1 Ratio, Proportion, Indices and Logarithm

1.0 RATIO

1.1 Meaning of Ratio: If a and b are two quantities of the same kind (in same units), then the fraction a/b is called the ratio of a to b. It is written as $a : b$. Thus, the ratio of a to b = a/b or $a : b$. The quantities a and b are called the terms of the ratio, a is called the **first term or antecedent** and b is called the **second term or consequent.**

For example, in the ratio 3 : 2, the numbers 3 and 2 are **terms** of the ratio. 3 is the **first term** or **antecedent** and 2 is the **second term** or **consequent**.

1.2 Features of Ratio

(i) Ratio is a fraction. It has no unit.

(ii) The quantities to be compared to form a ratio should be of the **same kind.** We cannot have a ratio between 3 m and 2 kg.

(iii) The quantities to be compared to form a ratio must be in the **same units**.
For example, ratio between 3 kg and 300 g = ratio between 3000 g and 300 g = 3000 : 300 = 10 : 1

(iv) Ratios are very similar to fractions. In fact a ratio is usually expressed in the form of 3 : 2, but it can also be expressed in fractional form as 3/2.

(v) **The order of the terms in a ratio is important.**

$x : y$ is equivalent to $\frac{x}{y}$ and $y : x$ is equivalent to $\frac{y}{x}$.

1.3 Ratio in the Simplest Form or in the Lowest Terms: A ratio $a : b$ is said to be in the simplest form if a and b has no factor in common. Thus 3 : 2 is in the simplest form while 20 : 25 is not in the simplest form.

To convert a ratio $x : y$ in the simplest form divide both x and y by the HCF of x and y.

1.4 Comparison of Ratios: A ratio $a : b$ is said to be greater than a ratio $c : d$ if $\frac{a}{b} > \frac{c}{d}$. or if $ad - bc > 0$.

1.5 Some Important Ratios

1. A ratio $a : b$ is said to be of greater inequality if $a > b$ and of less inequality if $a < b$.
Note: (i) If $a > b$ and some positive number is added to each term of $a : b$, then the ratio is diminished. (ii) If $a < b$ and some positive number is added to each term of $a : b$, then the ratio is increased.

2. The ratio compound of the two ratios $a : b$ and $c : d$ is $ac : bd$.
For example compound ratio of 3 : 4 and 5 : 7 is 15 : 28.
Compound ratio of 2 : 3, 5 : 7 and 4 : 9 is 40 : 189.

3. (i) $a^2 : b^2$ is called duplicate ratio of $a : b$.
   (ii) $a^3 : b^3$ is called triplicate ratio of $a : b$.
   (iii) $\sqrt{a} : \sqrt{b}$ is called sub-duplicate ratio of $a : b$.
   (iv) $a^{1/3} : b^{1/3}$ is called sub-triplicate ratio of $a : b$.
   For example, duplicate ratio of 2 : 3 is 4 : 9. Triplicate ratio of 2 : 3 is 8 : 27.
   sub duplicate ratio of 4 : 9 is $\sqrt{4} : \sqrt{9} = 2 : 3$.
   and sub triplicate ratio of 8 : 27 is $\sqrt[3]{8} : \sqrt[3]{27} = 2 : 3$.
4. One ratio is the inverse of another if their ratio compound is 1 : 1.
   Thus $a : b$ is the inverse of $b : a$ and vice-versa.
5. If the ratio of two similar quantities can be expressed as a ratio of two integers, the quantities are said to be commensurable; otherwise, they are said to be incommensurable.
   $\sqrt{3} : \sqrt{2}$ cannot be expressed as the ratio of two integers and therefore $\sqrt{3}$ and $\sqrt{2}$ are incommensurable quantities.
6. Continued ratio is the relation (or compassion) between the magnitudes of three or more quantities of the same kind. The continued ratio of three similar quantities a, b, c is written as $a : b : c$.
   For example: The continued ratio of ₹ 200, ₹ 400 and ₹ 600 is ₹ 200 : ₹ 400 : ₹ 600 = 1 : 2 : 3.

2.0 PROPORTION

2.1 Meaning of Proportion: The equality of two ratios is called proportion.

If $a : b = c : d$, we say that a, b, c, d are proportional and, we write $a : b : : c : d$.

Here a and d are known as extremes and b, c are known as means, d is called the called fourth proportional to a, b and c.

We always have: **Product of Means = Product of Extremes**

Tutorial Note: In a ratio $a : b$, both quantities must be of the same kind while in a proportional $a : b = c : d$, all the four quantities need not be of the same type. The first two quantities should be of the same kind and last two quantities should be of the same kind.

For example: ₹ 6 : ₹ 8 = 12 toffees : 16 toffees are in a proportion.

Here 1st two quantities are of same kind and last two are of same kind.

2.2 Continued Proportion: The quantities a, b, c, d, ... are said to be in continued proportion if

$$a : b = b : c = c : d = \dots$$

i.e. if
$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \dots$$

If a, b, c are in continued proportion then

$$a : b = b : c$$

i.e.
$$\frac{a}{b} = \frac{b}{c} \quad \text{or} \quad b^2 = ac.$$

then b is called mean proportional between a and c. Also c is called third proportional to a and b.

2.3 Some important relations derived from the definition of proportion which helps us in solving problems.

1. If $a : b :: c : d$, then $b : a = d : c$

For $$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$$

This operation is called invertendo.

2. If $a : b :: c : d$ then $a : c :: b : d$

For $$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$$

This operation is called alternendo.

3. If $a : b :: c : d$, then $a + b : b :: c + d : d$

i.e. if $$\frac{a}{b} = \frac{c}{d} \text{ then } \frac{a+b}{b} = \frac{c+d}{d}$$

For $$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} + 1 = \frac{c}{d} + 1 \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$$

This operation is called componendo.

4. If $a : b :: c : d$, then $a - b : b :: c - d : d$

i.e. if $$\frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a-b}{b} = \frac{c-d}{d}.$$

For $$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} - 1 = \frac{c}{d} - 1 \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$$

This operation is called dividendo.

5. If $a : b :: c : d$, then $a + b : a - b :: c + d : c - d$

i.e. if $$\frac{a}{b} = \frac{c}{d} \text{ then } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \text{(i) } \frac{a+b}{b} = \frac{c+d}{d} \text{ (by componendo)}$$

$$\text{(ii) } \frac{a-b}{b} = \frac{c-d}{d} \text{ (by dividendo)}$$

On dividing (i) by (ii)

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

This operation is called componendo and dividendo.

3.1 LAWS OF INDICES

(i) $a^m \times a^n = a^{m+n}$

(ii) $\dfrac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $(ab)^n = a^n \times b^n$

(v) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

(vi) $a^0 = 1$

3.2 SOME BASIC RESULTS

1. $(a + b) = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)^3 = a^3 + 3ab(a + b) + b^3$
4. $(a - b)^3 = a^3 - 3ab(a - b) - b^3$
5. $a^2 - b^2 = (a + b)(a - b)$
6. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
7. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
8. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

4.0 LOGARITHM

4.1 Meaning of Logarithm

If $a^x = n$, a and n are positive real number such that $a \neq 1$, then x is said to be the logarithm of the number n to the base 'a' symbolically it can be expressed as follows:

$$\log_a n = x$$

For Example

(i) $2^3 = 8 \Rightarrow \log_2 8 = 3$

ie logarithm of 8 to the base 2 is 3

(ii) $10^4 = 10000 \Rightarrow \log_{10} 10000 = 4$

ie logarithm of 1000 to the base 10 is 4

(iii) $3^{-4} = \frac{1}{81} \Rightarrow \log_3\left(\frac{1}{81}\right) = -4$

4.2 Types of Logarithm

(a) **Natural Logarithm.** Logarithm of numbers to base e are known as nature logarithm.

(b) **Common Logarithm.** Logarithms of numbers to the base 10 are known as common logarithm.

TUTORIAL NOTES

(i) Two equations $ax = n$ and $x = \log_a n$ are only transformation of each other and should be remembered to change one form of the relation into the other.

(ii) The logarithm of 1 to any base is zero. This is because any number raised to the power zero is one.

Since $a^0 = 1$, $\log_a 1 = 0$

(iii) The logarithm of any quantity to the same base is unity. This is because any quantity raised to the power 1 is that quantity only

Since $a^1 = a$, $\log_a a = 1$

4.3 Fundamental Laws of Logarithm

I. (i) $\log_a (mn) = (\log_a m) + (\log_a n)$.

(ii) $\log_a \left(\frac{m}{n}\right) = (\log_a m) - (\log_a n)$.

(iii) $\log_a (m^n) = n(\log_a m)$.

(iv) $\log_a m = \frac{1}{\log_m a}$.

(v) $\log_a m = \frac{\log_b m}{\log_b a}$.

II. If $\log_a N = x$, then

(i) $\log_{(1/a)} N = -x$

(ii) $\log_a \left(\frac{1}{N}\right) = -x,$

(iii) $\log_{\frac{1}{a}} \left(\frac{1}{N}\right) = x.$

4.4 Characteristic and Mantissa For Common Logarithms

The logarithm of a number consists of two pars, the whole part or the integral part is called the characteristic and the decimal part is called the mantissa where the former can be known by mere inspection, the latter has to be obtained from the logarithm tables.

(a) **Characteristic.** (The characteristic of the logarithm of any number greater than 1 is positive and is one less than the number of digits to the left of the decimal points in the given number). The characteristic of the logarithm of any number less than one (1) is negative and numerically one more than the number of zeros to the right of the decimal point. If there is no zero then obviously it will be –1. The following table will illustrate it.

Number	*Characteristic*	
4 8	1	One less than the
3 1 4	2	3 number of digits to
3.12	0	the left of the decimal point
.7	–1	One more than the
.9	–2	number of zeros on
.006012	–3	the right immediately
.0003210	–4	after the decimal point.

(b) **Mantissa**. The mantissa is the fractional part of the logarithm of a given number

Number	*Mantissa*	*Logarithm*
Log 1873	= (........... 2725)	= 3.2725
Log 187.3	= (........... 2725)	= 2.2725
Log 18.73	= (........... 2725)	= 1.2725
Log 1.873	= (........... 2725)	= 0.2725
Log .1873	= (........... 2725)	= $\bar{1}.2725$

Thus with the same figures there will be difference in the characteristic only. It should be remembered, that the mantissa is always a positive quantity. The other way to indicate this Log .001873 = – 3 + .2725 = $\bar{3}.2725$ = –(2.7275).

TUTORIAL NOTES

The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has

(i) Exactly one *i.e.* a unique solution if $a_1b_2 - a_2b_1 \neq 0$

(ii) If $a_1b_2 - a_2b_1 = 0$ *i.e.* if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, then system has

(a) infinites many solutions provided $c_1 = kc_2$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(b) no solution if $c_1 \neq kc_2$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

TUTORIAL NOTES

I. A system of two linear equations in x and y has

(i) a unique solution if the graph of the lines intersect at a point.

(ii) infinities many solution if the graph of both the lines coincide.

(iii) no solution if graph of the linear are parallel

II. (i) Verification is a must, if you are using the graphical method.

(ii) Try to locate the integer solutions so that you may plot the points in correct position.

5.0 QUADRATIC EQUATIONS

5.1 Meaning of Quadratic Equations

An equation of the form

$$ax^2 + bx + c = 0, \; a \neq 0$$

Where a, b, c are real numbers, is called a quadratic equation.

5.2 Roots of the equation

The values of x satisfying the equation $ax^2 + bx + c = 0$ are called its roots. These are given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

5.3 Discriminant

$D = b^2 - 4ac$ is called discriminant of the equation $ax^2 + bx + c = 0$

5.4 Nature of the Roots

The roots of the equation

$ax^2 + bx + c = 0, \; a \neq 0$ are

(i) real and equal if $D = 0$

(ii) real, unequal and rational if $D > 0$ and a perfect square

(iii) real, unequal and irrational if $D > 0$ and not a perfect square

(iv) imaginary if $D < 0$

(v) if $p+\sqrt{q}$ is a root, then $p-\sqrt{q}$ is also a root

(vi) if $p + iq$ is a root, then $p - iq$ is also a root ($i^2 = -1$)

5.5 RELATION BETWEEN ROOTS AND COEFFICIENTS

(i) If α, β are the roots of the equation $ax^2 + bx + c = 0$, $a \neq 0$ then

sum of roots $= \alpha + \beta = \frac{-b}{a}$

product of roots $= \alpha\beta = \frac{c}{a}$

(ii) An equation whose roots are α and β is given by

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

or $x^2 - (\text{sum})x + (\text{product}) = 0$

6.0 CUBIC EQUATIONS

6.1 Meaning of Cubic Equation

An equation of the form

$$ax^3 + bx^2 + cx + d = 0, \; a \neq 0,$$

where a, b, c, d are all real numbers, is called a cubic equation.

6.2 Relation Between Roots and Coefficients

If α, β, γ are the roots of the cubic equation

$$ax^3 + bx^3 + cx + d = 0, \; a \neq 0,$$

then

(i) $\alpha + \beta + \gamma = \dfrac{-b}{a}$

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{c}{a}$

(iii) $\alpha\beta\gamma = \dfrac{-d}{a}$

7.0 STRAIGHT LINES

7.1 Distance Between Two Points

The distance between any two points in the plane is the length of the line segment joining them. The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(\text{difference of } x \text{ coordinates})^2 + (\text{different of } y \text{ coordinates})^2}$$

If O is the origin and P(x, y) is any point, then

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}.$$

7.2 Section Formula

(a) Formula for Internal Division. The coordinates of a point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m : n$ are given by

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

(b) Formula for External Division. The coordinats of a point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) externally in the ratio $m : n$ are given by

$$x = \frac{mx_2 - nx_1}{m-n}, \quad y = \frac{my_2 - ny_1}{m-n}$$

TUTORIAL NOTES

In order to prove that a given figure is a

(i) **Square :** Prove that the four sides are equal and the diagonals are also equal.

(ii) **Rhombus (but not a Square) :** Prove that the four sides are equal but diagonels are not equal.

(iii) **Rectangle :** Prove that opposite sides are equal and diagonals are also equal.

(iv) **Parallelogram (but not Ractangle) :** Prove that the opposite sides are equal but diagonals are not equal.

Note that in each case diagonals bisect each other.

7.3 Area of a Triangle

The area of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by

$$\Delta = \frac{1}{2}[(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))].$$

7.4 Condition of Collinerarity of Three Points

Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear if all the three points lie on the same line ie if area of $\Delta ABC = 0$

ie if $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

OR

$AB + BC = AC$ or $AC + BC = AB$ or $AC + AB = BC$

8.0 GENERAL EQUATION OF A LINE

An equation of the form $ax + by + c = 0$, where a, b, c are any real numbers not all zero, always represents a straight line.

9.0 SLOPE OF A LINE

9.1 Slope of the line joining two points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

9.2 The slope of the line $ax + by + c = 0$ $(a \neq 0)$ is

$$m = -\frac{b}{a}$$

9.3 The slope of the line parallel to x-axis is 0 and perpendicular to x-axis is undefined

9.4 If three points A, B and C are collinear, then

Slope of AB = Slope of BC = Slope of CA

10.0 INTERCEPT OF A LINE ON THE AXES

Intercept of a line on x-axis. If a line cuts x-axis at A and y-axis at B, then OA and OB are known as intercepts of the line on x-axis and y-axis respectively.

The intercepts are positive or negative according as the line meets with positive or negative direction of the coordinate axes.

11.0 EQUATION OF LINES PARALLEL TO COORDINATE AXES

11.1 The Equation of x-axis is $y = 0$

11.2 The equation of y-axis is $x = 0$

11.3 The equation of a line parallel to y-axis at a distance of 'a' from it is $x = \pm a$

The sign is taken positive or negative according as the distance a is on positive or negative side of x-axis.

11.4 The equation of the line parallel to x-axis at a distance of 'b' from it is $y = \pm b$. The sign is taken positive or negative according as the distance b is on positive or negative side of y-axis.

12.0 EQUATION OF A LINE IN DIFFERENT FORMS

12.1 Slope-Interapt Form The equation of a line with slope m and making an intercept C on y-axis is $y = mx + C$.

12.2 Point-Slope Form The equation of a line passing through the point (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$

12.3 Two-Point Form The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

12.4 Intercept-Form The equation of a line which cuts off intercepts a and b on x-axis and y-axis respectively is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$

13.0 CONDITIONS FOR TWO LINES TO BE COINCIDENT, PARALLEL PERPENDICULAR OR INTERSECTING

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

(i) Coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(ii) Parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

(iii) Perpendicular if $a_1a_2 + b_1b_2 = 0$

(iv) Intersecting if they are neither coincident nor parallel ie if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ ie $a_1b_2 - a_2b_1 \neq 0$.

14.0 EQUATION OF A LINE PARALLEL TO A GIVEN LINE

The equation of a line parallel to a given line $ax + by + c = 0$ is $ax + by + \lambda = 0$, where λ is a constant.

Thus to write the equation of any line parallel to a given line we keep the expression containing x and y same and replace the constant term by λ which is determined by the given condition.

15.0 EQUATION OF A LINE PERPENDICULAR TO A GIVEN LINE

The equation of a line perpendicular to a given line $ax + by + c = 0$ is $bx - ay + \lambda = 0$, where λ is a constant.

Thus to write a line perpendicular to a given line we proceed as follows :

(i) Interchange the coefficients of x and y

(ii) Change the sign of one of the coefficients.

(iii) Replace the constant term by another constant λ which is determined by a given condition.

16.0 DISTANCE OF A POINT FROM A LINE

The length of perpendicular from the point (x_1, y_1) to the line $ax + by + c = 0$ is given by

$$p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

17.0 DISTANCE BETWEEN TWO PARALLEL LINES

To find distance between two parallel lines take an arbitrary point on one line and find the length of perpendicular from this point to the other line. To choose a point on one line give any value to x or y and find the values the other variable.

18.0 FAMILY OF LINES THROUGH THE INTERSECTION OF TWO GIVEN LINES

The equation of any line passing through the intersection of the lines

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0 \text{ is}$$

$$a_1x + b_1y + c_1 + \lambda(a_2x + b_2y + c_2) = 0$$

Where λ is a parameter

19.0 SOME IMPORTANT POINTS OF A TRIANGLE

19.1 Centroid The point of intersection of the medians of a triangle is called its centroid. If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of a triangle, then the coordinates of its centroid are

$$\left(\frac{ax_1 + bx_2 + cx_3}{3}, \frac{ay_1 + by_2 + cy_3}{3}\right)$$

19.2 Incentre The point of intersection of internal bisectors the angles of a triangle is called its incentre. If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle such that $BC = a$, $CA = b$ and $AB = c$, then the coordinates of its incentre is

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

ILLUSTRATION 24

Determine the ratio in which the line $x + 2y - 9 = 0$ divides the segment joining the points $(1, 3)$ and $(2, 5)$.

3 Matrices

MEANING OF MATRIX

A set of $m \times n$ numbers arranged in the form of rectangular array of m rows and n columns is called a $(m \times n)$ matrix (to be read as 'm' by 'n' matrix) and the whole array is denoted by A.

An $m \times n$ matrix is usually written as $A = [a_{ij}]_{m \times n}$ where a_{ij} denote ith row & jth column element.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

Notes:

1. The individual numbers in a matrix are called the elements.
2. A horizontal arrangement of the numbers in a matrix is called a row.
3. A vertical arrangement of the numbers in a matrix is called a column.
4. The number of rows and the number of columns in a matrix is called the dimensions of a matrix.
5. Any matrix that has the same number of rows as it has columns is called a square matrix.

