TIME VALUE FOR MONEY

Why is Interest Paid?

- **Time Value of money** : Sum of Money received in future will have less value than at present.
- **Opportunity Cost** : Lending incurs opportunity cost due to the possibilities of alternative use.
- **Inflation** : Given amount of money buys fewer goods in future than it will now.
- **Liquidity Preference** : People prefer to have resources that are cash convertible.
- **Risk Factor** : Borrower may go bankrupt, Thus it is determinable factor for Rate of Interest.

Important Definitions.

- **Interest :** Cost of borrowing or the return on investment, expressed as a percentage.
- **Principal** : Initial amount of money borrowed or invested before interest or returns.
- **Rate of Interest** : It is the percentage at which money grows or the cost of borrowing.
- **Accumulated Amount** : The total sum of principal and interest after a specified period.

SIMPLE INTEREST (S.I)

- It is a fixed percentage of the principal amount, paid or earned over time without compounding.
- Directly Proportional to Principal Amount (P), Rate of Interest (i or R/100) and Time (T).

$SI = P\left(\frac{R}{100}\right)T$	$A. = P\left(1 + \frac{R}{100}\right)T$

Q. *Certain sum amounts to Rs. 15748 in 3 years at simple interest at r% p.a. The same sum amount to Rs. 16,510 at (r + 2)% pa. simple interest in the same time. What is r?*

Way of Approaching question Let the Principal be P

1st Case : P for 3 Years at r% with A = 15748

$15748 = P(1 + \frac{r}{100}).3$

2nd Case : P for 3 Years at (r+2)% with A = 16510

$16510 = P(1 + \frac{r+2}{100}).3$

Divide both the cases

$$\frac{15748 = P(1 + \frac{r}{100}).3}{16510 = P(1 + \frac{r+2}{100}).3} = \frac{15748 = (100 + r)}{16510 = (100 + r+2)}$$

Solve for r

COMPOUND INTEREST VS SIMPLE INTEREST

- Let Principal = 1000, Interest = 5% (or 0.05) & T = 2.
- Simple Interest Calculated
  - First year Interest = 1000 x 0.05 = 50
  - Second year Interest = 1000 x 0.05 = 50
- Compound Interest Calculated
  - First year Interest = 1000 x 0.05 = 50
    - Principal now becomes 1050
  - Second year Interest = 1050 x 0.05 = 52.5
- Total **Simple Interest = 100** whereas total **Compound Interest = 102.5.**

HOW TO CALCULATE COMPOUND INTEREST

The accumulated money is	$A = P(1 + i)^n$

$$i = \frac{\text{Annual Interest Rate}}{\text{Conversions Per Year (m)}} = \frac{R}{100.m}$$

$$\text{Interest} = A_n - P = P\left[(1+i)^n - 1\right]$$

Conversion Per year

n is total conversions, i.e. **T x conversions per year**

Conversion period	Conversion Per year (m)
• 1 Day	• 365 (Compounded daily)
• 1 Month	• 12 (Compounded Monthly)
• 3 Months	• 4 (Compounded Quarterly)
• 6 Months	• 2 (Compound Semi Annually)
• 1 Year	• 1 (Compounded Annually)

Q. *In what time will ₹4,000 amount to ₹4,410 at 10% per annum interest compounded half-yearly.*

Way of Approaching question Let the Time Period be T

We know, n = T x conversions per year = 1 x 2

$$4410 = 4000\left(1 + \frac{10}{100} \times \frac{1}{2}\right)^{T \times 2}$$

Accumulated money — Principal — Interest — Conversions per year

EFFECTIVE RATE OF INTEREST (E)

- If interest is compounded more than once a year, then effective rate of interest exceeds per annum interest rate.

