MATHEMATICS FORMULA SHEET

CA Nishant Kumar

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Chapter 1 – Ratio, Proportion, Indices, Logarithms

Topic 1 – Ratio

1. Ratio exists only between quantities of same kind.
2. Quantities to be compared must be in the same units.
3. If a quantity increases or decreases in the ratio $a : b$, then new quantity = b of the original quantity/a.

4. **Inverse Ratio –** The inverse ratio of a/b is b/a.

5. **Compound Ratio –** The multiplication of two or more ratios is called compound ratio. The compound ratio of $a : b$ and $c : d$ is $ac : bd$.

6. **Duplicate Ratio –** A ratio compounded of itself is called a Duplicate Ratio. The duplicate ratio of $a : b$ is $a^2 : b^2$.

7. **Sub-Duplicate Ratio –** The sub-duplicate ratio of $a : b$ is $\sqrt{a} : \sqrt{b}$.

8. **Triplicate Ratio –** The triplicate ratio of $a : b$ is $a^3 : b^3$.

9. **Sub-Triplicate Ratio –** The sub-triplicate ratio of $a : b$ is $\sqrt[3]{a} : \sqrt[3]{b}$.

Topic 2 – Proportion

1. Cross Product Rule: If $\frac{a}{b}=\frac{c}{d}$, then $ad=bc$.
2. Invertendo: If $\frac{a}{b}=\frac{c}{d}$, then $\frac{b}{a}=\frac{d}{c}$.
3. Alternendo: If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a}{c}=\frac{b}{d}$, or, $\frac{d}{b}=\frac{c}{a}$
4. Componendo: If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{b}=\frac{c+d}{d}$.
5. Dividendo: If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a-b}{b}=\frac{c-d}{d}$

6. Componendo and Dividendo: If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$.

7. Addendo: If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots$, then each of these ratios is equal to $\frac{a+c+e+\ldots}{b+d+f+\ldots}$, i.e.,

$$\frac{a}{b}=\frac{a+c+e+\ldots}{b+d+f+\ldots};\ \frac{c}{d}=\frac{a+c+e+\ldots}{b+d+f+\ldots};$$

8. Subtrahendo: If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots$, then each of these ratios is equal to $\frac{a-c-e-\ldots}{b-d-f-\ldots}$,

i.e., $\frac{a}{b}=\frac{a-c-e-\ldots}{b-d-f-\ldots};\ \frac{c}{d}=\frac{a-c-e-\ldots}{b-d-f-\ldots};\ \frac{e}{f}=\frac{a-c-e-\ldots}{b-d-f-\ldots}$

Topic 3 – Indices

1. $a^n = a \times a \times a \times a \times \ldots \times a$ (n times)
2. $a^{-n} = \frac{1}{a^n}$
3. $a^0 = 1$
4. $a^m \times a^n = a^{m+n}$
5. $\frac{a^m}{a^n} = a^{m-n}$
6. $(a^m)^n = a^{mn} = (a^n)^m$
7. $(ab)^n = a^n b^n$; or, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

8. $a^{m/n} = \left(a^m\right)^{1/n}$, i.e., $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Topic 4 – Logarithms

1. $2^3 = 8$ is expressed in terms of Logarithms as $\log_2 8 = 3$. It is read as log 8 to the base 2 is 3.
2. $\log_a 1 = 0$; $\log_a a = 1$
3. $\log_a (mn) = \log_a m + \log_a n$
4. $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$
5. $\log_a (m^n) = n \log_a m$
6. $\log_a m = \frac{\log_b m}{\log_b a}$

7. $\frac{1}{\log_a m} = \log_m a$

8. $a^{\log_a n} = n$

9. $\log_{a^q} n^p = \frac{p}{q} \log_a n$

Chapter 2 – Equations

1. Quadratic Formula $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$

3. $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

4. Sum of Roots $(\alpha + \beta) = -\dfrac{b}{a}$

5. Product of Roots $\alpha\beta = \dfrac{c}{a}$

6. If α and β are the roots of the equation, the equation is given by: $x^2-(\alpha+\beta)x+\alpha\beta=0$

7. $(a+b)^2=a^2+b^2+2ab$

8. $(a-b)^2=a^2+b^2-2ab$

9. $a^2-b^2=(a+b)(a-b)$

10. $(a+b)^3=a^3+b^3+3ab+3(a+b)$

11. $(a-b)^3=a^3-b^3-3ab-3(a-b)$

12. $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

13. If $b^2-4ac=0$, the roots are real and equal.

14. If $b^2-4ac>0$, the roots are real and unequal.

a. If $b^2 - 4ac$ is a perfect square, the roots are real, rational, and unequal.

b. If $b^2 - 4ac$ is not a perfect square, the roots are real, irrational, and unequal.

