CHANAKYA

FOR CA FOUNDATION

11-14 marks

MATHEMATICS OF FINANCE
(Time Value Of Money)

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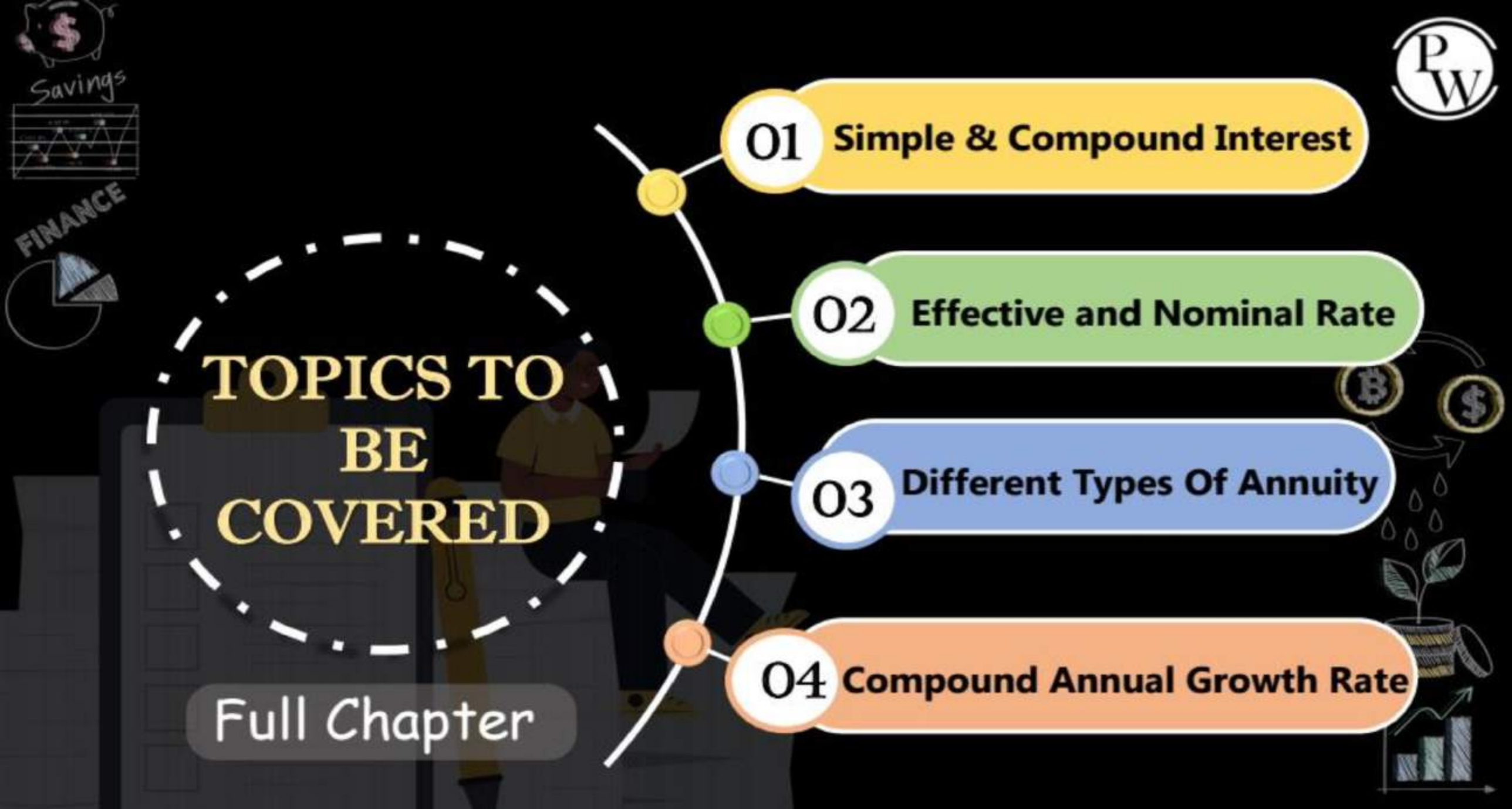

Savings
FINANCE
TOPICS TO BE COVERED
Full Chapter
01 Simple & Compound Interest
02 Effective and Nominal Rate
03 Different Types Of Annuity
04 Compound Annual Growth Rate

What is Finance ??

Money

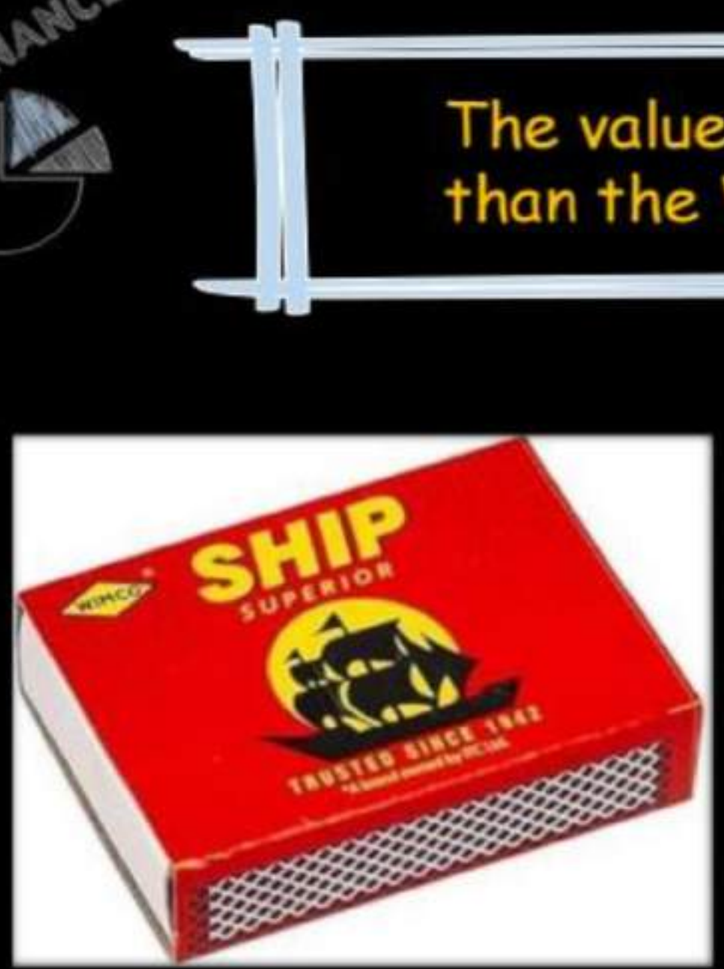
वित

Art & Science of Managing money

- Raising Money
- Spending
- Lending
- Investing
- Accounting
- Forecasting

Time Value Of Money

The value of 1 rupee is Worth more than the Value Of 1 rupee in the future



1990 – 4 FOR 1 RUPEE
2007 - 2 FOR 1 RUPEE
2024 - 1 FOR 2 RUPEE

Value Of Money Decreases Over time

Concept Of Interest (ब्याज)

5000

1Q → S/. P.a.

Consideration (Cost Incurred) Payable
For Using Other's Money

₹10

2016 → 2024

₹ Rice

₹10/kg → ₹20/kg

10 kg → 5 kg

- **Time value of money (Inflation)**
- **Opportunity Cost**
- **Liquidity**
- **Risk Factor**

Some Important Terms

(Lender) Mr. X (Rich) → ₹10,000 (10% P.a.) → (Borrower) Mr. (Y) (Poor)

1 year (₹11000) ←

- Principle = 10,000
- Interest = ₹1000
- % Rate of interest = 10%
- Decimal Rate of Interest = $\frac{10}{100} = 0.01$
- Accumulated amount = 11000
- Time Period = 1 year

$10,000 \times \frac{1}{100}$

$= 1000$

$$A = P + I$$

$$I = A - P$$

Methods Of Interest Calculation

- Simple Int. (S.I.)
- Compound Int. (C.I.)

Simple Interest

(सरल ब्याज)

⇓

Interest is calculated on principle value over a period of time

$I = PRT$

P ⇒ Principle

R ⇒ Decimal Rate of interest

T ⇒ Time Period

$A = P + I$

$= P + PRT$

$A = P[1 + RT]$

10% S.I. P.a.

Mr. X (Rich) —₹10,000→ Mr. Y

10,000
+ 1000
11000
+ 1000
12000
+1000
13000

8%
$= \frac{8}{100}$
$= 0.08$

Q) $P = 100,000$
$t = 5$ years
$r = 8\%$ P.a. S.I
find Interest & amount.

Sol: $I = P.r.t$
$= 100,000 \times 0.08 \times 5$
$I = 40,000$

Amount $= P + I$
$= 100000 + 40,000$
$= 1,40,000$

Q

$P = 50,000$

$t = 3$ years

$r = 7\%$ P.a. SI

Find Amount

Sol: $I = P.r.t$

$= 50,000 \times 0.07 \times 3$

$= 10,500$

$A = P + I$

$= 50,000 + 10,500 = A = 60,500$

Or

$A = P[1 + rt]$

$= 50,000\{1 + (0.07)(3)\}$

$= 60,500$

What is the interest on a sum of Rs 20,000 at 6% simple interest for 2 years ?

A 2200

C 2400

B 22400

D 23400

$P = 20,000$

$r = 6\%$ SI

$t = 2$ years

$I = P \times t$

$= 20,000 \times \frac{6}{100} \times 2$

$= 2400$

At what interest rate will Rs 75,000 yield Rs 3375 as simple interest in six months ?

(A) 8%
(C) 10%
(B) 9%
(D) None

$P = 75000$

$I = 3375$

$t = 6 \text{ months} = \frac{6}{12} \text{ years}$

$3375 = 75000 \times r \times \frac{6}{12}$

$3375 = 37500\, r$

$\frac{3375}{37500} = r$

$r = 0.09$ or 9%

$I = Prt$

$3375 = 75000 \times r \times \frac{6}{12}$

$\frac{3375 \times 12}{75000 \times 6} = r \Rightarrow r = 0.09$ or 9%

Square

⇓

x^2

e.g. $(2)^2 = 4$

e.g. $(3)^2 = 9$

e.g. $10^2 = 100$

Reciprocal $= \frac{1}{n} = ?$

$\div$ or $/$

$\frac{1}{2} = 0.50$

$\frac{1}{3} = 0.33$

$\frac{1}{5} = 0.20$

e.g. $\frac{3}{5}$

$= 3 \times \frac{1}{5}$

$= 0.6$

What principal will amount to Rs 645 in $1\frac{1}{2}$ years at 5%simple interest ?

(A) Rs 600 (C) Rs 550

(B) Rs 610 (D) Rs 500

$P = P$

$A = 645$

$t = 1\frac{1}{2} = \frac{3}{2}$ years $= 1.5$ years

$r = 5\%$ SI

$1\frac{1}{2} = \frac{3}{2}$

1.5 yrs

$A = P[1 + rt]$

$645 = P[1 + 0.05 \times 1.5]$

$P = \frac{645}{1.075} = 645 \times \frac{1}{1.075}$

$P = 600$

Q

Find the rate of interest if the amount owed after 6 months is Rs 1050 & borrowed amount being Rs 1000

(A) 5%

(C) 10%

(B) 8%

(D) None

$P = 1000$

$A = 1050$

$I = 50$

$t = 6$ months

$I = P \times r \times t$

$50 = 1000 \times r \times \frac{6}{12}$

$r = 0.1$

or

10%

Q

A sum of money lent out at simple interest amounts to Rs 720 after 2 years and to Rs 1020 after a further period of 5 years. The principal amount is

