Marathon 4

CA Nishant Kumar

Chapter 1 – Ratio, Proportion, Indices, Logarithms

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S. No.	Ratio	Proportion	Indices	Logarithms
1.	Ratio exists only between quantities of same kind.	Cross Product Rule If $\frac{a}{b}=\frac{c}{d}$, then $ad=bc$.	$a^n = a \times a \times a \times a \times \ldots \times a$ (n times)	$2^3=8$ is expressed in terms of Logarithms as $\log_2 8=3$. It is read as log 8 to the base 2 is 3.
2.	Quantities to be compared must be in the same units.	Invertendo If $\frac{a}{b}=\frac{c}{d}$, then $\frac{b}{a}=\frac{d}{c}$.	$a^{-n}=\frac{1}{a^n}$	$\log_a 1=0$
3.	To compare ratios, use calculator.	Alternendo	$a^0 = 1$	$\log_a a=1$

		If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a}{c}=\frac{b}{d}$, or, $\frac{d}{b}=\frac{c}{a}$		
4.	If a quantity increases or decreases in the ratio $a : b$, then new quantity = b of the original quantity/a. The fraction by which the original quantity is multiplied to get a	Componendo If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{b}=\frac{c+d}{d}$.	$a^m \times a^n = a^{m+n}$	$\log_a(mn) = \log_a m + \log_a n$

	new quantity is called the factor multiplying ratio. (This is basically unitary method.)			
5.	**Inverse Ratio** – The inverse ratio of a/b is b/a.	Dividendo If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a-b}{b}=\frac{c-d}{d}$	$\frac{a^m}{a^n}=a^{m-n}$	$\log_a\left(\frac{m}{n}\right)=\log_a m-\log_a n$
6.	**Compound Ratio** – The multiplication of two or more ratios	Componendo and Dividendo	$\left(a^m\right)^n=a^{mn}$ $=\left(a^n\right)^m$	$\log_a\left(m^n\right)=n\log_a m$

	is called compound ratio. The compound ratio of $a : b$ and $c : d$ is $ac : bd$.	If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.		
7.	**Duplicate Ratio** – A ratio compounded of itself is called a Duplicate Ratio. The duplicate ratio of $a : b$ is $a^2 : b^2$.	Addendo If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$, then each of these ratios is equal to $\frac{a+c+e+\ldots}{b+d+f+\ldots}$, i.e.,	$(ab)^n = a^n b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\log_a m = \frac{\log_b m}{\log_b a}$

		$\frac{a}{b}=\frac{a+c+e+\ldots}{b+d+f+\ldots};$ $\frac{c}{d}=\frac{a+c+e+\ldots}{b+d+f+\ldots};$ $\frac{e}{f}=\frac{a+c+e+\ldots}{b+d+f+\ldots}.$		
8.	**Sub-Duplicate Ratio** – The sub-duplicate ratio of $a : b$ is $\sqrt{a} : \sqrt{b}$.	Subtrahendo If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots,$ then each of these ratios is equal to $\frac{a-c-e-\ldots}{b-d-f-\ldots}$, i.e.,	$a^{m/n}=\left(a^{m}\right)^{1/n},$ i.e., $a^{m/n}=\sqrt[n]{a^{m}}$ $=\left(\sqrt[n]{a}\right)^{m}$	$\frac{1}{\log_a m}=\log_m a$

		$\frac{a}{b}=\frac{a-c-e-\ldots}{b-d-f-\ldots};$ $\frac{c}{d}=\frac{a-c-e-\ldots}{b-d-f-\ldots};$ $\frac{e}{f}=\frac{a-c-e-\ldots}{b-d-f-\ldots}$		
9.	**Triplicate Ratio –** The triplicate ratio of $a : b$ is $a^3 : b^3$.			$a^{\log_a n}=n$
10.	**Sub-Triplicate Ratio –** The sub-			$\log_{a^q} n^p=\frac{p}{q}\log_a n$

	triplicate ratio of a : b is $\sqrt[3]{a} : \sqrt[3]{b}$.			

Question

If A : B = 3 : 5, B : C = 5 : 4, C : D = 2 : 3, and D is 50% more than E, find the ratio between A and E.

