IMPORTANT FORMULAE MATHS

1. If a quantity increases or decreases in the ratio $a:b$ then

$$\text{new quantity} = \frac{b}{a} \times \text{original quantity}$$

Ex Ronak weighs 56.7 kg. If he reduces his weight in the ratio 7:6, find new ~~root~~ weight.

Sol. $\frac{6}{7} \times 56.7 = \boxed{48.6 \text{ kg}}$

2. Inverse ratio of $a:b$ is $b:a$

3. Ratio compounded of two ratios $a:b$ and $c:d$ is $ac:bd$

4. • $a^2 : b^2$ is the duplicate ratio of $a:b$
   • $a^3 : b^3$ is the triplicate ratio of $a:b$

5. • $\sqrt{a} : \sqrt{b}$ is the sub-duplicate ratio of $a:b$
   • $\sqrt[3]{a} : \sqrt[3]{b}$ is the sub-triplicate ratio of $a:b$

6. Continued Ratio: If given ratios are $a:b$ and $b:c$ we can make continued ratio $a:b:c$ if we make term b as same in both ratios.

Ex If $x:y = 2:3$, $y:z = 4:3$, then $x:y:z$ is

Sol. $\frac{x}{y} = \frac{2\times4}{3\times4} \Rightarrow \frac{8}{12}$, $\frac{y}{z} = \frac{4\times3}{3\times3} = \frac{12}{9} \Rightarrow \boxed{8:12:9}$

7. Continuous Proportion: $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$

a = first Proportion
b = Mean Proportion
c = Third Proportion

Ex Third proportional to 12, 18 is

Sol $a = 12$, $b = 18$, $c = ?$ $\quad \frac{12}{18} = \frac{18}{c} \Rightarrow \boxed{c = 27}$

8. Invertendo: If $a:b = c:d$, then $b:a = d:c$

9. Alternendo: If $a:b = c:d$, then $a:c = b:d$

10. Componendo: If $a:b = c:d$, then $\frac{a+b}{b} = \frac{c+d}{d}$

11. Dividendo: If $a:b = c:d$, then $\frac{a-b}{b} = \frac{c-d}{d}$

12. Componendo & Dividendo: If $a:b = c:d$, then
$\frac{a+b}{a-b} = \frac{c+d}{c-d}$ and $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

13. Addendo: If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$, then $\frac{a+c+e}{b+d+f} = k$

14. Subtrahendo: If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$, then $\frac{a-c-e}{b-d-f} = k$

15. Indices :
- Any base raised to power zero is 1
  i.e. $a^0 = 1$
- Roots can be expressed in power, i.e.
  $\sqrt[n]{a} = a^{1/n}$

16. Law.1 (Sum of Power)
$$a^m \times a^n = a^{m+n}$$

17. Law.2 (Diff. of Powers)
$$a^m \div a^n = a^{m-n}$$

18. Law.3 (Power of Power)
$$(a^m)^n = a^{m \times n}$$

19. Law.4
$$(a \times b)^n = a^n \times b^n$$

20. Calculator Trick for power (Integer) of any number:

base x = = = =
↳ square

21. Calculator Trick for Reciprocal of any number:

number ÷ =

22. Calculator Trick for n^{th} root of a number:

Base "√" 12 times −1 ÷ n +1 "x =" 12 times

23. Calculator Trick for Power (non integer)

Base "√" 12 times −1 x n +1 "x =" 12 times

Ex i) $7^5 \Rightarrow 7 \times = = = =$ 16807

ii) $\frac{10}{(1.06)^3} \Rightarrow 1.06 \times = = \div = \times 10 =$ 8.3961

iii) $\sqrt[5]{264} \Rightarrow 264$ "$\sqrt{\ }$" 12 times $-1 \div 5 + 1$ "$x =$" 12 times
3.052

iv) $3^{5.7} \Rightarrow 3$ "$\sqrt{\ }$" 12 times $-1 \times 5.7 + 1$ "$x =$" 12 times
522.25

