" means that out of 9 members of the jury of the Supreme Court, 6 people have upheld the decision of the lower court (meaning thereby that they are agreeing with the decision of the lower court), and 3 people have voted against the decision of the lower court.

A majority decision reversing the lower court can be obtained if out of the 9 members of the jury, at least 5 vote against the decision of the lower court.

This can be done in ${}^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8 + {}^9C_9$ ways.

$${}^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8 + {}^9C_9 = {}^9C_4 + {}^9C_3 + {}^9C_2 + {}^9C_1 + {}^9C_9$$

$$= \frac{9\times8\times7\times6}{1\times2\times3\times4} + \frac{9\times8\times7}{1\times2\times3} + \frac{9\times8}{1\times2} + 9 + 1 = 256 \text{ ways}$$

Question 75

A committee of 7 members is to be chosen from 6 Chartered Accountants, 4 Economists and 5 Cost Accountants. In how many ways can this be done if in the committee, there must be at least one member from each group and at least 3 Chartered Accountants?

(a) 3,570 (b) 3,750 (c) 7,350 (d) None

Solution

(a)

Case	CAs (6)	Economists (4)	CMAs (5)	Calculation	Total
1.	3	3	1	${}^{6}C_{3} \times {}^{4}C_{3} \times {}^{5}C_{1}$	400
2.	3	2	2	${}^{6}C_{3} \times {}^{4}C_{2} \times {}^{5}C_{2}$	1200

3.	3	1	3	${}^6C_3 \times {}^4C_1 \times {}^5C_3$	800
4.	4	2	1	${}^6C_4 \times {}^4C_2 \times {}^5C_1$	450
5.	4	1	2	${}^6C_4 \times {}^4C_1 \times {}^5C_2$	600
6.	5	1	1	${}^6C_5 \times {}^4C_1 \times {}^5C_1$	120
					3,570

Question 76

A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics Part II unless Mathematics Part I is also borrowed. In how many ways can he choose the three books to be borrowed?

(a) 41 (b) 51 (c) 61 (d) 71

Solution

(a)

There could be two cases:

Case I → Mathematics Part II is borrowed, OR

Case II → Mathematics Part II is not borrowed.

Case I → Mathematics Part II is borrowed

If Mathematics Part II is borrowed, that means Mathematics Part I is already borrowed. So, now, the two books that he has already chosen are Mathematics Part I, and Mathematics Part II. The selection of these two books can be made in only 1 way. Now, the number books that he can choose from is 6. He has to choose 1 book from 6 books, and this can be done in 6 ways.

Case II → Mathematics Part II is not borrowed

If Mathematics Part II is not borrowed, this means that the boy has to select 3 books from 7 books, and this can be done in ${}^7C_3 = \dfrac{7\times6\times5}{1\times2\times3} = 35$ ways.

Therefore, total number of ways = 6 + 35 = 41

Question 77

The ways of selecting 4 letters from the word 'EXAMINATION' is

(a) 136 (b) 130 (c) 125 (d) None

Solution

(a)

Distinct Letters: E, X, A, M, I, N, T, O = 8 Letters

Alike Letters: (A, A), (I, I), (N, N) = 3 Groups of 2 Letters each

Case 1: All Distinct Letters are Selected

Question 78

The number of 4-digit numbers formed with the digits 1, 1, 2, 2, 3, 4 is:

(a) 100 (b) 101 (c) 201 (d) None

Solution

(d)

Question 79

A person borrows ₹8,000 at 2.76% Simple Interest per annum. The principal and the interest are to be paid in the 10 monthly instalments. If each instalment is double the preceding one, find the value of the first and the last instalment.

(a) 8; 4,095 (b) 2; 4,096 (c) 8; 4,096 (d) None

Solution

(c)

Total amount to be paid $= 8,000 + \left(8,000 \times 0.0276 \times \frac{10}{12}\right) = 8,184$

Since each instalment is to be double the preceding one, it is clearly a GP with r = 2.

