QUANTITATIVE APTITUDE (PAPER 3)

Index

Chapter 1- Ratios, Proportions, Indices and Logarithm

Ratio

Ratio is a relationship between two quantities of the same kind and denotes how many times one of the quantities is Contained in the other.

If there are two quantities a & b, then the relationship between them can be expressed as *a:b*
('a' is to 'b')
where 'a' is called as *antecedent* and 'b' is called as the *Consequent*.

Inverse Ratio

b:a is the Inverse ratio of a:b

Explanation

Inverse ratio is of a:b is obtained by taking reciprocals of a & b

i.e,

Inverse ratio of a:b is of $\frac{1}{a} : \frac{1}{b}$

$= b : a$

Similarly inverse ratio of a:b:c $= \frac{1}{a} : \frac{1}{b} : \frac{1}{c}$ = bc:ac:ab

Compound Ratio

Compound ratio of two or more ratios is obtained by taking the ratio between the product of antecedents and product of consequent .

If a:b, c:d & e:f are the ratios given then compound ratio is (a x c x e) : (b x d x f)

Duplicate Ratio

Duplicate ratio of a:b is $a^2:b^2$

Triplicate Ratio

Triplicate ratio a:b is $a^3:b^3$

Sub duplicate ratio

Sub duplicate ratio of a:b is $\sqrt{a} : \sqrt{b}$

Sub Triplicate ratio

Sub Triplicate ratio of a:b is $\sqrt[3]{a} : \sqrt[3]{b}$

$$= a^{1/3} : b^{1/3}$$

Worked Examples:

Example1:

Find the Inverse ratio of 6:7

Solution: WKT the inverse ratio of a:b is b:a.

Therefore inverse ratio of 6:7 is 7:6

Example2:

Find the Inverse ratio of 5:8:9

Solution: WKT that inverse ratio of a:b:c = bc:ac:ab

Therefore Inverse ratio of 5:8:9 = 72:45:40

Example3:

The ratio compounded of 4:5 and Sub duplicate ratio of a:9 is 8:15 then 'a' is
a) 2 b) 3 c) 4 d) 5

Solution: Given compound ratio of 4:5 and $\sqrt{a} : \sqrt{9} = 8:15$

i.e $(4\times \sqrt{a}) : (5 \times 3) = 8:15$

Equating the antecents we get $(4\times \sqrt{a}) = 8$

$$\sqrt{a} = 2$$

a=4

Example4:

Find the compound ratio of 4:3, 5:2 & 10:7

Solution: Compound ratio = Product of antecedents: product of consequents i.e $(4\times 5 \times 10) : (3 \times 2 \times 7)$ = 100:21

Example5:

Find the Duplicate & Triplicate ratio of 5:6

Solution : Duplicate ratio of 5:6 = 5^2: 6^2 =25:36

Triplicate ratio of 5:6 = 5^3: 6^3 = 125 : 216

Example6:

Find the sub Duplicate ratio of 225:289

Solution: Sub duplicate ratio of 225: 289 = $\sqrt{225} : \sqrt{289} = 15: 17$

Example7:

If P:Q is the sub duplicate ratio of P- x^2 : Q-x^2, then x^2 is

a) $\frac{P}{P+Q}$ b) $\frac{Q}{P+Q}$ c) $\frac{PQ}{P+Q}$ d) none

Solution: Given $\sqrt{\mathrm{P}-x^2} : \sqrt{\mathrm{Q}-x^2}$ = P : Q

i.e $\frac{\sqrt{\mathrm{P}-x^2}}{\sqrt{\mathrm{Q}-x^2}} = \frac{P}{Q}$

square on both sides

$$\frac{\mathrm{P}-x^2}{\mathrm{Q}-x^2} = \frac{P^2}{Q^2}$$

$\mathrm{P}Q^2 - x^2 Q^2 = QP^2 - P^2 x^2$

$P^2 x^2 - x^2 Q^2 = QP^2 - \mathrm{P}Q^2$

$$x^2 (P^2 - Q^2) = PQ(P-Q)$$

$$x^2 (P+Q)(P-Q) = PQ(P-Q)$$

$$x^2 = \frac{PQ}{P+Q}$$

An amount divided in Certain ratio

let the total amount of Rs 30000 is divided amongst A, B & C in the ratio 2:3:5 then, The share of A is taken as 2x and that of B & C as 3x and 5x

Sum of their shares is equal to the Amount
ie. $2x + 3x + 5x = 30000$
$10x = 30000$
$x = 3000$

:. A's share is $2x = 2(3000) = 6000$
B's share is $3x = 3\ (3000) = 9000$

C's share is $5x = 5(3000) = 15000$

Example1:
A bag Contains Rs.115 in the form of 1 rupee, 50 paisa and 10 paisa coins in the ratio 1:2:3 Find the number of each type of coins

Solution: Total amount in the bag = Rs.115 = 11500 paise

Let the number of 1 rupee coins = 1x

Number of 50 paisa coins=2x

And number of 10 paisa coins = 3x

Now $(1x \times 100) + (2x \times 50) + (3x \times 10) = 11500$

$230x = 11500$

$X=50$

There fore the number of 1 rupee coins = 1x = 50

Number of 50 paisa coins=2x =100

And number of 10 paisa coins = 3x=150

Example2:

Rs 286 are to be divided amongst A, B&C so that their shares are in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. The respective shares of A, B , C are

Solution: Given ratio of shares $\frac{1}{2}:\frac{1}{3}:\frac{1}{4} = 12:8:6 = 6:4:3$

Let A's share be 6x

B's and C's share be 4x and 3x respectively

Now $6x+4x+3x = 286$

13x = 286

X=22

Therefore A's share = 6x = 132

B's share = 4x =88

C's share = 3x = 66

Problems on ratio between 2 numbers and 3 numbers with a Condition

Example1:

Two numbers are in the ratio 5:8. If 10 is added to each of them their ratio become 3:4 The numbers are.

Solution: let the 2 numbers be 5x and 8x

Given $\frac{5x+10}{8x+10} = \frac{3}{4}$

20x+40 = 24x+30

10=4x

X=5/2

Example 2:

The ages of two persons are in the ratio of 2:3. 5 years years hence their ages ratio will be 5:7. Their present ages are.

let the present ages be $2x$ & $3x$

5 years hence $\frac{2x+5}{3x+5} = \frac{5}{7}$

$14x + 35 = 15x + 25$

$\Rightarrow \boxed{x = 10}$

∴ Their present ages are $2\times10 = 20$ years & $3\times10 = 30$ years

Example3:

The two numbers are in the ratio 5:8. The difference b/w their squares is 39. find. The greater number

a) 5 b) 8 c) 16 d) 10

Let the 2 no's be $5x$ & $8x$

Given $(8x)^2 - (5x)^2 = 39$

$64x^2 - 25x^2 = 39$

$39x^2 = 39$

$x = 1$

$\therefore$ The greater no is $8x = \underline{\underline{8}}$

Example 4:

Find three numbers in the ratio 1:3:5 so that the sum of their cubes is equal to 1224

a) 2,6,10 b)1,3,5 c) 3,9,15 d) none

Let the 3 nos be x, $3x$ & $5x$

Given $x^3 + (3x)^3 + (5x)^3 = 1224$

$153x^3 = 1224$

$x^3 = 8$

$\therefore x = 2$

$\therefore$ The no's are 2, 6, 10

Comparative problems

Example1 :

If the salary of 'p' is 35% lower than that the Q Salary of R is 40% higher than that of Q, then the ratio of the salary of P & R will be.

Let Q's salary be 100

Since P's salary is 35% lower than Q.

=> P's salary = 100-35 = 65

R's Salary = 100+40 (∵ 40% higher than Q

= 140

Now P : R => 65 : 140

13 : 28

Continued ratio

Example1 :

If A:B = 3:5 & B:C = 6 : 7 then find A:B:C

Given A:B = 3:5

B:C = 6:7

A:B:C = ?

We shall use short cut to solve this

A → Product of numbers along left vertical

B → product of no's along the diagonal from lower left to upper right

C → product of no's along the right vertical

ie. A : B = 3 : 5

B : C = 6 : 7

A:B:C = (3×6) : (6×5) : (5×7) = 18 : 30 : 35

Example2:

If A:B = 2 : 3 & B:C = 4 : 7 find A:B:C

$A : B = 2 : 3$

$B : C = 4 : 7$

$A : B : C = (2 \times 4) : (4 \times 3) : (3 \times 7)$

$= 8 : 12 : 21$

Example3 :

If A:B = 2:3, B:C = 4 : 5 & C:D = 4 : 7 then find A:B:C:D

$A : B = 2 : 3$

$B : C = 4 : 5$

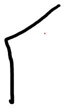
$C : D = 4 : 7$

$A : B : C : D = 32 : 48 : 60 : 105$

Short cut

A → product of nos along left vertical

B →



C →

D → product of nos along Right vertical

Example4:

If A:B = 5 : 6 , B:C = 7 : 8 & C:D =3:4 find A:B:C:D

A:B = 5:6

B:C = 7:8

C:D = 3:4

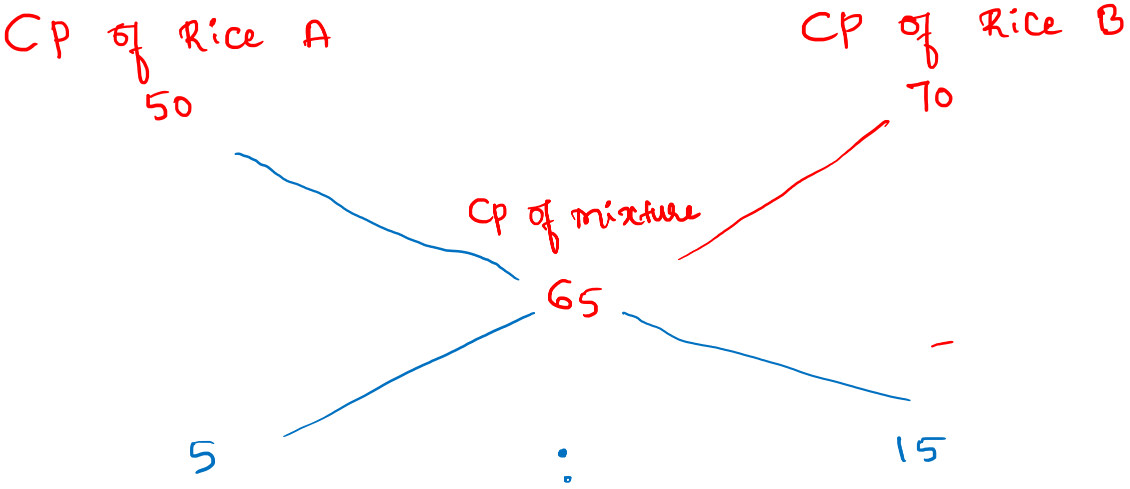
A:B:C:D = 105:126:144:192

= 35:42:48:64

Problems on Mixture

Example1

The cost price of Rice A is Rs.50 per kg and that of Rice B is Rs.70 per kg. In what ratio they have to be mixed so that the Cost price of the mixture may be Rs.65 per Kg.



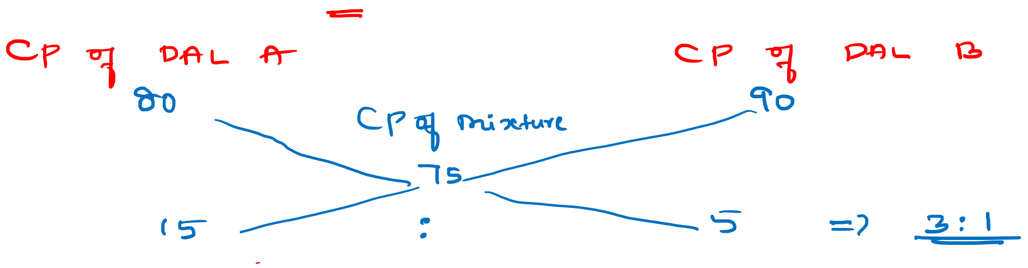
5 : 15 => 1:3

Example2:

The cost price of Dal A is Rs.80 per Kg and that of Dal B is Rs.90 per Kg. If the selling price of the mixture is Rs.100 and the profit percentage is 25 percent on the selling price then find the ratio in which the two varieties have to be mixed

Given Selling price of mixture = 100
Profit percentage = 25

Cp = Sp - profit on S.P
= 100 - 25
= 75



Proportion

The term proportion may be defined as equality of two ratios.

a:b=c:d

which is usually expressed as

$\frac{a}{b} = \frac{c}{d}$ or ad = bc or a:b::c:d

where 'd' is 4th proportional (highest proportional)

Note: for a proportion a:b::c:d ,

Product of extremes = product of means

i.e , ad = bc

Continued proportion

The proportion a:b :: b:c is Said to be a Continued proportion.

where 'c' is the 3rd proportional (highest proportional)

'b' is the mean proportional

Note:- For a Continued proportion
a:b :: b:c

We can write $\frac{a}{b} = \frac{b}{c}$

$\therefore b^2 =$ ac => b = $\sqrt{ac}$

Properties of proportion

- Alternendo
  If a:b :: c:d
  then by Alternendo a:c: :: b:d
  (obtained by interchanging Consequent of the first ratio & antecedent of the 2nd ratio)
- Invertendo
  If a:b :: c:d
  Then by Invertendo b:a :: d:c
  (obtained by interchanging antecedent & consequent both the ratios)
- Componendo
  If a:b :: c:d
  then by componendo, (a+b): b :: (c+d):d
- Dividendo
  If a:b :: c:d
  Then by dividendo (a-b): b :: (c-d): d
- Componendo and dividendo
  If a:b :: c:d
  Then by componendo & dividendo.
  (a+b): (a-b) :: (c+d): (c-d)

- Addendo
  If a:b = c:d = e:f = K
  Then by addendo

(a+c+e): (b+d+f) = k

- Subtrahendo
  If a: b = c : d = e : f = K
  Then by subtrahendo
  (a-c-e):(b-d-f) = K

Example1:

The mean proportion between 24 & 54

Solution: Given a=24 c=54

Therefore mean proportional b= $\sqrt{ac}$

b=$\sqrt{24 \times 54}$ = *36*

Example2:

If $\frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{x}$ are in proportion then x= ?

Solution: We know that ad = bc for 4 terms to be in proportion

$$\frac{1}{3} \times \frac{1}{x} = \frac{1}{5} \times \frac{1}{4}$$

$X = \frac{20}{3}$

Example3:

The 3rd proportion to 20 & 30

Solution: WKT third proportion means c,

It is given by c = b^2/a

$= 900/20$

$= 45$

Example4

If $\frac{a}{3} = \frac{b}{4} = \frac{c}{5}$ $then$ $\frac{a+b+c}{b}$ is

Solution: Let $\frac{a}{3} = \frac{b}{4} = \frac{c}{5} = k$

a=3k , b=4k and c=5k

Now $\frac{a+b+c}{b} = \frac{3k+4k+5k}{4k} = \frac{12k}{4k} = 3$

Example5

The fourth proportional of 3,5,9 is

a) 15 b) 16 c) 48 d) none of the above

Solution: We know that ad = bc for 4 terms to be in proportion

d=$\frac{bc}{a}$

=$\frac{5\times9}{3}$*= 15*

Example6

If $\frac{x}{y} = \frac{z}{w}$ implies $\frac{x}{z} = \frac{y}{w}$ then
the process is called

a) Dividendo b) Addendo c) Alternendo d) Invertendo

Solution: Answer is (c) as We know that If a:b=c:d then by Alternendo a:c=b:d

Example7

If $\frac{p}{q} = \frac{r}{s} = \frac{p+r}{q+s}$, the process is called

a) Addendo b) Subtrahendo c) Componendo d) dividendo

Solution: Answer is (a) , as we know that if a:b=c:d =k then by addendo (a+c): (b+d) = k

Example8

If $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$ $then$ $\frac{2x-5y+4z}{2y}$ is

a) 3/4 b) 4/3 c) 3/2 d) none

Solution: Let $\frac{x}{3}=\frac{y}{4}=\frac{z}{5} \quad = k$

Therefore $x=3k$, $y=4k$ *and* $z=5k$

Now $\frac{2(3k)-5(4k)+4(5k)}{2(4k)}=\frac{6k}{8k}=\frac{3}{4}$

Indices

It is the plural for of Index, which means power.

Laws of Indices

- a^m x a^n = a^{m+n} [Base must be same]
- $a^{-m}=\frac{1}{a^m}$
- $\frac{a^m}{a^n}$ = a^{m-n}
- a^o = 1
- If a^x = a^y then

x = y

- $\sqrt[m]{a}$ = $a^{1/m}$
- If x^a = y^a

then x=y

- $((a^m)^n)^p$ = a^{mnp}

Some Important Algebraic Formulae

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a+b)(a-b)$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Note :- If $a^{1/3} + b^{1/3} + c^{1/3} = 0$
Then $(a+b+c)^3 = 27abc$

Example1

The value of $\frac{2^{n+1} + 2^n}{2^{n+3} - 2^{n+1}}$ is

a) ½ b)1/3 c)1/6 d)none

$$\frac{2^{n+1} + 2^n}{2^{n+3} - 2^{n+1}} = \frac{\cancel{2^n}(2+1)}{\cancel{2^n}(2^3 - 2^1)}$$

$$= \frac{3}{8-2} = \frac{3}{6}$$

$$= \frac{1}{2}$$

Example2

If x · $x^{1/4}$ = $(x^{1/4})^x$ then x is

a)4 b)5 c) 6 d)none

$$x \cdot x^{\frac{1}{4}} = (x^{\frac{1}{4}})^x$$

$$x^{1+\frac{1}{4}} = x^{\frac{x}{4}}$$

$$x^{5/4} = x^{x/4}$$

$$\Rightarrow \frac{5}{4} = \frac{x}{4}$$

$$\Rightarrow \boxed{x = 5}$$

Example3

If $(5)^{150}$ = $(5x)^{50}$ then x =?

a) 125 b)625 c)25 d) none

$$(5)^{150} = (5x)^{50}$$

$$(5^3)^{50} = (5x)^{50}$$

$$5^3 = 5x$$

$$125 = 5x$$

$$\boxed{x = 25}$$

Example4

The value of$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2}$ x $\left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2}$ x $\left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$is

a)0 b)1 c)2 d)none

$$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \cdot \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \cdot \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$$

$$\Rightarrow \left(x^{a-b}\right)^{a^2+ab+b^2} \cdot \left(x^{b-c}\right)^{b^2+bc+c^2} \cdot \left(x^{c-a}\right)^{c^2+ca+a^2}$$

$$\Rightarrow x^{a^3-b^3} \cdot x^{b^3-c^3} \cdot x^{c^3-a^3} \quad \left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$$

$$\Rightarrow x^{\cancel{a^3}-\cancel{b^3}+\cancel{b^3}-\cancel{c^3}+\cancel{c^3}-\cancel{a^3}}$$

$$\Rightarrow x^0 = \underline{\underline{1}}$$

<u>Example5</u>

If $a^x=b$, $b^y=c$ & $c^z=a^3$ then the value of xyz is

a)1 b)2 c)3 d)none

Given $a^x=b,\quad b^y=c,\quad c^z=a^3$

Consider $c^z=a^3$

$$(b^y)^z = a^3 \quad [\because c=b^y]$$

$$\left[(a^x)^y\right]^z = a^3 \quad [\because b=a^x]$$

$$a^{xyz} = a^3$$

$$\Rightarrow \boxed{xyz=3}$$

<u>Example6</u>

If $3^{1/x} = 4^{1/y} = 12^{1/z}$ then x+y = ?

a)1/z b)z c)2z d)none

Let $3^{1/x} = 4^{1/y} = 12^{1/z} = k$

$3^{1/x} = k$

Raise the power to x on B.S

$3 = k^x$

IIIly $4 = k^y$ and $12 = k^z$

Now,

$$12 = k^z$$

$$\Rightarrow (3 \times 4) = k^z$$

$$\Rightarrow k^x \times k^y = k^z$$

$$\Rightarrow k^{x+y} = k^z$$

$$\rightarrow x + y = z$$

<u>Example7</u>

The value of $\frac{5^n+5^{n-1}}{5^{n+1}-5^n}$ is

a) $\frac{6}{21}$ b) $\frac{5}{21}$ c) $\frac{2}{3}$ d) *none*

$$\frac{5^n + 5^{n-1}}{5^{n+1} - 5^n} = \frac{\cancel{5^n}(1+5^{-1})}{\cancel{5^n}(5-1)} = \frac{1+\frac{1}{5}}{4}$$

$$= \frac{\frac{6}{5}}{4} = \underline{\underline{\frac{6}{20}}}$$

∴ Answer is (d)

<u>Example8</u>

The value of $\frac{2^{m+1}\ x\ 3^{2m-n+3}\ x\ 5^{n+m+4}\ x\ 6^{2n+m}}{6^{2m+n}\ x\ 10^{n+1}\ x\ 15^{m+3}}$ is

a) 3^{2m+2n} b) 3^{2n+2m} c) 1 d) none

$$\frac{2^{m+1} \times 3^{2m-n+3} \times 5^{n+m+4} \times 6^{2n+m}}{6^{2m+n} \times 10^{n+1} \times 15^{m+3}}$$

Plug in $m=0$ & $n=1$

we get $\underline{1}$.

Example9

$[1-\{1-(1-x^2)^{-1}\}^{-1}]^{-1/2}$ is equal to

a) x b) 1/x c) 1 d) none

$$\left[1-\{1-(1-x^2)^{-1}\}^{-1}\right]^{-\frac{1}{2}}$$

$$\Rightarrow \left[1-\left\{1-\frac{1}{1-x^2}\right\}^{-1}\right]^{-\frac{1}{2}}$$

$$\Rightarrow \left[1-\left\{\frac{1-x^2-1}{1-x^2}\right\}^{-1}\right]^{-\frac{1}{2}}$$

$$\Rightarrow \left[1-\frac{1-x^2}{-x^2}\right]^{-\frac{1}{2}}$$

$$\Rightarrow \left[1+\frac{1-x^2}{x^2}\right]^{-1/2}$$

$$\Rightarrow \left[\frac{\cancel{x^2}+1-\cancel{x^2}}{x^2}\right]^{-\frac{1}{2}} = \left[x^{-2}\right]^{-\frac{1}{2}} = \underline{\underline{x}}$$

Example10

$\left[(x^n)^{\frac{1}{n}+1}\right]$ is equal to

a) x^n b) x^{n+1} c) x^{n-1} d) none

$$(x^n)^{\frac{1}{n}+1}$$

$$\Rightarrow (x^n)^{\frac{1+n}{n}}$$

$$\Rightarrow x^{\cancel{n}\times\frac{n+1}{\cancel{n}}}$$

- $\Rightarrow x^{n+1}$

<u>Example11</u>

If a = 5+3√2 then the value of $a^{1/2} + a^{-1/2}$ is

a) 3.36 b) 4.155 c) 6.155 d) none

Given $a = 5+3\sqrt{2}$

$= 9.2426 \Rightarrow \sqrt{a} = 3.0401$

Now $a^{1/2} + a^{-1/2} = \sqrt{a} + \frac{1}{\sqrt{a}}$

$= 3.0401 + \frac{1}{3.0401}$

$= \underline{\underline{3.36}}$

Logarithm

If N = a^x

then 'x' is said to the logarithm of the number N to base 'a' ie x = $\log_a N$

Characteristic & Mantissa

The integral part of a common logarithm is called the characteristic & the non negative decimal part called the mantissa.

Antilogarithm

If x is said to be a logarithm of N to a given base, then N is said to be an antilogarithm of x to that base

i.e. If $\log_a N$ = x then N= antilog x

Laws of logarithm

- $\log_a x + \log_a y = \log_a xy$
  where x & y are positive number
- $\log_a x - \log_a y = \log_a(\frac{x}{y})$
- Log m^k = k log m
- Change of base property
  $\log_n m$= logm/logn
- $\log_a 1$= 0
- Logarithm of a number to the same base is equal to 1
  Ex :- 1) $\log_5 5$= 1

  2) $\log_{10} 10$= 1
- $a^{\log_a x} = x$
- If log x = log y
  Then x = y
- **Number of digits in a given numeral = log(numeral)**

Example: find the number of digits in the numeral 2^{12}

Given log2 = 0.3010

Solution: Number of digits $= \log(2^{12})$

$= 12 \log 2$

$= 12 \,.\, 0.3010$

$= 3.612$

$= 4$

Conversion from logarithm to exponential form

1. Base remains as the base
2. Value becomes the power

Conversion from Exponential to logarithm form

1. Power becomes the value
2. Base remains as the base

Problems on Conversion

Example1:

If $\text{Log}_{10000} x = \frac{-1}{4}$ then x = _____

$$\log_{10,000} x = -\frac{1}{4}$$

$$\Rightarrow \quad x = (10,000)^{-\frac{1}{4}}$$

$$= (10^4)^{-\frac{1}{4}}$$

$$x = 10^{-1}$$

$$\boxed{x = \frac{1}{10}}$$

Example2:

If $\text{Log}_2 x + \text{Log}_4 x = 6$ then the value of x

$$\log_2 x + \log_4 x = 6$$

By change of base rule

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 4} = 6$$

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 2^2} = 6$$

$$\frac{\log x}{\log 2} + \frac{\log x}{2\log 2} = 6$$

$$\log_2 x + \frac{1}{2}\log_2 x = 6$$

$$\log_2 x \left(1 + \frac{1}{2}\right) = 6$$

$$\log_2 x \left(\frac{3}{2}\right) = 6 \quad \Rightarrow \log_2 x = 4$$

$$\Rightarrow x = 2^4 = \underline{16}$$

Example3

If $\log_2[\log_3(\log_2 x)] = 1$, the x equals:

a) 128 b) 256 c) 512 d) None

$$\log_2 \log_3 \log_2 x = 1$$

$$\Rightarrow \log_3 \log_2 x = 2^1$$

$$\Rightarrow \log_3 \log_2 x = 2$$

$$\Rightarrow \log_2 x = 3^2$$

$$\Rightarrow \log_2 x = 9$$

$$\Rightarrow x = 2^9$$

$$\Rightarrow x = 512$$

Example4

$\log_4(x^2 + x) - \log_4(x + 1) = 2$. Find x

a) 16 b) 0 c) -1 d) None of these

$$\log_4(x^2+x) - \log_4(x+1) = 2$$

$$\Rightarrow \log_4\left(\frac{x^2+x}{x+1}\right) = 2 \qquad [\because \log m - \log n = \log \tfrac{m}{n}]$$

$$\Rightarrow \log_4\left(\frac{x(\cancel{x+1})}{\cancel{x+1}}\right) = 2$$

$$\Rightarrow \log_4 x = 2$$

$$\Rightarrow x = 4^2$$

$$\boxed{x = 16}$$

Example5

Solve: $\left(\frac{\log_{10} x - 3}{2}\right) + \left(\frac{11 - \log_{10} x}{3}\right) = 2$

a) 10^{-1} b) 10^2 c) 10 d) 10^3

$$\frac{\log_{10} x - 3}{2} + \frac{11 - \log_{10} x}{3} = 2$$

$$\Rightarrow \frac{3[(\log_{10} x) - 3] + 2[11 - \log_{10} x]}{6} = 2$$

$$\Rightarrow 3\log_{10} x - 9 + 22 - 2\log_2 x = 12$$

$$\Rightarrow \log_{10} x + 13 = 12$$

$$\Rightarrow \log_{10} x = -1$$

$$\Rightarrow x = 10^{-1}$$

Example6

If $\log_x y = 100$ and $\log_2 x = 10$, then the value of 'y' is:

a) 2^{10} b) 2^{100} c) 2^{1000} d) 2^{10000}

$\log_x y = 100$ $\log_2 x = 10$

$\Rightarrow x^{100} = y$ & $x = 2^{10}$

$\Rightarrow (2^{10})^{100} = y$ $(\because x = 2^{10})$

$\Rightarrow y = 2^{1000}$

Problems on Properties

Example1

If log (2a – 3b) = log a – log b then a= ____

$\log(2a-3b) = \log a - \log b$

$\Rightarrow \log(2a-3b) = \log\left(\frac{a}{b}\right)$ $\left[\because \log m - \log n = \log\frac{m}{n}\right]$

$\Rightarrow 2a-3b = \frac{a}{b}$ $\left[\because \text{If } \log x = \log y \text{ then } x = y\right]$

$\Rightarrow 2ab - 3b^2 = a$

$\Rightarrow 2ab - a = 3b^2$

$\Rightarrow a(2b-1) = 3b^2$

$\Rightarrow a = \frac{3b^2}{2b-1}$

Example 2:

$\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc}$ is equal

$$\overline{\log_{ab} abc} + \overline{\log_{bc} abc} \quad \log_{ac} abc$$

$$\Rightarrow \log_{abc} ab + \log_{abc} bc + \log_{abc} ac$$

$$\left[\because \frac{1}{\log_m n} = \log_n m \text{ by change of Base rule}\right]$$

$$\Rightarrow \log_{abc}(ab \times bc \times ac) \quad \left[\because \log m + \log n + \log p = \log(m \times n \times p)\right]$$

$$\Rightarrow \log_{abc}(abc)^2$$

$$\Rightarrow 2\log_{abc} abc \Rightarrow 2 \times 1 \quad \left[\because \log m^k = k\log m \text{ \& } \log_a a = 1\right]$$

$$\Rightarrow 2$$

Example3

The value of $\frac{\log_3 8}{\log_9 16 \log_4 10}$ is

$$\frac{\log_3 8}{\log_9 16 \ \log_4 10}$$

By change of base rule

$$\Rightarrow \frac{\frac{\log 8}{\log 3}}{\frac{\log 16}{\log 9} \times \frac{\log 10}{\log 4}} = \frac{\frac{\log 2^3}{\log 3}}{\frac{\log 2^4}{\log 3^2} \times \frac{\log 10}{\log 2^2}}$$

$$\Rightarrow \frac{\frac{3\log 2}{\log 3}}{\frac{\cancel{4}\log \cancel{2}}{\cancel{2}\log 3} \times \frac{1}{\cancel{2}\log \cancel{2}}} = \underline{\underline{3\log 2}}$$

Example4

If Log $\left(\frac{a+b}{4}\right) = \frac{1}{2}$ (Log a+ Log b) then $\frac{a}{b} + \frac{b}{a}$ is

$$\log\left(\frac{a+b}{4}\right) = \frac{1}{2}(\log a + \log b)$$

To find $\frac{a}{b} + \frac{b}{a} \Rightarrow \frac{a^2+b^2}{ab}$

$$\log\left(\frac{a+b}{4}\right) = \frac{1}{2}\log ab \quad [\because \log m + \log n = \log mn]$$

$$\log\left(\frac{a+b}{4}\right) = \log(ab)^{\frac{1}{2}} \quad [\because k\log m = \log m^k]$$

$$\frac{a+b}{4} = \sqrt{ab} \quad [\because \text{If } \log x = \log y \text{ then } x = y]$$

Sq. on B.S

$$\left(\frac{a+b}{4}\right)^2 = (\sqrt{ab})^2$$

$$\frac{a^2+b^2+2ab}{16} = ab \Rightarrow a^2+b^2+2ab = 16ab$$

$$\rightarrow a^2+b^2 = 14ab \Rightarrow \frac{a^2+b^2}{ab} = \underline{\underline{14}}$$

Exercise questions:

1. $\frac{3x-2}{5x+6}$ is is the duplicate ratio of 2/3 then x is

(a) 2 (b)6 (c)5 (d)9

2. If $\frac{a}{7} = \frac{b}{4} = \frac{c}{11}$ $then$ $\frac{a+b+c}{c}$ is

a) 2 b)3 c)7 d)none

3. $\log_2 \log_2 \log_2 16$ = ?

(a)0 (b)3 (c) 1 (d) 2

4. If a : b = 9 :4 then $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$

(a) 2/3 (b) 3/2 (c) 6/13 (d) 13/6

5. If a : b = 3: 7 then 3a + 2b : 4a + 5b = ?

(a) 27 : 43 (b)23 : 47 (c) 24 : 51 (d) 29 : 53

6. The ratio of no. of boys and the no. of girls in a school is found to be 15:32. How many boys and equal no. of girls should be added to bring the ratio to 2/3?

(a) 20 (b)19 (c) 23 (d)27

7. log 9 + log 5 is expressed as ________

a) log (9/5) (b)log 4 (c) log (5/9) (d) log 45

8. If $\log_a \sqrt{3}$ = 1/6 then the value of a is

(a) 81 (b)9 (c) 27 (d) 3

10. Mean proportion between 25 and 64 is
a)28 b)40 c)32 d)none
11. If A:B = 4:5 & B: C is 6:7 then A:B:C
a) 24:30:35 b) 6:9:10 c) 10:9:6 d) none

12. The fourth proportional of 6,8,9 is
a) 12 b) 32 c) 48 d) none of the above
13. If the ratio of two numbers is 5:7. If 5 is added to each number then the new ratio will be 11:15 then the numbers are
a)50, 70 b)25,35 c)21,33 d)none
14. If x : y : z = 7 : 4 : 11 then (x+y+z)/z is

a)2 b)3 c)4 d)none

15. If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$ then the value of $\left(\frac{a+b+c}{3}\right)^3$

a) abc b) 27abc c) $\frac{1}{abc}$ d) $\frac{1}{9abc}$

16.The value of $\log_4 9 . \log_3 2$ is :

a) 3 b) 9 c) 2 d) 1

17. The value of $\left(\log_y x . \log_z y . \log_x z\right)^3$ is:

a) 0 b) -1 c) 1 d) 3

18. If $x^2 + y^2 = 7xy$, then log $\frac{(x+y)}{3}$ =

a) (logx + logy) b) $\frac{1}{2}$ (logx + logy) c) $\frac{1}{3}$ (logx / logy) d) $\frac{1}{3}$ (logx + logy)

19. If log x = a+b, log y = a-b then the value of $\log\left(\frac{10x}{y^2}\right) =$

a) 1-a+3b b) a-1+3b c) a+3b+1 d) 1-b+3a

20. If p^x = q, q^y = r and r^Z= p^3, the the value of xyz will be:

a) 0 b) 1 c) 3 d) 6

21. If $3^x = 5^y = 75^z$, then

a) x+y-z=0 b) $\frac{2}{x} + \frac{1}{y} = \frac{1}{z}$ c) $\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$ d) $\frac{2}{x} + \frac{1}{z} = \frac{1}{y}$

22. If x = $3^{1/3} + 3^{-1/3}$ then find value of $3x^2 - 9x$

a) 3 b) 9 c) 12 d) 1

Chapter 2- Equations:

Equation:

It is a mathematical statement that connects 2 expressions by an equals sign.

Ex:-

$\frac{x+3}{6} = \frac{2x+5}{3}$

Simple equation :

→ An equation having One unknown (variable). A Simple equation is of the form ax + b = 0

Where, x is a variable

Example1

If twice the age of Rajesh 10 years hence is subtracted from 6 times his present age then the result would be equal to twice his present age. Find his present age.

Solution: Let the present age of Rajesh be x years

Given 6x- 2(x+10) = 2x

4x-20=2x

2x=20

x=10

Therefore the Present age of Rajesh is 10 years

Example2:

The numerator of a fraction exceeds the denominator by 3 And if 3 is added to to both numerator & denomination for then the fraction becomes $\frac{10}{7}$. Find the original fraction.

Let the Denomenator be x

$\Rightarrow$ Numerator $= x+3$

Original fraction $= \frac{x+3}{x}$

Given $\frac{x+3+3}{x+3} = \frac{10}{7}$

$7x+42 = 10x+30$

$12 = 3x$

$\boxed{x=4}$

$\therefore$ The original fraction $= \frac{4+3}{4} = \frac{7}{4}$

Example3

The number of students in each section of a school is 30. After admitting 60 new students, 5 new sections were started. If total number of students in each section now is 24, then the number of sections initially were:

(a) 6 (b) 10 (c) 14 (d) 18

Let the no. of sections initially be x

$\Rightarrow$ Total no. of students = $30x$ ($\because$ 30 students / section)

After adding 60 new students,

The total no. of students = $30x+60$ = $24(x+5)$

$$30x+60 = 24x+120$$
$$6x = 60$$
$$\boxed{x=10}$$

Simultaneous Linear equations in 2 unknown's

General form

$a_1 x + b_1 y + c_1 = 0$
$a_2 x + b_2 y + c_2 = 0$

Example

$3x + 5y = 20 \rightarrow$ ①
$5x + 6y = 32 \rightarrow$ ②

① x 5 & ② x 3

$15x + 25y = 100$
$15x + 18y = 96$
(-) (-) (-)

$7y = 4 \Rightarrow y=\frac{4}{7}$

① => $3x + 5y = 20$

$3x = 20-5y = 20 - 5\left(\frac{4}{7}\right)$

$3x = \frac{120}{7} \Rightarrow x = \frac{40}{7}$

Short cut

$3x+5y = 20$
$5x+6y = 32$

$$x = \frac{b_1c_2-b_2c_1}{a_2b_1-a_1b_2} = \frac{40}{7}$$

$$y = \frac{c_1a_2-c_2a_1}{a_2b_1-a_1b_2} = \frac{4}{7}$$

Example1

The value of k for which the System of equations Kx +2y=5
10x +2 y = 1 has no Solution is

a) 5 b) 10 c) 6 d) *none*

Condition for system of equations to have no solution is

$a_2b_1 - a_1b_2 = 0$

Given. $kx+2y=5$
$10x+2y=1$ $\Rightarrow$ $20-2k = 0$

$20=2k$

$\boxed{k=10}$

Example2:

A man sells 6 radios & 4 televisions for Rs 18,480. If 14 radios & 2 televisions are sold for the same amount, what is the price of television.

a) Rs 1848 b) Rs 840 c) Rs 1680 d) Rs 3360

Given $6x+4y = 18480$
$14x+2y = 18,480$

where $x \to$ price of Radio
$y \to$ price of Television

W.K.T $y = \frac{c_1a_2 - c_2a_1}{a_2b_1 - a_1b_2} = \frac{147840}{44} = \underline{3360}$

Quadratic Equation:

→ A polynomial in 'x' with degree 2.

General form

$ax^2 + bx + c = 0$

where a, b, c are constants

→ There are 2 roots for x for a quadratic equation

Solution to a quadratic equation

a) Factorization method

b). Formula method.

a) Factorization method

Example:-

$x^2 + 7x + 12 = 0$

$x^2 + 4x + 3x + 12 = 0$

$x(x+4) + 3(x+4)$

$(x+4)(x+3) = 0$

$x + 4 = 0$ or $x + 3 = 0$

x=-4 or x = -3

<u>Solution to quadratic equation using formula</u>

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Example:

$x^2+7x+12=0$

a = 1 b = 7 c = 12

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-7\pm\sqrt{7^2-4(1)(12)}}{2(1)}$$

$$= \frac{-7\pm\sqrt{1}}{2} \qquad = \frac{-7\pm1}{2}$$

$$x = \frac{-7\pm1}{2}$$

$$X = \frac{-7+1}{2} \text{ or } \frac{-7-1}{2}$$

X = -3 or x = -4

<u>Nature of the root for the equation $ax^2+bx+c=0$</u>

1) If $b^2 - 4ac = 0$, then the roots are real & equal.

2) If $b^2-4ac> 0$ & a perfect square, then the roots are real, distinct & rational

3) If $b^2-4ac> 0$ & not a perfect square then the roots are real, distinct & Irrational

4) If $b^2-4ac < 0$ then the roots are imaginary.

Example1:

Find the nature of the roots for the equation $x^2+6x+8=0$

Given $x^2+6x+8=0$

$a=1,\ b=6\ \ c=8$

$b^2-4ac = 6^2-4(1)(8)$

$= 36-32$

$= 4 > 0$ and perfect square

$\therefore$ The roots are real, distinct & rational

Example2

The quadratic equation $x^2-4kx+32=0$ will have equal roots, then the value k is.

Given $x^2-4kx+32=0$, $a=1$, $b=-4k$ $c=32$

Roots are equal

i.e $b^2-4ac=0$

$\Rightarrow (-4k)^2-4(1)(32)=0$

$16k^2=128$

$k^2=8$

$k=\pm\sqrt{8}$

Construction of quadratic equation when the roots α & β are given

$x^2-(\alpha+\beta)x+\alpha\beta=0$

Example:-

Find the quadratic equation whose roots are 4 & -3.

Given $\alpha=4$. $\beta=-3$

$\therefore$ The quadratic equation is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-(4-3)x+4(-3)=0$

$x^2-x-12=0$

Note.:-

1. Sum of the roots of a quadratic equation

$\alpha + \beta = \frac{-b}{a}$

Where α & β are the roots.

2. The product of the roots =) $\alpha\beta = \frac{c}{a}$

3. If One root the quadratic equation is P+ $\sqrt{q}$ then other root will be P- $\sqrt{q}$

4) If one root is reciprocal to the other then [$\alpha\beta = 1$] (:. $\alpha \frac{1}{\alpha} = 1$)

5)If two roots are in the ratio $\alpha:\beta$ then

$\alpha\beta b^2 = (\alpha + \beta)^2 ac$

Note: Same formula can be used for one root double the other (where the ratio is 1:2)

One root is triple the other (where the ratio is 1:3) and so on…

9) The difference between the roots

$\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

Example1

If one root of a quadratic equation is 2-√3, then the quadratic equation is
a) $x^2+4x-1=0$ b) $x^2-4x-1=0$
c) $x^2+4x+1=0$ c) none

Given one root is $2-\sqrt{3}$
=> othe root will be $2+\sqrt{3}$

∴ Required quadratic equation is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-(2-\sqrt{3}+2+\sqrt{3})x+(2-\sqrt{3})(2+\sqrt{3})=0$

$x^2-4x+(2)^2-(\sqrt{3})^2=0$

$x^2-4x+1=0$

Example2

If one root is reciprocal to the other for the equation
$2x^2-14x+k=0$ then K= ___

Since the roots are reciprocal to each other

$\Rightarrow \quad \alpha\beta = 1 \rightarrow ①$

Given $\quad 2x^2-14x+k=0$

$\alpha\beta = \frac{c}{a} = \frac{k}{2}$

$\frac{k}{2} = 1$ [∵ from ①]

$\boxed{k=2}$

Example3:

If α & β the roots of the equation

$2x^2 + 3x + 7 = 0$, then the value of $\alpha\beta^{-1} + \beta\alpha^{-1}$ is

a)2 b) 3/7 c) 7/2 d)- 19/14

Given $\quad 2x^2+3x+7=0$

To find $\quad \alpha\beta^{-1}+\beta\alpha^{1} = \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

W.K.T $\quad a^2+b^2=(a+b)^2-2ab$

For the given equation

$\alpha+\beta = \frac{-b}{a} = -\frac{3}{2} = -1.5$

$\alpha\beta = \frac{c}{a} = \frac{7}{2} = 3.5$

$\therefore \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{(-1.5)^2-2(3.5)}{3.5} = -1.357$

Option (d) is the correct answer.

Example 4

If α & β are the roots of the equation $x^2+7x+12=0$

then the equation whose roots are $(\alpha-\beta)^2$ & $(\alpha+\beta)^2$ is.

a) $x^2-14x+49=0$ b) $x^2-24x+144=0$
c)$x^2-50x+49=0$ d) $x^2-19x+144=0$

Given $\quad x^2+7x+12=0$

By factorizing $\quad \alpha=-3 \quad \beta=-4$

Required Roots are $(\alpha-\beta)^2$ and $(\alpha+\beta)^2$

$[-3-(-4)]^2$ and $(-3-4)^2$

1 and 49

Required equation $\Rightarrow x^2-(1+49)x+(1\times 49)=0$

$x^2-50x+49=0$

Example5:

If α & β are the equation
$2x^2 - 4x - 1 = 0$ then find the value of

$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is

Given

$2x^2 - 4x - 1 = 0$

$\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{2} = 2$

$\alpha\beta = \frac{c}{a} = \frac{-1}{2} = -0.5$

To find $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$

$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$= \frac{(2)^3 - 3(-0.5)(2)}{-0.5}$

$= \underline{\underline{-22}}$

<u>Problems on infinite series</u>

<u>Example1:-</u>

The value of $\sqrt{42 + \sqrt{42 + \sqrt{42 + \cdots \ldots \ldots}}}\infty$

a)7 b) 8 c) 9 d) none

Conventional method:

Let $x = \sqrt{42+\sqrt{42+\sqrt{42+\cdots\infty}}}$

$x = \sqrt{42+x}$

Sq. on B.S

$x^2 = 42 + x$

$x^2 - x - 42 = 0$

-42

Factors -7 6

Roots $7, \underline{\underline{-6}}$

∴ The answer is 7 (as the value must be +ve)

Short cut

Get the 2 no's in such a way the difference b/w them is 1 & the product is the number given within root.

And the greatest among the 2 no's is the value.

Example2:-

The value of $\sqrt{30+\sqrt{30+\sqrt{30+\cdots\ldots\ldots.\infty}}}$

a)-7 b) -8 c) 6 d) none

Let $x = \sqrt{30+\sqrt{30+\sqrt{30+\cdots\infty}}}$

Using short cut

30

⑥ 5

∴ Answer is $\underline{\underline{6}}$

Cubic equation

→ A polynomial in x having degree 3 (highest power of x is 3).
General form
$ax^3 + bx^2 + cx + d = 0$

The Sum the roots = $\frac{-b}{a}$

The product of roots = - $\frac{d}{a}$

Example1:

The roots of the Cubic equation.
$x^3 - 7x + 6 = 0$ are

a) (1,-2,3) b) (1,-2,-3)

c)(1,2,-3) d) (1,-2,-3)

Given $x^3 - 7x + 6 = 0$

$\Rightarrow x^3 + 0x^2 - 7x + 6 = 0$

$x^3 + 0x^2 - x - 6x + 6 = 0$

$\underline{x^3 - x^2} + \underline{x^2 - x} - \underline{6x + 6} = 0$

$x^2(x-1) + x(x-1) - 6(x-1) = 0$

$\Rightarrow (x-1)(x^2 + x - 6) = 0$

$\Rightarrow x - 1 = 0$ or $x^2 + x - 6 = 0$

$x = 1$ or $x = -3, \ x = 2$

-6

Factors : $3 \quad -2$

Roots : $-3, \ 2$

Short cut

Sum of roots $= -\frac{b}{a} = 0$

Product of roots $= -\frac{d}{a} = -6$

Option (c) Satisfy for above conditions

as Sum of roots $\Rightarrow 1 + 2 - 3 = 0$

Product of roots $\Rightarrow 1 \times 2 \times -3 = -6$

Example2:

The roots of the of equation $x^3 + x^2 - x - 1 = 0$ are

a) (1,1,1) b) (1, -1,1) c) (1,-1, -1) d) none

Given $x^3+x^2-x+1=0$

Sum of the roots $= -\frac{b}{a} = -1$

product of the roots $= \frac{-d}{a} = -1$

Option (c) Satisfy the above requirements

as Sum of the roots => $1-1-1=-1$

Product of the roots $= 1\times -1 \times -1 = -1$

Example3:

Find the roots the equation

$x^3 - 6x^2 + 11x - 6 = 0$

a) (1,2,3) b) (-1,2,5)

c) (1, -2,3) d) (1,2,-3)

Given $x^3-6x^2+11x-6=0$

Sum of the roots $= -\frac{b}{a} = 6$

product of the roots $= -\frac{d}{a} = 6$

Option (a) Satisfy the above requirements

as Sum of the roots => $1+2+3=6$

product of the roots => $1\times 2\times 3 = 6$

PRACTICE PROBLEMS

1. On solving $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$, we get one value of x as:

(a) $\frac{4}{13}$(b) $\frac{1}{13}$ (c)$\frac{2}{13}$ (d) $\frac{3}{13}$

2. If roots of the equation $x^2 + x + r = 0$ are 'α' and 'β'and $\alpha^3 + \beta^3 = -6$. Find the value of 'r'?

(a) $\frac{-5}{3}$(b) $\frac{7}{3}$ (c)$\frac{-4}{3}$ (d) 1

3. If one root of the Equation p$x^2 + qx + r = 0$ is r, then other root of the equation will be:

(a) 1/q (b) 1/r (c) 1/p (d) 1/(p+q)

4. If the ratio of the roots of the Equation $4x^2 - 6x + p = 0$ is 1 : 2, then the value of p is:

(a) 1 (b) 2 (c) -2 (d) -1

5. The present age of a man is 8 years more than thrice the sum of the ages of his two grandsons who are twins. After 8 years, his age will be 10 years more than twice the sum of the ages of his grandsons. The age of a man when his grandsons were born was:

(a) 86 years (b) 73 years (c) 68 years (d) 63 years

6.If the roots of the equation $4x^2-12x+k=0$ are equal, then the value of k is:

(a) -3 (b) 3 (c) -9 (d) 9

7.If α + β = -2 and α β = -3, then α, β are the roots of the equation, which is:

(a) $x^2-2x-3=0$ (b) $x^2+2x-3=0$

(c) $x^2+2x+3=0$ (d) $x^2-2x+3=0$

8.If α, β are the roots of the equation $x^2+7x+12=0$ then the value of $\alpha^2/\beta + \beta^2/\alpha$ is

a)(144/49) +(49/144) b)(7/12) +(12/7) c)-91/12 d)none

9.If the root of the equation are α and 1/ α then the quadratic equation will be

a) $\alpha x^2 - (\alpha^2 + 1)x + \alpha = 0$ b) $\alpha x^2 - (\alpha^2 + 1)x + 1 = 0$

c) $\alpha x^2 - \alpha^2 x + 1 = 0$ d) none

10.If x =$\frac{1}{5+2\sqrt{6}}$ then the value of $x^2-10x+1$

a)0 b)10 c)1 d)none

11.Find the value of K so that x =2 is a root of the equation $3x^2 - 2kx + 5 = 0$

(a) 17/4 (b) 4/17

(c) –17/4 (d) –4/17

12.The roots of the equations $4^x\, 8^y$ =128 $3^x\, /27^y$ =1/3

a)2,1 b)-2,1 c) 2,-1 d)none

13.The roots of the equations $x^3\ + 9x^2$ -x-9=0 are

a)1,-1,-9 b)1,1,9 c)1,-1,9

14Find the condition that one roots is double the other of $a\, x^2\ + bx + c = 0$

a)2 $b^2 = 3ac$ (b) $b^2\ = 3ac$

(c) 2 $b^2 = 9ac$ (d) 2 $b^2 > 9ac$

15.On solving m+$\sqrt{m}$ =6/25 m works out to be

a)2/25 (b) 1/25

(c) 3/25 (d) 1

16.Solving equation 3 g^2 –14g + 16 = 0, we get roots as

(a) 0 (b) ± 5

(c) 8 and 2/3 (d) 2 and 8/3

17. If 2 x^2 –(a+6)2X+12a = 0 then roots are

(a) 4 & a2 (b) 6 & a

(c) 3 & 2a (d) 6 & 3a

18.The rational root of the equation 0 = 2 $p^3\ -\ p^2$ – 4p + 2 is

(a) –2 (b) 2 (c) ½ (d) – ½

Chapter 3- Linear Inequalities

Inequality :-

These are special mathematical Symbols used to define boundaries for a variable. For example, In India the eligibility for citizens to vote in an elections is minimum age of 18 years (i.e atleast 18 years).

This can be expressed as $x \geq 18$ (where x is age).

Linear Inequality with two variables

$ax+by+c=0$ is a linear equation

where as $ax+by+c<0$, $ax+by+c>0$, $ax+by+c \leq 0$ and $ax + by + c \geq 0$ are linear inequalities

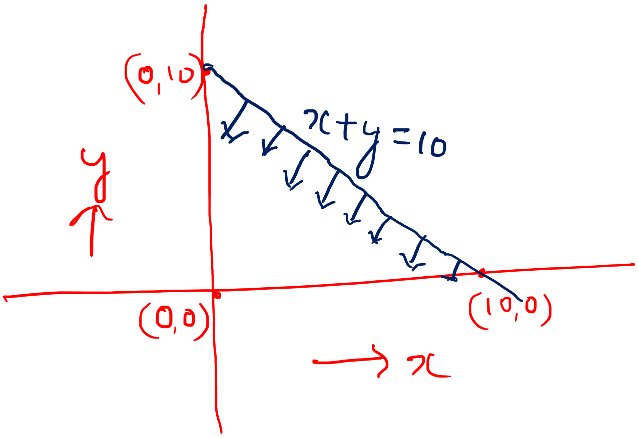
To show a given inequality using graph:

Note :-
1. We use arrows either above the line or below the line depending upon the nature of the inequality.

2. If the coefficient of y is positive then for lesser than inequalities the arrows should be marked below the line &. for greater than inequalities the arrows should be marked above the line.

3. If the coefficient of y is negative then for lesser than inequalities the arrows should be marked above the line & for greater than inequalities the arrows should be marked below the line.

Example1:



The inequality $x+y \leq 10$ Can be shown as

Let us first consider the Line

$x+y = 10$
To find x intercept put y=0

ie $x + 0 = 10$
$x = 10$

:. X intercept is (10,0)

To find y' intercept put x=0
ie $0+ y = 10$
$y = 10$

:. Y intercept is (0,10)

Note:-
In the given inequality $x+y \leq 10$, 'y' coefficient is positive.
:. $\leq$ is shown below the line.

Example 2

The inequality $3x - 4y \leq 12$ can be Shown as

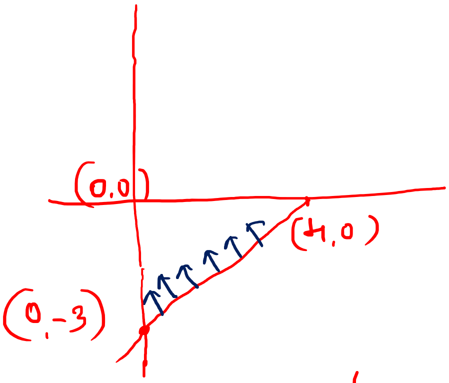
Consider $3x-4y=12$

put $x=0$ $\Rightarrow -4y=12$ $\Rightarrow (0,-3)$

$\Rightarrow y=-3$

put $y=0$ $\Rightarrow 3x=12$ $\Rightarrow (4,0)$

$\Rightarrow x=4$



Since y is -ve the arrows are shown above the line

Common region for the given inequalities.

Note:

- If there exists a Common region for the given inequalities then it is said to be feasible, otherwise infeasible.
- If the common region is within a boundary then it is said to be feasible & bounded Other wise feasible and unbounded.

Example1:-

Mark the inequalities Common region for the inequalities

$x + y \leq 10$ & $x + y \geq 5$.

Consider $x+y=10$

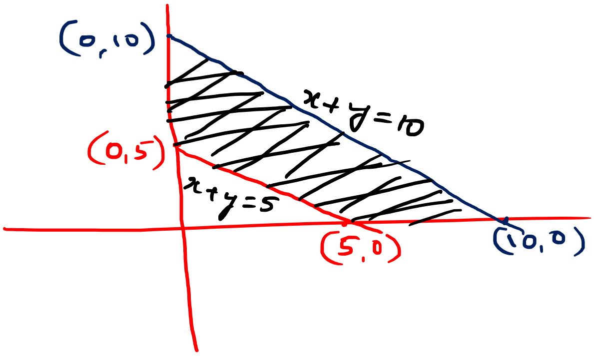
put $x=0$ $\Rightarrow y=10$ } $(0,10)$

put $y=0$ $\Rightarrow x=10$ } $(10,0)$

Consider $x+y=5$

put $x=0$ $\Rightarrow y=5$ $\Rightarrow (0,5)$

put $y=0$ $\Rightarrow x=5$ $\Rightarrow (5,0)$



Example 2

Mark the common region for the inequalities 2x + 3y ≥ 6 and 3x – y ≥ 0

Consider $2x+3y=6$

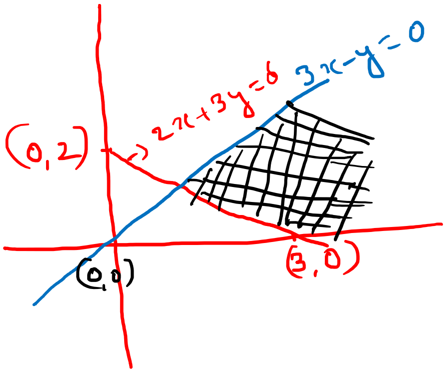
Put $x=0 \Rightarrow y=2 \Rightarrow (0,2)$

put $y=0 \Rightarrow x=3 \Rightarrow (3,0)$

Consider $3x-y=0$

Put $x=0 \Rightarrow y=0 \Rightarrow (0,0)$

=> The line passes through origin



Problems on solution of Linear inequalities

Note :-

When an inequality is multiplied by a negative sign the ‘<’ change to ‘>’ & ‘>’ changes to ‘<’.

Example 1:

The solution of the inequality $\frac{(7-3x)}{2} \leq \frac{x}{4} - 3$

$$\frac{7-3x}{\cancel{2}} \leq \frac{x-12}{\cancel{4}\ 2}$$

$$14-6x \leq x-12$$

$$26 \leq 7x$$

$$\frac{26}{7} \leq x$$

$$\Rightarrow x \geq \frac{26}{7}$$

or Solution for $x \Rightarrow \left(\frac{26}{7}, \infty\right)$

Example 2.

The Solution of the inequality 6x+8 < 10x+16

$$6x+8 < 10x+16$$

$$-8 < 4x$$

$$-2 < x$$

$$\Rightarrow x > -2$$

or $(-2, \infty)$

Example 3

Find the range of real values of x Satisfying the inequalities 3x-3>12 and 4x - 6 > 10

Consider $3x - 3 > 12$

$3x > 15$

$x > 5$

Consider $4x - 6 > 10$

$4x > 16$

$x > 4$

∴ the solution that satisfies both inequalities is

$x > 5$

Word problems on Inequalities

Note :-

1) For time we shall use '≤'
2) For work we shall use '≥

Example 1

On an average an experience Person does 5 units of work while a fresh person does 3 units of work daily. The employer has to maintain an output of at least 30 units of work per day. This Situation can be expressed as.

Let the number of experienced persons be x

& the number of fresh persons be y

$\Rightarrow$ $5x + 3y \geq 30$ (from the given information)

Example 2

The rules & regulations demand that the employer should employ not more than 5 experienced hands to every fresh hand. This can be represented as.

Given the ratio of experienced to fresh hands shouldn't be more than 5

$\Rightarrow$ $\frac{x}{y} \leq 5$ (By letting $x \rightarrow$ no. of experienced hands)

$y \rightarrow$ no. of fresh hands

$\Rightarrow \frac{x}{5} \leq y \Rightarrow y \geq \frac{x}{5}$

Example 3

The rules & regulations forbids the employer to employ less than 2 experienced hands to every fresh hand. This can be represented as.

Since the rules and regulations forbids the employer to choose less than 2 experienced hands to every fresh hand

∴ the employer has to employ atleast 2 experienced hands to every fresh hand

ie $\frac{x}{y} \geq 2 \Rightarrow \frac{x}{2} \geq y \Rightarrow y \leq \frac{x}{2}$

Problems on boundary points

Example

On solving the inequalities 2x+5y≤20 , 3x+2y ≤12 , x ≥0 , y ≥0 , we get the following situation

a)(0,0) , (0,4) ,(4,0) and (20/11 , 36/11) b)(0,0), (10 ,0),(0,6) and (20/11 , 36/11)

c) (0,0) , (0,4) ,(4,0) and (2,3) d) (0,0), (10 ,0),(0,6) and (2,3)

Consider

$2x+5y=20$

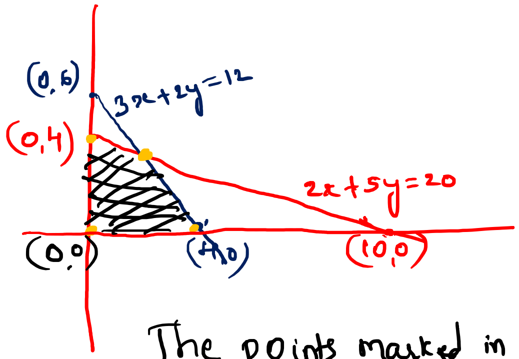
put $x=0 \Rightarrow y=4 \Rightarrow (0,4)$

put $y=0 \Rightarrow x=10 \Rightarrow (10,0)$

Consider $3x+2y=12$

put $x=0 \Rightarrow y=6 \Rightarrow (0,6)$

put $y=0 \Rightarrow x=4 \Rightarrow (4,0)$



The points marked in yellow are boundary points

One of the unknown boundary points is obtained by solving the 2 lines $2x+5y=20$, $3x+2y=12$ ie $x=\frac{20}{11}$, $y=\frac{36}{11}$

Therefore the answer is option a

Practice questions:

1. The common region represented by the following in equalitiesL1 = X1 + X2 $\leq$ 4; L2 = 2X1 + X2 $\geq$ 6 (June 2019)

A) Triangle ABE b) Outside the line EB c) OAED d) none

2.On Solving the Inequalities $5x + y \leq 100$, $x + y \leq 60$, $x \geq 0$, $y \geq 0$, we get the following situation:

(a) (0, 0), (20, 0), (10, 50) & (0, 60)(b) (0, 0), (60, 0), (10, 50) & (0, 60)

(c) (0, 0), (20, 0), (0, 100) & (10, 50) (d) None of these

3. solutions of the set of inequations $2x+y \geq 12$, $5x + 8y \geq 74$, $x + 6y \geq 24$, $x \geq 0$, $y \geq 0$ are

(a) (24,. 0), (126/11, 23/11) , (2,8) , (0 ,12)	(b)	(0, 24), (2, 8), (0, 12),
(c) (8, 4), (2, 8), (0, 12), (0, 24)	(d)	(8,4), (0, 0) (0,6) (2,0)

4. The solution set of the inequalities $x+2>0$ and $2x-6>0$ is

a) $x>3$ b)$x>-2$ c)$x<-2$ d) $x<-3$

5. The union forbids employer to employ less than 2 experienced person to each fresh person. This situation can be expressed as

(a) $x \leq y/2$ (b) $y \leq x/2$ (c) $y \geq x/2$ (d) $x > 2y$

6. An employer recruits experienced (x) and fresh workmen (y) for his firm under the condition that he cannot employ more than 9 people. x and y can be related by the inequality

$x + y = 9$ (b) $x + y \leq 9$ $x \geq 0, y \geq 0$ (c) $x + y \geq 9$ $x \geq 0, y \geq 0$ (d) none of these

7. On the average experienced person does 5 units of work while a fresh one 3 units of work daily but the employer has to maintain an output of at least 30 units of work per day. This situation can be expressed as

(a) $5x + 3y \leq 30$ (b) $5x + 3y > 30$ (c) $5x+3y \geq 30$ $x \geq 0, y \geq 0$ (d) none of these

8.The rules and regulations demand that the employer should employ not more than 5 experienced hands to 1 fresh one and this fact can be expressed as

a) $y \geq x/5$(b) $5y \leq x$ (c) $5y \geq x$ (d) none of these

Chapter 4- Mathematics of Finance

Time value of Money :

Money has value with respect to time. i.e the value of money changes with respect to time.

If we consider the worth of Rs 10000 today & the worth of Rs 10,000 after 3 years, then the worth of Rs 10,000 is more than the worth of Rs 10,000 after 3 years. as the value of money decreases with time due to inflation.
:. Keeping money with us for longer duration is not advisible as it is not safe & also the value decreases due to time. So the concept of investment was introduced to provide the Compensation (along with little profit) for the reduction in the value of money.

The first of this investment schemes was based on Simple Interest.

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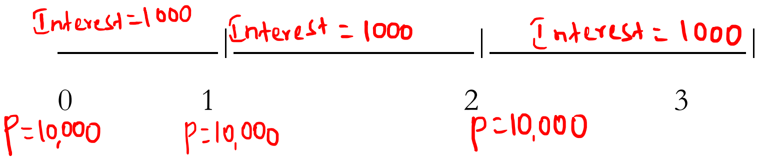
Simple Interest (SI),

Here the Sum of money invested (principal) doesn't change

Consider an example of Rs 10,000 invested for 3 years at SI rate of 10% pa.



From the above figure we can Observe that the principal (p) at the beginning is 10,000 & the SI obtained for the first year is 1000 (10% of 10,000).
At the end of 1st year the principal (P) = 10,000 (as it remains same) and SI for the Second year is 1000 (10% of 10,000 & also so on.

- The total SI accrued for 3 years is 1000 +1000 + 1000 = 3000 and Amount received at the end of 3 years = 10,000+ 3000
  = 13,000

Formula for SI

SI = $PTR/100$

Where P → principal

T → Time (in years)

R → SI rate pa (per annum)

Amount

$$\boxed{\text{Amount (A)} = P + SI}$$

Problems on SI

Type 1:-

To find SI and Amount when P, T & R are given.

Example 1:

P=25000 R=13.5 % and T= 2 years 8 months. Find SI and amount

Given P= 25000 T= 2 years 8 months R = 13.5%

W.K.T SI = PTR%

Since Time is given interms of years & months, we shall type time first i.e

$$\underbrace{\left(\frac{8}{12}+2\right)}_{\text{Time}} \times \underset{P}{\underset{\downarrow}{25,000}} \times \underset{R}{\underset{\downarrow}{13.5}}\,\% = 9000$$

Amount = P+SI = 25000+9000=34000

Example 2:

P=30000 R=8.5 % and T= 6 years 3 months. Find SI

Given P= 30,000 R=8.5% T= 6 years 3 months

SI = PTR%

In calculator, SI = $\left(\frac{3}{12}+6\right) \times 30,000 \times 8.5\%$

$= 15937.5$

Type 2:-

To find P, when A , T and R are given

Example1 :

Find the sum of money which amounts to Rs.3720 in 4 years at 6% pa SI

Given A = 3720 T=4 R = 6%

Short cut to find P when A.T & R are given

$$P = \frac{A}{1+TR\%}$$

$$= \frac{3720}{1+(4\times 6\%)}$$

$$= \underline{\underline{3000}}$$

Example 2

A Certain sum of money amounts to Rs. 650 in 6 years at 5% pa SI. Find the sum

$A = 650$. $T = 6$ $R = 5\%$

$$P = \frac{A}{1 + TR\%} = \frac{650}{1 + (6 \times 5\%)}$$

$$P = \underline{\underline{500}}$$

Example 3

A Certain sum of money amounts to Rs. 15,120 in 4 years at 6.5% pa SI. Find the sum

Given $A = 15{,}120$ $T = 4$ $R = 6.5\%$ $P = ?$

$$P = \frac{A}{1 + TR\%}$$

$$= \frac{15120}{1 + (4 \times 6.5\%)}$$

$$= \underline{\underline{12000}}$$

TYPE-3

a)To find R. when A, P&T are given.

b) To find T, when A , P & R are given

Example 1

Rs 8000 amounts to Rs 10000 in 2 years at a certain SI rate, Find the SI rate pa.