(A) Rs 500
(C) Rs 600
(B) Rs 520
(D) Rs 480

$\frac{A_2}{A_7} = \frac{720}{1020} = \frac{P[1+2r]}{P[1+7r]}$

$72 + 504r = 102 + 204r$

$300r = 30$

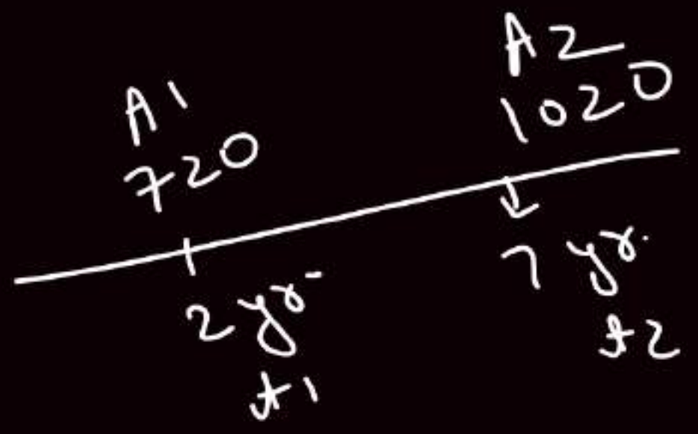
$r = \frac{1}{10} = 0.1$ or 10%

$A_2 = P(1 + r/2)] = 720$

$P(1 + 0.1 \times 2] = 720$

$P = \frac{720}{1.2}$

$P = 600$

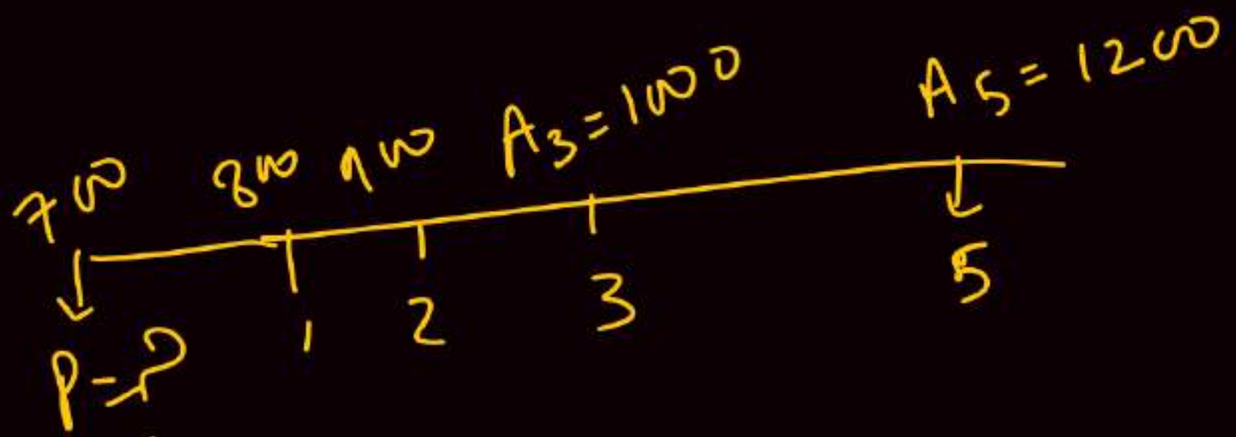


S.I. $r = ?$

$$r = \frac{A_2 - A_1}{A_1 t_2 - A_2 t_1}$$

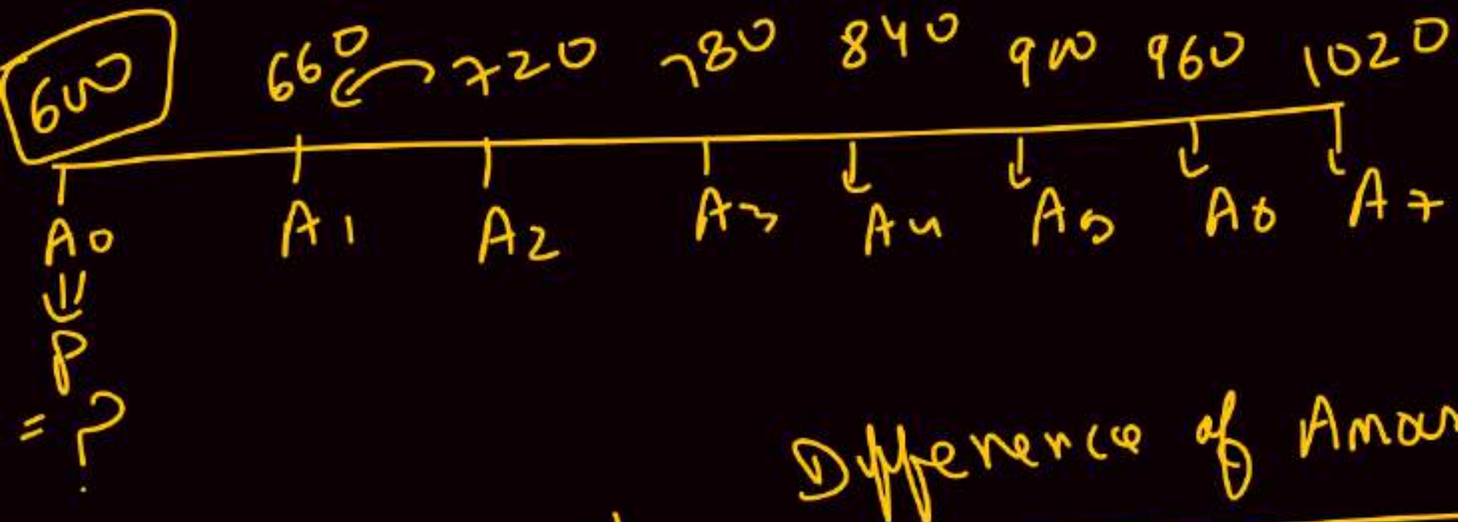
S.I. Rate of interest

$$r = \frac{1020 - 720}{720(7) - (1020)(2)} = \frac{300}{5040 - 2040} = \frac{300}{3000} = \frac{1}{10} =$$



$SW \Rightarrow 200$ in 2 yrs

$SW = \frac{200}{2} = 100/\text{yr}$



$$\text{Interest} = \frac{\text{Difference of Amount}}{\text{Diff. of time}}$$

$$= \frac{1020 - 720}{7 - 2}$$

$$= \frac{300}{5}$$

$$\text{Interest} = 60$$

$A_2 = 720$

5 yr ⟶ Amt ⇓ 300

$\frac{300}{5/60}$

A Sum of money double itself in 5 years
Find the simple rate of interest

(A) 5%
(C) 15%
(B) 10%
(D) 20%

$A = P[1 + rt]$

$2 = 1[1 + (r)(5)]$

$2 = 1 + 5r$

$1 = 5r \Rightarrow r = \frac{1}{5} = 0.20$ or 20%

S.I.

Double amount
$r = \frac{1}{t}$

Tripple amount
$r = \frac{2}{t}$

$r = \frac{1}{5} = 0.2$ or 20%

A Sum of money double itself in 10 years , The number Of years it would triple itself is

(A) 15 years (C) 25 Years

(B) 20 Years (D) 30 Years

$A = P[1 + rt]$

$2 = 1[1 + r(10)]$

$2 = 1 + 10r$

$1 = 10r$

$r = \frac{1}{10} = 0.1$ or 10%

Now

$A = P(1 + rt)$

$3 = 1[1 + 0.1t]$

$3 = 1 + 0.1t$

$2 = 0.1t$

$t = \frac{2}{0.1} = 20$ years.

Double

$r = \frac{1}{t} = \frac{1}{10} = 0.1$

Now Triple

$r = \frac{2}{t}$

$t = \frac{2}{r}$

$= \frac{2}{0.1}$

$= 20$ years.

If Simple interest on a certain sum of money is $\frac{1}{25}$ times Of principle, if numbers of years is equal to the rate percent of interest ,Then rate is ?

(A) 2%
(C) 25%
(B) 20%
(D) 35%

$I = \frac{1}{25} \times P$

$P \times \frac{r}{100} \times t = \frac{1}{25} \times P$

$t = r$

Rate in %

$\frac{r}{100} \times r = \frac{1}{25}$

$r^2 = \frac{100}{25}$

$r^2 = 4 \Rightarrow r = 2$

In what time a certain sum of money amounts to Rs 400 at 10% pa S.I. & to Rs 200 at 4% pa S.I.

(A) 15 years
(C) 40 Years
(B) 30 Years
(D) 50 Years

$r = 10\%$
$A = 400$
$P[1 + rt] = 400$
$P[1 + 0.10t] = 400$

$r = 4\%$
$A = 200$
$P[1 + rt] = 200$
$P[1 + 0.04t] = 200$

$$\frac{P(1 + 0.10t)}{P(1 + 0.04t)} = \frac{400}{200} = 2$$

$1 + 0.10t = 2 + 0.08t$

$0.02t = 1$

$t = \frac{1}{0.02}$

$t = 50$ yrs

$$t = \frac{A_2 - A_1}{A_1\gamma_2 - A_2\gamma_1}$$

$$= \frac{200 - 400}{400(0.04) - 200(0.10)}$$

$$= \frac{-200}{16 - 20}$$

$$t = \frac{-200}{-4} = 50$$

$$= \frac{P[1 + \gamma_1(t)]}{P[1 + \gamma_2(t)]}$$

$\gamma_1 = 0.10$ | $\gamma_2 = 0.04$
$A_1 = 400$ | $A_2 = 200$

γ_1 & γ_2
A_1 & A_2
$\Downarrow$
$t = ?$

Compound Interest

Interest on Interest

10%

S.I:

: 100000
+10,000

1,10,000
10,000

1,20,000
+10,000

1,30,000

CI

10% CI

100000 × 10%
= 10,000

100000 × 10% = 10,000

10,000 × 10% = 1000

11000

$A_1 = 100000 + 10,000$
$= 1,10,000$

$A_2 = 1,10,000 + 11000$
$= 1,21,000$

100,000

+ 10,000 → Interest for first year

1,10,000 → Amount after 1st year

+ 11000 → Interest for second year

1,21,000 → Amount after 2nd year ← A_2

+ 12,100 → Interest for 3rd year

1,33,100 → Amount after 3 years. ← A_3

$P = 100,000$

$A = 1,33,10$

Interest of 3 ye

$= A - P$

$= 33,100$

Compound Interest

$C.I = A - P$

$\# \quad Amount(A) = P\left[1 + \frac{r}{m}\right]^{t \times m}$

$m \Rightarrow$ No. of conversion
(No of compounding)

- Annually $\Rightarrow m = 1$
- Semiannually $\Rightarrow m = 2$
- Quarterly $\Rightarrow m = 4$
- Monthly $\Rightarrow m = 12$

Daily $\Rightarrow m = 365$

✗ Continously $\Rightarrow m \Rightarrow \infty$

$$A = P\left[1 + \frac{r}{m}\right]^{t \times m}$$

$$C.I = A - P$$

$$= P\left[1 + \frac{r}{m}\right]^{t \times m} - P$$

$$C.I. = P\left\{\left(1 + \frac{r}{m}\right)^{t \times m} - 1\right\}$$

Find the compound amount if Rs 20,000 invested for 4 years at 6% compounded annually

A. 25249.53
C. 25240.82
B. 24238.84
D. 25290.48

$$A = P\left[1 + \frac{r}{m}\right]^{t \times m}$$

$P = 20,000$

$t = 4$ years

$r = 0.06$

$m = 1$

$$A = 20,000\left\{1 + \frac{0.06}{1}\right\}^{4 \times 1}$$

$$= 20,000(1.06)^4$$

$$A = 25249.53$$

Now $I = A - P$

$= 5249.53$

$(x)^n = ?$

→ 'x' on the screen

→ Press [x]

→ Press [=] (n-1) times

$(2)^2 = 4$

$(2)^3 = 8$

$(2)^4 = 16$

$(5)^6 = 15625$

Find the compound amount if Rs 20,000 invested for 4 years at 6% compounded semiannually

(A) 25300.28 (C) 25240.28

(B) 26238.84 (D) 25335.40

$$A = P\left[1 + \frac{r}{m}\right]^{t \times m}$$

$$= 20{,}000\left[1 + \frac{0.06}{2}\right]^{4 \times 2}$$

$$= 20{,}000\,(1.03)^8$$

$$= 25{,}335.40$$

$$I = 5{,}335.40$$

Q: $P = 50{,}000$

$r = 12\%$ monthly

$t = 3$ years

$A = ?$

$I = ?$

Some Important Calculator Tricks

Reciprocal

$\div$ =

Ex $\frac{1}{5}$

$= 0.2$

Square

$\times$ =

Ex 11^2

$(x)^n$

→ 'x' on screen

→ Press [×]