(a) 2 : 3 (b) 3 : 4 (c) 3 : 5 (d) 4 : 5

(MTP November, 2021)

Question

Find the value of $\sqrt{6561}+\sqrt[4]{6561}+\sqrt[8]{6561}$

(a) 81 (b) 93 (c) 121 (d) 243

(MTP November, 2021)

Question

Find the value of $\log\frac{x^n}{y^n}+\log\frac{y^n}{z^n}+\log\frac{z^n}{x^n}$.

(a) –1 (b) 0 (c) 1 (d) 2

(MTP November, 2021)

Question

If $\frac{8^n \times 2^3 \times 16^{-1}}{2^n \times 4^2} = \frac{1}{4}$, then the value of n

(a) 1 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

(MTP November, 2021)

Question

If $\log_{10} 5 + \log_{10}(5x+1) = \log_{10}(x+5) + 1$, then x is equal to:

(a) 1 (b) 3 (c) 5 (d) 10

(MTP November, 2021)

Question

If $xy + yz + zx = -1$, then the value of $\left(\frac{x+y}{1+xy} + \frac{z+y}{1+zy} + \frac{x+z}{1+zx}\right)$ is:

(a) xyz (b) $-\frac{1}{yz}$ (c) $\frac{1}{xyz}$ (d) $\frac{1}{x+y+z}$

(MTP November, 2021; July, 2021)

Question

The salaries of *A*, *B* and *C* are in the ratio 2 : 3 : 5. If increments of 15%, 10% and 20% are allowed respectively to their salary, then what will be the new ratio of their salaries?

(a) 23 : 33 : 60 (b) 33 : 23 : 60 (c) 23 : 60 : 33 (d) 33 : 60 : 23

(MTP November, 2021)

Question

If $A : B = 5 : 3$, $B : C = 6 : 7$, and $C : D = 14 : 9$, then the value of $A : B : C : D$ is:

(a) 20 : 14 : 12 : 9 (b) 20 : 9 : 12 : 14 (c) 20 : 9 : 14 : 12 (d) 20 : 12 : 14 : 9

(MTP November, 2021)

Question

X and *Y* have their present ages in the ratio 6 : 7. 14 years ago, the ratio of the ages of the two was 4 : 5. What will be the ratio of their ages 21 years from now?

(a) 7 : 11 (b) 9 : 10 (c) 8 : 11 (d) 11 : 13

(MTP November, 2021)

Question

If $x = \sqrt{3} + \frac{1}{\sqrt{3}}$, then $\left(x - \frac{\sqrt{126}}{\sqrt{42}} \right) \left(x - \frac{1}{x - \frac{2\sqrt{3}}{3}} \right) = ?$

(a) 5/6 (b) 6/5 (c) 2/3 (d) –3/5

(November, 2019)

Question

Find the value of a from the following: $\left(\sqrt{9}\right)^{-5} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$

(a) 11 (b) 13 (c) 15 (d) 17

(November, 2020)

Question

If $\log_a(ab) = x$, then $\log_b(ab) = ?$

(a) $1/x$ (b) $\frac{x}{1+x}$ (c) $\frac{x}{x-1}$ (d) None

(January, 2021)

Question

A vessel contained a solution of acid and water in which water was 64%. Four litres of the solution were taken out of the vessel and the same quantity of water was added. If the resulting solution contains 30% acid, the quantity (in litres) of the solution, in the beginning in the vessel, was:

(a) 12 (b) 36 (c) 24 (d) 27

(July, 2021)

Question

If $\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$, then the value of x is:

(a) 64 (b) 4 (c) 16 (d) 2

Question

If $x^2 + y^2 = 7xy$, then $\log \frac{1}{3}(x + y) = ?$

(a) $(\log x + \log y)$ (b) $\frac{1}{2}(\log x + \log y)$ (c) $\frac{1}{3}(\log x + \log y)$ (d) $3(\log/ \log y)$

Question

Value of $\left[9^{n+\frac{1}{4}} . \frac{\sqrt{3.3^n}}{3.\sqrt{3^{-n}}} \right]^{\frac{1}{n}}$

(a) 9 (b) 27 (c) 81 (d) 3

Question

If $3^x = 5^y = 75^z$, then:

(a) $x + y - z = 0$ (b) $\frac{2}{x} + \frac{1}{y} = \frac{1}{z}$ (c) $\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$ (d) $\frac{2}{x} + \frac{1}{z} = \frac{1}{y}$

Question

A bag contains ₹187 in the form 1 rupee, 50 paise and 10 paise coins in the ratio 3:4:5. Find the number of each type of coins.

