# Marathon 11

CA Nishant Kumar

# Marathon Schedule

| Date (Day) | Topic |
|---|---|
| 18-12-2023 (Monday) | Time Value of Money |
| 19-12-2023 (Tuesday) | Logical Reasoning |
| 20-12-2023 (Wednesday) | Measures of Central Tendency and Dispersion |
| 21-12-2023 (Thursday) | Ratio, Proportion, Indices, Logarithms; Linear Inequalities |
| 22-12-2023 (Friday) | Equations; Statistical Description of Data |
| 23-12-2023 (Saturday) | Sequence and Series |
| 25-12-2023 (Monday) | Sets, Relations, and Functions |
| 26-12-2023 (Tuesday) | Correlation and Regression |
| Taken Before | Index Numbers |
| 27-12-2023 (Wednesday) | Permutations and Combinations |
| 28-12-2023 (Thursday) | Probability; Theoretical Distributions |

EKAGRATA

न भूतो न भविष्यति!

# CA. NISHANT KUMAR

## Co-Founder – Ekagrata Eduserv

- **Author of Various Books**
- Taught thousands of students since **7+** years
- **Former Educator – Unacademy**
- Instagram: canishantkumar
- **Subjects Teaching** CA-Foundation Maths; CA-Intermediate Taxation
- Website: www.kumarnishant.com

# My Schedule

EKAGRATA
न भूतो न भविष्यति!

**4:45 PM**

YOUTUBE
Ekagrata CA
Marathon

Monday to
Sunday

December, 2023

**7:00 PM**

Ekagrata
APP
Ekagrata
Batch

Monday to
Wednesday

June, 2024

EKAGRATA

न भूतो न भविष्यति!

# WARM-UP BATCH

## For CA-Intermediate Nov 2024

Special Batch for those students who have appeared for CA-Foundation Dec 2023 examination.

Conducted both OFFLINE at Ekagrata Indore Center and ONLINE on Ekagrata Live App from 22nd Jan'24

EKAGRATA
न भूतो न भविष्यति!

WARM-UP BATCH for CA-INTERMEDIATE

# OUR TEAM FOR WARM UP BATCH FOR CA-INTERMEDIATE (NEW SYLLABUS)


**CA. CS. ANSHUL AGRAWAL**
*Advanced Accounting*
*Cost & Mgt. Accounting*
FACE-TO-FACE | ONLINE


**CA. NISHANT KUMAR**
*Taxation*
ONLINE


**MR. JATIN DEMBLA**
*FM & SM*
FACE-TO-FACE | ONLINE

It will be Beginning for OTHERS.... Not for YOU!!!

EKAGRATA

न भूतो न भविष्यति!

WARM-UP BATCH for CA-INTERMEDIATE

## OUR TEAM FOR WARM UP BATCH FOR CA-INTERMEDIATE (NEW SYLLABUS)

**CA. ABHISHEK BANSAL**

*Corporate & Other Laws*

*Auditing & Ethics*

FACE-TO-FACE | ONLINE

**CA. ADARSH JOSHI**

*Corporate & Other Laws*

FACE-TO-FACE | ONLINE

**CA. CS. DARSHAN JAIN**

*FM & SM*

ONLINE

**CA. MOHIT PATIDAR**

*Taxation*

FACE-TO-FACE | ONLINE

It will be Beginning for OTHERS.... Not for YOU!!!

EKAGRATA

न भूतो न भविष्यति!

WARM-UP BATCH for CA-INTERMEDIATE

## HIGHLIGHTS OF WARM UP BATCH FOR CA-INTERMEDIATE (NEW SYLLABUS)

- **Specially curated 4-week batch that introduces you to all subjects of CA-Intermediate**
- Best way to utilize time after exam and before result
- **Daily 5-6 Hours of classes conducted from Monday to Friday**
- Introduction to all subjects - Adv. Acc., Law, Tax, Cost & Mgt. Acc., Auditing & FM-SM
- **Target is to crack CA-Intermediate November 2024 exam**
- Regular Batch for CA-Inter Nov'24 to start after Foundation result
- **Content taught in this batch will be taught again later in Regular Batch**
- This will give chance to warm up students to revise those topics again/cover backlogs
- **Batch will be conducted Face-to-Face at Indore and on Online on Ekagrata Live App**
- Batch Fees ~~Rs. 9,999~~ (50% Discount) **Rs.4,999**
- **The above fees shall be adjusted while taking admission in Regular batch, after result**
- Regular batch for CA-Inter will also be conducted Face-to-Face at Indore & Online
- **PDF Books will be provided in Warm Up Batch and Hard Copies in Regular Batch**

It will be Beginning for OTHERS.... Not for YOU!!!

# Chapter 15 – Probability

# Introduction

The result of a random experiment is known as an event or an outcome. Probability is the chance of an outcome.

$$\text{Probability} = \frac{\text{No. of Favourable Cases/Events/Outcomes}}{\text{Total No. of Cases/Events/Outcomes}}$$

# Computation of total number of outcomes when an experiment is repeated a certain number of times:

When an experiment with total number of events $a$ is repeated $b$ number of times, the total number of outcomes is given by $a^b$.

**Question**

A coin is tossed three times. What is the probability of getting 2 heads?

(a) 3/8 (b) 2/3 (c) ¾ (d) None

**Question**

A coin is tossed three times. What is the probability of getting at least 2 heads?

(a) 1/2 (b) 2/3 (c) ¾ (d) None

**Question**

Two dice are thrown simultaneously. Find the probability that the sum of points on the two dice would be 7 or more.

*OR*

What is the chance of throwing at least 7 in a single cast with 2 dice?

(a) 5/12 (b) 7/12 (c) ¼ (d) 17/36

**Question**

There are three boxes with the following composition:

Box I: 5 Red + 7 White + 6 Blue balls

Box II: 4 Red + 8 White + 6 Blue balls

Box III: 3 Red + 4 White + 2 Blue balls

If one ball is drawn at random, then what is the probability that they would be of same colour?

(a) 89/729 (b) 97/729 (c) 82/729 (d) 23/32

**Question**

If two letters are taken at random from the word HOME, what is the Probability that none of the letters would be vowels?

(a) 1/6 (b) ½ (c) 1/3 (d) ¼

**Question**

Two balls are drawn from a bag containing 5 white and 7 black balls at random. What is the probability that they would be of different colours?

(a) 35/66 (b) 30/66 (c) 12/66 (d) None

**Question**

A lot of 10 electronic components is known to include 3 defective parts. If a sample of 4 components is selected at random from the lot, what is the probability that this sample does not contain more than one defective?

(a) 2/3 (b) 1/3 (c) ¼ (d) None

**Question**

A bag contains 12 balls which are numbered from 1 to 12. If a ball is selected at random, what is the probability that the number of the ball will be a multiple of 5 or 6?

(a) 0.30 (b) 0.25 (c) 0.20 (d) 1/3

**Question**

What is the probability that a leap year selected at random would contain 53 Saturdays?

(a) 1/7 (b) 2/7 (c) 1/12 (d) ¼

**Question**

If two unbiased dice are rolled, what is the probability of getting points neither 6 nor 9?

(a) 0.25 (b) 0.50 (c) 0.75 (d) 0.80

**Question**

What is the chance of picking a spade or an ace not of spade from a pack of 52 cards?

(a) 4/13 (b) 5/13 (c) 6/13 (d) 7/13

**Question**

Four digits 1, 2, 4 and 6 are selected at random to form a four-digit number. What is the probability that the number so formed, would be divisible by 4?

(a) ½ (b) 1/5 (c) ¼ (d) 1/3

**Question**

A pair of dice is thrown together and the sum of points of the two dice is noted to be 10. What is the probability that one of the two dice has shown the point 4?

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) 2/3 (d) None

**Question**

It is given that a family of 2 children has a girl, what is the probability that the other child is also a girl?

