Thank you

1. Ratio & Proportion

For a : b we have, a → Antecedent, b → Consequent

Operations on Ratio

a : b

same kind

same unit

$a \times \frac{b}{a} = b$

FMR

1. Inverse ratio → $\frac{a}{b} \to \frac{b}{a}$
2. Duplicate ratio → $\frac{a}{b} \to \frac{a^2}{b^2}$
3. Triplicate ratio → $\frac{a}{b} \to \frac{a^3}{b^3}$
4. Sub-duplicate ratio → $\frac{a}{b} \to \frac{\sqrt{a}}{\sqrt{b}}$
5. Sub triplicate ratio → $\frac{a}{b} \to \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$
6. Compound ratio → $\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f}$

a : b

b : c

a : b : c

a : b

c : d

e : f

Proportion

a : b = c : d

1. Invertendo → $\frac{a}{b} = \frac{c}{d} \to \frac{b}{a} = \frac{d}{c}$
2. Alternendo → $\frac{a}{b} = \frac{c}{d} \to \frac{a}{c} = \frac{b}{d}$
3. Componendo → $\frac{a}{b} = \frac{c}{d} \to \frac{a+b}{b} = \frac{c+d}{d}$
4. Dividendo → $\frac{a}{b} = \frac{c}{d} \to \frac{a-b}{b} = \frac{c-d}{d}$
5. Componendo and Dividendo → $\frac{a}{b} = \frac{c}{d} \to \frac{a+b}{a-b} = \frac{c+d}{c-d}$
6. Adendo → $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \to \frac{a+c+e}{b+d+f}$
7. Subtra Hendo → $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \to \frac{a-c-e}{b-d-f}$

2. Logarithms

Property

1. $\log_a m = n \to a^n = m$
2. $\log_a m + \log_a n \to \log_a mn$
3. $\log_a m - \log_a n \to \log_a \frac{m}{n}$
4. $\log_a(m^n) = n.\log_a m$
5. $\log_a 1 = 0$
6. $\log_a a = 1$
7. $\log_a b = \frac{\log x^b}{\log x^a}$ → change of base

* Base by default is taken as 10 which is also called as common logarithm

$\log_a x = \frac{1}{\log_x a}$

$\log_{a^m} x = \frac{1}{m} \log_a x$

3. Indices

Properties of Indices

$a^m \times a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

$a^0 = 1$

$a^{-m} = \frac{1}{a^m}$

$a^m = \frac{1}{a^{-m}}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Formula

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^2 - b^2 = (a+b)(a-b)$

Roots

$2^1 = 2$	$3^1 = 3$	$5^1 = 5$
$2^2 = 4$	$3^2 = 9$	$5^2 = 25$
$2^3 = 8$	$3^3 = 27$	$5^3 = 125$
$2^4 = 16$	$3^4 = 81$	$5^4 = 625$
$2^5 = 16$	$3^5 = 243$	$5^5 = 3125$
$2^6 = 64$	$3^6 = 729$	$5^6 = 15,625$
$2^7 = 128$	$3^7 = 2187$	
$2^8 = 256$	$3^8 = 6561$	
$2^9 = 512$	$3^9 = 19683$	
$2^{10} = 1024$	$3^{10} = 59,049$	

Factorials

0! = 1
1! = 1
2! = 2
3! = 6
4! = 24
5! = 120
6! = 720
7! = 5040
8! = 40320
9! = 362880

4. Equations

ABC

Quadratic Equation → $ax^2 + bx + c = 0$

The roots of a quadratic equation:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$ax^2 + bx + c = 0$

Sum of roots = $-\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

$\alpha + \beta = -\frac{b}{a}$

$\alpha\beta = \frac{c}{a}$

Product of the roots = $\frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

To construct a quadratic equation for we have
:- x^2 -(Sum of the roots)
x + Product of the roots = 0

Some results to remember

$a + b = -\frac{b}{a}$

$a.b = \frac{c}{a}$

$a^2 + b^2 = (a+b)^2 - 2ab$

$(a-b)^2 = a^2 + b^2 - 4ab$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$a^3 - b^3 = (a+b)(a^2 + b^2 - ab)$

$a^3 - b^3 = (a-b)(a^2 + b^2 - ab)$

In case of resiprocal

$a^3 = 5$ so $b = \frac{1}{5}$

Equations

Nature of the roots $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

i) If $b^2-4ac = 0$ the roots are real and equal; $= -\frac{b}{2a}, -\frac{b}{2a}$

ii) If $b^2-4ac > 0$ then the roots are real and unequal (or distinct)

iii) If $b^2-4ac < 0$ then the roots are imaginary;

iv) If $b^2-4ac > 0$ is a perfect square ($\neq 0$) the roots are real, rational and unequal (distinct);

v) If $b^2-4ac > 0$ but not a perfect square the rots are real, irrational and unequal.