(a) 102, 136, 170 (b) 136, 102, 170 (c) 170, 102, 136 (d) None

Question

$\log_e x + \log(1+x) = 0$ is equivalent to:

(a) $x^2 + x + e = 0$ (b) $x^2 + x - e = 0$ (c) $x^2 + x + 1 = 0$ (d) $x^2 + x - 1 = 0$

Question

If $x = 3^{1/4} + 3^{-1/4}$, and $y = 3^{1/4} - 3^{-1/4}$, then the value of $3\left(x^2 + y^2\right)^2$ will be:

(a) 12 (b) 18 (c) 46 (d) 64

Question

Find the value of $(x+y)$, if $\left(x+\frac{y^3}{x^2}\right)^{-1}-\left(\frac{x^2}{y}+\frac{y^2}{x}\right)^{-1}+\left(\frac{x^3}{y^2}+y\right)^{-1}=\frac{1}{3}$.

(a) 1/3 (b) 3 (c) ½ (d) 2

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Question

If $pqr = a^x$, $qrs = a^y$, $rsp = a^z$, then find the value of $(pqrs)^{1/2}$.

(a) a^{x+y+z} (b) $a^{\sqrt{x+y+z}}$ (c) $a^{\sqrt[4]{x+y+z}}$ (d) $\left(a^{x+y+z}\right)^{1/4}$

Question

The ratio of the earnings of two persons 3:2. If each saves $1/5^{th}$ of their earnings, the ratio of their savings is:

(a) 2 : 3 (b) 3 : 2 (c) 4 : 5 (d) 5 : 4

Question

If $x = 5^{1/3} + 5^{-1/3}$, then $5x^3 - 15x$ is given by:

(a) 25 (b) 26 (c) 27 (d) 30

Question

The value of $\log_5\left(1+\frac{1}{5}\right)+\log_5\left(1+\frac{1}{6}\right)+-----+\log_5\left(1+\frac{1}{624}\right)$

(a) 2
(b) 3
(c) 5
(d) 0

Question

$\log_{2\sqrt{2}}(512) : \log_{3\sqrt{2}} 324 =$

(a) 128 : 81
(b) 2 : 3
(c) 3 : 2
(d) None

Question

$\log_{0.01} 10,000$

(a) 2 (b) –2 (c) 4 (d) –4

Question

The value of $\frac{64\left(b^4a^3\right)^6}{\left[4\left(a^3b\right)^2\times\left(ab\right)^2\right]}$

(a) $16a^{10}b^{20}$ (b) $4a^{20}b^{10}$ (c) $8a^{10}b^{20}$ (d) $4a^{10}b^{20}$

(MTP June, 2023)

Question

Four persons A, B, C, D wish to share a sum in the ratio of 5:2:4:3. If D gets ₹1,000 less than C, then the share of B?

(a) ₹2,000 (b) ₹1,200 (c) ₹2,400 (d) ₹3,000

(December, 2022; MTP June, 2023)

Question

The mean proportional between $12x^2$ and $27y^2$ is:

(a) $18xy$ (b) $81xy$ (c) $8xy$ (d) $9xy$

(MTP June, 2023)

Question

If thrice of A's age 6 years ago be subtracted from twice his present age, the result would be equal to his present age. Find A's present age.

(a) 7 (b) 8 (c) 9 (d) 6

(MTP June, 2023)

Question

If $\log_3 4.\log_4 5.\log_5 6.\log_6 7.\log_7 8.\log_8 9 = x$, then find the value of x.

