Chapter 7
Sets, Function and Relation

1. Out of 20 members in a family, 11 like to take tea and 14 like coffee. Assume that each one likes at least one of the two drinks. Find how many like **both** coffee and tea [2006]

(a) 2 (b) 3 (c) 4 (d) 5

n (TUC) = n (T) + n (C) – n (T∩C) , 20 = 11 + 14 – n (T∩C), n (T∩C) = 5

2. In a group of 70 people, 45 speak Hindi, 33 speak English and 10 speak neither Hindi nor English. Find how many can speak **both** English as well as Hindi :[2007]

(a) 13 (b)19 (c) 18 (d) 28

n (EUH) = n (E) + n (H) – n (E∩H) , 60 = 45 + 33 – n(E∩H) , n (E∩H) = 45 + 33 – 60 = 18

3. Let R is the set of real numbers, such that the function f:R → R and g:R→R are defined by $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$. Find (fog)[2007]

(a) $4x^2 + 6x + 1$ (b) x^2+6x+1 (c) $4x^2 - 6x + 1$ (d) x^2-6x+1

(fog)x = f {g(x)}

$= f(2x-3) = (2x-3)^2 + 3(2x-3) + 1 = 4x^2 + 9 - 12x + 6x - 9 + 1 = 4x^2 - 6x + 1$

4. In a survey of 300 companies, the number of companies using: different media Newspapers (N), Radio (R) and Television (T) are as follows: n(N)=200, n(R)=100,n(T) = 40, n(N∩R)=50, (R∩T) 20, n(N∩T)=25 and n(N∩R∩T)=5

Find the numbers of companies using none of these media: [2007]

(a) 20 companies (b) 250 companies (c) 30 companies (d) 50 companies

n(NURUT)= n (N) + n (R) + n (T) – n (N∩R) – n (N∩T) – n (R∩T) + (N∩R∩T)

= 200 + 100 + 40 – 50 – 25 – 20 + 5 = 250

∴ Number of companies not using any media = n (S) – n (NURUT) = 300 – 250 = 50

5. If R is the set of real numbers such that the function f : R→R is defined by $f(x) = (x+1)^2$, then find (fof) [2007]

(a) $(x+1)^2+1$ (b) X^2+1 (c) $\{(x+1)^2+1\}^2$ (d) None

$(fof)x = f\{f(x)\} = f(x+1)^2 = \{(x+1)^2 + 1\}^2$

6.if f:R→R, f(x)=2x+7, then the inverse of f is ___ [2007]

(a) $F^{-1}(x) = (x-7)/2$ (B) $F^{-1}(x) = (x+7)/2$ (c) $f^{-1}(x) = (x-3)/2$ (d) None.

Let f(x) = y, y = 2x + 7 , $X = \frac{y-7}{2}$, $y = \frac{x-7}{2}$ i.e. $f^{-1}(x) = \frac{x-7}{2}$

7. In a town of 20,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then the number of families which buy A only is ___ [2007]

(a) 6600 (b) 6300 (c) 5600 (d) 600

Given n(A) = 40% ; n(B) = 20% ; n(C) = 10% n(A ∩B)=5% n(B∩C)=4% n(C ∩A)=4%, n(A∩B∩C)=2%
No. of families which buy only A = n(A) - n(A∩B) - n(A∩C)+n(A∩B∩C) = 40-5-4+2=33%
=20,000x 33% = 6600

8. Let f: R→R be such that $f(x) = 2^x$, then f(x +y) equals ____ [2007]
(a) fx) + f(y) (b) f(x). fy) (c) f(X)÷ f(Y) (d) None of these

Given $f(x) = 2^x$, so $f(y) = 2^y$
$f(x) = 2^x$, $f(x+y) = 2^{x+y} = 2^x \cdot 2^y = f(x). f(y)$

9. Out of total 150 students, 45 passed in Accounts, 30 in Economics and 50 in Maths, 30 in both Accounts and Maths, 32 in both Maths and Economics, 35 in both Accounts and Economics, 25 students passed in all the three subjects. Find the numbers who passed **atleast** in anyone of the subjects [2008]
(a) 63 (b) 53 (c) 73 (d) None.

