Theoritical Distribution - 4 marks

Meaning :- in theoritical Distribution, we calculate probability Based on Expected observation from Past Exprience.

Theoritical Probability Distribution

- Discrete Probability Distributions.
  - Binomial Distribution
  - Poisson Distribution.
- Continious Probability Distributions
  - Normal Distribution.

Binomial Distribution :-

History – it is Discovered by James Bernouli.

☆ it is Based on 2 outcomes.

- Sucess (P)
- failure (q)

$\boxed{P + q = 1}$

☆ variable should be discreate. Eg, {1, 2 .. 10} not decimal.

formula :- ☆ $\boxed{P(x) = {}^{N}C_{x} \cdot (P)^{x} \cdot (q)^{N-x}}$ ☆

N = No. of trials.

x = Actual no. of Sucess

P = probability of Sucess

q = probability of failure.

Eg. → A coin is tossed 5 times. find probability of 2 head.

Sol → $P(x) = {}^{N}C_{x} \cdot (P)^{x} \cdot (q)^{N-x}$

$5 = N$, $x = 2$ | $P = \frac{1}{2}$, $P + q = 1$, $q = 1 - \frac{1}{2} = \frac{1}{2}$

$P(2) = {}^{5}C_{2} \cdot \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^3$

$= \frac{5 \times \cancel{4}}{\cancel{2}!} \times \frac{1}{4} \times \frac{1}{8} = \cancel{10}^{5} \times \frac{1}{\cancel{4}_{2}} \times \frac{1}{8} = \boxed{\frac{5}{16}}$

Eg. → A coin is tossed 6 times - find Probility of 3 tail.

Sol → $N = 6$, $x = 3$ | $P = \frac{1}{2}$, $P + q = 1$, $q = 1 - \frac{1}{2} = \frac{1}{2}$

$P(3) = {}^{6}C_{3} \cdot \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^3$

$= \frac{6 \times 5 \times 4}{3!} \times \frac{1}{8} \times \frac{1}{8}$

$= \frac{\cancel{6} \times 5 \times \cancel{4}}{\cancel{6}} \times \frac{1}{\cancel{8}_{2}} \times \frac{1}{8} = \frac{5}{16}$ Ans

Properties of Bionomial :- (Q)

1.) Mean = $N \times P$

2.) variance = $(SD)^2 = N \times P \times q$

3.) SD = $\sqrt{NPq}$

Mode :-

- Unimodel → only 1 Mode
- Bi-Model → More than 1 Mode.

Case-1 :- if $(n+1)\times P$ is non integer (In decimal), then only one Mode.

eg - $N = 10, P = \frac{1}{2}$

$(10+1)\times\frac{1}{2} = 5.5$

$\boxed{\text{Mode}} = 5$

Case-2 :- if $(n+1)\times P$ is integer, then 2 mode.

eg - $N = 9, P = \frac{1}{2}$

$= (9+1)\times\frac{1}{2} = 10\times\frac{1}{2} = 5$

Ist mode = 5

2nd Mode = 5 - 1 = 4

$\boxed{\begin{array}{l} \text{1st} = (N+1)\times P \\ \text{2nd} = (n+1)\times P - 1 \end{array}}$

* Features of Binomial :-

1) if $P = \frac{1}{2}$ or 0.5 — graph will symmetrical i.e, left = Right value.
2) if $P < 0.5$ → Graph - Skewed to right or positively Skewed.
3) if $P > 0.5$ → Graph - Skewed to left or Negatively skewed.
4) Maximum value of variance when $\boxed{P = q = 0.5}$

Q1) $N = 48$, $P = 0.75$

$P + q = 1$

$0.75 + q = 1$

$q = 1 - 0.75 = 0.25$

* $SD = \sqrt{NPq} = \sqrt{48\times0.75\times0.25}$

SD = 3 Ans → (a)

Q2) $N = 20$, $P = 0.5$ → as per properties

$\bar{x} = N\times P$

$= 20\times0.5 = 10$ (b)

Q3) Variance $= N\times P\times q$

$= 16\times0.5\times0.5 = 4 =$ (b) Ans

Q4) $N = 15$, $P = \frac{1}{3}$

Mode $= (n+1)\times P = (15+1)\times\frac{1}{P} = 16\times\frac{1}{3} = 5.33$

5 = mode

Q5) Mean $= N\times P = 3$

SD $= \sqrt{N\times P\times q} = 1.5$

$\sqrt{3\times q} = 1.5$

$3q = 2.25$

$q = \frac{2.25}{3} = 0.75$

$P + q = 1$

$P + 0.75 = 1$

$P = 0.25$

* $n\times0.25 = 3$

$n = \frac{3}{0.25}$, $n = 12$

Q6) $P(x) = {}^{N}C_{x}(P)^{x}\cdot(q)^{N-x}$

$N = 6$

$x = 3$

$P = \frac{1}{2}$

$q = \frac{1}{2}$

$^{6}C_{3}\cdot\left(\frac{1}{2}\right)^{3}\cdot\left(\frac{1}{2}\right)^{6-3}$

$\frac{6\times5\times4}{3!}\times\frac{1}{8}\times\frac{1}{8} = \frac{5}{16} = 0.3125$