(a) 4 (b) 2 (c) 3 (d) 1

(MTP June, 2023)

Question

If $\frac{1}{2}\log_{10} 4 = y$, and if $\frac{1}{2}\log_{10} 9 = x$, then find the value of $\log_{10} 15$.

(a) $x - y + 1$ (b) $x + y - 1$ (c) $x + y + 1$ (d) $y - x + 1$

(MTP June, 2023)

Question

In a hostel, ration is stocked for 400 students upto 31 days. After 28 days 280 students were vacated the hostel. Find the number of days for which the remaining ration will be sufficient for the remaining students.

(a) 5 (b) 4 (c) 7 (d) 10

(December, 2022; MTP June, 2023)

Question

Two vessels containing water and milk in the ratio 2 : 3 and 4 : 5 are mixed in the ratio 1 : 2. The ratio of milk and water in the resulting mixture is:

(a) 58 : 77 (b) 77 : 58 (c) 68 : 77 (d) None

(MTP June, 2023)

Question

If $(x-9):(3x+6)$ is the duplicate ratio of 4 : 9, find the value of x.

(a) $x = 9$ (b) $x = 16$ (c) $x = 36$ (d) $x = 25$

(MTP June, 2023)

Question

Value of $\left(a^{1/8}+a^{-1/8}\right)\left(a^{1/8}-a^{-1/8}\right)\left(a^{1/4}+a^{-1/4}\right)\left(a^{1/2}+a^{-1/2}\right)$ is:

(a) $a+\frac{1}{a}$ (b) $a-\frac{1}{a}$ (c) $a^2+\frac{1}{a^2}$ (d) $a^2-\frac{1}{a^2}$

(MTP June, 2023)

Question

If $(25)^{150} = (25x)^{50}$, then the value of x will be:

(a) 5^3 (b) 5^4 (c) 5^2 (d) 5

(MTP June, 2023)

Solution

(b)

$$(25)^{150} = (25x)^{50}$$

$$\Rightarrow 25^{150} = 25^{50} \times x^{50}$$

$$\Rightarrow x^{50} = \frac{25^{150}}{25^{50}}$$

$\Rightarrow x^{50} = 25^{150-50}$

$\Rightarrow x^{50} = 25^{100}$

$\Rightarrow x^{50} = \left(5^2\right)^{100}$

$\Rightarrow x^{50} = 5^{200}$

Now, try the options.

Option (b) $\rightarrow 5^4$

LHS: $\left(5^4\right)^{50} = 5^{4\times 50} = 5^{200} = \text{RHS}$

Therefore, option (b) is the answer.

Question

$7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)$ is equal to:

(a) 0 (b) 1 (c) $\log 2$ (d) $\log 3$

(MTP June, 2023)

Solution

(c)

$7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)$

$$\Rightarrow \log\left(\frac{16}{15}\right)^{7} + \log\left(\frac{25}{24}\right)^{5} + \log\left(\frac{81}{80}\right)^{3}$$

$$\Rightarrow \log\left(\frac{16^{7}}{15^{7}}\right) + \log\left(\frac{25^{5}}{24^{5}}\right) + \log\left(\frac{81^{3}}{80^{3}}\right)$$

$$\Rightarrow \log\left(\frac{16^{7}}{15^{7}} \times \frac{25^{5}}{24^{5}} \times \frac{81^{3}}{80^{3}}\right)$$

$$\Rightarrow \log 2$$

Question

$\log_4\left(x^2+x\right)-\log_4\left(x+1\right)=2$. Find x.

(a) 16 (b) 0 (c) –1 (d) None

(MTP June, 2023)

Solution

(a)

$\log_4\left(x^2+x\right)-\log_4\left(x+1\right)=2$

$\Rightarrow \log_4\left(\frac{x^2+x}{x+1}\right)=2$

$$\Rightarrow \log_4\left(\frac{x(x+1)}{x+1}\right)=2$$

$$\Rightarrow \log_4 x=2$$

$$\Rightarrow x=4^2=16$$

Chapter 3 – Linear Inequalities

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Question

On the average, an experienced person does 7 units of work while a fresh one work 5 units of work daily but the employer has to maintain an output of at least 35 units of work per day. The situation can be expressed as:

