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CA FOUNDATION

MATHEMATICS AND LOGICAL REASONING

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PREFACE

Dear Student,

Welcome to the World of Knowledge - **J.K. Shah Classes !**

I have the pleasure of presenting this study material to you. It contains good number of good problems, selected so carefully from wide-ranging sources. It covers the problems which will bring in to focus all important concepts that you need to study in order to fortify yourself for your examination.

The subject will be taught by eminent professors who are highly experienced and well-versed with the job.

The coaching is very exhaustive and wholly concept based. The conceptual explanations are entirely supported by good problems that cover the past and the problems which peep into the future. Also, the coaching is very systematic, well - planned and absolutely time bound. For a change, say good - bye to mechanical learning. I am sure you will feel that the study is a pleasurable job and not a painful exercise.

I wish you a very happy study time. **BEST OF LUCK !**

Prof. J.K. Shah.
Chartered Accountant

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CA FOUNDATION MATHEMATICS

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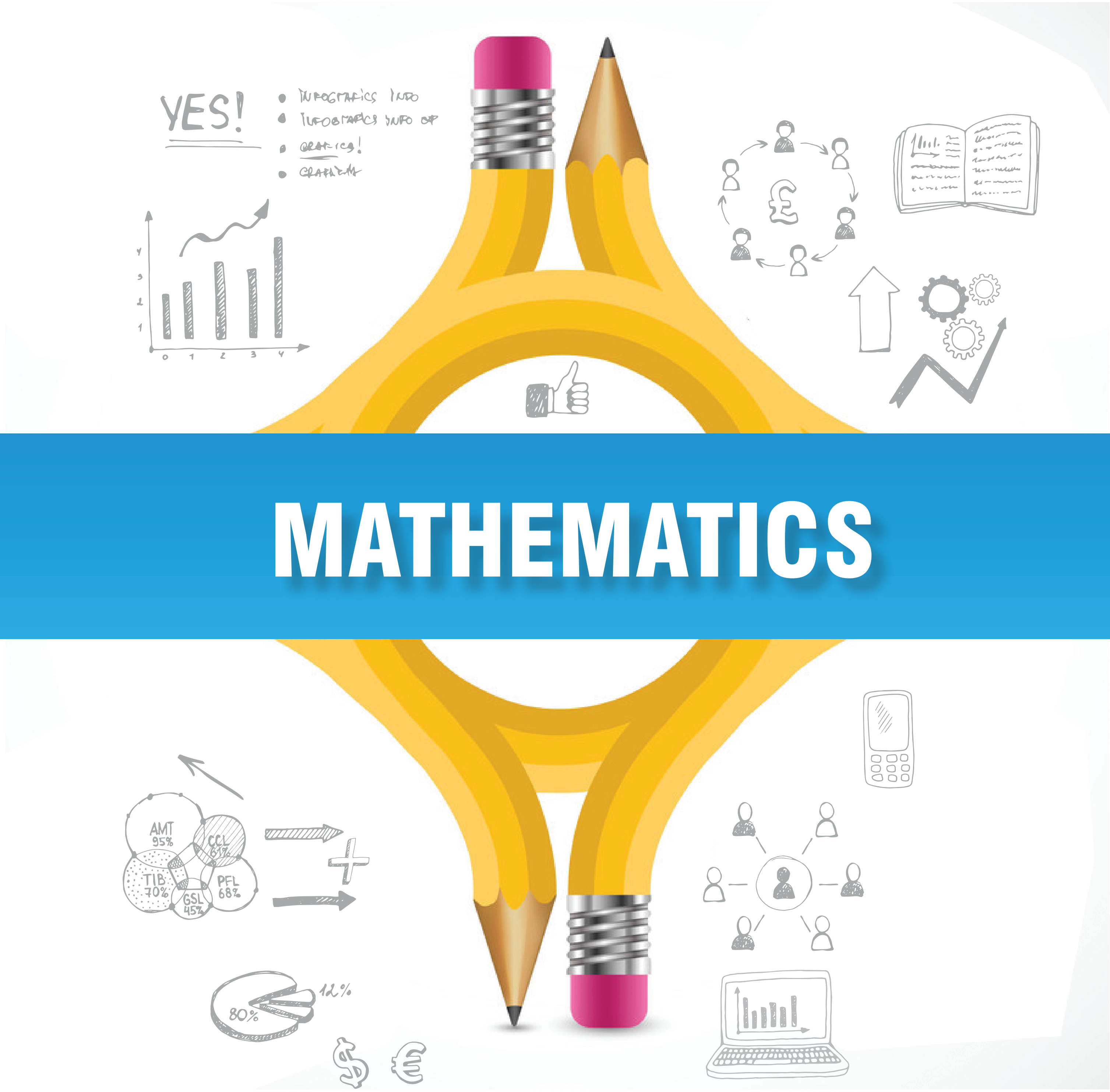
MATHEMATICS



1A Ratio, Proportion & Partnership

THEORY

Ratio

- A ratio is a fraction (either proper or improper) which compares two or more quantities of similar kind, which enables us to understand as to how many times one quantity is involved in the other.

- If A : B ($\frac{A}{B}$) is a ratio, then the numerator A is called "Antecedent" and the denominator B is called the "Consequent".

- Ratios must be expressed in the simplest possible form and we can calculate ratios only when the quantities are commensurable (fully quantifiable).

- Two or more ratios can be bridged in order to have a continuous comparison between more than two variables.

- Rule for bridging more than two ratios :

  If ,a,b,c,d,e are five Quantities, and

$$\frac{a}{b}=\frac{N_1}{D_1};\frac{b}{c}=\frac{N_2}{D_2};\frac{c}{d}=\frac{N_3}{D_3};\frac{d}{e}=\frac{N_4}{D_4}$$

Then, a:b:c:d:e= $N_1N_2N_3N_4 : D_1N_2N_3N_4 : D_1D_2N_3N_4 : D_1D_2D_3N_4 : D_1D_2D_3D_4$

Let a : b is a ratio, then:

- $\frac{a}{b}>1$ (Ratio of Greater Inequality)
- $\frac{a}{b}<1$ (Ratio of Lesser Inequality)
- $\frac{a}{b}=1$ (Ratio of Equality)

- $a^2 : b^2$ (Duplicate Ratio)
- $a^3 : b^3$ (Triplicate Ratio)
- $\sqrt{a} : \sqrt{b}$ (Sub-Duplicate Ratio)
- $\sqrt[3]{a} : \sqrt[3]{b}$ (Sub-Triplicate Ratio)
- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots\ldots$ If then the value of each ratio can be obtained by mean of any one of the following two operations;

a. Each ratio = $\frac{a+c+e+\ldots}{b+d+f+\ldots}$ (ADDENDO)

Or

b. Each ratio = $\frac{a-c-e-\ldots}{b-d-f-\ldots}$ (SUBTRANDENDO)

INVERSE RATIO:

- IR of a:b is b : a
- IR of a:b:c is bc : ac : ab
- IR of a:b:c:d is bcd : acd : abd : abc

COMPOUND RATIO:

The multiplying effect of all ratios given is known as compound ratio. If a:b and c:d are two ratios, then ac : bd is called the compounded ratio of the two.

Proportion

- Proportion is defined as the equality of two or more ratios. If $\frac{a}{b} = \frac{c}{d}$, in such a case the quantities a,b,c,d are said to be proportional, here 'd' is called the fourth proportional.
- If $\frac{a}{b} = \frac{b}{c}$, then a,b,c are said to be in continued proportion, where 'b' is called the mean proportional and 'c' is called third proportional.
- If $\frac{a}{b} = \frac{b}{c}$ or b^2 = ac $\therefore$ b = $\sqrt{ac}$

IF	THEN	PROPERTY
$\frac{a}{b}=\frac{c}{d}$	ad = bc	PRODUCT OF EXTREMES = PRODUCT OF MEANS
	$\frac{b}{a}=\frac{d}{c}$	INVERTENDO
	$\frac{a}{c}=\frac{b}{d}$	ALTERNENDO
	$\frac{a+b}{b}=\frac{c+d}{d}$	COMPONENDO
	$\frac{a-b}{b}=\frac{c-d}{d}$	DIVIDENDO
	$\frac{a+b}{a-b}=\frac{c+d}{c-d}$	COMPONENDO & DIVIDENDO
	$\frac{a-b}{a+b}=\frac{c-d}{c+d}$	DIVIDENDO & COMPONENDO

CLASSWORK SECTION

1. Two numbers are in the ratio 5 : 6. If 5 is subtracted from each number, the ratio becomes 4 : 5. The numbers are:
   a) 15, 20 b) 5, 10
   c) 10, 15 d) 25, 30

2. Two numbers are in the ratio 3 : 4. If 6 be added to each terms of the ratio, then the new ratio will be 4 : 5. The two numbers are:
   a) 24, 32 b) 18, 24
   c) 15, 20 d) 9, 12

3. Daily earnings of two persons are in the ratio 4 : 5 and their daily expenses are in the ratio 7 : 9. If each saves ₹ 50 per day, their daily incomes are ₹
   a) (40, 50) b) (50, 40)
   c) (400, 500) d) None of these

4. The sum of the ages of 3 persons is 150 years. 10 years ago their ages were in the ratio 7 : 8 : 9. Their present ages are:
   a. 40, 60, 50 b. 50, 45, 55
   c. 55, 35, 60 d. 45, 50, 55

5. Moi earns ₹ 80 in 7 hours and Zen earns ₹ 90 in 12 hours. The ratio of their earnings is:
   a) 32 : 21 b) 23 : 12
   c) 8 : 9 d) None of the above

6. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 kms in 5 hrs, what is the speed of the first train?
   a) 10 km per hour b) 70 km per hour
   c) 50 km per hour d) None of the above

7. The ratio of the speeds of two trains is 2 : 5. If the distances they travel are in the ratio 5 : 9, find the ratio of the times taken by them.
   a. 18 : 25 b. 5 : 4
   c. 25 : 18 d. 1:1

8. If x/2 = y/3 = z/7, then find the value of (2x – 5y + 4z) / 2y.
   a) 6/23 b) 23/6 c) 3/2 d) 17/6

9. The ratio of the number of 50 paise, Re. 1 and ₹ 5 coins with Mr. Zen is 5 : 2 : 1. If the amount with him is ₹ 38, then the number of Re. 1 coins with him is:
   a) 4 b) 8 c) 12 d) 16

10. A boy has five rupee coins, two rupee coins and one rupee coins in the ratio 3:4:5. If he has an amount of ₹ 224, then find the numbers of one rupee coins with the boy.
   a) 40 b) 35 c) 20 d) 30

11. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$. Then find the value of each ratio.
   a. 1 b. $\frac{1}{2}$ c. $\frac{1}{20}$ d. None of the above

12. A precious stone worth ₹ 15,600 is accidently dropped and broken into three pieces, the weights of which are respectively proportional to 2 : 3 : 5. The value of the stone of this variety varies as the cube of its weight. Calculate the percentage loss thus incurred by this breakage.
   a) 80% b) 90%
   c) 84% d) 78%

13. An employer reduces the number of employees in the ratio of 19 : 16 and increases their wages in the ratio of 4 : 5. What is the ratio of the wage bill of the employer initially and now?
   a. 20 : 19 b. 17 : 16
   c. 16 : 17 d. 19 : 20

Types of Ratios:

14. The duplicate ratio of 3 : 4 is
   a) $\sqrt{3}:2$ b) 4 : 3
   c) 9 : 16 d) None of these

15. The sub-duplicate ratio of 25 : 36 is

a) 6 : 5 b) 36 : 25
c) 50 : 72 d) 5 :6

16. The triplicate ratio of 2 : 3 is

a) 8 : 27 b) 6 : 9
c) 3 : 2 d) None of these

17. The sub-triplicate ratio of 8 : 27 is

a) 27 : 8 b) 24 : 81
c) 2 : 3 d) None of these

18. If p : q is the sub-duplicate ratio of $(p - x^2) : (q - x^2)$, then find the value of x^2.

a) $p / (p + q)$ b) $q / (p + q)$
c) $pq / (p - q)$ d) $pq / p+q$

Compound Ratio

19. The ratio compounded of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is

a) 1 : 1 b) 1 : 5
c) 3 : 8 d) None of these

20. The ratio compounded of 4 : 9, the duplicate ratio of 3 : 4, the triplicate ratio of 2 : 3 and 9 : 7 is

a) 2 : 7 b) 7 : 2
c) 2 : 21 d) None of these

21. Find the compounded ratio of 275 : 31, inverse of 729 : 1331, duplicate ratio of 2 : 5, triplicate ratio of 9 : 11, sub-duplicate ratio of 961 : 1296, sub-triplicate ratio of 729 : 1331.

a. 1 : 1 b. 1 : 2
c. 275 : 11 d. 31 : 25

Inverse Ratio

22. The Inverse ratio of 11 : 15 is
    a) 15:11 b) $\sqrt{11}:\sqrt{15}$
    c) 121 : 255 d) None of these

23. The ratio of the quantities is 5 : 7. If the consequent of its inverse ratio is 5, the antecedent is
    a) 5 b) $\sqrt{5}$ c) 7 d) None of these

Joint Ratio

24. If $\frac{a}{b}=\frac{2}{3}$ and $\frac{b}{c}=\frac{4}{5}$, the ratio a : b : c = ?
    a) 4 : 6 : 15 b) 4 : 8 : 15
    c) 8 : 12 : 15 d) 8 : 16 : 25

25. If A : B = 2 : 3, B : C = 4 : 5 and C : D = 3 : 7, find A : B : C : D
    a) 4 : 6 : 15 : 35 b) 4 : 12 : 15 : 35
    c) 8 : 12 : 15 : 35 d) 8 : 16 : 25 : 35

26. If a : b = 3 : 5, b : c = 5 : 4, c : d = 2 : 3 and d is 50% more than e, find the ratio between a and e.
    a) 2 : 3 b) 3 : 4
    c) 3 : 5 d) 4 : 5

27. Aoi, Boi and Coi work in a company. The ratio of Aoi's age to Boi's age is 11 : 13 and Boi's age to Coi's age is 13:14. If the sum of their ages is 76, what are their respective ages?
    a) 33, 39, 42 b) 23, 27, 32
    c) 22, 26, 28 d) 24, 28, 30

28. If $\frac{a}{b}=\frac{2}{3}$ and $\frac{b}{c}=\frac{4}{5}$, then find the value of $\frac{a+b}{b+c}$
    a) 8 : 15 b) 20 : 27
    c) 3 : 4 d) 27 : 20

29. Ahmedabad, Bombay and Calcutta are three cities. The ratio of average temperature between Ahmedabad and Bombay is 11 : 12 and the average between Ahmedabad and Calcutta is 9 : 8. Then the ratio between the average temperature of Bombay and Calcutta is:

a) 22 : 27 b) 27 : 22
c) 32 : 33 d) None of the above

30. A man distributes his property of ₹ 6,00,000 among his three sons. The share of his first son is thrice that of the second son's share and the share of the second son is twice that of the third son. Find the ratio in which sons share the property.

a) 1 : 2 : 6 b) 3 : 4 : 5
c) 6 : 2 : 1 d) 2 : 4 : 6

Proportion

31. The fourth proportional to 4, 6, 8 is

a) 12 b) 32
c) 48 d) None of these

32. The third proportional to 12, 18 is

a) 24 b) 27
c) 36 d) None of these

33. The mean proportional between 25, 81 is

a) 40 b) 50
c) 45 d) None of these

34. The fourth proportional to 2a, a^3 & c is

a) ac/2 b) ac
c) 2/ac d) $a^2c/2$

35. If four numbers 1/2, 1/3, 1/5, 1/x are proportional then x is

a) 6/5 b) 5/6
c) 15/2 d) None of these

36. The mean proportional between $12x^2$ and $27y^2$ is

a) 18xy b) 81xy
c) 8xy d) None of these

37. If x / y = z / w, implies y /x = w / z, then the process is called

a) Dividendo b) Componendo
c) Alternendo d) None of these.

38. If p /q = r / s = p - r / q - s, the process is called

a) Subtrahendo b) Addendo
c) Invertendo d) None of these.

39. If a/b = c/d, then the process (a+b)/(a-b) = (c+d)/(c-d), is called

a) Componendo
b) Dividendo
c) Componendo and Dividendo
d) None of these.

40. If u / v = w / p. then the process (u-v) / (u+v) = (w-p) / (w+p), is called

a) Invertendo
b) Alternendo
c) Addendo
d) None of these.

41. If $\frac{a}{4}=\frac{b}{5}$ then

a) $\frac{a+4}{a-4}=\frac{b-5}{b+5}$ b) $\frac{a+4}{a-4}=\frac{b+5}{b-5}$

c) $\frac{a-4}{a+4}=\frac{b+5}{b-5}$ d) None of these

42. What should be added to each of 3, 15, 38 and 134 so that the number become proportionate to each other.

a) 3 b) 5 c) 7 d) 2

43. In what proportion must rice @ ₹ 3.10/kg be mixed with rice @ ₹ 3.60/kg to make the mixture worth ₹ 3.25/kg?

a. 3 : 5 b. 5 : 3
c. 3 : 7 d. 7 : 3

44. On combining two groups of students having 30 and 40 marks respectively in an exam, the resultant group has an average score of 34. Find the ratio of the number of students in the first group to the number of students in the second group.

a. 2 : 3 b. 3 : 5
c. 5 : 3 d. 3 : 2

45. A merchant has 100 kg of sugar, part of which he sells at 7% profit and the rest at 17% profit. He gains 10% on the whole. Find how much is sold at 7% profit.

a. 30 kg b. 70 kg
c. 55 kg d. 45 kg

PAST YEAR QUESTIONS

46. An alloy is to contain copper and zinc in the ratio 9 : 4. The zinc required to melt with 24 kg of copper is

(a) $10\frac{2}{3}$ kg (b) $10\frac{1}{3}$ kg

(c) $9\frac{2}{3}$ kg (d) 9 kg

47. A box contains ₹ 56 in the form of coins of one rupee, 50 paise and 25 paise. The number of 50 paise coin is double the number of 25 paise coins and four times the numbers of one rupee coins. The numbers of 50 paise coins in the box is

(a) 64 (b) 32 (c) 16 (d) 14

48. Eight people are planning to share equally the cost of a rental car. If one person withdraws from the arrangement and the others share equally entire cost of the car, then the share of each of the remaining persons increased by:

(a) 1/9 (b) 1/8 (c) 1/7 (d) 7/8

49. The incomes of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5 : 3. If each saves ₹ 1,500, then B's income is:

(a) ₹ 6000 (b) ₹ 4500

(c) ₹ 3000 (d) ₹ 7500

50. In 40 litres mixture of glycerine and water, the ratio of glycerine and water is 3 : 1. The quantity of water added in the mixture in order to make this ratio 2 : 1 is

(a) 15 litres (b) 10 litres (c) 8 litres (d) 5 litres

51. The third proportional between $(a^2 - b^2)$ and $(a + b)^2$ is :

(a) $\frac{a+b}{a-b}$ (b) $\frac{a-b}{a+b}$

(c) $\frac{(a-b)^2}{a+b}$ (d) $\frac{(a+b)^3}{a-b}$

52. In a film shooting, A and B received money in a certain ratio and B and C also received the money in the same ratio. If A gets ₹ 1,60,000 and C gets ₹ 2,50,000. Find the amount received by B?
(a) ₹ 2,00,000 (b) ₹ 2,50,000
(c) ₹ 1,00,000 (d) ₹ 1,50,000

53. The ratio compounded of 4:5 and sub-duplicate of a : 9 is 8 : 15. Then value of a is:
(a) 2 (b) 3 (c) 4 (d) 5

54. Find two numbers such that mean proportional between them is 18 and third proportional between them is 144
(a) 9, 36 (b) 8, 32 (c) 7, 28 (d) 6, 24

55. If the salary of P is 25% lower than that of Q and the salary of R is 20% higher than that of Q, the ratio of the salary of R and P will be:
(a) 5 : 8 (b) 8 : 5 (c) 5 : 3 (d) 3 : 5

56. A dealer mixes rice costing ₹ 13.84 per kg. with rice costing ₹ 15.54 and sells the mixture at ₹ 17.60 per kg. So, he earns a profit of 14.6% on his sale price. The proportion in which he mixes the two qualities of rice is:
(a) 3 : 7 (b) 5 : 7 (c) 7 : 9 (d) 9 : 11

57. X, Y, Z together starts a business. If X invests 3 times as much as Y invests and Y invests two third of what Z invests, then the ratio of capitals of X, Y, Z is
(a) 3 : 9 : 2 (b) 6 : 3 : 2
(c) 3 : 6 : 2 (d) 6 : 2 : 3

58. There are total 23 coins of ₹ 1, ₹ 2 and ₹ 5 in a bag. If their value is ₹ 43 and the ratio of coins of ₹ 1 and ₹ 2 is 3 : 2. Then the number of coins of ₹ 1 is :
(a) 12 (b) 5 (c) 10 (d) 14

59. The ratio of the number of ₹ 5 coins and ₹ 10 coins is 8 : 15. If the value of ₹ 5 coins is ₹ 360, then the number of ₹ 10 coins will be:
(a) 72 (b) 120 (c) 135 (d) 185

HOMEWORK SECTION

1. The inverse ratio of 11 : 15 is
   (a) 15 : 11 (b) $\sqrt{11} : \sqrt{15}$
   (c) 121 : 225 (d) none of these

2. The ratio of two quantities is 3 : 4. If the antecedent is 15, the consequent is
   (a) 16 (b) 60 (c) 22 (d) 20

3. The ratio of the quantities is 5 : 7. If the consequent of its inverse ratio is 5, the antecedent is
   (a) 5 (b) $\sqrt{5}$ (c) 7 (d) none of these

4. The ratio compounded of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is
   (a) 1 : 1 (b) 1 : 5 (c) 3 : 8 (d) none of these

5. The duplicate ratio of 3 : 4 is
   (a) $\sqrt{3}$: 2 (b) 4 : 3 (c) 9 : 16 (d) none of these

6. The sub-duplicate ratio of 25 : 36 is
   (a) 6 : 5 (b) 36 : 25 (c) 50 : 72 (d) 5 : 6

7. The triplicate ratio of 2 : 3 is
   (a) 8 : 27 (b) 6 : 9 (c) 3 : 2 (d) none of these

8. The sub-triplicate ratio of 8 : 27 is
   (a) 27 : 8 (b) 24 : 81 (c) 2 : 3 (d) none of these

9. The ratio compounded of 4 : 9 and the duplicate ratio of 3 : 4 is
   (a) 1 : 4 (b) 1 : 3 (c) 3 : 1 (d) none of these

10. The ratio compounded of 4 : 9, the duplicate ratio of 3 : 4, the triplicate ratio of 2 : 3 and 9 : 7 is
    (a) 2 : 7 (b) 7 : 2 (c) 2 : 21 (d) none of these

11. The ratio compounded of duplicate ratio of 4 : 5, triplicate ratio of 1 : 3, sub duplicate ratio of 81 : 256 and sub-triplicate ratio of 125 : 512 is
(a) 4 : 512 (b) 3 : 32 (c) 1 : 12 (d) none of these

12. If a : b = 3 : 4, the value of (2a+3b) : (3a+4b) is
(a) 54 : 25 (b) 8 : 25 (c) 17 : 24 (d) 18 : 25

13. Two numbers are in the ratio 2 : 3. If 4 be subtracted from each, they are in the ratio 3 : 5. The numbers are
(a) (16, 24) (b) (4, 6) (c) (2, 3) (d) none of these

14. The angles of a triangle are in ratio 2 : 7 : 11. The angles are
(a) (20°, 70°, 90°)
(b) (30°, 70°, 80°)
(c) (18°, 63°, 99°)
(d) none of these

15. Division of ₹ 324 between X and Y is in the ratio 11 : 7. X & Y would get Rupees
(a) (204, 120) (b) (200, 124)
(c) (180, 144) (d) none of these

16. The ratio of two numbers is 7 : 10 and their difference is 105. The numbers are
(a) (200, 305) (b) (185, 290)
(c) (245, 350) (d) none of these

17. P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 12 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is
(a) 22 : 27 (b) 27 : 22 (c) 32 : 33 (d) none of these

18. If x : y = 3 : 4, the value of $x^2y + xy^2 : x^3 + y^3$ is
(a) 13 : 12 (b) 12 : 13 (c) 21 : 31 (d) none of these

19. If p : q is the sub-duplicate ratio of $p-x^2 : q-x^2$ then x^2 is
(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$ (c) $\frac{pq}{p+q}$ (d) None of these

20. If 2s : 3t is the duplicate ratio of 2s − p : 3t − p then
(a) $p^2 = 6st$ (b) p = 6st (c) 2p = 3st (d) none of these

21. If p : q = 2 : 3 and x : y = 4 : 5, then the value of 5px + 3qy : 10px + 4qy is
(a) 71 : 82 (b) 27 : 28 (c) 17 : 28 (d) none of these

22. The number which when subtracted from each of the terms of the ratio 19 : 31 reducing it to 1 : 4 is
(a) 15 (b) 5 (c) 1 (d) none of these

23. Daily earnings of two persons are in the ratio 4:5 and their daily expenses are in the ratio 7 : 9. If each saves ₹ 50 per day, their daily earnings in ₹ are
(a) (40, 50) (b) (50, 40) (c) (400, 500) (d) none of these

24. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 kms. in 5 hours, the speed of the first train is
(a) 10 Km/hr (b) 50 Km/hr (c) 70 Km/hr (d) none of these

25. The fourth proportional to 4, 6, 8 is
(a) 12 (b) 32 (c) 48 (d) none of these

26. The third proportional to 12, 18 is
(a) 24 (b) 27 (c) 36 (d) none of these

27. The mean proportional between 25, 81 is
(a) 40 (b) 50 (c) 45 (d) none of these

28. The number which has the same ratio to 26 that 6 has to 13 is
(a) 11 (b) 10 (c) 21 (d) none of these

29. The fourth proportional to 2a, a^2, c is
(a) ac/2 (b) ac (c) 2/ac (d) none of these

30. If four numbers 1/2, 1/3, 1/5, 1/x are proportional then x is
(a) 6/5 (b) 5/6 (c) 15/2 (d) none of these

31. The mean proportional between $12x^2$ and $27y^2$ is

(a) 18xy (b) 81xy (c) 8xy (d) none of these

(Hint: Let z be the mean proportional and z = $\sqrt{(12x^2 \times 27y^2)}$

32. If A = B/2 = C/5, then A : B : C is

(a) 3 : 5 : 2 (b) 2 : 5 : 3 (c) 1 : 2 : 5 (d) none of these

33. If a/3 = b/4 = c/7, then a + b + c/c is

(a) 1 (b) 3 (c) 2 (d) none of these

34. If p/q = r/s = 2.5/1.5, the value of ps : qr is

(a) 3/5 (b) 1:1 (c) 5/3 (d) none of these

35. If x : y = z : w = 2.5 : 1.5, the value of (x + z)/(y + w) is

(a) 1 (b) 3/5 (c) 5/3 (d) none of these

36. If (5x – 3y)/(5y – 3x) = 3/4, the value of x : y is

(a) 2 : 9 (b) 7 : 2 (c) 7 : 9 (d) none of these

37. If A : B = 3 : 2 and B : C = 3 : 5, then A : B : C is

(a) 9 : 6 : 10 (b) 6 : 9 : 10 (c) 10 : 9 : 6 (d) none of these

38. If x/2 = y/3 = z/7, then the value of (2x – 5y + 4z)/2y is

(a) 6/23 (b) 23/6 (c) 3/2 (d) 17/6

39. If x : y = 2 : 3, y : z = 4 : 3 then x : y : z is

(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 2 : 4 (d) none of these

40. Division of ₹ 750 into 3 parts in the ratio 4 : 5 : 6 is

(a) (200, 250, 300) (b) (250, 250, 250)

(c) (350, 250, 150) (d) 8 : 12 : 9

41. The sum of the ages of 3 persons is 150 years. 10 years ago their ages were in the ratio 7 : 8 : 9. Their present ages are

(a) (45, 50, 55) (b) (40, 60, 50)

(c) (35, 45, 70) (d) none of these

42. The numbers 14, 16, 35, 42 are not in proportion. The fourth term for which they will be in proportion is
(a) 45 (b) 40 (c) 32 (d) none of these

43. If x/y = z/w, implies y/x = w/z, then the process is called
(a) Dividendo (b) Componendo
(c) Alternendo (d) none of these

44. If p/q = r/s = p – r/q – s, the process is called
(a) Subtrahendo (b) Addendo
(c) Invertendo (d) none of these

45. If a/b = c/d, implies (a + b)/(a – b) = (c + d)/(c – d), the process is called
(a) Componendo (b) Dividendo
(c) Componendo and Dividendo (d) none of these

46. If u/v = w/p, then (u – v)/(u + v) = (w – p)/(w + p). The process is called
(a) Invertendo (b) Alternendo
(c) Addendo (d) none of these

47. 12, 16, *, 20 are in proportion. Then * is
(a) 25 (b) 14 (c) 15 (d) none of these

48. 4, *, 9, 13½ are in proportion. Then * is
(a) 6 (b) 8 (c) 9 (d) none of these

49. The mean proportional between 1.4 gms and 5.6 gms is
(a) 28 gms (b) 2.8 gms (c) 3.2 gms (d) none of these

50. If $\frac{a}{4} = \frac{b}{5} = \frac{c}{9}$ then $\frac{a+b+c+}{c}$ is
(a) 4 (b) 2 (c) 7 (d) none of these.

51. Two numbers are in the ratio 3 : 4; if 6 be added to each terms of the ratio, then the new ratio will be 4 : 5, then the numbers are
(a) 14, 20 (b) 17, 19 (c) 18 and 24 (d) none of these

52. If $\frac{a}{4} = \frac{b}{5}$ then

(a) $\frac{a+4}{a-4} = \frac{b-5}{b+5}$ (b) $\frac{a+4}{a-4} = \frac{b+5}{b-5}$

(c) $\frac{a-4}{a+4} = \frac{b+5}{b-5}$ (d) none of these

53. If a : b = 4 : 1 then + is $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$ is

(a) 5/2 (b) 4 (c) 5 (d) none of these

54. If $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$ then

(b - c)x + (c - a)y + (a - b)z is

(a) 1 (b) 0 (c) 5 (d) none of these

HOMEWORK SOLUTION

1. (a) 15 : 11

2. (d) 20

Ratio = $\frac{3}{4}$, antecedent = 3 × 5 = 15

∴ consequent = 4 × 5 = 20

3. (c) 7

4. (a) 1 : 1

5. (c) 9 : 16

6. (d) 5 : 6

7. (a) 8 : 27

8. (c) 2 : 3

9. (a) 1/4

Compound ratio = $\frac{4}{9} \times \frac{9}{16} = \frac{1}{4}$

10. (c) 2 : 27

Compound ratio = $\frac{4}{9} \times \frac{9}{16} \times \frac{8}{27} \times \frac{9}{7} = \frac{2}{21}$

11. (d) None of these

Compound ratio = $\frac{16}{25} \times \frac{1}{27} \times \frac{9}{16} \times \frac{5}{8} = \frac{1}{120}$

12. (d) 18 : 25

Here, a : b = 3 : 4

∴ a = 3, b = 4

∴ Value of 2a + 3b : 3a + 4b

= 2(3) + 3(4) : 3(3) + 4(4)

= 18 : 25

13. (a) 16, 24

Let numbers are 2 : 3 = 2x : 3x

∴ If 4 subtract from each

$$\therefore \frac{2x - 4}{3x - 4} = \frac{3}{5}$$

∴ 5(2x − 4) = 3(3x − 4)

∴ 10x − 20 = 9x − 12

∴ x = 8

∴ the numbers are = 2x, 3x

= 16, 24

14. (c) (18°, 63°, 99°)

Angles of triangle = 2 : 7 : 11

= 2x, 7x, 11x

Let 2x + 7x + 11x = 180

∴ 20x = 180

∴ x = 9

∴ Angles of triangle = 2x, 7x, 11x

= 18, 63, 99

15. (d) None of these Trial and error

16. (c) (245, 350) Trial and error

17. (b) 27 : 22

Here, P : Q = 11 : 12, P : R = 9 : 8

∴ Q : P = 12 : 11

Joint ratio = Q : P	P : R
9 × (12 : 11)	(9 : 8) × 11
= 108 : 99	99 : 88

∴ Ratio of Q : R = 108 : 88

= 27 : 22

18. (b) 12 : 13

Here, $x : y = 3 : 4$

$\therefore x = 3, y = 4$

the value of $x^2y + xy^2 : x^3 + y^3$

$= (3)^2(4) + 3(4)^2 : (3)^3 + (4)^3$

$= 36 + 48 : 27 + 64$

$= 84 : 91$

$= 12 : 13$

19. (c) pq / p + q

$$\frac{p}{q} = \frac{\sqrt{p - x^2}}{\sqrt{q - x^2}}$$

$$\therefore \frac{p^2}{q^2} = \frac{p - x^2}{q - x^2}$$

$\therefore p^2(q - x^2) = q^2(p - x^2)$

$\therefore p^2q - p^2x^2 = pq^2 - q^2 . x^2$

$\therefore p^2q - pq^2 = p^2x^2 - q^2x^2$

$\therefore pq(p - q) = x^2(pp - qq)$

$\therefore pq(p - q) = x^2(p - q)(p + q)$

$$\therefore x^2 = \frac{pq}{p + q}$$

20. (a) $p^2 = 6$ st

$$\frac{2s}{3t} = \frac{(2s - p)^2}{(3t - p)^2}$$

$$\therefore \frac{2s}{3t} = \frac{4s^2 - 4sp + p^2}{9t^2 - 6tp + p^2}$$

$\therefore 2s(9t^2 - 6tp + p^2) = 3t(4s^2 - 4sp + p^2)$

$\therefore 18t^2s - 12ps + 2p^2s = 12ts^2 - 12tps + 3pt$

$\therefore 18t^2s - 12 + s^2 = 3p^2t - 2p^2s$

$\therefore 6ts(3t - 2s) = p^2(3t - 2s)$

$\therefore p^2 = 6st$

21. (c) 17 : 28

$p : q = 2 : 3, x : y = 4 : 5$

$\therefore$ the value of $5px + 3qy : 10px + 4qy$

$= 5(2)(4) + 3(3)(5) : 10(2)(4) + 4(3)(5)$

$= 85 : 140 = 17 : 28$

22. (a) 15 Trial and error

$$\frac{19-15}{31-15} = \frac{4}{16} = \frac{1}{4}$$

23. (c) (400, 500) Trial and error

24. (c) 70 km/hrs

Speed d 2^{nd} train $= \frac{\text{Distance}}{\text{Time}}$

$$= \frac{000}{5}$$

$\therefore$ Speed of 2^{nd} train = 80 km/hr

Speed ratio $= \frac{S_1}{S_2} = \frac{7}{8}$

$$\therefore S_1 = \frac{7}{8} \times S_2$$

$$= \frac{7}{8} \times 80$$

$\therefore S_1 = 70$ km/hr.

25. (a) 12

$$\frac{4}{6} = \frac{8}{x}$$

$$\therefore x = \frac{8 \times 6}{4}$$

$\therefore x = 12$

26. (b) 27

$$\frac{12}{18} = \frac{18}{x}$$

$$\therefore x = \frac{18 \times 18}{12}$$

$$\therefore x = 27$$

27. (c) 45

Mean proportion = $\sqrt{25 \times 81}$

$= 45$

28. (d) None of these

$$\frac{x}{26} = \frac{6}{13}$$

$$\therefore x = \frac{6 \times 26}{13}$$

$$\therefore x = 12$$

29. (a) $\frac{ac}{2}$

$$\frac{2a}{a^2} = \frac{c}{x}$$

$$\therefore x = \frac{ca^2}{2a}$$

$$x = \frac{ac}{2}$$

30. (c) $\frac{15}{2}$

$$\frac{\frac{1}{2}}{\frac{1}{3}} = \frac{\frac{1}{5}}{\frac{1}{x}}$$

$$\therefore \frac{3}{2} = \frac{x}{5}$$

$$\therefore x = \frac{3 \times 5}{2} = \frac{15}{2}$$

31. (a) 18xy

Mean proportion = $\sqrt{12x^2 + 27y^2}$

$= \sqrt{324 . x^2 . y^2}$

= 18xy

32. (c) 1 : 2 : 5

Here A = $= \frac{B}{2} = \frac{C}{5}$

A = 1, B = 2, C = 5

∴ A : B : C = 1 : 2 : 5

33. (c) 2

Here, $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$

∴ a = 3, b = 4, c = 7

$\frac{a + b + c}{c} = \frac{3 + 4 + 7}{7} = \frac{14}{7} = 2$

34. (b) 1 : 1

$\frac{p}{q} = \frac{r}{s} = \frac{2.5}{1.5}$

∴ p = r = 2.5, q = s = 1.5

$\frac{ps}{qr} = \frac{(2.5)(1.5)}{(1.5)(2.5)} = \frac{1}{2}$

35. (c) $\frac{5}{3}$

$\frac{x}{y} = \frac{z}{2} = \frac{2.5}{1.5}$

x = z = 2.5, y = w = 1.5

the value of $\frac{x + z}{y + w} = \frac{2.5 + 2.5}{1.5 + 1.5} = \frac{5}{3}$

36. (d) $\frac{27}{29}$

Here, $\frac{5x - 3y}{5y - 3x} = \frac{3}{4}$

$\therefore$ 4(5x − 3y) = 3(5y − 3x)

$\therefore$ 20x − 12y = 15y − 9x

$\therefore$ 29x = 27y

$\therefore \frac{x}{y} = \frac{27}{29}$

37. (a) 9 : 6 : 10

A : B B : C

3 × (3 : 2) (3 : 5) × 2

$\therefore$ 9 : 6 6 : 10

$\therefore$ A : B : C = 9 : 6 : 10

38. (d) $\frac{17}{16}$

$\frac{x}{2} = \frac{y}{3} = \frac{z}{7}$

$\therefore$ x = 2, y = 3, z = 7

$\therefore$ The value of $\frac{2x - 5y + 4z}{2y}$

$= \frac{2(2) - 5(3) + 4(7)}{2(3)}$

$= \frac{17}{6}$

39. (d) 8 : 12 : 9

x : y y : z

4 × (2 : 3) (4 : 3) × 3

$\therefore$ 8 : 12 12 : 9

$\therefore$ x : y : z = 8 : 12 : 9

40. (a) (200, 250, 300) Trial and error

41. (a) (45, 50, 55) Trial and error

42. (b) 40

$$\frac{14}{16} = \frac{35}{x}$$

$$\therefore x = \frac{35 \times 16}{14}$$

x = 40

43. (d) None of these

44. (a) Subtrahendo

45. (c) Componendo & Dividendo

46. (d) None of these

47. (c) 15

$$\frac{12}{16} = \frac{x}{20}$$

$$\therefore x = \frac{12 \times 20}{16}$$

x = 15

48. (a) 6

$$\frac{4}{x} = \frac{9}{13.5}$$

$$\therefore x = \frac{4 \times 13.5}{9}$$

x = 6

49. (b) 2.8

Mean proportion = $\sqrt{1.4 \times 5.6}$

= 2.8

50. (b) 2

$$\frac{a}{4} = \frac{b}{5} = \frac{c}{9}$$

$\therefore$ a = 4, b = 5, c = 9

$$\therefore \frac{a+b+c}{c} = \frac{4+5+9}{9} = 2$$

51. (c) 18, 24

$$\frac{3x+6}{4x+6} = \frac{4}{5}$$

$\therefore$ 5(3x + 6) = 4(4x + 6)

$\therefore$ 15x + 30 = 16x + 24

$\therefore$ 6 = x

$\therefore$ numbers = 3x = 3(6) = 18

4x = 4(6) = 24

52. (b) $\frac{a+4}{a-4} = \frac{b+5}{b-5}$

53. (a) $\frac{5}{2}$

Here, $\frac{a}{b} = \frac{4}{1}$

$\therefore \frac{\sqrt{a}}{\sqrt{b}} = \frac{2}{1}, \frac{\sqrt{b}}{\sqrt{a}} = \frac{1}{2}$

$\therefore$ Value of $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = \frac{2}{1} + \frac{1}{2}$

$$= \frac{4+1}{2}$$

$$= \frac{5}{2}$$

54. (b) 0

Cyclical terms in base.

Partnership

CLASSWORK SECTION

1. Anand and Deepak started a business investing Rs. 22,500 and Rs. 35,000 respectively. Out of a total profit of Rs. 13,800, Deepak's share is

(a) Rs. 5,400 (b) Rs. 7,200 (c) Rs. 8,400 (d) Rs. 9,600

2. A, B, C enter into a partnership investing Rs. 35,000, Rs. 45,000 and Rs. 55,000 respectively. The respective shares of A,B,C in an annual profit of Rs. 40,500 are:

(a) Rs. 10,500 Rs. 13,500, Rs. 16,500 (b) Rs. 11,500 Rs. 13,000 Rs. 16,000
(c) Rs. 11,500 Rs. 14,000 Rs. 15,500 (d) Rs. 11,500 Rs. 12,500 Rs. 16,500

3. Reena and Shaloo are partner in a business. Reena invests Rs. 35,000 for 8 months and Shaloo invests Rs. 42,000 for 10 months. Out of a profit of Rs. 31,570, Reena's share is:

(a) Rs. 9471 (b) Rs. 12628 (c) Rs. 18040 (d) Rs. 18942

4. Simran Started a software business by investing Rs.50, 000 . After six months , Nanda joined her with capital of Rs. 80,000. After three years , they earned a profit of Rs.24,500. What was Simran's share in the profit ?

(a) Rs.9423 (b) Rs.10500 (c) Rs.12,500 (d) Rs.14,000

5. Aman started a business investing Rs. 70000 . Rakhi joined him after six months with an amount of Rs. 1,05,000 and Sagar joined them with Rs.1.4 lakhs after another six months. The amount of profit earned should be distributed in what ratio among six months. The amount profit earned should be distributed in what ratio among Aman, Rakhi, and Sagar respectively, 3 years after Aman started the business ?

(a)7:6:10 (b) 12:15: 16 (c) 42:45:56 (d) None of these

6. A began a business with Rs.85,000 and is joined afterwards by B with Rs.42,500. For how much period does B join , if profits at the end of the year are divided in the ratio of 3:1 ?

(a) 4 months (b) 5 months (c) 6 months (d) 8 months

HOMEWORK SUMS

7. Kanchan started a business investing Rs. 9000. After five months, Sameer joined with a capital of Rs. 8000. If at the end of the year, they earn a profit of Rs. 6970, then what will be the share of Sameer in the profit?

   (a) Rs. 1883.78 (b) Rs. 2380 (c) Rs. 3690 (d) Rs. 3864

8. A and B started a business in partnership by investing Rs.20, 000 and Rs.15,000 respectively. After six months , C joined them with Rs.20000. What will be B;s share in the total profit of Rs.20,000 earned at the end of 2 tears from the starting of the business ?

   (a) Rs. 7500 (b) Rs.6000 (c) Rs.9500 (d) Rs. 10,000

9. A, B and C enter into partnership by investing in the ratio of 3:2:4 . After one year , B invests another Rs.2,70, 000 and C , at the end of 2 years, also invests Rs.2,70, 000. At the end of three years , profits are shared in the ratio of 3:4: 5 . Find the initial investment of each .

   (a) 2,70,000 : 1,80,000; 3, 60,000 (b) 2,70,000 : 1,50,000; 3, 60,000
   (c) 2,50,000 : 1,80,000; 3, 60,000 (d) 2,70,000 : 1,80,000; 3, 00,000

10. A , B and C enter into partnership . A invests 3 times as much as B invests and B invests 2/3rd of what C invests . At the end of the year , the profit earned is Rs.6600. What is the share of B ?

    (a) Rs.1200 (b) Rs.1500 (c) Rs.1800 (d) Rs.2000

Time & Work

11. A and B can do a work in 8 days , B and C can do the same work in 12 days . A, B and C together can finish it in 6 days . A and C together will do it in :

    (a) 4 days (b) 6 days (c) 8 days (d) 12 days

1B Indices, Surds and Logarithms

THEORY

$a^x = N$

a = base

x = Power/Exponent/Index

N = Product

[But, $a \neq 0, 1, \pm\infty$]

Theory of Indices deals with the various changes in power, during various mathematical operations.

Basic Rules

1. $a^m \times a^n = a^{m+n}$

2. $\dfrac{a^m}{a^n} = a^{m-n}$

3. $(a^m)^n = a^{mn}$; m is added n times

4. $(ab)^m = a^m x b^m$

5. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

6. $a^0 = 1$

7. $a^{-n} = \dfrac{1}{a^n}$

8. If $a^m = a^n \Rightarrow m = n$; where, $a \neq 0, 1, -1, \pm\infty$

9. For $a^m = b^m$ if m $\neq$ 0 then

(i) $a = b$ (when m is odd)

(ii) $a = \pm b$ (when m is even)

10. $a^x = N$

$\Rightarrow a = N^{\frac{1}{x}} = \sqrt[x]{N}$

11. $(i) 0^a = 0$

$(ii) 1^a = 1$

$(iii) a^1 = a$

$(iv) a^0 = 1$

$(v) 0^0$ *has* no meaning

Basic Formulae

1. $(a+b)^2 = a^2 + 2ab + b^2$

2. $(a-b)^2 = a^2 - 2ab + b^2$

3. $a^2 - b^2 = (a+b)(a-b)$

4. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

5. $(a+b)^2 - (a-b)^2 = 4ab$

6. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

7. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$

8. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$

9. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

10. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

11. If $a+b+c=0$, then $a^3 + b^3 + c^3 = 3abc$

12. If $a^3 + b^3 + c^3 = 3abc, then\ \ either\ a + b + c = 0$ or a = b = c
but both the results cannot hold true simultaneously

Rational Numbers, Irrational Numbers & Surds

- A Rational Number is a number which can be expressed in the form p/q, where q ≠ 0; p & q are integers and p and q are prime to each other, i.e., there is no common factor between p & q, other than 1.
- Any terminating and recurring decimals are rational numbers.
- Thus any non-recurring and non-terminating decimals are irrational numbers, and when the irrational numbers are expressed in radical form (root form), it is known as "Surds".
- Thus all the surds are irrational, but all irrational numbers are not surds.
- The numbers whose perfect root can be evaluated are rational quantities and numbers for which perfect roots cannot be evaluated are irrational quantities.

Order of Surds

If $\sqrt[k]{m} = (m)^{\frac{1}{k}}$ is a surd, then, it is said to be a surd of order "k".

Pure Surds and Mixed Surds

In case of pure surds, entire expression is kept within the radical sign. In mixed surds, it is expressed as a product of one rational and one irrational quantity.

Example:
$\sqrt{7}$ is a pure surd; $\sqrt{12} = \sqrt{4x3} = 2\sqrt{3}$ is a mixed surd.

Conjugate of a Surd

If $(a + \sqrt{b})$ or $(\sqrt{a} + \sqrt{b})$ are surds, their respective conjugates would be given by,

$(a - \sqrt{b})$ or $(\sqrt{a} - \sqrt{b})$ and vice-versa.

Rationalization of Surds

Rationalization is a process, where we convert the irrational part of the surd into a rational quantity, with help of its conjugate.

Note: 1

- Rational + Rational = Rational
- Rational − Rational = Rational
- Rational x Rational = Rational
- Rational ÷ Rational = Rational

Note: 2

- Irrational + Irrational = Irrational
- Irrational − Irrational = Rational (only when the quantities are equal); otherwise −
- Irrational − Irrational = Irrational
- Irrational x Irrational = May be Rational or Irrational
- Irrational ÷ Irrational = May be Rational or Irrational

Note: 3

- Rational + Irrational = Irrational
- Rational − Irrational = Irrational
- Rational x Irrational = Irrational
- Rational ÷ Irrational = Irrational

Square Root of Surds

- The square root of a surd is always a surd.
- Every answer for square root must contain +ve or −ve sign and in the absence of +/- sign, "none of these" will be marked as answer.
- If the given surd, whose square root is to be evaluated is in the form $\left(a \pm \sqrt{b}\right)$, then the answer will also be in the form $\pm\left(x \pm \sqrt{y}\right)$.
- Square the options, in order to get the question back.

INDICES

1. The value of $4/(32)^{1/5}$ is

(a) 8 (b) 2
(c) 4 (d) none of these

2. $2^{1/2} \cdot 4^{3/4}$ is equal to

(a) a fraction (b) a positive integer
(c) a negative integer (d) none of these

3. The value of $y^{a-b} \times y^{b-c} \times y^{c-a} \times y^{-a-b}$ is

(a) y^{a+b} (b) y
(c) 1 (d) $1/y^{a+b}$

4. The value of $(8/27)^{-1/3} \times (32/243)^{-1/5}$ is

(a) 9/4 (b) 4/9
(c) 2/3 (d) none of these

5. $[\{(2)^{1/2} \cdot (4)^{3/4} \cdot (8)^{5/6} \cdot (16)^{7/8} \cdot (32)^{9/10}\}^4]^{3/25}$ is

(a) A fraction (b) an integer
(c) 1 (d) none of these

6. $[1 - \{1 - (1 - x^2)^{-1}\}^{-1}]^{-1/2}$ is equal to

(a) x (b) 1/x
(c) 1 (d) none of these

7. If $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, then the simplified form of

$$\left[\frac{x^l}{x^m}\right]^{l^2+lm+m^2} \times \left[\frac{x^m}{x^n}\right]^{m^2+mn+n^2} \times \left[\frac{x^n}{x^l}\right]^{l^2+ln+n^2}$$

(a) 0 (b) 1 (c) x (d) none of these

8. The value of $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

(a) 1 (b) 0 (c) 2 (d) none of these

9. The value of $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \times \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \times \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$

(a) 1 (b) 0 (c) −1 (d) none of these

10. Using $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ tick the correct of these when $x = p^{1/3} - p^{-1/3}$

(a) $x^3 + 3x = p + 1/p$

(b) $x^3 + 3x = p - 1/p$

(c) $x^3 + 3x = p + 1$ (d) none of these

11. If $x = 3^{\frac{1}{3}} + 3^{\frac{-1}{3}}$, then $3x^3 - 9x$ is

(a) 15 (b) 10

(c) 12 (d) none of these

12. If $a^x = b$, $b^y = c$, $c^z = a$, then xyz is

(a) 1 (b) 2

(c) 3 (d) none of these

13. If $x^{1/p} = y^{1/q} = z^{1/r}$ and $xyz = 1$, then the value of $p + q + r$ is

(a) 1 (b) 0

(c) 1/2 (d) none of these

14. On simplification, $1/(1 + a^{m-n} + a^{m-p}) + 1/(1 + a^{n-m} + a^{n-p}) + 1/(1 + a^{p-m} + a^{p-n})$ is equal to

(a) 0 (b) a (c) 1 (d) 1/a

15. If $2^x = 3^y = 6^{-z}$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is

(a) 1 (b) 0 (c) 2 (d) none of these

16. On simplification $\frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times 6^{y+1}}{6^{x+1} \times 10^{y+3} \times 15^x}$ reduces to

(a) −1 (b) 0 (c) 1 (d) 10

17. If $\dfrac{9^y.3^2.(3^{-y})^{-1} - 27^y}{3^{3x}.2^3} = \dfrac{1}{27}$ then x − y is given by

(a) −1 (b) 1 (c) 0 (d) none

18. Show that $\left(\dfrac{x^a}{x^b}\right)^{a+b} \times \left(\dfrac{x^b}{x^c}\right)^{b+c} \times \left(\dfrac{x^c}{x^a}\right)^{c+a}$ is given by

(a) 0 (b) −1 (c) 3 (d) 1

19. Show that reduces to $\left(\dfrac{x^b}{x^c}\right)^{a} \times \left(\dfrac{x^c}{x^a}\right)^{b} \times \left(\dfrac{x^a}{x^b}\right)^{c}$

(a) 1 (b) 3 (c) 0 (d) 2

20. The value of z is given by the following if $z^{z\sqrt{z}} = (z\sqrt{z})^z$

(a) 2 (b) $\dfrac{3}{2}$ (c) $-\dfrac{3}{2}$ (d) $\dfrac{9}{4}$

21. If $(5.678)^x = (0.5678)^y = 10^z$ then

(a) $\dfrac{1}{x} - \dfrac{1}{y} + \dfrac{1}{z} = 1$ (b) $\dfrac{1}{x} - \dfrac{1}{y} - \dfrac{1}{z} = 0$

(c) $\dfrac{1}{x} - \dfrac{1}{y} + \dfrac{1}{z} = -1$ (d) None

22. If $3^a = 5^b = (75)^c$, then the value of ab − c(2a + b) reduces to

(a) 1 (b) 0 (c) 3 (d) 5

23. If $2^a = 4^b = 8^c$ and abc = 288 then the value $\dfrac{1}{2a} + \dfrac{1}{4b} + \dfrac{1}{8c}$ is given by

(a) $\dfrac{1}{8}$ (b) $-\dfrac{1}{8}$ (c) $\dfrac{11}{96}$ (d) $-\dfrac{11}{96}$

24. If $ax^{2/3} + bx^{1/3} + c = 0$ then the value of $a^3x^2 + b^3x + c^3$ is given by

(a) 3abcx (b) −3abcx (c) 3abc (d) −3abc

25. $x^{a^2b^{-1}c^{-1}}.x^{b^2c^{-1}a^{-1}}.x^{c^2a^{-1}b^{-1}} - x^3$ would reduce to zero a + b + c is given by

(a) 1 (b) −1 (c) 0 (d) None

26. If $a^b = b^a$, then the value of $\left(\frac{a}{b}\right)^{\frac{a}{b}} - a^{\frac{a}{b}-1}$ reduces to

(a) a (b) b (c) 0 (d) None

SURDS

27. If $a = 3 + 2\sqrt{2}$ then the value of $a^{1/2} + a^{-1/2}$ is

(a) $\sqrt{2}$ (b) $-\sqrt{2}$
(c) $2\sqrt{2}$ (d) $-2\sqrt{2}$

28. If $a = \sqrt{\frac{7 + 4\sqrt{3}}{7 - 4\sqrt{3}}}$ then the value of $[a(a - 14)]^2$ is

(a) 14 (b) 7 (c) 2 (d) 1

29. The square root of $3 + \sqrt{5}$ is

(a) $\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}$ (b) $-\left(\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}\right)$

(c) Both the above (d) None

30. If $a = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$, $b = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$, then the value of $a^2 + b^2$ is

(a) 10 (b) 100 (c) 98 (d) 99

PAST YEAR QUESTIONS

31. Value of $(a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$ is:

(a) $a + \frac{1}{a}$ (b) $a - \frac{1}{a}$ (c) $a^2 + \frac{1}{a^2}$ (d) $a^2 - \frac{1}{a^2}$

32. If $2^x - 2^{x-1} = 4$ then x^x is equal to:

(a) 7 (b) 3 (c) 27 (d) 9

33. $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

(a) 1/2 (b) 3/2 (c) 2/3 (d) 1/3

34. If $2^x \times 3^y \times 5^z = 360$. Then what is the value of x, y, z ?

(a) 3, 2, 1 (b) 1, 2, 3 (c) 2, 3, 1 (d) 1, 3, 2

35. If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$ then the value of $\left(\frac{a+b+c}{3}\right)^3$

(a) abc (b) 9abc

(c) $\frac{1}{abc}$ (d) $\frac{1}{9abc}$

36. If $(25)^{150} = (25x)^{50}$, then the value of x will be:

(a) 5^3 (b) 5^4 (c) 5^2 (d) 5

37. If $p^x = q$, $q^y = r$ and $r^z = p^6$, then the value of xyz will be :

(a) 0 (b) 1 (c) 3 (d) 6

SPECIAL TYPE OF QUESTIONS

1. (i) The value of $\sqrt{20+\sqrt{20+\sqrt{20+.....to\infty}}}$ is

(a) 5 (b) 4 (c) 20 (d) None

(ii) The value of $\sqrt{20-\sqrt{20-\sqrt{20-.....to\infty}}}$ is

(a) 5 (b) 4 (c) 20 (d) None

(iii) $\sqrt{20\sqrt{20\sqrt{20........ \; to\infty}}}$

(a) 5 (b) 4 (c) 20 (d) None

(iv) The value of $\sqrt{8\div\sqrt{8\div\sqrt{8\div........ \; to\infty}}}$

(a) 2 (b) 8 (c) 6 (d) None

(v) The value of $\sqrt{8\sqrt{8\sqrt{8}}}$

(a) 8 (b) 8 (c) 8 (d) None

2. If xyz = 1 then the value of $\frac{1}{1+x+y^{-1}}+\frac{1}{1+y+z^{-1}}+\frac{1}{1+z+x^{-1}}$ is

(a) 1 (b) 0 (c) −2 (d) None

3. If x = $\sqrt{2-\sqrt{2-\sqrt{2-.....\infty}}}$ the value of X is given by

(a) − 2 (b) 1 (c) 2 (d) 0

4. If $x=\sqrt{7\sqrt{7\sqrt{7......\infty}}}$ the value of X is given by

(a) − 3 (b) 3 (c) 12 (d)

5. Simplify $\sqrt{a\sqrt{a\sqrt{a\sqrt{a}}}}$ for a = $3^{16/15}$

(a) 0; (b) 2; (c) 3; (d) None

Logarithms

THEORY

If $a^x = N$, then $x = \log_a N$; * $a \neq 0, 1, \pm\alpha$ and for the purpose of log, any negative quantity.

* x is called the logarithm of N (product) to the base "a".

Base "a"

- The base "a" of log can be any positive real number except 1.
- The base of log can be clearly divided into two parts:
- $0 < a < 1$ (the proper fraction)
- $a > 1$ (positive integer / mixed fraction)
- Unless otherwise specified, the base of log is always taken to be 10 and this is known as Common Logarithm.
- For theoretical purpose, the base is always taken to be "e", where "e" is a constant and this is known as "Natural Logarithm".
- Common Logarithms are used for numerical calculations and Natural Logarithms are used in calculus.