$2 + \sqrt{3}$

$2 - \sqrt{3}$

Lin Ineq

$2x + 5 > 0$

$x > -\frac{5}{2}$

$-2x + 5 > 0$

$-2x > -5$

$x < \frac{-5}{-2}$

$x < \frac{5}{2}$

Cubic Equation → $ax^3 + bx^2 + cx + d = 0$

Sum of Roots and Product of Roots:-

$\alpha + \beta + \gamma = -\frac{b}{a}$ | $\alpha\beta + \beta.\gamma + \alpha.\gamma = \frac{c}{a}$ | $\alpha\,\beta\,\gamma = -\frac{d}{a}$

Mathematics of Finance

Mathematics of Finance

Simple interest: It is the interest computed on the principal for the entire period of borrowing.

$I = Pit$

$SI = P \times \frac{r}{100} \times t$

$A = P + I$

$I = A - P$

$i = \frac{r}{100}$

Here, A = Accumulated amount (final value of an investment)

P = Principal (initial value of an investment)

i = Annual interest rate in decimal.

I = Amount of Interest

t = Time in years

$\boxed{A = P + n\,SC_1}$

Compound interest: as the interest that accrues when earnings for each specified period of time added to the principal thus increasing the principal base on which subsequent interest is computed.

Formula for compound interest: $A = P\left(1 + \frac{r}{100c}\right)^{tc}$

$A_n = P(1 + i)^n$

$C.I. = A_n - P = P(1 + i)^n - P$ $i = \frac{r}{100 \cdot c}$ $n = t c$

where, P = Principal i = Annual rate of interest

n = Number of total conversion period i.e. t x no. of conversions per year

6m → c = 2

3m → c = 4

1m → c = 12

Effective Rate of Interest: The effective interest rate can be computed directly by following formula:

$E = ((1 + I)^n - 1) \times 100$ I. Rate $= \frac{Int}{P.A} \times 100$

Future value of a single cash flow can be computed by above formula. Replace A by future value (F) and P by single cash flow (C.F.) therefore

$F = C.F.\,(1 + i)^n$

Sink Fund → FVAR

CE → PV → Gen, PV AR

Leasin → PV

Val. of Bond $= \frac{A((1+r)^n - 1)}{r(1+r)^n} + \frac{A_n}{(1+r)^n}$

$CAGR = \left(\left(\frac{V_n}{V_0}\right)^{\frac{1}{t_n - t_0}} - 1\right) \times 100$

Annuity

(same amount in same internal)

Annuity regular

(I set payment end of set period)

Future value

$FV = A\left(\frac{(1+i)^n - 1}{i}\right)$

(FV calculated on last annuity paid)

Present value

$PV = A\left(\frac{(1+i)^n - 1}{i(1+i)^n}\right)$

(PV calculated on start and last annuity after paid 1 period)

Annuity Immediate

(I set pay short of set period)

Future value

$FV = A\left(\frac{(1+i)^n - 1}{i}\right) \times (1+i)$

(FV calculated on after last annuity payment)

Present value

$PV = A\left(\frac{(1+i)^{n-1} - 1}{i(1+i)^{n-1}}\right) + A$

(PV calculated on day of first annuity paid)

Permutation and Combination

Fundamental principles of counting

$m \times n$

Multiplication Rule: If certain thing may be done in 'm' different ways and when it has been done, a second thing can be done in 'n' different ways then total number of ways of doing both things simultaneously = m × n.

Addition Rule : It there are two alternative jobs which can be done in 'm' ways and in 'n' ways respectively then either of two jobs can be done in (m + n) ways.

$m + n$

Permutation

$^nP_r = \frac{n!}{(n-r)!}$

The number of permutations of n things chosen r at a time is given by - $^nPr = n!/ n\text{-}r\,!$

Arranging n things in circular arrangement is given by :(n-1)!

•The number of necklaces formed with n beads of different colours $= \frac{1}{2}(n\text{-}1)!$

Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is $^{n-1}p_r$.

Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is $r.\,^{n-1}P_{r-1}$.