Relation of Nominal & Effective interest	$I = P.E.T$
Other way of computing m = conversion per year	$E = (1+i)^m - 1$

Q. *The effective rate of return for 24% per annum convertible monthly is given as:*

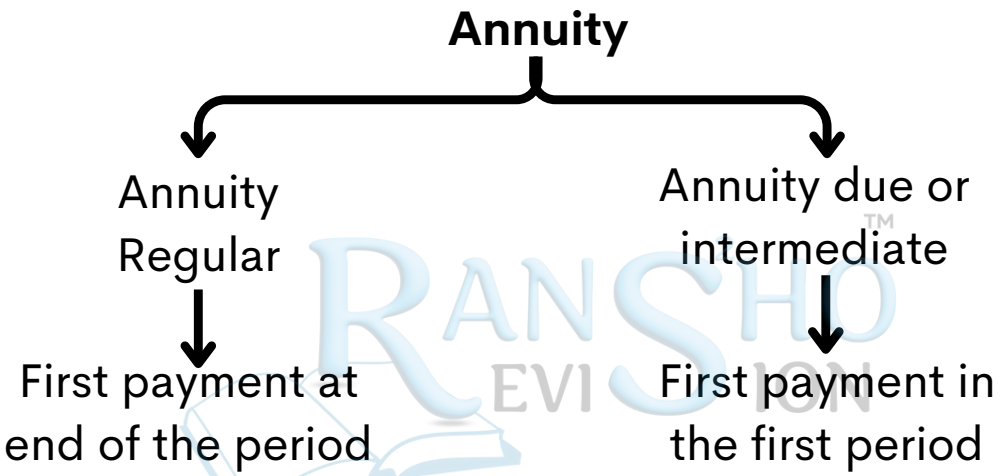
$$E = \left(1 + \frac{24}{100} \times \frac{1}{12}\right)^{12} - 1$$

Conversions per year — Conversions per year

ANNUITY

- When we pay (or receive) a fixed amount of money periodically over a specified period of time.
- When Payment takes place forever, it's **Perpetuity.**

- For a recurring payment to be Annuity,
  - Amount must be constant
  - Time interval of two payments must be same.



FUTURE VALUE

- Cash value of an investment in future

Future value & Single cashflow are related as	$F.V. = C.F.(1+i)^T$

T = Time Period ;C.F. = Cashflow ;F.V. = Future value

Q. You invest $10,000 at 5% for 2 years than what will be the Future value of money invested after 2 year.

$$F.V. = 10000\left(1 + \frac{5}{100}\right)^{2}$$

Cash Flow — Interest — Time Period

Way of Approaching question

FUTURE VALUE OF ANNUITY REGULAR

If A be the periodic payments, the future value A(n, i) of the annuity is given by

$$A(n, i) = A\left[\frac{(1+i)^{n} - 1}{i}\right]$$

*i should be used in decimals

n = no. of payments

Q. *$500 is invested at the end of each month in an account paying interest 12% per year compounded monthly. What is the future value after 9th payment?*

$$i = \frac{12}{100} \times \frac{1}{12} = 0.01 \qquad t = \frac{9}{12}\ \text{yrs} \qquad n = \frac{9}{12} \times 12 = 9$$

$$A(9, 0.01) = 500\left(\frac{(1+0.01)^{9} - 1}{0.01}\right)$$

FUTURE VALUE OF ANNUITY DUE OR INTERMEDIATE

Future value of annuity due or intermediate = Future value of annuity regular **x (i+1)**

Step-1 Calculate the future value as though it is an ordinary annuity.

Step-2 Multiply the result by (1+ i)

$$A(n,i) = A\left[\frac{(1+i)^n - 1}{i}\right] \times (i+1)$$

- *To distinguish between annuity regular and annuity due/intermediate, search the question for keywords.*
  - **Starting** of the year/month : Annuity **regular**
  - **End** of the year/month : Annuity **Intermediate**

PRESENT VALUE

- Value of future money in the present.