Given $P = 8000$ $A = 10{,}000$ $T = 2$

Conventional method

W.K.T $A = P + SI$

$$A = P + \frac{PTR}{100}$$

$$10{,}000 = 8000 + \frac{8000 \times 2 \times R}{100}$$

$$2000 = \frac{160\cancel{0}\cancel{0} \times R}{1\cancel{0}\cancel{0}}$$

$R = 12.5\%$

Short cut $R = \left(\frac{A-P}{PT}\right) \times 100 = 12.5\%$

Example 2

In what time Rs 15000 amounts to Rs 18000 at 5% pa SI.

Given $P = 15000$ $A = 18000$ $R = 5\%$ $T = ?$

$T = \left(\frac{A-P}{PR}\right) \times 100$ (Short cut)

$= \left(\frac{18000 - 15000}{15000 \times 5}\right) \times 100$

$T = 4$ years

Example 3

At what SI rate per annum will Rs 6000 amounts to Rs.9780 in 7 years

Given $P = 6000$ $A = 9780$ $T = 7$ $R = ?$

$R = \left(\frac{A-P}{PT}\right) \times 100 = \left(\frac{9780 - 6000}{6000 \times 7}\right) \times 100$

$= \underline{\underline{9\%}}$

Type 4

→ When 2 amounts (in terms of principal) and Time in the first Case are given then to find the time in the second case.

Example 1

A Certain sum of money doubles itself in 6 year as per SI. In what time it becomes 4 times of itself.

Given $A_1 = 2P$ $A_2 = 4P$ $T_1 = 6$ $T_2 = ?$

Conventional method

$A_1 = P + SI$

$2P = P + \frac{PT_1R}{100}$

$\cancel{P} = \frac{\cancel{P}\times 6\times R}{100}$

$R = \frac{100}{6}\%$

$A_2 = P + SI$

$4P = P + \frac{PT_2R}{100}$

$3\cancel{P} = \frac{\cancel{P}\,T_2\times\cancel{100}}{6\times\cancel{100}}$ $\left[\because R = \frac{100}{6}\right]$

$T_2 = 18$ years.

Short cut $T_2 = \frac{A_2-1}{A_1-1}\times T_1$ We shall sub $A_1 = 2$, $A_2 = 4$

$= \frac{4-1}{2-1}\times 6 = 3\times 6 = 18$ years

Example 2:

A certain sum of money becomes 4 times of itself in 15 years as per SI.
In what time it become 8 times of itself

Given $A_1 = 4$ $T_1 = 15$ $A_2 = 8$ $T_2 = ?$

$$T_2 = \frac{A_2-1}{A_1-1}\times T_1$$

$$= \frac{8-1}{4-1}\times 15$$

$$= \frac{7}{3}\times 15$$

$$= 35 \text{ years}$$

Example 3:

A certain sum of money becomes 3.5 times of itself in 10 years as per SI.
In what time it become 8.5 times of itself

Given

$A_1 = 3.5$ $T_1 = 10$ $A_2 = 8.5$ $T_2 = ?$

$$T_2 = \frac{A_2 - 1}{A_1 - 1} \times T_1$$

$$= \frac{8.5 - 1}{3.5 - 1} \times 10$$

$= 30$ years

Type 5:-

When 2 Amounts (interms of Rs) and time durations are give then to find P & SI rate pa

Note :- SI rate pa = $\frac{\textit{SI for 1 year}}{P}$ **x 100**

Example 1

A certain sum of money amounts to Rs 6900 in 3 year & Rs 7500 in 5 years as per SI. Find the sum and SI rate Pa.

Given $A_1 = 6900$ $T_1 = 3$ $A_2 = 7500$ $T_2 = 5$

$P = ?$ $R = ?$

Conventional method

$A_1 = P + SI_1$ $A_2 = P + SI_2$

6900 = P + SI for 3 years → ① 7500 = P + SI for 5 years → ②

② − ① ⇒ 7500 − 6900 = SI for 2 years

SI for 1 year = 300

① ⇒ 6900 = P + (3 × 300)

$P = \underline{6000}$

$$R = \frac{\text{SI for 1year}}{P} \times 100 = \frac{300}{6000} \times 100$$

$= 5\%$

Example 2

A Certain Sum of money amounts to Rs 26,400 in 4 years and Rs 31,200 in 7 years as per SI. Find the Sum and SI rate pa.

Short cut

$$P = \left| \frac{A_2 - A_1}{T_2 - T_1} \times T_1 - A_1 \right|$$

$$= \left| \frac{31200 - 26400}{7 - 4} \times 4 - 26400 \right|$$

$$P = 20000$$

$$\text{SI for 1year} = \frac{A_2 - A_1}{T_2 - T_1} = 1600$$

$$R = \frac{1600}{20000} \times 100 = 8\%$$

Example 3

A Certain Sum of money amounts to Rs 692 in 2 years and Rs 800 in 5 years as per SI. Find the Sum and SI rate pa.

$$P = \left| \frac{A_2 - A_1}{T_2 - T_1} \times T_1 - A_1 \right|$$

$$= \left| \frac{800 - 692}{5 - 2} \times 2 - 692 \right|$$

$$= 620$$

$$\text{SI for 1year} \Rightarrow \frac{A_2 - A_1}{T_2 - T_1} = 36$$

$$R = \frac{36}{620} \times 100 = 5.8\%$$

Type 6:

when SI rate pa are different for different years. Then to find SI

(i) when principal is same

Total SI $= \frac{P}{100}(T_1R_1 + T_2 R_2 + T_3 R_3 + \ldots\ldots\ldots)$

ii) when principal are different

Total SI $= \frac{P_1T_1R_1 + P_2T_2 R_2 + --}{100}$

Example 1

Mr. X invested Rs 30000 at 7% pa SI for first 3 years, 8% Pa SI for next 2 years, and 10% pa for the next 2 years. Find the SI accrued..

$P = 30,000$ $T_1 = 3$ $R_1 = 7\%$

$T_2 = 2$ $R_2 = 8\%$

$T_3 = 2$ $R_3 = 10\%$

$$\text{Total SI} = \frac{P}{100}(T_1R_1 + T_2R_2 + T_3R_3)$$

$$= \underline{\underline{17100}}$$

Example 2

Out of Rs 1,00,000 Mr. Y invested $\frac{1}{5}$th of the money at 5% pa SI for 4 years, ½ of the money at 6% pa SI for 3 years & remaining at 8% pa SI for 2 year. Find the for SI accrued.

Solution: $P_1 = 1/5 \times 10000 = 20000$, $T_1 = 4$ $R_1 = 5\%$

$P_2 = 1/2 \times 10000 = 50000$ $T_2 = 3$ $R_1 = 6\%$

$P_1 = 30000$*(remaining amount)* $T_3 = 2$ $R_1 = 8\%$

$$\text{Total SI} = \frac{P_1T_1R_1 + P_2T_2R_2 + P_2T_2R_2}{100}$$

$= 17800$

Type 7

To find 'T when an amount (in terms of P) & SI rate pa are given.
Formula

$$T = \frac{\text{Total SI rate}}{\text{SI rate pa}}$$

Note:

For this type of Problems P is taken as 100%

The Total SI rate is always 100%. less than amount.

If A=2P=200%
Then the total SI rate is 100%.

If A= 4P = 400%
Then the total SI rate = 300%..

Example 1:

A certain sum of money triples itself at 5% pa SI. Find the time taken for the same.

Given $A = 3P = 300\%$

Total SI = 200% (as it is 100% less than A)

$$T = \frac{\text{Total SI}}{R} = \frac{200}{5} = 40 \text{ years}$$

Example 2:

The SI on a certain sum becomes 0.125 times of itself at 6.25% pa. Find the time taken

Total SI $= 0.125 \times P = 12.5\%$

$R = 6.25\%$

$$T = \frac{\text{Total SI}}{R} = \frac{12.5}{6.25} = 2 \text{ years}$$

Example 3:

A certain sum of money 5 times of itself at 16% pa SI. Find the time taken for the same.

$A = 5P = 500\%$

Total SI = 400%

$R = 16\%$

$$T = \frac{\text{Total SI}}{R}$$

$$= \frac{400}{16} = 25 \text{ years}$$

Example 4:

The SI on a certain sum becomes 9/16 times of itself at certain rate of SI. Find the SI rate pa if R=T.

Total SI $= \frac{9}{16} \times P = \frac{900}{16}$

Given $T = R$ To find R

$R = \frac{\text{Total SI}}{T}$

$R^2 = \text{Total SI} \quad [\because R = T]$

$R = \sqrt{\text{Total SI}} = \sqrt{\frac{900}{16}}$

$= \underline{\underline{7.5\ \%}}$

Type 8:

Problems on two SI's

Example 1

The SI on Rs 1000 at 6% pa for a certain time duration is 30 rupees more than SI on the same sum at 5% pa for the same time duration. Find the time duration.

Given $SI_1 - SI_2 = 30$

$\frac{PT}{100}(R_1 - R_2) = 30$

$1000 \times T (6 - 5) = 3000$

$T = \underline{\underline{3 \text{ years}}}$

Example 2

The SI on a on certain sum at 6% pa 4 years is 450 rupees for more than the SI on same sum at 5% pa for 3 years.
Find the sum.

Given

$SI_1 - SI_2 = 450$

$\frac{P}{100}(T_1 R_1 - T_2 R_2) = 450$

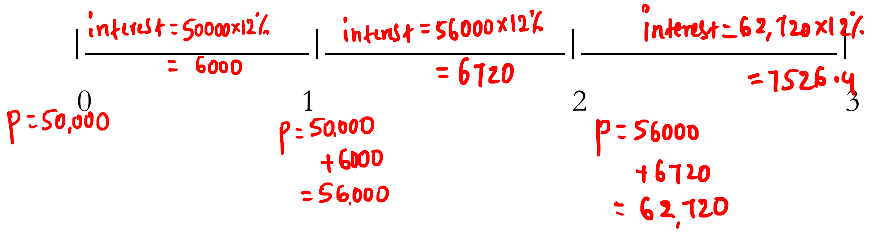
$P\left[(4 \times 6) - (3 \times 5)\right] = 45000$

$P = \underline{\underline{5000}}$

Compound Interest (CI)

Note: P changes at every compounding

Consider an example of Rs 50,000. invested for 3 years at CI rate of 12% at pa Compounded annually



Total Interest for 3 years = 6000 + 6720 + 7526.4

= 20.246.4

Total amount = P + Interest = 50000 + 20,246.4

= 70,246.4

Formula for CI

$CI = P\,[(1 + i)^n - 1\,]$

Where,

P → principal.

i → interest rate (fractional value)

n → no. of Compoundings or number of Conversion periods

Formula for amount

$A = P + CI$

$A = P + P\,[(1+i)^n - 1]$

$= P+P\,(1 + i)^n - P$

$\underline{\underline{A = P(1+i)^n}}$

This formula is also referred as appreciation formula.

Types of compoundings & Corresponding Conversions

(i) Annual (yearly) Compounding

→P changes once in a year.

n = no. of years
i = interest rate p.a.

(ii) Semi annual (half yearly) Compounding

→P. changes twice in a year
n = 2 x no. of years

i = $\frac{interest\ rate\ pa}{2}$

(iii) Quarterly compounding

→P changes 4 times in a year.

n = 4 X No. of years
i = $\frac{interest\ rate\ pa}{4}$

(iv) Monthly compounding

→P changes 12 times in a year.

n =12 x no. of years

i = $\frac{interest\ rate\ pa}{12}$

Problems on CI

Type 1

Problems on CI &.A. (when P. i & n are given)

Example 1

Find the CI & A if

P = 30,000 i= 12% pa time = 3 years

Compounded (i) Annually

(ii) Semi-annually
(iii) Quarterly
(iv) Monthly

Given $P = 30{,}000$ $i = 12\% = 0.12$ pa
Time = 3 years

(i) Annual Compounding

$n = 3$ $i = 0.12$

$CI = P[(1+i)^n - 1]$
$= 12147.84$
$A = P + CI = 42147.84$

(ii) Semi Annual

$n = 2 \times 3 = 6$ $i = \frac{0.12}{2} = 0.06$

$CI = P[(1+i)^n - 1]$
$= 12555.57$
$A = 42555.57$

(iii) Quarterly compounding

$n = 4 \times 3 = 12$ $i = \frac{0.12}{4} = 0.03$

$CI = P[(1+i)^n - 1] = 12772.82$
$A = 42772.82$

(iv) Monthly Compounding

$n = 12 \times 3 = 36$ $i = \frac{0.12}{12} = 0.01$

$CI = P[(1+i)^n - 1] = 12923.06$
$A = \underline{42923.06}$

Example2

A time by which a sum of money would double itself at 5% pa CI is

a)14.2 years b)14 years c)12 years d)15

$i = 0.05$. Annual Comp (By default)

$A = 2P$ (Given)

$A = P(1+i)^n$
$2\cancel{P} = \cancel{P}(1.05)^n$
$2 = (1.05)^n$

By option hitting method.
we find that for $n = 14$
$(1.05)^{14} = 1.98$
For $n = 15$ $(1.05)^{15} = 2.07$
∴ the correct answer is 14.28 years

Example3:

In what time will Rs.1,00,000 amount to Rs.1,26,677 at 6% per annum, when the interest is compounded semi-annually?(Given :$(1.03)^8 = 1.26677$]

(a) 8 years (b) 4 years (c) 6 years (d) none

Given $(1.03)^8 = 1.26677$

to

$(1+i)^n$

On Comparing $n = 8 = 2 \times$ no. of years

(∵ semi annual comp)

$\Rightarrow$ no. of years $= \frac{8}{2} = \underline{\underline{4}}$

Example4:A sum of money compounded annually becomes Rs.8450 in two years and Rs.10985 in three years. Find the rate of interest per annum.

(a) 30% (b) 20% (c)40% (d) 10%

Given $A_1 = 8450$ $n_1 = 2$ years

$A_2 = 10985$ $n_2 = 3$ years

$A_1 = P(1+i)^{n_1}$ $A_2 = P(1+i)^{n_2}$

$8450 = P(1+i)^2 \rightarrow ①$ $10985 = P(1+i)^3 \rightarrow ②$

$\frac{②}{①} \Rightarrow \frac{10985}{8450} = \frac{\cancel{P}(1+i)^3}{\cancel{P}(1+i)^2} = (1+i)$

$1.3 = 1+i$

$i = 0.3 \Rightarrow 30\%$

Short cut (for difference b/w time periods = 1 year)

$i = \frac{A_2}{A_1} - 1 = \frac{10.985}{8450} - 1 = 0.3 \Rightarrow 30\%$

Note:- for difference b/w time Periods = 2 years

$i = \sqrt{\frac{A_2}{A_1}} - 1$

Note: If the difference between the time periods is more than 1 year, then the short cut is

$$\frac{A_2}{A_2} = (1 + option)^{n_2 - n_1}$$

Example5:

A sum of money compounded annually becomes Rs.17280 in two years and Rs.20736 in three years. Find the rate of interest per annum.

(a) 30% (b) 20% (c)40% (d) 10%

Given $A_1 = 17280$ $A_2 = 20736$

$$i = \frac{A_2}{A_1} - 1 = \frac{20736}{17280} - 1$$

$$= 1.2 - 1$$

$$i = 0.2 \Rightarrow 20\%$$

Type 2

To find A when P, different rate of interests for different years are given..

Formula

$A=P(1+i_1)(1+i_2)(1+i_3)$……

Example:-

The present population of a town is 70,000. If it grows at 4%, 5% & 6% pa for 1st, 2nd & 3rd year respectively. then find the population of the town at the end of 3rd year.

Given $P = 70000$ $i_1 = 4\% = 0.04$

$i_2 = 5\% = 0.05$ $i_3 = 6\% = 0.06$

$A = ?$

$$A = p(1+i_1) \times (1+i_2) \times (1+i_3)$$

$$= 70000\,(1.04) \times (1.05) \times (1.06)$$

$$= \underline{\underline{81026}}$$

Alternatively

$$A = p + i_1\% + i_2\% + i_3\% = 81026$$

Type 3

When 2 amounts (in terms of P) and time period of first Case is given then to find time period for the 2nd case.

Example 1

A Certain sum of money doubles itself in 4 years as per CI. In how many years it will become 16 times of itself.

$n_1 = 4$ years

$n_2 = ?$

Conventional Method

$A_1 = P(1+i)^{n_1}$

$2P = P(1+i)^4$

$2 = (1+i)^4 \rightarrow ①$

$A_2 = P(1+i)^{n_2}$

$16P = P(1+i)^{n_2}$

$16 = (1+i)^{n_2} \rightarrow ②$

$2^4 = (1+i)^{n_2}$

$[(1+i)^4]^4 = (1+i)^{n_2}$ [∵ from ①]

$(1+i)^{16} = (1+i)^{n_2}$

$\therefore \underline{\underline{n_2 = 16}}$

Short cut

$A_2 = A_1^T$

$\boxed{n_2 = T \times n_1}$

where T is the value such that

i.e $16 = 2^4 \Rightarrow T = 4 \therefore n_2 = T \times n_1 = 4 \times 4 = 16$

Example 2

A Certain Sum of money becomes 8 times of itself in 12 year as per CI. In how many year it would become 128 times of itself.

$A_2 = 128 \quad A_1 = 8$

∵ A_2 is the direct power of A_1

we shall express both amounts interms of powers of a common no.

ie 8 times in 12 years $\Rightarrow 2^3 \Rightarrow 12$

$\therefore n_1 = \frac{12}{3} = 4$ years (Time for double)

Now $A_2 = 128 = 2^7$

$T = 7$

$\therefore n_2 = T \times n_1$

$= 7 \times 4$

$= 28$ years

Effective rate of interest (ERI)

It is the rate of interest Calculated for 1 year considering the given. type of compounding so that we can compare it with nominal rate of interest (NRI)

ERI=NRI for annual Compounding.

ERI > NRI for semi-annual quarterly and for monthly compounding

Formula for ERI

$ERI = (1+i)^n - 1$

$\% ERI = [(1+ i)^n – 1] X 100$

Note:-

ERI is independent of principal..

Example1 :-

Find the Effective rate of interest corresponding to a nominal interest rate of 12% pa Compounded monthly

Given: Monthly Compounding

$$i = \frac{0.12}{12} = 0.01$$

$$n = 12 \times 1 = 12$$

$$\therefore ERI = \left[(1+i)^n - 1 \right] \times 100$$

$$= 12.68\%$$

Example2:

The effective rate equivalent to nominal rate of 8% compounded quarterly is :

Given: Quarterly compounding

$$n = 4 \times 1 = 4$$

$$i = \frac{8\%}{4} = 0.02$$

$$\therefore ERI = \left[(1+i)^n - 1 \right] \times 100$$

$$= 8.24\%$$

Depreciation:

The value of assets like machinery, furniture, buildings etc. will reduce with time Such assets are said to be depreciating assets.

For depreciating assets the Future value (A) will be less than the present value (P).
i,e $A<P$.

Formula

$A = P\,(1- i)^n$

where, A → Future value or Scrap value

P → present value or Cost price

Example 1:

A machine with cost price of Rs 3,00,000 depreciates at 10% pa Find its scrap value if the life of the machine is 6 years.

Given $P = 3,00,000$

$n = 6$ $i = 10\% = 0.1$

For Depreciation

$A = P(1-i)^n$

$= 3,00,000\,(1-0.1)^6$

$= 1,\underline{59432.3}$ (scrap value)

Example 2:

A machine depreciates at 20% of its value at the beginning of the year. The cost and scrap value realized at the time of sale being ₹12000 and ₹3145.728 respectively. For how many years the machine can be put into service.

a)5 years b) 7 years c)6 years d)none

Given $i = 20\% = 0.2$

$P = 12000$ $A = 3145.728$

$A = P(1-i)^n$

$3145.728 = 12000\,(1-0.2)^n$

$\frac{3145.728}{12000} = (0.8)^n$

$0.2621 = (0.8)^n$

By option hitting

$n = 6$

ie $(0.8)^6 = 0.\underline{2621}$

Problems on SI and CI

Example 1:

The difference between SI and CI for 2 years on a certain sum is Rs.25.2 at 6% pa.

Find the sum

Given Difference b/w C.I & SI for 2 years = 25.2

$i = 6\% = 0.06$

$CI - SI = 25.2$

$P[(1+i)^n - 1] - PTR\% = 25.2$

$P[(1+i)^n - 1 - TR\%] = 25.2$

$P[(1.06)^2 - 1 - 2 \times 6\%] = 25.2$

$P = \underline{\underline{7000}}$

Short cut for difference b/w CI & SI problems

We shall use $P[(1+i)^n - 1 - TR\%] = Diff$

One can get the answer in one step using memory Calculation.

Example 2:

The difference between SI and CI for 2 years on a certain sum is Rs.64.8 at 9% pa.

Find the sum

Given Difference = 64.8 $i = 9\%$

Time = 2 years

$P[(1+i)^n - 1 - TR\%] = Diff$

Using memory Calculation

$P = \underline{\underline{8000}}$

Example 3

The difference between SI and CI for 2 years on a certain sum is Rs.172.8 at 12% pa.

Find the sum

Given Difference = 172.8 $i = 12\%$

Time = 2 years

$P[(1+i)^n - 1 - TR\%] = $ Difference

$P = \underline{12000}$

Annuity

It is a fixed amount to be paid regularly for specified number of periods.

Types of Annuity

(i) Annuity Regular (ordinary Annuity)

It is a fixed amount to be paid regularly at the end of each year / half year | quarter month, for specified Number of periods
Ex:- Rent, EMI etc

(ii) Annuity Immediate (Annuity due)

It is a fixed amount to be paid regularly at the beginning of the of year / half year / quarter / month for specified number of periods.
Ex- RD, Insurance premiums, etc

Future value an annuity :-

If a is the fixed amount paid regularly for 'n' number periods at rate of 'i' % per annum. then its future value
a) For annuity Regular

Future value of Annuity Regular (FV of AR) is = $\frac{a[(1+i)^n-1]}{i}$

b) For annuity Due (AD)

FV of A.D = $\frac{a[(1+i)^n-1]}{i}$ x (1+i)

Relationship between Future value & present value

$FV = PV(1+i)^n$ or $PV = \frac{FV}{(1+i)^n}$

Present value of an annuity

If 'a' is the fixed amount to be paid regularly for 'n' number of periods at 'i' % pa. then
(a) present value of annuity regular

$$PV \text{ of } AR = a\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$$

b) Present value of Annuity due

PV of AD = a $\left[\frac{(1+i)^{n-1}-1}{i(1+i)^{n-1}}\right]$ + a

Default annuity is A.R (If not mentioned)

Problems on FV, PV & relationship between FV & PV

Example 1:

Rs 5000 is invested every year at 12% pa for 10 years. Find its Future value.

Given $a = 5000$ $i = 12\% = 0.12$ $n = 10$

It is Annuity Regular (AR) by default

Fv. of A.R = $a \left[\frac{(1+i)^n - 1}{i}\right]$

$= 87743.67$

Example2:

Mr. Bhuvan invests Rs 10000 at the beginning of every year for the next 11 years. Find the PV of this annuity. If the interest rate pa is 8% pa CI.

Given $a = 10.000$ & Annuity due (AD)

$n = 11$ $i = 8\% = 0.08$

PV of A.D = $a \left[\frac{(1+i)^{n-1} - 1}{i(1+i)^{n-1}}\right] + a$

By doing memory calculation in Calculator

We get P.V of A.D = $\underline{77100.81}$

Example 3:

If the Refrigerator Costs Rs 40,000 today. Then what would be its Cost at the end of 2 year if the rate of interest is 6% pa.

Given $PV = 40000$. $n = 2$ $i = 6\% = 0.06$

$FV = ?$

W.K.T $FV = PV(1+i)^n$

$= 40000(1.06)^2$

$= \underline{\underline{44944}}$

Example 4:

If a car costs Rs 20,00,000 after 10 years. Then what would be its today's cost if the rate of interest is 7% pa

Given $FV = 20.00.000$ $n = 10$ $i = 7\% = 0.07$

$PV = ?$

$$PV = \frac{FV}{(1+i)^n} = \frac{20.00000}{(1.07)^{10}}$$

$= 10,16698.58$

Sinking fund

→ It is the fixed amount to be kept apart (as an investment) from the Income or profit in order to accumulate a lump sum amount required for the future.

Example1

Arun wishes to buy a machine after ten years, whose estimated cost is Rs 5,00,000. If the rate of interest for his investment is 8% pa. Find how much he has invest every year.

$n = 10$ FV of A.R $= 5.00.000$

$i = 8\% = 0.08$

$a = ?$

$$\text{FV of A.R} = a\left[\frac{(1+i)^n - 1}{i}\right]$$

$a = \underline{\underline{34514.74}}$ (By memory calculations)

Example2:

Karan wants to accumulate Rs.20 lakhs to purchase a plot at the end of 12 years. How must he has to invest every year at 12 % to achieve this goal

Given Fv of A.R = 20,00,000

$n = 12$ $i = 12\% = 0.12$

$$F.v \text{ of } A.R = a\left[\frac{(1+i)^n - 1}{i}\right]$$

$a = 82873.615$ (by memory calculations)

Applications of Annuity

Leasing :-

Leasing actually means renting.

If we have to make a decision between purchasing a machine & taking a machine on leasing basis.

Then we need to Compare their Present values.

i.e purchasing cost of a machine & pv of leasing.

If PV of leasing < PV of purchasing then leasing is preferred

If PV of leasing > PV of purchasing then purchasing is preferred

If PV of leasing = PV of purchasing then both are equally good

Example1:-

A machine can be obtained either by purchasing at Rs 75,000 or by leasing, at a rental of 22000 per year for the next 4 years (useful life of machine) If the rate of interest is 11 % pa.
Find whether the machine can be Purchased or not

Given

Pv. of purchase = 75000

$a = 22000$

$n = 4$ $i = 11\% = 0.11$

PV of leasing (A.R) $= a\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

$= 68253.8$

∵ PV of leasing < PV of purchase

∴ machine shouldn't be purchased as leasing is better.

Example2:-

A person wants to lease out a machine costing Rs. 8 lakhs for a period of 12 years at a annual rental of Rs. 1 lakh . Suppose the rate of interest is 12% pa, compounded annually. To whom this agreement is favourable? a) For lessee b) For lessor c) Not for both d) Can't be determined

Given

Pv of purchase = 800000

$n = 12$ $i = 12\% = 0.12$

$a = 1,00,000$

Pv of leasing (A.R) $= a\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

$= 619437.42$

∵ leasing is better => It is favourable for lessee

Investment Decision:

If PV of cash inflow (returns) ≥ PV of cash outflow (Investment), then the investment is good otherwise it's a loss.

Net present value (NPV)

NPV = PV of Cash inflow - PV of cash out flow

An Investment is said to be good

If NPV ≥ 0

Example1:-

A machine is purchased at Rs 80,000. If it Contributes Rs 20,000 every year. At 12% pa for 5 years. Is it a good investment?

Pv of cash outflow = 80000

$a = 20000$ $i = 12\% = 0.12$

$n = 5$

Pv of Cash inflow (A.R) $= a\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

$= 72095.52$

∵ PV of cash inflow < PV of cash outflow

∴ the machine shouldn't be purchased as it is a loss.

Example2:-

A machine can be purchased for Rs 65,000. Machine will contribute Rs.15,000 per year for next 6 years. Assume borrowing cost is 10% per annum. Determine whether machine should be purchased or not : (a) Should be purchased (b) Should not be purchased

(c) Can't say about purchase (d) None of the above

pv of cash outflow = 65000

$a = 15000 \quad n = 6 \quad i = 10\% = 0.1$

pv of Cash inflow (A.R) $= a\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

$= 65328.91$

Since pv of cash inflow > pv of cash outflow

$\Rightarrow$ Machine should be purchased

Valuation of bond:

A bond is a debt security in which the issuer owes the holder a debt & is obliged to repay the principal & interest. Bonds are generally issued for a fixed term longer than one year.

Example

An investor intends to purchase a 3 year Rs 3000 par value bond having nominal interest rate of 8% pa. At what price the bond may be Purchased now if it matures at par & investor requires a rate of return of 10% pa.

Par value => fv of the bond = 3000

n = 3

fv of each year int = 3000×8% = 240

Note :- 1) We shall consider nominal rate for calculating fv of interest. fv of interest is calculated on par value.

2) We shall consider rate of return for converting fv into pv.

$$\text{Purchasing cost of bond (pv)} = \frac{\text{fv of 1st year interest}}{(1+i)} + \frac{\text{fv of 2nd year interest}}{(1+i)^2} + \frac{\text{fv of 2nd year interest}}{(1+i)^3} + \frac{\text{par value}}{(1+i)^3}$$

$$= \frac{240}{(1.1)} + \frac{240}{(1.1)^2} + \frac{240}{(1.1)^3} + \frac{3000}{(1.1)^3} = 2850.78$$

Perpetuity

If is a fixed amount (receipt amount) paid regularly indefinitely.

Types of perpetuity

(1) Normal perpetuity (multi period perpetuity)

$$\text{PV of multiperiod perpetuity} = \frac{R}{i}$$

where

R → Fixed payment (Receipt amount)

i → interest rate.

2) Growing perpetuity

A stream of cash flow that grows at a constant rate forever is said to be a growing perpetuity.

$$\text{PV of growing perpetuity} = \frac{R}{i-g}$$

where. g is growth rate

Example:-

Find the present value of perpetuity of Rs 6000 payable monthly at 8 % p

Given R = 6000 pm => R = 6000×12

= 72000 pa

i = 8% = 0.08

$$\therefore \text{Pv of perpetuity} = \frac{R}{i} = \frac{72000}{0.08}$$

$$= \underline{\underline{900000}}$$

Compound annual growth rate (CAGR)

CAGR is of term used to describe the growth over a period of time of Some element of the business, for example the revenue, income etc.

$$\text{CAGR} = \left[\frac{V_n}{V_o}\right]^{\frac{1}{t_n - t_o}} - 1$$

$$\%\ \text{CAGR} = \left[\left[\frac{V_n}{V_o}\right]^{\frac{1}{t_n - t_o}} - 1\right] \text{x } 10$$

where

t_n → End period
t_o → Beginning period
V_n → value Corresponding t_n

V_o → Value corresponding t_o

Note: Students are required to do option hitting method using the formula as below

$$(1 + option)^{t_n - t_o} = \frac{V_n}{V_0}$$

Example1:-

Find CAGR

Year	**2013**	**2014**	**2015**	**2016**	**2017**
Revenue (in lakhs)	200	250	260	350	400

a)8.92% b)18.92% c)28.92% d)38.92%

Given : $V_n = 400$ $t_n = 2017$
$V_o = 200$ $t_o = 2013$
$t_n - t_o = 4$

W.K.T $(1 + CAGR)^{t_n - t_o} = \frac{V_n}{V_o}$

$(1 + option)^{t_n - t_o} = \frac{V_n}{V_o} = \frac{400}{200} = 2$

By option hitting option b => 18.92% = 0.1892

$(1.1892)^4 = 1.9999 \simeq \frac{V_n}{V_o}$

∴ Option (b) is the correct answer.

Example:-

Find CAGR

Year	2018	2019	2020	2021	2022
Income (in lakhs)	60	65	75	90	100

a)13.62%
b)18.62%
c)23.62%
d)33.62%

Given $t_0 = 2013$ $V_0 = 60$

$t_n = 2017$ $V_n = 100$

$\Rightarrow$ $t_n - t_0 = 4$

$$[1+\text{option}]^{t_n - t_0} = \frac{V_n}{V_0} = \frac{100}{60} = 1.6666$$

By option hitting method

option (a) 13.62% = 0.1362

ie $(1.1362)^4 = 1.\underline{666} = \frac{V_n}{V_0}$

$\therefore$ option (a) is the correct answer.

Practice questions

1. The future value of an annuity of Rs.5000 is made annually for 8 years at interest rate of 9% compounded annually (given that $(1.09)^8$= 1.99256) is _____
(a)Rs.55142.22 (b) Rs.65142.22 (c)Rs.65532.22 (d)Rs.57425.22

2. The effective annual rate of interest corresponding to nominal rate 6%p.a payable half yearly is (a) 6.06% (b) 6.07% (c) 6.08% (d) 6.09%

3. The cost of machinery is Rs.1,25,000/-. If it's useful life is estimated to be 20 years and the rate of depreciation of its cost is 10%p.a, then the scrap value of the machinery is [given that $(0.9)^{20}$ = 0.1215]

(a) Rs.15,187 (b) Rs.15,400 (c) Rs.15,300 (d) Rs.15,250

4. Mr. X invests 'P' amount at simple interest rate 10% and Mr. Y invests 'Q' amount at compound interest rate 5% compounded annually. At the end of two years both get the same amount of interest, then the relation between two amounts P and Q is given by:

(a) P = $\frac{41Q}{80}$ (b) P = $\frac{41Q}{40}$ (c) P = $\frac{41Q}{100}$ (d) P = $\frac{41Q}{200}$

5. If the difference of S.I and C.I is Rs.72 at 12% for 2 years. Calculate the amount.

(a) Rs.8,000 (b) Rs.6,000 (c) Rs.5,000 (d) Rs.7,750

6. If a simple interest on a sum of money at 6% p.a for 7 years is equal to twice of simple interest on another sum for 9 years at 5% p.a. The ratio will be:

(a) 2:15 (b) 7:15 (c) 15:7 (d)1:7

7. By mistake a clerk, calculated the simple interest on principal for 5 months at 6.5%p.a instead of 6 months at 5.5% p.a. If the error in calculation was Rs.25.40. The original sum of principal was____

(a) Rs.60,690 (b) Rs.60,960 (c) Rs.90,660 (d) Rs.90,690

8. If the simple interest on Rs.1,400 for 3 years is less than the simple interest on Rs.1,800 for the same period by Rs.80, then the rate of interest is:

(a) 5.67% (b) 6.67% (c) 7.20% (d) 5.00%

9. Nominal rate of interest is 9.9% p.a. If interest is compounded monthly, What will be the effective rate of interest [given$\left(\frac{4033}{4000}\right)^{12}$ = 1.1036 (approx)] ?

(a) 10.36% (b) 9.36% (c) 11.36% (d) 9.9%

10. The S.I on a sum of money is 4/9 of the principal and the number of years is equal to the rate of interest per annum. Find the rate of interest per annum?

(a) 5% (b) 20/3% (c) 22/7% (d) 6%

11. Simple interest on Rs.2,000 for 5 months at 16%p.a is _____

(a) Rs.133.33 (b) Rs.133.26 (c) Rs.134.00 (d) Rs.132.09

12. How much investment is required to yield an Annual income of Rs.420 at 7%p.a simple interest.

(a) Rs.6,000 (b) Rs.6,420 (c) Rs.5,580 (d) Rs.5,000

13. Mr. X invests Rs.90,500 in post office at 7.5%p.a simple interest. While calculating the rate was wrongly taken as 5.7% p.a. The difference in amounts at maturity is Rs.9,774. Find the period for which the sum was invested:

(a) 7 years (b) 5.8 years (c) 6 years (d) 8 years

14. The difference between compound and simple interest on certain sum of money for 2 years at 4%p.a is Rs.1. The sum (in Rs) is:

(a) 625 (b) 630 (c) 640 (d) 635

15. A sum of money compounded annually becomes Rs.1,140 in two years and Rs.1,710 in three years. Find the rate of interest per annum.

(a) 30% (b) 40% (c)50% (d) 60%

16. On what sum, difference between compound interest and simple interest for two years at 7%p.a interest is Rs.29.4

(a) Rs.5,000 (b) Rs.5,500 (c) Rs.6,000 (d) Rs.6,500

17. In what time will a sum of money double itself at 6.25%p.a simple interest?

(a) 5 years (b) 8 years (c) 12 years (d) 16 years

18. What principal will amount to Rs.370 in 6 years at 8%p.a at simple interest?

(a) Rs.210 (b) Rs.250 (c) Rs.310 (d) Rs.350

19. The partners A and B together lent Rs.3,903 at 4% per annum interest compounded annually. After a span of 7 years, A gets the same amount as B gets after 9 years. The share of A in the sum of Rs.3,903 would have been:

(a) Rs.1,875 (b) Rs.2,280 (c) Rs.2,028 (d) Rs.2,820

20. If a sum triples in 15 years at simple rate of interest, the rate of interest per annum will be:

(a) 13.0% (b) 13.3% (c) 13.5% (d) 18.0%

Chapter 5- Permutations and Combinations

In simple terms

- Permutation:- Arrangement
- Combinations – Selection

Factorial of n (n!)

It is the product of first n natural numbers.

n! = n x (n-1) x (n-2) x ………. x 1

ex :- 5! = 5 x 4 x 3 x 2 x 1 = 120

The value of $0! = 1$
$1! = 1$
$2! = 2$
$3! = 6$

$4! = 24$
$5! = 120$
$6! = 720$
$7! = 5040$
$8! = 40,320$

PERMUTATION

Arrangement of 'n' things taken 'r' at a time

$${}^nP_r = \frac{n!}{(n-r)!}$$

Note :-

- **$r \leq n$**
- **n cannot be 0. It must be natural number**
- **r must be a whole number.**
- ${}^nP_n = n!$
- ${}^nP_0 = 1$
- ${}^nP_n = {}^nP_{n-1}$
- ${}^nP_n = 2!\ \ {}^nP_{n-2}$
- ${}^nP_n = 3!\ \ {}^nP_{n-3}$
- ${}^nP_n = 4!\ \ {}^nP_{n-4}$
- **last term of the expansion ${}^nP_r$ is $(n-r+1)$**

Linear Permutation

Type – 1

Permutation (without condition) of letters of the given word

$$= \frac{n!}{n_1\,!\,X\ n_2!\,X\,n_3!\,X \ldots\ldots\ldots}$$

where,

n is no. of letters

n_1, n_2, n_3 *are number of times a particular letter repeats*

Note: If there is no letter which is repeated then the number of arrangements of the letters of the given word would be= $\boldsymbol{n}!$

Example1

Find the number of arrangements of letters of the word, SANSKRIT

Total number of letters = 8

Total arrangements = 8! / (2!)

= 40320/2

=20160

Example: Find the number of arrangements of letters of the word, MATHEMATICS

$n = 11$

Repeated letters

M → $n_1 = 2$

A → $n_2 = 2$

T → $n_3 = 2$

Total no. of arrangements $= \frac{n!}{n_1! \times n_2! \times n_3!}$

$= \frac{11!}{2! \times 2! \times 2!} = \underline{\underline{4989600}}$

Type 2

Problems with condition (Letters or digits)

Key points:

- Condition should be fulfilled first
- Or means +
- And means x

Exmaple1:

Find the number of arrangements of the letters of the word 'SUCCESS' such that the vowels may occupy odd positions only.

Given word 'SUCCESS'

Condition :- Vowels at odd positions only

ie. No. of vowels = 2

No. of odd positions = 4

Arrangements = $4P_2 = 4\times3 = 12$

Remaining letters = 5

Remaining places = 5

Arrangements = $5P_5 = 5! = 120$

Total arrangements = $\frac{12\times120}{\underbrace{3!\times2!}_{\text{for repetition of 'S and 'C}}} = \underline{\underline{120 \text{ ways}}}$

Example2:

The letters of the word SILENT are arranged so that the vowels occupy even places only. Find the number of permutations

Condition :- Vowels = 2

Even positions = 3

Arrangements = $3P_2 = 6$

Remaining letters: 4

Remaining places: 4

Arrangements = $4P_4 = 4! = 24$

Total arrangements = 6×24

$= \underline{\underline{144}}$

Example3

How many numbers greater than a lakh be formed with the digits 4,6 ,6,7,0,8

Given digits : 4.6, 6. 7, 0. 8

Condition :- No's greater than 1 Lakh

=> 6 digit no's or more than 6 digits

but with 6 digits given we can get

max of 6 digit no's only.

i.e.

$\boxed{5}\boxed{5}\boxed{4}\boxed{3}\boxed{2}\boxed{1}$

'0' can't be used

Total no. of no's greater than 1 Lakh $= \frac{5\times5\times4\times3\times2\times1}{2!}$

↑ for repeated digit

$= \underline{\underline{300}}$

Example4

How many 4 digit numbers greater than 5000 be formed using the digits 1,2,3,5,6,7,9

Condition :- 4 digit no's greater than 5000

$\boxed{4}\boxed{6}\boxed{5}\boxed{4}$

1, 2, 3

Can't be used

Total arrangements $= 4\times6\times5\times4$

$= \underline{\underline{480}}$

Example5

How many numbers not more than 10000 can be formed using the digits 1,2,3,4,5,6,7,8,9

Condition :- No's not more than 10,000

=> No's can be of 4 digits or 3 digits or 2 digits or 1 digit

4 digit no's => $\boxed{9}\boxed{8}\boxed{7}\boxed{6}$ => $9\times8\times7\times6 = \underline{\underline{3024}}$

3 digit no's => $\boxed{9}\boxed{8}\boxed{7}$ => $9\times8\times7 = 504$

2 digit no's => $\boxed{9}\boxed{8}$ => $9\times8 = 72$

1 digit no => $\boxed{9}$ => 9

Total no's => $3024+504+72\times9 = 3609$

-

Type 3 :-

Arrangements of n_1 & n_2 number of things in a row with a condition.

Example1:

There are 5 men and 5 women. In how many ways they can be arranged in a row such that no 2 women are together.

Condition :- No two women are together
∴ We shall arrange men first
=> 5 men can be arranged in 5! ways
as below

$$\times\ M_1 \times M_2 \times M_3 \times M_4 \times M_5 \times$$

Now there are 6 places in which 5 women can be arranged in $6P_5 = 6P_6 = 6!$ ways

Total arrangements $= 5! \times 6! = 120 \times 720$
$= 86400$

Exampe2:

5 men and 4 women to sit a row in such a manner that women always occupy even places. The number of such arrangements will be

Condition :- Women occupy even places
There are 4 women & 4 even places
No. of arrangements $= 4P_4 = 4! = 24$ ways
Now there are 5 men & 5 places.
No. of arrangements $= 5P_5 = 5! = 120$ ways
Total arrangements $= 24 \times 120 = \underline{\underline{2880}}$

Type 4:-

a)Arrangements of 'n' things such that 2 particular things are always together.

$= (n-1)!\ 2!$

Note :- If 3 things are always together $= (n-2)!\ 3!$

b)

Arrangements of 'n' things such that 2 particular things are never together,

$=(n-1)!\ (n-2)$

Note: Never together = Total arrangements(without condition) – always together

Example1

Find the number of ways of arrangements of the letters of the word CAUGHT such that the Vowels always together.

Given word 'CAUGHT'

Condition :- Vowels 'AU' are always together

∴ 'AU' must be taken as one single thing.

& there are 4 things remaining (C,G,H,T)

So. Total no. of things $= 1+4 = 5$

These 5 things can be arranged in $5! \times 2!$ ways

(For rearranging A & U)

Total arrangements $= 120 \times 2$

$= 240$

Example2

Find the number of arrangements of the letters of the word POVERTY such that the Vowels are never together

W.K.T

Never together = Total arrangements – always together

Now. the vowels O & E are considered to be a single thing

So total no. of things $= 1+5 = \underline{6}$

Now arrangements for always together $= 6! \times 2!$

∴ Never together $= 7! - 6! \times 2!$

$= \underline{\underline{3600}}$

Example3:

There are 5 books on Economics, 4 books on Statisics and 3 books on Computer science. In how many ways can these be arranged on a shelf such that the books on respective subjects are always together.

$\underbrace{\text{5 Economics}}_{1}$ $\underbrace{\text{4 stats}}_{1}$ $\underbrace{\text{3 CS}}_{1}$

So Total no. of things = $1+1+1 = 3$

Total arrangements = $3! \times \underbrace{5! \times 4! \times 3!}_{\text{For re arranging}}$

$= \underline{\underline{103680}}$

Example4

Find the number of arrangements of the word CALCULATOR such that vowels are never together.

W.K.T

Never together = Total arrangements − Always together

There are 4 vowels '$\underline{\text{AUAO}}$' to be taken as 1 thing

Total no. of things = $(1+6) = 7$

Arrangements for always together = $7! \times 4!$

∴ Never together = $\frac{10! - 7! \times 4!}{2! \times 2! \times 2!} = \underline{\underline{4,38,480}}$

Example5

Find the number of arrangements of the letters of the word GIRAFFE such that the vowels are never together.

Total letters for always together = $\underbrace{1}_{\text{Vowels}} + \underbrace{4}_{\text{Consonants}} = 5$

Arrangements for always together = $5! \times 3!$

∴ Never together = Total − always together

$= \frac{7! - 5! \times 3!}{2!} = \underline{\underline{2160}}$

Type 5 :-

a)number of ways of answering 'n' questions.

$= 2^n$

Where, n is number of questions

b)Number of ways of answering at least 1 questions out of n questions = $2^n - 1$

c)Number of ways of answering n questions with one alternative = $(2+1)^n$

Example1

There are 8 questions to be answered yes (or) No, In how many ways can these be answered ?

No. of ways of answering n questions $= 2^n$

given $n = 8$

$\therefore$ No. of ways $= 2^8 = \underline{\underline{256}}$

Example2

A question paper consists of 12 questions, 7 of them are of accounts and the remaining are Economics. In how many ways can these be answered such that the students needs to answer atleast 1 question from each part

7 acc

5 Eco

Condition :- Atleast 1 from acc and atleast 1 from Eco

ie $(2^7-1) \times (2^5-1)$

$= \underline{\underline{3937}}$

Circular Permutation [without condition]

We know that the arrangement of n things in a row(Linear arrangement)

= n!

Now the number of arrangements of n things in a circular manner

= (n-1)!

Example:

Find the number of ways of arrangement of 8 persons at a round table

No. of arrangements $= (n-1)!$ ($\because$ Circular)

$= (8-1)!$

$= 7! = \underline{\underline{5040}}$

Circular Permutation with condition

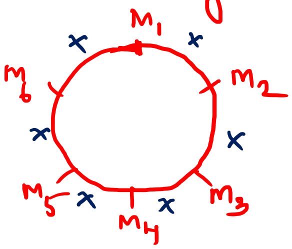
Example1:

In how many ways can 6 women & 6 men be seated at a circular table so that no 2 women are adjacent (together)

Condition :- No two women are adjacent

∴ let us arrange 6 men at round table

No. of such arrangements = $(6-1)! = 5! = 120$



Now there are 6 gaps & 6 women

They can be arranged in $^6P_6 = 6!$ ways $= 720$ ways

Total arrangements $= 120 \times 720 = \underline{\underline{86400}}$

Note:

1. Number of arrangements of 'n' pearls in a necklace (n flowers in a garland)

$$=\frac{(n-1)!}{2!}$$

2. Number of ways of arranging ‘r' things taken from n things such that one particular thing is always included.

$= r.\ ^{(n-1)}P_{(r-1)}$

3. Number of ways of arranging ‘r' things taken from n things such that one particular thing is always excluded.

$=^{(n-1)}P_r$

Example1:

Find the number of arrangements of 5 things taken out of 12 things in which one particular thing is always included.

$n = 12$, $r = 5$ (Given)

One particular thing is always included

i.e. Total arrangements $= r.\ ^{(n-1)}P_{r-1}$

$= 5.\ (^{11}P_4)$

$= \underline{\underline{39.600}}$

Example2:

Find the number of arrangements of 4 things taken out of 10 things such that 1particular thing is always excluded.

$n = 10, \quad r = 4$

One particular thing is always excluded

i.e. Total arrangements $= {}^{(n-1)}P_r$

$= {}^{9}P_4 = 9 \times 8 \times 7 \times 6$

$= 3024$

COMBINATION

Selection of 'r' things from 'n' things

$^nC_r = \frac{^nP_r}{r!}$ (Note: $r \le n$)

Some important results (Properties of combination)

- $^nC_o = 1$
- $^nC_1 = n$
- $^nC_n = 1$
- $^nC_r = {}^nC_{n-r}$
- $^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r$
- Number of straight lines formed using 'n' points of which 'm' are collinear.
  $= {}^nC_2 - {}^mC_2 + 1$
- The number of triangles formed using 'n points of which 'm' are collinear
  $= {}^nC_3 - {}^mC_3$
- Number of matches played by n teams such that each team plays one match each against other teams.
  $= {}^nC_2$
- Note: If each team plays two matches each against other teams then total number of matches would be $= 2. {}^nC_2$
- Number of handshake made by 'n' persons, Shaking hands with each other $= {}^nC_2$

- Number of quadrilaterals (or) parallelograms formed using set of ‘n’ parallel lines intersecting with set of 'm' parallel lines.
  $= {}^nC_2 \text{ X } {}^mC_2$

Note:

Permutation

If $^nP_r = {}^nP_s$

Then $r+s = 2n - 1$

Combination

If $^nC_r = {}^nC_s$

Then $r+s = n$

Note:

No. of diagonals in a polygon = $^nC_2 - n$

Where, n is number of sides

Example:

A polygon as 44 diagonal find the no sides

a) 8 b) 9 c) 10 d) 11

Given no. of diagonals $= 44 = nc_2 - n$

By option hitting $n = 11$

$\Rightarrow$ $44 = 11C_2 - 11$

$44 = 55 - 11$

$\underline{44 = 44}$

$\therefore$ $\underline{n = 11}$

Note:

Number of chords of a circle using n points = $^nC_2$

Number of points of intersection of n circles = $^nP_2$

Problems on Combination without condition

Example1:

A building contractor needs three helpers out of 10 men. in how many ways can these selections take place?

Given $n = 10$ $r = 3$

No. of Selections (Combinations) $= 10C_3$

$= \frac{10 \times 9 \times 8}{3!} = \underline{\underline{120}}$

Example2

The value of $\sum_{r=1}^{5}$ ${}^{6}C_{r}$ 5

Given $\sum_{r=1}^{5} 6C_r$

$= 6C_1 + 6C_2 + 6C_3 + 6C_4 + 6C_5$

$= 6 + 15 + 20 + 15 + 6$

$= \underline{\underline{62}}$

Example3:

5 seats of articled clerks are vacant in a chartered accountant firm. How many different batches of candidates can be choosen out of 8 candidates.

$n = 8$ $r = 5$

No. of combinations $= 8C_5 = 8C_3$ $[\because nC_r = nC_{n-r}]$

$= \underline{\underline{56}}$

Problem with Condition

Example1:

There are 5 men and 3 women in a group. Find the number of ways in which the committee of 4 persons can be formed of them such that, the committee is to include at Least 2 women

5 men 3 women

4 member Committee

Condition :- At least 2 women $\Rightarrow$ 2W or 3W

$\Rightarrow$ (2W & 2M) or (3W & 1M)

$\Rightarrow (3C_2 \times 5C_2) + (3C_3 \times 5C_1)$

$\Rightarrow (3 \times 10) + (1 \times 5)$

$\Rightarrow \underline{\underline{35}}$ ways

Example2:

In how many ways can a selection of 8 persons can be made out of 4 teachers & 10 students Such that the selected members must include at least two teachers?

10 students 4 Teachers

8 persons are selected

Condition :- At least 2 Teachers

=> 2T or 3T or 4T

=> (2T & 6S) or (3T & 5S) or (4T & 4S)

=> $({}^4C_2 \times {}^{10}C_6) + ({}^4C_3 \times {}^{10}C_5) + ({}^4C_4 \times {}^{10}C_4)$

=> 2478 ways

Practice problems

Problems on Permutation & Combination

1. If 6Pr = 24 6Cr, find r.
   a) 4b) 6 c) 2 d) 1
2. If ${}^nPr = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$ Then n = ?
   a) 2b) 3 c) 4 d) 5
3. A Code word is to consist of 2 Alphabets followed by 2 distinct digits from 1 to 9. how many such codes are possible.
   a) 6,15, 800 b) 46,800 c) 719500 d) none

4. How many words can be formed with the letters of the word ORIENTAL such that A&E always occupy odd places
   a) 540 b) 8770 c) 8640 d) 8480
5. If ${}^{18}C_r = {}^{18}C_{r+2}$, find the value of ${}^rC_5$?
   a) 155 b) 50 c) 56 d) none
6. Number of ways of painting a face of cube by 6 Colours is
   a) 36 b) 6 c) 24 d) 1

7. Find the number of arrangements in which letters of the word MONDAY be arranged so that the words always begin with M and do not end with N.
   a) 720 b) 120 c) 96 d) None

8. If six times the number of permutations of 'n' items taken 3 at a time is equal to seven times the number of permutation of (n-1) items taken 3 at a time, then the value of 'n' will be. a) 7 b) 9 c) 13 d) 21

9. Find the sum of all 4 digits number formed using the digits 2, 4, 6,8 number formed using the digit 2,4,6,8.

10. The number of words from the letters of the word BHARAT, in which B and H will never come together is
a)120 (b)360 c)240 (d) None of the above **(2018 Dec)**

11. If ${}^{n}P_{r} = 720$ and ${}^{n}C_{r} = 120$ then *r* is **(2018 Dec)**
(a) 4 (b)5 c)3 (d) 6

12. The value of N in
(a) 81 (b) 64
(c) 78 (d) 89 **(2018 Dec)**

13. A bag contains 4 red, 3 black and 2 white balls. In how many ways 3 balls can be drawn from this bag so that they include at least one black ball? **(2018 Dec)**
(a) 46 (b) 64
(c) 86 (d) None of the above

14. If ${}^{11}C_{x} = {}^{11}C_{2x-4}$ then ${}^{7}C_{x}$ will be **(June 2019)**
(a) 20 (b) 21
(c) 22 (d) 23

1. Which of the following is not a correct statement **(June 2019)**
a) ${}^{n}P_{n} = {}^{n}P_{n-1}$
b) ${}^{n}P_{n} = 2\,!\,{}^{n}P_{n-2}$
c) ${}^{n}P_{n} = 3\,!\,{}^{n}P_{n-3}$
d) ${}^{n}P_{n} = n\,!\ {}^{n-1}P_{n-1}$

16. How many number divisible by 5 of 6 digit can be made from the digit 2, 3, 4, 5, 6, 7 (2019 Nov)
(a) 120 (b) 600
(c) 240 (d) none

17. 5 boys and 3 girls are to be seated together such that no two girls are together (2019 Nov)

(a) 14,400 (b) 2400
(c) 720 (d) None of these

18. Out of 6 Boys & 4 girls, Find the number of ways for selecting 5 member committee in which there is exactly two girls ? (2019 Nov)
(a) 120 (b) 1440
(c) 720 (d) 71

19. if np5 :np3 is 2:1 than value of n is(2019 Nov)
(a) 2 (b) -5
(c) -2 (d) 5

20. A fruit basket contains 7 apples, 6 bananas and 4 mangoes. How many selections of 3 fruits can be made so that all 3 are apples? **(2020 Dec)**
120 ways (b) 35 ways (c) 168 ways (d) 70 ways

21. Out of 7 boys and 4 girls a team of a debate club of 5 is to be chosen. The number of teams such that each team includes at least one girl is __________ **(2020 Dec)**
(a) 429 (b) 439 c) 419 (d) 441

22. From a group of 8 men and 4 women, 4 persons are to be selected to form a committee so that at least 2 women are there on the committee. In how many ways can it be done? **(2020 Dec)**
(a) 201 (b) 168 c)202 (d) 220

23. If $^nP_4 = 20\ ^nP_2$ were denotes the number of permutations n = ________ **(2020 Dec)**
(a) 4 (b) 2 (c) 5 (d) 7

24. If $^nP_6 = 20\ ^nP_4$ were denotes the number of permutations n = ________**(2021 July)**
a)5 b) 3 c) 9 d) 8

25.How many numbers of 7 digits can be formed using the digits 3,4,5,6,7,8,9 no digis being repeated and are not divisible by 5 **(2021 July)**
a) 4320 b) 4690 c)3900 d) 3890

26.A person can go from A to B by 11 different modes of transport but is allowed to return back to A by any mode other than the one earlier. The number of different ways, the entire journey can be completed is…. **(2021 July)**

a)110 b) 10^{10} c) 9^5 d) 10^9

27.The number of ways 5 boys and 5 girls can be seated at a round table, so no two boys are adjacent is **(2021 July)**

a)2550 b) 2880 c) 625 d) 2476

Chapter 6- Sequence and Series

Sequence :-

Orderly arranged numbers are said to be in sequence.

Ex 1 :- 5 10 15 20 25

Ex 2 :- 2 4 8 16 32

Series :-

Sum of the orderly arranged numbers is said to be a series.

Ex :- 5 + 10 + 15 + 20 + ……..

Arithmetic Progression (AP)

A Sequence in which there exists a common difference is said to be an arithmetic progression

General from of an Ap:

a, a+d, a+2d, ……a+(n-1)d

n^{th} term of an AP

$T_n = l = a+(n-1)d$

Ex :- $T_5 = a+(5-1)d$

$= a + 4d$

Sum of n term in AP:-

a) When last term is not given

$S_n = \frac{n}{2}[2a+(n-1)d]$

b) when first term & last term are given

$S_n = \frac{n}{2}(a+l)$

where 'l' is last term

PROBLEMS OF VARIOUS TYPES

TYPE1:

To find 'n' when n^{th} term, a & d are given.

Example1:

Find the number of terms of an AP having first term as 10, common difference as 5 & last term as 200

Given $a = 10$ $l = 200$ $d = 5$

W.K.T T_n or $l = a + (n-1)d$

$200 = 10 + (n-1)5$

$190 = 5n - 5$

$195 = 5n$

$n = 39$

Short cut $n = \frac{l-a}{d} + 1 = \frac{200-10}{5} + 1$

$= 39$

Example2:

Find the number of terms of an AP having first term as 200 , common difference as 5 and last term as 400

$$n = \frac{l-a}{d} + 1 = \frac{400-200}{5} + 1$$

$$= \underline{\underline{41}}$$

TYPE2:

Sum of all natural numbers between the 2 given terms with a condition

Example 1:

Find the Sum of all natural numbers, between 1 and 300 which are exactly divisible by 5

No's exactly divisible by 5 b/w 1 & 300 are

5, 10, 15, 20, 25, - - . . . 295

$$n = \frac{l-a}{d} + 1 = \frac{295-5}{5} + 1 = \underline{\underline{59}}$$

$$\therefore S_n = \frac{n}{2}(a+l)$$

$$= \frac{59}{2}(5+295)$$

$$= \underline{\underline{8850}}$$

Example 2;

Find the sum of all natural number between 10 and 500 which are exactly divisible by 4.

Required no's

12, 16, 20, 24, - - - - 496

$a = 12$, $d = 4$. $l = 496$

$$n = \frac{l-a}{d} + 1 = \underline{\underline{122}}$$

$$\therefore S_n = \frac{n}{2}(a+l) = \underline{\underline{30,988}}$$

TYPE3:

To find rth term of an AP , when nth & mth terms are given.

Example:

The mth term of an Ap is n. nth term is m. Find its rth term.

a) m + n – r b) m + n + r c) m + n –2 r d) none of these

Given $T_m = n = a+(m-1)d \rightarrow ①$

$T_n = m = a+(n-1)d \rightarrow ②$

① − ② $\Rightarrow$ $n-m = [(m-1)-(n-1)]d$

$-(m-\cancel{n}) = (m-\cancel{n})d$

$\boxed{d=-1}$

① $\Rightarrow$ $a+(m-1)(-1) = n$

$\boxed{a = n+m-1}$

$\therefore$ $T_r = a+(r-1)d$

$= n+m-1+(r-1)(-1)$

$= n+m-1-r+1$

$\Rightarrow \underline{\underline{m+n-r}}$

Short cut :-

let $m=1$. $n=2$. $r=3$

To find, $T_r = T_3 = a+2d$

Given $T_m = T_1 = n = 2 \Rightarrow a$

$T_n = T_2 = m = 1$

$d = T_2 - T_1 = 1-2 = -1$

$\therefore T_r = T_3 = a+2d = 2+2(-1) = \underline{0}$

Sub $m=1$. $n=2$ $r=3$ in options.

whichever option gives zero, will be the correct answer.

Clearly option (a) $m+n-r = 0$

TYPE 4:

To find Sum of n terms when the middle term is given

Or

To find the middle term when the sum of n terms is given

Example 1:

The 6th term of an Ap is 15. Find the Sum of its first 11 terms

Given 6th term $T_6 = 15 = a+5d \rightarrow ①$

To find $S_{11} = \frac{n}{2}[2a+(n-1)d]$

$= \frac{11}{2}[2a+10d]$

$= \frac{11}{\cancel{2}} \times \cancel{2}(a+5d)$

$= 11 \times 15 = \underline{165}$

Short cut :

Given middle term = 15.

Sum of n terms in Ap = middle term × n

= 15 × 11

= $\underline{\underline{165}}$

Example 2:

The 16^{th} term of an Ap is 40. Find the Sum of its first 31 terms

Given middle term = 40

Sum of 31 terms = middle term × 31

= 40 × 31

= $\underline{\underline{1240}}$

Example3:-

If the Sum of 7 terms in Ap is 35, then find the fourth term

Given $S_7 = 35$

To find 4^{th} term (middle term)

Middle term $= \frac{\text{Sum}}{n}$

$= \frac{35}{7} = \underline{\underline{5}}$

Example4:-

If the Sum of 5 terms in Ap is 60, then find the third term

Given $S_5 = 60$

T_3 (middle term) $= \frac{S_5}{5}$

$= \frac{60}{5}$

$= \underline{12}$

TYPE 5:

To find the sum of n terms when the sum of the two terms which are equidistant from the middle term is given:

Example1 :

The Sum 5^{th} & 7^{th} terms of an AP is 30. Find the Sum of its 11 terms.

Given :- $T_5 + T_7 = 30$

$$a+4d+a+6d = 30$$

$$2a+10d = 30 \longrightarrow ①$$

Now $S_{11} = \frac{11}{2}[2a+(11-1)d]$

$$= \frac{11}{2}[2a+10d]$$

$$S_{11} = \frac{11}{2} \times 30$$

$$= \underline{\underline{165}}$$

Short cut: $T_6 = \frac{T_5+T_6}{2} = \underline{15}$ (Middle term)

$$S_{11} = \text{Middle term} \times 11$$

$$= 15 \times 11 = \underline{\underline{165}}$$

Example 2:

The Sum 9th & 11th terms of an AP is 50. Find the Sum of its 19 terms.

Middle term $\frac{T_9 + T_{11}}{2} = \frac{50}{2}$ (10th term)

Sum of 19 terms $= 25 \times 19$

$$= \underline{\underline{475}}$$

TYPE 6

To find n^{th} term when sum of 'n' terms (in terms of n) is given :

$T_n = S_n – S_{n-1}$

$T_1 = S_1$

$T_2 = S_2 - S_1$

$T_3 = S_3 – S_2$ ……

Ex :- The sum of 'n' term in AP is $3n^2 + 4n$. Find

a) First term

b) Eighth term

Given $S_n = 3n^2+4n$

a) $T_1 = S_1 = 3(1)^2+4(1)$

$$= 3+4$$

$$= \underline{\underline{7}}$$

b) $T_8 = S_8 - S_7$

$$= 3(8)^2+4(8) - [3(7)^2+4(7)]$$

$$= \underline{49}$$

TYPE 7:

To Insert 'n' Am's between 2 terms :

Let the n Am's be a, a+d, a+2d, …….. a+(n-1)d

:. The 2 terms given would be

a-d & a+nd

Example: Insert 3 Am's between 6 & 18.

let the 3 Am's be a, a+d, a+2d

$$\underset{\overset{\downarrow}{a-d}}{6} \quad \underline{a} \quad \underline{a+d} \quad \underline{a+2d} \quad \underset{\overset{\downarrow}{a+3d}}{18}$$

$$\begin{aligned} \cancel{a}-d &= 6 \\ \underset{(-)}{\cancel{a}}+\underset{(-)}{3d} &= \underset{(-)}{18} \\ \hline -4d &= -12 \\ \boxed{d=3} \end{aligned}$$

$$\begin{aligned} a-d &= 6 \\ a &= 6+d \\ a &= 9 \end{aligned}$$

∴ 3 Am's are 9, 12, 15

Type8:

To find common difference and first term when two particular terms in AP are given.

Ex:

If the 10th term and 19th terms of an AP are 72 and 108 respectively. Find the common difference and the first term.

Geometric progression (GP)

→ A Sequence in which there exists a Common ratio is said to be a Geometric progression.

→ General form of GP

$a, ar, ar^2, ar^3, \ldots\ldots\ldots ar^{n-1}$

where, n ⟶ no. of terms.

a ⟶ first term

r ⟶ Common ratio = $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \ldots\ldots\ldots$

n^{th} term of GP:

n^{th} term GP. $T_n = ar^{n-1}$

Ex:- $T_3 = ar^{3-1}$

$= ar^2$

similarly,

$T_5 = ar^4$

$T_{10} = ar^9$

Sum of n terms in GP:

$S_n = a\left(\frac{r^n-1}{r-1}\right)$ for $r > 1$

$S_n = a\left(\frac{1-r^n}{1-r}\right)$ for $r < 1$

Note:- Sum is not defined for r=1

PROBLEMS:

Type1: (Based on S_n)

Example :- Find the Sum of first 10 terms of the Sequence.

3,6,12,…….

Given $a=3, r=2$

To find S_{10}.