$r \cdot {}^{n-1}P_{r-1}$

$\frac{1}{2}(n-1)!$

Combinations

$^nC_r = n!/\,r!\,n\text{-}r\,!$

only select

$\boxed{^nC_r = {}^nC_{n-r}}$

$^nC_0 = 1$ and $^nC_n = 1.$

$^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$

$^nC_r = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$

Permutation and Combination

$^nP_r = {}^{n-1}P_r + r\,^{n-1}P_{r-1}$

Permutations when some of the things are alike, taken all at a time

$P = n!/\,n_1!\,n_2!\,n_3!$

Permutations of r things out of n when each thing may be repeated once, twice,… upto r times in any arrangement $= n^r$

The total number of ways in which it is possible to form groups by taking some or all of n things $(2^n - 1)$.

$^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$

The total, number of ways in which it is possible to make groups by taking some or all out of n (=n1 + n2 + n3 +…) things, where n1 things are alike of one kind and so on, is given by

$\{(n_1 + 1)(n_2 + 1)(n_3 + 1)\dots\} - 1$

The combinations of selecting r1 things from a set having n_1 objects and r2 things from a set having n_2 objects where combination of r_1 things, r_2 things are independent is given by

$^{n_1}C_{r_1} \times {}^{n_2}C_{r_2}$

am, mode, med → $y = a + bx$

[redacted]

UMP

Arithmetic Progression

A sequence $a_1, a_2, a_3, \ldots\ldots, a_n$ is called an Arithmetic Progression (A.P.) when

$a_2 - a_1 = a_3 - a_2 = \ldots.. = a_n - a_{n-1} = d$

n^{th} term $(t_n) = a + (n-1)d$,

Sum of n terms of AP:- $S_n = \frac{n}{2}[2a+(n-1)d]$

$t_n = S_n - S_{n-1}$

Sum of the first n terms

Sum of 1st n natural or counting numbers: $S = n\frac{(n+1)}{2}$

Sum of 1st n odd number : $S = n^2$

Sum of 1st n even number : $S = n(n+1)$

Sum of the Squares of the first, n natural numbers0: $\frac{n(n+1)(2n+1)}{6}$

Sum of the squares of the first, n natural numbers is $\left\{\frac{n(n+1)}{2}\right\}^2$

Geometric Progression (G.P)

n^{th} term of a GP= ar^{n-1}

Sum of first n terms of a G P

$S_n = a\frac{(1-r^n)}{(1-r)}$ / when $r < 1$

$S_n = a\frac{(r^n-1)}{(r-1)}$ / when $r > 1$

Sum of infinite geometric series

$S_\infty = \frac{a}{1-r}, r < 1$

Set, Function & Relation

Sets

no. of subset → 2^n

~~No. of sets for n number of elements :- 2^n subsets.~~

Power Set :- The collection of all possible subsets of a given set A is called the power set of A, to be denoted by P(A).

Proper Subset and Super Set :- $n(P(A)) = 2^n$

A set containing n elements has 2^n-1 proper subsets

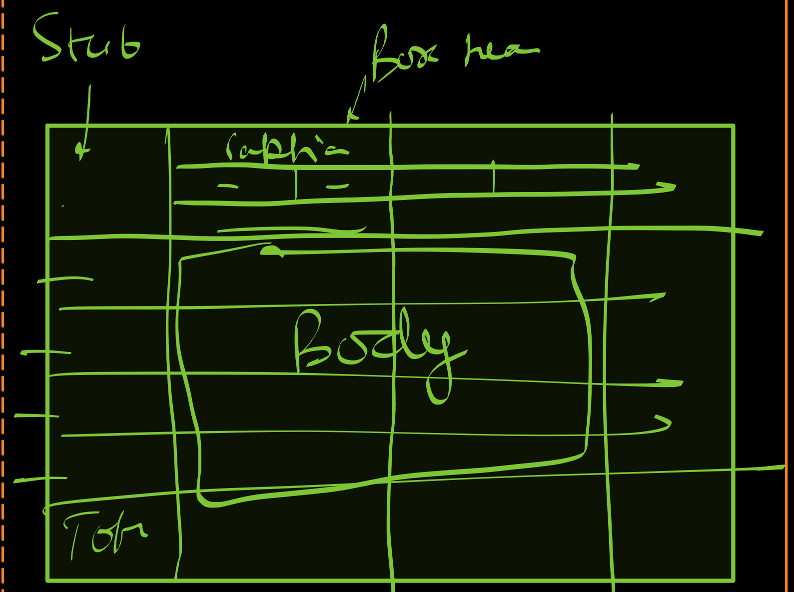
Cardinal Number:-

(1) $n(A \cup B) = n(A) + n(B) - \cancel{n(A \cap C)}$ $n(A \cap B)$

(2) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C)$ $- n(B \cap C) + n(A \cap B \cap C)$