→ Press [=] (n-1) times

Ex $(4)^9 = 262144$

Ex $(2)^{10} = 1024$

$\frac{1}{(x)^n} = (x)^{-n}$

→ 'x' on the screen

→ Press $\boxed{\div}$

→ Press $\boxed{=}$ n times

g $\frac{1}{(2)^3}$

$= 0.125$

g $\frac{1}{(3)^5}$

$= 0.004115$

$(x)^{1/n} = ?$

⇒ 'x' on the screen

⇒ $\sqrt{}$ 12 times

⇒ Subtract 1

⇒ $\div$ n

⇒ Add 1

⇒ $\boxed{X=}$ 12 times

g $(16)^{1/4} = 2$

g $(125)^{1/3}$

$= 5$

Q $(2)^x = 16$

$(2)^x = (2)^4$

$\boxed{x = 4}$

Q $(2)^x = 30$

$x = ?$

Sol: $(2)^x = 30$

$\log (2)^x = \log (30)$

$x \log 2 = \log 30$

$x = \dfrac{\log 30}{\log 2}$

$= \dfrac{1.4771}{0.3010}$

$\boxed{x = 4.9073}$

Logarithmic function

$$y = \log_{10}(x)$$

$\log(x)^n = n\log(x)$

§ $\log(x^2) = 2\log x$

§ $\log(x^{10}) = 10\log x$

log using Calculator

→ $\sqrt{}$ 19 times

→ -1

→ $\times 227695$

§ $\log(30) = 1.4771$

§ $\log(2) = 0.3010$

Antilog

$\Rightarrow \quad \div \; 227695$

$\Rightarrow \quad +1$

$\Rightarrow \quad \boxed{X=} \; 19 \text{ times}$

$2 \xrightarrow{\log} 0.3010$

$A.L. \leftarrow$

$\S \quad \log(2) = 0.3010$

$AL(0.3010) = 2$

$AL(1.4771) = 30$

Q

A certain sum of money is invested at 4 % compounded annually & the interest for the second year is Rs 25. Find the interest for the third year

A Rs 24

C Rs 25

B Rs 26

D Rs 27

$A = P(1+$

$r = 4\%$ Annually

I.I. for second year $= 25$

$A_2 - A_1 = 25$

$P(1+r)^2 - P(1+r) = 25$

$P(1+r)[(1+r)-1] = 25$

$P(1+r)(r) = 25$

$P(1+0.04)(0.04) = 25$

$P = 600.96$

Now I.I. for III rd year

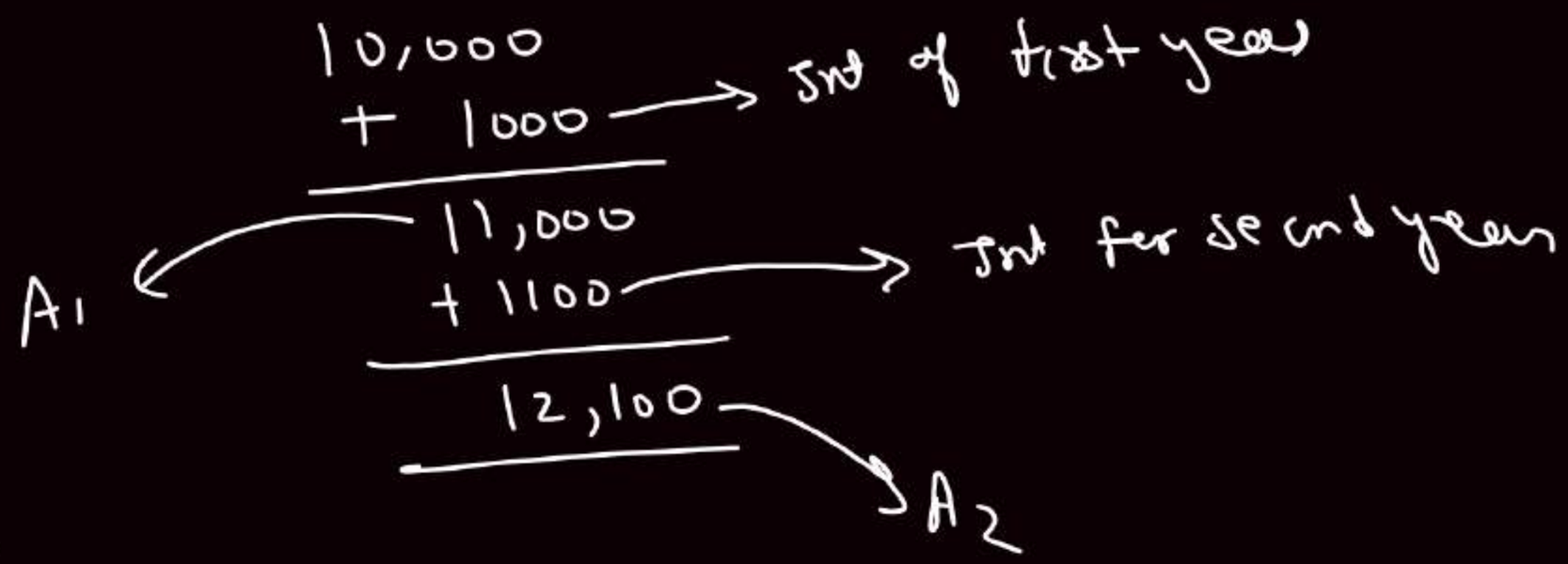
$= A_3 - A_2$

$= P(1+r)^3 - P(1+r)^2$

$= P(1+r)^2[(1+r)-1]$

$= 600.96(1+0.04)^2(0.04)$

10%



Int for 2^{nd} year $= A_2 - A_1$

Int for III^{rd} year $= A_3 - A_2$

IV^{th} year $= A_4 - A_3$

The Difference between simple interest and compound interest on Rs 2400 for 2 years at 5 % p.a. is

(A) Rs 5
(C) Rs 16
(B) Rs 10
(D) Rs 6

$P = 2400$
$t = 2$ years
$r = 5\%$ p.a.

$CI - SI = P r^2$
$= 2400 \times (0.05)^2$
$= 6$

$CI - S.I$
$= P[1+r]^t - P - Prt$
$= P[(1+r)^2 - 1 - rt]$
$= P[(1+r)^2 - 1 - 2r]$
$= P[1 + r^2 + 2(1)(r) - 1 - 2r]$
$= Pr^2$

for 2 years

$C.I - S.I = P r^2$

for 3 years

$CI - SI = P r^2 (r$

$CI - SI$

$P(1+r)^3 - P - Pr(3)$

$P[(1+r)^3 - 1 - 3r]$

$P[1 + r^3 + 3r + 3r^2 - 1 - 3r]$

$P[r^3 + 3r^2] = Pr^2(r+3)$

Q

The Difference between simple interest and compound interest on Rs P for 3 years at 6 % p.a. is 110.16. Then the value of P ?

(A) Rs 3000

(C) Rs 12000

(B) Rs 3700

(D) Rs 10000

$$CI - SI = P r^2 (r+3)$$

$$110.16 = P \times (0.06)^2 [0.06+3]$$

$$110.16 = P(0.011016)$$

$$P = 10,000$$

Q

A man made a deposit of Rs 2,500 in a saving account. The deposit was left to accumulate at 6% compound rate quarterly for the first 5 year and at 8% compounded semi annually for the next 8 years find the compound amount at the end of the 13 years

$A = P\left(1+\frac{r}{n}\right)$

A. Rs 6306.55

C. Rs 6200.48

B. Rs 6407.85

D. Rs 6398.58

$P = 2500$

for 5 years	for 8 years
6% quarterly	8% semiannually

$$A = 2500\left[1+\frac{0.06}{4}\right]^{5\times4} \times \left[1+\frac{0.08}{2}\right]^{8\times2}$$

$$= 2500(1.3468)(1.8729)$$

$$= 6306.32$$

How long will it take Rs 20000 to amount to Rs 25249.5392 at 6% compounded annually ?

(A) 3 years (C) 4 years ✓

(B) 5 years (D) 6 years

Sol:

$$A = P[1+r]^t$$

$$25249.5392 = 20{,}000[1+0.06]^t$$

$$1.262477696 = (1.06)^t$$

A National Saving Certificate costs Rs 15 and realizes Rs 20 after 10 years Find the rate of C.I. if it is compounded annually

(A) 2.12%
(C) 3.24 %
(B) 2.92%
(D) None

$$20 = 15(1+\gamma)^{10}$$

$$\frac{20}{15} = (1+\gamma)^{10}$$

$$\left(\frac{20}{15}\right)^{1/10} = 1+\gamma$$

$$\left(\frac{20}{15}\right)^{1/10} - 1 = \gamma$$

$$1.0291 - 1 = \gamma$$

$$0.0291 = \gamma$$

$$\gamma = 2.91\%$$

$(x)^{1/n}$

→ 5 12 times
→ −1
→ ÷n
→ +1
→ x = 12 times

Approximate how long will it take to double an investment at 6% compounded annually ?

(A) 11.89 yrs (C) 10.52 yrs

(B) 12.45 yrs (D) 11.24 yrs

Sol:

$$A = P[1 + r]^t$$

$$2 = 1(1 + 0.06)^t$$

$$2 = (1.06)^t$$

$$\log 2 = \log (1.06)^t$$

$$\log 2 = t \log (1.06)$$

$$t = \frac{\log 2}{\log (1.06)}$$

$$t = \frac{0.3010}{0.0253}$$

$$= 11.89 \text{ years}$$

Approximate how long will it take to triple an investment at 10% compounded annually ?

(A) 11.52 yrs
(C) 11.15 yrs
(B) 11.88 yrs
(D) 12 Years

$$A = P[1+r]^t$$
$$3 = 1[1+0.10]^t$$
$$3 = (1.10)^t$$

$$\log 3 = \log(1.10)^t$$
$$\log 3 = t \log(1.10)$$
$$t = \frac{\log 3}{\log(1.10)}$$

Q

The population of a town is 8,00,000 .during the first year ,the population increased by 25%.during the second year, the population increased by 20%.During the third year ,the population increased by 10%.Find the population after 3 years

(A) 13 lakh

(C) 14.5 lakh

(B) 13.20 lakh

(D) 14.8 lakh

$8l(1+0.25)^1(1+0.20)^1(1+0.10)^1$

=

Q

The annual birth and death rates per 1,000 are 30 and 10 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration

(A) 34 yrs
(C) 32 years
(B) 35 years
(D) None

$$\text{Birth Rate} = \frac{30}{1000} \times 100 = 3\%$$

$$\text{Death Rate} = \frac{10}{1000} \times 100 = 1\%$$

$$\text{Growth Rate} = 3\% - 1\% = 2\%$$

$$2 = 1\left[1 + 0.02\right]^{t}$$

$$2 = (1.02)^{t}$$

$$\log 2 = t \ln(1.02)$$

$$t = \frac{\log 2}{\log(1.0}$$

Depreciation

Reduction in the value of a tangible asset

Straight Line method

Dep. 10%

100,000
(-) 10,000
90,000
(-) 10,000
80,000

Written down value

10% Dep

₹
100000
(-) 10,000
90,000
(-) 9000
81000
(-) 8100

Cost = C

Rate of Depriciation = d (in decimals)

Life = t

Value of Asset after 't' years = $C(1-d)^t$

$P[1+r]^t$

A machine costing Rs.50,000 depreciates at a constant rate of 8%. If the estimated useful life of machine is 10 years ?

(A) 21719.41
(C) 216820,25
(B) 19981.86
(D) None

Value of Machine

$= 50,000\,[1-0.08]^{10}$

$= 50,000\,(0.92)^{10}$

$= 21,719.42$

Total Dep

$= 50,000 - 21719.42 = 28280.58$

A machine costing Rs.50,000 depreciates at a constant rate of 8%. If the estimated useful life of machine is 10 years ?

A. 21719.41

C. 216820,25

B. 19981.86

D. None

A machine is purchased for Rs. 10,000. It is depreciated at a constant rate 6% for the first 4 year and after that at 10% for the next 6 year. Find the value for the machine after a period of 10 years?

(A) 4149.22
(C) 4250.65
(B) 4365.28
(D) None

Value of machine

$= 10{,}000\,[1-0.06]^4\,[1-0.10]^6$

$= 10000\,(0.94)^4\,(0.90)^6$

$= 4149.22$

An asset costing Rs.2,000 will depreciate to a scrap value of Rs.160 in 10 years. Find the rate of depreciation

(A) 22%
(C) 22.31%
(B) 22.68 %
(D) None

$(x)^{1/n} = ?$

$\sqrt{}$ 12 times

$\to -1 \div n$

$\to +1 \to$ $x = 12$ times

$$160 = 2000(1-d)^{10}$$

$$\frac{160}{2000} = (1-d)^{10}$$

$$\left(\frac{160}{2000}\right)^{1/10} = 1-d$$

$$d = 1 - \left(\frac{160}{2000}\right)^{1/10}$$

$$= 1 - 0.77885$$

$$= 0.2231$$

$$\% = 22.31\%$$

EFFECTIVE RATE

Effective Rate is an annual Rate of interest which is Equivalent to the Rate interest where interest is compounde more than once in a year.