Example: $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \\ 2 & 6 \end{bmatrix}$

In the above matrix,

Elements of Matrix = 2, 1, 3, 4, 2, 6

Elements in Row 1 : $a_{11} = 2$, $a_{12} = 1$

Row 2 : $a_{21} = 3$, $a_{22} = 4$

Row 3 : $a_{31} = 2$, $a_{32} = 6$,

Elements in Column 1 : $a_{11} = 2$, $a_{21} = 3$, $a_{31} = 2$

Column 2 : $a_{12} = 1$, $a_{22} = 4$, $a_{32} = 6$

m = Total No. of Rows in Matrix = 3

n = Total No of Columns in Matrix = 2

Order of Matrix = $m \times n = 3 \times 2$

TYPES OF MATRICES

1. **Row Matrix:** A matrix having only one row and n columns is called row matrix, i.e., $[a_{11}\ a_{12}\ a_{13} \ldots a_{1n}]_{1\times n}$ Example $[2\ 3\ 4\ 5]_{1\times 4}$

2. **Column Matrix:** A matrix having m rows and only one column is called column matrix.

   i.e. $\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ **Example:** $\begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}_{4\times 1}$

3. **Square Matrix:** A matrix having same numbers of rows and columns is called square matrix i.e. $m = n$, and is called of order n, i.e. $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

   Example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$, $\begin{bmatrix} 2 & 1 & 5 \\ 3 & 4 & 7 \\ 2 & 6 & 8 \end{bmatrix}_{3\times 3}$

4. **Upper Triangular Matrix:** A square matrix is said to be an upper triangular matrix if all elements below the main diagonal (from left to right) are zero, i.e. $\begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix}$ where, $a_{11}, a_{12}, a_{22} \neq 0$.

   Example: $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}_{2\times 2}$, $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix}_{3\times 3}$

5. **Lower Triangular Matrix:** A square matrix is said to be an lower triangular matrix if all elements above the main diagonal (from left to right) are zero, i.e. $\begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix}$ where, $a_{11}\ a_{21}, a_{22} \neq 0$.

   Example: $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}_{3\times 3}$

6. **Diagonal Matrix:** A square matrix is said to be diagonal matrix if all main diagonal (from left to right) elements are **non zero** and other elements are zero, i.e. $\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$ where, a_{11} & $a_{22} \neq 0$.

   Example: $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}_{2\times 2}$, $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix}_{3\times 3}$

7. **Scalar Matrix:** A square matrix is said to be **Scalar Matrix** if all main diagonal (from left to right) elements are **equal** and other elements are zero , i.e. $\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$ where,

$a_{11} = a_{22}$ but $\neq 0$.

Example: $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}_{2\times 2}, \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{3\times 3}$

Note: *All Scalar Matrices are Diagonal Matrices but all Diagonal Matrices need not be Scalar Matrices.*

8. **Identity or Unit Matrix:** A square matrix is said to be **Identity Matrix** if all main diagonal (from left to right) elements are **equal to 1** and other elements are zero. It is denoted by I_n, i.e. $\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$ *where,* $a_{11} = a_{21} = 1$.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Notes: *(i) All Identity or Unit Matrices are Diagonal Matrices but all Diagonal Matrices need not be Identity or Unit Matrices.*

(ii) All Identity or Unit Matrices are Scalar Matrices but all Scalar Matrices need not be Identity or Unit Matrices.

(iii) The determinant of the identity matrix is 1.

(iv) The product of two orthogonal matrices is itself an orthogonal matrix.

(v) The determinant of any orthogonal matrix is either +1 or −1.

9. **Zero or Null Matrix:** A matrix is said to be Null Matrix if all elements are zero and it is denoted by O. i.e.

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3} \qquad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$$

10. **Equality of Matrix:** Two matrices are said to be equal if they are of same order and their corresponding elements are equal.

Example: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$

11. **Addition or Subtraction of two matrices:** For addition/subtraction, matrices should be of the same order.

Properties of Matrix Addition

1. $A + B = B + A$ *(Commutative law)*
2. $(A + B) + C = A + (B + C)$ *(Associative law)*
3. $k(A + B) = KA + KB$ *(K is a Scalar)*
4. $A + (-A) = (-A) + A = 0$ *(Additive Inverse) [0 is Null Matrix]*
5. $A + 0 = 0 + A = A$ *(Additive Identity) [0 is Null Matrix]*

Addition: Addition is done by adding the element of 1st Matrix to the corresponding element of 2nd Matrix. The resulting matrix will be of the same order.

Example: $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$$A + B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}$$

Example: $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & 2 \\ -1 & 6 & 8 \end{bmatrix}$, thus, $A + B = \begin{bmatrix} 3 & 6 & 6 \\ 4 & 12 & 15 \end{bmatrix}$

Substraction: Substraction is done by deducting the element of 2nd matrix from the corresponding element of 1st matrix. The resulting matrix will be of the same order.

Example: $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, thus $A + B = \begin{bmatrix} a_{11}-b_{11} & a_{12}-b_{12} \\ a_{21}-b_{21} & a_{22}-b_{22} \end{bmatrix}$

Example: $A = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}, B = \begin{bmatrix} 2 & 6 \\ 8 & 2 \end{bmatrix}$, thus, $A - B = \begin{bmatrix} 1 & -1 \\ -1 & 7 \end{bmatrix}$

12. Scalar Multiplication of a Matrix: If a scalar quantity (say k) multiplied by a matrix A of order $m \times n$, then k is multiplied by each element of matrix A.

i.e. $kA = ka_{ij}$ where $A = a_{ij}$

Properties of Scalar Multiplication

1. $k(A + B) = kA + kB$
2. $(k_1 + k_2)A = k_1A + k_2A$
3. $(k_1 k_2)A = k_1(k_2A) = k_2(k_1A)$

If $A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ & $k = 2$

then $kA = 2A = 2\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 4 & 2 \end{bmatrix}$

13. **Multiplication of two matrices:** The Product AB of two matrices A and B is defined only if the number of columns in A is equal to the number of rows in B. If A is of order $m \times n$ and B is of order $n \times s$. Then order of AB is $m \times s$.

To multiply A with B, elements of the ith row of A are to be multiplied by corresponding elements of jth column of B and then their sum is taken

i.e., If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ & $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

Then $AB = \begin{matrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{matrix}$

Properties of Matrix Multiplication

1. $A(BC) = (AB)\,C$ *[Associative law] (Where $A_{m\times n}$ $B_{n\times s}$ & $C_{s\times t}$)*
2. $A(B + C) = AB + AC$ *(Distributive law)*
3. $AI = IA = A$ *(where A is square matrix and I is unit matrix of same order)*
4. $A \times O = O \times A = O$

   (where A is matrix or order $m \times n$ & O is matrix of order $n \times m$)
5. In general $AB \neq BA$ (Not Commutative)

   (Note: $AB = BA$ only where B is equal to Adjoint of A)
6. If $AB = 0$, does not imply that $A = 0$ or $B = 0$ or both $= 0$

3.1 In this case, principal keeps on changing at the end of each period of time. For instance, the interest earned in the first period of time is added to the (original) principal to calculate the interest for the second period and so on.

3.2 Conversion Period

The fixed interval of time (or period) at the end of which the interest is calculated and added to the principal at the beginning of the next period is called the conversion period.

Some popular conversion periods are given below:

Conversion period	*Description*	*Number of conversion period in a year*
1 day	Compounded daily	365
1 month	Compounded monthly	12
3 months	Compounded quarterly	4
6 months	Compounded semi annually	2
12 months	Compounded annually	1

3.3 Formula for Compound Interest

The accrued amount A_n on a principal P after n conversion periods at i (in decimal) rate of interest per conversion period is given by

$$A_n = P(1 + i)^n$$

where $$i = \frac{\text{Annual rate of interest}}{\text{number of conversion periods in a year}}$$

$$\text{Interest} = A_n - P = P(1 + i)^n - P = P\left\lfloor (1+i)^n - 1 \right\rfloor$$

3.4 Compound Amount at Changing Rate

In for a principal P, rate of interest is R_1% p.a. for the first year, R_2% for the second year, and so on, then amount due after nth years

$$= P(1 + R_1)\ (1 + R_2 \ldots (1 + R_n)$$

Also interest for n years $= P(1 + R_1)\ (1 + R_2) \ldots (1 + R_n) - P$.

ILLUSTRATION 13

Rate of interest is 12% p.a. Calculate the effective annual rate of interest if the compounding is done (a) Half yearly (b) quarterly (c) monthly.

SOLUTION Effective Rate if Interest = $(1 + r)^n - 1$

(a) $\left(1+\frac{12}{2}\right)^2 - 1 = (1.06)^2 - 1 = 12.36\%$

(b) $\left(1+\frac{12}{4}\right)^4 - 1 = (1.03)^4 - 1 = 12.55\%$

(c) $\left(1+\frac{12}{12}\right)^{12} - 1 = (1.01)^{12} - 1 = 12.68\%$

5.0 ANNUITY

5.1 Meaning of Annuity

5.1 Meaning of Annuity A series of payments, usually equal in size, made at equal intervals of times is called an **annuity.**

Monthly Rental payments; premiums of life insurance; deposits into a recurring account in a bank; equal monthly payments got by a retired government servant as pension and loan installments of houses or automobiles.

5.2 Some terms related with annuities

A series of payments, usually equal in size, made at equal intervals of times is called an **annuity.**

1. Periodic Payment	The size of each payment of an annuity is called the *periodic payment* (or *periodic rent*) of the annuity.
2. Annual Rent	The sum of all payments made in one year of an annuity is called is its *annual rent.*
3. Payment Period	The time between two successive payments of an annuity is called the *payment period* (or *payment interval*) of the annuity

4. Term	The total time from the beginning of the first payment period to the end of the last payment period is called the *term* of the annuity.
5. Amount of an Annuity	The total worth of all the payments at the conclusion of an annuity is called the *amount* (or *future value*) of the annuity.
6. Present Value of an Annuity	The sum of the present values of all the payments of an annuity is called the *present value* (or *capital value*) of the annuity.

5.3 Types of Annuity

(a) Ordinary Annuity: When all the periodic payments are made at the end of each payment period, it is called *regular annuity* (or *ordinary annuity*). In this case, first, second, third, ... payments respectively fall due at the end of first, second third, ... intervals.

(b) Annuity Due: When all the periodic payments are made at the beginning of each payment period, it is called an *annuity due*. In this case, first second, third, ... payments respectively fall due at the beginning of first, second, third, ... intervals.

(c) Deferred Annuity: An annuity which commences after a certain period is called a *deferred annuity*. In this case, the money is allowed to accumulate for a certain period of time equivalent to a fixed number of payment intervals and the payments begin after the lapse of that period.

6.0 AMOUNT OF AN ORDINARY ANNUITY

6.1 Meaning *The amount (or the future value) of an annuity* is the value at the end of the term, of all the payments, *i.e.*, it is the sum of the compound amounts of all the payments.

6.2 Formula The amount, S, of the annuity is given by

$$S = A(1 + i)^{n-1} + A(1 + i)^{n-2} + A(1 + i)^{n-3} + ... + A(1 + i) + A$$

$$= A + A(1 + i) + ... + A(1 + i)^{n-2} + A(1 + i)^{n-1}$$

[Writing the series in reverse order]

$$= A\left[\frac{(1+i)^n - 1}{(1+i) - 1}\right]$$ [∵ This is a G.P. with first term = A and common ratio = $1 + i$]

$$\Rightarrow \quad S = \frac{A}{i}[(1 + i)^n - 1]$$

This is the formula giving the amount of an ordinary annuity consisting of a n payments of ₹ A at the rate of i per period.

3.0 PERMUTATIONS

3.1 Permutation Means Arrangement

Each of the different arrangement that can be made with a given number of objects by taking all or some of the objects at a time without repetition of any object is called permutation. The number of permutations of n different objects taken $r(\leq n)$ at a time will be denoted by the symbol ${}^nP_r$ or $P(n, r)$.

3.2 Some Important Results

1. The number of permutations of n distinct objects taken r at a time, when repetition of objects is allowed, is $n^r (r > 0)$.
2. The number of permutations of n distinct objects taken $r (0 \leq r \leq n)$ at a time, when repetition is not allowed, is given by

$$ {}^nP_r = \frac{n!}{(n-r)!} = n(n-1)(n-2) \ldots (n-r+1) $$

Obviously,

(i) ${}^nP_r = 0$ for $r > n$.

(ii) $P(n, n) = \dfrac{n!}{(n-n)!} = \dfrac{n!}{0!} = n!$

(iii) $P(n, 1) = \dfrac{n!}{(n-1)!} = n.$

3. The number of permutations of n objects taken all together, when p_1 of the objects are alike and of one kind, p_2 of them are alike and of the second kind, ..., p_r of them are alike and of the rth kind, where $p_1 + p_2 + \ldots + p_r = n$ is given by

$$ \frac{n!}{p_1!\,p_2!\ldots p_r!} $$

4. Number of permutations of n distinct objects taken r $(0 \leq r \leq n)$ at a time, when a particular object is not taken in any arrangement is ${}^{n-1}P_r$.
5. Number of permuations of n distinct objects taken r $(0 \leq r \leq n)$ at a time, when a particular object is always included in any arrangement is $r.\ {}^{n-1}P_{r-1}$.

4.0 CIRCULAR PERMUTATIONS

4.1 The permutations of object along the circumference of a circle are called circular permutations.

4.2 In such permuations, there is neither a beginning nor an end.

4.3 Since the number of the circular permutations depends only on the relative positions of the objects, we fix the position of one object and then arrange the remaining $(n - 1)$ objects in all possible ways. This can be done in $(n - 1)!$ ways.

4.4 Hence the number of ways of arranging n distinct objects around a circle is $(n - 1)!$.

5.0 COMBINATIONS

5.1 Combination Mean Selection (or Forming Groups)

A group or a selection which can be formed by taking some or all of a number of objects irrespective of their arrangements is called a combination. Here we are concerned only with the number of things in each selection and not with the order of things.

The number of combinations (or selections) of a n different things taken r at a time ($r \leq n$) is denoted by the symbol ${}^nC_r$ or $C(n, r)$.

5.2 Some Important Results

1. ${}^nC_r = \dfrac{{}^nP_r}{r!} = \dfrac{1}{r!}\dfrac{n!}{(n-r)!}$
2. ${}^nC_0 = 1 = {}^nC_n$.
3. ${}^nC_r = {}^nC_{n-r}$ $(0 \leq r \leq n)$.
4. ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$.
5. ${}^nC_r = {}^nC_s$ implies $r = s$ or $r + s = n$.
6. ${}^nC_r = \dfrac{n-r+1}{r}\,{}^nC_{r-1}$ $(1 \leq r \leq n)$.
7. ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$.
8. ${}^nC_0 + {}^nC_2 + \ldots = {}^nC_1 + {}^nC_3 + \ldots = 2^{n-1}$.
9. ${}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n = 2^{2n}$.
10. The number of combinations of n distinct objects taken r $(\leq n)$ at a time, when $k(0 \leq k \leq r)$ particular objects always occur, is ${}^{n-k}C_{r-k}$.
11. The number of combinations of n distinct objects taken r at a time, when $k(1 \leq k \leq n-r)$ never occur, is ${}^{n-k}C_r$.

TUTORIAL NOTES

1. In the formula $S_n = \frac{n}{2}[2a+(n-1)d]$ four quantities are involved viz S_n, a, n and d. If any three of these are known, the fourth can be determined.
2. If the sum S_n of first n terms of a sequence is given, then the nth term a_n of the sequence can be determined by

$$a_n = S_n - S_{n-1}$$

3. If the sum of first n terms of a_n A.P. is of the form $An^2 + Bn$, where A and B are constant, then common difference is $2A$.

4.5 Properties of A.P.

1. If a constant is added or subtracted from each term of an A.P., then the resulting sequence is also in A.P. with same common difference.
2. If each term of an A.P. is multiplied or divided by a non-zero constant k, then the resulting sequence is also an A.P. with common difference kd or $\frac{d}{k}$, where d is the common difference of the given A.P.
3. If $a_1, a_2, a_3, \ldots$ and $b_1, b_2, b_3, \ldots$ are two arithmetic progessions, then the sequence $a_1 + b_1, a_2 + b_2, a_3 + b_3, \ldots$ is also an A.P.
4. In a finite A.P. the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of first and last term

   ie $a_k + a_{n-(k-1)} = a_1 + a_n$, $k = 1, 2, 3, \ldots n - 1$.
5. Three numbers a, b, c are in A.P. if $2b = a + c$.

4.6 Insertion of Arithmetic Means **Single Arithmetic Mean :** A number A is said to be the arithmetic mean between two given numbers a and b if a, A, b are in A.P. Therefore the A.M. between a and b is given by

$$A = \frac{a+b}{2}$$

For example since 1, 3, 5 are in A.P. therefore 3 is A.M. between 1 and 5. Also $3 = \frac{1+5}{2}$.

***n*-Arithmetic Means :** The numbers $A_1, A_2 \ldots A_n$ are said to be n arithmetic means between two given numbers a and b if $a, A_1, A_2 \ldots A_n, b$ are an A.P.

5.0 GEOMETRIC PROGRESSION (G.P.)

5.1 Meaning of Geometric Progression A sequence of non-zero numbers is called in geometric progression if the ratio of a term and the term preceding to it is always a constant quantity.

The constant ratio, generally denoted by r, is called common ratio.

For example the sequence 1, 2, 4, 8, ... is in G.P. whose first term is 1 and common ratio is 2.

5.2 General Term of G.P. If a is the first term and r is the common ratio of a G.P., then the terms of G.P. are

$$a, ar, ar^2, ar^3 .....$$

and the nth term t_n is given by

$$t_n = ar^{n-1}.$$

5.3 Selection of Terms in G.P.

Number of terms	*Terms*	*Common ratio*
3	$\frac{a}{r}, a, ar$	r
4	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$	r^2
5	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$	r

5.4 Properties of G.P.

1. If all the terms of a G.P. be multiplied or divided by the same non-zero number, then it remains a G.P. with the same common ratio.
2. The reciprocals of the terms of a G.P. form a G.P.
3. If each term of a G.P. be raised to the same power, then the resulting sequence also form a G.P.

4. If $a_1, a_2, a_3, ...$ and $b_1, b_2, b_3, ...$ are two geometric progressions then sequence $a_1b_1, a_2b_2, a_3b_3, ...$ is also a G.P.
5. If $a_1, a_2, a_3, ...$ is a G.P. ($a_i > 0$), then $\log a_1, \log a_2, \log a_3, ...$ is an A.P. and conversely.
6. The non-zero numbers a, b, c are in G.P. if $b^2 = ac$.

5.5 Sum of *n* Terms of a G.P. The sum of first n terms of a G.P. with first term a and common ratio r is given by

$$S_n = \begin{cases} \dfrac{a(r^n - 1)}{r - 1} & \text{if } r \neq 1 \\ na & \text{if } r = 1 \end{cases}$$

5.6 Sum of an Infinite G.P. The sum of an infinite G.P. with first term a and common ration r ($|r| < 1$) is

$$S_\infty = \frac{a}{1-r}$$

5.7 Geometric Mean

Single Geometric Mean : A number G is said to be the geometric between two non-zero numbers a and b if a, G, b are in G.P. ie if $G^2 = ab$.

For Example : Since 2, 4, 8 are in G.P., therefore 4 is G.M. at between 2 and 8.

***n*-Geometric Means :** The number, $G_1, G_2, ... G_n$ are said to be n-geometric means between two non-zero numbers a and b if $a, G_1, G_2, ... G_n, b$ are in G.P.

TUTORIAL NOTES

1. To insert n geometric means between two non-zero numbers a and b, take common ratio

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

Then geometric means are given by

$$G_1 = ar = a \cdot \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_2 = ar^2 = a \cdot \left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$G_3 = ar^3 = \vdots \; a \cdot \left(\frac{b}{a}\right)^{\frac{3}{n+1}}$$

HARMONIC PROGRESSION (H.P.)

A sequence of non-zero numbers $a_1, a_2, a_3, \ldots$ is said to be a harmonic progression if the sequence

$$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots\ldots \text{ in an A.P.}$$

For example the sequence $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ is a H.P. because the sequence 1, 3, 5, 7, ... is an A.P.