(a) 0.50 (b) 0.75 (c) 1/3 (d) 2/3

# Odds

$$Odds\ in\ Favour\ of\ Event\ A = \frac{Number\ of\ Favourable\ Outcomes}{Number\ of\ Unfavourable\ Outcomes}$$

$$Odds\ Against\ Event\ A = \frac{Number\ of\ Unfavourable\ Outcomes}{Number\ of\ Favourable\ Outcomes}$$

**Question**

If $p : q$ are the odds in favour of an event, then the probability of that event is:

(a) $\frac{p}{q}$ (b) $\frac{p}{p+q}$ (c) $\frac{q}{p+q}$ (d) None

**Question**

If $P(A) = 5/9$, then the odds against the event $A$ is:

(a) 5 : 9 (b) 5 : 4 (c) 4 : 5 (d) 5 : 14

**Question**

If an unbiased die is rolled once, the odds in favour of getting a point which is a multiple of 3 is:

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

# Types of Events

# Equally Likely Events or Mutually Symmetric Events or Equi-Probable Events

- If two or more events have the same probability, the events are said to be equally likely events.
- For example, when a coin is tossed, the probability of getting heads is ½; also, the probability of getting tails is also ½.
- Therefore, these two events are said to be equally likely.
- Similarly, when a dice is thrown, the probability of getting either 1, or 2, or 3, or 4, or 5, or 6 is 1/6.
- Therefore, these events are known as equally likely events.
- The events which have different probabilities are said “Not Equally Likely” events.

**Question**

If $P(A) = P(B)$, then the two events $A$ and $B$ are:

(a) Independent (b) Dependent (c) Equally Likely (d) Both (a) and (c)

# Impossible Events

- Events which have zero probability are known as “Impossible Events”.
- For example, let today be Monday. Now, the probability that tomorrow is going to be Wednesday is zero.
- Therefore, this event is an impossible event.

# Sure/Certain Events

- Events which have 100% (or 1) probability are known as "Sure/Certain Events".
- For example, let today be Wednesday. Now, the probability that tomorrow is going to be Thursday is 100%, i.e. 1.
- Therefore, this event is a sure/certain event.
- From the above discussion on impossible and certain events, it can be seen that the probability ranges from 0 to 1 (both inclusive).
- Probability can never be a negative number.

**Question**

If $P(A)=1$, then the event is known as:

(a) Symmetric Event (b) Dependent Event (c) Improbable Event (d) Sure Event

# Mutually Exclusive Events or Incompatible Events

- The events which cannot occur simultaneously are called mutually exclusive events.
- For example, when a coin is tossed, there are a total of two outcomes – Heads, and Tails.
- However, these two events cannot occur at the same time.
- If heads occur, tails would not occur; and if tails occur, heads would not occur.
- Therefore, these two events are said to be mutually exclusive events.
- "Mutually exclusive events" is technically defined as follows: when the occurrence of one event prevents the occurrence of other event, such events are known as mutually exclusive events.
- The events which can occur simultaneously are called "Not Mutually Exclusive" events.

- For example, when a dice is rolled, the events “odd number occurs”, and “number 5 occurs” can occur together.
- Therefore, these events are called “not mutually exclusive” events.

**Question**

Which of the following pairs of events are mutually exclusive?

| | | |
|---|---|---|
| (a) | A: The student reads in School | B: He studies Philosophy |
| (b) | A: Raju was born in India | B: He is a fine Engineer |
| (c) | A: Ruma is 16 years old. | B: She is a good Singer |
| (d) | A: Peter is under 15 years of age | B: Peter is a voter of Kolkata |

# Simple (or Elementary) and Composite (or Compound) Events

- An event which cannot be split into two or more parts is known as a simple event.
- For example, when a dice is thrown, the event “5 occurs” cannot be broken down into any more parts.
- An event which can be broken down into two or more simple events is known as a composite event.
- For example, when a dice is thrown, the event “odd number occurs” can be broken down into two or more parts.
- This is because, if the numbers 1, 3, or 5 occur, they correspond to our event “odd number occurs”.
- Therefore, the event “odd number occurs” can be broken down into 3 parts –
    - “1 occurs”,
    - “3 occurs”, and

    - “5 occurs”.
- Similarly, on throwing of a dice, the event “number more than 2 occurs” can be split into 4 parts –
    - “3 occurs”,
    - “4 occurs”,
    - “5 occurs”, and
    - “6 occurs”.
- Therefore, the event “number more than 2 occurs” is also a composite event.

**Question**

An event that can be split into further events is known as:

(a) Complex Event (b) Mixed Event (c) Simple Event (d) Composite Event

# Exhaustive Events

- All the possible events of an experiment are combinedly known as Exhaustive Events.

**Question**

If an unbiased coin is tossed once, then the two events Head and Tail are:

(a) Mutually Exclusive (b) Exhaustive (c) Equally Likely (d) All

**Question**

If $P(A) = P(B)$, then

(a) A and B are the same events

(b) A and B must be same events

(c) A and B may be different events

(d) A and B are mutually exclusive events

# Operations on Events – Set Theoretic Approach to Probability

- Sample space represents the Universal Set, denoted by $S$ or $\Omega$.
- An event $A$ is defined as a non-empty subset of $S$.
- Then, probability of event $A$ is given by: $P(A)=\dfrac{n(A)}{n(S)}$, where, $n(A)$ is the cardinal number of the set $A$; and $n(S)$ is the cardinal number of the set $S$.

# Points to be Noted

1. If two events $A$ and $B$ are not mutually exclusive, then, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
2. Two events $A$ and $B$ are mutually exclusive, if $A \cap B = \phi$. Therefore, $P(A \cap B) = 0$, or $P(A \cup B) = P(A) + P(B)$.
3. If three events $A$, $B$, and $C$ are not mutually exclusive, then $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
4. Three events $A$, $B$, and $C$ are mutually exclusive, if $P(A \cup B \cup C) = P(A) + P(B) + P(C)$.
5. Two events $A$ and $B$ are exhaustive, if $P(A \cup B) = 1$.
6. Three events $A$, $B$, and $C$ are exhaustive, if $P(A \cup B \cup C) = 1$.

7. Three events *A*, *B*, and *C* are equally likely if $P(A) = P(B) = P(C)$.

8. Probability that only event *A* occurs: $P(A - B) = P(A) - P(A \cap B)$

9. Probability that only event *B* occurs: $P(B - A) = P(B) - P(A \cap B)$

**Question**

If $P(A \cap B) = 0$, then the two events $A$ and $B$ are:

(a) Mutually Exclusive (b) Exhaustive (c) Equally Likely (d) Independent

**Question**

If, for two events $A$ and $B$, $P\left(A \cup B\right) = 1$, then $A$ and $B$ are:

(a) Mutually Exclusive (b) Equally Likely (c) Exhaustive (d) Dependent

**Question**

If $A$, $B$ and $C$ are mutually exclusive and exhaustive events, then $P(A)+P(B)+P(C)$ equals to:

(a) $\frac{1}{3}$ (b) 1 (c) 0 (d) any value between 0 and 1

**Question**

Three events *A*, *B* and *C* are mutually exclusive, exhaustive, and equally likely. What is the probability of the complementary event of A?

(a) 6/11 (b) 3/11 (c) 1/6 (d) 2/3

**Question**

A number is selected from the first 25 natural numbers. What is the probability that it would be divisible by 4 or 7?

(a) 6/25 (b) 8/11 (c) 9/25 (d) None

**Question**

The probability that an Accountant's job applicant has a B. Com. Degree is 0.85, that he is a CA is 0.30 and that he is both B. Com. and CA is 0.25 out of 500 applicants, how many would be B. Com. or CA?

(a) 450 (b) 500 (c) 900 (d) 950

**Question**

If $P(A-B)=1/5$, $P(A)=1/3$ and $P(B)=1/2$, what is the probability that out of the two events *A* and *B*, only *B* would occur?