(r) nominal Rate

- Monthly
- Quaterly
- Semiannually
- Continously
- Daily

Effective Rate (r_e)

- Annually Rate

Q. 10% Semiannualy

Nominal Rate (r)

$P = 100$

$t = 1$ year

$A = ?$

$I = ?$

Sol:

$$A = 100\left[1 + \frac{0.10}{2}\right]^{1\times 2}$$

$A = 110.25$

Interest $= 10.25$

Eg 10.25% Annually

Effective Rate

$P = 100$

$t = 1$ year

$A = ?$

Sol:

$$A = 100\left[1 + 0.1025\right]^{1}$$

$= 110.25$

Interest $= 10.25$

nominal Rate

10%	Semiannually	=	10.25% Annually
12%	Quarterly	=	x% Annually
5%	monthly	=	y% Annually

Effective Rat

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

10% Semiannuly

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

$$= \left[1 + \frac{0.10}{2}\right]^2 - 1$$

$$= 0.1025$$

or

$$r_e = 10.25\%$$

& 12% monthly

$$r_e = \left[1 + \frac{0.12}{12}\right]^{12} - 1$$

$$= 0.12682$$

or

$$r_e = 12.682\%$$

Q

What effective rate is equivalent to a nominal rate of 8% converted quarterly?

A. 8.2432%

C. 7.9521%

B. 8.6842 %

D. 8.8495%

$$r_e = \left[1 + \frac{0.08}{4}\right]^4 - 1$$

$$= 0.0824$$

$$8.24\%$$

To what amount will Rs 12,000 accumulate in 12 years if invested at an effective rate of 5%?

Annual

A. 21200

C. 21550

B. 22650

D. 21956

$$P = 12000\left[1+\frac{0.05}{1}\right]^{12\times 1}$$

$$= 12000\,(1.05)^{12}$$

E.g.

Option A

10% P.a. semiannualy

$$r_e = \left[1 + \frac{0.10}{2}\right]^2 - 1$$

$$= 0.1025$$

or

10.25%

Option B

9.8% P.a. quarterly

$$r_e = \left[1 + \frac{0.098}{4}\right]^4 - 1$$

$$= 0.10166$$

or

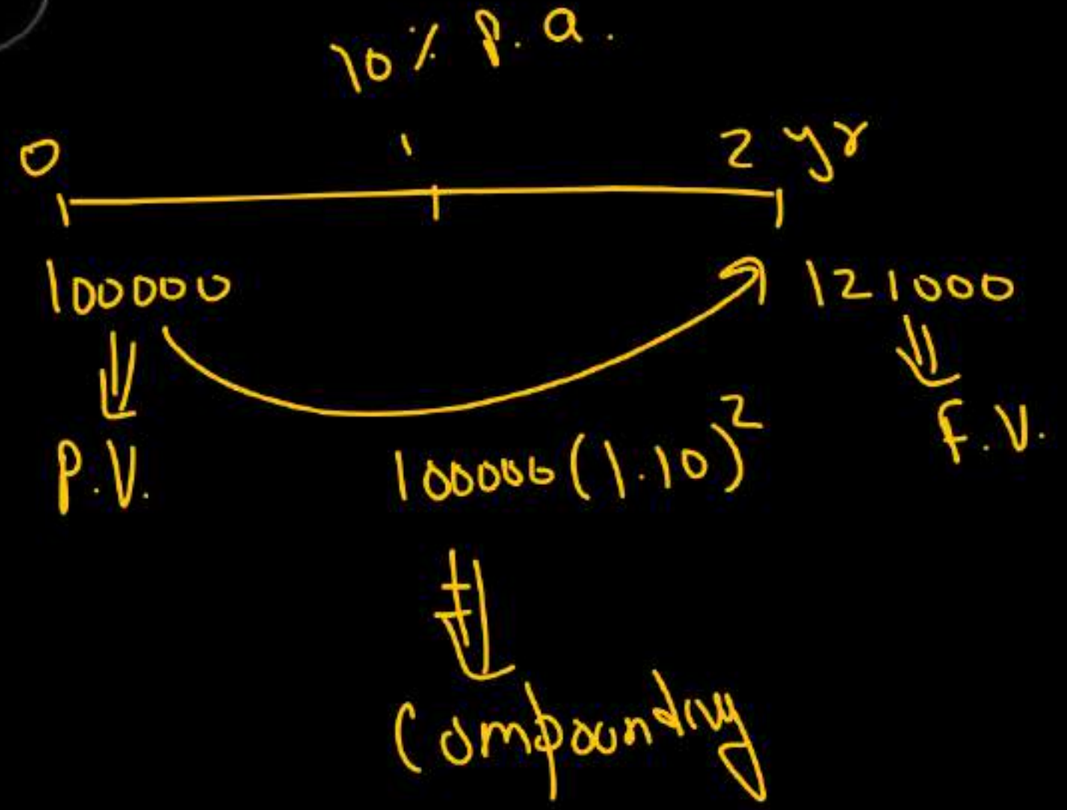
10.16%

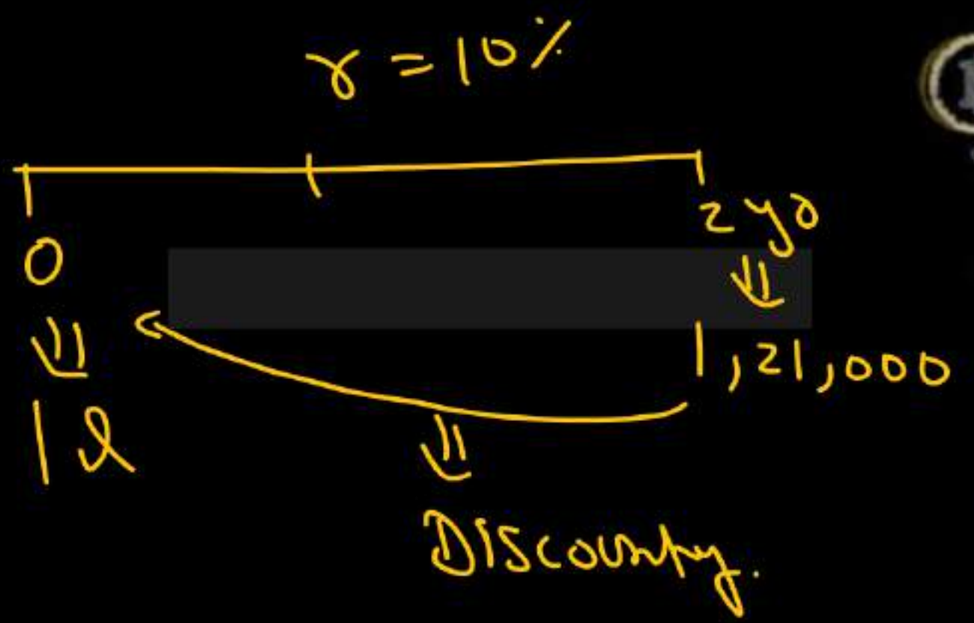
Option C

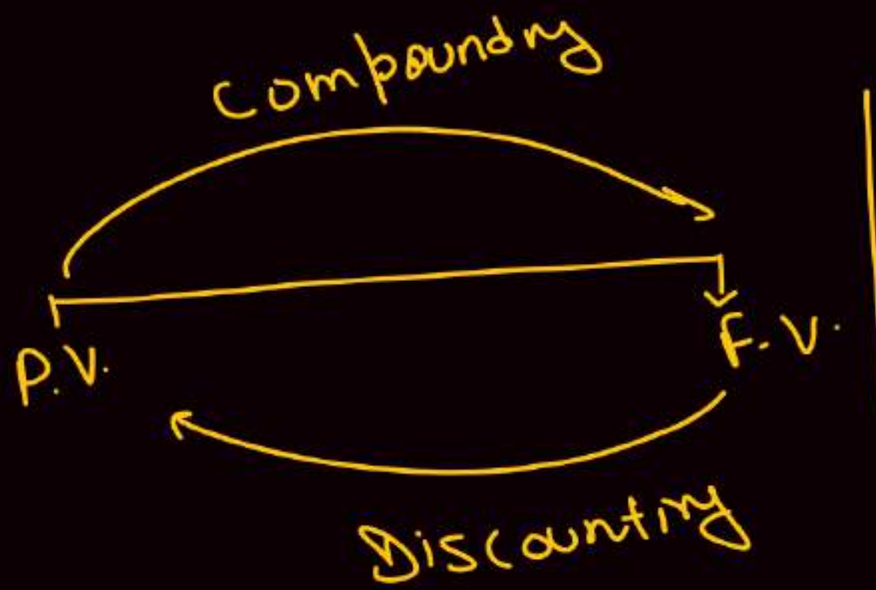
10.2% Annu

$$r_e = 10.2$$

Present Value
Future Value







$$A = P[1+r]^t$$

$$F.V. = P.V.(1+r)^t$$

$$P.V. = \frac{F.V.}{(1+r)^t}$$

$$\frac{121000}{(1+0.10)^2}$$

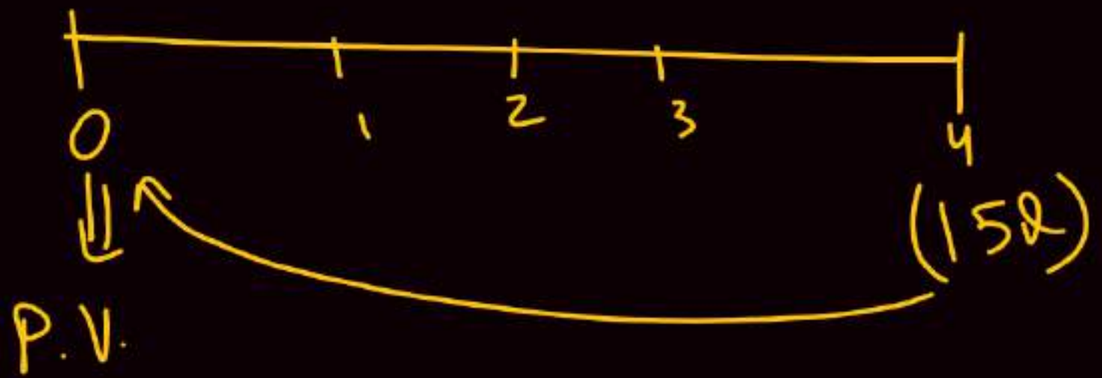
$$= \frac{121000}{(1.10)^2} =$$

$$= 100000$$

$r = 5\%$

India Govt.

modiji

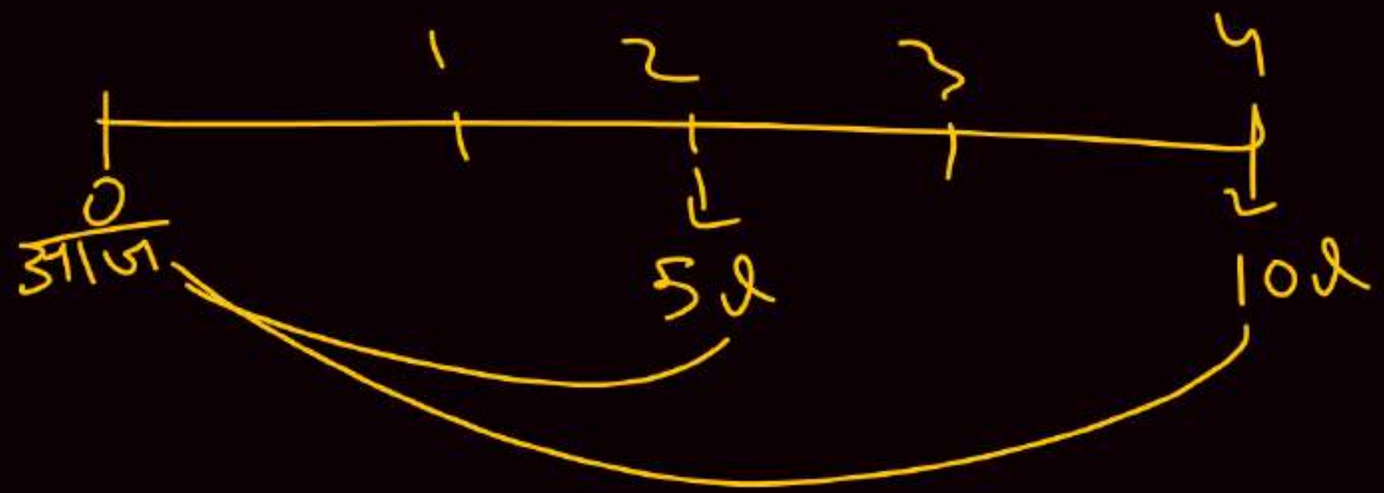


$$P.V = \frac{1500000}{[1+0.05]^4}$$

$$= \frac{1500000}{(1.05)^4} = 12,34,053$$

Rahul Ji

$r = 6\%$



$$P.V. = \frac{500000}{(1+0.06)^2} + \frac{10,00,000}{(1+0.06)^4}$$

$$= 4,44,998 + 7,92,093 = 12,37,091$$

How much should be invested at 6% per annum so that after 4 years the amount will be Rs 2,500 when the interest is annually

A. 1580

C. 1980.23

B. 1840.67

D. 2100.50

$$P.V = \frac{2500}{(1+0.06)^4}$$

Find the present value of Rs. 500 due 10 years hence when the interest of 10% is compounded half yearly

A. 306.95

C. 198.55

B. 188.44

D. None

$$\frac{500}{\left[1+\frac{0.10}{2}\right]^{10\times 2}}$$

$$= \frac{500}{(1.05)^{20}} = 188.44$$

Mr. *X* borrows Some money from Mr. Y and agrees to pay Rs 5,000 in two years , Rs 7,000 in four years and a final payment of Rs 2779 at the end of five years if the interest rate is 6.183 % compounded annually, how much did he borrow ?