W.K.T $S_n = a\left(\frac{r^n-1}{r-1}\right)$ $[\because r>1]$

$S_{10} = 3\left(\frac{2^{10}-1}{2-1}\right)$

$= \underline{\underline{3069}}$

Example 2 :-

The Sum of 'n' terms of the Series

3,9, 27 ………. is 9840. Find the number of terms (n)

a)8 b)9 c)10 d)none

$a = 3$ $r = 3$ $S_n = 9840$

$$S_n = a\left(\frac{r^n-1}{r-1}\right)$$

By option hitting method

for $n = 8$

$$S_8 = 3\left(\frac{3^8-1}{3-1}\right)$$

$= \underline{\underline{9840}}$

∴ correct answer is option ⓐ

TYPE 2

Sum of Infinite terms in GP: (S_∞)

$S_\infty = \frac{a}{1-r}$ (r<1)

Note:- S_∞ is not defined for r>1

Example 1

Find the sum of infinite terms of the series

$$1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots$$

Given $a = 1$ $r = \frac{1}{4}$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}}$$

$$= \underline{\underline{\frac{4}{3}}}$$

Example 2

Find the sum of infinite term of the series

$3 + \frac{9}{y} + \frac{27}{y^2} + \ldots\ldots..$

Given $a = 3 \quad r = \frac{3}{y}$

$$\therefore S_\infty = \frac{a}{1-r} = \frac{3}{1-\frac{3}{y}} = \frac{3}{\frac{y-3}{y}} = \underline{\underline{\frac{3y}{y-3}}}$$

TYPE 3

Sum of infinite terms with Combination of Ap & GP.

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Example.

The sum of the series.

$$1 + \frac{3}{5} + \frac{5}{5^2} + \frac{7}{5^3} + \cdots$$

$a = 1 \quad d = 2 \quad r = \frac{1}{5}$

$$\therefore S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

$$= \frac{1}{1-\frac{1}{5}} + \frac{2 \times \frac{1}{5}}{\left(1-\frac{1}{5}\right)^2} = \frac{1}{\frac{4}{5}} + \frac{\frac{2}{5}}{\left(\frac{4}{5}\right)^2}$$

$$= \frac{5}{4} + \frac{\frac{2}{5}}{\frac{16}{25}} = \frac{5}{4} + \frac{25 \times 2}{16 \times 5} = \underline{\underline{1.875}}$$

TYPE 4:

Special Series

Example 1 The sum of n terms of the series

$2 + 22 + 222 + \dots$ is

a) $\frac{2}{81}(10^{n+1}-10) - \frac{2}{9}n$ b) $\frac{2n}{9} - \frac{2}{81}(10^{n+1}-10)$

c) $\frac{2}{81}(10^{n+1}-10) + \frac{2}{9}n$ d) none.

Solution :- Such questions are to be solved by option hitting method by plugging in $n=2$

For the given example we get $S_2 = 2+22 = 24$

Now plug in $n=2$ in options

ie Option ⓐ $\Rightarrow \frac{2}{81}(10^{2+1}-10) - \frac{2}{9}\times 2 = 24$

∴ option (a) is the correct answer.

Example 2

Find the sum of n terms of the series

$0.4 + 0.44 + 0.444 + \dots$

a) $\frac{4}{9}n - \frac{4}{81}[1-(0.1)^n]$ b) $\frac{4}{9}n + \frac{4}{81}[1-(0.1)^n]$

c) $\frac{4}{81}[1-(0.1)^n]$ d) none

Solution :- put $n=2 \Rightarrow S_2 = 0.4+0.44 = \underline{0.84}$

Option ⓐ $\Rightarrow \frac{4}{9}\times 2 - \frac{4}{81}[1-(0.1)^2] = 0.84$

∴ Option ⓐ is the correct answer.

TYPE 5:

To find the product of n terms using the middle term of GP

Example

If the 5th Term of a GP is 3 then find the product of its first 9 terms

W.K.T Product of n terms of GP = $(G_m)^n$
and G_m = middle term of GP.

Given middle term = 3

$\therefore$ Product of first 9 terms = $\underline{\underline{3^9}}$

TYPE 6:

Problems on the Combination of AP, GP & HP:

Note:

→ For the 3 terms a, b, c to be in AP
b= (a+c)/2

Ex: 1, 2 , 3

→ For 3 terms a, b, c to be in GP b= $\sqrt{ac}$

Ex: 2 , 4 , 8

→ For 3 term a, b, c to be in HP
b = $\frac{2ac}{a+c}$

Ex: 6, 8, 12 or
12, 8, 6

→ If a^2 b^2 c^2 are in AP Then let a, b, c as 1, 5, 7

Example1 :

If x, y, z are in GP then the terms
(x^2+y^2), $(xy+yz)$, (y^2+z^2) are in
a) AP b) GP c) Hp d) none

let $x = 2$, $y = 4$, $z = 8$ [$\because$ x, y, z are in GP]

Now (x^2+y^2), $(xy+yz)$, (y^2+z^2)

$\Rightarrow$ $(4+16)$, $(8+32)$, $(16+64)$

$\Rightarrow$ 20, 40, 80 are in G.P

Example 2

If a, b, c are AP Then the value of a-b+c

a) a b) -b c) b d) c

let $a=1$, $b=2$. $c=3$ as a.b c are in A.p

$\therefore$ $a-b+c = 1-2+3$

$= \underline{2} = b$

Example 3

If a^2, b^2, c^2 are in AP then b+c, c+a & a+b are in

a)AP b)HP c)GP d) none

Given a^2, b^2, c^2 are in A.P

$\Rightarrow$ $a=1$. $b=5$. $c=7$

$\therefore$ $(b+c)$, $(c+a)$, $(a+b)$

$\Rightarrow$ $(5+7)$, $(7+1)$ $(1+5)$

$\Rightarrow$ 12, 8, 6 are in H.P

Example4:

A man employed in a company is promised a salary of Rs.3000 every month for the first year and an increment of Rs.1000 in his monthly salary every succeeding year. How much does the man earn from the company in 20 years?

a) Rs. 30,00,000 b) Rs. 27,50,000 c) Rs. 19,10,000 d) Rs. 7,90,000

First year salary, 2nd year salary, - - -

$\Rightarrow$ (3000×12), (4000×12), (5000×12)

$\Rightarrow$ 36000, 48000, 60000, - - .

$a = 36.000$ $d = 12000$ $n = 20$

$\therefore$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{20}{2}[(2\times36000) + 19d]$

$= \underline{30,00,000}$

Example5:

If a,b,c are in A.P and x,y,z are in GP, then the value of $x^{(b-c)}. y^{(c-a)}.z^{(a-b)}$ is

a) 1 b) 0 c) b(c-a) d)none

let $a=1, b=2, c=3$ [$\because$ a,b.c are in Ap)

let $x=2, y=4. z=8$ [$\because$ x.y.z are in GP]

Now,
$$x^{b-c} \times y^{c-a} \times z^{a-b}$$
$$\Rightarrow 2^{2-3} \times (2^2)^{3-1} \times (2^3)^{1-2}$$
$$\Rightarrow 2^{-1} \times 2^{4} \times 2^{-3}$$
$$\Rightarrow 2^{-1+4-3} = 2^0 = \underline{\underline{1}}$$

Example6:

The arithmetic mean of the squares of first 2n natural numbers is :

a) $\frac{1}{6}$ (2n+1) (4n-1) b) $\frac{1}{6}$ (2n-1) (4n-1) c) $\frac{1}{6}$ (2n-1) (4n+1) d) $\frac{1}{6}$ (2n+1) (4n+1)

Let $2n=2 \Rightarrow$ 2 terms

$\boxed{n=1}$

$\therefore$ Required sum $= 1^2+2^2 = 5$

now $A.m = \frac{\text{sum}}{2} = \frac{5}{2} = 2.5$

plug in n=1. in the options. whichever option gives 2.5, will be the correct answer

clearly option (d) $\Rightarrow \frac{1}{6}[2(1)+1][4(1)+1]$
$$= \frac{15}{6} = \underline{\underline{2.5}}$$

Example7:

If S be the sum, P the product and R is the sum of reciprocals of n terms in GP then P^2R^n = ….

a)S^{2n} b) S^n c) S^{-2n} d) S^{-n}

let $n = 3$

& let the 3 terms in G.P be 2, 4. 8

Given $S = 2+4+8 = 14$

$P = 2 \times 4 \times 8 = 64$

$R = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 0.875$

Now $P^2R^n = P^2R^3 = (64)^2 \times (0.875)^3 = 2744$

Option (b) $\Rightarrow S^n = S^3 = 14^3 = 2744$

$\therefore P^2R^n = S^n$

Practice problems on AP & GP

1. The sum of n terms of the series 1 + (1+3) +(1+3+5) +……. Is

a) $\frac{n(n+1)(2n+1)}{6}$ b) $\frac{n(n+1)(n+2)}{6}$ c) $\frac{n(n+1)(2n+1)}{3}$ d)none of these

2. The third term of GP is 2/3 and the 6th term is 2/81.. Find its first term

a)2 b)6 c)9 d)1/3

3. The sum of the series –8, –6, –4,... terms is 52. The number of terms is

(a) 10 (b) 11

(c) 13 (d) 12

4.The value of K for which 7K+3 , 4k-5 and 2K+10 are in AP is

a)-13 b)13 c)23 d)-23

5.The p^{th} term of an Ap is q. q^{h} term is p. Find its r^{th} term.

a) p + q – r b) p+q + r c) p + r –2 r d) none of these

6.If 2 + 6 + 10 + 14 + 18 + + x = 882 then the value of x (June 2019)

(a) 78 (b) 80 (c) 82(d) 86

7.In a G.P, If the fourth term is '3' then the product of first seven terms is (June 2019)

8. The Ratio of sum of n terms of the two AP's is (n +1) : (n–1) then the Ratio of their mth terms is(June 2019)

(a) (m + 1) : 2m (b) (m + 1) : (m –1)

(c) (2m –1 :(m + 1) (d) m : (m – 1)

9.If Y = 1+x + x2 +…….. ∞ then x = (June 2019) a)(y-1)/y b)(y+1) /y c)y/(y+1) d) y/(y-1)

10.The 20th terms of arithmetic progression whose 6th term is 38 and 10th term is 66 is ______

(a) 136 (b) 118 c)178 (d) 210

11.Divide 69 into 3 parts which are in A. P and are such that the product of first two parts is 460

(a) 20, 23, 26 (b) 21, 23, 25 c)19, 23, 27 (d) 22, 23, 24

12.Three numbers is G.P. with their sum is 130 and their product is 27,000 are ____

(a) 90, 30, 10 (b) 10, 30, 90 (c) (a)& (b) Both (d) 10 , 20 ,30

13.The number of terms of the series 5+7+9+…. Must be taken such that the sum may be equal to 480

a) 20 b)10 c)15 d)25

14.If the sum of n terms of AP is 2 n^2, the fifth term is

a)20 b)50 c)18 d)25

15.The sum of 3 numbers is GP is 28, when 7,2 and 1 are subtracted from the first, second and third numbers respectively, then the resulting numbers are in AP. What is the sum of the squares of the original numbers

a)510 b)456 c)400 d)336

Chapter 7- Sets, Relations, Functions, Limits and Continuity

Sets

A set is a collection of well defined elements.

There are two forms of sets. Namely,

a)**Roaster form:**

Ex :- A ={ a,e,i,o,u}

b)Set builder form:

Ex: A ={ x: x is a vowel}

Cardinal number of set :-

Number of elements enclosed within a set

If A is a set then n(A) denotes its cardinal number.

Null set:-

An empty set is said to be a null set(A set having no elements
A = { } (or) A = φ

Cardinal number of a null set is zero.

Complement of a set :-

If 'A' is a set, then A^1 is complement of 'A'

Example :- If U = {1,2,3,4} , A = {1,2,} find A^1

A^1 = U – A →elements in U but not in A

A^1 = {3,4}

Types of sets

Equal Sets:

A and B are said to be equal if they enclose same and equal number of elements

Ex: A ={1,2,3}

B ={1,2,3}

Equivalent sets:

If n(A) = n(B) then A and B are said to be equivalent sets

Note: All equal sets are equivalent but all equivalent sets are not equal sets.

Finite set:

A set having finite number of elements. Ex: A ={1,2,3, 4,5,6}

Infinite set:

A set having infinite number of elements.

Ex: set of all natural numbers, set of all even natural numbers, set of all real numbers etc.

Operations on 2 sets:-

- Union Operation.

To find union of A & B it is required to combine the elements in A & B

If A = {1,2,3,4,5}
B = {3,4,6,7}
Then, AUB = {1,2,3,4,5,6,7}

Note: [AUB →union →(or)→(+)]

- **Intersection Operation**

To find intersection of two sets A & B it is required to write the common elements b/w A & B

If A= {1,2,3,4} , B = {3,5,6,7}

then A∩B = {3}

Note: (A∩B→ Intersection →and →X)

- **A – B**

It means elements in A but not in B

If A = {1,2,3}
B = {1,2,3,4} then fine (i) A-B (ii) B-A

(i) $A - B = \{ \ \}$ →null set

(ii) $B - A = \{4\}$

Note :- <u>1) For any 2 sets A and B</u>

$$n(AUB) = n(A) + n(B) - n(A\cap B)$$

<u>2)For any 3 sets A, B and C</u>

$$n(AUBUC) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$$

→n(AUB) and n(AUBUC) are used for atleast 1

<u>Demorgan's laws</u>

1) $(AUB)^{I} = A^{I}\cap B^{I}$

2) $(A\cap B)^{I} = A^{I}UB^{I}$

Note :-

- $(AUB)^{I} = U - (AUB)$
- $n(AUB)^{I} = n(U) - n(AUB)$
- $n(AUB)^{I} = \text{total} - n(AUB)$ ***(to be used for neither A nor B)***

<u>Subset</u>

A set A is said to be a subset of set B if all elements in set A are present in set B

Ex: If $A = \{1, 2, 3\}$

$B = \{1,2,3,4\}$

A is a subset of set B, since all elements in A are in B

Note:

- Any set is a subset of itself

- A null set is a subset of any set
- Number of subsets of a set having n elements = 2^n
- Number of proper subsets of a set having n elements = 2^n -1
- **Power set:** It is a set of all subsets of a given set

  Ex: If A={1,2}

  Power set = {{1},{2},{1,2},{} }

Example1:

Find the number of proper subsets of the set A = { 1, 2, 3 ,4 , 5 , 6 ,7,8}

No. of elements, $n = 8$

$\therefore$ No. of proper subsets $= 2^n - 1$

$= 2^8 - 1 = \underline{\underline{255}}$

Example2:

Find the number of subsets of the set A={3,5,7,9}

a) 16 b)15 c)14 d)none

$n = 4$

No. of subsets $= 2^n = 2^4 = \underline{\underline{16}}$

Example3:

A ∩E' is equal to (E is a super set of A)

a)E b) φ c) A d) none

Let $U = \{1, 2, 3, 4\}$

$E = \{1, 2, 3\}$

$A = \{1, 2\}$

Now $E' = U - E = \{4\}$

$\therefore A \cap E' = \{1, 2\} \cap \{4\}$

$= \{\}$ or ϕ

Example4:

A ∩φ =

a) A b) φ c) E d) none

Let $A = \{1, 2\}$

$\phi = \{\ \}$

$\therefore A \cap \phi = \{\ \}$ or ϕ

Example5:

AUA’ is equal to (E is super set of A)

a) A b) E c) φ d) none

Let $U = E = \{1, 2, 3\}$

$A = \{1, 2\}$

$A' = U - A = \{3\}$

$\therefore A \cup A' = \{1, 2, 3\} = U = \underline{\underline{E}}$

Example6:

I is a set of Isosceles triangles and E is the set of Equilateral triangles. Then

a) I c N b)E c I c) E= I d) none

Given 'I' is a set of Isosceles triangles

'E' is the set of equilateral triangles

$\therefore$ All equilateral triangles (E) are Isosceles triangles (I) $\Rightarrow \underline{\underline{E \subset I}}$

Problems on 2 sets and 3 sets

Example1

Out of 30 members in a family, 21 like tea and 24 like coffee. Assume that each one like at least one of the 2 drinks. Find how many like both coffee and tea

Let Tea $\rightarrow$ A . Coffee $\rightarrow$ B

Given $n(A) = 21$ $n(B) = 24$ $n(A \cap B) = ?$

$n(A \cup B) = 30$ (given)

W.K.T $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 21 + 24 - 30$

$= \underline{\underline{15}}$

Example2

Out of 250 students, 65 passed in accounts, 50 in economics ,80 in maths , 20 in both accounts and maths, 25 in both maths and economics and 45 in both accounts and economics and 15 students passed in all the three subject. Find the number of student who passed at least in one of the subjects.

Let Accounts → A. Economics → B Maths → C

Given $n(A) = 65$ $n(B) = 50$ $n(C) = 80$

$n(A \cap C) = 20$ $n(B \cap C) = 25$ $n(A \cap B) = 45$ $n(A \cap B \cap C) = 15$

At least 1 $\Rightarrow n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 65 + 50 + 80 - 45 - 25 - 20 + 15 = \underline{\underline{120}}$

Problems using Ven diagram

Note :- To be used whenever it is required to find only 1 event.

Example1

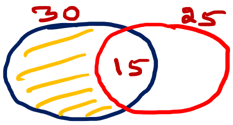
There are 40 students, 30 of them passed in English, 25 of them passed in maths and 15 of them passed in both. How many students passed in English only but not in maths?

a) 15 b) 20 c) 10 d) 25

Let English → A. Maths → B

$n(A) = 30$ $n(B) = 25$ $n(A \cap B) = 15$

Using Ven diagram



The shaded portion represents only english. It is obtained by subtracting the common portion from total english ie. only english $= 30 - 15 = \underline{\underline{15}}$

Short cut

only english $\Rightarrow$ only A $= n(A) - n(A \cap B)$

$= 30 - 15 = \underline{15}$

Example2

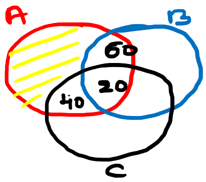
For a group of 400 persons, 200 are interested in music, 140 in photography, 80 in swimming, further 80 are interested in both music & photography, 60 in both music & swimming, 40 in both photography and swimming and 20 in all. How many are interested in music but not in photography & swimming

Let Music → A. photography → B swimming → C

$n(A) = 200$ $n(B) = 140$ $n(C) = 80$ $n(A \cap B) = 80$ $n(B \cap C) = 40$

$n(A \cap C) = 60$ $n(A \cap B \cap C) = 20$

Using Ven diagram



The shaded portion represents only music (only A) $= 200 - 60 - 20 - 40$

$= 80$

Using short cut only A $\Rightarrow n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$\Rightarrow 200 - 80 - 60 + 20$

$\Rightarrow \underline{\underline{80}}$

Cartesian product of sets

It is of all possible order pairs obtained on two sets

Ex1:If A = {1,2}   B = {3,4}

Then,

A X B = {(1,3), (1,4), (2,3), (2,4)}

Ex2:If A = {1,2,3}   B = {5,6}

A X B = {(1,5), (1,6), (2,5), (2,6), (3,5), (3,6)}

Then,

B X A = {(5,1), (5,2), (5,3), (6,1), (6,2), (6,3)}

Note:

Cardinal number of A × B is n(A) × n(B)

Relations

It is a subset of of Cartesian product of two sets.

R = {(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)}

Ex :- R = {(1,6) (2,5) (3,7) (3,6)}

Note: The number of Relations from A to B is equal to $2^{n(A)\ X\ n(B)}$

Types of relations

1) **Reflexive relation :-**

For all a Є A

=> (a,a) ЄR

2)Symmetric relation :-

For any (a,b) Є R

=> (b,a) Є R

a & b are 2 different elements

3)Transitive relation

For any (a,b) Є R, (b,c) Є R

=> (a,c) Є R

(a,b) & (b,c) are pairs of 2 different elements

Equivalence relation

If a relation is reflexive, symmetric and transitive then it is said to be equivalence.

Note: a → element , Є → belongs to, A → set & R → relation

Partial order relation

A relation which is reflexive, anti symmetric and transitive is said to be a partial order relation

Example1 :-

A= {1, 2, 3}

A R A = {(1,1), (1,2), (2,1) (2,2), (3,3)}

Given $A = \{1,2,3\}$

$R = \{(1,1)\ (1,2), (2,1), (2,2), (3,3)\}$

→ It is reflexive as for all $a \in A$ $(a,a) \in R$

→ It is symmetric as $(\underset{(1}{a}, \underset{2)}{b}) \in R \Rightarrow (\underset{(2,}{b}, \underset{1)}{a}) \in R$

→ It is Transitive as $(\underset{(1}{a}, \underset{2)}{b}) \in R$, $(\underset{(2,}{b}, \underset{1)}{c}) \in R \Rightarrow (\underset{(1,}{a}, \underset{1)}{c}) \in R$

∴ the given relation is equivalence

Example2:

If A = {1,2,3}

R = {(1,1), (1,2), (2,1) (2,2) }

Given $A = \{1,2,3\}$

$R = \{(1,1), (1,2)\ (2,1), (2,2)\}$

→ It is not reflexive as $(3,3) \notin R$

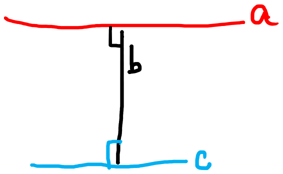
→ It is symmetric as $(\underset{(1}{a}, \underset{2)}{b}) \in R \Rightarrow (\underset{(2,}{b}, \underset{1)}{a}) \in R$

→ It is Transitive as $(\underset{(1,}{a}, \underset{2)}{b}) \in R$, $(\underset{(2,}{b}, \underset{1)}{c}) \in R \Rightarrow (\underset{(1,}{a}, \underset{1)}{c}) \in R$

Example3: On the sets of lines in a plane the Relation "is perpendicular to" is

a) Reflexive b) Symmetric c) Transitive d) none of these

Let a,b,c be 3 lines which are perpendicular to eachother



→ It is not reflexive as $a \not\perp a$

→ It is Symmetric as $a \perp b \Rightarrow b \perp a$

→ It is not transitive as $a \perp b$, $b \perp c$ but $a \not\perp c$

Example4:

If a relation R = {(1,1), (2,2), (1,2), (2,1)} is symmetric on A = {1,2,3} then R is

a) Reflexive but not Transitive b) Transitive but not Reflexive

c) Reflexive and Transitive d) Neither Reflexive nor Transitive

Given $A = \{1,2,3\}$

$R = \{(1,1),(2,2),(1,2)(2,1)\}$

→ It is not reflexive as $(3,3) \notin R$

→ It is Symmetric as $(1,2) \in R \Rightarrow (2,1) \in R$

→ It is Transitive as $(1,2) \in R, (2,1) \in R \Rightarrow (1,1) \in R$

Example5:

On the set of eggs "is smaller than" is

a)Reflexive b) Transitive c)Symmetric d)Equivalence

Let a,b,c be 3 eggs of different sizes

→ It is not reflexive as $a \nless a$

→ It is not Symmetric as $a < b \Rightarrow b \nless a$

→ It is Transitive as $a < b$, $b < c \Rightarrow a < c$

Function

Any relation is said to be function if the domain element occurs only once in the order pairs [i.e no domain element must occur more than once]

and all domain elements have images.
Consider :- R = {(1,1), (1,2), (2,1) (2,2) (3,3)}

It is not a function because the domain elements 1 & 2 have repeated.
R = {(1,2), (2,3), (3,4) (4,1) } It is a function because the domain element are not repeated.
Note :- All functions are relations but all relations are not function
First element in each order pairs is said to be a domain element
2nd element in each order pairs is said to be a range.

Type of functions

1.One – one function :-

If all domain elements have distinct images, then the function is said to be one - one function

2.Onto function :-

If all codomain elements have a distinct pre image in domain, then the function is Onto
Note :- A Function which is not ONTO , is said to be INTO function

3)One – one and onto [Bijective function]

A function which is both one – one and onto , is said to be bijective function.

4)Many – one function :-

If more than one domain element having same image then the function is said to be many – one

5).Constant function :-

If all domain element are having the same image then the function is said to be constant function

Note :- All constant functions are many one but all many one functions are not constant function

Inverse of a Function

- If f(x) is a function given then f^{-1} (x) is said to be the inverse of f(x)

To obtain inverse of f(x) it is required to interchange the domain & codomain elements given in the order pairs.

ex :- If f(x) = {(1,2), (2,3), (4,5), (3,6)}

Then f^{-1} (x) = {(2,1), (3,2), (5,4), (6,3)}

To find f^{-1} (x) when f(x) interms of x is given

Procedure:

Step 1 :- let y = f(x)

Step 2 :- express x in terms of y

Example1:

Find f^{-1} (x) If f(x) = 5x +9

Given $f(x) = 5x+9$

let $y = 5x+9$

$y-9 = 5x$

$\Rightarrow x = \boxed{\frac{y-9}{5}}$

$\therefore f^{-1}(x) = \underline{\underline{\frac{x-9}{5}}}$

Example2:

If $f(x) = \frac{3+x}{3-x}$, find f^{-1} (x)

Given $f(x) = \frac{3+x}{3-x}$

Let $y = \frac{3+x}{3-x} \Rightarrow 3y-xy = 3+x$

$3y-3 = xy+x$

$3(y-1) = x(y+1)$

$\Rightarrow x = \boxed{\frac{3(y-1)}{y+1}}$

$\therefore f^{-1}(x) = \frac{3(x-1)}{x+1}$

Function of a function

Example1:

If f(x) = 5x + 12 find f(3) and f(5x)

Given $f(x) = 5x+12$

(i) $f(3) = 5(3)+12$

$= \underline{\underline{27}}$

(ii) $f(5x) = 5(5x)+12 = \underline{\underline{25x+12}}$

Example2:

If f(x) = 2x + 7   g(x) = 4x -10 then find fog

$$f(x) = 2x+7 \qquad g(x) = 4x-10$$

$$fog = f[g(x)] = 2[g(x)] + 7$$

$$= 2[4x-10) + 7$$

$$= 8x - 20 + 7 = \underline{\underline{8x-13}}$$

Example3

If f(x) = 5x – 7, g(x) = x^2 + 5x + 5 find gof

$$f(x) = 5x-7 \qquad g(x) = x^2+5x+5$$

$$gof = g[f(x)] = [f(x)]^2 + 5[f(x)] + 5$$

$$= (5x-7)^2 + 5(5x-7) + 5$$

$$= 25x^2 + 49 - 70x + 25x - 35 + 5$$

$$= \underline{\underline{25x^2 - 45x + 19}}$$

Example4

If f(x) = x^2 + x – 1 and 4f(x) = f(2x) then x=?

$$f(x) = x^2 + x - 1 \;, \qquad 4f(x) = f(2x)$$

$$\Rightarrow 4(x^2+x-1) = (2x)^2 + (2x) - 1$$

$$\cancel{4x^2} + 4x - 4 = \cancel{4x^2} + 2x - 1$$

$$2x = 3$$

$$x = \frac{3}{2}$$

CONCEPT OF LIMIT

I) We consider a function f(x) = 2 x. If x is a number approaching to the number 2 then f(x) is anumber approaching to the value 2x2= 4

The following table shows f(x) for different values of x approaching 2

X	f(x)
1.90	3.8
1.99	3.98
1.999	3.998
1.9999	3.9998
2	4

Here x approaches 2 from values of x<2 and for x being very close to 2 f(x) is very close 4. This situation is defined as left-hand limit of f(x) as x approaches 2 and is written as lim f(x) = 4, $x \to 2-$

X	f(x)
2.0001	4.0002
2.001	4.002
2.01	1.02
2.0	4

Here x approaches 2 from values of x greater than 2 and for x being very close 2f(x) is very close to 4. This situation is defined as right-hand limit of f(x) as x approaches 2 andis written as lim f(x)= 4 as x $\to$ 2+

So we write

$$\lim_{x\to a-+} f(x) = \lim_{x\to a} f(x)$$

Useful Rules on limits

1. $\lim_{x \to a}[f(x) + g(x)]$ = $\lim_{x \to a} f(x)$ + $\lim_{x \to a} g(x)$
2. $\lim_{x \to a}[f(x) - g(x)]$ = $\lim_{x \to a} f(x)$ - $\lim_{x \to a} g(x)$
3. $\lim_{x \to a}[f(x).g(x)]$ = $\lim_{x \to a} f(x)$. $\lim_{x \to a} g(x)$
4. $\lim_{x \to a}[f(x)/g(x)]$ = $\lim_{x \to a} f(x)$ / $\lim_{x \to a} g(x)$
5. $\lim_{x \to a}(constant)$ = $constant$
6. $\lim_{x \to a}[C.f(x)]$ =C. $\lim_{x \to a} f(x)$ (where C is a constant)

Example1:

$\lim_{x \to 3}(4x + 5)$ = 4. $\lim_{x \to 3}(x)$ + 5

=(4.3)+5

=12+5

=17

Example2:

$\lim_{x \to 3} \frac{1}{x-2}$ = 1/(3-2) = 1

Example3:

$\lim_{x \to -3} \frac{x^2+7x+12}{x+3}$ = $\lim_{x \to -3} \frac{(x+3)(x+4)}{x+3)}$

= $\lim_{x \to -3}(x + 4)$

=-3+4

=1

Some important limits:

1. $\lim_{x \to a} \frac{x^n - a^n}{x-a}$ = na^{n-1}
2. $\lim_{x \to 0} \frac{a^x - 1}{x}$ = $\log_e a$ (a>0)
3. $\lim_{x \to 0} \frac{e^x - 1}{x}$ = 1

4. $\lim_{x \to 0} \frac{\log(1+x)}{x}$ = 1

5. $\lim_{x \to \infty} (1 + \frac{1}{x})^x$ = e

6. $\lim_{x \to 0} (1 + x)^{\frac{1}{x}}$ = e

7. $\lim_{x \to 0} \frac{(1+x)^n - 1}{x}$ = 1

Practice questions:

1. $\lim_{n \to \infty} (1 + \frac{2}{n})^n$
2. $\lim_{x \to 0} (1 + 3x)^{\frac{1}{x}}$
3. $\lim_{x \to 0} (1 + \frac{x}{3})^{\frac{1}{x}}$
4. $\lim_{x \to 0} \frac{e^{-3x} - 1}{x}$
5. $\lim_{x \to 0} \frac{2^x - 1}{3x}$
6. $\lim_{x \to \infty} \frac{3x^2 - 4x + 5}{2x^2 + 5x - 1}$
7. $\lim_{x \to \infty} \frac{3x^2 - 4x + 5}{2x^2 + 5x - 1}$
8. $\lim_{x \to \infty} \frac{3x^3 - 4x + 7}{2x^4 - 3x + 6}$
9. $\lim_{x \to \infty} \frac{(3x+4)(4x+3)}{(2x-7)(x+4)}$

10. $\lim_{x\to\infty} \frac{(2x-1)^{30}(3x-1)^{30}}{(2x+4)^{60}}$

CONTINUITY:

By the term continuous we mean some thing which goes on without interruption and without abrupt changes. Here in mathematics the term continuous carries the same meaning. Thus, we define continuity of a function in the following way.

A function f(x) is said to be continuous at x= a if and only if

(i) f(x) is defined x= a

ii) LHL =RHL

iii) $f(x) = f(a)$ as limit x tends to a

1. S.T f(x) defined by

$$f(x) = \begin{cases} \dfrac{x^2-25}{x-5}, & \text{when } x \neq 5 \\ 10, & \text{when } x = 5 \end{cases}$$

is Continuous at $x=5$

5. Find k for which $f(x) = \begin{cases} k+x, & x=1 \\ 4x+3 & x \neq 1 \end{cases}$

is Continuous at $x=1$

Practice questions:

1. A is {1,2,3,4} and B is {1,4,9,16,25} if a function f is defined from set A to B where f(x) = x^2 then the range of f is:

(a) {1,2,3,4} (b) {1,4,9,16}
(c) {1,4,9,16,25} (d) None of these

2. If A = {1,2} and B = {3, 4}. Determine the number of relations from A and B:

(a) 3 (b) 16
(c) 5 (d) 6

3. If A = {1,2,3,4,5,6,7} and B = {2, 4,6,8}. Cardinal number of A - B is:

a)4 b)3 c)9 d)7

4. If *A* = {1,2,3,4,5,6,7,8,9,}

B = {1,3,4,5,7,8};C = {2,6,8,} then find (A – B) U C =

(a) {2,6,} (b) {2,6,8}
c) {2,6,8,9} (d) None of these

5. If $f(x) = x^2$ and $g(x) = \sqrt{x}$

then

(a) go*f*(3)= 3 (b) go*f* (–3) = 9
(c) go*f*(9) = 3 (d) go*f* (–9) = 3

6. A= {1,2,3,4, ... 10} a relation on A,R ={(x, y)/x+ y =10, *x* belongs to A and y belongs to A and x is greater than equal to y}

then domain of R inverse is

(a) {1,2,3,4,5} (b) {0,3,5,7,9}
(c) {1,2,4,5,6,7} (d) None of these

7. The set of cubes of natural numbers is

Null set (b) Finite set
(c) Infinite set (d) A finite set of three numbers

8. Two finite sets respectively have x and y number of elements. The total number of sub sets of the first is 56 more than the total no. of sub sets of the second. The value of x, y are respectively ____________

a) 4 and 2 (b) 6 and 3
(c) 2 and 4 (d) 3 and 6

9. If f(x)=3x then its inverse is
a)x/3 b)1/x c)1/3x d) -3x

10. number of items in the set A is 40, in the Set B is 32; in the Set C is 50; in both A and B is 4; in both A and C is 5; in both B and C is 7; in all the set is 2. How many are in only one set?

a) 65
b) 110
c) 106
d) 84

11. If n(U) =650 n(A)=310 n(A ∩B)=95 and n(B)=190 then n(A' ∩B') is
a) 400 b) 200 c)300 d)245

12. The range of function f(x) =$\sqrt{(16-x^2}$ is
a) [-4,0] b) [-4,4] c)[0,4] d) (-4,4)

13. If f(x) =x^2-1 and g(x) =$|2x+3|$ then fog(3) – gof(-3) is
a)71 b)61 c) 41 d)25

14. If f(x) = $\frac{x-2}{x-3}$ then f^{-1}(1/2) is
a) 2/3 b) ¾ c) 1 d) -1

15. *f*(*x*) = *f*(*x*–1)+*f*(*x*–2) if f (0)= 0, f(1) = 1, x = 2, 3, 4,……. then what is *f* (7)
(a) 8 (b) 13
(c) 3 (d) 5

16. If f(x) = $2x^3$+1 then f^{-1}(x) is

17. If A ={*a, b, c, d*}; B = {*p, q, r ,s*} which of the following relation is a function from A to B
(a) R1= {(*a*, *p*), (*b*, *q*),(*c*, *s*)}
(b) R2 ={(*p*, *a*}, (*b*, *r*),(d, s)}
(c) R3 ={(*b*, p),(c, s),(b, r)}
(d) R4 = {(*a*, p)(*b*, *r*)(*c*, *q*), (d, s)}

Chapter 8- Differential and integral calculus

Calculus

Calculus means calculation

Differential calculus :

→ Differentiation : It is the process of finding the derivative of a function.

Derivative is rate at which the function changes.

Ex: $y= x^2 + 2x$

If y is a function of x then $\frac{dy}{dx}$ is said to be derivative of the function.

Derivatives of some standard functions

- $\frac{\boldsymbol{d(x^n)}}{\boldsymbol{dx}} = n.\ x^{n-1}$

'n' is a constant
ex : $\frac{d}{dx}(x^5) = 5.x^4$

- $\frac{d}{dx}(a^x) = a^x .\log a$
  'a' is a constant
  'x' is variable

ex : $\frac{d}{dx}(2^x) = 2^x.\ \log 2$

- $\frac{d}{dx}(\log x) = \frac{1}{x}$

Note: Here the base is e

- $\frac{d}{dx}(\log_a x) = \ = \frac{1}{xloga}$

Ex : $\frac{d}{dx}\ (\log_3 x) = \frac{1}{xlog3}$

- $\frac{d}{dx}\ (e^x) = e^x$

- $\frac{d}{dx}\ (\text{Constant}) = 0$

- $\frac{d}{dx}\ (\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Sum rule : {When separated with + or -}

If you have $\frac{d}{dx}$ (u + v) ‘ where u & v are functions of x,

$\frac{d}{dx}$ (u ± v) = $\frac{d(u)}{dx} \pm \frac{d(v)}{dx}$

Eg :- y = $e^x + 2^x + x^2$ find $\frac{dy}{dx}$

$$y = e^x + 2^x + x^2$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}(e^x) + \frac{d}{dx}(2^x) + \frac{d}{dx}(x^2)$$

$$= \underline{e^x + 2^x \log 2 + 2x}$$

Example1:

If y = $2x^3 - 5e^x + 2 \log x - 3^x + 10$ find $\frac{dy}{dx}$

$$y = 2x^3 - 5e^x + 2\log x - 3^x + 10$$

Diff w.r.t x

$$\frac{dy}{dx} = 2\frac{d}{dx}(x^3) - 5\frac{d}{dx}(e^x) + 2\frac{d}{dx}(\log x) - \frac{d}{dx}(3^x) + \frac{d}{dx}(10)$$

$$= 2(3x^2) - 5e^x + 2\left(\frac{1}{x}\right) - 3^x \log 3 + 0$$

$$= \underline{6x^2 - 5e^x + \frac{2}{x}} - 3^x \log 3$$

Example2:

If y = $\frac{3}{x}$ -4$\sqrt{x}$ + 6 $\log_2 x$. find $\frac{dy}{dx}$

$$y = \frac{3}{x} - 4\sqrt{x} + 6\log_2 x$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{-3}{x^2} - 4\left(\frac{1}{2\sqrt{x}}\right) + 6\left(\frac{1}{x\log 2}\right)$$

$$= \frac{-3}{x^2} - \frac{2}{\sqrt{x}} + \frac{6}{x\log 2}$$

Product Rule :

$\frac{d}{dx}$ (uv) = $\frac{u\, d(v)}{dx}$ + $\frac{v\, d(u)}{dx}$ (where u and v are functions of x)

OR

$\frac{d}{dx}$ (I x II) = I $\frac{d}{dx}$ (II) + II $\frac{d}{dx}$ (I)

Example1:

If y = $\sqrt{x}$ e^x, find $\frac{dy}{dx}$

$y = \sqrt{x}\ e^x$

Diff. w.r.t x

$$\frac{dy}{dx} = \sqrt{x}\ \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(\sqrt{x})$$

$$= \sqrt{x}\, e^x + e^x \cdot \frac{1}{2\sqrt{x}}$$

$$= e^x \left(\sqrt{x} + \frac{1}{2\sqrt{x}} \right) = e^x \left(\frac{2x+1}{2\sqrt{x}} \right)$$

Example2:

If y = x^5 5^x, find $\frac{dy}{dx}$

$y = x^5 \cdot 5^x$

Diff w.r.t x

$$\frac{dy}{dx} = x^5 (5^x \cdot \log 5) + 5^x (5x^4)$$

$$= 5^x x^4 (x \log 5 + 5)$$

Example3:

If y = x .e^x. log x then find $\frac{dy}{dx}$

$$y = x \cdot e^{x} \cdot \log x$$

Diff w.r.t x

$$\frac{dy}{dx} = x e^{x} \frac{d}{dx}(\log x) + x \log x \frac{d}{dx}(e^{x}) + e^{x} \cdot \log x \frac{d}{dx}(x)$$

$$= x e^{x} \cdot \frac{1}{x} + x \log x \, e^{x} + e^{x} \cdot \log x \, (1)$$

$$= e^{x} \left(1 + x \log x + \log x \right)$$

Quotient rule

If y = $\left(\frac{u}{v}\right)$, where u & v are functions of x

Then,

$$\frac{dy}{dx} = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$$

Example1:

Find $\frac{dy}{dx}$ if y = $\frac{2x^2+5}{5x^2+6}$

$$y = \frac{2x^2+5}{5x^2+6}$$

Diff w.r.t x

$$\frac{dy}{dx} = \frac{(5x^2+6)\frac{d}{dx}(2x^2+5) - (2x^2+5)\frac{d}{dx}(5x^2+6)}{(5x^2+6)^2}$$

$$= \frac{(5x^2+6)(4x) - (2x^2+5)(10x)}{(5x^2+6)^2}$$

$$= \frac{\cancel{20x^3} + 24x - \cancel{20x^3} - 50x}{(5x^2+6)^2} = \frac{-26x}{(5x^2+6)^2}$$

Short cut

If $y = \frac{af(x)+b}{cf(x)+d}$

Then $\frac{dy}{dx} = \frac{(ad-bc)}{[cf(x)+d]^2} \cdot f'(x)$

Given

$$y = \frac{2x^2+5}{5x^2+6}$$

$$\frac{dy}{dx} = \frac{[(2\times6)-(5\times5)] \cdot 2x}{(5x^2+6)^2} = \frac{-26x}{(5x^2+6)^2}$$

Example2:

If y = $\frac{3\log x+6}{4\log x-10}$ find $\frac{dy}{dx}$

$$y = \frac{3\log x+6}{4\log x-10}$$

$$\frac{dy}{dx} = \frac{(-30-24)}{(4\log x-10)^2} \cdot \frac{1}{x} = \frac{-54}{x(4\log x-10)^2}$$

Example3:

If y = $\frac{x+1}{x-1}$ find $\frac{dy}{dx}$

$$y = \frac{x+1}{x-1}$$

$$\frac{dy}{dx} = \frac{(-1-1) \cdot 1}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

Derivatives of composite functions (functions within a function):-

Chain Rule

For the function

$Y = (x^2 + 5x + 6)^5$

$(x^2 + 5x + 6) \rightarrow$ will be treated as x

i.e

Note:

- ***Priority is given for the constant power.***
- ***If there is no constant power then the next priority is given to the outer function***

Example1:

Find $\frac{dy}{dx}$ if y = log (log x)

$$y = \log(\underline{\log x})$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{1}{\log x} \frac{d}{dx}(\log x)$$

$$= \frac{1}{\log x} \cdot \left(\frac{1}{x}\right)$$

$$= \frac{1}{x \cdot \log x}$$

Example2:

Find $\frac{dy}{dx}$ if $Y = e^{x^2+3x+10}$

$$y = e^{\underline{x^2+3x+10}}$$

Diff w.r.t x

$$\frac{dy}{dx} = e^{x^2+3x+10} \frac{d}{dx}(x^2+3x+10)$$

$$= \underline{\underline{e^{x^2+3x+10} (2x+3)}}$$

Example3:

Find $\frac{dy}{dx}$ if Y = 2^{-3x}

$y = 2^{\underset{\sim}{-3x}}$

Diff w.r.t x

$\frac{dy}{dx} = 2^{-3x} \cdot \log 2 \frac{d}{dx}(-3x)$

$= \underline{\underline{-3 \cdot 2^{-3x} \cdot \log 2}}$

Example4:

Find $\frac{dy}{dx}$ if Y = e^{-3x}

$y = e^{\underset{\sim}{-3x}}$

Diff w.r.t x

$\frac{dy}{dx} = e^{-3x} \frac{d}{dx}(-3x)$

$= \underline{\underline{-3e^{-3x}}}$

Example5:

Find $\frac{dy}{dx}$ if $5^{x^2+3x+10}$

$y = 5^{\underset{\sim}{x^2+3x+10}}$

Diff w.r.t x

$\frac{dy}{dx} = 5^{x^2+3x+10} \cdot \log 5 \frac{d}{dx}(x^2+3x+10)$

$= \underline{\underline{5^{x^2+3x+10} \cdot \log 5 \, (2x+3)}}$

Parametric differentiation (2 parameters will be given)

Example1:x = $t^2 + 5t + 1$ y = 2t + 10 find $\frac{dy}{dx}$

$x = t^2+5t+1$

Diff wrt t

$\frac{dx}{dt} = 2t+5 \rightarrow$ ①

$y = 2t+10$

Diff wrt t

$\frac{dy}{dt} = 2 \rightarrow$ ②

$\frac{②}{①} \Rightarrow \frac{\frac{dy}{\cancel{dt}}}{\frac{dx}{\cancel{dt}}} = \frac{2}{2t+5}$

$\Rightarrow \frac{dy}{dx} = \underline{\underline{\frac{2}{2t+5}}}$

Example2:

If x = 2t + 5 and y =t^2– 5 find $\frac{dy}{dx}$

a) t, b) -1/t, c) – 1/t, d) none

$x = 2t+5$

$\frac{dx}{dt} = 2 \rightarrow$ ①

$y = t^2-5$

$\frac{dy}{dt} = 2t \rightarrow$ ②

$\frac{②}{①} \Rightarrow \frac{dy}{dx} = \frac{\cancel{2}t}{\cancel{2}} = \underline{\underline{t}}$

Example3:

If x = at^2 , y = 2at find dy/dx = ?

a) 1/t, b) -1/t, c) t d) none

$x = at^2$

$\frac{dx}{dt} = 2at \rightarrow$ ①

$y = 2at$

$\frac{dy}{dt} = 2a \rightarrow$ ②

$\frac{②}{①} \Rightarrow \frac{dy}{dx} = \frac{2a}{2at} = \underline{\underline{\frac{1}{t}}}$

Example4:

If x = log t , y = e^t find $\frac{dy}{dx}$

a) 1/t, b) t xe^t c) -1/t^2 d) none

$x = \log t$ $\qquad\qquad$ $y = e^t$

$\frac{dx}{dt} = \frac{1}{t} \rightarrow ①$ $\qquad\qquad$ $\frac{dy}{dt} = e^t \rightarrow ②$

$$\frac{②}{①} \Rightarrow \frac{dy}{dx} = \frac{e^t}{\frac{1}{t}} = \underline{\underline{t.e^t}}$$

Implicit functions :- (x & y on same side)

Example1 :- x^2 + y^2 + xy = 5

$x^2+y^2+xy = 5$

Diff w.r.t x

$2x + 2y\frac{dy}{dx} + \left[x\frac{dy}{dx} + y(1)\right] = 0$

$2y\frac{dy}{dx} + x\frac{dy}{dx} = -2x-y$

$\frac{dy}{dx}\left[2y+x\right] = -(2x+y)$

$\frac{dy}{dx} = \frac{-(2x+y)}{(2y+x)}$

Short cut

$$\frac{dy}{dx} = -\left[\frac{\text{Diff } x \text{ w.r.t } x \text{ taking } y \text{ as constant}}{\text{Diff } y \text{ w.r.t } x \text{ taking } x \text{ as constant}}\right]$$

Given $x^2+y^2+xy = 5$

$\Rightarrow x^2+y^2+xy-5 = 0$

$$\therefore \frac{dy}{dx} = -\left[\frac{2x + y(1)}{2y + x(1)}\right] = -\left(\frac{2x+y}{2y+x}\right)$$

Example2:

If x^3 + y^3 + log x + log y = 25 find $\frac{dy}{dx}$

$$x^3+y^3+\log x+\log y-25=0$$

$$\frac{dy}{dx}=-\left[\frac{3x^2+\frac{1}{x}}{3y^2+\frac{1}{y}}\right]=-\left[\frac{\frac{3x^3+1}{x}}{\frac{3y^3+1}{y}}\right]=-\frac{y}{x}\left[\frac{3x^3+1}{3y^3+1}\right]$$

Example3:

If $e^{xy} - 4xy = 4$ find $\frac{dy}{dx}$

a) y/x b) –y/x c) x/y d) –x/y

$$e^{xy}-4xy-4=0$$

$$\frac{dy}{dx}=-\left[\frac{e^{xy}.y-4y(1)}{e^{xy}.x-4x(1)}\right]$$

$$=-\frac{y\,[\cancel{e^{xy}-4}]}{x\,(\cancel{e^{xy}-4})}=\underline{\underline{-\frac{y}{x}}}$$

Example4:

If $xy(x-y) = 0$, find $\frac{dy}{dx}$

$$xy(x-y)=0$$

$$\Rightarrow x^2y-xy^2=0$$

$$\frac{dy}{dx}=-\left[\frac{2xy-y^2}{x^2(1)-2yx}\right]$$

$$=-\frac{y}{x}\frac{(2x-y)}{(x-2y)}=\underline{\frac{y}{x}\frac{(2x-y)}{(2y-x)}}$$

Logarithmic differentiation

(i) $Y = x^5$ ii) $y = 5^x$

Base is variable Base is a constant

Power is a constant Power is a variable

Y = x^x Here both base and powers are variables so (i) & (ii) cant be applied → that's when we use logarithmic differentiation

Procedure:

1) Apply log on both sides
2) Differentiate both sides w.r.t x to find $\frac{dy}{dx}$.

Example1:

If y = x^x then find $\frac{dy}{dx}$

$y = x^x$

Apply log on B.S

$\log y = \log x^x$

$\log y = x \log x \quad (\because \log m^k = k \log m)$

Diff w.r.t x

$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x\,(1)$

$\frac{dy}{dx} = y(1+\log x)$

$= x^x(1+\log x) \quad (\because y = x^x)$

Short cut

$\frac{dy}{dx} = y\left[\frac{P}{B}\frac{d}{dx}(B) + \log(B)\frac{d}{dx}(P)\right]$

P → power B → Base

Given $y = x^x$

$\frac{dy}{dx} = y\left[\frac{x}{x}(1) + \log(x)\,(1)\right]$

$= x^x(\underline{1+\log x})$

Example2:

If y = $\left(\sqrt{x}\right)^{\log x}$ find $\frac{dy}{dx}$

$$y = (\sqrt{x})^{\log x}$$

$$\frac{dy}{dx} = y\left[\frac{\log x}{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right) + \log\sqrt{x}\left(\frac{1}{x}\right)\right]$$

$$= y\left[\frac{\log x}{2x} + \frac{\log x^{1/2}}{x}\right]$$

$$= y\left[\frac{\log x}{2x} + \frac{1}{2}\frac{\log x}{x}\right]$$

$$= y\left[\cancel{2}\cdot\frac{\log x}{\cancel{2}x}\right] = y\cdot\frac{\log x}{x}$$

Example3:

If y = (log x)^x find $\frac{dy}{dx}$

$$y = (\log x)^x$$

$$\frac{dy}{dx} = y\left[\frac{\cancel{x}}{\log x}\left(\frac{1}{\cancel{x}}\right) + \log(\log x)\,(1)\right]$$

$$= y\left[\frac{1}{\log x}\underline{\underline{+\log}}(\log x)\right]$$

Successive Differentiation

(Higher order derivative)

Y = x^3 => $\frac{dy}{dx}$ = 3 x^2 ← Ist order derivative

Diff $\frac{dy}{dx}$ w.r.t x →$\frac{d^2y}{dx^2}$ = 6x ← II order derivative

Y can be expressed as f(x) → function of x.

$\frac{dy}{dx}$ => $f^1(x)$

$\frac{d^2y}{dx^2}$ => $f^{11}(x)$

Example1:

If $y = ae^{mx} + be^{-mx}$ then $\frac{d^2y}{dx^2}$ = ?

a) m^2y b) $-m^2y$ c) my d) none

$$y = ae^{mx} + be^{-mx}$$

Diff w.r.t x

$$\frac{dy}{dx} = a(me^{mx}) + b(-me^{-mx})$$

Diff w.r.t x

$$\frac{d^2y}{dx^2} = am(me^{mx}) - bm(-me^{-mx})$$

$$= m^2ae^{mx} + m^2be^{-mx}$$

$$= m^2(ae^{mx} + be^{-mx})$$

$$= \underline{m^2y} \qquad [\because y = ae^{mx} + be^{-mx}]$$

Example2:

If y= $2x + \frac{4}{x}$ then $\frac{x^2\, d^2y}{dx^2} + x\frac{dy}{dx} - y$ =

a) 3 b) 1 c) 0 d) 4

$$y = 2x + \frac{4}{x}$$

Diff w.r.t x

$$\frac{dy}{dx} = 2 - \frac{4}{x^2}$$

Diff w.r.t x

$$\frac{d^2y}{dx^2} = 0 - \left(-\frac{8}{x^3}\right) = \frac{8}{x^3}$$

Now $$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2\left(\frac{8}{x^3}\right) + x\left(2 - \frac{4}{x^2}\right) - 2x - \frac{4}{x}$$

$$= \frac{8}{x} + \cancel{2x} - \frac{4}{x} - \cancel{2x} - \frac{4}{x} = \cancel{\frac{8}{x}} - \cancel{\frac{8}{x}} = \underline{0}$$

Derivatives of infinite series :-

Example1:

If Y = $x^{x^{x^{\cdots\cdots\infty}}}$

$$y = x^{x^{x^{x^{\cdots\infty}}}}$$

$$y = x^{y} \qquad \left[\because y = x^{x^{x^{\cdots\infty}}}\right]$$

Take log on B.S

$$\log y = \log x^{y}$$

$$\log y = y\log x \qquad [\because \log m^{k} = k\log m]$$

Diff w.r.t x

$$\frac{1}{y}\frac{dy}{dx} = y\left(\frac{1}{x}\right) + \log x\,\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$$

$$\frac{dy}{dx}\left(\frac{1-y\log x}{y}\right) = \frac{y}{x} \Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}$$

Short cut

If $y = f(x)^{f(x)^{f(x)^{\cdots\infty}}}$

Then $\frac{dy}{dx} = \frac{y^2}{1-y\log f(x)} \cdot \frac{f'(x)}{f(x)}$

Given $y = x^{x^{x^{\cdots\infty}}} \Rightarrow f(x) = x$ & $f'(x) = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2}{1-y\log x} \cdot \frac{1}{x} = \frac{y^2}{x(1-y\log x)}$$

Example2:

If y = $\sqrt{x}^{\sqrt{x}^{\cdots\cdots\infty}}$ Find $\frac{dy}{dx}$

$$y = \sqrt{x}^{\sqrt{x}^{\cdots\infty}}$$

$$f(x) = \sqrt{x}. \qquad f'(x) = \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{y^2}{1 - y\log\sqrt{x}} \cdot \frac{1}{2\sqrt{x}\cdot\sqrt{x}} = \frac{y^2}{1 - \frac{1}{2}y\log x} \cdot \frac{1}{2x}$$

$$= \frac{y^2}{\frac{2 - y\log x}{2}} \cdot \frac{1}{\cancel{2}x} = \underline{\underline{\frac{y^2}{x(2 - y\log x)}}}$$

Note:-

- **If x^m y^n = $(x + y)^{m+n}$**

Then,

$\frac{dy}{dx} = \frac{y}{x}$

$eg:-$ **i) $x^3 y^2 = (x + y)^5$**

$\frac{dy}{dx} = \frac{y}{x}$

ii) $x^p y^q = (x+y)^{p+q}$

$\frac{dy}{dx} = \frac{y}{x}$

Application of Derivatives

- **Slope of the tangent (Gradient to the cure)**

  eg : The slope of the tangent to the curve y = $\sqrt{4 + x^2}$ at the points where the ordinate & abscissa are equal.

a) -1, b) 1, c) 0, d) None

Given $x = y$

$y = \sqrt{4+x^2}$

Slope of the tangent $\Rightarrow \frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{4+x^2}}(\cancel{2}x)$

$= \frac{\cancel{x}}{\cancel{y}} \quad [\because y = \sqrt{4+x^2}]$

$= \underline{\underline{1}}$

- **<u>Average cost, marginal cost, Average revenue & marginal Revenue</u>**

If c(x) is a cost function given by
=> c(x) = x^2 + 5x +10

x → Number of units of production

Aveg cost = $\frac{C(x)}{x}$

Marginal cost = $\frac{d}{dx}$ [c(x)]

If R(x) is a Revenue function given by

R(x) = x^3 + $5x^2$ + 6x + 15

Avg Revenue = $\frac{R(x)}{x}$

Marginal cost = $\frac{d}{dx}$ [R(x)]

<u>Note:</u>

- Revenue function= Demand function× x
- The profit is maximum at equilibribum point under perfect competition which occurs when Marginal revenue = Marginal cost
- Profit is zero at break even point which occurs when total cost = total revenue

Maximum & minimum value of a function :

Step 1 :- find $\frac{dy}{dx}$ & equate it to 0

Find the value for x.

Step 2 :- find $\frac{d^2y}{dx^2}$ & substitute the above value of x

If $\frac{d^2y}{dx^2} < 0$ then the function will have max value at that value of x.

If $\frac{d^2y}{dx^2} > 0$ then the functions will have min value at that value of x.

If $\frac{d^2y}{dx^2} = 0$ then the function has neither a max nor a min value.

Example

If $y = 2x^3 - 15x^2 + 36x + 15$ then find value of x at which the function will have min value, also find min value of a function

$$y = 2x^3 - 15x^2 + 36x + 15$$

$$\frac{dy}{dx} = 6x^2 - 30x + 36 = 0$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

6

$-3 \quad -2$

$x = 3$ or $\underline{2}$

$$\frac{d^2y}{dx^2} = 12x - 30$$

$$\left.\frac{d^2y}{dx^2}\right|_{\text{at } x=3} = 12(3) - 30 = 6 > 0$$

∴ The function has minimum value at $x=3$.

Now $y_{min} = 2(3)^3 - 15(3)^2 + 36(3) + 15$

$= \underline{42}$

Integral Calculus

$\frac{d}{dx}(e^x) = e^x$	$\int e^x \, dx = e^x + c$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{e^{ax}}{a} + c$
$\frac{d}{dx}(\log x) = \frac{1}{x}$	$\int 1/x \, dx = \log x + c$
$\frac{d}{dx}(a^x) = a^x \times \log a$	$\int a^x dx = \frac{a^x}{\log a} + c$
$\frac{d}{dx}(x) = 1$	$\int 1 dx = x + c$
$\frac{d}{dx}(x^n) = n \; x^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$

PROBLEMS ON SUM RULE

Example1

$$\int (e^{-3x} + 2x + 3x^3 - \frac{4}{x}) \, dx$$

$$\int (e^{-3x} + 2x + 3x^3 - \frac{4}{x})\, dx$$

$$= \int e^{-3x} dx + 2\int x\, dx + 3\int x^3\, dx - 4\int \frac{1}{x}\, dx$$

$$= \frac{e^{-3x}}{-3} + \cancel{2}\cdot\frac{x^2}{\cancel{2}} + 3\,\frac{x^4}{4} - 4 \log x + c$$

$$= -\frac{e^{-3x}}{3} + x^2 + \frac{3}{4}x^4 - 4\log x + c$$

Example2:

$\int (3x^2 - 2e^{2x} + 5 - 3^x)dx$

$$\int (3x^2 - 2e^{2x} + 5 - 3^x)\, dx$$

$$= \cancel{3}\frac{x^3}{\cancel{3}} - \cancel{2}.\frac{e^{2x}}{\cancel{2}} + 5x - \frac{3^x}{\log 3} + c$$

$$= x^3 - e^{2x} + 5x - \frac{3^x}{\log 3} + c$$

DEFINITE INTEGRALS

Eg:

$\int_1^4 x^2 dx =$

$$\int_1^4 x^2\, dx = \left.\frac{x^3}{3}\right]_1^4 = \frac{3^4}{3} - \frac{1^4}{3}$$

$$= \frac{81-1}{3}$$

$$= \underline{\frac{80}{3}}$$

Property of definite integral

$$\int_a^b f(x)dx = \int_a^b f\,[(a+b) - x]dx$$

Example1:

$\int_0^1 (e^x + e^{-x})dx$ is:

$$\int_0^1 (e^x + e^{-x})\,dx$$
$$= e^x\Big]_0^1 - e^{-x}\Big]_0^1$$
$$= (e^1 - e^0) - (e^{-1} - e^0)$$
$$= e^1 - \cancel{1} - e^{-1} + \cancel{1}$$
$$= \underline{\underline{e - \frac{1}{e}}}$$

Example2:

$\int_1^2 7x^6\,dx$:

$$\int_1^2 7x^6\,dx = \cancel{7}\cdot\frac{x^7}{\cancel{7}}\Big]_1^2$$
$$= 2^7 - 1^7$$
$$= 128 - 1 = \underline{\underline{127}}$$

Example3:

$$\int_2^3 f(x)dx - \int_2^3 f(5-x)dx = \ ?$$

$$\int_2^3 f(x)\,dx - \int_2^3 f(5-x)\,dx = 0$$

$$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

Integration by substitution:

Example1:

$$\int_1^2 \frac{2x}{1+x^2}dx:$$

a) $\log_e \frac{5}{2}$ b) $\log_e 5 - \log_e 2 + 1$

c) $\log_e \frac{2}{5}$ d) None of these

$$\int_1^2 \frac{2x}{1+x^2}\,dx = \log(1+x^2)\Big]_1^2$$

$$\left[\because \int \frac{f'(x)}{f(x)}\,dx = \log f(x)\right]$$

$$= \log(1+2^2) - \log(1+1^2)$$

$$\Rightarrow \log 5 - \log 2$$

$$\Rightarrow \log\left(\frac{5}{2}\right) \text{ or } \log_e\left(\frac{5}{2}\right)$$

Example2:

$$\int \frac{8x^2}{(x^3+2)^3}dx$$

a) $-\frac{4}{3}(x^3+2)^2+C$ b) $-\frac{4}{3}(x^3+2)^{-2}+C$

c) $\frac{4}{3}(x^3+2)^2+C$ d) None of these

$$\int \frac{8x^2}{(x^3+2)^3}dx = \frac{8}{3}\int \frac{3x^2}{(x^3+2)^3}dx = \frac{8}{3}\int (x^3+2)^{-3}\cdot 3x^2\,dx$$

$$= \frac{8}{3}\cdot\frac{(x^3+2)^{-2}}{-2} + C \qquad \left[\because \int [f(x)]^n\cdot f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C\right]$$

$$= -\frac{4}{3}(x^3+2)^{-2} + C$$

Example3:

$\int \frac{1}{x(1+\log x)^2}dx$is equal to

a) $-\frac{1}{2(1+\log x)^2}+C$ b) $\frac{1}{(1+\log x)}+C$

c) $-\frac{1}{(1+\log x)}+C$ d) None of these

$$\int \frac{1}{x(1+\log x)^2} dx = \int (1+\log x)^{-2} \cdot \frac{1}{x} dx$$

$$= \frac{(1+\log x)^{-1}}{-1} + c \quad \left[\because \int [f(x)]^n \cdot f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c\right]$$

$$= \frac{-1}{1+\log x} + c$$

-

Example4:

Solve: $\int \frac{(\log x^x)^2}{x^3} dx$

a) $\frac{3}{2}(\log x)^3 + C$ b) $\frac{1}{3}(\log x)^3 + C$

c) $\frac{1}{6}(\log x)^3 + C$ d) $\frac{3}{7}(\log x)^3 + C$

$$\int \frac{(\log x^x)^2}{x^3} dx$$

$$= \int \frac{(x \log x)^2}{x^3} dx = \int \frac{x^2}{x^3} \cdot (\log x)^2$$

$$= \int \frac{1}{x} \cdot (\log x)^2 dx$$

$$= \frac{(\log x)^3}{3} + c$$

Integration by parts:

Integrals which can not be solved using substitution be solved using parts.

$\int u\, v$ dx = u $\int v$ - $\int \int v\, du$

Ex:

$\int x$.e^x dx

Note: One must follow the Priority LAE (L: log , A: Algrebaic and E : Exponential)

According to the priority x gets the priority over e^x . Therefor x is taken as u where as e^x is treated as v

$$\int \underset{\substack{\downarrow \\ u}}{x}\,\underset{\substack{\downarrow \\ v}}{e^x}\,dx = u\int v - \int\int v\,du \rightarrow ①$$

$$\boxed{\begin{array}{l} u = x \Rightarrow du = 1 \\ \int v = \int e^x = e^x \end{array}}$$

$$① \Rightarrow \int xe^x\,dx = x\,e^x - \int e^x \cdot 1$$
$$= xe^x - \underline{\underline{e^x + c}}$$

Short <u>cut</u> :- $\int uv = uv_1 - u'v_2 + u''v_3 - \ldots$

where $u' \rightarrow$ derivative of u

$u'' \rightarrow$ Derivative of u'

$v_1 \rightarrow$ Integral of v_1 & $v_2 \rightarrow$ Integral of v_1

$$\therefore \int x\,e^x = (x)(e^x) - (1)\,e^x + c$$
$$= \underline{\underline{xe^x - e^x + c}}$$

<u>Example 1</u>

$\int X^2\, e^{2x}\, dx$

$$\int \underset{\substack{\downarrow \\ u}}{x^2}\,\underset{\substack{\downarrow \\ v}}{e^{2x}}\,dx = (x^2)\left(\frac{e^{2x}}{2}\right) - (2x)\left(\frac{e^{2x}}{4}\right) + (2)\left(\frac{e^{2x}}{8}\right) + c$$
$$= x^2\frac{e^{2x}}{2} - x\frac{e^{2x}}{2} + \frac{e^{2x}}{4} + \underline{\underline{c}}$$

<u>Example2</u>:

$\int_1^4 xe^{3x}\,dx$

$$\int_1^4 x e^{3x}\, dx = x\,\frac{e^{3x}}{3}\Big]_1^4 - 1\cdot\frac{e^{3x}}{9}\Big]_1^4$$

$$= \frac{4e^{12}}{3} - \frac{1\cdot e^{3}}{3} - \left[\frac{e^{12}}{9} - \frac{e^{3}}{9}\right]$$

$$= e^{12}\left(\frac{4}{3}-\frac{1}{9}\right) - e^{3}\left(\frac{1}{9}-\frac{1}{3}\right]$$

$$= e^{12}\left(\frac{11}{9}\right) + \frac{2}{9}e^{3}$$

Example3:

$\int$ logx dx

$$\int \log x\, dx = \int \underset{u}{\underset{\downarrow}{\log x}} \cdot \underset{v}{\underset{\downarrow}{1}}\, dx\ .$$

$$\int \underset{u}{\underset{\downarrow}{\log x}} \cdot \underset{v}{\underset{\downarrow}{1}}\, dx = u\int v - \int\int v\, du \rightarrow ①$$

$$\boxed{\begin{array}{l} u = \log x \Rightarrow du = \frac{1}{x} \\ \int v = \int 1 = x \end{array}}$$

$$① \Rightarrow \int \log x = \log x \cdot x - \int \cancel{x}\cdot\frac{1}{\cancel{x}}$$

$$= x\log x - \underline{\underline{x + c}}$$

Integration by Partial fraction

Type1:

Example1

$$\int \frac{1}{(x+3)\,(x+2)}\, dx$$

$$\int \frac{1}{(x+3)(x+2)} = \int \frac{A}{x+3} + \int \frac{B}{x+2}$$

$$= A\log(x+3) + B\log(x+2)$$

To find A put $x=-3$ on LHS except $(x+3)$

we get $A = -1$

To find B, put $x=-2$ except $(x+2)$

$$B = \frac{1}{1} = \underline{\underline{1}}$$

$$\therefore \int \frac{1}{(x+3)(x+2)} = -\log(x+3) + \log(x+2) = \log\left(\frac{x+2}{x+3}\right) + c$$

Example2

$$\int \frac{8x+3}{(x+3)\,(x+5)}\,dx$$

$$\int \frac{8x+3}{(x+3)(x+5)}\,dx = \int \frac{A}{x+3} + \int \frac{B}{x+5}$$

$$= A\log(x+3) + B\log(x+5)$$

$A = -\frac{21}{2}$ on sub $x=-3$

$B = \frac{-37}{-2} = \frac{37}{2}$ on sub $x=-5$

$$\int \frac{8x+3}{(x+3)(x+5)}\,dx = -\frac{21}{2}\log(x+3) + \frac{37}{2}\log(x+5)$$

Example3:

$$\int \frac{6x+4}{(x-2)\,(x-3)}$$

$$\int \frac{6x+4}{(x-2)(x-3)} = \int \frac{A}{x-2}\,dx + \int \frac{B}{x-3}\,dx$$

$$= A\log(x-2) + B\log(x-3) + c$$

$$= \frac{16}{-1}\log(x-2) + 22\log(x-3) + c$$

$$= -16\log(x-2) + 22\log(x-3) + c$$

Type2:

Example1:

$$\int \frac{3x+5}{(x-4)\,(x-5)^2} dx$$

$$\int \frac{3x+5}{(x-4)(x-5)^2} = \int \frac{A}{x-4} dx + \int \frac{B}{x-5} dx + C \int \frac{1}{(x-5)^2} dx$$

put $x = 4$ to find B

$B = \frac{17}{1} = 17$ $\quad A = -17 \quad (\because A = -B)$

put $x = 5$ to find C

$C = 20$

$$\therefore \int \frac{3x+5}{(x-4)(x-5)^2} dx = -17 \log(x-4) + 17 \log(x-5) - 20 \frac{1}{(x-5)} + c$$

$$= 17 \log\left(\frac{x-5}{x-4}\right) - \frac{20}{x-5} + c$$

Example2:

$$\int \frac{2x+3}{(x-3)\,(x-5)^2} dx$$

$$\int \frac{2x+3}{(x-3)(x-5)^2} dx = \int \frac{A}{x-3} dx + \int \frac{B}{x-5} dx + \int \frac{C}{(x-5)^2} dx$$

$$= \frac{9}{4} \log\left(\frac{x-5}{x-3}\right) - \frac{13}{2}\left(\frac{1}{x-5}\right) + c$$

Type3:

$$\int \frac{3x^2-2x+5}{(x-1)\,(x^2+5)} dx$$

a) Log[(x-1)((x^2 +5)] +c
b) log(x^2 +5)+c
b) Log(x-1) +c
d) none

$$\int \frac{3x^2-2x+5}{(x-1)(x^2+5)}\,dx$$

We shall Solve this by option hitting method

Option a) $\log (x-1)(x^2+5)$

On Differentiating we get $\Rightarrow \frac{1 \quad (x-1)(2x)+(x^2+5)}{(x-1)(x^2+5)}$

$$= \frac{2x^2-2x+x^2+5}{(x-1)(x^2+5)} = \frac{3x^2-2x+5}{(x-1)(x^2+5)}$$

$\therefore$ Option (a) is the Correct answer.

