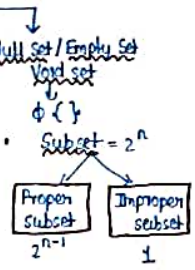
6. Set, Function & Relation

- Cardinal number = $n(A) = x$
- Types of set →



- Subset

e.g. A = {2,3,5}
B = {3,5}
B ⊂ A
No. of subset = 2^n

- Subset = 2^n

- Operation of Set

1) Union → (A ∪ B)
2) Intersection → (A ∩ B)
3) Subtraction → A = {2,3,7,8,9}
B = {1,5,7,9,10}
A − B = {2,3,8}

★ Theorem of addition :-

1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
3) $n(A \cap B') = n(A) - n(A \cap B)$
4) $n(A' \cap B) = n(B) - n(A \cap B)$
5) Total = Atleast + Nothing.

★ Formula for three sets :-

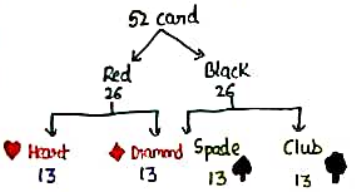
① $n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C)$
② $n(A \cap B' \cap C) = n(A \cap C) - n(A \cap B \cap C)$
③ $n(A' \cap B \cap C) = n(B \cap C) - n(A \cap B \cap C)$
④ $n(A \cap B' \cap C') = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
⑤ $n(A' \cap B \cap C') = n(B) - n(B \cap A) - n(B \cap C) + n(A \cap B \cap C)$

★ Domain & Co-Domain

e.g. {(3,8) (5,2) (7,5) (6,1)}
Domain = input = {3,5,7,6}
Co-domain = Output = {8,2,5,1}

PROBABILITY

Formula = $\frac{\text{Special way}}{\text{Normal way}}$



- $P(A) = 0.3$ $P'(A) = 1 - 0.3 = 0.7$
- $P(B) = 0.45$ $P'(B) = 1 - 0.45 = 0.55$
- $P(C) = 0.75$ $P'(C) = 1 - 0.75 = 0.25$

★ ODD's in favour of event & ODD's against event.

- ODD's in favour of event = $\frac{\text{Favourable}}{\text{Unfavourable}}$
- ODD's against = $\frac{\text{Unfavourable}}{\text{Favourable}}$
- $P(A) = \frac{F}{F+U} = \frac{3}{10}$

★ Types of event

1) Sure event → $P(A) = 1$
2) Impossible event → $P(A) = 0$
3) Exclusive event → $P(A \cap B) = 0$
4) Exhaustive event → $P(A \cup B) = 1$
5) Equally likely event → $P(A) = P(B)$
6) Dependent event = $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$P(B/A) = \frac{P(A \cap B)}{P(B)}$

7) Independent event = $P(A \cap B) = P(A) \cdot P(B)$

★ Problems of expected value

Expected value → [avg. value] [mean]

$E(x) = \Sigma x \cdot P$

$E(x^2) = \Sigma x^2 \cdot P$

Variance = $E(x^2) - [E(x)]^2$

INDEX NUMBER

1) Price Index Number :-

$P_{on} = \frac{P_n}{P_o} \times 100$

2) Quantity Index Number :-

$Q_{on} = \frac{Q_n}{Q_o} \times 100$

3) Value Index Number :-

$V_{on} = \frac{V_n}{V_o} \times 100$

4) Simple Aggregative method :-

$P_{on} = \frac{\Sigma P_n}{\Sigma P_o} \times 100$

5) Simple relative method :-

$P_{on} = \frac{\Sigma \frac{P_n}{P_o}}{N} \times 100$

6) Weighted relative method :-

$P_{on} = \frac{\Sigma \frac{P_n}{P_o} \cdot W}{\Sigma W} \times 100$

$= \frac{\Sigma I \cdot W}{\Sigma W}$

7) Weighted aggregative method :-

$P_{on} = \frac{\Sigma P_n \cdot W}{\Sigma P_o \cdot W} \times 100$

8) Laspeyre's method = $\frac{\Sigma P_n \cdot q_o}{\Sigma P_o \cdot q_o} \times 100$

9) Paasche's method = $\frac{\Sigma P_n \cdot q_n}{\Sigma P_o \cdot q_n} \times 100$

10) Fisher = $P_{on} = \sqrt{L \times P}$

11) Bowley = $P_{on} = \frac{L+P}{2}$

12) Marshall-Edgeworth method → $P_{on} = \frac{\Sigma P_n \cdot \left[\frac{q_o + q_n}{2}\right]}{\Sigma P_o \cdot \left[\frac{q_o + q_o}{2}\right]} \times 100$

13) CLI = $\frac{\Sigma I \cdot W}{\Sigma W}$

RSION

* Properties of Dispersion

1) If all observation are same then Range = QD = MD = SD are also same.