Basic Rules

1. $\log_a mn = \log_a m + \log_a n$
2. $\log_a \frac{m}{n} = \log_a m - \log_a n$
3. $\log_a m^n = n \log_a m$
4. $\log_a a = 1$
5. $\log_a 1 = 0$
6. $\log_a 0 =$ Undefined
7. $\log_a -ve =$ Undefined

8. $\log_a m = \log_a n \Rightarrow m = n$

Change of Base in Logarithms

1. $\log_b a = \dfrac{\log_m a}{\log_m b}$ (m can be any common base) ($m \neq 0, 1, \pm\alpha$,-ve value)

2. $\log_a b = \dfrac{1}{\log_b a}$

3. $a^{\log_a x} = x$

Nature of Log Values

- All the values which are obtained from log tables are irrational numbers provided the numbers are not 10 or in the form of 10^n.
- $\log_b a$ is a rational quantity only when, $\dfrac{\log a}{\log b}$ is rational.
- If K is a number, then its log value, logK can be divided into two parts: a) Integral Part, b) Fractional Part.
- The integral part is called "Characteristics" and the fractional part is called "Mantissa".
- The integral characteristics part can be positive or negative or zero but not a fraction.
- The values of mantissa are always positive fractions.
- The values for mantissa are obtained from log tables.
- Characteristics are to be calculated before we evaluate mantissa from the log table.
- Value of characteristics = number of significant digits before decimal – 1

CLASSWORK SECTION

1. $\log_{10} 10 + \log_{10} 100 + \log_{10} 1000 + \log_{10} 10000 + \log_{10} 100000$ is
   a) 15
   b) $\log_{10} 11111$
   c) $\log_{10} 1111$
   d) $14\log_{10} 100$

2. If $\log\left(\frac{a}{b}\right)+\log\left(\frac{b}{a}\right)=\log(a+b)$, then which of the following is true?
   a) a + b = 1
   b) a + b = 0
   c) a = b
   d) a − b = 1

3. Find the value of $\log_{10}\left(\frac{4}{25}\right)+\log_{10}\left(\frac{125}{7}\right)-\log_{10}\left(\frac{2}{7}\right)$.
   a) 1 b) 4
   c) 41 d) None of the above

4. $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$ equals
   a) 18
   b) 1
   c) 3
   d) None of the above

5. $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80} =$
   a) log 2
   b) log 3
   c) log 5
   d) None of the above

6. If $\log_{10}[98 + \sqrt{x^2-12x+36}] = 2$, then $x =$
   a) 4
   b) 8
   c) 12
   d) 4, 8

7. If $\log_5(x^2+x)-\log_5(x+1)=2$; then find the value of x.

a) 5

b) 1/5

c) 5^2

d) None of the above

8. If $\left(\frac{21}{10}\right)^x$ = 2, then x = ?

a) $\frac{\log 2}{\log 3+\log 7+1}$ b) $\frac{\log 2}{\log 3+\log 7-1}$

c) $\frac{\log 2}{\log 7+\log 3+2}$ d) None of the above

9. Evaluate: $x^{\log y-\log z}.\ y^{\log z-\log x}.\ z^{\log x-\log y}$.

a. 0 b. 1 c. 2 d. − 1

10. The value of is $a^{\log b/c}.\ b^{\log c/a}.\ c^{\log a/b}$

a) 0 b) 1 c) -1 d) None

11. Given log2 = 0.3010 and log3 = 0.4771, find the value of log6.

a) 0.9030 b) 0.9542

c) 0.7781 d) None of the above

12. Given that $\log_{10}2$ = x and $\log_{10}3$ = y, the value of $\log_{10} 60$ is expressed as:

a) x + y + 1 b) x − y + 1

c) x − y − 1 d) None of the above

13. Given logx = m + n and logy = m − n, the value of log $(10x/y^2)$ is expressed in terms of m and n as:

a) 1- m + 3n b) m − 1 + 3n

c) m + 3n + 1 d) None of the above

14. If $\log(\frac{x+y}{5})=\frac{1}{2}(\log x+\log y)$, then $\frac{x}{y}+\frac{y}{x}$ =

a) 20 b) 23 c) 22 d) 21

15. If $\log a=\frac{1}{2}\log b=\frac{1}{5}\log c$, then find the value of $a^4b^3c^{-2}$.

a) 0 b) − 1 c) 1 d) None of the above

16. Find the value of $\log_{2\sqrt{3}} 1728$.

a) 2 b) 6 c) 1 d) None of the above

17. On solving the equation logt + log (t - 3) = 1 we get the value of t as

a) 5 b) 2 c) 3 d) 0

18. For any three consecutive integers x, y, z. the equation log (1 + xz) − 2logy = 0 is:

a) True b) False

c) Sometimes true d) Cannot be determined in case of cyclic order

19. If $\log_2 (\log_3 (\log_2 x)) = 1$, then x =

a) 512 b) 128 c) 12 d) 0

20. If $\log_{0.5} (\log_x (\log_4 32)) = 2$, then x =

a) 5/2 b) 625/16 c) 25/4 d) None of the above

21. If $x = \log_a bc$; $y = \log_b ca$; $z = \log_c ab$, then the value of xyz − x − y − z is:

a) 1 b) 2 c) − 1 d) 0

22. Find the value of $\log_5 5.\log_4 9.\log_3 2$.

a) 1 b) 2 c) 5 d) None of the above

23. Find the value of $(\log_b a \times \log_c b \times \log_a c)^3$

a). 1 b) 2 c) 3 d) None of the above

24. If $\log_4 x + \log_2 x = 6$, then the value of x is

a) 2 b) 4 c) 8 d) 16

25. If $\log_{10} \sqrt{x} = 2 \log_x 10$, then a possible value of x is given by:

a) 10 b) $\frac{1}{100}$ c) $\frac{1}{1000}$ d) None of the above

26. Evaluate : $a^{\frac{1}{\log_b a}}$

a) a b) b c) a + b d) None of the above

27. Find the value of the following expression: $a^{\log_a b.\log_b c.\log_c d.\log_d t}$

a) t b) abcdt c) a+b+c+d+t d) None of the above

28. The value of $\left[\frac{1}{\log_{\frac{p}{q}} x}+\frac{1}{\log_{\frac{q}{r}} x}+\frac{1}{\log_{\frac{r}{p}} x}\right]$ is ...?

a) 3 b) 2 c) 1 d) None of the above

29. $\log_2 \log_{\sqrt{2}} \log_3 81 = ?$

a) 3 b) 2 c) 1 d) 0

30. If MOI = $\log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$, then MOI equals:

a) 3 b) 5 c) 7 d) 25

31. Given log2 = 0.30103, the number of digits in 2^{50} is

a) 14 b) 16 c) 18 d) 25

32. $\log_2 5$

a) An integer
b) A rational number
c) An irrational number
d) A prime number

33. $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$

a) log 2 b) 1 c) 0 d) None of the above

34. The value of $\log_2 [\log_2 [\log_3 (\log_3 27^3)]]$ is equal to

(a) 1 (b) 2 (c) 0 (d) none of these

35. If $\log_2 x + \log_4 x + \log_{16} x = 21/4$, these x is equal to

(a) 8 (b) 4 (c) 16 (d) none of these

36. Given that $\log_{10} 2 = x$, $\log_{10} 3 = y$, then $\log_{10} 1.2$ is expressed in terms of x and y as

(a) x + 2y − 1 (b) x + y − 1 (c) 2x + y − 1 (d) none of these

37. The value of $\log_8 25$ given log 2 = 0.3010 is

(a) 1 (b) 2 (c) 1.5482 (d) none of these

38. If $a = b^2 = c^3 = d^4$ then the value of $\log_a (abcd)$ is

(a) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$ (b) $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}$

(c) $1 + 2 + 3 + 4$ (d) None

39. The sum of the series $\log_a b + \log_{a^2} b^2 + \log_{a^3} b^3 + ... + \log_{a^n} b^n$ is given by

(a) $\log_a b^n$ (b) $\log_{a^n} b$ (c) $\log_{a^n} b^n$ (d) None

40. If $a^2 + b^2 = 7ab$ then the value of is $\log \frac{a+b}{3} - \frac{\log a}{2} - \frac{\log b}{2}$

(a) 0 (b) 1 (c) −1 (d) 7

41. If $a^3 + b^3 = 0$ then the value of $\log(a + b) - \frac{1}{2}(\log a + \log b + \log 3)$ is equal to

(a) 0 (b) 1 (c) −1 (d) 3

PAST YEAR QUESTIONS

42. The value of $2 \log x + 2 \log x^2 + 2 \log x^3 + . . . + 2 \log x^n$ will be:

(a) $\frac{n(n+1)\log x}{2}$ (b) $n(n + 1) \log x$

(c) $n^2 \log x$ (d) none of these

43. If n = m! where ('m' is a positive integer ≥ 2) then the value of :

$$\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+....+\frac{1}{\log_m n}$$

(a) 1 (b) 0 (c) −1 (d) 2

44. Which of the following is true. If $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=\frac{1}{abc}$

(a) $\log(ab + bc + ca) = abc$

(b) $\log\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc$

(c) $\log (abc) = 0$

(d) $\log (a + b + c) = 0$

45. For what value of x, the equation $(\log_{\sqrt{x}} 2)^2 = \log_x 2$ is true?

(a) 16 (b) 32 (c) 8 (d) 4

46. If $x = \log_{24} 12$, $y = \log_{36} 24$ and $z = \log_{48} 36$, then $xyz + 1 =$ ______

(a) 2xy (b) 2xz (c) 2yz (d) 2

47. The value of $\log(1^3 + 2^3 + 3^3 + \ldots n^3)$ is equal to:

(a) $3 \log 1 + 3 \log 2 + \ldots + 3 \log n$

(b) $2 \log n + 2 \log (n + 1) - 2 \log 2$

(c) $\log n + \log (n + 1) + \log (2n + 1) - \log 6$

(d) 1

SPECIAL TYPE OF QUESTIONS

1. Find the simplest value of $\log_{17}\sqrt{17\sqrt{17\sqrt{17.....\infty}}}$

   (a) 1; (b) − 1; (c) 0; (d) None

2. If $\log_{1000} x = \frac{-1}{4}$, then x is given by:

   (a) 1/100 (b) 1/10 (c) 1/20 (d) None of these

HOMEWORK SECTION (INDICES & LOG)

1. $4x^{-1/4}$ is expressed as

   (a) $-4x^{1/4}$ (b) x^{-1} (c) $4/x^{1/4}$ (d) none of these

2. The value of $8^{1/3}$ is

   (a) $3\sqrt{2}$ (b) 4 (c) 2 (d) none of these

3. The value of $2 \times (32)^{1/5}$ is

   (a) 2 (b) 10 (c) 4 (d) none of these

4. The value of $4/(32)^{1/5}$ is

   (a) 8 (b) 2 (c) 4 (d) none of these

5. The value of $(8/27)^{1/3}$ is

   (a) 2/3 (b) 3/2 (c) 2/9 (d) none of these

6. The value of $2(256)^{-1/8}$ is

   (a) 1 (b) 2 (c) 1/2 (d) none of these

7. $2^{1/2}.\ 4^{3/4}$ is equal to

   (a) a fraction (b) a positive integer
   (c) a negative integer (d) none of these

8. $\left(\dfrac{81x^4}{y^{-8}}\right)^{\frac{1}{4}}$ has simplified value equal to

   (a) xy^2 (b) x^2y (c) $9xy^2$ (d) $3xy^2$

9. $x^{a-b} \times x^{b-c} \times x^{c-a}$ is equal to

   (a) x (b) 1 (c) 0 (d) none of these

10. The value of $\left(\dfrac{2p^2q^3}{3xy}\right)^0$ where $p,q,x,y \neq 0$ is equal to

    (a) 0 (b) 2/3 (c) 1 (d) none of these

11. $\{(3^3)^2 \times (4^2)^3 \times (5^3)^2\} / \{(3^2)^3 \times (4^3)^2 \times (5^2)^3\}$ is

    (a) 3/4 (b) 4/5 (c) 4/7 (d) 1

12. Which is True ?

(a) $2^0 > (1/2)^0$ (b) $2^0 < (1/2)^0$

(c) $2^0 = (1/2)^0$ (d) none of these

13. If $x^{1/p} = y^{1/q} = z^{1/r}$ and $xyz = 1$, then the value of $p + q + r$ is

(a) 1 (b) 0 (c) 1/2 (d) none of these

14. The value of $y^{a-b} \times y^{b-c} \times y^{c-a} \times y^{-a-b}$ is

(a) ya+b (b) y (c) 1 (d) $1/y^{a+b}$

15. The True option is

(a) $x^{2/3} = \sqrt[3]{x^2}$ (b) $x^{2/3} = \sqrt{x^3}$

(c) $x^{2/3} > 3\sqrt{x^2}$ (d) $x^{2/3} < 3\sqrt{x^3}$

16. The simplified value of $16x^{-3}y^2 \times 8^{-1}x^3y^{-2}$ is

(a) 2xy (b) xy/2 (c) 2 (d) none of these

17. The value of $(8/27)^{-1/3} \times (32/243)^{-1/5}$ is

(a) 9/4 (b) 4/9 (c) 2/3 (d) none of these

18. log 6 + log 5 is expressed as

(a) log 11 (b) log 30 (c) log 5/6 (d) none of these

19. $\log_2 8$ is equal to

(a) 2 (b) 8 (c) 3 (d) none of these

20. log 32/4 is equal to

(a) log 32/log 4 (b) log 32 − log 4

(c) 23 (d) none of these

21. log (1 × 2 × 3) is equal to

(a) log 1 + log 2 + log 3 (b) log 3

(c) log 2 (d) none of these

22. The value of log 0.0001 to the base 0.1 is

(a) −4 (b) 4 (c) 1/4 (d) none of these

23. If 2 log x = 4 log 3, the x is equal to
(a) 3 (b) 9 (c) 2 (d) none of these

24. $\log_{\sqrt{2}} 64$ is equal to
(a) 12 (b) 6 (c) 1 (d) none of these

25. $\log_{2\sqrt{3}} 1728$ is equal to
(a) $2\sqrt{3}$ (b) 2 (c) 6 (d) none of these

26. log (1/81) to the base 9 is equal to
(a) 2 (b) ½ (c) −2 (d) none of these

27. log 0.0625 to the base 2 is equal to
(a) 4 (b) 5 (c) 1 (d) none of these

28. Given log2 = 0.3010 and log3 = 0.4771 the value of log 6 is
(a) 0.9030 (b) 0.9542 (c) 0.7781 (d) none of these

29. The value of $\log_2 \log_2 \log_2 16$
(a) 0 (b) 2 (c) 1 (d) none of these

30. The value of log 1/3 to the base 9 is
(a) − ½ (b) ½ (c) 1 (d) none of these

31. If log x + log y = log (x+y), y can be expressed as
(a) x−1 (b) x (c) x/x−1 (d) none of these

32. The value of $\log_2 [\log_2 \{\log_3 (\log_3 27^3)\}]$ is equal to
(a) 1 (b) 2 (c) 0 (d) none of these

33. If $\log_2 x + \log_4 x + \log_{16} x = 21/4$, these x is equal to
(a) 8 (b) 4 (c) 16 (d) none of these

34. Given that $\log_{10} 2 = x$ and $\log_{10} 3 = y$, the value of $\log_{10} 60$ is expressed as
(a) x − y + 1 (b) x + y + 1
(c) x − y − 1 (d) none of these

35. Given that $\log_{10}2 = x$, $\log_{10}3 = y$, then $\log_{10}1.2$ is expressed in terms of x and y as
   (a) $x + 2y - 1$ (b) $x + y - 1$
   (c) $2x + y - 1$ (d) none of these

36. Given that $\log x = m + n$ and $\log y = m - n$, the value of $\log 10x/y^2$ is expressed in terms of m and n as
   (a) $1 - m + 3n$ (b) $m - 1 + 3n$
   (c) $m + 3n + 1$ (d) none of these

37. The simplified value of $2 \log_{10}5 + \log_{10}8 - ½ \log_{10}4$ is
   (a) 1/2   (b) 4   (c) 2   (d) none of these

38. $\log [1 - \{1 - (1 - x^2)^{-1}\}^{-1}]^{-1/2}$ can be written as
   (a) $\log x^2$ (b) $\log x$
   (c) $\log 1/x$ (d) none of these

39. The simplified value of $\log \sqrt[4]{729 \sqrt[3]{9^{-1}}.27^{-4/3}}$ is
   (a) log 3   (b) log 2   (c) log ½   (d) none of these

40. The value of $(\log_b a \times \log_c b \times \log_a c)^3$ is equal to
   (a) 3   (b) 0   (c) 1   (d) none of these

41. The logarithm of 64 to the base $2\sqrt{2}$ is
   (a) 2   (b) $\sqrt{2}$ (c) ½   (d) none of these

42. The value of $\log_8 25$ given $\log 2 = 0.3010$ is
   (a) 1   (b) 2
   (c) 1.5482   (d) none of these

43. Show that $\left(\frac{x^b}{x^c}\right)^{1/bc} \times \left(\frac{x^c}{x^a}\right)^{1/ca} \times \left(\frac{x^a}{x^b}\right)^{1/ab}$ reduces to
   (a) -1   (b) 0   (c) 1   (d) None

44. Show that $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \times \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \times \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$ is given by
   (a) 1   (b) -1   (c) 0   (d) 3

45. On simplification $\frac{1}{1+z^{a-b}+z^{a-c}}+\frac{1}{1+z^{b-c}+z^{b-a}}+\frac{1}{1+z^{c-a}+z^{c-b}}$ would reduces to

(a) 0 (b) -1 (c) 1 (d) 2

46. If $x=5^{1/3}+5^{-1/3}$ prove that $5x^3-15x$ is given by

(a) 25 (b) 26 (c) 27 (d) 30

47. On simplification $\frac{x^{\frac{a}{a-b}}}{x^{\frac{a}{a+b}}} \times \frac{x^{\frac{b}{b-a}}}{x^{\frac{b}{b+a}}}$ reduces to

(a) 1 (b) -1 (c) 0 (d) None

48. On simplification $\left[\frac{x^{ab}}{x^{a^2+b^2}}\right]^{a+b} \times \left[\frac{x^{b^2+c^2}}{x^{bc}}\right]^{b+c} \times \left[\frac{x^{ca}}{x^{c^2+a^2}}\right]^{c+a}$ reduces to

(a) x^{-2a3} (b) x^{2a3} (c) $x^{-2(a^3+b^3+c^3)}$ (d) $x^{2(a^3+b^3+c^3)}$

49. If $a=\sqrt[3]{\sqrt{2}+1}-\sqrt[3]{\sqrt{2}-1}$ then the value of a^3+3a-2 is

(a) 3 (b) 0 (c) 2 (d) 1

50. If $a=\frac{1}{2}(5-\sqrt{21})$ then the value of $a^3+a^{-3}-5a^2-5a^{-2}+a+a^{-1}$ is

(a) 0 (b) 1 (c) 5 (d) - 1

51. $\frac{1}{1+log_a(bc)}+\frac{1}{1+log_b(ca)}+\frac{1}{1+log_c(ab)}$

(a) 0 (b) 1 (c) 3 (d) -1

52. If $\frac{loga}{y-z}=\frac{logb}{z-x}=\frac{logc}{x-y}$ the value of abc is

(a) 0 (b) 1 (c) -1 (d) None

53. If $\frac{1}{2}loga=\frac{1}{3}logb=\frac{1}{5}logc$ the value of a^4-bc is

(a) 0 (b) 1 (c) -1 (d) None

54. If $\frac{1}{4}log_2a=\frac{1}{6}log_2b=-\frac{1}{24}log_2c$ the value of a^3b^2c is

(a) 0 (b) 1 (c) -1 (d) None

55. If $\frac{1}{log_at}+\frac{1}{log_bt}+\frac{1}{log_ct}=\frac{1}{log_zt}$ then the value if z is given by

(a) abc (b) a+b+c (c) a(b+c) (d) (a+b)c

56. If $l = 1+\log_a bc$, $m = 1+\log_b ca$, $n = 1+\log_c ab$ then the value of $\frac{1}{l}+\frac{1}{m}+\frac{1}{n}-1$ is

(a) 0 (b) 1 (c) -1 (d) 3

57. If $(4.8)^x = (0.48)^y = 1,000$ then the value of $\frac{1}{x}-\frac{1}{y}$ is

(a) 3 (b) -3 (c) 1/3 (d) -1/3

58. If $x^{2a-3}y^{2a} = x^{6-a}\,y^{5a}$ then the value of $a\log\left(\frac{x}{y}\right)$ is

(a) 3 log x (b) log x (c) 6 log x (d) 5 log x

HOMEWORK SOLUTION

1. (c) $\dfrac{4}{x^{1/4}}$

2. (c) 2

3. (c) 4

4. (b) 2

5. (a)

6. (a) 1

7. (b) A positive integer

$$2^{1/2}.(4)^{3/4} = 2^{1/2}.(2^2)^{3/4}$$

$$= 2^{1/2}.(2)^{3/2}$$

$$= 2^{\frac{1}{2}+\frac{3}{2}}$$

$$= 2^2$$

$$= 4$$

8. (d) $3xy^2$

$$\left(\frac{81.x^4}{y-8}\right)^{1/4} = \left(\frac{3^4.x^4}{y^{-8}}\right)^{1/4}$$

$$= \frac{3x}{y^{-2}} = 3xy^2$$

9. (b) 1 Cyclical terms (in power)

10. (c) 1

11. (d) 1

$$\frac{3^6 \times 4^6 \times 5^6}{3^6 \times 4^6 \times 5^6} = 1$$

12. (c) $2^{\circ} = \left(\frac{1}{2}\right)^{o}$

13. (b) 0

$x^{1/p} = y^{1/q} = z^{1/r} = k$

$\therefore\ x^{1/p} = k \quad \Rightarrow x = k^{p}$

$\therefore\ y^{1/q} = k \quad \Rightarrow x = k^{q}$

$\therefore\ z^{1/r} = k \quad \Rightarrow z = k^{r}$

$xyz = 1$

$\therefore\ k^{p}.k^{q}.k^{r} = k^{o}$

$\therefore\ k^{p+q+r} = k^{o}$

$\therefore\ p + q + r = 0$

14. (d) $\frac{1}{y^{a+b}}$

$y^{a-b} \times y^{b-c} \times y^{c-a} \times y^{-a-b}$

$= y^{a-b+b-c+c-a-a-b}$

$= y^{-(a+b)}$

$= \frac{1}{y^{a+b}}$

15. (a) $x^{2/3} = \sqrt[3]{x^{2}}$

16. (c) 2

$= 16 . x^{-3} . y^{2} . 8^{-1} . x^{3} . y^{-2}$

$= \frac{16 . y^{2} . x^{3}}{8 . x^{3} . y^{2}}$

$= 2$

17. (a) $\frac{9}{4}$

$\left(\frac{8}{27}\right)^{-1/3} . \left(\frac{32}{243}\right)^{-1/5} = \left(\frac{27}{8}\right)^{1/3} . \left(\frac{243}{32}\right)^{1/5}$

$= \frac{3}{2} . \frac{3}{2}$

$= \frac{9}{4}$

18. (b) log 30

$\log(6 \times 5) = \log 30$

19. (c) 3

$\log_2 8 = \log_2 2^3 = 3\log_2 2 = 3$

20. (b) log 32 − log 4

21. (a) log 1 + log 2 + log 3

22. (b) 4

$\log_{0.1} 0.0001 = \log_{0.1} (0.1)^4 = 4\log_{0.1} 0.1 = 4$

23. (b) 9

$\text{Log } x^2 = \log 3^4$

$\therefore X^2 = 3^4$

$\therefore X = 3^2 = 9$

24. (a) 12

$\text{Log}_{\sqrt{2}} 64 = \log_{\sqrt{2}} \left(\left(\sqrt{2}\right)^2\right)^6$

$= \log_{\sqrt{2}} \left(\sqrt{2}\right)^{12}$

$= 12$

25. (c) 6

$\text{Log}_{258} 1728 = \log_{2\sqrt{3}} \left(2\sqrt{3}\right)^6$

$=6$

26. (c) -2

$\text{Log}_9 \left(\frac{1}{81}\right) = \log_9 (81)^{-1}$

$= \log_9 (9)^{-2}$

$= -2$

27. (d) None of these trial and error

28. (c) 0.7781

$\therefore$ Log 6 $= \log(2 \times 3) = \log 2 + \log 3$

$= 0.3010 + 0.4771$

$= 0.7781$

29. (c) 1

$\log_2 \log_2 \log_2 16 = \log_2 \log_2 \log_2 2^4$

$= \log_2 \log_2 4$

$= \log_2 \log_2 2_2$

$= \log_2 2$

$= 1$

30. (a) $\frac{-1}{2}$

$\log_9 1/3 = \frac{\log(3)^{-1}}{\log 9} = \frac{-1 \log 3}{2 \log 3} = \frac{-1}{2}$

31. (c) $\frac{x}{x-1}$

$\log x + \log y = \log(x + y)$

$\therefore \log(xy) = \log(x + y)$

$\therefore xy = x + y$

$\therefore xy - y = x$

$\therefore y(x - 1) = x$

$\therefore y = \frac{x}{x-1}$

32. (c) 0

$\log_2 [\log_2 \{\log_3 (\log_3 27^3)\}]$

$= \log_2 [\log_2 \{\log_3 (\log_3 (3^3)^3)\}]$

$= \log_2 [\log_2 \{\log_3 9\}]$

$= \log_2 [\log_2 2]$

$= \log_2 1$

33. (a) 8

$\text{Log}_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$

$\therefore \frac{\log x}{\log 2} + \frac{\log x}{\log 4} + \frac{\log x}{\log 16} = \frac{21}{4}$

$\therefore \frac{\log x}{\log 2}\left[1 + \frac{1}{2} + \frac{1}{4}\right] = \frac{21}{4}$

$\therefore \log_2 x \left[\frac{4+2+1}{4}\right] = \frac{21}{4}$

$\therefore \log_2 x = 3$

$\therefore 2^3 = x$

$\therefore x = 8$

34. (b) x + y + 1

$\log_{10} 60 = \log_{10} (2 \times 3 \times 10)$

$= \log 2 + \log 3 + \log 10$

$= x + y + 1$

35. (c) 2x + y - 1

$\log_{10} 12 = \log_{10} \frac{12}{10}$

$= \log 12 - \log 10$

$= \log (2 \times 2 \times 3) - \log 10$

$= \log 2 + \log 2 + \log 3 - \log 10$

$= x + x + y - 1$

$= 2x + y - 1$

36. (a) 1 − m + 3n

$\log \frac{10x}{y^2} = \log 10 + \log x - \log y^2$

$= \log 10 + \log x - 2 \log y$

$= 1 + m + n - 2(m - n)$

$= 1 + m + n - 2m - 2n$

$= 1 - m + 3n$

37. (c) 2

$2\log_{10}5 + \log_{10}8 - \log_{10}4$

$= \log_{10}5^2 + \log_{10}8 - \log_{10}4^{\frac{1}{2}}$

$= \log_{10}\left(\frac{25 \times 8}{2}\right)$

$= \log_{10}100$

$= 2$

38. (b) log x

$= \log [1 - \{1 - (1 - x^2)^{-1}\}^{-1}]^{\frac{-1}{2}}$

$= \log [1 - \{1 - \frac{1}{1 - x^2}\}^{-1}]^{\frac{-1}{2}}$

$= \log [1 - \{\frac{1 - x^2 - 1}{1 - x^2}\}^{-1}]^{\frac{-1}{2}}$

$= \log [1 - \{\frac{x^2}{1 - x^2}\}^{-1}]^{\frac{-1}{2}}$

$= \log \left[\frac{1}{x^2}\right]^{\frac{-1}{2}}$

$= \log (x^{-2})^{\frac{-1}{2}}$

$= \log x$

39. (a) log 3

$\log \sqrt[4]{729 \sqrt[3]{9^{-1}(27)^{\frac{-4}{3}}}}$

$= \log \left[729 (9^{-1} (27)^{\frac{-4}{3}})^{\frac{1}{3}}\right]^{\frac{1}{4}}$

$= \log [3^6 (3^{-2} \times (3^3)^{\frac{-4}{3}})^{\frac{1}{3}}]^{\frac{1}{4}}$

$= \log [3^6 (3^{-2-4})^{\frac{1}{3}}]^{\frac{1}{4}}$

$= \log [3^6\, 3^{-2}]^{\frac{1}{4}}$

$= \log [3^4]^{\frac{1}{4}}$

$= \log 3$

40. (c) 1

The value of ($\log_b a \times \log_c b \times \log_a c$)

$= \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right)^3$

$= (1)^3$

$= 1$

41. (d) 4

$\log_{2\sqrt{2}} 64 = \log_{2\sqrt{2}} (2\sqrt{2})^4$

$= 4$

42. (c) 1.5482

$\log_8 25 = \log_8 \left(\frac{25 \times \cancel{4}}{4}\right) = \cancel{\log_8}\left(\frac{100}{4}\right)$

$= \log_8\left(\frac{100}{4}\right)$

$= \frac{\log 100 - \log 4}{\log 8}$

$= \frac{\log 20 - \log 2}{\log 2}$

43. Show that $\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \cdot \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$.

M – 1 $\left(x^{b-c}\right)^{\frac{1}{bc}} \cdot \left(x^{c-a}\right)^{\frac{1}{ca}} \cdot \left(x^{a-b}\right)^{\frac{1}{ab}}$.

$= x^{\frac{b-c}{bc} + \frac{c-a}{ca} + \frac{a-b}{ab}}$

$= x^{\frac{(b-c)+b(c-a)+c(a-b)}{abc}}$ [Taking L.C.M. in the power]

$= x^{\frac{ab-ac+bc-ac+ac-ab}{abc}}$

$= x^{\frac{0}{abc}}$

$= x^\circ = 1$

M – II = $\frac{x^{\frac{\cancel{b}}{\cancel{b}c}}}{x^{\frac{\cancel{c}}{b\cancel{c}}}} \frac{x^{\frac{\cancel{c}}{\cancel{c}a}}}{x^{\frac{\cancel{a}}{c\cancel{a}}}} \frac{x^{\frac{\cancel{a}}{\cancel{a}b}}}{x^{\frac{\cancel{b}}{a\cancel{b}}}}$

$= \frac{x^{\frac{1}{\cancel{c}}}}{x^{\frac{1}{\cancel{b}}}} \frac{x^{\frac{1}{\cancel{a}}}}{x^{\frac{1}{\cancel{c}}}} \frac{x^{\frac{1}{\cancel{b}}}}{x^{\frac{1}{\cancel{a}}}} = 1$

44. S.T $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$

$= x^{(a-b)(a2+ab+b2)} \cdot x^{(b-c)(b2+bc+c2)} \cdot x^{(c-a)(c2+ac+a2)}$

$= x^{a3-b3}\ x^{b3-c3} \cdot x^{c3-a3}$

$= x^{a3-b3+b3-c3+c3-a3}$

$= x^{\circ} = 1$

45. on simplification $\frac{1}{1+{}^{a-b}+Z^{a-c}} + \frac{1}{1+Z^{b-c}+Z^{b-a}} + \frac{1}{1+Z^{c-a}+Z^{c-b}}$

Multiply 1st term by Z^{-a} (both no and Dr.)

2nd term by Z^{-b} (______,_______)

3rd term by Z^{-c} (______,_______)

$= \frac{Z^{-a}}{Z^{-a}+Z^{-b}+Z^{-c}} + \frac{Z^{-b}}{Z^{-b}+Z^{-c}+Z^{-a}} + \frac{Z^{-c}}{Z^{-c}+Z^{-a}+Z^{-b}}$

$= \frac{Z^{-a}+Z^{-b}+Z^{-c}}{Z^{-a}+Z^{-b}+Z^{-c}} = 1$

46. $x = 5^{\frac{1}{3}} + 5^{\frac{-1}{3}}$ find $5x^3 - 15x = ?$

$x^3 = \left(5^{\frac{1}{3}} + 5^{\frac{-1}{3}}\right)^3$ [Cubing on the sides]

$x^3 = \left(5^{\frac{1}{3}}\right)^3 + \left(5^{\frac{-1}{3}}\right)^3 + 3\left(5^{\frac{1}{3}}\right) \times 3\left(5^{\frac{-1}{3}}\right) \times \left(5^{\frac{1}{3}} + 5^{\frac{-1}{3}}\right)$

[$\because$ $(a+b)^2 - a^3 + b^3 - 3ab\,(a+b)$

$x^3 = 5 + 5^{-1} + 3(5^{\circ})\,(x)$ [$\because$ $x = 5^{\frac{1}{3}} + 5^{\frac{-1}{3}}$]

$x^3 = 5 + \frac{1}{5} + 3x$

$x^3 - 3x = \frac{26}{5}$

$\therefore 5x^3 - 15x = 26$

47. a. $\left[\frac{\frac{a}{x^{a-b}}}{\frac{a}{x^{a+b}}} \times \frac{\frac{b}{x^{b-a}}}{\frac{b}{x^{b+a}}}\right]$

$= \left[\frac{\frac{a}{x^{a-b}} - \frac{b}{a-b}}{\frac{a}{x^{a+b}} + \frac{b}{b+a}}\right] = \left[\frac{\frac{a-b}{x^{a-b}}}{\frac{a+b}{x^{a+b}}}\right] = \frac{x}{x} = 1$

48. $\left(\dfrac{x^{ab}}{x^{a^2+b^2}}\right)^{a+b}\left(\dfrac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c}\left(\dfrac{x^{ca}}{x^{c^2+a^2}}\right)^{a+b}$

$= (x^{ab-a^2-b^2})^{a+b}\ x^{(b^2+c^2-bc)(b+c)}\ x^{(ca-c^2-a^2)(c+a)}$

$= x^{-(a^2+b^2-ab)(a+b)}\ x^{(b^2+c^2-bc)(b+c)}\ x^{-(c^2-a^2-ca)(c+a)}$

$= x^{-(a^3+b^3)}\ x^{b^3+c^3}\ x^{-(c^3+a^3)}$

$= x^{-a^3-\cancel{b^3}+\cancel{b^3}+\cancel{c^3}-\cancel{c^3}-a^3}$

$= x^{-2a^3}$

49. If a = $a = \sqrt[3]{\sqrt{2}+1} - \sqrt[3]{\sqrt{2}-1}$

Then the value of $a^3 + 3a-2$ is

$a = (\sqrt{2}+1)^{\frac{1}{3}} - (\sqrt{2}+1)^{\frac{1}{3}}$

Cubing on both sides, we get,

$a^3 = \left[(\sqrt{2}+1)^{\frac{1}{3}} - (\sqrt{2}-1)^{\frac{1}{3}}\right]^3$

$= \cancel{\sqrt{2}}+1-(\cancel{\sqrt{2}}-1) - 3\left[(\sqrt{2}+1)^{\frac{1}{3}}\left(\sqrt{2}-1\right)^{\frac{1}{3}}\right]\ (\sqrt{2}+1)^{\frac{1}{3}} - (\sqrt{2}-1)^{\frac{1}{3}}$

$= \sqrt{2}+1-(\sqrt{2}-1) - 3\left[(\sqrt{2}+1)^{\frac{1}{3}}\left(\sqrt{2}-1\right)^{\frac{1}{3}}\right]\ (a)$

$= \cancel{\sqrt{2}}+1-\cancel{\sqrt{2}}+1-3a\,[(\cancel{\sqrt{2}})^2-(1)^2]^{\frac{1}{3}}$

$a^3 = 2-3a[2-1]^{\frac{1}{3}}$

$a^3 = 2-3a$

$\therefore a^3+3a-2=0$

50. $a = \dfrac{1}{2}(5 - \sqrt{21})$

$a = \dfrac{(5-\sqrt{21})}{2};\ \dfrac{1}{a} = \dfrac{1}{2}(5+\sqrt{21})$

$\therefore a + \dfrac{1}{a} = \dfrac{5\sqrt{21}+5+\sqrt{21}}{2} = \dfrac{10}{2} = 5$

51. $\dfrac{1}{1+\log_a bc}+\dfrac{1}{1+\log_b ca}+\dfrac{1}{1+\log_c ab}=?$

M – I

Let, a = 2, b = 4, c = 8

$$=\frac{1}{1+\log_2{}^{32}}+\frac{1}{1+\log_4{}^{16}}+\frac{1}{1+\log_8{}^{8}}$$

$$=\frac{1}{1+5\log_2{}^{2}}+\frac{1}{1+2\log_4{}^{4}}+\frac{1}{1+1}$$

$$=\frac{1}{1+5}+\frac{1}{1+2}+\frac{1}{2}$$

$$=\frac{1}{6}+\frac{1}{3}+\frac{1}{2}$$

$$=\frac{1+2+3}{6}$$

$$=\frac{6}{6}$$

M-II

$$\frac{1}{\log_a{}^{a}+\log_a{}^{bc}}+\frac{1}{\log_b{}^{b}+\log_b{}^{ac}}+\frac{1}{\log_c{}^{c}+\log_a{}^{ab}}$$

$$=\frac{1}{\log_a abc}+\frac{1}{\log_b abc}+\frac{1}{\log_c abc}$$

$$=\cancel{\log}_a{}^{\cancel{abc}}+\cancel{\log}_b{}^{\cancel{abc}}+\cancel{\log}_c{}^{\cancel{abc}}$$

$$=\log_{abc} a+\log_{abc} b+\log_{abc} c$$

$$=\log_{abc} abc=1$$

52. $\dfrac{\log a}{y-z}=\dfrac{\log b}{z-x}=\dfrac{\log c}{x-y}=k$

log a = k (y - z)

log b = k (z - x)

$\underline{\text{log c = k (x - y)}}$

log a + log b + log c = 0

log abc = 0 = log 1

$\therefore$ abc = 1

Hence, x, y, z are in cyclic order, $\therefore$ their sums is 0

M-II

$\log_{10}$ abc = 0

10^{o} = abc = 1

53. $a^4 - bc = ?$

$\frac{1}{2}\log a = \frac{1}{3}\log b = \frac{1}{5}\log c = k$

$\log_{10}a = 2k ; \log_{10}b = 3k ; \log_{10}c = 5k$

$10^{2k} = a ; 10^{3k} = b ; 10^{5k} = c$

$a^4 = 10^{8k} ;$

$10^{8k} - 10^{3k} \times 10^{5k}$

$10^{8k} - 10^{8k}$

$= 0$

54. $\frac{1}{4}\log_2 a = \frac{1}{6}\log_2 b = = \frac{-1}{24}\log_2 c = k$ $\quad a^3 b^2 c = ?$

$\log_2 a = 4k ; \log_2 b = 6k ; \log_2 c = -24k$

$2^{4k} = a ; 2^{6k} = b ; 2^{-24k} = c$

$a^3 = (2^{4k})^3 = 2^{12k} ; b^2 = (2^{6k})^2 = 2^{12k}$

$\therefore a^3b^2c = 2^{12k} \; 2^{12k} \; 2^{-24k}$

$= 2^{24k - 24k}$

$= 2^0 = 1$

55. $\frac{1}{\log_a t} + \frac{1}{\log_b t} + \frac{1}{\log_c t} = + \frac{1}{\log_z t}$ $\quad z = ?$

$\log_t a + \log_t b + \log_t c = \log_t z$

$\log_t abc = \log_t z$

$abc = z$

56. $L = 1 + \log_a bc$

$M = 1 + \log_b ac$

$M = 1 + \log_c ab$

M- I

Take, $a = 2 ; b = 2^2 ; c = 2^3 = 8$

$L = 1 + \log_2{}^{32} = 1 + 5 \log_2{}^{2} = 6$

$M = 1 + \log_4{}^{16} = 1 + 2 = 3$

$N = 1 + \log_8{}^{8} = 1 + 1 = 2$

$= \frac{1}{L} + \frac{1}{M} + \frac{1}{N} - 1$

$= \frac{1}{6} + \frac{1}{3} + \frac{1}{2} - 1$

$= \frac{1+2+3}{6} - 1$

$= 1 - 1 = 0$

M- II

Let a = b = c

$L = 1 + \log_a bc = 1 + \log_a a^2 = 1 + 2 = 3$

M = 3 ; N = 3.

$= \frac{1}{L} + \frac{1}{M} + \frac{1}{N} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} - 1 = 1 - 1 = 0$

57. $(4.8)^x = (0.48)^y = 1,000$ $\quad \frac{1}{x} + \frac{1}{y} = ?$

$(4.8)^x = (0.48)^y = (10)^3$

$(4.8)^x = 10^3 \quad ; (0.48)^y = 10^3$

$4.8 = 10^{\frac{3}{x}} \quad ; 0.48 = 10^{\frac{3}{y}}$

$0.48 \times 10 = 10^{\frac{3}{Y}}\ 10$

$4.8 = 10^{\frac{3}{Y}+1}$

$\therefore 10^{\frac{3}{Y}} = 10^{\frac{3}{Y}+1}$

$\therefore \frac{3}{x} = \frac{3}{y} + 1$

$\therefore \frac{3}{x} - \frac{3}{y} = 1$

$= 3\left(\frac{1}{x} - \frac{1}{y}\right) = 1$

$= \frac{1}{x} - \frac{1}{y} = \frac{1}{3}$

58. $x^{2a-3}y^{2a} = x^{6-a}\,y^{5a}$ $\qquad a\log\left(\frac{x}{y}\right) = 3\log^{x}$

$$\frac{x^{2a-3}}{x^{6-a}} = \frac{y^{5a}}{y^{2a}}$$

$$x^{2a-3-6} = y^{3a}$$

$$x^{30-9}\;y^{3a}$$

$$\frac{x^{3a}}{x^{9}} = y^{3a}$$

$$\frac{x^{3a}}{y^{3a}} = x^{9}$$

$$\left(\frac{x}{y}\right)^{3a} = x^{9}$$

$$\cancel{3}a\log\frac{x}{y} = \cancel{9}^{3}\log x$$

SELF ASSESSMENT TEST 1
Ratio, Proportion and Mixtures
20 Question, 20 Marks

1. The ratio of A to B is 4 : 5 and that of C to B is 3 : 2. If A = 800, C = ?

a) 1000 b) 1200 c) 1500 d) 2000

2. Three numbers A, B and C are in the ratio 1 / 2 : 2 / 3 : 3 / 4. The difference between the greatest and the smallest number is 36. Find A.

a) 60 b) 72 c) 84 d) None of the above

3. Ratio of land and water on earth is 1 : 2. In northern hemisphere, the ratio is 2 : 3. What is the ratio in the southern hemisphere?

a) 3 : 11 b) 2 : 11 c) 4 : 11 d) 5 : 11

4. The ratio of copper and zinc in a 63 kg alloy is 4 : 3. Some amount of copper is extracted from the alloy and the ratio becomes 10 : 9. How much copper is extracted?

a) 6 kg b) 8 kg c) 12 kg d) 10 kg

5. A bag contains Rs. 55 in the denominations of Re 1, 50 paise and 25 paise coins. The coins are in the ratio 1 : 2 : 3. Find the number of 50 paise coins.

a) 15 b) 30 c) 40 d) 45

6. A person cover certain distance by train, bus and car in the ratio 4 : 3 : 2. The ratio of fare is 1 : 2 : 4 per km. The total expenditure as fare is Rs. 720. Find the total expenditure as fare on train.

a) Rs. 140 b) Rs. 150 c) Rs. 160 d) Rs. 170

7. The ratio of the expenditure of A, B and C are 16 : 12 : 9 respectively and their savings are 20%, 25% and 40% of their incomes. If the sum of their income is Rs. 15,300; find B's salary.

a) Rs. 4800 b) Rs. 5000 c) Rs. 4900 d) Rs. 5100

8. One year ago, the ratio between A and B salary was 3 : 5. The ratio of their individual salaries of last year and present year are 2 : 3 and 4 : 5 respectively. If their total salary for the present year is Rs. 4300, find the present salary of A.

a) Rs. 1200 b) Rs. 1800 c) Rs. 1600 d) Can't be determined

9. The income of A and B is in the ratio 5 : 3. The expenses of A, B and C are in the ratio 8 : 5 : 2. If C spends Rs. 2000 and B saves Rs. 700, then how much did A saves?

a) Rs. 500 b) Rs. 1500 c) Rs. 1000 d) Rs. 250

10. The ratio of total amount distributed in all the males and females as salary is 6 : 5. The ratio of salary of each male and female is 2 : 3. Find the ratio of the number of male and female.

a) 5 : 9 b) 5 : 7 c) 9 : 5 d) 7 : 5

11. Rs. 56000 is to be divided among A, B, C and D in such a way that the ratio of share of A : B is 1 : 2, B : C is 3 : 1 and C : D is 2 : 3. Find the sum of share of A & C and B & C.

a) Rs. 24000, Rs. 30000 b) Rs. 20000, Rs. 32000
c) Rs. 24000, Rs. 32000 d) Rs. 20000, Rs. 30000

12. A, B and C works on a project for 30, 50 and 60 days respectively. The ratio of the salary of each day is 4 : 3 : 2 respectively. If the total amount received by A is Rs. 14,400, find total amount received by B.

a) Rs. 18000 b) Rs. 19000 c) Rs. 18500 d) Rs. 19500

13. Two numbers are in the ratio 4 : 5. If each number is reduced by 25, the ratio becomes 3 : 4. Find the second number.

a) 120 b) 130 c) 125 d) 135

14. The price of gold is directly proportional to square of its weight. A person broke down the gold in the ratio of 3 : 2 : 1 and faces a loss of Rs. 46,200. Find the initial price of the gold.

a) Rs. 75,200 b) Rs. 75,400 c) Rs. 75,300 d) Rs. 75,600

15. Rs. 78,000 is distributed among A, B and C such that the share of A = 3 / 4 share of B and share of B = 2 / 3 of the share of C. What is the difference between the shares of B and C?

a) Rs. 9000 b) Rs. 10000 c) Rs. 11000 d) Rs. 12000

16. A dog chases a rabbit. The dog takes 6 leaps for every 7 leaps of the rabbit. The rabbit takes 6 leaps for every 5 leaps of the dog. What is the ratio of speed of dog and rabbit?

a) 36 : 35 b) 36 : 40 c) 35 : 36 d) None of the above

17. A mixture contains milk and water in the ratio of 4 : 3 respectively. If 6 litres of water is added to this mixture, the respective ratio of water and milk becomes 7 : 8. What is the quantity of milk in the original mixture?

a) 96 litres b) 84 litres c) 36 litres d) 48 litres

18. There are X members in a club, whose average age is 26 years. 3 more persons join them at the 35th Annual General Meeting, and thus the average age of members now increases by 1 year. If the average age of 3 new members joining at the 35th AGM is 29 years, find X.

a) 6 b) 7 c) 8 d) None of the above

19. If the ratio of volume of two cubes with Amit and Ajay is 125 : 8, then find the ratio of the total surface area of the cubes with Ajay and Amit.

a) 25 : 4 b) 4 : 25 c) 5 : 2 d) 2 : 5

20. Mr. Azon divides $ 51,300 among his four partners A, B, C and D such that 3 times A's share = 4 times B's share = 5 times C's share = 6 times D's share. What is the share of B?

a) $ 11,000 b) $ 13,500 c) $ 15,300 d) $ 12,350

EXPLANATORY ANSWERS

1. A : B = 4 : 5

   B : C = 2 : 3

   A : B : C = 8 : 10 : 15

   C = 800/8 * 15 = 1500

   Option C

2. A : B : C = 1 / 2 : 2 / 3 : 3 / 4 = 6 : 8 : 9

   9x − 6x = 3x = 36; x = 12

   A = 6 * 12 = 72

   Option B

3. Let total = 300. At earth, Land = 100, Water = 200

   In northern hemisphere = 150, Land = 150 * 2/5 = 60, Water = 90

   In southern hemisphere: Land = 100 − 60 = 40, Water = 200 − 90 = 110

   Ratio = 40 : 110 = 4 : 11

   Option C

4. Alloy: Copper = 4/7 * 63 = 36; Zinc = 3/7 * 63 = 27

   (36 − X) / 27 = 10 / 9

   36 − X = 30; X = 6

   Option A

5. X + (2X)*1/2 + (3X)*1/4 = 55

   11X/4 = 55

   X = 20

   No. of 50 paise coins = 2X = 40

   Option C

6. 4x * 1 + 3x * 2 + 2x * 4 = 18x = 720; x = 40

   Fare expenditure on train = 4 * 40 = 160; Option C

7. Expenditure = 100% - Savings%

   16x/80% + 12X/75% + 9X/60% = 20x + 16x + 15x = 51x = 15300; x = 300

   B's income = 16 * 300 = 4800; Option A

8. One year ago = 3x, 5x

Present salary = (3x)3/2 : (5x)5/4 = 18 : 25

A's present salary = 18/43 * 4300 = 1800; Option B

9. Income = 5x, 3x

Expenses = 8y, 5y, 2y

2y = 2000; y = 1000

3x − 5000 = 700; x = 1900

A saves = (5 * 1900) − (8 * 1000) = 9500 − 8000 = 1500

Option B

10. Person = Total salary/Salary per person

M : F = 6/2 : 5/3 = 9 : 5

Option C

11. A : B = 1 : 2

B : C = 3 : 1

C : D = 2 : 3

A : B : C : D = 6 : 12 : 4 : 6 = 3 : 6 : 2 : 3

A & C = 5/14 * 56000 = 20000

B & C = 8/14 * 56000 = 32000

Option B

12. 30x(4) + 50x(3) + 60x(2) = 390x

120x = 14400; x = 120

B's amount = 150x = 150 * 120 = 18000

Option A

13. (4x − 25) : (5x − 25) = 3 / 4

16x − 100 = 15x − 75

Or, x = 25

Second number = 5 * 25 = 125

Option C

14. P = k.w²

New P = k (9 + 4 + 1) = 14k

Old P = k(6)2 = 36k

Loss = 36k − 14k = 22k = 46200; k = 2100

Old P = 36 * 2100 = 75600; Option D

15. A : B = 3 : 4

B : C = 2 : 3

A : B : C = 6 : 8 : 12

6x + 8x + 12x = 78000; x = 78000 / 26 = 3000

B − C difference = 4 * 3000 = 12000

Option D

16. 1 leap of dog = 1 units

6 leaps of dog = 6 units = 7 leaps of rabbit; 1 leap of rabbit = 6/7 units

In a given time: 6 rabbit = 5 dog; 6(6/7) : 5 (1) = 36 : 35

Speed of dog and rabbit = 35 : 36

Option C

17. 3x/7 + 6 = 7/15 (x + 6)

45x + 630 = 49x + 294

4x = 336, x = 84

Milk in original mixture = 4/7 * 84 = 48

Option D

18. Total age of 26 members = 26X

Total age of 29 members = 26X + (29x3) = 27(X + 3)

26X + 87 = 27X + 81

X = 6

Option A

19. Amit : Ajay

V = 125 : 8

Side = 5 : 2

Total Surface = 25 : 4

Option B

20. A : B : C : D = 1 / 3 : 1 / 4 : 1 / 5 : 1 / 6 = 120 : 90 : 72 : 60

B's share = 90 / 342 * 51300 = 13500

Option B

SELF ASSESSMENT TEST 2
Surds, Indices & Logarithms
30 Question, 30 Marks

1. The value of is: $\sqrt{30+\sqrt{30+\sqrt{30+.....to\infty}}}$

   a) 6 b) 5 c) 3 d) None of these

2. If $8^x = 4^y$, what is the value of $K^{(3x-2y)}$?

   a) K b) K2 c) 0 d) 1

3. If $X = \log_2 \log_3 \log_2 512$; find the value of $(X - 1)!$

   a) 1 b) 4 c) 6 d) 24

4. If $2\log\frac{4}{3} - \log\frac{x}{10} + \log\frac{63}{160} = 0$, find the value of x.

   a) 3 b) 4 c) 7 d) 9

5. If log 2 = 0.30103; then find the number of digits in 25^{10}

   a) 13 b) 15 c) 14 d) 25

6. Find the value of $9^{1/3}.9^{1/9}.9^{1/27}.9^{1/81}$.

   a) 3 b) 9 c) 81 d) None of the above

7. If X = cube root of 2, Y = 6th root of 3, Z = 9th root of 4; then which of the following is true?

   a) X = Y = Z b) X > Y > Z c) X < Y < Z d) X = Y < Z

8. If $(AB)^{1/2} = 6$ and A & B are positive integers, then which of the following could not be a value of C = (A – B)?

   a) 0 b) 5 c) 8 d) 9

9. If $x = 3 + (8)^{1/2}$, then what is the value of $x^4 + x^{-4}$?

   a) 1154 b) 1145 c) 1164 d) 1146

10. If $a^z = b^y = c^x$ and $b^2 = ca$, then find the value of $y(x + z)$.

    a) 2 b) 2xz c) xz d) None of these

11. If $\log\left(\frac{a+b}{7}\right)=\frac{1}{2}(\log a+\log b)$, then which of the following is true?

a) a + b = 49 b) $a^2 + b^2 = 47ab$

c) $a^2 - b^2 = 49ab$ d) None of these

12. Given X = $\frac{1}{6}\cdot\sqrt{\frac{3\log(1728)}{1+\frac{1}{2}\log(0.36)+\frac{1}{3}\log 8}}$, find the value of (2X − 1).

a) 4 b) 1 c) 0 d) − 1

13. If $x=\frac{6ab}{a+b}$, then the value of $\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}$ is equal to:

a) 0 b) 1 c) 2 d) None of the above

14. If X = $3\sqrt{2} + 7\sqrt{8} + \sqrt{27} + 5\sqrt{3}$. Find X.

a) $17\sqrt{2} + \sqrt{3}$ b) $17\sqrt{2} + 8\sqrt{3}$

c) $15\sqrt{2} + 8\sqrt{3}$ d) None of the above

15. Given $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\frac{5}{2}$. The value of x is:

a) 1 b) 331/5 c) 263/20 d) 17/21

16. If a : b = 3 : -1, find $\frac{ab}{a+b}$

a) 1 b) 3 c) − 3 d) None of the above

17. If $\log\frac{12}{13}-\log\frac{7}{25}+\log\frac{91}{3}=x$, find the value of x.

a) 0 b) 1 c) 2 d) 3

18. $\frac{1}{1+\frac{1}{x}}$ is equal to which of the following?

a) (x + 1) b) (x + 1)-1

c) $x(x + 1)^{-1}$ d) None of the above

19. If $\frac{a^3-b^3}{a^3+b^3}=\frac{13}{14}$; then find the value of $\frac{a+b}{a-b}$.

a) 1 b) 2 c) 1.5 d) None of the above

20. If $(x + x^{-1}) = 3$, then the value of $(x^6 + x^{-6})$ is

a) 927 b) 364 c) 414 d) 322

21. Which of the following relations is true?

a) $\sqrt{4} + \sqrt{3} < \sqrt{5} + \sqrt{2}$

b) $\sqrt{4} + \sqrt{3} = \sqrt{5} + \sqrt{2}$

c) $\sqrt{4} + \sqrt{3} > \sqrt{5} + \sqrt{2}$

d) None of the above is true

22. The expression simplifies to: $\frac{(y-1)(y-2)(y^2-9y+14)}{(y-7)(y^2-3y+2)}$

a) $(y - 1)$ b) $(y - 7)$

c) $(y - 2)$ d) $(y - 7)^{-1}$

23. If $x=\frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y=\frac{\sqrt{3}-1}{\sqrt{3}+1}$, then find the value of $\frac{x^2+xy+y^2}{x^2-xy+y^2}$.

a) 13/15 b) 15/13

c) 2/13 d) 11/13

24. Find the value of $\frac{1}{\sqrt{11-2\sqrt{30}}}-\frac{3}{\sqrt{7-2\sqrt{10}}}-\frac{4}{\sqrt{8+4\sqrt{3}}}$

a) $\sqrt{6} + \sqrt{2}$ b) $\sqrt{6} + \sqrt{5}$

c) $\sqrt{5} + \sqrt{2}$ d) None of the above

25. If X = 3 log5 + 2 log4 − log2, Find the value of (X + 3).

a) 0 b) 6 c) 3 d) None of the above

26. If logX = log1.5 + log12, Find the value of X/3.

a) 0 b) 6 c) 3 d) None of the above

27. Find the value of X, if: Log (X − 13) + 3 Log2 = Log(3X + 1).

a) 21 b) 22 c) 20 d) 24

28. If log3 = 0.4771, find the value of log $(0.81)^2$ * log $(27/10)^{2/3}$ ÷ log9.

a) 2.702 b) − 0.0552 c) 2.2402 d) − 2.689

29. $E = \frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}$. Find the value of (E − 3).

a) 0 b) 1 c) − 1 d) None of the above

30. If $\log\frac{75}{35} + 2\log\frac{7}{5} - \log\frac{105}{x} - \log\frac{13}{25} = 0$ find the value of x.

a) 13 b) 45 c) 50 d) 65

EXPLANATORY ANSWERS

1. 30 = 6*5. If A = K(K +1) and you find "+" sign in it, answer is always (k + 1). Option A

2. $8^x = 4^y$

   $3x = 2y$

   $(3x - 2y) = 0.\ K^0 = 1$; Option D

3. $X = \log_2 \log_3 \log 2512$

   $= \log_2 \log_3 \log 2(2)^9$

   $= \log_2 \log^3 9$

   $= \log_2 \log_3 (3)^2$

   $= \log_2 2 = 1$

   $(X - 1)! = 0! = 1$; Option A

4. $2\log\frac{4}{3} - \log\frac{x}{10} + \log\frac{63}{160} = 0$

   $2\log\frac{4}{3} + \log\frac{63}{160} = \log\frac{x}{10}$

   $\log\left(\frac{4*4*7*3*3}{3*3*4*4*10}\right) = \log\frac{7}{10} = \log\frac{x}{10}$

   x = 7; Option C

5. $X = 25^{10}$

   LogX = 10Log25 = 20Log5 = 20(1 − Log2) = 20(1 − 0.30103) = 13.97

   Characteristics of LogX = 13

   Thus number of digits in X = C + 1 = 13 + 1 = 14; Option C

6. $9^{1/3}.9^{1/9}.9^{1/27}.9^{1/81}.$

   $= 9^{1/3+1/9+1/27+1/81} = 9^{40/81} = 3^{80/81}$

   Option D

7. $X = 2^{1/3} = 2^{6/18} = 64^{1/18}$

   $Y = 3^{1/6} = 3^{3/18} = 27^{1/18}$

   $Z = 4^{1/9} = 4^{2/18} = 16^{1/18}$

   X > Y > Z. Option B

8. AB = 36. Possible pairs (1, 36) (2, 18) (3, 12) (4, 9) (6, 6)

C = |A − B| = 35, 16, 9, 5, 0

Option C

9. $X = 3 + \sqrt{8}$

$X^{-1} = (3 - \sqrt{8})/(9-8) = 3 - \sqrt{8}$

$X + X^{-1} = 6$

$X^2 + X^{-2} = 36 - 2 = 34$

$X^4 + X^{-4} = 34^2 - 2 = 1156 - 2 = 1154$

Option A

10. $A^z = B^y = C^x = K$

$K^{2/y} = K^{1/z+1/x}$

2xz = y(x + z); Option B

11. $\log\left(\frac{a+b}{7}\right) = \frac{1}{2}(\log a + \log b)$

$2\log\left(\frac{a+b}{7}\right) = \log(ab)$

$(a+b)^2 = 49ab$

$a^2 + b^2 + 2ab = 49ab$; Option B

12. $X = \frac{1}{6}\cdot\sqrt{\frac{3\log(1728)}{1+\frac{1}{2}\log(0.36)+\frac{1}{3}\log 8}}$

$X = \frac{1}{6}\cdot\sqrt{\frac{3\log(1728)}{1+\log(0.6)+\log 2}} = \frac{1}{6}\cdot\sqrt{\frac{9\log 12}{\log 12}} = ½$

(2X − 1) = 0. Option C

13.