Special integrals

(Degree of x is same in both numerator and denominator)

Example1:

$$\int \frac{x+1}{x+2}\,dx$$

$$\int \frac{x+1}{x+2}\,dx = \int \frac{(x+2)-1}{x+2}\,dx$$

$$= \int \left(\frac{x+2}{x+2}\right)dx - \int \frac{1}{x+2}\,dx$$

$$= \int 1\,dx - \int \frac{1}{x+2}\,dx$$

$$= x - \log(x+2) + c$$

Example2:

$$\int \frac{x-1}{x+3}\,dx$$

$$\int \frac{x-1}{x+3}\,dx = \int \frac{(x+3)-4}{x+3}\,dx$$

$$= \int \left(\frac{x+3}{x+3}\right)dx - 4\int \frac{1}{x+3}\,dx$$

$$= x - 4\log(x+3) + c$$

Some standard integrals

1. $\int \frac{1}{\sqrt{x^2-a^2}}\, dx$ = log(x +$\sqrt{x^2-a^2}$) + c
2. $\int \frac{1}{\sqrt{x^2+a^2}}\, dx$ = log(x +$\sqrt{x^2+a^2}$) + c
3. $\int \sqrt{x^2+a^2}$ dx = $\frac{x}{2}\sqrt{x^2+a^2}$ + $\frac{a^2}{2}$ log(x +$\sqrt{x^2+a^2}$) + c
4. $\int \sqrt{x^2-a^2}$ dx = $\frac{x}{2}\sqrt{x^2-a^2}$ - $\frac{a^2}{2}$ log(x +$\sqrt{x^2+a^2}$) + c
5. $\int \frac{1}{x^2-a^2}$ dx = $\frac{1}{2a}\log\frac{x-a}{x+a}$ +c
6. $\int \frac{1}{a^2 \cdot x^2}$ dx = $\frac{1}{2a}\log\frac{a+x}{a-x}$ +c

Practice questions:

1. $\int_0^2 |1-x|$ dx = ……
   a) 0 b) ½ c) 3/2 d) 1
2. $\int_0^{1/2} \frac{dx}{\sqrt{3-2x}}$ is
   a) 1 b) 1- $\sqrt{\frac{3}{2}}$ c) $\sqrt{3}$ -$\sqrt{2}$ d) $\sqrt{2}$ - $\sqrt{3}$
3. If u = 5t^4 + 4t^2 +t+1 , then the value of du/dt at t = 1 is
   a) 28 b) 27 c) 25 d) 26
4. The slope of the tangent to the curve y = $\frac{x-1}{x+2}$ at x = 3 is
   a) 3/25 b) -3/25 c) ¼ d) -1/4
5. If x = at^3 +bt^2-t and y = at^2-2bt, then the value of dy/dx at t= 0 is:
   a) 2b b) -2b c) 1/2b d) -1/2b
6. The value of $\int_0^2 xe^{x^2}$ dx is
   a) 1 b) e -1 c) (e/2) -1 d) $\frac{1}{2}$ (e^4-1)
7. If y = $\log_3 x$, then dy/dx is
   a) 1/x b) $\frac{1}{log3}(\frac{1}{x})$ c) x d) 0
8. If y = x^3-3x, then the value of d^2y/dx^2 is
   a) 6 b) 3x c) 6x d) 3
9. If x = ct , y =c/t then dy/dx is equal to

a) 1/t b) t.e^t c) -1/t^2 d) none of these

10. $\int_{-1}^{1} e^x$ dx is

a) e – (1/e) b) e + (1/e) c) 2e d) none of these

11. If x^3y^2 = $(x-y)^5$, find dy/dx at (1,2)

a) -7/9 b) 7/9 c) 9/7 d) -9/7

12. Differentiate e^{x^x}

a) (1+logx) b) x^x (1+logx) c) e^{x^x} x^x (1+logx) d) e^{x^x} (1+logx)

13. $\int_1^e \frac{1+logx}{x}$ dx is

a) ½ b) 3/2 c) 1 d) 5/2

14. The value of $\int \frac{1}{x^2}$ dx is

a) 1/x b) -1/x c) -2/x^3 d) none of these

15. $\int xlogx$ dx is

a) $\frac{x^2}{2}\ logx - \frac{1}{4}\ x^2 +\ C$ b) $\frac{x^2}{2}\ logx + \frac{1}{4}\ x^2 +\ C$

c) $\frac{x^2}{4}\ logx - \frac{1}{2}\ x^2 +\ C$ d) none of these

16. If $x^2 + y^2$ = 4 , then dy/dx at (1,1) is

a) 1 b) – 1 c) 2 d) none of these

17. If y = xe^xlogx , then dy/dx is

a) $e^x\ (1 + logx + xlogx)$ b) logx (1+ $e^x + x$) c) $(1 + logx + xlogx)$ d) none

18. If y = log(log(x)) , then dy/dx is

a) 1/logx b) 1/x c) 1/(xlogx) d) none

19. If y = 3^x, then dy/dx at x = 0 is

a) 3^xlog3 b) 3^x c) log3 d) none

20. $\int x^2e^x$ dx is

a) $x^2e^x - 2xe^x + 2e^x$+C b) $x^2e^x + 2xe^x - 2e^x$+C c) $x^2e^x - 2xe^x - 2e^x$+C d) none of these

Chapter 9 : Number series, coding, decoding and Odd man out

Key points

Types of series and series identification

- Natural numbers: 1,2,3,4,5………
- Even numbers: 2,4,6,8,10……..
- Odd numbers: 1,3,5,7,9 ……..
- Prime numbers: 2,3,5,7,11,13,17,…….
- Squares of first n natural numbers: 1,4,9,16,25,36,49,……
- Squares of odd natural numbers: 1,9,25,49,81,…..
- Squares of Even natural numbers: 4,16,36,64,100,…..
- Cubes of first n natural numbers: 1,8,27,64,125,216………
- Cubes of odd natural numbers: 1,27,125,343,729,….
- Cubes of Even natural numbers: 8,64,216,512,1000,…..
- Combination of squares and cubes (alternating): 1,1,4,8,9,27,16,64,…..
- AP : 1,3,5,7,9…. (there exists a common difference)
- GP: 4, 16,64,256,…. (there exists a common ratio of 4)
- + and – series: 1, 4, 2, 5, 3, 6, 4…….
- + and X series: 1, 2 ,2 ,4 ,8, 11, 33,…… (+1 X1 +2 X2 +3 X3 ……..)
- Observe the sequence to check whether there exists a odd number pattern, even number pattern, prime number pattern or square , cubes , square + or - , cube + or –
- Observe the numbers, if the difference is not by large then it is additive pattern or difference pattern.
- Sometimes you may have to write two series of differences to get the right sequence
- Ex: 2 14 32 62 110 182

  First series of difference : 12 18 30 48 72 (no pattern found)

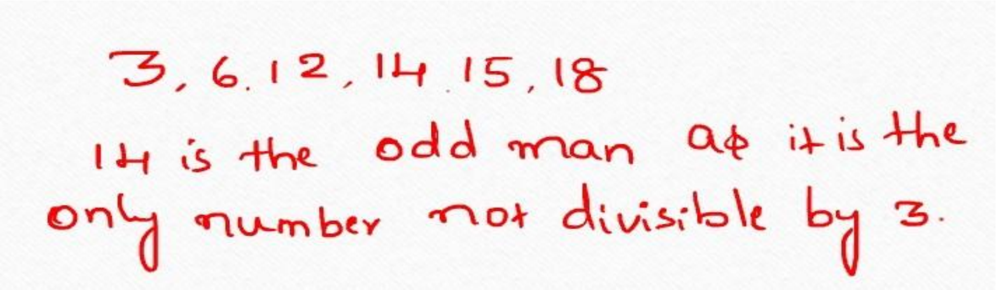
  Second series of difference 6 12 18 24 (common difference of 6)

- If difference between first number and the last number is very large then look for multiplication pattern

Problems on Number series & Odd man out

1)Find the odd man out 3,6,12,14,15,18

a) 18 b) 15 c) 14 d)None



3, 6. 12, 14 . 15, 18

14 is the odd man as it is the only number not divisible by 3.

2)Find the odd man out 7,26,124,342,1331

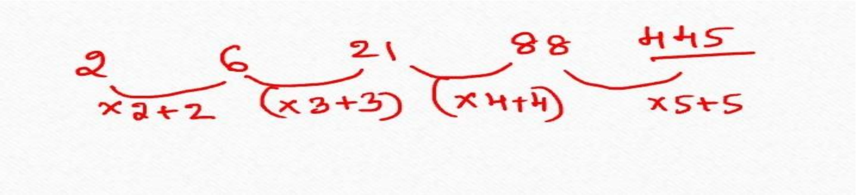
a)342 b)124 c)1331 d)None

Solution: The pattern is (prime number cube-1)

Therefore the odd man out is 1331 as 1331 is just a cube of 11

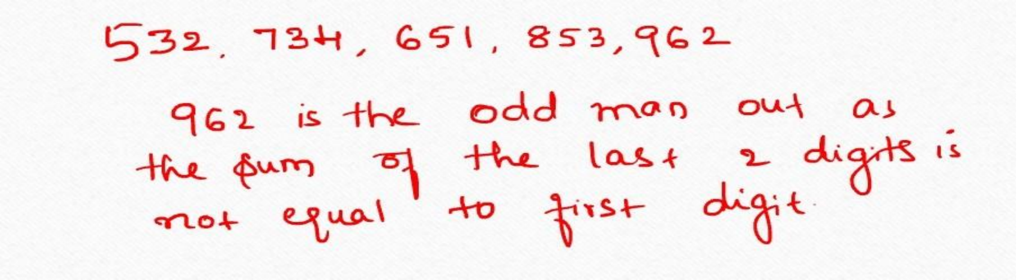
3)2,6,21,88,?

a) 440 b)445 c)450 d)None



4)Find the odd man out 532, 734, 651,,853,962

a)962 b)853 c)651 d)None

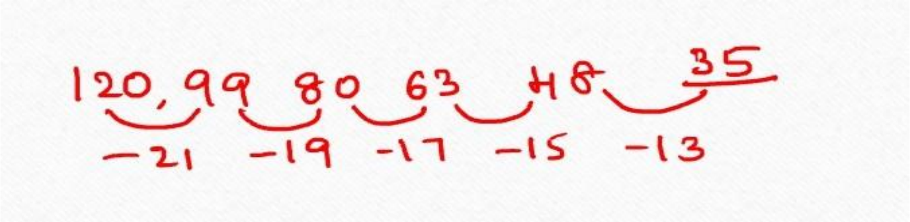


532, 734, 651, 853, 962

962 is the odd man out as the sum of the last 2 digits is not equal to first digit.

5)120,99,80,63,48 ,?

a)35 b)36 c)45 d)40

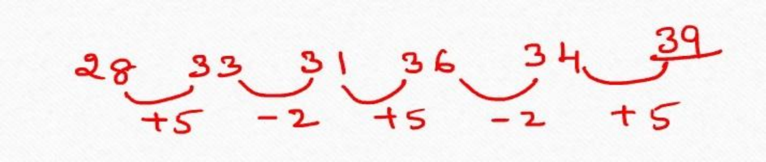


120, 99 80 63 48 35

−21 −19 −17 −15 −13

6)28, 33, 31, 36, 34?

a)38 b)39 c)40 d)None

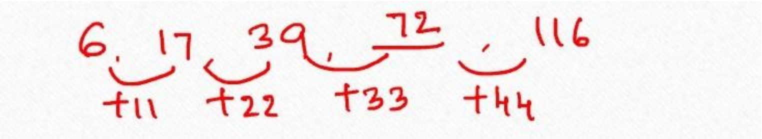


28 33 31 36 34 39

+5 −2 +5 −2 +5

7)6, 17 , 39 , ? , 116

a)72 b)75 c)85 d)80



6, 17, 39, 72, 116

+11 +22 +33 +44

8)1, 4 , 10 , 22 , ? 94

a)46 b)48 c)49 d)47

$$1, 4, 10, 22, \underline{46}, 94$$

$$+3 \quad +6 \quad +12 \quad +24 \quad +48$$

9)3, 7 , 15 , 31 , ? , 127

a)62 b)63 c)61 d)none

$$3, 7, 15, 31, \underline{63}, 127$$

$$+4 \quad +8 \quad +16 \quad +32 \quad +64$$

10)0,3,8,15,24,35,?

a)48 b)47 c)46 d)45

$$0, 3, 8, 15, 24, 35, \underline{48}$$

$$(1^2-1)\ (2^2-1)\ (3^2-1)\ (4^2-1)\ (5^2-1)\ (6^2-1)\ (7^2-1)$$

<u>Coding and Decoding</u>

<u>Key points</u>

- Forward Position of alphabets :

Remember the following letters with respective forward positions

EJOTY

E-5

J-10

O-15

T-20

Y-25

- Backward Positions of alphabets:

Backward position = 27- Forward position

Ex: Backward position of I = 27- 9 =18

Note: Sum of backward positions of given letters of the word

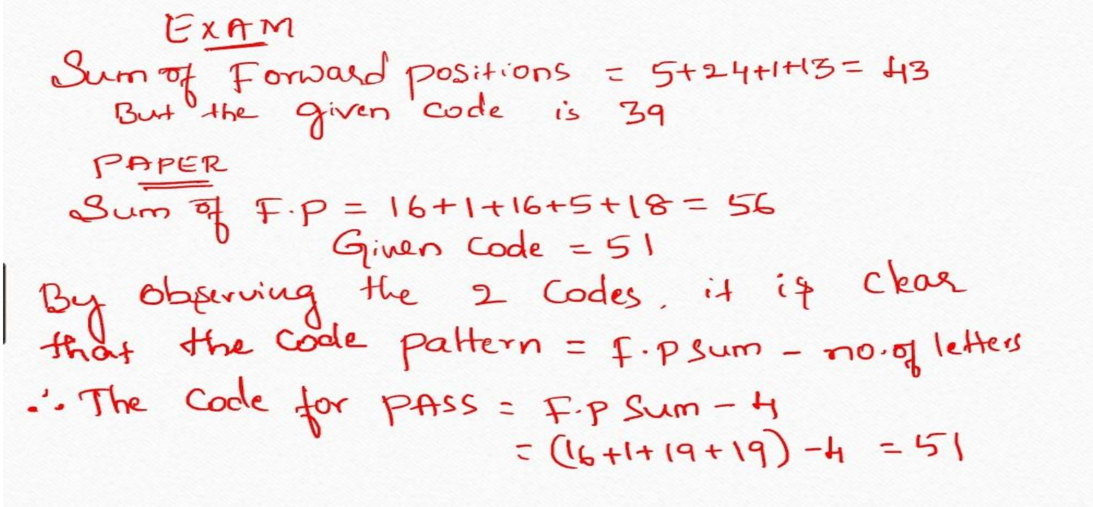
=($27\times no\ of\ letters$) - *sum of forward positions*

Type1

Based on sum of forward or backward postions

1)In a certain code language "EXAM" is coded as 39 'PAPER' is coded at 51 then PASS is coded as

a) 47 b) 51 c) 50 d)52



EXAM

Sum of Forward positions = 5+24+1+13= 43

But the given code is 39

PAPER

Sum of F.P = 16+1+16+5+18 = 56

Given Code = 51

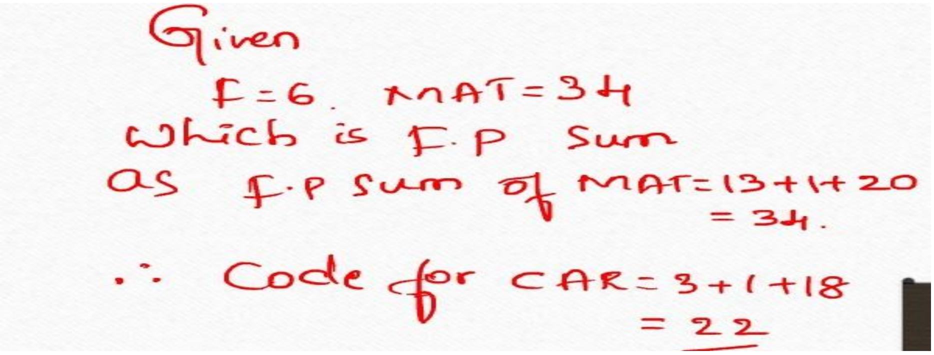
By observing the 2 Codes, it is clear that the code pattern = F.P sum − no. of letters

∴ The Code for PASS = F.P Sum − 4

= (16+1+19+19) −4 = 51

2)If F=6 , MAT = 34, then how much is CAR?

a) 21 b) 22 c) 25 d)28



Given

F=6. MAT=34

which is F.P Sum

as F.P sum of MAT=13+1+20

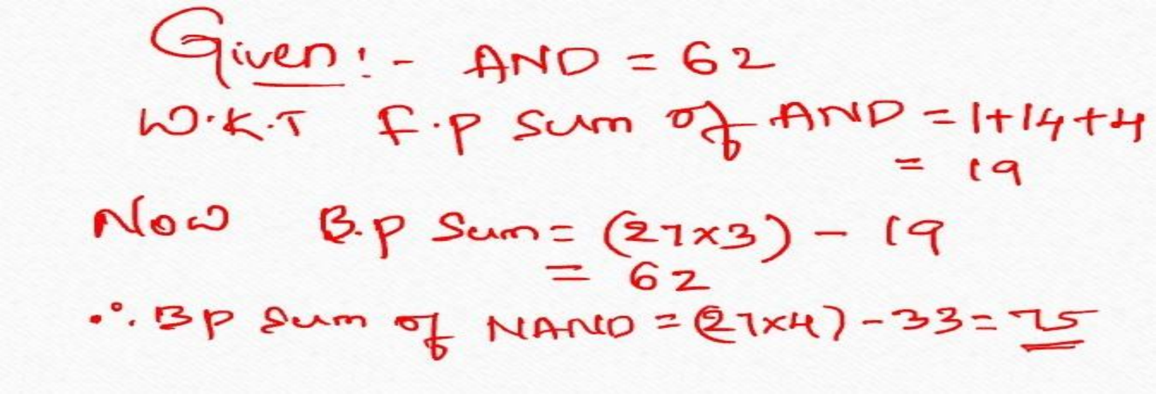
= 34.

∴ Code for CAR = 3+1+18

= 22

3)In a certain code language AND = 62 Then NAND be coded as

a) 75 b) 33 c) 78 d) none



Given :- AND = 62

W.K.T f.p sum of AND = 1+14+4

= 19

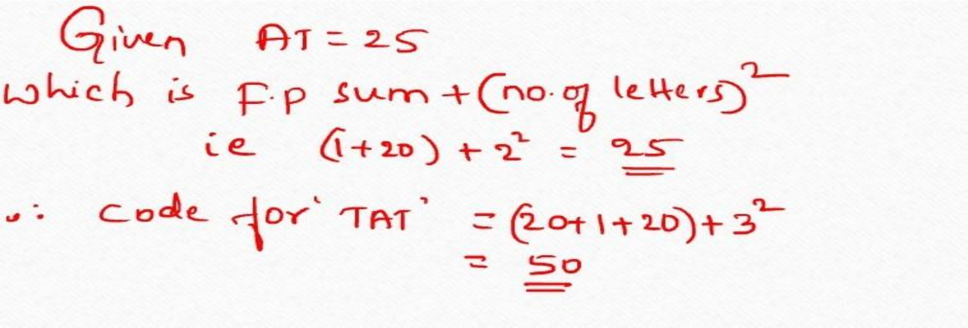
Now B.p Sum = (27×3) − 19

= 62

∴ Bp sum of NANO = (27×4) − 33 = 75

4)In certain code language AT= 25. WET = 57. Then how TAT be coded?

a) 47 b) 46 c)41 d) 50



Given AT = 25

which is F.p sum + $(\text{no. of letters})^2$

ie $(1+20) + 2^2 = 25$

∴ code for 'TAT' $= (20+1+20) + 3^2$

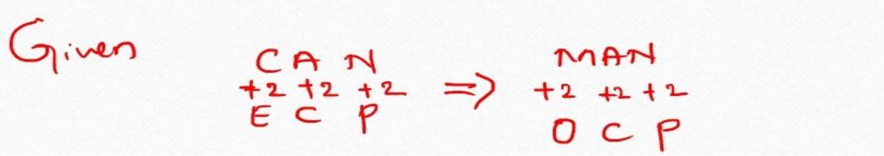
$= 50$

Type2

Letter to letter coding

1)In a certain coding language 'CAN' is coded as 'ECP' then how 'MAN' be coded

a) NBO b) OCP c) PCP d) none



Given

CAN ⇒ MAN

+2 +2 +2 ⇒ +2 +2 +2

ECP ⇒ OCP

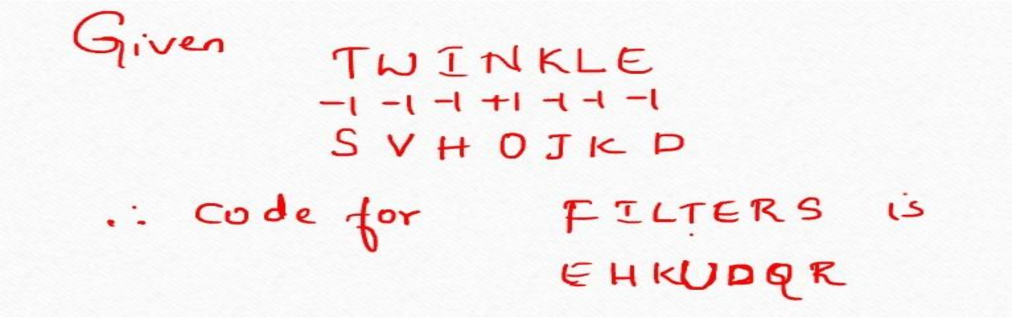
2)In a certain coding language SISTER is coded as SFUTJT then how BROTHER be coded?

a) REHTORB b) SFIUPSC c) SFUIPCS d) none

Given SISTER which follows
SFUTJT Reverse +1
pattern
∴ Code for BROTHER is
SFIUPSC

3)In a certain language TWINKLE is written as SVHOJKD, then how would FILTERS be written in the same code?

a) EHKUDQR b)ITNFKD c)KVOHMF d)TIMFKD



Given TWINKLE
−1 −1 −1 +1 −1 −1 −1
SVHOJKD
∴ Code for FILTERS is
EHKUDQR

4) If ‘MEAT’ is written as ‘TEAM’, then ‘BALE’ is written as

(a) ELAB b) EABL c) EBLA d) EALB

Given MEAT => BALE
TEAM EALB

Type3

Letter to digit coding

1)If GARDEN is coded as 325764 and WATER as 92165 , how can we code the word WARDEN in the same way?

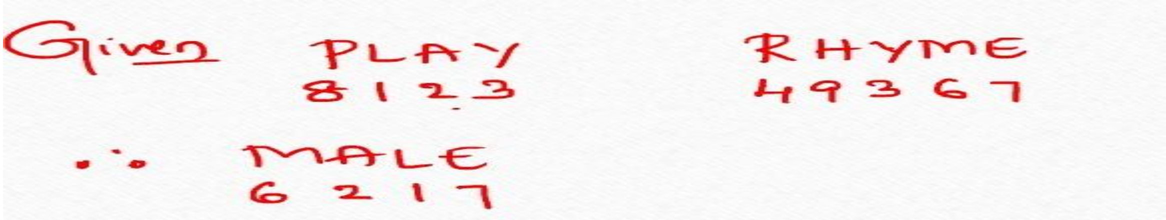
a)925764 b)295764 c)952764 d) 957264

Given GARDEN WATER
325764 92165

∴ WARDEN
925764

2)If PLAY is coded as 8123 and RHYME is coded as 49367. What will be code of MALE?

a) 6285 b) 6217 c) 6395 d) 6198



Given PLAY RHYME
8123 49367

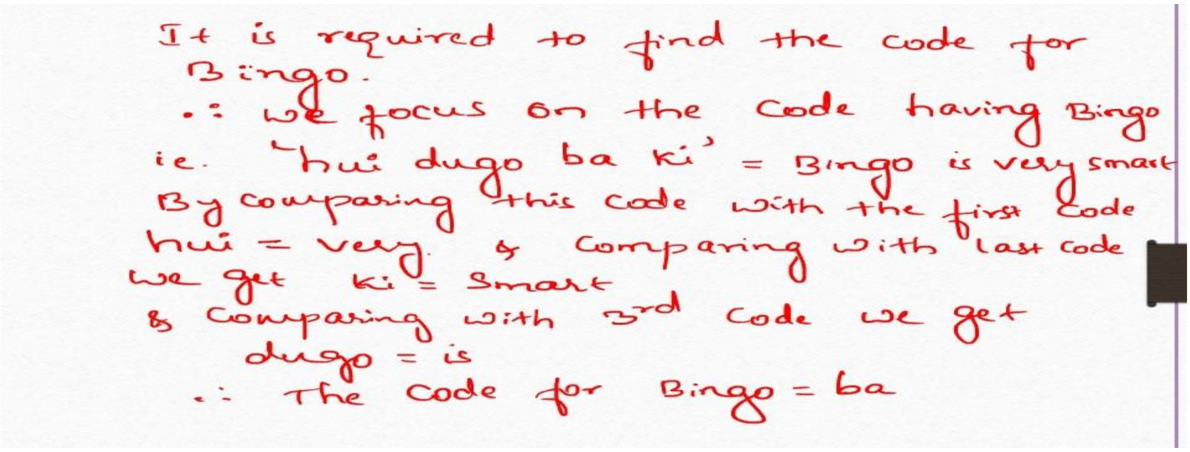
∴ MALE
6217

Type 4:

Word to word / word to character / word to digit coding

1)In a certain code language : 'dugo hui mul zo' stands for 'work is very hard' 'hui dugo ba ki' for 'Bingo is very smart'; 'nano mul dugo' for 'cake is hard'; and 'mul ki gu' for 'smart and hard' Which of the following word stand for Bingo ?

a)Jalu b)Dugo c)Ki d)Ba



It is required to find the code for Bingo.

∴ we focus on the code having Bingo

i.e. 'hui dugo ba ki' = Bingo is very smart

By comparing this code with the first code

hui = very. & Comparing with last code

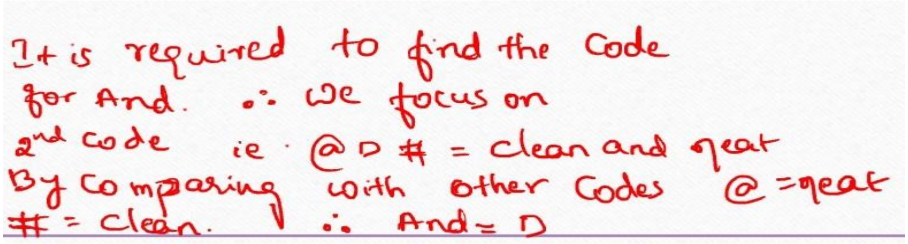
we get ki = Smart

& Comparing with 3rd code we get

dugo = is

∴ The code for Bingo = ba

2)In a certain code language $#* means 'Shirt is clean', @ D# means 'Clean and neat' and @? means 'neat boy', then what is the code for 'and' in that language a)# b)D c)@ d)Data inadequate



It is required to find the Code for And. ∴ We focus on 2nd code ie. @ D # = clean and neat

By Comparing with other Codes @ = neat

= Clean. ∴ And = D

Practice Questions:

1) If HONEY is coded as JQPGA then which word is coded as VCTIGVU

- a) TRAPETS
- b) TARGETS
- c) CARPETS
- d) NONE

2) In a certain language, MADRAS is coded NBESBT, how BOMBAY is coded in that code?

a) CPNCBZ b)CNPCBZ c)CPNCZB d) none

3) Which of the following is odd one

- a) CEHL
- b) KMPT
- c) OQTX
- d) NPSV

4) If SYSTEM is coded as 131625 then TERMS is coded as

- a) 62251
- b) 62451
- c) 64951

d) 62415

5) In a certain language, MADRAS is coded NBESBT, how DELHI is coded in that code?

a) EFMIJ b)CDKGH c)EFKIJ d) none

6) If in a certain language HEALTH is coded as IFBMUI then what is the code for NORTH is

(a) OPSUI (b) OPUSI (c) OUSPI (d) OIPSU

7) IF DELHI is coded as EFMIJ then JAIPUR is coded as

a) KBGQVS
b) KBJQVS
c) KBQJVS
d) none

8) IF FRAME is coded as 0618011305 then ARISE is coded as

a) 1189195
b) 01181905
c) 01171805
d)none

9) If CLOCK is coded as 34235 and TIME as8679 then how MOTEL is coded

a) 72894
b) 73894
c) 74892
d) none

10) 7 11 13 17 19 23 25 29 ?

a)31 b)32 c)33 d)none

11) Find the odd man out 15 21 63 81 69

a)81 b)63 c)21 d)none

12) Find the odd man out 7 9 13 17 19

a) 13 b)9 c)7 d)none

13) Find the odd man out 4 12 44 176 890

a) 44 b)12 c) 176 d)none

14) 7 23 47 119 167 ?

a) 194 b)289 c) 223 d)none

15)Find the odd man out 1 5 14 30 51 55 91

a) 14 b)51 c)55 d)none

16) 4 16 36 64 100 ?

a) 121 b) 144 c)196 d)none

17) 0 2 3 6 10 17 28 ?

a) 45 b)46 c)47 d) none

18) Find the odd man out 6 9 12 18 21 26 30

a) 12 b)21 c) 26 d)none

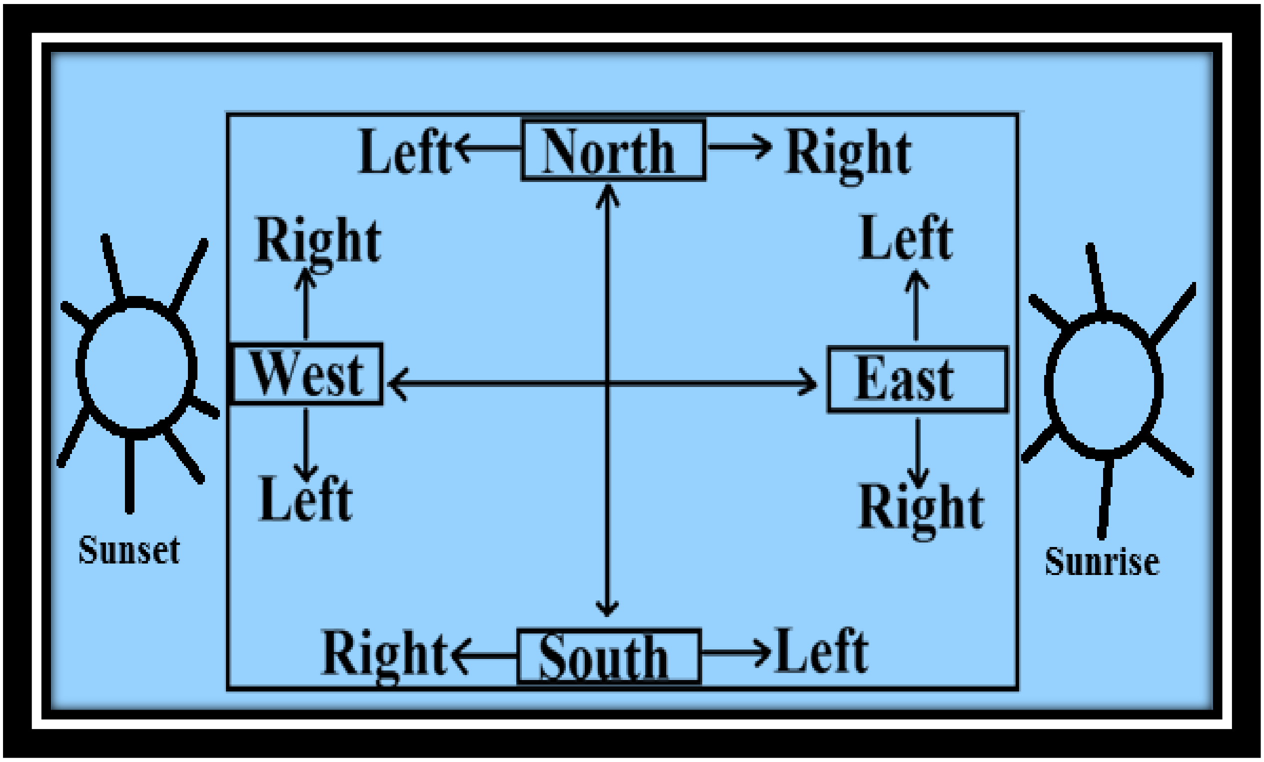
19) 1 1 8 4 27 ? 64 16

a) 8 b) 9 c) 35 d) none

20) In a certain code '256' means 'you are good', '637' means 'we are bad' and '358' means 'good and bad'. Which of the following represents 'and' in that code?

(a) 2 (b) 5 (c) 8 (d) 3

Chapter 10 - Direction Test



Key Points

Turning right or left means making 90 degree angle with the reference line

- A person going towards North →

His left is our left & his right is our right

- A person going towards South →

His left is our right & his right is our left

- A person going towards East→

His left is north & his right is south

- A person going towards West→

His left is south & his right is nortg

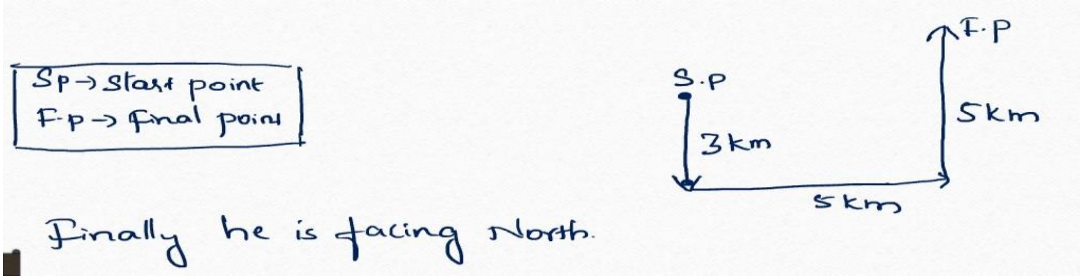
- Mark the distances travelled in each direction proportionally
- Use Pythogorus theorem to find the distance in special cases

Problem Set1:

Based on Final Position and Final position with respect to initial position

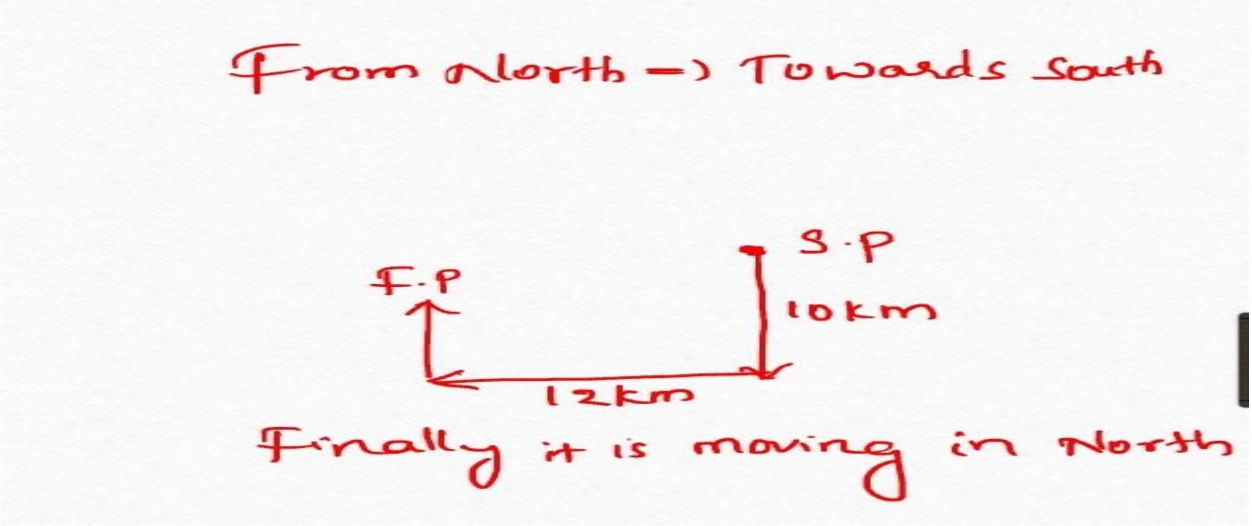
1)Gagan starts from point X and walks 3km towards south, turns left and walks 5 km. Then he turns left again and walks 5 km. Now he is facing

a) East b)West c)North d)South – West



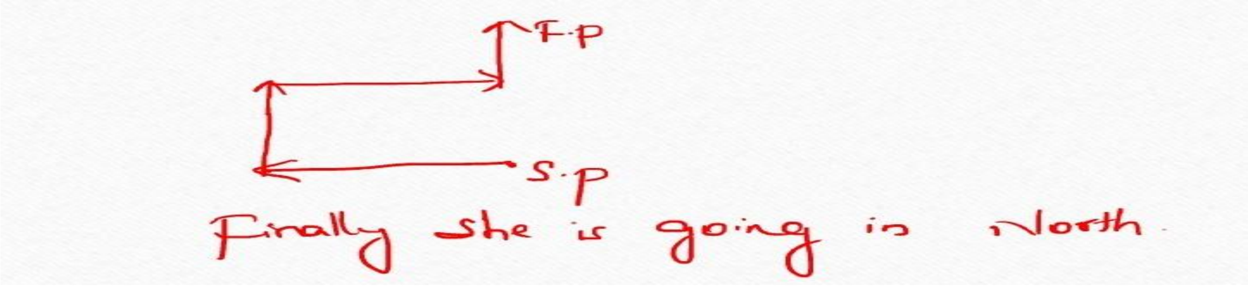
2)A vehicle travelling from North covers a distance of 10 kms, then turns right and runs another 12kms and again turns to the right and was stopped. Which direction does it face now?

a)South b)North c)West d)East



3)Sharmila is going towards West. She turns right, moves on same distance and again turns to her right After walking some distance, she turns to her left and moves on. In which direction she is going now?

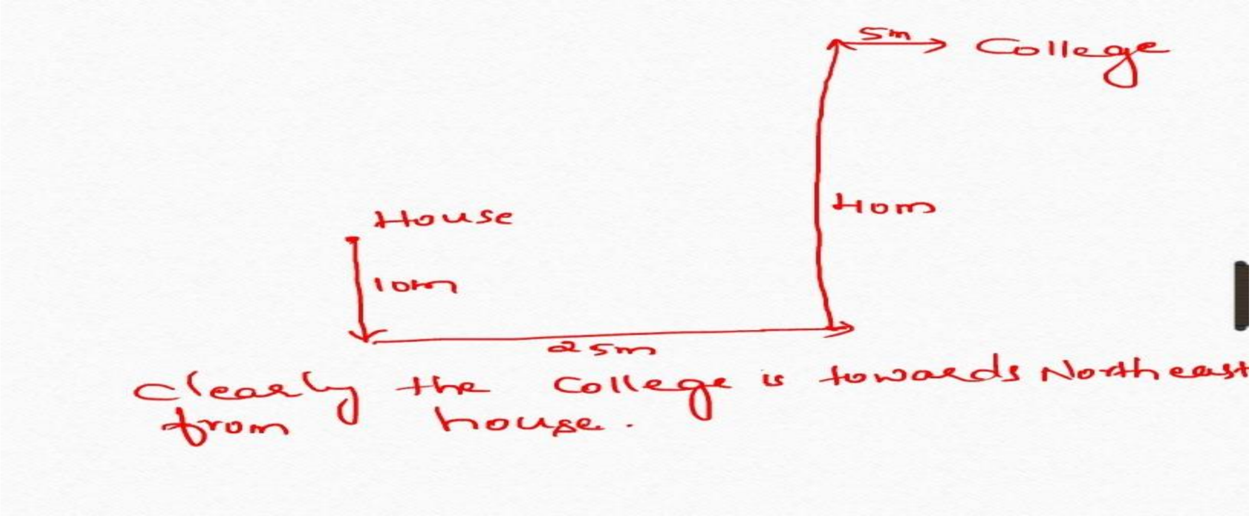
a)North b)South c)North – West d)West



Finally she is going in North.

4)Rajiv walks 10m South from his house, turns left and walks 25 m ,again turns left and walks 40m and then turns right and walks 5m to reach the college.In which direction the college from his house.

a)North b)South – West c)North – East d)East



Clearly the College is towards Northeast from house.

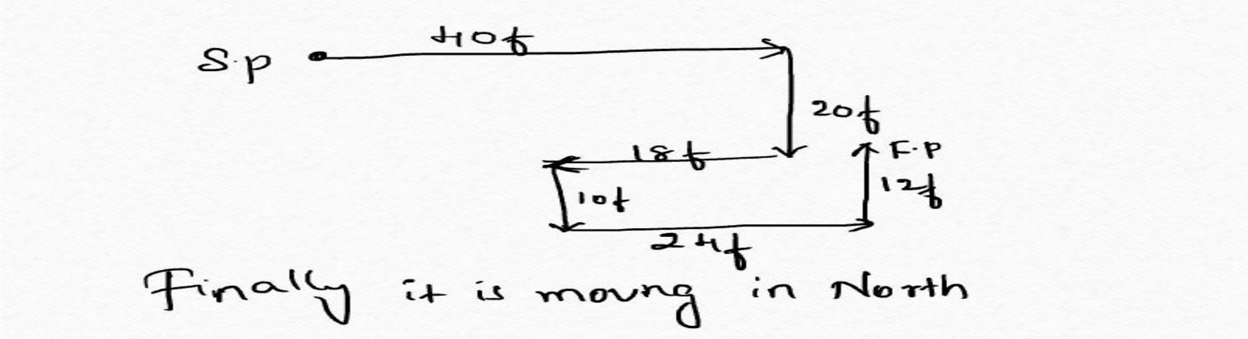
5) **A** rabbit run 40 feet towards East and turns to right runs 20 feet and turns to right runs 18 feet and again turns to left runs 10 feet and then turns to left runs 24 feet and finally turns to left and runs 12 feet .Now what direction is the rabbit facing.

a)East

b)North

c)West

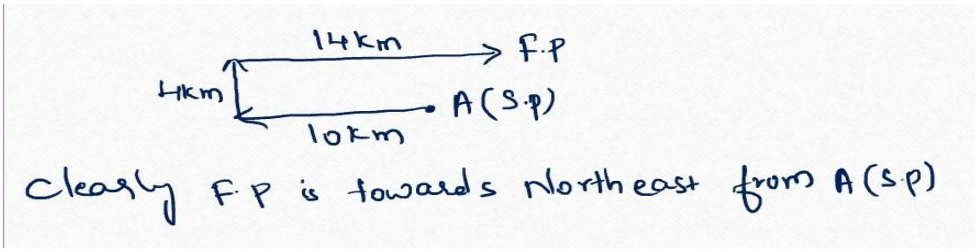
d)South



Finally it is moving in North

6)Mahesh started from point A and walked straight 10 km West, then turned right and walked straight 4km and again turned right and walked straight 14 km. In which direction is he from the point A?

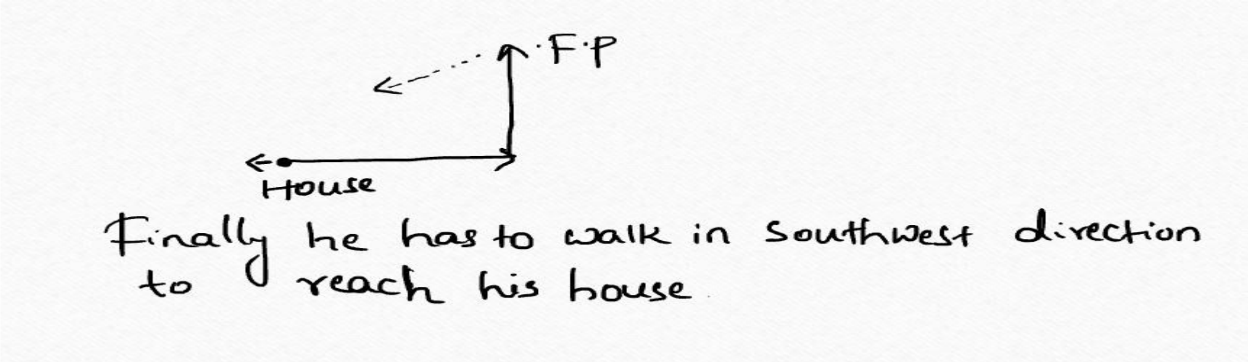
(a) North-East (b) South-East (c) South-West (d) North-West



Clearly F.P is towards Northeast from A(S.p)

7)A man takes his Grandfather for a walk (whose is facing West). They walk first towards East and then walk towards North. In which direction they have to walk now to reach home?

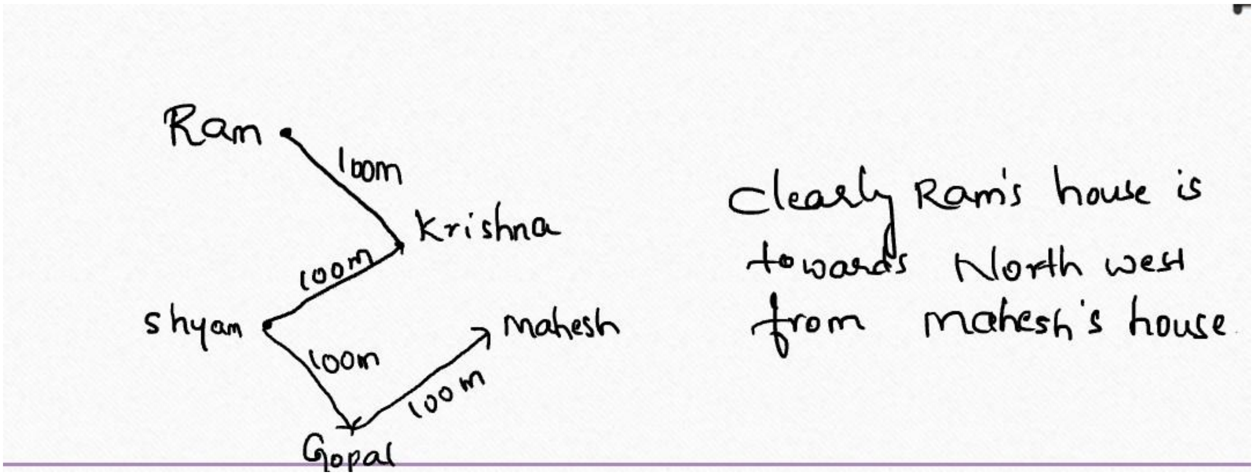
(a) North East (b) West (c) South (d) South West



Finally he has to walk in southwest direction to reach his house

8)Ram is Krishna's neighbour and his house is 100 meters away in the north-west direction. Shyam is Krishna's neighbour and his house is located 100 meter away in the south-west direction. Gopal is Shyam's neighbour and he stays 100 meters away in the south-east direction. Mahesh is Gopal's neighbour and his house is located 100 meters away in the north-east direction. Then where is the position of Ram' s house in relation to Mahesh's ?

(a) South-east (b) south-west (c) North –west (d) North-east

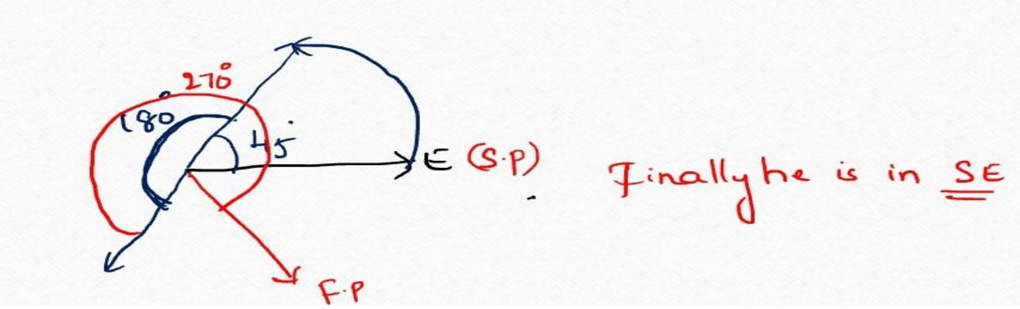


Problem Set 2

Based on angular movement

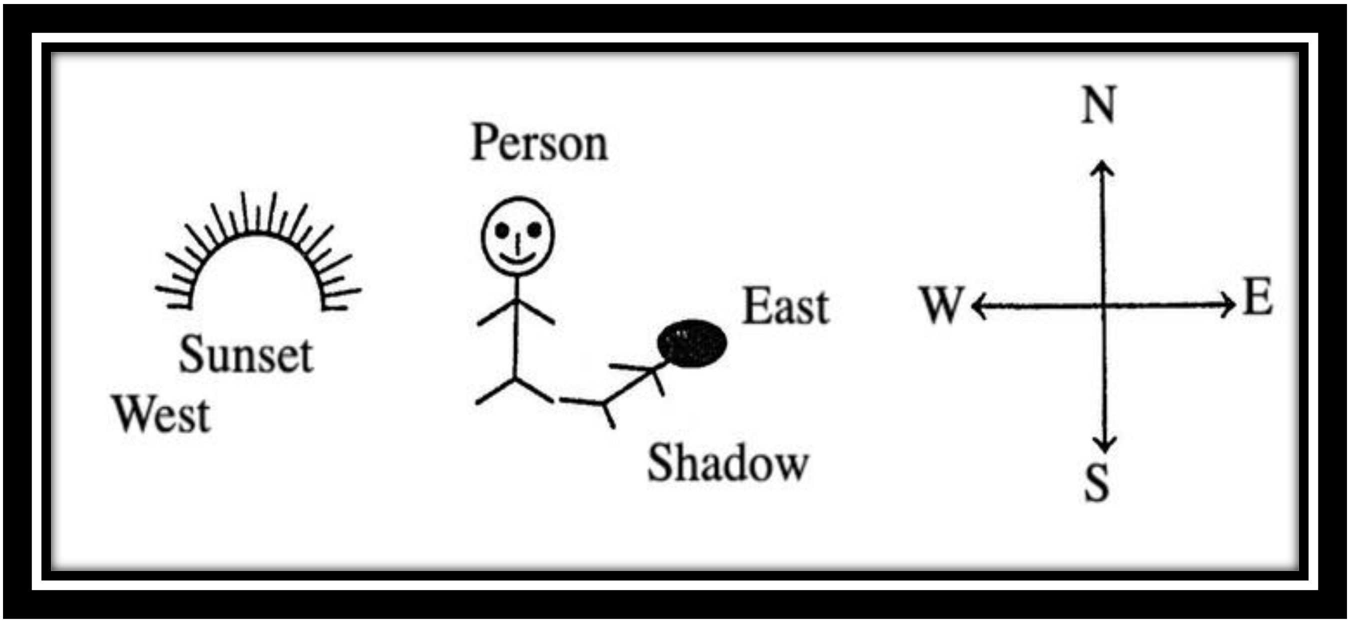
1)A man is facing East. He turns 45 degrees in the anti clockwise direction and then another 180 degrees in the same direction and then 270 degrees in the clockwise direction. Which direction is he facing now?

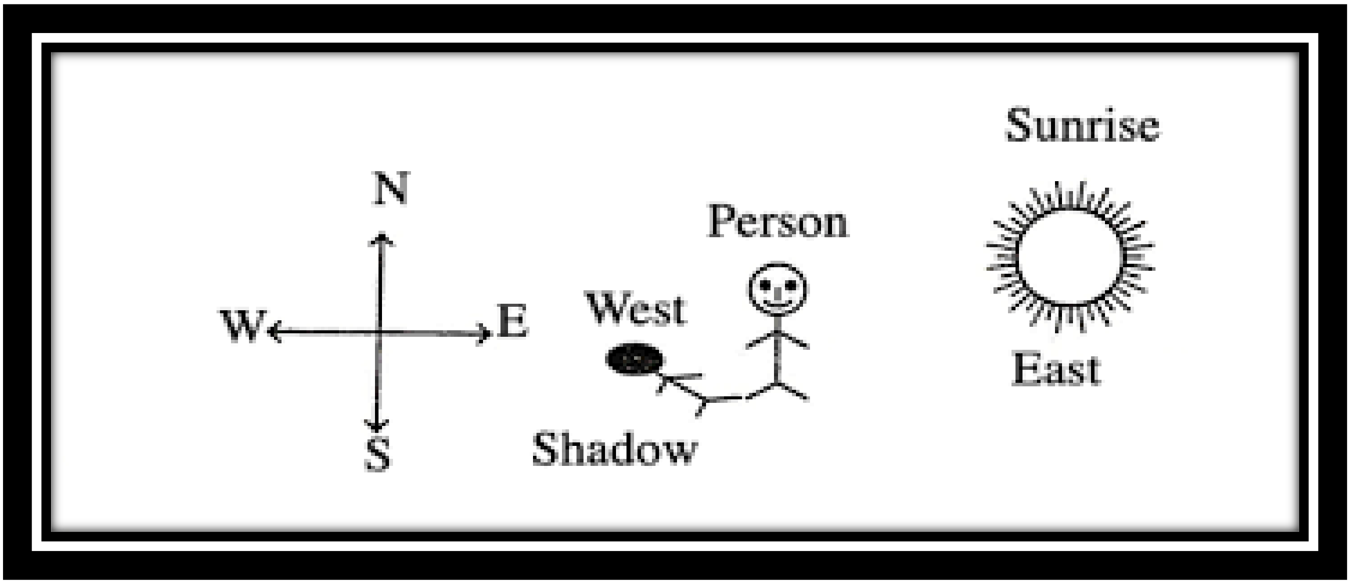
a) South-west b)North-west c)West d)South-east



Problem Set 3:

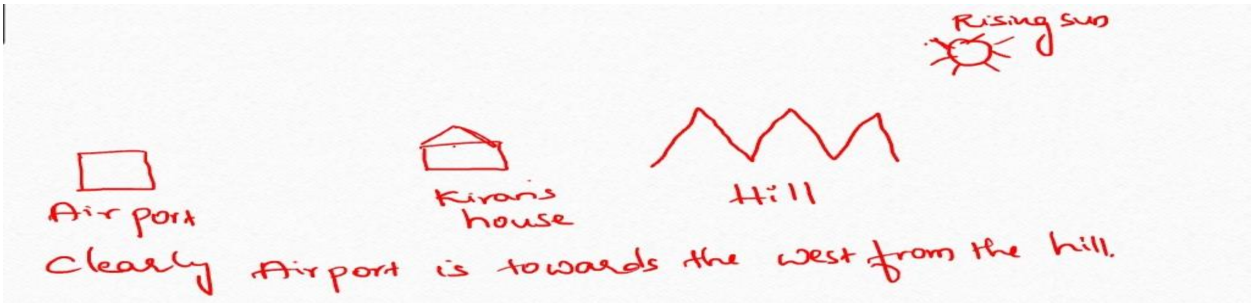
Based on Sunset , sunrise and shadow





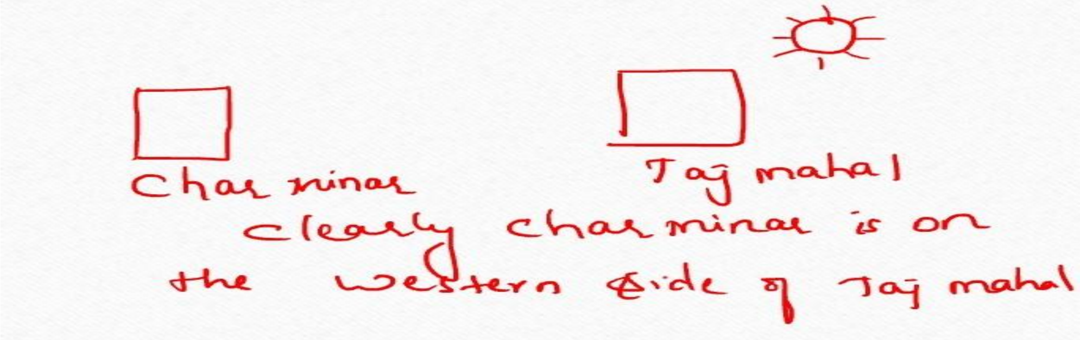
1)If Kiran sees the rising sun behind the Hill and setting sun behind the Airport from his house. What is the direction of Airport from the Hill?

(a) South (b) North (c) West (d) East



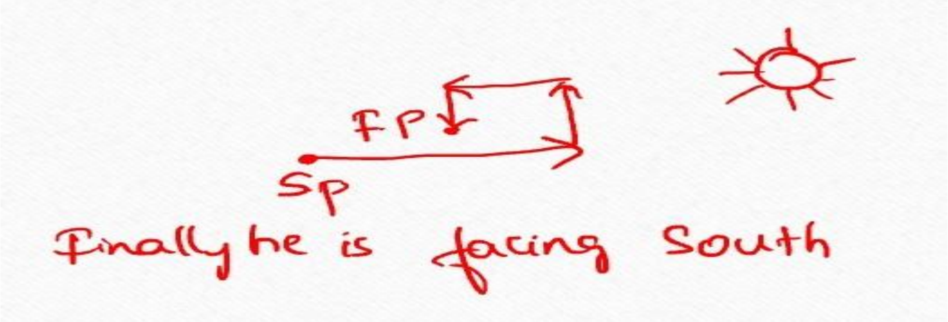
2)If daily in the morning the shadow of Taj mahal falls on Char Minar and in the evening the shadow of Char Minar falls on Taj mahal exactly. So in which direction is Char Minar to Taj mahal?

(a) Easter side (b) Western side (c) Northern side (d) Southern side



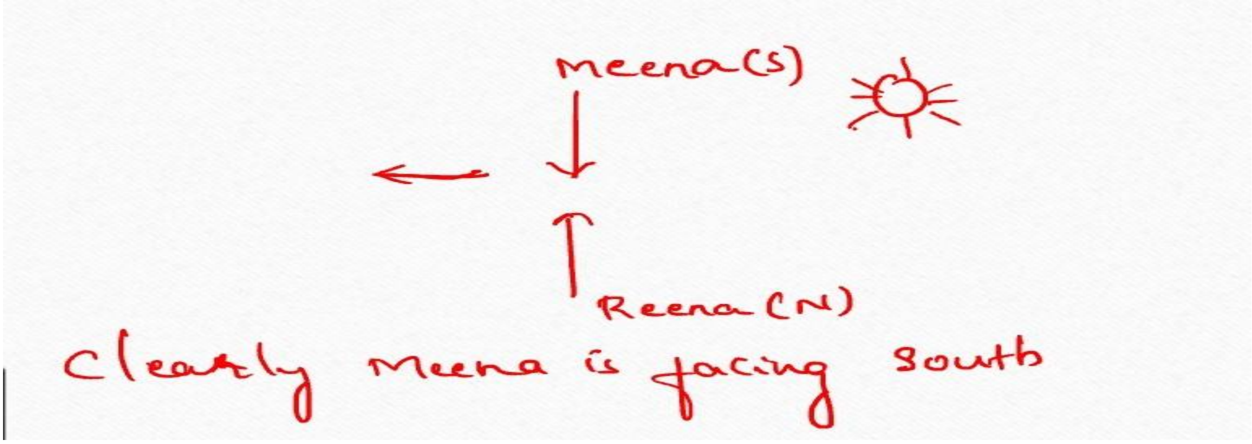
3)One morning, Robin started to walk toward the Sun. After walking a while, he turned to his left and again to his left. After walking a while, he again turned left. In which direction is he facing?

(a) South (b) East (c) West (d) North



4)One morning after sunrise Meena and Reena were standing in a bus stand facing each other. Reena's Shadow fell exactly towards Meena's right hand side of Meena . Which direction was Meena facing?

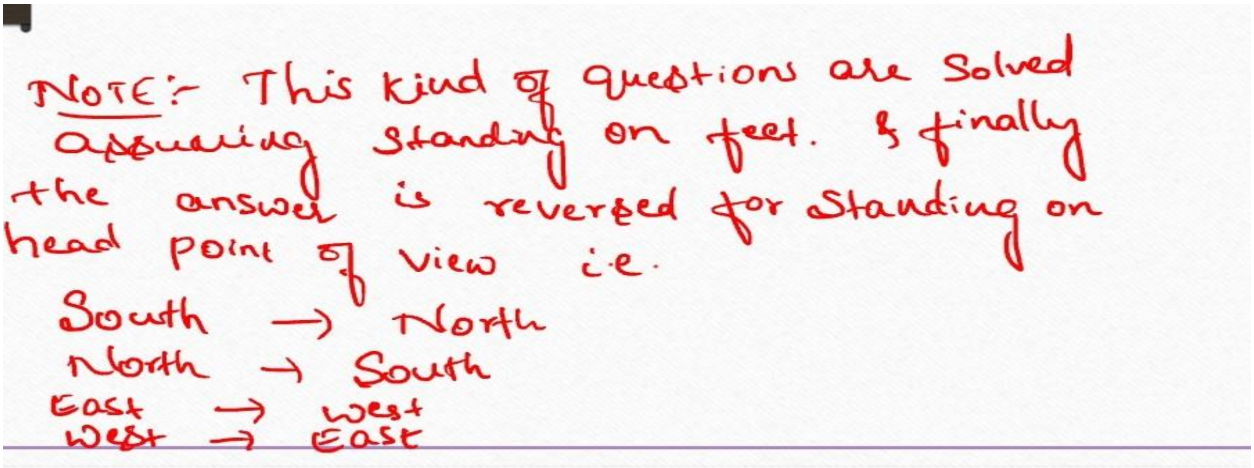
a)South b)North c)south west d)South-East



Clearly Meena is facing south

Problem set 4

Based on standing on head

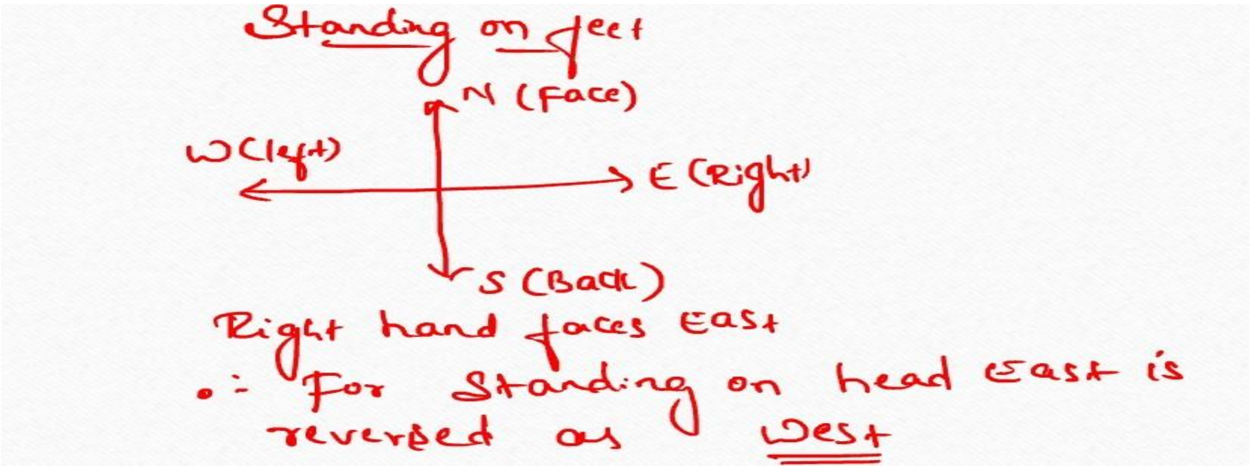


NOTE:- This kind of questions are solved assuming standing on feet. & finally the answer is reversed for standing on head point of view i.e.

