Marathon 6

CA Nishant Kumar

Chapter 6 – Sequence and Series

Arithmetic Progression

Meaning

A series in which the difference between any two consecutive terms is the same

Formulas

nth Term

$$t_n = a + (n-1)d$$

Arithmetic Mean

Simple Average of the Numbers

Sum of the first n terms

When the First and Last terms are Known

$$S_n = \frac{n}{2}(a + l)$$

Formula for Calculating n when a and l are known

$$n = \frac{l-a}{d} + 1$$

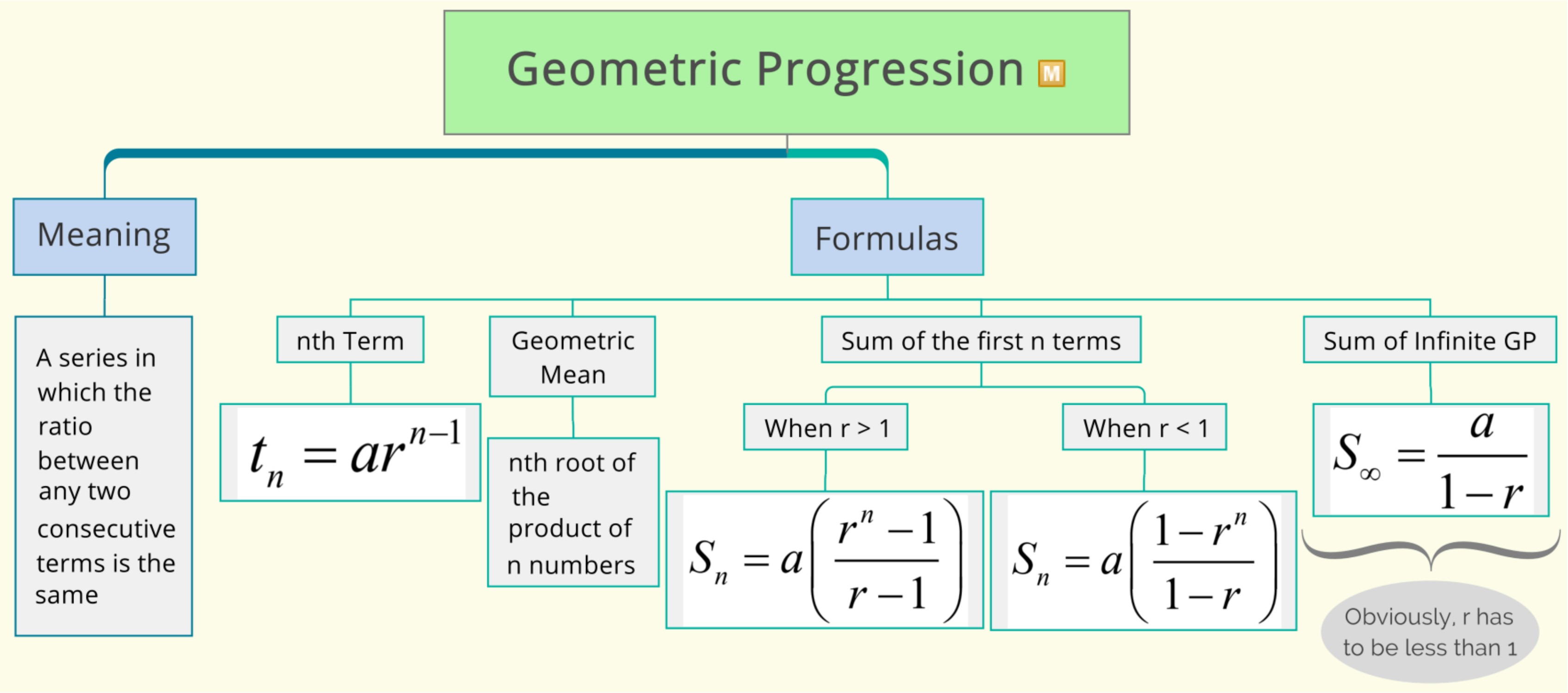
Other Cases

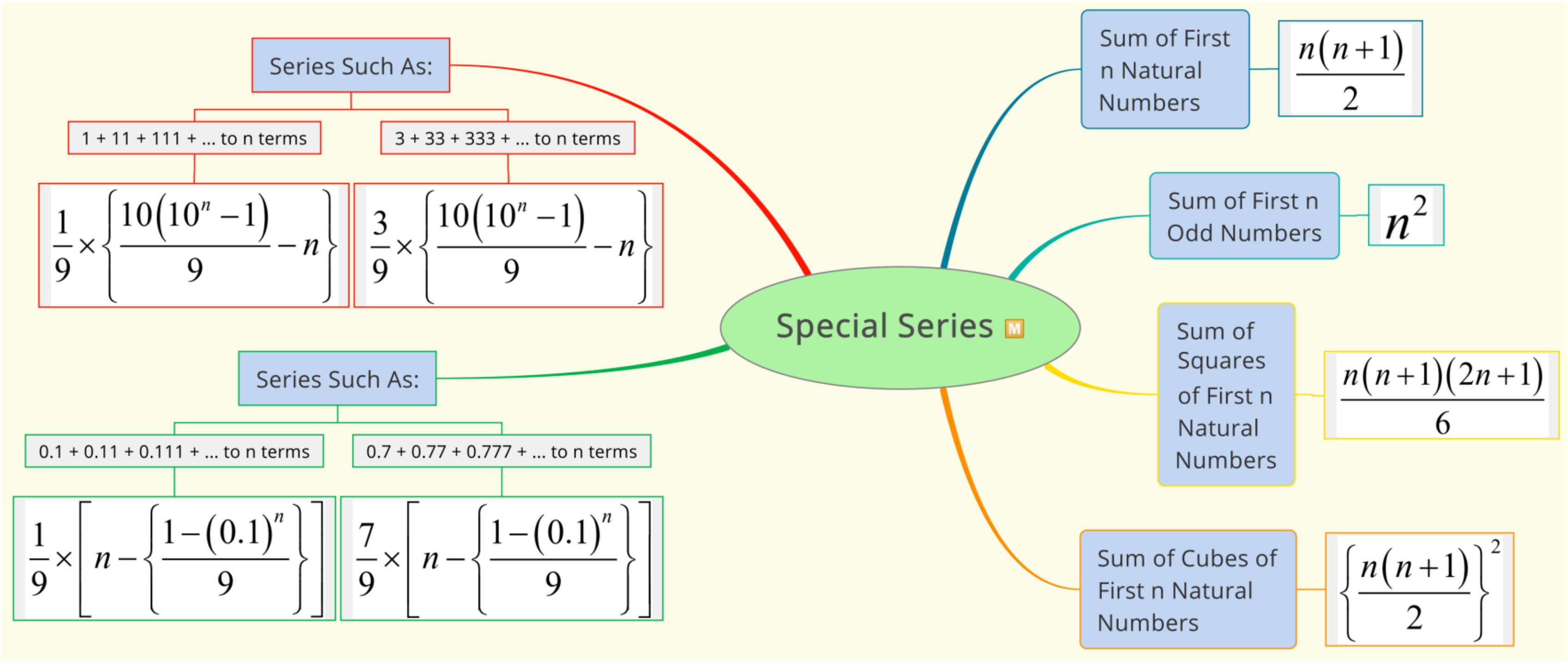
$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Meaning of Terms