Therefore, we have $n=10;\ r=2;\ S_{10}=8{,}184$

Since $r>1,\ S_n=a\left(\dfrac{r^n-1}{r-1}\right)$

$$a=\frac{S_n}{\left(\dfrac{r^n-1}{r-1}\right)}=\frac{8{,}184}{\left(\dfrac{2^{10}-1}{2-1}\right)}=8$$

Therefore, the first instalment is 8.

Now, let's calculate the last instalment.

$$t_{10}=ar^9=8\times 2^9=4{,}096$$

Question 80

Find the sum to n terms of the series 3 + 33 + 333 + 3333 + …

(a) $\frac{1}{27}\times\left(10^{n+1}-9n-10\right)$

(b) $\frac{1}{27}\times\left(10^{n+1}-9n+10\right)$

(c) $\frac{1}{27}\times\left(10^{n+1}+9n+10\right)$

(d) None

Solution

(a)

The sum of such type of series is given by $\frac{Number}{9}\times\left\{\frac{10\left(10^{n}-1\right)}{9}-n\right\}$

Therefore, sum of 3 + 33 + 333 + 3333 + … is given by: $\frac{3}{9}\times\left\{\frac{10\left(10^n-1\right)}{9}-n\right\}$

$$\Rightarrow \frac{3}{9}\times\left\{\frac{10\left(10^n-1\right)-9n}{9}\right\}$$

$$\Rightarrow \frac{3}{81}\times\left\{10\left(10^n-1\right)-9n\right\}$$

$$\Rightarrow \frac{1}{27}\times\left\{10\times10^n-10-9n\right\}$$

$$\Rightarrow \frac{1}{27}\times\left(10^{n+1}-10-9n\right)$$

$$\Rightarrow \frac{1}{27} \times \left(10^{n+1} - 9n - 10\right)$$

Question 81

Find the sum to n terms of 6 + 27 + 128 + 629 + …

(a) $\{5(5^n - 1)\} + \{n(n+1)\}$

(b) $\left\{\frac{5}{4}(5^n - 1)\right\} + \left\{\frac{n(n+1)}{2}\right\}$

(c) $\{5(5^n - 1)\} + \left\{\frac{n(n+1)}{2}\right\}$

(d) None

Solution

(b)

6 + 27 + 128 + 629 + …

$$\Rightarrow (5+1)+(25+2)+(125+3)+(625+4)+\ldots$$

$$\Rightarrow (5+25+125+625+\ldots)+(1+2+3+4+\ldots)$$

$$\Rightarrow \left(5+5^2+5^3+5^4+\ldots+5^n\right)+(1+2+3+4+\ldots+n)$$

The first bracket is a Geometric Progression with a = 5, and r = 5

$$\Rightarrow \left\{5\left(\frac{5^n-1}{5-1}\right)\right\}+\left\{\frac{n(n+1)}{2}\right\}$$

$$\Rightarrow \left\{5\left(\frac{5^n-1}{4}\right)\right\}+\left\{\frac{n(n+1)}{2}\right\}$$

$$\Rightarrow \left\{\frac{5}{4}\left(5^n - 1\right)\right\} + \left\{\frac{n\left(n+1\right)}{2}\right\}$$

Question 82

The series $1+10^{-1}+10^{-2}+10^{-3}...$ to ∞ is:

(a) 9/10 (b) 1/10 (c) 10/9 (d) None

Solution

(c)

Given series $1+10^{-1}+10^{-2}+10^{-3}...$

$$\Rightarrow 1+\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+...\infty$$

Here, $a=1;\ r=\frac{1}{10}$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{10}} = \frac{1}{\frac{9}{10}} = \frac{10}{9}$$

Question 83

The sum of the first two terms of an infinite geometric series is 15 and each term is equal to the sum of all the terms following it; then the sum of the series is:

(a) 20 (b) 15 (c) 25 (d) None

(a)

Let the first term of the GP be *a*, and the second term of the GP be *ar*.

Given:

$a + ar = 15$

$\Rightarrow a(1 + r) = 15$

$$\Rightarrow a = \frac{15}{1+r} \ldots \text{Eq. (1)}$$

Also, we are given that every term is equal to the sum of all the terms following it. This means that $t_2 = S_\infty - S_2$.

Now, we know that $S_\infty = \frac{a}{1-r}$, and S_2 is given as 15.