A. 12000

C. 13000

B. 11500

D. None

Annuity

Series Of Payments

Payments are Equal

Equal Time Interval

LIC
भारतीय जीवन बीमा निगम
LIFE INSURANCE CORPORATION OF INDIA

(On the Basis of payment time)

Types Of Annuities

60,000
↓
12 x 5000

Ordinary Annuity (Regular Annuity)

⇓

Payment at the end of Period

eg Loan installment

Annuity Due (Immediate Annuity)

⇓

Payment in the beginning of Period

eg Health insurance Premium

Mr X

⇓

आज Bitiya

⇓

A/C

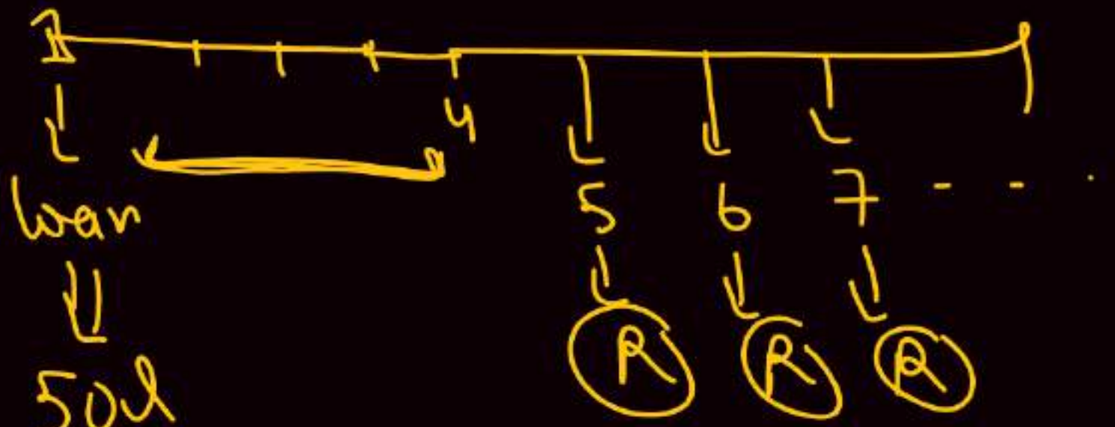
⇓

50,000

Deffered Annuity

⇓

Student loan



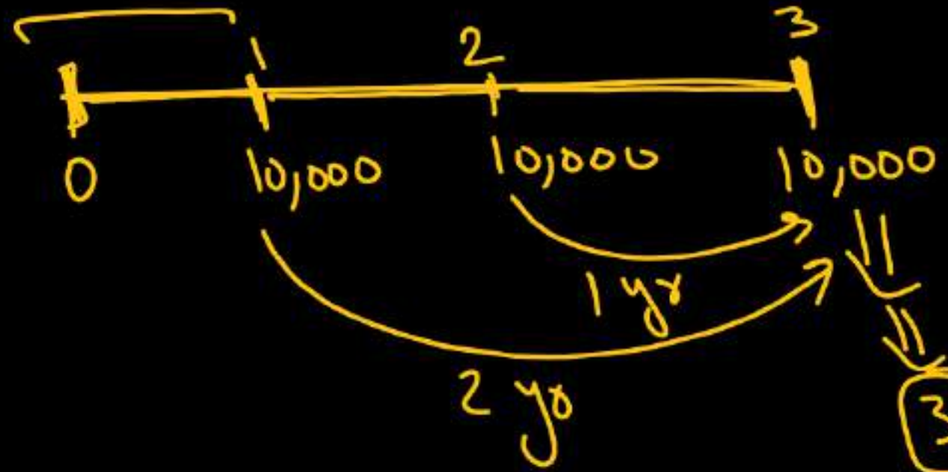
Future Value Of Regular Annuity

Eg

Mr. X

Beta

$\gamma = 10\%$



F.V. of Regular Annuity

$$= 10{,}000(1+0.10)^2 + 10{,}000(1+0.10)^1 + 10{,}000$$

$$= 12100 + 11000 + 10{,}000$$

$$= 33{,}100$$

F.V. of ordinary Annuity (Regular)

$$F.V. = R\left\{\frac{(1+i)^n - 1}{i}\right\}$$

R => Regular payments

$i = \frac{r}{m}$

n => Total payments
$= t \times m$

$R = 10,000$

$i = 10\%$

$n = 3$

$$F.V. = 10,000\left\{\frac{(1+0.10)^3 - 1}{0.10}\right.$$

$= 33,100$

Q

Find the amount of an regular annuity Of Rs 1000 payable at the end of year for 3 years at 10 % compounded annually

(A) 3120

(C) 3310

(B) 3420

(D) None

$R = 1000$

$t = 3\text{ yr}$

$r = 10\%$

$$F.V = R\left\{\frac{(1+i)^n - 1}{i}\right\}$$

$$= 1000\left\{\frac{(1+0.10)^3 - 1}{0.10}\right\}$$

$$= 3,310$$

Q

Find the amount of an regular annuity Of Rs 500 payable at the end of every quarter for 10 years at 8 % compounded quarterly

(A) 30150 (C) 30289

(B) 30201 (D) None

$R = 500$

8% quarterly

$t = 10$ years.

$$F.V. = R\left\{\frac{(1+i)^n - 1}{i}\right\}$$

$$= 500\left\{\frac{\left(1+\frac{0.08}{4}\right)^{10\times 4} - 1}{\frac{0.08}{4}}\right\}$$

$$= 500\left\{\frac{(1.02)^{40} - 1}{0.02}\right\}$$

$$= 30,201$$

$(1.02)^4$

$= (1.02^2)^2$

Q

To save for a child's education, a family decides to invest Rs 3000 at the end of each 6 month in a fund paying 8 % per year compounded semiannually .Find The amount of investment at the end of 18 years

(A) 231150 (C) 232794

(B) 233450 (D) 238450

R = 3000

8% Semi

t = 18 yrs

$$F.V = R\left\{\frac{(1+i)^n - 1}{i}\right\}$$

$$= 3000\left\{\frac{\left(1+\frac{0.08}{2}\right)^{18\times 2} - 1}{\frac{0.08}{2}}\right\}$$

$$= 3000\left\{\frac{(1.04)^{36} - 1}{0.04}\right\}$$

$$= 232794$$

(1.04)

$(1.04)^{47}$

Q

A person wants to accumulate Rs 50,000 by making equal payments at the end of each quarter for the next 5 years, What will be the size of these investments if money is worth 6 % converted quarterly

(A) 2140.22 (C) 2162.28

(B) 2222.22 (D) 2312.28

$$F.V = R\left\{\frac{(1+i)^n - 1}{i}\right\}$$

$t = 5$ yr. & 6% quarterly

R R R . . . . F.V 50,000

$$50,000 = R\left\{\frac{\left(1+\frac{0.06}{4}\right)^{5\times4} - 1}{\frac{0.06}{4}}\right\}$$

$$50,000 = R\left\{\frac{(1.015)^{20} - 1}{0.015}\right\}$$

$$50,000 = R(23.12)$$

$$R = \frac{50,0}{23}$$

$$= 2$$

Present Value Of Regular Annuity

$r = 10\%$ p.a.

PV = ?

$$PV = \frac{50000}{(1+0.10)} + \frac{50,000}{(1+0.10)^2} + \frac{50,000}{(1+0.10)^3}$$

$$= 45454.54 + 41322.31 + 37565.74$$

$$= 1,24,342.59$$

$$50,000 \left\{ \frac{1}{1.10} + \frac{1}{(1.10)^2} \right.$$

$$= 124342.59$$

F.V. of Regular Annuity $\Rightarrow FV = R\left\{\frac{(1+i)^n - 1}{i}\right\}$

P.V. of Regular Annuity $\Rightarrow P.V. = R\left\{\frac{1-(1+i)^{-n}}{i}\right\}$

$= R\left\{\frac{1-\frac{1}{(1+i)^n}}{i}\right\}$

$$P.V = R\left\{\frac{1-(1+i)^{-n}}{i}\right\}$$

$$= 50{,}000\left\{\frac{1-(1.10)^{-3}}{0.10}\right\}$$

$$\div 1{,}24{,}342.59$$

$$(1.10)^{-3} = \frac{1}{(1.10)^3}$$

9 Pension ⇒ ₹ 150,000

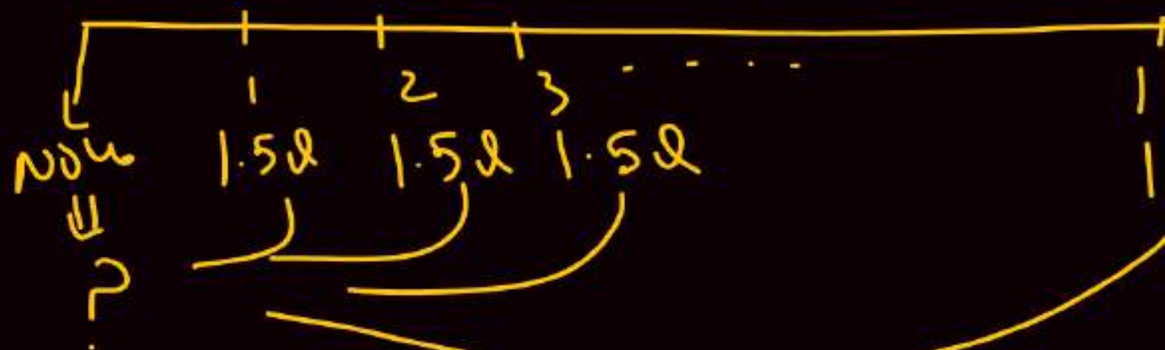
⇓

at the end of each year

⇓

for 10 years

$r = 5\%$ p.a.



Sol: $P.V = R\left\{\frac{1-(1+i)^{-n}}{i}\right\}$

$= 150,000\left\{\frac{1-(1.05)^{-10}}{0.05}\right\}$

$= 11,58,260$

$1.5Q\left\{\frac{1}{1.05} + \frac{1}{1.05^2} + \cdots + \frac{1}{(1.05)^{}}\right\}$

Mr. A borrows Rs 5lakh to buy a house. If He pays equal annual installments for 20 years and 10% interest on outstanding balance , then what will his annual installment ?