a	First Term
d	Common Difference
n	Total Number of Terms
l	Last Term

CA NISHANT KUMAR


Geometric Progression
Meaning
Formulas
A series in which the ratio between any two consecutive terms is the same
nth Term
$t_n = ar^{n-1}$
Geometric Mean
nth root of the product of n numbers
Sum of the first n terms
When r > 1
$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$
When r < 1
$S_n = a\left(\frac{1 - r^n}{1 - r}\right)$
Sum of Infinite GP
$S_\infty = \frac{a}{1 - r}$
Obviously, r has to be less than 1


Special Series
Series Such As:
1 + 11 + 111 + ... to n terms
$\frac{1}{9}\times\left\{\frac{10(10^n-1)}{9}-n\right\}$
3 + 33 + 333 + ... to n terms
$\frac{3}{9}\times\left\{\frac{10(10^n-1)}{9}-n\right\}$
Series Such As:
0.1 + 0.11 + 0.111 + ... to n terms
$\frac{1}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$
0.7 + 0.77 + 0.777 + ... to n terms
$\frac{7}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$
Sum of First n Natural Numbers
$\frac{n(n+1)}{2}$
Sum of First n Odd Numbers
n^2
Sum of Squares of First n Natural Numbers
$\frac{n(n+1)(2n+1)}{6}$
Sum of Cubes of First n Natural Numbers
$\left\{\frac{n(n+1)}{2}\right\}^2$

Questions Based on Arithmetic Progression

Question

The value of x such that $8x + 4$, $6x – 2$, $2x + 7$ will form an AP is:

(a) 15 (b) 2 (c) 15/2 (d) None

Question

The last term of the A.P. 0.6, 1.2, 1.8, … to 13 terms is:

(a) 8.7 (b) 7.8 (c) 7.7 (d) None

Question

Which term of the progression –1, –3, –5, … is –39?

(a) 21^{st} (b) 20^{th} (c) 19^{th} (d) None

Question

The number of numbers between 74 and 25,556 divisible by 5 is:

(a) 5090 (b) 5097 (c) 5095 (d) None

Question

The n^{th} element of the sequence –1, 2, –4, 8, … is:

(a) $(-1)^n 2^{n-1}$ (b) 2^{n-1} (c) 2^n (d) None

Question

The n^{th} term of the series 3 + 7 + 13 + 21 + 31 + … is:

(a) $4n-1$ (b) n^2+2n (c) n^2+n+1 (d) n^3+2

Question

The two arithmetic means between –6 and 14 is:

(a) 2/3, 1/3

(b) 2/3, $7\frac{1}{3}$

(c) $-2/3, -7\frac{1}{3}$

(d) None

Question

The 4 arithmetic means between –2 and 23 are

(a) 3, 13, 8, 18 (b) 18, 3, 8, 13 (c) 3, 8, 13, 18 (d) None

Question

The sum of the series 9, 5, 1, … to 100 terms is:

(a) –18,900 (b) 18,900 (c) 19,900 (d) None

Question

The sum of all natural numbers between 500 and 1000 which are divisible by 13, is:

(a) 28,405 (b) 24,805 (c) 28,540 (d) None

Question

The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 and 5 is:

(a) 2,200 (b) 2,000 (c) 2,220 (d) None

Question

The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 or 5 is:

(a) 10,200 (b) 15,200 (c) 16,200 (d) None

Question

A person is employed in a company at ₹3,000 per month and he would get an increase of ₹100 per year. Find the total amount which he receives in 25 years and the monthly salary in the last year.