[$\bar{x}$ = med = mode = GM = HM
R = MD = S.D = Q.D = 0]

2) For 2 numbers a and b
Range = b − a
$\sigma = \frac{b-a}{2} = \frac{R}{2}$

3) Measures of Dispersion

- Absolute measure → has some unit → R, MD, SD, QD
- Relative measure → No unit → co-eff. of range, co-eff. of MD, co-eff. of SD, co-eff. of QD

4) Application of C.V
Less C.V → more consistent, more stable

5) Combined S.D
Steps
① $\bar{x}_{12} = \frac{N_1\bar{x}_1 + N_2\bar{x}_2}{N_1+N_2}$
② $d_1 = \bar{x}_{12} - \bar{x}_1$
$d_2 = \bar{x}_{12} - \bar{x}_2$
③ $\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2+d_1^2)+N_2(\sigma_2^2+d_2^2)}{N_1+N_2}}$

6) Change of Scale & Change of Origin :-

Central tendency	Dispersion
↓	↓
AM, Median, Mode	Range, S.D, M.D, Q.D
↓	↓
(×, ÷) Scale → ✓	Scale → ✓
(+, −) origin → ✓	Origin → ×
↓	↓
$y = a + bx$	$y = a + bx$
$\bar{y} = a + b\bar{x}$	$R_y = \lvert b\rvert . R_x$
$me_y = a + b\, me_x$	$\sigma_y = \lvert b\rvert . \sigma_x$
$mo_y = a + b\, mo_x$	$M.D_y = \lvert b\rvert . M.D_x$
	$S.D_y = \lvert b\rvert . S.D_x$
	$Q.D_y = \lvert b\rvert . Q.D_x$

1) CENTRAL TENDENCY

1. If all obsev. are same then AM, GM & HM are also same.
AM = GM = HM

2. Relationship between AM, GM & HM

Case A :- for 2 numbers
$AM = \frac{a+b}{2}$
$GM = \sqrt{ab}$
$HM = \frac{2ab}{a+b}$ $GM^2 = AM \times HM$

Case B :- for any number
AM ≥ GM ≥ HM

3. Application of GM & HM
GM & HM are used for finding average rate & average interest.

4. Problems of average speed
$Speed = \frac{Distance}{Time}$
Case I] Distance is same → Use HM
Case II] Time is same → Use AM
Case III] Distance & time both are different → Average speed = Total Dis / Total Ti

5. Combine Harmonic mean
$H_{12} = \frac{N_1 + N_2}{\frac{N_1}{H_1} + \frac{N_2}{H_2}}$

8. DERIVAITIVE & INTEGRATION

	f(x)		f'(x)
1)	x^n	→	$n.x^{n-1}$
2)	a^x	→	$a^x.\log a$
3)	x^2	→	$2x$
4)	x	→	1
5)	e^x	→	e^x
6)	$\log x$	→	$\frac{1}{x}$
7)	$\frac{1}{x}$	→	$-\frac{1}{x^2}$
8)	$\sqrt{x}$	→	$\frac{1}{2\sqrt{x}}$
9)	k	→	0

10) $y = u \cdot v$
$\frac{dy}{dx} = \frac{d}{dx}(u).v + u\frac{d}{dx}(v)$

11) $y = \frac{u}{v}$
$\frac{dy}{dx} = \frac{\frac{d}{dx}u(v) \overset{(-)}{\bullet} u\frac{d}{dx}(v)}{v^2}$

Note :- Agr variable ke upr variable hai to both the side log leneka.
e.g. $x^x = \log x^x = x.\log x$

BELIEVE IN YOURSELF

1. Ratio & Proportion

$\frac{a}{b}$ → Antecedent
$\frac{a}{b}$ → Consequent

OPERATIONS RATIO

1) Inverse Ratio → $\frac{a}{b} \to \frac{b}{a}$
2) Duplicate " → $\frac{a}{b} \to \frac{a^2}{b^2}$
3) Triplicate " → $\frac{a}{b} \to \frac{a^3}{b^3}$
4) Sub-duplicate " → $\frac{a}{b} \to \frac{\sqrt{a}}{\sqrt{b}}$
5) Sub-triplicate " → $\frac{a}{b} \to \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$
6) Compound " → $\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f}$