$x = \frac{6ab}{a+b}; \frac{x}{3a} = \frac{2b}{a+b}; \frac{x+3a}{x-3a} = \frac{a+3b}{b-a}$

$x = \frac{6ab}{a+b}; \frac{x}{3b} = \frac{2a}{a+b}; \frac{x+3b}{x-3b} = \frac{3a+b}{a-b}$

$\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b} = \frac{a+3b}{b-a} + \frac{3a+b}{a-b} = \frac{-a-3b+3a+b}{(a-b)} = \frac{2a-2b}{a-b} = 2$

Option C

14. $3\sqrt{2} + 7\sqrt{8} + \sqrt{27} + 5\sqrt{3} = 3\sqrt{2} + 14\sqrt{2} + 3\sqrt{3} + 5\sqrt{3} = 17\sqrt{2} + 8\sqrt{3}$

Option B

15. $$\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\frac{5}{2}$$

$$\frac{2\sqrt{x+4}}{2\sqrt{x-10}}=\frac{5+2}{5-2}=\frac{7}{3}$$

$$\frac{x+4}{x-10}=\frac{49}{9};\frac{2x-6}{14}=\frac{58}{40};\frac{x-3}{7}=\frac{29}{20}$$

20x − 60 = 203; x = 263/20. Option C

16. a : b = 3 : -1 find $\frac{ab}{a+b}$

a & b = $\frac{ab}{(a+b)}$, find 3 & (& -1)

3 & -1 = $\frac{3(-1)}{3-1}=\frac{-3}{2}$

Then, 3 & (3 &- 1) = $\frac{(3)\left(\frac{-3}{2}\right)}{3-\left(\frac{3}{2}\right)}$ = $\frac{\frac{-9}{\cancel{2}}}{\frac{3}{\cancel{2}}}$ = -3

17. $$\log\frac{12}{13}-\log\frac{7}{25}+\log\frac{91}{3}=\log\left(\frac{2*2*3}{13}x\frac{5*5}{7}x\frac{13*7}{3}\right)=\log 100$$

x = log100 = 2. Option C

18. $$\frac{1}{1+\frac{1}{x}}=\frac{x}{x+1}=x(x+1)^{-1}$$

Option C

19. $$\frac{a^3-b^3}{a^3+b^3}=\frac{13}{14};\frac{2a^3}{-2b^3}=\frac{27}{-1};\frac{a^3}{b^3}=\frac{27}{1};\frac{a}{b}=\frac{3}{1};\frac{a+b}{a-b}=\frac{4}{2}=2$$

Option B

20. $x + x^{-1} = 3$

$x^2 + x^{-2} = 9 - 2 = 7$

$x^3 + x^{-3} = (3)(7-1) = 18$

$x^6 + x^{-6} = 18^2 - 2 = 324 - 2 = 322$. Option D

21. 2 + √3 = 2 + 1.732 = 3.732

√5 + √2 = 2.23 + 1.41 = 3.64

Option C

22. $\frac{(y-1)(y-2)(y^2-9y+14)}{(y-7)(y^2-3y+2)}=\frac{(y-1)(y-2)(y-2)(y-7)}{(y-7)(y-2)(y-1)}=(y-2)$

Option C

23. (x + y) = 8/2 = 4; xy = 1

$\frac{x^2+xy+y^2}{x^2-xy+y^2}=\frac{(x+y)^2-xy}{(x+y)^2-3xy}=\frac{16-1}{16-3}=\frac{15}{13}$

Option B

24. $\frac{1}{\sqrt{11-2\sqrt{30}}}-\frac{3}{\sqrt{7-2\sqrt{10}}}-\frac{4}{\sqrt{8+4\sqrt{3}}}$

$=\frac{1}{\sqrt{6}-\sqrt{5}}-\frac{3}{\sqrt{5}-\sqrt{2}}-\frac{4}{\sqrt{6}+\sqrt{2}}$

$=\frac{\sqrt{6}+\sqrt{5}}{6-5}-\frac{3(\sqrt{5}+)\sqrt{2}}{(5-2)}-\frac{4()\sqrt{6}-\sqrt{2}}{(6-2)}$

$=\sqrt{6}+\sqrt{5}-\sqrt{5}-\sqrt{2}-\sqrt{6}+\sqrt{2}=0$

Option D

25. X = Log(5*5*5*16/2) = Log(1000) = 3

(X + 3) = 3 + 3 = 6. Option B

26. LogX = Log(1.5*12) = Log18.

X = 18. X/3 = 18/3 = 6. Option B

27. (X − 13).8 = (3X + 1)

8X − 104 = 3x + 1; 5X = 105; X = 21. Option A

28. $\log\left(\frac{81}{100}\right)^2*\log\left(\frac{27}{10}\right)^{2/3}\div\log 9$

$=(8\log 3-4)(2\log 3-2/3)*(1/2\log 3)$

= (3.8168 − 4) (0.9542 − 0.6667)(1/0.9542) = (- 0.1832)(0.2875)/(0.9542) = - 0.0552

Option B

29. E= $\frac{1}{\log_{xy}(xyz)}+\frac{1}{\log_{yz}(xyz)}+\frac{1}{\log_{zx}(xyz)}=\frac{\log xy}{\log xyz}+\frac{\log yz}{\log xyz}+\frac{\log zx}{\log xyz}=\log_{xyz}(xyz)^2=2$

(E − 3) = (2 − 3) = - 1

Option C

30. $\log\frac{75}{35}+2\log\frac{7}{5}-\log\frac{105}{x}-\log\frac{13}{25}=0$

$$\log\left(\frac{75*49*25}{35*25*13}\right)=\log\frac{105}{13}=\log\frac{105}{x}$$

x = 13

Option A

2 EQUATIONS

THEORY

Equations

An equation is defined as a mathematical statement of equality.

Types of Equations

a) Linear equation in one variable.

b) Linear simultaneous equations in 2 or 3 variables.

c) Quadratic equations.

d) Cubic equations.

e) Bi-quadratic equations.

f) Exponential equations.

Quadratic Equations

- A quadratic equation is defined as polynomial equation of degree 2.
- A quadratic equation can be expressed in the following general form:

$$ax^2 + bx + c = 0;\ (a \neq 0)$$

- A quadratic equation can also be expressed in the factor form as follows:

$$a(x-\alpha)(x-\beta) = 0$$

Here, α and β are the roots or solutions of quadratic equations.

- The general solution of the quadratic equation can be obtained as follows:

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\text{Sum of roots} = \alpha + \beta = -\frac{b}{a}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a}$$

Structure of Quadratic Equations

If Sum (S) $(\alpha + \beta)$ and Product (P) $(\alpha\beta)$ of the roots are known, then the quadratic equation is

$$x^2 - Sx + P = 0$$

Sign of Roots of a Quadratic Equation

- When c=0, one root of the equation must be 0.
- When b and c are 0, then both the roots must be 0.
- If a, b, c all are of same sign, both roots are negative.
- If a and c are of same sign, opposite to that of b, then both the roots will be positive.
- If a and c are of opposite signs, one root is positive and another root is negative.

Nature of Roots

The expression "$b^2 - 4ac$" is called the "Discriminant (D)" of the quadratic equation.

- When D > 0, Roots are real and distinct.
- When D = 0, Roots are real and equal.
- When D < 0, Roots are imaginary.
- When D $\geq$ 0, Roots are real.
- When D is a perfect square, Roots are real, rational and unequal.
- When D is not a perfect square, Roots are real, irrational and unequal.
- If roots are equal use $b^2 = 4ac$.
- If roots are reciprocal of each other, use **a = c**
- If roots are equal but of opposite sign, use **b = 0**
- If roots are reciprocal but opposite in sign, use **c = -a**

Note

- Irrational roots will always appear in conjugate pairs.

$$\alpha = \left(a - \sqrt{b}\right) \text{ and } \beta = \left(a + \sqrt{b}\right)$$

- Imaginary roots will always appear in conjugate pairs

$$\alpha = (a - ib) \text{ and } \beta = (a + ib)$$

Cubic Equations

- A cubic equation is a polynomial equation of degree 3, and the general form is represented as follows:

$$ax^3 + bx^2 + cx + d = 0; \ (a \neq 0)$$

- The factor form of a cubic equation is given as follows:

$$a(x - \alpha)(x - \beta)(x - \gamma) = 0$$

Here, α, β, and γ are the roots or solutions of the cubic equation.

- Sum of roots = $\alpha + \beta + \gamma$ = -b/a
- Product of the roots = $\alpha\beta\gamma$ = -d/a

Bi-Quadratic Equations

- A bi-quadratic equation is a polynomial of degree 4, and the general form is represented as follows:

$$ax^4 + bx^3 + cx^2 + dx + e = 0; \ (a \neq 0)$$

- The factor form of a cubic equation is given as follows:

$$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta) = 0$$

Here, α, β, γ and δ are the roots or solutions of the bi-quadratic equation.

- Sum of roots = $\alpha + \beta + \gamma + \delta$ = -b/a
- Product of the roots = $\alpha\beta\gamma\delta$ = e/a

CLASSWORK

Choose the most appropriate option (a), (b), (c) or (d).

1. The sum of two numbers is 52 and their difference is 2. The numbers are

   a) 17 and 15 b) 12 and 10
   c) 27 and 25 d) none of these

2. The diagonal of a rectangle is 5 cm and one of its sides is 4 cm. Its area is

   a) 20 sq.cm. b) 12 sq.cm.
   c) 10 sq.cm. d) none of these

3. Divide 56 into two parts such that three times the first part exceeds one third of the second by 48. The parts are.

   a) (20, 36) b) (25, 31)
   c) (24, 32) d) none of these

4. The sum of the digits of a two digit number is 10. If 18 be subtracted from it the digits in the resulting number will be equal. The number is

   a) 37 b) 73
   c) 75 d) none of these numbers.

5. The fourth part of a number exceeds the sixth part by 4. The number is

   a) 84 b) 44
   c) 48 d) none of these

6. Ten years ago, the age of a father was four times of his son. Ten years hence, the age of the father will be twice that of his son. The present ages of the father and the son are.

   a) (50, 20) b) (60, 20)
   c) (55, 25) d) none of these

7. The product of two numbers is 3200 and the quotient when the larger number is divided by the smaller is 2. The numbers are

   a) (16, 200) b) (160, 20) c) (60, 30) d) (80, 40)

8. One student is asked to divide a half of a number by 6 and other half by 4 and then to add the two quantities. Instead of doing so the student divides the given number by 5. If the answer is 4 short of the correct answer then the number was

a) 320 b) 400 c) 480 d) none of these.

9. If a number of which the half is greater than 1/5 th of the number by 15 then the number is

a) 50 b) 40 c) 80 d) none of these.

10. Monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenses are in the ratio 7 : 9. If each saves ₹ 50 per month find their monthly incomes.

a) (500, 400) b) (400, 500) c) (300, 600) d) (350, 550)

11. The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age.

a) 60 years b) 52 years c) 51 years d) 50 years

12. A number between 10 and 100 is five times the sum of its digits. If 9 be added to it the digits are reversed find the number.

a) 54 b) 53 c) 45 d) 55

13. The wages of 8 men and 6 boys amount to ₹ 33. If 4 men earn ₹ 4.50 more than 5 boys determine the wages of each man and boy.

a) (₹ 1.50, ₹ 3) b) (₹ 3, ₹ 1.50) c) (₹ 2.50, ₹ 2) d) (₹ 2, ₹ 2.50)

14. y is older than x by 7 years 15 years back, x's age was 3/4 of y's age. Their present ages are:

a) (x=36, y=43) b) (x=50, y=43)

c) (x=43, y=50) d) (x=40, y=47)

15. The sum of the digits in a three digit number is 12. If the digits are reversed the number is increased by 495 but reversing only of the ten's and unit digits increases the number by 36.The number is

a) 327 b) 372 c) 237 d) 273

16. The demand and supply equations for a certain commodity are 4q + 7p = 17 and $p = \frac{q}{3} + \frac{7}{4}$. respectively where p is the market price and q is the quantity then the equilibrium price and quantity are:

(a) $2, \frac{3}{4}$ (b) $3, \frac{1}{2}$ (c) $5, \frac{3}{5}$ (d) None of these.

17. The sum of two numbers is 8 and the sum of their squares is 34. Taking one number as x form an equation in x and hence find the numbers. The numbers are

a) (7, 10) b) (4, 4) c) (3, 5) d) (2, 6)

18. Five times of a positive whole number is 3 less than twice the square of the number. The number is

a) 3 b) 4 c) −3 d) 2

19. Two squares have sides p cm and (p + 5) cms. The sum of their squares is 625 sq. cm. The sides of the squares are

a) (10 cm, 30 cm) b) (12 cm, 25 cm)
c) 15 cm, 20 cm) d) none of these

20. Divide 50 into two parts such that the sum of their reciprocals is 1/12. The numbers are

a) (24, 26) b) (28, 22) c) (27, 23) d) (20, 30)

21. There are two consecutive numbers such that the difference of their reciprocals is 1/240.The numbers are

a) (15, 16) b) (17, 18) c) (13, 14) d) (12, 13)

22. The sum of two numbers is 45 and the mean proportional between them is 18. The numbers are

a) (15, 30) b) (32, 13) c) (36, 9) d) (25, 20)

23. The sides of an equilateral triangle are shortened by 12 units 13 units and 14 units respectively and a right angle triangle is formed. The side of the equilateral triangle is

a) 17 units b) 16 units c) 15 units d) 18 units

PAST YEAR QUESTIONS

24. Area and perimeter of rectangle is 6000 cm^2 and 340 cm length of rectangle is

    (a) 140 (b) 120 (c) 170 (d) 200

25. If length of rectangle is 5 cm more than the breadth and if perimeter of rectangle is 40 cm, length and breadth of rectangle will be

    (a) 7.5 cm, 2.5 cm (b) 10 cm, 5 cm
    (c) 12.5 cm, 7.5 cm (d) 15.5 cm, 10.5 cm

26. Number of students in each section of a school is 36. After admitting 12 new students, four new sections are started. If total number of students in each section now is 30, then number of section initially were

    (a) 6 (b) 10 (c) 14 (d) 18

QUADRATIC EQUATIONS

1. Equation : $x^2 + x + 1 = 0$ roots are
   (a) Real and equal (b) Real and unequal
   (c) Imaginary (d) Real and rational

2. For what value of 'c', the roots of the equation $2x^2 - 10x + c = 0$ are real and equal
   (a) 25/2 (b) 25/4 (c) 25/3 (d) none

3. If '−4' is a root of the equation $x^2 + ax - 4 = 0$ and the equation $x^2 + ax + b = 0$ has equal roots, the value of 'a' & 'b' are
   (a) $a = 2, b = \frac{5}{4}$ (b) $a = 3, b = \frac{9}{4}$
   (c) $a = \ , b = \frac{5}{2}$ (d) none

4. If the roots of equation $x^2 + (2k - 1)x + k^2 = 0$ are real, condition is
   (a) $k \geq 1$ (b) $k \leq 4$
   (c) $k \geq \frac{1}{4}$ (d) $k \leq \frac{1}{4}$

5. If the equation $x^2 - (b + 4)x + 2b + 5 = 0$ has equal roots, then the values of 'b'
   (a) −2 (b) 2 (c) ±2 (d) ±1

6. If α and β are roots of $x^2 + 2x + 1 = 0$, $\alpha^3 + \beta^3 =$
   (a) 2 (b) −2 (c) 4 (d) −4

7. If $p + q + r = 0$ and p, q, r are rational nos. the roots of equation $(q + r - p)x^2 + (r + p - q)x + (p + q - r) = 0$
   (a) real and irrational (b) real & equal
   (c) imaginary (d) real & rational

8. If one root of the equation $x^2 - 8x + k = 0$ exceeds the other by 4, value of k is
   (a) k = 10 (b) k = 11 (c) k = 9 (d) k = 12

9. If one root is double the other for the equation $ax^2 + bx + c = 0$, then
   (a) $b^2 = 4ac$ (b) $2b^2 = 9ac$ (c) $3b^2 = 10ac$ (d) $4b^2 = 9ac$

10. If roots of the equation $ax^2 + bx + c = 0$ are in the ratio $\frac{l}{m}$, then value of b^2/ac is

(a) $\frac{(l+m)^2}{lm}$ (b) $\frac{l+m}{lm}$ (c) $\left(\frac{l-m}{lm}\right)^2$ (d) $\frac{l-m}{lm}$

11. If α, β are roots of equation $x^2 - 5x + 6 = 0$, $\alpha > \beta$, then equation with roots $\alpha + \beta$, $\alpha - \beta$ as

(a) $x^2 - 6x + 5 = 0$ (b) $2x^2 - 6x + 5 = 0$

(c) $2x^2 - 5x + 6 = 0$ (d) $x^2 - 5x + 6 = 0$

12. The values of $4 + \cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\ldots\infty}}}$

(a) $1 \pm \sqrt{2}$ (b) $2 + \sqrt{5}$ (c) $2 \pm \sqrt{5}$ (d) None of these

13. If the sum of the roots of the quadratyic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then $\frac{b^2}{ac} + \frac{bc}{a^2}$ is equal to

a) 2 b) -2 c) 1 d) -1

14. If $p \neq q$ and $p^2 = 5p - 3$ and $q^2 = 5q - 3$ the equation having roots as $\frac{p}{q}$ and $\frac{q}{p}$ is

(a) $x^2 - 19x + 3 = 0$ (b) $3x^2 - 19x - 3 = 0$

(c) $3x^2 - 19x + 3 = 0$ (d) $3x^2 + 19x + 3 = 0$

15. If one root of $5x^2 + 13x + p = 0$ be reciprocal of the other then the value of p is

a) -5 b) 5 c) 1/5 d) $-1/5$

PAST YEARS QUESTIONS

1. On solving $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$, we get one value of x as:

(a) $\frac{4}{13}$ (b) $\frac{1}{13}$ (c) $\frac{2}{13}$ (d) $\frac{3}{13}$

2. Find the positive value of k for which the equation : $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots

(a) 12 (b) 16 (c) 18 (d) 22

3. If one root of an equation is $3 + \sqrt{5}$, then the quadratic equation is

(a) $x^2 - 6x + 4 = 0$ (b) $x^2 + 6x - 4 = 0$
(c) $x^2 + 6x + 4 = 0$ (d) $x^2 - 6x - 4 = 0$

4. If $(2 + \sqrt{3})$ is a root of a quadratic equation $x^2 + px + q = 0$ then find value of p and q

(a) (4, −1) (b) (4, 1) (c) (−4, 1) (d) (2, 3)

5. On root of the equation $x^2 - 2(5 + m)x + 3(7 + m) = 0$ is reciprocal of the other. Find the value of m

(a) $\frac{-64}{7}$ (b) 7 (c) 1/7 (d) $\frac{-20}{3}$

6. If one root of the equation $x^2 - 3x + k = 0$ is 2 then value of k will be

(a) −10 (b) 0 (c) 2 (d) 10

7. If roots of equation $x^2 + x + r = 0$ are α and β and $\alpha^3 + \beta^3 = -6$. Find value of r

(a) $-\frac{5}{3}$ (b) $\frac{7}{3}$ (c) $-\frac{4}{3}$ (d) 1

8. If one root of the equation $px^2 + qx + r = 0$ is r then other root of the equation will be

(a) 1/q (b) 1/r (c) 1/p (d) $\frac{1}{p+q}$

9. If the ratio of the root of the equation $4x^2 - 6x + p = 0$ is 1 : 2 then the value of p is

(a) 1 (b) 2 (c) −2 (d) −1

10. If p and q are the root of equation $x^2 - bx + c = 0$ then what is the equation whose roots are $(pq + p + q)$ and $(pq - p - q)$

(a) $x^2 - 2cx + c^2 - b^2 = 0$ (b) $x^2 - 2cx + c^2 + b^2 = 0$

(c) $cx^2 - 2(a + c)x + c^2 = 0$ (d) $x^2 + 2bx - (c^2 - b^2) = 0$

11. If α, β are the roots of the quadratic equation $2x^2 - 4x = 1$ then the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

(a) -11 (b) 22 (c) -22 (d) 11

12. If difference between the roots of the equation $x^2 - kx + 8 = 0$ is 4 then the value of k is

(a) 0 (b) ± 4 (c) $\pm 8\sqrt{3}$ (d) $\pm 4\sqrt{3}$

CUBIC EQUATION

Choose the most appropriate option (a), (b), (c) or (d)

1. The solution of the cubic equation $x^3-6x^2+11x-6=0$ is given by the triplet:

   a) -1, 1 -2 b) 1, 2, 3 c) -2, 2, 3 d) 0, 4, -5

2. The cubic equation $x^3+2x^2-x-2=0$ has 3 roots namely.

   a) 1, -1, 2 b) -1, 1, -2 c) -1, 2, -2 d) 1, 2, 2

3. x,x-4,x+5 are the factors of the left-hand side of the equation.

   a) $x^3 + 2x^2-x-2=0$ b) $x^3+x^2-20x=0$
   c) $x^3-3x^2-4x+12=0$ d) $x^3-6x^2+11x-6=0$

4. The equation $3x^3+ 5x^2=3x+5$ has got 3 roots and hence the factors of the left-hand side of the equation $3x^3+5x^2-3x-5=0$ are

   a) x-1,x-2,x-5/3 b) x-1,x+1, 3x +5
   c) x+1,x-1,3x-5 d) x-1,x+1,x-2

5. The roots of the equation $x^3+7x^2-21x-27=0$ are

   a) (-3,-9,-1) b) (3,-9,-1) d) (3,9,1) e) (-3,9,1)

6. If $4x^3+8x^2-x-2=0$ than value of (2x+3)is given by

   a) 4, -1, 2 b) -4, 2, 1 c) 2, -4, -1 d) None of these

7. The rational root of the equation $2x^3-x^2-4x+2=0$ is

   a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) -2

BI QUADRATIC EQUATION

1. Solving equation $6x^4 + 11x^3 - 9x^2 -11x + 6 = 0$ following roots are obtained

   a) $\frac{1}{2}, -2, \frac{-1\pm\sqrt{37}}{6}$ b) $-\frac{1}{2}, 2, \frac{-1\pm\sqrt{37}}{6}$
   c) $\frac{1}{2}, -2, \frac{5}{6}, \frac{-7}{6}$ d) None

2. Find the roots of the equation: $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$

a) 1, 2, ½ b) − 1, 2, ½ c) − 2, 1, ½ d) − ½, 1, 2

CONSISTENCY OF EQUATION

1. The system of equation $5x - 4y = 7$ and $3x - 2y = 15$ have
   (a) unique solution (b) infinite solution
   (c) no solution (d) none

2. The system of equation $9x - 17y = 34$ and $36x - 68y = 115$ have
   (a) unique Solution (b) infinite Solution
   (c) no solution (d) none

3. The system of equation $6x + 5y = 11$ and $9x + (15/2)y = 21$ have
   (a) unique Solution (b) infinite Solution
   (c) no solution (d) none

4. The system of equation $4x + 7y = 10$ and $10x + (35/2)y = 25$ have
   (a) unique solution (b) infinite solution
   (c) no solution (d) none

5. The value of k for which the system of equations: $7x - y = 5$; $21x - 3y = k$ have infinite solution is:
   (a) $k = 4$ (b) $k = 15$ (c) $k = 3$ (d) $k = 7$

6. Determine the values of a and b for which the following system of linear equations has consistent infinite many solutions: $2x - (a-4)y = 2b + 1$ and $4x - (a-1)y = 5b - 1$
   (a) $a = -7, b = 3$ (b) $a = 7, b = 3$
   (c) $a = -7, b = -3$ (d) None

EQUATION HOMEWORK

1. The solution of the equation (p+2) (p-3)+(p+3) (p-4)= p(2p-5) is
   (a) 6 (b) 7 (c) 5 (d) none of these

2. The equation $\frac{12x+1}{4} = \frac{15x-1}{5} + \frac{2x-5}{3x-1}$ is true for
   (a) x=1 (b) x = 2 (c) x = 5 (d) x = 7

3. Solve for x and y : $\frac{4}{x} - \frac{5}{y} = \frac{x+y}{xy} + \frac{3}{10}$ and 3xy= 10 (y-x)
   (a) (5,2) (b) (-2,-5) (c) (2,-5) (d) (2,5)

4. The simultaneous equations 7x−3y = 31, 9x−5y = 41 have solutions given by
   (a) (−4, −1) (b) (−1, 4) (c) (4, −1) (d) (3, 7)

5. $\frac{xy}{x+y} = 20,\ \frac{yz}{y+z} = 40,\ \frac{zx}{z+x} = 24$
   (a) (120, 60, 30) (b) (60, 30, 120)
   (c) (30, 120, 60) (d) (30, 60, 120)

6. $\frac{xy}{y-x} = 110,\ \frac{yz}{z-y} = 132,\ \frac{zx}{z+x} = 60/11$
   (a) (12, 11, 10) (b) (10, 11, 12)
   (c) (11, 10, 12) (d) (12, 10, 11)

7. If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal then value of m is
   (a) − 3 (b) − 1 (c) 1 (d) − 2

8. If $2^{2x+3} - 3^2 . 2^x + 1 = 0$ then values of x are
   (a) 0, 1 (b) 1, 2 (c) 0, 3 (d) 0, − 3

9. A solution of the quadratic equation (a+b−2c)x2 + (2a−b−c)x + (c+a−2b) = 0 is
   a) x = 1 b) x = −1 c) x = 2 d) x = − 2

10. If the root of the equation $x^2-8x+m=0$ exceeds the other by 4 then the value of m is
    a) m = 10 b) m = 11 c) m = 9 d) m = 12

11. The values of x in the equation
$7(x+2p)^2 + 5p^2 = 35xp + 117p^2$ are
(a) $(4p, -3p)$ (b) $(4p, 3p)$ (c) $(-4p, 3p)$ (d) $(-4p, -3p)$

12. The solutions of the equation $\frac{6x}{x+1} + \frac{6(x+1)}{x} = 13$ are
(a) (2, 3) (b) (3, −2) (c) (−2, −3) (d) (2, −3)

13. The satisfying values of x for the equation $\frac{1}{x+p+q} = \frac{1}{x} + \frac{1}{p} + \frac{1}{q}$ are
a) (p, q) b) (−p, −q) c) (p, −p) d) (−p, q)

14. The values of x for the equation $x^2 + 9x + 18 = 6 - 4x$ are
a) (1, 12) b) (−1, −12) c) (1, −12) d) (−1, 12)

15. The values of x satisfying the equation
$\sqrt{(2x^2+5x-2)} - \sqrt{(2x^2+5x-9)} = 1$ are
(a) (2, −9/2) (b) (4, −9) (c) (2, 9/2) (d) (−2, 9/2)

16. The solution of the equation $3x^2 - 17x + 24 = 0$ are
(a) (2,3) (b) $\left(2, 2\frac{2}{3}\right)$ (c) $\left(3, 2\frac{2}{3}\right)$ (d) $\left(3, \frac{2}{3}\right)$

17. The equation $\frac{3(3x^2+15)}{6} + 2x^2 + 9 = \frac{2x^2+96}{7} + 6$ has got the solution as
a) (1, 1) b) (1/2, −1) c) (1, −1) d) (2, −1)

18. Solving equation $x^2 - (a+b)x + ab = 0$ are, value(s) of x
(a) a, b (b) a (c) b (d) None

19. Solving equation $x^2 - 24x + 135 = 0$ are, value(s) of x
(a) 9, 6 (b) 9, 15 (c) 15, 6 (d) None

20. If $\frac{x}{b} + \frac{b}{x} = \frac{a}{b} + \frac{b}{a}$ the roots of the equation are
(a) $a, b^2/a$ (b) $a^2, b/a^2$ (c) $a^2, b^2/a$ (d) a, b^2

21. Solving equation $3x^2 - 14x + 16 = 0$ we get roots as
(a) ± 1 (b) 2 and $\frac{8}{3}$ (c) 0 (d) None

22. Solving equation $3x^2 - 14x + 8 = 0$ we get roots as

(a) ±4 (b) ± 2 (c) $4, \frac{2}{3}$ (d) None

23. Solving equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ following roots are obtained

(a) $\frac{a-b}{b-c}, 1$ (b) (a-b) (c-a),1 (c) $\frac{b-c}{a-b}, 1$ (d) None

24. Solving equation $7\sqrt{\frac{x}{1-x}} + 8\sqrt{\frac{1-x}{x}} = 15$ following roots are obtained

(a) $\frac{64}{113}, \frac{1}{2}$ (b) $\frac{1}{50}, \frac{1}{65}$ (c) $\frac{49}{50}, \frac{1}{65}$ (d) $\frac{1}{50}, \frac{64}{65}$

25. If α, β are the roots of equation $x^2-5x+6=0$ and $\alpha > \beta$ then the equation with roots $(\alpha\beta + \alpha + \beta)$ and $(\alpha\beta - \alpha - \beta)$ is

(a) $x^2 - 12x + 11 = 0$ (b) $2x^2 - 6x + 12 = 0$

(c) $x^2 - 12x + 12 = 0$ (d) None

26. Solving $x^3 + 9x^2 - x - 9 = 0$ we get the following roots

(a) ±1, -9 (b) ±1, ±9 (c) ±1, 9 (d) None

27. Solve $x^3 - 7x^2 + 14x - 8 = 0$ given that the roots are in geometrical progression

(a) ½, 1, 2 (b) 1, 2, 4 (c) ½ , -1, 2 (d) -1, 2, -4

28. Solve $x^3 - 6x^2 + 5x + 12 = 0$ given that the product of the two roots is 12

(a) 1, 3, 4 (b) -1, 3, 4 (c) 1, 6, 2 (d) 1, -6, -2

HOMEWORK SOLUTIONS

1. The solution of the equation (p + 2) (p − 3) + (p + 3)(p − 4) = p(2p − 5) = ?

$(p + 2)(p - 3) + (p + 3)(p - 4) = p(2p - 5)$

$\Rightarrow (p^2 + 2p - 3p - 6) + (p^2 + 3p - 4p - 12) = 2p^2 - 5p$

$\Rightarrow 2p^2 - 2p - 18 = 2p^2 - 5p$

$\Rightarrow 5p - 2p = 18$

$\Rightarrow 3p = 18 \Rightarrow p = 6$

2. The equation $\frac{12x+1}{4} = \frac{15x-1}{5} + \frac{2x-5}{3x-1}$ is true for?

$\Rightarrow \frac{12x}{4} + \frac{1}{4} = \frac{15x}{5} - \frac{1}{5} + \frac{2x-5}{3x-1}$

$\Rightarrow 3x + \frac{1}{4} = 3x - \frac{1}{5} + \frac{2x-5}{3x-1}$

$\Rightarrow \frac{1}{4} + \frac{1}{5} = \frac{2x-5}{3x-1}$

$\Rightarrow \frac{9}{20} = \frac{2x-5}{3x-1}$

Now, if x = 7, $\frac{9}{20} = \frac{2(7)-5}{3(7)-1} = \frac{14-5}{21-1} = \frac{9}{20}$

3. (d)

4. (c)

5. $\frac{xy}{x+y} = 20$; $\frac{yz}{y+z} = 40$; $\frac{zx}{z+x} = 24$

Here, (x > y > z) ∴ We pick option (d) 30, 60, 120

also check, $\frac{(30)(60)}{30+60} = \frac{180}{90} = 20$

(Ascending order) $\frac{(60)(120)}{60+120} = \frac{7200}{180} = 40$

6. $\frac{xy}{y-x} = 110$; $\frac{yz}{z-y} = 132$; $\frac{zx}{z+x} = \frac{60}{11}$ $x < y < z$

(Ascending order) $\therefore$ 10, 11, 12

also, $\frac{(10)(11)}{11-10} = 110$; $\frac{(11)(12)}{12-11} = 132$

7. If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal, then value of m is:

When the roots are equal $b^2 = 4ac$

Here, $a = 2$; $b = 8$; $c = -m^3$

$\therefore 64 = (4)(2)(-m^3)$

$\Rightarrow \frac{64}{8} = -m^3$

$\therefore m = -2$

$\Rightarrow -8 = -m^3$

$m = -2$

8. If $2^{2x+3} - 3^2 . x + 1 = 0$; $x = ?$

M–I

$2^{2x} . 2^3 - 3^2 . 2^x + 1 = 0$

$\Rightarrow 8x^{2x} - 9 . 2^x + 1 = 0$

$\Rightarrow 8t^2 - 9t + 1 = 0$

$\Rightarrow \underline{8t^2 - 8t^2} - 1\underline{t + 1} = 0$

$\Rightarrow 8t(t-1) - 1(t-1) = 0$

$\Rightarrow t = \frac{1}{8}$; $t = 1$

Let $2^x = t$

Now, $2^x = \frac{1}{8} = \frac{1}{2^3}$

$2^x = 2^{-3}$

$\therefore x = -3$

$2^x = 1 = 2^{\circ}$

$\therefore x = 0$

M–II

Plug in option (d) (0, –3)

Put x = 0, $2^{0+3} - 3^2 . 2^0 + 1$

$= 8 - 9 + 1 = 0$

$x = -3 \quad 2^{-6+3} - 3^2 . 2^{-3} + 1$

$\Rightarrow \frac{1}{8} - \frac{9}{8} + 1$

$-1 + 1 = 0$

9. (b)

10. (d) $\alpha + \beta = 8, \alpha - \beta = 4$

$2\alpha = 12, \alpha = 6, \beta = 2$

$m = \alpha\beta = 12$

11. (a)

12. (d)

13. (b)

14. (b) Equation $x^2 + 13x + 12 = 0$

$\alpha\beta = 12, \alpha + \beta = -13$

15. (a)

16. (c) $\alpha\beta = 8, \alpha + \beta = 17$

17. (c)

18. (a) $\alpha + \beta = a + b, \alpha\beta = ab$

19. (b) $\alpha + \beta = 24, \alpha\beta = 135$

20. (a)

21. (b) $\alpha + \beta = \frac{14}{3}$, $\alpha\beta = \frac{16}{3}$,

22. (c) $\alpha + \beta = \frac{14}{3}$, $\alpha\beta = \frac{8}{3}$

23. (a) Since, '1' is the roots in all 3 options

Let $\beta = 1$. We know that, $\alpha\beta = \frac{c}{a}$

$(\alpha)(1) = \frac{a-b}{b-c}$ $\therefore$ the other toot

$\alpha = \frac{a-b}{b-c}$

24. (a) $7\sqrt{\frac{x}{1-x}} + 8\sqrt{\frac{1-x}{x}} = 15$

M–I:

Let, $\sqrt{\frac{x}{1-x}} = k$, then $\sqrt{\frac{1-x}{x}} = \frac{1}{k}$

$\therefore\ 7k + \frac{8}{k} = 15 \Rightarrow$'s $7k^2 + 8 = 15k$

$7k^2 - 15k + 8 = 0$

$\underline{7k^2 - 7k} - \underline{8k + 8} = 0$

$7k(k-1) - 8(k-1) = 0$

$k = 1$; $k = \frac{8}{7}$

Now, $\sqrt{\frac{x}{1-x}} = 1$; $\sqrt{\frac{x}{1-x}} = \frac{8}{7}$

$\frac{x}{1-x} = 1$; $\frac{x}{1-x} = \frac{64}{49}$

$x = 1 - x$; $49x = 64 - 64x$

$2x = 1$ $x = \frac{1}{2}$; $113x = 64$

$\frac{64}{113} = x$

25. (a) If α, β are the roots of $x^2 - 5x + 6 = 0$, $\alpha > \beta$ then equation with $(\alpha\beta + \alpha + \beta)$ and $(\alpha\beta - \alpha - \beta)$ = ?

$x^2 - 5x + 6 = 0$, $x = 2$ and $x = 3$ [on factorization]

$\therefore \alpha = 3$ and $\beta = 2$ $[\because \alpha > \beta]$

Now, $(\alpha\beta + \alpha + \beta) = [3 \times 2 + 3 + 2] = [6 + 5] = 11$

and $(\alpha\beta - \alpha - \beta) = [3 \times 2 - 3 - 2] = [6 - 5] = 1$

$\therefore$ the equation $(x^2 - 5x + p)$

$\Rightarrow x^2 - 12x + 11 = 0$

26. (a) Solving $x^3 + 9x^2 - x - 9 = 0$ we get; the foll roots.

Sol. We know that, if $\frac{a}{b} = \frac{c}{d}$, in $ax^3 + bx^2 + cx + d = 0$ then, we factorise,

$\underline{x^3 + 9x^2} - \underline{x - 9} = 0$

$\Rightarrow x^2(x + 9) - 1(x + 9) = 0$

$\Rightarrow (x^2 - 1)(x + 9) = 0$

$\Rightarrow x = \pm 1 ; x = -9$

27. Solve $x^3 - 7x^2 + 14x - 8 = 0$, roots are in G.P.

Sol. Among the options, (b) has 1, 2, 4. a, b, c

$\sqrt{ac} = b \quad \therefore 2 = \sqrt{1 \times 4}$

28. Solve $x^3 - 6x^2 + 5x + 12 = 0$, given the product of 2 roots is 12.

We know that, $\alpha\beta\gamma = \frac{-d}{a} = \frac{-12}{1}$

Here, a = 1 ; b = −6 ; c = 5 ; d = 12

Only option (b) satisfies with −1, 3, 4 as $-1 \times 3 \times 4 = -12$

Again, chk, sum of the roots,

$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-6)}{1} = 6$

$4 + 3 - 1 = 7 - 1 = 6.$

SELF ASSESSMENT TEST 3
Equations 1
13 Question, 13 Marks

1. If sum of three numbers is 25, sum of product of numbers in pairs is 250, what is the sum of square of numbers?

   a) 250 b) 125 c) 375 d) 300

2. Find the value of $(1.348)^3 + 3(1.348)(1.304) + (0.652)^3$

   a) 2 b) 8 c) 2.258 d) 8.258

3. Solve for X and Y: $1.5X + 2.4Y = 1.8$ and $2.5(X + 1) = 7Y$

   a) 0.5, 0.5 b) 0.4, 0.4 c) 0.4, 0.5 d) 0.5, 0.4

4. If 3 chairs and 2 tables cost Rs. 12000 and 5 chairs and 3 tables cost Rs. 19000, then the cost of 2 chairs and 2 tables is:

   a) Rs. 9000 b) Rs. 7000 c) Rs. 10000 d) Rs. 11000

5. Find the value of K for which the system of equations $2x + 2y = 5$ and $3x + Ky = 7$ has no solution.

   a) 9 b) 5 c) 7 d) 3

6. Find the value of k for which the system of equations $2x + ky = 1$; $3x - y = 7$ has a unique solution.

   a) k = - 2/3 b) $\neq \frac{2}{3}$
   c) $k \neq -\frac{2}{3}$ d) None of the above

7. Find the value of $(1.729)^3 + 3(1.729)(0.542) + (0.271)^3$

   a) 2 b) 8 c) 2.271 d) 1.458

8. A man has some hens and cows. If the number of heads be 48 and number of feet equals 140, the number of hens will be:

   a) 22 b) 23 c) 24 d) 26

9. Ramesh bought a horse for Rs. X. He sold it at 0.9X, thereby registering 10% loss. Had the horse been sold at Rs. 4500 more Ramesh would have make a profit of 12.5%. Find X.

a) Rs. 20,000 b) Rs. 18,000 c) Rs. 200,000 d) Rs. 150,000

10. Given $x \in \{-3, -4, -5, -6\}$ and $9 \leq 1 - 2x$, find the possible values of x.

a) $\{-3, -4, -5, -6\}$ b) $\{-4, -5, -6\}$

c) $\{-3, -5, -6\}$ d) None of the above

11. Give the solution set for $3 - 2x \geq x - 32$, given that $x \in N$.

a) $\{1, 2, 3, 4, 5\}$ b) $\{-3, -2, 1, 2, 3, 4, 5, 6, 7\}$

c) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ d) None of the above

12. Dipti needs a minimum of 360 marks in four tests in a Statistics course to obtain an A grade. On her first three tests, she scored 88, 96, 79 marks. What should her score (X) be in the fourth test so that she can make an A grade?

a) $X > 95$ b) $X \geq 95$ c) $X \geq 97$ d) None of the above

13. Which of the following is the solution set for $|x + 2| \geq 5$?

a) $\{x : x \in R, x < -7 \text{ or } x \geq 3\}$

b) $\{x : x \in R, x \leq -7 \text{ or } x \geq 3\}$

c) $\{x : x \in R, x \leq -7 \text{ or } x > 3\}$

d) None of the above

EXPLANATORY ANSWERS

1. $(A + B + C) = 25, (AB + BC + CA) = 250$

   $(A^2 + B^2 + C^2) = (A + B + C)^2 - 2(AB + BC + CA) = 625 - 500 = 125$; Option B

2. $(1.348)^3 + 3(1.348)(1.304) + (0.652)^3$

   $= (1.348)^3 + (0.652)^3 + 3(1.348)(0.652)(1.348 + 0.652)$

   $= (1.348 + 0.652)^3 = 2^3 = 8$; Option B

3. Option C (Using options).

4. $3C + 2T = 12000; 5C + 3T = 19000$

   $2C + T = 7000$

   $C + T = 5000$

   $2C + 2T = 10000$. Option C

5. $2/3 = 2/K$. $K = 3$. Option D

6. $2/3 \neq k/-1$, Option C

7. $(1.729)^3 + 3(1.729)(0.542) + (0.271)^3$

   $= (1.729)^3 + (0.271)^3 + 3(1.729)(0.271)(1.729 + 0.271)$

   $= (1.729 + 0.271)^3 = 23 = 8$

   Option B

8. $H + C = 48; 2H + 4C = 140; H = 26$. Option D

9. $CP = X$

   New $SP = 0.9X + 4500$

   Profit $= 0.9X + 4500 - X = 4500 - 0.1X$

   $(4500 - 0.1X)/X = 0.125$

   $4500 - 0.1X = 0.125X$

   $0.225X = 4500$

   $X = 4500/0.225 = 20000$; Option A

10. $9 \le 1 - 2x$

$x \le -4$

$x = \{-4, -5, -6\}$

Option B

11. $3 - 2x \ge x - 32$

$35 \ge 3x$

$x \le 11.67$

$x = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

Option D

12. $88 + 96 + 79 = 263$

$x \ge (360 - 263) = 97$

Option C

13. $(x + 2) \le -5;\ x \le -7$

$(x + 2) \ge 5;\ x \ge 3$

Option B

SELF ASSESSMENT TEST 4
Equations 2
15 Question, 15 Marks

1. If A and B are the roots of the equation $16x^2 - 8x + 1 = 0$, then which of the following is true?
   a) A, B are real
   b) A, B are real and $A \neq B$
   c) A, B are real and $A = B$
   d) A and B, both are imaginary

2. If M and N are the roots of the equation $2(A^2 + B^2)X^2 + 2(A + B)x + 1 = 0$, then which of the following is true?
   a) M, N are real
   b) M, N are Imaginary
   c) M, N are Distinct
   d) Both b) & c) above

3. If one root of the equation $4x^2 + 5x + K = 0$ be reciprocal of another root, but negative in sign, then what can be the value of K?
   a) 4
   b) − 4
   c) 2.75
   d) − 3.25

4. If the roots of the equation $1/(x + 2) + 1/(x + 3) = 2/5$, are equal in magnitude but opposite in sign, then the product of the roots is:
   a) 1
   b) 2.5
   c) −13
   d) − 6.5

5. 'A' meters of cloth costs \$35. If this piece of cloth would had been 4 m longer and each meter costs \$ 1 less, the cost of the cloth would had been \$35. What can be the value of A?
   a) 8
   b) 10
   c) 12
   d) 14

6. Find the maximum value of the expression $x^2 - 4x + 7$, for real value of x.
   a) 3
   b) 8
   c) 9
   d) Undeterminable

7. For the given bi-quadratic equation: $4x^4 - 16x^3 + 7x^2 + 16x + 4 = 0$, what is the value of product of all possible roots of the equation?
   a) − 4
   b) 4
   c) − 16
   d) 1

8. For the given equation: $(x^2 + 2)^2 + 8x^2 - 6x(x^2 + 2) = 0$, what is the sum of the roots of the equation?

a) 8 b) 6 c) -6 d) 0

9. If one root of the equation $14x^2 + 5^3x + K = 0$ be reciprocal of another root, find the value of K.

a) 7 b) 14 c) 14/125 d) None of the above

10. For the given equation: $(x^2 + 2)^2 + 8x^2 - 6x(x^2 + 2) = 0$, what is the product of the roots of the equation?

a) -4 b) 4 c) 3 d) None of the above

11. Which of the following equation in variable X have two roots, 2 and 4?

a) $X^2 - 6X - 8 = 0$ b) $X^2 + 6X - 8 = 0$

c) $X^2 + 6X + 8 = 0$ d) $X^2 - 6X + 8 = 0$

12. Two roots of a quadratic equation $x^2 + x - 6 = 0$ are A and B respectively. If A > B, then which of the following quadratic equation will have roots $-$A and B?

a) $x^2 - 5x + 6 = 0$ b) $x^2 + 5x - 6 = 0$

c) $x^2 + 5x + 6 = 0$ d) $x^2 - 5x - 6 = 0$

13. Which of the following is one of the factor of the equation $x^4 - 19x^2 + 6x + 72 = 0$?

a) $(x + 3)$ b) $(x - 2)$ c) $(x - 4)$ d) $(x - 3)$

14. Which of the following cubic equation have factors $(x - 2)$, $(2x + 3)$ and $(x - 3)$?

a) $2x^3 - 7x^2 - 9x + 18 = 0$ b) $x^3 - 7x^2 - 9x + 18 = 0$

c) $x^3 - 7x^2 + 9x + 18 = 0$ d) $2x^3 + 7x^2 - 9x + 18 = 0$

15. If the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ $(a \neq b)$ have a common root, then:

a) $a + b = ab$ b) $a + b = a^2 + b^2$ c) $a^2 + b^2 = ab$ d) $a + b + 1 = 0$

EXPLANATORY ANSWERS

1. Discriminant (D) = 64 – 4*16*1 = 0. Roots are Real and Equal. Option C

2. D = $4(A + B)^2 - 4*2(A^2 + B^2)*1 = 4[A^2 + B^2 + 2AB - 2A^2 - 2B^2] = - 4(A - B)^2 < 0$

   Thus M and N are imaginary and distinct. Option D

3. Product of roots = K/4 = -1. K = - 4. Option B

4. 5(x + 2 + x + 3) = 2(x + 2) (x + 3)

   $10x + 25 = 2x^2 + 10x + 12$

   2x2 – 13 = 0

   Product of roots = -13/2 = - 6.5

   Option D

5. Cost per meter = 35/A

   (A + 4)(35/A – 1) = 35

   35 – A + 140/A – 4 = 35

   $A^2 + 4A - 140 = 0$

   A = 10

   Option B

6. $x^2 - 4x + 7$

   $= x^2 - 2*2*x + 2^2 + 3$

   $= (x - 2)^2 + 3$

   The expression is positive for any value of x> = 2

   Maximum value is undeterminable. Option D

7. Product of roots = +e/a = 4/4 = 1. Option D

8. $x^4 + 4 + 4x^2 + 8x^2 - 6x^3 - 12x = 0$

   $x^4 - 6x^3 + 12x^2 - 12x + 4 = 0$

   Sum of roots = -b/a = -(-6)/1 = 6

   Option B

9. Product = 1 = K/14. K = 14. Option B

10. $(x^2 + 2)^2 + 8x^2 - 6x(x^2 + 2) = 0$

$x^4 + 4x^2 + 4 + 8x^2 - 6x^3 - 12x = 0$

$x^4 - 6x^3 + 12x^2 - 12x + 4 = 0$

Product of roots = 4/1 = 4

Option B

11. $(X - 2)(X - 4) = 0$

$X^2 - 6X + 8 = 0$; Option D

12. $x^2 + x - 6 = 0$

$(x - 2)(x + 3) = 0$

$A = 2, B = -3$

$B = -3$ and $-A = -2$

Required equation is $(x + 3)(x + 2) = 0$

$x^2 + 5x + 6 = 0$

Option C

13. $F(x) = x^4 - 19x^2 + 6x + 72$

$F(3) = 3^4 - 19(3)^2 + 6*3 + 72 = 81 - 171 + 18 + 72 = 0$

$(x - 3)$ is one of the factor. Option D

14. $(x - 2)(2x + 3)(x - 3) = 0$

$(2x^2 - x - 6)(x - 3) = 0$

$2x^3 - 7x^2 - 9x + 18 = 0$

Option A

15. $k^2/(a^2 - b^2) = k/(b - a) = 1/(b - a)$

$k = (a + b)$ or 1

$1 + a + b = 0$

Option D

3 LINEAR INEQUALITIES

CLASS WORK

1. Draw the graph for the following set of in-equalities and indicate the feasible region:

   a) $3x + 4y \leq 12$

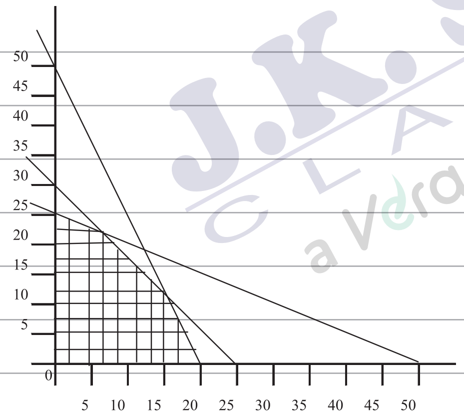
   b) $2x - 3y \geq 6$

   c) $x + 2y \leq 6,\ 5x + 3y \leq 15,\ x \geq 0,\ y \geq 0$

   d) $x + y \leq 4,\ 3x + y \geq 3,\ x + 4y \geq 4,\ x \leq 3,\ y \leq 2$

   e) $x - y \geq 4,\ 2x + 3y \geq 18,\ y \leq 4,\ x \leq 7$

2. Graph of five linear in-equations are given below. These are $15x + 6y = 300$, $5x + 4y = 120$, $x + 2y = 50$, $x \geq 0$ and $y \geq 0$. Identify the set of five in-equations which satisfy the common region as shown in the figure:



a) $\left.\begin{array}{l} 15x + 6y \leq 300 \\ 5x + 4y \leq 120 \\ x + 2y = 50 \\ x \geq 0, y \geq 0 \end{array}\right\}$

b) $\left.\begin{array}{l} 15x + 6y \leq 300 \\ 5x + 4y \leq 120 \\ x + 2y \leq 50 \\ x \geq 0, y \geq 0 \end{array}\right\}$

c) $\left.\begin{array}{l} 15x + 6y \leq 300 \\ 5x + 4y \leq 120 \\ x + 2y \leq 50 \\ x > 0, y > 0 \end{array}\right\}$

d) None of the given

3. A car manufacturing company manufactures cars of two types A and B. Model A requires 150 man-hours for assembling, 50 man-hours for painting and 10 man-hours for checking and testing. Model B requires 60 man-hours for assembling, 40 man-hours for painting and 20 man-hours for checking and testing. There are available 30 thousand man-hours for assembling, 13 thousand man-hours for painting and 5 thousand man-hours for testing and checking. Let the company manufacture x units of type A model of car and y units of type B model of the car. Then, the inequalities are:

a) $5x + 2y = 1000$, $5x + 4y \leq 1300$, $x + 2y \leq 500$, $x \geq 0$, $y \geq 0$

b) $5x + 2y \leq 1000$, $5x + 4y \leq 1300$, $x + 2y \leq 500$, $x \geq 0$, $y \geq 0$

c) $5x + 2y \leq 1000$, $5x + 4y = 1300$, $x + 2y = 500$, $x \geq 0$, $y \geq 0$

d) $5x + 2y \leq 1000$, $5x + 4y \geq 1300$, $x + 2y \geq 500$, $x \geq 0$, $y \geq 0$

4. A dealer wishes to purchase a number of fans and sewing machines. He has only ₹ 5760 to invest and has space for at most 20 items. A fan costs him ₹ 360 and sewing machine ₹ 240. Express the above situation in terms of linear inequalities.

a) $\left.\begin{array}{l} x + y \leq 30 \\ 360x + 240y \leq 5760 \end{array}\right\}$

b) $\left.\begin{array}{l} x + y \leq 20 \\ 360x + 240y \leq 5760 \end{array}\right\}$

c) $\left.\begin{array}{l} x + y \leq 30 \\ 36x + 24y \leq 576 \end{array}\right\}$

d) None of the above

5. A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrient constituents, it is necessary to buy two additional products, say A and B. The contents of the various products (per unit) in nutrient constituent (eg., vitamins, proteins, etc.) is given in the following table:

Nutrient	Nutrient content in product		Minimum amount of Nutrient
	A	B	
M1	36	6	108
M2	3	12	36
M3	20	10	100

The last column of the above table gives the minimum amounts of nutrients constituents M1, M2 and M3 which must be given to the pigs. Express the above situation in terms of linear inequalities.

a) $\left.\begin{array}{l} 36x+6y\geq 108 \\ 3x+12y\geq 36 \\ 20x+10y\geq 100 \\ x\geq 0, y\geq 0 \end{array}\right\}$

b) $\left.\begin{array}{l} 6x+y\geq 18 \\ x+4y\geq 12 \\ x+0.5y\geq 5 \\ x\geq 0, y\geq 0 \end{array}\right\}$

c) $\left.\begin{array}{l} 36x+6y\leq 108 \\ 3x+12y\leq 36 \\ 20x+10y\leq 100 \\ x\geq 0, y\geq 0 \end{array}\right\}$

d) (a) & (b) both

6. The rules and regulations demand that the employers should employ not more than 5 experienced hands to 1 fresh one and this fact is represented by: (Taking experienced person as x and fresh person as y)

a) $y \geq x/5$ b) $y \geq x$

c) $y \leq x/5$ d) None of the above

7. Which of the following represents the linear relationship between two variables in an in-equality:

a) $ax + by \leq c$ b) $ax \times by \leq c$

c) $axy + by \leq c$ d) $ax + bxy \leq c$

8. The solution of the in-equality $\frac{(5-2x)}{3} \leq \frac{x}{6} - 5$ is:

a) $x \leq 8$ b) $x = 8$ c) $x \geq 8$ d) None of the above

9. Solution space of the in-equalities $2x + y \leq 10$ and $x - y \leq 5$ are:

I. Includes the origin

II. Includes the point (4, 3)

Which one of the following is correct?

a) Only I b) Only II

c) Both I and II d) Neither I nor II

10. On an average, experienced person does 5 units of work while a fresh person does 3 units of work daily but the employer has to maintain the output of at least 30 units of work per day. The situation can be expressed as:

a) $5x + 3y \leq 30$ b) $5x + 3y < 30$

c) $5x + 3y \geq 30$ d) $5x + 3y = 30$

11. Find the range of real values of x satisfying the in-equalities $3x - 2 > 7$ and $4x - 13 > 15$.

a) $x > 3$ b) $x < 7$

c) $x > 7$ d) $x < 3$

12. If $\left|x + \frac{1}{4}\right| > \frac{7}{4}$, then which of the following is correct?

a) $x < -\frac{3}{2}$ or $x > 2$ b) $x < -2$ or $x > \frac{3}{2}$

c) $-2 < x < \frac{3}{2}$ d) None of the above

13. If a > 0 and b < 0, then which of the following follows:

a) $1/a > 1/b$ b) $1/a < 1/b$

c) $1/a = 1/b$ d) None of the above

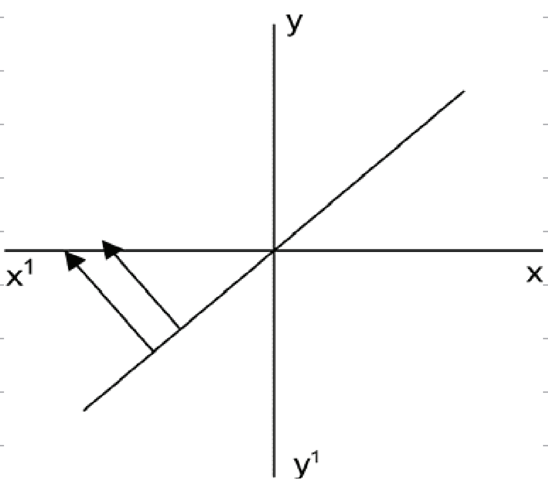
14. For 14 Do as Directed

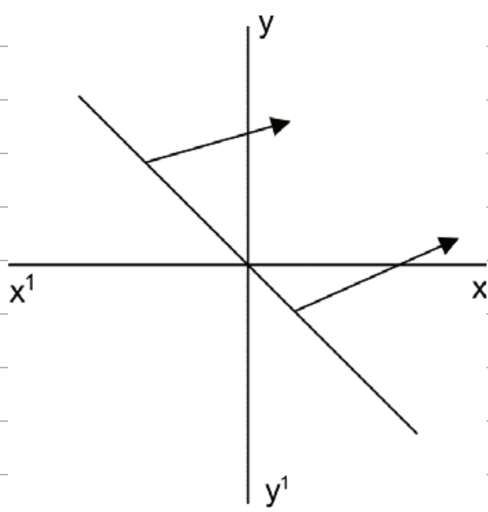
Mark (a) if $x - y \leq 0$

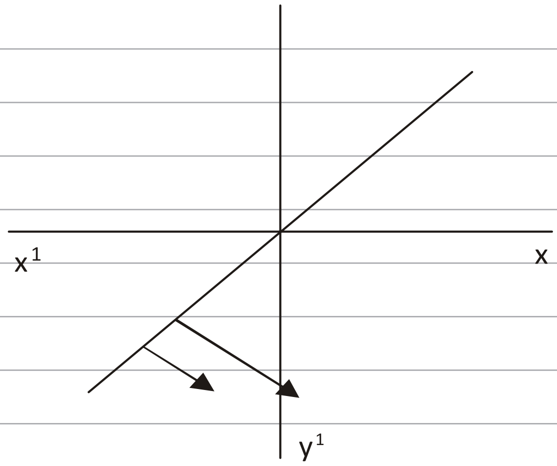
Mark (b) if $x - y \geq 0$

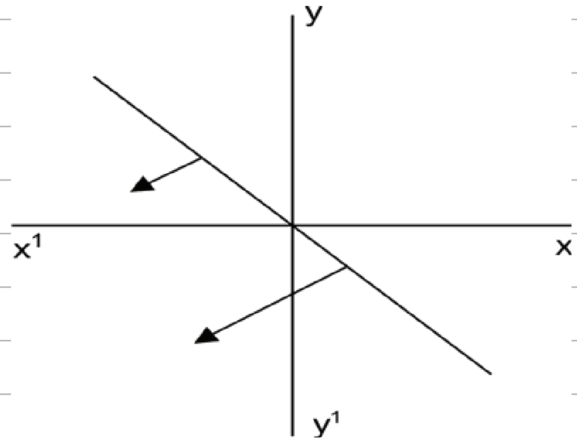
Mark (c) if $x + y \leq 0$

Mark (d) if $x + y \geq 0$









15. A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is at least 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D. The vitamin content per Kg. of each food is shown below:

	A	B	C	D
Food I :	2	1	1	2
Food II:	1	1	2	3

Assuming x units of food I is to be mixed with y units of food II the situation can be expressed as :

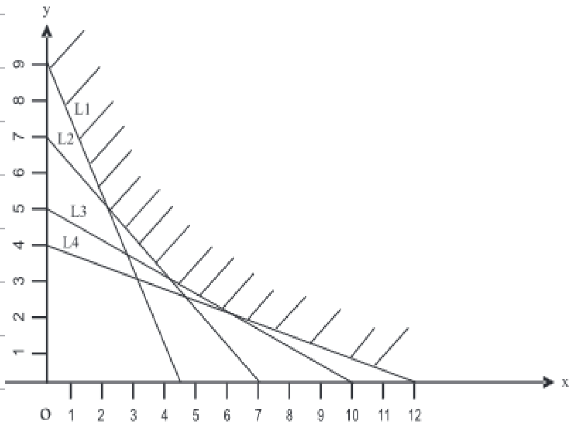
(a) $2x + y \le 9$
$x + y \le 7$
$x + 2y \le 10$
$2x + 3y \le 12$
$x \ge 0, y \ge 0$

(b) $2x + y \ge 30$
$x + y \le 7$
$x + 2y \ge 10$
$x + 3y \ge 12$

(c) $2x + y \ge 9$
$x + y \ge 7$
$x + y \le 10$
$2x + 3y \ge 12$

(d) $2x + y \ge 9$
$x + y \ge 7$
$x + 2y \ge 10$
$2x + 3y \ge 12$
$x \ge 0, y \ge 0$

16.