(A) 57605
(C) 60210
(B) 59802
(D) 58729

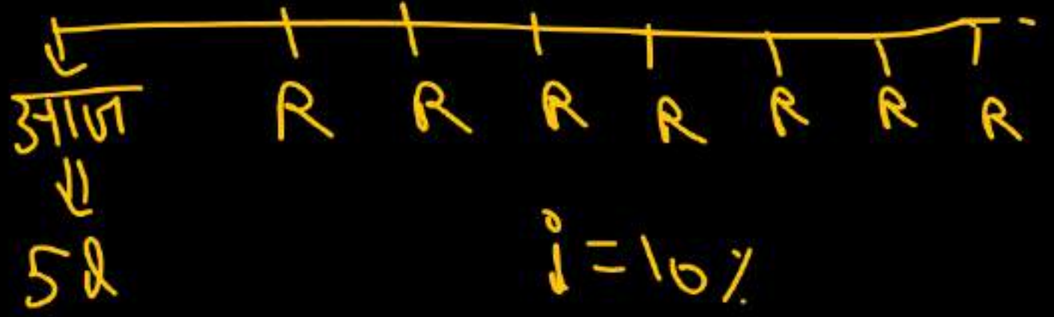
$$P.V = R\left\{\frac{1-(1+i)^{-n}}{i}\right\}$$

$$5000 = R\left\{\frac{1-(1.10)^{-20}}{0.10}\right\}$$

$$5000 = R(8.513)$$

$$R = 58729$$

$n = 20$ years



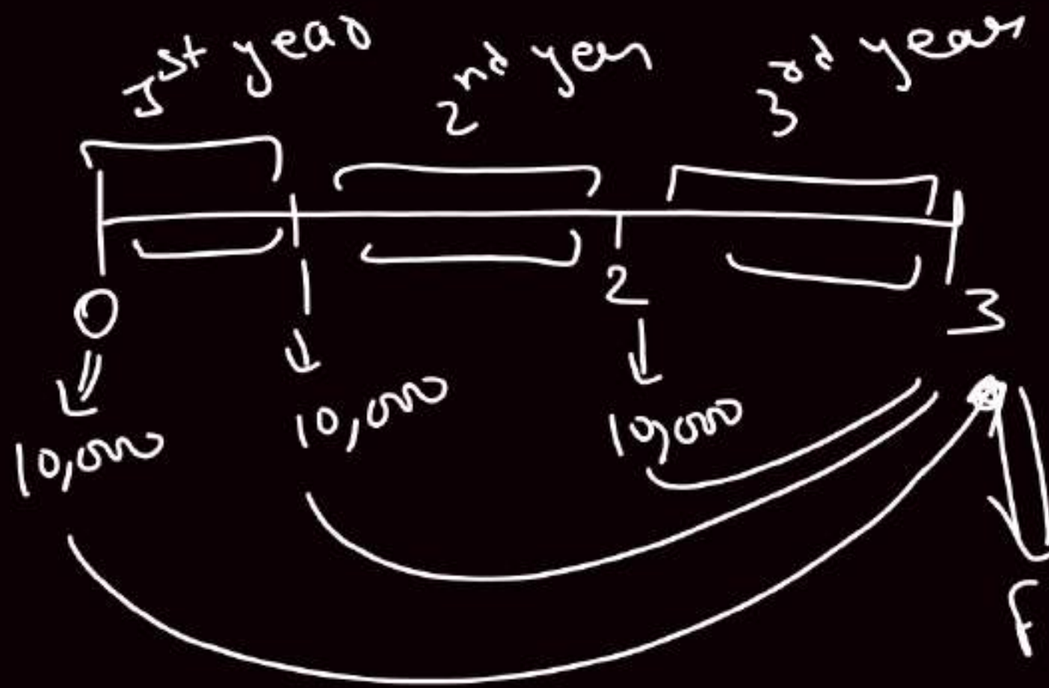
Future Value Of Annuity Due

Payment in the beginning of period

$$F.V = R\left\{\frac{(1+i)^n - 1}{i}\right\}(1+i)$$

$$P.V. = R\left\{\frac{1-(1+i)^{-n}}{i}\right\}(1+i)$$

$i = 10\%$



$$F.V = R\left\{\frac{(1+i)^n - 1}{i}\right\}(1+i)$$

$$= 10{,}000\left\{\frac{(1+0.10)^3 - 1}{0.10}\right\}(1+$$

$$=$$

$$F.V = 10000(1.10)^3 + 10{,}000(1.10)^2 + 10{,}000(1.1)^1$$

$$= 10000\{(1.10)^3 + (1.10)^2 + 1.1\}$$

$$= 10000\{1.331 + 1.21 + 1.1\}$$

$$= 10000 \times 3.641 = 36410$$

Q

Suppose your father decides to gift you Rs. 10,000 every year starting from today for the next 10 years, you deposit this amount in a bank as and when you receive and get 8% per annum interest rate compounded annually. What is the future value of this annuity?

(A) 156454
(C) 156494
(B) 156484
(D) None

$$F.V = 10,000 \left\{ \frac{(1.08)^{10} - 1}{0.08} \right\} (1+0.08)$$

$$= 156454$$

Present Value Of Annuity Due

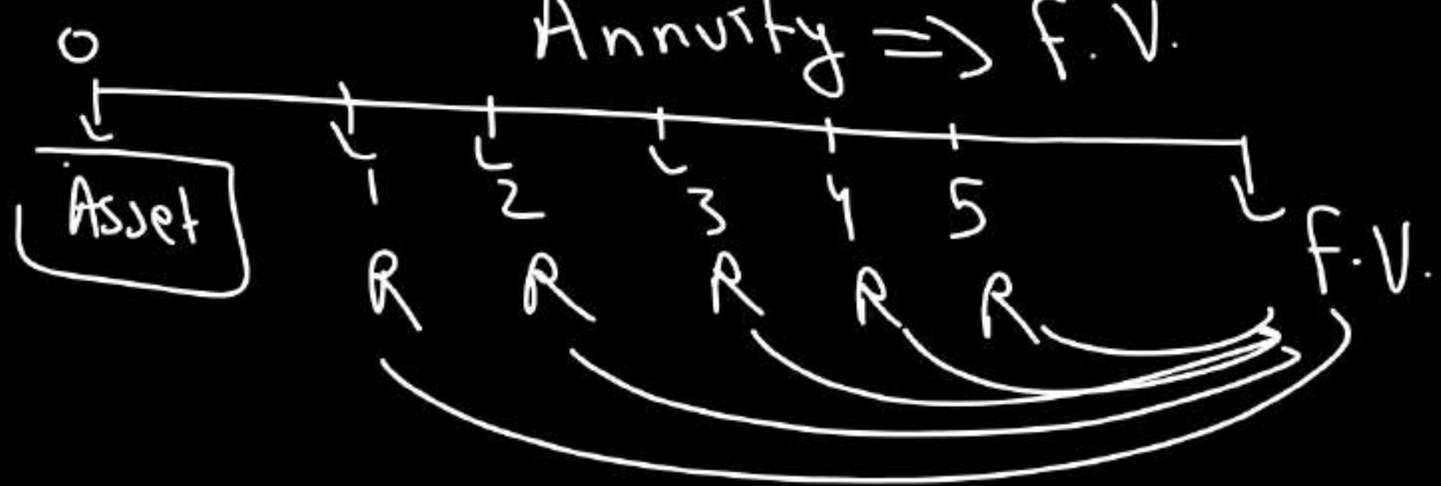
$$P.V. = R \left\{ \frac{1-(1+i)^{-n}}{i} \right\} (1+i)$$

Sinking Fund

It is the fund created for a specified purpose by way of sequence of periodic payments over a time period

factory

Annuity ⇒ F.V.



Q

A machine cost ₹520000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realisation of₹25000.what amount should be set aside every year if sinking fund investments accumulate at 3.5% compound interest p.a.

A. 16000
C. 16500
B. 16050
D. 16005

25 Yrs

₹ 520,000
+25%

New Machine = 520,000 + 130,000
= 650,000
(-) S.V = (25000)
6,25,000

$$f.V. = R\left\{\frac{(1+i)^n - 1}{i}\right\}$$

$$6,25,000 = R\left\{\frac{(1+0.035)^{25} - 1}{0.035}\right\}$$

$$R = 16046$$

Net Present Value

Net present value (NPV) is the difference between the present value of cash inflows and the present value of cash outflows over a period of time.

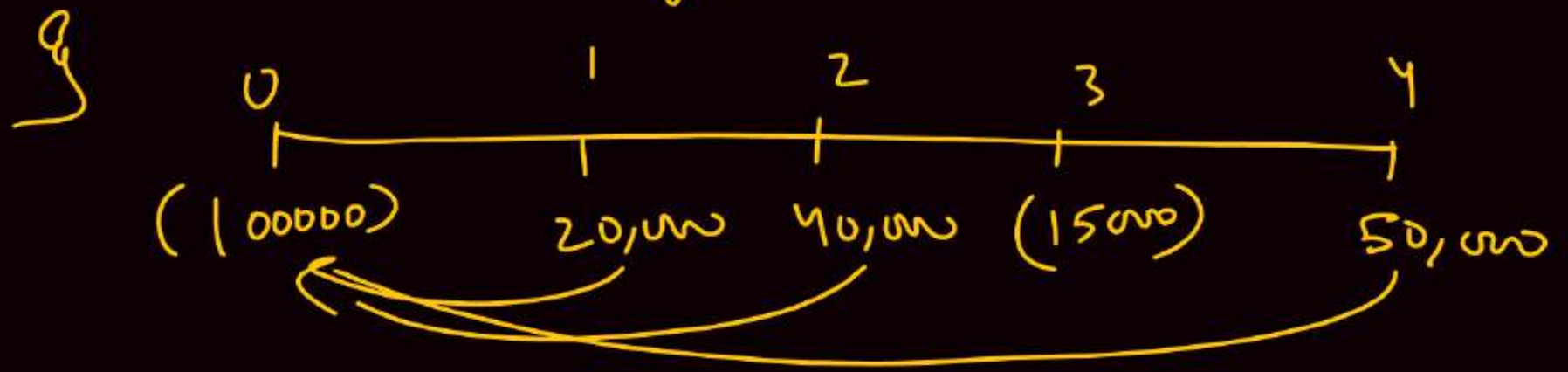
NPV is used in Capital budgeting and Investment planning to analyze the profitability of a projected investment or project.

NPV = P.V of all cash inflows
− PV of all cash outflow

NPV is negative ⇒ Reject

NPV is Positive ⇒ Accept

$r = 12\%$

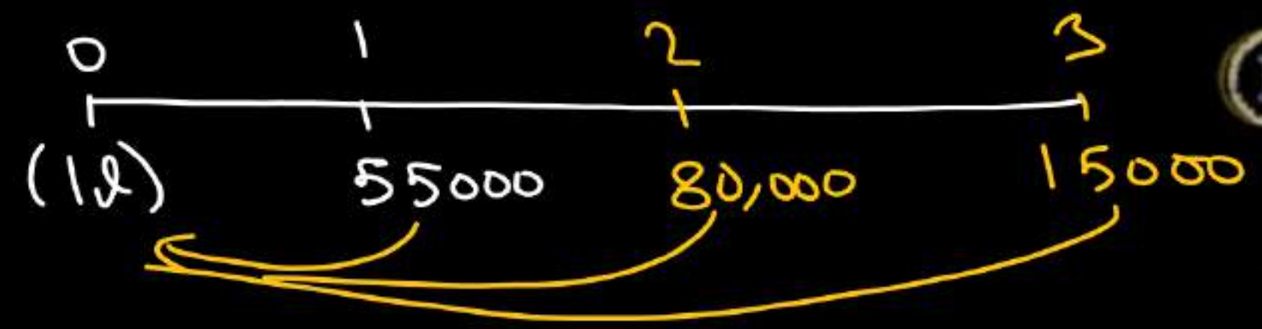


$$NPV = \frac{20,000}{(1.12)^1} + \frac{40,000}{(1.12)^2} + \frac{50,000}{(1.12)^4} - 100000 - \frac{15000}{(1.12)}$$

$$= ?$$

Compute the NPV for a project with a net investment of Rs 1 lakh and net cash flows for year one is Rs 55000, for year 2 Rs 80000 and For Year Three is Rs 15000, Further the company's cost of capital is 10%

A. 27350
C. 27480
B. 27110
D. 27514



$r = 10\%$

$$NPV = \frac{55000}{(1.10)^1} + \frac{80000}{(1.10)^2} + \frac{15000}{(1.10)^3} - \frac{100000}{(1.10)^0}$$

$$= 50000 + 66115.70 + 11269.72 - 100000$$

$$= 27385.42$$

Leasing (पट्टा)