(a) 10/30 (b) 11/30 (c) 9/30 (d) None

**Question**

There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

(a) 0.30 (b) 0.90 (c) 0.45 (d) None

**Question**

If a card is drawn at random from a pack of 52 cards, what is the chance of getting a Spade or an ace?

(a) 4/13 (b) 5/13 (c) 0.25 (d) 0.20

**Question**

If $P\left(\overline{A} \cup \overline{B}\right) = 5/6$, $P(A) = 1/2$ and $P\left(\overline{B}\right) = 2/3$, what is $P(A \cup B)$?

(a) 1/3 (b) 5/6 (c) 2/3 (d) 4/9

# Conditional Probability and Compound Theorem of Probability

# Independent and Dependent Events

- For practical questions, just remember the following:
  - OR means Union of Sets
    - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - AND means Intersection of Sets
    - $P(A \cap B) = P(A) \times P(B)$

**Question**

A box contains 5 white and 7 black balls. Two successive draws of 3 balls are made with replacement. The probability that the first draw would produce white balls and the second draw would produce black balls are respectively:

(a) 6/321 (b) 1/20 (c) 35/144 (d) 7/968

**Question**

A box contains 5 white and 7 black balls. Two successive draws of 3 balls are made without replacement. The probability that the first draw would produce white balls and the second draw would produce black balls are respectively:

(a) 3/926 (b) 1/30 (c) 35/108 (d) 5/264

**Question**

Tom speaks truth in 30 percent cases and Dick speaks truth in 25 percent cases. What is the probability that they would contradict each other?

(a) 0.325 (b) 0.400 (c) 0.925 (d) 0.075

**Question**

A problem in probability was given to three CA students A, B and C whose chances of solving it are 1/3, 1/5 and 1/2 respectively. What is the probability that the problem would be solved?

(a) 4/15 (b) 7/8 (c) 8/15 (d) 11/15

**Question**

For two events $A$ and $B$, $P(B)=0.3$, $P(A \text{ but not } B)=0.4$, and $P(\text{not } A)=0.6$. The events $A$ and $B$ are:

(a) exhaustive (b) independent (c) equally likely (d) mutually exclusive

# Rules of Probability When Events are Independent

1. $P(A \cap B) = P(A) \times P(B)$

2. Probability of event *A* given that event *B* has already occurred is given by *P*(*A*/*B*):

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

3. Probability of event *B* given that event *A* has already occurred is given by *P*(*B*/*A*):

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

4. $P(A'/B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{P(B) - P(A \cap B)}{P(B)}$

5. $P(A/B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{P(A) - P(A \cap B)}{1 - P(B)}$

6. $P(A'/B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{P(A \cup B)'}{1 - P(B)} = \dfrac{1 - P(A \cup B)}{1 - P(B)}$

7. Probability that only event $A$ or only event $B$ occurs: $P(A) + P(B) - 2P(A \cap B)$

**Question**

In connection with a random experiment, it is found that $P(A)=\frac{2}{3}$, $P(B)=\frac{3}{5}$, $P(A\cup B)=\frac{5}{6}$. Evaluate $P(A/B)$.

(a) 13/18 (b) 13/20 (c) 7/12 (d) 5/18

**Question**

Given that $P(A)=1/2$, $P(B)=1/3$, $P(A\cap B)=1/4$, what is $P(A'/B')$?

(a) ½ (b) 7/8 (c) 5/8 (d) 2/3

**Question**

For a group of students, 30%, 40% and 50% failed in Physics, Chemistry, and at least one of the two subjects respectively. If an examinee is selected at random, what is the probability that he passed in Physics if it is known that he failed in Chemistry?

(a) ½ (b) 1/3 (c) ¼ (d) 1/6

# Random Variable – Probability Distribution

# Expected Value of a Random Variable

## Important Formulae

1. Expected value $(\mu)$ of a random variable $(x)$ is given by: $\mu = E(x) = \sum p_i x_i$
2. Expected value of $(x^2)$ is given by: $E(x^2) = \sum\left[p_i(x_i^2)\right]$
3. Expected value of a monotonic function $\left[g(x)\right]$ is given by: $E\left[g(x)\right] = \sum\left[p_i\{g(x)\}\right]$
4. Variance $(\sigma^2)$ of a random variable $(x)$ is given by: $V(x) = \sigma^2 = E(x-\mu)^2 = E(x^2) - \mu^2$

5. Standard Deviation $(\sigma)$ of a random variable $(x)$ is given by the positive square root of the variance.
6. If $a$ and $b$ are two constants related with two random variables $x$ and $y$ as $y = a + bx$, then the mean, i.e., the expected value of $y$ is given by: $\mu_y = a + b\mu_x$.
7. If $a$ and $b$ are two constants related with two random variables $x$ and $y$ as $y = a + bx$, then the standard deviation of $y$ is given by: $\sigma_y = |b| \times \sigma_x$.
8. If $a$ and $b$ are two constants related with two random variables $x$ and $y$ as $y = a + bx$, then the variance of $y$ is given by: $(\sigma_y)^2 = (|b| \times \sigma_x)^2 = (b)^2 \times (\sigma_x)^2$.

## Properties of Expected Value

1. Expectation of a constant $k$ is $k$, i.e., $E(k)=k,$ for any constant $k$.
2. Expectation of sum of two random variables is the sum of their expectations, i.e., $E(x+y)=E(x)+E(y),$ for any two random variables $x$ and $y$.
3. Expectation of the product of a constant and a random variable is the product of the constant and the expectation of the random variable, i.e., $E(kx)=k.E(x),$ for any constant $k$.
4. Expectation of the product of two random variables is the product of the expectation of the two random variables, provided the two variables are independent, i.e., $E(x\times y)=E(x)\times E(y)$. This holds true whenever $x$ and $y$ are independent.

**Question**

In a business venture, a man can make a profit of ₹50,000 or incur a loss of ₹20,000. The probabilities of making profit or incurring loss, from the past experience, are known to be 0.75 and 0.25 respectively. What is his expected profit?

(a) ₹42,500 (b) ₹32,500 (c) ₹35,000 (d) None

**Question**

A bag contains 6 white and 4 red balls. If a person draws 2 balls and receives ₹10 and ₹20 for a white and red balls respectively, then his expected amount is:

(a) ₹25 (b) ₹26 (c) ₹29 (d) ₹28

**Question**

If a random variable $x$ assumes the values 0, 1 and 2 with probabilities 0.30, 0.50 and 0.20, then its expected value is:

(a) 1.50 (b) 3 (c) 0.90 (d) 1

**Question**

The probability distribution of a random variable $x$ is given below:

| | | | | | |
|---|---|---|---|---|---|
| $x$: | 1 | 2 | 4 | 6 | 8 |
| $P$: | $k$ | $2k$ | $3k$ | $3k$ | $k$ |

The variance of $x$ is:

(a) 2.1 (b) 4.41 (c) 2.32 (d) 2.47

# Theoretical Distributions

# Binomial Distribution

Binomial Distribution is used to find out the probability where the total no. of outcomes is huge. The probability is given by the following formula:

$P(x) = {}^nC_x p^x q^{n-x}$, for $x = 0, 1, 2, 3, \ldots, n$

Here,
$n$ = number of times the experiment is repeated
$x$ = the requirement of the question
$p$ = probability of success in each trial
$q$ = probability of failure in each trial = $1 - p$

Sometimes, $P(x)$ is also written as $f(x)$. $f(x)$ is called "Probability Mass Function".

## Conditions

Binomial distribution is applicable only if the following conditions are satisfied:

1. All the trials are independent, and
2. Each trial has only two outcomes.

**Question 1**

The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. What is the probability that out of 5 workmen, 3 or more will contract the disease?