(3) $n(A - B) = n(A) - n(A \cap B)$

(4) $n(B - A) = n(B) - n(A \cap B)$



Footnote :-

Demorgan's

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

Measure of Central Tendancy and Dispersion

$\bar{x}_c = \frac{\bar{x}_w + (c_1 + c_2 \ldots) - w_1 - w_2 \ldots}{n}$

Measure of Central Tendancy

	ARITHMETIC MEAN	GEOMETRIC MEAN	HARMONIC MEAN	MODE
Individual Observation	$\bar{x} = \frac{(x_1+x_2+x_3+\ldots+x_n)}{n}$ $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$	$GM = (x_1 \times x_2 \times x_3 \ldots. \times x_n)^{1/n}$ Logarithm of G for a set of observations is the AM of the logarithm of the observations; i.e. $\log GM = \frac{\Sigma \log x}{n}$ G.M. = Antilog $\frac{\Sigma \log x}{n}$	$HM = \frac{n}{\Sigma\left(\frac{1}{x_i}\right)}$	The value that occurs the maximum number of times
Frequency Distribution	$\bar{x} = \frac{(x_1f_1 + x_2f_2 + x_3f_3 + \ldots + x_nf_n)}{(f_1+f_2+f_3+\ldots+f_n)}$	$GM = (x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \ldots. \times x_n^{f_n})^{1/n}$	$HM = \frac{n}{\Sigma\left(\frac{f_i}{x_i}\right)}$	Mode = $L + \frac{(f_1 - f_0) \times h}{2f_1 - f_0 - f_2}$ l_1 = LCB of the modal class i.e. the class containing mode. f_0 = frequency of the modal class f_{-1} = frequency of the pre-modal f_1 = frequency of the post modal class C = class length of the modal class
Relationship variables	$\bar{y} = a + b\bar{x}$	if z = xy then GM z = (GM of x) × (GM of y) if z = x / y then GM of z= (GM of x) /(GM of y)		$y_{mo} = a + bx_{mo}$
Weighted Mean	Weighted A.M = $\frac{\Sigma x_i w_t}{\Sigma w_t}$	Weighted G.M = Antilog $\frac{\Sigma w_t \log x_i}{\Sigma w_i}$	Weighted H.M= $\frac{\Sigma w_i}{\Sigma\left(\frac{w_i}{x_i}\right)}$	
Combined Mean	Combined A· M. $\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$		Combined H·M.= $\frac{n_1+n_2}{\frac{n_1}{H_1}+\frac{n_2}{H_2}}$	

Relation among Average

For Given two positive numbers $(A.M) \times (H.M) = (G.M)^2$ → disgrind → AM > GM > HM

Same → AM = GM = HM

$AM = \frac{a+b}{2}$ $\quad GM = \sqrt{ab}$ $\quad HM = \frac{2ab}{a+b}$

AM ≥ GM ≥ HM The equality sign occurs, as we have already seen, when all the observations are equal.

Mode = 3 Median – 2 Mean or Mean – Mode =3 (Mean – Median)