(a) $7x + 5y < 35$ (b) $7x + 5y \leq 35$ (c) $7x + 5y > 35$ (d) $7x + 5y \geq 35$

Question

The solution space of the inequalities $2x + y \leq 10$ and $x - y \leq 5$:

1. Includes origin
2. Includes the point (4, 3)

Which one is correct:

(a) Only 1 (b) Only 2 (c) Both 1 and 2 (d) None

Question

The solution of the inequality $\frac{(5-2x)}{3} \leq \frac{x}{6} - 5$ is:

(a) $x \geq 8$ (b) $x \leq 8$ (c) $x = 8$ (d) None

Question

On the average, an experienced person does 5 units of work while a fresh one work 3 units of work daily but the employer has to maintain an output of at least 30 units of work per day. The situation can be expressed as:

(a) $5x + 3y \leq 30$ (b) $5x + 3y \geq 30$ (c) $5x + 3y = 30$ (d) None

Question

The solution set of the inequation $x+2>0$ and $2x-6>0$ is:

(a) $(-2,\infty)$ (b) $(3,\infty)$ (c) $(-\infty,-2)$ (d) $(-\infty,-3)$

Question

A company produces two products A and B, each of which requires processing in two machines. The first machine can be used at most for 60 hours, the second machine can be used at most for 40 hours. The product A requires 2 hours on machine one and one hour on machine two. The product B requires one hour on machine one and two hours on machine two. Express above situation using linear inequalities.

(a) $2x + y \leq 60$ and $x + 2y \geq 40$

(b) $2x + y \geq 60$ and $x + 2y \geq 40$

(c) $2x + y \leq 60$ and $x + 2y \leq 40$

(d) $2x + y \geq 60$ and $x + 2y \leq 40$

Question

Mr. A plans to invest up to ₹30,000 in two stocks *X* and *Y*. Stock *X* (*x*) is priced at ₹175 and Stock *Y* (*y*) at ₹95 per share. This can be shown by:

(a) $175x + 95y < 30{,}000$ (b) $175x + 95y > 30{,}000$ (c) $175x + 95y = 30{,}000$ (d) None

Question

The solution of the inequality $8x + 6 < 12x + 14$ is:

(a) $(-2, 2)$ (b) $(0, -2)$ (c) $(2, \infty)$ (d) $(-2, \infty)$

Question

The rules and representations demand that employer should employ not more than 8 experienced leads to 1 fresh one and this fact can be expressed as:

(a) $y \geq x/8$ (b) $8y \leq x$ (c) $8y = x$ (d) $y = 8x$

Question

A manufacturer produces two items A and B. He has ₹10,000 to invest and a space to store 100 items. A table costs him ₹400 and a chair ₹100. Express this in the form of linear inequalities.

(a) $x + y \leq 100$, $4x + y \leq 100$, $x \geq 0$, $y \geq 0$
(b) $x + y \leq 1000$, $2x + 5y < 1000$, $x \geq 0$, $y \geq 0$
(c) $x + y > 100$, $4x + y \geq 100$, $x \geq 0$, $y \geq 0$
(d) None

Question

The common region in the graph of the inequalities $x + y \leq 4,\ x - y \leq 4,\ x \geq 2$ is

(a) Equilateral triangle
(b) Isosceles triangle
(c) Quadrilateral
(d) Square

Solution

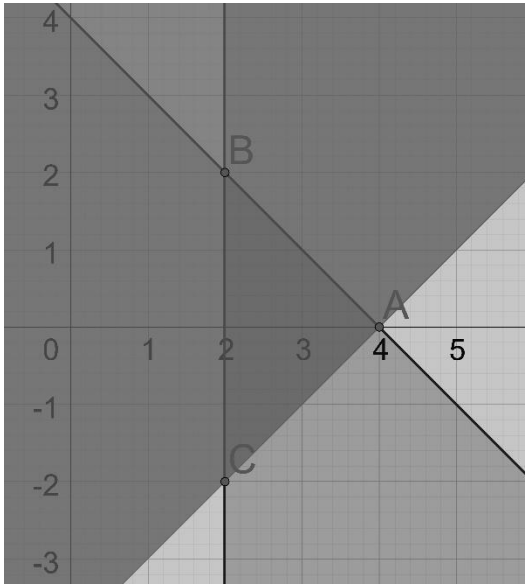
(b)

Inequalities graph for

$x + y \leq 4,$ $x - y \leq 4$ $x \geq 2$

x	0	4
y	4	0

x	0	4
y	–4	0



Common Area in the graph is Δ ABC

Clearly, it is an isosceles triangle.