Asset on Rent
Long term

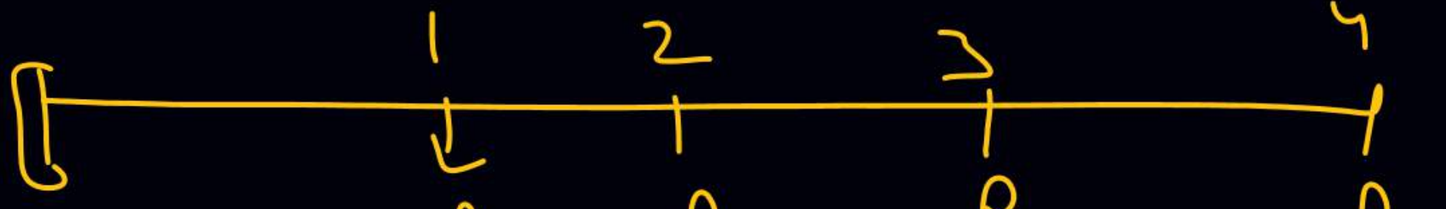
Leasing is a financial arrangement under which the owner of the asset (lessor) allows the user of the asset (lessee) to use the asset for a defined period of time(lease period) for a consideration (lease rental) payable over a given period of time

User
Asset
Lessee

Lessor
Owner

P.V. of all Rents = cost of Asset

Machine
100,000

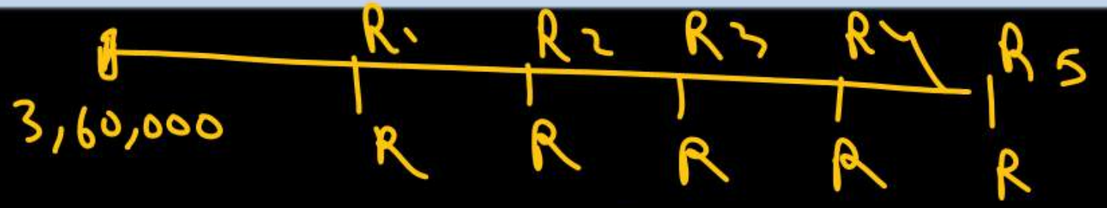


आज
1 lakh

$$\text{Cost of Asset} = R\left\{\frac{1-(1+i)^{-n}}{i}\right\}$$

ABC Ltd wants to lease out an machine costing Rs.3,60,000 for a five year period , it has a fixed rental of Rs.1,05,000 per annum payable annually starting from the end of first year , supposed rate of interest is 14% per annum compounded annually on which money can be invested in the company .is this agreement favorable to the company?

(A) yes
(C) Data Inadequate
(B) No
(D) None



$$3,60,000 = R\left\{\frac{1-(1+0.14)^{-5}}{0.14}\right\}$$

$$3,60,000 = R(3.4330)$$

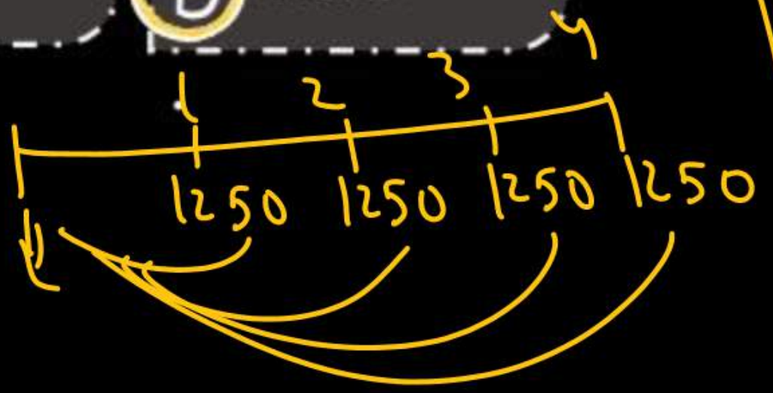
$$R = 1,04,862$$

A company is considering the proposal of purchasing a machine either by making full payment of RS 4000 or by leasing it for 4 years at an annual rate of RS 1250. Which course of action is preferable if the company can borrow money at 14% compounded annually ?

(A) Purchase
(C) Both
(B) Lease
(D) None

Purchase ⇒ ₹ 4000

Lease ⇒ 4 years ₹ 1250/-



P.V. of future rents

$= 1250\left\{\frac{1-(1.14)^{-4}}{0.14}\right\}$

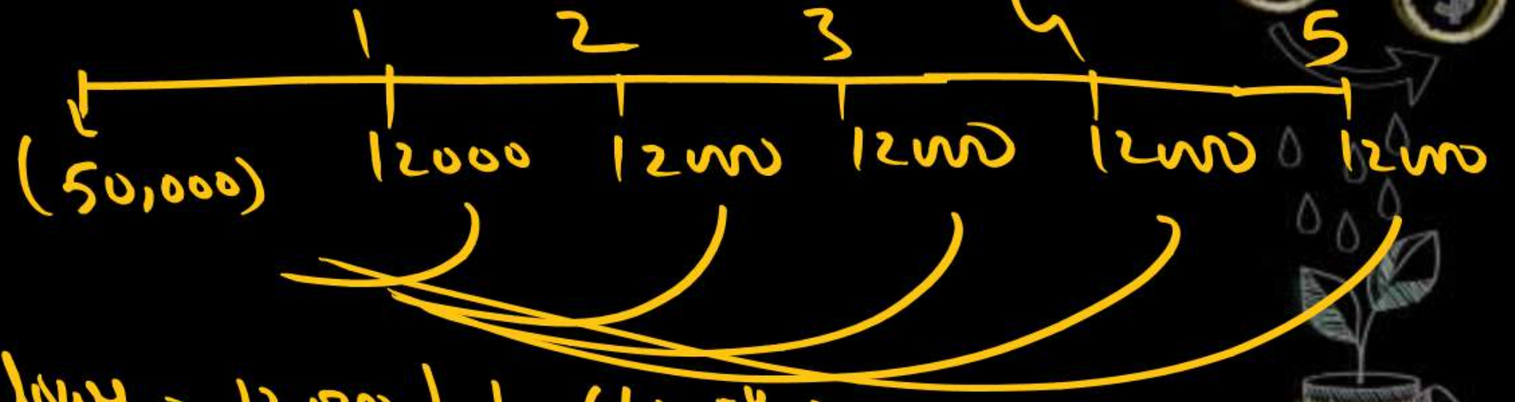
$= 1250 \times 2.9137 = 3642$

Lease

Capital Expenditure

A machine can be purchased for Rs50000. Machine will contribute Rs12000 per year for the next five years. Assume borrowing cost is 10% per annum compounded annually. Determine whether machine should be purchased or not

(A) yes
(C) Data Inadequate
(B) No
(D) None

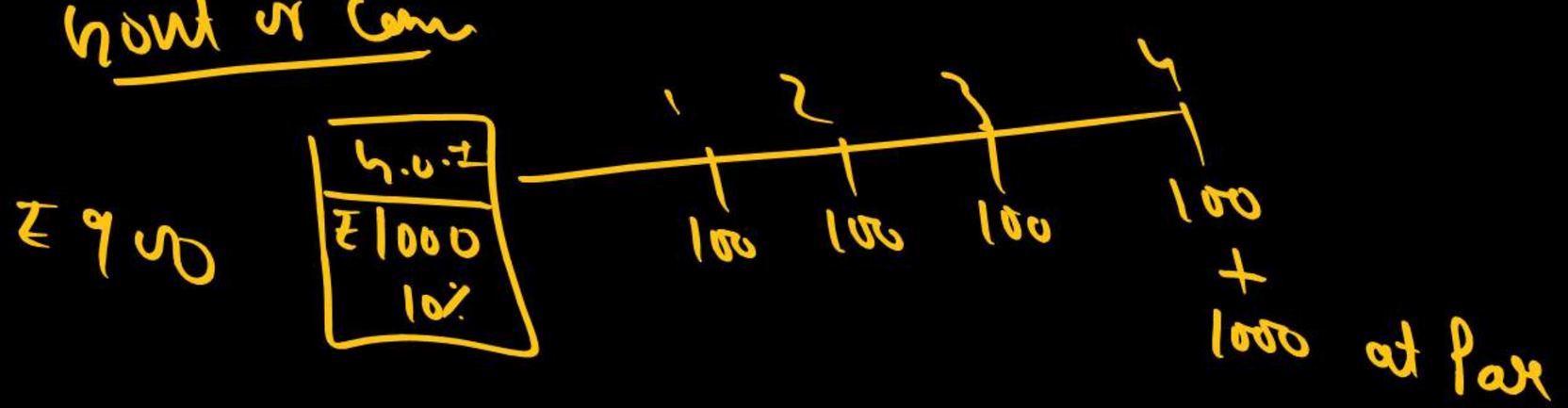


P.V. of all future cash inflows $= 12000 \left\{ \frac{1-(1.10)^{-5}}{0.10} \right\} = 38038.38$

NPV = P.V of Cash inflow − P.V. of outflow
$= 38038 - 50000 = (-)$

Bond Valuation

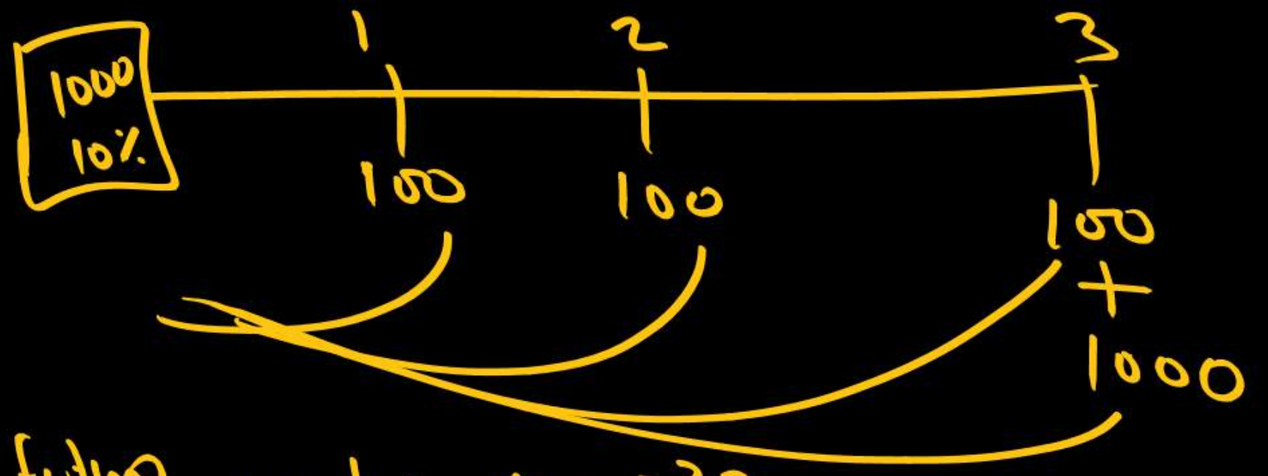
-Bonds are long-term Debt Securities
-Issue By Govt or Companies To Raise Funds
-Investor Receive periodic interest payments
-At maturity investor also gets redemption price (Usually face value)



Arjun intends to purchase a three year Rs 1000 par value bond having nominal interest rate of 10% .At what price the bond should be purchased now if it matures at par and the investor expects a return of 14% ?