OPERATION ON PROPORTION

1) Inverendo → $\frac{a}{b} = \frac{c}{d} \to \frac{b}{a} = \frac{d}{c}$
2) Alternendo → $\frac{a}{b} = \frac{c}{d} \to \frac{a}{c} = \frac{b}{d}$
3) Componendo → $\frac{a}{b} = \frac{c}{d} \to \frac{a+b}{b} = \frac{c+d}{d}$
4) Dividendo → $\frac{a}{b} = \frac{c}{d} \to \frac{a-b}{b} = \frac{c-d}{d}$
5) Componendo - dividendo → $\frac{a}{b} = \frac{c}{d} \to \frac{a+b}{a-b} = \frac{c+d}{c-d}$
6) Adendo → $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \to \frac{a+c+e}{b+d+f}$
7) Subtra Hendo → $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a-c-e}{b-d-f}$

2. LOGARITHMS

★ Property

1) $\log_a m = n \to a^n = m$
2) $\log_a m + \log_a n = \log_a mn$
3) $\log_a m - \log_a n = \log_a \frac{m}{n}$
4) $\log_a (m^n) = n \cdot \log_a m$
5) $\log_a 1 = 0$ 6) $\log_a a = 1$
7) $\log_a b = \frac{\log_x b}{\log_x a}$ → Change of base

★ Base by default (10) hota hai

3. INDICES

★ Properties of Indices

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{-m} = \frac{1}{a^m}$
- $a^m = \frac{1}{a^{-m}}$
- $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

★ FORMULA

- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $a^2 - b^2 = (a+b)(a-b)$

★ Roots

$2^1 = 2$	$3^1 = 3$	$5^1 = 5$
$2^2 = 4$	$3^2 = 9$	$5^2 = 25$
$2^3 = 8$	$3^3 = 27$	$5^3 = 125$
$2^4 = 16$	$3^4 = 81$	$5^4 = 625$
$2^5 = 32$	$3^5 = 243$	$5^5 = 3125$
$2^6 = 64$	$3^6 = 729$	$5^6 = 18,750$
$2^7 = 128$	$3^7 = 2187$	
$2^8 = 256$	$3^8 = 6561$	
$2^9 = 512$	$3^9 = 19683$	
$2^{10} = 1024$	$3^{10} = 59,049$	

4. EQUATION

Quadratic equation → $ax^2 + bx + c = 0$

Formula → $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

★ Properties of roots

1) $\alpha + \beta = -\frac{b}{a}$
2) $\alpha \cdot \beta = \frac{c}{a}$
3) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
4) $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$
5) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta)$
6) $\alpha^3 - \beta^3 = (\alpha-\beta)(\alpha^2 + \beta^2 + \alpha\beta)$

★ In Case of reciprocal

If $\alpha = 5$ so $\beta = \frac{1}{5}$

FACTORIALS

$0! = 1$
$1! = 1$
$2! = 2$
$3! = 6$
$4! = 24$
$5! = 120$
$6! = 720$
$7! = 5040$
$8! = 40320$
$9! = 362880$

5. TIME VALUE OF MONEY

1. Simple Interest

$I = \frac{P \times n \times r}{100}$
$A = P + I$
$A = P\left[1 + \frac{nr}{100}\right]$

2. Compound Interest

$A = P(1+i)^n$
$I = A - P$
$i = \frac{\text{rate of interest}}{1/2/4/12}$ → [as per given in question]

- Yearly = $n = \times 1$
- Half-yearly = $n = \times 2$
- Quaterly = $n = \times 4$
- Monthly = $n = \times 12$

3. Application of compound interest

★ In the problems of population

Increase, $A = P(1+i)^n$
Decrease, $A = P(1-i)^n$

P = Initial population
A = Final population
n = no. of years
i = Rate of growth = Birth rate - death rate

4. In Problem of Depreciation

$S.V = C.P(1-i)^n$

S.V → Scrap value, C.P → Cost price
i → Rate of dep., n → effective life

5. Effective rate of interest

$ie = (1+i)^n - 1$

ie → Effective rate, i → Nominal rate
n → It is always 1 year.

6. Future value of annuity

a) Regular annuity = F.V → [Payment at end]
$F.V = \frac{A[(1+i)^n - 1]}{i}$
A = annuity

b) Annuity due = F.V → [Payment at start]
$F.V = \frac{A[(1+i)^n - 1](1+i)}{i}$

[If nothing is mention in sum to hamesha sum of (Regular) Ka hi consider Krneka]

7. Present value

V → Present value
$V = \frac{A[(1+i)^n - 1]}{i[(1+i)^n]}$

Short formula, A × PVF
PVF → $\frac{1}{1+i}$ → MR + (Calculation)
MR +
MR +
⋮
MRC -[Jitna (n) diya hoga utne time (MR+)]

CENTRAL TENDENCY

1. Arithmetic Mean
- Average

$\bar{x} = \frac{\Sigma x}{N}$ .... discrete without frequency

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ .... discrete with frequency

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ .... continuous data, x = midpoint