n(AUMUE) = n(A) + n(M) + n(E) – n(A∩M) – n(A∩E) – n(M∩E) – n(A∩M∩E) = 45 + 50 + 30 – 30 – 35 – 32 + 25 n (AUMUE) = 53

10. If $f(x) = \frac{2+x}{2-x}$ then $f^{-1}(x)$ [2008]
(a)$\frac{2(x-1)}{x+1}$ (b)$\frac{2(x+1)}{x-1}$ (c)$\frac{x+1}{x-1}$ (d) $\frac{x-1}{x+1}$

Let f(x) = y
$\frac{2+x}{2-x} = y$, 2 + x = 2y – xy , X + xy = 2y – 2
X (1 + y) = 2 (y – 1), $X = \frac{2(y-1)}{(y+1)}$.
Therefore $f^{-1}(x) = \frac{2(x-1)}{(x+1)}$

11.If A={1,2,3,4) : B={2,4,6,8} f(1) = 2, f(2) = 4, f(3) = 6 and f(4) = 8 and f:A→B then f^{-1} is ____ [2008]
(a) {(2,1),(4,2),(6,3),(8,4)} (b) {1, 2), (2,4), (3,6), (4, 8)}
(c) {(1,4),(2,2),(3,6),(4,8)} (d) None of these

f^{-1}implies f: B →A. f^{-1} = {(2, 1), (4, 2), (6, 3), (8, 4)}

12. If $f(x) = x^2+x-1$ and 4f(x) = f(2x) then find 'x'[2008]
(a) 4/3 (b) 3/2 (c) -3/4 (d) None of these

4 f(x) = f (2x) , 4 $[x^2 + x - 1] = (2x)^2 + (2x) - 1$, $4x^2 + 4x - 4 = 4x^2 + 2x - 1$, 2x = 3 , X = 3/2

13. If A={p,q,r,s},B= {q,s t}, C= (m,q,n} Find C - (A∩B)[2008]
(a) {m,n} (b) {p,q} (c) {r,s} (d) {p,r}

A∩B = {q, s} , C – (A∩B) = {m, n}

14. X ={x,y,w,z} y={1,2,3,4} H= {(x, 1); (y, 2); (y, 3); (z,4); (x,4} [2009]

(a) H is a function from x to y (b) H is not a function from x to y
(c) H is a relation from y to x (d) None of these

H is not a function from x to y because x has 2 images 1 & 4 and z has also image 4. So, there is no unique image

15. Given the function f(x) = (2x+3), then the value of f(2x) - 2f(x) +3 will be ___ [2009]
(a) 3 (b)2 (c) 1 (d) 0

$f(2x) - 2f(x) + 3 = [2(2x) + 3] - [2(2x + 3)] + 3 = 4x + 3 - 4x - 6 + 3 = 4x - 4x + 6 - 6 = 0$

16. If f(x) = 2x + h then find f(x+h) - 2f(x) [2009]
(a) h-2x (b) 2x-h (c) 2x+h (d) None of these

$F(x + h) - 2f(x) = [2(x + h) + h] - [2(2x + h)] = 2x + 2h + h - 4x - 2h = -2x + h = h - 2x$

17. If A={x:x^2-3x +2=0] B={x:X^2+4x-12=0} Then, B-A is equal to ___ [2010]
(a) {-6} (b) {1} (c) {1.2} (d) {2,- 6}

$x^2-3x+2=0$, $x^2-2x-x+2=0$, x(x-2)-1(x-2)=0, (x-2) (x-1)=0, x =1,2
A= {1,2}
$x^2+4x-12=0$, $x^2+6x-2x-12=0$, x(x +6) -2 (x + 6) = 0, (x+6)(x-2)=0, x=-6,2
B={-6,2}
B-A={-6,2} - {1, 2} = {-6}