24. Basic logarithm: if $a^x = n$ then $\log_a n = x$

Conditions: $n > 0$, $a > 0$, $a \neq 1$

25. Log Standard Result

- Log with same base as number is equal to 1
$$\log_a a = 1$$
- Log of 1 for any base is equal to 0
$$\log_a 1 = 0$$

26. Law of Log.(1):
$$\log_a mn = \log_a m + \log_a n$$

27. Law of Log.(2):
$$\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$$

28. Log Law (3):
$$\log_a m^n = n \log_a m$$

Change of Base: $\log_b m = \frac{\log m}{\log b} = \frac{\log_a m}{\log_a b}$

30. form of Quadratic Equation: $ax^2 + bx + c = 0$

31. Solution of Quadratic Equation: $\frac{-b \pm \sqrt{b^2 - 4a}}{2a}$

32. Sum of roots: $\alpha + \beta = -b/a$

33. Product of roots: $\alpha\beta = c/a$

34. Construction of Quadratic Equation: $x^2 + (\alpha+\beta)x + \alpha\beta = 0$

35. Discriminant: $D = b^2 - 4ac$

36. Conjugate pairs: If one root of the equation is $m + \sqrt{n}$ then other is $m - \sqrt{n}$

37. form of Simple Equation (One Variable). $ax + b = 0$

38. form of Simultaneous Linear Equations
$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$

39. Cross Multiplication: $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - a_1c_2} = \frac{z}{a_1b_2 - a_2b_1}$

40. form of Cubic Equation, $ax^3 + bx^2 + cx + d = 0$

41. Simple Interest: $SI = \frac{Prt}{100}$
where P = pricipal, r = rate of interest, t = time

42. Amount under S.I. :

$$A = P + S.I = P + \frac{P.r.t}{100} = P\left(1 + \frac{rt}{100}\right)$$

43. Number of Conversion Period:

• Compounded daily	365
• Compounded monthly	12
• Compounded quarterly	4
• Compounded semi-annually	2
• Compounded annually	1

44. Amount Under C.I. (A) $= P(1+i)^n$

Where, $i = \frac{r\%}{\text{no. of C.Period}}$, $n = t \times$ no. of C.P

45. Calculator trick for C.I. ka amount (A)

$P + i\% + i\% \ldots$ n times

46. $C.I. = A - P = P[(1+i)^n - 1]$

47. $E.R.I. = [(1+i)^n - 1] \times 100$

48. f.V. of single C.f. : $f.V. = C.f. \times (1+i)^n$

49
~~48~~. F.V. Annuity Regular :

$$FVAR = A\left[\frac{(1+i)^n - 1}{i}\right]$$

50.
~~49~~. f.V. Annuity due :

$$FVAD = A\left[\frac{(1+i)^n - 1}{i}\right] \times (1+i)$$

51. P.V. of single C.f. : $PV = \frac{Cf}{(1+i)^n}$

52. Compounding factor = $(1+i)^n$

Discounting factor = $\frac{1}{(1+i)^n}$

53. P.V. Annuity Regular :

$$PVAR = A\left[\frac{(1+i)^n - 1}{(1+i)^n \times i}\right]$$

54. Calculator trick

$(1+i)$ x' = = '... n times, M+ , -1 = ÷ i ÷ MRC = x A =

55. P.V. Annuity due :

$$PVAD = A\left[\frac{(1+i)^{n-1} - 1}{i\,(1+i)^{n-1}}\right] + A$$

56. P.V. of Perpetuity : $PVP = \frac{A}{i}$

57. P.V. of Growing Perpetuity : $PVGP = \frac{A}{i-g}$ → growth rate

58. Net P.V. :

NPV = P.V. Cash Inflow − P.V. Cash Outflow

59. Real Rate of Return (RRR) = NRR − Rate of inflation

60 CAGR = Annual rate used in C.I.

61. Multiplication (And) Addition (OR) Rule :

If one thing can be done in "m" ways and another thing can be done in "n" ways then

- No. of ways <u>both</u> things together = $(m \times n)$ ways
- No. of ways <u>either</u> of the thing = $(m+n)$ ways.