A general H.P. is

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$$

7.0 SOME SPECIAL SEQUENCES

1. The sum of the first n natural numbers

$$\Sigma n = 1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

2. The sum of the squares of first n natural numbers

$$\Sigma n^2 = 1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3. The sum of the cubes of the first n natural numbers

$$\Sigma n^3 = 1^3 + 2^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

4. If the nth term of a sequence is

$$T_n = An^3 + Bn^2 + Cn + D$$

Then the sum of n terms is given by

$$S_n = \Sigma T_n = \mathrm{A}\Sigma n^3 = \mathrm{B}\Sigma n^2 + \mathrm{C}\Sigma n + nD.$$

8A Sets

1.0 MEANING OF SET

A set is any well-defined list, collection or class of objects. By well-defined collection of objects, we mean that there is a rule (or rules) by means of which it is possible to say without ambiguity, whether or not a particular object belongs to the set. The objects or members of a set may be anything: numbers, people, letters, rivers, countries *etc.*

Sets are usually denoted by capital letters: $A, B, S, T, X, Y, \ldots$ and their elements by small letters $a, b, s, t, x, y, \ldots$ If an object a is a member or element of set A, we write $a \in A$. If a is not an element of set A, we write $a \notin A$.

Example: (i) The collection of all English Alphabets is a set.

(ii) The collection of all integers between –4 and 4 is a set.

(iii) The collection of all tall students in a school is not a set since the term 'tall' is not well defined.

2.0 METHODS OF WRITING A SET

There are two method of writing a set:

1. Roster Method (Tabular form)
2. Rule Method (Set builder form)

2.1 Roster Method

In 'Tabular Form' or 'Roaster method', the elements of a set are listed one by one within braces { } and one separated by each other by commas,

Example: (i) A = {Lucknow, Patna, Bhopal, Itanagar, Shillong}

(ii) $B = \{3, 4, 5, 6, 7, 8, 9, 10\}$

Note that the order in which the elements appear is immaterial. Two identical elements are considered to be one element.

2.2 Rule Method

In 'set-builder form' we write between the braces { } a variable x which stands for each of the elements of the set, then we state the properties possessed by x. We denote this property by $p(x)$ and separate x and $p(x)$ by a symbol: or 1 (read as 'such that')

$$A = \{x : p(x)\}$$

Example: (i) $A = \{x : x \text{ is Capital of States}\}$

(ii) $B = \{x : x \text{ is a natural number and } 2 < x < 11\}$

As described above, we can write the set of four fourth roots of unity in either of the following two ways:

$$\{1, -1, i, -i\} \text{ or } \{z : z^4 = 1\}.$$

3.0 TYPES OF SETS

3.1 Null Set (or Empty Set or Void Set)

A set having no element is called as an empty set or void set. It is denoted by ϕ or { }

Examples: $A = \{x : x \text{ is an even number not divisible by } 2\}$

$B = \{x : x \text{ is a real number and } x^2 = -1\}$

3.2 Singleton Set

A set having only one element is called a singleton set.

Examples: $A = \{x : x$ is present Prime Minister of India)

$B = \{2\}$

3.3 Pair Set

A set having two elements is called a pair set.

Example: $\{1, 2\}$, $\{0, 3\}$, $\{4, 9\}$ etc.

3.4 Finite Set

A set having a finite number of elements *i.e.* a set, where counting of elements is possible is called as a finite set.

Examples: $A = \{1, 2, 4, 6\}$ is a finite set because it has four elements.

B = a null set ϕ, is also a finite set because it has zero number of elements.

3.5 Infinite Set

A set having infinite number of elements *i.e.* a set where counting of elements is impossible, is called an infinite set

Examples: $A = \{x : x \text{ is a set of all natural numbers}\}$

B = Set of all points on the arc of a circle

3.6 Equal Sets

Two sets A and B are said to be equal, if every element of A is in B and every element of B is in A and we write, $A = B$. Otherwise, the sets are said to be *unequal* and we write $A \neq B$.

Examples: (i) The elements of a set may be listed in any order. Thus, $\{1, 2, 3\} = \{1, 3, 2\} = \{2, 3, 1\}$ etc.

(ii) The repetition of elements in a set is meaningless. Thus, $\{1, 2, 3, 2, 3, 1, 1, 2, 3, 3, 2, 1\} = \{1, 2, 3\}$.

(iii) $\phi \neq \{0\}$, since ϕ is a set containing no element at ail, $\{0\}$ is a singleton set.

3.7 Equivalent Sets

Two finite sets A and B are said to be *equivalent* if they have the same number of elements. We write $A \approx B$.

For example, let $A = \{a, b, c, d, e\}$ and $B = \{1, 3, 5, 7, 9\}$. Then A and B are equivalent sets.

Obviously, all equal sets are equivalent, but all equivalent sets are not equal.
The sets {1, 2, 3} and {*a*, *b*, *c*} are equivalent but not equal.

3.8 Sub Set

If every element of a set A is also an element of a set B, then A is called a *subset* of B or A is contained in B or B is a **superset** of A. We write it as $A \subseteq B$.
If at least one element of A does not belong to B, then A is not a subset of B. We write it as $A \not\subseteq B$.

Example:

(i) Let $A = \{4, 5, 6\}$ and $B = \{4, 5, 7, 8, 6\}$
Then $A \subset B$

(ii) Let $A = \{1, 2, 3\}$, and $B = \{3, 2, 9\}$
Then $A \not\subset B$.

3.9 Proper Subset

Set A is said to be a proper subset of a set B if

(a) every element of set A is an element of set B, and

(b) set B has at least one element which is not an element of set A, and we write $A \subset B$.

Example: {1, 2, 3} is a proper subset of {1, 2, 3, 4}.

3.10 Power Set

The set of all subsets of a given set A is called the power set of A, denoted by $P(A)$. If A has n elements, $P(A)$ has 2^n elements.
Example: The power set of $A = \{1, 2, 3\}$ is
$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.

3.11 Universal Set

Any set which is super set of all the sets under consideration is known as the universal set and is either denoted by Ω or S or $\cup$.
For example: Let $A = \{1, 2, 3\}$, $B = \{3, 4, 6, 9\}$, and $C = \{0, 1\}$
We can take

$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ as Universal Set.

4.0 SOME OPERATIONS ON SETS

4.1 Union of Sets

The union of two sets A and B is the set of all those elements which are either in A or in B or in both. This set is denoted by $A \cup B$ and read as 'A union B'.
Symbolically, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Examples:

1. If $A = \{a, b, c, d\}$ and $B = \{b, d, e, f, g\}$, then $A \cup B = \{a, b, c, d, e, f, g\}$
2. If A is the set of all positive integers and B is the set of all negative integers, then clearly $A \cup B$ is the set of all integers except zero.

4.2 Intersection of Sets

The intersection of two sets A and B is the set of all the elements, which are common in A and B. This set is denoted by $A \cap B$ and read as 'A intersection B'.

i.e. $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Examples:

1. If $A = \{b, d, e, f\}$ and $B = \{b, e, g, h\}$, then $A \cap B = \{b, e\}$
2. If $A = \{x : x \in N, x \text{ is a multiple of } 4\}$
   And $B = \{x : x \in N, x \text{ is a multiple of } 6\}$, *then*
   $A \cap B = \{x : x \in N, x \text{ is a multiple of } 12\}$
   Note that l.c.m. of 4 and 6 is 12.

4.3 Difference of Sets

The difference of sets A and B, in this order, is the set of elements which belong to A but not to B. Symbolically, we write $A - B$ and read as 'A difference B'.

Thus $\quad A - B = \{x : x \in A \text{ and } x \notin B\}$

Similarly $\quad B - A = \{x : x \in B \text{ and } x \notin A\}$

For example, $\quad$ if $A = \{4, 5, 6, 7, 8, 9\}$, and $B = \{3, 5, 2, 7\}$

then $\quad A - B = \{4, 6, 8, 9\}$ and $B - A = \{3, 2\}$.

In general, $\quad A - B \neq B - A$

4.4 Symmetric Difference of Sets

The symmetric difference of two sets A and B is defined by:

$A \Delta B = (A - B) \cup (B - A)$

For example: If $A = \{a, b, c, d\}$ and $B = \{b, d, e, f\}$, then

$A - B = \{a, c\}$ and $B - A = \{e, f\}$

$\therefore A \Delta B = (A - B) \cup (B - A) = [a, c, e, f]$

4.5 Complement of a Set

Let A be a subset of the universal set U, then the complement of A, denoted by A^C is defined by:

$A^C = \{x : x \in U, x \notin A\}$

Clearly, $x \in A^C \Leftrightarrow x \notin A$

Examples:

1. If $U = \{a, b, c, d, e, f\}$ and $A = \{b, d, e\}$, then $A^C = \{a, c, f\}$
2. If U is the set of all letters in English Alphabet and A is the set of all vowels, then A^C is the set of all consonants.

5.0 SOME RESULTS ON COMPLEMENTATION

1. $\phi^C = U$ $\quad$ 2. $U^c = \phi$ $\quad$ 3. $A \cup A^C = U$ $\quad$ 4. $A \cap A^C = \phi$

6.0 LAWS OF OPERATIONS

6.1 (i) $A \cup A = A$ $\quad$ (ii) $A \cup \phi = A$ $\quad$ (iii) $A \cap A = A$ $\quad$ (iv) $A \cap \phi = \phi$

6.2 Commutative Laws

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

6.3 Associative Laws

(i) $(A \cup B) \cup C = A \cup (B \cup C)$ (ii) $(A \cap B) \cap C = A \cap (B \cap C)$

6.4 Distributive Laws

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6.5 De-Morgan's Laws

(i) $(A \cup B)^C = A^C \cap B^C$ (ii) $(A \cap B)^C = A^C \cup B^C$

7.0 SOME MORE IMPORTANT RESULTS

7.1 $A \cup B = \phi \Leftrightarrow A = \phi$ and $B = \phi$

7.2 $A - B = \phi \Leftrightarrow A \subseteq B$

7.3 $A - B = A \cap B^C$

7.4 $A \subseteq B \Leftrightarrow B^C \subseteq A^C$

7.5 $A - (B \cup C) = (A - B) \cap (A - C)$

7.6 $A - (B \cap C) = (A - B) \cup (A - C)$

7.7 $A \cap (B - C) = (A \cap B) - (A \cap C)$

8.0 VENN DIAGRAM AND SOME OF THE APPLICATIONS OF SET THEORY

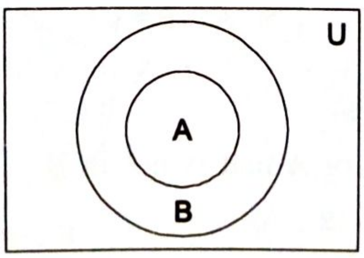
8.1 Venn Diagrams

We represent sets by closed bounded figures, known as Venn diagrams.
The universal set is denoted by a rectangular region and each subset of it is shown by a closed bounded figure within this rectangle.

8.2 Venn Diagrams in Different Situations

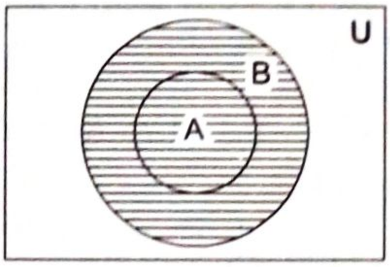
First of all we will represent the set or a statement regarding sets with the help of the diagram or Venn Diagram. The shaded area represents the set written.

(a) **Subset**

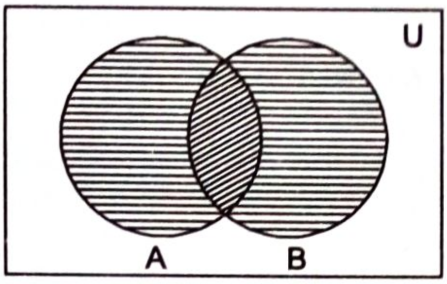


$A \subset B$

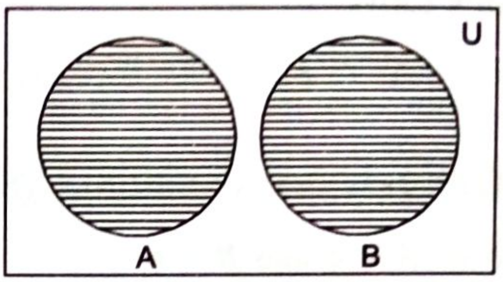
(b) **Union of Sets:** Let $A \cup B = B$. Here, whole area represented by B represents $A \cup B$.



A ∪ B, when A ⊂ B

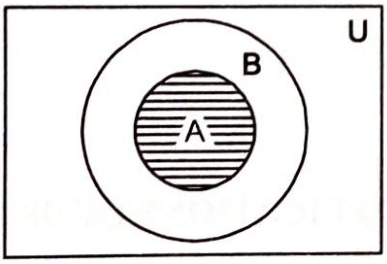


A ∪ B when neither A ⊂ B nor B ⊂ A

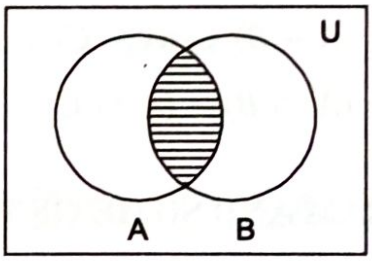


A ∪ B when A and B are disjoint events

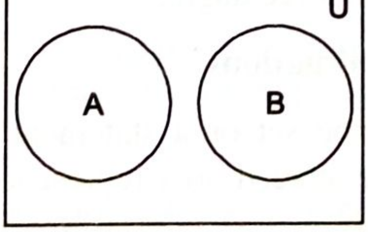
(c) **Intersection of Sets ($A \cap B$):** $A \cap B$ represents the common area of A and B.



A ∩ B when A ⊂ B and A ∩ B = A

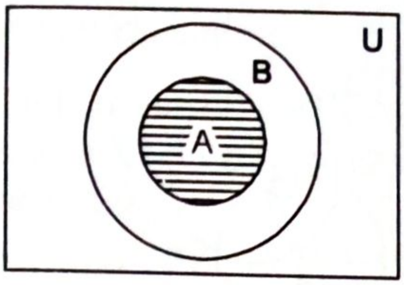


A ∩ B when neither A ⊂ B nor B ⊂ A

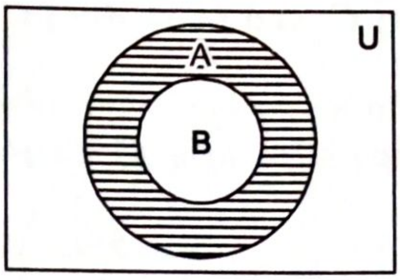


A ∩ B = ϕ

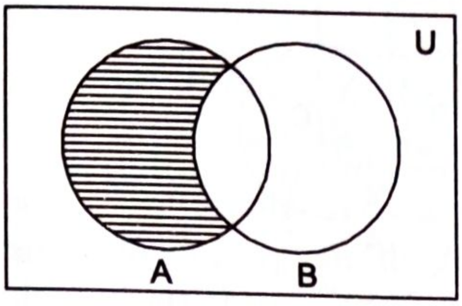
(d) **Difference of Sets:** ($A - B$).
$A - B$ represents the area of A that is not in B.



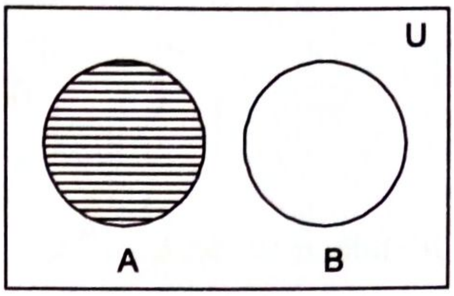
A – B, when A ⊂ B
(A – B = ϕ)



A – B, when B ⊂ A

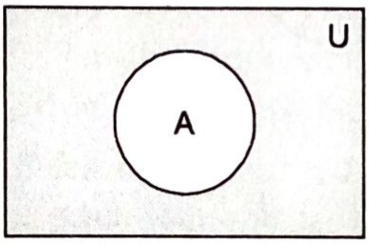


A – B when neither
A ⊂ B nor B ⊂ A



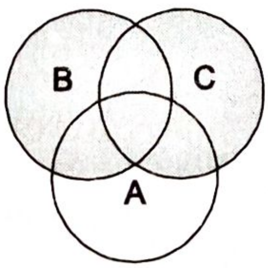
A – B, when A and B are
disjoint sets
(A – B ⊂ A)

(e) **Complement of Sets (A')**: A' or A^c is the set of those points of Universal Sets U which are not in A.

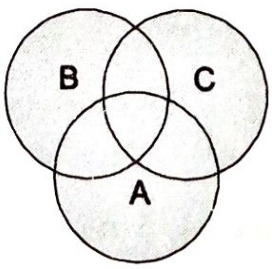


A′

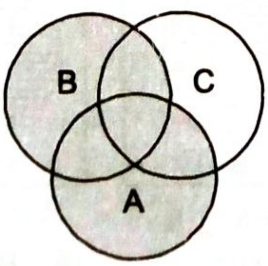
(f) $A \cup (B \cup C)$ **and** $(A \cup B) \cup C$



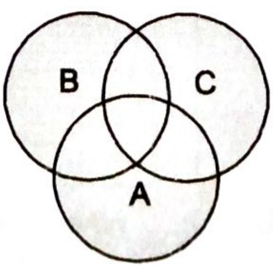
B ∪ C



A ∪ (B ∪ C)

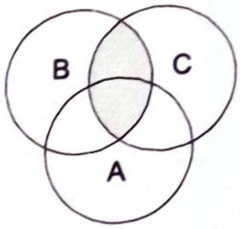


A ∪ B

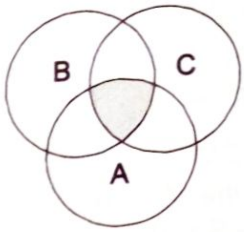


(A ∪ B) ∪ C

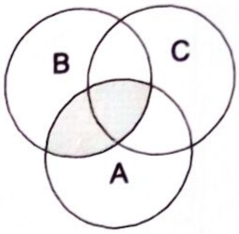
(g) $A \cap (B \cap C)$ and $(A \cap B) \cap C$



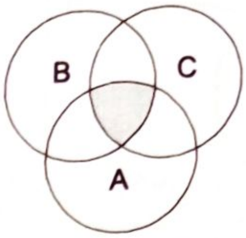
B ∩ C



A ∩ (B ∩C)

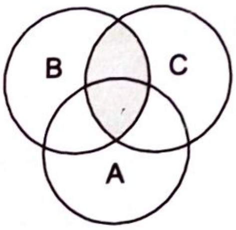


A ∩ B

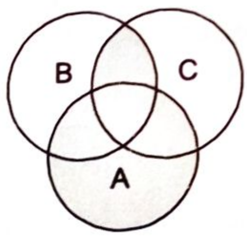


(A ∩ B) ∩C

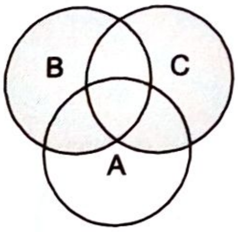
(h) $A \cup (B \cap C)$ and $A \cap (B \cup C)$



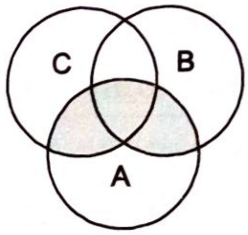
B ∩ C



A ∩ (B ∩ C)

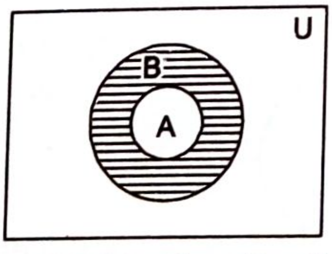


B ∪ C

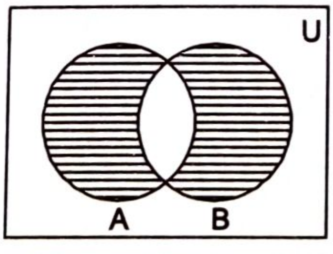


A ∩ (B ∪ C)

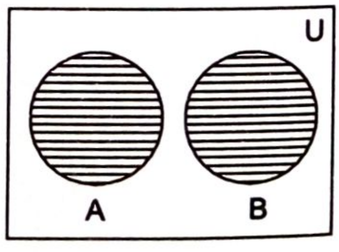
(i) **Symmetric difference** $(A \Delta B)$



A ΔB = (A – B) ∪ (B – A)



When neither A ⊆ B nor B ⊆ A and A and B are not disjoint

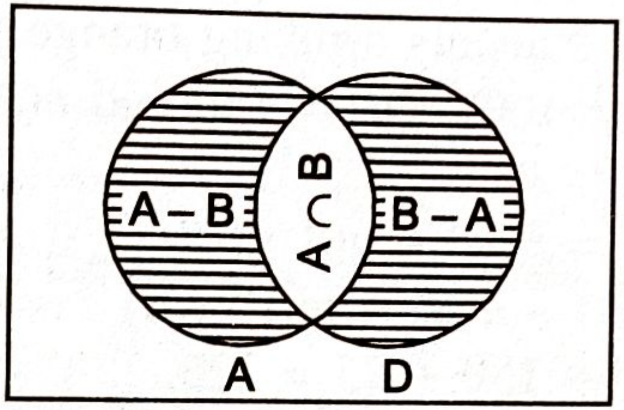


A Δ B = (A – B) ∪ (B – A) = A ∪B When A and B are disjoint

8.3 Application of Set Theory

Let A and B be two intersecting sets. While counting the elements of $A \cup B$, we count some elements of $A \cup B$ twice, once in counting the elements of A and again in counting the elements of B. So

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



In case A and B are disjoint sets, so we have $A \cap B = \phi$,

From the above Venn-Diagram, the following results appears, immediately. It will help in the application of set theory.