(a) 0.0906 (b) 0.0086 (c) 0.8006 (d) None

**Solution**

(b)

Here $n$ = 5; $p$ = 0.10; $q$ = 1 – 0.10 = 0.90; $x \geq 3$

We know that $P(x) = {}^nC_x p^x q^{n-x}$

$$P(x \geq 3) = P(x = 3) + P(x = 4) + P(x = 5)$$

$$\Rightarrow P(x \geq 3) = {}^5C_3(0.10)^3(0.90)^2 + {}^5C_4(0.10)^4(0.90)^1 + {}^5C_5(0.10)^5(0.90)^0$$

$$\Rightarrow P(x \geq 3) = 0.00856 \approx 0.0086$$

## Question 2

If the overall percentage of success in an exam is 60, what is the probability that out of a group of 4 students, at least one has passed?

(a) 0.6525 (b) 0.9744 (c) 0.8704 (d) 0.0256

## Solution

(b)

Pass percentage is 60. This means 60%. This means that $p$ = 0.6. Therefore, $q$ = 0.4.

Therefore, we have, $n = 4;\ p = 0.6;\ q = 0.4;\ x \geq 1$

$$P(x) = {}^{n}C_{x} p^{x} q^{n-x}$$

$$P(x \ge 1) = 1 - P(x < 1)$$

$$= 1 - P(x = 0)$$

$$= 1 - {}^{4}C_{0}(0.6)^{0}(0.4)^{4-0}$$

$$= 1 - 0.0256$$

$$= 0.9744$$

**Question 3**

An experiment succeeds thrice as after it fails. If the experiment is repeated 5 times, what is the probability of having no success at all?

(a) 1/1024 (b) 2/3 (c) 1/1025 (d) None

**Solution**

(a)

We have $n = 5$; $p = 3q$; $x = 0$

$p = 3q$

$\Rightarrow p = 3(1 - p)$

$$\Rightarrow p = 3 - 3p$$

$$\Rightarrow 4p = 3$$

$$\Rightarrow p = \frac{3}{4}$$

$$\Rightarrow q = \frac{1}{4}$$

$$P\left(x = 0\right) = {}^{5}C_{0}\left(\frac{3}{4}\right)^{0}\left(\frac{1}{4}\right)^{5} = \frac{1}{1024}$$

# Important Points

1. Binomial Distribution is applicable when the random variable ($x$) is discrete.
2. As $n > 0$, $p, q > 0$, therefore, $f(x) \geq 0$ for every $x$.

   Also, $\sum f(x) = f(0) + f(1) + f(2) + f(3) + ... + f(n) = 1$
3. Binomial distribution is known as bi-parametric distribution as it is characterised by two parameters $n$ and $p$. This means that if the values of $n$ and $p$ are known, then the distribution is known completely.
4. The mean of the binomial distribution is given by $\mu = np$.
5. A binominal distribution is symmetrical when $p = q$.

## Question 4

$x$ is a binomial variable with $n = 20$. What is the mean of $x$ if it is known that $x$ is symmetric?

(a) 5 (b) 10 (c) 2 (d) 8

## Solution

(b)

If $x$ is symmetric, $p = q = 0.5$.

Therefore, mean = $np = 20 \times 0.5 = 10$

# Important Points (Contd.)

6. Mode of a Binomial Distribution is given by $\mu_0 = (n+1)p$

   a. If the value of $(n+1)p$ is an integer (i.e., without decimal part), then the binomial distribution is said to have two modes. It is called a bi-modal binomial distribution. The two modes are given by:

      i. $(n+1)p$, and

      ii. $[(n+1)p]-1$

   For example, if, in a binomial distribution, $n=11$, and $p=\frac{1}{2}$, then $(n+1)p$ = $(11+1)\times\frac{1}{2}$ = $\frac{12}{2}$ = 6 (Integer). Therefore, this binomial distribution will have two modes:

i. $(n+1)p=6$

ii. $[(n+1)p]-1=6-1=5$

b. If the value of $(n+1)p$ is a fraction (i.e., with a decimal part), then the binomial distribution is said to have one mode. It is called a unimodal binomial distribution. Its mode is given by the largest integer contained in $(n+1)p$.

For example, if, in a binomial distribution, $n=12$, and $p=\frac{1}{3}$, then $(n+1)p$ = $(12+1)\times\frac{1}{3}$ = $\frac{13}{3}$ = 4.33

Since the answer is a fraction, this binomial distribution has only one mode. Its mode is given by the largest integer contained in $(n+1)p$. Therefore, the mode is 4.

## Question 5

If $x$ is a binomial variate with parameter 15 and 1/3, what is the value of mode of the distribution?

(a) 5 and 6 (b) 5 (c) 5.50 (d) 6

## Solution

(b)

Mode = $(n+1)p = (15+1)\times\frac{1}{3} = 5.33$

Since this is a fraction, mode is the highest integer, i.e., 5.

# Important Points (Contd.)

7. The variance of the binomial distribution is given by $\sigma^2 = npq$.
    a. Variance of a binomial distribution is always less than its mean.
    b. If $p = q = 0.5$, variance is the maximum, and is given by $\frac{n}{4}$.
8. Standard Deviation of a binomial distribution is given by $\sigma = \sqrt{npq}$.

**Question 6**

If $X \sim B(n, p)$, what would be the greatest value of the variance of $x$ when $n = 16$?

(a) 2 (b) 4 (c) 8 (d) $\sqrt{5}$

**Solution**

(b)

In a binomial distribution, the value of the variance is maximum when $p = q = 0.5$.

Variance is given by $n/4 = 16/4 = 4$.

## Question 7

What is the standard deviation of the number of recoveries among 48 patients when the probability of recovering is 0.75?

(a) 36 (b) 81 (c) 9 (d) 3

## Solution

(d)

Here, $n$ = 48; $p$ = 0.75

Standard Deviation of a Binomial Distribution = $\sigma = \sqrt{npq} = \sqrt{48 \times 0.75 \times 0.25} = 3$

## Question 8

What is the number of trials of a binomial distribution having mean and SD as 3 and 1.5 respectively?

(a) 2 (b) 4 (c) 8 (d) 12

## Solution

(d)

Mean of a Binomial Distribution is given by $np = 3$

SD of a Binomial Distribution is given by $\sqrt{npq} = 1.5$

Putting the value of $np = 3$ above, we get $\sqrt{3q} = 1.5$

$$\Rightarrow \left(\sqrt{3q}\right)^2 = (1.5)^2$$

$$\Rightarrow 3q = 2.25$$

$$\Rightarrow q = \frac{2.25}{3} = 0.75$$

If $q$ = 0.75, $p$ = 1 – 0.75 = 0.25

Therefore, we have $n \times 0.25 = 3$

$$\Rightarrow n = \frac{3}{0.25} = 12$$

**Question 9**

What is the mode of the distribution for which mean and SD are 10 and $\sqrt{5}$ respectively?

(a) 10 (b) 11 (c) 10 and 11 (d) None

**Solution**

(a)

Mean = $np = 10$

Standard Deviation = $\sqrt{npq} = \sqrt{5}$

Squaring both sides:

$$\left(\sqrt{npq}\right)^2 = \left(\sqrt{5}\right)^2$$

$$\Rightarrow 10q = 5$$

$$\Rightarrow q = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow p = \frac{1}{2}$$

Putting the value of $p$ from above in the equation $np = 10$

$$n \times \frac{1}{2} = 10$$

$$\Rightarrow n = 2 \times 10 = 20$$

Mode is dependent on the value of $(n+1)p$

$$(n+1)p = (20+1)\times\frac{1}{2} = \frac{21}{2} = 10.5$$

Since it is fractional, Mode is the largest integer contained in it.