L1 : $2x + y = 9$ L2 : $x + y = 7$ L3 : $x + 2y = 10$ L4 : $x + 3y = 12$

The common region (shaded part) indicated on the diagram is expressed by the set of inequalities

(a) $2x + y \leq 9$	(b) $2x + y \geq 9$	(c) $2x + y \geq 9$
$x + y \geq 7$	$x + y \leq 7$	$x + y \geq 7$
$x + 2y \geq 10$	$x + 2y \geq 10$	$x + 2y \geq 10$
$x + 3y \geq 12$	$x + 3y \geq 12$	$x + 3y \geq 12$

(d) none of these

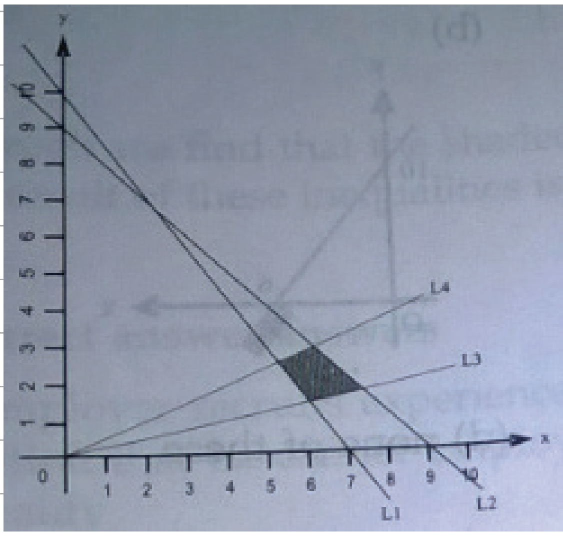
17. A company produces two types of leather belts, say A and B. Belt A is of superior quality and belt B is of lower quality. Each belt of type A requires twice as much time as required by a belt of type B. If all belts were of type B, the company could produce 1000 belts per day. But the supply of leather is sufficient only for 800 belts per day. Belt A requires fancy buckles and only 400 fancy buckles are available per day. For belt of type B only 700 buckles are available per day.

Constraints can be formulated by assuming that the company produce x units of belt A and y units of belt B as :

(a) $2x + y \leq 1000$	(b) $2x + y \leq 1000$	(c) $2x + y \geq 1000$
$x + y \geq 800$	$x + y \leq 800$	$x + y \leq 800$
$x \leq 400$; $y \leq 700$	$x \leq 400$; $y \leq 700$	$x \leq 400$; $y \leq 700$
$x \geq 0$; $y \geq 0$	$x \geq 0$; $y \geq 0$	$x \geq 0$; $y \geq 0$

d) None of these

18.



L1 : $5x + 3y = 30$ L2 : $x+y = 9$ L3 : $y = x/3$ L4 : $y = x/2$

The common region (shaded part) shown in the diagram refers to

(a) $5x + 3y \leq 30$
$x + y \leq 9$
$y \leq 1/5\, x$
$y \leq x/2$

(b) $5x + 3y \geq 30$
$x + y \leq 9$
$y \geq x/3$
$y \leq x/2$
$x \geq 0,\ y > 0$

(c) $5x + 3y \geq 30$
$x + y > 9$
$y \leq x/3$
$y \geq x/2$
$x \geq 0,\ y \geq 0$

(d) $5x + 3y \geq 30$
$x + y \leq 9$
$y \geq 9$
$y \leq x/2$
$x \geq 0,\ y \geq 0$

HOMEWORK

1.

(i) An employer recruits experienced (x) and fresh workmen (y) for his firm under the condition that he cannot employ more than 9 people, x and y can be related by the inequality.

(a) $x + y \neq 9$ (b) $x + y \leq 9$ $x \geq 0, y \geq 0$

(c) $x + y \geq 9$ $x \geq 0, y \geq 0$ (d) none of these

(ii) On the average experienced person does 5 units of work while a fresh one 3 units of work daily but the employer has to maintain an output of at least 30 units of work per day. This situation can be expressed as

(a) $5x + 3y \leq 30$ (b) $5x + 3y > 30$

(c) $5x + 3y \geq 30$ $x \geq 0, y \geq 0$ (d) none of these

(iii) The rules and regulations demand that the employer should employ not more than 5 experienced hands to 1 fresh one and this fact can be expressed as

(a) $y \geq x/5$ (b) $5y \leq x$

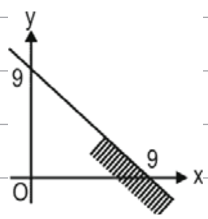
(c) $5y \geq x$ (d) none of these

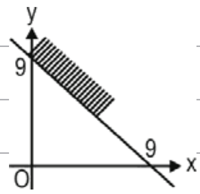
(iv) The union however forbids him to employ less than 2 experienced person to each fresh person. This situation can be expressed as

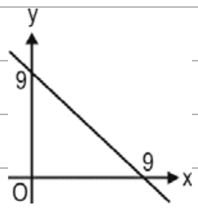
(a) $x \leq y/2$ (b) $y \leq x/2$

(c) $y \geq x/2$ (d) $x > 2y$

(v) The graph to express the inequality $x + y \leq 9$ is

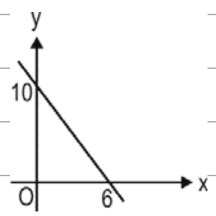
a) 

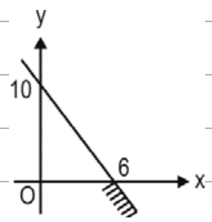
b) 

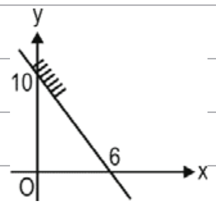
c) 

(d) none of these

(vi) The graph to express the inequality $5x + 3y \geq 30$ is

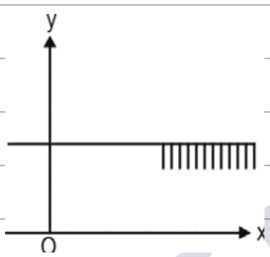
a) 

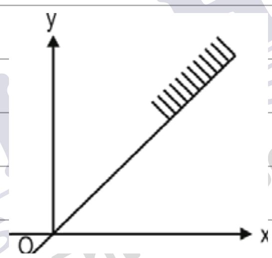
b) 

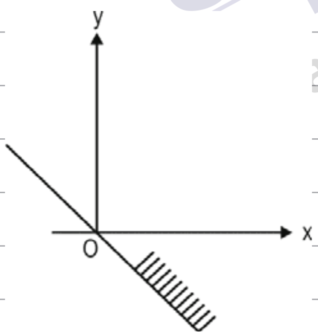
c) 

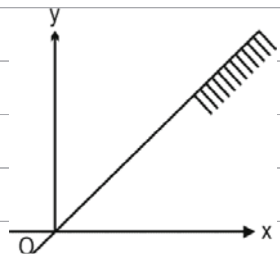
(d) none of these

(vii) The graph to express the inequality $y \leq \frac{1}{2}x$

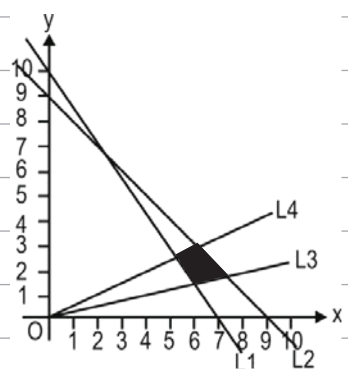
a) 

b) 

c) 

d) 

(viii)



$L_1 : 5x + 3y = 30$ $L_2 : x+y = 9$ $L_3 : y = x/3$ $L_4 : y = x/2$

The common region (shaded part) shown in the diagram refers to

(a) $5x + 3y \leq 30$
$x + y \leq 9$
$y \leq 1/5\ x$
$y \leq x/2$

(b) $5x + 3y \geq 30$
$x + y \leq 9$
$y \geq x/3$
$y \leq x/2$
$x \geq 0, y \geq 0$

(c) $5x + 3y \geq 30$
$x + y \geq 9$
$y \leq x/3$
$y \geq x/2$
$x \geq 0, y \geq 0$

(d) $5x + 3y \geq 30$
$x + y \leq 9$
$y \geq 9$
$y \leq x/2$
$x \geq 0, y \geq 0$

2. A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is at least 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D. The vitamin content per Kg. of each food is shown below:

	A	B	C	D
Food I:	2	1	1	2
Food II:	1	1	2	3

Assuming x units of food I is to be mixed with y units of food II the situation can be expressed as

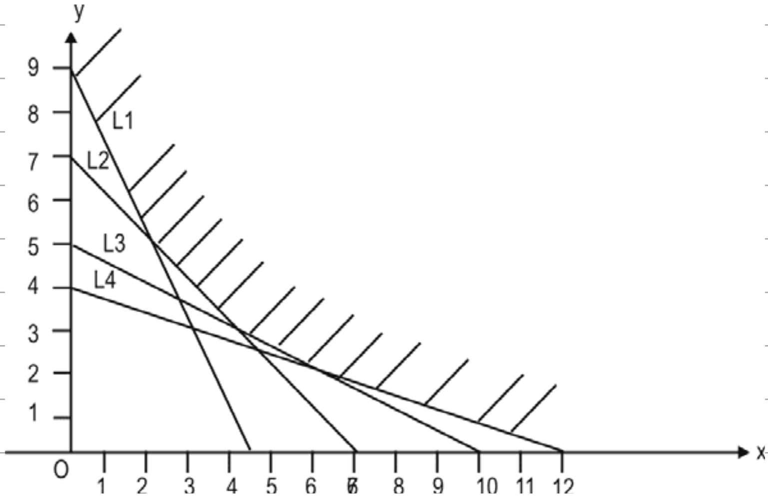
(a) $2x + y \leq 9$
$x + y \leq 7$
$x + 2y \leq 10$
$2x + 3y \leq 12$
$0 > 0, y > 0$

(b) $2x + y \geq 30$
$x + y \leq 7$
$x + 2y \geq 10$
$x + 3y \geq 12$

(c) $2x + y \geq 9$
$x + y \geq 7$
$x + y \leq 10$
$x + 3y \geq 12$

(d) $2x + y \geq 9$
$x + y \geq 7$
$x + 2y \geq 10$
$2x + 3y \geq 12$
$x \geq 0, y \geq 0$

3. Graphs of the inequations are drawn below :



L1 : $2x + y = 9$ L2 : $x + y = 7$ L3 : $x + 2y = 10$ L4 : $x + 3y = 12$

The common region (shaded part) indicated on the diagram is expressed by the set of inequalities

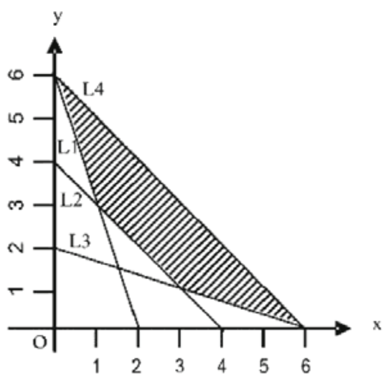
(a) $2x + y \leq 9$
$x + y \geq 7$
$x + 2y \geq 10$
$x + 3y \geq 12$

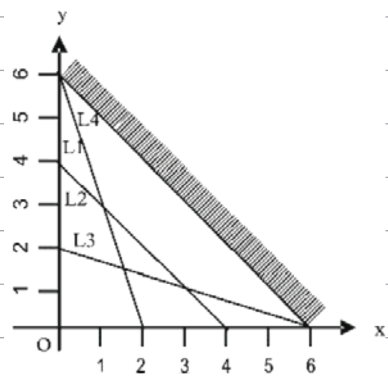
(b) $2x + y \geq 9$
$x + y \leq 7$
$x + 2y \geq 10$
$x + 3y \geq 12$

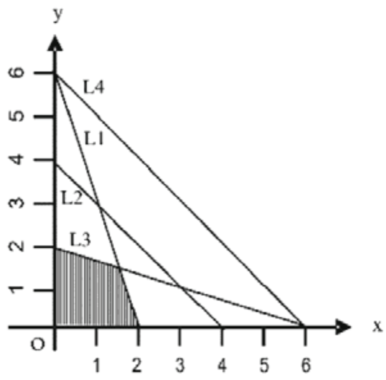
(c) $2x + y \geq 9$
$x + y \geq 7$
$x + 2y \geq 10$
$x + 3y \geq 12$
$x \geq 0, y \geq 0$

(d) none of these

4. The common region satisfied by the inequalities L1 : $3x + y \geq 6$, L2 : $x + y \geq 4$, L3 : $x + 3y \geq 6$ and L4 : $x + y \leq 6$ is indicated by

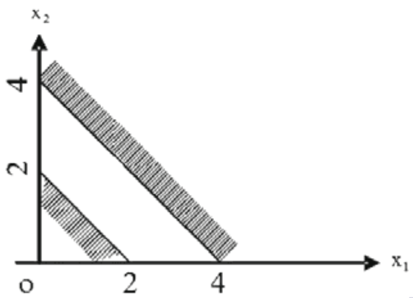
a) 

b) 

c) 

(d) none of these

5. The region indicated by the shading in the graph is expressed by inequalities



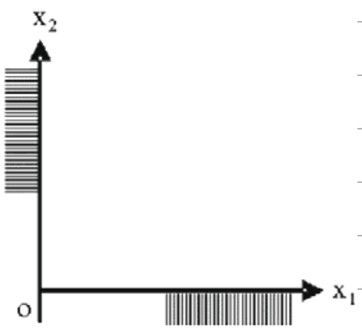
a) $x_1 + x_2 \le 2$
$2x_1 + 2x_2 \ge 8$
$x_1 \ge 0, x_2 \ge 0$

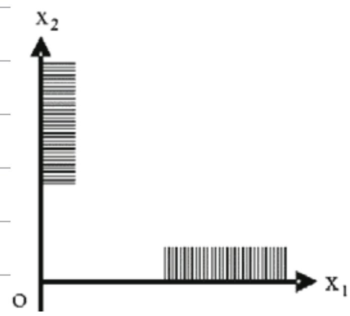
(b) $x_1 + x_2 \le 2$
$x_2x_1 + x_2 \le 4$

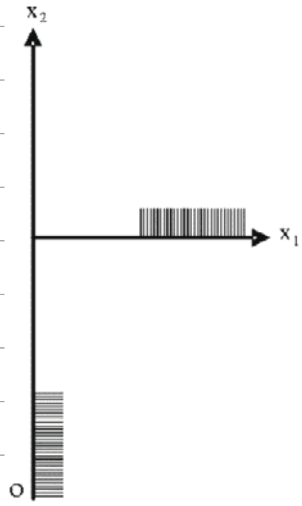
(c) $x_1 + x_2 \ge 2$
$2x_1 + 2x_2 \ge 8$

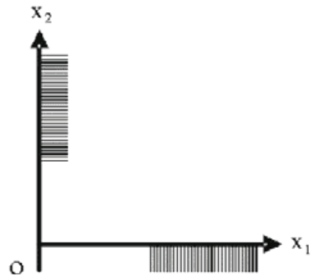
(d) $x_1 + x_2 \le 2$
$2x_1 + 2x_2 > 8$

6.(i) The inequalities $x_1 \ge 0$, $x_2 \ge 0$ are are presented by one of the graphs shown below:

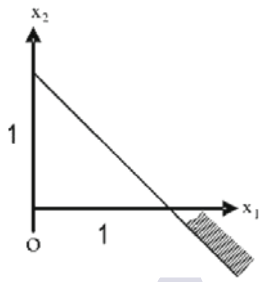
a) 

b) 

c) 

d) 

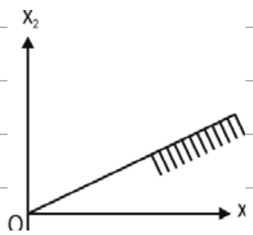
(ii) The region is expressed as

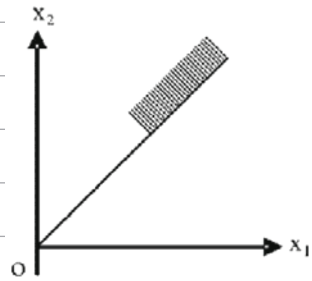


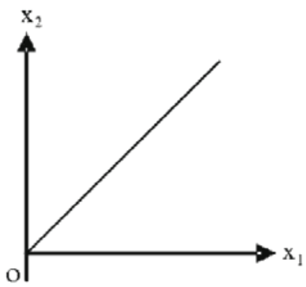
(a) $x_1 - x_2 \geq 1$ (b) $x_1 + x_2 \leq 1$

(c) $x_1 + x_2 \geq 1$ (d) none of these

(iii) The inequality $-x_1 + 2x_2 \leq 0$ is indicated on the graph as

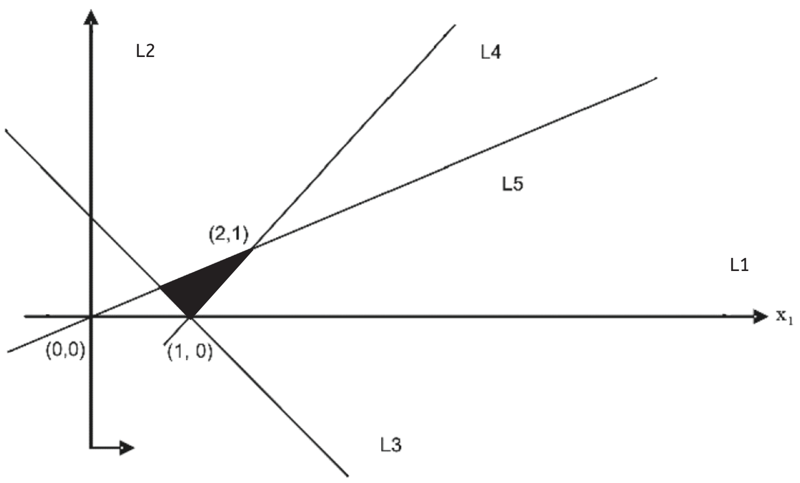
a) 

b) 

c) 

(d) none of these

7.



The common region indicated on the graph is expressed by the set of five inequalities

(a) $L_1 : x_1 \geq 0$
$L_2 : x_2 \geq 0$
$L_3 : x_1 + x_2 \leq 1$
$L_4 : x_1 - x_2 \geq 1$
$L_5 : -x_1 + 2x_2 \leq 0$

(b) $L_1 : x_1 \geq 0$
$L_2 : x_2 \geq 0$
$L_3 : x_1 + x_2 \geq 1$
$L_4 : x_1 - x_2 \geq 1$
$L_5 : -x_1 + 2x_2 \leq 0$

(c) $L_1 : x1 \leq 0$
$L_2 : x2 \leq 0$
$L_3 : x_1 + x_2 \geq 1$
$L_4 : x_1 - x_2 \geq 1$
$L_5 : -x_1 + 2x_2 \leq 0$

(d) none of these

8. A firm makes two types of products : Type A and Type B. The profit on product A is Nu. 20 each and that on product B is Nu. 30 each. Both types are processed on three machines M1, M2 and M3. The time required in hours by each product and total time available in hours per week on each machine are as follows:

Machine	Product A	Product B	Available Time
M1	3	3	36
M2	5	2	50
M3	2	6	60

The constraints can be formulated taking x_1 = number of units A and x_2 = number of unit of B as

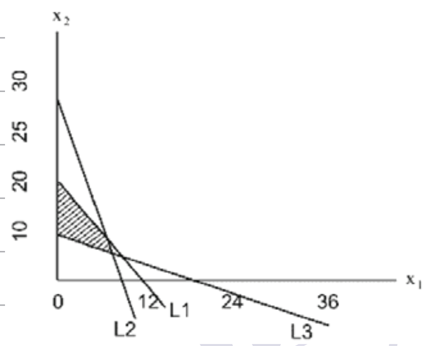
(a) $x_1 + x_2 \leq 12$
$5x_1 + 2x_2 \leq 50$
$2x_1 + 6x_2 \leq 60$

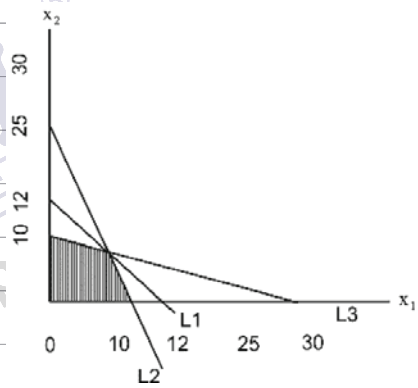
(b) $3x_1 + 3x_2 \geq 36$
$5x_1 + 2x_2 \leq 50$
$2x_1 + 6x_2 \geq 60$
$x_1 \geq 0, x_2 \geq 0$

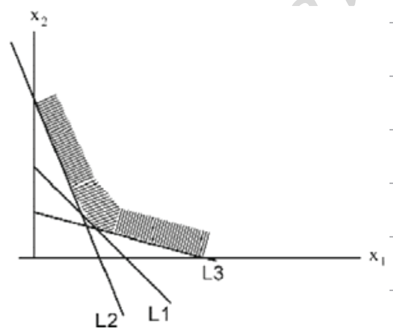
(c) $3x_1 + 3x_2 \leq 36$
$5x_1 + 2x_2 \leq 50$
$2x_1 + 6x_2 \leq 60$
$x_1 \geq 0, x_2 \geq 0$

(d) none of these

9. The set of inequalities $L_1 : x_1 + x_2 \leq 12$, $L_2 : 5x_1 + 2x_2 \leq 50$, $L_3 : x_1 + 3x_2 \leq 30$, $x_1 \geq 0$ and $x_2 \geq 0$ is represented by

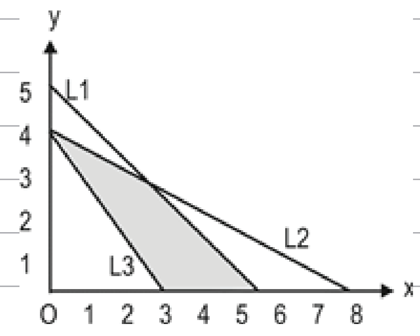
a) 

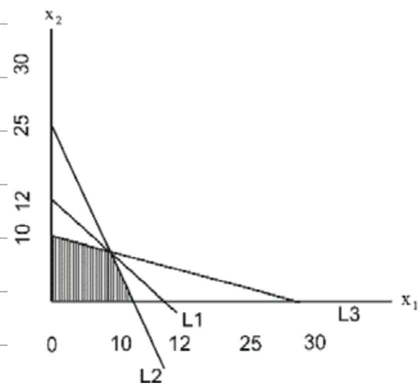
b) 

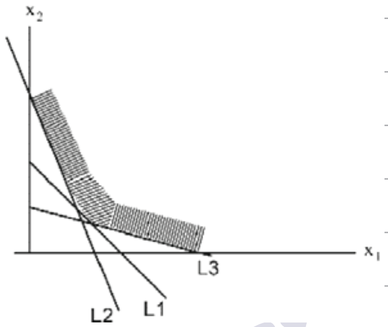
c) 

(d) none of these

10. The common region satisfying the set of inequalities $x \geq 0$, $y \geq 0$, $L_1 : x + y \leq 5$, $L_2 : x + 2y \leq 8$ and $L_3 : 4x + 3y \geq 12$ is indicated by

a) 

b) 

c) 

(d) none of these

HOMEWORK SOLUTION

1) (i) (b)

(ii) (c)

(iii) x $\longrightarrow$ Experienced person

y $\longrightarrow$ fresher

$\frac{x}{5} \leq y$ or $y \geq \frac{x}{5}$

Ans. (a)

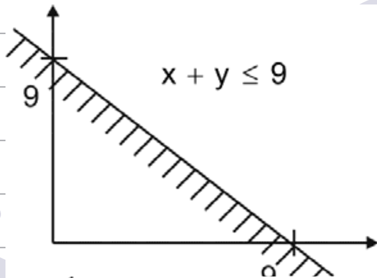
(iv) $\frac{x}{2} < y$

Oppose / Forbids $\frac{x}{2} \geq y$ OR $y \leq \frac{x}{2}$

Ans. (b)

(v) $x + y \leq 9$

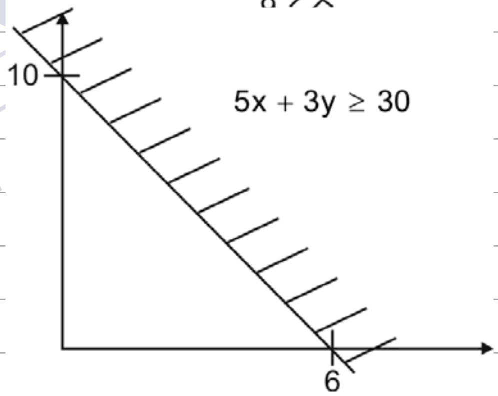
x	y	(x, y)
0	9	(0, 9)
9	0	(9, 0)



Ans. (a)

(vi) $5x + 3y \geq 30$

x	y	(x, y)
0	10	(0, 10)
6	0	(6, 0)



Ans. (c)

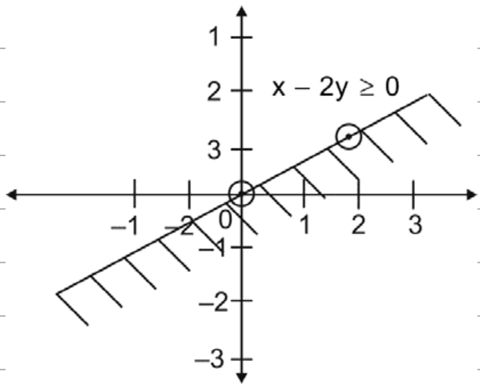
(vii) $y \leq \frac{1}{2}x$

$\therefore 2y \leq x$

$\therefore 0 \leq x - 2y$

$\therefore x - 2y \geq 0$

x	y	(x, y)
0	0	(0, 0)
2	1	(2, 1)



Ans. (d)

(viii) (b)

2. (d) x – Food I Y – Food II

	Vitamin			
	A	B	C	D
Food I	2	1	1	2
Food II	1	1	2	3
Atleast req.	9	7	10	12

$2x + y \geq 9$

$x + y \geq 7$

$x = 2y \geq 10$

$2x + 3y \geq 12$

$\because$ Food cannot be consumed in negative

$x \geq 0$ & $y \geq 0$

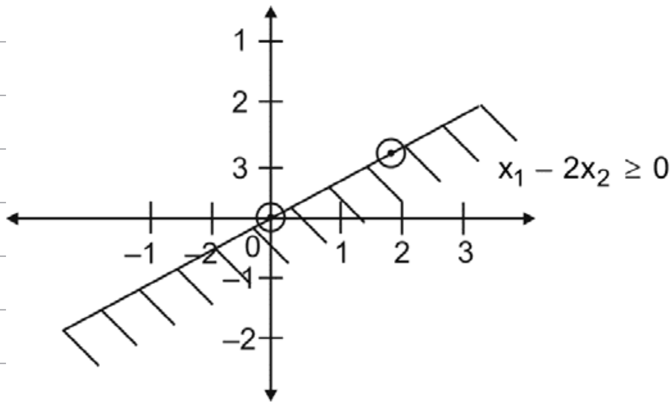
3. (c) By option method
4. (a) By observation & option method
5. (a) By observation & option method
6. (i) Ans. (b)

(ii) By option method Ans. (c)

(iii) $-x_1 + 2x_2 \leq 0$

$x_1 - 2x_2 \geq 0$

x	y	(x, y)
0	0	(0, 0)
2	1	(2, 1)



Ans. (a)

7. (d)

8. (c) As per given information

$3x_1 + 3x_2 \leq 36$

$5x_1 + 2x_2 \leq 50$

$2x_1 + 6x_2 \leq 60$

$x_1 \geq 0, x_2 \geq 0$

9. (b) $L_1 : x_2 + x_2 \leq 12$

x_1	x_2	(x_1, x_2)
0	12	(0, 12)
12	0	(12, 0)

$L_2 : 5x_1 + 2x_2 \leq 50$

x_1,	x_2	(x_1, x_2)
0	25	(0, 25)
10	0	(10, 0)

$L_3 : x_1 + 3x_2 \quad 30$

x_1	x_2	(x_1, x_2)
0	10	(0, 10)
30	0	(30, 0)

$\because x_1 \quad 0$ & $x_2 \quad 0$

Sol. lies in

1st Quadrant

Now by observation

10. (a) By observation & option method.

SELF ASSESSMENT TEST 5
LINEAR INEQUALITIES
12 Question, 12 MARKS

1. Find the range of values of x, which satisfy the inequality $-\frac{1}{5} \le \frac{3x}{10} + 1 < \frac{2}{5}, x \in R$

   a) $\{x : x \in R, -4 < x < 2\}$

   b) $\{x : x \in R, -4 < x < -2\}$

   c) $\{x : x \in R, -4 \le x < 2\}$

   d) $\{x : x \in R, -4 \le x < -2\}$

2. List the elements of the solution set of the in-equation $-3 < (x - 2) \le (9 - 2x)$, $x \in N$.

   a) {0, 1, 2}

   b) {1, 2, 3}

   c) {1, 2, 3.67}

   d) None of the above

3. An animal feed company must produce 200 kg of a mixture consisting on ingredients X1 and X2. Not more than 80 Kg of X1 can be used and at least 60 kg of X2 must be used. The ingredient X1 costs Rs. 30 per kg and X2 costs Rs. 50 per kg. To determine the quantities of X1 and X2, give a mathematical formulation.

   a) $\left.\begin{array}{l} X1 + X2 = 200 \\ X1 \le 80 \\ X2 \ge 60 \\ X1 \ge 0, X2 \ge 0 \end{array}\right\}$

   b) $\left.\begin{array}{l} X1 + X2 \le 200 \\ X1 \le 80 \\ X2 \ge 60 \\ X1 \ge 0, X2 \ge 0 \end{array}\right\}$

   c) $\left.\begin{array}{l} X1 + X2 \le 200 \\ X1 \le 80 \\ X2 \le 60 \\ X1 \ge 0, X2 \ge 0 \end{array}\right\}$

   d) $\left.\begin{array}{l} X1 + X2 \ge 200 \\ X1 \ge 80 \\ X2 \ge 60 \\ X1 \ge 0, X2 \ge 0 \end{array}\right\}$

4. A machine producing either product A or B can produce A by using 2 units of chemical and 1 unit of a compound and can produce B by using 1 unit of chemical and 2 units of the compound. Only 800 units of chemical and 1000 units of the compound are available. Express this information by linear inequalities.

a) $\left.\begin{array}{l} 2A+B \geq 800 \\ A+2B \geq 1000 \\ A \geq 0, B \geq 0 \end{array}\right\}$ b) $\left.\begin{array}{l} 2A+B \leq 800 \\ A+2B \geq 1000 \\ A \geq 0, B \geq 0 \end{array}\right\}$

c) $\left.\begin{array}{l} 2A+B \geq 800 \\ A+2B \leq 1000 \\ A \geq 0, B \geq 0 \end{array}\right\}$ d) $\left.\begin{array}{l} 2A+B \leq 800 \\ A+2B \leq 1000 \\ A \geq 0, B \geq 0 \end{array}\right\}$

5. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space at most for 20 items. A fan costs him Rs. 360 and a sewing machine Rs. 240. Express this information by linear inequalities.

a) $\left.\begin{array}{l} x+y \leq 20 \\ 360x+240y \leq 5760 \\ x \geq 0, y \geq 0 \end{array}\right\}$ b) $\left.\begin{array}{l} x+y \leq 20 \\ 360x+240y \geq 5760 \\ x \geq 0, y \geq 0 \end{array}\right\}$

d) $\left.\begin{array}{l} x+y \geq 20 \\ 360x+240y \leq 5760 \\ x \geq 0, y \geq 0 \end{array}\right\}$ d) $\left.\begin{array}{l} x+y \leq 20 \\ 360x+240y \leq 5760 \\ x < 0, y < 0 \end{array}\right\}$

6. A firm manufactures two products A and B. Each product is processed on two machines X and Y. The product A requires one minute of processing time on X and two minutes on Y while B requires one minute on X and one minute on B. Machine X us available for use for not more than 7 hours 30 minutes, while machine Y is available for 10 hours during any working day. Give the mathematical formulation to this linear problem.

a) $\left.\begin{array}{l} A+B \leq 450 \\ 2A+B \leq 600 \\ A \geq 0, B \geq 0 \end{array}\right\}$ b) $\left.\begin{array}{l} A+B \leq 7.30 \\ 2A+B \leq 10 \\ A \geq 0, B \geq 0 \end{array}\right\}$

c) $\left.\begin{array}{l} A+B \leq 7.5 \\ 2A+B \leq 10 \\ A \geq 0, B \geq 0 \end{array}\right\}$ d) $\left.\begin{array}{l} A+B \leq 45 \\ 2A+B \leq 60 \\ A \geq 0, B \geq 0 \end{array}\right\}$

7. A carpenter has 90, 80 and 50 running feet respectively of teak, plywood and rosewood. The product A requires 2, 1 and 1 running feet and the product B requires 1, 2 and 1 running feet of teak, plywood and rosewood respectively. Give the mathematical formulation to this linear problem.

a) $\left.\begin{array}{l} x+y \le 50 \\ x+2y \le 90 \\ 2x+y \le 80 \\ x \ge 0, y \ge 0 \end{array}\right\}$

b) $\left.\begin{array}{l} x+y \le 50 \\ x+2y \le 80 \\ 2x+y \le 90 \\ x \ge 0, y \ge 0 \end{array}\right\}$

c) $\left.\begin{array}{l} x+y \le 80 \\ x+2y \le 90 \\ 2x+y \le 50 \\ x \ge 0, y \ge 0 \end{array}\right\}$

d) $\left.\begin{array}{l} 2x+y \le 50 \\ 2x+2y \le 80 \\ 2x+y \le 90 \\ x \ge 0, y \ge 0 \end{array}\right\}$

8. A firm manufactures two types of electrical items A and B. Both A and B make use of two essential components, a motor and a transformer. Each unit of A requires 3 motors and 2 transformers, and each unit of B requires 2 motors and 4 transformers. The total supply of components per month is restricted to 210 motors and 300 transformers. Type B is an export model requiring a voltage stabilizer, restricted to 65 units per month. Give a mathematical formulation of this linear problem.

a) $\left.\begin{array}{l} 3x+2y \le 210 \\ 2x+4y \le 300 \\ x \le 65 \\ x \ge 0, y \ge 0 \end{array}\right\}$

b) $\left.\begin{array}{l} 3x+2y \le 210 \\ 2x+4y > 300 \\ x \le 65 \\ x \ge 0, y \ge 0 \end{array}\right\}$

c) $\left.\begin{array}{l} 3x+2y \ge 210 \\ 2x+4y \ge 300 \\ y \le 65 \\ x \ge 0, y \ge 0 \end{array}\right\}$

d) $\left.\begin{array}{l} 3x+2y \le 210 \\ 2x+4y \le 300 \\ y \le 65 \\ x \ge 0, y \ge 0 \end{array}\right\}$

9. A confectioner manufactures two types of biscuits – One Regular type and the other Non-regular type. The biscuits are processed in three main operations: Blending, Cooking and Packaging. The average time taken (in minutes) for each box for each of the processing operations is given below:

	Blending	Cooking	Packaging
Regulars	1 min	5 min	3 min
Non-Regulars	2 min	4 min	1 min

The Blending equipment is available for a maximum of 12 hours, the Cooking facilities for 30 hours at the most, and Packaging equipment for maximum 15 hours. Give a mathematical formulation to this linear problem.

a) $\left.\begin{array}{l} x+2y \le 720 \\ 5x+4y \le 1800 \\ 3x+y \le 900 \\ x \ge 0, y \ge 0 \end{array}\right\}$

b) $\left.\begin{array}{l} x+5y+3z \le 720 \\ 2x+4y+z \le 1800 \\ 3x+y \le 900 \\ x \ge 0, y \ge 0, z \ge 0 \end{array}\right\}$

c) $\left.\begin{array}{l} x+2y+z \le 720 \\ 5x+4y+z \le 1800 \\ 3x+y+z \le 900 \\ x \ge 0, y \ge 0, z \ge 0 \end{array}\right\}$

d) $\left.\begin{array}{l} x+2y \le 720 \\ 5x+4y < 1800 \\ 3x+y \le 900 \\ x < 0, y \ge 0 \end{array}\right\}$

10. The standard weight of a special purpose brick is 5 kg and it contains two basic ingredients B1 and B2. Strength considerations dictate that the brick contain not more than 4 Kg of B1 and at least 2 kg of B2. To determine the amount of ingredients B1 and B2, give a mathematical formulation.

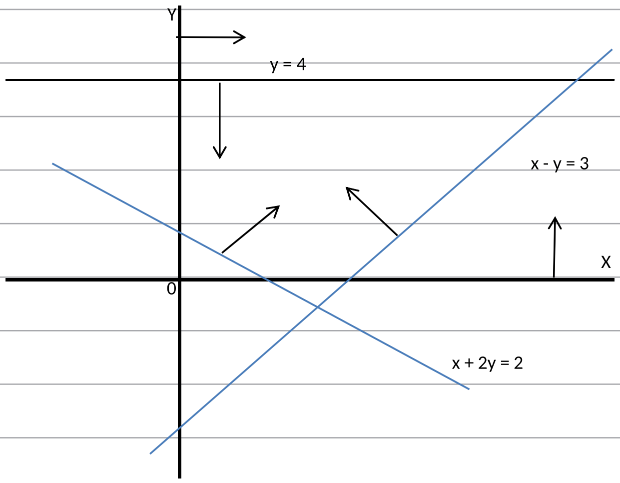
a) $\left.\begin{array}{l} x+y = 5 \\ x \le 4 \\ y \ge 2 \\ x \ge 0, y \ge 0 \end{array}\right\}$

b) $\left.\begin{array}{l} x+y \le 5 \\ x \le 4 \\ y \le 2 \\ x \ge 0, y \ge 0 \end{array}\right\}$

b) $\left.\begin{array}{l} x+y \ge 5 \\ x \le 4 \\ y \le 2 \\ x \ge 0, y \ge 0 \end{array}\right\}$

d) $\left.\begin{array}{l} x+y = 5 \\ x \le 4 \\ y \le 2 \\ x \ge 0, y \ge 0 \end{array}\right\}$

11. Following is the solution of which of the set of linear in-equations:



a) $x + 2y \geq 2, x - y \leq 3, y \leq 4, x \geq 0, y \geq 0$

b) $x + 2y > 2, x - y < 3, y \leq 4, x \geq 0, y \geq 0$

c) $x + 2y \leq 2, x - y \geq 3, y \leq 4, x \geq 0, y \geq 0$

d) $x + 2y \geq 2, x - y \leq 3, y < 4, x > 0, y > 0$

12. Find the solution set for $\frac{1}{|2x - 3|} < 1$

a) $\{x : x \in R, x < 1\}$

b) $\{x : x \in R, x > 2\}$

c) $\{x : x \in R, x > 1 \text{ or } x < 2\}$

d) $\{x : x \in R, x < 1 \text{ or } x > 2\}$

EXPLANATORY ANSWERS

1. $-\frac{1}{5} \leq \frac{3x}{10} + 1 < \frac{2}{5}, x \in R$

$-2 \leq 3x + 10 < 4$

$\Rightarrow -2 - 10 \leq 3x + 10 + (-10) < 4 + (-10)$

$\Rightarrow -12 \leq 3x < -6$

$\Rightarrow -4 \leq x < -2$ Option D

2. $-3 < (x - 2) \leq (9 - 2x), x \in N$

$-3 < (x - 2)$ and $(x - 2) \leq (9 - 2x)$

$-1 < x$ and $(3x - 2) \leq 9$

$-1 < x$ and $x \leq 11/3$

$-1 < x \leq 11/3$ but $x \in N$

The solution set is {1, 2, 3}

Option B

3. An animal feed is the mixture of X1 kg of type X1 and X2 kg of type X2.

X1 + X2 = 200 (as weight of the mix is 200 kg)

X1 ≤ 80 (as not more than 80 kg of X1 can be used)

X2 ≥ 60 (as at least 60 kg of X2 must be used)

X1, X2 ≥ 0 (Non Negativity Constraint, as both ingredients should be used)

Option A

4. Let A units of product A and B units of product B are produced.

2A + B ≤ 800 (Only 800 units of chemical are available)

A + 2B ≤ 1000 (Only 1000 units of compound is available)

A, B ≥ 0 (Non Negativity Constraint)

Option D

5. Let x units of fans and y units of sewing machines are purchased.

$360x + 240y \leq 5760$ (Maximum investment allowed is Rs. 5760)

$x + y \leq 20$ (Maximum units of x and y that can be stored are 20)

$x, y \geq 0$ (Non Negativity Constraint)

Option A

6. Let A units of product A and B units of product B are produced.

$A + B \leq 450$ (7*60 + 30) (Maximum time available on Machine X is 7 hrs 30 minutes)

$2A + B \leq 600$ (10*60) (Maximum time available on Machine Y is 10 hours)

$A, B \geq 0$ (Non Negativity Constraint)

Option A

7. Let x units of Product A and y units of Product B are produced

$2x + y \leq 90$ (Maximum 90 feet of teak is available)

$x + 2y \leq 80$ (Maximum 80 feet of plywood is available)

$x + y \leq 50$ (Maximum 50 feet of rosewood is available)

$x, y \geq 0$ (Non Negativity Constraint)

Option B

8. Let x units of item A and y units of item B are manufactured.

$3x + 2y \leq 210$ (Maximum 210 motors are available)

$2x + 4y \leq 300$ (Maximum 300 transformers are available)

$y \leq 65$ (Maximum 65 voltage stabilizers are available)

$x, y \geq 0$ (Non Negativity Constraint)

Option D

9. Let x boxes of Regulars and y boxes of Non-Regulars are produced.

$x + 2y \leq 720$ (12*60) (Maximum time available with blending equipment is 12 hours)

$5x + 4y \leq 1800$ (30*60) (Maximum time available with cooking facilities is 30 hours)

$3x + y \leq 900$ (15*60) (Maximum time available with Packaging equipment is 15 hours)

$x, y \geq 0$ (Non Negativity Constraint)

Option A

10. Let x kg of ingredient B1 and y kg of ingredient B2 is used

$x + y = 5$ (The standard weight of the brick is 5 kg)

$x \leq 4$ (Maximum B1 to be used is 4 Kg)

$y \geq 2$ (Minimum B2 to be used is 2 kg)

$x, y \geq 0$ (Non Negativity Constraint)

Option A

11. $x \geq 0$ & $y \geq 0$, as solution set is in the first quadrant
    $y = 4$ line shading is towards the origin. Thus, $y \leq 4$
    $x + 2y = 2$ area is away from origin. Thus $x + 2y \geq 2$
    $x - y = 3$ shaded portion is towards the origin (it lies to the right of origin). Thus $x - y \leq 3$
    Option A

12. First, we note that $(2x - 3) \neq 0$, i.e. $x \neq 3/2$
    Also, for $x \neq 3/2$, $|2x - 3| > 0$

    Thus, $\frac{1}{|2x-3|} < 1 \quad \rightarrow 1 < |2x - 3|$
    $\rightarrow |2x - 3| > 1 \rightarrow (2x - 3) < -1$ or $(2x - 3) > 1$
    $\rightarrow 2x < 2$ or $2x > 4$
    $\rightarrow x < 1$ or $x > 2$
    Hence the solution set is $\{x : x \in R, x < 1 \text{ or } x > 2\}$
    Option D

4 TIME VALUE OF MONEY

Simple Interest

Simple interest is charged on the principal amount and hence it is same for every year.

A = Amount, P = principal, n = number of years, R = interest rate

$$SI = \frac{PTR}{100}$$

$$A = P + SI = P + \frac{PTR}{100} = P\left(1 + \frac{TR}{100}\right)$$

Notes:

- If rate of interest is known, then sum of money will double itself in 100/r years.
- If number of years is known, then sum of money will double itself @ 100/n %.
- A sum of money will become "n" times in $\frac{(n-1) \times 100}{R}$ years.

Example:

In how many years a sum of money @10% p.a. SI will become (a) double, (b) triple, (c) N times.

(a) Double	(b) Triple	(c) N times
$\frac{(2-1) \times 100}{10} = 10$ years	$\frac{(3-1) \times 100}{10} = 20$ years	$\frac{(N-1) \times 100}{10} = 10(N-1)$ years

- If the sum of money becomes "n_1" times in T_1 years and "n_2" times in T_2 years, then the ratio of their times is: $\frac{T_1}{T_2} = \frac{n_1 - 1}{n_2 - 1}$.

Compound Interest

- In case of compound interest, the interest is calculated on the amount of the succeeding years, i.e., principal keeps changing every year.

- Here interest on interest is also earned, thus money grow faster when Compounding is done

- If P is the principal, n = number of years for which interest is calculated and "i" (R/100) is the rate of interest, then, the amount A after n years will be given by:

$$A=P(1+i)^n$$

- In case of depreciation by diminishing balance method (WDV), if C = Cost of the machinery, I = rate of depreciation per annum and n = effective life of the machinery, then the depreciated value D after n years is :

$$D = C\,(1 - i)^n$$

  D is also known as the scrap value of the machinery.

- Compound Interest thus would be calculated as follows:

$$CI = A - P = P\left[(1+i)^n - 1\right]$$

- Depending upon the compounding style of interest rate, the effective formula for calculating Amount would be as follows:

Half Yearly or Semi Annually	Quarterly	Monthly
$A = P\left(1+\frac{i}{2}\right)^{2n}$	$A = P\left(1+\frac{i}{4}\right)^{4n}$	$A = P\left(1+\frac{i}{12}\right)^{12n}$

- When differential interest rates are charged (i_1, i_2, i_3,... ... i_n), then:
  $A = P(1+i_1)(1+i_2)(1+i_3)\ldots\ldots\ldots\ldots(1+i_n)$

- Relationship between CI and SI
  a) For the first year, CI = SI, i.e. for the first year difference is zero.
  b) For two years, CI − SI = Pi^2
  c) For three years, CI − SI = $Pi^2(i + 3)$

Notes:

1. A sum of money will double itself in approximately 72/r years (known as Rule 72), where r is the rate of interest per annum.

2. A sum of money will triple itself in approximately 114/r (known as Rule 114), where r is the rate of interest per annum.

3. If a sum of money becomes "n" times in "t" years, then, it will become n^m times in "mt" years.
   Example: If sum of money doubles itself in 3 years, then it will be 8 times (2^3) in 3x3 = 9 years at CI.

Concept of Effective Rate of Interest

1. When the compounding is done more than once a year, then, the net annual rate of interest is found to be slightly higher than the given annual rate of interest.

2. This new rate of interest is known as the effective rate of interest and the given annual rate is called the nominal rate of interest.

3. Effective rate of interest is denoted by E and is given by the formula:

$$E = \left\{ (1+i)^n - 1 \right\} \text{ x } 100$$

   Where "i" is rate of interest, converted monthly, quarterly, half yearly and n is the number of conversion period per annum.

4. Effective rate of interest are particularly useful in making investment decisions when various options are given with differential interest rates.

5. Amongst various investment options, we shall choose that investment option, where effective rate of interest is maximum.

Concept of Present Value

Present Value is defined as the present worth of the money that would yield an amount A after n years at a specified rate of interest i.

$$\text{If } A = P(1+i)^n$$

$$\therefore\ P = PV = \text{Principal} = \frac{A}{(1+i)^n}$$

$$or, PV = A(1+i)^{-n}$$

Annuities

- Annuity is defined as a series of payments (usually equal) which are made at regular intervals of time (usually a year).

- The period for which the payment continues is called the status or the term of the annuity.

- Unless otherwise stated, the first payment will fall due at the end of every year. This is known as "Ordinary Annuity".

- When the payment falls due at the beginning of every year, i.e., immediately, it is called "Immediate Annuity".

- When the status or term of the annuity is not fixed, i.e., the payment is to be continued for an indefinite period, these are known as "Perpetual Annuity or Perpetuity".

- Hence forth, we shall maintain the following notation throughout. The regular annual payment i.e., annuity = P, rate of interest = "i" and the period for which payment is made = n (status or term of the annuity).

- The amount of the ordinary annuity is given by:

$$A = \frac{P}{i}\left\{ (1+i)^{n} - 1 \right\}$$

- The amount of immediate annuity is obtained by multiplying amount obtained for ordinary annuity by (1 + i); hence the formula becomes:

$$A = \frac{P}{i}\left\{ (1+i)^{n} - 1 \right\}(1+i)$$

- Note:
  1. When half yearly or quarterly or monthly payment is "P", in such a case change "i" to i/2 or i/4 or i/12 and change "n" to 2n or 4n or 12n respectively.

  2. When half yearly, quarterly or monthly rate of interest is "i", in such a case, change P to P/2, P/4 or P/12 and change n to 2n or 4n or 12n respectively.

- The present value of an annuity payable over a period of n years is defined as the sum of the present value of all the future payments.

- The present value of an ordinary annuity is represented by V and given as follows:

$$V = \frac{P}{i}\left\{1-(1+i)^{-n}\right\}$$

- If the term of the annuity is n years, then for evaluating the present value of the immediate annuity, first calculate the present value of the annuity for (n−1) years and then add to it the initial or first payment.

$$v = \frac{P}{i}\left\{1-(1+i)^{-n}\right\}(1+i)$$

- Present value of the perpetual annuity is given by,

V = P/i

Important concepts related to CA Inter and CA Final

Financial Management

1. Sinking Fund

It is the fund credited for a specified purpose by way of sequence of periodic payments over a time period at a specified interest rate. Interest is compounded at the end of every period. Size of the sinking fund deposit is computed from A = P.A(n, i) where A is the amount to be saved, P the periodic payment, n the payment period.

2. Leasing

Leasing is a financial arrangement under which the owner of the asset (lessor) allows the user of the asset (lessee) to use the asset for a defined period of time(lease period) for a consideration (lease rental) payable over a given period of time. This is a kind of taking an asset on rent.

3. Capital Expenditure (investment decision)

Capital expenditure means purchasing an asset (which results in outflows of money) today in anticipation of benefits (cash inflow) which would flow across the life of the investment. For taking investment decision we compare the present value of cash outflow and present value of cash inflows. If present value of cash inflows is greater than present value of cash outflows decision should be in the favour of investment.

4. Valuation of Bond

A bond is a debt security in which the issuer owes the holder a debt and is obliged to repay the principal and interest. Bonds are generally issued for a fixed term longer than one year.

5. Perpetuity

Perpetuity is an annuity in which the periodic payments or receipts begin on a fixed date and continue indefinitely or perpetually. Fixed coupon payments on permanently invested (irredeemable) sums of money are prime examples of perpetuities.

The formula for evaluating perpetuity is relatively straight forward. Two points which are important to understand in this regard are:.

(a) The value of the perpetuity is finite because receipts that are anticipated far in the future have extremely low present value (today's value of the future cash flows).

(b) Additionally, because the principal is never repaid, there is no present value for the principal.

Therefore, the price of perpetuity is simply the coupon amount over the appropriate discount rate or yield.

Calculation of multi period perpetuity:

The formula for determining the present value of multi-period perpetuity is as follows:

$$PVA\infty = \frac{R}{(1+i)^1} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \ldots\ldots + \frac{R}{(1+i)} = \sum_{n=1}^{\infty} \frac{R}{(1+i)^n} = \frac{R}{i}$$

Where:

R = the payment or receipt each period

i = the interest rate per payment or receipt period

6. Calculation of Growing Perpetuity

A stream of cash flows that grows at a constant rate forever is known as growing perpetuity. The formula for determining the present value of growing perpetuity is as follows:

$$PVA = \frac{R}{(1+i)^1} + \frac{R(1+g)}{(1+i)^2} + \frac{R(1+g)^2}{(1+i)^3} + \ldots.. + \frac{R(1+g)^\infty}{(1+i)^\infty}$$

$$\sum_{n=1}^{\infty} \frac{R(1+g)^{n-1}}{(1+i)^n} = \frac{R}{i-g}$$

7. Net Present Value

Net present value = Present value of net cash inflow – Total net initial investment

Since it might be possible that some additional investment may also be required during the life time of the project then appropriate formula shall be:

Net present value = Present value of cash inflow – Present value of cash outflow

The steps to calculate net present value are:-

1. Determine the net cash inflow in each year of the investment.
2. Select the desired rate of return or discounting rate or Weighted Average Cost of Capital.
3. Find the discount factor for each year based on the desired rate of return selected.
4. Determine the present values of the net cash flows by multiplying the cash flows by respective the discount factors of respective period called Present Value (PV) of Cash flows
5. Total the amounts of all PVs of Cash Flows

Decision Rule:

If NPV > 0 Accept the Proposal

If NPV < 0 Reject the Proposal

8. Nominal Rate of Return

The nominal rate is the stated interest rate. If a bank pays 5% annually on a savings account, then 5% is the nominal interest rate. So if you deposit ₹ 100 for 1 year, you will receive ₹ 5 in interest.

However, that Rs. 5 will probably be worth less at the end of the year than it would have been at the beginning. This is because inflation lowers the value of money. As goods, services, and assets, such as real estate, rise in price.

The nominal interest rate is conceptually the simplest type of interest rate. It is quite simply the stated interest rate of a given bond or loan. It is also defined as a stated interest rate. This interest works according to the simple interest and does not take into account the compounding periods.

Real Rate of Return: The real interest rate is so named because it states the "real" rate that the lender or investor receives after inflation is factored in; that is, the interest rate that exceeds the inflation rate.