62. factorial : $n! = n(n-1)(n-2)\ldots\ldots 3.2.1$

63. Special formula in factorial :

- $n! = n(n-1)(n-2)! = n(n-1)!$
- $0! = 1$

64. Permutation Theorem:

$${}^nP_r = \frac{n!}{(n-r)!}$$, when r object are chosen out of n different objects.

trick: जितनी r की value होगी उतना reverse में x

Ex. ${}^5P_2 = 5 \times 4 = 20$

65. Permutation of all object:

$${}^nP_n = n!$$

66. Special formula: $(n+1)! - n! = n \times n!$

67. Number of Circular Permutation when all objects are chosen out of n diff. objects.

$$(n-1)!$$

68. No. of C. Per. when all objects are chosen out of n objects such that no two person have same two neighbors

$$\frac{(n-1)!}{2}$$

69. Permutation with Restrictions (Theorem.1)

- When a particular object is not taken $\boxed{{}^{n-1}P_r}$

70. Permutation with Restrictions (Theorem.2)

- When a particular object is always included $\boxed{r.\ {}^{n-1}P_{r-1}}$

71. ${}^{n-1}P_r$ (one always included) + $r \cdot {}^{n-1}P_{r-1}$ (One always excluded)

$= {}^{n}P_r$ (total)

72. No. of ways when objects are never together =

total − No. of ways when objects are always together

73. Combination:

$${}^{n}C_r = \frac{n!}{r!(n-r)!}$$

74. Linkage of P & C theorem:

$${}^{n}C_r = \frac{{}^{n}P_r}{r!}$$

75. Standard results:

• ${}^{n}C_0 = 1$, • ${}^{n}C_n = 1$

76. Complimentary Combination:

$${}^{n}C_r = {}^{n}C_{n-r}$$

77. Special formula: ${}^{n+1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$

78. Combination of one or more out of n things
[when there are two choices]

$$2^n - 1$$

79. Formulas in Geometry using Combination:

- No. of straight lines with n points = ${}^nC_2$
- No. of triangles with n points = ${}^nC_3$
- No. of triangles with n points where m points are collineare = ${}^nC_3 - {}^mC_3$
- No. of parallelogram with two sets of m and n parallel lines = ${}^mC_2 \times {}^nC_2$
- No. of diagonals out of n lines of a polygon = ${}^nC_2 - n$

80. Common difference in AP:

$$d = t_2 - t_1 = t_3 - t_2 = \ldots = t_n - t_{n-1}$$

81. General term of an AP:

$$t_n = a + (n-1)d$$

82. Calculator trick of General term of an AP:

$$a \pm d = = \ldots = =$$

[1st equal press will gives 2nd term)

83. Sum of first n terms of an AP:

$$S_n = \frac{n}{2}\left[a + l\right] \quad \text{or} \quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

84. Calculator trick for Sum of n terms:

$$a \pm d = = \ldots = GT + a$$

85. Sum of 1st n natural number: $\frac{n(n+1)}{2}$

86. Sum of 1st n odd number: n^2

87. Sum of 1st square of n natural no.: $\frac{n(n+1)(2n+1)}{6}$

88. Sum of cube of 1st n natural no.: $\left[\frac{n(n+1)}{2}\right]^2$

89. Common Ratio of G.P.

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_n}{t_{n-1}}$$

90. General term of GP:

$$t_n = ar^{n-1}$$

91. Calculator trick for General term of GP:

$$r \times a = = \ldots = n \text{ times.}$$

92. Sum of first n terms of GP:

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

93. Calculator trick for Sum of terms of GP:

$$r \times a = = \ldots = GT + a$$

94. Sum of infinite GP:

$$S_\infty = \frac{a}{1-r}$$

95.
- No. of subsets of a set containing n elements: 2^n
- No. of proper subset of set containing n element: $2^n - 1$

96. De Morgan's law:
- $(P \cup Q)' = P' \cap Q'$
- $(P \cap Q)' = P' \cup Q'$

97. 2 Sets Operation formula:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

98. 3 Sets Operation formula:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

99. Composite function

$$fog = fog(x) = f[g(x)]$$

$$gof = gof(x) = g[f(x)]$$