Question

Solve for x of the Inequalities

$$2 \leq \frac{3x-2}{5} \leq 4 \text{ where } x \to N$$

(a) {5, 6, 7}
(b) {3, 4, 5, 6}
(c) {4, 5, 6}
(d) None

Solution

(d)

Given:

$$2 \le \frac{3x-2}{5} \le 4$$

Multiplying the entire equation with 5, we get:

$$(2\times 5) \le \left\{\frac{(3x-2)}{5}\times 5\right\} \le (4\times 5)$$

$$10 \le 3x-2 \le 20$$

Adding 2 to the entire equation, we get:

$$10+2 \le 3x-2+2 \le 20+2$$

$$12 \le 3x \le 22$$

Dividing the entire equation by 3, we get:

$$\frac{12}{3} \leq \frac{3x}{3} \leq \frac{22}{3}$$

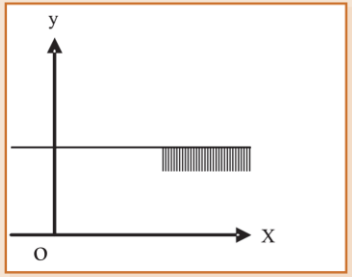
$$4 \leq \times \leq 7.33$$

So, solution set is $x = \{4, 5, 6, 7\}$

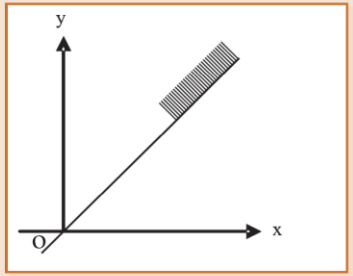
Question

(vii) The graph to express the inequality $y \leq \left(\frac{1}{2}\right) x$ is indicated by

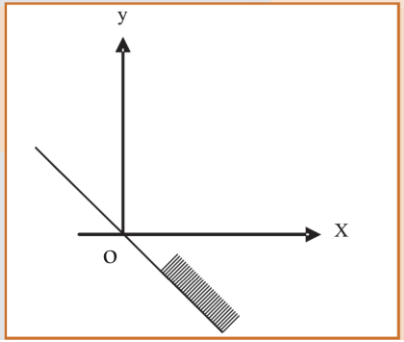
(a)



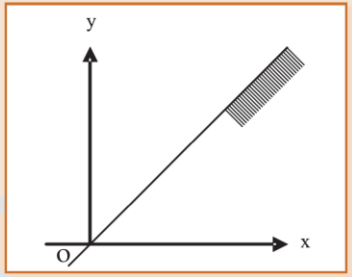
(b)



(c)



(d)

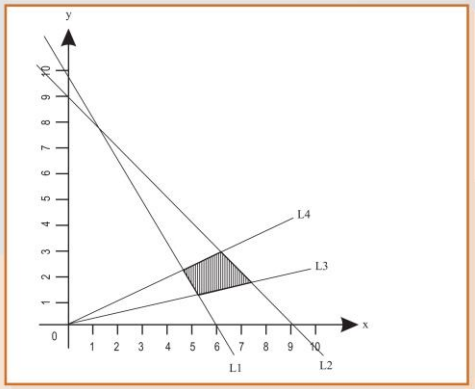


Solution

(d)

Question

(viii)