Therefore, Mode = 10

9. Additive property of binomial distribution:

Let $x$ and $y$ be two independent binomial distributions where $x$ has the parameters $n_1$ and $p$, and $y$ has the parameters $n_2$ and $p$. Then $(x+y)$ will be a binomial distribution with parameters $(n_1+n_2)$ and $p$.

## Question 10

If $x$ and $y$ are 2 independent binomial variables with parameters 6 and ½ and 4 and ½ respectively, what is $P\left(x+y\geq 1\right)$?

(a) 1023/1024 (b) 1024/1023 (c) Both (d) None

## Solution

(a)

We have $n_1 = 6;\ n_2 = 4;\ p = \frac{1}{2}$

Let $z = x + y$

The parameters of $z$ will be: $n_1 + n_2 = 6 + 4 = 10$ and $p = \frac{1}{2}$

$$P(z \geq 1) = 1 - P(z < 1)$$

$$\Rightarrow P(z \geq 1) = 1 - P(z = 0)$$

$$\Rightarrow P(z \geq 1) = 1 - \left[ {}^{10}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{10} \right]$$

$$\Rightarrow P(z \geq 1) = 1 - 0.0009765625 = 0.9990234375$$

10. Sometimes, Binomial Distribution is also written as $B(n, p)$. So, if, in a question you find something like "$X \sim B(5, 0.4)$", it means that $n = 5$, and $p = 0.4$. Here, $X$ denotes the requirement of the question.

# Poisson Distribution

Poisson Distribution is used to find out the probability where the total no. of outcomes is too huge and the probability of success is extremely small. The probability is given by the following formula:

$$P(x) = \frac{e^{-m} \times m^{x}}{x!}, \text{ for } x = 0, 1, 2, 3, \ldots, n$$

Here,

$e$ = exponential constant = 2.71828

$m$ = mean = $np$

$x$ = the requirement of the question

Sometimes, $P(x)$ is also written as $f(x)$. $f(x)$ is called “Probability Mass Function”.

**Question 11**

If 1.5 per cent of items produced by a manufacturing unit are known to be defective, what is the probability that a sample of 200 items would contain no defective item?

(a) 0.05 (b) 0.15 (c) 0.20 (d) 0.22

**Solution**

(a)

Here $n = 200;\ p = \frac{1.5}{100} = 0.015$

Therefore, $m = np = 200 \times 0.015 = 3$

$$P\left(x=0\right)=\frac{\left(2.71828\right)^{-3}\times\left(3\right)^{0}}{0!}$$

$$P\left(x=0\right)=\frac{1}{\left(2.71828\right)^{3}}=0.0497$$

**Question 12**

If 2 per cent of electric bulbs manufactured by a company are known to be defectives, what is the probability that a sample of 150 electric bulbs taken from the production process of that company would contain more than 2 defective bulbs?

(a) 0.33 (b) 0.58 (c) 0.15 (d) None

**Solution**

(b)

Here, $n = 150;\ p = \frac{2}{100} = 0.02$

$m = np = 150 \times 0.02 = 3$

$$P(x)=\frac{e^{-m}.m^{x}}{x!}$$

$$P(x>2)=1-P(x\le 2)$$

$$P(x>2)=1-\left[P(x=0)+P(x=1)+P(x=2)\right]$$

$$P(x>2)=1-\left[\frac{2.71828^{-3}\times 3^{0}}{0!}+\frac{2.71828^{-3}\times 3^{1}}{1!}+\frac{2.71828^{-3}\times 3^{2}}{2!}\right]$$

$$P(x>2)\approx 0.58$$

## Question 13

The manufacturer of a certain electronic component is certain that two per cent of his product is defective. He sells the components in boxes of 120 and guarantees that not more than two per cent in any box will be defective. Find the probability that a box, selected at random, would fail to meet the guarantee? Given that $e^{-2.40} = 0.0907$.

(a) 0.43 (b) 0.58 (c) 0.15 (d) None

## Solution

(a)

Here, $n = 120;\ p = \dfrac{2}{100} = 0.02$

$$m = np = 120 \times 0.02 = 2.40$$

As per Poisson Distribution, $P(x) = \dfrac{e^{-m}.m^{x}}{x!}$

A box, selected at random would fail to meet the guarantee if more than 2.40 components turn out to be defective.

$$P(x > 2.40) = 1 - P(x \le 2.40)$$

$$P(x > 2.40) = 1 - \left[P(x=0) + P(x=1) + P(x=2)\right]$$

$$P(x > 2.40) = 1 - \left[\frac{e^{-2.40}.(2.40)^{0}}{0!} + \frac{e^{-2.40}.(2.40)^{1}}{1!} + \frac{e^{-2.40}.(2.40)^{2}}{2!}\right]$$

$$P\left(x>2.40\right)=1-\left[\frac{0.0907\times 1}{1}+\frac{0.0907\times 2.40}{1}+\frac{0.0907.\left(2.40\right)^{2}}{2}\right]$$

$$P\left(x>2.40\right)\approx 0.43$$

# Important Points

1. Poisson Distribution is applicable when the random variable ($x$) is discrete.
2. Since $e^{-m} = \frac{1}{e^m} > 0,$ whatever may be the value of $m$ (>0), it follows that $f(x) > 0$ for every $x$.
   Also, $\sum f(x) = f(0) + f(1) + f(2) + f(3) + \ldots + f(n) = 1.$
3. Poisson distribution is known as a uniparametric distribution as it is characterised by only one parameter $m$.
4. The mean of Poisson distribution is given by $m$, i.e., $\mu = m = np$.
5. The variance of Poisson distribution is given by $\sigma^2 = m = np$.
6. The standard deviation of Poisson distribution is given by $\sigma = \sqrt{m} = \sqrt{np}$.

**Question 14**

If the mean of a Poisson variable *x* is 1, what is *P* (*x* = takes the value at least 1)?

(a) 0.456 (b) 0.821 (c) 0.632 (d) 0.254

**Solution**

(c)

$$P(x) = \frac{e^{-m} \times m^{x}}{x!}$$

Here, we have $m = 1;\ x \geq 1$

$$P(x \geq 1) = 1 - P(x < 1)$$

$$= 1 - P(x = 0)$$

$$= 1 - \frac{(2.71828)^{-1}(1)^{0}}{0!}$$

$$= 1 - \frac{1}{2.71828}$$

$$= 0.6321$$

**Question 15**

For a Poisson variate $x$, $P$ ($x$ = 1) = $P$ ($x$ = 2). What is the mean of $x$?

(a) 1.00 (b) 1.50 (c) 2.00 (d) 2.50

**Solution**

(c)

$$P(x=1)=P(x=2)$$

$$\frac{e^{-m}m^1}{1!}=\frac{e^{-m}m^2}{2!}$$

$$m = \frac{m^2}{2}$$

$$2m = m^2$$

$$m = 2$$

## Question 16

Find the mean and standard deviation of $x$ where $x$ is a Poisson variate satisfying the condition $P(x=2)=P(x=3)$.

(a) 3.00 (b) 1.50 (c) 2.00 (d) 2.50

## Solution

(a)

$$P(x=2)=P(x=3)$$

$$\frac{e^{-m}.m^{2}}{2!}=\frac{e^{-m}.m^{3}}{3!}$$

$$\frac{m^2}{2} = \frac{m^3}{6}$$

$$6m^2 = 2m^3$$

$$\frac{6}{2} = \frac{m^3}{m^2}$$

$$m = 3$$

Therefore, mean = 3.

Standard Deviation $(\sigma) = \sqrt{m} = \sqrt{3}$

## Question 17

If the standard deviation of a Poisson variate $x$ is 2, what is $P\ (1.5 < x < 2.9)$?

(a) 0.231 (b) 0.158 (c) 0.15 (d) 0.144

## Solution

(d)

Standard Deviation = $\sqrt{m} = 2 \Rightarrow m = 4$

We know that $P(x) = \dfrac{e^{-m} \times m^{x}}{x!}$, for $x = 0, 1, 2, 3, \ldots, n$

Since $x$ can only take integral values, $1.5 < x < 2.9 \Rightarrow x = 2$.