(a) $n(A) = n(A - B) + n(A \cap B)$ (b) $n(B) = n(B - A) + n(A \cap B)$

(c) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

The result, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ can be generalised as,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

5.0 TOTAL NUMBER OF DISTINCT RELATION FROM A SET A TO A SET B

Let the number of elements of A and B be m and n respectively. Then the number of element of $A \times B$ is mn. Therefore, the number of elements of the power set of $A \times B$ is 2^{mn}. Thus, $A \times B$ has 2^{mn} different subsets. Now every subset of $A \times B$ is a relation from A to B. Hence th number of different relations from A to B is 2^{mn}.

6.0 DIFFERENT TYPES OF RELATIONS

6.1 Inverse Relation

Let R be a relation from the set A to the set B, then the inverse relation R^{-1} from the se B to the set A is defined by

$$R^{-1} = \{(b, a) : (a, b) \in R\}.$$

In other words, the inverse relation R^{-1} consists of those ordered pairs which when reverse belong to R. Thus every relation R from the set A to the set B has an inverse relation R^{-1} from B to A.

Example 1: Let $A = \{1, 2, 3\}$, $B = \{a, b\}$ and $R = \{(1, a), (1, b), (3, a), (2, b)\}$ be relation from A to B.

The inverse relation of R is

$$R^{-1} = \{(a, 1), (b, 1), (a, 3), (b, 2)$$

Example 2: Let $A = \{2, 3, 4\}$, $B = \{2, 3, 4\}$ and $R = \{(x, y) : |x - y| = 1)$ be a relatio from A to B. That is, $R = \{(3, 2), (2, 3), (4, 3), (3, 4)\}$.

The inverse relation of R is

$$R^{-1} = \{(3, 2), (2, 3), (4, 3), (3, 4)\}.$$

6.2 Identity Relation

A relation R in a set A is said to be identity relation, generally denoted by I_A, if

$$I_A = \{(x, x) : x \in A\}.$$

Example: Let $A = \{2, 4, 6\}$ then $I_A = \{(2, 2), (4, 4), (6, 6)\}$ is an identity relation in A

6.3 Universal Relation

A relation R from A to B is said to be universal relation if $R = A \times B$.

Example: Let $A = \{1, 2, 3\}$ then

$R = A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$ is a universa relation in A.

6.4 Void Relation

A relation R from A to B is said to be a void relation if R is a null set, *i.e.*, if $R = \phi$.

Example: Let $A = \{3, 5\}$ and $B = \{7, 11\}$. Let $R = \{(a, b) : a \in A, b \in B, a-b \text{ is odd}\}$.

Since none of the numbers (3 – 7), (3 – 11), (5 – 7), (5 – 11) is an odd number, R is an empty relation.

6.5 Reflexive Relations

Let R be a relation in a set A. Then R is called a reflexive relation if $(a, a) \in R$, for all $a \in A$.

In other words, R is reflexive if every element in A is related to itself.

Thus, R is reflexive if aRa holds for all $a \in A$.

A relation R in a set A is not reflexive if there is at least one element $a \in A$, such that $(a, a) \notin R$.

Example: Let $A = \{1, 2, 3, 4\}$. Then

(i) The relation $R_1 = \{(1, 1), (2, 4), (3, 3), (4, 1), (4, 4)\}$ in A is not reflexive since $2 \in A$ but $(2, 2). \notin R_1$

(ii) The relation $R_2 = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (2, 3)\}$ is reflexive.

6.6 Symmetric Relations

Let R be a relation in a set A. Then R is said to be symmetric relation if $(a, b) \in R \Rightarrow (b, a) \in R$.

Thus R is symmetric if bRa holds whenever aRb holds.

A relation R in a set A is not *symmetric* if there exist two distinct elements $a, b \in A$, such that $(a, b) \in R$, but $(b, a) \notin R$.

Example 1: Let L be the set of all straight lines in a plane. The relation R in L defined by 'x is parallel to y' is symmetric, since if a straight line a is parallel to a straight line b, then b is also parallel to a. Thus, $(a, b) \in R \Rightarrow (b, a) \in R$.

Example 2: Let R be a relation in the natural numbers N which is defined by '$x - y > 0$'. Then R is not symmetric since, $4 - 2 > 0$ but $2 - 4 \not> 0$.

Thus, $(4, 2) \in R$ but $(2, 4) \notin R$.

6.7 Transitive Relations

Let R be a relation in a set A. Then R is said to be a transitive relation if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

A relation R in a set A is not transitive if there exist elements $a, b, c \in A$, not necessarily distinct, such that $(a, b) \in R$ $(b, c) \in R$ but $(a, c) \notin R$.

Example 1: Let L be the set of all straight lines in a plane and R be the relation in L defined by 'x is parallel to y'. If a is parallel to b and b is parallel to c then obviously a is parallel to c. Thus $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$. Hence R is transitive.

Example 2: Let A be the set of all Indian. Let R be the relation in A defined by 'x loves y'. If a loves b and b loves c, it does not necessarily follow that a loves c. So, R is not a transitive relation.

6.8 Equivalence Relations

Let R be a relation in a set A. Then R is an equivalence relation in A if

(i) R is reflexive, *i.e.*, for all $a \in R$, $(a, a) \in R$.

(ii) R is symmetric, *i.e.*, $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$.

Let $f : N \to N$ be defined by $f(x) = 3x$. Show that f is not an onto function

SOLUTION Range of f is $\{3n : n \in N\}$ which is not N. So, f is not an onto function.

8.4 Into Functions

A function $f : A \to B$ is said to be an into function, if there exist an element in B having no pre-image in A. That is if $f(A)$ is a proper subset of B.

Example 1: Let $A = \{2, 3, 5, 7\}$ and $B = \{0, 1, 3, 5, 7\}$. Let us consider $f : A \to B; f(x) = x - 2$. Then $f(2) = 0; f(3) = 1; f(5) = 3$ and $f(7) = 5$.

It is clear that f is a function from A to B.

Here there exists an element 7 in B, having no pre-image in A.

So, f is an into function.

Example 2: Let $f : Z \to Z$ (the set of integers) be defined by $f(x) = 2x, x \in Z$. Then f is an into mapping, because $f(Z)$ (the set of all even integers) is a proper subset of the codomain set Z.

8.5 Bijective Function

A one-one and onto function is said to be bijective.

A bijective function is also known as a one-to-one correspondence.

8.6 Identity Function

Let A be a non-empty set. Then, the function I defined by $I : A \to A : I(x) = x$ for all $x \in A$ is called an identity function on A.

It is a one-to-one onto function with domain A and range A.

8.7 Constant Function

Let $f : A \to B$, defined in such a way that all the elements in A have the same image in B, then f is said to be a constant function. The range set of a constant function is a singleton set.

Example: Let $A = \{1, 2, 3\}$ and $B = \{5, 7, 9\}$. Let $f : A \to B : f(x) = 5$ for all $x \in A$.

Then, all the elements in A have the same image namely 5 in B.

So, f is a constant function.

8.8 Equal Functions

Two functions f and g are said to be equal, written as $f = g$ if they have the same domain and they satisfy the condition $f(x) = g(x)$, for all x.

3.9 Composite Function

Let $f : A \to B$ and $g : B \to C$ be two functions. The function from A to C which maps an element $x \in A$ into $g(f(x)) \in C$ is called composite of functions f and g and is written as gof.

Example: Let $A = \{1, 3, 5\}$, $B = \{3, 9, 15, 21\}$, $C = \{2, 8, 14, 20, 24\}$

$f = \{(1, 3), (3, 9), (5, 15)\}$, $g = \{(3, 2), (9, 8), (15, 14), (21, 20)\}$

Then gof is the function $\{(1, 2), (3, 8), (5, 14)\}$

10 Inverse Function

Let f be a one-one onto function from A to B. Let y be an arbitrary element of B. We may define a function, denoted by f^{-1} as:

$f^{-1} : B \to A : f^{-1}(y) = x$ if and only if $f(x) = y$.

The above function f^{-1} is called the inverse of f. Clearly $f^{-1}of$ is identity function on A.

A function is invertible if and only if f is one-one onto.

Remarks: If f is one-one onto then f^{-1} is also one-one onto.

Example: Let $A = \{1, 2, 3\}$, $B = \{3, 6, 9\}$

$f = \{(1, 3), (2, 6), (3, 9)\}$, $g = \{(3, 1), (6, 2), (9, 3)\}$

Here gof and fog are identity functions on A and B respectively. Thus functions f and g are inverse of each other.

2.0 SOME FORMULAE OF DIFFERENTIATION

Let $f(x)$ and $g(x)$ be differentiable functions and $\alpha \in \mathbf{R}$.

2.1 Sum and Difference Rule

$$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

2.2 Scalar Multiple Rule

$$\frac{d}{dx}(\alpha f(x)) = \alpha \frac{d}{dx} f(x)$$

2.3 Product Rule

$$\frac{d}{dx}(f(x)\, g(x)) = g(x)\frac{d}{dx}(f(x)) + f(x)\frac{d}{dx}(g(x))$$

2.4 Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$

provided $g(x) \neq 0$

2.5 Chain Rule

If $y = h(u)$ and $u = f(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

2.6 Test of Constancy

If all points of a certain interval, $f'(x) = 0$, then the function $f(x)$ has a constant value within this interval.

3.0 SOME STANDARD RESULTS (FORMULAS)

3.1 $\frac{d}{dx}(x^n) = nx^{n-1}$

3.2 $\frac{d}{dx}(e^x) = e^x$

3.3 $\dfrac{d}{dx}(a^x) = a^x \log_e a$

3.4 $\dfrac{d}{dx}\,(\text{constant}) = 0$

3.5 $\dfrac{d}{dx}\,(e^{ax}) = ae^{ax}$

3.6 $\dfrac{d}{dx}\,(\log x) = \dfrac{1}{x}$

3.7 $\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x}\,\log_a e$

11.0 DEFINITION OF SOME FUNCTIONS IN BUSINESS

1.1 Total Cost Function

$C(x)$: **Total Cost** at any level of output is the sum of fixed cost and the variable cost at that level.

Total cost = Fixed Cost + Variable Cost

1.2 Total Revenue Function

$R(x)$. It is the amount received by selling the x units produced by the firm. It has direct relationship with the sales turnover of a firm. Higher the sales turnover, more would be the revenue of the firm. Thus, in mathematical form, it can be given as

$$R(x) = p \times x,$$

where p is the selling price per unit and x is the number of units sold.

Average Revenue: $$AR = \frac{R(x)}{x}.$$

1.3 Profit Function

Profit is a function of number of units sold and depend upon its cost and sale price. Thus profit function can be derived by subtracting cost function from revenue function, as unde

$$P(x) = R(x) - C(x).$$

11.4 Break-Even Point

Break-even point is defined as the point at which revenue is equal to the cost.

∴ At break-even point.

$$R(x) = C(x).$$

(a) when $R(x) = C(x)$, no profit-no loss.

(b) when $R(x) > C(x)$, profit is earned.

4.0 SOME STANDARD RESULTS

4.1 $\int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq 1)$

4.2 $\int \frac{1}{x} dx = \log x$

4.3 $\int e^x dx = e^x$

4.4 $\int a^x dx = \frac{a^x}{\log a}$

4.5 $\int dx = x$

4.6 $\int e^{ax} dx = \frac{e^{ax}}{a}$

4.7 $\int a^{mx} dx = \frac{a^{mx}}{m \log a}$

1. If $\int f(x)\,dx = g(x)$, then

$$\int f(ax+b)\,dx = \frac{g(ax+b)}{a} + c$$

Put $ax + b = t \Rightarrow a\,dx = dt \Rightarrow dx = \frac{dt}{a}$

$$\therefore \quad \int f(ax+b)\,dx = \int f(t)\frac{dt}{a} = \frac{1}{a}g(t) = \frac{g(ax+b)}{a} + c$$

2. (i) $\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1}$ if $n \neq -1$

(ii) $\int \frac{f'(x)}{f(x)}dx = \log(f(x))$

Put $f(x) = t \Rightarrow f'(x)\,dx = dt$

$\therefore$ (i) $\int (f(x))^n f'(x)\,dx = \int t^n\,dt = \frac{t^{n+1}}{n+1} = \frac{(f(x))^{n+1}}{n+1}$, $n \neq -1$

(ii) $\int \frac{f'(x)}{f(x)}dx = \int \frac{1}{t}dt = \log(t) = \log f(x)$.

Using the first form, we have

(i) $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(x+1)}$ (ii) $\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b}$

(iii) $\int \frac{dx}{ax+b} = \frac{1}{a}\log(ax+b)$ (iv) $\int a^{lx+m}\,dx = \frac{1}{l\log a}a^{lx+m}$

SOME SPECIAL INTEGRALS

1. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right) \quad (x > a)$

2. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left(\frac{a+x}{a-x}\right) \quad (x < a)$

3. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left[x + \sqrt{x^2 - a^2}\right]$

4. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left[x + \sqrt{x^2 + a^2}\right]$

5. $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left[x + \sqrt{x^2 + a^2}\right]$

6. $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log\left[x + \sqrt{x^2 - a^2}\right]$

7. $\int e^x [f(x) + f'(x)]\, dx = e^x f(x).$

1 Number Series, Coding & Decoding & Odd Man Out

NATURAL NUMBERS

Meaning	A natural number is a whole and non-negative number.
Example	1,2,3,4,5,6,7,8,9,...... Next Natural Number = 10

PRIME NUMBERS

Meaning	A prime number is a whole number greater than 1 whose only factors are 1 and itself.
Example	2,3,5,7, 11, 13, 17, 19, ? Next Prime Number = 23

ODD NUMBERS

Meaning	A number that cannot be divided exactly by 2 .
Example	1,3,5,7,9, 11, 13, 17, 19, ? Next Odd Number = 21

EVEN NUMBERS

Meaning	A number that can be divided exactly by 2 .
Example	2,4,6,8,10,12,14,16,18,20, ? Next Even Number = 22

SEQUENCE

Meaning	An arrangement of numbers in a definite order according to some rule is called a sequence.
Example	**1. Finite Sequence (which** has finite terms): 2, 4, 6, 8, 10, 12 is a finite sequence. **2. Infinite Sequence (which** has infinite terms): 2, 4, 6, 8, 10, 12.... is an infinite sequence whose *n*th term is given by the formula $a_n = 2n - 1$, where n is a natural number.

SERIES

Meaning	A series is obtained by adding all the terms of a sequence.
Example	**1. Finite Series (which** has finite terms): 2 + 4 + 6 + 8 + 10 + 12 is a finite sequence. **2. Infinite Series (which** has infinite terms): 2 + 4 + 6 + 8 + 10 + 12.... is an infinite sequence whose *n*th term is given by the formula $a_n = 2n - 1$, where n is a natural number.

PROGRESSION

If the terms of a sequence follow certain pattern, then the sequence is called a progression.

ARITHMETIC PROGRESSION

Meaning	A sequence in which the difference between the term and its previous term is always a constant quantity. The next term of AP is obtained by adding/subtracting a constant number to its previous term. The constant number is called the common difference of the A.P. In an A.P. the first term is generally denoted by a, the common difference by d and the nth term by t_n. $d = t_n - t_{n-1}$ **Note:** Three numbers a, b, c are in A.P. if $2b = a + c$.
Example	I. 1, 3, 5, 7, ... is an A.P. whose first term is 1 and common difference is 2. The next term $= 7 + 2 = 9$ II. 7, 4, 1, –2, –5, –8, ... is an A.P. whose first term is 7 and common difference is –3. The next term $= -8 + (-3) = -11$
Common Difference	(a) in case of odd numbers of terms d (b) in case of even numbers of terms $2d$
nth term t_n of an A.P.	If a is the first term and d is the common difference of an A.P., then its nth term t_n is given by $t_n = a + (n - 1)d$
Sum to first n Terms of an A.P.	$S_n = n/2[a + l]$, where l is the last term $a + (n - 1)d$.

GEOMETRIC PROGRESSION

Meaning	A sequence in which the ratio of a term and its previous term is always a constant quantity. The next term of GP is obtained by dividing or multiplying its previous number by a non–zero constant number. The constant ratio, generally denoted by r, is called common ratio. **Note:** The non–zero numbers a, b, c are in G.P. if $b^2 = ac$.
Example	I. 1, 2, 4, 8, ... is G.P. whose first term is 1 and common ratio is 2. Hence, Next Term $= 8 \times 2 = 16$ II. 1, 3, 9, 27, ... is G.P. whose first term is 1 and common ratio is 3. Hence, Next Term $= 27 \times 3 = 81$ III. 3, 18, 108, 864 ... is G.P. whose first term is 3 and common ratio is 6. Hence, Next Term $= 864 \times 6 = 5184$ IV. 256,64,16,4 is G.P. whose first term is 256 and common ratio is 1/4. Hence, Next Term $= 4 \times 1/4 = 1$

nth term t_n of an G.P.	**If a is the first term and r is the common ratio of a G.P., then the nth term t_n is given by** $t_n = ar^{n-1}$.
Sum to first n Terms of an G.P.	$S_n = \begin{cases} \frac{a(r^n-1)}{r-1} & \text{if } r \neq 1 \\ na & \text{if } r \neq 1 \end{cases}$
Sum of an Infinite G.P	$S_\infty = \frac{a}{1-r}$

MULTIPLICATION SERIES

Meaning	It is series in which the next term is obtained by multiplying its previous number by a non–zero constant/variable number.
Example	**Example:** 3, 3, 6, 18, 72, ? **Analysis** 3×1= 3 3×2= 6 6×3= 18 18×4= 72 Hence, Next term = 72×5= 360

DIVISION SERIES

Meaning	It is series in which the next term is obtained by dividing its previous number by a non–zero constant/variable number.
Example	**Example:** 360, 72, 18, 6, 3 ? **Analysis** 360/72=5 72/18= 4 18/6=3 6/3= 2 Hence, Next term = 3/1=3

MIXED SERIES

Meaning	It contains more than one different pattern in a series which is arranged alternatively in a single series or follow any non–conventional rule.

Example	**Example :** 3, 4, 10, 33, 136, ? **Analysis** 3×1+1= 4 4×2+2= 10 10×3+3= 33 33×4+4= 136 Hence, Next term = 136×5+5= 685 The missing term is 685.