Measure of Central Tendancy

	MEDIAN	QUARTILES (Q_1, Q_2 & Q_3)	DECILES ($D_1, D_2, D_3....D_9$)	PERCENTILES ($P_1, P_2, P_3....P_{99}$)
Discrete Series/Unclassified Data	Median $=$ Size of $\left(\frac{N+1}{2}\right)^{th}$ item	Q_1 quartile is given by the $\frac{1}{4}(N+1)$ th value the Q_n quartile is given by the $\frac{n}{4}(N+1)$ th value	The D_1 Decile is given by the $\frac{1}{10}(N+1)$ th value D_n Decile is given by the $\frac{n}{10}(N+1) th$ value	The P_1 Percentile is given by the $\frac{1}{100}(N+1)$ th value P_n Percentile is given by the $\frac{n}{100}(N+1)$ th value
Group Frequency Distribution	Median $= l_1 + \left(\frac{\frac{N}{2} - CF}{f}\right) XC$ l_1 = lower class boundary of the median class i.e. "" the class containing median. N= total frequency. CF = less than cumulative frequency corresponding to l_1. (Pre median class) f = frequency of the median class $C = l_2 - l_1$ = length of the median class.	$Q_n = l_1 + \left(\frac{N \cdot p - CF_l}{f}\right) \times C$ l^1 = lower class boundaty of the Quartile class i.e. the class containing Quartile. N = total frequency. $p = \frac{1}{4}, \frac{2}{4}, \frac{3}{4}$ for Q_1, Q_2, Q_3 respectively CF = less than cumulative frequency corresponding to l_1 (Pre Quartile class) F = frequency of the Quartile class. $C = l_2 - l_1$ = length of the Quartile class.	$D_n = l_1 + \left(\frac{N \cdot p - CF_l}{f}\right) \times C$ l_1 = lower class boundary of the Decile class i.e. the class containing Decile. N = total frequency. $p = \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, ..., \frac{9}{10}$ for $D_1, D_2, D_3, ..., D_9$ respectively CF = less than cumulative frequency corresponding to l_1. (Pre Decile class) F = frequency of the Decile class. $C = l_2 - l_1$ = length of the Decile class.	$P_n = l_1 + \left(\frac{N \cdot p - CF_l}{f}\right) \times C$ l_1 = lower class boundary of the Percentile class i.e. the class containing Percentile. N = total frequency $p = \frac{1}{100}, \frac{2}{100}, \frac{3}{100}, ..., \frac{99}{100}$ for $P_1, P_2, P_3, ..., P_{99}$ respectively CF = less than cumulative frequency corresponding to l_1. (Pre Percentile class) F = frequency of the Decile class. $C = l_2 - l_1$ = length of the Percentile class.

Note:- 1. $y_{me} = a + bx_{me}$ 2. $S|(x_i - A)|$ is minimum if we choose A as the median

$\Sigma(x_i - \bar{x}) = 0$ $\Sigma|x_i - x_{med}| \rightarrow min$

Measure of Dispersion

	Absolute	Relative	If y = a + bx
RANGE (R)	Range = Largest (L) – Smallest (S)	Co efficient of Range $= \frac{L-S}{L+S} \times 100$	$R_y = \|b\| \times R_x$
MEAN DEVIATION (M.D) about A	$M.D_A = \frac{1}{n}\sum\|x - A\|$	Co efficient of M.D from A $= \frac{M.D\ about\ A}{A} \times 100$	$M.D_y = \|b\| \times M.D D_x$
MEAN DEVIATION (M.D) about A.M ($\bar{x}$)	M.D about Mean $= \frac{1}{n}\sum\|x_i - \bar{X}\|$	Co efficient of M.D from A.M $= \frac{\text{M.D about } \bar{x}}{\bar{x}} \times 100$	$M.D_y = \|b\| \times M \cdot D_x$
MEAN DEVIATION (M.D) about Median	M.D about Median $= \frac{1}{n}\sum \| x_i - \text{Med} \|$	Co efficient of M.D from Median $= \frac{\text{M.D about } Med}{Med} \times 100$	$M.D_y = \|b\| \times M \cdot D_x$

Measure of Dispersion

	Absolute	Relative	If y = a + bx
Standard Deviation (s)	$\sigma = \sqrt{\frac{\sum(x_i-\bar{X})^2}{n}}$ $\sigma = \sqrt{\frac{\sum x_i^2}{n} - \bar{X}^2}$	Co efficient of Variation $= \frac{\sigma}{\bar{x}} \times 100$	$\sigma_y = \lvert b \rvert \times \sigma_x$
	Standard Deviations for first n Natural numbers, $\sigma = \sqrt{\frac{n^2-1}{12}}$	Combined Standard Deviation, $\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2+n_2\sigma_2^2+n_1d_1^2+n_2d_2^2}{n_1+n_2}}$ Where $d_1 = \bar{x}_1 - \bar{x}_{12}$, $d_2 = \bar{x}_2 - \bar{x}_{12}$	
Quartile Deviation (QD)	QD= $\frac{Q_3-Q_1}{2}$	Co-efficient of Q.D $= \frac{Q_3-Q_1}{Q_3+Q_1} \times 100$ Or Co-efficient of Q.D $= \frac{Q.D}{\text{Median}} \times 100$(for Symm. Distribution)	
Variance (s^2)	Variance means Square of standard Deviation		