A comparison of real and nominal interest rates can therefore be summed up in this equation:

Nominal Rate of Return – Inflation = Real Rate of Return

Nominal Interest Rate = Real Interest Rate + Inflation

9. Compound Annual Growth Rate (CAGR)

Compound Annual Growth Rate (CAGR) is a business and investing specific term for the smoothed annualized gain of an investment over a given time periodit is not an accounting term, but remains widely used, particularly in growth industries or to compare the growth rates of two investments because CAGR dampens the effect of volatility of periodic returns that can render arithmetic means irrelevant. CAGR is often used to describe the growth over a period of time of some element of the business, for example revenue, units delivered, registered users, etc.

$$\text{CAGR}(t_0, t_n) = \left(\frac{V(t_n)}{V(t_0)}\right)^{\frac{1}{t_n - t_0}} - 1$$

Where $V(t_0)$ = Beginning Period ; $V(t_n)$ = End Period

CLASSWORK SECTION

SIMPLE INTEREST

1. How much interest will be earned on Rs. 4000 at 6% p.a. simple interest for 2 yrs?
   (a) 450 (b) 480 (c) 500 (d) 540

2. A deposited 1,00,000 in a bank for 2 years with the interest at 5.5% p.a. What will be the final value of investment?
   (a) 1,00,000 (b) 1,11,000 (c) 1,20,000 (d) 1,30,000

3. Find rate of interest if the amount owed after 6 months is 2100, borrowed amount being Rs. 2000.
   (a) 10% (b) 8% (c) 9% d 11%

4. P = 5000, N = 1, I = 300, R will be
   (a) 5% (b) 4% (c) 6% (d) none

5. 46875 was lent out at SI and at the end of 1 yr 8 months, total amount was 50000. Find rate of int per annum?
   (a) 2% (b) 4% (c) 6% (d) 8%

6. Sum required to earn quarterly interest of 3600 at 18% p.a. is
   (a) 50,000 (b) 60,000 (c) 80,000 (d) none

7. In how much time would SI on a certain sum be 0.125 times the principal at 10% p.a.?
   (a) 1 ¼ years (b) 1.5 years (c) 1 ¾ years (d) 2 ¼ years

8. A sum of 3402 amounts to 6804 on 20 yrs. What sum will amount to 5200 in 6 yrs at same rate?
   (a) 3000 (b) 4000 (c) 5000 (d) 600

9. 30000 is invested in two parts : partly at 10% p.a. and partly at 15% p.a. Total interest earned is 3300. How much is invested at lower rate?
   (a) 20000 (b) 24000 (c) 26000 (d) 28000

10. A bike is purchased by making a down payment of 15000 and balance to be paid alongwith interest at 5% p.a. for 2 yrs. Total amount paid is 28200. Find cash price of the bike.
(a) 28000 (b) 26000 (c) 27000 (d) 25000

11. A certain sum amounts to 7400 in 3 yrs and 8600 in 4 yrs. Find the sum and rate of interest
(a) 3800, 31.57% (b) 3,000, 25%
(c) 3,500, 20% (d) none

12. A certain sum doubles itself in 20 yrs. In how many years it will become 7 times
(a) 100 (b) 120 (c) 140 (d) none

13. Mr. X takes loan of 7000 for 8 yrs. After 3 yrs he taken 3000 more. Total interest paid at the end of 8 years is 3550. Find the rate of interest.
(a) 4% p.a. (b) 5% p.a. (c) 6% p.a. (d) none

PAST EXAM QUESTIONS

14. ₹ 8,000 becomes ₹ 10,000 in two years at simple interest. The amount that will become ₹ 6,875 in 3 years at the same rate of interest is:
(a) ₹ 4,850 (b) ₹ 5,000 (c) ₹ 5,500 (d) ₹ 5,275

15. The rate of simple interest on a sum of money is 6% p.a. for first 3 years, 8% p.a. for the next five years and 10% p.a. for the period beyond 8 years. If the simple interest accrued by the sum for a period for 10 years is ₹ 1,560. The sum is:
(a) ₹ 1,500 (b) ₹ 2,000 (c) ₹ 3,000 (d) ₹ 5,000

16. A sum of money doubles itself in 10 years. The number of years it would treble itself is :
(a) 25 years (b) 15 years (c) 20 years (d) none

17. If ₹ 1,000 be invested at interest rate of 5% and the interest be added to the principal every 10 years, then the number of years in which it will amount to ₹ 2,000 is:

(a) $16\frac{2}{3}$ years (b) $16\frac{1}{4}$ years

(c) 16 years (d) $16\frac{1}{3}$ years

18. Two equal sums of money were lent at simple interest at 11% p.a. for $3\frac{1}{2}$ years and $4\frac{1}{2}$ years respectively. If the difference in interests for two periods was ₹ 412.50, then each sum is:

(a) ₹ 3,250 (b) ₹ 3,500 (c) ₹ 3,750 (d) ₹ 4,350

19. Find the numbers of years in which a sum doubles itself at the rate of 8% per annum.

(a) $11\frac{1}{2}$ (b) $12\frac{1}{2}$

(c) $9\frac{1}{2}$ (d) $13\frac{1}{2}$

20. If a simple interest on a sum of money at 6% p.a. for 7 years is equal to twice of simple interest on another sum for 9 years at 5% p.a. The ratio will be

(a) 2 : 15 (b) 7 : 15 (c) 15 : 7 (d) 1 : 7

21. If the simple interest on ₹ 1,400 for 3 years is less than the simple interest on ₹ 1,800 for the same period by ₹ 80, then the rate of interest is

(a) 5.67% (b) 6.67% (c) 7.20% (d) 5.00%

22. The S.I. on a sum of money is $\frac{4}{9}$ of the principal and the no. of years is equal to the rate of interest per annum. Find the rate of interest per annum?

(a) 5% (b) 20/3% (c) 22/7% (d) 6%

23. Mr. X invests ₹ 90,500 in post office at 7.5% p.a. simple interest. While calculating the rate was wrongly taken as 5.7% p.a. The difference in amounts at maturity is ₹9,774. Find the period for which the sum was invested:

(a) 7 years (b) 5.8 years (c) 6 years (d) 8 years

24. A sum of ₹ 44,000 is divided into three parts such that the corresponding interest earned after 2 years, 3 years and 6 years may be equal. If the rates of simple interest are 6% p.a., 8% p.a. and 6% p.a. respectively, then the smallest part of the sum will be :
(a) ₹ 4,000 (b) ₹ 8,000 (c) ₹ 10,000 (d) ₹ 12,000

25. A person borrows ₹ 5,000 for 2 years at 4% per annual simple interest. He immediately lends to another person at $6\frac{1}{4}$ %. Per annual for 2 years find his gain in the transaction for per year:
(a) ₹ 112.50 (b) ₹ 225 (c) ₹ 125 (d) ₹ 107.50

26. A man invests an amount of ₹ 15,860 in the names of his three sons A, B and C in such a way that they get the same SI after 2, 3 and 4 years respectively. If the rate of interest is 5%, then the ratio of amount invested in the name of A, B and C is
(a) 6 : 4 : 3 (b) 3 : 4 : 6 (c) 30 : 12 : 5 (d) none of these

COMPOUND INTEREST

27. Find amount for a sum of 4000 at 8% p.a. for 5 yrs compounded annually?
(a) 5877 (b) 6577 (c) 8577 (d) 5677

28. Find C–I for a sum of 8000 at 4% p.a. for 6 yrs compounded half yearly?
(a) 2146 (b) 2416 (c) 2164 (d) 2641

29. Find amount and C–I for a sum of 6000 at 12% p.a. for 3 years compounded quarterly?
(a) 8554.5, 2554.5 (b) 7554.5,1554.5
(c) 9554.5, 3554.5 (d) 6554.5,554.5

30. Find amount for a sum of 10000 at 6% p.a. for 2 years compounded monthly?
(a) 12171 (b) 11712 (c) 11271 (d) 12117

31. Find present value of 10000 due in 2 yrs at 5% p.a. compound interest paid annually?
(a) 9050 (b) 9070 (c) 9080 (d) 9090

32. Find present value of 20000 due in 3 yrs at 6% p.a. C–I paid half yearly?
(a) 16570 (b) 16500 (c) 16750 (d) 16075

33. A machinery is depreciated at 10% p.a. for 3 yrs costing Rs. 50000.Find scrap value?
(a) 36400 (b) 36450 (c) 36500 (d) 36550

34. Find depreciation if machinery worth 12000 is depreciated at 6% p.a. for 4 yrs?
(a) 2631.8 (b) 2613.8 (c) 2361.8 (d) 2316.8

35. A machinery worth 10000 is depreciated at the rate of 10% p.a. for first 3 yrs. 8% p.a. for next 2 yrs. Find its value after 5 yrs.
(a) 5170.25 (b) 7170.25 (c) 6170.25 (d) 8170.25

36. An investment of 20000 sums on interest of 6% p.a. for first 4 yrs 5% p.a. for next 3 yrs and 4% p.a. for next 2 yrs. Find value of investment after 9 yrs where interest is compound annually
(a) 31441.62 (b) 31614.62 (c) 31416.62 (d) 31641.62

37. Difference between C–I and SI at 5% p.a. for 2 yrs on a sum of 6000 is
(a) 10 (b) 12 (c) 15 (d) 18

38. Difference between CI and SI on 10000 at 5% p.a. for 4 yrs is
(a) 150 (b) 155 (c) 160 (d) 165

39. A sum of money at 5% p.a. CI double in
(a) 14 yrs (approx.) (b) 16 yrs (approx.)
(c) 18 yrs (approx.) (d) 19 yrs (approx.)

40. In how many years a sum of money trebles at 5% p.a. CI payable on half yearly basis
(a) 20 yrs 3 months (b) 21 yrs 3 months
(c) 22 yrs 3 months (d) 24 yrs 3 months

41. A sum at a certain rate of interest compounded annually doubles in 5 yrs. In how many yrs will it become 8 times
(a) 10 (b) 15 (c) 18 (d) 20

PAST EXAM QUESTIONS

42. The difference between the simple and compound interest on a certain sum for 3 year at 5% p.a. is ₹ 228.75. The compound interest on the sum for 2 years at 5% p.a. is

(a) ₹ 3,175 (b) ₹ 3,075 (c) ₹ 3,275 (d) ₹ 2,975

43. In what time will ₹ 3,90,625 amount to ₹ 4,56,976 at 8% per annum, when the interest is compounded semi-annually?

(a) 2 years (b) 4 years (c) 5 years (d) 7 years

44. The annual birth and death rates per 1000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is:

(a) 35 years (b) 30 years (c) 25 years (d) none of these

45. A person deposited ₹ 5000 in a bank. The deposit was left to accumulate at 6% compounded quarterly for the first five years and at 8% compounded semi-annually for the next eight years. The compound amount at the end of 13 years is:

(a) ₹ 12621.50 (b) ₹ 12613.10 (c) ₹ 13613.10 (d) none

46. A sum amount to ₹ 1,331 at a principal of ₹ 1,000 at 10% compounded annually. Find the time.

(a) 3.31 years (b) 4 years (c) 3 years (d) 2 years

47. Mr. X invests 'P' amount at simple interest rate 10% and Mr. Y invests 'Q' amount at compound interest rate 5% compounded annually. At the end of two years both get the same amount of interest, then the relation between two amounts P and Q is given by

(a) $P = \frac{41Q}{80}$ (b) $P = \frac{41Q}{40}$

(c) $P = \frac{41Q}{100}$ (d) $P = \frac{41Q}{100}$

48. A sum of money compounded annually becomes ₹ 1,140 in two years and ₹ 1,710 in three years. Find the rate of interest per annum.

(a) 30% (b) 40% (c) 50% (d) 60%

49. A sum of money invested of compound interest doubles itself in four years. It becomes 32 times of itself at the same rate of compound interest in

(a) 12 years (b) 16 years (c) 20 years (d) 24 years

50. A compound interest on a sum for 2 years is ₹ 30 more than the simple interest at the rate of 5% per annum then the sum is

(a) ₹ 11,000 (b) ₹ 13,000 (c) ₹ 12,000 (d) ₹ 15,000

51. If compound interest on any sum at the rate of 5% for two years is ₹ 512.50 then the sum would be:

(a) ₹ 3,000 (b) ₹ 4,000 (c) ₹ 5,000 (d) ₹ 6,000

52. If compound interest on a sum for 2 years at 4% per annum is ₹ 102, then the simple interest on the same sum for the same period at the same rate will be

(a) ₹ 99 (b) ₹ 101 (c) ₹ 100 (d) ₹ 95

EFFECTIVE RATE OF INTEREST

53. Effective annual rate of interest compounding to a nominal rate of 6% p.a payable half yearly is

(a) 6.07 (b) 6.08 (c) 6.09 (d) none

54. Effective rate of interest of 8% p.a. converted monthly is

(a) 8% (b) 8.34% (c) 8.43% (d) 8.30%

55. Which is a better investment?

(i) 9% p.a. compounded half yrly.

(ii) 9.23% p.a. S.I.

(a) (i) (b) (ii) (c) both (d) none

ANNUITY (FUTURE VALUE)

56. The amount of annuity of 6,000 payable at the end of each 3 months for 4 years compounded Quarterly at 8% p.a.

(a) 111836 (b) 110836 (c) 112836 (d) 113836

57. The amount of annuity of Rs. 2000 payable at the end of each year for 5 years at 8% p.a. is

(a) 11733.86 (b) 14502.6 (c) 21005.8 (d) 16721.31

58. A company requires 20,00,000 at the end of 10 yrs to replace one of its assets. It is decided to create a sinking fund by investing a fixed amount every year in securities which gives 10% CI. Yearly investment is

(a) 124590 (b) 125490 (c) 154290 (d) 145290

59. A company issued 10% cumulative debentures of Rs. 100 each, 5000 cumulative debentures are to be redeemed with 10% of interest for 5 yrs. For this a Sinking Fund is created and invested at 12% rate of C.I. Sum to be transferred every year to sinking fund is

(a) 805500 (b) 126834.64 (c) 207382 (d) 126755

60. The value of the amount at the end of 12 years of an annuity of 1200 payable at the beginning of each year for 12 yrs at 8% p.a. C.I is

(a) 26879.32 (b) 3432.11 (c) 24594.35 (d) none

61. A machine costing 5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by new model at 25% higher cost after 25 years with a scrap value realization of 25000. What amount should be set aside every year if sinking fund investment at 3.5% C.I p.a.?

(a) 16000 (b) 16500 (c) 16050 (d) 16005

62. Ratan aged 45 wishes his wife Ratna to have 40 lacs at his death. His expectation of life is another 30 years and he starts making equal annual investments commencing now at 3% p.a., how much should he invest annually?

(a) 84077 (b) 81628 (c) 84450 (d) none

ANNUITY (PRESENT VALUE)

63. A loan of 30,000 at the interest rate of 6% compounded annually is to be amortized by equal payments at the end of each year for 5 years. Find annual payment.

(a) 6121.89 (b) 7121.89 (c) 8121.89 (d) 9121.89

64. Present value of an annuity which pays 200 at the end of each 3 months for 10 years, assuming money to be worth 5% p.a. converted quarterly.
(a) 3809.20 (b) 3109.60 (c) 6265.38 (d) none

65. Pravin buys a house paying Rs. 50,000 in cash and balance in 20 installments of Rs. 8,000 each at the end of each year. If interest is at 16% p.a., how much he should have paid if had purchased it cash down.
(a) 85250 (b) 94730 (c) 97430 (d) 87589

66. P.V. of an annuity of Rs. 80 made at the end of each 6 months forever, if money worth 4% p.a. is compounded semi-annually
(a) 2000 (b) 3000 (c) 4000 (d) 5000

67. A man purchased house valued at 3,00,000 by making a payment of 2,00,000 at the time of purchase and agreed to pay balance with interest at 12% p.a. compounded half yearly in 20 equal half yearly installments. If first installment is paid after 6 months from the date of purchase then amount of each installment is
(a) 8719 (b) 8679 (c) 7719 (d) 8769

68. John wants to create a fund to donate 1800 every month to a deprived family. Rate of unit is 12% p.a. Find amount to be deposited
(a) 360000 (b) 180000 (c) 90000 (d) none

69. A company borrows 10000 on condition to repay it with C.I. at 5% p.a. by annual installments of 1000 each. The number of years by which debt will be cleared is
(a) 14.2 (b) 10 (c) 12 (d) 17

70. A person retires at 60 years receiving a pension of 14,400 a year paid in half yearly installments for the rest of his life with his life expectation to be 13 years and interest at 4% p.a. payable half yearly. What single sum is equivalent to his pension?
(a) 144000 (b) 144900 (c) 144600 (d) 144300

71. If discount rate is 7% p.a., how much would you pay to receive 500, growing at 5% annually forever?
(a) 25000 (b) 250000 (c) 2500 (d) none

PAST YEARS QUESTION

72. A machine can be purchased for ₹50,000. Machine will contribute ₹12,000 per year for the next five years. Assume borrowing cost is 10% per annum. Determine whether machine should be purchased or not:

(a) Should be purchased (b) Should not be purchased
(c) Can't say about purchase (d) None of the above

73. A company considering proposal of purchasing a machine either by, making full payment of ₹4000 or by leasing it for four years at an annual rate of ₹1,250. Which course of action is preferable, if the company can borrow money at 14% compounded annually?

[Given: $(1.14)^4 = 1.68896$]

(a) Leasing is preferable (b) Should be Purchased
(c) No difference (d) None of these

74. Vipul Purchases a car for ₹5,50,000. He gets a loan of ₹5,00,000 at 15% P.a. from a bank and balance ₹50,000 he pays at the time of purchase. He has to pay the whole amount of loan in 12 equal monthly instalments with interest starting from the end of the first month. The money he has to pay at the end of every month is:

[Given $(1.0125)^{12} = 1.16075452$]

(a) ₹ 45,130.43 (b) ₹ 45,230.43 (c) ₹ 45,330.43 (d) ₹ 45,430.43

75. A company establishes a sinking fund to provide for the payment of ₹ 2,00,000 debt maturing in 20 years. Contributions to the fund are to be made at the end of every year. Find the amount of each annual deposit if Interest is 5% per annum.

(a) ₹ 6,142 (b) ₹ 6,049 (c) ₹ 6,052 (d) ₹ 6,159

76. A company may obtain a machine either by leasing it for 5 years (useful life) at an annual rent of ₹2,000 or by purchasing the machine for ₹8,100. If the company can borrow money at 18% per annum, which alternative is preferable?

(a) Leasing (b) Purchasing (c) Can't say (d) None of these

77. A sinking fund is created for redeeming debentures worth ₹5 lacs at the end of 25 years. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a?

(a) ₹ 12,006 (b) ₹ 12,040 (c) ₹ 12,039 (d) ₹ 12,035

78. Find the present value of an annuity of ₹1,000 payable at the end of each year for 10 years. If rate of interest is 6% compounding per annum (Given $(1.06)^{-10} = 0.5584$):

(a) ₹ 7,360 (b) ₹ 8,360 (c) ₹ 12,000 (d) None of these

79. The future value of an annuity of ₹ 5,000 is made annually for 8 years at interest rate of 9% compounded annually. [Given $(1.09)^8 = 1.99256$] is ______

(a) ₹ 55,142.22 (b) ₹ 65,142.22
(c) ₹ 65,532.22 (d) ₹ 57,425.22

80. A person wants to lease out a machine costing ₹ 5,00,000 for a 10 year period. It has fixed a rental of ₹ 51, 272 per annum payable annually starting from the end of first year. Suppose rate of interest is 10% per annum compounded annually on which money can be invested. To whom this agreement is favourable?

(a) Favour of lessee (b) Favour of lessor
(c) Not for both (d) Can't be determined

Typical Sums related to Important Concepts

81. If the nominal rate of growth is 17% and inflation is 9% for five years. Let P be the Gross Domestic Product (GDP) amount at the present year then the projected real GDP after 6 years is

(A) 1.587 P (B) 1.921 P (C) 1.403 P (D) 2.51 P

82. Let the operating profit of a manufacturer for five years is given as:

Year	1	2	3	4	5	6
Operating profit (in lakh *)	90	100	106.4	107.14	120.24	157.35

Then the operating profit of Compound Annual Growth Rate (CAGR) for year 6 with respect to year 2 is given at

(A) 9% (B) 12% (C) 11% (D) 13%

83. If the cost of capital be 12% per annum, then the net present value (in nearest *) from the given cash flow is given as

Year	0	1	2	3
Operating profit (in lakh *)	(100)	60	40	50

(A) 31048 (B) 34185 (C) 21048 (D) 24187

84. Madhu takes a loan of 50,000 from ABC Bank LTD.The rate of interest is 10% per annum. The first instalmennt will be paid at the end of five year. Determine the amount (in) of equal instalments, if Madhu wishes to repay the amount in five years.

(a) ₹ 19,510 (b) ₹ 19,430 (c) ₹ 19,310 (d) ₹ 16,630

HOMEWORK SECTION

1. S.I on ₹ 3,500 for 3 years at 12% per annum is
   (a) ₹ 1,200 (b) ₹ 1,260 (c) ₹ 2,260 (d) none of these

2. P = 5,000, R = 15, T = 4 ½ using I = PRT/100, I will be
   (a) ₹ 3,375 (b) ₹ 3,300 (c) ₹ 3,735 (d) none of these

3. If P = 5,000, T = 1, I = ₹ 300, R will be
   (a) 5% (b) 4% (c) 6% (d) none of these

4. If P = ₹ 4,500, A = ₹ 7,200, than Simple interest i.e. I will be
   (a) ₹ 2,000 (b) ₹ 3,000 (c) ₹ 2,500 (d) ₹ 2,700

5. P = ₹ 12,000, A = ₹ 16,500, T = 2 ½ years. Rate percent per annum simple interest will be
   (a) 15% (b) 12% (c) 10% (d) none of these

6 P = ₹ 10,000, I = ₹ 2,500, R = 12 ½% SI. The number of years T will be
   (a) 1 ½ years (b) 2 years (c) 3 years (d) none of these

7. P = ₹ 8,500, A = ₹ 10,200, R = 12 ½ % SI, t will be.
   (a) 1 yr. 7 mth. (b) 2 yrs. (c) 1 ½ yr. (d) none of these

8. The sum required to earn a monthly interest of ₹ 1,200 at 18% per annum SI is
   (a) ₹ 50,000 (b) ₹ 60,000
   (c) ₹ 80,000 (d) none of these

9. A sum of money amount to ₹ 6,200 in 2 years and ₹ 7,400 in 3 years. The principal and rate of simple interest are
   (a) ₹ 3,800, 31.57% (b) ₹ 3,000, 20%
   (c) ₹ 3,500, 15% (d) none of these

10. A sum of money doubles itself in 10 years at simple interest. The number of years it would triple itself is

(a) 25 years. (b) 15 years. (c) 20 years (d) none of these

11. If P = ₹ 1,000, R = 5% p.a, n = 4; What is Amount and C.I. is
(a) ₹ 1,215.50, ₹ 215.50 (b) ₹ 1,125, ₹ 125
(c) ₹ 2,115, ₹ 115 (d) none of these

12. ₹ 100 will become after 20 years at 5% p.a compound interest amount of
(a) ₹ 250 (b) ₹ 205 (c) ₹ 265.50 (d) none of these

13. The effective rate of interest corresponding to a nominal rate 3% p.a payable half yearly is
(a) 3.2% p.a (b) 3.25% p.a (c) 3.0225% p.a (d) none of these

14. A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was ₹ 1,00,000 and its ultimate scrap value was ₹ 30,000. The effective life of the machine is
(a) 4.5 years (appx.) (b) 5.4 years (appx.)
(c) 5 years (appx.) (d) none of these

15. If A = ₹ 1,000, n = 2 years, R = 6% p.a compound interest payable half-yearly, then principal (P) is
(a) ₹ 888.48 (b) ₹ 885 (c) 800 (d) none of these

16. The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is
(a) 7 years (b) 10 years (c) 17 years (app) (d) none of these

17. The difference between C.I and S.I on a certain sum of money invested for 3 years at 6% p.a is ₹ 110.16. The sum is
(a) ₹ 3,000 (b) ₹ 3,700 (c) ₹ 12,000 (d) ₹ 10,000

18. The useful life of a machine is estimated to be 10 years and cost ₹ 10,000. Rate of depreciationis 10% p.a. The scrap value at the end of its life is
(a) ₹ 3,486.78 (b) ₹ 4,383 (c) ₹ 3,400 (d) none of these

19. The effective rate of interest corresponding a nominal rate of 7% p.a convertible quarterly is

(a) 7% (b) 7.5% (c) 5% (d) 7.18%

20. The C.I on ₹ 16000 for 1 ½ years at 10% p.a payable half -yearly is

(a) ₹ 2,222 (b) ₹ 2,522 (c) ₹ 2,500 (d) none of these

21. The C.I on ₹ 40000 at 10% p.a for 1 year when the interest is payable quarterly is

(a) ₹ 4,000 (b) ₹ 4,100 (c) ₹ 4,152.51 (d) none of these

22. The difference between the S.I and the C.I on ₹ 2,400 for 2 years at 5% p.a is

(a) ₹ 5 (b) ₹ 10 (c) ₹ 16 (d) ₹ 6

23. The annual birth and death rates per 1,000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is

(a) 35 years. (b) 30 years. (c) 25 years (d) none of these

24. The C.I on ₹ 4,000 for 6 months at 12% p.a payable quarterly is

(a) ₹ 243.60 (b) ₹ 240 (c) ₹ 243 (d) none of these

25. The present value of an annuity of ₹ 3000 for 15 years at 4.5% p.a CI is

(a) ₹ 23,809.41 (b) ₹ 32,218.63

(c) ₹ 32,908.41 (d) none of these

26. The amount of an annuity certain of ₹ 150 for 12 years at 3.5% p.a C.I is

(a) ₹ 2,190.28 (b) ₹ 1,290.28 (c) ₹ 2,180.28 (d) none of these

27. A loan of ₹ 10,000 is to be paid back in 30 equal instalments. The amount of each installment to cover the principal and at 4% p.a CI is

(a) ₹ 587.87 (b) ₹ 587 (c) ₹ 578.30 (d) none of these

28. A = ₹ 1,200 n = 12 years i = 0.08, V = ?

Using the formula $V = \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$ value of v will be

(a) ₹ 3,039 (b) ₹ 3,990 (c) ₹ 9930 (d) none of these

29. p = ₹ 100 n = 10, i = 5% find the FV of annuity

(a) ₹ 1,258 (b) ₹ 2,581 (c) ₹ 1,528 (d) none of these

30. If the amount of an annuity after 25 years at 5% p.a C.I is ₹ 50,000 the annuity will be

(a) ₹ 1,406.90 (b) ₹ 1,047.62 (c) ₹ 1,146.90 (d) none of these

31. Given annuity of ₹ 100 amounts to ₹ 3137.12 at 4.5% p.a C. I. The number of years will be

(a) 25 years (appx.) (b) 20 years (appx.)
(c) 22 years (d) none of these

32. A company borrows ₹ 10,000 on condition to repay it with compound interest at 5% p.a by annual installments of ₹ 1000 each. The number of years by which the debt will be clear is

(a) 14.2 years (b) 10 years (c) 12 years (d) none of these

33. Mr. X borrowed ₹ 5,120 at 12 ½ % p.a C.I. At the end of 3 yrs, the money was repaid along with the interest accrued. The amount of interest paid by him is

(a) ₹ 2,100 (b) ₹ 2,170 (c) ₹ 2,000 (d) none of these

34. Mr. Paul borrows ₹ 20,000 on condition to repay it with C.I. at 5% p.a in annual installments of ₹ 2000 each. The number of years for the debt to be paid off is

(a) 10 years (b) 12 years (c) 11 years (d) none of these

35. A person invests ₹ 500 at the end of each year with a bank which pays interest at 10% p. a C.I. annually. The amount standing to his credit one year after he has made his yearly investment for the 12th time is.

(a) ₹ 11,761.36 (b) ₹ 10,000 (c) ₹ 12,000 (d) none of these

36. The present value of annuity of ₹ 5,000 per annum for 12 years at 4% p.a C.I. annually is

(a) ₹ 46,000 (b) ₹ 46,850 (c) ₹ 15,000 (d) none of these

37. A person desires to create a fund to be invested at 10% CI per annum to provide for a prize of ₹ 300 every year. Using V = a/I find V and V will be

(a) ₹ 2,000 (b) ₹ 2,500 (c) ₹ 3,000 (d) none of these

38. A = ₹ 5,200, R = 5% p.a., T = 6 years, P will be

(a) ₹ 2,000 (b) ₹ 3,880 (c) ₹ 3,000 (d) none of these

39. If P = 1,000, n = 4 years., R = 5% p.a then C. I will be

(a) ₹ 215.50 (b) ₹ 210 (c) ₹ 220 (d) none of these

40. The time in which a sum of money will be double at 5% p.a C.I is

(a) ₹ 10 years (b) 12 years (c) 14.2 years (d) none of these

41. If A = ₹ 10,000, n = 18yrs., R = 4% p.a C.I, P will be

(a) ₹ 4,000 (b) ₹ 4,900 (c) ₹ 4,500 (d) none of these

42. The time by which a sum of money would treble it self at 8% p. a C. I is

(a) 14.28 years (b) 14 years (c) 12 years (d) none of these

43. The present value of an annuity of ₹ 80 a years for 20 years at 5% p.a is

(a) ₹ 997 (appx.) (b) ₹ 900 (c) ₹ 1,000 (d) none of these

44. A person bought a house paying ₹ 20,000 cash down and ₹ 4,000 at the end of each year for 25 yrs. at 5% p.a. C.I. The cash down price is

(a) ₹ 75,000 (b) ₹ 76,000 (c) ₹ 76,392 (d) none of these.

45. A man purchased a house valued at ₹ 3,00,000. He paid ₹ 2,00,000 at the time of purchase and agreed to pay the balance with interest at 12% per annum compounded half yearly in 20 equal half yearly instalments. If the first instalment is paid after six months from the date of purchase then the amount of each instalment is [Given log 10.6 = 1.0253 and log 31.19 = 1.494]

(a) ₹ 8,719.66 (b) ₹ 8,769.21 (c) ₹ 7,893.13 (d) none of these.

46. The difference between compound and simple interest at 5% per annum for 4 years on ₹ 20,000 is ₹ __________

(a) 250 (b) 277 (c) 300 (d) 310

47. The compound interest on half-yearly rests on ₹ 10,000 the rate for the first and second years being 6% and for the third year 9% p.a. is ₹.______________.

(a) 2,200 (b) 2,287 (c) 2,285 (d) 2290

48. The present value of ₹ 10,000 due in 2 years at 5% p.a. compound interest when the interest is paid on yearly basis is ₹ _________.

(a) 9,070 (b) 9,000 (c) 9,061 (d) None

49. The present value of ₹ 10,000 due in 2 years at 5% p.a. compound interest when the interest is paid on half-yearly basis is ₹ _________.

(a) 9,070 (b) 9,069 (c) 9,059.50 (d) None

50. Johnson left ₹ 1,00,000 with the direction that it should be divided in such a way that his minor sons Tom, Dick and Harry aged 9, 12 and 15 years should each receive equally after attaining the age 25 years. The rate of simple interest being 3.5%, how much each son receive after getting 25 years old?

(a) 50,000 (b) 51,994 (c) 52,000 (d) 48332

51. In how many years will a sum of money double at 5% p.a. compound interest?

(a) 15 years 3 months (b) 14 years 2 months
(c) 14 years 3 months (d) 15 years 2 months

52. In how many years a sum of money trebles at 5% p.a. compound interest payable on halfyearly basis?

(a) 18 years 7 months (b) 18 years 6 months
(c) 18 years 8 months (d) 22 years 3 months

53. A machine depreciates at 10% of its value at the beginning of a year. The cost and scrap value realized at the time of sale being ₹ 23,240 and ₹ 9,000 respectively. For how many years the machine was put to use?

(a) 7 years (b) 8 years (c) 9 years (d) 10 years

54. A machine worth ₹ 4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹ 2,00,000?

(a) 4 years 6 months (b) 4 years 7 months
(c) 4 years 5 months (d) 5 years 7 months approximately

55. A machine worth ₹ 4,90,740 is depreciated at 15% of its opening value each year. When its value would reduce by 90%?

(a) 11 years 6 months
(b) 11 years 7 months
(c) 11 years 8 months
(d) 14 years 2 months approximately

HOMEWORK SOLUTION

1. B

$I = \frac{3500 \times 3 \times 12}{100}$

I = 1260

2. A

$I = \frac{5000 \text{ x } 15 \text{ x } 4.5}{100} = 3375$

3. C

$300 = \frac{5000 \text{ x } 1 \text{ x } r}{100} = 6\%$

4. D

I = A –P = 7200 – 4500

= 2700

5. A

I = A – P = 16,500 -12,000

= 4,500

$45{,}000 = \frac{12{,}000 \text{ x R x } 2.5}{100} = 15\%$

6. B

$2500 \ \frac{10{,}000 \times 12.5 \times N}{100} = $ 2 years

7. A

I = 10,200 – 8,500 = 1700

$1700 = \frac{85{,}00 \times 12.5 \times N}{100}$

=1.6 years

= (1.6 x 12) = 19.2 month

=1 year 7 months

8. C

$1200 = \frac{P(1)(18)}{100 \times 12} = 80{,}000$

9. A

After	3 years	=	7400
	2 years	=	6200
	1 year	=	1200

After 3 years	7400
(1200 x 3)	3600
Principle	3800

$12{,}00 = \frac{3800 \text{ x } 1 \text{ x R}}{100}$

R = 31.57%

10. C

$R = \frac{(2-1)x\ 100}{10} = 10\%$

$N = \frac{(3-1)x\ 100}{10} = 20$ years

11. (a) P = 100, R = 5%, N = 4

$$A = P\left(1+\frac{R}{100}\right)^N$$

$$= 1000\left(1+\frac{5}{100}\right)^4$$

A = 1215.50

$\therefore$ CI = A – P

= 1215.50 – 1000

$\therefore$ CI = 215.50

12. (c) P = 100, R = 5%, r = 20 years

$$A = P\left(1+\frac{R}{100}\right)^N$$

$$= 100\left(1+\frac{5}{100}\right)^{20}$$

A = 265.50

13. (c) $E = [(1 + i)^n – 1] \times 100$

$$= \left[\left(1+\frac{3}{200}\right)^2 - 1\right] \times 100$$

E = 3.0225%

14. (b) $A = P\left(1+\frac{R}{100}\right)^N$

$$\therefore\ 30{,}000 = 1{,}00{,}000\left(1-\frac{20}{100}\right)^N$$

$\therefore$ $0.3 = (0.8)^N$

$\therefore$ N = 5.4 years approx.

15. (a) $A = P\left(1+\frac{R}{100}\right)^N$

$$\therefore\ 1000 = P\left(1+\frac{6}{200}\right)^{2\times 2}$$

$\therefore$ P = 888.48

16. (c) $A = P\left(1+\frac{R}{100}\right)^N$

$$\therefore\ 140 = 100\left(1+\frac{2}{100}\right)^N$$

$\therefore$ $1.40 = (1.02)^N$

$\therefore$ N = 17 years approx

17. (d) $D = Pi^2(i + 3)$

$\therefore$ $110.16 = P(0.06)^2\ (0.06 + 3)$

$\therefore$ 110.16 = P(0.011016)

$\therefore$ P = 9999.999 $\approx$ 10,000.

18. (a) $A = P\left(1-\frac{R}{100}\right)^N$

$$\therefore\ A = 10{,}.000\left(1-\frac{10}{100}\right)^{10}$$

$\therefore$ A = 3486.78

19. (d) $E = [(1 + i)^N – 1] \times 100$

$$= \left[\left(1+\frac{7}{400}\right)^4 - 1\right] \times 100$$

E = 7.18%

20. (b) $CI = P\left[\left(1+\frac{R}{100}\right)^N - 1\right]$

$$= 16{,}000\left[\left(1+\frac{10}{200}\right)^{1.5\times 2} - 1\right]$$

= 16,000 [0.157623]

CI = 2522.

21. (c) $CI = P\left[\left(1+\frac{R}{100}\right)^{N} - 1\right]$

$= 40{,}000\left[\left(1+\frac{10}{400}\right)^{1\times 4} - 1\right]$

$= 40{,}000\ [0.1038]$

$CI = 4152.51$

22. (d) $D = Pi^{2}$

$D = 2400\ (0.05)^{2}$

$D = 6$

23. (a) Rate of growth $= \frac{39.4 - 19.4}{1000} \times 100$

$= 2\ \%$

$A = P\left(1+\frac{R}{100}\right)^{N}$

$2 = 1\left(1+\frac{2}{100}\right)^{N}$

$\therefore$ N = 35 yrs. Approx.

OR

Approximate

No. of years $= \frac{72}{\text{Rate}} = 36 \approx 35$ years.

24. (a)

$CI = P\left[\left(1+\frac{R}{100}\right)^{N} - i\right]$

$= 4000\left[\left(1+\frac{12}{400}\right)^{\frac{1}{2}\times 4} - 1\right]$

$CI = 243.6$

25. (b) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^{n}}\right]$

$= \frac{3000}{\frac{4.5}{100}}\left[1 - \frac{1}{\left(1+\frac{4.5}{100}\right)^{15}}\right]$

$= \frac{3000}{0.045}\left[1 - \frac{1}{(1.045)^{15}}\right]$

$\therefore$ P = 32,218.63

26. (a) $A = \frac{a}{i}\left[(1+i)^{n} - 1\right]$

$= \frac{150}{\frac{3.5}{100}}\left[\left(1+\frac{3.5}{100}\right)^{12} - 1\right]$

$= \frac{150}{0.035}\left[(1.035)^{12} - 1\right]$

$A = 2190.28$

27. (c) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$\therefore 10{,}000 = \frac{9}{0.04}\left[1 - \frac{1}{(1.04)^{30}}\right]$

$\therefore 10{,}000 = 9(17.2920)$

$\therefore a = 578.87$

28. (d) Value of $V = \frac{A}{i}\left[1 - \frac{1}{(1+i)^N}\right]$

Hence, A = 1200, n = 12 years, i = 0.08

$V = \frac{1200}{0.08}\left[1 - \frac{1}{(1.08)^{12}}\right]$

V = 9043.29

29. (a) $FV = \frac{a}{i}[(1+i)^n - 1]$

$= \frac{100}{0.05}[(1+0.05)^{10} - 1]$

FV = 1258

30. (b) $A = \frac{a}{i}[(1+i)^n - 1]$

$50{,}000 = \frac{a}{0.05}[(1+0.05)^{25} - 1]$

$\therefore 50{,}000 = \frac{a}{0.05}[2.3864]$

$\therefore a = 1047.62$

31. (b) $A = \frac{a}{i}[(1+i)^n - 1]$

$\therefore 3137.12 = \frac{100}{0.045}[(1+0.045)^n - 1]$

$\therefore 1.4117 = (1.045)^n - 1$

$\therefore n = 20$ years.

32. (a) $P = \frac{a}{i}\left[i - \frac{1}{(1+i)^n}\right]$

$\therefore 10{,}000 = \frac{1000}{0.05}\left[1 - \frac{1}{(1+0.05)^n}\right]$

$\therefore 0.5 = 1 - \frac{1}{(1.05)^n}$

$\therefore \frac{1}{(1.05)^n} = 0.5$

$\therefore (1.05)^n = 2$

$\therefore N = 14.2$ years

33. (b) $CI = P\left[\left(1 + \frac{R}{100}\right)^N - 1\right]$

$= 5120\left[\left(1 + \frac{12.5}{100}\right)^3 - 1\right]$

$\therefore CI = 2170$

34. (d) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$20,000 = \frac{2000}{0.05}\left[1 - \frac{1}{(1+0.05)^n}\right]$

$\therefore 0.5 = 1 - \frac{1}{(1.05)^n}$

$\therefore \frac{1}{(1.05)^n} = 0.5$

$\therefore (1.05)^n = 2$

$\therefore$ n = 14.2 years

35. (a) $A = \frac{a}{i}[(1+i)^n - 1](1+i)$

$= \frac{500}{0.10}[1+0.10)^{12} - 1](1+0.10)$

A = 11,761.36

36. (d) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$= \frac{5000}{0.04}\left[1 - \frac{1}{(1+0.04)^{12}}\right]$

P = 46,925.37

37. (c) $P = \frac{a}{i}$

$= \frac{300}{0.10}$

P = 3000

38. (b) $A = P\left(1+\frac{R}{100}\right)^{N}$

$\therefore 5200 = P\left(1+\frac{5}{100}\right)^{6}$

$\therefore$ P = 3880.

39. (a) $CI = P\left[\left(1+\frac{R}{100}\right)^{N} - 1\right]$

$CI = 1000\left[\left(1+\frac{5}{100}\right)^{4} - 1\right]$

CI = 215.50

40. (c) $A = P\left(1+\frac{R}{100}\right)^{N}$

$\therefore 2 = 1\left(1+\frac{5}{100}\right)^{N}$

$\therefore (1.05)^{N} = 2$

$\therefore$ N = 14.2 year approved.

41. (d) $A = P\left(1+\frac{R}{100}\right)^{N}$

$\therefore 10{,}000 = P\left(1+\frac{4}{100}\right)^{18}$

$\therefore$ P = 4936.28

42. Year = $\frac{114}{\text{Rate}}$

$= \frac{114}{8}$

Year = 14.25

$\approx$ 14.28 year.

43. (a) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$= \frac{80}{0.05}\left[1 - \frac{1}{(1+0.05)^{20}}\right]$

P = 997.

44. (c) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$= \frac{4000}{0.05}\left[1 - \frac{1}{(1+0.05)^{25}}\right]$

P = 56375

Cash price = cash down + P

= 20,000 + 56,375

= 76,375

$\approx$ 76392

45. (a) P = 3,00,000 – 2,00,000

P = 1,00,000

$$P = \frac{a}{i}\left[1-\frac{1}{(1+i)^n}\right]$$

$$1,00,000 = \frac{a}{\frac{12}{200}}\left[1-\frac{1}{\left(1+\frac{12}{200}\right)^{20}}\right]$$

$$1,00,000 = \frac{a}{0.06}\left[1-\frac{1}{(1.06)^{20}}\right]$$

$\therefore$ a = 8719.66

46. (d) S.I. = $\frac{20,000 \times 5\% \times 4}{100} = 4000$

C.I. = 20,000 $\{(1.05)^4-1\}$ = 4310.125

D = 310.125

47. (d) A = 10,000 $(1.03)^4\,(1.045)^2$

= 12,290

$\therefore$ C.I. = 2290

48. (d) A = $p(1+i)^N$

10,000 = P $(1.05)^2$

= 9070

49. (c) A = $P(1+i)^N$

10,000 = $P(1.025)^4$

= 9059.50

50. (d) Assuming simple interest

Let the amount = x

$$\frac{x}{\left(1+\frac{3.5\times16}{100}\right)}+\frac{x}{\left(1+\frac{3.5\times13}{100}\right)}+\frac{x}{\left(1+\frac{3.5\times10}{100}\right)}=1L$$

$$\frac{x}{1.56}+\frac{x}{1.455}+\frac{x}{1.35}=1L$$

x(0.6410 + 0.6873 + 0.7407) = 1 L

x(2.0690) = 1 L

$x = \frac{100000}{2.0690}$ x = 48332

51. (b) $2 = 1\ (1.05)^N$

For perfect answer taking log

log2 = N log 1.05

0.3010 = N(0.0212)

N = 14.2

52. (c) $A = P\ (1+i)^N$

$3 = 1\ (1.025)^N$

Trial and Error

(c) 22 years 23 month

53. (c) $A = P(1+i)^N$

$9000 = 23240\ (0.9)^N$

$0.3873 = (0.9)^N$

N = 9 years [Trial and Error]

54. (d) $A = P(1 - i)^N$

$2,00,000 = 490,740\ (0.85)^N$

$0.4075 = (0.85)^N$

5 years 7 months [Trial and Error]

55. (d) $49074 = 490740\,(0.85)^N$

$0.1 = (0.85)^N$

N = 14 years 2 month

[Trial and Error]

SELF ASSESSMENT TEST 6
ANNUITIES
25 Question, 25 Marks

1. Find the amount of an annuity of Rs. 100 in 10 years allowing compound interest at 5%, given $(1.05)^{10} = 1.6289$.
   a) Rs. 1527.80 b) Rs. 1257.80
   c) Rs. 1357.90 d) None of the above

2. Find the present value of an annuity certain of Rs. 300 for 5 years reckoning compound interest at 4% per annum. Given $(1.04)^5 = 1.2167$
   a) Rs. 1335.78 b) Rs. 1353.87
   c) Rs. 1533.98 d) None of the above

3. Gargi borrows Rs. 20000 at 4% compound interest and agrees to pay both the principal and the interest in 10 equal annual instalments at the end of each year. Find the amount of these in-stalments. Given $(1.04)^{10} = 1.4802$.
   a) Rs. 2470 b) Rs. 2570 c) Rs. 2740 d) Rs. 2760

4. Find the amount of an annuity of Rs. 50 payable quarterly for 15 years at 5%, compounded quarterly. Given $(1.0125)^{60} = 2.1028$
   a) Rs. 4422 b) Rs. 4411 c) Rs. 5544 d) Rs. 4433

5. Find the present value of an annuity of Rs. 50 payable quarterly for 15 years at 5%, compound-ed quarterly. Given $(1.0125)^{60} = 2.1028$
   a) Rs. 2000 b) Rs. 2098 c) Rs. 2108 d) Rs. 2100

6. Find the value of perpetuity of Rs. 105 a year at 5.25% per annum.
   a) Rs. 2000 b) Rs. 2098 c) Rs. 2108 d) Rs. 2100

7. A wagon is purchased on instalment basis, such that Rs. 5000 is to be paid on the signing of the contract and four yearly instalment of Rs. 3000 each payable at the end of the first, second, third and fourth years. If interest is charged at 5% p.a., what would be the cash down price? Given $(1.05)^{-4} = 0.82270$
   a) Rs. 16358 b) Rs. 15638 c) Rs. 10638 d) Rs. 14500

8. A government constructed housing flat costs Rs. 136000; 40% is to be paid at the time of possession and the balance, reckoning compound interest of 9% p.a. is to be paid in 12 equal annual instalments. Find the amount of each such instalment. Given $(1.09)^{-12} = 0.3558$

   a) Rs. 11000 b) Rs. 12000 c) Rs. 11400 d) Rs. 12400

9. What sum will buy an annuity of Rs. 1000 payable half-yearly for 5 years, the rate of interest being 8% p.a. compounded half-yearly? Given $(1.04)^{-10} = 0.6756$

   a) Rs. 8000 b) Rs. 9000 c) Rs. 9110 d) Rs. 8110

10. Dipti borrowed Rs. 40000 at 6% compound interest promising to repay Rs. 9000 at the end of each of the first four years and to pay the balance at the end of the fifth year. Ascertain how much she would pay as the final instalment. Given $(1.06)^{-4} = 0.79206$

    a) Rs. 8809 b) Rs. 11789 c) Rs. 12897 d) Rs. 10589

11. The present vale and amount of an annuity certain of Rs. 180 at a fixed rate per cent p.a. com-pound are Rs. 2000 and Rs. 3000 respectively. Find the rate of interest.

    a) 4% b) 3.5% c) 5% d) 3%

12. A man retires at the age of 60 years and his employer gives him a pension of Rs. 3600 a year paid in half-yearly instalments for the rest of his life. If the expectation of his life is taken to be 10 years and interest is 6% per annum payable half-yearly, determine the present value of the pension. Given $(1.03)^{-20} = 0.55362$

    a) Rs. 26,783 b) Rs. 28,768 c) Rs. 26,893 d) Rs. 28,763

13. A freehold estate was worth Rs. 50,000. If the annual rent of the property be Rs. 2,000, find the rate per cent p.a.

    a) 3% b) 3.5% c) 4% d) 5%

14. A machine costs a company Rs. 52000 and its effective life is estimated to be 25 years. A sinking fund is created for replacing the machine by a new model at the end of its life-time, when its scrap realizes a sum of Rs. 2500 only. The price for the new model is estimated to be 25% higher than the price of the present one. Find what amount should be set aside every year out of profits for the sinking fund, if it accumulates at 3.5% p.a. compound. Given $(1.035)^{25} = 2.3659$

    a) Rs. 1590 b) Rs. 1650 c) Rs. 1602 d) Rs. 1592

15. A man decides to deposit Rs. 10000 at the end of each year in a bank which pays 10% per annum compound interest. If the instalments are allowed to accumulate, what will be total accumulation at the end of 9 years? Given $(1.1)^9 = 2.2583$

a) Rs. 124000 b) Rs. 125000 c) Rs. 125830 d) Rs. 124930

16. The annual rent of a perpetual annuity is Rs. 4000. Find its value, the interest being compounded at 8% p.a.

a) Rs. 40000 b) Rs. 45000 c) Rs. 50000 d) None of the above

17. The value and annual rent of perpetuity are Rs. 12500 and Rs. 1000 respectively. Find the rate of compound interest.

a) 7% b) 8% c) 10% d) None of the above

18. Find the value of a deferred perpetuity of Rs. 500 p.a. to commence 10 years hence at 6% p.a. compound interest. Given $(1.06)^{10} = 1.791$

a) Rs. 4653 b) Rs. 4563 c) Rs. 4356 d) Rs. 4365

19. A loan of Rs. 10,000 is to be repaid in 30 equal annual instalments of Rs. X. Find X, if the CI charged is at the rate of 4% p.a. (Annuity is an annuity immediate). Given $(1.04)^{30} = 3.2434$

a) Rs. 878.80 b) Rs. 758.40 c) Rs. 578.40 d) Rs. 598.80

20. A man buys a house for Rs. 60,000 on condition that he will pay Rs. 30000 cash down and the balance in 10 equal annual instalments, the first to be paid one year after the date of purchase. Calculate the amount of each instalment, compound interest being computed at the rate of 5% p.a. Given $(1.05)^{-10} = 0.6139$

a) Rs. 5883 b) Rs. 3885 c) Rs. 8583 d) Rs. 3588

21. The annual subscription for the membership of a club is Rs. 25 and a person may become a life-member by paying Rs. 1000 in a lump sum. Find the rate per cent per annum.

a) 2% b) 3% c) 2.5% d) 3.5%

22. An overdraft of Rs. 50,000 is to be paid back in equal instalments over a period of 20 years. Find the value of the instalment, if interest is compounded annually at 14% per annum. Given $(1.14)^{20} = 13.74349$

a) Rs. 7549 b) Rs. 7561 c) Rs. 7571 d) Rs. 7539

23. What is the value of an annuity at the end of 5 years, if Rs. 100 per month is deposited into an account earning interest 9% per year compounded monthly? Given log10075 = 4.003245055 and antilog 4.1947033 = 15656.81067

a) Rs. 7542 b) Rs. 7892 c) Rs. 7598 d) Rs. 7498

24. A man borrows Rs. 20,000 at interest rate 4% per annum compounded annually and agrees to pay both the principal and the interest in 10 equal annual instalments at the end of each year. Find the amount of these instalments. Given log104 = 2.0170 and log6761 = 3.8300

a) Rs. 2400 b) Rs. 2470 c) Rs. 2489 d) Rs. 2459

25. Rs. 12,000 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the amount of this annuity after 10th payment? Given $(1.005)^{10} = 1.0511$

a) Rs. 122000 b) Rs. 120680 c) Rs. 122980 d) Rs. 122640

EXPLANATORY ANSWERS

1. $A = 100/0.05[(1.05)^{10} - 1] = 2000(1.6289 - 1) = 1257.80$, Option B

2. $PV = 300/0.04[1-(1.04)^{-5}] = 7500[1 - 1/1.2167] = 1335.78$. Option A

3. $20000 = P/0.04[1 - (1.04)^{-10}]$

   $800 = P[1 - 1/1.4802]$

   $P = (800 * 1.4802)/0.4802 = 2470$

   Option A

4. $\text{Amount} = 50/0.0125[(1.0125)^{60} - 1] = 4000(1.1028) = 4411$. Option B

5. $PV = 50/0.0125[1 - (1.0125)^{-60}] = 4000(1.1028/2.1028) = 2098$. Option B

6. Perpetuity $PV = 105/5.25\% = 2000$. Option A

7. PV of instalment $= 3000/0.05[1 - 0.82270] = 10638$

   Cash down price $= 5000 + 10638 = 15638$. Option B

8. Loan amount $= 136000 * .60 = 81600$

   $81600 = P/0.09\ [1 - 0.3558]$

   $7344 = P(0.6442)$

   $P = 11400$. Option C

9. $PV = 1000/0.04[1 - 0.6756] = 25000(0.3244) = 8110$. Option D

10. PV of 4 instalments $= 9000/0.06[1 - 0.79206] = 31191$

    PV of loan amount remaining to be paid $= 40000 - 31191 = 8809$

    FV of Rs. 8809 payable after 5 years $= 8809(1.06)5 = 11789$

    Option B

11. $2000 = 180/r[1 - (1+r)^{-n}] = 180/r[(1+r)^n - 1]/(1+r)^n = 3000/(1+r)^n$

$3000 = 180/r[(1+r)^n - 1]$

$3/2 = (1+r)^n$

$3000 = 180/r[3/2 - 1]$

$r = 180/3000[1/2] = 0.03$

Interest rate = 3%

Option D

12. $PV = 1800/0.03[1 - (1.03)^{-20}] = 60000[1 - 0.55362] = 26783$. Option A

13. $R/100 = 2000/50000 = 0.04$. R = 4%. Option C

14. Amount required = 52000 * 1.25 − 2500 = 62500

$62500 = A/.035[(1.035)^{25} - 1] = A(39.0257)$

$A = 62500/39.0257 = 1602$. Option C

15. $FV = 10000/0.10 [(1.1)^9 - 1] = 100000[2.2583 - 1] = 125830$. Option C

16. Value = 4000/0.08 = 50000. Option C

17. $R/100 = 1000/12500 = 0.08$. R = 8%. Option B

18. Value = 500/0.06 = 8333

PV of Value $= 8333/(1.06)^{10} = 8333/1.791 = 4653$. Option A

19. $10000 = X/0.04 [1 - 1/3.2434] = X(17.292)$

$X = 10000/17.292 = 578.40$. Option C

20. $30000 = X/0.05[1 - 0.6139] = X(7.722)$

$X = 30000/7.722 = 3885$. Option B

21. $R/100 = 25/1000 = 0.025$. R = 2.5%. Option C

22. $50000 = X/0.14[1 - 1/13.74349] = X(6.6231)$

$X = 50000/6.6231 = 7549$. Option A

23. Amount = $100/0.0075[(1.0075)^{60} - 1]$

$X = (1.0075)^{60}$

LogX = 60 log(1.0075) = 60[0.003245055] = 0.1947033 = Log 1.565681067

X = 1.565681067

Amount = 13333.33(1.565681067 − 1) = 7542. Option A

24. $20000 = P/0.04[1 - (1.04)^{-10}]$

Let X = (1.04)10

Log X = 10 log(1.04) = 10(0.0170) = 0.170 = 4 − 3.83 = log 10000 − log 6761 = log 1.4791

X = 1.4791

20000 = P/0.04[1 − 1/1.4791] = P(8.098)

P = 20000/8.098 = 2470

Option B

25. Amount = $12000/0.005[(1.005)^{10} - 1]$ = 2400000(0.0511) = 122640. Option D

SELF ASSESSMENT TEST 7
INTEREST

24 Question, 24 Marks

1. Mahesh lent Rs. 100000, partly at 12% and partly at 10% simple interest. After three years he got Rs. 31500 as total simple interest. How much did he lend at the 12%?

   a) 25000 b) 75000 c) 50000 d) 40000

2. At what rate will a sum of money Rs. 205000, becomes Rs. 410000 with simple interest in 20 years?

   a) 5% b) 6.67% c) 5.5% d) 7.14%

3. A sum of money amounts to Rs. 44700 in 14 years at 8.33% simple interest. When will it double itself at the same rate?

   a) 16 years b) 12 years c) 11 years d) 17 years

4. A sum of Rs. 57000 is lent out in two parts A and B in such a way that interest on A @ 8% per annum simple interest for 5 years is equal to that on B at 0.5% per annum sim-ple interest for 15 years. Find the value of A.

   a) 9000 b) 6000 c) 48000 d) 45000

5. In what time will Rs. 4,500 amount to 22,500 at 4% per annum simple interest?

   a) 50 b) 60 c) 110 d) 100

6. Mr. M takes a loan of Rs. 525000 at 4% p.a. Compound Interest from Mr. J. He pays Mr. J Rs. 250000 at the end of 1st year. How much should he pay at the end of the 2nd year in order to clear his dues?

   a) 300000 b) 300800 c) 307840 d) 370480

7. A person bought a robot under the following scheme: Down payment of Rs. 150,000 and the rest amount at 8% per annum for 2 years. In this way, he paid Rs. 289,200 in total. Find the actual price of the robot. (Assume simple interest).

   a) 270000 b) 300000 c) 280000 d) 275000

8. Manohar Bhaiya borrows Rs. 140,000 at simple interest from the village co-operative society bank. At the end of 3 years, he again borrows Rs. 60,000 and closes his account after paying Rs. 92,300 as interest after 8 years from the time he made the first bor-rowing. What is the simple interest rate of interest charged by the bank?

a) 6.5% b) 5.5% c) 5% d) 7%

9. At R% per annum simple interest, simple interest on Rs. X for 20 years is equal to 4/9 th of its amount for that period. Find R.

a) 4% b) 5% c) 4.5% d) 6.5%

10. Rs. X lent out at compound interest amounts to Rs. 484000 in 2 years at 10% p.a. Find X.

a) 400000 b) 410000 c) 390000 d) 380000

11. A sum of money doubles itself in 5 years @ R% per annum compounded annually. In how many years will it become four times at the same compound rate on interest?

a) 10 b) 8 c) 12 d) 17

12. Rs. X doubles itself and becomes Rs. 2X at a certain rate of compound interest in 3 years. In how many years will the ratio of the principal to the compound interest be 1:3?

a) 6 b) 5 c) 8 d) 7.5

13. Rs. 1,25,000 placed at compound interest becomes Rs. 250000 in 3 years @ R% per annum compound interest? In how many years will it amount becomes 1000000?

a) 9 years b) 10 years c) 11 years d) 7 years

14. Rs. M grows up to Rs. 80000 in 2 yrs. and up to Rs. 85000 in 3 yrs @ R% per annum compound interest. Find the value of R.

a) 6.25% b) 6% c) 6.75% d) 5.5%

15. An amount of money grows upto Rs. 86400 in 2 years and upto Rs. 103680 in 3 years on compound interest. What is the sum?

a) Rs. 51000 b) Rs. 61000 c) Rs. 58000 d) Rs. 60000

16. The difference between Compound interest and simple interest on a certain sum of money is Rs. 400 for first two years and Rs. 1220 for first three years. Find the sum, if the rate is same in both the cases.

a) 6400 b) 8000 c) 12000 d) 9400

17. The Compound Interest @ R% per annum on a certain sum of money Rs. X for 2 yrs. is Rs. 357 and the Simple Interest on the same sum of money Rs. X at the same rate of in-terest is Rs. 350. What is the value of R?

a) 4% b) 5% c) 4.5% d) 5.5%

18. The value of a residential flat constructed at a cost of Rs. 15,00,000 is depreciating at the rate of 9% per annum. Find its value after two years of construction.

a) 12,42,150 b) 12,50,000 c) 12,75,850 d) 12,95,540

19. A money lender borrows a sum from market at 3% per annum simple interest and lent it out to another person at 6% per annum compounded half-yearly. If after one year, he gets a profit of Rs. 618, then find out the sum borrowed by the money lender.

a) Rs. 25,000 b) Rs. 20,000
c) Rs. 18,000 d) Can't be determined

20. The simple interest on a certain sum of money for 2 years at 12% per annum is Rs. 3120. What would be the compound interest at the same rate and for the same time?

a) Rs. 3507.80 b) Rs. 3407.20
c) Rs. 3207.20 d) Rs. 3307.20

21. What would be the compound interest on Rs. 10000 for three years, if the rate of inter-est is 5% for the first year, 6% for the second year and 7% for the third year?

a) Rs. 1900.10 b) Rs. 1919.10
c) Rs. 1909.10 d) Rs. 1809.10

22. Rs. 4000 becomes Rs. 5000 in 4 years at a certain rate of compound interest. What will be the sum at the end of 12 years?

a) Rs. 7812.50 b) Rs. 7612.50
c) Rs. 7712.50 d) Rs. 7512.50

23. Simple interest on a sum for 3 years at any rate of interest is Rs. 225 while compound interest on the same sum at the same rate for 2 years is Rs. 153. Find the sum and rate percent.

a) Rs. 1875, 4% b) Rs. 1875, 5%

c) Rs. 1785, 4% d) Rs. 1785, 5%

24. A man borrows Rs. 6000 at 10% compound rate of interest. At the end of each year he pays back Rs. 2000. How much amount should be pay at the end of third year to clear all his dues?

a) Rs. 3636 b) Rs. 3663 c) Rs. 3366 d) None of the above

EXPLANATORY ANSWERS

1. X * 0.12 * 3 + (100000 – X) * 0.10 * 3 = 31500
   0.36X + 30000 – 0.30X = 31500
   0.06X = 1500
   X = 25000; Option A

2. 410000/205000 = 2; Doubled; Rate = 100/20 = 5%. Option A

3. Time = 100/8.33 = 12 years. Option B

4. A * 0.08 * 5 = (57000 – A) * 0.005 * 15
   0.4A = 4275 – 0.075A
   A = 4275 / 0.475 = 9000; Option A

5. 22500 – 4500 = 18000 = 4500 * 0.04 * T
   T = 18000/180 = 100 years; Option D

6. 525000 * 1.04 = 546000 – 250000 = 296000 x 1.04 = 307840. Option C

7. 150000 + X(1.16) = 289200
   X = 120000; Actual price = 150000 + 120000 = 270000; Option A

8. 140000 * R/100 * 3 + 200000 * R/100 * 5 = 92300
   4200R + 10000R = 92300
   R = 6.5%; Option A

9. X * R/100 * 20 = 4X (1 + 20R/100)/9
   9R/5 = 4 + 4R/5
   R = 4, Option A

10. 484000 = X $(1.1)^2$
    X = 484000/1.21 = 400000; Option A

11. Doubles = 5 years, Four times = 5 * 2 = 10 years. Option A

12. Double 3 years
    1:3 means Sum now becomes 4 times.
    Four times 3*2 = 6 years; Option A

13. Double 3 years
    Eight times = 3*3 = 9 years; Option A

14. 5000 = 80000 * R/100
    R = 5000/800 = 6.25%; Option A

15. (103680 – 86400) = 17280 = Interest on 86400 for 1 years
    R = 17280 / 86400 * 100 = 20%
    P = 86400 / (1.2)2 = 60000
    Option D

16. PII = 400
    PII (I/100 + 3) = 1220
    I/100 = 0.05
    I = 5%
    P (25/100) (25/100) =400
    P = 400 * 4 * 4 = 6400
    Option A

17. X * R/100 * R/100 = 7
    2X * R/100 = 350
    XR/100 = 175
    175 * R/100 = 7
    R = 4
    Option A

18. Value = 1500000 $(1 - 0.09)^2$ = 1242150.
    Option A

19. 3% pa CI is equivalent to $(1.03)^2 - 1 = 6.09\%$ pa SI

Profit = 3.09% = 618

100% = Sum borrowed = 618/3.09% = 20000. Option B

20. P = 3120 / 24% = 13000

CI = $13000 [(1.12)^2 - 1] = 13000 * 0.2544 = 3307.20$.