Q7) $P \Rightarrow 0.60$, $q \Rightarrow 0.40$

P.(at least on passed) $\Rightarrow 1 - P(x \Rightarrow 0)$

$\Rightarrow 1 - {}^{4}C_{0} \cdot (0.60)^{0} \times (0.40)^{4-0}$

$\Rightarrow 1 - 1 \times 1 \times 0.0256$

$\Rightarrow 1 - 0.0256 \Rightarrow 0.9744$ - (b) Ans

Q8) $N \Rightarrow 5, x \Rightarrow 3, p \Rightarrow \frac{1}{2}, q \Rightarrow \frac{1}{2}$

${}^{5}C_{3} \cdot (\frac{1}{2})^{3} \times (\frac{1}{2})^{5-3}$

$\Rightarrow \frac{5 \times \cancel{4} \times \cancel{3}}{\cancel{6} \cancel{2}} \times \frac{1}{\cancel{8}} \times \frac{1}{4} \Rightarrow \frac{5}{16} \Rightarrow 0.3125$ - (a) Ans

Poission Distribution :-

* it is discoverd by Simon Denis Poisson.

$\boxed{\begin{array}{l} N \rightarrow \text{large} \\ P \rightarrow \text{very small} \end{array}}$

* only one outcome - either sucess or failure.

formule :- $P(x) \Rightarrow \frac{e^{-M} \cdot M^{x}}{x!}$

$\varepsilon \Rightarrow$ Exponent $\Rightarrow 2.7183$

$M \Rightarrow$ Mean (Avg. Sucess)

$x \Rightarrow$ Actual No. of Sucess.

Properties :-

1.) Mean $\Rightarrow np$

2) Variance NP.

3.) $SD \Rightarrow \sqrt{NP}$

Mode → Uni Model, Bi Model.

Uni Model →

* NP is integer - Two mode

* NP is non integer - one Mode.

Q9.) $\sqrt{NP} \Rightarrow 2$ - given.

$x \Rightarrow 2$, $M \Rightarrow NP \Rightarrow 4$:

$P(x) \Rightarrow \frac{e^{-m} \cdot M^{x}}{x!} \Rightarrow \frac{\varepsilon^{-4} \cdot (4)^{2}}{2!} \Rightarrow \frac{\varepsilon^{-4} \times 1\cancel{6}^{8}}{\cancel{2} \times 1} \Rightarrow \frac{8}{\varepsilon^{4}}, \frac{8}{(2.7183)^{4}}$

~~$\Rightarrow 0.3553$~~

$\Rightarrow \frac{8}{54.599}$

$\Rightarrow 0.1645$ Ans (d)

3)

Poission $\boxed{P(x) \Rightarrow \frac{e^{-m} \cdot m^{x}}{x!}}$

Q10.) $m \Rightarrow NP \Rightarrow 1$

$\Rightarrow 1 - P(x=0)$

$\Rightarrow P(x=0) \Rightarrow$

$P(x=0) \Rightarrow \frac{e^{-m} \cdot m^{x}}{x!}$

$\Rightarrow \frac{e^{-1} \times (1)^0}{0!} \Rightarrow e^{-1} \Rightarrow \frac{1}{e}$

$\Rightarrow \frac{1}{2.7183} \Rightarrow 0.3678$

$\therefore 1 - 0.3678 \Rightarrow 0.632$ - (C) Ans

Q11.) $C.V \Rightarrow \frac{SD}{\bar{y}} \times 100$

$\cancel{50} \Rightarrow \frac{\sqrt{NP} \times \cancel{100}^{2}}{NP}$

$NP \Rightarrow 2\sqrt{NP}$

$(NP)^2 \Rightarrow 4 \times NP$, $\boxed{NP \Rightarrow 4 \Rightarrow M}$

Now, $P(x=0) \Rightarrow \frac{e^{-m} \cdot m^{x}}{x!} \Rightarrow \frac{e^{-4} \cdot m^{0}}{0!}$

$\Rightarrow e^{-4} \Rightarrow \frac{1}{(2.7183)^4} \Rightarrow 0.018$ ~~(a) Ans~~

$1 - 0.018 \Rightarrow 0.9817$ - (D) Ans

Q12) $m \Rightarrow N \times P$

$\Rightarrow 200 \times \frac{1.5}{100}$

$m \Rightarrow 3$

$P(x=0)$?