Therefore, $t_2 = \frac{a}{1-r} - 15$

Also, we know that $t_2 = ar$

Therefore, $ar = \frac{a}{1-r} - 15 \ldots \text{Eq. (2)}$

Putting the value of *a* from Eq. (1) to Eq. (2), we get:

$$\frac{15}{1+r}\times r=\frac{\frac{15}{1+r}}{1-r}-15$$

$$\frac{15r}{1+r}=\left(\frac{15}{1+r}\div 1-r\right)-15$$

$$\frac{15r}{1+r}=\left(\frac{15}{1+r}\times\frac{1}{1-r}\right)-15$$

$$\frac{15r}{1+r}=\frac{15}{(1+r)(1-r)}-15$$

$$\frac{15r}{1+r}=\frac{15-15(1+r)(1-r)}{(1+r)(1-r)}$$

$$15r=\frac{15\{1-(1+r)(1-r)\}}{1-r}$$

$$r=\frac{\{1-(1-r^2)\}}{1-r}$$

$$r(1-r)=1-1+r^2$$

$$r-r^2=r^2$$

$$r^2+r^2-r=0$$

$$2r^2 - r = 0$$

$$r(2r - 1) = 0$$

Therefore, either $r = 0$, or $r = \frac{1}{2}$

Since r cannot be 0, it'll be ½.

Putting the value of r in Eq. (1), we get:

$$a = \frac{15}{1 + \frac{1}{2}} = 10$$

Therefore, we have $a = 10$, and $r = ½$.

$$S_\infty = \frac{a}{1-r} = \frac{10}{1-\frac{1}{2}} = 20$$

Therefore, option (a) is the answer.

Question 84

If the p^{th} term of a GP is x and the q^{th} term is y, then find the n^{th} term.

(a) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]$ (b) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]^{(p-q)}$ (c) 1 (d) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]^{\frac{1}{p-q}}$

Solution

(d)

$t_p = ar^{p-1} = x$ …Eq. (1)

$t_q = ar^{q-1} = y$ …Eq. (2)

Dividing Eq. (1) by Eq. (2)

$$\frac{ar^{p-1}}{ar^{q-1}} = \frac{x}{y}$$

$$r^{p-1-(q-1)} = \frac{x}{y}$$

$$r^{p-1-q+1} = \frac{x}{y}$$

$$r^{p-q} = \frac{x}{y}$$

$$r = \left(\frac{x}{y}\right)^{\frac{1}{p-q}}$$

$$t_n = ar^{n-1}$$

Adding p and subtracting p in the power of r, we get:

$$t_n = ar^{n-1+p-p}$$

$$t_n = ar^{(n-p)+(p-1)}$$

$$t_n = ar^{(p-1)}r^{(n-p)}$$

We know that $ar^{p-1} = x$ and $r = \left(\frac{x}{y}\right)^{\frac{1}{p-q}}$. Putting these values above, we get:

$$t_n = x\left[\left(\frac{x}{y}\right)^{\frac{1}{p-q}}\right]^{n-p}$$

$$t_n = x\left(\frac{x}{y}\right)^{\frac{n-p}{p-q}}$$

$$t_n = x\left(\frac{x^{\frac{n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}\right)$$

$$t_n = \frac{x.x^{\frac{n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \frac{x^{1+\frac{n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \frac{x^{\frac{p-q+n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \frac{x^{\frac{n-q}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \left(\frac{x^{n-q}}{y^{n-p}}\right)^{\frac{1}{p-q}}$$

Question 85

The sum of three numbers in a geometric progression is 28. When 7, 2, and 1 are subtracted from the first, second, and the third numbers respectively, the resulting numbers are in Arithmetic Progression. What is the sum of squares of the original three numbers?

(a) 510 (b) 456 (c) 400 (d) 336

Solution

(d)

Let the numbers in GP be $\frac{a}{r}$, a, and ar respectively.

Given that the sum is 28.

Therefore, $\frac{a}{r}+a+ar=28$

$$\Rightarrow a\left(\frac{1}{r}+1+r\right)=28\ldots\text{Eq. (1)}$$

Also, given that if we subtract 7, 2, and 1 from the first, second and third terms respectively, we get an AP.