root mean squared deviation around the mean

uniform, hom

cov $= \frac{\sigma}{\bar{x}} \times 100$

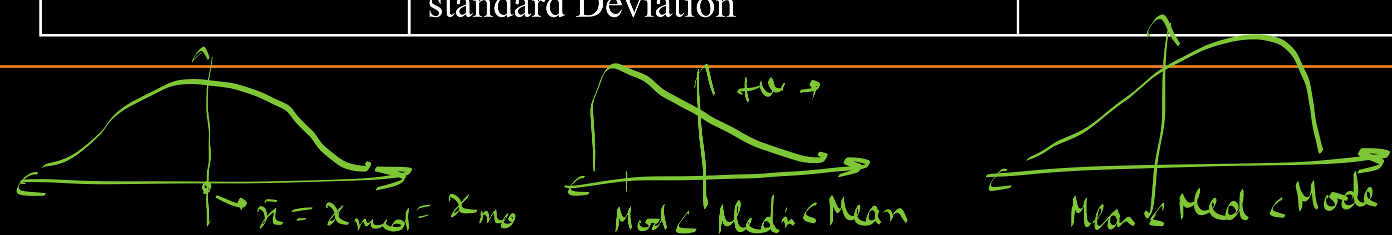
a, b → SD = $\frac{|a-b|}{2}$

n_1, n_2; σ_1, σ_2; $\bar{x}_1, \bar{x}_2$ → $\bar{x} = \frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$

* Properties of Dispersion

Change of Scale & Change of Origin:

Central tendency	Dispersion
↓	↓
AM, Median, Mode	Range, SD, MD, QD
↓	↓
(x,÷) scale → ✓	Scale →
(+,–) origin → ✓	Origin →
↓	↓
y = a + bx	y = a + bx
y = a + bx	$R_y = \lvert b \rvert . R_x$
$me_y = a + b\, me_x$	$s_y = \lvert b \rvert .\, s_x$
$mo_y = a + b\, mo_x$	$M.D_y = \lvert b \rvert . M.D_x$
	$S.D_y = \lvert b \rvert . S.D_x$
	$Q.D_y = \lvert b \rvert . Q.D_x$



Probability

$$P(A) = \frac{n_A}{n} = \frac{\text{No. of eqally likely events favourable to A}}{\text{Total no. of equally likely events}}$$

- The probability of an event lies between 0 and 1, both inclusive. i.e., $0 \le P(A) \le 1$
- When P(A) = 0, A is known to be an impossible event and when P(A) = 1, A is known to be a sure event.

  Odds in favour of A = $m_A : (m - m_A)$
  and Odds in against A = $(m - m_A) : m_A$

$0 \le P(A) \le 1$

$P(A) + P(A') = 1$

- For any two events A and B, i.e., $P(A \cup B)=P(A)+P(B)-P(A\cap B)$
- For any three events A, B and C, the probability that at least one of the events occurs is given by

$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$

Conditional Probability

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}$$

$P(A/B) = \frac{P(A\cap B)}{P(B)}$

Compound Probability or Joint Proability

$P(A \cap B) = P(A) \times P(B/A)$ Provided $P(A) > 0$

Expected value

$E(x) = \Sigma x_i p_i$ or $P(x)$

x →	0	1	2	3
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

→ $\Sigma P(x) = 1$

- When x is a discrete random variable with probability mass function $f(x)$, then its expected value is given by $\mu = \Sigma x f(x)$

and its variance is

$\sigma^2 = E(x^2) - \mu^2$

Where $E(x^2) = \Sigma x^2 f(x)$

$E(x)$ or $\mu = \Sigma x_i . P$

$E(x^2) = \Sigma x_i^2 . P$

$\sigma = \sqrt{E(x^2) - (E(x))^2}$

Properties of Expected Values

- Expectation of a constant k is k i.e.
- E(k) = k for any constant k.
- E(x+y) = E(x)+E(y) for any two random variables x and y.
- E(kx) = k.E(x) for any constant k
- E(xy) = E(x)×E(y)

Whenever x and y are independent.