(a) ₹14,60,000 (b) ₹13,60,000 (c) ₹12,60,000 (d) None

Question

A sum of ₹6240 is paid off in 30 instalments such that each instalment is ₹10 more than the preceding installment. The value of the 1st instalment is:

(a) ₹36 (b) ₹30 (c) ₹60 (d) None

Question

A person saved ₹16,500 in ten years. In each year after the first year, he saved ₹100 more than he did in the preceding year. The amount of money he saved in the 1st year was:

(a) ₹1,000 (b) ₹1,500 (c) ₹1,200 (d) None

Question

The sum of a certain number of terms of an AP series –8, –6, –4, … is 52. The number of terms is:

(a) 12 (b) 13 (c) 11 (d) None

Question

The first and the last term of an AP are –4 and 146. The sum of the terms is 7171. The number of terms is:

(a) 101 (b) 100 (c) 99 (d) None

Question

The number of terms of the series $10+9\frac{2}{3}+9\frac{1}{3}+9+\ldots$ will amount to 155 is:

(a) 30 (b) 31 (c) 32 (d) Both (a) and (b)

Question

If 8^{th} term of an AP is 15, the sum of its first 15 terms is:

(a) 15 (b) 0 (c) 225 (d) 225/2

Question

A person pays ₹975 by monthly instalment each less than the former by ₹5. The first instalment is ₹100. The time by which the entire amount will be paid is:

(a) 10 months (b) 15 months (c) 14 months (d) None

Question

The n^{th} term of the series whose sum to n terms is $5n^2 + 2n$ is:

(a) $3n - 10$ (b) $10n - 2$ (c) $10n - 3$ (d) None

Question

The p^{th} term of an AP is $(3p - 1)/6$. The sum of the first n terms of the AP is:

(a) $n(3n+1)$ (b) $n(3n+1)/12$ (c) $n/12(3n-1)$ (d) None

Question

If 5th and 12th terms of an AP are 14 and 35 respectively, find the AP.

(a) 2, 5, 8, 11 (b) 2, 5, 8, 9 (c) 2, 5, 9, 13 (d) None

Question

$\sum_{i=4}^{7}\sqrt{2i-1}$ can be written as:

(a) $\sqrt{7}+\sqrt{9}+\sqrt{11}+\sqrt{13}$

(b) $2\sqrt{7}+2\sqrt{9}+2\sqrt{11}+2\sqrt{13}$

(c) $2\sqrt{7}+2\sqrt{9}+2\sqrt{11}+2\sqrt{13}$

(d) None

Question

The sum to ∞ of the series –5, 25, –125, 625, … can be written as:

(a) $\sum_{k=1}^{\infty}(-5)^k$ (b) $\sum_{k=1}^{\infty}5^k$ (c) $\sum_{k=1}^{\infty}-5^k$ (d) None

Question

The m^{th} term of an AP is n and the n^{th} term is m. The r^{th} term of it is:

(a) $m+n+r$ (b) $n+m-2r$ (c) $m+n+r/2$ (d) $m+n-r$

Question

The first term of an A.P is 14 and the sums of the first five terms and the first ten terms are equal in magnitude but opposite in sign. The 3^{rd} term of the AP is:

(a) $6\frac{4}{11}$ (b) 6 (c) 4/11 (d) None

Question

If unity is added to the sum of any number of terms of the A.P. 3, 5, 7, 9, … the resulting sum is:

(a) a perfect cube (b) a perfect square (c) a number (d) None

Question

The sum of the progression $(a+b),\ a,\ (a-b)\ldots n$ terms is:

(a) $\frac{n}{2}\left[2a+(n-1)b\right]$ (b) $\frac{n}{2}\left[2a+(3-n)b\right]$ (c) $\frac{n}{2}\left[2a+(3-n)\right]$ (d) $\frac{n}{2}\left[2a+(n-1)\right]$

Question

Find the sum of first twenty-five terms of A.P. series whose n^{th} term is $\left(\frac{n}{5}+2\right)$.

(a) 105 (b) 115 (c) 125 (d) 135

Question

The sum of the first 3 terms in an AP is 18 and that of the last 3 is 28. If the AP has 13 terms, what is the sum of the middle three terms?

(a) 23 (b) 18 (c) 19 (d) None

Question

The first term of an A.P. is 100 and the sum of whose first 6 terms is 5 times the sum of the next 6 terms, then the c.d. is:

(a) –10 (b) 10 (c) 5 (d) None

Question

If $\frac{1+3+5+...+n \text{ terms}}{2+4+6+...+50 \text{ terms}} = \frac{2}{51}$, the value of n is:

(a) 9 (b) 10 (c) 12 (d) 13

Question

The sum of n terms of an A.P. is $3n^2 + n$; then its p^{th} term is:

(a) $6p+2$ (b) $6p-2$ (c) $6p-1$ (d) None

Questions Based on Geometric Progression

Question

t_{12} of the series –128, 64, –32, … is:

(a) –1/16 (b) 16 (c) 1/16 (d) None

Question

The last term of the series 1, –3, 9, –27 up to 7 terms is:

(a) 297 (b) 729 (c) 927 (d) None

Question

The last term of the series x^2, x, 1, …. to 31 terms is:

(a) x^{28} (b) $1/x$ (c) $1/x^{28}$ (d) None

Question

Which term of the progression 1, 2, 4, 8, … is 256?

(a) 9^{th} (b) 10^{th} (c) 11^{th} (d) None

Question

Insert 3 geometric means between 1/9 and 9.

(a) 1/3, 1, 3 (b) 1/9, 1, 9 (c) ¼, 1, 4 (d) None

Question

The sum of the series –2, 6, –18, …. to 7 terms is:

(a) –1094 (b) 1094 (c) –1049 (d) None

Question

The sum of the series 243, 81, 27, …. to 8 terms is:

(a) 36 (b) $\left(36\frac{13}{30}\right)$ (c) $36\frac{1}{9}$ (d) None

Question

The sum of the series $\frac{1}{\sqrt{3}}+1+\frac{3}{\sqrt{3}}+\ldots$ to 18 terms is:

(a) $9841\frac{\left(1+\sqrt{3}\right)}{\sqrt{3}}$ (b) 9841 (c) $\frac{9841}{\sqrt{3}}$ (d) None

Question

If you save 1 paise today, 2 paise the next day 4 paise the succeeding day and so on, then your total savings in two weeks will be:

(a) ₹163 (b) ₹183 (c) ₹163.83 (d) None

Question

The sum of the series 1 + 2 + 4 + 8 + … to n terms is:

(a) $2^n - 1$ (b) $2n - 1$ (c) $1/2^n - 1$ (d) None

Question

The number of terms to be taken so that 1 + 2 + 4 + 8 + will be 8191 is:

(a) 10 (b) 13 (c) 12 (d) None

Question

The sum of the infinite GP 14, – 2, + 2/7, – 2/49, + … is:

(a) $4\frac{1}{12}$ (b) $12\frac{1}{4}$ (c) 12 (d) None

Question

The sum of the infinite G. P. $1 - 1/3 + 1/9 - 1/27 + \ldots$ is:

(a) 0.33 (b) 0.57 (c) 0.75 (d) None

Question

The n^{th} term of the series 16, 8, 4, … is $1/2^{17}$. The value of n is:

(a) 20 (b) 21 (c) 22 (d) None

Question

The sum of $1 + 1/3 + 1/3^2 + 1/3^3 + \ldots + 1/3^{n-1}$ is:

(a) 2/3 (b) 3/2 (c) 4/5 (d) None

Question

The sum of 1.03 + $(1.03)^2$ + $(1.03)^3$ + … to n terms is:

(a) $103\{(1.03)^n - 1\}$ (b) $103/3\{(1.03)^n - 1\}$ (c) $(1.03)^n - 1$ (d) None

Question

The sum of the infinite series 1 + 2/3 + 4/9 + … is:

(a) 1/3 (b) 3 (c) 2/3 (d) None

Question

Find the G.P where 4th term is 8 and 8th term is 128/625:

(a) 125, 50, 20, … (b) –125, 50, –20 (c) 120, 60, 30, … (d) Both (a) and (b)

Question

If x, y, and z are in GP, then:

(a) $y^2 = xz$ (b) $y\left(z^2 + x^2\right) = x\left(z^2 + y^2\right)$ (c) $2y = x + z$ (d) None

Question

In a G.P., the product of the first three terms 27/8. The middle term is:

(a) 3/2 (b) 2/3 (c) 2/5 (d) None

Question

The sum of the first 20 terms of a G.P. is 244 times the sum of its first 10 terms. The common ratio is:

(a) $\pm\sqrt{3}$ (b) ± 3 (c) $\sqrt{3}$ (d) None

Question

The sum of the first two terms of a G.P. is 5/3 and the sum to infinity of the series is 3. The common ratio is:

(a) 1/3 (b) 2/3 (c) –2/3 (d) Both (b) and (c)

Question

If $y = 1 + x + x^2 + ... + \infty$, then $x =$

(a) $\frac{y-1}{y}$ (b) $\frac{y+1}{y}$ (c) $\frac{y}{y+1}$ (d) $\frac{y}{y-1}$

Solution

(a)

$y = S_\infty$

$\Rightarrow y = \frac{a}{1-r}$

$$\Rightarrow y = \frac{1}{1-x}$$

$$\Rightarrow y(1-x) = 1$$

$$\Rightarrow y - xy = 1$$

$$\Rightarrow xy = y - 1$$

$$\Rightarrow x = \frac{y-1}{y}$$

Question

Sum upto infinity of series: $\frac{1}{2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}+\frac{1}{2^5}+...$

(a) 19/24 (b) 24/19 (c) 5/24 (d) None

Solution

(a)

This is a combination of two separate series:

$$\left(\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}+...\infty\right)+\left(\frac{1}{3^2}+\frac{1}{3^4}+...\infty\right)$$

$$= \frac{1/2}{1-(1/4)} + \frac{1/3^2}{1-(1/3^2)}$$

$$= \frac{1/2}{3/4} + \frac{1/9}{8/9}$$

$$= \left(\frac{1}{2} \times \frac{4}{3}\right) + \left(\frac{1}{9} \times \frac{9}{8}\right)$$

$$= \frac{2}{3} + \frac{1}{8} = \frac{16+3}{24} = \frac{19}{24}$$

Question

If 2 + 6 + 10 + 14 + 18 + … + x = 882 then the value of x

(a) 78 (b) 80 (c) 82 (d) 86

Solution

(c)

We have $a = 2;\ d = 4;\ S_n = 882$

$$S_n = \frac{n}{2}\left\{2a + (n-1)d\right\}$$

$$\Rightarrow 882 = \frac{n}{2}\left\{(2\times 2) + (n-1)(4)\right\}$$

$$\Rightarrow 882 \times 2 = n\{4 + 4n - 4\}$$

$$\Rightarrow 882 \times 2 = n\{4n\}$$

$$\Rightarrow 882 \times 2 = 4n^2$$

$$\Rightarrow n^2 = \frac{882 \times 2}{4}$$

$$\Rightarrow n = \sqrt{\frac{882 \times 2}{4}} = 21$$

$$x = t_{21}$$

$$\Rightarrow t_{21} = a + 20d$$

$$\Rightarrow t_{21} = 2 + (20 \times 4) = 82$$

$$\Rightarrow x = 82$$

Question

The sum of n terms of a G.P. whose first term is 1 and the common ratio is 1/2, is equal to $1\frac{127}{128}$. The value of n is:

(a) 7 (b) 8 (c) 6 (d) None

Solution

(b)

We have $a = 1$; $r = \frac{1}{2}$; $S_n = 1\frac{127}{128} = \frac{255}{128}$

$$S_n = a\left[\frac{1-r^n}{1-r}\right]$$

$$\Rightarrow \frac{255}{128} = 1\left[\frac{1-(1/2)^n}{1-1/2}\right]$$

$$\Rightarrow \frac{255}{128} = \frac{1}{1/2}\left[1-\left(\frac{1}{2}\right)^n\right]$$

$$\Rightarrow \frac{1/2\times 255}{128} = 1-\left(\frac{1}{2}\right)^n$$

$$\Rightarrow 0.99609375 = 1 - \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \left(\frac{1}{2}\right)^n = 1 - 0.99609375 = 0.00390625$$

Now, try the options.

$$\left(\frac{1}{2}\right)^8 = 0.00390625$$

$$\Rightarrow n = 8$$

Question

In a G.P., if the fourth term is '3' then the product of first seven terms is:

(a) 3^5 (b) 3^7 (c) 3^6 (d) 3^8

Solution

(b)

$t_4 = 3$

$t_4 = ar^3$

$\Rightarrow ar^3 = 3$

$t_1 \times t_2 \times t_3 \times t_4 \times t_5 \times t_6 \times t_7$

$$= a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6$$

$$= a^{1+1+1+1+1+1+1} r^{1+2+3+4+5+6}$$

$$= a^7 r^{21}$$

$$= \left(ar^3\right)^7$$

$$= 3^7$$

Question

If t_4 of a GP is x, $t_{10} = y$, and $t_{16} = z$, then,

(a) $x^2 = yz$ (b) $z^2 = xy$ (c) $y^2 = zx$ (d) None

Solution

(c)

$ar^3 = x$; $ar^9 = y$; $ar^{15} = z$

Try the options.