PERFECT SQUARES SERIES

Meaning	It is based on the perfect squares of the numbers in a specific order.
Example	529,576,625,676,... **Analysis** $529 = 23^2$, $576 = 24^2$, $625 = 25^2$, $676 = 26^2$, $729 = 27^2$, **Ans** 729

PERFECT CUBES SERIES

Meaning	It is based on the perfect cubes of the numbers in a specific order.
Example	216,343,512,729,... **Analysis** $216 = 6^3$, $343 = 7^3$, $512 = 8^3$, $729 = 9^3$, $1000 = 10^3$, **Ans** 1000

ALTERNATING SERIES

Meaning	It is a combination of two different series in which successive terms increase and decrease alternatively. Two different operations are performed on successive terms alternatively.
Example	**Example I.** 1, 3, 6, 7, 11, 11, ? ? **Analysis** **Pattern of First Series** 1, 6, 11, there is difference of 5 in each term. Hence, Next term = 11+5=16 **Pattern of Second Series** 3, 7, 11, there is difference of 4 in each term. Hence, Next to next term = 11+4=15 **Ans** 16,15

Example II. 1, 3, 5, 12, 25, 48, 125, ?, ?

Analysis

Pattern of First Series 1, 5, 25, 125, = Previous Term × 5

Hence, Next term = 125 × 5 = 625

Pattern of Second Series 3, 12, 48

= Previous Term × 4. Hence Next Term = 48 × 4 = 192

Ans. 192, 625

Example III. 1, 3, 5, 6, 9, 12, ?, ?

Analysis

Pattern of First Series 1, 5, 9, ... Increases by 4. Hence, Next Term = 9 + 4 = 13

Pattern of Second Series 3, 6, 12,

= Previous Term × 2. Hence Next Term = 12 × 2 = 24

Ans. 13, 24

SOME IMPORTANT FORMULE

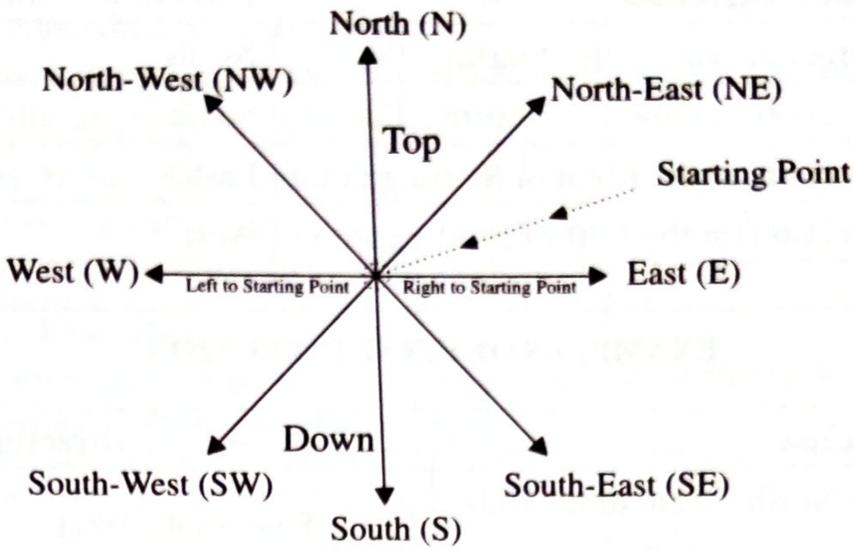
Sum of the first *n* natural numbers	$\Sigma n = 1 + 2 + ... + n = \frac{n(n+1)}{2}$
Sum of the squares of first *n* natural numbers	$\Sigma n^2 = 1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$
Sum of the cubes of the first n natural numbers	$\Sigma n^3 = 1^3 + 2^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

TABLE SHOWING SQUARES OF NUMBERS 1-50

No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Square	1	4	9	16	25	36	49	64	81	100
No.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Square	121	144	169	196	225	256	289	324	361	400
No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Square	441	484	529	576	625	676	729	784	841	900
No.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Square	961	1024	1089	1156	1225	1296	1369	1444	1521	1600
No.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
Square	1681	1764	1849	1936	2025	2116	2209	2304	2401	2500

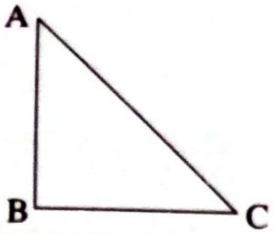
2

To solve the questions relating to direction, the students are required to observe and understand the following direction map:



POINTS TO REMEMBER:
ALWAYS PREPARE DIRECTION CHART TO ASCERTAIN THE FINAL DIRECTION
1. Moving from starting point towards North means moving towards Top.
2. Moving from starting point towards South means moving towards Down.
3. Moving from starting point towards East means moving towards Right.
4. Moving from starting point towards West means moving towards Left.
5. Moving Right from North means towards East.
6. Moving Left from North means towards West.
7. Moving Right from South means towards West.
8. Moving Left from South means towards East.
9. Moving Right from East means towards South.
10. Moving Left from East means towards North.
11. Moving Right from West means towards North.
12. Moving Left from West means towards South.
13. For calculating distance, remember that the opposite sides of a rectangle are always equal.

14. **For calculating the shortest distance in case of a right angled triangle route, use Pythagoras Theorem as follows:**



$$AC = \sqrt{AB^2 + BC^2}$$

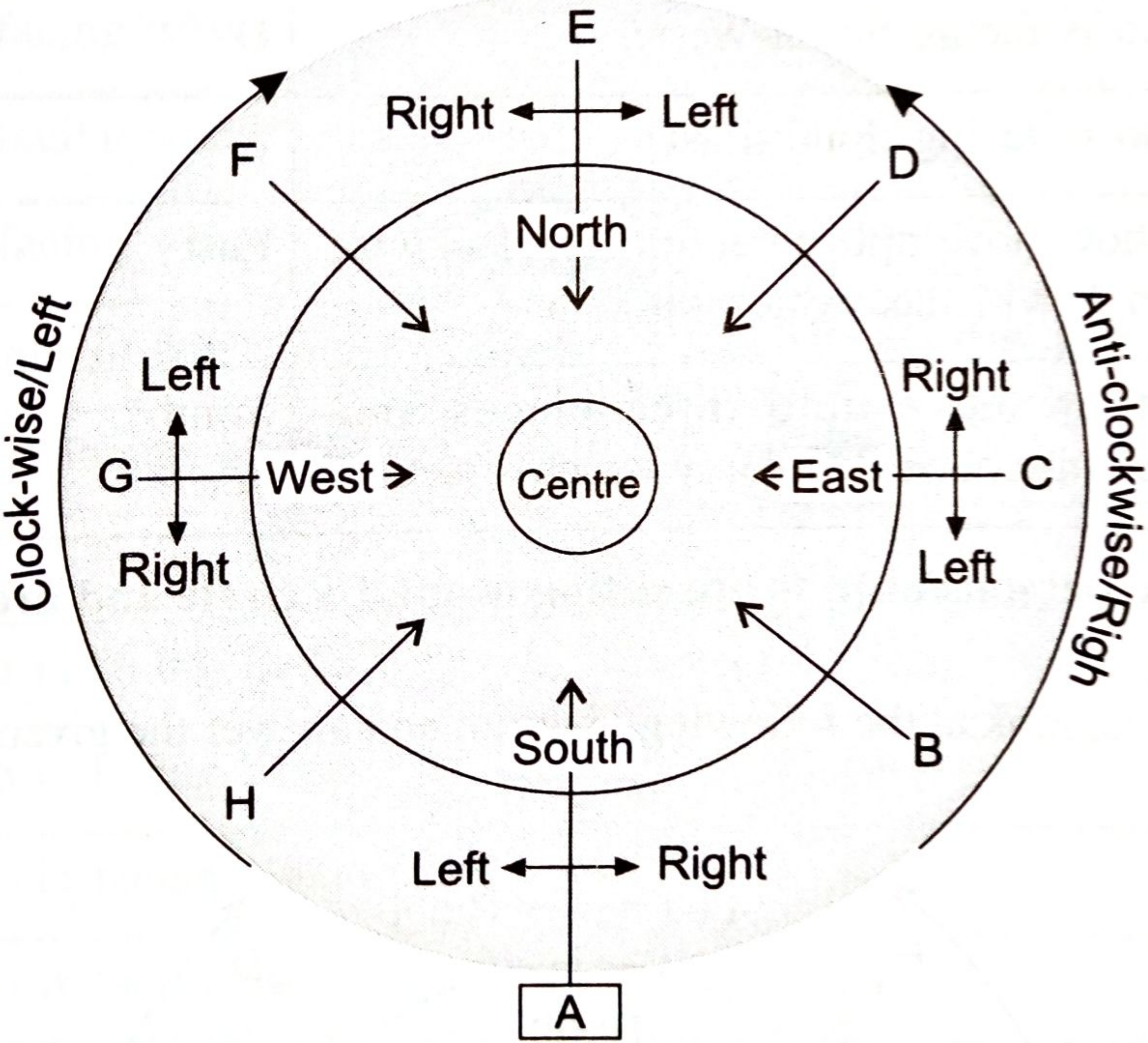
15. **Decide Direction from the Starting Point as follows:**

Location of Final Direction	**Direction from the Starting Point**
(a) If Final Direction above the Starting Point	North
(b) If Final Direction below the Starting Point	South
(c) If Final Direction on the Right of Starting Point	East
(d) If Final Direction on the Left of Starting Point	West

EXAMPLES OF FINAL DIRECTION

Case	Direction
1. *L* moves towards North, then turns right, again turns right, and finally again turns right.	North-East-South-West
2. *M* moves towards North, then turns right, again turns right, further turns right, and finally again turns right.	North-East-South-West-North
3. *N* moves towards North, then turns right, again turns right, further turns right, again turns right and finally again turns right.	North-East-South-West-North-East
4. *O* moves towards North, then turns left, again turns left, further turns left, again turns left and finally again turns left.	North-West-South-East-North-West
5. *P* moves towards South, then turns left, again turns left, further turns left, again turns left and finally again turns left.	South-East-North-West- South-East
6. *Q* moves towards West, then turns right, again turns right, further turns right, again turns right and finally again turns right.	West-North-East-South-West-North
7. *R* moves towards East, then turns right, again turns right, further turns right, again turns right and finally again turns right.	East-South-West-North-East-South

8. *S* moves towards West, then turns right, again turns right, thereafter turns left, again turns left and finally again turns left.	West-North-East-North-West- South
9. *T* moves towards East, then turns left, again turns left, thereafter turns right, again turns right and finally again turns right.	East-North-West-North-East- South
10. *U* moves towards South, then turns right, then turns left, further turns left, thereafter turns right and finally again turns right.	South-West- South-East-South-West
11. *V* moves towards West, then turns left, then turns right, further turns right, thereafter turns left and finally again turns left.	West-South-West-North-West- South
12. *W* moves towards North, then turns left, again turns left, thereafter turns right, then turns left and finally again turns left.	North-West-South-West-South-East
13. *X* moves towards East, then turns right, again turns right, thereafter turns left, then turns right and finally again turns right.	East-South-West-South-West-North


E
Right
Left
F
D
North
Clock-wise/Left
Anti-clockwise/Righ
Left
Right
G
West
Centre
East
C
Right
Left
South
B
H
Left
Right
A

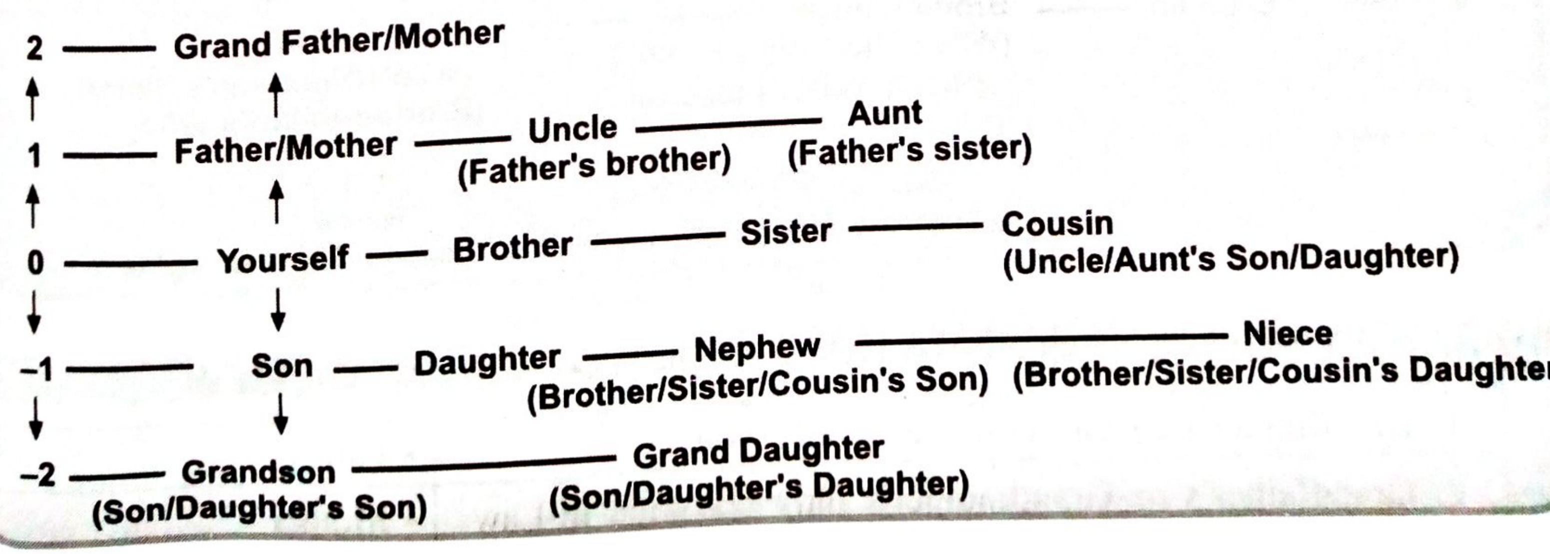
MEANING OF BLOOD RELATIONS

Blood Relation means blood relation among family members.

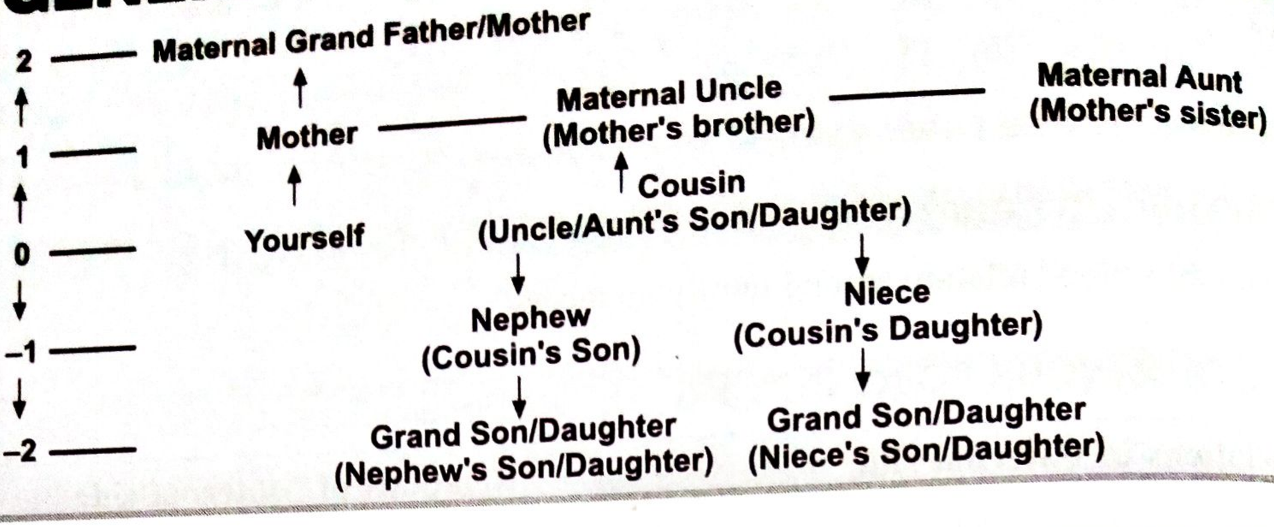
RELATIONS FROM YOUR POINT OF VIEW

Relations of Paternal side	Relations of Maternal side
1. Father's Father → Grandfather	1. Mother's Father → Maternal Grandfather
2. Father's Mother → Grandmother	2. Mother's Mother → Maternal Grandmother
3. Father's Brother → Uncle	3. Mother's Brother → Maternal Uncle
4. Father's Sister → Aunt	4. Mother's Sister → Maternal Aunt
5. Father's Sister's Husband → Uncle	5. Mother's Sister's Husband → Maternal Uncle
6. Father's Brother's Wife → Aunt	6. Mother's Brother's Wife → Maternal Aunt
7. Children of Uncle/Aunt → Cousin	7. Children of Maternal Uncle/Aunt → Cousin
8. Father's Daughter → Sister	
9. Father's Daughter's Husband → Brother-In-Law	
10. Father's Son → Brother/Self	
11. Father's Son's Wife → Wife/Sister-In-Law	
12. Son of Brother /Sister →Nephew	
13. Daughter of Brother /Sister →Niece	

GENERATION FROM PATERNAL SIDE



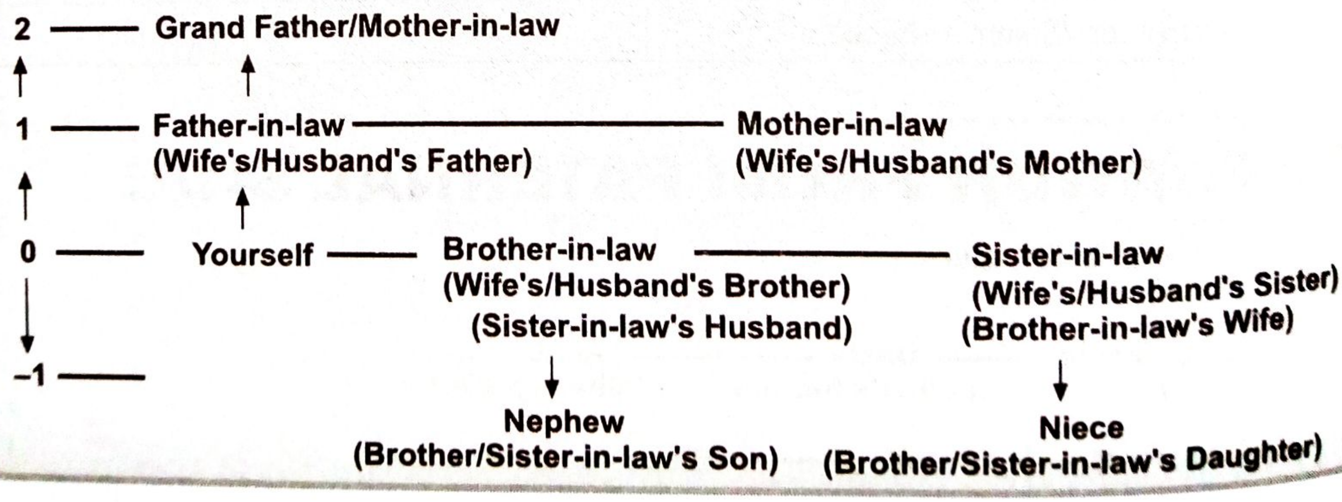
GENERATION FROM MATERNAL SIDE



SOME OTHER IMPORTANT RELATIONS FROM YOUR POINT OF VIEW

1.	Wife's or Husband's Father	• Father-in-law.
2.	Wife's or Husband's Mother	• Mother-in-law.
3.	Wife's or Husband's Brother	• Brother-in-law.
4.	Wife's or Husband's Sister	• Sister -in-law.
5.	Brother's Wife	• Sister -in-law.
6.	Sister's Husband	• Brother-in-Law
7.	Son's Wife	• Daughter-in-Law
8.	Daughter's Husband	• Son-in-Law

GENERATION FROM IN-LAWS SIDE



INDIRECTLY DEFINED RELATIONSHIPS FROM YOUR POINT OF VIEW

1. Grandfather's *or* Grandmother's only Son	• Father
2. Grandfather's *or* Grandmother's only Daughter-in-Law	• Mother

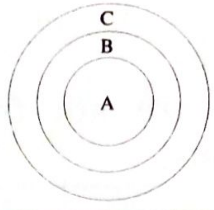
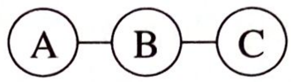
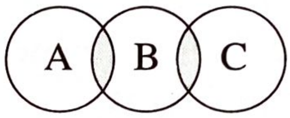
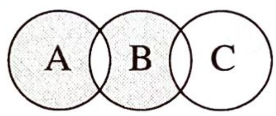
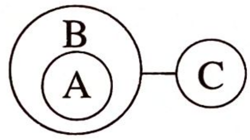
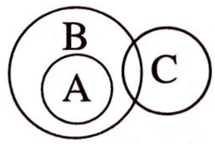
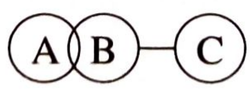
3. Maternal Grandfather's *or* Grandmother's only Daughter	• Mother
4. Maternal Grandfather's *or* Grandmother's only Son-in-Law	• Father
5. Grandfather's *or* Grandmother's only Grandson	• Myself
6. Maternal Grandfather's *or* Grandmother's only Grandson-in-Law	• Myself
7. Father's *or* Mother's Grandson	• Son/Nephew
8. Father's *or* Mother's Grand Daughter	• Daughter /Niece
9. Father's *or* Mother's only Grandson	• My Son
10. Father's *or* Mother's only Grand Daughter	• My Daughter
11. Grandson's or Granddaughter's Daughter	• Great grand Daughter
12. Grandson's or Granddaughter's Son	• Great grand Son
13. Cousin's Son	• Nephew
14. Cousin's Daughter	• Niece
15. Father's Brother's only Grand Daughter	• Niece
16. Father's Brother's only Grandson	• Nephew
17. Father's *or* Mother's only Grandson's Wife	• My Daughter-in-Law
18. Father's *or* Mother's only Grand Daughter's Husband	• My Son-in-Law

PRACTICAL STEPS TO SOLVE BLOOD RELATION QUESTIONS

[Note: Use + sign to indicate female and - sign to indicate male.]