On subtracting 7, 2, and 1 from first, second and third terms, we get:

$$\left(\frac{a}{r}-7\right),\ (a-2),\ \text{and}\ (ar-1)$$

Since these numbers are in AP, we have $(a-2)-\left(\frac{a}{r}-7\right)=(ar-1)-(a-2)$

$$\Rightarrow a-2-\frac{a}{r}+7=ar-1-a+2$$

$$\Rightarrow a-\frac{a}{r}+5=ar-a+1$$

$$\Rightarrow a-\frac{a}{r}-ar+a=1-5$$

$$\Rightarrow 2a-\frac{a}{r}-ar=-4$$

$$\Rightarrow a\left(2-\frac{1}{r}-r\right)=-4 \ldots \text{Eq. (2)}$$

Dividing Eq. (1) by Eq. (2), we get:

$$\frac{a\left(\frac{1}{r}+1+r\right)}{a\left(2-\frac{1}{r}-r\right)}=\frac{28}{-4}$$

$$\Rightarrow \frac{\frac{1+1r+r^2}{r}}{\frac{2r-1-r^2}{r}}=-7$$

$$\Rightarrow \frac{1+r+r^2}{2r-1-r^2}=-7$$

$$\Rightarrow 1+r+r^2=-7\left(2r-1-r^2\right)$$

$$\Rightarrow 1+r+r^2=-14r+7+7r^2$$

$$\Rightarrow 7r^2+7-14r-1-r-r^2=0$$

$$\Rightarrow 6r^2-15r+6=0$$

Here, $a=6;\ b=-15;\ c=6$

$$\alpha+\beta=-\frac{b}{a}=-\frac{-15}{6}=\frac{15}{6}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{6} = 1$$

As per fastest method, $\left(\frac{15}{6\times 2} + x\right)\left(\frac{15}{6\times 2} - x\right) = 1$

$$\Rightarrow \left(\frac{15}{12}\right)^2 - x^2 = 1$$

$$x^2 = \left(\frac{15}{12}\right)^2 - 1 = 1.5625 - 1 = 0.5625$$

$$x = \sqrt{0.5625} = 0.75$$

$$\alpha = \frac{15}{12} + 0.75 = 2$$

$$\beta = \frac{15}{12} - 0.75 = 0.5$$

Therefore, common ratio could either be 2, or 0.5

Taking the common ratio to be 2, let's find out the value of *a*.

Putting the value of $r = 2$ in Eq. (1), we'll get:

$$a\left(\frac{1}{2} + 1 + 2\right) = 28$$

$$\Rightarrow a(3.5) = 28$$

$$\Rightarrow a = \frac{28}{3.5} = 8$$

Therefore, the GP will be $\frac{8}{2}, 8, 8 \times 2 = 4, 8, 16$

We can see that the sum of these numbers = 4 + 8 + 16 = 28

Subtracting 7, 2, and 1 from first, second, and third terms, we'll get 4 – 7 = –3, 8 – 2 = 6, 16 – 1 = 15.

These terms are clearly in AP as 15 – 6 = 6 – (–3) = 9

The sum of squares of the numbers 4, 8, and 16 = $4^2 + 8^2 + 16^2 = 336$

Now, taking 0.5 as the common ratio, let's find out the value of *a*.

Putting the value of $r = 0.5$ in Eq. (1), we'll get:

$$a\left(\frac{1}{r}+1+r\right)=28$$

$$\Rightarrow a\left(\frac{1}{0.5}+1+0.5\right)=28$$

$$\Rightarrow a(3.5)=28$$

$$\Rightarrow a=\frac{28}{3.5}=8$$

Therefore, the GP will be $\frac{8}{0.5}$, 8, $8\times 0.5=16, 8, 4$

We can see that the sum of these numbers = 16 + 8 + 4 = 28

Subtracting 7, 2, and 1 from first, second, and third terms, we'll get 16 – 7 = 9, 8 – 2 = 6, 4 – 1 = 3.