Option D

21. CI = 10000 [(1.05)(1.06)(1.07) − 1] = 10000 * 0.19091 = 1909.10. Option C

22. In 4 years amount increases by 1000/4000 * 100 = 25%

In next 4 years 5000 shall become 5000 * 1.25 = 6250

In next 4 years 6250 shall become 6250 * 1.25 = 7812.50

Option A

23. Simple interest for 1 year = 225/3 = 75

Thus CI for 2 years = 75 + 75 + 75*R% = 153

75 * R/100 = 3

R = 300/75 = 4%

P = 225/12% = 1875; Option A

24. Year 1 − 6000 * 1.10 = 6600 − 2000 = 4600

Year 2 − 4600 * 1.1 = 5060 − 2000 = 3060

Year 3 − 3060 * 1.1 = 3366

Option C

5 PERMUTATION AND COMBINATION

THEORY

Permutation

- Permutation is defined as the arrangement of things by taking some or all at a time
- Permutation is order dependent
- Fundamental principle of counting;

  If one operation can be performed in 'm' ways and another operation can be performed in 'n' ways, then the total number of ways in which both the operation can be performed will be given by 'm n' ways
- Definition of Factorial 'n', i.e., n! or $\lfloor n$

  Factorial n (n!) is defined as the continued product of first n natural numbers or first n positivel integers and is expressed as n! = 1 x 2 x 3 x 4 ... . x n

- $\lfloor n = n \times \lfloor n-1 = n \times (n-1) \lfloor n-2 = \ldots\ldots$
-

1!	= 1		6!	=	720
2!	= 2		7!	=	5040
3!	= 6		8!	=	40320
4!	= 24		9!	=	362880
5!	= 120		10!	=	3628800

- Mathematical definition of Permutation (Repetition not allowed):

  Total number of arrangements of 'n' different things taking "r" at a time will be given by

  nPr or P (n, r) = $\frac{\lfloor n}{\lfloor n-r}$, where r $\leq$ n.

Note:

- When r = n, it is known as "all at a time"
- When r < n, it is known as "some at a time"
- r can never exceed n
- n and r must be positive integers

- ${}^nP_1 = n$
- ${}^nP_0 = 1$
- ${}^nP_n = n!$
- ${}^nP_n = {}^nP_{(n-1)} = n!$
- But ${}^nP_r \neq {}^nP_{(r-1)}$

- **Permutation or arrangements of 'n' different things in which few are alike (Repetition not allowed)**

  The total number of arrangements of n different things in which p are alike and of one kind, q are alike and a second kind, r are alike and yet of another kind and the rest are different, will be given by $\dfrac{\underline{|n}}{\underline{|p}\,\underline{|q}\,\underline{|r}}$

- **Permutation when repetitions are allowed**

  The total number of arrangements of n things taken r at a time when each thing may be repeated once, twice, thrice, ...to r number of times will be given by n^r

- **Rules for restricted Permutation**

a) Whenever the arrangements should begin or end or begin and end with a particular letter or object keep the objects fixed at the respective places and arrange the rest.

b) When in the arrangement of n things, r things are together, the total arrangements will be given by: $(n - r + 1)!\, r!$

c) When in the arrangements of n things, r things are together in a specified order, the total arrangement will be given by $(n - r + 1)!$

d) Total number of ways in which out of n things, r things are never together = total ways – number of ways when they are always together, i.e., $n! - (n - r + 1)!\, r!$

e) When the relative positions of few objects are to be kept unaltered it implies that the objects can be interchanged or arranged in their respective place only.

f) In problems involving re-arrangements always subtract 1 from the total arrangements.

g) When in the arrangement of n things, r alike things are together, then total number of arrangements will be given by $(n - r + 1)!$

Circular Permutation (When the things are arranged in a ring or circle)

- Total ways in which n things can be arranged in a ring or circle is $\lfloor n-1$

- Total ways in which n things can be arranged in a ring or circle with respect to any object will be given by $\lfloor n$

- When the clockwise or anti clock wise position cannot be disguised (for example: arrangements of different flowers in garland or arrangement of different beads in a necklace etc), in such a case the total number of circular arrangements will be given by (n – 1)! / 2

Arrangements of digits

- There are 10 random digits: 0, 1, 2, 3, 4, 5, 6, 7 ,8, 9

- 5 odd digits (1, 3, 5, 7, 9) and

- 5 even digits (0, 2, 4, 6, 8)

- Unless otherwise mentioned no number can start with '0'

- If there are 'n' different digits (0 is included) then the total number of n digit numbers not beginning with 0 will be given by: $\lfloor n$ - $\lfloor n-1$

- If there are n different digits (0 is included) and we are to form a number with r different digits then the total number of r digit numbers not beginning with 0 will be given by: ${}^{n}P_{r}$ - ${}^{n-1}P_{r-1}$

CLASSWORK SECTION

Fundamental Principle of Counting

1. There are 10 trains moving between Calcutta and Delhi. In how many ways can a man go from Calcutta to Delhi and return by a different one?

a) 100 b) 90 c) 120 d) None of the above

2. There are four bus lines between A & B and there are 3 bus lines between B & C. In how many ways can a man take round trips by bus from A to C by way of B, if he does not want to use a bus line more than once?

a) 36 b) 24 c) 72 d) 120

3. There are 26 stations on a railway line. How many different kinds of tickets of class II must be printed in order that a passenger may go from any one station to another by purchasing a ticket.

a) 65 b) 240 c) 650 d) 1300

4. In a class after every student had sent greeting cards to the other student, it was found that 1,640 cards were exchanged. Find the number of students in the class.

a) 40 b) 39 c) 41 d) 42

Formula Pattern - ^nP_r

5. $^nP_5 = 20 \times {}^nP_3$, then n =

a) 7 b) 6 c) 9 d) 8

6. $^nP_3 : {}^nP_2 = 3:1$, then the value of n is:

a) 4 b) 5 c) 6 d) 7

7. $^{n+r}P_2 = 110$, $^{n-r}P_2 = 20$, then find n & r.

a) 8, 4 b) 8, 3 c) 4, 8 d) 8, 5

8. The value of 1.1! + 2.2! + 3.3! +... ... ... + n. n! is:

a) $\underline{|(n+1)}-1$ b) $\underline{|(n+2)}-1$

c) $\underline{|(n-1)}-1$ d) None of the above

General Linear Permutation – Repetition not allowed

9. Find how many words can be formed by taking letters from the following word DAUGHTER taking (i) all at a time (ii) 4 at a time (iii) 3 at a time?

    a) 40320,1680, 336 b) 1680, 40320, 336
    c) 336, 1680, 40320 d) None of the above

10. How many (i) 5 digit no (ii) 6 digit no (iii) 8 digit no can be formed with the digit 1, 2, 3, 4, 5, 6, 7, 8?

    a) 6720, 20160, 40320 b) 5720, 20161, 40324
    c) 4720, 20164, 5040 d) None of the above

There are 5 letters and 5 addressed envelopes

11. In how many ways the letters can be put into the envelopes?

    a) 3! b) 4! c) 5! d) None of These

12. In how many arrangements all the letters are correctly placed?

    a) 0 b) 1 c) 2 d) None of the above

13. Find the total no of ways in which exactly one letter is wrongly placed?

    a) 0 b) 1 c) 2 d) None of the above

Alike Items – Repetition not allowed

Find the number of arrangements that can be made out of the letters of the following words:

14. MATHEMATICS

    a) 4989600 b) 4998960 c) 5987590 d) 4545450

15. INSTITUTION

    a) 554499 b) 445588 c) 554400 d) None of the above

Repetition Allowed

In how many ways can 5 prizes be given to

16. 4 boys

    a) 5^4 b) 5^5 c) 4^5 d) None of the above

17. 5 boys
    a) 5^5 b) 4^5 c) 5^4 d) None of the above

18. 6 boys
    a) 5^6 b) 6^5 c) 6^4 d) None of the above

19. In how many ways can 20 different dates can any of the 7 days of the week?
    a) 20^7 b) 7^{20}
    c) Both of a) and b) above d) None of the above

In how many ways the results of:

20. 10 matches be declared when each match ends in a win, loss or a draw?
    a) 3^{10} b) 10^3 c) 22^3 d) 3^{22}

21. A telegraph post has 5 arms and each arm is capable of 4 distinct positions including the position for rest. What is the total number of signals that can be made?
    a) 1024 b) 2048 c) 720 d) 1023

22. In how many ways can 3 persons enter into 4 hotels if (i) repetition is allowed, and (ii) repetition is not allowed?
    a) $3^4, 3^4$ b) 4^3,P(4,3)
    c) 3^4,P(4,3) d) None of the above

23. How many 3 digit nos. can be formed with the 3,1 and 9?
    a) 9 b) 12 c) 27 d) 81

24. There are 'm' men and 'n' monkeys. If a man has any number of monkeys, but a monkey may have only one master, then in how many ways may every monkey have a master?
    a) Mn b) n^n c) m^n d) n^m

25. Find the number of different signals that can be made with 20 flags, 4 flags each of 5 different colours, if 4 flags are required to make a signal?
    a) 560 b) 625 c) 480 d) 240

Restricted Permutation

You are given the letters of the word "MONDAY". Find the number of arrangements in the following cases:

26. Without any restriction.
    a) 120 b) 144 c) 720 d) 360

27. Words beginning with M
    a) 120 b) 240 c) 360 d) 720

28. Words ending with Y
    a) 24 b) 120 c) 96 d) 144

29. Words beginning with M & ending with Y
    a) 24 b) 96 c) 144 d) 240

30. Words beginning with M & not ending with Y
    a) 24 b) 96 c) 360 d) 144

31. M & Y are at the two extremes
    a) 24 b) 48 c) 96 d) 144

32. Vowels are together
    a) 120 b) 144 c) 240 d) 360

33. Vowels are together in the given order
    a) 144 b) 240 c) 120 d) 480

34. Vowels are never together
    a) 480 b) 240 c) 360 d) 144

35. Vowels occupy odd places
    a) 480 b) 240 c) 120 d) 144

36. Vowels occupy even places
    a) 240 b) 156 c) 144 d) 360

37. Relative position of the vowels and the consonants are to be kept untouched

a) 96 b) 48 c) 56 d) 144

38. Consonants are together

a) 96 b) 120 c) 144 d) 240

39. How many words can be formed by taking four letters at a time?

a) 240 b) 120 c) 360 d) 700

40. In how many of these (given in previous question) M is always included?

a) 240 b) 120 c) 360 d) 144

41. In how many of these (given in Q. No. 50) M is excluded?

a) 240 b) 360 c) 120 d) 144

42. Find the number of rearrangements of the word "Monday".

a) 720 b) 719 c) 360 d) 717

43. How many words can be formed of the letters in the word COSTING, the vowels being not separated?

a) 144 b) 1440 c) 1280 d) 2880

Consider words formation with the letters of the word DELHI

44. How many arrangements of the letters of the word 'COMRADE" can be made (i) if the vowels are never separated; (ii) if the vowels are to occupy only odd places.

a) 600,576 b) 600,120 c) 120,144 d) None of the above

45. In how many ways can 8 sweets of different size be distributed among 8 boys of different ages, so that the largest sweet always goes to the youngest boy? [Assume that each boy gets a sweet]

a) 2520 b) 1240 c) 5040 d) None of the above

You are given the letters of the word BALLOON. Find the following arrangements:

46. Without any restriction.

a) 960 b) 1060 c) 1160 d) 1260

47. Two LL will always come together.
a) 720 b) 360 c) 180 d) 120

48. Two LL and two OO will always come together.
a) 360 b) 480 c) 180 d) 120

49. All the O's & the L's will come together.
a) 120 b) 184 c) 144 d) 168

50. Vowels are together.
a) 180 b) 120 c) 360 d) 240

51. B & N are together.
a) 180 b) 120 c) 360 d) 240

52. B & N are never together.
a) 900 b) 980 c) 160 d) 720

53. B, N & O's are together.
a) 288 b) 142 c) 144 d) 368

All different words formed by the letters of the word BHARAT:

54. How many different words can be formed with the letters of the word BHARAT?
a) 720 b) 360 c) 180 d) 240

55. In how many of these B and H are never together?
a) 360 b) 180 c) 240 d) 120

56. How many of these begin with B and end with T?
a) 15 b) 12 c) 18 d) 21

57. How many different words can be formed with the letters of the word CAPTAIN? In how many of these C and T are never together?
a) 2520, 1600 b) 2520, 1890 c) 2520, 1800 d) 3250, 1800

58. In how many different ways can the letters of the word "CONSTITUTION" be arranged? How many of these will have the letter N both at the beginning and at the end?

a) 9979200, 151200 b) 9989920, 152150

c) 9979000, 151000 d) None of the above

59. In how many ways can the letters of the word AGARTALA be arranged?

a) 1600 b) 1800 c) 1980 d) 1680

60. Taking data from the previous question, In how many of these will the 4 A's (i) come together, (ii) not come together?

a) 120, 1560 b) 360, 1620 c) 300, 1500 d) 220, 1380

61. In how many different ways can the letters of the word VIDYAPITH be arranged ? How many of these arrangements begin with V but do not end with H?

a) 181000,17600 b) 181400,17640

c) 181440,17640 d) 182000,18600

62. In how many ways can 5 dots (.) and 3 crosses (x) be arranged in a row?

a) 36 b) 46 c) 66 d) 56

63. A library has 5 copies of 1 book, 4 copies of each of the 2 books, 6 copies of each of the 3 books and single copies of 8 books. In how many ways can all the books be arranged?

a) $\frac{39!}{5!(4!)^2(6!)^3}$ b) $\frac{39!}{8!(4!)^2(6!)^3}$

c) $\frac{39!}{(4!)^2 \times (6!)^3}$ d) None

Circular Permutation

64. In how many ways 8 persons can be seated at a round table?

a) 5040 b) 40320 c) 2020 d) 2520

65. In how many ways can 7 persons be arranged at a round table so that 2 particular persons can be together?

a) 180 b) 240 c) 360 d) None of the above

66. In how many ways can 4 MBA & 4 MCA be seated at the round table so that no 2 MBA students are adjacent?

a) 12 b) 24 c) 96 d) 144

67. Find the no. of ways in which 5 beads can be arranged to form a necklace

a) 12 b) 24 c) 36 d) 48

68. In how many ways 8 stones of different colours be arranged on a ring? In how many of these arrangements red and yellow beads being separated?

a) 2520, 900 b) 2520, 1800
c) 1800, 2520 d) 1800, 1260

69. 20 persons are invited to a party. In how many ways can they and host be seated at a circular table? In how many of these 2 particular persons be seated on either side of the host?

a) $(18!), 2(20!)$ b) $(18!), (20!)$ c) $(20!), (18!)$ d) $(20!), 2(18!)$

70. In how many ways can 6 ladies and 6 gents be arranged at a round table, if the two particular ladies Miss X and Miss Y refuse to sit next to Mr. Z, all men being separated?

a) 1278 b) 1730 c) 1728 d) 34560

Problem Involving Digits

71. How many (i) 8 digit no (ii) 5 digit no (iii) 4 digit no (iv) 3 digit no can be formed with the digit 0,1,2,3,4,5,6,7?

a) 35280, 5880, 1470, 294 b) 5880, 1740, 4294, 294
c) 294, 1470, 5880, 35280 d) None

72. How many 6 digit nos. can be formed with 3, 4, 5, 6, 7, & 8 (no digits are repeated)? How many of these are (i) divisible by 5 (ii) not divisible by 5.

a) 720, 120, 600 b) 720, 600, 120
c) 780, 600, 180 d) 780, 180, 600

73. How many 4 digit nos. can be formed with the digits 0, 1, 2, 3, 4, 5, ... and 9 (repetition not allowed)?

a) 9000 b) 4536 c) 3654 d) 2635

74. In the above question, if repetition is allowed then find the number of ways it can be done?

a) 4536 b) 3654 c) 2598 d) 9000

75. How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1, so that odd digits are at odd places?

a) 18 b) 19 c) 20 d) None

76. How many four digits numbers can be formed with the digits 3, 4, 5, 6? Find the sum of all the numbers thus formed.

a) 24, 1420 b) 24, 1520 c) 24, 4742 d) 24, 119988

77. How many 3 digit nos. are there, with distinct digits, with each digit odd?

a) 30 b) 60 c) 90 d) 120

78. Find the total number of numbers divisible by 2 which can be formed with the six digits 1, 2, 4, 5, 6 and 7.

a) 180 b) 360 c) 720 d) 120

79. How many 5 digit nos. can be formed with the digits 2, 3, 5, 7, and 9 which are (i) greater than 30000; (ii) less than 70000: (iii) lies between 30000 and 90000?

a) 96, 72, 72 b) 96, 96, 72 c) 96, 96, 96 d) 72, 72, 96

80. How many numbers greater than 4000 can be formed with the digits 2, 3, 4, 5 and 6 when no digit is repeated?

a) 216 b) 212 c) 192 d) 180

81. How many numbers of four different digits each greater than 4000 can be formed from the digits 2, 3, 4, 6, 7 and 0?

a) 160 b) 180 c) 150 d) 240

82. How many nos. between 300 and 3000 can be formed with the digits 0, 1, 2, 3, 4 and 5(no digit being repeated in any number)?

a) 180 b) 120 c) 160 d) 240

83. How many numbers can be formed with the digits 1, 2, 3, 4 and 5; which are greater than 3400?

a) 180 b) 120 c) 160 d) 360

84. How many numbers greater than a million (10 lakhs) can be formed with the digits 2, 3, 0, 3, 4, 2, and 3?

a) 240 b) 180 c) 540 d) 360

85. How many even numbers greater than 300 can be formed with the digits 1, 2, 3, 4 and 5 (no digit being repeated)?

a) 121 b) 111 c) 222 d) 124

Miscellaneous

86. How many ways can 3 boys and 5 girls be arranged in row so that no 2 boys are together?

a) 14400 b) 604800 c) 2880 d) 28800

87. In how many ways can 5 boys and 4 girls be arranged in a row so that the boys and the girl stand alternately?

a) 28800 b) 14400 c) 2880 d) 60480

88. A dinner is arranged for 11 guests in which there are 4 children, 1 old man and 6 adults. They are to be arranged in a row for a dinner. The 4 children wish to occupy 2 seats at each end corner seats and the old man refuses to have a child on his either side. In how many ways can all guests be arranged?

a) 28800 b) 43200 c) 86400 d) 14400

89. In how many ways can 5 boys and 5 girls be arranged in a row so that they stand alternately?

a) 14400 b) 2880 c) 604800 d) 28800

90. A, B, C, D, E are suppose to speak in a meeting. In how many ways can they be arranged so that A & B speak next to each other.

a) 48 b) 49 c) 56 d) None

91. A Speaks immediately before B

a) 24 b) 120 c) 12 d) None

92. A speaks always before B

a) 60 b) 6 c) 120 d) None

93. In how many ways can the letters of the word PARNECIOUS be arranged without changing the order of the vowels?

a) 33590 b) 30239 c) 14400 d) 28800

94. One arrange 10 letters taken 7 at a time. In how many of these 3 particular letters (i) always occur, and (ii) never occurs?

a) 720, 30240 b) 30240, 5040
c) 5040, 5040 d) 176400, 5040

95. If the number of permutation of "n' different things taken 4 at a time, in which 1 particular thing does not occur, is equal to the number in which it does occur, find the value of "n".

a) 2 b) 4 c) 6 d) 8

96. Find the rank of word CHALK?

(a) 31 (b) 32 (c) 33 (d) 34

97. Find rank of word 'ZENITH'.

(a) 613 (b) 614 (c) 615 (d) 616

98. Find rank of word GOOGLE

(a) 86 (b) 87 (c) 88 (d) 89

COMBINATION

- Combination is the selection of different items from a given number of items
- Combination is order independent
- The total number of combinations or selections of r items from n different items will be given by;

$$^nC_r \text{ or } C(n, r) = \frac{\underline{|n}}{\underline{|r} \times \underline{|n-r}} \quad \text{where } r \leq n$$

- No arrangement (Permutation) is possible without selection (Combination) but selection (Combination) process can take place independently
- Thus $^nC_r < {^nP_r}$, except when r=0 or 1
- Relation between ^nP_r and ^nC_r
  - $^nP_r = {^nC_r} \times \underline{|r}$
  - $\frac{^nP_r}{^nC_r} = \underline{|r}$
  - $^nC_0 = {^nP_0} = 1$
  - $^nC_1 = {^nP_1} = n$
  - $^nC_n = 1, \ {^nP_n} = \underline{|n}$

Complementary Combination

- $^nC_r = {^nC_{n-r}}$ (Use this result, when $r > \frac{n}{2}$)
- If $^nC_x = {^nC_y}$ then either
  a. x = y or
  b. x+y=n or
  c. both the results can hold true simultaneously
- $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

- $\frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}$

Application of combinations

Case 1:

Total number of straight lines that can be formed out of 'n' points on a plane when no three of them are co-linear will be given by

$$^nC_2$$

Case 2:

Total number of triangles that can be formed out of these 'n' points is

$$^nC_3$$

Case 3:

Total Number of circles that can be formed out of these 'n' points is

$$^nC_3$$

Case 4:

Total number of lines that can be formed with 'n' points when p of them are collinear will be given

$$^nC_2 - {}^pC_2 + 1$$

Case 5:

Total number of triangles that can be formed with 'n' points when p of them are collinear will be given by

$$^nC_3 - {}^PC_3$$

Case 6:

The total number of points of intersection that can be obtained from 'n' straight lines are $^nC_2$ when,

i. No two of them are parallel and

ii. No three of them are concurrent

Case 7:

To find the number of diagonals in a polygon having 'n' sides

No of diagonals = $^nC_2 - n$

Where ${}^{n}C_{2}$ = total number of lines by joining 2 vertices in pairs and

'n' number of sides = number of vertices

$${}^{n}C_{2} - n$$

$$= \frac{n(n-1)}{2} - n = n\left(\frac{n-1}{2} - 1\right) \Rightarrow n\left(\frac{n-1-2}{2}\right) \Rightarrow n\left(\frac{n-3}{2}\right)$$

Case 8:

Total number of selections or combinations of 'n' different things taking one or more at a time (i.e., at least 1) will be given by

$${}^{n}C_{1} + {}^{n}C_{2} + {}^{n}C_{3} + \ldots\ldots\ldots + {}^{n}C_{n} = 2^{n} - 1$$

Case 9:

Combinations or selections of things which are alike.

Total number of combinations or selection of p, q, r items by taking one or more(atleast one) will be given by, (p+1)(q+1)(r+1)-1

When p are alike and of one kind, q are alike and of a second kind and r are alike and of yet of another kind.

Note:

Total number of selections of p alike, q alike and r different items by taking at least one will be given by $(p+1)(q+1)2^{r}-1$

Case 10: Division into groups

- The total number of ways in which (m+n) items can be divided into two distinct groups containing m & n items respectively will be given by:

$$\frac{\underline{|m+n}}{\underline{|m}\,\underline{|n}}$$

- Total ways in which m+n+p items can be divided into 3 distinct groups containing m, n & p items respectively will be given by,

$$\frac{\underline{|m+n+p}}{\underline{|m}\,\underline{|n}\,\underline{|p}}$$

Case i :

When m = n or m=n=p then 2m or 3m items can be equally distributed into two or three distinct groups in,

$$\frac{\lfloor 2m}{(\lfloor m)^2} \text{ or } \frac{\lfloor 3m}{(\lfloor m)^3} \text{ ways}$$

Case ii:

When the identities of the groups are not distinct i.e, the groups are alike in such a case 2m or 3m items can be distributed equally into 2 or 3 identified groups in

$$\frac{\lfloor 2m}{(\lfloor m)^2} \times \frac{1}{2!} \text{ or } \frac{\lfloor 3m}{(\lfloor m)^3} \times \frac{1}{3!} \text{ ways}$$

CLASSWORK SECTION

Basic Meaning

1. Find the value of n when ${}^{20}C_{3n} = {}^{20}C_{2n+5}$

a) 3 b) 5
c) Both a) and b) above d) Neither a) nor b) above

2. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, find the value of r.

a) 3 b) 4
c) Both a) and b) above d) None of a) or b) above

3. If ${}^{2n}C_{r} = {}^{2n}C_{r+2}$, find the value of r.

a) n + 1 b) n − 1 c) 2n + 1 d) None of the above

4. If ${}^{n}P_{r} = 336$ and ${}^{n}C_{r} = 56$, find n and r. Find also ${}^{n+2}C_{r+1}$

a) 8, 3, 120 b) 8, 3, 210 c) 8, 5, 210 d) None of the above

5. Find the value of: ${}^{12}C_{4} + {}^{12}C_{3}$

a) 720 b) 715 c) 815 d) 820

6. If ${}^{13}C_{6} + 2.\ {}^{13}C_{5} + {}^{13}C_{4} = {}^{15}C_{x}$, what is the value of X?

a) 6 b) 9
c) Either a) or b) d) Both a) and b)

7. If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$, find x

a) 1000 b) 998 c) 997 d) 999

8. Find the value of ${}^{47}C_{4} + \sum_{r=1}^{5} {}^{52-r}C_{3}$

a) ${}^{52}C_{6}$ b) ${}^{52}C_{5}$ c) ${}^{52}C_{4}$ d) None

9. If $\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{1}{4}$ and $\frac{{}^{n}C_{r}}{{}^{n}C_{r+1}} = \frac{1}{3}$ then find the value of n and r?

a) 35,7 b) 53,8 c) 35,8 d) 19,4

"n" different things, "r" to be selected ($r \leq n$) – Without Restrictions

10. In how many ways can a committee of 6 men and 2 women be formed out of 10 men and 5 women?
    a. 1200 b. 2100 c. 1300 d. 3100

"n" different things, "r" to be selected ($r \leq n$) – With Restrictions

In how many ways can four students be selected out of twelve students if

11. 2 particular students are not included at all.
    a. 120 b. 210 c. 340 d. 320

12. 2 particular students are always included?
    a. 54 b. 56 c. 57 d. 45

13. From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include at least one lady?
    a. 642 b. 259 c. 246 d. 586

In how many ways can a committee of 5 members be formed from 10 candidates so as to

14. Include both the youngest and the oldest candidates
    a. 196 b. 65 c. 54 d. 56

15. Exclude the youngest if it includes the oldest?
    a. 196 b. 165 c. 156 d. 157

16. In a class of 16 students, there are 5 lady students. In how many ways can 10 students be selected from them so as to include at least 4 lady students?
    a. 2770 b. 2772 c. 7227 d. 7272

17. The question paper on Mathematics and Statistics contains 10 questions divided into two groups of 5 questions each. In how many ways can an examinee select 6 questions taking at least two questions from each group?
    a. 200 b. 150 c. 100 d. 250

18. In an examination paper there are two groups each containing 7 questions. A candidate is required to attempt 9 questions but not more than five questions from any group. In how many ways can 9 questions be selected?
    a) 1470 b) 1570 c) 1680 d) 1970

19. A candidate is required to answer 6 out of 12 questions which are divided into two groups each containing 6 questions and he is permitted to attempt not more than four from any group. In how many different ways can he make up his choice?
    a. 580 b. 680 c. 850 d. 950

"n" different things, any number can be selected at a time

20. In how many ways can a person choose one or more of the four electrical appliances: T.V., Refrigerator, Washing Machine, Radiogram?
    a. 16 b. 31 c. 32 d. 15

21. In how many ways a man can invite 5 friends to a dinner so that two or more of them remain present?
    a. 24 b. 25 c. 26 d. 32

22. At an election there are 5 candidates and 3 members are to be elected and a voter is entitled to vote for any number of candidates but not more then to be elected. In how many ways may a voter choose a vote?
    a. 24 b. 23 c. 26 d. 25

A man has 5 German, 4 Spanish and 3 French friends. Find:

23. Total ways in which at least one friend can be invited.
    a. 4096 b. 4095 c. 2048 d. None of the above

24. Total ways of invitation so that there is at least 1 German friend.
    a. $(2^5-1).2^4$ b. $(2^5-1).2^3$
    c. $(2^5-1).2^7$ d. None of the above

25. At least 1 German and at least 1 French friend.

a. $(2^5-1).(2^4-1).2^3$ b. $(2^5-1).(2^3-1).2^4$

c. $(2^4-1).2^3.2^5$ d. None of the above

26. At least one friend from each country.

a. 3165 b. 3290 c. 3255 d. 3250

27. Ten electric bulbs, of which 3 are defective, are to be tried in three different light points in a dark room. In how many out of the total trial the room shall be lighted?

a. 120 b. 121 c. 119 d. 122

28. Five bulbs of which three are defective are to be tried in two light points in a dark room. In how many trials the room shall be lighted?

a. 7 b. 8 c. 9 d. 11

29. There are 10 lamps in a hall. Each of them can be switched on independently. What is the number of ways in which the hall can be illuminated?

a. 1024 b. 100 c. 1023 d. 120

30. A Supreme Court Bench consists of five judges. In how many ways the bench can give a decision in majority?

a. 16 b. 15 c. 31 d. 32

Application of Combination in Geometry

31. How many straight lines can be obtained by joining 16 points on a plane, no three points being on the same line?

a. 120 b. 240 c. 119 d. 480

32. How many triangles can be drawn, by joining 16 points on a plane, no three points being on the same line?

a. 240 b. 360 c. 560 d. 480

33. Find the number of straight lines formed by joining 10 different points on a plane, no three of them being collinear (with the exception of 4 points which are collinear).

a. 41 b. 45 c. 39 d. 40

34. Find the number of triangles formed by joining 10 different points on a plane, no three of them being collinear (with the exception of 4 points which are collinear).

a. 120 b. 116 c. 121 d. 126

35. If 20 straight lines be drawn in a plane, no two of them being parallel and no three of them being concurrent, how many points of intersection will there be?

a. 200 b. 180 c. 210 d. 190

Find the number of diagonals in:

36. Quadrilateral

a. 2 b. 3 c. 4 d. 6

37. Octagon

a. 10 b. 35 c. 20 d. 9

38. A polygon has 44 diagonals. Find the number of its sides.

a. 10 b. 11 c. 12 d. 14

Selections of p,q,r alike items taken any number at a time

There are 8 mangoes, 4 apples and 5 oranges. If any number of fruits can be selected, find:

39. Total number of selections.

a. 269 b. 270 c. 275 d. 300

40. Total number of selections so that there is at least 1 mango.

a. 269 b. 270 c. 240 d. None

41. At least 1 mango and at least 1 apple.

a. 129 b. 108 c. 40 d. 192

42. At least 1 fruit of each type.

a. 160 b. 170 c. 190 d. 210

How many selections can be made by taking any letters from the words

43. DADDY DID A DEADLY DEED

a. 1920 b. 1921 c. 1290 d. 1919

44. A person has in his bag 14 notes of Rs. 10 each, 9 notes of Rs. 5 each, 4 notes of Rs. 2 each and 7 notes of Re. 1 each. In how many different ways can he contribute to a charitable fund?

a) 3000 b) 6000 c) 5999 d) 2999

Division into Groups – either distinct or alike

45. The three different sections of a Library need the services of 3, 4 and 5 workers respectively. If 12 workers are available, in how many ways can they be allotted to different sections?

a. 27720 b. 22770 c. 27270 d. 72270

46. Divide 12 items in two groups so that each containing 8 and 4 items.

a) $\frac{12!}{8!}$ b) $\frac{12!}{4!8!}$

c) $\frac{8!4!}{12!}$ d) None of the above

In how many ways can 20 books be equally divided amongst:

47. 2 Boys.

a. $\frac{20!}{(10!)^2}$ b. $\frac{20!}{(5!)^4}$ c. $\frac{20!}{(4!)^5}$ d. $\frac{20!}{(2!)^{10}}$

48. 4 Boys.

a. $\frac{20!}{(10!)^2}$ b. $\frac{20!}{(5!)^4}$ c. $\frac{20!}{(4!)^5}$ d. $\frac{20!}{(2!)^{10}}$

49. In how many ways 9 students be divided equally into 3 groups?

a) 260 b) 270 c) 280 d) 300

"n" things, few are alike, "r" things chosen at a time and then arranged

How many different words can be formed taking 4 letters from the following words?

50. PROPORTION

a. 758 b. 875 c. 587 d. 578

51. EXAMINATION

a. 2434 b. 2454 c. 1424 d. 6424

Mixed Bag

52. There are 25 candidates, which include 5 from the Scheduled Castes for 12 vacancies. If 3 vacancies are reserved for Scheduled Caste candidates and the remaining vacancies are open to all, find the number of ways in which the selection can be made

a. ${}^{5}C_{3}.{}^{20}C_{9}$ b. ${}^{5}C_{3}.{}^{22}C_{9}$ c. ${}^{25}C_{12}$ d. None

53. A league match in football ends in a win, loss or draw. How many different forecasts can give exactly 18 correct results out of 22 matches played?

a. ${}^{22}C_{18}$ b. ${}^{22}C_{18}.2^{4}$ c. ${}^{22}C_{18}.4^{2}$ d. None

54. In how many ways can the letters of the word FORECAST taken 3 at a time and the word MILKY taken 2 at a time be arranged?

a. 62700 b. 67000 c. 68720 d. 67200

55. A "number lock" consists of 3 rings each marked with 10 different numbers. In how many ways the lock cannot be opened?

a) 1000 b) 999 c) 998 d) 997

56. How many different factors can 210 have?

a) 15 b) 16 c) 31 d) 32

57. How many different factors can 2160 have?

a) 40 b) 39 c) 37 d) 45

58. A captain and a vice-captain are to be chosen out of a team having 11 players. How many ways are there to achieve this?

a) 100 b) 110 c) 120 d) None of the above

HOMEWORK SECTION

1. In ^nP_r = n (n−1) (n−2) … … … … (n−r+1), the number of factors is

   a) n b) r−1 c) n−r d) r

2. n articles are arranged in such a way that 2 particular articles never come together. The number of such arrangements is

   (a) $(n-2)\lfloor n-1$ (b) $(n-2)\lfloor n-2$ (c) $\lfloor n$ (d) None of these

3. The number of ways the letters of the word `TRIANGLE' to be arranged so that the word 'angle' will be always present is

   (a) 20 (b) 60 (c) 24 (d) 32

4. The sum of all 4 digit number containing the digits 2, 4, 6, 8, without repetitions is

   a) 1,33,330 b) 1,22,220 c) 2,13,330 d) 1,33,320

5. If the letters word 'DAUGHTER' are to be arranged so that vowels occupy the odd places, then number of different words are

   (a) 2,880 (b) 676 (c) 625 (d) 576

6. 3 ladies and 3 gents can be seated at a round table so that any two and only two of the ladies sit together. The number of ways is

   (a) 70 (b) 27 (c) 72 (d) none of these

7. The number of arrangements of 10 different things taken 4 at a time in which one particular thing always occurs is

   (a) 2015 (b) 2016 (c) 2014 (d) none of these

8. The number of permutations of 10 different things taken 4 at a time in which one particular thing never occurs is

   (a) 3,020 (b) 3,025 (c) 3,024 (d) none of these

9. The total number of ways in which six '+' and four '−' signs can be arranged in a line such that no two '−' signs occur together is

   (a) $\lfloor 7/\lfloor 3$ (b) $\lfloor 6/\lfloor 7/\lfloor 3$ (c) 35 (d) none of these

10. 5 persons are sitting in a round table in such way that Tallest Person is always on the right-side of the shortest person; the number of such arrangements is
(a) 6 (b) 8 (c) 24 (d) none of these

11. The total number of 9 digit numbers of different digits is
(a) $10\underline{|9}$ (b) $8\underline{|9}$ (c) $9\underline{|9}$ (d) none of these

12. There are 5 speakers A, B, C, D and E. The number of ways in which A will speak always before B is
(a) 24 (b) $\underline{|4} \times \underline{|2}$ (c) $\underline{|5}$ (d) none of these

13. The number of ways in which 8 sweats of different sizes can be distributed among 8 persons of different ages so that the largest sweat always goes to be younger assuming that each one of then gets a sweat is
(a) $\underline{|8}$ (b) 5040 (c) 5039 (d) none of these

14. If $^{n}C_{r-1} = 56$, $^{n}C_{r} = 28$ and $^{n}C_{r}+1$ +1 = 8, then r is equal to
(a) 8 (b) 6 (c) 5 (d) none of these

15. If $^{28}C_{2r} : {}^{24}C_{2r-4} = 225 : 11$, then the value of r is
(a) 7 (b) 5 (c) 6 (d) none of these

16. The number of ways in which 12 students can be equally divided into three groups is
(a) 5775 (b) 7575 (c) 7755 (d) none of these

17. The number of ways in which 15 mangoes can be equally divided among 3 students is
(a) $\dfrac{\underline{|15}}{(\underline{|5})^4}$ (b) $\dfrac{\underline{|15}}{(\underline{|5})^3}$ (c) $\dfrac{\underline{|15}}{(\underline{|5})^2}$ (d) none of these

18. The Supreme Court has given a 6 to 3 decision upholding a lower court; the number of ways it can give a majority decision reversing the lower court is
(a) 256 (b) 276 (c) 245 (d) 226.

19. Out of 7 gents and 4 ladies a committee of 5 is to be formed. The number of committees such that each committee includes at least one lady is
(a) 400 (b) 440 (c) 441 (d) none of these

20. If ${}^{500}C_{92} = {}^{499}C_{92} + {}^{n}C_{91}$ then n is

(a) 501 (b) 500 (c) 502 (d) 499

21. Five bulbs of which three are defective are to be tried in two bulb points in a dark room.
Number of trials the room shall be lighted is

(a) 6 (b) 8 (c) 5 (d) 7.

22. The number of different words that can be formed with 12 consonants and 5 vowels by taking 4 consonants and 3 vowels in each word is

(a) ${}^{12}C_4 \times {}^{5}C_3$ (b) ${}^{17}C_7$ (c) $4950 \times \lfloor 7!$ (d) none of these

23. Eight guests have to be seated 4 on each side of a long rectangular table.2 particular guests desire to sit on one side of the table and 3 on the other side. The number of ways in which the sitting arrangements can be made is

(a) 1732 (b) 1728 (c) 1730 (d) 1278.

24. The number of words that can be made by rearranging the letters of the word APURNA so that vowels and consonants appear alternate is

(a) 18 (b) 35 (c) 36 (d) none of these

25. The results of 8 matches (Win, Loss or Draw) are to be predicted. The number of different forecasts containing exactly 6 correct results is

(a) 316 (b) 214 (c) 112 (d) none of these

26. The number of 4 digit numbers formed with the digits 1, 1, 2, 2, 3, 4 is

(a) 100 (b) 101 (c) 201 (d) none of these

27. ${}^{(n-1)}P_r + r.\ {}^{(n-1)}P_{(r-1)}$ is equal to

(a) ${}^{n}C_r$ (b) $\dfrac{\lfloor n}{(\lfloor r \lfloor n-r)}$ (c) ${}^{n}P_r$ (d) none of these

28. The number of even numbers greater than 300 can be formed with the digits 1, 2, 3, 4, 5 without repetion is

(a) 110 (b) 112 (c) 111 (d) none of these

29. $\underline{|2n}$ can be written as

(a) 2^n {1.3.5.... (2n-1)} (b) $2^n \underline{|n}$

(c) {1.3.5....(2n-1)} (d) none of these

30. How many numbers higher than a million can be formed with the digits 0,4,4,5,5,5,3?

(a) 420 (b) 360 (c) 7! (d) None

31. The total number of numbers less than 1000 and divisible by 5 formed with 0,1,2, ...9 such that each digit does not occur more than once in each number is

(a) 150 (b) 152 (c) 154 (d) None

32. The number of ways in which 8 examination papers be arranged so that the best and worst papers never come together is

(a) 8! – 2 x 7! (b) 8! – 7! (c) 8! (d) None

33. If you have 5 copies of one book, 4 copies of each of two books, 6 copies each of three books and single copy of 8 books you may arrange it in __________number of ways.

(a) $\frac{39!}{5! \times (4!)^2 \times (6!)^3}$ (b) $\frac{39!}{5! \times 8! \times (4!)^2 \times (6!)^3}$

(c) $\frac{39!}{5! \times 8! \times 4! \times (6!)^2}$ (d) $\frac{39!}{5! \times 8! \times 4! \times 6!}$

34. A family comprised of an old man, 6 adults and 4 children is to be seated is a row with the condition that the children would occupy both the ends and never occupy either side of the old man. How many sitting arrangements are possible?

(a) 4! x 5! x 7! (b) 4! x 5! x 6! (c) 2! x 4! x 5! x 6! (d) None

35. A boat's crew consist of 8 men, 3 of whom can row only on one side and 2 only on the other. The number of ways in which the crew can be arranged is ___________.

(a) ${}^3C_1 \times (4!)^2$ (b) ${}^3C_1 \times 4!$ (c) ${}^3C_1$ (d) None

36. In a cross word puzzle 20 words are to be guessed of which 8 words have each an alternative solution. The number of possible solution is __________.

(a) $(2 \times 8)^2$ (b) ${}^{20}C_{16}$ (c) ${}^{20}C_8$ (d) None

37. In how many ways the vowels of the word 'ALLAHABAD' will occupy the even places?

(a) 120 (b) 60 (c) 30 (d) None

38. How many numbers between 3000 and 4000 can be formed with 1, 2, ...6?

(a) 3,024 (b) 60 (c) 78 (d) None

39. How many numbers greater than 23,000 can be formed with 1, 2, ...5?

(a) 3,024 (b) 60 (c) 78 (d) None

40. The total number of sitting arrangements of 7 persons in a row if 3 persons sit together in a particular order is __________.

(a) 5! (b) 6! (c) 2! x 5! (d) None

41. In how many ways 6 men can sit at a round table so that all shall not have the same neighbours in any two arrangements?

(a) 5! ÷ 2 (b) 5! (c) $(7!)^2$ (d) 7!

42. A team of 12 men is to be formed out of n persons. Then the number of times 2 men 'A' and 'B' are together is __________.

(a) ${}^{n}C_{12}$ (b) ${}^{n-1}C_{11}$ (c) ${}^{n-2}C_{10}$ (d) None

43. The number of combinations that can be made by taking 4 letters of the word 'COMBINATION' is ________.

(a) 70 (b) 63 (c) 3 (d) 136

44. In how many ways 21 red balls and 19 blue balls can be arranged in a row so that no two blue balls are together?

(a) 1540 (b) 1520 (c) 1560 (d) None

HOMEWORK SOLUTIONS

1. (d) ${}^nP_r$ is the product of "r" consecutive factors beginning with "n". Thus for ${}^nP_r$, the number of factors is "r".

2. (a) Never together = Total possible arrangements – Number of arrangements when things are together

   Total arrangement of n things = n!

   Arrangements, when 2 particular things are always together = (n -2 + 1)! X 2! = (n -1)! * 2!

   Arrangements, where 2 particulars things never come together:
   = n! – (n – 1)! * 2!
   = n(n – 1)! – 2(n–1)!
   = (n – 1)! (n – 2)

3. (c) Word "ANGLE" will always be present, means 5 things always occur in the given order.

   Number of ways this can be done = (8 – 5 + 1)! * 5!/5! = 4! = 24

4. (d) Sum of numbers = (Sum of digits) * (n – 1)! * (111111..... n times)
   Here, n = 4, Sum of digits = (2 + 4 + 5 + 8) = 20
   Thus, sum of all numbers = 20 * (4 – 1)! * (1111) = 20 * 6 * 1111 = 133,320

5. (a) Vowels = A, U, E. No. Of odd places = 4
   Vowels arranged at the odd places in ${}^4P_3$ ways.
   Remaining 5 consonants are placed on the remaining 5 places in ${}^5P_5$ ways.
   Total number of different words this formed = ${}^4P_3$ * ${}^5P_5$ = 24 * 120 = 2880

6. (c) 3 Gents arranged at the round table in $(3 - 1)! = 2! = 2$ ways
2 Ladies selected out of 3 ladies in ${}^3C_2 = 3$ ways
These selected ladies are placed in 3 gaps among the 3 gents in ${}^3P_1 = 3$ ways
These 2 ladies shall arrange themselves in $2! = 2$ ways
Remaining 1 lady can be placed at remaining 2 places in ${}^2P_1 = 2$ ways
Total ways = 2 * 3 * 3 * 2 * 2 = 72 ways

7. (b) One particular things shall be arranged first in ${}^4P_1 = 4$ ways
Remaining 9 things can be arranged in remaining 3 places in ${}^9P_3 = 504$ ways
Total number of arrangements = 4 * 504 = 2016.

8. (c) One particular thing never occurs shall be kept aside for the arrangement.
Total number of arrangements shall be with remaining 9 things in ${}^9P_4 = 3024$ ways.

9. (c) All "+" and "-" sigs are alike.
Six '+' signs can be arranged in 6!/6! = 1 way
Now four '-' signs shall be placed among the 7 places generated between and on sides of '+' signs in ${}^7P_4/4! = 35$ ways
Total number of arrangements = 1 * 35 = 35.

10. (a) Tallest and the shortest person shall be grouped together. But they shall not arrange themselves, as tallest is always on the right-side of the shortest person.
Total arrangements = $(5 - 2 + 1 - 1)! = 3! = 6$

11. (c) 9-digit different numbers (without restriction of '0') = ${}^{10}P_9 = 10!$
Numbers starting with '0' = 1 * ${}^9P_8 = 9!$
Total numbers = $10! - 9! = 9!(10 - 1) = 9*9!$

12. (a) As A shall always speak before B, the order of A and B needs to be first and hence both shall be treated as alike objects.
Total arrangements = 5!/2! = 120 / 2 = 60
OR, we can also solve it (in detail) as follows:
A is the 1st speaker, B can speak at 2nd, 3rd, 4th or 5th position in
1! * 4! = 24 ways

A is the 2nd speaker, B can speak only at 3rd, 4th or 5th position in

${}^3P_1 * 1! * {}^3P_3 = 18$ ways

A is the 3rd speaker, B can speak only at 4th or 5th position in

${}^3P_2 * 1! * {}^2P_2 = 12$ ways

A is the 4th speaker, B can speak only at the 5th position in

${}^3P_3 * 1! * {}^1P_1 = 6$ ways

Total arrangements = 24 + 18 + 12 + 6 = 60 ways

13. (b) Larger sweet goes to the youngest member. Thus we are left with 7 sweets which need to be distributed among remaining 7 persons in ${}^7P_7 = 5040$ ways.

14. (b) $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n!}{r!(n-r)!} * \frac{(r-1)!(n-r+1)!}{n!} = \frac{n-r+1}{r} = \frac{28}{56} = \frac{1}{2}$ Or, 2n − 2r + 2 = r. Or, 2n − 3r + 2 = 0 **(i)**

$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n!}{(r+1)!(n-r-1)!} * \frac{r!(n-r)!}{n!} = \frac{n-r}{r+1} = \frac{8}{28} = \frac{2}{7}$ Or, 7n − 7r = 2r + 2. Or, 7n − 9r − 2 = 0 **(ii)**

Solving (i) and (ii) we get, n = 8, r = 6.

15. (a) $\frac{{}^{28}C_{2r}}{{}^{24}C_{2r-4}} = \frac{28!}{2r!(28-2r)!} * \frac{(2r-4)!(24-2r+4)!}{24!} = \frac{28.27.26.25}{2r(2r-1)(2r-2)(2r-3)} = \frac{28}{2r} \cdot \frac{27}{(2r-1)} \cdot \frac{26}{(2r-2)} \cdot \frac{25}{(2r-3)} = \frac{225}{11}$

$\frac{28}{2r} \cdot \frac{3}{(2r-1)} \cdot \frac{26}{(2r-2)} \cdot \frac{1}{(2r-3)} = \frac{1}{11}$

Or, 2r(2r − 1)(2r − 2)(2r − 3) = 14*13*12*11

Or, 2r = 14. r = 7.

16. (a) 12 students can be equally divided in three groups, such that 4 students go to each group. Here groups are identical in nature.

Thus, number of selection = $12!/(4!)^3 * 1/3! = 5775$.

17. (b) In this case each student shall get 15/3 = 5 mangoes. Students are not identical in nature.

Number of ways = $15!/(5!)^3$,

18. (a) Total number of judges = 6 + 3 = 9

Majority decision = 9C9 + 9C8 + 9C7 + 9C6 + 9C5 (i.e., at least 5 judges should be in favour)

= 1 + 9 + 36 + 84 + 126 = 256

19. (c) Total number of committees = 11C5 = 462

Number of committees where no lady is present = ${}^{7}C_{5}$ = 21

Thus, at least one lady is present in 462 − 21 = 441 committees

20. (d) $^{500}C_{92} = {}^{499}C_{92} + {}^{n}C_{91}$

$^{500}C_{92} = {}^{499+1}C_{91+1} = {}^{499}C_{91} + {}^{499}C_{92}$

$\therefore {}^{499}C_{91} = {}^{n}C_{91}$

n = 499.

21. (d) Room shall be lighted in (Total selection) − (Selection of all defective bulbs)

= ${}^{5}C_{2} - {}^{3}C_{2}$ = 10 − 3 = 7 ways.

22. (c) Selection of consonants in 12C4 ways

Selection of vowels in ${}^{5}C_{3}$ ways

7 alphabets thus chosen cab be arranged in ${}^{7}P_{7}$ = 7! Ways

Total number of words = 12C4 * ${}^{5}C_{3}$ * 7! = 4950 * 7!

23. (b) 2 on one side and 3 on other side. Thus five persons are fixed.

Of the remaining (8 − 5) = 3 guests, 1 shall be seated along with 3 fixed and other two on the side with 2 fixed guests in ${}^{3}C_{1}$ * ${}^{2}C_{2}$ = 3 ways.

Total number of arrangements = 3 * 4!! * 4! = 1728

24. (c) A P U R N A − Vowels (A U A) (2 alike) Consonants (P R N)

Arrangement shall be of the form CVCVCV or VCVCVC

Total arrangements = 2 (3! * 3!)/2! = 36

Again APURNA is of type VCVCCV, so this word shall not feature in our arrangement

Total number of re-arrangements = 36 − 0 = 36

25. (c) 6 correct forecasts = ${}^{8}C_{6}$ * 1^{6} ways

Remaining 2 incorrect forecasts in 2^{2} ways

Total number of different forecasts = ${}^{8}C_{6}$ * 1^{6} * 2^{2} = 28 * 4 = 112.

26. (d) 1, 1, 2, 2, 3, 4 = (1, 1) (2, 2) 3, 4

We have 4 different digits, where 2 of it occurs twice

a) Choosing 4 different digits and arranging it in ${}^4C_4$ * 4! = 24 ways

b) Choosing 2 same and 2 different digits and arranging it in ${}^2C_1$ * ${}^3C_2$ * 4!/2! = 72 ways

c) Choosing 2 same and 2 same digits and arranging it in ${}^2C_2$ * 4!/2!2! = 6 ways

Total number of 4 digits numbers formed = 24 + 72 + 6 = 102

27. (c) $^{n-1}P_r + r.^{n-1}P_{r-1} = \frac{(n-1)!}{(n-r-1)!} + r.\frac{(n-1)!}{(n-r)!} = (n-1)!\left[\frac{1}{(n-r-1)!} + \frac{r}{(n-r)(n-r-1)!}\right]$

$= (n-1)!\left[\frac{1}{(n-r-1)!} + \frac{r}{(n-r)(n-r-1)!}\right] = (n-1)!\left[\frac{n}{(n-r).(n-r-1)!}\right] = \frac{n.(n-1)!}{(n-r)!} = {}^nP_r$

28. (c) Digits – 1, 2, 3, 4, 5

3 X (2, 4) = 1 * ${}^3P_1$ * ${}^2P_1$ = 6

4 X (2) = 1 * ${}^3P_1$ * 1 = 3

5 X (2, 4) = 1 * ${}^3P_1$ * ${}^2P_1$ = 6

X X X (2, 4) = ${}^4P_3$ * ${}^2P_1$ = 48

X X X X (2, 4) = ${}^4P_4$ * ${}^2P_1$ = 48

Total numbers = 6 + 3 + 6 + 48 + 48 = 111

29. (d) N! contains n terms, thus (2n)! Shall contain (2n) terms

(2n)! = (2n) (2n – 1) (2n – 2) (2n – 3) 3.2.1

$= [(2n)(2n-2)(2n-4).....6.4.2]_{n\text{ terms}} .[(2n-1)(2n-3).....5.3.1]_{n\text{ terms}}$

$= 2^n [n(n-1)(n-2)....3.2.1] .[(2n-1)(2n-3).....5.3.1]_{n\text{ terms}}$

$= 2^n.n! [1.3.5.....(2n-3)(2n-1)]$

30. (b) 1 Million = 10, 00, 000 = 7 digit number starting with digit 1.