18. If F: A → R is a real valued function defined by f(x) = $\frac{1}{x}$, then A = ___ [2010]
(a)R (b)R-{1} (c) R-{0} (d)R-N

f(x) = $\frac{1}{x}$ is defined at all Real numbers except x=0
A=R - {0}

19. In the set N of all natural numbers the relation R defined by a R b 'if and only if, **a divide b**", then the relation R is ___ [2010]
(a) Partial order relation (b) Equivalence relation
(c) Symmetric relation (d) None of these

"a divides b" satisfies the above 3 relations as follows:
1. a/a – Correct ∴Reflexive
2. a/b and b/a – Not Correct ∴Not - symmetric
3. a/b, b/c ∴a/c – Correct ∴Transitive
a/b is not a symmetric function and hence, not an equivalence relation i.e. partial order relation

20. For any two sets A and B, A∩(A'UB)= ______, where A' represent the compliment of the set A [2010]
(a) A∩B (b) AUB (c) AUB (d) None of these

Take an example
Let U={0,1,2,3,4,5}
A={0,1,2,3}

B={2,3,4,5}
A'=U-A = {4,5}
A'UB={4,5}U{2,3,4,5} ={2,3,4,5}
A∩(A' UB) ={0,1,2,3}∩{2,3,4,5} = {2,3} =A∩B

21. If f:R →R, $f(x)=x+1$, g:R→R $g(x)=x^2+1$, then fog(-2) equals to ___ [2010]
(a) 6 (b) 5 (c) 2 (d) None

$fog = x^2+1+1 = x^2+2 = (-2)^2+2 = 6$

22. If A ⊂ B, then following is true [2010]
(a) A∩B=B (b) AUB=B (c) A∩B=A' (d) A∩B=ϕ

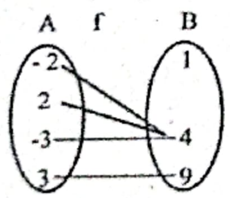
23. If $f(x-1)=x^2 - 4x + 8$, then $f(x+1)=$ ___ [2010]
(a) x^2+8 (b) x^2+7 (c) x^2+4 (d) x^2-4x

$f(x-1) = x^2 - 4x + 8 = (\mathbf{x-1+1})^2 - 4(\mathbf{x-1+1}) + 8$
$f(x+1) = (\mathbf{x+1+1})^2 - 4(\mathbf{x+1+1}) + 8$
$=(x+2)^2-4(x+2)+8 = x^2+4x+4-4x-8+8 = x^2+4$

24. There are 40 students, 30 of them passed in English, 25 of them passed in Maths and 15 of them passed in both. Assuming that every Student has passed atleast in one subject. How many student's passed in English only but not in Maths.
(a) 15 (b)20 (c) 20 (d) 10

n(E-M) = n(E) – n(E∩M) = 30 – 15 = 15

25.If A= {±2, ±3} B={1,4,9} and F={(2,4),(-2,4)(3,9) (-3,4) then 'f' is defined as:
(a) One to one function from A in to B
(b) One to one function from A onto B
(c) Many to one function from A onto B.
(d) Many to one function from A into 8.



26. If $f(x)=\frac{x}{\sqrt{1+x^2}}$ $g(x)=\frac{x}{\sqrt{1-x^2}}$. Find fog?[2011]
(a)x (b)$\frac{1}{x}$ (c) $\frac{x}{\sqrt{1-x^2}}$ (d)$x\sqrt{1-x^2}$

fog(x) = f{g(x)}
$$=\{\frac{x}{\sqrt{1+x^2}}\} = \frac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{1+(\frac{x}{\sqrt{1-x^2}})^2}} = \frac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{1+\frac{x^2}{1-x^2}}} = \frac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{\frac{1-x^2+x^2}{1-x^2}}} = \frac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{\frac{1}{1-x^2}}} = \frac{\frac{x}{\sqrt{1-x^2}}}{\frac{1}{\sqrt{1-x^2}}} = x$$

27. f(x) = 3+x, for -3 < x < 0 and 3-2x for 0 < x < 3, then value of f(2) will be ___ [2011]