South → North
North → South
East → West
West → East

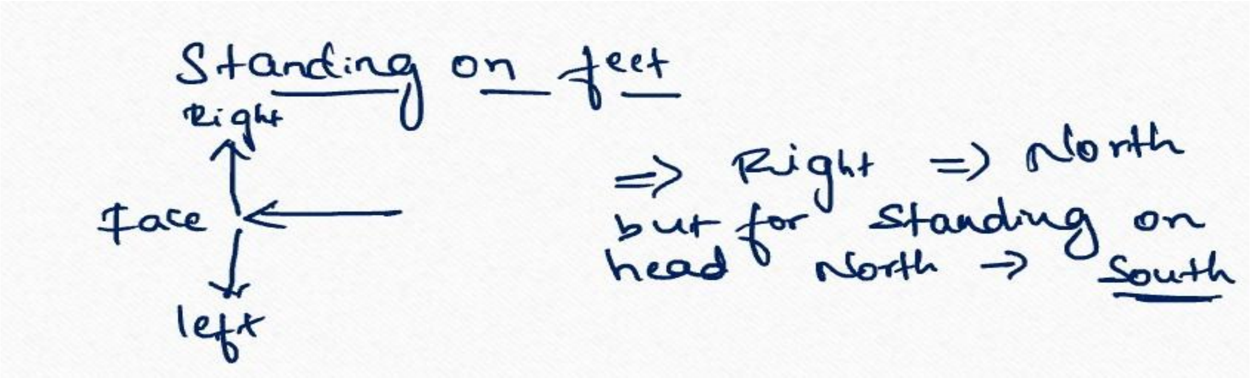
1)If a man stands on his head with his face towards North, to which direction will his right hand point ?

(a) East (b) West (c) North (d) South



2)If Mr. Sanketh stands on his head with his face towards West. In which direction will his right hand point ?

(a) South (b) North (c) East (d) none

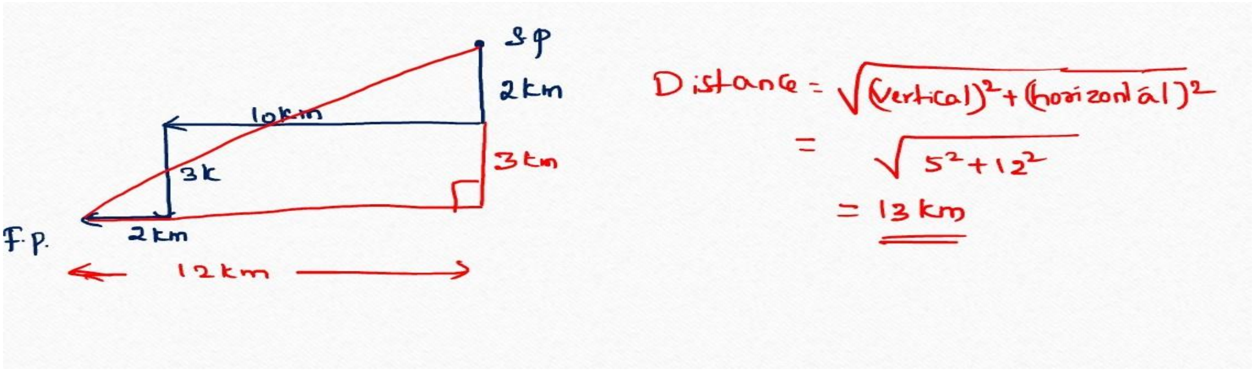


Problem Set 5

Based on distance between starting point and final point

1)Disha walks 2km towards South then she turns west and walks 10km. After this she turns South and walks 3km. Again she turns towards west and walks 2 km. How far is Disha from the starting point?

a) 10km b)13km c)15km d)17km



Practice questions:

1. Rahim started from point X and walked straight 5 km. West, then turned left and walked straight 2 km and again turned left and walked straight 7 km. In which direction is he from the point X?

(a) North-East

(b) South-East

(c) South-West

(d) North-West

2. Anoop Starts walking towards South after walking 15 meters he turns towards North. After walking 20 meters he turns towards East and walks 10 meters. He then turns towards south and walks 5 meters. In which direction is he from the original position.

(a) East

(b) South

(c) West

(d) North

3. Manu wants to go to the market. He starts from his house towards North reaches at a crossing after 30m. He turns towards East, goes 10m till the second crossing and turns again, moves towards South straight for 30m where marketing complex exits. In which direction is the market from his house? **(2018 june)**

(a) North

(b) West

(c) South

(d) East

4. A man started to walk towards east. After moving a certain distance he turns to this right. After moving some distance he turns to his right again. After moving a little he turns now to his left. Currently he is going in …… direction **(2018 june)**

a) North

b) East

c) West

d) South

5. When a person faces north and walk 25 m and turns left and walk 20 m and again turns right and walk 25m, turns right 25m and turns right and walks 40 m. In which direction is he now from starting point? **(2019 june)**

a) NW

b) NE

c) SE

d) SW

6. Madhuri moved a distance of 75m towards north, She then turned to the left and walks about 25m, turned left again and walks 80m, finally she turned to the right at an angle of 45 degrees. In which direction was she moving finally? **(2019 june)**

a) SE

b) SW

c) NW

d) NE

7. A man facing North turns 70 degrees clockwise and turns by 300 degrees in the same direction. Which direction is he facing now? **(2019 june)**

a)South-west

b)North-east

c)North- west

d) None

8. Sangeetha leaves from her home . She first walks 30m in northwest direction and then 30 m in south west direction, next she walks 30 m in south east direction. Finally she turns towards her home. In which direction she is moving now? **(2019 june)**

a) NW

b) NE

c) SE

d) None

9. A man is moving on cycle and move 4 km South then turns left and move 2km and turns again to the right to move to go more. In which direction is he moving?

(a) North

(b) West

(c) East

(d) South

10. A man takes his dog for a walk whose house is facing East. He walks first towards west and then walks towards south. In which direction he has to walk now to reach home?

(a) North East

(b) West

(c) South

(d) North West

11. A boy starts walking towards West, he turns right and again he turns right and then turns left at last. Towards which direction is he walking now? **(2019 Nov)**

(a) West (b) North (c) South (d) East

12.If Shyam sees the rising sun behind the tower and setting sun behind the Railway station from his house. What is the direction of tower from the Railway station? **(2019 Nov)**

(a) South

(b) North

(c) West

(d) East

13. If Mohan travels towards north from his house and then turns to left then to south covering equal distances in each direction to reach Sohan’s house. In which direction Mohan’s house is from Sohan’s house ? **(2019 Nov)**

a) East

b) South

c) West

d) North

14. A man is facing west. He turns 45 degrees in the clockwise direction and then another 180 degrees in the same direction and then 270 degrees in the anticlockwise direction. Which direction is he facing now? **(2020 Dec)**

(a) South – West

(b) North – West

c)West

(d) South

15. A man can walk by having long, medium and short steps. He can cover 60 meters by 100 long steps, 100 meters by 200 medium steps and 80 meters by 200 short steps. Initially he is facing north and starts walking by 5,000 long steps, then he turns left and walk by taking 6,000 medium steps. He then turns right and walk by taking 2,500 short steps. How far (in meters) is he away from his starting point? **(2020 Dec)**

(a) 5,000 m

(b) 4,000 m

(c) 6,000 m

(d) 7,000 m

16. Rahim faces towards north turning to his right he walks 25 mtrs he then turns to his left and walks 30 mtrs. Next he moves 25 mtrs. To his right then he turns to his right again and walks 55 mtrs. Finally he turns to the right and moves 40 mtrs. In which direction is he now from the starting point? **(2020 Dec)**

a) South – West (b) South

c)North – West (d) South – East

17.One day, Ram left home and cycled 10 km southwards, then he turns right and cycled 5 km, then he turns right and cycled 10 km and then he turns left and cycled 10 km. How many kilometers will he have to cycle to reach his home straight? **(2020 Dec)**

a) 15 km (b) 10 km

(c) 20 km (d) 25 km

18. You are facing North – east and moved forward 10 cms and turned left for 7.5 cm what is your position ? **(2020 Dec)**

a) North from initial (b) South from initial

(c) East from initial (d) None of these

19. One morning after sunrise Vivek and Srinath were standing in a lawn with their backs towards each other. Srinaths's Shadow fell exactly towards vivek's left hand side . Which direction was Srinath facing? **(2021 July)**

a) North b)West c)South west d)South-East

20.A and B start moving towards each other from two places 200 m apart. After walking 60 m, B turns left and goes 20 m, then he turns right and goes 40m. He then turns right again and comes back to the road on which he had started walking. If A and B walk with the same speed, what is the distance between them now? **(2021 July)**

a) 80m

b) 70m

c) 40 m

d) None

21.There are 4 towns P, Q , R and T. Q is to the South west of P, R is to the east of Q and south east of P, and T is to the north of R in line with Q and P. In which direction of P is T located? **(2021 July)**

a)North b)NE c) East d) SE

22.Five friends A, B , C , D and E are staying in the same locality. B’s house is to the east of A’s house and to the north of C’s house. C’s house is to the west of D’s house. D’s house is in which direction with respect to A’s house? **(2021 July)**

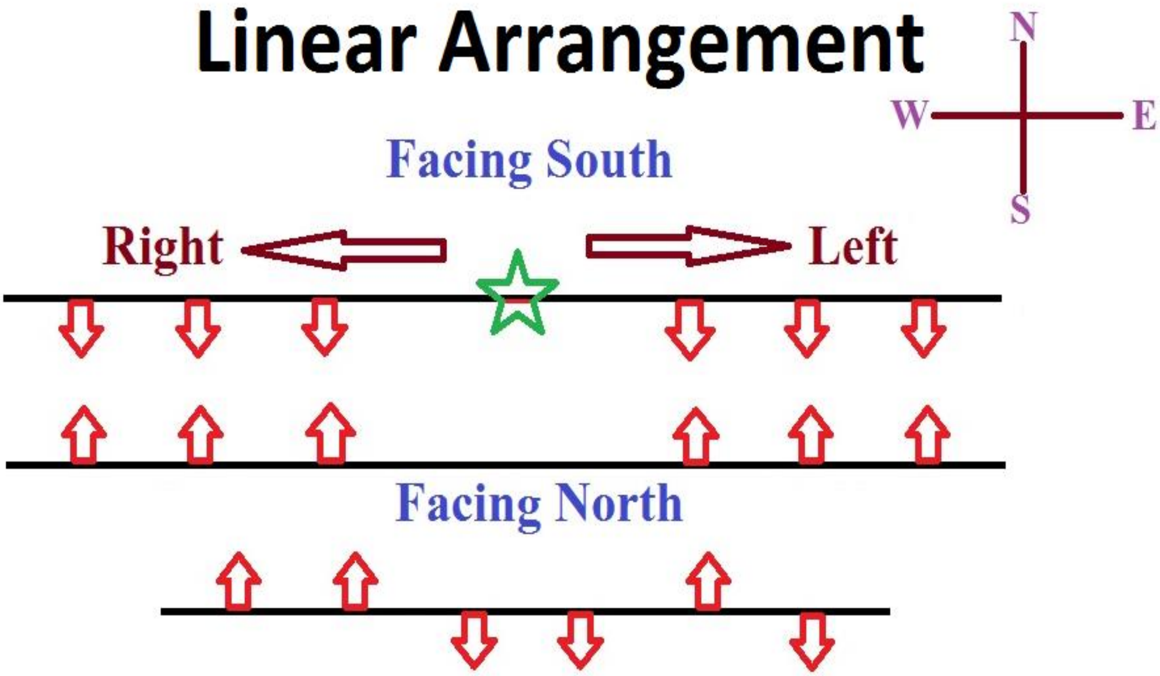
a) NE

b) SE

c) NW

d) SW

Chapter 11 : Seating arrangement



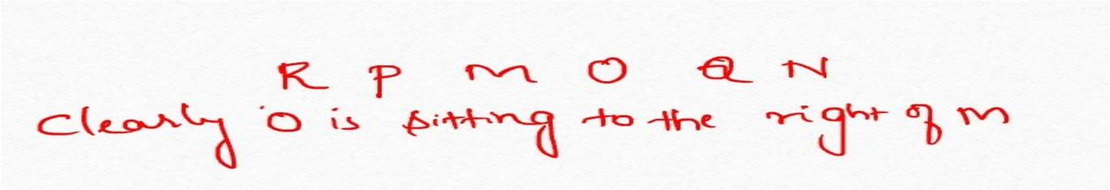
Key points:

1. Direction
2. North (Default direction)
3. And , but refers to first person
4. Who refers to 2nd person
5. The sure one is to be marked first
6. If there is no sure statement , then the one which is having more references to be considered first.
7. Always keep an eye on number of postions marked and number of positions to be marked

Problems on Linear seating arrangement

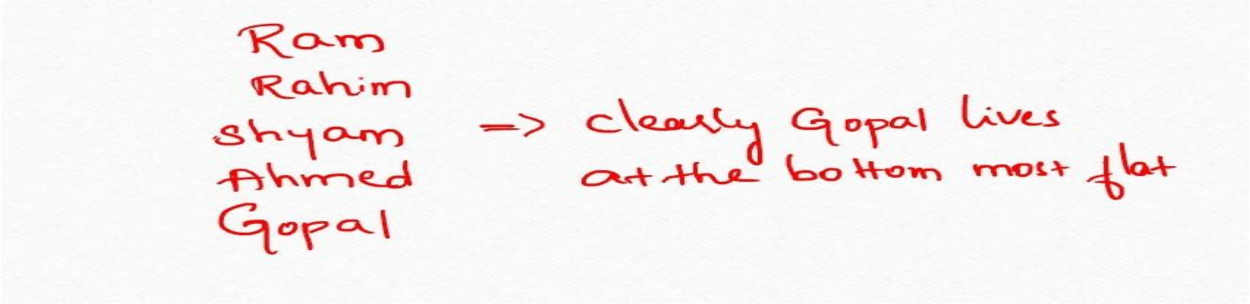
1)Six friends M, N , O , P, Q and R are sitting towards a row and facing towards North, O is sitting between M and Q, P is not at the end, N is sitting at immediate right of Q , R is not at the right end, P is sitting at 3rd left of Q . Which of the following is sitting to the right of M?

(a) R b)Q c)P d)O


R P M O Q N
Clearly O is sitting to the right of M

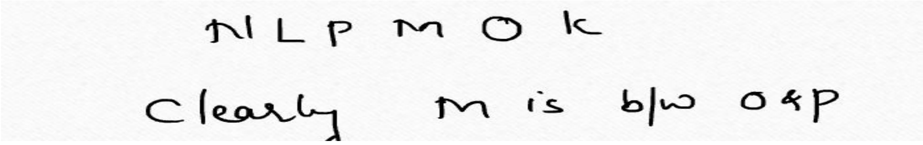
2)Five senior citizens are living in apartments. Mr. Ram lives in a flat above Mr. Rahim, Mr. Ahmed in a flat below Mr. Shyam, Mr. Rahim lives in a flat above Mr. Shyam and Mr. Gopal lives in a flat below Mr. Ahmed. Who lives at the bottom most flat?

(a) Mr. Gopal (b) Mr. Shyam (c) Mr. Ram (d) none


Ram
Rahim
shyam => clearly Gopal lives
Ahmed at the bottom most flat
Gopal

2) Six men K, L,M,N,O and P are sitting in a row. L is between P and N. O is between K and M. However, K does not stand next to P or N. M does not stand next to N. M is between which of the following pairs of men?

(a) O and P (b)P and N (c) M and P (d) none


N L P M O K
Clearly M is b/w O & P

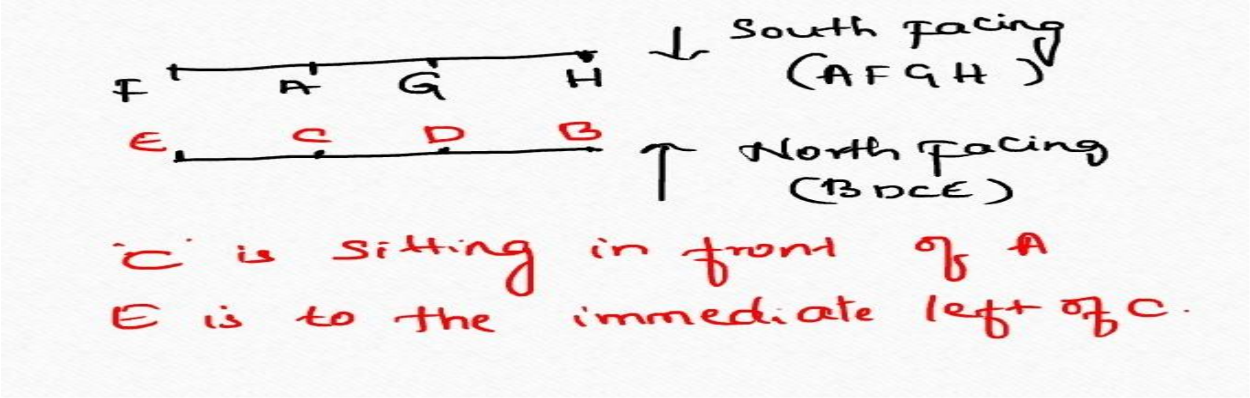
4) Study the following information carefully to answer the given questions. Eight person's A to H are sitting in front of one another in two rows. Each row has four persons. A is between F and G and facing South. B, who is to the immediate right of D is facing H. C is between E and D and H is to the immediate left of G.

1. Who is sitting in front of A?

(a) H (b) G (c) C (d) E

2. Who is to the immediate left of C?

(a) E (b) D (c) A (d)none

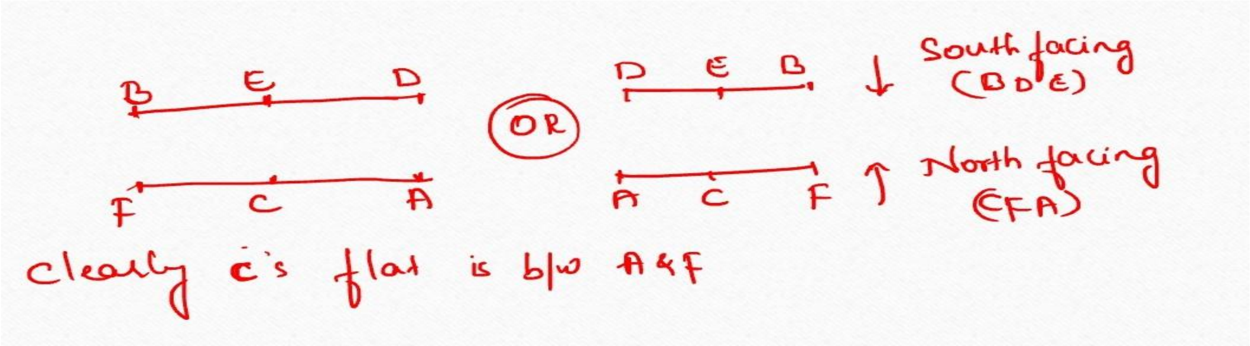


'C' is sitting in front of A

E is to the immediate left of C.

5)Six flats in two rows facing North and South are allotted to A,B,C,D,E and F. If B gets a south facing flat and is not next to D. D and F get diagonally opposite flats. C next to F gets a North facing flat and E gets a south facing flat. Whose flat is between A and F?

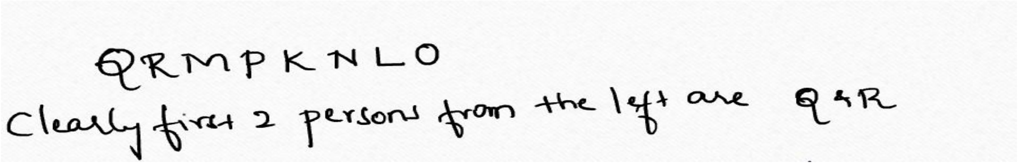
(a) C (b) B (c) D (d) none



Clearly C's flat is b/w A & F

6) Eight persons K,L,M,N,O,P, Q and R are sitting in a line. O Sits second right to N. R sits fourth left to N. M and P are immediate neighbors, but M is not immediate neighbor of K. Q is not neighbor of O. Only two persons sit between K and O. The first two persons from the left are

a) Q and R b) L and O c)P and K d)none

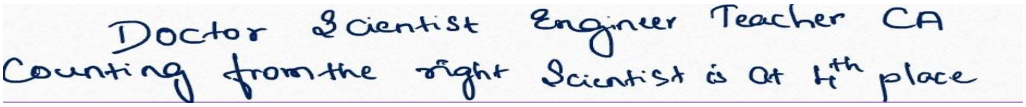


QRMPKNLO

Clearly first 2 persons from the left are Q & R

7) 5 persons are standing in a line one of the 2 persons at the extreme ends is a Doctor and the other a CA. An Engineer is standing to the right of Scientist. A Teacher is to the left of the CA. The Scientist is standing between the Doctor and Engineer. Counting from the right The Scientist is at which place?

(a) 2nd (b) 3rd (c) 4th (d) None of these



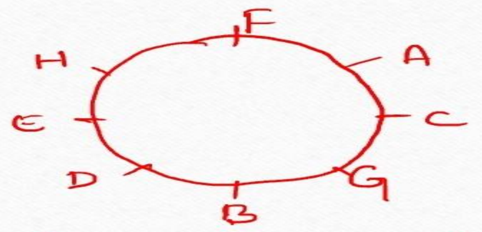
Doctor Scientist Engineer Teacher CA
Counting from the right Scientist is at 4th place

Circular Seating arrangement

Problems on Circular arrangement

1) A,B,C,D,E,F,G and H are 8 friends sitting in a circle facing the centre. B is sitting between G and D. H is third to the left of B and second to the right of A. C is sitting between A and G. B and E are not sitting opposite to each other.D is sitting between

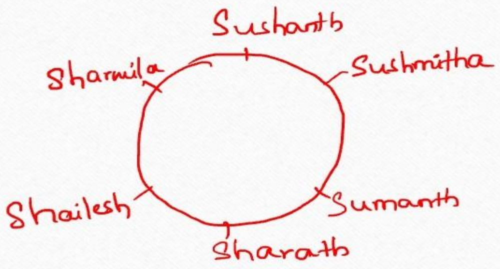
a) E and B b) A and B c) C and F d) none of these



Clearly D is sitting b/w E&B

2) Sharath is sitting between Sumanth and Shailesh, Sharmila is to the left of Shailesh, Sushanth is sitting between Sharmila and Sushmitha. They all sitting around a circle facing the centre, then who is sitting to the left of Sushanth?

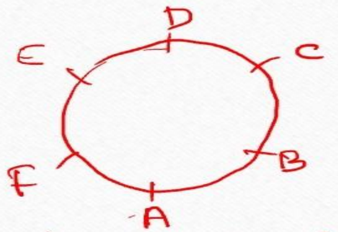
a) Sumanth b) Sushmitha c) Shailesh d) none of these



Clearly Sushmitha is to the left of Sushanth

3) Six persons A,B,C,D,E and F are standing in such a way that they form a circle, facing the centre. B is to the left of C, A is between F and B, D is between C and E. Who is to the right of D?

a)A b)E c) C d)none



Clearly E is to the right of D.

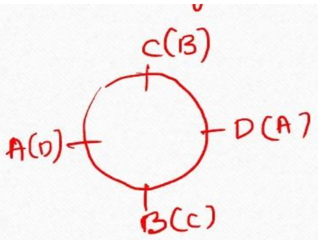
4) Four girls are A, B, C and D are sitting around a circle facing the centre. B and C are in front of each other, which of the following is definitely true?

a)A and D are in front of each other

b) A is not between B and C

c) D is to the left of C

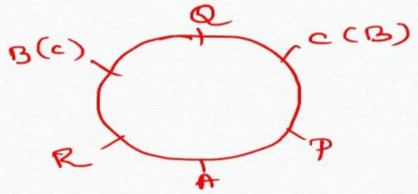
d) A is to the left of C



Clearly A&D are infront of eachother

5) 3 gents A, B , C and 3 ladies P, Q , R are sitting round a circle, such that no two gents sit together. Q is facing A. P is to the immediate right of A. Who sits to the immediate left of R?

a)C b) B c) A d) Can't determine



As the positions (exact) of B&D can't be determined => we can not determine who sits to the immediate left of R.

Practice Questions:

1. Five students A, B, C, D and E are standing in a row. D is on the right of E, B is on the left of E but on the right of A. D is next to C on his left. The student in middle is **(2018 June)**

a)B (b) A

(c)E (d) C

2.Six flats on a floor in two rows facing North and South are allotted to P, Q, R, S, T and U. If Q gets a North facing flat and is not next to S. S and U get diagonally opposite flat. R next to U gets a South facing flat and T gets a North facing flat. Whose flat is between Q and S? **(2018 June)**

a)P (b) T

(c)R (d) U

3. Eight persons A, B, C, D, E, F, G and H are sitting in a line. E Sits second right to D. H sits fourth left toD. C and F are immediate neighbors, but C is not immediate neighbor of A. G is not neighbor of E.Only two persons sit between A and E. The persons on left and right end respectively are **(2018 June)**

(a)G and B (b) G and E

C)H and E (d) B and E

4.Six children A, B, C, D, E and F are sitting in a row. B is between F and D. E is between A and C. However, A does not sit next to F or D. C does not sit next to D. Then, F is sitting between. **(2018 June)**

a)B and D (b) B and C

(c)E and C (d) None of the above

5. Five boys A, B, C, D, E are sitting in a row A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. who is second from the left end? **(June 2019)**

(a) D (b) A

(c) E (d) B

5 children are sitting in a row. S is sitting next to P .K is sitting next to R but not T. R is sitting on extreme left. T is not sitting next to P. Who are sitting adjacent to S. **(June 2019)**

a)K & P (b) R & P

c)Only P (d) P & T

6.Four girls are seated for a photograph. Shikha is left of Reena. Manju is to the right of Reena. Rita is between Reena and Manju. Who is the second left in photograph. **(June 2019)**

a) Reena (b) Manju

(c) Rita (d) Shikha

7. 5 persons are standing in a line one of the 2 persons at the extreme ends is a professor and the other a business man. An advocate is standing to the right of student. An author is to the left of the business man. The student is standing between the professor and advocate. Counting from the left. The author is at which place?(**Nov 2019)**

a) 2nd (b) 3rd

(c) 4th (d) None of these

8.Parikh is sitting between narendra and babita, charu is to the left of babita, pankaj she's sitting between charu and ashma they all sitting around a circle facing the center then who is sitting to the right of babita? **(Nov 2019)**

a) Parikh (b) Ashma

(c) Charu (d) Narendra

9. Five girls G,H, I, J, K are sitting in a row facing south not necessarily in the same order. H is sitting between G and K; I is immediate right to K; J is immediate left to G. Which of the following is true? **(Dec 2020)**

a) j is third to the left of (b)G is second to the left of I

c)H is to the right of K (d)H is to the left of G

10. Eight friends I, J, K, L, M, N, O and P are sitting in a circle facing the centre. J is sitting between O and L; P is third to the left of J and Second to the right of I; K is sitting between I and O; J & M are not sitting opposite to each other. Which of the following statements is NOT correct? **(Dec 2020)**

a) K is sitting third to the right of L

(b)I is sitting between K and N

(c) L and I are sitting opposite to each other

(d)M is sitting between N and L

11.A,B,C,D andE are sitting on a bench. A is sitting next to B, C is siting next to D, D is not sitting with E who is on the left end of the bench. C is on the second position from the right. A is to the right of B and E. A and C are sitting together. A is sitting between **(2021 July**)

a)C and D b) D and E

c) B and C d) B and D

12.5 girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary. who is sitting immediate right of Reeta? **(2021 July**)

a) Seema b) Rani c) Bindu d) Mary

13.Six friends P,Q,R,S,T and Uare sitting around the hexagonal table each at one corner and are facing the centre. P is second to the left of U. Q is neighbor of R and S. T is second to the left of S. Which one is sitting opposite to S? **(2021 July**)

a)R b)P c) Q d) T

14.A,B,C,D,E,F, and G are sitting in a row facing north.

- F is to the immediate right of E
- E is 4th to the right of G
- C is neighbor of B and D
- Person who is third to the left of D is at one of the ends

Who are to the right of D? **(2021 July**)

a)E and F only b) G, B and C

c)E,F and A d) G and B only

15.Four ladies A, B, C and D and four gentlemen E, F, G and H are sitting in circle around a table facing each other

No two ladies or gentlemen are sitting side by side

C, who is sitting between G and E , facing D

F is between D and A and facing G and H is to the right of B

Who is sitting left of A

a)F

b)E

c)C

d)D

16.Five girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary. Who is sitting immediate right to Reeta ?

a)Bindu

b)Rani

c)Mary

d)Seema

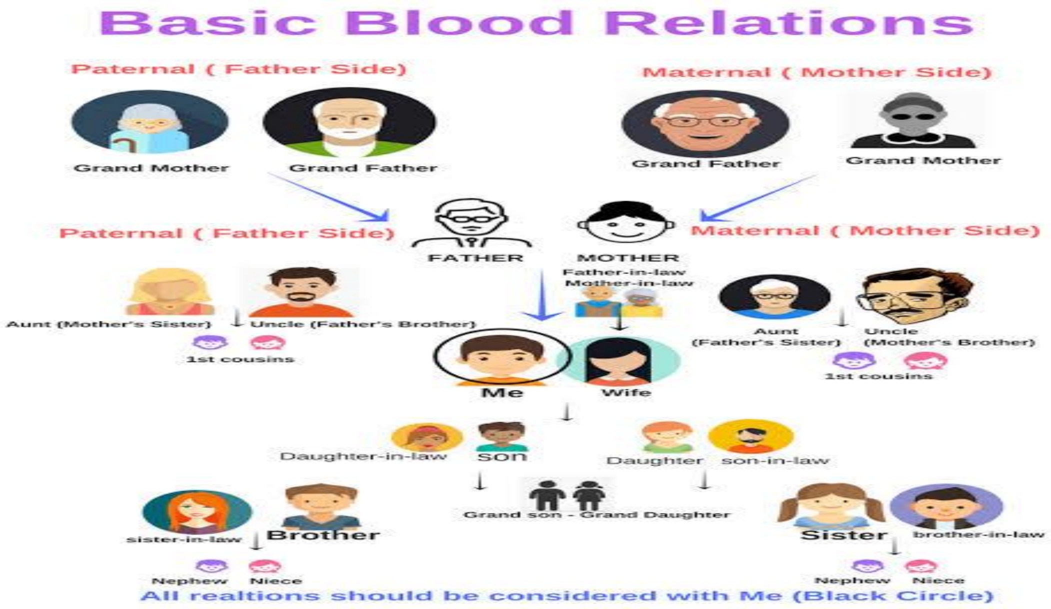
17.Five students A, B, C, D, and E are standing in a row. D is right on the E; B is on the left of E but on the right of A. D is next to C on his left. The student in middle is

a)B b)E c)C d)A

18.Five children are sitting in row. S is sitting next to P but not T. K is sitting next to R, who is sitting on the extreme left and T is not sitting next to K . Who are adjacent to S.

a)K+P b)R+P c)Only P d)P and T

Chapter12: Blood relations



Important blood relations:

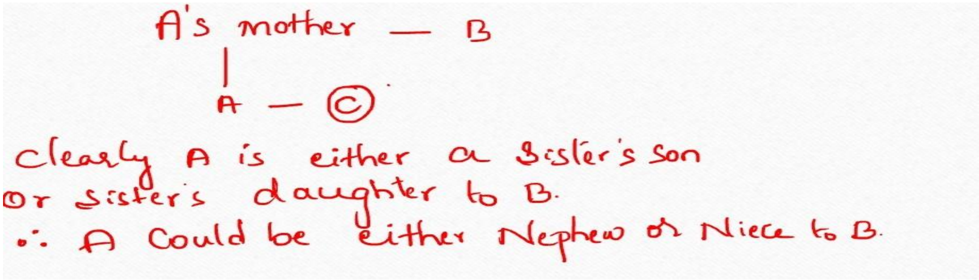
- Paternal uncle : Father's brother
- Paternal aunt : Father's sister
- Maternal uncle: Mother's brother
- Maternal aunt: Mother's sister
- Niece : Brother's/Sister's daughter
- Nephew : Brother's /Sister's son
- Son in law : Daughter's husband
- Daughter in law: Son's wife
- Father in law : wife's/Husband's father
- Mother in law: wife's/husband's mother
- Siblings : brother-brother, brother-sister, sister-sister
- Cousin :Aunt's/Uncle's daughter/Son
- Brother in law : Wife's /Husband's brother

- Sister in law : Wife's/ Husband's sister
-
- **Conventions to be followed to solve problems**
- Female → ○
- Male → □
- Sibings → —
- Married couple → ↔
- Generations Before → ↑
- Next Generation → ↓

Problems on Blood Relations (TYPE1)

1)A's mother is sister of B and has a daughter C. How can A be related to B from among the following?

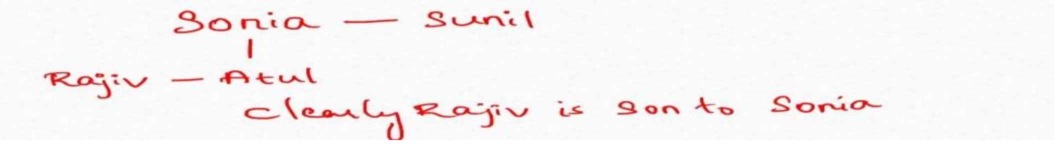
(a) Niece (b) Uncle (c) Daughter (d) Father



clearly A is either a sister's son or sister's daughter to B.
∴ A could be either Nephew or Niece to B.

2) Rajiv is the brother of Atul. Sonia is the sister of Sunil. Atul is the son of Sonia. How is Rajiv related to Sonia?

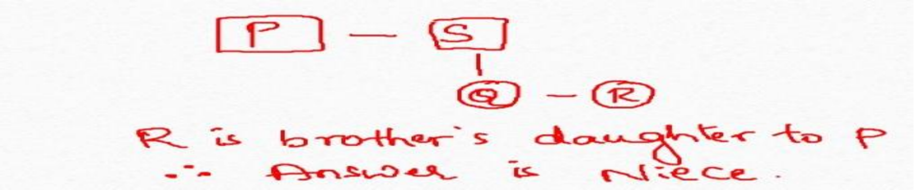
(a) Nephew (b) Son (c) Brother (d) Father



clearly Rajiv is son to Sonia

3) P is S's brother. S is Q's father. R and Q are sisters. How is R related to P?

a) Sister b) niece c) aunt d) none

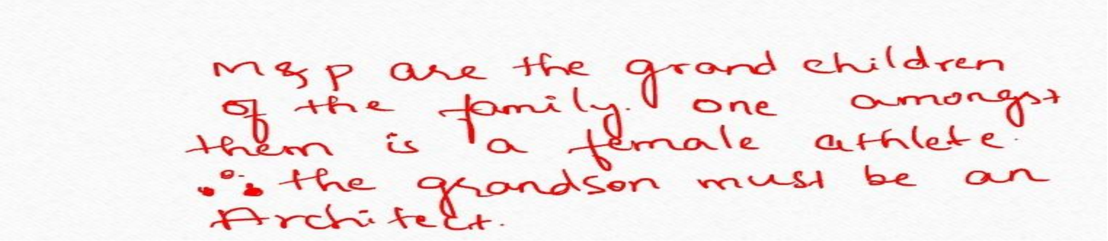


R is brother's daughter to P

∴ Answer is Niece.

4) K, L, M, N, O and P are 6 members of a family in which there are two married couples. L the lawyer married to K. K has one son and one grandson. O, a doctor married to a engineer who is the mother of M and P. Of the two married ladies, one is a housewife. There is also one architect and one female athlete in the family. Which of the following is true about the grandson of the family?

a) Athlete b)Architect c)doctor d)none



M&P are the grand children of the family. one amongst them is a female athlete.

∴ the grandson must be an Architect.

5) Read the following information carefully to answer the questions that follow.

'P + Q' means 'P is father of Q'

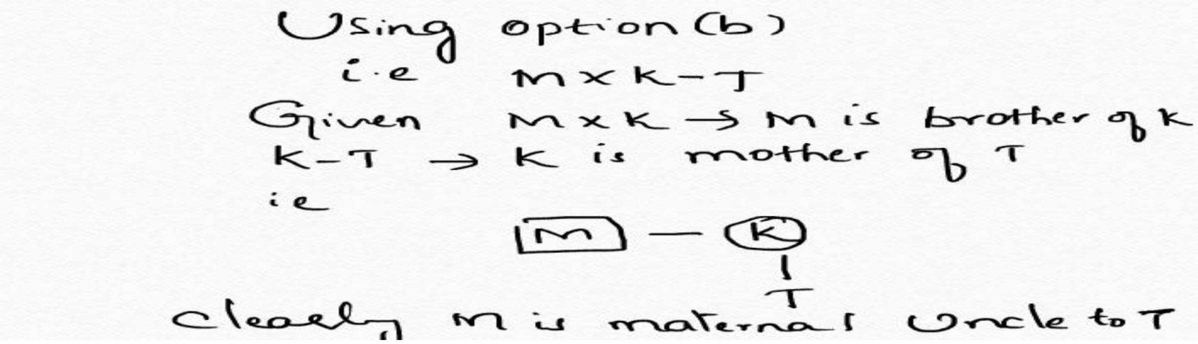
'P - Q' means 'P is mother of Q'

'P × Q' means 'P is brother of Q'

'P ÷ Q' means 'P is sister of Q'

Which of the following means 'M' is maternal uncle of T?

a)M÷K-T b)M×K-T c)M×K+T d)M÷K+T

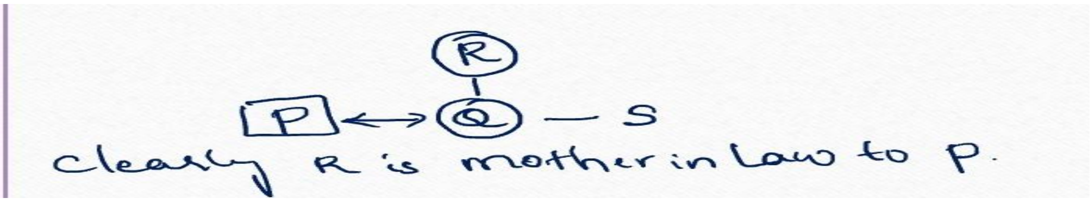


Using option (b)
i.e M × K − T
Given M × K → M is brother of K
K − T → K is mother of T
i.e

Clearly M is maternal Uncle to T

6) If P is the husband of Q and R is the mother of S and Q. What is R to P?

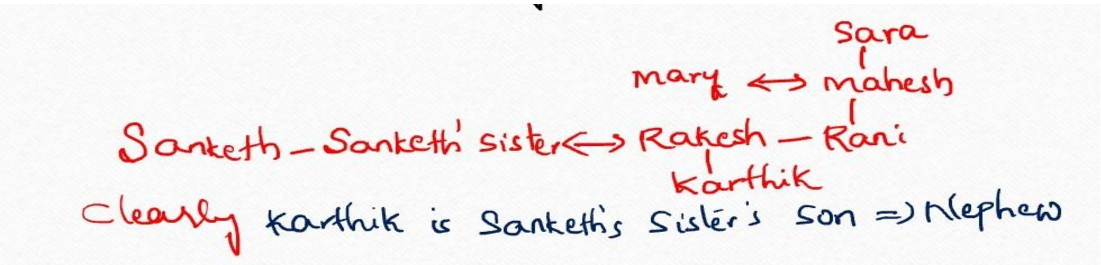
(a) Mother (b) Sister (c) Aunt (d) Mother-in-law



Clearly R is mother in law to P.

7) Sanketh's sister is the wife of Rakesh. Rani's brother is Rakesh. Mahesh is Rani's father. Mary is the daughter-in-law of Sara who is Rakesh's grandmother. Karthik is Rani's brother's son. Who is Karthik to Sanketh?

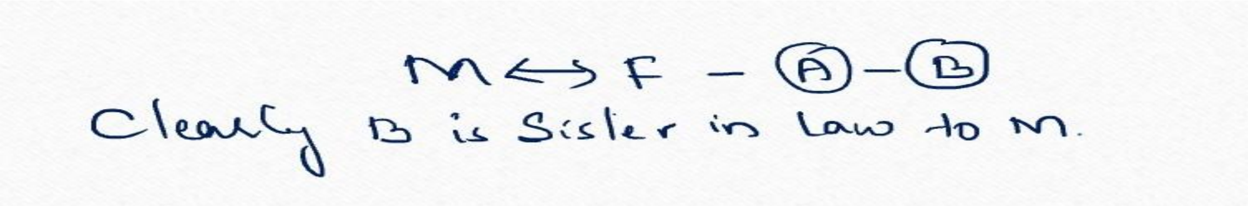
a)Nephew b) Uncle c)Father d)none



Clearly Karthik is Sanketh's Sister's son => Nephew

8) M and F are a married couple. A and B are sisters. A is the sister of F. Who is B to M?

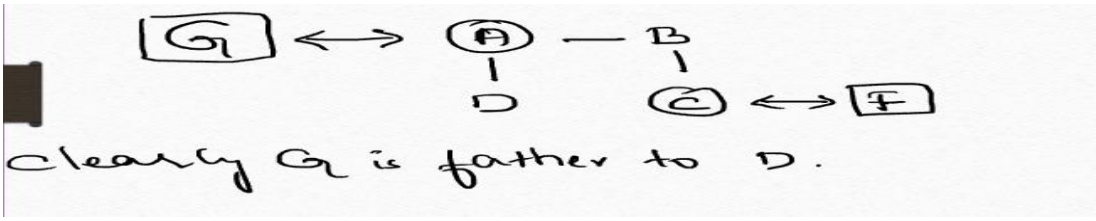
(a) Sister (b) Sister-in-law (c) Niece (d) Daughter



Clearly B is Sister in law to M.

9) A is the mother of D and sister of B. B has a daughter C who is married to F. G is the husband of A. How is G related to D?

(a) Uncle (b) Husband (c) Son (d) Father



Clearly G is father to D.

TYPE2:

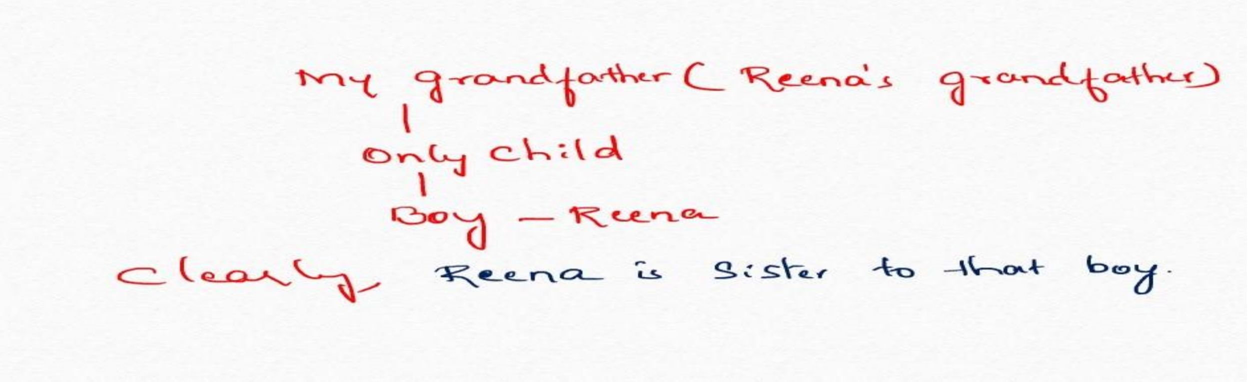
INTRODUCING A PERSON AND POINTING OUT A PERSON IN THE PHOTOGRAPH

Note:

- ***The person who is introducing : Speaker***
- ***The person who is getting introduced: Reference person***
- ***The person who is listening: Listener***
- ***If the statement is within the quotes then, I/mine/ myself refers to speaker, he/she/her/his refers to reference person and you/your refers to the listener.***
- ***If the statement is without the quotes then, he/his/she/her refers to the speaker***

1)Pointing to a boy in the photograph Reena said, "He is the only son of the only child of my grandfather." How Reena is related to that boy?

Mother b)Sister c)aunt d) none



My grandfather (Reena's grandfather)

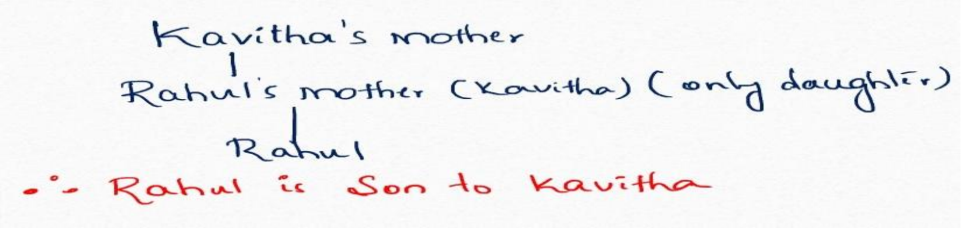
only child

Boy — Reena

Clearly Reena is Sister to that boy.

2) Kavitha says, "Rahuls's mother is the only daughter of my mother". How is Rahul related to Kavitha?

Nephew b)mother c) aunt d) Son



Kavitha's mother

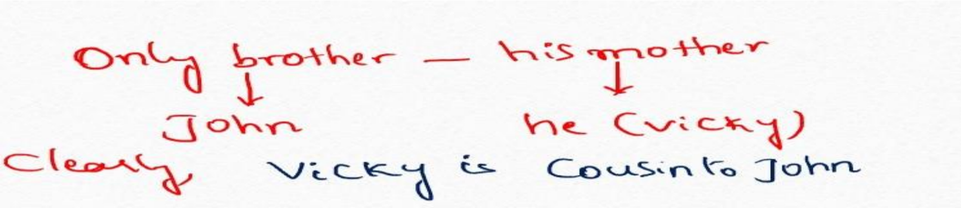
Rahul's mother (Kavitha) (only daughter)

Rahul

∴ Rahul is Son to Kavitha

3) Vicky introduces John as the son of the only brother of his father's wife. How is Vicky related to John?

a) Cousin b) Son c) Brother d) Uncle



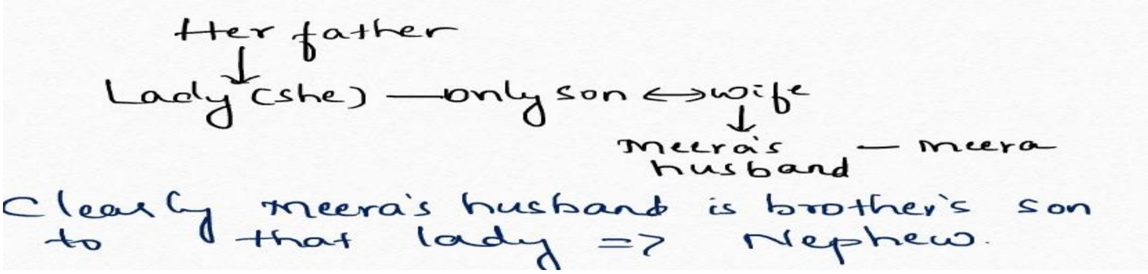
Only brother — his mother

John he (vicky)

Clearly Vicky is Cousin to John

4) Pointing to a lady in a photograph. Meera said. "Her father's only son's wife is my mother-in-law "How is Meera's husband related to that lady in the photo?

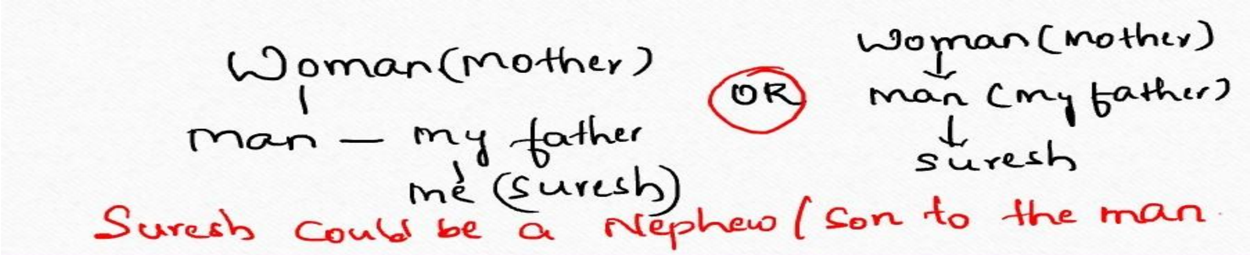
(a) Nephew (b) Uncle (c) Son (d) Father



Clearly Meera's husband is brother's son to that lady => Nephew.

5) Suresh introduces a man as “He is the son of the woman who is the mother of the husband of my mother”. How is Suresh related to the man?

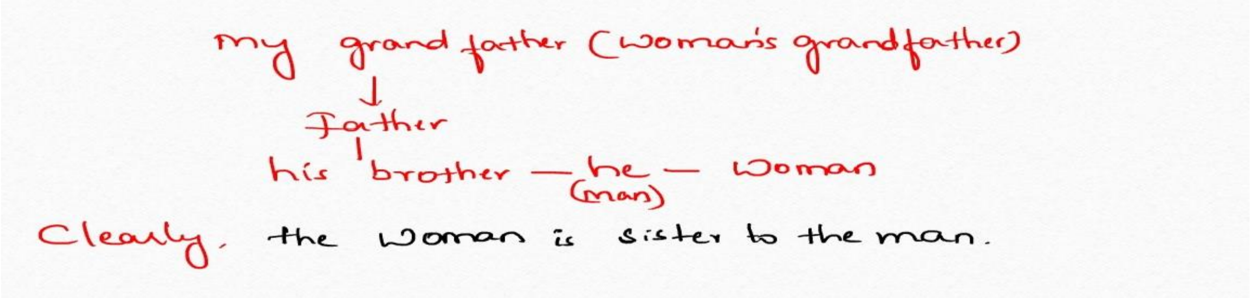
(a) Uncle (b) Son (c) Cousin (d) Grandson



Suresh Could be a Nephew (Son to the man.

6) Pointing to a man in photograph, a woman said “ the father of his brother is the only son of my grandfather”, how is the woman related to the man in the photograph?

a) Sister b) aunt c) Mother d) none



Clearly. the Woman is sister to the man.

Practice problems

1. A is B’s brother. C is A’s mother. D is C’s father, E is B’s son. How is D related to A?

a)Son (b) Grandson (c) Grandfather (d) Great Grandfather

2)As is B’s brother. C is A’s father. D is C’s sister and E is D’s mother. How is B related to E?

a)Grand-daughter (b) Great grands daughter (c) Grandaunt (d) Daughter

3) A is B’s Sister. C is B’s Mother. D is C’s Father. E is D’s Mother. Then how is A related to D?

a)Grandmother (b) Grandfather (c) Daughter (d) Grands-daughter

4)A is the father of B. C is the daughter of B. D is the brother of B. E is the son of A. What is the relationship between C and E?

a)Brother and sister (b) Cousins (c) Niece and uncle (d) Uncle and aunt

5) X is the husband of Y. W is the daughter of X. Z is husband of W. N is the daughter of Z. What is the relationship of N to Y?

a)Cousin (b) Niece (c) Daughter (d) Grand-daughter

6)A reads a book and find the name of the author familiar. The author 'B' is the paternal uncle of C. C is the daughter of A. How is B related to A?

a)Brother (b) Sister (c) Father (d) Uncle

7) Ram and mohan are brothers , Shankar is Mohan's father. Chaya is Shankar's sister. Priya is Shankar's niece. Shubhra is Chaya's grand daughter . Then Ram is Shubhra's

a)Brother b)Uncle c)Cousin d)Nephew

8)If P +Q means P is the mother of Q. P/Q means P is the father of Q. P-Q means P is the sister of Q. then which of the following releationship shows that M is the daughter of R?

a) R/M + N b)R+N/M c) R-M/N d) none

9)Pointing to an old man Kailash said " his son is my son's uncle" how is Kailash related to old man**)**

a)Brother b)Either son or son in law c)Father d)Grand father

10)6members of a family namely A, B, C, D, E and F are travelling together. 'B' is the son of C but C is not the mother of B. A and C are married couple. E is the brother of C. D is the daughter of A. F is the brother of B.

1.How many female members are there in the family?

(a) 3 (b) 2 (c) 4 (d) 1

2. How many children does A have?

a) 1 b)2 c)3 d)4

3)Who is E to D?

a)Uncle b)brother c) Father d) none

4.Who is the mother of B?

11)Pointing to a person photograph , a man said " His mother's husband's sister is my aunt", then who is that man to the person in the photograph

a) Son
b) Uncle
c) Nephew
d) Brother

12)Read the following information carefully and answer the question below. In a family of six persons A, C, E, F, I, K there are two married couples. G is grandmother of A and mother C. E is wife of C and mother of K, K is the granddaughter of I. What is E to A?

a)Daughter (b) Mother c)Grandmother (d) Aunt

13)A man said to a lady "Your mother's husband's sister is my aunt." How is the man related to the lady?

a)Father (b)Grandfather c)Son (d)Brother

14)Pointing towards a person a man said to a woman. "His mother is the only daughter of your father. How is the woman related to that person?

a)Daughter (b)Mother c)Sister (d)Wife

15)Pointing to a lady, A said "that woman is my nephew' maternal grandmother." How is that woman related to A's sister who has no sister?

a)Cousin (b)Son-in-law (c) Mother (d)Mother-in-law

16)Pointing out to a lady sohil said ""she is the daughter of woman who is the mother of the husband of my mother". Who is the lady to Sohil ?

a)Sister (b)Aunt (c) Daughter (d)Sister-in-law

17) Pointing towards A , B said "your mother is the younger sister of my mother". A is related to B as

a)Uncle b)Cousin c) Nephew d) father

18)Shyam's mother said to Shyam , "My mother has a son whose son is Ram". Shyam is related to Ram as

a)Uncle b)Cousin c) Nephew d)Grand father

19)Amit said “This girl is the wife of the grandson of my mother”, How Amit is related to the girl

a)Father in law b) Grandson c)Father d) Son

20)A is son of C. C and Q are sisters. Z is the mother of Q and P is the son of Z. Which of the following is true

a) A and P are Cousins
b) C and P are sisters
c) P is the maternal uncle of A
d) A is the maternal uncle of P

Chapter 13- Statistical description of data and Sampling Theory

Statistics

It is a branch of mathematics that deals with collection of numerical data, analysis, interpretation and presentation of the same in a systematic manner.

It is derived from an Italian word Statista. Which means ‘Statesman’

It is also derived from Latin word Status which means ‘political state’

Statistics in Singular sense:

In singular sense we can define statistics as the science that provides techniques or methods for collecting, analyzing, interpreting and presenting the data

Statistics in plural sense:

In plural sense we can define statistics as the numerical statements of facts relating to any field of enquiry such as data relating to production, income , population , prices etc

Type of data:

There are two basic types:

1. Primary data

2. Secondary data

1. **Primary data** :

A data collected directly from the source

Methods of primary data collection

1. Personal interview

2. Telephonic interview

3.Email questionnaire

4.Observation method : (quantities line height and weight measured using measuring instruments)

Note:

- For population census and during natural calamities, personal interview is the best method of primary data collection

- When the source is far away, telephonic interview is the most inexpensive and effective method of primary data collection:
- Ex: Information collected over phone about railway accidents
- Email questionnaire has the widest area coverage, but the non responses are more.

2. Secondary data

Data collected indirectly from the source, via intermediate agencies

Some important sources of Secondary data

- Govt. & international Agencies
- Internet
- Books and Magazines etc

Data Classification:

1. Temporal(Chronological) Data:

Time related data

Ex: Sales of a company in various years

2.Spatial(Geographical) data

Area related data

Ex: Weather report of various cities

3.Qualitative data

Data related to the characteristic or an attribute

Ex: knowledge, habit of a person, skill, nationality etc

4. **Quantitative data (measurable)**

Any measurable data is said to be quantitative

Ex: Marks, Height, weight, age etc

FREQUENCY DISTRIBUTION

- A systematic presentation of the values taken by a variable and the corresponding frequencies are called frequency distribution of that variable.

- A tabular presentation of frequency distribution is called Frequency Table.

Types:

1. **Continuous (grouped) frequency distribution (class intervals are consi**

a) Inclusive Classes

- Lower limit and upper limits of the a class are inclusive
- Class limits and class boundaries are different
- Inclusive classes can be converted into exclusive by setting up new class boundaries
  i.e the class limits of exclusive classes are 4.5-14.5 , 14.5-24.5 , …….

Marks	**5-14**	**15-24**	**25-34**	**35-44**	**45-54**	**55-64**
No. of students	10	18	32	26	14	10

b) Exclusive classes

- **Class limits and class boundaries are the same.**
- **Only lower class limit is inclusive whereas upper limit is exclusive.**

Marks	**0-20**	**20-40**	**40-60**	**60-80**	**80-100**
No. of students	5	18	5	12	5

2. Discrete frequency Distribution

A distribution of variable which assumes only whole numbers.

X	**1**	**2**	**3**	**4**	**5**
F	5	10	15	10	5

Terminologies used in frequency distribution

- **Class interval –** If the range of a frequency distribution is large, then it is divided into mutually exclusive sub-ranges called class-intervals.

- **Class limit –** Each class interval is specified by 2 limits – Upper class limit & lower class limit
- **Class Width(Class length) –** Difference between the class limits of a class interval is the width of class interval (for exclusive classes). Where as for inclusive classes it is the difference between the class boundaries.
- **Class mark or class mid-value –** The central value of a class interval is called class mid-value.
- **Class frequency –** Number of observations in any class is the class frequency
- **N –** It is the sum of all frequencies in a frequency distribution
- **Inclusive Class Interval –** A class in which upper limit and lower limits are included in the class
- **Exclusive Class Interval –** A class in which lower limit of a class is included in the same class where as the upper limit is included in the succeeding class
- **UCB (upper class boundary**) -It's an upper limit to an upper class limit
- **LCB (Lower class boundary) –** it is a lower limit to a lower limit class
- **Frequency density**

  It is the ratio of class frequency to the class length. i.e.

  Frequency density = $\frac{Class\ frequency}{Class\ length}$
- **Relative frequency (RF)**

  It is the ratio of class frequency to the total frequency. i.e

  RF = $\frac{Class\ frequency}{Total\ frequency}$

  RF ranges between 0 and 1 (both exclusive)
- **Percentage frequency** = RF x 100

Basics rules to be observed for any frequency distribution

- As far as possible the classes must be of equal lengths
- The classes must be unambiguously defined
- The data must be homogeneous
- Classes must be mutually exclusive
- Classes must be Exhaustive
-

LCF & MCF

LCF: Less than cumulative frequency:

- It is required to find Median and partitions for a given data
- For Class boundaries we shall consider UCB of each class
- The last cumulative frequency is N
- If the class intervals are inclusive then we shall convert them into exclusive classes by setting up new boundaries

Example:
Find LCF

CI	**0-9**	**10-19**	**20-29**	**30-39**
F	**5**	**15**	**10**	**20**

Solution:

Less than claas boundary	**9.5**	**19.5**	**29.5**	**39.5**	**49.5**
CF	**5**	**20**	**30**	**50**	**60**

MCF: More than cumulative frequency:

- It is required (alongwith LCF) to plot ogive
- For Class boundaries we shall consider LCB of each class
- The first cumulative frequency is N
- If the class intervals are inclusive then we shall convert them into exclusive classes by setting up new boundaries

Ex: Find MCF

CI	0-10	10-20	20-30	30-40
F	5	15	10	20

Solution:

More than	0	10	20	30	40

claas boundary					
CF	60	55	40	30	10

Note: The sum of LCF and MCF for a given class boundary is always equal to N

Methods of data presentation

1. Textual (Data in the form of a text)
2. **Tabulation (Data arranged in rows &columns)**
   The best method of data presentation

3. **Diagramatic (Graphical) Data presentation**

   → Most attractive method of data presentation

TABULATION

- **Tabulation is a systematic arrangement of classified data in rows and columns of a table.**

Advantages of Tabular presentation

- **It facilitates comparison between rows and columns**
- **Complicated data can also be represented using tabulation**
- **It is a must for diagrammatic representation**
- **Without tabulation, statistical analysis of data is not possible**

Parts of Tabulation:

There are 4 main parts. Namely

1. **Caption: It is the uppermost part of the tabulation which describes the columns or sub columns**
   It is also refered as Column heading
2. **Box head: It is the entire upper part of the tabulation which include Caption, columns, Sub Columns and also units of measurement**
3. **Stubs: It is the left most part of the tabulation which is referred as row headings**
4. **Body: It is the main part of the tabulation which consists of information (numbers) arranged in rows and columns**
   Footnote: It is the lower most part of the tabulation which gives the source of information and also any missed out information

Diagrammatic Presentation of data

It is the representation of statistical data in the form of charts, diagrams and pictures.

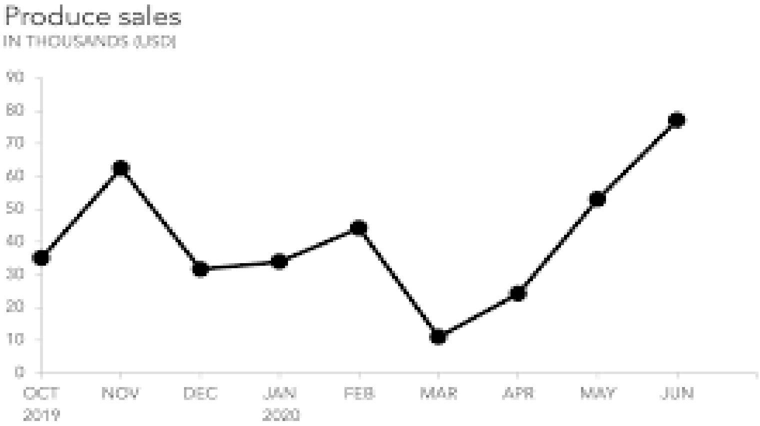
It may be one-dimensional, two dimensional or three dimensional.

The different types of diagrams are

- **Line Diagram**
- **Bar diagram**
- **Pie chart**
- **Histogram**
- **Frequency Polygon**
- **Ogives**
- **Frequency curves**

Line Diagram

- It is one dimensional



Bar graph

→ Bar graphs are one dimensional

Types of bar graphs

→ **Horizontal** : Used for qualitative data

→**Vertical:** Used for quantitative data

→**Multiple Bar graph**: In order to Compare two or more related series

→**Divided bar graph:** For comparing various components of a variable and relating different components to the whole..