Present value Formula A_n = Amount due at end of n period at rate i.	$P.V. = \frac{A_n}{(1+i)^n}$

Q. *Find the present value of 5000 to be required after 4 years if the interest was 7%?*

$$P.V. = \frac{5000}{(1+0.07)^4}$$

PRESENT VALUE OF ANNUITY REGULAR

$$V = \frac{A(n,i)}{(i+1)^n} = A\left[\frac{(1+i)^n - 1}{i.(i+1)^n}\right] = A.P(n,i)$$

Q. The present value of an annuity of ₹80 a year for 20 years at 5% p.a is

Way of Approaching question A = ₹80; i = 0.05; n = 20

$$V = 80\left[\frac{(1+0.05)^{20} - 1}{0.05.(0.05+1)^{20}}\right]$$

PRESENT VALUE OF ANNUITY INTERMEDIATE/DUE

Step-1 Calculate the Present value of **n-1 period** of annuity regular

Step-2 Add the initial cash payment (A) to the step 1

Q. Your Papa gives you ₹10000 every year starting from today for next 5 years as a gift. So, you invest it at the interest rate of 10% in mutual funds today morning. What should be the present value of this annuity?

Way of Approaching question A = 10000; i = 0.15; n = 5

Step 1 : Present value for 4 Years i.e n–1

V = 10000 x P (4, 0.10) = 31698.70

Step 2 : Add one cash payment to above

V = 10000 + 31698.70 = 41698.70

Sinking Fund : Fund credited for a specific purpose

Sinking fund deposited is **A = P.A(n,i)**

Here, P = Periodic Payment and A = amount to be saved

Application : Leasing

- Financial arrangement under which the owner of the asset (lessor) allows the user of the asset (lessee) to use the asset for a defined period of time(lease period) for a consideration given period of time.

Easy Example : *A company has a machine worth 5 Lacs, it can lease out at 2 Lacs P.A. for 4 years, If company invests the rent at 14% P.A., is leasing favourable?*

Application : Capital Expenditure

- Capital expenditure means purchasing an asset today in result of benefits of tomorrow which would flow across the life of the investment.

Easy Example : *You buy a factory worth 10 Crore by borrowing money at 10% interest, if you generate a return of 3 Crore every year,* ***Will you be able to recover the cost in 4 years?***

To Check whether Buying an asset/Leasing is favourable or not, Check for the present value of the amount with given interest and periodic cash for fixed period of time

Application : Valuation of a bond

- A bond is a debt security in which the issuer owes the holder a debt and is obliged to repay the principal and interest. Bonds are generally issue

To Calculate present value of bond,

$$\frac{I}{(1+i)}+\frac{I}{(1+i)^2}+\frac{I}{(1+i)^3}+\cdots\frac{I}{(1+i)^n}+\frac{B.I.}{(1+i)^n}$$

- Here,$\mathbf{I}$ = Interest amount provided by issuer,
- B.I. = Bond value
- i = Interest Percentage that investor requires

Perpetuity

- When you get annuity for unlimited amount of time

$$\mathrm{PVA}_{\infty}=\frac{\mathrm{A}}{i}$$

A=Particular amount received
PVA = Amount to be Paid to get A

Growing perpetuity means the periodic installment is increasing with fixed interest rate

$$\mathrm{PVA}_{\infty}=\frac{\mathrm{A}}{i-\mathrm{g}}$$

$$\mathrm{g}=\frac{\mathrm{G}}{\mathrm{m}\times 100}$$

Rate of Return

- Net Present Value (NPV)= Present value of cash inflow – Present value of cash outflow
- RULE : To make decision
  - If NPV > 0 Accept the Proposal
  - If NPV < 0 Reject the Proposal

- Easy Example : You invest 1,00,000/- in a machine and expect a return of 30000 in 1st year, 80000 in 2nd, 50000 in 3rd, **is it worth investing?**

Check for the present value of the all the returns with given interest and add them all to get NPV.

Compound Annual Growth Rate (CAGR)

To find the annualized gain of an investment over a given time period

$$\text{CAGR} = i = \left(\frac{\text{F. V.}}{\text{P. V.}}\right)^{\frac{1}{\text{n}}} - 1$$

In Other Words, $\text{CAGR}(t_0, t_n) = \left(\frac{V_{t_n}}{V_{t_0}}\right)^{\frac{1}{t_n - t_0}} - 1$

V_{t_n} = End Period	V_{t_0} = Beginning Period