Step 1: Represent Top to Bottom relations vertically. ***For example*****,**

Great Grand Father
↑
Grand Father
↑
Father
↑
Self
↓
Son/Daughter
↓
Grand Son/Daughter
↓
Great Grand Son/Daughter

Type of Proposition	Propositions	Venn Diagram	Valid Mediate Inference
A	1. All Animals are Birds. 2. All Birds are Cows.		All Animals are Cows. **Reason: A is subset of C.**
E	1. No Animals are Birds. 2. No Birds are Cows.		No Conclusion **Reason: There is no direct connection between A &C.**
I	1. Some Animals are Birds. 2. Some Birds are Cows.		No Conclusion **Reason: There is no direct connection between A &C.**
O	1. Some Animals are not Birds. 2. Some Birds are not Cows.		No Conclusion
A + E	1. All Animals are Birds. 2. No Birds are Cows.		No Animals are Cows.
A + I	1. All Animals are Birds. 2. Some Birds are Cows.		No Conclusion
I + E	1. Some Animals are Birds. 2. No Birds are Cows.		Some Animals are not Cows.

IMPORTANT POINTS TO REMEMBER

1. **Croxton and Cowden** has defined statistics in **singular sense** as statistical method.
2. **Prof. Horace Secrist** has defined statistics in **plural sense** as statistical data.
3. **P C Mahalanobis** is an Indian statistician who has made significant contribution in the development of statistics.
4. The real giant in the development of the theory of statistics is R.A. Fisher.
5. Kilometers – Tonnes is an example of complex statistical unit.
6. Primary data is collection of original data for the first time.
7. Secondary data is basically compilation of existing data.
8. Primary data can be collected by Interview Method, Observation Method and Questionnaire Method.
9. Data collected on religion from Census Report is secondary data.
10. Questionnaire Method can be used when informants are literates.
11. Where the field of enquiry is wide and the informants are literate, the suitable method of collecting primary data is Questionnaires sent through enumerators.
12. Quickest method to collect personal data is Telephone interview.
13. Best method to collect data in case of a natural calamity is personal interview.
14. In case of rail accident, the appropriate method of data collection is indirect interview.
15. The term 'Error' in statistics refers to bias.
16. Sampling Errors are present only in sampling survey and usually decreases with increase in sample size.
17. Non – sampling errors include both bias and mistakes and may occur in both sample survey and complete enumeration survey.
18. Marks of a student is a Discrete Variable but Weight of a student is a Continuous Variable.
19. Nationality and Reading habit of a student are attributes.

2 Classification and Tabulation

IMPORTANT POINTS TO REMEMBER

1. The methods of presentation of data are textual, tabulation and diagrammatic.
2. In tabulation, row designations are called **Stubs** and column headings are called **Captions**.
3. The entire upper part of the a table is known as **Box head**.
4. The unit of measurement in tabulation is shown in **Box head**.
5. In tabulation source of the data, if any, is shown in the **Foot Note.**
6. A table showing Height and Weight is an example of Quantitative classification.
7. A table showing population of Delhi during 2008 to 2018 is an example of Temporal classification.
8. A table showing state-wise wheat production in India is an example of Spatial classification.
9. Continuous Random Variable is a real valued function on an infinite sample space and Discrete Random Variable is a real valued function on an finite sample space
10. Exclusive class limits are suitable for continuous variable and Inclusive class limits are suitable for discrete variable.
11. In exclusive data series the upper limit class is not included
12. In inclusive data series the upper limit class is included.
13. Class width is Range/No of classes decided.
14. The midpoint of a class is obtained by adding upper and lower limits and dividing by 2.
15. Mutually exclusive classification is usually meant for continuous variable.
16. The distribution of shares is an example of the frequency distribution of a discrete variable and the distribution of profits is an example of the frequency distribution of a continuous variable.

1. One Dimensional Diagrams	One Dimensional Diagram is a diagram which is prepared on the basis of only one dimension i.e. **length.** This type of diagram takes the shape of bars.
(a) Simple Bar Diagrams	Simple Bar Diagram is a one dimensional diagram in which bar is constructed to represent one value of a given variable. The length of various bars is in the ratio of the magnitude of the given data.
(b) Multiple Bar Diagrams	Multiple Bar Diagram is one dimensional diagram in which two or more bars adjoining each other are constructed to represent the values of different variables or the values of various components of the same variable.
(c) Sub-divided Bar Diagrams (Component Bar Diagrams)	Sub-divided Bar Diagram is one dimensional diagram in which one bar is constructed for the total value of the variable and this bar is sub-divided in proportion to the values of the various components of that variable.
2. Two Dimensional Diagrams (Area Diagrams)	Two Dimensional Diagram is a diagram which is prepared on the basis of two dimensions i.e. **length and width.** As the product of length and width indicates the area, this type of diagram is also called **Area Diagram.**
(a) Rectangle Diagrams	Rectangle Diagram is a two dimensional diagram in which a rectangle is prepared representing one variable by length, another variable by width and value of given variable by area.
(b) Sub-Divided Rectangle Diagrams	Sub-divided Rectangle is a two dimensional diagram in which one rectangle is constructed for the total value of the variable and then this rectangle is sub-divided in proportion to the values of the various components of that variable.
(c) Square Diagrams	Square Diagram is a two dimensional diagram in which a square is prepared and the side of square is determined on the basis of minimum square root value of the variable.
(d) Circle Diagrams	Circle Diagram is a two dimensional diagram in which a circle is prepared and the radius of circle is determined on the basis of minimum square root value of the variable.
(e) Pie Diagrams	Pie Diagram is a circular diagram whose area is proportionately divided among the various components of a given variable.

2.0 TYPES OF GRAPHS

1. Time Series Graph **(a) One Dependent Variable Historigram**	One Dependent Variable Historigram is graph of time series data which is prepared to show the value of one dependent variable over different periods of time.
(b) More Than One Dependent Variable Historigram	More than one Dependent Variable Historigram is a graph of time series data which is prepared to show the value of more than one dependent variable over different periods of time.
(c) Mixed Graph	Mixed Graph is a type of historigram which is prepared to show the two dependent variables with two different units of measurement.
(d) Range Graph	Range Graph is a type of historigram which is prepared to show the range of the data between two extreme values of a dependent variable at different points of time.
2. Frequency Distribution Graphs **(a) Histogram**	The graphical representation of vertical adjacent rectangles with class intervals on x - *axis* and corresponding **simple frequencies** on y - *axis*.
(b) Frequency Polygon	The figure obtained by joining the mid - points of tops of adjacent rectangles of the histogram by straight lines. It is **used to locate mode.**
(c) Frequency Curve	The free hand smooth curve drawn through the points of frequency polygon.
(d) Ogive	The graphical representation of cumulative frequency distribution with class intervals on x - *axis* and corresponding **cumulative frequencies** on y - *axis*. It is **used to locate median.**

4 Measures of Central Tendency

1.0 COMPARATIVE STUDY OF MEASURES OF CENTRAL TENDANCY

		Arithmetic Mean (AM)	*Median*	*Mode*	*Geometric Mean (GM)*	*Harmonic Mean (HM)*
1	**Meaning**	It is obtained by dividing the sum of values of all items of a series by the no. of items of that series.	It is the central value of the variable that divides the series into two equal parts in such a way that half of the items lie above this value and the remaining half lie below this value.	It is that value in a series which is the greatest frequency.	GM of *n* items is the n^{th} root of their product.	HM of various items of a series is the reciprocal of the AM of their reciprocals.
2	**Symbol used**	$\bar{X}$	M_d	M_o	G.M.	H.M.
3	**Whether based on All Items of Series.**	Yes	No	No	Yes	Yes
4	**Can its formula be extended to calculate Combined Average of two or more related series?**	Yes	No	No	Yes	Yes
5	**Whether it requires arrangement of data in ascending / descending order?**	No	Yes	No No		No

		Arithmetic Mean (AM)	*Median*	*Mode*	*Geometric Mean (GM)*	*Harmonic Mean (HM)*
6	**Whether affected by Sampling Fluctuations.**	Least	Affected more than AM	Affected more than AM	Affected more than AM	Affected more than AM
7	**Whether affected by extreme values.**	Yes	No	No	Yes (gives more weight to small item)	Yes (gives largest weight to smallest item)
8	**Suitable for**	Other cases	Open-ended distribution	Qualitative data	Average Rate of Increase / Decrease, Average Ratios / Percentages	For Rates and Ratios involving Speed, Time, Distance, Price and Quantity.
9	**Can it be determined graphically?**	No	Yes	Yes	No	No
10	**Is it Independent of Choice of Origin?**	No	No	No	No	No
11	**Is it Independent of Choice of Scale?**	No	No	No	No	No
12	**Mathematical Property.**	1.Sum of Deviations from AM is always zero. 2.The Sum of Squared Deviations from AM is minimum.	The Sum of Absolute Deviations from Median is minimum.		1.The product of the values of series will remain unchanged when the value of geometric mean is substituted for each individual value. 2. The sum of the deviations of the logarithms of the original observations above or below the logarithm of the geometric mean is equal.	If each value of the variate is replaced by harmonic mean, the total of reciprocals of values of the variate remains the same.

IMPORTANT POINTS TO REMEMBER

1. Arithmetic Mean, Geometric Mean and Harmonic Mean are based on all items of series but Median and Mode are not based on all items of series.

2. The formula of Arithmetic Mean, Geometric Mean and Harmonic Mean can be extended to compute the combined average of two or more related series but the formula of Median and Mode cannot be so extended.

3. Median requires arrangement of items in ascending / descending order but Arithmetic Mean, Geometric Mean ,Harmonic Mean and Mode do not require such order.

4. Median, Mode, Geometric Mean and Harmonic Mean are more affected by sampling distribution than Arithmetic Mean.

5. Median and Mode are not affected by extreme values but Arithmetic Mean, Geometric Mean and Harmonic Mean are so affected.

6. The suitable average in case of open end distribution is the Median.
7. Median and Mode can be determined graphically but Arithmetic Mean, Geometric Mean and Harmonic Mean can not be so determined.
8. Median can be obtained from Ogives and Mode can be obtained from Frequency Polygon.
9. No measure of central tendency is independent of choice of origin and scale.
10. The sum of deviations is ***zero*** and sum of squared deviations is ***minimum*** **when taken from mean.**
11. The sum of absolute deviations from median is *minimum*.
12. For a symmetrical distribution, Median = $(Q_1 + Q_3)/2$
13. If weights are multiplied/divided by a constant, weighted arithmetic mean shall remain unchanged.
14. **Relationship between Mean, Median and Mode**

1. *Symmetrical Distribution*	Mean = Median = Mode
2. *Positively Skewed Distribution*	Mean > Median > Mode
3. *Negatively Skewed Distribution*	Mean < Median < Mode
4. *Moderately Skewed Distribution*	Mode = 3 Median – 2 Mean

15. **Relationship between AM, GM and HM**

1. *Symmetrical Distribution*	AM = GM = HM
2. *Moderately & Asymmetrical Distribution*	AM > GM > HM
3. *For a set of 2 observations*	GM^2 = AM × HM

16. Geometric Mean is the geometric mean of arithmetic mean (AM) and Harmonic Mean (HM) and thus, G.M. = $\sqrt{AM \times HM}$

17. In a series of values if one value is 0, GM is 0 and HP is indeterminate.
18. Both GM and HM cannot be computed when there are both positive and negative values in a series.
19. GM is theoretically considered to be the best average in the construction of index number since it satisfies the time reversal test and gives weight to equal ratio of change.
20. GM < AM because GM gives more weight to small items and less weight to large items.
21. $(a + b)^2 - (a - b)^2 = 4ab$

LIST OF FORMULAE

Individual Series	*Discrete Series*	*Continuous Series*
1. *Arithmetic mean* ***Direct Method:*** $\bar{X} = \frac{\Sigma X}{N}$ ***Short-cut Method :*** $\bar{X} = A + \frac{\Sigma d}{N}$	***Direct Method :*** $\bar{X} = \frac{\Sigma fX}{N}$ ***Short-cut Method :*** $\bar{X} = A + \frac{\Sigma fd}{N}$	***Direct Method :*** $\bar{X} = \frac{\Sigma fm}{N}$ ***Short-cut Method :*** $\bar{X} = A + \frac{\Sigma fd}{N} \times i$
2. *Median* Size of $\frac{N+1}{2}$ th item	Size of $\frac{N+1}{2}$ th item	Size of $\frac{N}{2}$ th item $Med = L + \frac{N/2 - c.f.}{f} \times i$
3. *Mode*	Either by Inspection or Grouping method determining that value around which most of the frequencies are concentrated	$M_o = L + \frac{\lvert f_1 - f_0 \rvert}{\lvert f_1 - f_0 \rvert + \lvert f_1 - f_2 \rvert} \times i$ **Note:** Empirical Mode in case of a Moderately Skewed Distribution: Mode = 3 Median – 2 Mean
4. *Geometric Mean* $G.M. = AL\left(\frac{\Sigma \log X}{N}\right)$	$G.M. = AL\left(\frac{\Sigma f \log X}{N}\right)$	$G.M. = AL\left(\frac{\Sigma f \log m}{N}\right)$
5. *Harmonic Mean* $H.M. = \frac{N}{\Sigma (1/X)}$	$H.M. = \frac{N}{\Sigma (f/X)}$	$H.M. = \frac{N}{\Sigma (f/m)}$
6. *Weighted Arithmetic Mean* $\bar{X}_w = \frac{\Sigma WX}{\Sigma W}$	***Weighted Geometric Mean*** $G.M._w = AL\left[\frac{\Sigma (W \log X)}{\Sigma W}\right]$	***Weighted Harmonic Mean*** $H.M._w = \frac{\Sigma W}{\left(\frac{1}{a} \times W_1\right) + \left(\frac{1}{b} \times W_2\right)}$

5 Measures of Dispersion

1. COMPARATIVE STUDY OF MEASURES OF DISPERSION

		Range	*Inter – Quartile Range (Quartile Deviation)*	*Mean Deviation*	*Standard Deviation*
1	**Meaning**	It is the difference between the value of largest item and the value of smallest item.	It is half of the difference between upper quartile and lower quartile.	It is the arithmetic mean of the absolute deviations of all items from a measure of central tendancy.	It is the square root of the arithmetic mean of the squares of deviations of all items from arithmetic mean.
2	**Symbol used**	R	Q.D.	M.D.	σ
3	**Whether based on All items of Series**	No	No	Yes	Yes
4	**Whether affe-cted by extr-eme values**	Yes	Least affected	Affected less than S.D.	Affected more than M.D.
5	**Whether Suitable for Open-end distribution**	No	Yes	No	No
6	**Can its form-ula be extend-ed to calculate Combined Dispersion of two or more related series?**	No	No	No	Yes

7	**Signs of Deviations**	-	-	Actual ± Signs of Deviations are **ignored** and all deviations are taken as positive.	Actual ± Signs of Deviations are **taken into** consideration.
8	**Is it Independent of Choice of Origin?**	Yes	Yes	Yes	Yes
9	**Is it Independent of Choice of Scale?**	No	No	No	No
10	**Mathematical Property**	-	-	The Sum of Absolute Deviations from Median is minimum.	The Sum of Squared Deviations from AM is minimum.
11	**In a Symmetrical Distribution**	1. Highest Value = Mean + Half of Range 2. Lowest Value = Mean – Half of Range	(a) Q_1 & Q_3 are equidistant from the median. Hence, Med. – Q_1 = Q_3 – Med. (b) Q.D. = 2/3 σ	M.D. = 4/5 σ	1. S.D. is never less than M.D. / Q.D. 2. Mean ± 1 σ covers 68.27% of items 3. Mean ± 2 σ covers 95.45% of items 4. Mean ± 3 σ covers 99.73% of items

IMPORTANT POINTS TO REMEMBER

1. M.D. and S.D. are based on all items of series but Range and Q.D. are not based on all items of series.
2. All Measures of Dispersion are affected by extreme values. S.D. is **more affected** by extreme values than M.D. Q.D. is least affected by extreme values.
3. The formula of S.D. can be extended to compute the Combined Dispersion of two or more related series but the formula of Range, Q.D. and M.D cannot be so extended.
4. The suitable measure of dispersion in case of open end distribution is Quartile Deviation.

5. **Absolute Measures of Dispersion** (Range, Q.D., M.D., S.D.) are **Independent of Origin and not of Scale** of Measurement. These are independent of units of measurement.

6. **Relative Measures of Dispersion** (Coefficient of Range, Coefficient of M.D., Coefficient of Variation) are **Independent of Scale and not of Origin.**

7. M.D. is usually computed from median and Actual ± Signs of Deviations are **ignored** and all deviations are taken as positive. The sum of absolute deviations from median is *minimum*. M.D. about median < M.D. about mean or any other value. or MD is minimum when deviations are taken about median.

8. S.D. is computed from A.M. Actual ± Signs of Deviations are **taken into** consideration. The sum of deviations is ***zero*** and sum of squared deviations is ***minimum*** **when taken from mean.**

9. Root-Mean square deviation is minimum because deviations are taken from arithmetic mean.

10. S.D. and Variance (σ^2) can never be negative because the S.D. is the square root of the averages of squares of deviation of the values from their means.

11. Standard Deviation of one observation is zero.

12. Standard deviation of two observations = (Largest Item – Smallest Item)/2

13. SD > MD about Mean

14. S.D. of n natural numbers is $\sqrt{[(n^2 - 1)/12]}$ and Arithmetic Mean of natural numbers is $\frac{n+1}{2}$

15. If all observations are same, any absolute and relative measure of dispersion will be *zero*.

16. In case of Normal/Symmetrical Distribution:

    (a) Mean Deviation = 4/5th or more precisely 0.7979 of the Standard Deviation.