Statistics (Chapter – Theoretical Distribution)

Majj ama

[redacted]

Name	Condition	Probability Mass Function	Notation	Mean	Varinace	Mode	Remarks
Binomial Distribution	Trials are independent and each trail has only two outcomes Success & failure.	$P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$	$X \sim B(n, p)$	$\mu = np$	$\sigma^2 = npq$	Mode = (n+1)p if non integer If integer = (n+1)p and (n+1)p –1	p + q = 1
Poisson Distribution	Trials are independent and probability of occurrence is very small in give time.	$P(X=x) = \frac{e^{-m} m^{x}}{x!}$ For x=0,1,2,..,n	$X \sim P(m)$	$\mu = m$	$\sigma^2 = m$	Mode = m if non integer If integer = m and m–1	e = 2.71828
Normal or Gaussian Distribution	When distribution is symmetric	$P(X=x) = \frac{1}{s\sqrt{2p}} e^{-(x-m)}$ For $-\infty < x < +\infty$	$X \sim N(\mu, \sigma^2)$ $Z = \frac{x-m}{s}$	Mean = Median = Mode =	σ^2		Mean Deviation = 0.8σ First Quartile = μ–0.675σ Third Quartile = μ+0.675σ Quartile Deviation =0.675σ Point of Inflexion x= μ– σ and x= μ+ σ

Relationship for Normal Distribution: - 4SD = 5MD = 6QD

S.N.D → $\mu = 0, \sigma = 1$
MD = 0.8, QD = 0.675
$Q_1 = -0.675$, $Q_3 = 0.675$

Statistics (Chapter – Correlation and Regression)

Correlation

The following table shows degrees of correlation according to various values of r.

Degree of Correlation	Positive	Negative
Perfect correlation	+1	–1
Very high degree of correlation	+ 0.9 to + 1	– 0.9 to – 1
Fairly high degree of correlation	+ 0.75 to + 0.9	– 0.75 to – 0.9
Moderate degree of correlation	+ 0.50 to + 0.75	– 0.50 to – 0.75
Low degree of correlation	+ 0.25 to + 0.50	– 0.25 to – 0.5
Very low degree of correlation	0 to + 0.25	– 0.25 to 0
No correlation	0	0

Correlation

KARL PEARSON'S PRODUCT MOMENT CORRELATION COEFFICIENT	SPEARMAN'S RANK CORRELATION COEFFICIENT	COEFFICIENT OF CONCURRENT DEVIATIONS
$r = r_{xy} = \frac{Cov(x,y)}{\sigma_x \times \sigma_y}$ Where, $Cov(x,y) = \frac{\sum (x_i-\bar{x})(y_i-\bar{y})}{n}$ or $= \frac{\sum x_i y_i}{n} - \bar{x}\bar{y}$ $\sigma_x = \sqrt{\frac{\sum (x_i-\bar{x})^2}{n}}$ $\sigma_y = \sqrt{\frac{\sum (y_i-\bar{y})^2}{n}}$ $= \frac{n\sum x_i y_i - \sum x_i \times \sum y_i}{\sqrt{[n\sum x_i^2 - (\sum x_i)^2] \times [n\sum y_i^2 - (\sum y_i)^2]}}$ $r_{xy} = \frac{bd}{\lvert b \rvert \lvert d \rvert} r_{uv}$ where $u = \frac{x-a}{b}$ and $v = \frac{y-c}{d}$	$r_R = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$ For tied ranking, $r_R = 1 - \frac{6\left[\sum d_i^2 + \sum \frac{(t^3-t)}{12}\right]}{n(n^2-1)}$	$r_C = \pm\sqrt{\pm\frac{(2c-m)}{m}}$ Where c is concurrent deviation, m is one less than number of pairs of observations m = n-1

u = a + bx
v = c + dy
$r_{uv} = + r_{xy}$ $r_{uv} = -r_{xy}$
→ b & d → same sign

REGRESSION ANALYSIS

	Y depends on X	X depends on Y
Simple Regression Equation	$y = a_{yx} + b_{yx}\, x$	$x = a_{xy} + b_{xy}\, y$
Normal Equations	$\sum y_i = na + b_{yx}\sum x_i$ $\sum x_i y_i = a\sum x_i + b_{yx}\sum x_i^2$	$\sum x_i = na + b_{xy}\sum y_i$ $\sum x_i y_i = a\sum y_i + b_{xy}\sum y_i^2$
Regression Coefficient r, b_{yx} & b_{xy} → same sign	$b_{yx} = \frac{Cov(x,y)}{\sigma_x^2}$ $b_{yx} = \frac{r\sigma_y}{\sigma_x}$	$b_{xy} = \frac{Cov(x,y)}{\sigma_y^2}$ $b_{xy} = \frac{r\sigma_x}{\sigma_y}$
$r = \sqrt{b_{yx} \times b_{xy}}$	$b_{yx} = \frac{n\sum x_i y_i - \sum x_i \times \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$	$b_{xy} = \frac{n\sum x_i y_i - \sum x_i \times \sum y_i}{n\sum y_i^2 - (\sum y_i)^2}$