We have digits = 0, 4, 4, 5, 5, 5, 3 = 7 digits (O included)

Total numbers formed without any restrictions = 7!/2!3! = 420

Total numbers starting with "0" = 6!/2!3! = 60

Thus, total numbers greater than a million = 420 – 60 = 360

31. (c) Digits = 0, 1, 2, ..., 9 = 10 digits

1 digit number divisible by 5 = 1

2 digits number divisible by 5:

Ending with 5: $({}^{9}P_{1} - {}^{1}P_{1}) = 8$

Ending with 0: ${}^{9}P_{1} = 9$

Total = 8 + 9 = 17

3 digits number divisible by 5:

Ending with 5: $({}^{9}P_{2} - {}^{8}P_{1}) = 72 - 8 = 64$

Ending with 0: ${}^{9}P_{2} = 72$

Total = 64 + 72 = 136

Total numbers less than 1000 divisible by 5 = 1 + 17 + 136 = 154

32. (a) Total cases, without any restrictions = ${}^{8}P_{8} = 8!$

2 papers always come together = (8 − 2 + 1)! * 2! = 2.7!

Never comes together = 8! − 2.7!

33. (a) Total copies = (5*1) + (4*2) + (6 * 3) + (1 * 8) = 5 + 8 + 18 + 8 = 39 books

Out of these 39 books, 5, 4, 4, 6, 6, 6 copies of different kinds are alike.

Total arrangement = $39!/5!.(4!)^2.(6!)^3$

34. (d) Total person = 1 + 6 + 4 = 11

4 children shall be seated at the two ends in 4P4 = 4! = 24 ways

For old man (4 + 2) = 6 seats are not available. He shall be seated in ${}^{5}P_{1} = 5$ ways

Remaining 6 person can be seated in 6P6 = 720 ways

Total arrangements = 24 * 5 * 720 = 86400.

35. (a) 2 sides, i.e., 4 person on each side. (3 + 2 = 5 person fixed) (remaining 3 need to be arranged)

3 are fixed on row side. 1 more can be selected from 3 in ${}^{3}C_{1}$ ways and arranged in 4! Ways

2 are fixed on other side. 2 more selected from remaining 2 in 2C2 = 1 way and arranged in 4! ways.

Total number of arrangements = ${}^{3}C_{1} * 4! * 4! = {}^{3}C_{1} * (4!)^2$

36. (a) 8 words having alternative solution can be selected in $2*2*...*2_{8 \text{ times}} = 2^8$ ways

Remaining 12 words can be selected in $1^{12} = 1$ way

Total possible solutions = $2^{8}*1 = 2^8 = 256 = 16^2 = (2 * 8)^2$

37. (b) A L L A H A B A D – 9 letters word, with 4 even place and 5 odd place

Vowels = 4 (A A A A) (all alike)

Consonants = 5 (L L H B D) (2 are alike)

Vowels occupying even places in ${}^4P_4/4! = 1$ way

Consonants occupying the remaining 5 places in $5!/2! = 60$ ways

Total arrangements = 1 * 60 = 60.

38. (d) Digits = 1, 2, 3, 4, 5, 6 = 6 digits

If repetition is not allowed:

4 digits number starting with 3 = 3 X X X = $1 * {}^5P_3 = 60$

If repetition is allowed:

4 digits number starting with 3 = 3 X X X = 1 * 6 * 6 * 6 = 216

[NOTE: In the sum, it is not mentioned whether repetition is allowed or not]

39. (d) Digits – 1, 2, 3, 4, 5 = 5 digits

Numbers greater than 23000 shall be of 5-digit only

Assuming, repetition is not allowed:

2 3 X X X = ${}^3P_3 = 6$

2 4 X X X = ${}^3P_3 = 6$

2 5 X X X = ${}^3P_3 = 6$

(3, 4, 5) X X X X = ${}^3P_1 * {}^4P_4 = 72$

Total = 6 + 6 + 6 + 72 = 90

[NOTE: Here we can't assume, repetition is allowed, as in that case, answer shall be infinity]

40. (a) Out of 7 persons 3 persons sit together, but in particular order (i.e., 3 persons shall be treated as alike items).

Total number of sitting = (7 − 3 + 1) * 3!/3! = 5!

41. Since all shall not have same neighbours hence, we have to view the arrangement either from clockwise or anti-clockwise sense and not both

Required arrangement $= \frac{1}{2}(n-1)!$

$= \frac{1}{2}(6-1)!$

$= \frac{1}{2}5! = \frac{120}{2}$

= 60 ways

42. (c) 2 men out of n persons shall always feature in the team. It means, 2 particular objects are always included.

Total number of selection = ${}^{(n-2)}C_{(12-2)} = {}^{(n-2)}C_{10}$ ways.

43. (d) C O M B I N A T I O N

(O O), (I I), (N N), C, M, B, A, T

8 different letters, 3 letters are in alike pairs

Choosing 4 different letters = 8C4 = 70

Choosing 2 alike and 2 different letters = ${}^{3}C_{1} * {}^{7}C_{2}$ = 63

Choosing 2 same and 2 same letters = ${}^{3}C_{2}$ = 3

Total number of selection = 70 + 63 + 3 = 136

44. (a) Arrange 21 red balls (alike) in 21!/21! = 1 way

Now arrange 19 blue balls (alike) in 22 places (22 gaps generated by 21 red balls) in 22P19/19! = 1540

Total number of arrangements = 1 * 1540 = 1540

SELF ASSESSMENT TEST 8
PERMUTATION
20 Question, 20 Marks

1. How many different eight letters words can be formed out of the letters of the word DAUGHTER so that the word starts with D and ends with R?

a) 720 b) 5040 c) 6 d) None of the above

2. How many three digits odd numbers can be formed using 0, 1, 2, 3, 4, 5 with repetitions?

a) 3 b) 60 c) 90 d) 120

3. How many even numbers are there with three digits such that if 5 is one of the digits, then 7 is the next digit?

a) 5 b) 360 c) 365 d) None of the above

4. The number of all 3 digit numbers which have at least one digit as 7 is:

a) 252 b) 648 c) 392 d) None of the above

5. There are 5 gentlemen and 4 ladies to dine at a round table. In how many ways can they seat themselves so that no two ladies are together?

a) 4! * 5! b) 9! c) 5! * 3! * 2! d) None of the above

6. Find the value of n if four times the number of permutations of n things taken 3 together is equal to 5 times the number of permutations of (n − 1) things taken 3 together.

a) 13 b) 15 c) 16 d) None of the above

7. How many different eight letters words can be formed out of the letters of the word DAUGHTER so that the position of letter H remains unchanged?

a) 600 b) 5040 c) 4050 d) None of the above

8. The number of arrangements of 3 items taken from a group of "A" items is 12 times the num-ber of arrangements of 1 item taken from the same group of "A" items. Find A.

a) 2 b) 3 c) 4 d) 5

9. Three dice are rolled simultaneously. In how many of the outcomes will there be at least one "3"?

a) 6^3 b) 5^3 c) $6^3 - 5^3$ d) None of the above

10. How many even integers between 4000 and 7000 have all digits different?

a) 600 b) 720 c) 728 d) None of the above

11. The letters of the word SPACE are written in all possible order and if these words are written out in a dictionary then the rank of the word SPACE is:

a) 112 b) 113 c) 114 d) 115

12. How many different eight letters words can be formed out of the letters of the word DAUGHTER so that the relative position of vowels and consonants remains unaltered?

a) 720 b) 5040 c) 60 d) None of the above

13. A cat invites 3 rats and 4 cockroaches for dinner. How many seating arrangements are possible along a round table? Assume that animals of a species all look alike, though they will be deeply offended at this assumption.

a) 7! b) 35 c) 350 d) None of the above

14. How many different eight letters words can be formed out of the letters of the word DAUGHTER so that no two vowels are together?

a) 12200 b) 13200 c) 14400 d) None of the above

15. How many necklaces you can make of 14 beads, 6 being blue, 4 red, 3 green and 1 yellow?

a) 33000 b) 30300 c) 30030 d) None of the above

16. In how many ways can three girls and nine boys be seated in two vans, each having numbered seats, 3 in front and 4 at the back?

a) ${}^{14}P_{11}$ b) ${}^{14}P_{12}$ c) ${}^{11}P_{9}$ d) None of the above

17. How many different eight letters words can be formed out of the letters of the word DAUGHTER so that all vowels never occur together?

a) 40320 b) 4320 c) 36000 d) None of the above

18. How many 4 digit numbers divisible by 4 can be made with digits 4, 5, 6, 7, 8; if repetition of digits is not allowed?

a) 6 b) 36 c) 216 d) None of the above

19. Find the number of divisors of the number 36000.

a) 36 b) 216 c) 72 d) None of the above

20. In how many ways can three prizes be given to 20 boys when a boy may receive any number of prizes?

a) 20! b) $^{20}P_3$ c) 8000 d) None of the above

EXPLANATORY ANSWERS

1. Words starting with D and ending with R = ${}^{1}P_{1}$ * ${}^{6}P_{6}$ * ${}^{1}P_{1}$ = 720. Option A

2. First digit = 1, 2, 3, 4, 5 – 5 options
   Second digit = 0, 1, 2, 3, 4, 5 = 6 options
   Third / Last digit = 1, 3, 5 = 3 options
   No. of 3 digit odd numbers = 5 * 6 * 3 = 90; Option C

3. 57A – A can be 0, 2, 4, 6, 8 = 5 cases
   A57 – Not possible, as we are looking for even numbers
   Now, any 3 digit number not containing 5 and even = (8) * (9) * (5) = 360
   Total = 360 + 5 = 365; Option C

4. All 3 digit numbers = 9*10*10 = 900
   All 3 digit numbers, not including 7 = 8*9*9 = 648
   At least 1 digit is 7 = 900 – 648 = 252 Option A

5. 5 gentlemen can be arranged at a round table in (5 – 1)! = 4! ways
   Now in 5 gaps, 4 ladies can be arranged in 4! Ways
   Total = 4! * 4! Option D

6. $4 * {}^{n}P_{3} = 5 * {}^{(n-1)}P_{3}$
   $4n(n-1)(n-2) = 5(n-1)(n-2)(n-3)$
   $4n = 5n - 15$
   Therefore, n = 15. Option B

7. H is fixed. Remaining 7 letters will be arranged in 7! Ways = 5040 ways. Option B

8. ${}^{A}P_{3} = 12 * {}^{A}P_{1}$
   $A(A-1)(A-2) = 12A$
   $A^2 - 3A + 2 = 12$
   $A^2 - 3A - 10 = 0$
   $(A-5)(A+2) = 0$
   A = 5 Option D

9. Total outcomes = 6^3

Outcomes with no "3" = 5^3

At least one "3" = $6^3 - 5^3$ Option C

10. A) 4 - - (2, 6, 8, 0) = 1 * ${}^8P_2$ * 4 = 224

B) 5 - - (2, 4, 6, 8, 0) = 1 * ${}^8P_2$ * 5 = 280

C) 6 - - (2, 4, 8, 0) = 1 * ${}^8P_2$ * 4 = 224

Total cases = 224 + 280 + 224 = 728; Option C

11. SPACE – A, C, E, P, S

Starting with A = 1*${}^4P_4$ = 24

Starting with C = 1*${}^4P_4$ = 24

Starting with E = 1*${}^4P_4$ = 24

Starting with P = 1*${}^4P_4$ = 24

Starting with SA = 1*${}^3P_3$ = 6

Starting with SC = 1*${}^3P_3$ = 6

Starting with SE = 1*${}^3P_3$ = 6

Starting with SPACE = 1

Rank = 24+24+24+24+6+6+6+1 = 115 Option D

12. 3 vowels, 5 consonants. Arrangements = 3! * 5! = 6 * 120 = 720 ways. Option A

13. "Fix" the position of the cat. Now remaining 3 rats and 4 cockroaches can be seated in 7!/3!4! = 35 ways. Option B

14. Place 5 consonants in ${}^5P_5$ = 120 ways.

Now in 6 places generated between and on the sides of 5 consonants, 3 vowels can be placed in ${}^6P_3$ = 120 ways.

Total ways = 120 * 120 = 14400. Option C

15. Here we have one yellow bead which is not repeated. Hence we can "fix" its position. Now remaining 6 blue, 4 red and 3 green beads can be arranged in 13!/6!4!3! = 60060 ways. However, since a necklace can be turned over, actual number of arrangements is 60060/2 = 30030. Option C

16. As there are a total of 14 seats and 12 persons are to be seated, it can be done in ${}^{14}P_{12}$ ways. Option B

17. Total cases = 8! = 40320

    Vowels are always together = (8 − 3 + 1)! * 3! = 720 * 6 = 4320

    Vowels never occur together = 40320 − 4320 = 36000; Option C

18. A number is divisible by 4, when last 2 digits of the number are divisible by 4.

    The last 2 digits pairs shall be 48, 56, 64, 68, 76, 84 = 6 cases

    With each case, the first two digits can be arranged in 3P2 = 6 ways

    Total arrangements = 6 * 6 = 36

    Option B

19. Factorizing 36000 = $2^5 . 3^2 . 5^3$

    This means any divisor of 36000 is of the type 2a.3b.5c where a can take values 0, 1, 2, 3, 4, 5; b can take values 0, 1, 2; c can take values 0, 1, 2, 3. Hence number of divisors is 6 * 3 * 4 = 72. [Note, both 1 and 36000 are counted among 72 divisors].

    Option C

20. Ways = 20^3 = 20 * 20 * 20 = 8000. Option C

SELF ASSESSMENT TEST 9
COMBINATION
25 Question, 25 Marks

1. Determine the number of ways of obtaining 4 heads and 2 tails in 6 tosses of a coin.
   a) 21 b) 15 c) 10 d) None of the above

2. In how many ways a committee, consisting of 5 men and 6 women can be formed from 8 men and 10 women?
   a) 266 b) 5040 c) 11760 d) 86400

3. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there in the committee. In how many ways can it be done?
   a) 645 b) 756 c) 564 d) 735

4. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?
   a) 210 b) 1050 c) 21400 d) 25200

5. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?
   a) 205 b) 209 c) 194 d) 159

6. In how many ways can final eleven be selected from 15 cricket players if there is no restriction?
   a) 1365 b) 1001 c) 364 d) 572

7. In how many ways can final eleven be selected from 15 cricket players if one of them must be included?
   a) 1365 b) 1001 c) 572 d) 364

8. In how many ways can final eleven be selected from 15 cricket players if one of them, who is in bad form, must always be excluded?
   a) 1365 b) 1001 c) 364 d) 572

9. In how many ways can final eleven be selected from 15 cricket players if two of them being leg spinners, one and only one leg spinner must be included?

a) 364 b) 572 c) 1563 d) None of the above

10. A box contains 2 white balls. 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

a) 32 b) 48 c) 64 d) 96

11. How many words can be formed by taking 4 different letters of the word MATHEMATICS?

a) 756 b) 1680 c) 2454 d) 18

12. In how many ways can be mixed doubles tennis games be arranged from a group of 10 players consisting of 6 men and 4 women?

a) 80 b) 90 c) 180 d) 360

13. In an examination paper there are two groups, each containing 4 questions. A candidate is required to attempt 5 questions but not more than 3 questions from any group. In how many ways can 5 questions be selected?

a) 24 b) 48 c) 96 d) None of the above

14. There are 15 points in a plane, no three of which are in the same straight line, except 4 which are collinear. Find the ratio of total number of straight line to total number of triangles that can be formed on this plane.

a) 200 : 451 b) 99 : 449 c) 100 : 451 d) None of the above

15. Seven points lie on a circle. How many chords can be drawn by joining these points?

a) 21 b) 22 c) 23 d) 24

16. A convex polygon has twice as many diagonals as the number of sides. The number of sides of the polygon is:

a) 5 b) 6 c) 7 d) 8

17. Find the number of ways in which one can make a selection of at least one book on each of the three subjects P, C and M from a collection containing 3 different books on P, 4 different books on C and 5 different books on M.

a) 3522 b) 3255 c) 3525 d) None of the above

18. In how many ways Arjun can invite 5 of his friends, viz. Arman, Aakash, Anirudh, Alok and Aditya to a dinner so that two or more of them remain present?

a) 31 b) 32 c) 26 d) 25

19. From 8 boys and 6 girls 7 are to be selected for admission for a particular course. In how many ways can this be done if there must be exactly 2 boys?

a) 186 b) 172 c) 187 d) 168

20. In how many ways Ankita can invite 6 of her friends, viz. Aditi, Amita, Amrisha, Amrita, Aakansha and Aakriti to her birthday treat so that Aditi is always present?

a) 32 b) 63 c) 64 d) 31

21. How many five-letter words containing 3 vowels and 2 consonants can be formed using the letters of the word EQUATION so that the two consonants occur together?

a) 720 b) 1440 c) 360 d) None of the above

22. A is planning to give a birthday party at his place. In how many ways can he invite one or more of his five friends and make them sit at a circular table?

a) 744 b) 74 c) 89 d) None of the above

23. 2 balls need to be selected from a bag. In how many ways is it possible to choose a white ball and a black ball from a bag containing 5 white and 4 black balls?

a) 9 b) 12 c) 15 d) 20

24. In how many ways can 18 identical white and 16 identical black balls be arranged in a row so that no two black balls are together?

a) 969 b) 699 c) 996 d) None of the above

25. A tea party is arranged for 16 people along two sides of a long table with 8 chairs on each side. Four men wish to sit on one particular side and two on the other side. In how many ways can they be seated?

a) ${}^{10}C_4 * 7! * 7!$ b) ${}^{10}C_4 * 8! * 7!$

c) ${}^{10}C_4 * 8! * 8!$ d) None of the above

EXPLANATORY ANSWERS

1. Out of 6 tosses – 4 heads means automatically there will be 2 tails.
Ways = ${}^6C_4 * {}^2C_2 = 15 * 1 = 15$. Option B

2. Ways = ${}^8C_5 * {}^{10}C_6 = 56 * 210 = 11760$. Option C

3. $({}^7C_3 * {}^6C_2) + ({}^7C_4 * {}^6C_1) + ({}^7C_5) = (35 * 15) + (35 * 6) + 21 = 525 + 210 + 21 = 756$.
Option B

4. Words = ${}^7C_3 * {}^4C_2 * {}^5P_5 = 35 * 6 * 120 = 25200$. Option D

5. ${}^{10}C_4 - {}^4C_4 = 210 - 1 = 209$. Option B

6. 11 players can be selected out of 15 in ${}^{15}C_{11} = 1365$ ways. Option A

7. A particular player is always included. We have to select 10 more out of remaining 14 players in ${}^{14}C_{10} = 1001$ ways. Option B

8. We have to choose playing 11 from the remaining 14 players in ${}^{14}C_{11} = 364$ ways.
Option C

9. We need to select 1 from 2 leg spinners and other 10 from the remaining 13 in ${}^2C_1 * {}^{13}C_{10} = 2 * 286 = 572$ ways. Option B

10. ${}^9C_3 - {}^6C_3 = 84 - 20 = 64$. Option C

11. MATHEMATICS = (MM, AA, TT), (H, E, I, C, S)
$= {}^8C_4 * 4! + {}^3C_1 * {}^7C_2 * \frac{4!}{2!} + {}^3C_2 * \frac{4!}{2!2!} = 2454$ (C)

12. ${}^6C_2 * {}^4C_2 * 2! = 15 * 6 * 2 = 180$ ways. Option C

13. Number of selection = $({}^4C_3 * {}^4C_2) + ({}^4C_2 * {}^4C_3) = (4 * 6) + (6 * 4) = 48$. Option B

14. Straight Lines = ${}^{15}C_2 - {}^{4}C_2 + 1 = 100$
    Triangles = ${}^{15}C_3 - {}^{4}C_3 = 451$
    Ratio = 100 : 451. Option C

15. Number of chords = ${}^{7}C_2 = 21$. Option A

16. $2N = N(N - 3)/2$
    $4N = N^2 - 3N$
    $N^2 = 7N$; $N = 7$. Option C

17. $(2^3 - 1) * (24 - 1) * (2^5 - 1) = 7 * 15 * 31 = 3255$. Option B

18. ${}^{5}C_2 + {}^{5}C_3 + {}^{5}C_4 + {}^{5}C_5 = 10 + 10 + 5 + 1 = 26$. Option C

19. ${}^{8}C_2 * {}^{6}C_5 = 28 * 6 = 168$. Option D

20. ${}^{1}C_1 * 2^5 = 32$
    [Aditi is always present ${}^{1}C_1$, Others may come or may not come = 2^5]. Option A

21. EQUATION = (E, A, I, O, U) + (Q, T, N)
    Number of words = ${}^{5}C_3 * {}^{3}C_2 * (5 - 2 + 1)! * 2! = 10 * 3 * 24 * 2 = 1440$. Option B

22. ${}^{5}C_1 * (1-1)! + {}^{5}C_2 * (2-1)! + {}^{5}C_3 * (3-1)! + {}^{5}C_4 * (4-1)! + {}^{5}C_5 * (5-1)!$
    $= 5 + 10 + 20 + 30 + 24 = 89$. Option C

23. $({}^{5}C_1 * {}^{4}C_1) = 20$. Option D

24. When 18 identical white balls are put in a straight line, there will be 19 spaces created. Thus 16 black balls will have 19 places to fill in. This will give an answer of ${}^{19}C_{16} = 969$. (Here, the balls are identical, the arrangement is not important). Option A

25. Having seated 4 on side A and 2 on side B, we are left with 10 persons. We can choose 4 of them for side A in ${}^{10}C_4$ ways, and for side B we are automatically left with 6 persons. Now 8 persons on each side can be arranged among themselves is 8! Ways. Hence required number of seating arrangements = ${}^{10}C_4 * 8! * 8!$. Option C

6 SEQUENCES AND SERIES (PROGRESSIONS)

THEORY

- A sequence is defined as an array of numbers in such a manner so that there is a similarity in a given array, which enables us to determine the term or terms preceding or succeeding to such an array.

- A sequence can be categorized into 3 parts:
  a) Arithmetic Progression
  b) Geometric Progression
  c) Harmonic Progression

	Arithmetic Progression	Geometric Progression
Definition	Series which increases or decreases by a fixed quantity	Series which increases or decreases by a fixed proportion
First Term	a	a
Constant	Common Difference = d	Common Ratio = r
Last Term	$l = t_n = a + (n-1)d$	$l = t_n = a.r^{n-1}$
Sum	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_n = \frac{n}{2}(a + l)$	$S_n = a.\frac{1-r^n}{1-r}$ when $r < 1$ $S_n = a.\frac{r^n - 1}{r - 1}$ when $r > 1$

- If three numbers are in G.P., their Logarithms are always in A.P.

Infinite GP Series

$$a + ar + ar^2 + ar^3 + \ldots\ldots\ldots\ldots\ldots\alpha = \frac{a}{1-r} \quad \text{given } |r| < 1$$

Sum of Natural Numbers:

$$\sum n = 1+2+3+......+n = \frac{n(n+1)}{2}$$

$$\sum n^2 = 1^2+2^2+3^2+......+n^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum n^3 = 1^3+2^3+3^3+......+n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}$$

Harmonic Progression(H.P)

- Three numbers are in H.P, If their reciprocals are in A.P
- a,b,c are in H.P , if $\frac{1}{a}\ \frac{1}{b}\ \frac{1}{c}$ are in A.P.
- H.P fails when one of the terms of the A. P is Zero.

$$t_n \text{ of HP} = \frac{1}{t_n \text{ of the crresponding A.P}}$$

Concept of A.M , G.M and H.M

If a & b are any unequal real positive numbers then,

	A.M(A)	G.M(G)	H.M(H)
Definition	$\frac{a+b}{2}$	$+\sqrt{ab}$	$\frac{2ab}{a+b}$
Relation	i) A > ii) $A\times H$	G > = G^2	H

Things to remember

- **The ratio of the sum of X number of A.Ms to the sum of Y number of A.Ms is always X : Y**
- **Two numbers can have more than one A.M/G.M/H.M**
- **A.Ms/G.Ms/ H.Ms are also the members of A.P/G.P/ H.P**

CLASSWORK SECTION

ARITHMETIC PROGRESSION

Choose the most appropriate option (a), (b), (c) or (d).

1. The nth element of sequence 5, 7, 9, 11 . . . is
   (a) $3n + 2$ (b) $n + 4$ (c) $2n + 3$ (d) none of these

2. If −17, −13, −9, . . . in the progression then t_{10} =
   (a) 41 (b) 43 (c) 40 (d) 19

3. Which term of the progression −1, −3, −5, . . . is −57
   (a) 27^{th} (b) 29^{th} (c) 39^{th} (d) none of these

4. Which term of the series 7 + 11 + 15 + . . . is 467?
   (a) 116 (b) 190 (c) 119 (d) 125

5. The 10th term in $3, \frac{9}{2}, 6, \frac{15}{2}$ . . . is ____
   (a) 33 (b) $\frac{33}{2}$ (c) $-\frac{33}{2}$ (d) −33

6. If the ninth term of an AP is 30 then S_{17} = ____
   (a) 150 (b) 501 (c) 510 (d) 105

7. The a^{th} term of an AP is b and b^{th} term is a. Then c^{th} term of it is
   (a) $a + b + c$ (b) $b + a - 2c$
   (c) $a + b + c/2$ (d) $a + b - c$

8. Third term of an AP is 8 and the 17th term is 51/2. The 23rd term is
   (a) 37 (b) 33 (c) 41 (d) 31

9. The n^{th} term of the series whose sum to n terms $3n^2 + 2n$ is
   (a) $3n - 1$ (b) $8n - 2$ (c) $11n - 3$ (d) none of these

10. If 3 consecutive terms of AP are K + 2, 4K − 6 and K − 2 then k = ____
    (a) 1 (b) 2 (c) 3 (d) none of these

11. The 16th term of an AP is 99 and common difference is 8 then S_{21} is:
(a) 1230 (b) 1290 (c) 1239 (d) 1293

12. The sum of all odd numbers between 100 and 200 is
(a) 6200 (b) 6500 (c) 7500 (d) 3750

13. The sum of all positive integral multiples of 3 less than 100 is
(a) 1584 (b) 1665 (c) 1683 (d) none of these

14. The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 or 5 is
(a) 10200 (b) 15200 (c) 16200 (d) none of these

15. The sum of all numbers between 400 and 900 which are divisible by 13 is
(a) 22504 (b) 29405 (c) 25402 (d) 25350

16. The 4 arithmetic means between – 2 and 23 are
(a) 3, 13, 8, 18
(b) 18, 3, 8, 13
(c) 3, 8, 13, 18
(d) none of these

17. The r^{th} term of AP is $(3r - 1)/6$. The sum of first p terms of the series is
(a) $n(3p + 1)$
(b) $(p/12)(3p + 1)$
(c) $(p/12)(3p - 1)$
(d) none of these

18. A sum of Rs. 6240 is paid off in 30 installments such that each installment is Rs. 10 more than the preceding installment. The value of first installment is
(a) Rs. 36 (b) Rs. 30 (c) Rs. 60 (d) none of these

19. The 1st and the last term of an AP are −4 and 146. The sum of the terms is 7171. The number of terms
(a) 101 (b) 100 (c) 99 (d) none of these

PAST YEARS QUESTIONS

20. The sum of all natural numbers between 100 and 1000 which are multiple of 5 is
(a) 98450 (b) 96450 (c) 97450 (d) 95450

21. On 1st January every year a person buys national saving certificates of value exceeding that of his last years purchase by Rs. 100. After 10 years he finds that the total value of the certificates purchased by him is Rs. 54500. Find the value of certificates purchased by him in the first year

(a) 6000 (b) 4000 (c) 5000 (d) 5500

22. If in an AP, Tn represent nth term $t_7 : t_{10} = 5 : 7$ then $t_8 : t_{11}$ = _____

(a) 13 : 16 (b) 17 : 23 (c) 14 : 17 (d) 15 : 19

23. If sum of 3 arithmetic means between 'a' and 22 is 42 then 'a' = ____

(a) 14 (b) 11 (c) 10 (d) 6

24. If each month Rs. 100 increases in any sum then find out the total after 10 months if the sum of first month is Rs. 2000.

(a) 24500 (b) 24000 (c) 50000 (d) 60000

25. The 4th term of AP is three times the first term and the 7th term exceeds twice the third term by 1. Find the first term 'a' and common difference 'd'.

(a) a = 3, d = 2 (b) a = 4, d = 3
(c) a = 5, d = 4 (d) a = 6, d = 5

GEOMETRIC PROGRESSION

26. The last term of the series 0.5, 1, 2, 4 . . . to 8 term is

(a) 64 (b) 128 (c) 512 (d) none of these

27. Sum of three numbers x, y, z are in a GP is 39 and their product is 729. The values of x, y, z are

(a) 3, 27, 9 (b) 9, 3, 27 (c) 3, 9, 27 (d) none of these

28. If x, y, z are in GP, and xyz = 27/8. The value of y is

(a) 3/2 (b) 2/3 (c) 2/5 (d) none of these

29. The value of three numbers in GP, so that their sum is (57/2) and product is 729 are

(a) 2, 9, 27 (b) 6, 9, 27/2
(c) 4, 16, 64/3 (d) none of these

30. A ball is dropped from a height of 48 m and rebounds two third of the distance it falls. It continued to fall and rebound in this way, how far will it travel before coming to rest

(a) 240 m (b) 260 m (c) 380 m (d) none

31. If a, b, c are in GP, $a^p = b^q = c^r$ then 1/p, 1/q, 1/r are in

(a) AP (b) GP (c) HP (d) none of these

32. If x, y, z are pth, qth and rth terms of a GP then the value of $x^{q-r}\, y^{r-p}\, z^{p-q}$ is

(a) 0 (b) 1 (c) −1 (d) none of these

33. Let A be the A.M and G_1, G_2 be two GMs between two positive numbers. Then $G_1^3 + G_2^3$ is equal to

(a) $2AG_1G_2$ (b) AG_1G_2 (c) $(AG_1G_2)/2$ (d) none

34. If the pth term of the series 16, 8, 4, . . . is $\frac{1}{2^{17}}$. The value of p is

(a) 25 (b) 22 (c) 23 (d) none of these

35. If the first term and the common ratio of a GP are 1 and 1/2 respectively and sum of its n terms is equal to $\frac{255}{128}$. The value of n is

(a) 6 (b) 5 (c) 8 (d) none of these

36. If 'S' be the sum 'P' the product and 'R' the sum of the reciprocals of n terms in a GP then 'P' is . . .. of S^n and R^{-n}

(a) Arithmetic mean (b) Geometric mean

(c) Harmonic mean (d) none of these

37. The sum of 3 numbers in A.P. is 15. If 1, 4 and 19 added to them respectively the results are in G.P. The numbers are

(a) 26, 5, −16 (b) 2, 5, 8 (c) 5, 8, 2 (d) both (a) and (b)

38. Given x, y, z are in GP, $x^p = y^q = z^\sigma$ then $\frac{1}{p}, \frac{1}{q}, \frac{1}{\sigma}$ are in

(a) AP (b) GP

(c) Both AP and GP (d) none of these

39. If $x = a + a/r + a/r^2 + \ldots \infty$, $y = b - b/r + b/r^2 - \ldots \infty$; $z = c + c/r^2 + c/r^4 + \ldots \infty$, then value of $(xy)/z-(ab)/c$ is

(a) 0 (b) 1 (c) −1 (d) none of these

40. The value of $S = 2/3 + 5/9 + 2/27 + 5/81 + \ldots$ to infinite terms is

(a) 11/8 (b) 8/11 (c) 3/11 (d) none of these

41. The third term of GP is 2, the product of first five term is

(a) 2^5 (b) 2^3 (c) 5^3 (d) none of these

PAST YEARS QUESTIONS

42. Find n such that $\dfrac{a^{n+1}+b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b

(a) 1/2 (b) 1 (c) −1/2 (d) 0

43. If the first term of a GP exceeds the second term by 2 and the sum to infinity is 50; the series is

(a) $10, 8, \dfrac{32}{5} \ldots$ (b) $10, 8, \dfrac{5}{2}, \ldots$

(c) $10, \dfrac{10}{3}, \dfrac{10}{9}, \ldots$ (d) none

44. In a GP if the (p + q)th term is m and $(p - q)^{th}$ term is n then pth term is

(a) mn (b) $\sqrt{mn}$ (c) m^2 (d) n^2

45. If G be geometric mean between a and b then the value of $\dfrac{1}{G^2 - a^2} + \dfrac{1}{G^2 - b^2}$ is equal to

(a) G^2 (b) $3G^2$ (c) $1/G^2$ (d) $2/G^2$

46. Find the product of $243, 243^{1/6}, 243^{1/36}, \ldots$ to ∞

(a) 1024 (b) 27 (c) 729 (d) 246

47. Geometric mean of $p, p^2, p^3, \ldots p^n$ be

(a) p^{n+1} (b) $p^{\frac{n}{2}}$ (c) $p^{\frac{(n+1)}{2}}$ (d) none of these

48. A GP (Geometric Progression) consists of 2n terms. If the sum of the terms occupying the odd places is S_1 and that of the terms in even places is S_2. The common ratio of the progression is

(a) n (b) $2S_1$ (c) $\frac{S_2}{S_1}$ (d) $\frac{S_1}{S_2}$

SPECIAL SERIES

49. The sum of 'n' term of the series 1x4 + 3x7 + 5x10 + ___________

(a) $\frac{n}{2}[5n^2 + 4n - 1]$ (b) $\frac{n}{2}[4n^2 + 5n - 1]$

(c) $\frac{n}{2}[4n^2 + 5n + 1]$ (d) None

50. 7+77+777+................ n terms is equal to

(a) $\frac{7}{9}[10^{n+1} - 10] - \frac{7n}{9}$ (b) $\frac{7}{9}[10^{n+1} - 10] + \frac{7n}{9}$

(c) $\frac{7}{81}[10^{n+1} - 10] - \frac{7n}{9}$ (d) $\frac{7}{81}[10^{n+1} - 10] + \frac{7n}{9}$

51. $\frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + \ldots\ldots\ldots$ n terms is equal to

(a) $\frac{n}{2n+1}$ (b) $\frac{n}{n+1}$ (c) $\frac{1}{n+1}$ (d) None

52. Sum of 'n' terms whose t_n is $n^2 + 2^n$

(a) $\frac{n(n+1)(2n+1)}{6} + 2(2^n - 1)$ (b) $\frac{(n+1)(2n+1)}{6} + 2(2^n - 1)$

(c) $\frac{n(n+1)^2}{6} + 2(2^n - 1)$ (d) None

MIXED BAG

53. If 10 times the 10th term of an A.P. is equal to 15 times the 15th term, then 25th term of the A.P. is _____.

a) 1 b) 25 c) 0 d) −25

54. If the sum of p terms of an AP is same as the sum of its q terms, then the sum of the first (p + q) terms is:

a) 0 b) p + q c) p − q d) None of the above

55. If S_1, S_2, S_3 be the sums of n terms of three AP and the first term of each AP being 1 and the respective common difference are 1, 2, 3; then $S_1 + S_3 = ?$

a) S_2 b) $3S_2$ c) $0.5S_2$ d) $2S_2$

56. An AP consists of n terms. If the sum of its first three terms is x and the sum of the last three terms is y then the sum of all the terms of the AP is:

(a) $\frac{n}{6}(xy)$ (b) $\frac{n}{6}(x+y)$ (c) $n(x+y)$ (d) $\frac{n(x-y)}{6}$

57. 300 trees are planted in a regular pattern in rows in the shape of an isosceles triangle, the numbers in the successive rows diminishing by one from the base to the apex. How many trees are there in the row, which forms the base of the triangle

a) 30 b) 21 c) 27 d) 24

58. The first and the last term of an AP are "a" and "1" respectively. The sum of n^{th} term from the beginning and the n^{th} term from the end is:

a) a + 1 b) a − 1 c) a + 31 d) 2a + 1

59. If the sums of n, 2n and 3n terms of an AP be S_1, S_2 and S_3 respectively, then show that $S_3 = ?$

a) $3(S_2 - S_1)$ b) $(S_2 - S_1)$ c) $2(S_2 - S_1)$ d) $3(S_2 + S_1)$

60. If S_n be the sum of n consecutive terms of an AP, then the value of $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$ is:

a) 0 b) 1 c) 2 d) 3

61. The sum of first n terms of two AP are in the ratio (7n + 2) : (n + 4). Find the ratio of their 5th terms.

a) 1 : 5 b) 5 : 1 c) 2 : 3 d) 3 : 2

62. $31^3 + 32^3 + 33^3 + ... + 50^3$

a) 2010000 b) 3025000 c) 2870000 d) 1409400

63. The common ratio, last term, and the sum of a G.P. are 3, 486 and 728 respectively. The first term of the progression is:

a) 4 b) 6 c) 8 d) 2

64. When a certain golf ball is dropped on a piece of pavement, it bounces to a height of three-fifth the distance from which it falls. If the ball is dropped from a height of 100 cm, how far it has travelled when it hits the pavement for the tenth time ?

a) 397 cm b) 400 cm c) 450 cm d) 460 cm

65. If a, b, x, y, z are positive numbers such that a, x, b are in AP; a, y, b are in GP and $(a + b)z = 2ab$, then x, y, z are in:

a. Arithmetic Progression b. Geometric Progression
c. Harmonic Progression d. None of the above

66. If S_1, S_2, S_3 be respectively the sum of n, 2n and 3n terms of a GP, then $S_1 (S_3 - S_2) - (S_2 - S_1)^2$ is:

a) n b) 2n c) 3n d) 0

67. If "a" be the first term, "b" the nth term and "p" the product of the first n terms of a GP, then which of the following is true?

a) $p = ab$ b) $p = (ab)^n$
c) $p^2 = (ab)^n$ d) None of the above

68. The sum of 1st six terms of a G.P. is 9 times the sum of the first three terms. Find the common ratio.

a. 2 b. 3 c. 4 d. 8

69. The sum of the first three terms of a G.P. is to the sum of the first six terms as 125:152. Find the common ratio of the G.P.

a. 0.40 b. 0.50 c. 0.75 d. 0.60

70. The first, tenth and twenty-eighth term of an AP are three successive terms of a GP. Find the common ratio of the GP. given that the sum of the first 28 terms of the AP is 210, find its first term.

a. 2, 2 b. 2, 3 c. 3, 2 d. − 3, 2

71. An air pump used to extract air from a vessel removes one-tenth of the air at stroke each stroke. Find what fraction of original volume of air is left after the 5th stroke.

a) 0.54899 b) 0.54999 c) 0.59049 d) 0.60099

72. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP, then $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in :

a. Harmonic Progression
c. Geometric Progression
b. Arithmetic Progression
d. None of the above

73. If a^2, b^2, c^2 are in AP, the $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in :

a. Geometric Progression
c. Both a) and b) above
b. Arithmetic Progression
d. None of the above

74. If a, b, c are in AP, then $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in:

a. Geometric Progression
c. Arithmetic Progression
c. None of the above
d. Harmonic Progression

75. If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in AP, then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in :

a. Harmonic Progression
c. Arithmetic Progression
b. Geometric Progression
d. None of the above

76. Evaluate the following:

(i) $0.\overline{4}$
(ii) $0.\overline{42}$
(iii) $0.\overline{423}$
(iv) $0.4\overline{2}$
(v) $0.4\overline{23}$
(vi) $0.42\overline{3}$
(vii) $7.\overline{42}$

77. The ratio of the sum of x AM to y AM between two numbers is:

a) x : y
b) $x^2 : y^2$
c) 1 : 1
d) None of the above

78. If a, b, c are in GP and x, y be the arithmetic means between a, b and b, c respectively, then which of the following/s is/are true?

(a) $\frac{a}{x}+\frac{c}{y}=2$
(b) $\frac{1}{x}+\frac{1}{y}=\frac{2}{b}$
(c) Both a) and b) above
(d) Neither a) nor b) is true

HOMEWORK SECTION

1. The nth element of the sequence 1, 3, 5, 7, ...is
   (a) n (b) 2n – 1 (c) 2n +1 (d) none of these

2. The nth element of the sequence –1, 2, –4, 8 ... is
   (a) $(-1)^n 2^{n-1}$ (b) 2^{n-1} (c) 2^n (d) none of these

3. $\sum_{i=4}^{7} \sqrt{2i-1}$ can ne written as
   (a) $\sqrt{7} + \sqrt{9} + \sqrt{11} + \sqrt{13}$ (b) $2\sqrt{7} + \sqrt{9} + 2\sqrt{11} + 2\sqrt{13}2$
   (c) $2\sqrt{7} + \sqrt{9} + 2\sqrt{11} + 2\sqrt{13}2$ (d) None of these

4. The sum to ∞ of the series -5,25,-125,625, ...can be written as
   (a) $\sum_{k=1}^{\infty}(-5)^k$ (b) $\sum_{k=1}^{\infty}(5)^k$ (c) $\sum_{k=1}^{\infty}-5^k$ (d) None of these

5. The first three terms of sequence when nth term t_n is $n^2 - 2n$ are
   (a) –1, 0, 3 (b) 1, 0, 2 (c) –1, 0, –3 (d) none of these

6. Which term of the progression –1, –3, –5, ... is –39
   (a) 21^{st} (b) 20^{th} (c) 19^{th} (d) none of these

7. The value of x such that 8x + 4, 6x – 2, 2x + 7 will form an AP is
   (a) 15 (b) 2 (c) 15/2 (d) none of the these

8. The m^{th} term of an A. P. is n and n^{th} term is m. The rth term of it is
   (a) m + n +r (b) n + m – 2r (c) m + n + r/2 (d) m + n – r

9. The number of the terms of the series 10 + $9\frac{2}{3} + 9\frac{1}{3} + 9 +$......... will amount to 155 is
   (a) 30 (b) 31 (c) 32 (d) (a) and (b) both

10. The n^{th} term of the series whose sum to n terms is $5n^2 + 2n$ is
    (a) 3n – 10 (b) 10n – 2 (c) 10n – 3 (d) none of these

11. The 20^{th} term of the progression 1, 4, 7, 10................is
    (a) 58 (b) 52 (c) 50 (d) none of these

12. The last term of the series 5, 7, 9, ... to 21 terms is

(a) 44 (b) 43 (c) 45 (d) none of these

13. The last term of the A.P. 0.6, 1.2, 1.8, ... to 13 terms is

(a) 8.7 (b) 7.8 (c) 7.7 (d) none of these

14. The sum of the series 9, 5, 1, ... to 100 terms is

(a) −18,900 (b) 18,900 (c) 19,900 (d) none of these

15. The two arithmetic means between −6 and 14 is

(a) 2/3, 1/3 (b) 2/3, $7\frac{1}{3}$ (c) -2/3, $-7\frac{1}{3}$ (d) None of these

16. The sum of three integers in AP is 15 and their product is 80. The integers are

(a) 2, 8, 5 (b) 8, 2, 5 (c) 2, 5, 8 (d) 8, 5, 2

17. The sum of n terms of an AP is $3n^2 + 5n$. The series is

(a) 8, 14, 20, 26 (b) 8, 22, 42, 68

(c) 22, 68, 114, ... (d) none of these

18. The number of numbers between 74 and 25,556 divisible by 5 is

(a) 5,090 (b) 5,097 (c) 5,095 (d) none of these

19. The pth term of an AP is (3p − 1)/6. The sum of the first n terms of the AP is

(a) n (3n + 1) (b) n/12 (3n + 1)

(c) n/12 (3n − 1) (d) none of these

20. The arithmetic mean between 33 and 77 is

(a) 50 (b) 45 (c) 55 (d) none of these

21. The 4 arithmetic means between −2 and 23 are

(a) 3, 13, 8, 18 (b) 18, 3, 8, 13

(c) 3, 8, 13, 18 (d) none of these

22. The first term of an A.P is 14 and the sums of the first five terms and the first ten terms are equal in magnitude but opposite in sign. The 3rd term of the AP is

(a) $6\frac{4}{11}$ (b) 6 (c) 4/11 (d) none of these

23. The sum of a certain number of terms of an AP series −8, −6, −4, ... is 52. The number of terms is

(a) 12 (b) 13 (c) 11 (d) none of these

24. The first and the last term of an AP are −4 and 146. The sum of the terms is 7171. The number of terms is

a) 101 (b) 100 (c) 99 (d) none of these

25. The sum of the series 3½ + 7 + 10½ + 14 + ... to 17 terms is

(a) 530 (b) 535 (c) 535 ½ (d) none of these

26. The 7^{th} term of the series 6, 12, 24, ...is

(a) 384 (b) 834 (c) 438 (d) none of these

27. t_8 of the series 6, 12, 24,... is

(a) 786 (b) 768 (c) 867 (d) none of these

28. t_{12} of the series −128, 64, −32, ...is

(a) − 1/16 (b) 16 (c) 1/16 (d) none of these

29. The 4th term of the series 0.04, 0.2, 1, ... is

(a) 0.5 (b) 1/2 (c) 5 (d) none of these

30. The last term of the series 1, 2, 4,.... to 10 terms is

(a) 512 (b) 256 (c) 1024 (d) none of these

31. The last term of the series 1, −3, 9, −27 up to 7 terms is

(a) 297 (b) 729 (c) 927 (d) none of these

32. The last term of the series x^2, x, 1, ... to 31 terms is

(a) x^{28} (b) 1/x (c) $1/x^{28}$ (d) none of these

33. The sum of the series −2, 6, −18, ... to 7 terms is

(a) −1094 (b) 1094 (c) − 1049 (d) none of these

34. The sum of the series 243, 81, 27, ... to 8 terms is

(a) 364 (b) $364\frac{13}{30}$ (c) $364\frac{1}{9}$ (d) None of these

35. The sum of the series $1+\sqrt{3} + 1 + 3/\sqrt{3}$ + ... to 18 terms is

(a) $\frac{9841(1+\sqrt{3})}{\sqrt{3}}$ (b) 9841 (c) $\frac{9841}{\sqrt{3}}$ (d) None of these

36. The second term of a G P is 24 and the fifth term is 81. The series is

(a) 16, 36, 24, 54,.. (b) 24, 36, 53, ...

(c) 16, 24, 36, 54,.. (d) none of these

37. The sum of 3 numbers of a G P is 39 and their product is 729. The numbers are

(a) 3, 27, 9 (b) 9, 3, 27 (c) 3, 9, 27 (d) none of these

38. In a G. P, the product of the first three terms 27/8. The middle term is

(a) 3/2 (b) 2/3 (c) 2/5 (d) none of these

39. If you save 1 paise today, 2 paise the next day 4 paise the succeeding day and so on, then your total savings in two weeks will be

(a) ₹ 163 (b) ₹ 183 (c) ₹ 163.83 (d) none of these

40. Sum of n terms of the series 4 + 44 + 444 + ... is

(a) $4/9 \{ 10/9 (10^n - 1) - n \}$ (b) $10/9 (10^n - 1) - n$

(c) $4/9 (10^n - 1)^{-n}$ (d) none of these

41. Sum of n terms of the series 0.1 + 0.11 + 0.111 + ... is

(a) $(1/9) \{n - (1 - (0.1)^n)\}$ (b) $(1/9) \{n - (1-(0.1)^n)/9\}$

(c) $n - 1 - (0.1)^n/9$ (d) none of these

42. The sum of the first 20 terms of a G. P is 244 times the sum of its first 10 terms. The common ratio is

(a) $\pm\sqrt{3}$ (b) ± 3 (c) $\sqrt{3}$ (d) None of these

43. Sum of the series 1 + 3 + 9 + 27 +...is 364. The number of terms is

(a) 5 (b) 6 (c) 11 (d) none of these

44. The product of 3 numbers in G P is 729 and the sum of squares is 819. The numbers are
(a) 9, 3, 27 (b) 27, 3, 9 (c) 3, 9, 27 (d) none of these

45. The sum of the series 1 + 2 + 4 + 8 + .. to n term
(a) $2^n - 1$ (b) $2n - 1$ (c) $1/2^n - 1$ (d) none of these

46. The sum of the infinite GP 14, − 2, + 2/7, − 2/49, + ... is
(a) $4\frac{1}{12}$ (b) $12\frac{1}{4}$ (c) 12 (d) None of these

47. The sum of the infinite GP 14, − 2, + 2/7, − 2/49, + ... is
(a) $4\frac{1}{12}$ (b) $12\frac{1}{4}$ (c) 12 (d) none of these

48. The sum of the infinite G. P. 1 - 1/3 + 1/9 - 1/27 +... is
(a) 0.33 (b) 0.57 (c) 0.75 (d) none of these

49. The number of terms to be taken so that 1 + 2 + 4 + 8 + will be 8191 is
(a) 10 (b) 13 (c) 12 (d) none of these

50. Four geometric means between 4 and 972 are
(a) 12, 36, 108, 324 (b) 12, 24, 108, 320
(c) 10, 36, 108, 320 (d) none of these

51. Three numbers are in AP and their sum is 21. If 1, 5, 15 are added to them respectively, they form a G. P. The numbers are
(a) 5, 7, 9 (b) 9, 5, 7 (c) 7, 5, 9 (d) none of these

52. The sum of $1 + 1/3 + 1/3^2 + 1/3^3 +$............... upto infinite
(a) 2/3 (b) 3/2 (c) 4/5 (d) none of these

53. The sum of the infinite series 1 + 2/3 + 4/9 + .. is
(a) 1/3 (b) 3 (c) 2/3 (d) none of these

54. The sum of the first two terms of a G.P. is 5/3 and the sum to infinity of the series is 3. The common ratio is
(a) 1/3 (b) 2/3 (c) − 2/3 (d) (b) & (c) both

55. If p, q and r are in A.P. and x, y, z are in G.P. then x^{q-r}. y^{r-p}. z^{p-q} is equal to

(a) 0 (b) −1 (c) 1 (d) none of these

56. The sum of three numbers in G.P. is 70. If the two extremes by multiplied each by 4 and the mean by 5, the products are in AP. The numbers are

(a) 12, 18, 40 (b) 10, 20, 40
(c) 40, 20, 10 (d) (b) & (c) both

57. The sum of 3 numbers in A.P. is 15. If 1, 4 and 19 be added to them respectively, the results are is G. P. The numbers are

(a) 26, 5, −16 (b) 2, 5, 8 (c) 5, 8, 2 (d) (a) & (b) both

58. Given x, y, z are in G.P. and $x^p = y^q = z^\sigma$, then 1/p , 1/q, $1/\sigma$ are in

(a) A.P. (b) G.P.
(c) Both A.P. and G.P. (d) none of these

59. If the terms 2x, (x+10) and (3x+2) be in A.P., the value of x is

(a) 7 (b) 10 (c) 6 (d) none of these

60. If A be the A.M. of two positive unequal quantities x and y and G be their G. M, then

(a) A < G (b) A>G (c) A ≥ G (d) A ≤ G

61. The A.M. of two positive numbers is 40 and their G. M. is 24. The numbers are

(a) (72, 8) (b) (70, 10) (c) (60, 20) (d) none of these

62. Three numbers are in A.P. and their sum is 15. If 8, 6, 4 be added to them respectively, the numbers are in G.P. The numbers are

(a) 2, 6, 7 (b) 4, 6, 5 (c) 3, 5, 7 (d) none of these

63. The sum of four numbers in G. P. is 60 and the A.M. of the first and the last is 18. The numbers are

(a) 4, 8, 16, 32 (b) 4, 16, 8, 32 (c) 16, 8, 4, 20 (d) none of these

64. A sum of ₹ 6240 is paid off in 30 instalments such that each instalment is ₹ 10 more than the proceeding installment. The value of the 1st instalment is

(a) ₹ 36 (b) ₹ 30 (c) ₹ 60 (d) none of these

65. The sum of $1.03 + (1.03)^2 + (1.03)^3 + ...$ to n terms is

(a) $103 \{(1.03)^n - 1\}$ (b) $103/3 \{(1.03)^n - 1\}$

(c) $(1.03)^n - 1$ (d) none of these

66. If x, y, z are in A.P. and x, y, (z + 1) are in G.P. then

(a) $(x - z)^2 = 4x$ (b) $z^2 = (x - y)$

(c) $z = x - y$ (d) none of these

67. The numbers x, 8, y are in G.P. and the numbers x, y, −8 are in A.P. The value of x and y are

(a) (−8, −8) (b) (16, 4) (c) (8, 8) (d) (a) & (b) both

68. The nth term of the series 16, 8, 4,..... in $1/2^{17}$. The value of n is

(a) 20 (b) 21 (c) 22 (d) none of these

69. The sum of n terms of a G.P. whose first terms is 1 and the common ratio is 1/2 , is equal to $1\frac{127}{128}$. The value of n is

(a) 7 (b) 8 (c) 6 (d) none of these

70. $t_4 = x$, $t_{10} = y$ and $t_{16} = z$. are in G.P., Then

(a) $x^2 = yz$ (b) $z^2 = xy$ (c) $y^2 = zx$ (d) none of these

71. If x, y, z are in G.P., then

(a) $y^2 = xz$ (b) $y (z^2 + x^2) = x (z^2 + y^2)$

(c) $2y = x+z$ (d) none of these

72. The sum of all odd numbers between 200 and 300 is

(a) 11,600 (b) 12,490 (c) 12,500 (d) 24,750

73. The sum of all natural numbers between 500 and 1000 which are divisible by 13, is

(a) 28,405 (b) 24,805 (c) 28,540 (d) none of these

74. If unity is added to the sum of any number of terms of the A.P. 3, 5, 7, 9, ... the resulting sum is

(a) 'a' perfect cube (b) 'a' perfect square

(c) 'a' number (d) none of these

75. The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 or 5 is

(a) 10,200 (b) 15,200 (c) 16,200 (d) none of these

76. The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 and 5 is

(a) 2,200 (b) 2,000 (c) 2,220 (d) none of these

77. A person pays ₹ 975 by monthly instalment each less then the former by ₹ 5. The first instalment is ₹ 100. The time by which the entire amount will be paid is

(a) 10 months (b) 15 months (c) 14 months (d) none of these

78. A person saved ₹ 16,500 in ten years. In each year after the first year he saved ₹ 100 more than he did in the preceding year. The amount of money he saved in the 1st year was

(a) ₹ 1000 (b) ₹ 1500 (c) ₹ 1200 (d) none of these

79. At 10% C.I. p.a., a sum of money accumulate to ₹ 9625 in 5 years. The sum invested initially is

(a) ₹ 5976.37 (b) ₹ 5970 (c) ₹ 5975 (d) ₹ 5370.96

80. The population of a country was 55 crores in 2005 and is growing at 2% p.a C.I. the population is the year 2015 is estimated as

(a) 57.05 (b) 60.05 (c) 67.00 (d) none of these

HOMEWORK SOLUTIONS

1. (c) Here a = 3, cd = d = 3 − 1 = 2

   T(n) = a + (n − 1)d = 3 + (n − 1)2 = 2n + 1;

2. (a) Here a = - 1, cr = r = 2/-1 = -2

   $T(n) = a.r^{n-1} = (-1).(-2)^{n-1} = (-1).(-1)^{n-1}.2^{n-1} = (-1)^n.2^{n-1}$

3. (a)

   $$\sum_{i=4}^{7}\sqrt{2i-1}=\sqrt{7}+\sqrt{9}+\sqrt{11}+\sqrt{13}$$

4. (a) Here a = - 5, common ratio = r = 25/ - 5 = -5

   So the terms are: $(-5), (-5)^2, (-5)^3, (-5)^4, \ldots\ldots, (-5)^k, \ldots.$

   Sum to infinity can be written as: $\sum_{k=1}^{\alpha}(-5)^k$

5. (a) $T(n) = n^2 - 2n$

   $T(1) = 1^2 - 2(1) = -1$

   $T(2) = 2^2 - 2(2) = 0$

   $T(3) = 3^2 - 2(3) = 3$

6. (b) Here, a = -1, cd = -3 − (-1) = - 2, T(n) = - 39 = L

   $$N = \frac{L-a}{d}+1=\frac{-39-(-1)}{-2}+1=\frac{-38}{-2}+1=19+1=20$$

   Value of 20th term is − 39.

7. (c) Three numbers a, b, c are in AP, i.e., 2b = a + c

   2 (6x − 2) = (8x + 4) + (2x + 7) = 9x + 11

   Or, 2x = 11 + 4 = 15

   Thus, x = 15/2.

8. (d) For a AP series, if T(A) = B and T(B) = A, then T(C) = A + B − C

   So, if T(m) = n & T(n) = m, the rth term = T(r) = m + n − r

9. (d) A = 10, D = 29/3 – 10 = - 1/3, S(n) = 155

$$155 = \frac{n}{2}[2A + (n-1)D] = \frac{n}{2}\left[20 - \frac{n-1}{3}\right] = \frac{n}{6}[61 - n]$$

$$Or, 930 = n(61-n); \therefore n^2 - 61n + 930 = 0, (n-30)(n-31) = 0$$

n = 30 or 31.

10. (c) S(n) = $5n^2$ + 2n

S(1) = T(1) = 5 + 2 = 7 = A

S(2) = 20 + 4 = 24 = T(1) + T(2). T(2) = 24 – 7 = 17

CD = D = T(2) – T(1) = 17 – 7 = 10

T(n) = A + (n – 1)D = 7 + (n – 1)10 = 10n – 3

11. (a) A = 1, CD = 4 – 1 = 3, T(20) = A + 19D = 1 + 19(3) = 58.

12. (c) A = 5, D = 7 – 5 = 2, T(21) = A + 20D = 5 + 40 = 45.

13. (b) A = 0.6, D = 1.2 – 0.6 = 0.6, T(13) = A + 12D = 0.6 + 7.2 = 7.8.

14. (a) A = 9, D = 5 – 9 = - 4, S(100) = 100/2[18 – 99(4)] = 50[- 378] = - 18900.

15. (b) A = - 6, T(4) = 14 = A + 3D. Thus D = 20/3

The two AM are (-6 + 20/3) = 2/3 and (2/3 + 20/3) = 22/3 = 7 1/3

16. (c) or (d)

Option selection shall also help. Both (c) and (d) sum up to 15 and product is 80 and also are in AP. Both options are correct.

Or, one can take numbers (A – D), A, (A + D), and solve the following to equations to find the value of A and D.

(1) [A + D + A + A – D] = 15

(2) (A – D).A.(A + D) = 80

On solving one shall get numbers, 2-5-8 or 8-5-2.

17. (a) $S(n) = 3n^2 + 5n$

$S(1) = T(1) = 3 + 5 = 8$

$S(2) = 12 + 10 = 22 = T(1) + T(2)$, $T(2) = 22 - 8 = 14$

$CD = D = T(2) - T(1) = 14 - 8 = 6$

The AP series is: 8, 14, 20, 26, ...