$\Rightarrow \frac{e^{-m} \cdot m^{x}}{x!}$

$\Rightarrow \frac{e^{-3} \times (3)^0}{0!} \Rightarrow e^{-3} \Rightarrow \frac{1}{e^3}$

$\Rightarrow \frac{1}{(2.7183)^3} \Rightarrow 0.049$ - (a) - 0.005 Ans

Q13.) $P(x=1) \Rightarrow \frac{e^{-m} \cdot m^{x}}{x!} \Rightarrow \frac{e^{-m} \cdot m^{1}}{1!}$

$P(x=2) \Rightarrow \frac{e^{-m} \cdot m^{2}}{2!} \Rightarrow \frac{e^{-m} \cdot m^{2}}{2}$

$\Rightarrow \frac{\cancel{e^{-m}} \cdot \cancel{m}}{1} \Rightarrow \frac{\cancel{e^{-m}} \cdot m^{\cancel{2}}}{2}$

$\Rightarrow m \Rightarrow 2$ - (C) Ans

Q14.) $m \Rightarrow N \times P$

$\Rightarrow 100 \times \frac{1}{100}$, $m \Rightarrow 1$

$P(x=2)$

$\Rightarrow \frac{e^{-m} \cdot m^{x}}{x!}$, $\frac{e^{-1} \cdot (1)^2}{2!} \Rightarrow \frac{1}{2 \times 2.7183} \Rightarrow 0.1839$ - (B) Ans

Q(18) $P(x=2) \Rightarrow \dfrac{e^{-m}.m^{x}}{2!}$

$\Rightarrow \dfrac{e^{-m}.m^{2}}{2!}$

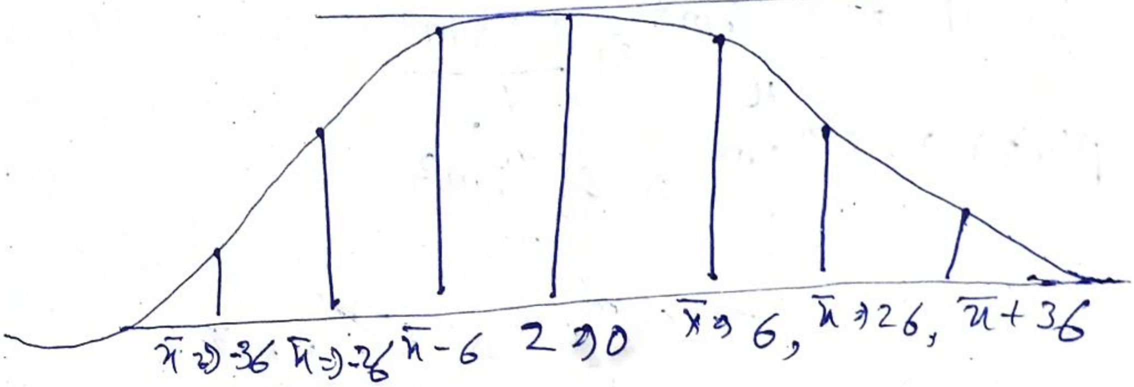
$P(x=4) \Rightarrow \dfrac{e^{-m}.m^{4}}{4!}$

$\Rightarrow \dfrac{\cancel{e^{-m}}.m^{2}}{2!} \Rightarrow \dfrac{3\times \cancel{e^{-m}}.m^{4}}{4!} \Rightarrow \dfrac{m^{\cancel{2}}}{\cancel{2}\times 1} \Rightarrow \dfrac{\cancel{3}\times m^{\cancel{4}}}{4\times\cancel{3}\times 2\times 1}$

$4 \Rightarrow m^2$

$m \Rightarrow 2$ (a) Ans

☆ Normal Distribution ☆



$\boxed{\bar{x} \Rightarrow \text{mode} \Rightarrow \text{Median}}$

0.5 + 0.5

$\boxed{\Rightarrow 1}$

☆ properties of Normal curve :-

1) QD : MD : SD
10 : 12 : 15

$\dfrac{QD}{MD} \Rightarrow \dfrac{10}{12}$ | $\dfrac{QD}{SD} \Rightarrow \dfrac{10}{15}$

$\dfrac{MD}{QD} \Rightarrow \dfrac{12}{15}$

$Q_1 \Rightarrow \mu - 0.675\sigma$

$Q_3 \Rightarrow \mu + 0.675\sigma$

Q18.) $Q_1 \Rightarrow \mu - 0.675\sigma \Rightarrow 14.6$

$Q_3 \Rightarrow \mu + 0.678\sigma \Rightarrow 25.4$

$\cancel{+}1.356 \Rightarrow \cancel{+}10.8$

$\sigma \Rightarrow \frac{10.8}{1.35} \Rightarrow 8$ Ans - d

Q19.) $MD \Rightarrow 16$, $QD \Rightarrow ?$ — $QD : MD : SD$

$10 : 12 : 15$

$\frac{QD}{MD} \Rightarrow \frac{10}{10}$, $\frac{QD}{16} \times \frac{10}{12} \Rightarrow 160 \Rightarrow 12QD$