These terms are clearly in AP as 6 – 9 = 3 – 6 = –3

The sum of squares of the numbers 16, 8, and 4 = $16^2 + 8^2 + 4^2 = 336$

Question 86

If p, q and r, are in A.P. and x, y, z are in G.P., then $x^{q-r}.y^{r-p}.z^{p-q}$ is equal to:

(a) 0 (b) –1 (c) 1 (d) None

Solution

(c)

Since p, q, and r, are in AP, we have $q-p=r-q=d$

$\therefore q-p=d \Rightarrow p-q=-d$

And $r-q=d \Rightarrow q-r=-d$

Also, $r-p=(r-q)+(q-p)=d+d=2d$

Also, since x, y, and z are in GP, we have $y^2 = xz$

Now, we have:

$$x^{q-r}.y^{r-p}.z^{p-q}$$

$$x^{-d}.y^{2d}.z^{-d} \text{ (Since } q-r=-d;\ r-p=2d;\ p-q=-d\text{)}$$

$$(xz)^{-d}.y^{2d}$$

$$(y^2)^{-d}.y^{2d} \text{ (Since } y^2 = xz\text{)}$$

$$y^{-2d}.y^{2d} = 1$$

Question 87

Given x, y, and z are in GP and $x^p = y^q = z^\sigma$, then $1/p$, $1/q$, $1/\sigma$ are in:

(a) AP (b) GP (c) Both (d) None

Solution

(a)

Question 88

The sum of the first 3 terms in an AP is 18 and that of the last 3 is 28. If the AP has 13 terms, what is the sum of the middle three terms?

(a) 23 (b) 18 (c) 19 (d) None

Solution

(a)

Let the first term be *a* and the common difference be *d*.

$t_1 + t_2 + t_3 = 18$

$\Rightarrow (a) + (a + d) + (a + 2d) = 18$

$$\Rightarrow a+a+d+a+2d=18$$

$$\Rightarrow 3a+3d=18$$

$$\Rightarrow 3(a+d)=18$$

$$\Rightarrow a+d=\frac{18}{3}=6$$

$$\Rightarrow a+d=6\ldots\text{Eq. (1)}$$

$$t_{11}+t_{12}+t_{13}=28$$

$$\Rightarrow (a+10d)+(a+11d)+(a+12d)=28$$

$$\Rightarrow a+10d+a+11d+a+12d=28$$

$$\Rightarrow 3a+33d=28$$

$$\Rightarrow 3(a+11d)=28$$

$$\Rightarrow a+11d=\frac{28}{3} \ldots \text{Eq. (2)}$$

Subtracting Eq. (2) from Eq. (1), we get:

$$d-11d=6-\frac{28}{3}$$

$$\Rightarrow -10d=\frac{18-28}{3}$$

$$\Rightarrow -10d=-\frac{10}{3}$$

$$\Rightarrow d = \frac{1}{3}$$

Putting this value in Eq. (1), we get:

$$a + \frac{1}{3} = 6$$

$$\Rightarrow a = 6 - \frac{1}{3} = \frac{18-1}{3} = \frac{17}{3}$$

Therefore, $a = \frac{17}{3}; \ d = \frac{1}{3}$

Middle three terms of the series are t_6, t_7, and t_8

$$t_6 + t_7 + t_8$$

$$=(a+5d)+(a+6d)+(a+7d)$$

$$=a+5d+a+6d+a+7d$$

$$=3a+18d$$

$$=\left(3\times\frac{17}{3}\right)+\left(18\times\frac{1}{3}\right)$$

$$=17+6=23$$

Question 89

The first term of an A.P. is 100 and the sum of whose first 6 terms is 5 times the sum of the next 6 terms, then the c.d. is:

(a) –10 (b) 10 (c) 5 (d) None

(a)

Try the options.

Option (a) → –10

If the common difference is –10, the series is:

100, 90, 80, 70, 60, 50, 40, 30, 20, 10, 0, –10

Sum of the first 6 terms = 100 + 90 + 80 + 70 + 60 + 50 = 450

Sum of the next 6 terms = 40 + 30 + 20 + 10 + 0 + (–10) = 90

Since 450 = 5 × 90, therefore, clearly sum of the first 6 terms, i.e., 450, is 5 times the sum of the next 6 terms, i.e. 90.