2. Median – (Middle most value)
a) Discrete without frequency →
- first arrange data in A·O
- Median = $\left(\frac{n+1}{2}\right)^{th}$ value

N = no. of observation

b) Discrete with frequency
- Find less than C·F
- Find $\frac{N+1}{2}$ ($N = \Sigma f$)
- Check frequency ∵ C·F ≥ $\frac{N+1}{2}$
- C·F → x → median

c) Median for continuous data
- Find less than C·F.
- Find $\frac{N}{2}$ ($N = \Sigma f$)
- Check C·f. ≥ $\frac{N}{2}$
- C·F → Class → median Class
- Median = $L + \left(\frac{N}{2} - C\cdot f\right) \times \frac{h}{f}$

☆ Mode
Value having maximum frequency repetition.

- Continuous data

$$\text{Mode} = L + \frac{(f_1 - f_0) \times h}{2f_1 - f_0 - f_2}$$

L = Lower Class boundary
h = UCB - LCB
f_0 = Previous Class frequency
f_1 = maximum frequency
f_2 = next class frequency.

4] Harmonic Mean
$HM = \frac{N}{\Sigma \frac{1}{x}}$ – [Discrete without frequency]

$HM = \frac{\Sigma f}{\Sigma \frac{f}{x}}$ – [Discrete with frequency]

$HM = \frac{\Sigma f}{\Sigma \frac{f}{x}}$ – [Continuous]

5] Geometric Mean
$GM = (x_1 \cdot x_2 \cdot x_3 \ldots x_n)^{\frac{1}{N}}$ – (Without frequency)

$GM = (x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \ldots)^{\frac{1}{\Sigma f}}$ – (With frequency)

$GM = (x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \ldots)^{\frac{1}{\Sigma f}}$ – (Continuous)

☆ Properties of mean, median & mode
1. If all observation are same then mean, median, mode are also same. [$\bar{x}$ = median = mode]
2. Relation between mean, median & mode

   For symmetric, $\bar{x}$ = median = mode

   For asymmetric, $\bar{x}$ – mode = 3 ($\bar{x}$ – median)
3. Combine arithmatic mean

$$\bar{x}_{12} = \frac{N_1\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2}$$

4. Change of scale & change of origin

   Mean, median, mode are affected by both
   Change of scale (×, ÷) ; $y = a + bx$
   Change of origin (+, −) ; $\bar{y} = a + b\bar{x}$
   Key = a + b mean z
5. Sum of deviation of all observation about Arithmatic mean is zero.
   i.e. $\Sigma(x - \bar{x}) = 0$
6. Sum of absolute deviation of all observation is minimum when taken above median.

   Absolute = Positive = Mod

Median = Q_2 = D_5 = P_{50} are always equal

Median
He

a) Discrete without frequency
- first arrange in A·O
- Median = $\left(\frac{n+1}{2}\right)^{th}$ value

b) Discrete with frequency
- Find less than C·F
- Find $\frac{N+1}{2}$
- Check that CF ≥ $\frac{N+1}{2}$
- Check → x → median

c) Continuous data
- Find less than c·f
- Find $\frac{N}{2}$
- C·F ≥ $\frac{N}{2}$
- C·F → Class → Median Class

Median = $L + \left(\frac{N}{2} - C\cdot f\right) \times \frac{h}{f}$

Quartile
Q_1 Q_2 Q_3

a) Discrete without freq.
- first arrange in A·O
- $Q_P = \left[\frac{(N+1)P}{4}\right]^{th}$ value

b) Discrete with freq.
- Find less than C·F
- Find → $\frac{(N+1)P}{4}$
- Check that C·F ≥ $\frac{(N+1)P}{4}$
- C·F → x → quartile

c) Continuous data
- Find less than C·F
- Find $\frac{NP}{4}$
- C·F ≥ $\frac{NP}{4}$
- C·F → Class → Quartile class

$Q_P = L + \left(\frac{NP}{4} - C\cdot f\right) \times \frac{h}{f}$

Decile
D_1 D_2 D_3 ... D_9

a) Discrete without freq.
- first arrange in A·O
- $D_P = \left[\frac{(N+1)P}{10}\right]^{th}$ value

b) Discrete with frequency
- → Find less than C·F
- → Find $\frac{(N+1)P}{10}$
- → Check that C·F ≥ $\frac{(N+1)P}{10}$
- → C·F → x → Decile

Percentile
P_1 P_2 P_3 ..... P_{99}

a) Discrete without freq.
- → First arrange in A·O
- → $P_P = \left[\frac{(N+1)P}{100}\right]^{th}$ value

b) Discrete with freq.
- → Find less than C·F
- → Find $\frac{(N+1)P}{100}$
- → Check that C·F ≥ $\frac{(N+1)P}{100}$
- → C·F → x → Percentile