    (b) Quartile Deviation is 2/3rd or more precisely 0.6745 of the Standard Deviation.

    (c) Q_1 and Q_3 are equidistant from the median and hence Q_1 = Mean – .6745 σ, Q_3 = Mean + .6745 σ **[Note:** Mean = Median = Mode]

    (d) Med. – $Q_1 = Q_3$ – Med.

    (e) Highest Value = Mean + Half of Range & Lowest Value = Mean – Half of Range

    (f) Mean ± 1σ covers 68.27% of the items

    (g) Mean ± 2σ covers 95.45% of the items

    (h) Mean ± 3σ covers 99.73% of the items

LIST OF FORMULAE

Measure	*Absolute Measure*	*Relative Measure*
1. **Range** (i) *Individual/Discrete* (ii) *Continuous*	$R = L - S$ $R = U_L - L_S$	Coefficient of Range = $\frac{L-S}{L+S}$ Coefficient of Range = $\frac{U_L - U_S}{U_L + U_S}$
2. **Inter-Qurtile Range**	Inter-Quartile Rnage = $Q_3 - Q_1$	
3. **Percentile Range**	Percentile Range = $P_{90} - P_{10}$	Coefficient of Percentile Range = $\frac{P_{90} - P_{10}}{P_{90} + P_{10}}$
4. **Quartile Deviation or** Semi-Inter Quartile Range	Quartile Deviation = $\frac{Q_3 - Q_1}{2}$	Coefficient of Quartile Deviation = $\frac{Q_3 - Q_1}{Q_3 + Q_1}$
5. **Mean Deviation** (i) *Individual Series* (ii) *Discrete Series* (iii) *Continuous Series*	M.D. = $\frac{\Sigma \lvert D \rvert}{N}$ M.D. = $\frac{\Sigma f \lvert D \rvert}{N}$ M.D. = $\frac{\Sigma f \lvert D \rvert}{N}$	Coefficient of M.D.about Mean/Median/Mode = $\frac{\text{M.D. about Mean/Median/Mode}}{\text{Mean/Median/Mode}}$
6. **Standard Deviation** (i) **Individual Series** (a) *Actual Mean Method* (b) *Assumed Mean Method*	$\sigma = \sqrt{\frac{\Sigma x^2}{N}}$ $\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$	Coefficient of Variation = $\frac{\text{Standard Deviation}}{\text{Arithmetic Mean}} \times 100$

Measure	Absolute Measure	Relative Measure
(ii) **Discrete Series**		
(a) *Actual Mean Method*	$\sigma = \sqrt{\frac{\Sigma fx^2}{N}}$	
(b) *Assumed Mean Method*	$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$	
(c) *Step-Deviation Method*	$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times c$	
(iii) **Continous Series**		
(a) *Actual Mean Method*	$\sigma = \sqrt{\frac{\Sigma fx^2}{N}}$	
(b) *Assumed Mean Method*	$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$	
(c) *Step-Deviation Method*	$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times c$	
7. **Variance**	Variance $= \sigma^2$	
8. **Combined Standard Deviation**	$\sigma_{12} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_1 d_1^2 + N_2 d_2^2}{N_1 + N_2}}$	

6 Correlation

1.0 COMPARATIVE STUDY OF MEASURES OF CORRELATION

	Co-Variance	*Karl Pearson's Coefficient of Correlation*	*Spearman's Rank Correlation Coefficient*	*Concurrent Deviation Method*
Who Originated	Karl Pearson	Karl Pearson	Spearman	-
Based on	Actual data	Actual data	Ranks	Direction of Change
Merit	It is direct measure of correlation.	It gives direction as well as degree of relationship between the variables.	It is suitable for qualitative data which can only be ranked in some order for abnormal data.	It is suitable for large no. of items
Is Independent of Choice of Origin?	Yes	Yes	Yes	Yes
Is Independent of Choice of Scale?	No	Yes	Yes	Yes
Is Independent of Units of Measurement	Yes	Yes	Yes	Yes
Limits within which lies	$-\infty$ to $+\infty$	-1 to $+1$	-1 to $+1$	-1 to $+1$
Formula	$COV = \frac{\Sigma xy}{N}$			

IMPORTANT POINTS TO REMEMBER

1. Co-variance can vary from $-\infty$ to $+\infty$
2. r can vary from -1 to $+1$.
3. r is independent both of choice of origin and scale.
4. The value of r between $-X$ and $-Y$ will be same as between X and Y.
5. Karl Pearson and Rank Method would give the same answer (+1) when both the variables are either increasing or decreasing.

6. **Karl Pearson and Rank Method would give the same answer (–1) when one variable is increasing and the other is decreasing.**

7. S.E. = $\dfrac{1-r^2}{\sqrt{N}}$

8. P.E. = .6745 S.E.

9. If $|r| > 6$ P.E, r is considered significant.

10. Limits of Population = $r \pm$ P.E.

11. Coefficient of Determination (r^2) is the ratio of explained variance to total variance.

12. Coefficient of Non-Determination ($1 - r^2$) is the ratio of unexplained variance to total variance.

13. Both Coefficient of Determination (r^2) and Coefficient of Non-Determination ($1 - r^2$) can never be negative and can never exceed 1.

14. Coefficient of Alienation ($\sqrt{1-r^2}$) is the square root of Coefficient of non-determination ($1 - r^2$).

15. If Sum of the Products of Deviations of X and Y series from their means (i.e. Σxy) is zero, r will be 0 since numerator is 0.

LIST OF FORMULAE

1. Karl Pearson's Correlation Coefficient (a) **When Deviations are taken from Actual Mean**	$r = \dfrac{\text{Cov.}(X,Y)}{\sigma_x \times \sigma_y} = \dfrac{\Sigma xy}{N\sigma_x\sigma_y}$ or $\dfrac{\Sigma xy}{\sqrt{\Sigma x^2 \times \Sigma y^2}}$ where, $x = (X - \bar{X})$ and $y = (Y - \bar{Y})$
(b) **When Deviations are taken from Assumed Mean**	$r = \dfrac{\Sigma d_x d_y - \dfrac{(\Sigma d_x)(\Sigma d_y)}{N}}{\sqrt{\Sigma d_x^2 - \dfrac{(\Sigma d_x)^2}{N}}\sqrt{\Sigma d_y^2 - \dfrac{(\Sigma d_y)^2}{N}}}$ where, $d_x = (X - A)$ and $d_y = (Y - A)$
(c) **In a Bivariate Frequency Distribution**	$r = \dfrac{\Sigma d_x d_y - \dfrac{\Sigma d_x \, \Sigma fd_y}{N}}{\sqrt{\Sigma fd_x^2 - \dfrac{(\Sigma fd_x)^2}{N}}\sqrt{\Sigma fd_y^2 - \dfrac{(\Sigma fd_y)^2}{N}}}$

(d) **When Actual Values of X and Y are considered**	$r = \dfrac{\Sigma XY - \dfrac{\Sigma X.\Sigma Y}{N}}{\sqrt{\Sigma X^2 - \dfrac{(\Sigma X)^2}{N}}\sqrt{\Sigma Y^2 - \dfrac{(\Sigma Y)^2}{N}}}$
2. Spearman's Rank Correlation Coefficient	
(a) **If Ranks are not repeated**	$R = 1 - \dfrac{6\,\Sigma D^2}{N^3 - N}$
(b) **If Ranks are repeated**	$R = 1 - \dfrac{6\left(\Sigma D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + ---\right)}{N^3 - N}$
3. Concurrent Deviation Method	$r_c = \pm\sqrt{\left(\dfrac{2C - n}{n}\right)}$
4. Standard Error (S.E.)	$\text{S.E.}r = \dfrac{1 - r^2}{\sqrt{N}}$
5. Probable Error (P.E.)	$\text{P.E.} = 0.6745\ \dfrac{1 - r^2}{\sqrt{N}}$
6. Coefficient of Determination = (r^2)	$r^2 = \dfrac{\text{Explained Variance}}{\text{Total Variance}}$
7. Coefficient of Non-Determination ($1 - r^2$)	$= 1 - r^2 = \dfrac{\text{Unexplained Variance}}{\text{Total Variance}}$
8. Coefficient of Alienation	$= \sqrt{1 - r^2}$
9. Limits of Population Correlation	$= r \pm$ Probable Error.

7 Regression Analysis

IMPORTANT POINTS TO REMEMBER

1. Regression Coefficients are Independent of Choice of **Origin and not of Scale.**
2. Coefficient of Correlation and both the Regression Coefficients have **same signs** (i.e. either positive or negative).
3. Both the Regression Coefficients can never exceed 1. (i.e. if one is > 1, the other has to be < 1 to the extent their product is less than *or* equal to 1).
4. Square Root of Product of Two Regression Coefficients is r i.e. $r = \sqrt{b_{xy} \times b_{yx}}$
5. A.M. of two Regression Coefficients is > r.
6. The Covariance, Correlation Coefficient and the two Regression Coefficients have the ***same origin.***
7. The Regression Lines cut each other at the point of the **Mean Values** of X and Y.
8. If the Regression Lines coincide, r will be +1 or –1.
9. If r is + 1 or –1, the two lines of regression are reversible.
10. If the two lines of regression cut each other making an angel of 90°, r will be 0.
11. The farther the two regression lines cut each other, the lesser will be the degree of correlation.
12. In a bivariate frequency distribution where width of class interval of X variable differs from that of Y variable, the formulae for calculating b_{xy} and b_{yx} are adjusted by multiplying the normal formulae by $\frac{i_x}{i_y}$ and $\frac{i_y}{i_x}$ respectively.

LIST OF FORMULAE

1. **Regression Equation of X on Y**	$X - \bar{X} = b_{xy}(Y - \bar{Y})$
Regression Coefficient of X on Y (b_{xy})	
(i) ***If Deviations are taken from Actual Means***	$b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{\Sigma xy}{\Sigma y^2}$
(ii) ***If Deviations are taken from Assumed Means***	$b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{\Sigma d_x d_y - \frac{(\Sigma d_x)\times(\Sigma d_y)}{N}}{\Sigma d_y^2 - \frac{(\Sigma d_y)^2}{N}}$
(iii) ***In a Bivariate Frequency Distribution***	$b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{\Sigma f d_x d_y - \frac{(\Sigma f d_x)(\Sigma d_x d_y)}{N}}{\Sigma f d_y^2 - \frac{(\Sigma f d_y)^2}{N}} \times \frac{i_x}{i_y}$

2. Regression Equation of *Y* on *X* **Regression Coefficient of *Y* on *X* (b_{yx})**	$Y - \bar{Y} = b_{yx}(X - \bar{X})$
(i) ***If Deviations are taken from Actual Means***	$b_{yx} = r\dfrac{\sigma_y}{\sigma_x} = \dfrac{\Sigma xy}{\Sigma x^2}$
(ii) ***If Deviations are taken from Assumed Means***	$b_{yx} = r\dfrac{\sigma_y}{\sigma_x} = \dfrac{\Sigma d_x d_y - \dfrac{(\Sigma d_x)\times(\Sigma d_y)}{N}}{\Sigma d_x^2 - \dfrac{(\Sigma d_x)^2}{N}}$
(iii) ***In a Bivariate Frequency Distribution***	$b_{yx} = r\dfrac{\sigma_y}{\sigma_x} = \dfrac{\Sigma f d_x d_y - \dfrac{(\Sigma f d_x)(\Sigma f d_y)}{N}}{\Sigma f d_x^2 - \dfrac{(\Sigma f d_x)^2}{N}} \times \dfrac{i_y}{i_x}$
3. Coefficient of Correlation (r)	$r = \sqrt{\text{(Regression coefficient of } X \text{ on } Y) \times \text{(Regression Coefficient of } Y \text{ on } X)}$ $r = \sqrt{b_{xy} \times b_{yx}}$
4. Standard Error of Estimate of *X* (S_{xy})	$S_{xy} = \sqrt{\dfrac{\Sigma(X - \bar{X}_c)^2}{N}}$ *or* $\sigma_x\sqrt{1 - r^2}$
5. Standard Error of Estimate of *Y* (S_{yx})	$S_{yx} = \sqrt{\dfrac{\Sigma(Y - \bar{Y}_c)^2}{N}}$ *or* $r = \sigma_y\sqrt{1 - r^2}$
6. Total Variation in *Y*	$\Sigma(Y - \bar{Y})^2$ *or* Σy^2
7. Unexplained Variation	$\Sigma(Y - Y_c)^2$
8. Explained Variation	= Total Variation – Unexplained Variation $= \Sigma(Y - \bar{Y})^2 - \Sigma(Y - Y_c)^2$
9. Coefficient of Determination (r^2)	$r^2 = \dfrac{\text{Explained variation}}{\text{Total variation}}$ $= \dfrac{\Sigma(Y - \bar{Y})^2 - \Sigma(Y - Y_c)^2}{\Sigma(Y - \bar{Y})^2}$
10. Coefficient of Non-determination ($1 - r^2$)	$1 - r^2 = \dfrac{\text{Unexplained Variation}}{\text{Total Variation}}$ $= \dfrac{\Sigma(Y - Y_c)^2}{\Sigma(Y - \bar{Y})^2}$

8 Index Numbers

IMPORTANT POINTS TO REMEMBER

1. Theoretically the best average in the construction of index numbers is Geometric Mean.
2. Fisher's Index is known as an ideal index because it satisfies Time Reversal Test and Factor Reversal Test.
3. **Fisher's Index** is the **Geometric Mean** of Laspeyre's Index and Paasche's Index.
4. **Dorbish Bowley Index** Number is the **Arithmetic Mean** of Laspeyre's and Paasche's Index.
5. Index number based on arithmetic mean is higher than the index number based on geometric mean.
6. Time Reversal Test is satisfied when $P_{01} \times P_{10} = 1$
7. Factor Reversal Test is satisfied when $P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$
8. Circular Test is satisfied when $P_{01} \times P_{12} \times P_{20} = 1$
9. Circular Test is not met by any of weighted aggregative with changing weights.
10. Circular Test is met by simple geometric mean of price relatives and weighted aggregative with fixed weights.
11. **Time Reversal Test** is the test which requires that the product of price index for year 1 on the base year 0 and price index for year 0 on the base year 1 should be equal to 1.
12. **Factor Reversal Test** is the test which requires that the product of a price index and the quantity index should be equal to the corresponding value index.
13. **Chain Base Index** is the index method which permits the introduction of new item and deletion of old items without necessitating the recalculation of entire series and in which the base changes from year to year
14. **Base Shifting** is The technique of changing the old base period to new base period.
15. **Splicing** is the technique of linking two or more index number series.
16. **Deflating** is the technique to eliminate effect of changing price levels.
17. **Consumer Price Index** is the index which measures how much the consumers of a particular class have to pay more or less for a certain basket of goods and services in a given period.

LIST OF FORMULAE

1. UNWEIGHTED INDEX NUMBERS	
(a) Simple Aggregative	$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$
(b) Simple Average of Relatives	
(i) When Arithmetic Mean is used	$P_{01} = \frac{\Sigma\left(\frac{p_1}{p_0} \times 100\right)}{N}$
(ii) When Geometric Mean is used	$P_{01} = \text{Antilog}\left[\frac{\Sigma \log\left(\frac{p_1}{p_0} \times 100\right)}{N}\right]$
2. WEIGHTED INDEX NUMBERS	
(a) Weighted Aggregative Indices	
(i) Laspeyres Method	$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$
(ii) Paasche Method	$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$
(iii) Dorbish & Bowley's Method	$P_{01} = \frac{L+P}{2} = \frac{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}{2} \times 100$
(iv) Fisher's Ideal Method	$P_{01} = \sqrt{L \times P} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$
(v) Marshall-Edgeworth Method	$P_{01} = \frac{\Sigma(q_0 + q_1) \times p_1}{\Sigma(q_0 + q_1) \times p_0} \times 100 = \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$
(vi) Kelly's Method	
(a) **If fixed quantities are given as weights**	$P_{01} = \frac{\Sigma p_1 q}{\Sigma p_0 q} \times 100$
(b) **If average of the quantities of two years is used as weights**	$P_{01} = \frac{\Sigma p_1 q}{\Sigma p_0 q} \times 100 \text{ where } q = \frac{q_0 + q_1}{2}$
(b) Weighted Average of Relatives	
(i) **If Arithmetic Mean is used**	$P_{01} = \frac{\Sigma PV}{\Sigma V}$ where, P = Price Relative = $\frac{P_1}{P_0} \times 100$; V = Value Weights = $p_0 q_0$
(ii) **If Geometric Mean is used**	$P_{01} = \text{Antilog}\left[\frac{\Sigma V \log p}{\Sigma V}\right]$

3. QUANTITY INDEX NUMBERS	
(a) Weighted Aggregative Indices	
(i) **Laspeyres Method**	$Q_{01} = \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times 100$
(ii) **Paasche Method**	$Q_{01} = \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1} \times 100$
(iii) **Dorbish & Bowley's Method**	$Q_{01} = \frac{L+P}{2} = \frac{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} + \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}{2} \times 100$
(iv) **Fisher's Ideal Method**	$Q_{01} = \sqrt{L \times P} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}} \times 100$
(v) **Marshall-Edgeworth Method**	$Q_{01} = \frac{\Sigma(p_0 + p_1) \times q_1}{\Sigma(p_0 + p_1) \times q_0} \times 100 = \frac{\Sigma q_1 p_0 + \Sigma q_1 p_1}{\Sigma q_0 p_0 + \Sigma q_0 p_1} \times 100$
(vi) **Kelly's Method**	
(a) **If fixed quantities are given as weights**	$Q_{01} = \frac{\Sigma q_1 p}{\Sigma q_0 p} \times 100$
(b) **If average of the quantities of two is used as weights**	$Q_{01} = \frac{\Sigma q_1 p}{\Sigma q_0 p} \times 100$ *where* $p = \frac{p_0 + p_1}{2}$
(b) Weighted Average of Relatives	
(i) If Arithmetic Mean is used	$Q_{01} = \frac{\Sigma PV}{\Sigma V}$ *where,* Q = Quantity Relatives $\frac{Q_1}{Q_0} \times 100$ V = Value Weights = $p_0 q_0$
(ii) If Geometric Mean is used	$Q_{01} = \text{Antilog}\left[\frac{\Sigma V \log Q}{\Sigma V}\right]$
4. VALUE INDEX NUMBER	$V = P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \times 100$
5. TESTS OF ADEQUACY	
(a) **Time Reversal Test**	$P_{01} \times P_{10} = 1$ **Note:** Fisher's Ideal Index and Marshall Edge worth Index satisfy Time Reversal Test.
(b) **Factor Reversal Test**	$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$ **Note:** The Factor Reversal Test is satisfied only by the Fisher's Ideal Index.
(c) **Circular Test**	$P_{01} \times P_{12} \times P_{20} = 1$ **Note:** The Circular Test is satisfied only by simple geometric mean of price relatives and the weighted aggregative fixed weights.