L1 : 5x + 3y = 30 L2 : x+y = 9 L3 : y = x/3 L4 : y = x/2

The common region (shaded part) shown in the diagram refers to

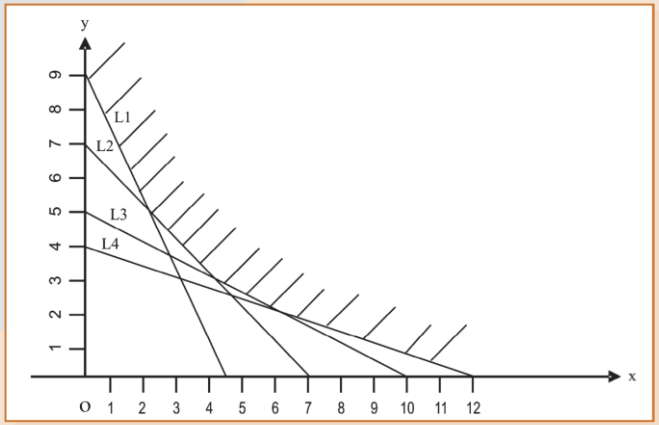
(a)	(b)	(c)	(d)	(e)
5x + 3y ≤ 30	5x + 3y ≥ 30	5x + 3y ≥ 30	5x + 3y > 30	None of these
x + y ≤ 9	x + y ≤ 9	x + y ≥ 9	x + y < 9	
y ≤ 1/5 x	y ≥ x/3	y ≤ x/3	y ≥ 9	
y ≤ x/2	y ≤ x/2	y ≥ x/2	y ≤ x/2	
	x ≥ 0, y≥ 0	x ≥ 0, y≥ 0	x ≥ 0, y≥ 0	

Solution

(b)

Question

3. Graphs of the inequations are drawn below :



L1 : 2x +y = 9 L2 : x + y = 7 L3 : x+2y= 10 L4 : x + 3y = 12

The common region (shaded part) indicated on the diagram is expressed by the set of inequalities

(a) $2x + y \le 9$, $x + y \ge 7$, $x + 2y \ge 10$, $x + 3y \ge 12$

(b) $2x + y \ge 9$, $x + y \le 7$, $x + 2y \ge 10$, $x + 3y \ge 12$

(c) $2x + y \ge 9$, $x + y \ge 7$, $x + 2y \ge 10$, $x + 3y \ge 12$, $x \ge 0, y \ge 0$

(d) none of these

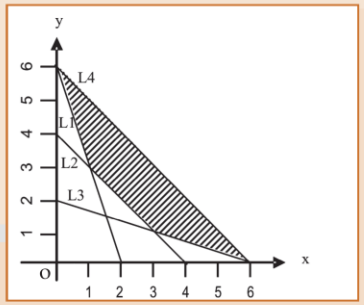
Solution

(c)

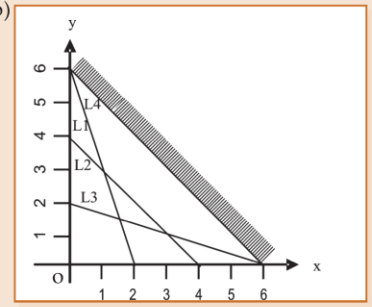
Question

4. The common region satisfied by the inequalities L1: $3x + y \geq 6$, L2: $x + y \geq 4$, L3: $x + 3y \geq 6$, and L4: $x + y \leq 6$ is indicated by

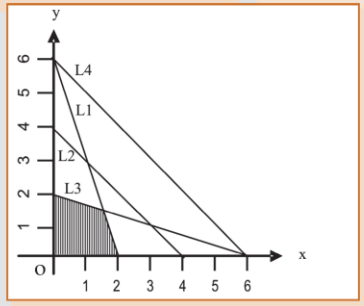
(a)



(b)



(c)



(d) none of these

Solution

(a)

Question

If $3x+2<2x+5$ and $4x-5\geq 2x-3$, then x can take from the following values:

(a) 3 (b) –1 (c) 2 (d) –3

Solution

(c)

$3x+2<2x+5$

$\Rightarrow 3x-2x<5-2$

$\Rightarrow x<3\ldots \text{Eq. (1)}$

$4x-5\geq 2x-3$

$\Rightarrow 4x - 2x \geq -3 + 5$

$\Rightarrow 2x \geq 2$

$\Rightarrow x \geq 1 \ldots$Eq. (2)

From Equations (1), and (2), x can take values between 1 and 3 (including 1, but excluding 3).

Therefore, option (c) is the answer.