Pie chart:

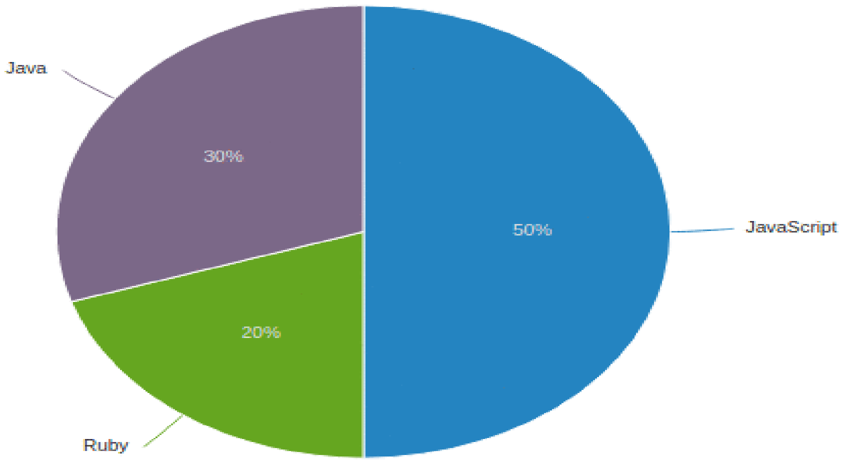
→Also called as circular diagram

→Used to represent family's monthly budget, 5 year planning of a country etc

→Pie charts are two dimensional

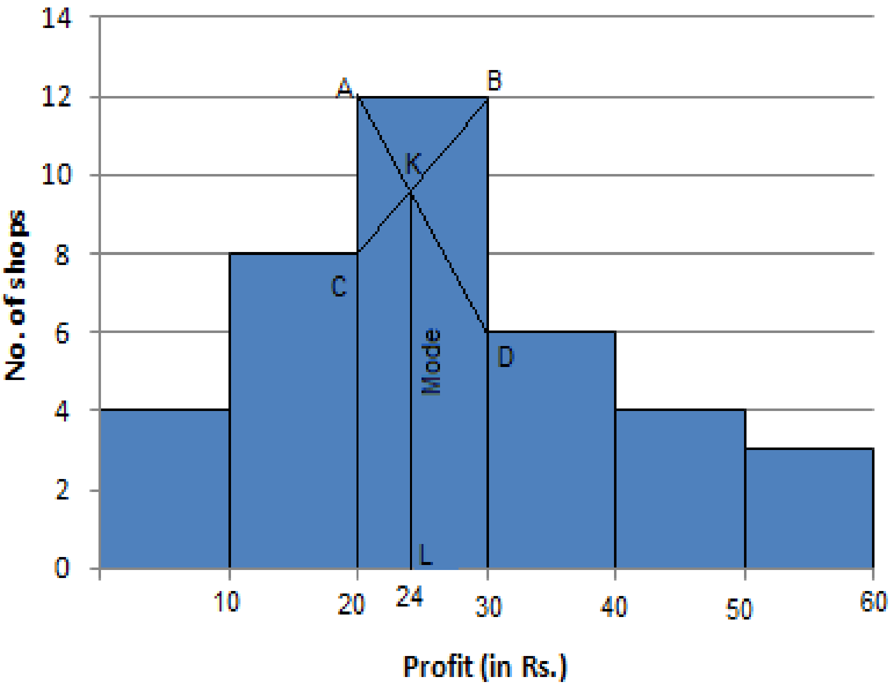
→Central angle = $\frac{x}{\sum x} \times 360$

Where x is observation



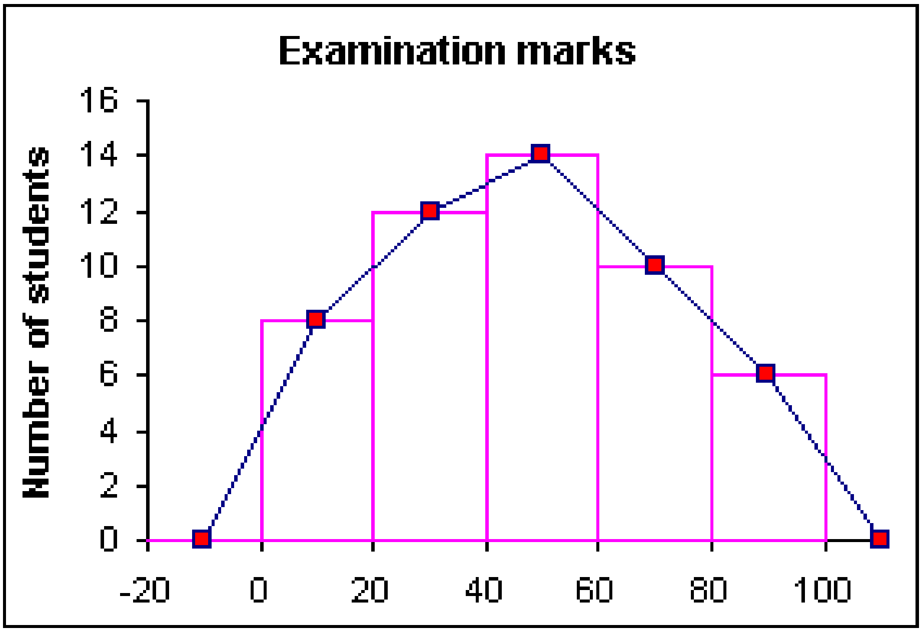
Histogram

- It is also called as area diagram
- It is two dimensional
- Used to measure mode graphically
- Most commonly used graph for continuous frequency distribution
- Can be used for classes with equal widths or unequal widths



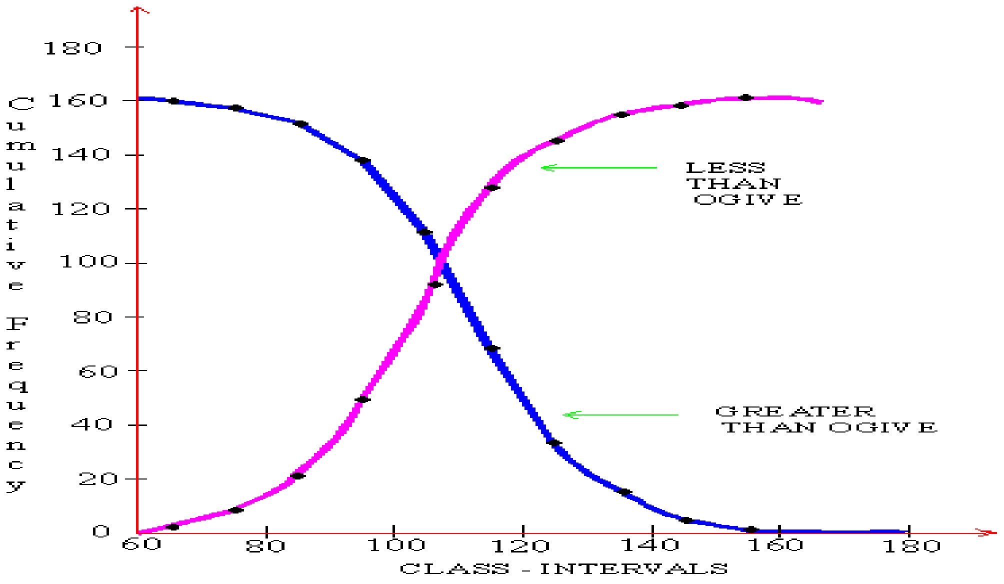
Frequency Polygon:

- It is two dimensional
- Can be obtained from Histogram
- For equal class widths, the area under histogram is same area under frequency polygon



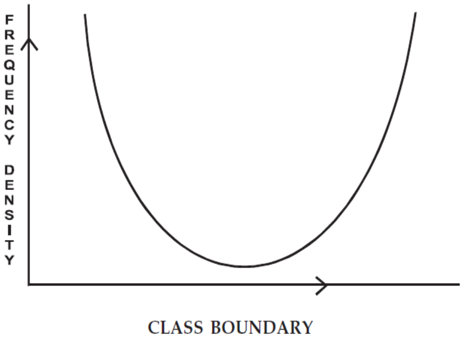
Ogives

- Ogives are cumulative frequency curve
- Ogives are one dimensional
- Used to measure Median and quartiles graphically
- The point of intersection of LCF and MCF gives Median



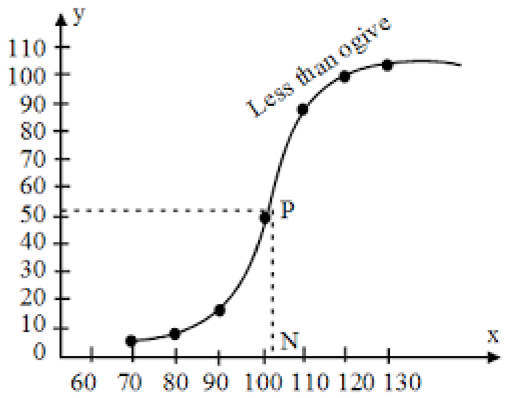
Frequency Curves

U shaped



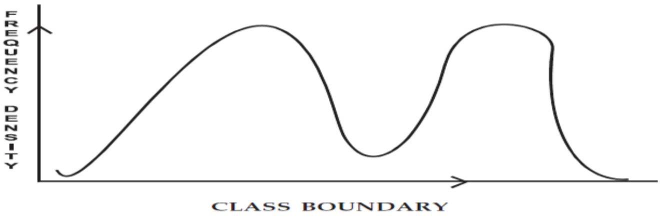
U-shaped curve

Inverted J shaped (s shaped)



Bell Shaped

Mixed



Mixed Curve

- **Frequency curves are one dimensional**
- **Bell shaped (inverted U shaped) frequency curves are most commonly used**
- **Bell shaped frequency curves are used for respresenting profit, height,weight**
- **Age , marks etc**

<u>To find number of classes when the range and Class length (Class width) are given</u>

Number of Classes = $\frac{Range}{Class\ length}$

Or

Number of classes = 1+3.322 logN
Where N is total frequency

Note:

- **Range = Largest observation – Smallest observation**
- **If the number of classes is found to be a non integer then we shall round of to the next immediate integer. I.e if the number of classes is found to be 5.2 then it must be rounded off to 6.**

Worked Examples:

1. The following table relates to the income of 90 persons:

Income in ₹.	1500-1999	2000-2499	2500-2999	3000-3499
No. of Persons:	13	32	20	25

What is the percentage of persons earning more than ₹. 2,500?

a) 45 b) 50 c) 52 d) 55

Solution

No. of persons earning more than 2500 = 20+25 = 45

Total no. of persons = 90

Now percentage of persons earning more than 2500 $= \frac{45}{90} \times 100$

$= \underline{\underline{50}}$

2. Cost of sugar in a month under the heads raw materials, labour, direct production and others were 12, 20, 35 and 23 units respectively. What is the difference between the central angles for the largest and smallest components of the cost of sugar?

a) 72^0 b) 48^0 c) 56^0 d) 92^0

SOLUTION :-

Largest observation = 35

Smallest observation = 12

Difference = 35-12 = 23 ⇒ x

$\Sigma x = 12+20+35+23 = 90$

Central angle corresponding to the difference

$$= \frac{x}{\Sigma x} \times 360$$

$$= \frac{23}{90} \times 360 = \underline{\underline{92^\circ}}$$

3.In 2000, out of total 1,750 workers of a factory, 1,200 were members of a trade union. The number of women employed was 200 of which 175 did not belong to a trade union. In 2004, there were 1,800 employees belong to a trade union and 50 who did not belong to a trade union. Of all the employees in 2004, 300 were women of whom only 8 did not belong to the trade union. On the basis of this information, the ratio of female members of the trade union in 2000 and 2004 is:

a) 292:25 b) 8:175 c) 175:8 d) 25:292

SOLUTION

It is required to find $\frac{\text{female members of union in 2000}}{\text{female members of union in 2004}}$

It is given that out of 200 female employees 175 did not belong to trade union in 2000.

∴ No. of female employees of trade union = 200-175 = $\underline{25}$

It is also given that in 2004, out of 300 female employees 8 did not belong to trade union.

∴ No. of female employees of trade union = 300-8 = 292.

⇒ The required ratio = $\underline{25 : 292}$

4. Out of 1000 persons, 25 percent were industrial workers and the rest were agricultural workers. 300 persons enjoyed world cup matches on T.V. 30 percent of the people who had not watched the world cup matches were industrial workers. What is the number of agricultural workers who had enjoyed world cup matches on TV?

a) 230 b) 250 c) 240 d) 260

SOLUTION :-

WT → Watch TV
DWT → Do not watch TV
IW → Industrial workers
AW → Agricultural Workers

	WT	DWT	Total
IW	40	210	250
AW	260	490	750
Total	300	700	1000

30% of 700 = 210

∴ No. of Agricultural Workers who enjoyed world cup watches on TV = 260

5. The data given below refers to the marks gained by a group of students:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
No. of Students	15	38	65	84	100

Then the no. of students getting marks more than 30 would be______________

a) 50 b) 53 c) 35 d) 62

SOLUTION :-

More than 30 = Below highest CB − Below given CB
= 100 − 65
= 35

CB → Class boundary

6. A pie diagram is used to represent the following data:

Source	Customs	Excise	Income tax	Wealth tax
Revenue in million rupees:	120	180	240	180

The central angles in the pie diagram corresponding to income tax and wealth tax respectively:

a) $(120^0, 90^0)$
b) $(90^0, 120^0)$
c) $(60^0, 120^0)$
d) $(90^0, 60^0)$

SOLUTION :

$\Sigma x = 120 + 180 + 240 + 180 = 720$

Central angle Corresponding to Income Tax = $\frac{240}{720} \times 360$

$= \underline{120^\circ}$

Central angle Corresponding to Wealth Tax = $\frac{180}{720} \times 360$

$= \underline{90^\circ}$

7. If the class interval is 10 – 14, 15 - 19, 20 – 24, then the first class is:

a) 10 – 15
b) 9.5 – 14.5
c) 10.5 – 15.5
d) 9 – 15

SOLUTION :

The given class 10-14, 15-19, 20-24 are Inclusive.

Now the exclusive classes are

9.5 - 14.5, 14.5 - 19.5, 19.5 - 24.5

$\therefore$ The first class is 9.5 - 14.5

8. Find the number of observations between 250 and 300 from the following data:

Value more than:	200	250	300	500
No. of observation:	56	38	15	0

a) 38
b) 23
c) 15
d) None of the above

SOLUTION :

Number of observations b/w 250 & 300

= Observations more than 250 − Observations more than 300

= 38 − 15

= 23

9. From the following data find the number of class intervals if class length is given as 5.

73, 72,65,41,54,80,50,46,49,53

a) 6 b) 5 c) 7 d) 8

SOLUTION

Given class length = 5

Range = 80 − 41

= 39

∴ No. of class intervals = $\frac{\text{Range}}{\text{class length}}$ = $\frac{39}{5}$ = 7.8

∴ No. of class intervals = 8 (Next higher digit to 7.8)

10. The following data relate to the marks of 25 students in statistics

55, 60, 62, 75, 78, 88, 85, 90, 92, 65, 56, 63, 95, 98, 84, 71, 51, 57, 68, 79, 80, 59, 53, 77, 79

What are the frequency densities for the class intervals 61-70 and 81-90

a) 0.4 and 0.4 b) 0.3 and 0.5 c) 0.4 and 0.5 d) none of these

SOLUTION :-

Frequency of a class 61-70 = 4

class length = 10

∴ Frequency density = $\frac{4}{10}$ = 0.4

Now, Frequency of a class 81-90 = 4

class length = 10

∴ Frequency density = $\frac{4}{10}$ = 0.4

11. In a study about male and female students of commerce and science departments of a college in 5 years, the following data were obtained

1995	**2000**
70% male students	75% male students
65% read commerce	40% read science
20% of female students read science	50% of male students read
commerce	
3000 total number of students	3600 total number of students

After combining 1995 and 2000 , if x represents the ratio of female commerce students to female science students and y represents the ratio of male commerce students to male science students, then

a) x=y b) x>y c)x<y d) none of these

SOLUTION :

Required to find. $x = \frac{\text{Female Commerce Students in 1999 \& 2000}}{\text{Female Science Students in 1999 \& 2000}}$

$y = \frac{\text{Male Commerce Students in 1999 \& 2000}}{\text{Male Science Students in 1999 \& 2000}}$

1999

	C	S	Total
M	1230	870	2100
F	720	180	900
Total	1950	1050	3000

2000

	C	S	Total
M	1350	1350	2700
F	810	90	900
Total	2160	1440	3600

$x = \frac{720+810}{180+90} = 5.66$ clearly $\underline{x > y}$

$y = \frac{1230+1350}{870+1350} = \frac{2580}{2220} = 1.66$

12. The number of accidents for seven days in a locality are given below

No of accidents :	0	1	2	3	4	5	6
Frequency :	15	19	22	31	9	3	2

What is the number of cases where 3 or more than 3 accidents occurred?

a) 45 b) 14 c) 34 d) none of these

Solution:

Frequecny of 3 or more accidents = 31+9+3+2 = 45

Note: Descriptive statistics: It will focus on all the observations of the data. For example the average salary of workers is descriptive as we need to know the salary of each worker.

Inferential statistics: It helps in making inference about population based on samples. Using the information about the small sample one can generalize for the large sample.

Practice questions:

1. Data collected on religion from the census reports are:
   a) Primary data b)secondary data c) sample data d) a or b
2. The chart that uses logarithm of variable is known as

a) Ratio chart b) line chart c) multiple line chart d) none of these

3. In collection of data, which of the following are interview methods

   a) Personal interview method b) telephonic interview method

   c)published data d) a and b

4. For constructing a histogram the class intervals of a frequency distribution must be of the following type

   a) Equal b) Unequal c) Equal or unequal d) none of these

5. Profits made by XYZ bank in different years refers to

   a) An attribute b) A discrete variable c) A continuous variable d) none of these

6. The point of intersection of less than ogive curve and more than ogive curve give us:

   a) Mean b) mode c) median d) none of these

7. Frequency density corresponding to the class interval is the ratio of

   a) Class frequency to the total frequency
   b) Class frequency to the class length
   c) Class length to the class frequency
   d) Class frequency to the cumulative frequency

8. Mode of presentation of data:

   a) Textual presentation b) Tabulation c) oral presentation d) a and b

9. Mode can be obtained from

   a) Frequency polygon b) histogram c) ogive d) all of the above

10. The most appropriate diagram to represent the data relating to the monthly expenditure on different items by a family is:

    a) Histogram b) Pie diagram c) Frequency polygon d) line diagram

11. When the two curves of ogive intersect, the point of intersection provides:

    a) First quartile b) second quartile c) Third quartile d) mode

12. The data obtained by the internet are:

    a)primary data b) secondary data c) both a and b c) none of these

13. Which of the following is not a two dimensional diagram

    a) Square diagram b) line diagram c) Rectangular diagram d) pie chart

14. Most extreme values which would ever be included in a class interval are called:

    a) Class interval b) class limits c) class boundaries d) none of these

15. In a study related to the labourers of a factory, the following data were revealed 40% of the total employees are females and 50% of them are married. 50 female workers are not the members of trade union. Compared to this, out of 1200 male workers 1000 are members of trade union. 60% of the male employees are married. 100 male workers are unmarried non members. The unmarried non member

employees are 120. On the basis of this information, the ratio of married female non members to the married male non members is

a) 3:10 b) 4:5 c) 3:8 d) none of these

16. The less than ogive is a

a) U shaped curve b) J- shaped curve c) S- shaped curve d) Bell shaped curve

17. The central angle corresponding to the largest observation given is,

150, 200, 180, 170, 20

a) 100° *b*) 200° *c*) 120° *d*)*none of these*

18. From the following data find the number of class intervals if class length is given as 5.

73, 72,65,41,54,80,50,46,49,53,90,65

a) 6 b) 5 c) 7 d) none

19. The best method of presentation of data is

a) Textual b) tabular c) diagrammatic d) none of these

20. Most commonly used frequency curve is

a) b) inverted J shaped c) bell shaped d) U shaped

21. The following data relate to the marks of 25 students in statistics

55, 60, 62, 75, 78, 88, 85, 90, 92, 65, 56, 63, 95, 98, 84, 71, 51, 57, 68, 79, 80, 59, 53, 77, 79

What are the frequency densities for the class intervals 51-60 and 71-80

a) 0.6 and 0.7 b) 0.3 and 0.5 c) 0.4 and 0.5 d) none of these

22. The number of accidents for seven days in a locality are given below

No of accidents :	0	1	2	3	4	5	6
Frequency :	15	19	22	31	9	3	2

What is the number of cases where 3 or more than 3 accidents occurred?

a) 45 b) 14 c) 34 d) none of these

23. The following data relate to the incomes of 86 persons

Income in Rs. :	500-999	1000-1499	1500-1999	2000-2499
No of persons:	15	28	36	7

What is the percentage of persons earning more than Rs. 1500 ?

a) 50 b) 45 c) 40 d) 60

24. Consecutive recatangles in histogram have no space in between
a) True b) false c) both d) none
25. The amount of non responses is maximum in
a) Mail questionnaire method b) interview method c) observation method d) all of these
26. In order to compare two or more related series, we consider:
    a) Multiple bar chart b) grouped bar chart c) a or b d) a and b
27. The lower class boundary is :
    a) An upper limit to lower class limit b) a lower limit to lower class limit c)both a and b d) none of these
28. Some important sources of secondary data are:
    a) International and government sources b) international and primary sources c)private and primary sources d) Government sources
29. In inclusive classification of data
a) Only LCB is inclusive b) only UCB is inclusive c)Both LCB and UCB are inclusive d) none of these

Sampling Theory

Sampling theory is a branch of statistics that deals with the selection of a subset of individuals from a larger population to make inferences about the whole population. Let's break it down with an example:

Imagine you want to know the average height of all students in a school, but measuring every student's height is impractical and time-consuming. Instead, you decide to use sampling. Here's how it works:

1. **Define the Population:** Your population is all the students in the school.
2. **Choose a Sampling Method:** You might use simple random sampling, where each student has an equal chance of being selected. Alternatively, you could use stratified sampling, dividing students into groups (strata) based on grade levels, and then randomly selecting from each stratum.
3. **Select the Sample:** Let's say you randomly choose 100 students from the school.
4. **Collect Data from the Sample:** Measure the height of the 100 selected students.
5. **Make Inferences:** Use the data from the sample to make predictions or inferences about the entire population's average height.

Sampling theory ensures that the sample you've selected is representative of the population, reducing the risk of biased results. The larger and more representative the sample, the more reliable your inferences about the population will be. It's like getting a taste of the whole pie without having to eat the entire thing!

Criteria for an ideal estimator:

- **Unbiasedness**
- **Consistency**
- **Efficiency**
- **Sufficiency**

Basic principles of Sample survey

1. **Law of Large Numbers:** This law states that as the size of a sample increases, the estimate derived from that sample will converge to the true population parameter. In other words, larger samples tend to provide more accurate representations of the population.
2. **Principle of Inertia (Nonresponse Bias):** This principle emphasizes the importance of minimizing nonresponse bias. Nonresponse occurs when selected individuals in the sample do not participate in the survey. The principle of inertia suggests that nonresponse can introduce bias, and efforts should be made to encourage participation to ensure a representative sample.
3. **Principle of Optimization (Efficiency):** This principle involves finding the balance between sample size and the resources (time, cost, etc.) required for the survey. It aims to optimize the survey process by choosing a sample size that provides sufficient precision without unnecessary costs.
4. **Principle of Validity (Validity):** Validity refers to the extent to which a survey accurately measures what it intends to measure. The principle of validity emphasizes the importance of designing survey questions and methodologies that truly capture the concepts or characteristics of interest in the population.

Comparison between Sample survey and Complete enumeration

1. **Definition:**
    - **Sample Survey:** In a sample survey, data is collected from a subset or sample of the entire population, and statistical techniques are applied to make inferences about the whole population.
    - **Complete Enumeration (Census):** In a complete enumeration, data is collected from every individual or unit in the entire population.
2. **Scope:**
    - **Sample Survey:** Surveys are often conducted when it's impractical or too costly to collect data from the entire population. The goal is to generalize findings from the sample to the larger population.
    - **Complete Enumeration:** A census aims to collect data from every individual in the population, leaving no one out.
3. **Time and Cost:**
    - **Sample Survey:** Generally requires less time and resources compared to a census, making it a more feasible option in many situations.
    - **Complete Enumeration:** Conducting a census is often time-consuming and can be costly, especially for large populations.

4. **Accuracy:**
    - **Sample Survey:** May introduce sampling error, but with proper sampling techniques, statistical methods can be used to estimate and control this error.
    - **Complete Enumeration:** Has the potential to be more accurate since it includes every unit in the population. However, it may still involve non-sampling errors due to issues like nonresponse or data collection errors.
5. **Representativeness:**
    - **Sample Survey:** The representativeness of the sample is crucial for generalizing findings to the entire population. Random sampling methods are often employed to ensure unbiased representation.
    - **Complete Enumeration:** Ensures complete representation of the population by including every unit.
6. **Flexibility:**
    - **Sample Survey:** Offers flexibility in terms of adjusting sample sizes, targeting specific subgroups, and accommodating changes in the research design.
    - **Complete Enumeration:** Less flexible due to the necessity to collect data from everyone in the population.
7. **Applicability:**
    - **Sample Survey:** Commonly used in social sciences, market research, and other fields where insights from a subset of the population can be generalized.
    - **Complete Enumeration:** Essential for official government censuses, where accurate and detailed information about the entire population is required.

In summary, the choice between a sample survey and complete enumeration depends on factors such as the research objectives, available resources, time constraints, and the level of accuracy needed for the study

Errors in sampling survey

1. **Sampling Errors:**
    - **Definition:** Sampling errors occur due to the inherent variability that arises when a sample, rather than the entire population, is surveyed.
    - **Causes:**
        - **Random Sampling Variability:** In simple random sampling, there's a chance that the selected sample does not perfectly represent the population, leading to differences between the sample and population characteristics.

        - **Systematic Sampling Bias:** If there's a pattern or systematic error in the way the sample is selected, it can lead to biased estimates.
    - **Control:** To minimize sampling errors, researchers can use random sampling methods, increase the sample size, and employ statistical techniques to quantify and control for the variability introduced by sampling.
2. **Non-sampling Errors:**
    - **Definition:** Non-sampling errors are not related to the act of sampling itself but can occur at various stages of the survey process.
    - **Causes:**
        - **Nonresponse Bias:** Occurs when selected individuals refuse to participate or cannot be reached, leading to a potential bias in the survey results.
        - **Measurement Errors:** Result from inaccuracies in the way survey questions are worded, how respondents interpret questions, or errors made by interviewers during data collection.
        - **Processing Errors:** Mistakes in data entry, coding, or analysis that can introduce errors into the final dataset.
        - **Coverage Errors:** Arise from issues related to defining the population or including/excluding certain groups.
    - **Control:** Non-sampling errors are challenging to eliminate entirely, but researchers can minimize them through careful survey design, rigorous training of interviewers, thorough data validation processes, and regular quality control checks.
3. **Total Survey Error (TSE):**
    - **Definition:** Total Survey Error is the sum of sampling and non-sampling errors in a survey.
    - **Management:** Researchers need to be aware of and manage TSE by considering and addressing all potential sources of error throughout the survey process—from the design phase to data analysis.

Types of Sampling:

1. **Random Sampling:**
    - **Simple Random Sampling:** Every individual in the population has an equal chance of being selected. This can be done using random number generators or randomization techniques.

    Stratified Random Sampling: The population is divided into subgroups (strata) based on certain characteristics, and then random samples are taken from each stratum. This ensures representation from all subgroups.

It is a sampling technique providing separate estimates for population means for different segments and also an overall estimate.

2. **Non-Random Sampling:**
    - **Convenience Sampling:** Individuals are chosen based on their availability and accessibility. This method is convenient but may lead to biased samples.
    - **Purposive Sampling:** Specific individuals are chosen intentionally based on certain characteristics. This is useful when researchers want to study a particular subgroup.
    - **Quota Sampling:** Similar to stratified sampling, but the samples are not chosen randomly. Researchers select individuals based on certain quotas, ensuring representation from different categories.
3. **Systematic Sampling:**
    - Individuals are selected at regular intervals from a list after a random starting point has been determined. This method is simple and efficient when a complete list of the population is available.
4. **Cluster Sampling:**
    - The population is divided into clusters, and then a random sample of clusters is selected. All individuals within the chosen clusters are included in the study. This method is useful when it's impractical to sample individuals individually.
5. **Snowball Sampling:**
    - This is a chain-referral sampling method where existing participants refer others for inclusion in the study. This is often used when the population is difficult to identify or locate.
6. **Multi-Stage Sampling:**
    - This involves a combination of various sampling methods at different stages. For example, a researcher might use stratified random sampling to choose clusters and then use simple random sampling within those clusters.

Each type of sampling has its own strengths and weaknesses, and the choice of method depends on the research objectives, available resources, and the nature of the population being studied. Researchers must carefully consider the implications of their chosen sampling method to ensure the generalizability of their findings to the larger population.

Important terms associated with Sampling

1. **Sample:**
   - *Definition:* A subset of the population selected for a study or analysis.
   - *Role:* The sample is chosen to represent the larger population, and conclusions are drawn based on observations or measurements within this subset.
   - *A sample having number of units less than 30 is called a small sample*
2. **Parameter:**
   - *Definition:* A numerical characteristic of the entire population.
   - *Role:* Parameters are the true, fixed values that describe the population. Examples include the population mean, standard deviation, or proportion. In most cases, it's impractical or impossible to measure the parameter for the entire population, so we estimate it using statistics from a sample.
3. **Statistic:**
   - *Definition:* A numerical characteristic of a sample.
   - *Role:* Statistics are calculated based on the data collected from the sample. Common examples include the sample mean, sample standard deviation, or sample proportion. These serve as estimators or approximations of the corresponding population parameters

Population mean

$$\boldsymbol{\mu} = \frac{\sum_{i=1}^{n} \boldsymbol{x_i}}{\boldsymbol{n}}$$

Where:

- μ is the population mean,
- n is the number of observations in the sample (population size), and
- xi represents each individual value in the sample.

Population variance

$$\boldsymbol{\sigma^2} = \frac{\sum(\boldsymbol{x_i} - \boldsymbol{\mu})^2}{\boldsymbol{n}}$$

Standard Deviation

$$\boldsymbol{\sigma} = \sqrt{\frac{\sum(\boldsymbol{x_i} - \boldsymbol{\mu})^2}{\boldsymbol{n}}}$$

Sampling Distribution and Standard Error of a statistic

1. Sampling Distribution:

- *Definition:* The distribution of a statistic (such as mean, proportion, standard deviation) calculated from multiple samples of the same size drawn from the same population.
- *Role:* It provides a theoretical probability distribution that describes how a statistic varies over all possible samples of a particular size from a population. Sampling distributions help us make inferences about the population based on sample data.
- **Example:** Imagine you repeatedly take random samples of 30 students from a school and calculate the mean height for each sample. The collection of all these sample means forms the sampling distribution of the sample mean.

2. Standard Error of a Statistic:

- *Definition:* A measure of the variability or spread of a statistic's sampling distribution.
- *Role:* It quantifies the precision of our estimate of the population parameter. A smaller standard error indicates that the sample statistic is likely to be closer to the true population parameter.
- *Standard deviation of a sampling distribution is known as standard error*

SE(for mean) $= \frac{\sigma}{\sqrt{n}}$ ***For simple random sampling with replacement***

$= \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$ ***for simple random sampling without replacement***

Where N is total population
n is sample size

Standard Error for proporation

SE(p) $= \sqrt{\frac{pq}{n}}$ ***for simple random sampling with replacement***

$= \sqrt{\frac{pq}{n}}\sqrt{\frac{N-n}{N-1}}$ ***for simple random sampling without replacement***

Where $\sqrt{\frac{N-n}{N-1}}$ $\boldsymbol{is\ known\ as\ finite\ population\ correction\ or}$

$\boldsymbol{finite\ population\ multiplier}$

It may be ignored as it tends to 1 if the sample size(n) is very large or population under consideration is infinite when the parameters are unknown, they may be replaced by the corresponding statistic.

Worked Examples:

1. **A population comprises the following units m,n,o,p,q,r. Draw all possible samples of size 5 without replacement**

Solution:

Given n=5

N=6

Therefore total number of possible samples = $6C_5$ = 6

2.)A population comprises 3 members 1,5,3. Draw all possible samples of size 2.

a) **With replacement**
b) **Without replacement**

Find the sampling distribution of sample mean in both cases.

Solution:

Total possible samples of size 2 with replacement = 3^2 = 9

Sl. No	**Sample of size 2 with replacement**	**Sample mean**
1	**1,1**	**1**
2	**1,5**	**3**
3	**1,3**	**2**
4	**5,1**	**3**
5	**5,5**	**5**
6	**5,3**	**4**
7	**3,1**	**2**
8	**3,5**	**4**
9	**3,3**	**3**

Sampling distribution of sample mean

Mean	1	2	3	4	5	Total
P	1/9	2/9	3/9	2/9	1/9	1

ii) Without replacement

Total possible samples= $3C_2$ = $3C_1$=3

Sl. No	Sample of size 2 without replacement	Sample mean
1	1,3	2
2	1,5	3
3	3,5	4

Sample distribution of mean

Mean	2	3	4	Total
P	1/3	1/3	1/3	1

3. Compute the standard deviation of sample mean for the last problem. Obtain the SE of sample mean and show that they are same for with replacement and without replacement

Practice questions:

1. Sampling can be described as a statistical procedure
   - (a) To infer about the unknown universe from a knowledge of any sample
   - (b) To infer about the known universe from a knowledge of a sample drawn from it
   - (c) To infer about the unknown universe from a knowledge of a random sample drawn from it
   - (d) Both (a) and (b).
2. The Law of Statistical Regularity says that
   - (a) Sample drawn from the population under discussion possesses the characteristics of the population
   - (b) A large sample drawn at random from the population would posses the characteristics of the population
   - (c) A large sample drawn at random from the population would possess the characteristics of the population on an average
   - (d) An optimum level of efficiency can be attained at a minimum cost.
3. A sample survey is prone to
   - (a) Sampling errors
   - (b) Non-sampling errors
   - (c) Either (a) or (b)
   - (d) Both (a) and (b)
4. population of roses in Salt Lake City is an example of
   - (a) A finite population
   - (b) An infinite population
   - (c) A hypothetical population
   - (d) An imaginary population.
5. Statistical decision about an unknown universe is taken on the basis of
   - (a) Sample observations
   - (b) A sampling frame
   - (c) Sample survey
   - (d) Complete enumeration
6. Random sampling implies
   - (a) Haphazard sampling
   - (b) Probability sampling
   - (c) Systematic sampling
   - (d) Sampling with the same probability for each unit.
7. A parameter is a characteristic of
   - (a) Population
   - (b) Sample
   - (c) Both (a) and (b)
   - (d) (a) or (b)
8. A statistic is

(a) A function of sample observations (b) A function of population units

(c) A characteristic of a population (d) A part of a population.

9. Sampling Fluctuations may be described as

(a) The variation in the values of a statistic

(b) The variation in the values of a sample

(c) The differences in the values of a parameter

(d) The variation in the values of observations.

10. The sampling distribution is

(a) The distribution of sample observations

(b) The distribution of random samples

(c) The distribution of a parameter

(d) The probability distribution of a statistic.

11. Standard error can be described as

(a) The error committed in sampling

(b) The error committed in sample survey

(c) The error committed in estimating a parameter

(d) Standard deviation of a statistic.

12. A measure of precision obtained by sampling is given by

(a) Standard error (b) Sampling fluctuation

(c) Sampling distribution (d) Expectation.

13. As the sample size increases, standard error

(a) Increases (b) Decreases

(c) Remains constant (d) Decreases proportionately.

14. If from a population with 25 members, a random sample without replacement of 2 members is taken, the number of all such samples is

(a) 300 (b) 625

(c) 50 (d) 600

15. A population comprises 5 members. The number of all possible samples of size 2 that can be drawn from it with replacement is

(a) 100 (b) 15

(c) 125 (d) 25

16. Simple random sampling is very effective if

(a) The population is not very large

(b) The population is not much heterogeneous

(c) The population is partitioned into several sections.

(d) Both (a) and (b)

17. Simple random sampling is

(a) A probabilistic sampling (b) A non- probabilistic sampling

(c) A mixed sampling (d) Both (b) and (c).

18. According to Neyman's allocation, in stratified sampling

(a) Sample size is proportional to the population size

(b) Sample size is proportional to the sample SD

(c) Sample size is proportional to the sample variance

(d) Population size is proportional to the sample variance.

19. Which sampling provides separate estimates for population means for different segments and also an over all estimate?

(a) Multistage sampling (b) Stratified sampling

(c) Simple random sampling (d) Systematic sampling

20. Which sampling adds flexibility to the sampling process?

(a) Simple random sampling (b) Multistage sampling

(c) Stratified sampling (d) Systematic sampling

21. Which sampling is affected most if the sampling frame contains an undetected periodicity?

(a) Simple random sampling (b) Stratified sampling

(c) Multistage sampling (d) Systematic sampling

22. Which sampling is subjected to the discretion of the sampler?

(a) Systematic sampling (b) Simple random sampling

(c) Purposive sampling (d) Quota sampling.

23. If a random sample of size 2 with replacement is taken from the population containing the units 3,6 and 1, then the samples would be

(a) (3,6),(3,1),(6,1)

(b) (3,3),(6,6),(1,1)

(c) (3,3),(3,6),(3,1),(6,6),(6,3),(6,1),(1,1),(1,3),(1,6)

(d) (1,1),(1,3),(1,6),(6,1),(6,2),(6,3),(6,6),(1,6),(1,1)

24. If a random sample of size two is taken without replacement from a population containing the units a,b,c and d then the possible samples are

(a) (a, b),(a, c),(a, d) (b) (a, b),(b, c), (c, d)

(c) (a, b), (b, a), (a, c),(c,a), (a, d), (d, a) (d) (a, b), (a, c), (a, d), (b, c), (b, d), (c,d)

Chapter 14- Measures of central tendency and dispersion

Central Tendency

Central tendency may be defined as

→A single value that represents the whole set of data

or

→ A value around which most of the observations get clustered or concentrated

Ex: An IPL team is recognized by high average wins

Ex: Educational institutions are recognized by average marks obtained by its students

Measures of Central Tendency

- Arithmetic mean (AM)
- Median (M)
- Partitions
  a) Quartile
  b) Decile
  c)Percentile
- Mode(Z)
- Geometric mean (GM)
- Harmonic Mean (HM)

Arithmetic Mean (AM) For Ungrouped data

AM = $\bar{x} = \frac{\sum x}{n}$

where, $\sum x = x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots + x_n$

n → number of observations

Eg :- Find AM of 5, 10, 15, 25, 35

Solution :-

$$AM = \frac{\sum x}{n} = \frac{5 + 10 + 15 + 25 + 35}{5}$$

$$= \frac{90}{5} = \underline{18}$$

Note :- AM is rigidly defined as it gives us a single value.

AM of observations having equal spacing

If the observation have equal spacing then

AM = AM of Extreme values

Example ;- Find the AM of 10, 20, 30, 40, 50, 60

Solution :-

For the given observations there exists an equal spacing of 10.
:. AM = AM of 10 & 60

= $\frac{10+60}{2}$ = 35

Note:
If each observation is added / subtracted / multiplied / divided by a common number k', then AM also gets added/ Subtracted / multiplied / divided by the same common number 'K'

Ex:1. The AM of 30 observations is 35. If each Observation is added by 5 then the new AM is
Solution:

K=5
New AM = old AM+K
= 35+5
=40

Ex :- The AM of 30 observations is 60. If each observation is divided by 6, then find the New AM.

***Solution:-* K=6, old AM=60 :New AM= (old Am)/K**
= (60)/6 = 10

Combined AM($\overline{X}_c$)

If there are 2 groups with n_1, &n_2, as the no of observations respectively & $\overline{x_1}$ & $\overline{x_2}$ as the respective AM's then,

$$\overline{x_c} = \frac{n_1\,\overline{x_1} + n_2\,\overline{x_2}}{n_1 + n_2}$$

Note : This formula Can be extended for any number of groups.

Type1:
Example:
1. There are 2 Groups with 60 & 40 as the respective no. observations 60 & 70 as the

respective AM's. Find combine

SOLUTION :

Given $n_1=60$, $n_2=40$ $\bar{x}_1 = 60$ $\bar{x}_2 = 70$

Combined AM = $\frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2} = \frac{(60\times60)+(40\times70)}{60+40}$

$\boxed{\bar{x}_c = 64}$

Type2:

Example :-

The AM of 15 observations is 20. If the AM of first 11 of them is 21. Then find the AM of remaining observations,

SOLUTION

Given :- $n_1+n_2=15$, $\bar{x}_c = 20$ $\bar{x}_2 = ?$

$n_1 = 11$, $\bar{x}_1 = 21$

$$\bar{x}_2 = \frac{(n_1+n_2)\bar{x}_c - n_1\bar{x}_1}{n_2} = \underline{\underline{17.25}}$$

Example:

The mean weight of 60 students of a class is 65kgs. If the mean weight of 20 of them is 70kgs, that of another 25 students is 60 kgs, then find the mean weight of remaining students.

SOLUTION :

Given $(n_1+n_2+n_3) = 60$ $\quad \bar{x}_c = 65$

$n_1 = 20$ $\quad \bar{x}_1 = 70$

$n_2 = 25$ $\quad \bar{x}_2 = 60$

$n_3 = 15$ (Remaining). $\quad \bar{x}_3 = ?$

$$\bar{x}_3 = \frac{(n_1+n_2+n_3)\,\bar{x}_c - n_1\bar{x}_1 - n_2\bar{x}_2}{n_3} = \underline{\underline{66.66}}$$

AM for Grouped data

(i) Without shift of origin & change of 'scale' (without step deviation)

Note :

a) shift of origin means adding or subtracting each observation by a common number

b) Change of scale means multiplying or dividing each observation by a common number

$\bar{X} = \frac{\sum f.x}{N}$

where N = $\sum f$

$\sum fx$ = $f_1\ x_1 + f_2\ x_2 + \ldots\ldots.. + f_n\ x_n$

(ii) with shift of origin & change of scale. (With step deviation)

$\bar{X} = A + \left(\frac{\sum fd}{N}\right) \times C$

Example1

Find AM for following data. data.

CI:	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
f :	**15**	**20**	**25**	**20**	**15**

SOLUTION :

$N = \Sigma f = 95$

x :	5	15	25	35	45
f :	15	20	25	20	15

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{2375}{95} = \underline{25}$$

Example2:

Find the value such that AM for the distribution is 4.

X:	**2**	**4**	**6**	**8**	**P**
f:	**5**	**10**	**15**	**20**	**25**

SOLUTION :-

$\bar{x} = 4$ (Given)

$N = \Sigma f = 75$

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{10 + 40 + 90 + 160 + 25P}{75}$$

$$4 = \frac{300 + 25P}{75}$$

$$300 = 300 + 25P$$

$$25P = 0$$

$$\therefore P = \underline{0}$$

Median (M)

For Ungrouped data

→ It is the middle most observation , provided the observation are sorted (arranged either in ascending or descending Order)

→ If the number observations (n) is odd then,

M = $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation

→ If ‘n’ is even Then,

M = Am of $\left(\frac{n}{2}\right)^{\text{th}}$ & $\left(\frac{n+2}{2}\right)^{\text{th}}$ Observations.

Note :-

Median is an appropriate measure of Central tendency for open end classes.

→ Like AM, median is also rigidly defined.

→ Median is regarded as a postitional average

Example1

Find the median of 30,20,40, 70, 10,50

SOLUTION :-

Sorted Observations => 10, 20 .30, 40, 50, 70

Median = AM of 30 & 40

$= \frac{30+40}{2} = \underline{35}$

Example2:

Find the value of x such that the median for the distribution

x, $\frac{x}{3}, \frac{x}{6}, \frac{x}{9}, \frac{x}{12}, \frac{x}{15}$ $is\ 25.$

SOLUTION :-

Given median = 25

Median = AM of $\frac{x}{6}$ & $\frac{x}{9}$

$$25 = \frac{\frac{x}{6} + \frac{x}{9}}{2}$$

$$50 = \frac{15x}{54}$$

$$2700 = 15x$$

$$\boxed{x = 180}$$

Median for discrete grouped data

M = $\left(\frac{N+1}{2}\right)$ th value

Procedure :-

1. Find less than cumulative frequencies (LCF)

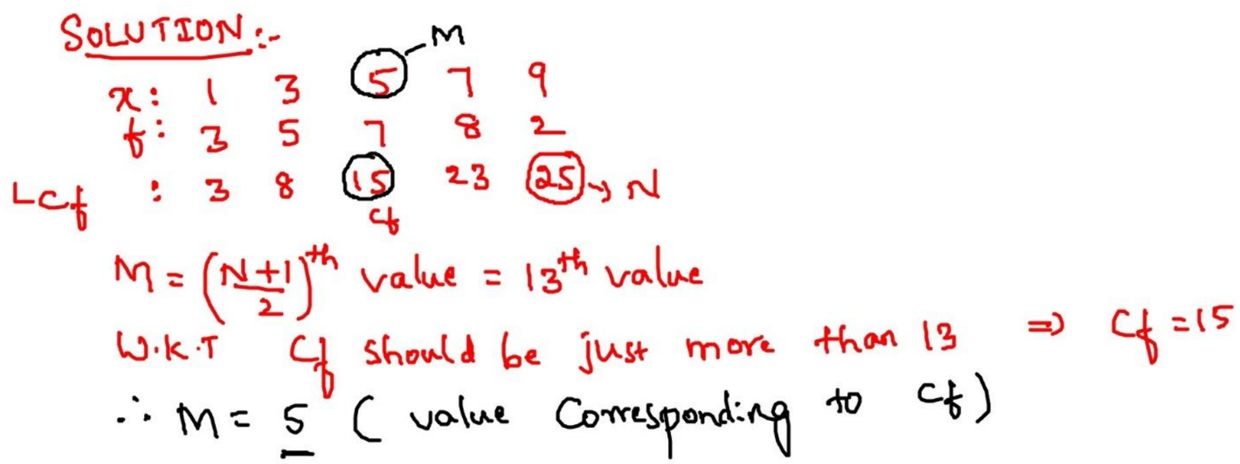
2. Mark cf such that, that cf > $\left(\frac{N+1}{2}\right)$

3. The Corresponding 'x' value will be the median

Example1

Find the median for the following frequency distribution

x :	**1**	**3**	**5**	**7**	**9**
f :	**3**	**5**	**7**	**8**	**2**



SOLUTION :-

x :	1	3	$\overset{M}{\boxed{5}}$	7	9
f :	3	5	7	8	2
Lcf :	3	8	$\underset{cf}{\boxed{15}}$	23	$\boxed{25} \rightarrow N$

$M = \left(\frac{N+1}{2}\right)^{th}$ value $= 13^{th}$ value

W.K.T cf should be just more than 13 $\Rightarrow$ cf $= 15$

$\therefore M = \underline{5}$ (value Corresponding to cf)

Example2:

Find the median for the following frequency distribution

x :	**4**	**8**	**12**	**16**	**20**
f :	**3**	**6**	**9**	**12**	**15**

SOLUTION :

x :	4	8	12	$\boxed{16} \rightarrow M$	20
f :	3	6	9	12	15
Lcf :	3	9	18	$\boxed{30}$ cf	$\boxed{45} \rightarrow N$

$M = \left(\frac{N+1}{2}\right)^{th}$ value $= 23^{rd}$ value

$= \underline{16}$

Median for Continuous grouped data

$$M = l_1 + \left(\frac{\frac{N}{2} - cf}{f}\right) \times c$$

where l_1 → LCB of a median class

1_2 → UCB of Median class

C→ length of the median class = $l_2 - l_1$

f→ Median class frequency

Note:-

1. Median class is the class next to the cf.
2. cf is the Lcf just less than $\frac{N}{2}$

Example1

Find median

CI	0-9	10-19	20-29	30-39	40-49
F	10	30	50	30	10

SOLUTION :-

l_1 ↖ 19.5 median class ↑ 29.5 → l_2

CI :	0-9	10-19	20-29	30-39	40-49
f :	10	30	50 f	30	10
Lcf :	10	40 cf	90	120	130 N

$c = l_2 - l_1 = \underline{10}$

$\frac{N}{2} = 65$

cf must be a value just less than $\frac{N}{2}$

$$M = l_1 + \left(\frac{\frac{N}{2} - cf}{f}\right) \times c = 19.5 + \left(\frac{65-40}{50}\right) \times 10 = \underline{24.5}$$

Example2:

Find median

CI :	**0-20**	**20-40**	**40-60**	**60-80**	**80-100**
f :	**15**	**20**	**25**	**20**	**15**

SOLUTION :

l_1 median class $\to l_2$

CI:	0-20	20-40	40-60	60-80	80-100
f :	15	20	25 f	20	15
Lcf :	15	35 cf	60	80	95 N

$\frac{N}{2} = 47.5$

$c = l_2 - l_1 = 60 - 40 = 20$

$$M = l_1 + \left(\frac{\frac{N}{2} - cf}{f}\right) \times c = 40 + \left(\frac{47.5-35}{25}\right) \times 20 = \underline{50}$$

Mode (Z)

For Ungrouped data

→ It is the observation that occurs most number of times.

Ex1:- Find mode for the observations 10, 20, 30, 40, 50

Solution: Since no observation is repeated. Therefore mode is not defined for this data

Ex2 : Find mode for the observations 10, 20, 10, 20, 30

Solution: Mode: 10 and 20

Ex 3: Find the mode for the observations
10, 20, 10, 20, 10

Solution: Mode: 10

Mode for grouped data

$$Z = l_1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times C$$

where,

l_1 → LCB of modal class

C → modal class length

$C = l_2 - l_1$

1_2 → UCB of modal class

f_1 → frequency of the modal class (highest frequency)

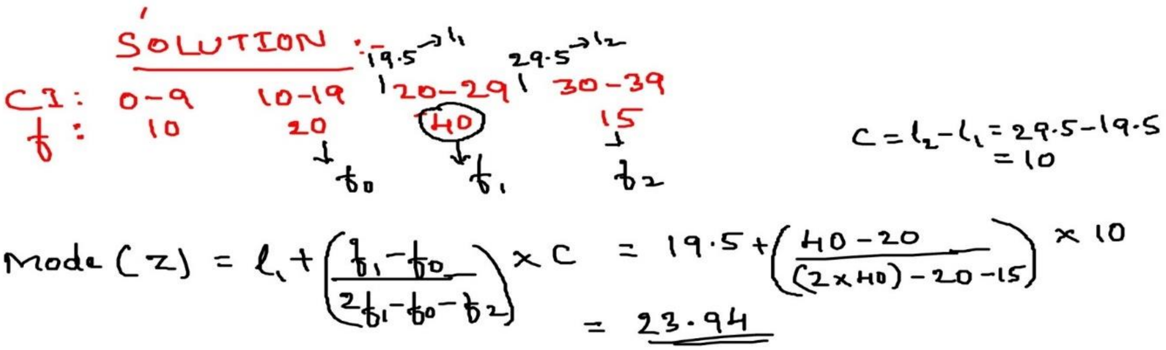
f_0 → frequency of pre modal class

f_2 → frequency of post modal class.

Example1

Find the mode of the distribution

CI	0-9	10-19	20-29	30-39
F	10	20	40	15



SOLUTION :-

$19.5 \to l_1$ $\quad 29.5 \to l_2$

CI :	0-9	10-19	20-29	30-39
f :	10	20	$\boxed{40}$	15
		f_0	f_1	f_2

$$c = l_2 - l_1 = 29.5 - 19.5 = 10$$

$$\text{Mode }(Z) = l_1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c = 19.5 + \left(\frac{40 - 20}{(2 \times 40) - 20 - 15}\right) \times 10$$

$$= \underline{\underline{23.94}}$$

Example2:

CI	0-20	20-40	40-60	60-80
F	10	50	20	5

SOLUTION :-

CI :	0-20	l_1 20-40 l_2	40-60	60-80
f :	10 $\downarrow$ f_0	$\boxed{50}$ $\downarrow$ f_1	20 $\downarrow$ f_2	05

$c = l_2 - l_1 = 40 - 20 = \underline{20}$

$$\text{mode}(Z) = l_1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c = 20 + \left(\frac{50-10}{(2\times50)-10-20}\right) \times 20$$

$$= \underline{31.42}$$

Relationship between mean, Median & mode

a) For symmetrical distribution
Mean= median= Mode

b) For positively skewed (skewed right) distribution

Mean> median > mode

c) For negatively skewed (skewed left) distribution
mode > median> mean

d) For moderately skewed distribution
Mean-mode = 3(mean-Median)
Or
Mode = 3 median-2 Mean

Example1

For moderately skewed distribution the difference between mean & mode is 75. Find the difference between mean & median.

SOLUTION :-

Given Mean − mode = 75 To find mean − median

W.K.T Mean − mode = 3 (Mean − Median)

75 = 3 (Mean − Median)

$\therefore$ Mean − Median = $\underline{\underline{25}}$

Example2:

For a moderately skewed distribution if mean = 6, median= 8 then find mode

SOLUTION :-

Given mean = 6 median = 8. To find, mode

W.K.T for a Moderately skewed distribution

Mode = 3 median − 2 mean

$= 3(8) - 2(6)$

$= 24 - 12$

$= \underline{12}$

Note:-

→ ***AM gets affected by extreme values & also by sampling fluctuations.***

→ ***Median is not affected by extreme values & sample fluctuation.***

→ ***Mode may or may not get affected due to extreme values.***

Note:-

→ ***The sum of the deviations taken from AM is zero.***
i.e. $\sum(x - \overline{x})$ ***= 0***

→ ***The sum of absolute deviations is minimum when taken from Median.***

i.e $\sum |x - M|$ ***is minimum***

Partitions

→ These are the values dividing the given set of observations into number of equal parts.

Types of partitions

a) Quartiles

→These are the values dividing the given set of observations into 4 equal parts.

→Number of partition points = 3

b) Deciles

→These are the values dividing the given set of observations into 10 equal parts.

→Number of partition points = 9

c) Percentiles

→These are the values dividing the given set of observations into 100 equal parts.

→Number of partition points =99

For Ungrouped data

$Q_i = \left[\frac{i}{4}\,(n+1)\right]^{th}$ value , $D_i = \left[\frac{i}{10}\,(n+1)\right]^{th}$ value , $P_i = \left[\frac{i}{100}\,(n+1)\right]^{th}$ value

where , n is number of observations
where Q_1→ lower quartile
Q_3 → upper quartile
Q_2 →Median

For discrete grouped data

$Q_i = \left[\frac{i}{4}\,(N+1)\right]^{th}$ value , $D_i = \left[\frac{i}{10}\,(N+1)\right]^{th}$ value , $P_i = \left[\frac{i}{100}\,(N+1)\right]^{th}$ value
where $N = \sum f$

For Continuous grouped data:-

$Q_i = l_1 + \left[\frac{\frac{iN}{4} - cf}{f}\right] \text{x C}$, $D_i = l_1 + \left[\frac{\frac{iN}{10} - cf}{f}\right] \text{x C}$, $P_i = l_1 + \left[\frac{\frac{iN}{100} - cf}{f}\right] \text{x C}$

where $\frac{iN}{4} > cf$, $\frac{iN}{10} > cf$, $\frac{iN}{100} > cf$

Partition class is a class next to cf.

f → The frequency of Partition class.
l_1,→ LCB of Partition class
l_2,→ UCB of Partition class
$C = l_2 - l_1$

Example1
1. Find Q_1 and D_8 for 15, 25, 35, 65, 55, 40

SOLUTION :-

Given observations $\Rightarrow$ 15, 25, 35, 65, 55, 40 , $\boxed{n=6}$

Observations in asceding order $\Rightarrow$ 15, 25, 35, 40, 55, 65

$Q_1 = \left[\frac{1}{4}(n+1)\right]^{th}$ value $\qquad \left[\because Q_i = \left[\frac{i}{4}(n+1)\right]^{th} \text{value}\right]$

$Q_1 = \left[\frac{1}{4}(6+1)\right]^{th}$ value $= 1.75^{th}$ value

$= 1^{st}$ value $+ 0.75\,(2^{nd}$ value $- I^{st}$ value$)$

$= 15 + 0.75\,(25-15) = \underline{22.5}$

$D_8 = \left[\frac{8}{10}(n+1)\right]^{th}$ value $= 5.6^{th}$ value

$= 5^{th} + 0.6\,(6^{th} - 5^{th})$

$= 55 + 0.6\,(65-55) = \underline{61}$

Example2:

Find P30

x	2	4	6	8	10
f	3	2	7	8	3

SOLUTION

Given

$$\begin{array}{llllll} x: & 2 & 4 & \boxed{6} \rightarrow P_{30} & 8 & 10 \\ f: & 3 & 2 & 7 & 8 & 3 \\ Lcf: & 3 & 5 & \underset{cf}{\boxed{12}} & 20 & \boxed{23} \rightarrow N \end{array}$$

$P_{30} = \left[\frac{30}{100}(N+1)\right]^{th}$ value. $\qquad \left[\because P_i = \left[\frac{i}{100}(N+1)\right]^{th} \text{value}\right]$

$= 7.2^{th}$ value

$= \underline{6}$

Example3

Find Q3.

CI	0-10	10-20	20-30	30-40	40-50
F	5	10	20	15	10

SOLUTION :-

Given

C I:	0-10	10-20	20-30	l_1 30-40 l_2	40-50
f:	5	10	20	$\boxed{15}$ f	10
Lcf :	5	15	$\boxed{35}$ cf	50	$\boxed{60}$ N

N_p for $Q_3 = \frac{3}{4} N = \frac{3}{4} \times 60 = \underline{45}$.

Now cf should be Lcf just less than 45

ie cf = 35

∴ partition class is 30-40. $c = l_2 - l_1 = 40-30 = 10$

Now $Q_3 = l_1 + \left(\frac{N_p - cf}{f}\right) \times c = 30 + \left(\frac{45-35}{15}\right) \times 10 = \underline{\underline{36.66}}$

Note :- Median in terms of partitions:

→ **M = Q_2 = $\frac{Q_1 + Q_3}{2}$**

Geometric Mean (GM)

It is an appropriate measure of central tendency when the observations are in terms of percentages or ratios.

If x & y are 2 variables then

- GM(xy) = GM of x × GM of y
- GM(x/y) = (GM of x)/(GM of y)

The logarithm of GM of given set of observation is the Arithmetic mean of the logarithm of those observation.

Log (GM) = $\frac{1}{n} \sum \log x$

GM For Un grouped data

GM = $(X_1 \times X_2 \times X_3 \times \ldots\ldots \times X_n)^{1/n}$

Or

$(GM)^n = (X_1 \times X_2 \times X_3 \times \ldots\ldots \times X_n)$

where

n → no of observations.

Note :-

Finding n^{th} root other than square root is not possible in simple calculators. So students are advised to use option hitting method to solve the questions on GM.

$(option)^n$ = product of observations

Example1

Find the GM of 8, 10 & 12

a)9.8648 b) 10.8648 c) 11.8648 d) none

<u>SOLUTION</u> :- 8, 10, 12 → n = 3.

We shall solve such questions using option hitting method.

i.e $(\text{option})^n$ = product of observations

$(\text{option a})^n = 8 \times 10 \times 12$

$(9.8648)^3 \approx \underline{960}$

∴ option a is the correct answer.

<u>Example2:</u>

Find GM of 10,15,20,25

a) 15.5487 b) 16.5487 c) 14.5487 d) 17.5487

<u>SOLUTION</u> :-

Given 10, 15, 20, 25

$(\text{Option})^n$ = product of observations

$(\text{option b})^4 = 10 \times 15 \times 20 \times 25$

$(16.5487)^4 = 75000$

$74999. \approx \underline{75,000}$ (approx equal)

∴ Option b is the correct answer

<u>GM of any two observation a & b</u>

GM = $(a \times b)^{1/2}$

GM = $\sqrt{ab}$

<u>Ex:-</u> Find the GM of 20 & 40

<u>Solution :-</u>

Gm = $\sqrt{ab}$

= $\sqrt{20 \, x \, 40}$ = *28.28*

<u>GM for grouped data</u>

GM = $(X_1^{f_1}. X_2^{f_2}. X_3^{f_3}. \ldots\ldots.. X_n^{f_n})^{\frac{1}{N}}$

Example1

Find GM of

x : 1 2 3

f : 1 2 3

a) 1.1822 b) 2.1822 c) 3.1822 c) 4.1822

SOLUTION:

Given $x: 1 \quad 2 \quad 3$ $N = 1+2+3 = \underline{\underline{6}}$

$f: 1 \quad 2 \quad 3$

W.K.T $(\text{Option})^N = x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \times \ldots$

using option(b) $\Rightarrow (2.1822)^6 = 1^1 \times 2^2 \times 3^3$

$107.98 = 108$

approx equal

∴ option (b) is the correct answer.

Example2:

Find GM

x : 2 4 8

f : 1 2 3

a) **4.0396 b)5.0396 c)3.0396 d)6.0396**

SOLUTION:

Given $x: 2 \quad 4 \quad 8$

$f: 1 \quad 2 \quad 3$ $\underline{\underline{N = 6}}$

W.K.T $(\text{option})^N = x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \times \ldots\ldots$

using option (b) $\Rightarrow (5.0396)^6 = 2^1 \times 4^2 \times 8^3$

$16{,}382.35 = 16{,}384$

approx equal

∴ option (b) is the correct answer.

Harmonic mean(HM)

- It is the reciprocal of the AM of reciprocals of the given set of Observations.
- It is also regarded as the ratio average

For Ungrouped data

HM = $\frac{n}{\Sigma\left(\frac{1}{x}\right)}$

Where

n→ no of observations

$$\sum \frac{1}{x} = \frac{1}{x_1} + \frac{1}{x_2} + \cdots \ldots \ldots + \frac{1}{x_n}$$

For Grouped data

HM = $\frac{N}{\Sigma\left(\frac{f}{x}\right)}$

Where, N = $\sum f$

$$\sum \left(\frac{f}{x}\right) = \frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots \ldots \ldots . + \frac{f_n}{x_n}$$

Combined HM

Combined HM = $\frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$

Where, n_1 & n_2 → no. of observations

H1 & H2 → Harmonic means

HM of any two observations

HM = $\frac{n}{\Sigma\frac{1}{x}}$ = $\frac{2}{\frac{1}{a} + \frac{1}{b}}$ = $\frac{2ab}{a+b}$

Note :- The above formula is used to find average speed for equal distances to be travelled.

Example1:

Find HM of 5, 6 , 9 , 12

SOLUTION

Given. observations 5,6,9,12

W.K.T. $HM = \frac{n}{\Sigma\left(\frac{1}{x}\right)} = \frac{4}{\frac{1}{5}+\frac{1}{6}+\frac{1}{9}+\frac{1}{12}} = \frac{4}{0.5611} = \underline{\underline{7.12}}$

Example2:

Find HM.

x :	**3**	**6**	**9**	**12**	**15**
f :	**6**	**8**	**4**	**3**	**4**

SOLUTION

Given

x:	3	6	9	12	15
f:	6	8	4	3	4

$N = \Sigma f = 25$

$$HM = \frac{N}{\Sigma\left(\frac{f}{x}\right)} = \frac{25}{\frac{6}{3}+\frac{8}{6}+\frac{4}{9}+\frac{3}{12}+\frac{4}{15}}$$

$$= \frac{25}{4.2944}$$

$$HM = \underline{5.82}$$

<u>Example3:</u>

There are 2 groups having 15 & 35 Observations, respectively.with 45 & 105 as the respective HM's. Find the combined HM.

SOLUTION

Given $n_1 = 15$ $n_2 = 35$

$H_1 = 45$ $H_2 = 105$

W.K.T Combined $HM = \frac{n_1+n_2}{\frac{n_1}{H_1}+\frac{n_2}{H_2}} = \frac{15+35}{\frac{15}{45}+\frac{35}{105}} = \frac{50}{0.3333+0.3333}$

$$\Rightarrow \frac{50}{0.6666} = \underline{\underline{75}}$$

<u>Example4:</u>

An aeroplane flies from Chennai to Bangalore at 400 kmph & returns from Bangalore to Chennai at 500 kmph find the average speed of the journey

SOLUTION:-

Given $a = 400$ kmph $b = 500$ kmph

Average Speed $= \frac{2ab}{a+b} = \frac{2 \times 400 \times 500}{400+500} = \frac{400000}{900} = \underline{\underline{444.44}}$ kmph

<u>Relationship between AM, GM and HM</u>

→If the observations are Positive and equal, then

AM = GM = HM

- → If the observations are Positive and distinct, then
  AM>GM>HM
  - → For any two observations a & b , GM is the GM of AM and HM

  i.e GM = $\sqrt{AM\ x\ HM}$

- → AM GM & HM Possess mathematical Properties (As their combined measures are defined)
  - → <u>Weighted AM</u>

  Weighted AM = $\frac{\sum w.x}{\sum w}$

  Where, w is weight

  Similarly

  Weighted HM = $\frac{\sum w}{\sum\left(\frac{w}{x}\right)}$

  and

  weighted GM = Antilog $\left(\frac{\sum w.\log x}{\sum w}\right)$

<u>Note:</u>

If x & y are 2 variables related as ax + by + c = 0 & given Central tendency of x. Then to find central tendency of y, replace x & y (in the equation ax + by + c = 0) by C.T of x & CT of y.

<u>Example1:</u>

x & y are related as 3x – 2y – 10 = 0 & AM of x is 15. Find AM of y.

<u>SOLUTION</u> :-

Given $3x-2y-10=0$, Am of x = 15

$3x-10=2y$

$\therefore y=\frac{3x-10}{2}$

Now replace x by Am of x & y by Am of y.

Am of y = $\frac{3(\text{Am of } x)-10}{2} = \frac{3(15)-10}{2} = \underline{\underline{17.5}}$

<u>Example2:</u>

If the mode of x is 8, then find the mode of (4x – 5)

SOLUTION :- Given Mode of $x = 8$

Let $y = 4x - 5$

$\therefore$ Mode of $y = 4$ (mode of x) $- 5$

$= 4(8) - 5$

$= 32 - 5$

Mode of $y = 27$

Dispersion:

- An amount of deviation in the observations from the measure of central tendency is referred to as dispersion
- Central tendency measures are said to be first order measures , whereas Dispersion measures are said to be 2nd order measures

Measures of Dispersion

There are 2 types of measures. Namely

a)Absolute measures b) Relative measures

Note :-

Absolute measures are dependent on unit of variables where as relative measures are unit independent

Absolute Measure	Relative measure
1) Range	1) Coefficient of Range.
2) Mean Deviation (MD)	2) coefficient of MD
3) standard deviation (SD)	3) coefficient of variation (cv)
4) Quartile Deviation (QD)	4) Coefficient of QD

Range:

Both for ungrouped and grouped data

Range = L –S

Where, L is largest observation

S is the smallest observation

Coefficient of Range = ($\frac{L-S}{L+S}$) x 100

Example1:

Find the range and its coefficient for the observations 10 , 20 , 30 , 40 , 50 , 60 , 70 , 80, 100, 120

SOLUTION :

Given observations 10, 20, 30, 40 50 60 70, 80, 100 120

Largest observation (L) = 120

Smallest Observation (S) = 10

∴ Range = L − S = 120 − 10 = 110

Now Coefficient of range $= \frac{L-S}{L+S} \times 100$

$= \frac{110}{130} \times 100$

$= 84.61$

Mean Deviation (MD)

For ungrouped data:

- MD about mean = $\frac{\sum |x - \bar{x}|}{n}$
- MD about Median = $\frac{\sum |x - M|}{n}$
- MD about Assumed mean (A) = $\frac{\sum |x - A|}{n}$

Where ,

n is no. of observations

For grouped (both for discrete & Continuous)

- MD about $\bar{x}$ = $\frac{\sum f.|x-\bar{x}|}{N}$
- MD about M = $\frac{\sum f.|x-M|}{N}$
- MD about A = $\frac{\sum f.|x-A|}{N}$ (A is assumed mean)

Coefficient of MD about $\bar{x}$ **=** $\frac{MD\ about\ \bar{x}}{\bar{x}}$ **x 100**

Coefficient of MD about A = $\frac{MD\ about\ A}{A}$ **x 100**

Coefficient of MD about M = $\frac{MD\ about\ M}{M}$ **x 100**

Example1:

what is the value of mean deviation about mean for the numbers 4 ,5 , 6 , 8 , 3

a) 5.2 b) 7.2 C) 1.44 d) 2.23

SOLUTION:-

Given observations 4, 5, 6, 8, 3 . $\boxed{n=5}$

Required. Mean deviation about mean = $\frac{\Sigma|x-\bar{x}|}{n}$

$\bar{x} = \frac{\Sigma x}{n} = \frac{26}{5} = 5.2$

$\therefore$ M.D about $\bar{x}$ = $\frac{\Sigma|x-\bar{x}|}{n} = \frac{1.2+0.2+0.8+2.8+2.2}{5}$

$= \frac{7.2}{5} = \underline{\underline{1.44}}$

Example2:

what is the value Mean deviation about median for the numbers

4, 9, 11, 14, 37

a) 11 b) 8.5 c) 7.6 d) 7.45

SOLUTION:

Given observations, 4, 9, 11, 14, 37 , $\boxed{n=5}$

Median = 11

$\therefore$ MD about median = $\frac{\Sigma|x-M|}{n} = \frac{7+2+0+3+26}{5}$

$= \frac{38}{5}$

$= \underline{\underline{7.6}}$

Example3:

Find MD about M for the data given

X: 2 4 6 8 10

f : 5 3 8 4 5

SOLUTION

Given : $x : \quad 2 \quad 4 \quad \overset{\to M}{\boxed{6}} \quad 8 \quad 10$

$f : \quad 5 \quad 3 \quad 8 \quad 4 \quad 5$

$Lcf : \quad 5 \quad 8 \quad \underset{cf}{\boxed{16}} \quad 20 \quad \boxed{25} \to N$

$M = \left(\frac{N+1}{2}\right)^{th}$ value $= 13^{th}$ value

$= 6$

Now MD about M $= \frac{\sum f \cdot |x - M|}{N}$

$= \frac{20 + 6 + 8 + 20}{25}$

$= 2.16$

Standard deviation (SD)

→The reference central tendency measure for SD is AM.