Option (a) $\rightarrow$ $x^2 = yz$

$$\text{LHS} \rightarrow \left(ar^3\right)^2 = a^2r^6$$

$$\text{RHS} \rightarrow ar^9 \times ar^{15} = a^2r^{9+15} = a^2r^{24}$$

$$\text{Option (b)} \rightarrow z^2 = xy$$

$$\text{LHS} \rightarrow \left(ar^{15}\right)^2 = a^2r^{30}$$

$$\text{RHS} \rightarrow ar^3 \times ar^9 = a^2r^{3+9} = a^2r^{12}$$

$$\text{Option (c)} \rightarrow y^2 = zx$$

$$\text{LHS} \rightarrow \left(ar^9\right)^2 = a^2r^{18}$$

$$\text{RHS} \rightarrow ar^{15} \times ar^3 = a^2r^{15+3} = a^2r^{18}$$

Therefore, option (c) is the answer.

Alternatively,

We can see that t_{10} is the middle term between t_4 and t_{16}. Therefore, t_{10} is the geometric mean. Therefore, $\left(t_{10}\right)^2 = t_4 \times t_{16} \Rightarrow y^2 = xz$

Question

If p, q and r, are in A.P. and x, y, z are in G.P., then $x^{q-r}.y^{r-p}.z^{p-q}$ is equal to:

(a) 0 (b) –1 (c) 1 (d) None

Solution

(c)

Since p, q, and r, are in AP, we have $q-p=r-q=d$

$\therefore q-p=d \Rightarrow p-q=-d$

And $r-q=d \Rightarrow q-r=-d$

Also, $r-p=\left(r-q\right)+\left(q-p\right)=d+d=2d$

Also, since *x*, *y*, and *z* are in GP, we have $y^2 = xz$

Now, we have:

$$x^{q-r}.y^{r-p}.z^{p-q}$$

$$x^{-d}.y^{2d}.z^{-d} \text{ (Since } q-r=-d;\ r-p=2d;\ p-q=-d\text{)}$$

$$(xz)^{-d}.y^{2d}$$

$$\left(y^2\right)^{-d}.y^{2d} \text{ (Since } y^2 = xz\text{)}$$

$$y^{-2d}.y^{2d} = 1$$

Question

Given x, y, and z are in GP and $x^p = y^q = z^\sigma$, then $1/p$, $1/q$, $1/\sigma$ are in:

(a) AP (b) GP (c) Both (d) None

Solution

(a)

Let $x^p = y^q = z^\sigma = k$

$$\Rightarrow x^p = k \Rightarrow x = k^{\frac{1}{p}}$$

$$\Rightarrow y^q = k \Rightarrow y = k^{\frac{1}{q}}$$

$$\Rightarrow z^{\sigma} = k \Rightarrow z = k^{\frac{1}{\sigma}}$$

Since x, y, and z are in GP, $y^2 = xz$

$$\Rightarrow \left(k^{\frac{1}{q}}\right)^2 = k^{\frac{1}{p}} \times k^{\frac{1}{\sigma}}$$

$$\Rightarrow k^{\frac{2}{q}} = k^{\frac{1}{p}+\frac{1}{\sigma}}$$

$$\Rightarrow \frac{2}{q} = \frac{1}{p} + \frac{1}{\sigma}$$

$$\Rightarrow \frac{1}{q} + \frac{1}{q} = \frac{1}{p} + \frac{1}{\sigma}$$

$$\Rightarrow \frac{1}{p} - \frac{1}{q} = \frac{1}{q} - \frac{1}{\sigma}$$

Therefore, they are in AP.

Question

If A be the A.M. of two positive unequal quantities x and y and G be their G.M., then:

(a) $A < G$ (b) $A > G$ (c) $A \geq G$ (d) $A \leq G$

Solution

(b)

Question

If x, y, z, are in A.P. and x, y, $(z + 1)$ are in G.P., then:

(a) $(x-z)^2 = 4x$ (b) $z^2 = x-y$ (c) $z = x-y$ (d) None

Solution

(a)

Since x, y, and z are in AP, $y = \frac{x+z}{2}$ …Eq. (1)

Also, since x, y, $(z + 1)$ are in G.P., $y^2 = x(z+1)$…Eq. (2)

Putting the value of y from Eq. (1) in Eq. (2), we have:

$$\left(\frac{x+z}{2}\right)^2 = xz + x$$

$$\frac{x^2+z^2+2xz}{4} = xz + x$$

$$x^2+z^2+2xz = 4xz + 4x$$

$$x^2+z^2+2xz-4xz = 4x$$

$$x^2+z^2-2xz = 4x$$

$$(x-z)^2 = 4x$$

Question

The numbers x, 8, y are in G.P. and the numbers x, y, –8 are in A.P. The value of x and y are:

(a) (–8, –8) (b) (16, 4) (c) (8, 8) (d) Both (a) and (b)

Solution

(d)

Try the options.