Therefore, $P(x=2)=\dfrac{(2.71828)^{-4}\times 4^{2}}{2!}=\dfrac{16}{(2.71828)^{4}\times 2}=0.1465$

# Important Points (Contd.)

7. Like binomial distribution, Poisson distribution could be also unimodal or bimodal depending upon the value of the parameter $m$.
   a. If $m$ is an integer, there are two modes:
      i. $m$
      ii. $m - 1$
   b. If $m$ is a fraction, the mode is given by the largest integer contained in $m$.

**Question 18**

The probability that a random variable $x$ following Poisson Distribution would assume a positive value is $\left(1-e^{-2.7}\right)$. What is the mode of the distribution?

(a) 2 (b) 3 (c) 4 (d) None

**Solution**

(a)

Given

$$P\left(x>0\right)=1-e^{-2.7}$$

$$1-P\left(x\leq 0\right)=1-e^{-2.7}$$

$$1-P(x=0)=1-e^{-2.7}$$

$$P(x=0)=e^{-2.7}$$

$$\frac{e^{-m}m^{0}}{0!}=e^{-2.7}$$

$$e^{-m}=e^{-2.7}$$

$$m=2.7$$

Since $m$ is fractional, mode will be the largest value of integer contained in it. Therefore, mode = 2.

# Important Points (Contd.)

8. Poisson approximation to Binomial distribution

   When $n$ is rather large and $p$ is rather small so that $m = np$ is moderate then $B(n, p) \cong P(m)$.

9. Additive property of Poisson distribution:

   Let $x$ and $y$ be two independent poisson distributions where $x$ has the parameter $m_1$, and $y$ has the parameter $m_2$. Then $(x+y)$ will be a poisson distribution with parameter $(m_1 + m_2)$.

# Normal or Gaussian Distribution

$$P(x) = f(x) = \frac{1}{\sigma\sqrt{2\pi}}.e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}, \text{ for } -\infty < x < \infty$$

Here,

$e$ = exponential constant = 2.71828

$x$ = random variable

$\mu$ = mean of the normal random variable $x$

$\sigma$ = standard deviation of the given normal distribution

Sometimes, $P(x)$ is also written as $f(x)$. $f(x)$ is called "Probability Density Function".

**Question 19**

What is the coefficient of variation of $x$, characterised by the following probability density function: $f\left(x\right)=\frac{1}{4\sqrt{2\pi}}e^{-\left(x-10\right)^2/32}$ for $-\infty < x < \infty$?

(a) 50 (b) 60 (c) 40 (d) 30

**Solution**

(c)

The standard form is $P\left(x\right)=f\left(x\right)=\frac{1}{\sigma\sqrt{2\pi}}.e^{\left(\frac{-\left(x-\mu\right)^2}{2\sigma^2}\right)}$

Given: $f(x)=\frac{1}{4\sqrt{2\pi}}e^{-(x-10)^2/32}$

The breakup of the power of $e$ is analysed as follows:

| **Standard Formula** | **Formula as per Question** |
|---|---|
| $\frac{-(x-\mu)^2}{2\sigma^2}$ | $\frac{-(x-10)^2}{32}$ |
| $\Rightarrow \frac{-1}{2\sigma^2}\times(x-\mu)^2$ | $\Rightarrow \frac{-1}{32}\times(x-10)^2$ |

Comparing the given equation with the standard form, we have $\mu=10$.

Also, we have $-\frac{1}{32} = -\frac{1}{2\sigma^2}$

$$2\sigma^2 = 32$$

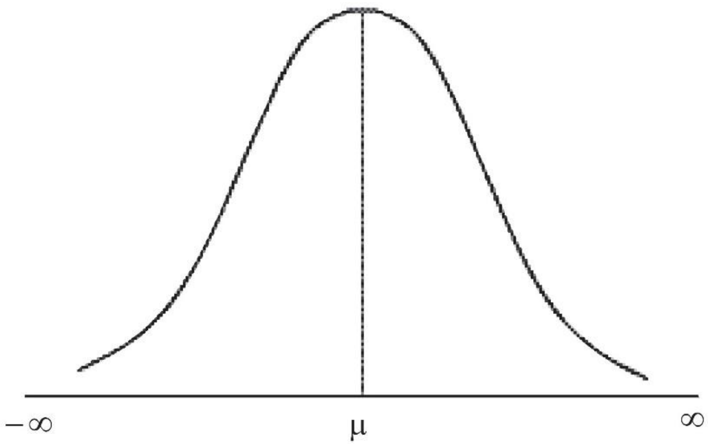
$$\sigma^2 = 16$$

$$\sigma = 4$$

$$CV = \frac{SD}{AM} \times 100 = \frac{4}{10} \times 100 = 40$$

# Important Points

1. Normal Distribution is applicable when the random variable ($x$) is continuous.
2. If we plot the probability function $y = f(x)$, then the curve, known as probability curve, takes the following shape:




The area under this curve gives us the probability.

3. The area between $-\infty$ and $\mu$ = the area between $\mu$ and $\infty = 0.5$

4. If $\mu = 0$, and $\sigma = 1$, we have $f(z) = \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-z^2}{2}\right)}$, for $-\infty < z < \infty$.

   The random variable $z$ is known as standard normal variate (or variable) or standard normal deviate. It is given by $z = \frac{x-\mu}{\sigma}$.

5. Normal distribution is bell shaped.
6. It is unimodal.
7. The normal distribution is known as biparametric distribution as it is characterised by two parameters $\mu$ and $\sigma^2$. Once the two parameters are known, the normal distribution is completely specified.
8. Since the normal distribution is symmetrical about its mean ($\mu$), Mean = Median = Mode.

9. Relationship between MD, SD, and QD → $4SD = 5MD = 6QD$

10. Mean Deviation = $0.8\sigma$.

11. Quartile Deviation = $0.675\sigma$.

**Question 20**

If the two quartiles of $N\left(\mu, \sigma^2\right)$ are 14.6 and 25.4 respectively, what is the standard deviation of the distribution?

(a) 9 (b) 6 (c) 10 (d) 8

**Solution**

(d)

We know that Quartile Deviation = 0.675$\sigma$

Quartile Deviation = $\dfrac{Q_3 - Q_1}{2} = \dfrac{25.4 - 14.6}{2} = 5.4$

Therefore, $\sigma = \dfrac{QD}{0.675} = \dfrac{5.4}{0.675} = 8$

**Question 21**

The quartile deviation of a normal distribution with mean 10 and SD 4 is:

(a) 0.675 (b) 67.50 (c) 2.70 (d) 3.20

**Solution**

(c)

QD = 0.675SD

QD = $0.675 \times 4 = 2.70$

**Question 22**

If the mean deviation of a normal variable is 16, what is its quartile deviation?

(a) 10 (b) 13.5 (c) 15 (d) 12.05

**Solution**

(b)

We know that $4SD = 5MD = 6QD$

$5 \times 16 = 6\text{QD}$

$\text{QD} = 80 \div 6 = 13.33$

**Question 23**

The mean deviation about median of a standard normal variate is:

(a) $0.675\sigma$ (b) 0.675 (c) $0.80\sigma$ (d) 0.80

**Solution**

(d)

A standard normal random variable is a normally distributed random variable with mean $\mu = 0$ and standard deviation $\sigma = 1$.