→ SD is the best measure of dispersion

For Ungrouped data

$$SD = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$$

Where $\bar{x} = \frac{\sum x}{n}$

OR

$$SD = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

S.D for grouped data

$$SD = \sqrt{\frac{\sum f.x^2}{N} - (\bar{x})^2}$$

where $\bar{x} = \frac{\sum fx}{N}$

OR

$$SD = \sqrt{\frac{\sum f(x - \bar{x})^2}{N}}$$

Example1:

Find SD of 2, 4, 6 , 8 , 10, 12

SOLUTION:

Given Observations 2, 4, 6, 8, 10, 12 . $n = 6$

$\boxed{\bar{x} = \frac{\sum x}{n} = 7}$

$$SD = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{25+9+1+1+9+25}{6}}$$

$$SD = \underline{\underline{3.41}}$$

Example2:

Find S.D of

CI:	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
f :	**10**	**20**	**30**	**20**	**10**

SOLUTION :-

Given

CI :	0-10	10-20	20-30	30-40	40-50
f :	10	20	30	20	10

$\boxed{N = 90}$

$$\bar{x} = \frac{\sum fx}{N} = \frac{2250}{90} = \underline{\underline{25}}$$

$$SD = \sqrt{\frac{\sum f.(x-\bar{x})^2}{N}} = \sqrt{\frac{12,000}{90}}$$

$$= \underline{\underline{11.54}}$$

Properties of S.D

SD of any 2 no's a & b

SD of a & b = $\left|\frac{a-b}{2}\right| = \frac{Range}{2}$

S.D remain unaltered on shift of origin but changes with change of scale

(Shift of origin -> Adding or subtracting each observation by the same number

change of scale -> Multiplying or dividing each observation by same number.

SD of Ist 'n' natural number [SD of n Consecutive natural nos]

SD of first 'n' natural number = $\sqrt{\frac{n^2-1}{12}}$

variance

variance = $(SD)^2$

$$= \left[\sqrt{\frac{\sum(x-\bar{x})^2}{n}}\right]^2 = \frac{\sum(x-\bar{x})^2}{n}$$

Coefficient of Variation (CV)

→ It is a statistical tool used to test the consistency of the data

→ Lesser the CV, greater will be the consistency and vice versa.

CV = $\frac{SD}{AM} \times 100$

Example1:

Find the SD ,

Given, CV = 45% & AM = 100.

SOLUTION:

Given CV = 45% AM = 100

W.K.T $CV = \frac{SD}{AM} \times 100$

$45 = \frac{SD}{\cancel{100}} \times \cancel{100}$

$\boxed{SD = 45}$

Example2:

Find CV if Variance is 36 and AM = 10

SOLUTION:

Given Variance = 36 ⇒ $SD = \sqrt{\text{variance}} = \underline{\underline{6}}$

AM = 10

W.K.T $CV = \frac{SD}{Am} \times 100$

$= \frac{6}{10} \times 100$

$\boxed{CV = 60\%}$

Combined SD

If there are 2 group with n_1 & n_2 as the respective number of Observations, $\overline{x_1}$& $\overline{x_2}$ as the respective Am's & S_1 & S_2 as the respective SD's then

Combined SD = $\sqrt{\frac{n_1 s_1{}^2 + n_2 s_2{}^2 + n_1 d_1{}^2 + n_2 d_2{}^2}{n_1+n_2}}$

where ,

$d_1^2 = (\overline{x_1} - \overline{x_c})^2$

$d_2^2 = (\overline{x_2} - \overline{x_c})^2$

$\overline{x_c}$ → Combined mean

$\overline{x_c} = \frac{n_1 \overline{x_1} + n_2 \overline{x_2}}{n_1 + n_2}$

Example1:

If two sample of sizes 35 & 25 have means as 50 & 60 & SD's as 3&4 respective then find the Combined SD.

SOLUTION:

Given $n_1 = 35$ $n_2 = 25$ $\bar{x}_1 = 50$ $\bar{x}_2 = 60$

$S_1 = 3$ $S_2 = 4$

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \underline{54.166}$$

$$d_1^2 = (\bar{x}_1 - \bar{x}_c)^2 = 17.36 \qquad d_2^2 = (\bar{x}_2 - \bar{x}_c)^2 = 34.02$$

Now Combined S.D $= \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$

$$= \sqrt{\frac{2173.1}{60}} = \underline{\underline{6.01}}$$

Example2:

The mean marks of 30 boys and 40 girls are 70 and 80 respectively. Their respective SD's are 5 and 6. Find the combined SD.

SOLUTION:

Given $n_1 = 30$ $n_2 = 40$, $\bar{x}_1 = 70$ $\bar{x}_2 = 80$, $s_1 = 5$, $s_2 = 6$

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \underline{75.71}$$

$$d_1^2 = (\bar{x}_1 - \bar{x}_c)^2 = 32.6 \qquad d_2^2 = (\bar{x}_2 - \bar{x}_c)^2 = 18.4$$

Combined S.D $= \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$

$$= \sqrt{\frac{3904}{70}} = \underline{\underline{7.46}}$$

Quartile Deviation (QD)

→ It is an appropriate measure of dispersion for open end classification.

→ QD = $\frac{Q_3 - Q_1}{2}$

→ $Q_3 - Q_1$ is called as the inter quartile range

∴ $\frac{Q_3 - Q_1}{2}$ = QD is called as the semi inter quartile range.

Coefficient of QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$ x 100

OR

Coefficient of QD = $\frac{QD}{M}$ x 100

Where M → Median.

Example1:

Find QD for the following data

x : 1 2 3 4 5

f : 6 8 5 6 4

SOLUTION :

Given

$$\begin{array}{llllll} x: & 1 & \overset{Q_1}{\textcircled{2}} & 3 & \overset{Q_3}{\textcircled{4}} & 5 \\ f: & 6 & 8 & 5 & 6 & 4 \\ Lcf: & 6 & \underset{cf}{\textcircled{14}} & 19 & \underset{cf}{\textcircled{25}} & \textcircled{29}\ N \end{array}$$

$$Q_1 = \left[\frac{1}{4}(N+1)\right]^{th} \text{ value} = 7.5^{th} \text{ value} \Rightarrow cf = 14$$

$$\therefore Q_1 = 2$$

$$Q_3 = \left[\frac{3}{4}(N+1)\right]^{th} \text{ value} = 22.5^{th} \text{ value} \Rightarrow cf = 25$$

$$\therefore Q_3 = 4$$

$$\text{Now} \quad QD = \frac{Q_3 - Q_1}{2} = \frac{4-2}{2} = 1$$

Relationship between SD, MD, & QD for Normal or symmetrical distribution

2SD= 2.5 MD = 3QD

Note: SD > MD > QD

If x & y are 2 variable related as ax + by + c =0, with dispersion of x given then,

Dispersion of y= $\left|\frac{a}{b}\right|$ × dispersion of x

Ex1:- If x & y are related as 5x + 4y + 10 =0 , & MD of x is 15. Find MD of y.

SOLUTION

Given MD of x = 15 & 5x+4y+10=0

$\Rightarrow a=5, \quad b=4$

W.K.T Dispersion of y = $\left|\frac{a}{b}\right|$ × dispersion of x

$$\therefore \text{MD of } y = \frac{5}{4} \times \text{MD of } x$$

$$= \frac{5}{4} \times 15$$

$$\text{MD of } y = \underline{\underline{18.75}}$$

Ex2 :- If the variance of x is 25. Then find the variance of (2x +10)

SOLUTION:

Given variance of $x = 25$

let $y = (2x+10)$

$a = 2$

W.K.T Variance of $(ax+b) = a^2 \cdot V(x)$

$\therefore$ Variance of $(2x+10) = 2^2 \cdot (25)$

$= \underline{\underline{100}}$

Practice questions:

1.For open-end classification, which of the following is the best measure of central tendency?

a. AM b) GM c) Median d) Mode

2. The difference between maximum and minimum value of the data is known as :

a) Range b) Size c) Width d) Class

3 If the range of a set of values is 65 and maximum value in the set is 83, then the minimum value in the set is

a) 74 b) 9 c) 18 d) None of these

4. If G.M = 5 and A.M = 7.5 then the two numbers are_

a) 10, 5 b) 11, 4 c) 13.09, 1.91 d) 12, 3

5. Which of the following statement is true?

a.Median is based on all observations
b.The Mode is the mid value
c.The Median is the 2nd Quartile
d.The Mode is the 5th decline

6. If for a normal distribution Q1= 54.52 and Q3 = 78.86, then the median of the distribution is

(a) 12.17 (b) 66.69 (c) 39.43 (d) None of these

7. If the mean of data is 55.6 and the mode is 46, then the median is

a. 50.4 b) 40.7 c) 52.4 d) None

8. GM of 8, 4, 2 is

a)4 b) 2 c) 8 d) None

9._&___are called ratio averages

a) H.M & G.M b) H.M & A.M c) A.M & G.M d) None

10. When the mean is 3.57 and mode is 2.13, then the value of median is

(a) 3.09 (b) 5.01 (c) 5.01 (d) none of these.

11. Which of the following is positional average?

a)Median (b) GM (c) HM (d) AM

12. Which one of the following is not a central tendency?

a)Mean Deviation (b) Arithmetic mean (c) Median (d)Mode

13. The means of 20 items of a data is 5 and if each item is multiplied by 3, then the new mean will be

(a) 20 (b) 5 (c) 15 (d) 10

14.

$$\sum_{i=1}^{m}(x-\bar{x}) = ?$$

a) 1 b) 0 c) -1 d) None of these

15. The mean of four observations is 10 and when a constant a is added to each observation, the mean becomes 13. The value of a is

a. 2 b) –3 c) 3 d) None of these

16. A Random variables X follows uniform distribution in the interval [-3, 7]. Then the mean of distribution is

a) 2 b) 4 c) 5 d) 6

17. The median of the data 5, 6, 7, 7, 8, 9, 10, 11, 11, 12, 15, 18, 18 and 19is

(a) 10 (b) 10.5 (c) 11.5 (d) 11

18. If the standard deviation for the marks obtained by a student in monthly test is 36, then the variance is

(a) 36 (b) 6 (c) 1296 (d) None of the above

19. If the values of all observations are equal then the Standard Deviation of the given observations is

a. 0 b) 2 c) 1 d) None of these

20. If variance= 100 and coefficient of variation = 20% then *AM is*

(a) 60 (b) 70 (c) 80 (d) 50

21. The mean and coefficient of variance is 20 and 80 find the value of variance

(a) 16 (b) 256 (c) 36 (d) none

22. If the mean of frequency distribution is 100 and coefficient of variation is 45% then standard deviation is.

a. 45 (b) 0.45 (c) 4.5 (d) 450

23. Which of the following measures of dispersion is used for finding consistency between the series?

a. Q.D b) S.D c) Coefficient of variation d) none

24. The sum of mean and SD of a series is a+b, if we add 2 to each observation of the series then the sum of mean and SD is

(a) a + b+ 2 (b) 6 + a + b (c) 4 + a – b (d) a + b + 4

25. The standard deviation for the set of numbers 1, 4,5,7,8, is 2.45 nearly. If 10 is added to each number then new standard deviation is

(a) 24.45 (b) 12.45 (c) 2.45 (d) 0.245

26. If every observation is increased by 5 then:

a. SD increase by 5 (b) MD increased by 5

(c) QD increases by 5 (d) none affected

27. The Standard deviation is independent of change of

a. Origin b) Scale c) Both d) none

28. Range of values 4, 3, 1, 6, 7, 10, 8 is______

a) 9 b) 7 c) 6 d) None

29. Q.D is

a. 2/3 S.D b) 4/5 S.D c) 5/6 S.D d) None

30. What will be the probable value of mean deviation? when Q_3 =40 and Q_1 =15

a) 17.50 b) 18.75 c) 15.00 d) None of the above

31. In normal distribution the relation between Q.D, S.D is

a) Q.D > S.D b) Q.D < S.D c) Q.D = S.D d) None

32. Measures of central tendency for a given set of observations measures

(a) The scatterness of the observations (b) The central location of the observations

(c) Both (a) and (b) (d) None of these.

33. the most commonly used measure of central tendency is

(a) AM (b) Median (c) Mode (d) Both GM and HM.

34. Which measure(s) of central tendency is (are) considered for finding the average rates?

(a) AM (b) GM (c) HM (d) Both (b) and (c)

35. The measure of central tendency which is most affected by extreme observations is—

(a) Mean (b) Median (c) Geometric mean (d) Mode

36. Which of the following averages would be more suitable for ascertaining average size of shoes—

(a) Arithmetic mean (b) Mode (c) Geometric mean (d) Median

Chapter 15-Probability

→ It is a chance of occurance of an event.

If 'A' is an event then the probability of occurance of A is given by

P(A) = $\frac{n(A)}{n(S)}$

Where n(A) is number of favourable outcomes

n(S) is total number of possible outcomes

Initially probability was a branch of mathematics

Classification of probability

Probability is broadly classified into

a) **Subjective probability:**

Is is based on one's experience and observation

b) Objective probability:

It is based on mathematical facts

Note:

→Probability of an event lies between 0 and 1 (both inclusive)

→If the probability of an event is equal to 0. Then it is said to be impossible or improbable event.

→If the probability of an event is equal to 1. Then it is said to be a sure event.

→If the probability of an event lies between 0 and 1 then it is said to be a possible or probable event.

→The sum of the probabilities of occurance of an event and non occurance of the same event is equal to 1.

i.e. $P(A) + P(A') = 1$

Where A' is non occurance of an event A.

→An event which can produce only one outcome is said to be a simple event

Ex: Getting a number 2 on tossing a die once, where the outcome is only 2

→An event which can produce more than one outcomes is said to be a composite or compound event. Ex: getting number multiple of 2 on tossing of a coin , where the outcomes are 2,4,6

Sample Space (S):

It is a set of all possible outcomes

Sample space for tossing of a coin

→ When a coin is tossed once

S = { H, T}

n(S) =2

→ When a coin is tossed twice

S = {HH, HT, TH, TT}

n(S) =4

→ When a coin is tossed thrice

S ={ HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

n(S) = 8

Note: n(S) = 2^n for tossing of a coin

n is no. of tosses

Note:

The probability of getting head and tail alternatively on tossing a coin 'n' times is

= $\frac{2}{2^n}$

Example:

Find the probalility of getting head and tail alternatively on tossing a coin 6 times

Sample space for tossing/rolling of a die (dice)

a) **When tossed once**

S = {1,2,3,4,5,6}

n(S) =6

b) When tossed twice

S = { (1,1), (1,2), (1,3) , (1,4) , (1,5) , (1,6)

(2,1) , ……………………….(2,6)

(3,1) , ……………………….(3,6)

(4,1) , ……………………….(4,6)

(5,1), ………………………..(5,6)

(6,1), ……………………….(6,6) }

n(S) = 36

When a die is rolled twice (2 dice rolled simultaneously

Sum of points	**Favourable outcomes**	**Probability**
2	1	1/36
3	2	2/36 = 1/18
4	3	3/36 =1/12
5	4	4/36 = 1/9
6	5	5/36
7	6	6/36 = 1/6
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36

Note: n(s) =6^n for tossing a die

Where n is number of tosses

Sample space for deck of cards

Total number of cards = 52

a)Red (26)

i)Hearts (13)

ii)Diamonds (13)

a)Black (26)

i)Spades (13)

ii)Clubs(13)

Each suit of 13 cards consists of :

Number cards = 9 (2-10)

King =1

Queen =1

Jack =1

Ace =1

No of face cards:

King, queen and jack are face cards.

Therefore the number of face cards = 4+4+4 = 12

Simple Problems on Tossing of a coin, Tossing of dice and Deck of cards

Example1:

A Coin is tossed twice what is the probability that atmost 1 head occurs.

SOLUTION :- $S=\{HH, HT, TH, TT\}$

$n(S) = 2^2 = 4$

A → Atmost 1 head ⇒ 1 head or more heads

ie A → 1 head or 2 head

$n(A) = 2 + 1 = 3$

[∵ As there are 2, one head cases & 1 one head case)

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$

Example2:

A coin is tossed thrice. What is the probability that exactly 2 heads occur

SOLUTION·

A Coin is tossed thrice

$S = \{ HHH, HHT, HTH, THH, THT, TTH, HTT, TTT \}$

$n(S) = 8$

$A \rightarrow$ Exactly 2 heads

$n(A) = 3$ [∵ As there are 3 two head cases)

$\therefore p(A) = \frac{3}{8}$

Example3:

A Coin is tossed thrice what is the probability that at least 1 head occuss.

SOLUTION:-

$n(S) = 2^3 = 8$ [∵ Refer to the sample space of example 2)

$A \rightarrow$ Atleast 1 head $\Rightarrow$ 1 head or more heads

$A \rightarrow$ 1 head or 2 heads or 3 heads

$n(A) = 3+3+1 = 7$

$\therefore p(A) = \frac{7}{8}$

Example4:

Two dice are tossed simultaneously, Find the probability of getting a doublet.

SOLUTION:-

Two dice are tossed Simultaneously

$n(S) = 6^2 = 36$

$A \rightarrow$ Doublet $\Rightarrow \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$n(A) = 6$

$$\therefore p(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \underline{\frac{1}{6}}$$

Example5:

Two dice are tossed simultaneously. What is the probability that the sum of two point is equal to 7.

SOLUTION :-

$n(S) = 6^2 = 36$ [∵ Two dice are tossed Simultaneously)

A → Sum of 2 points is equal to 7

$A \rightarrow \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$n(A) = 6$

$\therefore p(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

Example6:

A card is drawn at random from a well shuffled deck of 52 cards. What is the probability that the card is a face card of spades.

SOLUTION :-

$n(S) = 52C_1 = 52$ [∵ one card is drawn from a deck of 52 cards]

A → face card of Spade

$n(A) = 3$ [∵ as there are 3 face cards in spades)

$\therefore p(A) = \frac{3}{52}$

Example7:

A card is drawn at random from a deck of 52 cards what is the probability that the card is a numbered card of red .

SOLUTION :-

$n(s) = {}^{52}C_1 = 52$

A → Numbered card of red

$n(A) = 18$ [∵ As there are 18 numbered cards of red
i.e 9 in hearts + 9 in diamonds)

$\therefore p(A) = \frac{18}{52} = \underline{\frac{9}{26}}$

<u>For any 2 events A & B</u>

<u>1.The probability that either A or B occurs (probability that at least 1 event occurs)</u>

P(AUB) = P(A) + P(B) – P(A∩B) → **eqn1**

where, P(AUB) →probability of A or B

P(A∩B) → probability of A and B

<u>Note :-</u>
Equation ① is also called as addition theorem of probability.

<u>2. If A & B are mutually Exclusive</u>

Two events A&B are said to be mutually exclusive when there is nothing Common between the two events.

P(A∩B) = 0

P(AUB) = P(A) + P(B) → **eqn2**

3. A & B are said to be exhaustive when **n(AUB) = n(s)**.
P (AUB) = $\frac{\mathrm{n(AUB)}}{n(s)}$

P(AUB) = 1 → **eqn3**

<u>If A&B are mutually exclusive & exhaustive</u>

P(AUB) = 1 for exhaustive events

P(A∩B) = 0 for mutually exclusive event

Now P(AUB) = P(A) + P(B) – P(A∩B)

1= P(A)+P(B) -0

i.e P(A)+P(B) = 1

5.Conditional Probability

If A&B are 2 events, with B has already occurred then the probability of occurrence of A is P(A/B) = $\frac{P(A∩B)}{P(B)} = \frac{n(A∩B)}{n(B)}$

→Probability of Occurance B, given that event A has already occurred.

P(B/A) = $\frac{P(A∩B)}{P(A)} = \frac{n(A∩B)}{n(A)}$

Example:

There are 50 men & 30 women in a locality. Out of them 35 men & 15 women are service holders. If one person is selected at random, then what is the Probability that the selected person is a service holder given that the person selected is a woman.

SOLUTION:

Let Service holders → A

Woman → B

To find $P(A/B) = \frac{P(A∩B)}{P(B)} = \frac{n(A∩B)}{n(B)}$

$P(A/B) = \frac{15}{30}$

[∵ n(A∩B) = 15 as there are 15 women who are service holders)

5. **Independent events**

   → Two Events are said to be independent if occurrence or Non-occurrence of one event doesn't affect occurrence or non-occurrence of other event

   i.e

- P(A/B) = P(A)
- P(B/A) = P(B)
- P(A/B') = P(A)
- P(B/A') = P(B)

→ If the 2 events A & B are independent then P (A∩B) = P(A)X P(B) (Multiplication theorem of probability

→If the 2 events are mutually exclusive then they can't be independent & vice-versa.

7.Probability of occurrence of only A (out of A & B)

P(A-B) = P $(A∩B^{I})$ = P(A) -P(A∩B)

8.Probability of occurrence of only B

P(B-A)=P$(B∩A^{I})$ = P(B) -P(A∩B)

9.Probability of occurrence of only 1 event

only one Event means only A or only B

$\rightarrow P(A)-P(A\cap B] + P(B) - P(A\cap B)$
$\rightarrow P(A) + P(B) - 2P(A\cap B)$

10.If the events A&B are equally likely
→Equally likely events will have equal probabilities

$P(A) = P(B)=K$

Where K is a constant

For any 3 events A,B, & C
1. probability that at least one event occurs is
$P(AUBUC) = P(A) + P(B) + P(c) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$ →①

2. If A, B, C are mutually exclusive then

$P(A\cap B) = 0$
$P(B\cap C) = 0$

$P(A\cap C) = 0$

$P(A\cap B\cap C) = 0$

Now ① => $P(AUBUC) = P(A) + P(B) + P(C)$ → 2

3. If A, B, C are Exhaustive then n(AUBUC) = n(S)

$P(AUBUC) = 1$ → 3

4. If A.B, C are mutually exclusive & exhaustive then

$P(A) + P(B) + P(C) = 1$ [:. By combining ② & ③]

5. If A, B, C are equally likely then

$P(A)= P(B) = P(C) = K$

Where K is a constant.

De Morgan's Theorem

1) $P(A^{I}\cap B^{I}) = P(AUB)'$
2) $P(A'UB') = P(A\cap B)'$

Note :-

$P(AUB)' = 1- P(AUB)$
This formula is to be used when the Probability of occurrence of neither A nor B is asked.

Sample Space for leap year and non leap year problems

a) **Non Leap year:**

Total number of days in non leap year = 365

No. of odd days =1

The sample space for this odd day is S = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

n(S) = 7

Note: P(A∩B)=0 for non leap year problems

b) Leap year

Total number of days in a leap year = 366

No. of odd days =2

Sample space for these two odd days is

S ={Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thur, Thur-Fri, Fri-Sat, Sat-Sun }

n(S) = 7

Note:P(A∩B) = 0 if the 2 days given are non successive days

P(A∩B) =1/7 if the 2 days given are successive days.

Problems on 2 events & 3 events

Example1:

2 dice are tossed simultaneously. Find the probability of getting the sum neither 8 nor 9.

SOLUTION:

$n(S) = 6^2 = 36$

We know that $P(A' \cap B') = P(A \cup B)' \Rightarrow$ Neither A nor B

let $A \rightarrow$ Sum $= 8$

$B \rightarrow$ Sum $= 9$

$$P(A \cup B)' = 1 - P(A \cup B) \qquad \left[\begin{array}{l} \because P(A) + P(A') = 1 \\ \quad P(A') = 1 - P(A) \end{array} \right]$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - \left[\frac{5}{36} + \frac{4}{36} - 0\right]$$

$$= \frac{27}{36}$$

$$= \underline{\underline{0.75}}$$

Example2:

A card is drawn at random from a well shuffled deck 52 cards. what is the probability that the Card is either a face card or red

SOLUTION :

$n(S) = {}^{52}C_1 = 52$

let $A \rightarrow$ Face card

$B \rightarrow$ Red Card

To find $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{12}{52} + \frac{26}{52} - \frac{6}{52}$$

$$= \frac{32}{52}$$

$$= \frac{8}{13}$$

Example3:

The probabilities of a Person getting qualified for 2 different entrance exams are 1/3 & 3/4 .
Find the probability that he would get qualified in one of the 2 exams.

SOLUTION

Given $P(A) = \frac{1}{3}$

$P(B) = \frac{3}{4}$

& $P(A \cap B) = P(A) \times P(B)$ ($\because$ as A & B are independent events)

To find $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

One of the 2 exams
(atleast 1 exam)

$$= \frac{1}{3} + \frac{3}{4} - \left(\frac{1}{3} \times \frac{3}{4}\right)$$

$$= \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$$

$$= \frac{4+9-3}{12} = \frac{10}{12}$$

$$= \frac{5}{6}$$

Example4:

There are 3 events A,B,C having probabilities $\frac{1}{2}, \frac{1}{3} \& \frac{3}{4}$. If the Probabilities of Occurrence of A&B, B&C, A&C and A & B & C are $\frac{1}{3}, \frac{1}{5}, \frac{1}{6}$ and $\frac{1}{15}$ respectively. Then find the probability of occurrence of at least one event.

SOLUTION :

Given $P(A) = \frac{1}{2}$ $\quad P(A \cap B) = \frac{1}{3}$ $\quad P(A \cap B \cap C) = \frac{1}{15}$

$P(B) = \frac{1}{3}$ $\quad P(B \cap C) = \frac{1}{5}$

$P(C) = \frac{3}{4}$ $\quad P(A \cap C) = \frac{1}{6}$

To find $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Atleast 1 event

$= \underline{\underline{0.95}}$

Example5:

What is the probability of getting 53 sundays or 53 wednesdays in a non leap year

SOLUTION

Let $A \rightarrow$ 53 sundays

$B \rightarrow$ 53 wednesdays

To find $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{7} + \frac{1}{7} - 0$

$= \underline{\underline{\frac{2}{7}}}$

[∵ $P(A) = P(B) = \frac{1}{7}$ as prob of getting 53 sundays in a non leap year $= \frac{1}{7}$ & prob of getting 53 wed in a non leap year $= \frac{1}{7}$]

Example6:

What is the probability of getting 53 Fridays or 53 Saturdays in a leap year

SOLUTION :

let $A \rightarrow$ 53 Fridays

$B \rightarrow$ 53 Saturdays

To find $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{2}{7} + \frac{2}{7} - \frac{1}{7}$

$= \underline{\frac{3}{7}}$

[∵ As the 2 days given are successive days $P(A \cap B) = \frac{1}{7}$]

Random variable & probability Distribution

Random variable (x)
It is a function defined on a sample Space associated with a random experiment assuming any real number. It is denoted by x.

Probability Distribution :-
It is the distribution of a random variable with corresponding Probability

Note: $\sum P(x) = 1$

Mathematical Expectation of x

- → It is also called as expected value or mean
- → It is denoted by E(x)
- → $E(x) = \sum x.P(x)$
- → $E(x^2) = \sum x^2. P(x)$

Variance of X :

$V(x) = E(x^2) - [E(x)]^2$

Example1:

write down the probability distribution for getting head on tossing a coin twice.

SOLUTION:-

Coin is tossed twice $\Rightarrow n(S) = 2^2 = 4$

Random variable (x) → head

$\therefore x \rightarrow 0 \quad 1 \quad 2$

Probability Distribution

x	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

Example2:

Find the expected value & variance of getting heads on tossing Coin thrice

SOLUTION:

Given: A coin is tossed thrice $\Rightarrow n(s) = 2^3 = \underline{\underline{8}}$

Random variable $(x) \rightarrow$ head

$x \rightarrow 0 \quad 1 \quad 2 \quad 3$

Probability distribution

x:	0	1	2	3
f:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Now $E(x) = \Sigma x \cdot p(x)$

$= \underline{1.5}$

$E(x^2) = \Sigma x^2 \cdot p(x)$

$= \underline{\underline{3}}$

$V(x) = E(x^2) - [E(x)]^2 = 3 - (1.5)^2$

$= 0.75$

Example3:

Find the mathematical expectation of number on the upper face of the dice, when rolled once

SOLUTION:

Given: A die is rolled once $\Rightarrow n(s) = 6$

$x \rightarrow$ no. on the upper face of the die

$x \rightarrow 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

Probability distribution

x:	1	2	3	4	5	6
$p(x)$:	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Now Mathematical expectation of $x \Rightarrow E(x) = \Sigma x \cdot p(x)$

$= \underline{\underline{3.5}}$

Properties of E(x)

(i) E(K) = K

Where 'k' is a constant.

(ii) E (x+y) = E(x) + E(y)

(iii) E(XY) = E(x) x E(y)

(iv) E(a+bx) = a + b E(x)

<u>Ex:-</u>

E(3 + 2x) = 3+ 2 E(x)

Baye's theorem

If E_1, E_2, E3 ……….. En are the events which are mutually Exclusive
and 'A' is an event which has already occurred
Then probability of occurance of E_i given that 'A' has already Occurred is.

P(Ei/A) = $\frac{P(A/E_i), P(E_i)}{\sum P(A/E_i),P(E_i)}$

where i can be 1, 2, 3, 4……….n

$\sum$ P(A/Ei) .P(Ei)= P(A/E_1) P(E_1) + P(A/E_2). P(E_2) +...

Example:

There are 3 bags.

Bag 1 consists of 3 red & 4 white balls
Bag 2 consists of 4 red & 5 white balls
Bag 3 consists of 5 red & 2 white balls
If one ball is drawn at random. then what is the probability that the ball is selected from bag2 given the selected ball is red

<u>SOLUTION</u> :-

Let $E_1 \rightarrow$ Bag1 $E_2 \rightarrow$ Bag2 & $E_3 \rightarrow$ Bag3
$A \rightarrow$ occured event $\Rightarrow$ Red coloured ball

Now

$$\underset{\substack{\downarrow \\ \text{Required}}}{P(E_2/A)} = \frac{P(A/E_2) \times P(E_2)}{P(A/E_1) \times P(E_1) + P(A/E_2) \times P(E_2) + P(A/E_3) \times P(E_3}$$

$$= \frac{\frac{4}{9} \times \frac{1}{3}}{\left(\frac{3}{7} \times \frac{1}{3}\right) + \left(\frac{4}{9} \times \frac{1}{3}\right) + \left(\frac{5}{7} \times \frac{1}{3}\right)} = \frac{0.1481}{0.5291} = \underline{0.28}$$

odds in favour and against the occurrence of an event :-

- If the odds in favour of an event is given as a:b.

Then ‘a’ is the no. of favourable outcomes, where as ‘b’ is against it. & **a+b = n(S)**

- If the odds against of an event is given as a:b.

Then b is the number of favourable outcomes and a is number of against outcomes.

& **a+b = n(S)**

Example:-

If the odds in favour of an event 2:3 then find the

Probability of
(i) occurrence of an event
(ii) Non-occurrence an event

SOLUTION :

Given odds in favour of an event $= \underset{\text{favour}}{\underset{\downarrow}{2}} : \underset{\text{against}}{\underset{\downarrow}{3}}$

(i) Probability of occurance of an event $P(A) = \frac{n(A)}{n(S)}$

$$= \frac{2}{2+3} = \underline{\underline{\frac{2}{5}}}$$

(ii) Probability of non ocurance $\Rightarrow P(A') = 1 - P(A)$

$$= 1 - \frac{2}{5}$$

$$= \underline{\underline{\frac{3}{5}}}$$

Problems involving Combination

Example1:A bag consists of 5 red ball, 3 blue balls & 2 green balls.
If 4 balls are drawn at random, them what is the probability that all of them being red balls.

SOLUTION:

Given Bag : 5 Red, 3 blue & 2 green balls

$n(S) = {}^{10}C_4 = 210$

A → All of them being red

$n(A) = {}^{5}C_4 = {}^{5}C_1 = 5$

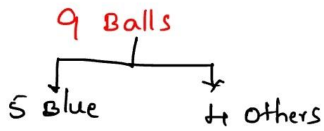
$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{5}{210} = \frac{1}{42}$$

Example2:

A bag Consists of 9 balls of which 5 are blue. If 3 balls are selected at random. Then what is the probability that out of 3 balls 2 are blue.

SOLUTION:

Given Bag :-



$n(S) = {}^{9}C_3 = 84$

A → 2 Blue Balls

ie A → 2 Blue balls & 1 other (∵ as 3 balls have to be drawn)

$n(A) = {}^{5}C_2 \times {}^{4}C_1$

$n(A) = 40$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{40}{84} = \underline{\frac{10}{21}}$$

Practice questions:

1) A dice is thrown once. What is the mathematical expectation of the number on the dice?
(a)16/6 (b)13/2 (c)3.5 (d)4.5

2) If P (A/B)= P (A), then A and B are
(a)Mutually exclusive events (b)Dependent events
(c)Independent events (d)Composite events

3) A bag contains 3 white and 5 black balls and second bag contains 4 white and 2 black balls. If one ball is taken from each bag, the probability that both the balls are white is ________

(a) 1/3 (b)1/4 (c)1/2 (d)None of the above

4) The odds in favour of A solving a problem is 5:7 and odds against B solving the same problem is 9:6. What is the probability that if both of them try, the problem will be solved?

(a) 117/180 (b)181/200 (c)147/180 (d)119/180

5) Consider
Urn I : 2 white balls, 3 black balls
Urn II: 4 white balls, 6 black balls
One ball is randomly transferred from first to second urn, then one ball is drawn from II urn. The probability that drawn ball is white is

(a)22/65 (b)22/46 (c)22/55 (d)21/45

6) If P(AUB) = P(A), Find P (A∩B).

(a) P(A).P(B) (b) P(A)+P(B) (c) 0 (d) P(B)

7) The probability of Girl getting scholarship is 0.6 and the same probability for Boy is 0.8. Find the probability that at least one of the categories getting scholarship.

(a)0.32 (b)0.44 (c) 0.92 (d) None of the above

8) Exactly 3 girls are to be selected from 5 girls and 3 boys. The probability of selecting 3 girls will be:

(a) $\frac{5}{28}$(b) $\frac{1}{56}$ (c)$\frac{15}{28}$ (d) None

9) Two unbiased dice are thrown. The expected value of the sum of numbers of the upper side is:

(a)3.5 (b)7 (c)12 (d)6

10) One card is drawn from pack of 52, what is the probability that it is a king or a queen?

(a) 11/13 (b)2/13 (c)1/13 (d)None of the abov

11) In a packet of 500 pens, 50 are found to be defective. A pen is selected at random. Find the probability that it is non-defective.

(a) 8/9 (b)7/8 (c)9/10 (d)2/3

12) Four married couples are gathered in a room. Two persons are selected at random amongst them. Find the probability that selected persons are a gentleman and a lady but not a couple.

(a) 1/7 (b)3/7 (c)1/8 (d)3/8

13) A team of 5 is to be selected from 8 boys and 3 girls. Find the probability that it includes two particular girls.

(a) 2/30 (b)1/5 (c)2/11 (d)8/9

14) Let A and B two events in a sample space S such that P(A) = 1/2, P($\overline{B}$) = 5/8, P(AUB) = 3/4; Find P($\overline{A} \cap \overline{B}$)

(a) 3/4 (b)1/4 (c)3/16 (d)None of the above

15) A card is drawn out of a standard pack of 52 cards. What is the probability of drawing a king or red colour?

(a) 1/4 (b)4/13 (c)7/13 (d)1/2

16) A player tosses two fair coins, he wins Rs.5 if 2 heads appear, Rs.2 if one head appears and Rs.1 if no head occurs. Find his expected amount of winning.

(a)2.5 (b)3.5 (c)4.5 (d)5.5

17) Arun and Tarun appear for an interview for two vacancies. The probability of Arun's selection is 1/3 and that of Tarun's selection is 1/5. Find the probability that only one of them will be selected.

(a)2/5 (b)4/5 (c)6/5 (d)8/5

18) Two dice are thrown together. Find the probability of getting a multiple of 2 on one dice and multiple of 3 on the other.

(a) 2/3 (b)1/6 (c)1/3 (d)None of the above

19) The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5. What is the probability that the problem will be solved if they both try?

(a) 15/21 (b)16/21 (c)17/21 (d)13/21

20) Find the expected value of the following probability distribution

x : -20 -10 30 75 80
p(x) : 3/20 1/5 1/2 1/10 1/20

(a) 20.5 (b) 21.5 (c) 22.5 (d) 24.5

21) A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball from the bag is twice that of red ball, Find the number of blue balls in the bag.

(a) 10 (b) 12 (c) 14 (d) 16

Chapter 16-Theoretical Distribution

- ➔ We may think of a probability distribution in the same way as we see frequency distribution. Just like distributing the total frequency to different class intervals, the total probability (i.e. one) is distributed to different mass points in case of a discrete random variable or to different class intervals in case of a continuous random variable. Such a probability distribution is known as Theoretical Probability Distribution, since such a distribution exists in theory.
- ➔ Theoretical probability distribution may be profitably employed to make short term projections for the future.
- ➔ Probability distribution possesses all the characteristics of an observed distribution, we can define mean median, mode, standard deviation etc.
- ➔ A probability distribution can be discrete or Continuous
- ➔ Some Important discrete probability distributions are

(i) Binomial distribution (BD)
(ii) Poisson distribution (PD)

- Important continuous probability distribution is

Normal Distribution

Note: A parameter is a characteristic of any population..

Population mean is an example of a Parameter

Binomial distribution

- ➔ It is based on Bernoulli trials.
- ➔ It is one of the important discrete probability distribution.
- ➔ It is a biparametric distribution.
- ➔ The 2 parameters are n & p

Where n → no. of trials
p = probability of success.

- ➔ The Binomial probability mass function (PMF)

$P(x) = nc_x\, p^x\, q^{n-x}$

where x → Binomial variate

q→ probability of failure

Note :-

- For a binomial distribution
  p+q=1

- If p=q the Binomial distribution will be Symmetric.
- If p<q then Binomial distribution is positively skewed
- If p>q then the Binomial distribution is negatively skewed
- An expression X~ B(n, p) is to be read as 'x' is a binomial variate with parameters n & p. x can be 0, 1, 2, ………. n

→ **Mean of a Binomial distribution.**

mean = np

➔ standard deviation of BD = $\sqrt{npq}$

Variance = $\left(\sqrt{npq}\right)^2$ = npq

- For a Binomial distribution mean > variance
- Variance is maximum when p=q=0.5
- Binomial distribution is unimodal if (n+1)P is a non integer.
  In that Case mode = Integral part of (n + 1)P
- Binomial distribution is bimodal if (n+1)p is an integer.

In that case, modes = (n+1)P and (n+1)p -1

➔ **Additive property**

X~ B(n_1 ,p) & Y ~ B (n_2, p) Then

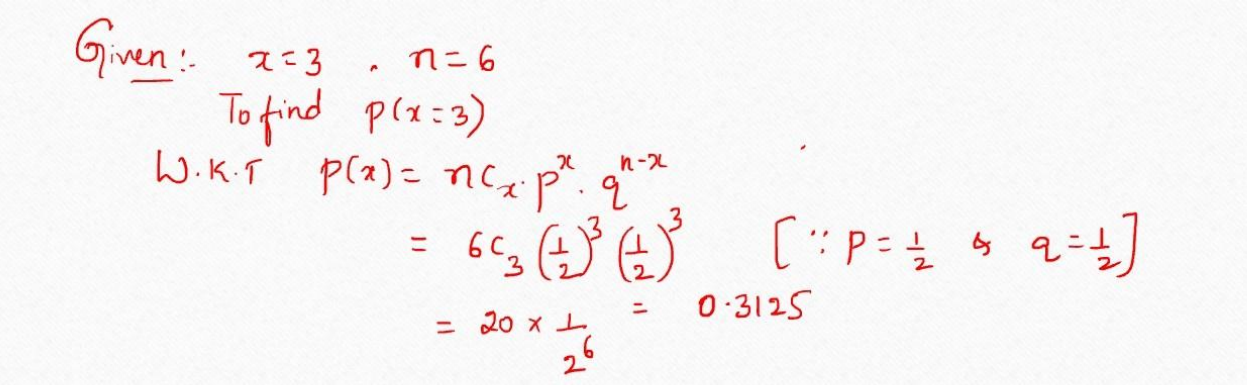
(X+Y) ~ B [(n_1 + n_2), p]

➔ BD is applicable when the trials are independent and each trial has just 2 Outcomes Success & failure. It is applied in coin tossing experiments, Sampling inspection plan, genetic experiments etc.

Examples on Binomial distribution

Example1:

A coin is tossed 6 times. what is the probability of getting exactly 3 heads



Given:- $x=3$. $n=6$

To find $p(x=3)$

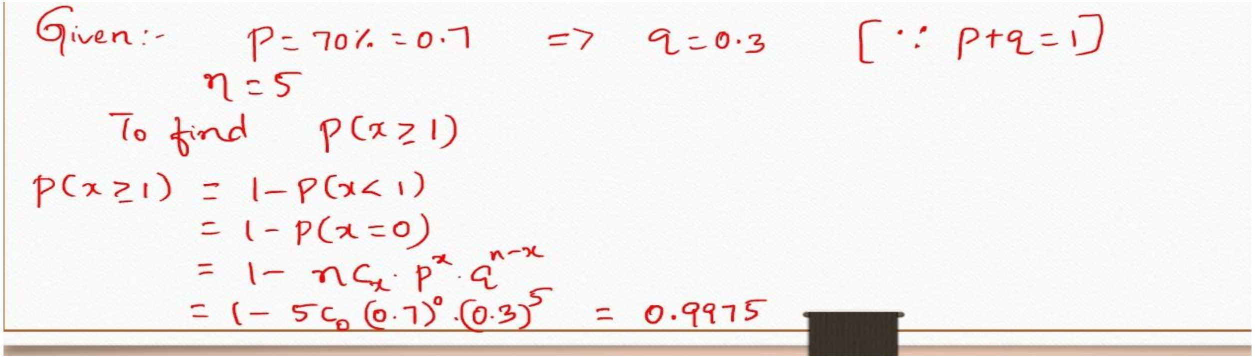
W.K.T $p(x) = nC_x \cdot p^x \cdot q^{n-x}$

$= 6C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3$ $\left[\because p=\frac{1}{2} \text{ \& } q=\frac{1}{2}\right]$

$= 20 \times \frac{1}{2^6} = 0.3125$

Exampe2:

If overall percentage of success in an exam is 70. what is the probability that out of a group of 5 students at least I has passed.



Given:- $p = 70\% = 0.7 \Rightarrow q = 0.3$ $[\because p+q=1]$

$n=5$

To find $p(x \geq 1)$

$p(x \geq 1) = 1 - p(x<1)$

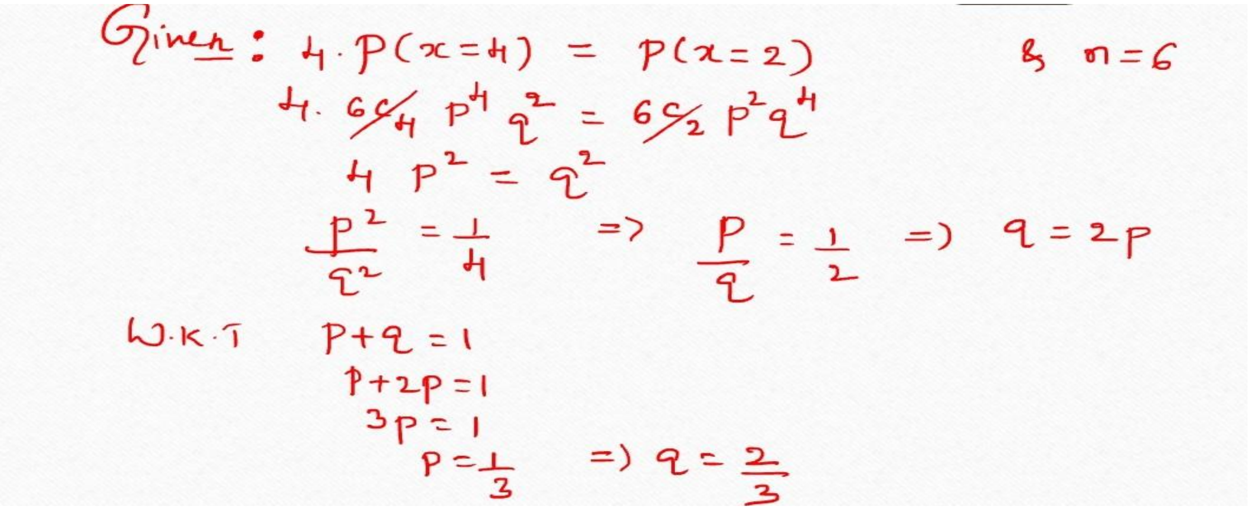
$= 1 - p(x=0)$

$= 1 - nC_x \cdot p^x \cdot q^{n-x}$

$= 1 - 5C_0 (0.7)^0 \cdot (0.3)^5 = 0.9975$

Example3:

For a Binomial distribution if 4p(x = 4) = p(x=2) with n = 6 then find q



Given: $4 \cdot P(x=4) = P(x=2)$ & $n=6$

$4 \cdot \cancel{6C_4}\, p^4 q^2 = \cancel{6C_2}\, p^2 q^4$

$4p^2 = q^2$

$\frac{p^2}{q^2} = \frac{1}{4} \Rightarrow \frac{p}{q} = \frac{1}{2} \Rightarrow q = 2p$

W.K.T $p+q=1$

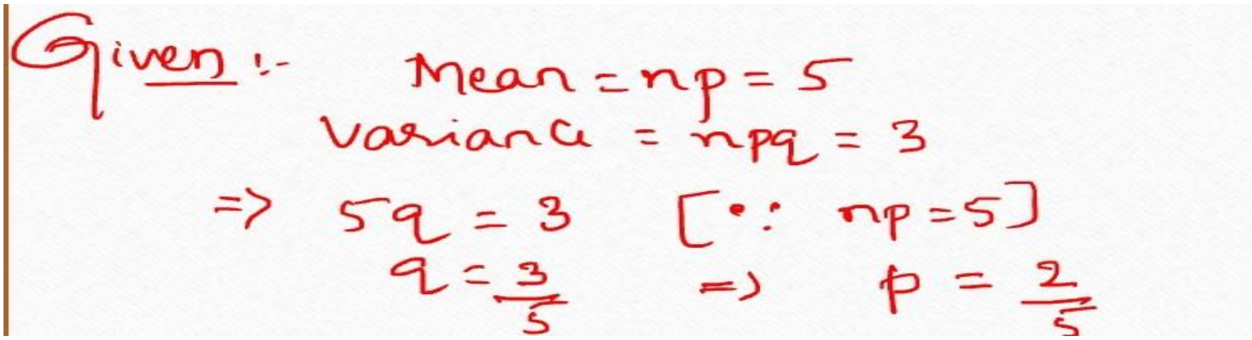
$p+2p=1$

$3p=1$

$p=\frac{1}{3} \Rightarrow q = \frac{2}{3}$

Exampe4:

For a Binomial distribution the mean & variance are 5 & 3 respectively then find. p & q



Given :- Mean $= np = 5$

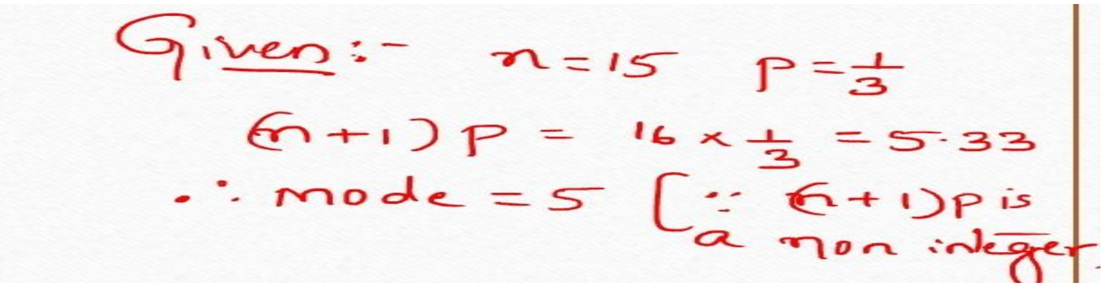
Variance $= npq = 3$

$\Rightarrow 5q = 3$ [$\because np = 5$]

$q = \frac{3}{5} \Rightarrow p = \frac{2}{5}$

Example5:

If ‘x’ is a Binomial variate with parameters 15 & 1/3 then the value of mode



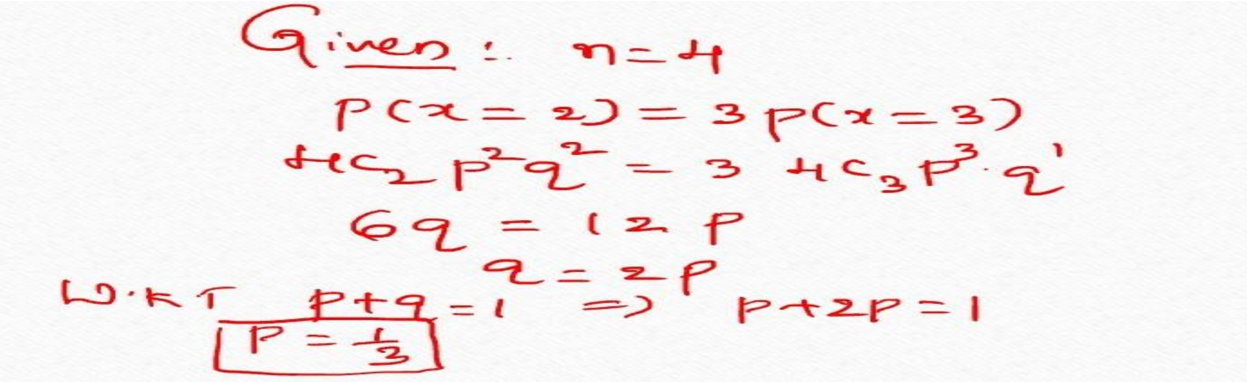
Given :- $n = 15$ $p = \frac{1}{3}$

$(n+1)p = 16 \times \frac{1}{3} = 5.33$

$\therefore$ mode $= 5$ [$\because (n+1)p$ is a non integer.

Exampe6:

For a Binomial distribution n=4 & P(x=2) = 3p(x=3) then P= ?



Given :- $n = 4$

$P(x=2) = 3P(x=3)$

$4C_2 p^2 q^2 = 3\ 4C_3 p^3 \cdot q^1$

$6q = 12p$

$q = 2p$

W.K.T $p+q=1 \Rightarrow p+2p=1$

$\boxed{P = \frac{1}{3}}$

Poisson's distribution (PD)

→ It is a preferred discrete theoretical distribution when n → ∞ (‘n’ tends to infinity & p → 0 ('p' tends to 0) & np → finite value,

→ It is uni parametric distribution & the parameter is ‘m’.

→ 'm is the mean'. 'm' itself is the variance.

i.e for a poisson’s distribution
mean = variance.

→ standard Deviation.= √m

→**Mode:**

If m is a non integer then PD is unimodal and mode = integral part of m

If m is an integer then PD is bimodal and modes = m and m-1

→ poisson's probability mass function is given by

P(x) = $\frac{e^{-m}\, m^{x}}{x!}$ For x = 0,1,2, ….. ∞

where x is poisson's variate

→ The value of e= 2.72

→ The expression x~p (m) can be read as
x is a poisson's variate with a parameter 'm'

Note: P(x=odd) = (1-e^{-2m})/2

P(x=even) = (1+e^{-2m})/2

Additive property of PD

If x ~ p(m_1) & y~p(m_2) Then,
(x+y) ~ P (m_1 +m_2)

Applications of PD:

It is used when the total number of events. or trials is large number & the probability of success is very small.

If can be used in following cases.

(i) The distribution of no. of printing mistakes per page of a large book

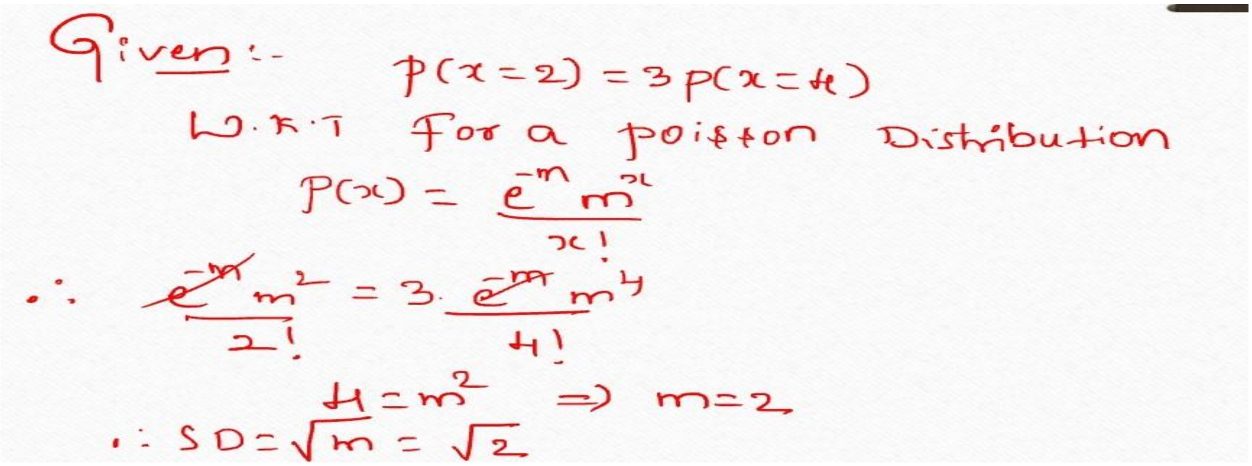
(ii) The distribution of no. of road accident on a busy road per minute etc.

Note: Poisson's distribution is always positively skewed.

Examples on Poisson's distribution

Example1:

For a PD P(x=2) = 3p(x=4) then SD is



Given :- $p(x=2) = 3\,p(x=4)$

W.K.T for a poisson Distribution

$$P(x) = \frac{e^{-m} m^{x}}{x!}$$

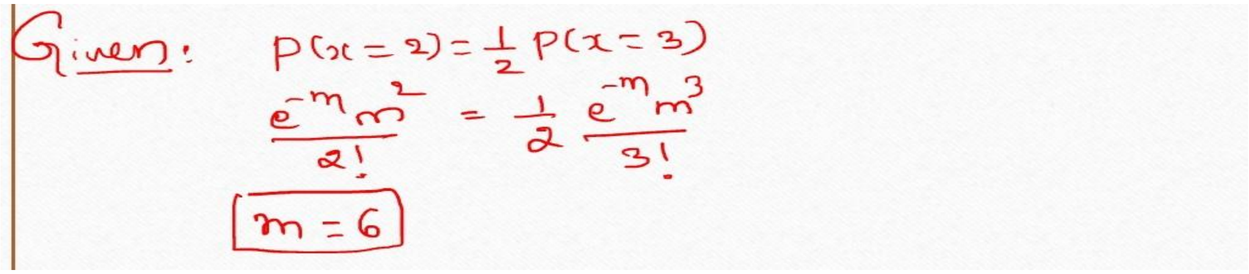
$$\therefore \frac{\cancel{e^{-m}}\, m^{2}}{2!} = 3 \cdot \frac{e^{-m}\, m^{4}}{4!}$$

$$4 = m^{2} \Rightarrow m = 2$$

$$\therefore SD = \sqrt{m} = \sqrt{2}$$

Exampe2:

In a certain PD the probability corresponding to 2 successes is half the probability corresponding to 3 successes the mean of the distribution is



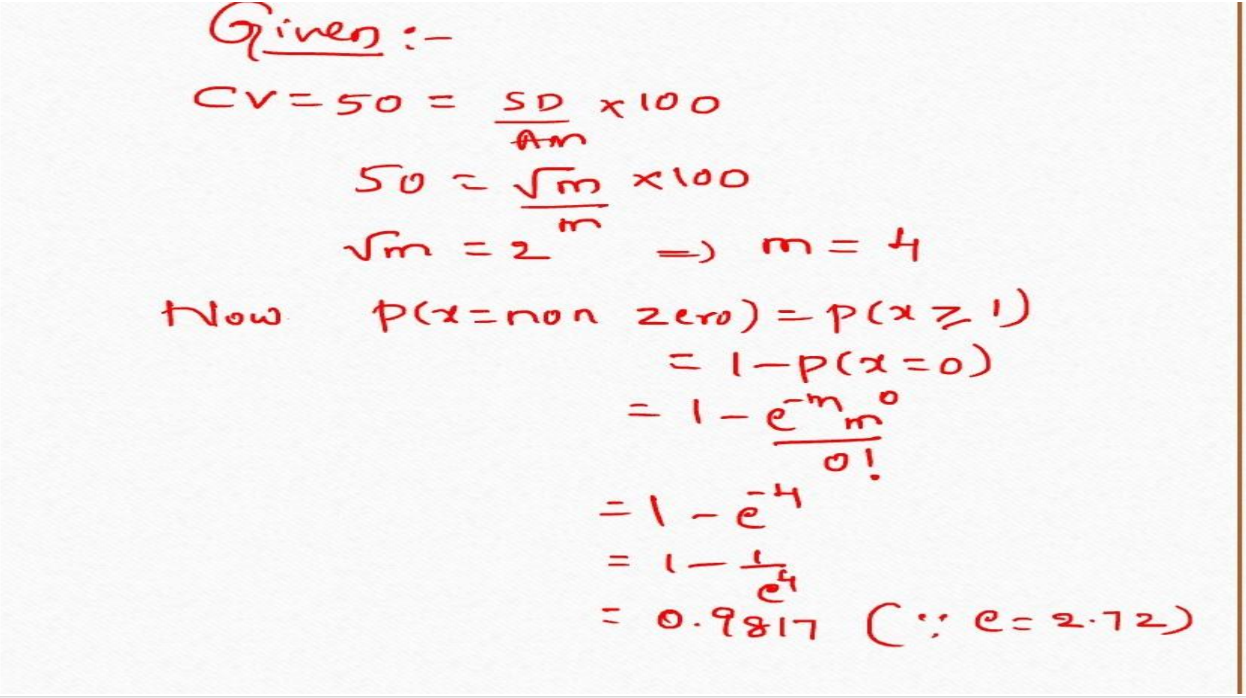
Given: $p(x=2) = \frac{1}{2}\,p(x=3)$

$$\frac{e^{-m} m^{2}}{2!} = \frac{1}{2}\,\frac{e^{-m} m^{3}}{3!}$$

$$\boxed{m = 6}$$

Example3:

If X ~ P (x) & its coefficient of variation is 50 then what is the probability that x would assume only non zero values



Given :-

$$CV = 50 = \frac{SD}{Am} \times 100$$

$$50 = \frac{\sqrt{m}}{m} \times 100$$

$$\sqrt{m} = 2 \Rightarrow m = 4$$

Now $p(x = \text{non zero}) = p(x \geq 1)$

$$= 1 - p(x=0)$$

$$= 1 - \frac{e^{-m} m^{0}}{0!}$$

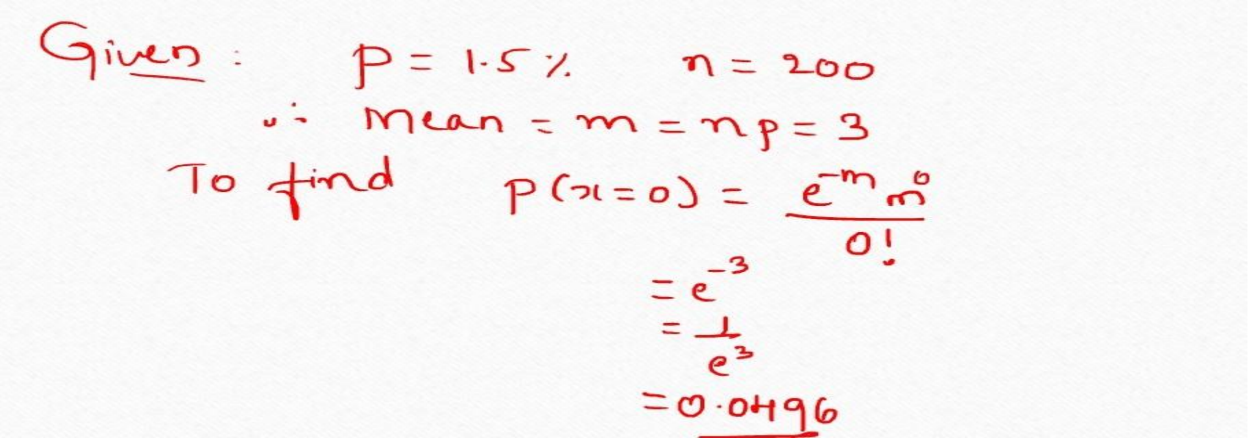
$$= 1 - e^{-4}$$

$$= 1 - \frac{1}{e^{4}}$$

$$= 0.9817 \quad (\because e = 2.72)$$

Exampe4:

If 1.5% of items produced by a manufacturing unit are known to be defective. what is the probability that a sample of 200 items would contain no defective item



Given : $p = 1.5\%$ $\quad n = 200$

$\therefore$ Mean $= m = np = 3$

To find $\quad P(x=0) = \dfrac{e^{-m} m^{0}}{0!}$

$= e^{-3}$

$= \dfrac{1}{e^{3}}$

$= 0.0496$

Normal Distribution

It is one the important continuous Probability distributions, applicable for the distribution of variables like height , weight, wages etc.

➔ It is a biparametric distribution.The 2 parameters are μ & σ^2
where μ → mean , σ^2 → Variance

➔ The normal probability mass function is given by

f(x) = $\dfrac{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$ for **- ∞ < x < ∞**

- It is a symmetrical distribution having Zero skewness.
- for a normal distribution mean=median = mode
- The shape of normal distribution doesn't depend on its parameters.
- Normal distribution is always unimodal

➔ **Points of inflexion**

➔ μ - σ & μ + σ are called as the Points of inflexion.
Here 'σ' is standard deviation.

➔ For a normal distribution the median is equi distant from 1st & 3rd quartiles

Q_1 = μ -0.6745 σ

Q_3 = μ + 0.6745 σ

- **Quartile deviation (QD) = $\frac{Q_3 - Q_1}{2}$ = 0.6745 σ**

➔ Mean deviation (MD) = 0.8 σ

- ➔ The area under normal curve ,
- ➔ i-e from (- ∞ to + ∞) is 1
- ➔ Area from - ∞ to 0 is 0.5 & Area from 0 and ∞ is 0.5.
- ➔ Area between. μ -3σ & μ +3σ is 0.9973 (99.73%)
- ➔ Area between. μ – 2 σ & μ + 2σ is 0.9546 (95.46%)
- ➔ Area between μ – σ & μ + σ is 0.68 28 (68.28%)
- ➔ **Additive property**

If $X \sim N(\mu_1, \sigma_1^2)$ & $y \sim N(\mu_2, \sigma 2^2)$
Then $(x+y) \sim N[(\mu_1 + \mu_2), ({\sigma_1}^2 + \sigma 2^2)]$

Note:- SD of (x + y) is $\sqrt{({\sigma_1}^2 + \sigma 2^2)}$

For area related problems

(i) $p(x<a) = p\left(\frac{x-\mu}{\sigma} < \frac{a-\mu}{\sigma}\right) = P(Z<K)$
(where K is a constant)

$P(Z<k) = \varphi(K)$
=>Area upto K

- ⇒ Area from -∞ to 0 + Area from 0 to K
- ⇒ (ii) $P(a<x<b) = P[\frac{a-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{b-\mu}{\sigma})$
- ⇒ $= P(K_1 < Z < K_2)$
- ⇒ => $\varphi(K_2) - \varphi(K_1)$
- ⇒ Note :- 1)K_1 & K_2 are constants
- ⇒ 2) $\varphi(-K) = 1 - \varphi(K)$

Standard normal Distribution:

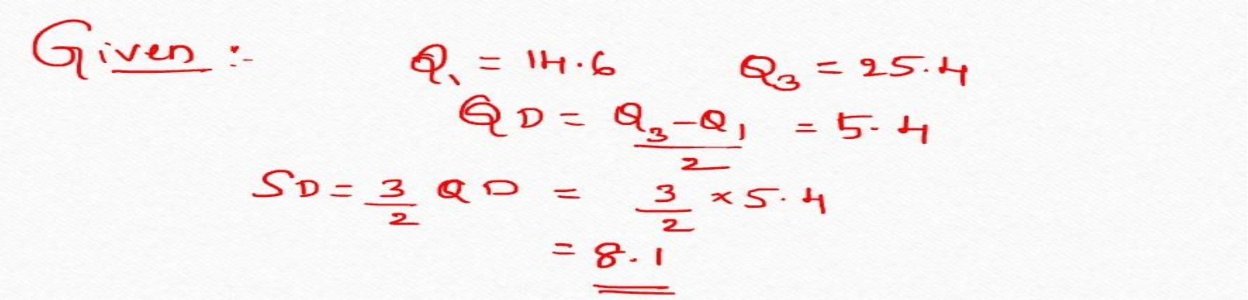
The mean of Standard normal distribution is 0

The SD of Standard normal distribution is 1

Examples on Normal distribution

Example1:

If the two quartiles of the normal distribution are 14.6 and 25.4 respectively then. find its standard deviation.



Given :- $Q_1 = 14.6$ $\quad Q_3 = 25.4$

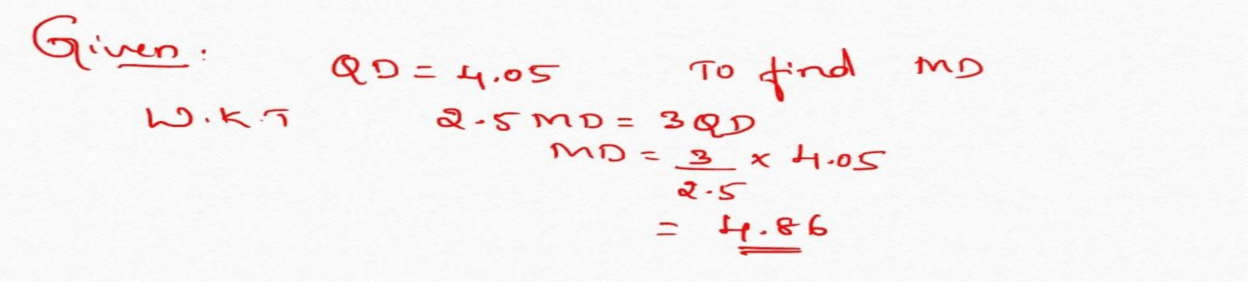
$$QD = \frac{Q_3 - Q_1}{2} = 5.4$$

$$SD = \frac{3}{2} QD = \frac{3}{2} \times 5.4$$

$$= \underline{\underline{8.1}}$$

Exampe2:

If the quartile deviation of a normal curve is 4.05 then the mean deviation is.