Question

The series $1+10^{-1}+10^{-2}+10^{-3}...$ to ∞ is:

(a) 9/10 (b) 1/10 (c) 10/9 (d) None

Solution

(c)

Given series $1+10^{-1}+10^{-2}+10^{-3}...$

$$\Rightarrow 1+\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+...\infty$$

Here, $a=1;\ r=\frac{1}{10}$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{10}} = \frac{1}{\frac{9}{10}} = \frac{10}{9}$$

Question

The sum of the first two terms of an infinite geometric series is 15 and each term is equal to the sum of all the terms following it; then the sum of the series is:

(a) 20 (b) 15 (c) 25 (d) None

(a)

Let the first term of the GP be *a*, and the second term of the GP be *ar*.

Given:

$$a + ar = 15$$

$$\Rightarrow a(1 + r) = 15$$

$$\Rightarrow a = \frac{15}{1+r} \ldots \text{Eq. (1)}$$

Also, we are given that every term is equal to the sum of all the terms following it. This means that $t_2 = S_\infty - S_2$.

Now, we know that $S_\infty = \frac{a}{1-r}$, and S_2 is given as 15.

Therefore, $t_2 = \frac{a}{1-r} - 15$

Also, we know that $t_2 = ar$

Therefore, $ar = \frac{a}{1-r} - 15 \ldots \text{Eq. (2)}$

Putting the value of *a* from Eq. (1) to Eq. (2), we get:

$$\frac{15}{1+r}\times r=\frac{\frac{15}{1+r}}{1-r}-15$$

$$\frac{15r}{1+r}=\left(\frac{15}{1+r}\div 1-r\right)-15$$

$$\frac{15r}{1+r}=\left(\frac{15}{1+r}\times\frac{1}{1-r}\right)-15$$

$$\frac{15r}{1+r}=\frac{15}{(1+r)(1-r)}-15$$

$$\frac{15r}{1+r} = \frac{15-15\left(1+r\right)\left(1-r\right)}{\left(1+r\right)\left(1-r\right)}$$

$$15r = \frac{15\left\{1-\left(1+r\right)\left(1-r\right)\right\}}{1-r}$$

$$r = \frac{\left\{1-\left(1-r^2\right)\right\}}{1-r}$$

$$r\left(1-r\right) = 1-1+r^2$$

$$r - r^2 = r^2$$

$$r^2 + r^2 - r = 0$$

$$2r^2 - r = 0$$

$$r(2r-1) = 0$$

Therefore, either $r = 0$, or $r = \frac{1}{2}$

Since *r* cannot be 0, it'll be ½.

Putting the value of *r* in Eq. (1), we get:

$$a = \frac{15}{1+\frac{1}{2}} = 10$$

Therefore, we have *a* = 10, and *r* = ½.

$$S_\infty = \frac{a}{1-r} = \frac{10}{1-\frac{1}{2}} = 20$$

Therefore, option (a) is the answer.

Question

If the p^{th} term of a GP is x and the q^{th} term is y, then find the n^{th} term.

(a) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]$ (b) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]^{(p-q)}$ (c) 1 (d) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]^{\frac{1}{p-q}}$

Solution

(d)

$t_p = ar^{p-1} = x \ldots$Eq. (1)

$t_q = ar^{q-1} = y \ldots$Eq. (2)

Dividing Eq. (1) by Eq. (2)

$$\frac{ar^{p-1}}{ar^{q-1}} = \frac{x}{y}$$

$$r^{p-1-(q-1)} = \frac{x}{y}$$

$$r^{p-1-q+1} = \frac{x}{y}$$

$$r^{p-q} = \frac{x}{y}$$

$$r = \left(\frac{x}{y}\right)^{\frac{1}{p-q}}$$

$$t_n = ar^{n-1}$$

Adding *p* and subtracting *p* in the power of *r*, we get:

$$t_n = ar^{n-1+p-p}$$

$$t_n = ar^{(n-p)+(p-1)}$$

$$t_n = ar^{(p-1)}r^{(n-p)}$$

We know that $ar^{p-1} = x$ and $r = \left(\frac{x}{y}\right)^{\frac{1}{p-q}}$. Putting these values above, we get:

$$t_n = x\left[\left(\frac{x}{y}\right)^{\frac{1}{p-q}}\right]^{n-p}$$

$$t_n = x\left(\frac{x}{y}\right)^{\frac{n-p}{p-q}}$$

$$t_n = x\left(\frac{x^{\frac{n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}\right)$$

$$t_n = \frac{x.x^{\frac{n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \frac{x^{1+\frac{n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \frac{x^{\frac{p-q+n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \frac{x^{\frac{n-q}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \left(\frac{x^{n-q}}{y^{n-p}} \right)^{\frac{1}{p-q}}$$

Question

The sum of three numbers in a geometric progression is 28. When 7, 2, and 1 are subtracted from the first, second, and the third numbers respectively, the resulting numbers are in Arithmetic Progression. What is the sum of squares of the original three numbers?

(a) 510 (b) 456 (c) 400 (d) 336

Solution

(d)

Let the numbers in GP be $\frac{a}{r}$, a, and ar respectively.

Given that the sum is 28.

Therefore, $\frac{a}{r}+a+ar=28$

$$\Rightarrow a\left(\frac{1}{r}+1+r\right)=28 \ldots \text{Eq. (1)}$$

Also, given that if we subtract 7, 2, and 1 from the first, second and third terms respectively, we get an AP.