Some Important Relation:- Intersection point of these two lines is $\bar{x}, \bar{y}$

2y + 3x = 8, 4x + 5y = 6

Some Important Points :-

$r = \pm\sqrt{b_{yx} \times b_{xy}}$

$b_{yx} = \frac{q}{p} \times b_{vu}$ where $u = \frac{x-a}{p}$ and $v = \frac{y-c}{q}$

Coefficient of Determination = r^2

Coefficient of Non – Determination = $1 - r^2$

u = a + bx
v = c + dy
→ $b_{vu} = \left(\frac{d}{b}\right) b_{yx}$
$b_{uv} = \frac{b}{d} b_{xy}$

Index Number

$V = P \times Q$

Index Number

(1) Price Index Number:- $P_{on} = \dfrac{P_n}{P_0} \times 100$

(2) Quantity Index Number:- $Q_{on} = \dfrac{Q_n}{Q_0} \times 100$

(3) Value Index Number:- $V_{on} = \dfrac{V_n}{V_0} \times 100$

Method	Price Index	Quantity Index
1. Simple Aggregate $P_{on} =$	$\dfrac{\sum P_n}{\sum P_0}$	$\dfrac{\sum Q_n}{\sum Q_0}$
2. Simple Average or Rel. $P_{on} =$	$\dfrac{\sum\left(\frac{P_n}{P_0}\right) \times 100}{n}$	$\dfrac{\sum\left(\frac{Q_n}{Q_0}\right)}{n}$
3. Weighted Aggregate		
(a) With base year weight (Laspeyre's index) Q_0	$\dfrac{\sum P_n Q_0}{\sum P_0 Q_0}$	$\dfrac{\sum Q_n P_0}{\sum Q_0 P_0}$
(b) With current year weight (Paasche's index) Q_n	$\dfrac{\sum P_n Q_n}{\sum P_0 Q_n}$	$\dfrac{\sum Q_n P_n}{\sum Q_0 P_n}$
(c) Fisher's Ideal [Geometric mean of Laspeyere's and Paasche's] $F = \sqrt{L \times P}$; ME - $w \to \frac{Q_n + Q_0}{2}$; $B \to \frac{L+P}{2}$	$\sqrt{\dfrac{\sum P_n Q_0}{\sum P_0 Q_0} \times \dfrac{\sum P_n Q_n}{\sum P_0 Q_n}}$	$\sqrt{\dfrac{\sum Q_n P_0}{\sum Q_0 P_0} \times \dfrac{\sum Q_n P_n}{\sum Q_0 P_n}}$
4. Weighted Average W = Weights = Base Year or Current Year Price Weights	$\dfrac{\sum\left(\frac{P_n}{P_0} W\right)}{\sum W}$	$\dfrac{\sum\left(\frac{Q_n}{Q_0} W\right)}{\sum W}$

Index Number

Bowley $= P_{on} = \dfrac{L+P}{2}$ CLI $= \dfrac{\Sigma I.W}{\Sigma W}$

Chain Index $= \dfrac{\text{Link relative of current year} \times \text{Chain Index of the previous year}}{100}$

Deflated Value $= \dfrac{\text{Current Value}}{\text{Price Index of the current year}} \times \text{Base}$ or Current Value $\times \dfrac{\text{Base Price } (P_0)}{\text{Current Price } (P_n)}$

Shifted Price Index $= \dfrac{\text{Original Price Index}}{\text{Price Index of the year on which it has to be shifted}} \times 100$

Marshall - Edge worth $\to P_{on} = \dfrac{\Sigma P_n \cdot \left[\dfrac{q_0 + q_n}{\cancel{2}}\right]}{\Sigma P_0 \cdot \left[\dfrac{q_0 + q_n}{\cancel{2}}\right]} \times 100$ CLI $= \dfrac{\Sigma I \cdot W}{\Sigma W}$

Index Number

Splicing of Index Number → new comm. add

Test of Adequacy

(1) Unit test → S.A.X

(2) Time reversal Test → $P_{01} \times P_{10} = 1$ → L X P S A X, FUMEV

(3) Factor reversal test → $P_{01} \times Q_{01} = V_{01}$ → FV

(4) Circular Test – Simple GM of Price Relatives and Weighted Agg. With fixed weights