6. **Chain Base Index Number** = $\frac{\text{Average Link Relative of Current Year} \times \text{Chain Index of Previous Year}}{100}$

Note: Link Relative = $\frac{\text{Current Year's Price}}{\text{Provious Year's Price}} \times 100$

7. **Conversion of Chain Base Index into Fixed Base Index**

Current year's F.B.I. = $\frac{\text{Current year's C.B.I.} \times \text{Previous year's F.B.I.}}{100}$

8. **Conversion of Fixed Base Index into Chain Base Index**

Current year's C.B.I. = $\frac{\text{Current year's F.B.I.}}{\text{Previous year's F.B.I.}} \times 100$

9. **Conversion of Link Relative to Price Relative**

Current year's Price Relative

$= \frac{\text{Current year's link Relative} \times \text{Previous year's Price Relative}}{100}$

10. **Base Shifting**

New Index Number using New Base = $\frac{\text{Old Index Number using old Base}}{\text{Index Number Corresponding to New Base period}} \times 100$

11. **Splicing**

(a) **Index No. of old series under forward splicing**

$= \frac{100}{\text{Overlapping year's Index No. of old series}} \times \text{Given Index No. of old series}$

(b) **Index No. of new series under backward splicing**

$= \frac{\text{Overlapping year's Index no. of old series}}{100} \times \text{Given Index No. of new series}$

12. **Deflating**

(i) Real Wage = $\frac{\text{Money Wage}}{\text{Price Index}} \times 100$

(ii) Money Wage Index = $\frac{\text{Real Wage}}{\text{Money Wage of the Base Year}} \times 100$

(iii) Real Wage Index = $\frac{\text{Meney Wage Index}}{\text{Price Index}} \times 100$

13. **Consumer Price Index**

(i) Consumer Price Index = $\frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$

(ii) Consumer Price Index = $\frac{\sum PW}{\sum W}$

(iii) Consumer Price Index = $\frac{\sum IW}{\sum W}$

Bowley's Price Index Number $= \dfrac{\dfrac{\sum p_1 q_0}{\sum p_0 q_0} + \dfrac{\sum p_1 q_1}{\sum p_0 q_1}}{2} \times 100 = \dfrac{\dfrac{100}{80} + \dfrac{96}{60}}{2} \times 100$

$= \dfrac{1.25 + 1.6}{2} \times 100 = 142.5$

Fisher's Price Index Number (P_{01}) $= \sqrt{\dfrac{\sum p_1 q_0}{\sum p_0 q_0} \times \dfrac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100 = \sqrt{\dfrac{100}{80} \times \dfrac{96}{60}} \times 100$

$= \sqrt{\dfrac{9600}{4800}} \times 100 = \sqrt{2} \times 100 = 141.42.$

Marshall Edgeworth's Price Index Number $= P_{01} = \dfrac{\sum (q_0 + q_1)\, p_1}{\sum (q_0 + q_1)\, p_0} \times 100$

$= \dfrac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100$

$= \dfrac{100 + 96}{80 + 60} \times 100 = \dfrac{196}{140} \times 100 = 140$

QUANTITY INDEX NUMBERS

Laspeyre's Quantity Index Number $= \dfrac{\sum q_1 p_0}{\sum q_0 p_0} \times 100 = \dfrac{60}{80} \times 100 = 75$

Paasche's Quantity Index Number $= \dfrac{\sum q_1 p_1}{\sum q_0 p_1} \times 100 = \dfrac{96}{100} \times 100 = 96$

Bowley's Quantity Index Number $= \dfrac{\dfrac{\sum q_1 p_0}{\sum q_0 p_0} + \dfrac{\sum q_1 p_1}{\sum q_0 p_1}}{2} \times 100$

$= \dfrac{\dfrac{60}{80} + \dfrac{96}{100}}{2} \times 100 = \dfrac{0.75 + 0.96}{2} \times 100 = 85.5$

Marshall Edgeworth's Quantity Index Number

$Q_{01} = \dfrac{\Sigma (p_0 + p_1) q_1}{\Sigma (p_0 + p_1) q_0} \times 100 = \dfrac{156}{180} \times 100 = 86.67$

Fisher's Quantity Index Number (Q_{01}) $= \sqrt{\dfrac{\sum q_1 p_0}{\sum q_0 p_0} \times \dfrac{\sum q_1 p_1}{\sum q_0 p_1}} \times 100 = \sqrt{\dfrac{60}{80} \times \dfrac{96}{100}} \times 100$

$= \sqrt{\dfrac{5760}{8000}} \times 100 = \sqrt{0.72} \times 100 = 84.853.$

9 Probability Theory

IMPORTANT POINTS TO REMEMBER

1. Probability ranges from 0 to 1.
2. The Classical School of probability assumes that all possible outcomes of an experiment are equally likely.
3. If an event cannot take place, probability will be 0.
4. $p + q$ would always be 1.
5. If two events A and B are independent, then their complements are also independent.
6. If two events A and B are mutually exclusive the probability of occurrence of either A or B is given by $P(A) + P(B)$.
7. If two events A and B are overlapping events, the probability of occurrence of either A or B is given by $P(A) + P(B) - P(A \text{ and } B)$.
8. If two events A and B are independent, the probability that they will both occur is given by $P(A) \times P(B)$.
9. If two events A and B are dependent, the conditional probability of B given A is given by $P(AB)/P(A)$.
10. Dependent events are those in which the outcome of one affects and is affected by the other.
11. Joint Probability is the probability of the joint or simultaneous occurrence of two or more events.
12. The probability obtained by following relative frequency is called ***posterior*** probability.
13. The order of arrangement is important in permutations.
14. Permutation exceeds combinations.
15. ${}^nc_r$ is equal to 1 when $r = n$
16. ${}^nc_r$ is equal to n when $r = 1$.
17. ${}^nc_r$ is equal to 1 when $r = 0$
18. ${}^nc_r$ is equal to n when $r = n - 1$
19. ${}^7c_4 = {}^7c_3$.

LIST OF FORMULAE

1. Range of probability of an event	0 to 1
2. Probability of an event which cannot take place	0

3. Probability of an event which is certain i.e., bound to occur	1
4. Probability of the entire sample space	$P(S) = 1$
5. Addition Rule of Probability (a) ***When events are Mutually Exclusive*** (b) ***When events are Not Mutually Exclusive***	**Probability of occurrence of either A or B—** $P(A \cup B) = P(A) + P(B)$ **Probability of occurrence of at least A or B—** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
6. Multiplication Rule of Probability When events are Independent	(i) **Probability of occurrence of both A & B—** $P(A \cap B) = P(A) \times P(B)$ (ii) **Probability of non-occurrence of both A & B—** $P(\bar{A} \cap \bar{B}) = P(\bar{A}) \times P(\bar{B})$ (iii) **Probability of occurrence of A and not B—** $P(A \cap \bar{B}) = P(A) \times P(\bar{B})$ (iv) **Probability of occurrence of B and not A—** $P(\bar{A} \cap B) = P(\bar{A}) \times P(B)$ (v) **Probability of occurrence of only one event—** $P(A \cap \bar{B}) + P(\bar{A} \cap B)$ (vi) **Probability of occurrence of at least one event** $= 1 - P(\bar{A} \cap \bar{B})$ (vii) **Probability of non-occurrence of at least one event** $= 1 - P(A \cap B)$ (viii) **Probability of occurrence of an event** $= P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B)$
7. Conditional Probability	$P(A \cap B) = P(A) \times P(B/A)$
8. Posterior Probability	$= \dfrac{\text{Joint Probability}}{\text{Sum of Joint Probabilities}}$ here, Joint Probability = Prior Probability × Conditional Probability
9. Expected Value $E(X)$	$E(X) = p_1X_1 + p_2X_2 + ... + p_kX_k$
10. Variance of Expectation	$E(X^2) - [E(X)]^2$ *where*, $E(X)^2 = \Sigma px^2$

10 Theoretical Distributions— Binomial Distribution

IMPORTANT POINTS TO REMEMBER

1. Binomial Distribution is associated with the name of the french Mathematician James Bernoulli.
2. Binomial Distributions are **discrete** probability distributions
3. In a Binomial Distribution, variance is always less than mean.
4. The Mean of Binomial Distribution is ***np*.**
5. The Variance of Binomial Distribution is ***npq***
6. The Variance of a Binomial Distribution can not exceed ***n*/4.**
7. If Value of $P = 0.5$, the Binomial Distribution is symmetrical.
8. If Value of $P > 0.5$, the Binomial Distribution is skewed to the left.
9. As p increases for fixed n, the Binomial Distribution shifts to the right.
10. As n increases for fixed p, the Binomial Distribution shifts to the right.
11. As p increases for a fixed n, both the mean and mode increase.
12. If in Binomial Distribution Mean is 10 and Standard Deviation is 3, q will be 0.9
13. The results calculated by a student for a Binomial Distribution as mean 10 and standard deviation 4, are not correct.
14. The Sampling Distribution of the Numbers of Successes follows a Binomial Probability Distribution.
15. The First Moment of Binomial Distribution is = 0.
16. The Second Moment of Binomial Distribution is = npq.
17. Binomial Distribution tends to be a Normal Distribution when (a) '*N*' is large (b) *Neither 'p' or 'q'* is close to zero. In practice, the approximation is very good if both np and nq are greater than σ.

LIST OF FORMULAE

1. **Probability Function**	$P(x = r) = {}^nC_r p^r q^{n-r}$, *where* $r = 0, 1, 2, ..., n$ P = Probability of success in a single trial $q = 1 - p$ n = *Number of trials*

	r = *Number of successes in n trials*
Probability of 0 event	$P(r = 0)\ ^nc_0\, p^0\, q^{n-0} = q^n$
Probability of 1 event	$P(r = 1)\ ^nc_1\, p^1\, q^{n-1}$
Probability of 2 events	$P(r = 2)\ ^nc_2\, p^2\, q^{n-2}$
Probability of 3 events	$P(r = 3)\ ^nc_3\, p^3\, q^{n-3}$
Probability of at least 1 event	$P(r = 1$ or more$) = 1 - P\ (r = 0)$
Probability of at least 2 events	$P(r = 2$ or more$) = 1 - [P\ (r = 0) + P\ (r = 1)]$
Probability of at the most 2 events	$P(r = 0, 1$ or $2) = P\ (r = 0) + P(r = 1) + P(r = 2)$
Probability of less than 2 events	$P(r = 0$ or $1) = P\ (r = 0) + P\ (r = 1)$
Probability of more than 2 events	$P(r = 3$ or more$) = 1 - [P\ (r = 0) + P(r = 1) + P(r = 2)]$
2. Expected Frequency Function	$N.\ P\ (r) = N.\ ^nC_r\, p^r\, q^{n-r}$ Thus, expected frequency = Probability of an event × Total no. of observations (N) **Note:** Fitting a binomial distribution means finding out the theoretical expected frequencies.
3. Mean	$= np$
4. Variance	$= npq$

11 Theoretical Distributions — Poisson Distribution

IMPORTANT POINTS TO REMEMBER

1. Poisson Distribution is associated with the name of French Mathematician Simon Denis Poisson.
2. Poisson Distributions are **discrete** probability distributions.
3. In case of Poisson Distribution **Mean & Variance are equal**.
4. In case of Poisson Distribution Mean & Variance is m.
5. In case of Poisson Distribution p is less than q.
6. In case of Poisson Distribution m must be greater than 0.
7. In case of Poisson Distribution $e = 2.7183$.
8. In case of Poisson Distribution as m or n increases, the distribution shifts to right.
9. In case of Poisson Distribution Mean, Variance, Second Moment and Third Moment are equal.
10. In case of Poisson Distribution Skewness (β_1) and Coefficient of Kurtosis (γ_2) are same and are equal to reciprocal of mean/variance
11. In case of Poisson Distribution Coefficient of skewness is square root of reciprocal of mean/variance.
12. The First Moment of Poisson Distribution is = 0.
13. The Second and Third Moment of Poisson Distribution is = m.
14. The Poisson Distribution can frequently be used to approximate Binomial Distribution when—
    (i) ***n***, *i.e.* number of trials, is **indefinitely large**, *i.e. n*
    (ii) ***p***, *i.e.* the probability of success for each trial is **indefinitely small**, *i.e.* $p \to 0$, and
    (iii) $np = m$ (say) is **finite**.

    Note: In practice, the Poisson distribution may be used in place of the binomial distribution, where $n \geq 20$ and $P \leq 0.1$.

LIST OF FORMULAE

1. **Probability Function**	$P(r) = \frac{e^{-m} m^r}{r!}$ *where,* $r = 0, 1, 2, \ldots, e = 2.7183$

	m = mean i.e. np or the average number of occurrences of an event.
Probability of 0 event	$P(r=0) = \dfrac{e^{-m}.m^0}{0!} = e^{-m}$
Probability of 1 event	$P(r=1) = \dfrac{e^{-m}.m^2}{2!}$
Probability of 2 events	$P(r=2) = \dfrac{e^{-m}.m^2}{2!}$
Probability of 3 events	$P(r=3) = \dfrac{e^{-m}.m^3}{3!}$
Probability of at least 1 event	$P(r = 1$ or more$) = 1 - P(r = 0)$
Probability of at least 2 events	$P(r = 2$ or more$) = 1 - [P(r = 0) + P(r = 1)]$
Probability of at the most 2 events	$P(r = 0, 1$ or $2) = P(r = 0) + P(r = 1) + P(r = 2)$
Probability of less than 2 events	$P(r = 0$ or $1) = P(r = 0) + P(r = 1)$
Probability of more than 2 events	$P(r = 3$ or more$) = 1 - [P(r = 0) + P(r = 1) + P(r = 2)]$
2. **Expected Frequency Function**	$N.P(r) = N. \dfrac{e^{-m}.m^r}{r!}$ Thus, expected frequency = Probability of an event × Total No. of Observations (N). **Note:** Fitting a Poisson distribution means finding out the theoretical expected frequencies.
3. **Mean**	$= m = np$
4. **Variance**	$= m$

IMPORTANT POINTS TO REMEMBER

1. Normal Distribution (t-distribution, x^2-distribution, F-distribution) is **continuous** distribution.
2. Normal Distribution is bell shaped and unimodal.
3. In case of Normal Distribution Mean = Median = Mode.
4. In case of Normal Distribution Skewness = 0
5. In case of Normal Distribution Mean Deviation = 4/5th or more precisely 0.7979 of the Standard Deviation.
6. In case of Normal Distribution Quartile Deviation is 2/3rd or more precisely 0.6745 of the Standard Deviation.
7. In case of Normal Distribution Q_1 and Q_3 are equidistant from the median and hence Q_1 = Mean – .6745 σ , Q_3 = Mean + .6745 σ **[Note:** Mean = Median = Mode]
8. In case of Normal Distribution Med. – $Q_1 = Q_3$ – Med.
9. In case of Normal Distribution Highest Value = Mean + Half of Range &
   Lowest Value = Mean – Half of Range
10. In case of Normal Distribution Mean ± 1 σ covers 68.27% of area,Mean ± 3 σ covers 95.45% of area,Mean ± 3 σ covers 99.73% of area
11. Under a normal distribution the area enclosed between Mean and 1 σ = 0.34135. Mean and 2 σ = 0.47725 and Mean and 3 σ = 0.49865.
12. Normal Distribution with $X = 0$ and $\sigma = 1$ is known as Standard Normal Distribution.
13. The height of normal curve is maximum at the Mean Value.

LIST OF FORMULAE

1. Mean	Mean = μ
2. Variance	Variance = σ^2
3. Probability Function (or Density Function)	$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (-\infty < x < \infty)$ *where,* μ = Mean of the normal random variable X. σ = Standard deviation of the given normal distribution. [Two parameters]

	$\sqrt{2}\pi = 2.5066$ $e = 2.7183$ [Two constants]
4. Density Function in Terms of Standard Normal Variable	$P(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(z)^2}$ *where,* $z = \frac{X-\mu}{\sigma}$ Mean = 0 , Standard deviation = 1

COMPARATIVE STUDY OF THEORETICAL DISTRIBUTIONS

	Basis of Comparison	*Binomial Distribution*	*Poisson Distribution*	*Normal Distribution*
1	**Type of Distribution**	Discrete Probability Distribution	Discrete Probability Probability	Continuous Distribution Distribution
2	**Parameters**	Biparametric *n, p*	Uniparametric *m*	μ, σ
3	**Restriction on Parameters**	$0 < p < 1$	$m > 0$	$\mu = 0$ $\sigma = 1$
4	**Mean $(\bar{X})$**	$\bar{X} = np$	$\bar{X} = m$	$\bar{X} = \mu$
5	**Variance (σ^2)**	$\sigma^2 = npq$	$\sigma^2 = m$	σ^2
6	**Probability Function** **(or Density Function)**	$P(r) = {}^nCr\, p^r\, q^{n-r}$	$P(r) = \frac{e^{-m} m^r}{r!}$ *where,* $r = 0, 1, 2$.......... m = Mean	$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $(-\infty < x < \infty)$ *where,* μ = Mean $e = 2.7183$ (the base of natural logarithms) σ = Standard Deviation
7	**Expected Frequency Distribution**	$N. P(r)$ $= N. {}^nCr\, p^r q^{n-r}$	$N.P(r) = N.\frac{e^{-m} m^r}{r!}$	
8	**Density Function in terms of Standard Normal Variate**			$P(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(z)^2}$ *where,* $z = \frac{X-\mu}{\sigma}$ Mean = 0 Standard deviation = 1
9	**Skewness/ Shape**	P = 0.5 Normal P < 0.5 Skewed to right P > 0.5 Skewed to left	Positively Skewed to right	Skewnes = 0 Bell–Shaped
10	**Mode**		Unimodal or bimodal	Unimodal

IMPORTANT POINTS TO REMEMBER

1. **A Time Series** is a set of observations taken at specified times, usually at 'equal intervals'. Mathematically, a time series is defined by the values Y_1, Y_2 ... or variable Y (temperature, closing price of a share, etc.) at times t_1, t_2.... Thus Y is a function of t, symbolised by $Y = F(t)$.

2. **Essential requirements of a Time Series** are:
   1. Time series must consist of a homogeneous set of values.
   2. It must consist of data for a sufficiently long period.
   3. Time elapsing between various observations must as far as possible be equal.
   4. The gaps if any in the data should be made up by interpolation.

3. **The main objectives of analysing Time Series are:**
   1. To study the past behaviour of the data.
   2. To predict the future on the basis of past experience.
   3. To segregate the effects of various factors which might affect time series.

4. **Four components of a Time Series are :**
   1. Secular Trend (T)
   2. Seasonal Variations (S)
   3. Cyclical Variations (C)
   4. Irregular/Random Movements (I)

5. **Two Models for decomposition of Time Series into its four components are:**
   1. **Additive Model** — Additive model assumes that all the components of the time series are independent of one another and describes all the components as absolute values. The original data (Y) is expressed as a sum of four components as follows:

      $$Y = T + C + S + I$$

      where, Y = Observed value in a given time series, T = Trend, C = Cyclical Variations, S = Seasonal Variations and I = Irregular Variations
   2. **Multiplicative Model** — Multiplicative model assumes that all the four components are due to different causes but they are not necessarily independent and they can affect one another. It describes only the trend (T) as an absolute value while other components (i.e., C, S & I) are expressed as rate or percentage. Thus, a seasonal index of 89% would indicate that the actual value is expected to be 11% lower than it would be without the seasonal influence. The original data (Y) is expressed as a product of four components as follows:

      $$Y = T \times C \times S \times I$$

6. **Secular Trend** is the long-term tendency of steady movements in a set of observations to move in an upward or downward or constant direction over a fairly long period of time. It does not include short term variations like seasonality, irregularity etc.

7. **Methods of Measuring Trend are:**
   (1) Freehand or graphic method,
   (2) Semi-average method,
   (3) Moving average method,
   (4) Least squares method.

8. **Extrapolation:** The process of extending the trend into the future is known as 'Extrapolation'.

9. **Elimination of Trend**: The elimination of trend provides the short term fluctuations (i.e. seasonal, cyclical and irregular). Elimination of trend under both the models can be shown as follows:

   Under Additive Model: $O - T = S + C + I$.

   Under Multiplicative Model: $O/T = S \times C \times I$

10. **Seasonal Variations(S)** are the regular periodic changes which occur within a period of less than a year. Weather and social customs are the most important factors causing seasonal variations.

11. **Cyclical Variations(C),** which are also generally termed as business cycles, are the periodic movements in the time series around the trend line which occur during intervals of time of more than one year, say, once in 3 to 7 years.

12. **Irregularities (I)** are irregular variations which occur on account of random external events. These variations either go very deep downward or too high upward to attain peaks abruptly. These variations may occur due to strikes, lockouts, floods, wars, elections etc.

13. **Free Hand Method** is the simplest method of measuring trend under which the trend line is drawn by an experienced statistician. It is highly subjective because it involves personal judgement of the statistician. It cannot be adopted in drawing non-linear trend lines.