15. If $b^2 - 4ac < 0$, the roots are imaginary and unequal.

16. Irrational roots occur in conjugate pairs, i.e., if $\left(m+\sqrt{n}\right)$ is a root, then $\left(m-\sqrt{n}\right)$ is the other root of the same equation.

17. If one root is reciprocal to the other root, then their product is 1 and so $\frac{c}{a}=1$, i.e. $c=a$.

18. If one root is equal to the other root but opposite in sign, then their sum = 0, i.e. $-\frac{b}{a}=0 \Rightarrow b=0$.

Chapter 4 – Mathematics for Finance

Topic 1 – Simple Interest

1. $I = Pit$
2. $A = P(1 + it)$
3. $i = \dfrac{A - P}{Pt}$
4. $t = \dfrac{A - P}{Pi}$

Topic 2 – Compound Interest

1. $A = P\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}$

2. $CI = P\left[\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY} - 1\right]$

3. Difference between Compound Interest and Simple Interest

$$CI - SI = P\left[\left\{(1+i)^{t} - 1\right\} - it\right]$$

4. Effective Rate of Interest $E = \left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY} - 1$

Topic 3 – Annuity

1. Future Value of Annuity Regular $FV = A\left[\dfrac{\left(1+\dfrac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\dfrac{i}{NOCPPY}}\right]$

2. Future Value of Annuity Due

$$FV = \left[A\left\{\frac{\left(1+\dfrac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\dfrac{i}{NOCPPY}}\right\}\right]\times\left(1+\frac{i}{NOCPPY}\right)$$

3. Present Value of Annuity Regular $PV = A \times \left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t \times NOCPPY} - 1}{\frac{i}{NOCPPY} \times \left(1+\frac{i}{NOCPPY}\right)^{t \times NOCPPY}} \right]$

4. Present Value of Annuity Due = *Initial Cash Payment/Receipt + P.V. of Annuity Regular (for n – 1 periods)*

Topic 4 – Perpetuity

1. Present Value of Perpetuity = $\frac{A}{i / NOCPPY}$

2. Present Value of Growing Perpetuity = $\frac{A}{i - g}$

Topic 5 – Miscellaneous Topics

1. Nominal Rate of Return = Real Rate of Return + Inflation Rate
2. Compound Annual Growth Rate = Formula of Amount in Compound Interest

Chapter 5 – Permutations and Combinations

1. The number of arrangements of n items in a straight line is given by $n!$.

2. Formula for selecting r items out of n items = $\frac{n!}{r!(n-r)!}$.

3. Formula for arranging r items out of n items = $\frac{n!}{(n-r)!}$

4. Obvious Relationship between ${}^nC_r$ and ${}^nP_r \rightarrow {}^nP_r = {}^nC_r \times r!$

5. The number of arrangements of n items in a circle is given by $(n-1)!$.

6. The number of necklaces formed with n beads of different colours is $\frac{1}{2}(n-1)!$.

7. Number of ways of selecting some or all items from a set of n items –

   a. When there are 2 choices for each item: $\left(2^n-1\right)$.

   b. When there are 3 choices for each item: $\left(3^n-1\right)$.

8. ${}^{n+1}C_r={}^nC_r+{}^nC_{r-1}$

9. $\frac{{}^nC_r}{{}^nC_{r+1}}=\frac{r+1}{n-r}$; $\frac{{}^nC_{r-1}}{{}^nC_r}=\frac{r}{n-r+1}$

10. If ${}^nC_x={}^nC_y$, and $x\neq y$, then $x+y=n$.

11. If ${}^nP_x={}^nP_y$, and $x\neq y$, then $x+y=2n-1$.

12. The number of diagonals in a polygon of n sides is $\frac{1}{2}n(n-3)$.

13. Division of Items in Groups –

a. Division of Distinct Items in Groups –

i. Equal items in every group – The number of ways to divide n students into k groups of h students each is given by $\frac{n!}{k!(h!)^k}$.

ii. Unequal items in every group – The number of ways to divide n items into 3 groups → one containing a items, the second containing b items, and the third containing c items, such that $a+b+c=n$, is given by $\frac{n!}{a!b!c!}$.

b. Division of Identical Items in Groups – The number of ways to divide n identical objects into k groups of h items each is given by $\frac{n!}{(h!)^k}$.

14. Number of Factors of a number – Factors of a number N refers to all the numbers which divide N completely.

Step 1 – Express the number N in the form of $N = p^a.q^b.r^c$, where p, q, and r are the prime factors of the number N.