Given : $QD = 4.05$ $\quad$ To find MD

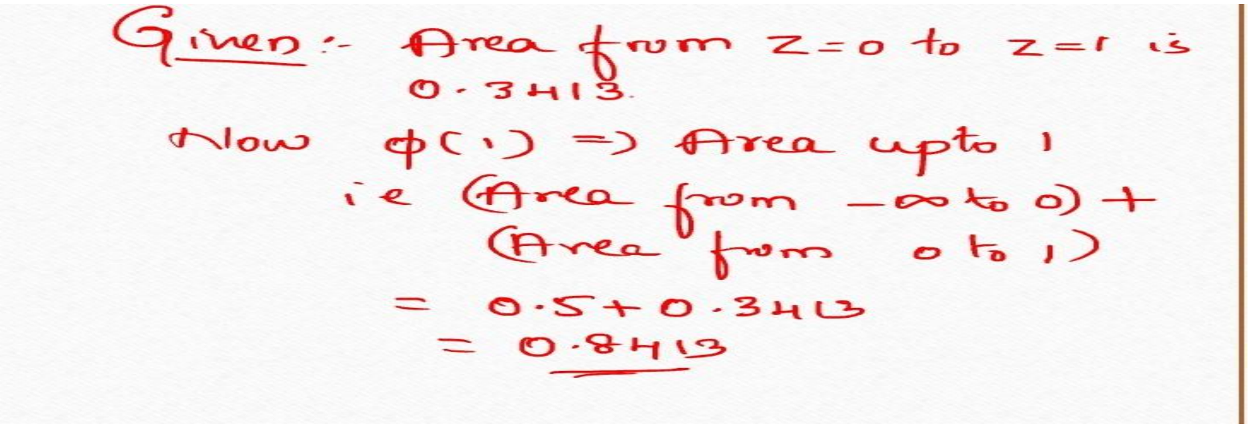
W.K.T $\quad 2.5\,MD = 3QD$

$$MD = \frac{3}{2.5} \times 4.05$$

$$= \underline{\underline{4.86}}$$

Example3:

If the area of standard normal curve between Z=0 to Z=1 is 0.3413, then find the value of Φ (1) ?



Given :- Area from $Z=0$ to $Z=1$ is 0.3413.

Now $\phi(1) \Rightarrow$ Area upto 1

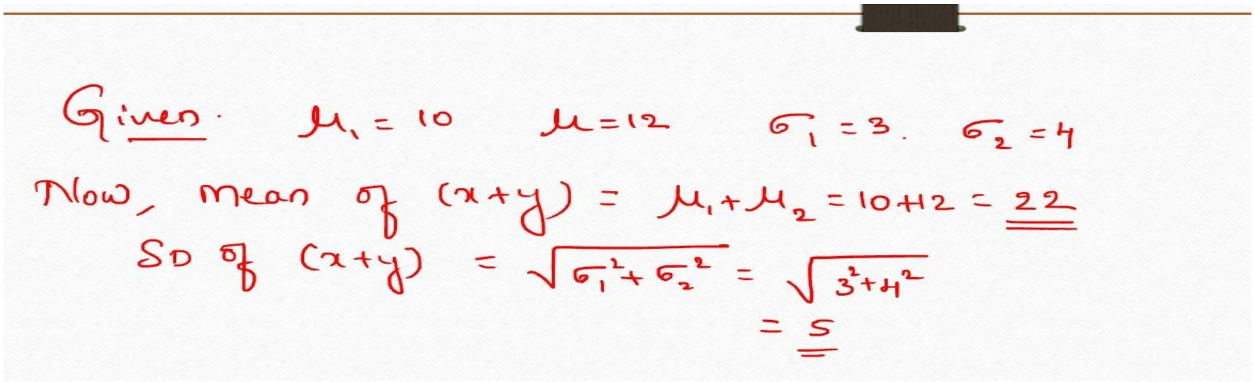
ie (Area from $-\infty$ to 0) + (Area from 0 to 1)

$$= 0.5 + 0.3413$$

$$= \underline{0.8413}$$

Example4:

If x and y are 2 independent normal Variables with means as 10 & 12 and SDs as 3 & 4, then (x+y) is normally distributed with mean =_____ and SD= _____



Given: $\mu_1 = 10$ $\mu = 12$ $\sigma_1 = 3$. $\sigma_2 = 4$

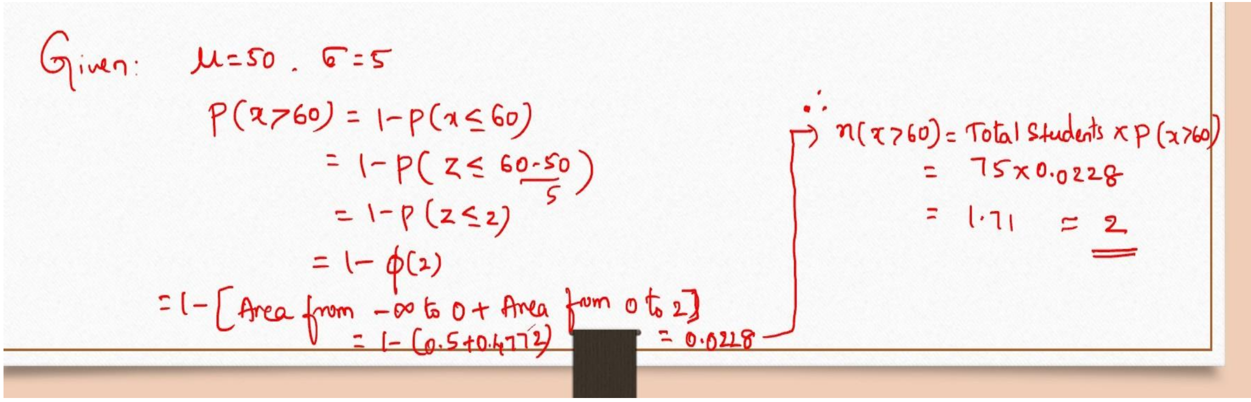
Now, mean of $(x+y) = \mu_1 + \mu_2 = 10+12 = \underline{\underline{22}}$

SD of $(x+y) = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{3^2+4^2}$

$= \underline{\underline{5}}$

Example5:

There are 75 students in a class and their average marks is 50 and SD of masks is 5. The number of students who Secured more than 60 marks is (Given that area from z=0 to Z=2 is 0.4772)



Given: $\mu = 50$. $\sigma = 5$

$P(x>60) = 1 - P(x \leq 60)$

$= 1 - P\left(z \leq \frac{60-50}{5}\right)$

$= 1 - P(z \leq 2)$

$= 1 - \phi(2)$

$= 1 - [\text{Area from } -\infty \text{ to } 0 + \text{Area from } 0 \text{ to } 2]$

$= 1 - (0.5 + 0.4772)$ $= 0.0228$

$\therefore n(x>60) = \text{Total Students} \times P(x>60)$

$= 75 \times 0.0228$

$= 1.71 \simeq \underline{\underline{2}}$

Central moments

Central Moments	Binomial distribution	Poisson's Distribution	Normal distribution
1	0	0	0
2	npq	m	σ^2
3	npq(q-p)	m	0

4	$3n^2p^2q^2$+npq(1-6pq)	m(3m+1)	$3\sigma^4$

Methods of curve fitting

- Method of moments is used to fit Binomial distribution and Poisson distribution curve
- Area method and ordinate methods are used to fit Normal distribution curve

Practice questions

1) The number of calls arriving at an internal switch board of an office is 96 per hour. Find the probability that there will be:

I) Not more than 3 calls on the board
II) Exactly 3 calls in a minute on the board. [Given:$e^{-1.6} = 0.2019$]
a) 0.08 and 0.92 respectively
b) 0.19 and 0.92 respectively
c) 0.92 and 0.13 respectively
d) 0.92 and 0.08 respectively

2) The overall percentage of failure in a certain examination is 0.30. What is the probability that out of a group of 6 candidates at least 4 passed the examination?
a) 0.74
b) 0.71
c) 0.59
d) 0.67

3) A manufacturer, who produces medicines bottles, find that 0.1 of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contains at least two defectives:
a) 7
b) 13
c) 9
d) 11

4) If 5% of the families in Kolkata do not use gas as a fuel. What will be the probability of selecting 10 families in a random sample of 100 families who do not use gas as fuel? [Given:$e^{-5} = 0.00671$]
a) 0.038
b) 0.028
c) 0.048
d) 0.018

5) The method usually applied for fitting a binomial distribution is known as:
a) Method of probability distribution
b) Method of deviations
c) Method of moments
d) Method of least squares

6) Examine the validity of the following:
Mean and standard deviation of a binomial distribution are 10 and 4 respectively.
a) Not valid
b) Valid
c) Both (a) & (b)
d) Neither (a) nor (b)

7) In Poisson distribution, probability of success is very close to:
a) -1
b) 0
c) 1
d) None

8) Shape of Normal Distribution Curve:
a) Depends on its parameter
b) Does not depend on its parameter

c) Either (a) or (b)　　d) Neither (a) nor (b)

9) If the inflexion points of a Normal Distribution are 6 and 14. Find its standard deviation?
a) 4　　b) 6　　c) 10　　d) 12

10) For binomial distribution
a) Variance < Mean　　b) Variance = Mean
c) Variance > Mean　　d) None of the above

11) If x and y are two independent normal random variables then the distribution of x + y is:
a) Normal　　b) T-distribution　　c) Chi-square　　d) F-distribution

12) If x is a binomial variable with parameters n and p, then x can assume
a) Any value between 0 and n　　b) Any value between 0 and n, both inclusive
c) Any whole number between 0 and n, both inclusive d) any number between 0 and infinity

13) Standard deviation of a binomial distribution is:
a)$\sqrt{np}$　　b) $(np)^2$　　c)$\sqrt{npq}$　　d) $(npq)^2$

14) The wages of workers of factory follows:
a) Binomial Distribution　　b) Poisson Distribution
c) Normal Distribution　　d) Chi-square Distribution

15) For a binomial distribution if variance = $(\text{Mean})^2$, then the values of n and p will be:
a) 1 and ½　　b) 2 and ½　　c) 3 and ½　　d) 1 and 1
c16. The area under the normal curve is
a) 1　　b) 0　　c) 0.5　　d) -1　　Type equation here.

17. For a normal distribution,
P($\mu - 2\sigma < x < \mu + 2\sigma$) is equal to
a) 0.9973　　b) 0.9546　　c) 0.9899　　d) 0.9788

18. For ta binomial distribution B(6,p) , P(x=2) = 9 P(x=4), then P is
a) ½　　b) 1/3　　c) 10/13　　d) ¼

19. If standard deviation of a poisson distribution is 2. Then its
a) mode is 2　　b) mode is 4　　c) modes are 3 and 4　　d) modes are 4 and 5

20. In a binomial distribution, if mean is k times the variance, then the value of k is,
a) p　　b) 1/p　　c) 1-p　　d)1/(1-p)

21. If x $\sim N(3,36) and\ y \sim N(5,64)$ are two independent normal variate with their standard parameters of distribution, then if (x+y) $\sim N(8, A)$ also follows normal distribution. The value of A will be……
a) 100　　b) 10　　c) 64　　d) 36

22.In a normal distribution quartile deviation is 6, the standard deviation will be
a) 4 b) 9 c)7.5 d) 6

23.For Poisson distribution:
a) mean and standard deviation are equal
b) mean and variance are equal
c) standard deviation and variance are equal
d) both a and b are correct

24. which of the following is not a characteristic of a normal probability distribution?
a) mean of the normally distributed population lies at the centre of its normal curve.
b) it is multi modal
c) the mean, median and mode are equal
d) it is symmetric curve

Chapter 17-Correlation and Regression

Correlation

It is a tool using which one can know an extent to which two variables (bivariate data) are related.

i.e how the value of one variable(y) changes on changing the value of other variable(x).

Conditional distributions:

If there are p classifications for x and q classifications for y, then there would be total of (p+q) conditional distributions

Note: For a bivariate frequency table having (p + q) classifications the total number of cells is equal to pq

Marginal distributions:

For p x q bivariate distributions, the number of marginal distributions is 2

Correlation coefficient (r)

- It is the measure of correlation
- By looking at the value of r, one can know whether there exists a high degree , moderate degree or low degree of correlation between two variable

Range of r

The value of r lies between -1 and +1 (both inclusive)

- If $r=1$, then the correlation is perfect positive
- If $r=-1$, then the correlation is perfect negative
- If $0 < r<0.25$, then the correlation is low degree positive
- If $0.25 < r < 0.75$, then the correlation is moderate degree positive
- If $0.75< r < 1$, then the correlation is high degree positive
- If $-1 < r< -0.75$, then the correlation is high degree negative
- If $-0.75 < r < -0.25$, then the correlation is moderate degree negative
- If $-0.25< r < 0$, then the correlation is low degree negative
- If $r=0$ then it is said to be no correlation or zero correlation

Spurious Correlation:

If there exists no casual relation between the variables, then they are said to be spuriously correlated

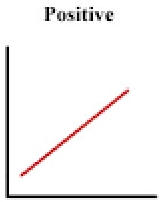
Ex: Age and height , height and weight etc.

Scatter diagram:

It is a diagrammatic representation of correlation

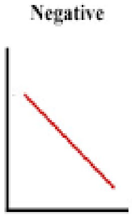
- **Perfect positive correlation:**

It is a straight line which extends from lower left to upper right



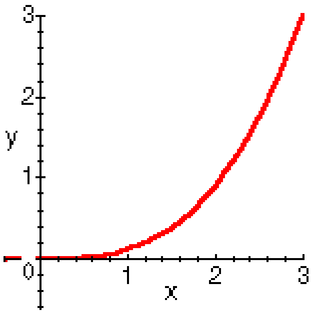
- **Perfect negative correlation**

It is a straight line which extends from upper left to lower right.



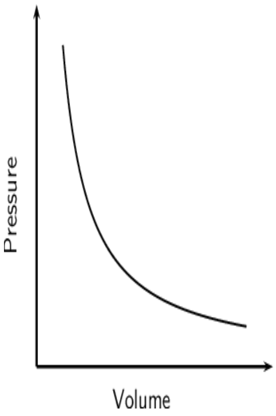
- **Simple positive correlation**

It is a curvi linear relation between the 2 variables, extending from lower left to upper right



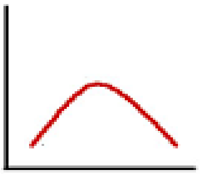
- **Simple negative correlation**

It is a curvi linear relation between the 2 variables, extending from upper left to lower right



- **No correlation (Zero correlation)**

A graph which shows increase or decrease of one variable upto an increase in the value other variable and thereafter it changes the other way.



Measures of Correlation Coefficient

Karl pearson's Correlation Coefficient (product moment correlation coefficient)

- When there exists a linear relation between the two variables then KPCC is the best measure
- If there exists a non linear relationship between x and y then r_{xy} = 0
- $r_{xy} = \frac{Cov(x,y)}{S_x.S_y}$

$$r_{xy} = \frac{n\sum xy - \sum x\sum y}{\sqrt{[\{n\sum x^2 - (\sum x)^2\}\{n\sum x^2 - (\sum x)^2\}]}}$$

Where r_{xy} is correlation coefficient between x and y

$Cov(x,y)$ is covariance between x and y

$S_x and\ S_y$ are SD's of x and y respectively

- Karl pearson's Correlation Coefficient (when deviation is taken from AM)

$\boldsymbol{r_{uv}} = \frac{\sum \boldsymbol{dx.dy}}{\sqrt{(\sum \boldsymbol{dx^2} \sum \boldsymbol{dy^2})}}$

$$\boldsymbol{r_{uv}} = \frac{\sum \boldsymbol{dx.dy}}{nS_x.S_y}$$

Where,

u =dx

v = dy

$$\sum dx.dy\ \text{is sum of the product of deviations in } x \text{ and } y$$

$$\sum dx^2\ \text{is sum of the squares of deviations in } x$$

$$\sum dy^2\ \text{is sum of the squares of deviations in } y$$

Properties of Correlation coefficient:

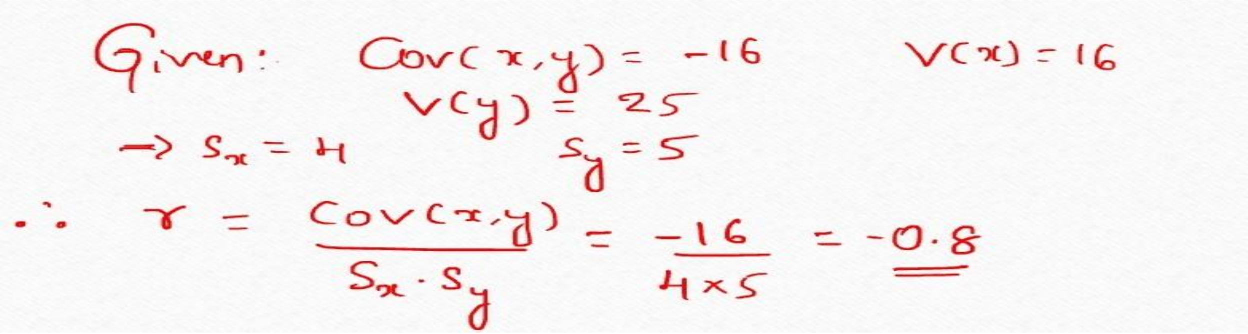
- Correlation coefficient is independent of unit of measurement
- It remains unchanged in terms of magnitude but may or may not change interms of sign due to shift of origin and change of scale
- If x and u are related by u = ax+b ----> eq1, y and u are related by v = cy+d ------> eq2

Then r_{xy} = r_{uv} (when signs of x and y in the equations 1 and 2 are same)

r_{xy} = -r_{uv} (when signgs of x and y in the equations 1 and 2 are different)

Example1:

Given Cov(x,y)= -16 , variance of x and y are 16 and 25 respectively. Find karl pearson's Correlation coefficient



Given: $Cov(x,y) = -16$ $\quad V(x) = 16$

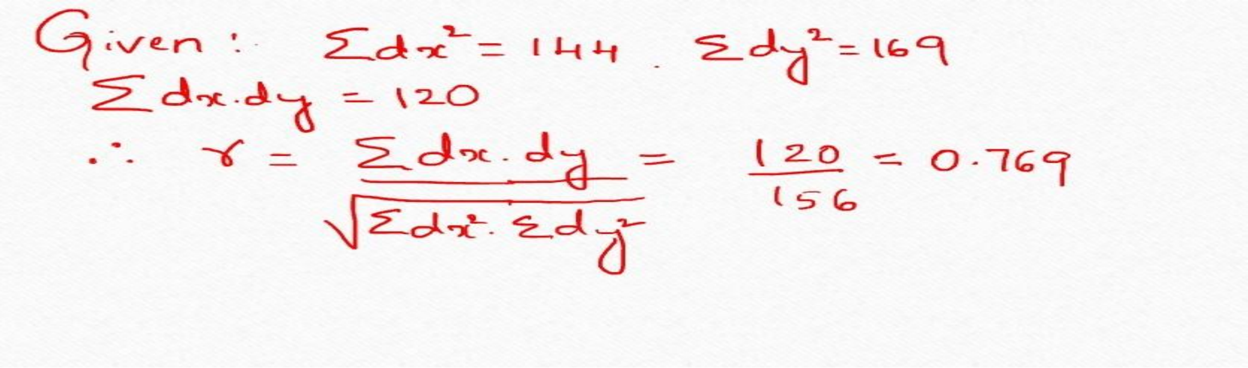
$V(y) = 25$

$\Rightarrow S_x = 4 \quad S_y = 5$

$$\therefore\ r = \frac{Cov(x,y)}{S_x \cdot S_y} = \frac{-16}{4\times5} = \underline{\underline{-0.8}}$$

Example2:

The sum of the squares of deviations of x and y are taken from respective AM's are 144 and 169 and the sum of the products of deviations is 120. Find the correlation coefficient between x and y



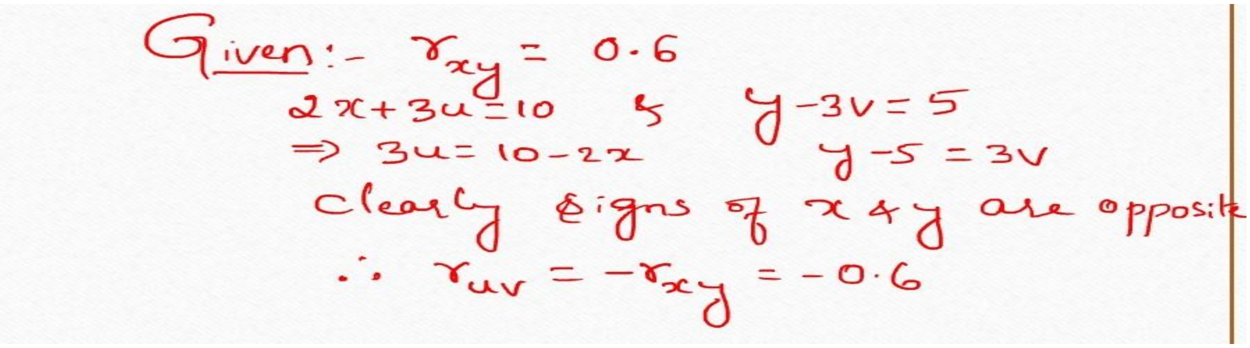
Given :- $\Sigma dx^2 = 144$. $\Sigma dy^2 = 169$

$\Sigma dx.dy = 120$

$\therefore \gamma = \frac{\Sigma dx.dy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}} = \frac{120}{156} = 0.769$

Example3:

The correlation coefficient between x and y is 0.6. Given 2x +3u =10 & y-3v =5. Find the correlation coefficient between u and v



Given :- $r_{xy} = 0.6$

$2x+3u=10$ & $y-3v=5$

$\Rightarrow 3u = 10-2x$ $y-5 = 3v$

clearly signs of x & y are opposit

$\therefore r_{uv} = -r_{xy} = -0.6$

<u>To find the nature of r when x and y are related by an equation ax+by+c = 0</u>

Note: The equation represents a straight line. Therefore the correlation can be either perfect positive or perfect negative based on the signs of x and y in the equation ax+by+c = 0

On increasing x value if y also increases then it is perfect positive correlation

On increasing x value if y decreases then it is said to be a perfect negative correlation

Example:

Examine the nature of correlation between x and y related as 2x+3y=10

Shortcut:

If the signs of x and y are similar in the equation y=a+bx , then the correlation is perfect Positive otherwise it is perfect negative

Spearman’s Rank Correlation Coefficient

- This is a measure used to see an extent of agreement of disagreement beween to persons in assessing the quantitative characteristics(expressed interms of ranks)
- It is easy to compute compared to KPCC.

Spearman’s Rank Correlation Coefficient (with tie)

$$\boldsymbol{r_r} = 1 - \left[\frac{6\left\{\sum d^2 + \frac{\sum(t^3 - t)}{12}\right\}}{n^3 - n} \right]$$

Where,

$\sum d^2$ is sum of the squares of rank differences

$i.e,\quad d^2 = (\, x_R - y_R)^2$

t is tie length

n is number of observations

Spearman’s Rank Correlation Coefficient (without tie)

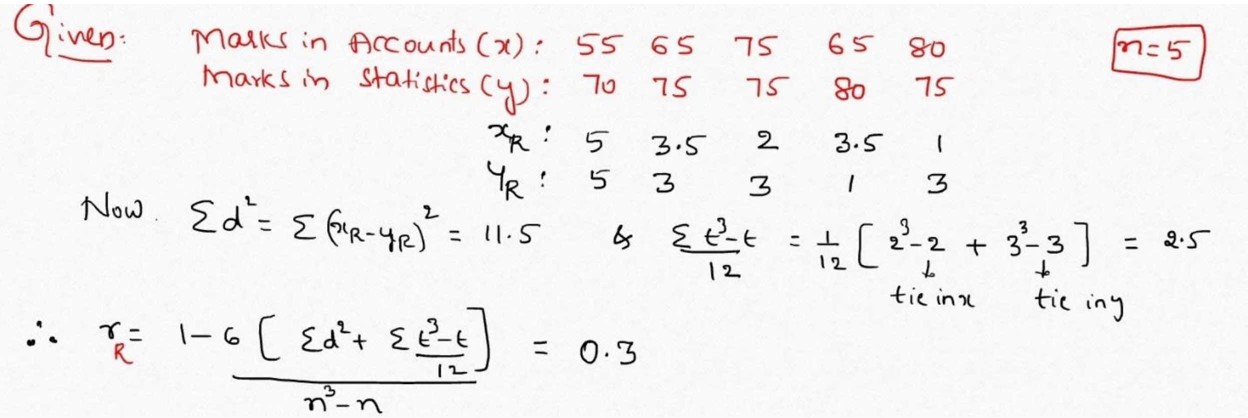
$$\boldsymbol{r_r} = 1 - \left(\frac{6\sum d^2}{n^3 - n} \right)$$

Example1:

Marks scored by 5 students in Accounts and statistics is as shown below. Find the rank correlation coefficient

Marks in Acc : 55 65 75 65 80

Marks in Stats: 70 75 75 80 75



Given:

Marks in Accounts (x):	55	65	75	65	80	$\boxed{n=5}$
Marks in Statistics (y):	70	75	75	80	75	
x_R:	5	3.5	2	3.5	1	
y_R:	5	3	3	1	3	

Now. $\sum d^2 = \sum (x_R - y_R)^2 = 11.5$ & $\frac{\sum t^3 - t}{12} = \frac{1}{12}\left[\underset{\substack{\downarrow \\ \text{tie in} x}}{2^3 - 2} + \underset{\substack{\downarrow \\ \text{tie in} y}}{3^3 - 3} \right] = 2.5$

$\therefore\ r_R = 1 - \frac{6\left[\sum d^2 + \frac{\sum t^3 - t}{12}\right]}{n^3 - n} = 0.3$

Example2:

If the sum of the squares of rank differences for 5 observations is 20 , then find the rank correlation coefficient

Given

$\sum d^2 = 20$ *and n=5*

r_r =1 – ($\frac{6\sum d^2}{n^3 - n}$) = 1-(120/120)

r_r =0

Some important notes on Spearman's rank Correlation Coefficient

- ***r_r = -1 when the ranks are in reverse order***
- ***The association between the two variables need not be linear.***
- ***The sum of the rank differences will be always zero***

Coefficient of Concurrent deviation:

When we are not concerned about the magnitude of the two variables, then Coefficient of concurrent deviation is the simplest measure of correlation

We assign a positive sign if the value is increasing from the previous value , negative sign is assigned if the value is decreasing from the previous and we assign = sign if there is no change from previous value

If the of signs of deviations for x and y are similar, then it is said to be a concurrent deviation

$$r_{cd} = \pm \sqrt{\pm\left(\frac{2c-m}{m}\right)}$$

Where,

c is number of concurrent deviations

m is number of pairs of deviation

m = n-1

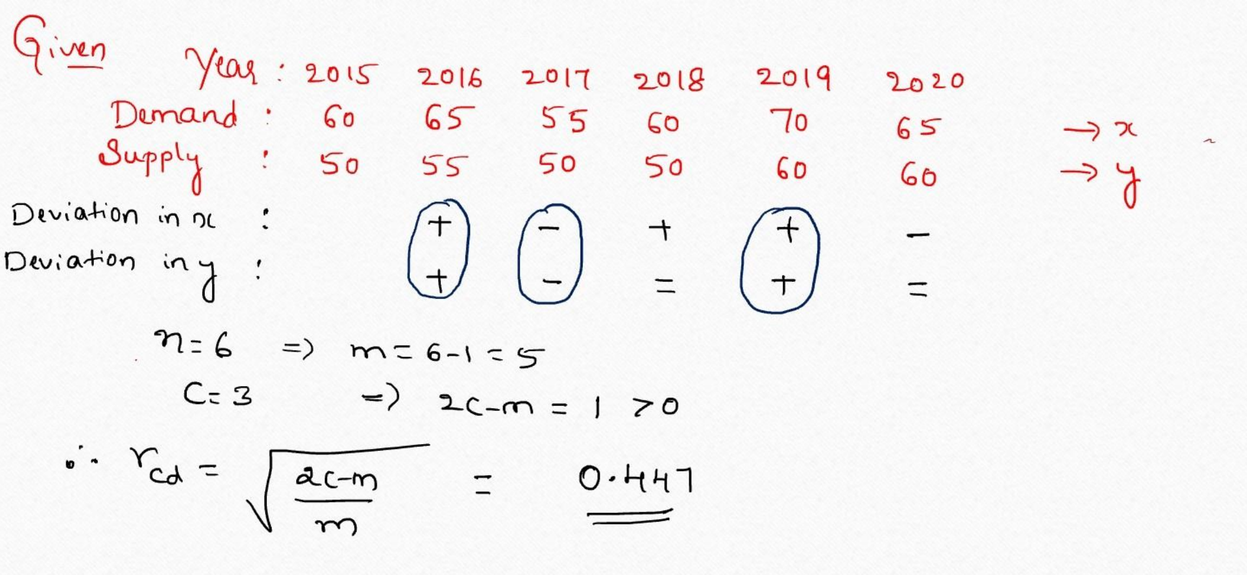
n is number of pairs of observations

Note: If 2c-m is positive then the signs are taken positive otherwise negative.

Example1:

Find the coefficient of concurrent deviations for the data given

Year :	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**
Demand:	**60**	**65**	**55**	**60**	**70**	**65**
Supply :	**50**	**55**	**50**	**50**	**60**	**60**



Given

Year :	2015	2016	2017	2018	2019	2020	
Demand :	60	65	55	60	70	65	$\rightarrow x$
Supply :	50	55	50	50	60	60	$\rightarrow y$
Deviation in x :		+	−	+	+	−	
Deviation in y :		+	−	=	+	=	

$n = 6 \Rightarrow m = 6-1 = 5$

$C = 3 \Rightarrow 2c-m = 1 > 0$

$$\therefore r_{cd} = \sqrt{\frac{2c-m}{m}} = \underline{\underline{0.447}}$$

Example2:

The number of concurrent deviations for the 10 pairs of observations is 4. Find the coefficient of concurrent deviation

Given n=10

m=9

C=4

2c-m = -1

$$r_{cd} = -\sqrt{-(\frac{2c-m}{m}})$$

$$= -1/3$$

Probable Error (PE)

- It is used to determine the limits of correlation coefficient
- The limits are given by l = r ± PE
- **PE = 0.6745** $\frac{(1-r^2)}{\sqrt{n}}$

Where n is number of observations

- PE is never negative
- If r is less than the probable error then there is no evidence of correlation

Standard Error (SE)

- SE = 1.5 PE

$= \frac{(1- r^2)}{\sqrt{n}}$

Significant value of r

The value of r is said to be significant if r > 6 PE

Coefficient of determination:

It describes the amount of explained variation and it is the square of correlation coefficient (r^2)

Coefficient of non determination:

It describes the amount of un explained variation and it is the given by ($1-r^2$)

Regression Analysis

This provides a linear relation between two variables (obtained by method of least squares) using which one can estimate the probable value of a variable given the value of another variable

Regression line of y on x

Note: To be used when the probable value of y is to be determined, given the value of x

y = a+ b_{yx} .x

(This equation is comparable with equation of a straight line in y intercept form i.e y = mx+C , where m is the slope and C is the y intercept)

Where,

b_{yx} is the regression coefficient of y on x and it is given by

b_{yx}= r. $\frac{s_y}{s_x}$

$= \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$

Note: slope of x axis or slope of any line parallel to x axis is 0.

Regression line of x on y

Note: To be used when the probable value of x is to be determined, given the value of y

x = a+ b_{xy} .y

b_{xy} is the regression coefficient of x on y and it is given by

b_{xy}= r. $\frac{s_x}{s_y}$

$= \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$

Properties of regression coefficients

- **Regression coefficient remain unaltered on shift of origin but do change with change of scale**

If u = $\frac{x-a}{b}$ & v = $\frac{y-c}{d}$

Then b_{uv} = $\frac{d}{b}$ b_{xy}

To find Regression line of y on x when b_{xy} , b_{yx} , $\overline{x}$ and $\overline{y}$ are given

(y- $\overline{y}$) = b_{yx} (x- $\overline{x}$)

Relationship between regression coefficients and correlation coefficient

- Correlation coefficient is the GM of regression coefficients
- r = **±** $\sqrt{b_{xy}.b_{yx}}$

Note: r is +ve when both b_{xy} & b_{yx} are +ve

r is –ve when both b_{xy} & b_{yx} are -ve

To find r when regression lines are given

Procedure:

- We shall assume one of the regression line as y on x and another line as x on y
- Proceed on to find b_{xy} & b_{yx} on comparing with y = a+b_{yx}.x and x = a+b_{xy}.y
- Then find r using the relation r = **±** $\sqrt{b_{xy}.b_{yx}}$
- If the value of r is within the range then the assumptions are correct the value so obtained for r is correct. Otherwise we shall reverse the assumptions, in that case b_{yx} = 1/b_{xy} and $b_{xy} = 1/b_{yx}$.

Point of intersection of Regression lines

The point of intersection of regression lines gives the mean values of x and y i.e ($\overline{x}$, $\overline{y}$)

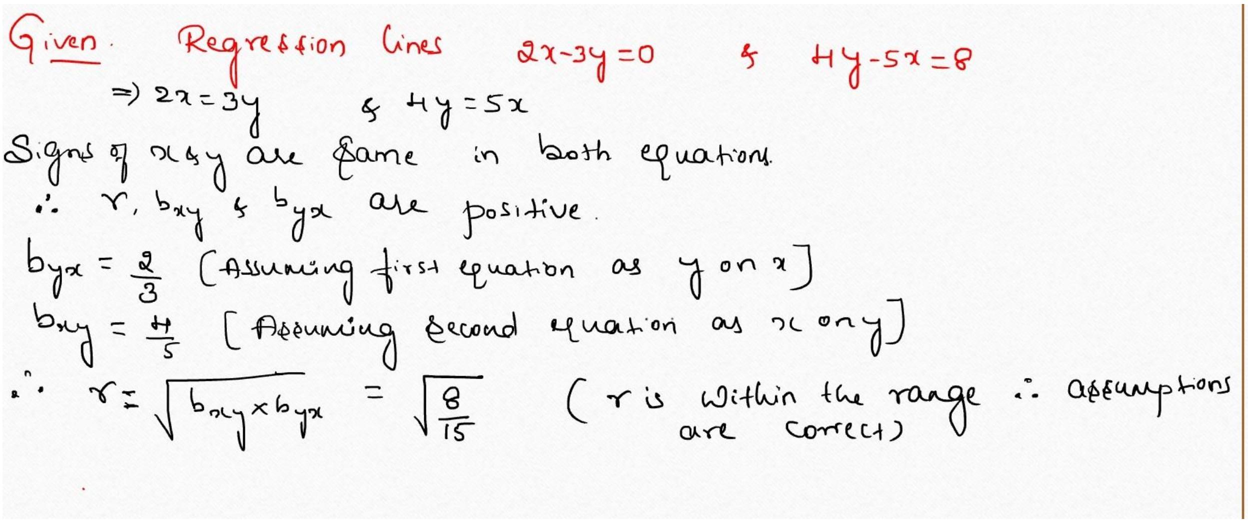
Value of r based on the angle between regression lines:

- r = 0 if the regression lines are at right angles (angle= 90 degrees)
- r= ± 1 if the regression lines are coincident or identical

Example1:

If the equation of the two regression lines are 2x -3y =0 and 4y – 5x= 8 then the correlation coefficient between x and y is equal to

a) $\sqrt{\frac{15}{8}}$ b)) $\sqrt{\frac{8}{15}}$ c)) $\sqrt{\frac{6}{15}}$ d) none of these



Given. Regression lines $2x-3y=0$ & $4y-5x=8$

$\Rightarrow 2x=3y$ & $4y=5x$

Signs of x & y are same in both equations.

$\therefore$ r, b_{xy} & b_{yx} are positive.

$b_{yx} = \frac{2}{3}$ (Assuming first equation as y on x]

$b_{xy} = \frac{4}{5}$ [Assuming second equation as x on y]

$\therefore$ $r = \sqrt{b_{xy} \times b_{yx}} = \sqrt{\frac{8}{15}}$ (r is within the range $\therefore$ assumptions are correct)

Example2:

The regression lines are 3x+2y=26, 6x+y =31. Find the regression line of x on y

Solution: r= - 0.5

Example3:

If the two lines of regression are *x* +2*y* – 5 = 0 and 2*x*+3y –8=0, then the regression line of y on x is

(a) x +2y – 5 = 0 (b) x +2y = 0

(c) 2x + 3y – 8 = 0 (d) 2x + 3y = 0

Solution:

Assuming first equation as y on x and 2nd equation as x on y

r= -0.75 (within the range)

Therefore assumptions are correct..

Therefore regression line of y on x is x +2y – 5 = 0

Practice questions:

1. The coefficient of correlation r between x and y when: Cov(x,y) = -16.5, Var(x) =2.89 and Var(y) = 100 is

a) -0.97 b) 0.97 c) 0.89 d) -0.89

2. Take 200 and 150 respectively as the assumed mean for x and y series of 11 values , then dx = x-200, dy = y-150 , $\sum dx = 13$, $\sum dx^2 = 2667$, $\sum dy^2 = 6964$, $\sum dy = 42$, $\sum dxdy = 3943$. The value of r is
   a) 0.77 b)0.98 c) 0.92 d) 0.82
3. For some bivariate data, the following results were obtained for the two variables x and y:
   $\bar{x}$ = 53.2 , $\bar{y}$ = 27.9 , b_{yx}= -1.5 , b_{xy}= -0.2
   The most probable value of y when x= :
   a) 0.267 b) 0.867 c) 0.92 d)none
4. Two random variables have the regression lines 3x+2y=26 and 6x+y =31. The coefficient of correlataion between x and y is:
   a) -0.25 b) 0.5 c) -0.5 d) 0.25
5. The coefficient of correlation between x and y is 0.6. U and V arwe two variables defined as U = $\frac{x-3}{2}$, V = $\frac{y-2}{3}$, then the coefficient of correlation between U and V is :
   a) 0.6 b) 0.4 c) 0.8 d)1
6. For the following data , the coefficient of rank correlation is:
   Rank in Botany : 1 2 3 4 5
   Rank in Chemistry : 2 3 1 5 4
   a) 0.93 b) 0.4 c) 0.6 d) none
7. The following data is given based on 450 students for marks in statistics and Economics at certain examination:
   Mean marks in statistics = 40
   Mean marks in Economics = 48
   S.D of Statistics Marks = 12
   Variance of Economics Marks = 256
   Sum of the product of deviations of marks
   From their respective mean = 42075
   The average marks in Economics of candidates who obtained 50 marks in statistics is:
   a) 45 b) 54.5 c)54 d)47.5
8. If the covariance between two variables is 20 and the variance of one of the variables is 16, what would be the variance of the other variable?
   a) More than 10 b) more than 100 c) more than 1.25 d) less than 10
9. Assume 69 and 112 as the mean values for X and Y respectively.
   dx = x-69, dy = y-112 , $\sum dx = 47$, $\sum dx^2 = 1475$, $\sum dy^2 = 3468$, $\sum dy = 108$, $\sum dxdy = 2116$. then the value of r is
   a)0.95 b) 0.65 c) 0.75 d) 0.85
10. In rank correlation, the association need not not be linear
    a) True b) False c) partly True d) partly false
11. The lines of Regression are as follows:
    5x-145= -10y : 14y – 208 = -8x .

The mean values ($\bar{x}$, $\bar{y}$)

a) (12,5) b) (5, 7) c) (7, 12) d) (5, 12)

12. The coefficient of rank correlation of marks obtained by 10 students, in English and Economics was found to be 0.5. it was later discovered that the ranks in the two subjects obtained by one student was wrongly taken as 3 instead of 7. The correct coefficient of rank correlation is:

    a) 0.32 b) 0.26 c) 0.49 d)0.93
13. If the correlation coefficient between two variables is 1. Then the two lines of regression are:

    a) Anti Parallel b) At right angles c) Identical d) none of these
14. Given b_{yx}= 1.6 and b_{xy}= 0.4 . the coefficient of determination is

    a) 0.74 b) 0.42 c) 0.58 d) 0.64
15. Then method applied for deriving regression equations is known as:

    a) Concurrent deviations b) product moment c) least squares d) normal equation
16. If the sum of squares of differences of rank is 50 and number of items is 8 then what is the value of rank correlation coefficient.

    a) 0.59 b) 0.4 c) 0.36 d)0.63
17. If coefficient of correlation between x and y is 0.46. find coefficient of correlation between x and y/2

    a) 0.46 b) 0.92 c) -0.46 d) -0.92
18. Given regression equations as 3x + y = 13 and 2x+ 5y = 20. Find regression equation of y on x.

    a) 3x + y = 13 b) 2x + y = 20 c) 3x + 5y = 13 d) 2x +5y = 20
19. The correlation coefficient between x and y is -1/2. The value of b_{xy}= -1/8. Find b_{yx}

    a) -2 b) -4 c) 0 d) 2
20. If the ranks given by 2 judges are in reverse order then the value of spearman rank correlation coefficient is

    a) -1 b) 0 c) 1 d) 0.75
21. If the rank correlation coefficient between marks in Management and Mathematics for a group of students 0.6 and the sum of the squares of the difference in ranks is 66. Then what is the number of students in the group?

    a) 9 b) 10 c) 11 d)12
22. Correlation coefficient between x and y will be negative when

    a) X and y are decreasing b) x is increasing, y is decreasing
    c)X and y are increasing d) none of these

Chapter 18-Index numbers

It is a statistical tool that helps in Comparing price/quantity/value of a Commodity in 2 different time periods (Base year and current year)

- → Index numbers are expressed interms of percentages.
- → Averages used for the Construction of Index numbers, are AM & GM
  - → GM is the best average for construction of Index numbers.

Cassification of Index numbers

Simple Index number,

Simple aggregative price Index :

$= \frac{\sum P_1}{\sum P_0} \times 100$

Where P_1 → Current year Price
P_0 → Base year price

Simple Relative price Index

$= \frac{\sum\left(\frac{P_1}{P_0}\right)}{N} \times 100$

where, N → no. of Commodities.

Note:

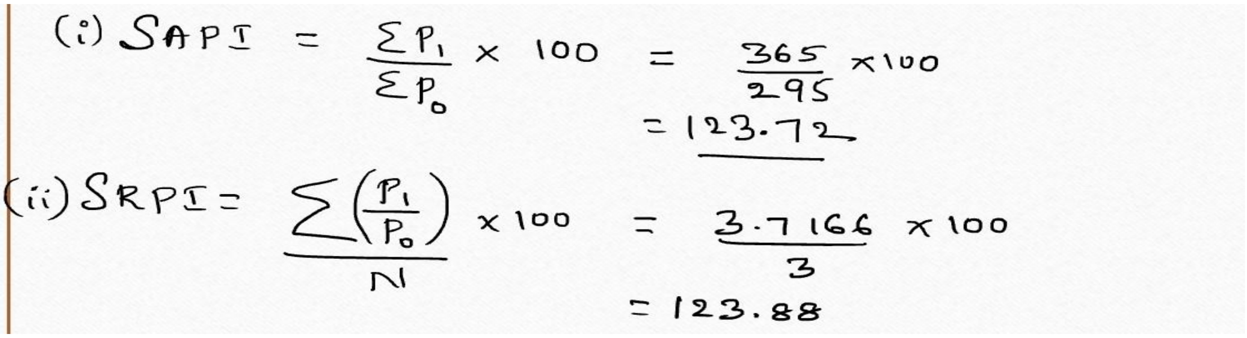
SAPI is unit dependent where as SRPI is unit independent.

Example:

Find (i) Simple aggregative & (ii) Simple relative price Index for the data

Given

Commodity	Base Year Price	Current year Price
Rice (Per kg)	75	95
Dal (Per Kg)	100	120
Sun flower oil (per Litre)	120	150



(i) SAPI $= \frac{\Sigma P_1}{\Sigma P_0} \times 100 = \frac{365}{295} \times 100$

$= 123.72$

(ii) SRPI $= \frac{\Sigma\left(\frac{P_1}{P_0}\right)}{N} \times 100 = \frac{3.7166}{3} \times 100$

$= 123.88$

weighted Index numbers..

→ Here weight means quantity

Types of weighted Index numbers

Laspeyre's Index (L)

$$L = \frac{\Sigma(P_1 q_0)}{\Sigma(P_0 q_0)} \times 100$$

where,

q_0 is base year quantity

Paasche's Index (P)

$$P = \frac{\Sigma(P_1 q_1)}{\Sigma(P_0 q_1)} \times 100$$

where q_1 → Current year quantity.

Note: If the prices or quantities change in the same ratio, then the laspeyre’s index will be equal to Pasche’s index.

Bowley's Index (B)

→ It is an AM of Laspeyre's & paasche's Index.

$$B = \frac{L+P}{2}$$

Fisher's Index (F)

→ It is a GM of Laspeyre's and Paasche's Index.

→ $F = \sqrt{L x P}$

$$F = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \, x \, \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}} \times 100$$

→ Fisher's Index is an Ideal Index number.

Marshall's Index (M)

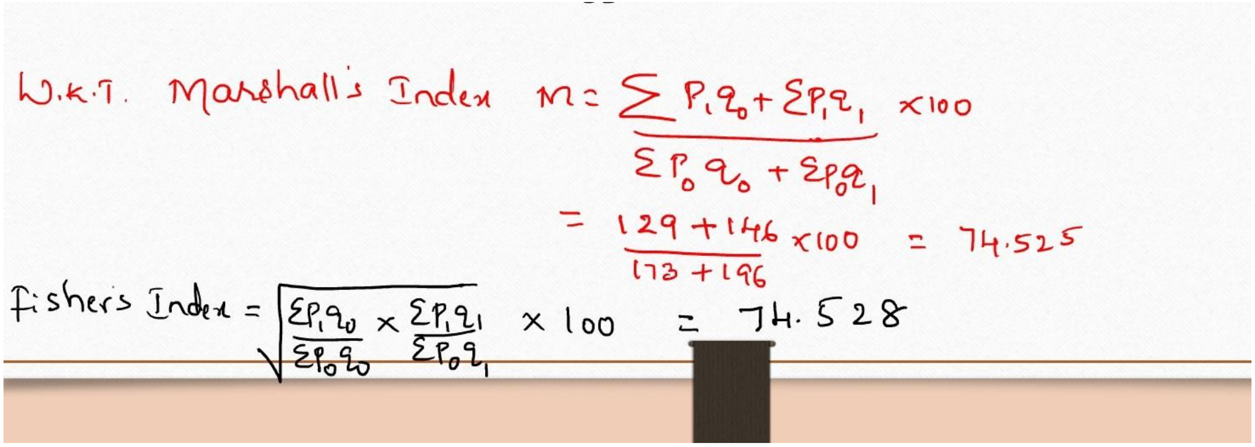
M = $\frac{\sum P_1(q_0 + q_1)}{\sum P_0(q_0 + q_1)}$ x100

→ Marshall's Index is a good approximation to Fisher's Index.

Examples:

Find Marshall's & Fisher's Index for the data given

Commodity	P_0	q_0	P_1	q_1
A	3	7	3	8
B	6	12	4	14
C	8	10	6	11



W.K.T. Marshall's Index $M = \frac{\sum P_1q_0 + \sum P_1q_1}{\sum P_0q_0 + \sum P_0q_1} \times 100$

$= \frac{129 + 146}{173 + 196} \times 100 = 74.525$

Fisher's Index $= \sqrt{\frac{\sum P_1q_0}{\sum P_0q_0} \times \frac{\sum P_1q_1}{\sum P_0q_1}} \times 100 = 74.528$

Consumer Price Index (CPI)

-> It is also referred as Cost of living Index

-> The CPI is a measure of average change in price over a given period of time the Consumer pays for the basket of goods and services.

→ The CPI helps in finding the rate of Inflation.

→ CPI = $\frac{\sum Iw}{\sum w}$

where I → group index

w → weight.

Deflated value:

→ Deflation of value occurs when retailers & Service providers Cut their costs & sell smaller packages, give out smaller portions or generally provide less for the same Price so as to maintain the same sticker price.

Deflated value = $\frac{\text{current value}}{\text{current year CPI}}$

Splicing of Index numbers

→ It means the construction of One Continuous series from two different index number series on the basis of Common base.

→It is to be used when a new commodity is to be added to the existing list of commodities.

Shifted Price Index

Shifted price Index = $\frac{\text{original Price Index}}{\text{Price Index of the year to which it has to be shifled}}$ x 100

Chain Index number (CIN)

A chain Index is an Index number in which the value of any given period is related to the value of immediate preceding period.
This in different from the fixed- base Index.

CIN = $\frac{\text{Link Relative of current year x CIN of Previous year}}{100}$

Here, link relative is the price relative.

Purchasing power of Money :-

→ It is the reciprocal of price index number

Purchasing power of money = $\frac{1}{\text{Price Index number}}$

→ Lesser the price index number, greater will be the purchasing Power of money.

Real wages

= $\frac{\text{current year wages}}{\text{Current year CPI}}$ x Base year CPI

- It is the wages in comparison with base year wage

Percentage increase in Real wages

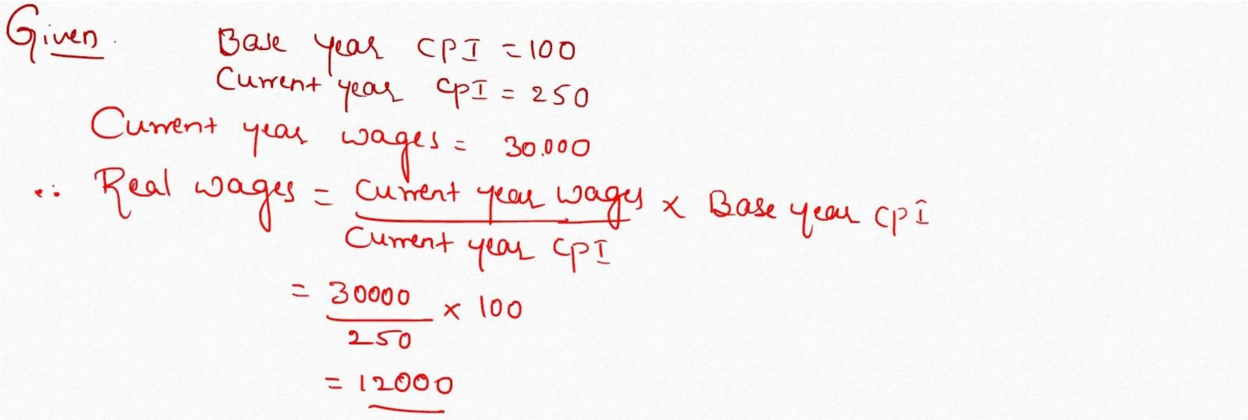
= $\left[1-\left(\frac{\text{I+ \% Increase in Price}}{\text{1+\% increase in wages}}\right)\right]$ x100

Where,
% Increase in price is rate of inflation

Examples:

the CPI goes up from 100 to 250 & the wages of a worker also raised from 10,000 to 30,000.

Find the real wages.



Given. Base year CPI = 100

Current year CPI = 250

Current year wages = 30,000

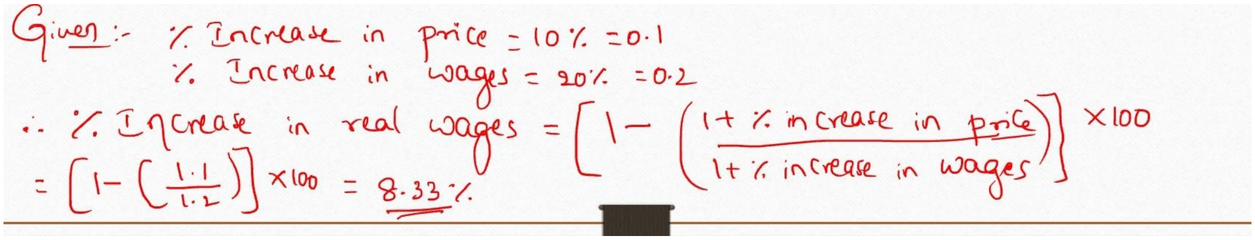
∴ Real wages = $\frac{\text{Current year wages}}{\text{Current year CPI}} \times \text{Base year CPI}$

$= \frac{30000}{250} \times 100$

$= 12000$

Example:

If with an increase of 10% in price, the rise in wages is 20% then the Real wages has increased by
(a) 20 % b) 10% c) less than 10% d) more than 10%



Given:- % Increase in price = 10% = 0.1

% Increase in wages = 20% = 0.2

∴ % Increase in real wages = $\left[1-\left(\frac{1+\%\text{ increase in price}}{1+\%\text{ increase in wages}}\right)\right] \times 100$

$= \left[1-\left(\frac{1.1}{1.2}\right)\right] \times 100 = 8.33\%$

Note:

%. Increase in real wages will be always Less than the difference b/w % increase in wages & % increase in price.

Tests of Adequacy (Tests of consistency)

1. Unit test

For Index no. to pass unit test, it must be independent of unit.

→ only simple aggregative Price Index doesn't satisfy unit test as it is not Independent it is units.

2). Time reversal test (TRT)

,Condition for satisfying TRT
$P_{01} \times P_{10} = 1$

P_{01} represents 1 on 0

Where as P_{10} represents 0 on 1

Fisher's index and Marshall's index satisfy

3) Factor Reversal Test (FRT)

→ Condition for satisfying factor reversal test is

$P_{01} \times q_{01} = V_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_0}$

Where

V_{01}, value index of 1 on 0

→ only Fisher's Index Satisfies FRT.

4) <u>circular Test</u>

→ Condition for satisfying circular test is $P_{01} \times P_{12} \times P_{20} = 1$

→ Fisher's Index fails to satisfy Circular test

→ It is satisfied by Simple GM of price relatives & weighted aggregative with fixed weights

→Circular test is an extension of TRT

<u>To find the current year salary and Dearness allowance(DA) when Base year salary, Base year CPI and Current year CPI are given.</u>

Note: If base year CPI is not given, then it is taken as 100

Current year salary: $\frac{Base\ year\ salary \times Curent\ year\ CPI}{Base\ year\ CPI}$

- → Dearness allowance= Current year salary – Base year salary

<u>To find the current year CPI when Percentage increase/ decrease in price is given</u>

Example:Index number of prices in the year 2008 is 225. With 2004 as the base year , What is the percentage change in price from base period?

Given

CPI

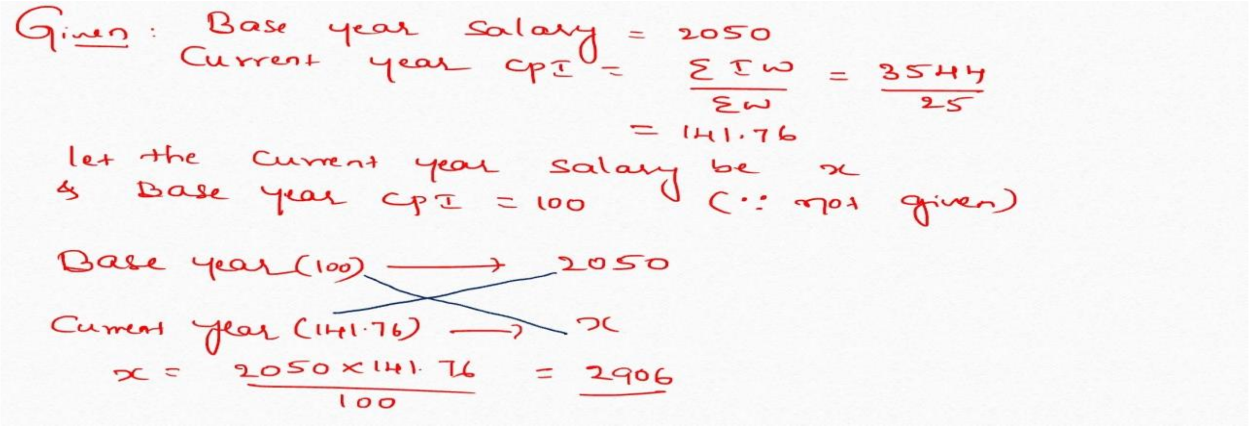
$$\left.\begin{array}{ll} \text{Base year} \rightarrow & 100 \\ \text{Current year} \rightarrow & 225 \end{array}\right\} +125\%$$

∴ The percentage Change in price from base year is 125%.

<u>Example1:</u>

Suppose a business executive was earning ₹. 2,050 in the base period, what should be his salary in the current period if his standard of living is to remain the same? Given $\sum IW = 3544$ $\sum W = 25$

a) ₹. 2096 b) ₹. 2906 c) ₹. 2106 d) ₹. 2306



Given: Base year salary = 2050

Current year CPI = $\frac{\Sigma IW}{\Sigma W} = \frac{3544}{25}$

$= 141.76$

let the current year salary be x

& Base year CPI = 100 (∵ not given)

Base year (100) $\longrightarrow$ 2050

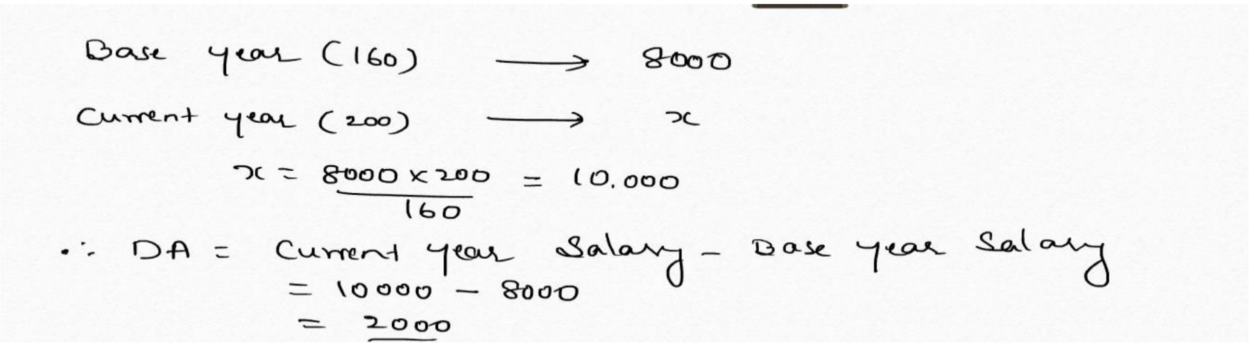
Current year (141.76) $\longrightarrow$ x

$x = \frac{2050 \times 141.76}{100} = \underline{2906}$

Example2:

The monthly income of an employee was Rs.8000 in 2014. The consumer price index number was 160 in 2014, which rose to 200 in 2017. If he has to be rightly compensated, the additional dearness allowance to be paid to him in 2017 would be:

a) Rs.2400 b) Rs.2750 c)Rs.2500 d) <u>none of these</u>



Base year (160) $\longrightarrow$ 8000

Current year (200) $\longrightarrow$ x

$x = \frac{8000 \times 200}{160} = 10{,}000$

∴ DA = Current year Salary − Base year Salary

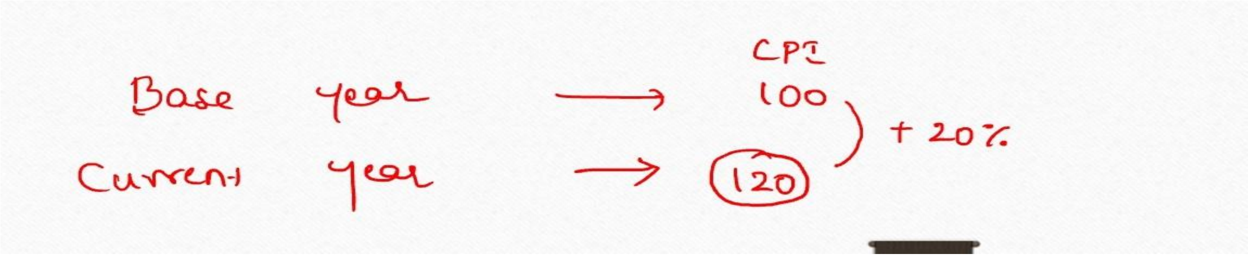
= 10000 − 8000

= $\underline{2000}$

Example3:

If the prices of all commodities in a place has increased 20% in comparison to the base period prices, then the index number of prices for the place is now….

a) 100 b) 120 c) 20 d) 150



CPI

Base year $\longrightarrow$ 100

Current year $\longrightarrow$ $\boxed{120}$ + 20%

Practice questions:

1. Chain index is equal to

(a) $\frac{\text{like relative of current year} \times \text{chain index of the current year}}{100}$

(b) $\frac{\text{link relative of previous year} \times \text{chain index of the current year}}{100}$

(c) $\frac{\text{link relative of current year} \times \text{chain index of the previous year}}{100}$

(d) $\frac{\text{link relative of previous year} \times \text{chain index of the previous year}}{100}$

2. The test of shifting the base is called
(a) Unit Test. (b) Time Reversal Test. (c) Circular Test. (d) None of these.

3. An index time series is a list of _______ numbers for two or more periods of time.
(a) Index (b) Absolute (c) Relative (d) Sample

4. P01 is the index for time
(a) 1 on 0 (b) 0 on 1 (c) 1 on 1 (d) 0 on 0

5. The index number of prices at a place in 1998 is 355 with 1991 as base. This means
(a) There has been on the average a 255% increase in prices
(b) There has been on the average a 355% increase in price
(c) There has been on the average a 250% increase in price
(d) None of these

6. If the price of all commodities in a place has increased by 1.25 times in comparison to the base period prices, then the index number of prices for the place is now
(a) 100 (b) 125 (c) 225 (d) None of the above.

7. If now the prices of all the commodities in a place have been decreased by 85% over the base period prices, then the index number of prices for the place is now (index number of prices of base period = 100)
(a) 100 (b) 135 (c) 65 (d) None of these

8. Simple Aggregative Method is used for computing a:
(a) Relative index. (b) Price index. (c) Value index. (d) None of these.

9. The ______ is satisfied when Pab × Pbc × Pca = 1
(a) Time reversal test (b) Factor reversal test (c) Circular test (d) Unit test

10. ______ is an extension of time reversal test.
(a) Factor reversal test (b) Circular test (c) Unit test (d) None of these

11. Time reversal test is satisfied when
(a) P01 x P10 = 0 (b) P01 x P10 = 1
(c) P01 x P10 < 1 (d)P01 x P10 > 1

12. The total sum of the values of a given year divided by the sum of the values of the base year is
(a) Price index. (b) Quantity index. (c) Value index. (d) None of these.

13. Fisher's ideal index is
(a) Arithmetic mean of Laspeyre's and Paasche's index.
(b) Median of Laspeyre's and Paasche's index.
(c) Geometric mean of Laspeyre's and Paasche's index.
(d) None of these.

14. Factor reversal test is satisfied by
(a) Laspeyre's index. (b) Paasche's index.
(c) Fisher's ideal index. (d) None of these.

15. Laspeyre's index is based on
(a) Base year quantities. (b) Current year quantities.

(c) Average of current year and base year. (d) None of these.

16. Laspeyre's and Pasche's method satisfy time reversal test
(a) True (b) False (c) Both (d) None of these

17. The index number is a special type of G.M.
(a) True (b) False (c) Both (d) None of these

18. The number of test of adequacy is ___________
(a) 2 (b) 5 (c) 3 (d) 4

19. Theoretically, A.M. is the best average in the construction of index nos. but in practice, mostly the G.M. is used:
(a) False (b) True (c) Both (d) None of these

20.P10 is the index for time
(a) 1 on 0 (b) 0 on 1 (c) 1 on 1 (d) 0 on 0

21. The index number is not a special type of average
(a) False (b) True (c) Both (d) None of these

22. Fisher's ideal idex no. is equal to
(a) Laspeyse's index × Pasche's Index (b$\sqrt{Laspeyse's\ index \times Pasche's\ Index}$
(c)(L+P)/2 (d) None of these

23. Fisher's Ideal formula does not satisfy_________ test
(a) Circular test (b) Unit test (c) Time Reversal test (d) None of these

24. The price level of a country in a certain year has increased by 20% over the base period. The Index number for that year is _____________
(a) 20 (b) 120 (c) 220 (d) None of these

25. In a circular test the _____________condition must be satisfied?
(a) P01 x P12 x P20 = 1 (b) P02 x P10 x P20 = 1 (c) P10 x P20 x P21 = 1 (d) None of these

26. For factor reversal test:P01 × Q01 = V01 = True Value Ratio (T.V.R.) This is
(a) False (b) True (c) Both (a) & (b) (d) None of these

27. During a certain period, the cost of living index number goes up from 110 to 200 and the salary of the worker is also raised from Rs. 3,250 to Rs. 5,000. Does the worker really gain?
(a) No (b) Yes (c) Cannot determine (d) None of these

28. During a certain period, the cost of living index number goes up from 110 to 200 and the salary of the worker is also raised from Rs. 3,250 to Rs. 5,000. What should be his salary in real terms?
(a) Rs. 5,800 (b) Rs. 5,909 (c) Rs. 5,900 (d) None of these

29. When the prices or quantities of all the goods are changing in the same ratio then the Laspeyre's and Paasche's Index Number will be
(a) Equal (b) Unequal (c) Either (a) or (b) (d) None of these

30. Between 1990 and 2000, the price of a commodity increased by 60% while the production decreased by 30%. By what percentage did the value index of production of commodity change in 2000 with respect to its value 1990.
(a) 10% (b) 15% (c) 12% (d) None of these

31. The consumer price index over a certain period increased from 120 to 215 and the wages of worker increased from Rs. 1,680 to Rs. 3000. What is the loss of the worker?
(a) 5.58 (b) 6.58 (c) 7.58 (d) None of these

32. The consumer price index for a group of workers was 250 in 1994 with 1980 as the base. Compute the purchasing power of a rupee in 1994 Compared to 1980.
(a) 0.40 (b) 0.50 (c) 0.60 (d) None of these