Therefore, mean deviation = $0.80 \times 1 = 0.80$

**Question 24**

If the quartile deviation of a normal curve is 4.05, then its mean deviation is:

(a) 5.26 (b) 6.24 (c) 4.24 (d) 4.80

**Solution**

(d)

4SD = 5MD = 6QD

5MD = 6QD

MD = 6QD/5 = $(6 \times 4.05)/5 = 4.86$

# Important Points (Contd.)

12. $Q_1$ and $Q_3$ are equidistant from the median, therefore,

    i. $Q_1 = \mu - 0.675\sigma$, and

    ii. $Q_3 = \mu + 0.675\sigma$

**Question 25**

What is the first quartile of *x* having the following probability density function?

$$f(x) = \frac{1}{\sqrt{72\pi}} e^{-(x-10)^2/72} \text{ for } -\infty < x < \infty$$

(a) 4 (b) 5 (c) 5.95 (d) 6.75

**Solution**

(c)

$$Q_1 = \mu - 0.675\sigma$$

The standard format is $P(x) = f(x) = \frac{1}{\sigma\sqrt{2\pi}}.e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}$

Comparing this with the given expression, we have $\mu = 10$.

Also, we have $-\frac{1}{72} = -\frac{1}{2\sigma^2}$

$2\sigma^2 = 72$

$\sigma^2 = 36$

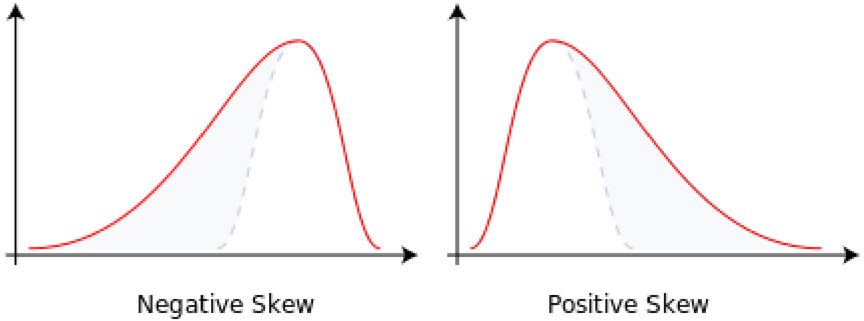
$\sigma = 6$

$Q_1 = \mu - 0.675\sigma = 10 - (0.675 \times 6) = 5.95$

# Important Points (Contd.)

13. Median $- Q_1 = Q_3 -$ Median.

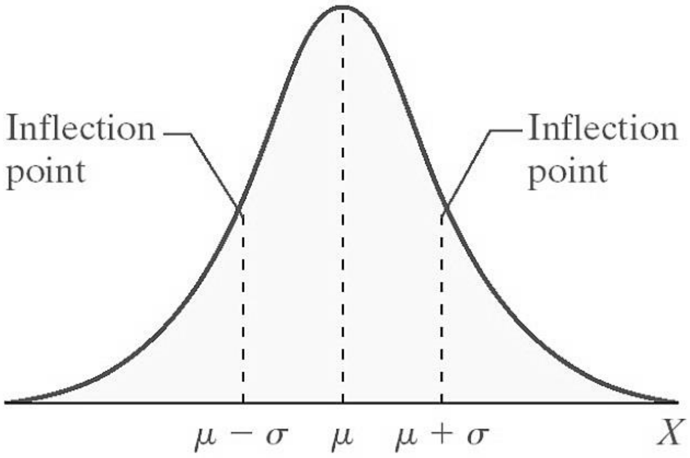
14. The normal distribution is symmetric about Therefore, its skewness is zero, i.e., the curve is neither tilted towards right (negatively skewed), nor towards left (positively skewed).



15. Points of inflexion – A normal curve has two inflexion points, i.e., the points where the curve changes its shape from concave to convex, and from convex to concave. These two points are given by:

i. $x = \mu - \sigma$, and

ii. $x = \mu + \sigma$



## Question 26

Find the points of inflexion of the normal curve $f(x) = \frac{1}{4\sqrt{2\pi}}.e^{-(x-10)^2/32}$ for $-\infty < x < \infty$.

(a) 6 and 14 (b) 7 and 15 (c) 8 and 16 (d) None

## Solution

(a)

The standard normal density function is given by $P(x) = f(x) = \frac{1}{\sigma\sqrt{2\pi}}.e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}$.

Given: $f(x)=\frac{1}{4\sqrt{2\pi}}.e^{-(x-10)^2/32}$

The breakup of the power of *e* is analysed as follows:

| **Standard Formula** | **Formula as per Question** |
|---|---|
| $\frac{-(x-\mu)^2}{2\sigma^2}$ | $\frac{-(x-10)^2}{32}$ |
| $\Rightarrow \frac{-1}{2\sigma^2}\times(x-\mu)^2$ | $\Rightarrow \frac{-1}{32}\times(x-10)^2$ |

Comparing it with the standard formula, we have:

$$(x-\mu)^2=(x-10)^2$$

Therefore, Mean $(\mu)=10$

Also, we have $\frac{-1}{2\sigma^2} = \frac{-1}{32}$

$$\Rightarrow 2\sigma^2 = 32$$

$$\Rightarrow \sigma^2 = \frac{32}{2} = 16$$

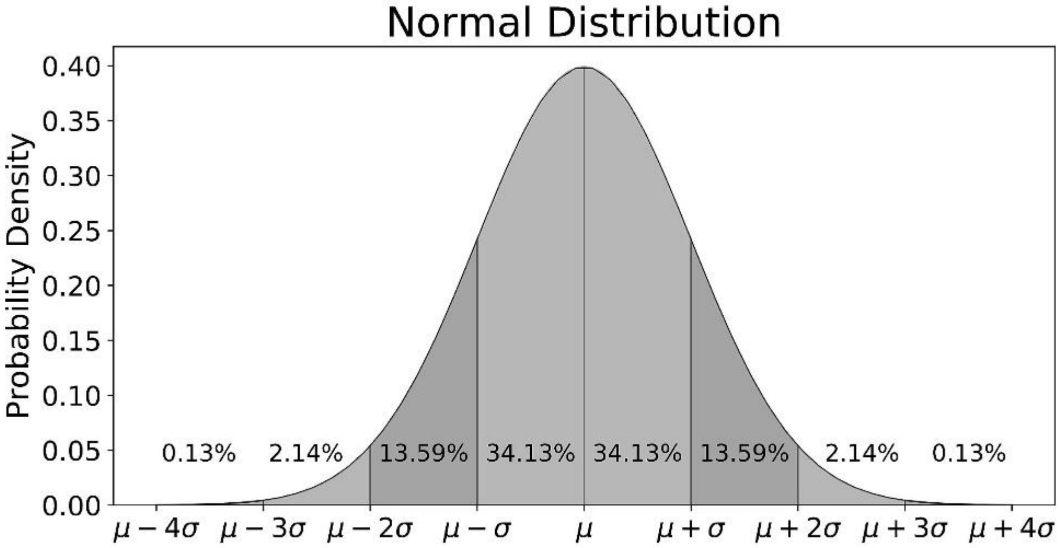
$$\Rightarrow \sigma = \sqrt{16} = 4$$

We know that points of inflexion are given by $\mu - \sigma$ and $\mu + \sigma$

Therefore, $\mu - \sigma = 10 - 4 = 6$; $\mu + \sigma = 10 + 4 = 14$

# Important Points (Contd.)

16. In a normal distribution, $\mu \pm 1\sigma$ covers 68.27% of area, $\mu \pm 2\sigma$ covers 95.45% of area, and $\mu \pm 3\sigma$ covers 99.73% of area.



## Question 27

The interval $\left(\mu - 3\sigma,\ \mu + 3\sigma\right)$ covers:

(a) 95% area of a normal distribution
(b) 96% area of a normal distribution
(c) 99% area of a normal distribution
(d) all but 0.27% area of a normal distribution

## Solution

(d)

# Important Points (Contd.)

17. Under a normal distribution, the area enclosed between mean ($\mu$) and $1\sigma$ is 0.34135; mean and $2\sigma$ is 0.47725; and mean and $3\sigma$ is 0.49865.