Step 2 – Use the formula: Number of factors of $N = (a+1)(b+1)(c+1)$.

15. The maximum number of points of intersection of n circles will be $n(n-1)$.

Chapter 6 – Sequence and Series

Topic 1 – Arithmetic Progression

1. $t_n = a + (n-1)d$
2. $n = \frac{l-a}{d} + 1$
3. Sum of first *n* terms of the series: $S_n = \frac{n}{2} \times \{2a + (n-1)d\}$
4. Sum of the series when first and last terms are known: $S_n = \frac{n}{2} \times (a + l)$

Topic 2 – Geometric Progression

1. $t_n = ar^{n-1}$

2. Sum of first n terms of the series when $r > 1$: $S_n = a\left(\dfrac{r^n - 1}{r - 1}\right)$

3. Sum of first n terms of the series when $r < 1$: $S_n = a\left(\dfrac{1 - r^n}{1 - r}\right)$

4. Sum of infinite series (provided $r < 1$): $S_\infty = \dfrac{a}{1 - r}$

Topic 3 – Special Series

1. Sum of first n natural or counting numbers $(1+2+3+4+...+n)=\frac{n(n+1)}{2}$

2. Sum of first n odd numbers $\{1+3+5+...+(2n-1)\}=n^2$

3. Sum of the Squares of first n natural numbers

$$\left(1^2+2^2+3^2+4^2+...+n^2\right)=\frac{n(n+1)(2n+1)}{6}$$

4. Sum of the Cubes of first n natural numbers $\left(1^3+2^3+3^3+4^3+...+n^3\right)=\left\{\frac{n(n+1)}{2}\right\}^2$

5. Sum of the series such as: 1 + 11 + 111 + … to n terms, or 2 + 22 + 222 + … to n terms, or 3 + 33 + 333 + … to n terms, and so on: $\frac{Number}{9} \times \left\{ \frac{10\left(10^n - 1\right)}{9} - n \right\}$. For example:

a. $1 + 11 + 111 + \ldots$ to n terms $= \frac{1}{9} \times \left\{ \frac{10\left(10^n - 1\right)}{9} - n \right\}$

b. $2 + 22 + 222 + \ldots$ to n terms $= \frac{2}{9} \times \left\{ \frac{10\left(10^n - 1\right)}{9} - n \right\}$

c. $3 + 33 + 333 + \ldots$ to n terms $= \frac{3}{9} \times \left\{ \frac{10\left(10^n - 1\right)}{9} - n \right\}$

6. Sum of the series 0.1 + 0.11 + 0.111 + … to n terms = $\frac{1}{9} \times \left[n - \left\{ \frac{1-(0.1)^n}{9} \right\} \right]$.

Example: Calculate the sum of 0.7 + 0.77 + 0.777 + … to n terms.

Solution:

0.7 + 0.77 + 0.777 + … to n terms = 7 × (0.1 + 0.11 + 0.111 + … to n terms)

Therefore, 0.7 + 0.77 + 0.777 + … to n terms = $\frac{7}{9} \times \left[n - \left\{ \frac{1-(0.1)^n}{9} \right\} \right]$

Similarly, sum of series 0.2 + 0.22 + 0.222 + … to n terms = $\frac{2}{9} \times \left[n - \left\{ \frac{1-(0.1)^n}{9} \right\} \right]$

Sum of series 0.4 + 0.44 + 0.444 + … to n terms = $\frac{4}{9}\times\left[n-\left\{\frac{1-(0.1)^{n}}{9}\right\}\right]$.

Chapter 7 – Sets, Relations, and Functions

Topic 1 – Sets

1. Number of subsets of a set with n elements: 2^n
2. Number of proper subsets of a set with n elements: $2^n - 1$
3. $(A \cup B)' = A' \cap B'$
4. $(A \cap B)' = A' \cup B'$
5. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
6. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Topic 2 – Relations

1. Number of elements in a product set: $n(A \times B) = n(A) \times n(B)$
2. Total number of relations from Set A to Set B containing m and n elements respectively: 2^{mn}
3. A relation R on the set A is a reflexive relation if $(a, a) \in R$ for all $a \in A$.
4. A relation R on the set A is a symmetric relation if $(a, b) \in R \Rightarrow (b, a) \in R$.
5. A relation R on the set A is a symmetric relation if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

Topic 3 – Functions

1. Inverse of a Function

Step 1 –	Write the function in the form of an equation, substituting *y* in place of $f(x)$.
Step 2 –	Rearrange the terms so that *x* comes on the LHS.
Step 3 –	Substitute $f^{-1}(x)$ in place of *x*, and *x* in place of *y*.