Therefore, option (a) is the answer.

Question 90

If $\frac{1+3+5+...+n \text{ terms}}{2+4+6+...+50 \text{ terms}} = \frac{2}{51}$, the value of n is:

(a) 9 (b) 10 (c) 12 (d) 13

Solution

(b)

Try the options.

Option (b) $\rightarrow$ 10

This becomes the sum of first 10 odd numbers

Numerator → 1 + 3 + 5 + … 10 terms

$$S_{10}=\frac{10}{2}\{(2\times1)+(10-1)2\}=100$$

Denominator → $S_{50}=\frac{50}{2}\{(2\times2)+(50-1)2\}=2550$

On calculator $\frac{100}{2550}=\frac{2}{51}$

Question 91

The inverse h^{-1} when $h(x) = \log_{10} x$ is:

(a) $\log_{10} x$ (b) 10^x (c) $\log_{10}(1/x)$ (d) None

Solution

(b)

Question 92

Let R be the set of real numbers such that the function $f:R\to R$ and $g:R\to R$ are defined by $f\left(x\right)=x^{2}+3x+1$ and $g\left(x\right)=2x-3$. Find $\left(fog\right)$.

(a) $4x^{2}+6x+1$ (b) $x^{2}+6x+1$ (c) $4x^{2}-6x+1$ (d) $x^{2}-6x+1$

Solution

(c)

Question 93

For the function $h(x)=10^{1+x}$, the domain of real values of x where $0\le x\le 9$, the range is:

(a) $10\le h(x)\le 10^{10}$ (b) $0\le h(x)\le 10^{10}$ (c) $0<h(x)<10$ (d) None

Solution

(a)

Question 94

If $f(x) = \left(\frac{x^2 - 4}{x - 2}\right)$, then $f(2)$ is:

(a) 0 (b) 2 (c) 4 (d) 1

Solution

(c)

Question 95

Let $A = \{1, 2, 3\}$, then $R_3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$

(a) Only Symmetric
(b) Reflexive & Symmetric
(c) Reflexive & Transitive
(d) Symmetric & Transitive

Solution

(b)

Question 96

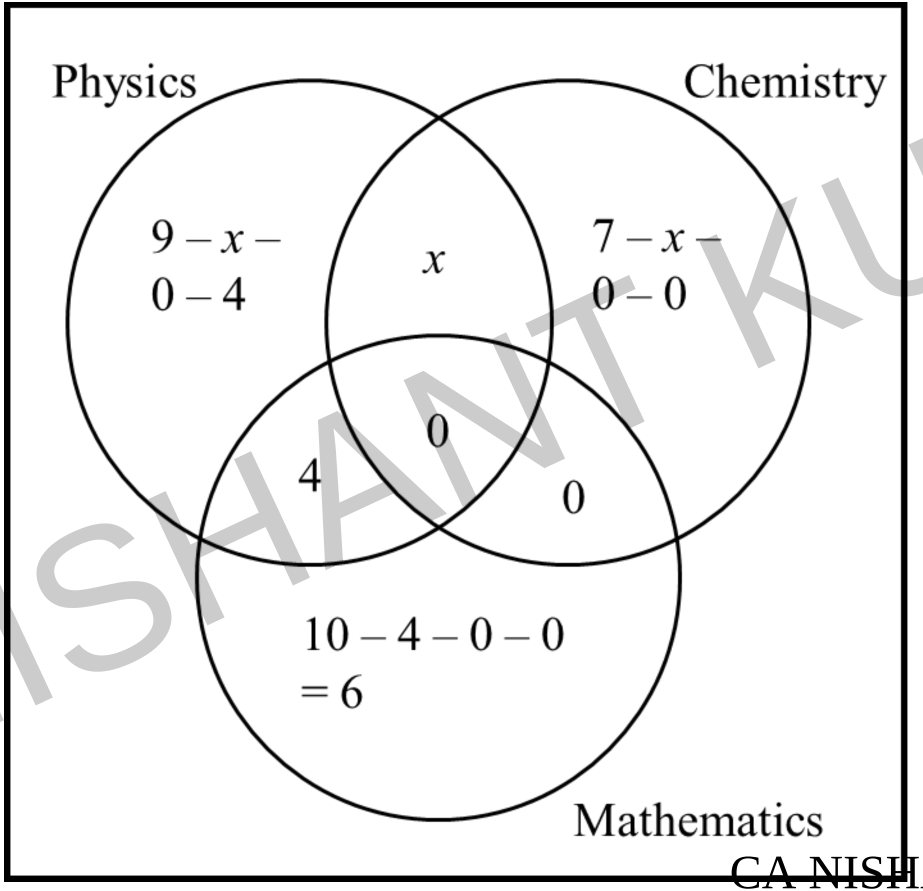
Out of a group of 20 teachers in a school, 10 teach Mathematics, 9 teach Physics and 7 teach Chemistry. 4 teach Mathematics and Physics but none teach both Mathematics and Chemistry. How many teach Chemistry and Physics? How many teach only Physics?