On subtracting 7, 2, and 1 from first, second and third terms, we get:

$\left(\frac{a}{r}-7\right)$, $(a-2)$, and $(ar-1)$

Since these numbers are in AP, we have $(a-2)-\left(\frac{a}{r}-7\right)=(ar-1)-(a-2)$

$$\Rightarrow a-2-\frac{a}{r}+7=ar-1-a+2$$

$$\Rightarrow a-\frac{a}{r}+5=ar-a+1$$

$$\Rightarrow a-\frac{a}{r}-ar+a=1-5$$

$$\Rightarrow 2a-\frac{a}{r}-ar=-4$$

$$\Rightarrow a\left(2-\frac{1}{r}-r\right)=-4 \ldots \text{Eq. (2)}$$

Dividing Eq. (1) by Eq. (2), we get:

$$\frac{a\left(\frac{1}{r}+1+r\right)}{a\left(2-\frac{1}{r}-r\right)}=\frac{28}{-4}$$

$$\Rightarrow \frac{\frac{1+1r+r^2}{r}}{\frac{2r-1-r^2}{r}}=-7$$

$$\Rightarrow \frac{1+r+r^2}{2r-1-r^2} = -7$$

$$\Rightarrow 1+r+r^2 = -7\left(2r-1-r^2\right)$$

$$\Rightarrow 1+r+r^2 = -14r+7+7r^2$$

$$\Rightarrow 7r^2+7-14r-1-r-r^2 = 0$$

$$\Rightarrow 6r^2-15r+6 = 0$$

Here, $a=6;\ b=-15;\ c=6$

$$\alpha+\beta = -\frac{b}{a} = -\frac{-15}{6} = \frac{15}{6}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{6} = 1$$

As per fastest method, $\left(\frac{15}{6\times 2} + x\right)\left(\frac{15}{6\times 2} - x\right) = 1$

$$\Rightarrow \left(\frac{15}{12}\right)^2 - x^2 = 1$$

$$x^2 = \left(\frac{15}{12}\right)^2 - 1 = 1.5625 - 1 = 0.5625$$

$$x = \sqrt{0.5625} = 0.75$$

$$\alpha = \frac{15}{12} + 0.75 = 2$$

$$\beta = \frac{15}{12} - 0.75 = 0.5$$

Therefore, common ratio could either be 2, or 0.5

Taking the common ratio to be 2, let's find out the value of *a*.

Putting the value of $r = 2$ in Eq. (1), we'll get:

$$a\left(\frac{1}{2} + 1 + 2\right) = 28$$

$$\Rightarrow a(3.5) = 28$$

$$\Rightarrow a = \frac{28}{3.5} = 8$$

Therefore, the GP will be $\frac{8}{2}, 8, 8 \times 2 = 4, 8, 16$

We can see that the sum of these numbers = 4 + 8 + 16 = 28

Subtracting 7, 2, and 1 from first, second, and third terms, we'll get 4 – 7 = –3, 8 – 2 = 6, 16 – 1 = 15.

These terms are clearly in AP as 15 – 6 = 6 – (–3) = 9

The sum of squares of the numbers 4, 8, and 16 = $4^2 + 8^2 + 16^2 = 336$

Now, taking 0.5 as the common ratio, let's find out the value of *a*.

Putting the value of $r = 0.5$ in Eq. (1), we'll get:

$$a\left(\frac{1}{r}+1+r\right)=28$$

$$\Rightarrow a\left(\frac{1}{0.5}+1+0.5\right)=28$$

$$\Rightarrow a(3.5)=28$$

$$\Rightarrow a=\frac{28}{3.5}=8$$

Therefore, the GP will be $\frac{8}{0.5}$, 8, $8\times0.5=16$, 8, 4

We can see that the sum of these numbers = 16 + 8 + 4 = 28

Subtracting 7, 2, and 1 from first, second, and third terms, we'll get 16 – 7 = 9, 8 – 2 = 6, 4 – 1 = 3.

These terms are clearly in AP as 6 – 9 = 3 – 6 = –3

The sum of squares of the numbers 16, 8, and 4 = $16^2 + 8^2 + 4^2 = 336$

Special Series

Following are some of the Standard Results:

1. Sum of first n natural or counting numbers $(1+2+3+4+...+n) = \frac{n(n+1)}{2}$
2. Sum of first n odd numbers $\{1+3+5+...+(2n-1)\} = n^2$
3. Sum of the Squares of first n natural numbers

   $$(1^2+2^2+3^2+4^2+...+n^2) = \frac{n(n+1)(2n+1)}{6}$$

4. Sum of the Cubes of first n natural numbers $(1^3+2^3+3^3+4^3+...+n^3) = \left\{\frac{n(n+1)}{2}\right\}^2$

5. Sum of the series such as: 1 + 11 + 111 + … to n terms, or 2 + 22 + 222 + … to n terms, or 3 + 33 + 333 + … to n terms, and so on: $\frac{Number}{9} \times \left\{ \frac{10\left(10^{n}-1\right)}{9} - n \right\}$. For example:

a. 1 + 11 + 111 + … to n terms = $\frac{1}{9} \times \left\{ \frac{10\left(10^{n}-1\right)}{9} - n \right\}$

b. 2 + 22 + 222 + … to n terms = $\frac{2}{9} \times \left\{ \frac{10\left(10^{n}-1\right)}{9} - n \right\}$

c. 3 + 33 + 333 + … to n terms = $\frac{3}{9} \times \left\{ \frac{10\left(10^{n}-1\right)}{9} - n \right\}$

6. Sum of the series 0.1 + 0.11 + 0.111 + … to n terms = $\frac{1}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$.

Example: Calculate the sum of 0.7 + 0.77 + 0.777 + … to n terms.

Solution:

0.7 + 0.77 + 0.777 + … to n terms = 7 × (0.1 + 0.11 + 0.111 + … to n terms)

Therefore, 0.7 + 0.77 + 0.777 + … to n terms = $\frac{7}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$

Similarly, sum of series 0.2 + 0.22 + 0.222 + … to n terms = $\frac{2}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$

Sum of series 0.4 + 0.44 + 0.444 + … to n terms = $\frac{4}{9}\times\left[n-\left\{\frac{1-(0.1)^{n}}{9}\right\}\right]$.