$QD \Rightarrow \frac{160}{12} \Rightarrow$ (b) 13.33

Q20.) $\bar{x} + 6 \Rightarrow 60$

$\bar{x} - 6 \Rightarrow 40$

$+2\sigma \Rightarrow 20$

$\boxed{\sigma \Rightarrow 10}$

$QD : MD : SD$

$\Rightarrow 10 : 12 : 15$

$\frac{MD}{10} \times \frac{12}{15} \Rightarrow 15MD \Rightarrow 120$

$\boxed{MD \Rightarrow 8}$

Q21.) $QD \Rightarrow 4.05$, $MD \Rightarrow ?$

$\frac{QD}{MD} \Rightarrow \frac{10}{12} \Rightarrow \frac{4.05}{MD} \Rightarrow \frac{10}{12}$, $MD \Rightarrow \frac{12 \times 4.05}{10} \Rightarrow 4.86$

(d) Ans

$Q_1 \Rightarrow 13.25$, $MD \Rightarrow 8$

$Q_1 \Rightarrow \mu - 0.675\sigma$
↓
Mean

$13.25 \Rightarrow \mu - 0.675 \times 10$

$13.25 \Rightarrow \mu - 6.75$

$\mu \Rightarrow 20$ Ans – (a)

Equation of Normal Distribution :-

$$\boxed{f(x) \Rightarrow \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{(\bar{x}-\mu)^2 / 2\sigma^2}}$$

Q16) $C.V \Rightarrow \frac{\sigma}{\bar{x}} \times 100 \Rightarrow \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{(\bar{x}-\mu)^2}{2\sigma^2}}$

So, $\sigma \Rightarrow 4$, $\bar{x} \Rightarrow 10$

$CV \Rightarrow \frac{4}{10} \times 100 \Rightarrow 40$ Ans – (d)

Q24.) Combined Mean $\Rightarrow 10 + 12 \Rightarrow 22$

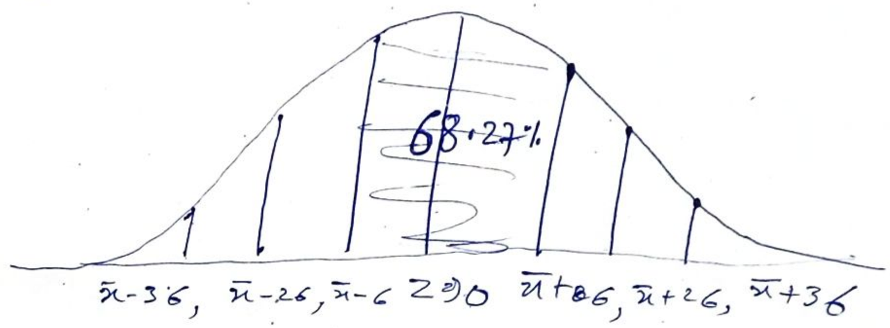
Combined SD $\Rightarrow \sqrt{S_1^2 + S_2^2} \Rightarrow \Rightarrow \sqrt{3^2+4^2}$

$\Rightarrow \sqrt{9+16}$

$\Rightarrow 5$

(c) 22 & 5 Ans

Emperical Rules :-

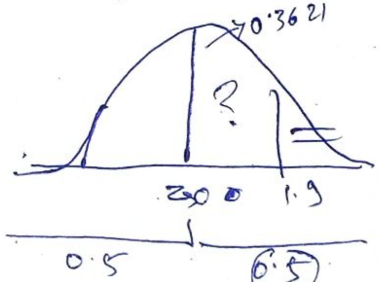


1) Upto 1st SD, Area covered 68.27%.
2) Upto 2nd SD Area covered – 95.45%.
3) Upto 3rd SD Area covered – 99.73%.

What is the probability that normal variant Z be :-
1) Greater than 1.9
2) Less then –1.65
3) between –1.25 & 2.75

Given :– $\phi(1.9) \Rightarrow 0.3621$
$\phi(1.65) \Rightarrow 0.4505$
$\phi(2.75) \Rightarrow 0.4970$
$\phi(1.25) \Rightarrow 0.3944$

Sol–



1) $0.5 - 0.3621 \Rightarrow 0.1379$
2) $0.5 - 0.4505 = 0.0495$
3) $0.4970 - 0.3944$
$\Rightarrow 0.1026$