(a) 3; 2 (b) 2; 3 (c) 4; 5 (d) None

Solution

(b)

Let the number of teachers teaching both Physics and Chemistry be x.


Physics
Chemistry
9 – x –
0 – 4
x
7 – x –
0 – 0
0
4
0
10 – 4 – 0 – 0
= 6
Mathematics

In the absence of information, it is safe to assume that all the teachers teach at least one of the subjects. Therefore,

$9 - x - 0 - 4 + x + 7 - x - 0 - 0 + 4 + 0 + 0 + 6 = 20$

$\Rightarrow 9 - 4 + 7 + 4 + 6 - x + x - x = 20$

$\Rightarrow 22 - x = 20$

$\Rightarrow x = 22 - 20 = 2$

Therefore, number of teachers teaching both Physics and Chemistry = 2.

Number of teachers teaching only Physics = 9 – 2 – 4 = 3

Question 97

Let Z be the universal set for two sets – A and B. If $n(A)=300$, $n(B)=400$ and $n(A\cap B)=200$, then $n(A'\cap B')$ is equal to 400 provided $n(Z)$ is equal to:

(a) 900 (b) 800 (c) 700 (d) 600

Solution

(a)

Given: $n(A)=300$; $n(B)=400$; $n(A\cap B)=200$; $n(A'\cap B')=400$; $n(Z)=?$

$n(A'\cap B')=n(A\cup B)'=n(Z)-n(A\cup B)$

$\Rightarrow n(A'\cap B')=n(Z)-n(A\cup B)$

$$\Rightarrow n(Z) = n(A' \cap B') + n(A \cup B)$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\Rightarrow n(A \cup B) = 300 + 400 - 200 = 500$$

$$n(Z) = n(A' \cap B') + n(A \cup B)$$

$$\Rightarrow n(Z) = 400 + 500 = 900$$

Question 98

The number of integers from 1 to 100 which are neither divisible by 3 nor by 5 nor by 7 is:

(a) 67 (b) 55 (c) 45 (d) 33

Solution

(c)

Question 99

At a certain conference of 100 people there are 29 Indian women and 23 Indian men. Out of these Indian people 4 are doctors and 24 are either men or doctors. There are no foreign doctors. The number of women doctors attending the conference is:

(a) 2 (b) 4 (c) 1 (d) None

Solution

(c)

We have $n(U)=100$, $n(W)=29$, $n(M)=23$, $n(D)=4$, $n(M \cup D)=24$

We have to find out the number of female doctors, i.e., $n(W \cap D)$. We have $n(W)$, and $n(D)$, but we don't have $n(W \cup D)$. Therefore, we cannot apply the formula $n(W \cap D) = n(W) + n(D) - n(W \cup D)$.

However, if we find out the number of Male Doctors, we can then subtract them from the total doctors to find out the number of female doctors.

$$n(M \cap D) = n(M) + n(D) - n(M \cup D)$$

$$\Rightarrow n(M \cap D) = 23 + 4 - 24 = 3$$

Therefore, number of female doctors = 4 – 3 = 1.

Question 100

If R is the set of isosceles right-angled triangles and I is set of isosceles triangles, then:

(a) $R = I$ (b) $R \supset I$ (c) $R \subset I$ (d) None

Solution

(c)