18. In case of normal distribution
    i. Highest Value = Mean + Half of Range, and
    ii. Lowest Value = Mean – Half of Range

19. Normal Distribution with $X = 0$, and $\sigma = 1$ is known as Standard Normal Distribution.

20. The height of normal curve is maximum at the Mean Value.

21. Additive Property: If there are two Independent Normal Distributions $x \sim N\left(\mu_1, \sigma_1^{\,2}\right)$ and $y \sim N\left(\mu_2, \sigma_2^{\,2}\right)$, then $z = x + y$ follows normal distribution with mean $\left(\mu_1 + \mu_2\right)$ and $SD = \sqrt{\sigma_1^{\,2} + \sigma_2^{\,2}}$ respectively.

**Question 28**

$x$ and $y$ are independent normal variables with mean 100 and 80 respectively and standard deviation as 4 and 3 respectively. What is the distribution of $(x + y)$?

(a) 180; 5 (b) 190; 10 (c) 180; 10 (d) None

**Solution**

(a)

Mean of $z$ = 100 + 80 = 180

$$SD = \sqrt{{\sigma_1}^2 + {\sigma_2}^2}$$

$$SD = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

**Question 29**

If $x$ and $y$ are 2 independent normal variables with mean as 10 and 12 and SD as 3 and 4, then $(x + y)$ is normally distributed with:

(a) Mean = 22 and SD = 7
(b) Mean = 22 and SD = 25
(c) Mean = 22 and SD = 5
(d) Mean = 22 and SD = 49

**Solution**

(c)

Mean = 10 + 12 = 22

$$SD = \sqrt{\sigma_1^{\ 2} + \sigma_2^{\ 2}}$$

$$SD = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

# Problems on Finding Probability Through Graph

**Question 30**

$x$ follows normal distribution with mean as 50 and variance as 100. What is $P\left(x \geq 60\right)$? Given $\phi\left(1\right) = 0.8413$.

(a) 0.16 (b) 0.26 (c) 0.36 (d) None

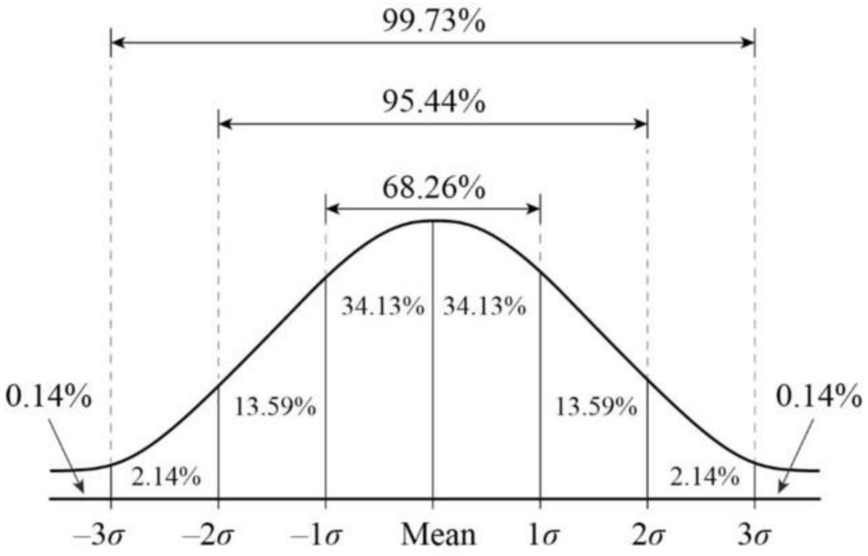
**Solution**

(a)

Given: $\mu = 50$; $\sigma = \sqrt{100} = 10$

For $x = 60$, $Z = \dfrac{x-\mu}{\sigma} = \dfrac{60-50}{10} = 1$

$$P(x \geq 60) = P(Z \geq 1)$$




Z = –3 Z = –2 Z = –1 Z = 0 Z = 1 Z = 2 Z = 3

As can be seen from the above diagram, the area to the right side of Z = 1 is 13.59 + 2.14 + 0.14 = 15.87% ≈ 16% or 0.16

However, this is not always so straightforward and simple. The $\phi(1) = 0.8413$ given in the question denotes the area from the left end to $Z$ = 1; and we know that the total area of the graph is 1. So, if we subtract 0.8413 from 1, we'll get the desired area.

Therefore, 1 – 0.8413 = 0.1587 ≈ 0.16

**Question 31**

In a sample of 500 workers of a factory, the mean wage and SD of wages are found to be ₹500 and ₹48 respectively. Find the number of workers having wages more than ₹600. Given that $\phi(2.08) = 0.9812$

(a) 0.0188 (b) 9 (c) 10 (d) None

**Solution**

(b)

Given: $\mu = 500$; $\sigma = 48$

$$Z = \frac{x - \mu}{\sigma}$$

For $x = 600$; $Z = \dfrac{600-500}{48} = 2.08$

$$P(x > 600) = P(Z > 2.08) = 1 - \phi(2.08) = 1 - 0.9812 = 0.0188$$

Therefore, number of workers $= 0.0188 \times 500 = 9.4 \approx 9$

## Question 32

In a sample of 500 workers of a factory, the mean wage and SD of wages are found to be ₹500 and ₹48 respectively. Find the number of workers having wages less than ₹450. Given that $\phi(1.04) = 0.8508$.

(a) 0.1492 (b) 75 (c) 10 (d) None

## Solution

(b)

Given: $\mu = 500$; $\sigma = 48$

$$Z = \frac{x - \mu}{\sigma}$$

For $x = 450$; $Z = \dfrac{450-500}{48} = -1.04$

Since the graph is symmetrical, $\phi(-k) = 1-\phi(k)$

$$P(x<450) = P(z<-1.04) = \phi(-1.04) = 1-\phi(0.14) = 1-0.8505 = 0.1492$$

Therefore, number of workers = $0.1492 \times 500 = 74.6 \approx 75$

**Question 33**

In a sample of 500 workers of a factory, the mean wage and SD of wages are found to be ₹500 and ₹48 respectively. Find the number of workers having wages between ₹548 and ₹600. Given that $\phi(2.08)=0.9812$; $\phi(1)=0.8413$.

(a) 70 (b) 75 (c) 0.1399 (d) None

**Solution**

(a)

Given: $\mu=500$; $\sigma=48$

$$Z=\frac{x-\mu}{\sigma}$$

For $x = 548$; $Z = \dfrac{548-500}{48} = 1$

For $x = 600$; $Z = \dfrac{600-500}{48} = 2.08$

$$P(548 < x < 600) = P(1 < z < 2.08) = \phi(2.08) - \phi(1) = 0.9812 - 0.8413 = 0.1399$$

Therefore, number of workers = $0.1399 \times 500 = 69.95 \approx 70$

## Question 34

For a normal distribution with mean as 500 and SD as 120, what is the value of *k* so that the interval [500, *k*] covers 40.32 percent area of the normal curve? Given $\phi(1.30) = 0.9032$.

(a) 740 (b) 750 (c) 656 (d) 800

## Solution

(c)

Given: $\mu = 500$; $\sigma = 120$

$$Z = \frac{x - \mu}{\sigma}$$

The interval given is [500, $k$]. 500 is the mean. $Z$ = 0 at mean. Hume isse right side ka thoda sa area chahiye, jisse mean se le ke us area tak ka area 40.32% ho jaaye. Hume $\phi(1.30) = 0.9032$ diya hua hai. Isme se agar $-\infty$ se z = 0 tak ka area minus karen, toh 0.4032 aayega…aur wohi toh hume chahiye.

Matlab, jo $z$ aayega, wohi $\phi$ diya hua hai. Matlab $z$ 1.30 aayega.

$$Z = \frac{x - \mu}{\sigma}$$

$$1.30 = \frac{k - 500}{120}$$

Try the options.