Question

The ratio of the sum of first n natural numbers to that of the sum of cubes of first n natural numbers is:

(a) 3 : 16 (b) $n(n+1)/2$ (c) $2/n(n+1)$ (d) None

Solution

(c)

Sum of first n natural numbers = $\frac{n(n+1)}{2}$

Sum of cubes of first n natural numbers = $\left\{\frac{n(n+1)}{2}\right\}^2$

$$\text{Ratio} = \frac{n(n+1)}{2} \div \left\{\frac{n(n+1)}{2}\right\}^2$$

$$= \frac{n(n+1)}{2} \div \left\{\frac{n(n+1)}{2} \times \frac{n(n+1)}{2}\right\}$$

$$= \frac{n(n+1)}{2} \div \left\{\frac{n^2(n+1)^2}{4}\right\}$$

$$= \frac{n(n+1)}{2} \times \frac{4}{n^2 (n+1)^2}$$

$$= \frac{2}{n(n+1)}$$

Question

Find the sum to *n* terms of 6 + 27 + 128 + 629 + …

(a) $\left\{5\left(5^n-1\right)\right\}+\left\{n\left(n+1\right)\right\}$

(b) $\left\{\frac{5}{4}\left(5^n-1\right)\right\}+\left\{\frac{n\left(n+1\right)}{2}\right\}$

(c) $\left\{5\left(5^n-1\right)\right\}+\left\{\frac{n\left(n+1\right)}{2}\right\}$

(d) None

Solution

(b)

6 + 27 + 128 + 629 + …

$$\Rightarrow (5+1)+(25+2)+(125+3)+(625+4)+\dots$$

$$\Rightarrow (5+25+125+625+\dots)+(1+2+3+4+\dots)$$

$$\Rightarrow \left(5+5^2+5^3+5^4+\dots+5^n\right)+(1+2+3+4+\dots+n)$$

The first bracket is a Geometric Progression with a = 5, and r = 5

$$\Rightarrow \left\{5\left(\frac{5^n-1}{5-1}\right)\right\}+\left\{\frac{n(n+1)}{2}\right\}$$

$$\Rightarrow \left\{5\left(\frac{5^n-1}{4}\right)\right\}+\left\{\frac{n(n+1)}{2}\right\}$$

$$\Rightarrow \left\{ \frac{5}{4}\left(5^{n}-1\right) \right\} + \left\{ \frac{n\left(n+1\right)}{2} \right\}$$

Question

Find the sum to *n* terms of the series 3 + 33 + 333 + 3333 + …

(a) $\frac{1}{27}\times\left(10^{n+1}-9n-10\right)$

(b) $\frac{1}{27}\times\left(10^{n+1}-9n+10\right)$

(c) $\frac{1}{27}\times\left(10^{n+1}+9n+10\right)$

(d) None

Solution

(a)

The sum of such type of series is given by $\frac{Number}{9}\times\left\{\frac{10\left(10^{n}-1\right)}{9}-n\right\}$

Therefore, sum of 3 + 33 + 333 + 3333 + … is given by: $\frac{3}{9}\times\left\{\frac{10\left(10^n-1\right)}{9}-n\right\}$

$$\Rightarrow \frac{3}{9}\times\left\{\frac{10\left(10^n-1\right)-9n}{9}\right\}$$

$$\Rightarrow \frac{3}{81}\times\left\{10\left(10^n-1\right)-9n\right\}$$

$$\Rightarrow \frac{1}{27}\times\left\{10\times10^n-10-9n\right\}$$

$$\Rightarrow \frac{1}{27}\times\left(10^{n+1}-10-9n\right)$$

$$\Rightarrow \frac{1}{27} \times \left(10^{n+1} - 9n - 10\right)$$

Question

Find the sum to n terms of the series 0.7 + 0.77 + 0.777 + 0.7777 + …

(a) $\frac{7}{81}\times\left\{9n-1-10^{-n}\right\}$

(b) $\frac{7}{81}\times\left\{9n-1+10^{n}\right\}$

(c) $\frac{7}{81}\times\left\{9n-1+10^{-n}\right\}$

(d) None

Solution

(c)

The sum to such series is given by $\frac{7}{9}\times\left[n-\left\{\frac{1-(0.1)^{n}}{9}\right\}\right]$

$$\Rightarrow \frac{7}{9} \times \left[\frac{9n - \left\{1 - (0.1)^n\right\}}{9} \right]$$

$$\Rightarrow \frac{7}{81} \times \left\{9n - 1 + (0.1)^n\right\}$$

$$\Rightarrow \frac{7}{81} \times \left\{9n - 1 + \left(\frac{1}{10}\right)^n\right\}$$

$$\Rightarrow \frac{7}{81} \times \left\{9n - 1 + \frac{1}{10^n}\right\}$$

$$\Rightarrow \frac{7}{81} \times \left\{9n - 1 + 10^{-n}\right\}$$

Question

Evaluate $0.21\dot{7}\dot{5}$ using the sum of an infinite geometric series.

(a) $\frac{357}{1650}$ (b) $\frac{358}{1650}$ (c) $\frac{359}{1650}$ (d) None

Solution

(c)

Try the options.

Question

A person borrows ₹8,000 at 2.76% Simple Interest per annum. The principal and the interest are to be paid in the 10 monthly instalments. If each instalment is double the preceding one, find the value of the first and the last instalment.

(a) 8; 4,095 (b) 2; 4,096 (c) 8; 4,096 (d) None

Solution

(c)

Total amount to be paid = $8,000+\left(8,000\times 0.0276\times \frac{10}{12}\right)=8,184$

Since each instalment is to be double the preceding one, it is clearly a GP with r = 2.

Therefore, we have $n=10;\ r=2;\ S_{10}=8{,}184$

Since $r > 1,\ S_n = a\left(\frac{r^n-1}{r-1}\right)$

$$a=\frac{S_n}{\left(\frac{r^n-1}{r-1}\right)}=\frac{8{,}184}{\left(\frac{2^{10}-1}{2-1}\right)}=8$$

Therefore, the first instalment is 8.

Now, let's calculate the last instalment.

$$t_{10}=ar^9=8\times 2^9=4{,}096$$