(A) 907 (C) 813

(B) 920 (D) 850



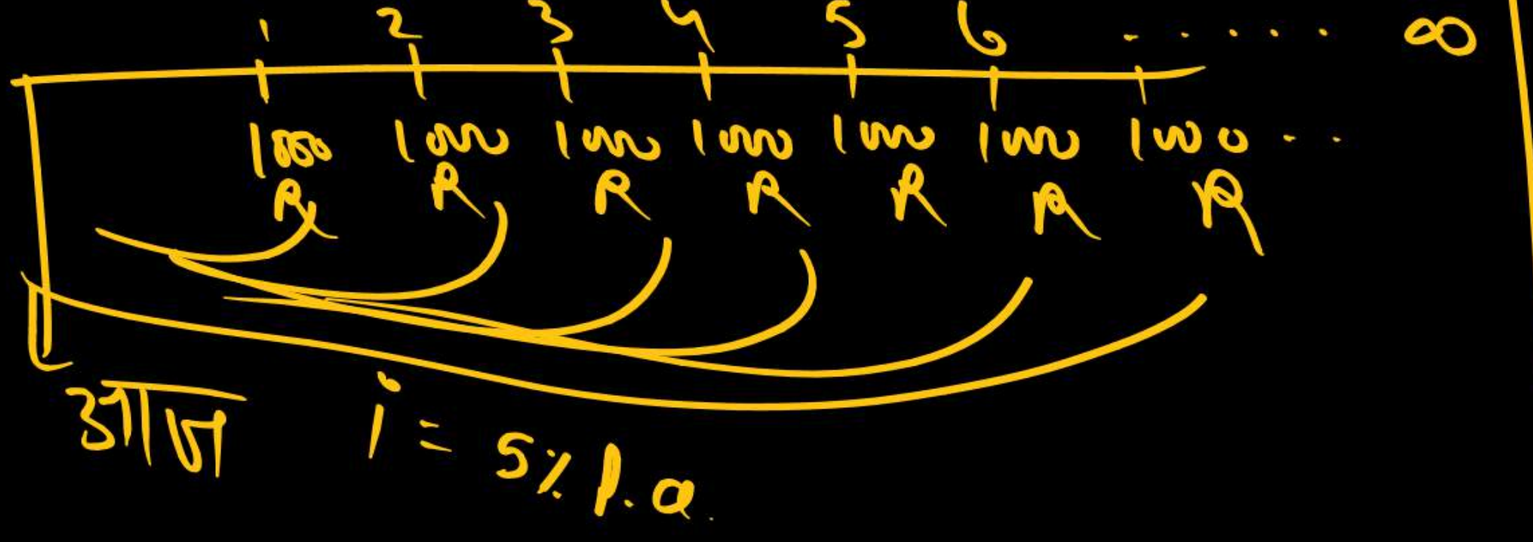
P.V. of all future inflows $= 100\left\{\frac{1-(1.14)^{-3}}{0.14}\right\} + \frac{1000}{(1.14)^3} = 232.16 + 674.97$

$= 907.13$

Perpetual Annuity

(Perpetuity)

- Also Known as perpetuity
- Sequence Of payments Continuing Forever
- No ending Date
- Only present value can be determined



$$P.V. = \frac{R}{i}$$

$$= \frac{1000}{0.05} = 20,000$$

Perpetual Annuity

- Ordinary
  - ⇓
  - Endny
  - ⇓
  - $P.V. = \frac{R}{i}$
- Due (Immediate)
  - ⇓
  - Beginning
  - ⇓
  - $P.V. = \frac{R}{i}(1+i) = \frac{R}{i} + R$

If Money is worth 6% per annum ,find the present value of a perpetuity of Rs 3300 payable annually

(A) 50000
(C) 58000
(B) 55000
(D) 60000

$$P.V. = \frac{R}{r} = \frac{3300}{0.06}$$

$$= 55000$$

If Money is worth 8% per annum compounded quarterly ,find the present value of a perpetuity of Rs 1000 payable at each quarter

(A) 50000
(C) 58000
(B) 55000
(D) 60000

$$\frac{R}{i} = \frac{1000}{\left(\frac{0.08}{4}\right)} = \frac{1000}{0.02} = 50,000$$

If Money is worth 4% per annum compounded annually ,find the present value of a perpetuity of Rs 4500 payable at the beginning of each year

(A) 112500 (C) 118000

(B) 117000 (D) none

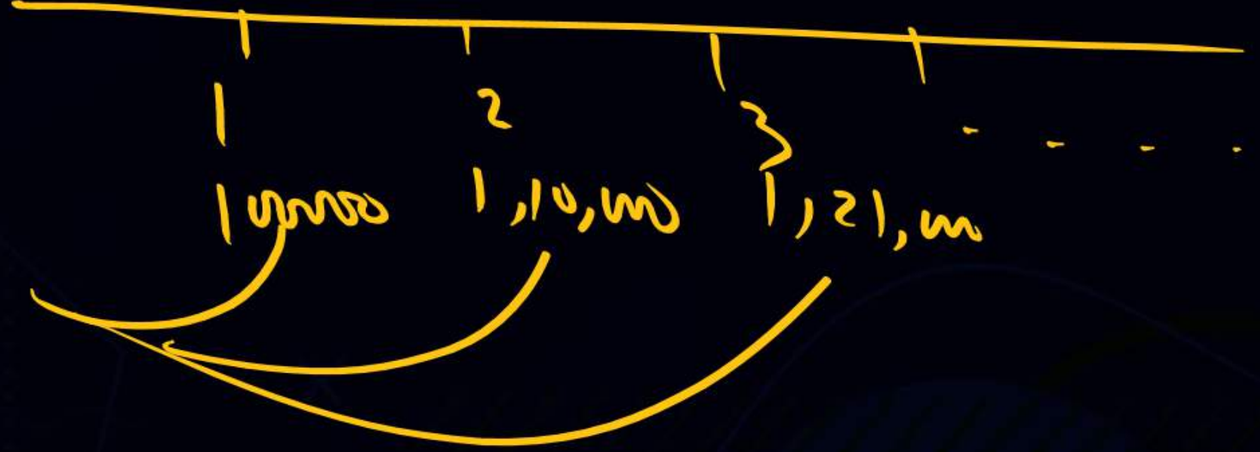
$$\frac{R}{i} + R$$

$$= \frac{4500}{0.04} + 4500$$

$$= 1,17,000$$

Growing Perpetuity

$g = 10\%$



$$PV = \frac{R}{i - g}$$

Growing Perpetuity

Periodic Payment increases at a constant rate

If Money is worth 4% per annum compounded annually ,find the present value of a perpetuity of Rs 4500 payable at the beginning of each year

A 112500

C 118000

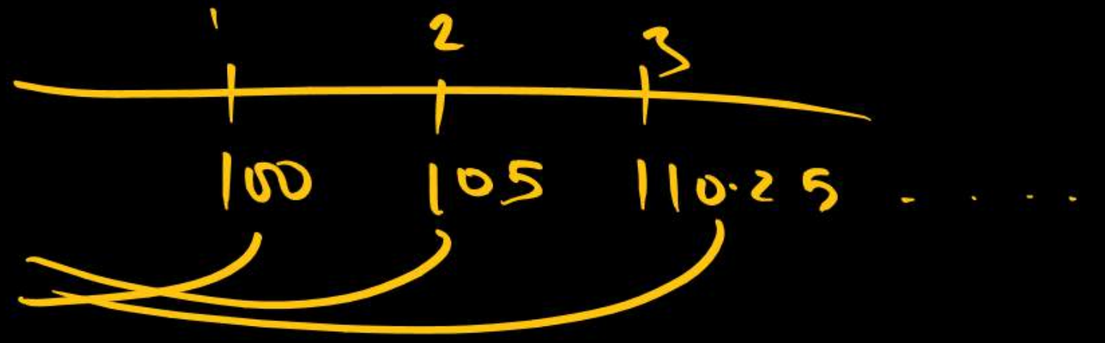
B 117000

D none

If the perpetuity entitles to a payment of amount Rs 100 at the end of first year that will grow at a rate of 5% annually. Find the present value if discount rate is 15 %

(A) 1000 (C) 9000

(B) 10000 (D) 11000



P.V of growing Perpetuity $= \frac{R}{i-g} = \frac{100}{0.15-0.05}$

$= \frac{100}{0.10} = 1000$

Compound Annual Growth rate

CAGR tells you the average rate at which an investment has grown over a specified period.

Time	2013	2014	2015	2016	2017
Sales	250	300	350	400	480
		20%	16.67	14.28	20%

$$CAGR = \left[\frac{V(T_n)}{V(T_0)}\right]^{\frac{1}{T_n - T_0}} - 1$$

$$= \left(\frac{480}{250}\right)^{\frac{1}{2017-2013}} - 1$$

$$= \left(\frac{480}{250}\right)^{\frac{1}{4}} - 1$$

$$= 1.1771 \quad = 0.1771 \text{ or } 17.71\%$$

$\sqrt{}$ 12 times
-1
$\div 4$
$+1$
$X =$ 12 times

$$480 = 250(1+r)^4$$

$$\left(\frac{480}{250}\right) = (1+r)^4$$

$$\left(\frac{480}{250}\right)^{1/4} = 1+r$$

$$r = \left(\frac{480}{250}\right)^{1/4} - 1$$

T	2015	2016	2017	2018
EMI	200	210	240	300

$$300 = 200[1+r]^3$$

$$\left(\frac{300}{200}\right)^{1/3} - 1 = r$$

Compound Annual Growth rate

CAGR tells you the average rate at which an investment has grown over a specified period.

Real Rate Of Return

4% Interest

Inflation ⇒ 6%

nominal Rate

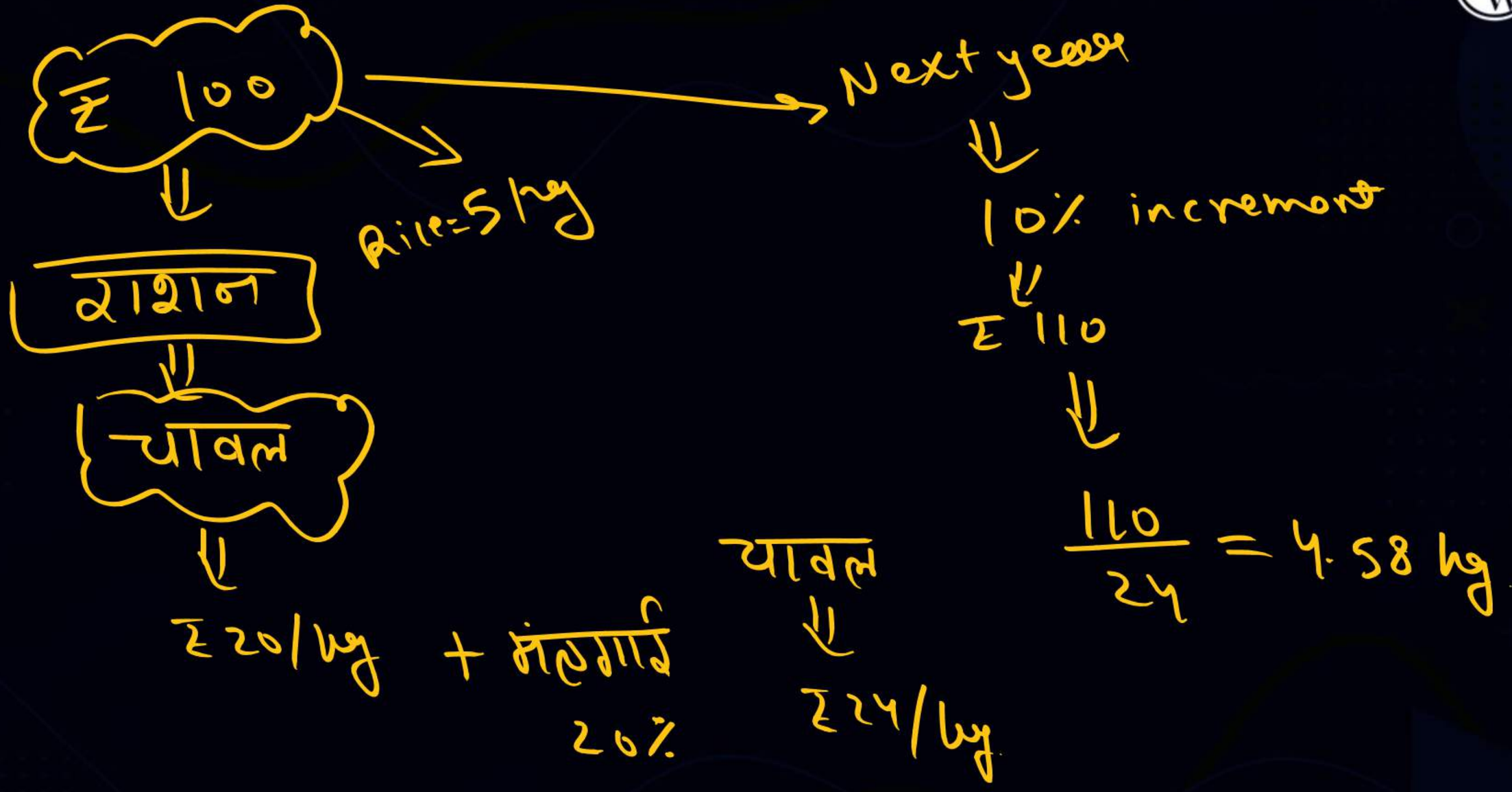
⇓

Return we get on an investment

4% − 6%

= ?

Nominal Rate − Inflation − Taxes = Real Rate of Return


₹ 100
Rice=5/kg
Next year
10% increment
₹ 110
राशन
चावल
₹20/kg + महंगाई 20%
चावल
₹24/kg
110/24 = 4.58 kg

$$10\% - 20\% = -10\%$$

Nominal Rate $-$ Inflation Taxes $=$ Real Return

Inflation Rate $\leq$ Nominal Rate


Savings
FINANCE
THANK YOU
KEEP REVISING
&
STAY MOTIVATED !!