18. (b) A= 75, L = 25555, D = 5

$$N = \frac{L-A}{D} + 1 = \frac{25555-75}{5} + 1 = \frac{25480}{5} + 1 = 5097$$

19. (b) T(p) = 1/6 [3p − 1]

$$S(n) = \sum_{p=1}^{n} T_p = \frac{1}{6}\left[3\sum n - n\right] = \frac{1}{6}\left[\frac{3n(n+1)}{2} - n\right] = \frac{1}{12}\left[3n^2 + n\right] = \frac{n}{12}(3n+1)$$

(c) AM between 33 and 77 = (33 + 77)/2 = 110/2 = 55.

21. (c) A = - 2, T(6) = 23 = A + 5D, D = 25/5 = 5

The 4 AM between -2 and 23 are:

(- 2 + 5)= 3; (3 + 5) = 8; (8 + 5) = 13; (13 + 5) = 18;

(a) A = 14

S(5) = - S(10)

$$\frac{5}{2}[28+4D] = -\frac{10}{2}[28+9D]$$

$$140 + 20D = -280 - 90D$$

$$110D = -420; D = -42/11$$

T(3) = A + 2D = 14 − 84/11 = 70/11 = 6 4/11.

23. (b) A = - 8, D = - 6 + 8 = 2, L = S(N) = 52

$$52 = \frac{N}{2}[-16 + (N-1)2]$$

$$104 = N(2N - 18)$$

$$52 = N(N-9)$$

Or, $N^2 - 9N - 52 = 0$

Or, (N − 13)(N + 4) = 0

N = 13;

24. (a) 7171 = N/2(-4 + 146) = 71N

N = 7171 / 71 = 101.

25. (c) A = 3.5, D = 7 − 3.5 = 3.5

S(17) = 17/2[7 + 16(3.5)] = 535.5.

26. (a) A = 6, R = 12/6 = 2. $T_7 = A.R^6 = 6(2)^6 = 384$.

27. (b) A = 6, R = 12/6 = 2. $T_8 = A.R^7 = 6(2)^7 = 768$.

28. (c) A = - 128, R = 64/-128 = - ½

$T_{12} = A.R^{11} = (-128).(-½)^{11} = 2^7/2^{11} = ½^4 = 1/16$.

29. (c) A = 0.04, R = 0.2/0.04 = 5. $T_4 = A.R^3 = 0.04(5)^3 = 5$.

30. (a) A = 1, R = 2/1 = 2, $T_{10} = A.R^9 = 1.(2)^9 = 512$.

31. (b) A = 1, R = -3/1 = -3. $T_7 = A.R^6 = (1).(-3)^6 = 729$.

32. (c) $A = x^2$, $R = x/x^2 = 1/x$, $T_{31} = A.R^{30} = x^2(1/x)^{30} = 1/x^{28}$.

33. (a) A = - 2, R = 6/-2 = -3

$$S_7 = (-2)\left[\frac{1-(-3)^7}{1-(-3)}\right] = \frac{-2}{4}\left(1+3^7\right) = -1094$$

34. (d) A = 243, R = 81/243 = 1/3

$$S_8 = (243)\left[\frac{1-\frac{1}{3^8}}{1-\frac{1}{3}}\right] = 3^5\left[\frac{3^8-1}{3^8}.\frac{3}{2}\right] = \frac{3^8-1}{18} = \frac{6560}{18} = \frac{3280}{9} = 364\frac{4}{9}$$

35. (d) A = 1/√3, R = 1/A = √3

$$S_{18} = \left(\frac{1}{\sqrt{3}}\right)\left[\frac{\left(\sqrt{3}\right)^{18}-1}{\sqrt{3}-1}\right] = \left(\frac{1}{\sqrt{3}}\right)\left[\frac{19682}{\sqrt{3}-1}\right]\frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)} = \frac{(9841)\left(\sqrt{3}+1\right)}{\sqrt{3}}$$

(c) T_2 = A.R = 24; T_5 = A.R^4 = 81

R^3 = T_5/T2 = 81/24 = (27/8) = $(3/2)^3$

R = 3/2

A = 24/R = 24/1.5 = 16

Series: 16, 24, 36, 54, 81, ...

37. (d) Options can be used.

a) 3, 27, 9 are not in GP. Rejected

b) 9, 3, 27 are not in GP, rejected

c) 3, 9, 27 are in GP. Sum = 39, Product = 729

But again, for 27, 9, 3, which are also in GP, same Sum = 39 and Product = 729 exists.

We thus have 2 set of answer: (3 – 9 – 27) & (27 – 9 – 3).

38. (a) Product of three numbers in GP = 27/8. Let the middle term is A.

A^3 = 27/8 = $(3/2)^3$

A = 3/2.

39. (c) A = 1, R = 2/1 = 2, N = 14

$$S_{14} = (1)\left[\frac{2^{14}-1}{2-1}\right] = 16383 = Rs.163.83$$

40. (a) S(3) = 4 + 44 + 444 = 492

Putting n= 3 in the options, the option which gives result 492 is the correct option.

(a) 4/9 [10/9(1000 – 1) – 3] = 492

41. (b) S(3) = 0.1 + 0.11 + 0.111 = 0.321

Putting n= 3 in the options, the option which gives result 0.321 is the correct option.

(a) 1/9{3 – (1 – $(0.1)^3$)} = 0.222

(b) 1/9{3 – (1 – $(0.1)^3$)/9} = 0.321

42. (a) S(20) = 243 S(10)

$$a\left[\frac{r^{20}-1}{r-1}\right]=244.a\left[\frac{r^{10}-1}{r-1}\right]$$

$$\left(r^{10}+1\right)=244$$

$r^{10} = 243 = (\sqrt{3})^{10}$; $r = \pm\sqrt{3}$

43. (b) A = 1, R = 3/1 = 3, S_n = 364

$$364=(1).\left[\frac{3^n-1}{3-1}\right]$$

$3^n = 729 = 3^6$; n = 6.

44. (d) Using options we get, 3,9,27 and 27,9,3 both fits in the given criteria.

45. (a) A = 1, R = 2/1 = 2, S(n) = $1(2^n - 1)/(2 - 1) = (2^n - 1)$.

46. (b) A = 14, R = -2/14 = -1/7, Sum to infinity = 14/(1 + 1/7) = 98/8 = 49/4 = 12 ¼.

47. (b) A = 14, R = -2/14 = -1/7, Sum to infinity = 14/(1 + 1/7) = 98/8 = 49/4 = 12 ¼.

48. (c) A = 1, R = -1/3 / 1 = -1/3, Sum to infinity = 1/(1 + 1/3) = ¾ = 0.75.

49. (b) A = 1, R = 2/1 = 2, S(n) = 8191

$$8191=(1).\left[\frac{2^n-1}{2-1}\right]$$

$2^n = 8192 = 2^{13}$. Thus, n = 13.

50. (a) A = 4, T_6 = A.R^5 = 4.R^5 = 972. Thus R = 3.
Four geometric means are: (4*3 = 12), (12*3 = 36), (36*3 = 108), (108*3=324)

51. (a) Of the options given, 5-7-9 are in AP. Again 5 + 7 + 9 = 21.
(5+1), (7+5), (9+15) = 6, 12, 24 and these are in GP.
Thus required numbers are 5,7,9.

52. (b) $A = 1$, $R = 1/3$, $S(\infty) = 1/(1 - 1/3) = 3/2$.

53. (b) $A = 1$, $R = 2/3$, $S(\infty) = 1/(1 - 2/3) = 3$.

54. (d) $A + AR = A(1 + R) = 5/3$

$A/(1 - R) = 3$

$(1 + R)(1 - R) = (1 - R^2) = 5/9$

$R^2 = 4/9 = (2/3)^2$

$R = \pm 2/3$

55. (c) AP: p(2), q(3), r(4)

GP: x(2), y(4), z(8)

$x^{q-r}.y^{r-p}.z^{p-q} = 2^{-1}.4^2.8^{-1} = 16/16 = 1$

56. (d) 10, 20, 40 are in GP. 40, 100, 160 are in AP. Option B fits in

Again, 40, 20, 10 are also in GP. 160, 100, 40 are also in AP. Option C also fits in.

57. (b) 2, 5, 8 are in AP and sum is 15. (2 + 1), (5 + 4), (8 + 19) = 3, 9, 27 are in GP

Again, 8, 5, 2 are also in AP. (8 + 1), (5 + 4), (2 + 19) = 9, 9, 21 are not in GP

58. (a) x, y, z are in GP. $y^2 = xz$

$x^p = y^q = z^r = k$

$k^{2/q} = k^{1/p}.k^{1/r}$

$2/q = 1/p + 1/r$

Thus, 1/p, 1/q, 1/r are in AP

59. (c) $2(x + 10) = 2x + 3x + 2$

$2x + 20 = 5x + 2$

$x = 18/3 = 6$;

60. (b) For unequal quantities, AM > GM.

61. (a) $A = 40$, $G = 24$, $A^2 - G^2 = 40^2 - 24^2 = 1024$

$X = A + \sqrt{(A^2 - G^2)} = 40 + 32 = 72$

$Y = A - \sqrt{(A^2 - G^2)} = 40 - 32 = 8$

62. (d) 3, 5, 7 are in AP, and sum = 15.

(3 + 8), (5 + 6), (7 + 4) = 11, 11, 11. But this is not in GP.

63. (d) 4, 8, 16, 32 are in GP. Sum = 4 + 8 + 16 + 32 = 60

AM of 4 and 32 = 36/2 = 18

But again, 32, 16, 8, 4 are also in GP and fits in the criteria.

64. (d) A = A, D = 10, N = 30, S(30) = 6240

6240 = 30/2 [2A + 290]

416 = 2A + 290

A = 63;

65. (b) Series is: $(1.03)^1, (1.03)^2, (1.03)^3, \ldots$

A = 1.03, R = 1.03

$S_n = (1.03)\,[(1.03)^n - 1]/(1.03 - 1) = 103/3\,[1.03^n - 1]$

66. (a) $2y = (x + z)$ and $y^2 = x(z + 1)$

$(x+z)^2/4 = x(z + 1)$

$x^2 + z^2 + 2xz = 4xz + 4x$

$x^2 + z^2 - 2xz = 4x$

$(x - z)^2 = 4x$

67. (b) $xy = 64$; $2y = (x - 8)$

$(2y + 8)y = 64$

$2y^2 + 8y - 64 = 0$

$y^2 + 4y - 32 = 0$

$(y + 8)(y - 4) = 0$

$y = 4, -8$

$x = 64/4 = 16,\ 64/-8 = -8$

$(x, y) = (16, 4), (-8, -8)$

But, -8, -8, -8 is not in AP.

68. (c) $A = 16$, $R = 8/16 = ½$, $T_n = ½17 = 16(1/2)^{n-1} = 2^{5-n}$

$5 - n = -17$

$22 = n$

69. (b) A = 1, CR = ½, S_n = 1 127/128

$$\frac{255}{128}=(1).\left[\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right]=\frac{2^n-1}{2^n}.\frac{2}{1}=\frac{2^n-1}{2^{n-1}}$$

$$\frac{256-1}{128}=\frac{2^8-1}{2^7}=\frac{2^n-1}{2^{n-1}}$$

n = 8

70. (c) T_4, T_{10}, T_{16} of a GP are also in GP

Y^2 = X.Z

71. (a) If X, Y, Z are in GP, Y^2 = XZ.

72. (c) A = 201, L = 299, D = 2, N = (299-201)/2 + 1 = 50

Sum = 50/2(201 + 299) = 12500

73. (a) A = 507, D = 13, L = 988, N = (988 − 507)/13 + 1 = 38

Sum = 38/2 [507 + 988] = 28405

74. (b) We know, 1 + 3 + 5 + 7 + ... nth term = n^2

Thus, when 1 is added to the sum of (3, 5, 7, ...), the resulting term is a perfect square.

75. (c) Divisible by 4

A = 100, D = 4, L = 300, N = (300 − 100)/4 + 1 = 51

Sum = 51/2 (100 + 300) = 10200

Divisible by 5

A = 100, D = 5, L = 300, N = (300 − 100)/5 + 1 = 41

Sum = 41/2 [100 + 300] = 8200

Divisible by both 4 and 5, i.e. 20

A = 100, D = 20, L = 300, N = (300 − 100)/20 + 1 = 11

Sum = 11/2(100 + 300) = 2200

Required sum = 10200 + 8200 − 2200 = 16200.

76. (a) Divisible by both 4 and 5, i.e. 20

$A = 100, D = 20, L = 300, N = (300 - 100)/20 + 1 = 11$

$\text{Sum} = 11/2(100 + 300) = 2200$

77. (b) $A = 100, D = -5. Sn = 975$

$975 = n/2 [200 - (n - 1)5]$

$1950 = n(205 - 5n)$

$390 = n(41 - n)$

$N2 - 41N + 390 = 0$

$(N - 26)(N - 15) = 0$

$N = 15$

78. (c) $A = A, D = 100, N = 10, \text{Sum} = 16500$

$16500 = 10/2 [2A + 900]$

$A = 1200.$

79. (a) Sum invested $= 9625 (10/11)^5 = 5976.37.$

80. (d) $P(2015) = 55 (1.02)^{10} = 67.05$ Crores

MIXED BAG (HOMEWORK)

1. If the p^{th} term of an A.P. is q and the q^{th} term is p the value of the $(p + q)^{th}$ term is________.

   (a) 0 (b) 1 (c) −1 (d) None

2. If S_1 , S_2 , S_3 be the respectively the sum of terms of n, 2n, 3n an A.P. the value of $S_3 \div (S_2 - S_1)$ is given by _______.

   (a) 1 (b) 2 (c) 3 (d) None

3. The sum of n terms of two A.P.s are in the ratio of (7n−5)/(5n+17) . Then the ________ term of the two series are equal.

   (a) 12 (b) 6 (c) 3 (d) None

4. If a, b, c are in A.P. then the value of $(a^3 + 4b^3 + c^3)/[b(a^2 + c^2)]$ is

   (a) 1 (b) 2 (c) 3 (d) None

5. If a, b, c are in A.P. then the value of $(a^2 + 4ac + c^2)/(ab + bc + ca)$ is

   (a) 1 (b) 2 (c) 3 (d) None

6. The Pth term of an A.P. is 1/q and the qth term is 1/p. The sum of the pq terms is ____________________

   (a) $\frac{1}{2}(pq + 1)$ (b) $\frac{1}{2}(pq - 1)$ (c) pq+1 (d) pq-1

7. The sum of p terms of an A.P. is q and the sum of q terms is p. The sum of p + q terms is __________.

   (a) − (p + q) (b) p + q (c) $(p - q)^2$ (d) $p^2 - q^2$

8. If S_1, S_2, S_3 be the sums of n terms of three A.P.s the first term of each being unity and the respective common differences 1, 2, 3 then $(S_1 + S_3) / S_2$ is _______.

   (a) 1 (b) 2 (c) −1 (d) None

9. 2^{4n} −1 is divisible by

   (a) 15 (b) 4 (c) 6 (d) 64

10. $3^n - 2n - 1$ is divisible by

(a) 15 (b) 4 (c) 6 (d) 64

11. The least value of n for which the sum of n terms of the series $1 + 3 + 3^2 + \ldots\ldots$ is greater than 7000 is _______.

(a) 9 (b) 10 (c) 8 (d) 7

12. If 'S' be the sum, 'P' the product and 'R' the sum of the reciprocals of n terms in a G.P. then 'P' is the ________ of S^n and R^{-n}.

(a) Arithmetic Mean (b) Geometric Mean

(c) Harmonic Mean (d) None

13. If $1+a+a^2+\ldots\ldots\infty = x$ and $1+b+b^2+\ldots\ldots\infty = y$ then $1 + ab + a^2b^2 + \ldots\ldots\infty$ is given by ____________.

(a) $(xy)/(x+y-1)$ (b) $(xy)/(x-y-1)$

(c) $(xy)/(x+y+1)$ (d) None

14. If a, b, c are in G.P. then the value of $a(b^2+c^2)-c(a^2+b^2)$ is _____

(a) 0 (b) 1 (c) -1 (d) None

15. If a, b, x, y, z are positive numbers such that a, x, b are in A.P. and a, y, b are in G.P. and $z=(2ab)/(a+b)$ then

(a) x, y, z are in G.P. (b) $x \geq y \geq z$

(c) both (d) None

16. If a, b−a, c−a are in G.P. and $a=b/3=c/5$ then a, b, c are in

(a) A.P. (b) G.P. (c) H.P. (d) None

17. If $S_1, S_2, S_3, \ldots\ldots\ldots S_n$ are the sums of infinite G.P.s whose first terms are 1, 2, 3 ....n and whose common ratios are 1/2, 1/3, ...1/(n+1) then the value of $S_1, S_2, S_3, \ldots\ldots\ldots S_n$ is

(a) $(n/2)(n+3)$ (b) $(n/2)(n+2)$

(c) $(n/2)(n+1)$ (d) n2 /2

MIXED BAG (HOMEWORK SOLUTION)

1. (a) T(1) = 2, T(2) = 1; A = 2, D = -1. T(3) = A + 2D = 2 − 2 = 0

Similarly, $(p + q)^{th}$ term in this case = 0.

2. (c) Let n = 1. The three terms of AP = 100, 200, 300

S1 = 100, S2 = 100 + 200 = 300; S3 = 100+200+300 = 600

S3 / (S2 − S1) = 600/200 = 3;

3. (b) Sum of n terms of two AP are in the ratio (7n − 5)/(5n + 17)

Equate ratio to 1, we get 7n − 5 = 5n + 17; n = 11

Required term is (11 − 1)/2 + 1 = 6^{th} term

$$\frac{S_{n1}}{S_{n2}} = \frac{\frac{n}{2}[2A_1 + (n-1)D_1]}{\frac{n}{2}[2A_2 + (n-1)D_2]} = \frac{A_1 + \left(\frac{n-1}{2}\right)D_1}{A_2 + \left(\frac{n-1}{2}\right)D_2}$$

4. (b) A, B, C are in AP. Let the numbers be 1, 2, 3

$(A^3 + 4B^3 + C^3) = (1 + 12 + 27) = 40$

$B(A^2 + C^2) = 2(1 + 9) = 20$

Required values = 40/20 = 2;

5. (b) A, B, C are in AP. Let the numbers be 1, 2, 3

$(A^2 + 4AC + C^2) = (1 + 12 + 9) = 22$

(AB + BC + CA) = (2 + 6 + 3) = 11

Required value = 22/11 = 2;

6. (a) T2 = 1/3, T3 = 1/2.

D = ½ - 1/3 = 1/6, A = 1/3 − 1/6 = 1/6

(P.Q) = 6

S_6 = 6/2 (2/6 + 5/6) = 7/2

Option (a): (PQ + 1)/2 = 7/2

7. (a) S1 = 3, S3 = 1 (P = 1, Q = 3)

A = 3

A + A + D + A + 2D = 1; 3A + 3D = 1; D = -8/3

S4 = 4/2 [6 – 8] = - 4

Option (a) = - (P + Q) = - 4;

8. (b) S1 = 1 + 2 + 3 + ... + n = 6 [Assuming n = 3)

S2 = 1 + 3 + 5 + ... = 9 (Assuming n = 3)

S3 = 1 + 4 + 7 + ... = 12 (Assuming n = 3)

(S1 + S3)/S2 = 18/9 = 2

9. (a) 2^{4n} – 1 is divisible by:

Let n = 1, Number = 16 – 1 = 15 (15 = 3 * 5)

Let n = 2, Number = 256 – 1 = 255 (127 = 3 * 5 * 17)

Common factors are 3*5 = 15;

10. (b) 3^n – 2n – 1 is divisible by:

When n = 1, Expression = (3 – 2 – 1) = 0

When n = 2, Expression = (9 – 4 – 1) = 4

When n = 3, Expression = (27 – 6 – 1) = 20

HCF of 4 and 20 is 4;

11. (a) Series is: 1, 3^1, 3^2, 3^3,....

A = 1, R = 3; Sn = 1 [3^n – 1]/(3 – 1) > 7000

3^n > 14001

3^9 = 19683, which is just greater than 14001

The least value of n is 9.

12. (b) Let n = 3. GP terms are 1, 2, 4

S = Sum = 1+2+4 = 7

P = Product = 1.2.4 = 8

R = Sum of reciprocals = 1 + ½ + ¼ = 7/4

Now, $S^3.R^{-3} = 7^3.4^3/7^3 = 4^3 = 64 = 8^2 = P^2$

P is GM between S^n and R^{-n}

13. (a) $X = 1/(1 - a)$; $X - aX = 1$; $a = (X - 1)/X$

$Y = 1/(1 - b)$; $Y - bY = 1$; $b = (Y - 1)/Y$

Required Sum $= 1/(1 - ab) = XY/(XY - XY + X + Y - 1) = XY/(X + Y - 1)$;

14. (a) A, B, C are in GP. Let $A = 1, B = 2, C = 4$

$A(B^2 + C^2) - C(A^2 + B^2) = 1(4 + 16) - 4(1 + 4) = 20 - 20 = 0$;

15. (a) Let $A = 2, B = 18, X = 10, Y = 6, Z = (2*2*18)/(2+18) = 3.6$

$X, Y, Z = 10, 6, 3.6$ are in GP

And $X > Y > Z$. (Equality shall not hold true)

16. (a) $A = B/3 = C/5 = K$

$A = K, B = 3K, C = 5K$

$A, (B - A), (C - A)$ are in GP; $K, 2K, 4K$ are in GP and that's true

$A, B, C = K, 3K, 5K$ are in AP.

17. (c) $S1 = 1/(1 - ½) = 2$

$S2 = 2/(1 - 1/3) = 3$

$Sn = (n + 1)$

$S1 + S2 + \dots Sn = 2 + 3 + \dots + (n + 1) = n(n+1)/2$;

SELF ASSESSMENT TEST 10
ARITHMETIC PROGRESSION

18 Question, 18 Marks

1. The sum of three numbers in A.P. is 33 and their product is 1155. Find the second term of the series.

a) 5 b) 7 c) 9 d) 11

2. The sum of 8^{th} and 18th terms of an A.P. is 144. Find the sum of the first 25 terms.

a) 1000 b) 1500 c) 1800 d) 2500

3. The sum of 16^{th} and 26^{th} terms of an A.P. is 200 and that of 18^{th} and 28^{th} terms is 600. Find the 22nd term.

a) 50 b) 200 c) 125 d) 175

4. If three prime numbers in AP are such that that their sum is 39, then the smallest of the prime number is:

a) 3 b) 7 c) 13 d) Data Insufficient

5. The sum of three numbers in AP is 24 and their product is 440. Find the second term of the series.

a) 5 b) 8 c) 11 d) 16

6. The sum of 4^{th} and 10^{th} term of an AP is 42. Find the sum of the first 13 terms.

a) 253 b) 263 c) 273 d) 293

7. The sum of 3^{rd} and 5^{th} term of an AP is 2 and that of 4th and 8th term is 10. Find the 2nd term.

a) 2 b) – 3 c) – 5 d) None of the above

8. Find the sum of all the numbers between 100 and 200 which are divisible by 7.

a) 2114 b) 2107 c) 2100 d) None of the above

9. What is the maximum sum of the AP: 15, 14.5, 14,...

a) 217.5 b) 218 c) 232.5 d) 233

10. The interior angles of a polygon are in AP. The smallest angle is 120° and the common differ-ence is 5°. Find the number of sides of the polygon.

a) 9 b) 16 c) Can't be determined d) None of the above

11. The sum of 4th and 7th term of an AP is 56 and that of 5th and 8th term is 68. Find the 6th term.

a) 25 b) 28 c) 31 d) 37

12. Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row of two balls and so on. If 669 more balls are added, then all the balls can be arranged in the shape of a square and each of the sides than contain 8 balls less than each side of the triangle did. Determine the initial number of balls.

a) 1540 b) 1210 c) 2878 d) 2209

13. The ratio between the sum of n terms of two arithmetic progressions is (7n + 1) : (4n + 27). The ratio of their 11th term is:

a) 124 : 105 b) 136 : 117 c) 148 : 111 d) None of the above

14. Find the sum of the following series till n terms: $1+5+12+22+35+.......+t_n$.

a) $\frac{1}{2}(n+1)$ b) $\frac{n}{2}(n+1)$ c) $\frac{n^2}{2}(n+1)$ d) $\frac{n^3}{2}(n+1)$

15. The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 34. What is the common difference of the A.P.?

a) 1.5 b) 2.5 c) 3.5 d) 5.5

16. The first and the last terms of an A.P. are A and L respectively. The sum of nth term from the beginning and nth term from the end is:

a) A + 2L b) A + 3L c) A + L d) 2A + L

17. The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.

a) 1, 7, 13 b) 7, 13, 19 c) 1, 5, 9 d) None of the above

18. The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. What is the sum of the first term, the common difference and the sum of first 20 terms?

a) 740 b) 742 c) 741 d) 743

EXPLANATORY ANSWERS

1. Sum = 33 = 3A; A = 11
   Numbers are: (11 – D), 11, (11 + D)
   Option D

2. A + 7D + A + 17A = 144
   2A + 24D = 144
   S(25) = 25/2[2A + 24D] = 25/2(144) = 1800
   Option C

3. A + 15D + A + 25D = 200; 2A + 40D = 200; A + 20D = 100
   A + 17D + A + 27D = 600; 2A + 44D = 600; A + 22D = 300
   D = 100, A = - 1900
   T22 = A + 21D = - 1900 + 2100 = 200
   Option B

4. 3A = 39, A = 13
   Numbers are (13 – D), 13, (13 + D)
   Two seta are possible: (3, 13, 23) and (7, 13, 19)
   Option D

5. 3A = 24, A = 8
   Numbers are (8 – D), 8, (8 + D)
   Second term of the series is 8. Option B

6. A + 3D + A + 9D = 42; 2A + 12D = 42
   S(13) = 13/2[2A + 12D] = 13/2(42) = 273
   Option C

7. A + 2D + A + 4D = 2; 2A + 6D = 2; A + 3D = 1
   A + 3D + A + 7D = 10; 2A + 10D = 10; A + 5D = 5
   D = 2, A = - 5
   T(2) = A + D = - 5 + 2 = - 3
   Option B

8. A = 105, L = 196, D = 7
N = (196 − 105)/7 + 1 = 14
S(14) = 14/2[105 + 196] = 2107
Option B

9. A = 15, D = - 0.5, L = 0.5
N = (0.5 − 15)/-0.5 + 1 = 30
S(30) = 30/2 [15 + 0.5] = 232.5
S(31) = 31/2 [15 + 0] = 232.5
Option C

10. Sum = (2N − 4)*90 = N/2[240 + (N − 1)5]; N = 9
Sum of interior angles of N sided polygon is (2N − 4)*90°
Option A

11. A + 3D + A + 6D = 56; 2A + 9D = 56
A + 4D + A + 7D = 68; 2A + 11D = 68
D = 6, A = 1
T(6) = A + 5D = 1 + 30 = 31
Option C

12. Total number of balls inside the triangle = 1 + 2 + 3 +... + N = N(N + 1)/2
Number of balls in each side of the square = (N − 8)
Thus, $(N − 8)^2$ = N(N + 1)/2 + 669; N = 55
Initial number of balls = 55*56/2 = 1540
Option A

13. $N/2[2A_1 + (N − 1)D_1] / N/2[2A_2 + (N − 1)D_2] = [A_1 + (N − 1)D_1/2] / [A_2 + (N − 1)D_2/2]$
Putting (N − 1)/2 = 10; we get N = 21
If we put N = 21, we get, [A1 + 10D1] / [A2 + 10D2] which becomes the ratio of their 11th term.
Ratio = (7*21 + 1) : (4*21 + 27) = 148 : 111
Option C

14. $T1 + T2 + T3 = 1 + 5 + 12 = 18$

Put $n = 3$, in the options.

a) $\frac{1}{2}(n + 1) = 2$

b) $n/2(n + 1) = 6$

c) $n^2/2(n+1) = 18$ – Option C

15. $A + 3D + A + 7D = 24$; $2A + 10D = 24$

$A + 5D + A + 9D = 34$; $2A + 14D = 34$

$4D = 10$; $D = 2.5$

Option B

16. Nth term from the beginning is the last term = L

Nth term from the end is the first term = A

Thus, Sum = A + L

Option C

17. $3A = 21$; $A = 7$

Now if numbers are 1, 7, 13 (as given in option A), we get:

Product of first and third term = $1 * 13 = 13$, which is 6 more than the second term, i.e. 6

The three terms are either 1, 7, 13 or 13, 7, 1

Option A

18. $A + 2D = 7$

$A + 6D = 3*7 + 2 = 23$

$4D = 16$; $D = 4$; $A = -1$; $S(20) = 20/2[-2 + 19*4] = 740$

$A + D + S(20) = -1 + 4 + 740 = 743$

Option D

SELF ASSESSMENT TEST 11
GEOMETRIC PROGRESSION
17 Question, 17 Marks

1. The 9th term of a G.P. is 27 times the 6th term. What is the first term of the G.P. if the 4^{th} term is 27?

   a) 1 b) 2 c) 3 d) 4

2. The third term of a G.P. is the square of its first term. If the second term is 8, determine the 6^{th} term.

   a) 32 b) 128 c) 64 d) 1024

3. In a G.P., the ratio of the second and the fourth terms is 1 : 4 and the sum of the first and the fourth terms is 108. What is the value of the third term?

   a) 42 b) 44 c) 48 d) 52

4. If $(x + 9)$, $(x - 6)$ and 4 are in G.P., then find the value of x.

   a) -16 b) -4 c) 4 d) 16

5. How many terms of the GP 3, 3/2, 3/4 ... are needed to give the sum 3069/512?

   a) 9 b) 10 c) 11 d) 12

6. How many terms of the GP $\sqrt{3}$, 3, $3\sqrt{3}$, ... add up to $39 + 13\sqrt{3}$?

   a) 3 b) 4 c) 6 d) 5

7. The third term of a GP is the square of its 1st term. If the 2nd term is 27, determine the 16^{th} term.

   a) 315 b) 317 c) 322 d) None of the above

8. If A, B, C are real and 5, A, B, C, 405 are in GP, find A.

   a) 45 b) 135 c) ± 15 d) None of the above

9. Four numbers form a GP in which the product of the extreme terms is 256 and the sum of the middle terms is 40. Find the sum of the four terms of the series.

   a) 32 b) 170 c) 160 d) 180

10. The second, the first and the third term of an AP whose common difference is non zero, form a GP in that order. Find its common ratio.

a) 2 b) − 2 c) 1 d) − 1

11. The number of bacteria in certain culture doubles every hour. If there were 25 bacteria present originally, how many bacteria will be present at the end of 6th hour?

a) 800 b) 1600 c) $25(2)^6$ d) $2\ (25)^6$

12. Mr. Shyam Das is entitled to receive an annual payment from his employer, which for each year is less by 1/10th of what it was for the previous year. If the first payment is Rs. 10,000; what is the maximum amount he can receive, however long he may live?

a) Rs. 80,000 b) Rs. 100,000 c) Rs. 90,000 d) Rs. 110,500

13. If $x = 1 + a + a^2 + a^3 + a^4 + ...\infty$ and $y = 1 + b + b^2 + b^3 + b^4 + ...\infty$, then what is the value of ?

$1 + ab + a^2b^2 + a^3b^3 + ...\infty$?

a) $\frac{xy}{x+y-1}$ b) $\frac{x}{y(x+y)}$ c) $\frac{xy}{(x+y+1)}$ d) None of the above

14. After striking a floor a certain ball rebounds 4/5th of the height from which it has fallen. Find the total distance that it travels before coming to rest, if it is gently dropped from a height of 600 metres.

a) 3600 m b) 5400 m c) 7200 m d) None of the above

15. If a, b, c, d are in Geometric Progression then $(a^2 + b^2)$, $(b^2 + c^2)$, $(c^2 + d^2)$ are in:

a) Geometric Progression b) Arithmetic Progression

c) Both a) and b) above d) None of the above

16. If a, b, c, d are in Geometric Progression, then $\frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+d}$ are in:

a) Harmonic Progression b) Arithmetic Progression

c) Geometric Progression d) All of the above

17. If p, q, r are in AP, q, r, s are in GP and r, s, t are in HP, then p, r, t are in:

a) Arithmetic Progression b) Geometric Progression

c) Harmonic Progression d) None of the above

EXPLANATORY ANSWERS

1. $AR^8 = 27.AR^5$; $AR^3 = 27$

   $R^3 = 27$; Thus $A = 1$

   Option A

2. $AR^2 = A^2$; $R^2 = A$

   $AR = 8$; $R^3 = 8$; $R = 2$; $A = 4$

   $T6 = AR^5 = 4*2^5 = 128$

   Option B

3. $AR/AR^3 = ¼$; $R^2 = 4$; $R = 2$

   $A + AR^3 = 108$

   $A(1 + 8) = 108$; $A = 12$

   $T3 = AR^2 = 12*2^2 = 48$

   Option C

4. $(x - 6)^2 = 4(x + 9)$; $x^2 - 16x = 0$; $x = 16$; Option D

5. $A = 3$, $R = ½$

   $3069/512 = 3\,[1 - (1/2)^n]/(1 - ½)$

   $1023/512 = (2^n - 1)/2^{n-1}$

   Putting $n = 10$, we get the required answer. Option B

6. $A = \sqrt{3}$, $R = \sqrt{3}$

   $39 + 13\sqrt{3} = \sqrt{3}\,[(\sqrt{3})^n - 1]/(\sqrt{3} - 1)$

   $78 + 26\sqrt{3} = \sqrt{3}\,[(\sqrt{3})^n - 1](\sqrt{3} + 1)$

   Putting $n = 6$, in RHS, we get $= 26\sqrt{3}(\sqrt{3} + 1) = 78 + 26\sqrt{3}$

   Option C

7. $AR^2 = A^2$, $R^2 = A$

   $AR = 27$; $R^3 = 27$; $R = 3$; $A = 9$

   $T(16) = AR^{15} = 9.3^{15} = 3^{17}$

   Option B

8. $B^2 = 405*5 = 2025; B = \pm 45$

$A^2 = 45 * 5 = 225, A = \pm 15$

Option C

9. $A.AR^3 = A^2R^3 = 256 = 2^2.4^3$

$AR + AR^2 = 40; AR(1 + R) = 40$, assumption fits in

$S(4) = A(1 + R + R^2 + R^3) = 2(1 + 4 + 16 + 64) = 2*85 = 170$

Option B

10. A, (A − D), (A + D) are in GP

$(A - D)^2 = A(A + D)$

$A^2 + D^2 - 2AD = A^2 + AD$

$3AD = D^2$

$D = 3A$

Terms are, A , -2A, 4A are in GP

Common ratio = -2A/A = - 2

Option B

11. $T(6) = 25(2)^5 = 800$. Option A

12. Series is: 10000, 9000, 8100, ...

A = 10000, R = 9/10, S = 10000/(1 − 9/10) = 100,000

Option B

13. $X = 1/(1 - a); X - aX = 1; a = (X - 1)/X$

$Y = 1/(1 - b); Y - bY = 1; b = (Y - 1)/Y$

Required Sum = $1/(1 - ab) = XY/(XY - XY + X + Y - 1) = XY/(X + Y - 1)$

Option A

14. Distance covered = H(1 + R)/(1 − R) = 600 (1+4/5)/(1-4/5) = 600*9 = 5400.

Option B

15. A = 1, B = 2, C = 4, D = 8

$(A^2 + B^2) = 5$

$(B^2 + C^2) = 20$

$(C^2 + D^2) = 80$

5, 20, 80 are in GP

Option A

16. A = 1, B = 2, C = 4, D = 8

1/(A+B) = 1/3

1(B+C) = 1/6

1/(C+D) = 1/12

1/3, (1/2*1/3), (1/4*1/3) are in GP

Option C

17. AP: 1, 2, 3

GP: 2, 3, 4.5

HP: 3, 4.5, 9

1, 3, 9 are in GP

Option B

7A SET THEORY RELATION AND FUNCTIONS

SET THEORY RELATIONS

STANDARD NOTATIONS

1) $\cup$ $\Rightarrow$ OR (Union)
2) $\cap$ $\Rightarrow$ and (Intersection)
3) $\Rightarrow$ $\Rightarrow$ Implies
4) $\in$ $\Rightarrow$ belongs to
5) $\notin$ $\Rightarrow$ does not belong to
6) $\forall x$ $\Rightarrow$ for all x
7) : $\Rightarrow$ such that
8) / $\Rightarrow$ such that
9) $\subset$ $\Rightarrow$ Subset OR Proper Subset.
11) $\not\subset$ $\Rightarrow$ (not a proper subset)
12) $\supset$ $\Rightarrow$ (Superset)
13) ~ $\Rightarrow$ (Difference)
14) $\varnothing$ or { } $\Rightarrow$ (nullset)
15) U or S $\Rightarrow$ (Universal set)

2. SET THEORY (Concepts)

1. A set is a collection of well-defined and distinct object. The objects are called the elements of the set.

2. Sets are denoted by A, B, C, D etc and the elements are kept within brackets.

   e.g $A = \{a, b, c, d\}$

   $A = \{1, 2, 3, 4\}$

3. METHOD OF DESIGNATING A SET

i. ROSTER METHOD / TABULAR METHOD / ENUMERATION METHOD

ii. PROPERTY METHOD / SELECTOR METHOD / RULE METHOD/SET BUILDER NOTATION.

1) Under Roster or Enumeration method the set is defined by listing all the elements.

   e.g A = {a, e, i, o, u}

2) Under Property Method the sets are indicated by their common characteristics which an object must possess in order to its elements.

   e.g. A = { x : x is a vowel}

TYPES OF SETS

1) A set is said to be finite when the elements can be exhausted by counting.

   A = {4,5,6}

2) A set is said to be infinite when its elements can not be exhausted by counting.

   Eg. A = {1, 2, 3.........}

3) SINGLETON SET : A set which has only 1 element is called Singleton set

   e.g A = {2}

3. A FEW STANDARD INFINITE SETS

1. I^+ = Sets of Positive integers = N = Set of natural numbers

   = { 1, 2, 3.....}

2. W = Set of whole nos.

   = {0, 1, 2.....}

3. I^- = Sets of Negative integers

   = {-1, -2, -3.....}

4. I = Set of Integers

   = {0, ±1, ±2, ±3.........}

5. Q = Sets of Rational nos.

6. R = Set of real nos

- NULL SET / EMPTY SET / VOID SET

  It is a set having no element in it. It is denoted by $\varnothing$ or { }

  A = {x : x is a real no. whose square is negative}

4. EQUAL SETS

Two sets are said to be equal if all the elements of A belong to B and all the elements of B belong to A

A = { S, T, R, A, N, D}

B = { S, T, A, N, D, A, R, D}

Note : Order of arrangement or repetition of elements does not affect the property of equality.

5. EQUIVALENT SETS

If the total no. of elements of one set is equal to the total no. of elements of another set, then the two sets are said to be equivalent. The elements may or may not be same always.

A = {1, 2, 3, 4}

B = {b, l, u, e}

$A \equiv B$

6. SUB SET

If each element of set A is an element of set B, then A is said to be a subset of B or A is contained in B or B is the Superset of A.

Symbolically, $A \subseteq B$

If a set has n elements than the number of subset are 2^n .

e.g. If A = {1, 2, 3}

then the subsets of A are $\varnothing$, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}

Therefore the total number of subsets are $2^3 = 8$

Note 1. : If a set has n elements then

i. TOTAL NUMBER OF SUBSETS = 2^n

ii. TOTAL NUMBER OF NON- EMPTY SUBSETS = $2^n - 1$

iii. TOTAL NUMBER OF PROPER SUBSETS = $2^n - 1$

iv. TOTAL NUMBER OF NON- EMPTY PROPER SUBSETS = $2^n - 2$

Note 2. : i. Every set is a subset of itself

ii. Φ is a subset of every set

iii. In subset element may be equal

iv. If $A \subseteq B$ and $B \subseteq A$ A = B

7. PROPER SUB SET

If each element of set A is an element of set B but there is atleast 1 element in B which is not in A, in such a case A is said to be proper subset of B and is symbolically denoted by :

$A \subset B$: for example, A={1,2,3}

To the above e. g. the proper subsets of A are {1}, {2}, {3}, {1,2}, {1,3}, {2,3} & $\varnothing$

{1,2,3} is the improper subset because all the element are equal.

8. UNIVERSAL SET (U \ S)

Universal set or the universe is the set which contains all the elements under investigation in a particular content.

Eg. U = {1, 2, 3, 4, 5}

A = {2, 3}

B = {1, 3, 5}

C = {4, 5}, etc

Here A, B, C are all subsets of U.

9. POWER SET

It is defined as the set of all possible subsets in a particular investigations. If a set contains n elements, its power set will contain 2^n elements.

A = {2, 3, 4} Total elements in the Power set will be $2^3 = 8$

[there are 3 elements in set A]

P(A) = {∅, {2}, {3}, {4}, {2,3}, {2, 4}, {3, 4}, {2,3,4}}

e.g. The power set of A contains 128 elements. Find the no. of elements in set A Let there be n elements in Set A

$\therefore 2^n = 128$

Or $2^n = 2^7$

Or n = 7 ∴ Set A has 7 elements

10. CARDINAL NO. IN A SET: n(A)

If a set A contains "X" no. of elements, then the cardinal no. in set A will be given by:

n(A) = x.

e.g. A {2, 3, 4, 5}

n(A) = 4

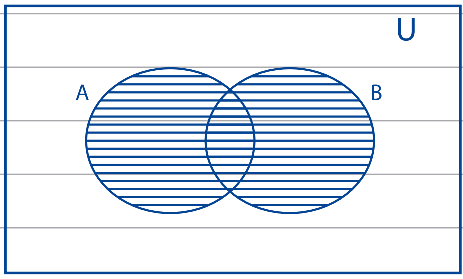
SET OPERATIONS

1. UNION OR JOIN OF 2 SETS

If A & B are 2 sets then the Union or Join of 2 sets is defined as, the set of all elements which belong either to A or to B or to both A & B.

Symbolically $A \cup B = \{ x : x \in A \text{ or } x \in B \}$

NOTE : Here 'UNION' ⇒ or



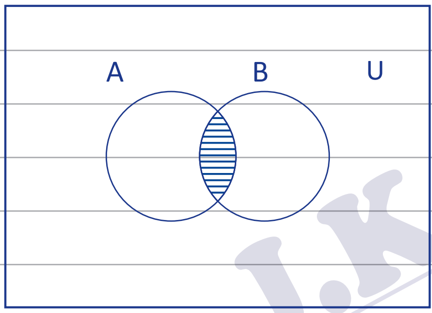
e.g

A = {1, 2, 3, 4, 5} : B = {2, 3, 5, 6, 7}

$A \cup B = \{1,2,3,4,5,6,7\}$

2. INTERSECTION OF 2 SETS

If A & B are 2 sets, then the intersection of the sets A & B is the set of those elements which belong to both A & B and is denoted by $A \cap B$.



Symbolically,

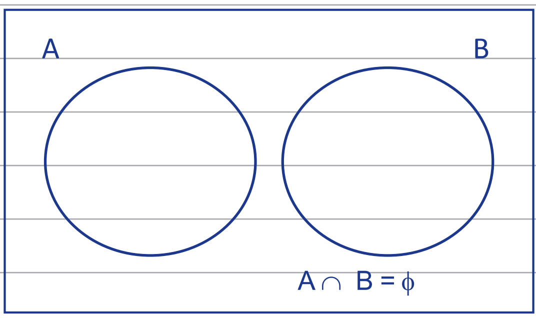
$A \cap B = \{\ x : x \in A \text{ and } x \in B\}$

A = { 1, 2, 3, 4}   B = (3, 4, 5}

$A \cap B = \{3, 4\}$

3. DISJOINT SETS

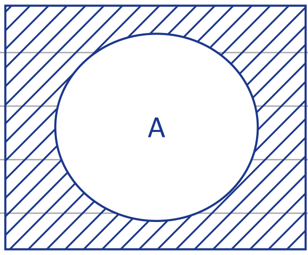
2 Sets are said to be disjoint when they have no elements in common i.e. their intersection is a Null Set.



e.g. If $A = \{1, 3, 5,\}$ $B = \{2, 4\}$ then $A \cap B = \phi$

therefore A & B are disjoints sets.

4. COMPLEMENT OR NEGATION SET



If U be the universal set and A be its subset, then the complement of set A in relation to U is the set whose element belong to U and not to A. This is denoted by :

$\hat{A}$ or A' or $A^c = (U - A)$

therefore $A^c = \{x : x \in U \text{ and } x \notin A\}$

e.g.

$U = \{ 1, 2, 3, 4, ...10\}$

$A = \{2, 3, 5, 7\}$

$B = \{1, 2, 9, 10\}$

$A^C = \{ 1, 4, 6, 8, 9, 10\}$

$B^C = (4, 3, 5, 7, 6, 8\}$

5. DIFFERENCE OF 2 SETS

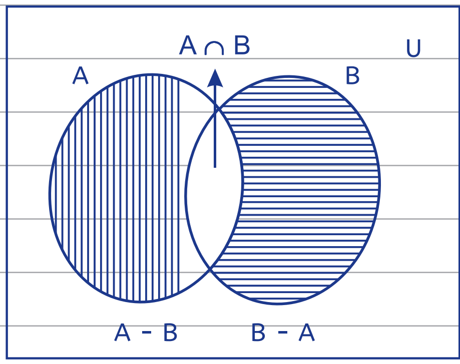
$A - B = \{ x : x \in A \text{ and } x \notin B\}$ Or

$A \sim B$

$B - A = \{ x : x \in B \text{ and } x \notin A\}$

Or

$B \sim A$



e. g. $A = \{ 1, 2, 3, 4, 5\}$

$B = \{3, 5, 6, 7\}$

$A - B = \{ 1, 2, 4\}$ and $B - A = \{ 6, 7\}$

6. CARTESIAN PRODUCT OF 2 SETS

If A and B are 2 sets, then the set of all ordered pairs (x, y) such that $x \in A$ and $y \in B$ is called Cartesian Product of A & B and it is denoted by A x B (read an A "cross" B)

Symbolically, $A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$

$A = \{1,2\}$ $B = (3, 4, 7\}$

$A \times B = \{(1, 3), (1, 4), (1, 7), (2,3), (2, 4), (2, 7)\}$

$B \times A = \{(3, 1), (3, 2), (4, 1), (4,2), (7, 1), (7, 2)\}$

$A \times B \neq B \times A$ but $A \times B \cong B \times A$ since $n(A \times B)$

$= n(B \times A)$

Note : 1. If $n(A) = m$ and $n(B) = n$ then the total number of elements in $A \times B = m \times n$

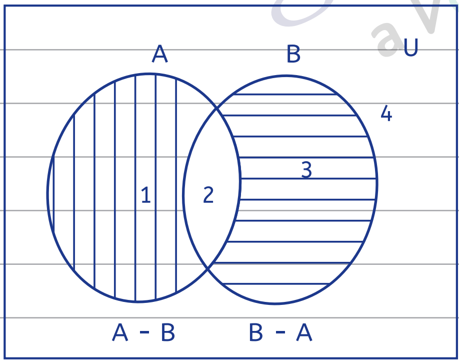
2. The total number of subsets of $A \times B = 2^{mn}$

Notes :

1. $\phi' = U$
2. $U' = \phi$
3. $(A^c)^c = A$
4. $A \cup A' = U$
5. $A \cap A' = \phi$
6. $A \subset B$ then $B' \subset A'$
7. $A \cup \phi = A$
8. $A \cap \phi = \phi$
9. $A \cup U = U$
10. $A \cap U = A$

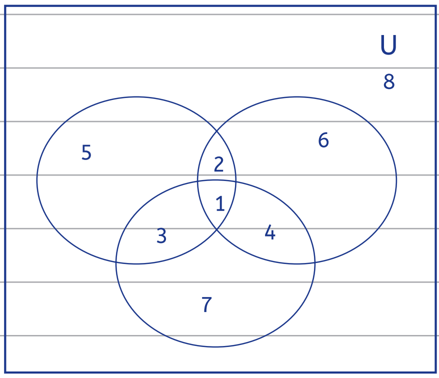
PARTITIONING OF SETS

Case 1



1. A-B or $A \cap B^c$ or A but not B = $n(A) - n(A \cap B)$
2. $(A \cap B)$ or (A and B)
3. B-A or $A^c \cap B$ or B but not A = $n(B) - n(A \cap B)$
4. $A^c \cap B^c$ or neither A nor B or $n(A \cup B)^c$ or $n(U) - n(A \cup B)$
5. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Case 2



1. $(A \cap B \cap C)$
2. $n(A \cap B \cap C^c) = n(A \cap B) - n(A \cap B \cap C)$
3. $n(A \cap B^c \cap C) = n(A \cap C) - n(A \cap B \cap C)$
4. $n(A^c \cap B \cap C) = n(B \cap C) - n(A \cap B \cap C)$
5. $n(A \cap B^c \cap C^c) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
6. $n(A^c \cap B \cap C^c) = n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$
7. $n(A^c \cap B^c \cap C) = n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
8. $n(A^c \cap B^c \cap C^c) = n(A \cup B \cup C)^c = n(U) - n(A \cup B \cup C)$
9. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Notes :

a) (2), (3), (4) are cases where only 2 items of the 3 are taken at a time.

b) (5), (6), (7) are cases where only 1 item of the 3 is taken at a time

c) (8) is the case where no item of the 3 are taken.

d) (1) is the case where all the items are taken i.e. the common part to all the 3.

LAWS

ASSOCIATIVE LAW

(a) $A \cup (B \cup C) = (A \cup B) \cup C$

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

DISTRIBUTIVE LAW

(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

DEMORGAN'S LAW

(a) $(A \cup B)^c = A^c \cap B^c$

(b) $(A \cap B)^c = A^c \cup B^c$

DEMORGAN'S LAW ON DIFFERENCE OF SETS

(a) $A - (B \cup C) = (A\text{-}B) \cap (A\text{-}C)$

(b) $A - (B \cap C) = (A\text{-}B) \cup (A\text{-}C)$

CARTESIAN PRODUCT

(a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(b) $A \times (B - C) = (A \times B) - (A \times C)$

RELATIONS

1. If A and B are two non empty sets, then any sub-set of A x B is called a relation from A to B. If R is a relation, then, $R \in A \times B$.

2. $A = \{1, 2, 3, 5\}$ $B = \{2, 4\}$

   Then, $A \times B = \{(1, 2), (1, 4), (2, 2), (2, 4), (3, 4), (3, 2), (5, 2), (5, 4)\}$

3. If we consider the relation 'is less than' then the set of all ordered pairs R in A x B, where

   (i) $R = \{(1,2), (1, 4), (2, 4), (3, 4)\} = \{(x, y) : x \in A, Y \in B, X R Y\}$

   (ii) Let A = (1, 2, 3, 4 ....32) R be the relation "one fourth of A"

   $R = \{(1, 4), (2,8), (3, 12), (4, 16), (5, 20), (6, 24), (7, 28), (8, 32)\}$

4. Number of Relation

If A and B are 2 sets containing m and n items respectively, then A x B will have mn ordered pairs, Total number of subsets of mn ordered pairs = 2^{mn}

Since each relation is subset of A x B.

$\therefore$ Total Relation = 2^{mn}

e.g. if $n(A) = 4$, $n(B) = 2$

Total relations = $2^8 = 256$.

5. Domain and Range of Relation

If A and B are 2 non-empty sets and R be the relation, then the set of first element in the ordered pair (x, y) is called the Domain of the relation and the set of second elements in the ordered pair is called the Range of the relation.

e.g. : $A = \{1, 3, 4, 5, 7\}$

B = (2, 4, 6, 8)

And R is the relation 'is one less than' from

A to B, then

R = {(1, 2), (3, 4), (5, 6), (7, 8)}

Domain of R = { 1, 3, 5, 7}

Range of R = (2, 4, 6, 8}

Co-domain of R = (2, 4, 6, 8}

Range $\subseteq$ Co-domain

TYPES OF RELATIONS

1. Note : A relation R in set A is a subset of A x A

2. A relation R in set A is said to be "Reflexive", if $(a, a) \in R$, for all $a \in A$
   where 'a' is the element of set A
   e.g. : A = {2, 4, 7} then the relation R =
   {(2, 2), (4, 4), (7, 7)} is reflexive.

3. A relation R in set A is called "Symmetric"
   if $(a, b) \in R$, then $(b, a) \in R$.
   e.g. A = {2, 4, 7}
   R = {(2,4), (4, 2), (2, 7), (7, 2)} is a
   symmetric relation.

4. A relative R in Set A is called "Transitive" relation if $(a, b), (b, c) \in R$, then $(a, c) \in R$
   e.g. : R = {(2, 4), (4, 7), (2, 7)} is transitive

5. A relation which is reflexive, symmetric and transitive is called an "Equivalence" relation.

Note :

1. Inverse of Equivalence relation is also an Equivalence relation.
2. Intersection of two Equivalence relation is also Equivalence relation.

Inverse Relation

Let, R be the relation from set A to B, then the inverse relation of R is denoted by R^{-1} is a relation from B to A.

$\therefore$ If R is a subset of A x B.

R^{-1} is a subset of B x A which consists of all the ordered pairs which when reversed belongs to R.

e.g. A = (2, 3, 5, 7), B = (4, 6, 9, 10, 11)

R be the relation "is a divisior of" from A to B

then, R = {(2, 4), (2, 6), (2, 10), (3, 6), (3, 9), (5, 10)}

$\therefore$ R^{-1} is a relation from B to A will be given by;

R^{-1} in this relation "is divisible by"

Domain of R^{-1} = {4, 6, 10, 9} = Range of R

Range of R^{-1} ={2, 3, 5) = Domain of R

Note :

D (R^{-1}) = R (R)

R (R^{-1}) = D(R)

FUNCTIONS

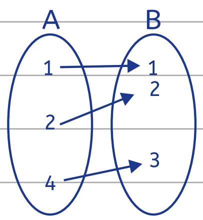
1. If A and B are 2 non-empty sets then, function is a rule or correspondence which associates every element 'X' of A to a unique element of 'Y' in B.
2. Symbolically we express it as $f : A \rightarrow B$

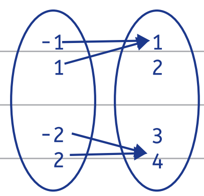
Note :

1. Set from which it is defined is called domain i.e. Set A
2. Set to which it is defined is called co-domain i.e. Set B
3. The set of images are the ranges of the function, Range $\subseteq$ Co-domain

Types of Functions

One – One (Injective) | Many – One | One – Many (does not exist)

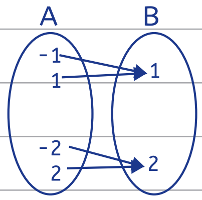




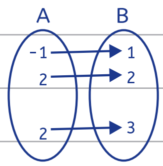
Each Element in A has only one image in B and each element in B has one pre-image in A

At least two elements in A has the same image in B and at least one element in B, has more than one pre-image in A

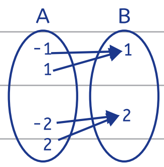
Onto Function
(Surjective Function)
Range = co-domain



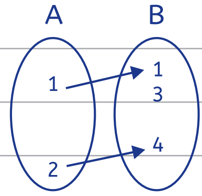
One one-onto (Bijective Function)



Many one-onto

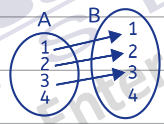


Into Function
Range $\subseteq$ co-domain

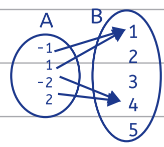


There should be at least one element in Set B which has no pre-image in A

One – one Into



Many Into



Domain : {-1, 1, -2, 2}
Co-Domain : {1, 2, 3, 4, 5}
Range : {1, 4)

CLASSWORK SECTION

(For Q. No. 1 to 6)

If A = { a, b, c, d, e}; B = { a, e, i, o, u} and C = {m, n, o, p, q, r, s, t, u}

1. $A \cup B$ has how many elements?

a) 8 b) 7 c) 9 d) 11

2. $B \cup C$ is equal to:

a) {a, e, i, o, u, m, n, p, q, r, s, t} b) {a, e, i, r, s, t}
c) {o, u, p, q, r, s} d) None of the above

3. $A \cup C$ is equal to:

a) {d, e, f, p, q, r} b) {a, b, c, d, e, m, n, o, p, q, r, s, t, u}
c) {a, b, c, s, t, u} d) None of the above

4. $B \cap C$ is equal to:

a) {a, e} b) {o, u} c) {o, p} d) None of the above

5. $A \cap B$ is equal to:

a) {a, e} b) {o, u} c) {o, p} d) None of the above

6. A - B is equal to:

a) {a, e, o} b) {m, n, p, q} c) {b, c, d} d) None of the above

7. The set of cubes of the natural numbers is:

a) A finite set b) An infinite set
c) A null set d) None of the above

8. If A and B are two sets containing 4 and 7 distinct elements respectively, find the minimum possible number and maximum possible number of elements $A \cup B$.

a) 5, 10 b) 4, 12 c) 7, 11 d) 8, 13

9. Set X and Y had 6 and 12 elements respectively, what can be the minimum number of elements in $X \cup Y$?

a) 14 b) 16 c) 12 d) 18

10. If A, B, C are three sets in which $n(A \cap B \cap C) = 8$, $n(A \cap B) = 15$, $n(A) = 22$, $n(B \cap C) = 11$, $n(B) = 19 = n(C)$, $n(A \cap C) = 10$, then what is $n(A \cup B \cup C)$?

a) 31 b) 33 c) 35 d) 32

11. K_1 & K_2 are two sets such that $n(K_1) = 17$, $n(K_2) = 23$, $n(K_1 \cup K_2) = 38$, then $(K_1 \cap K_2) =$?

a) 12 b) 2 c) 7 d) 9

12. If A = { 1, 2, 3}, B = {3, 4}, and C = {4, 5, 6} then $(A \times B) \cap (B \times C)$ is equal to :

a) { } b) {(3, 4)}

c) {(2, 3), (3, 2), (3, 4)} d) None of the above

13. Let A : {O, R, A, N, G, E} and B = { 0, 1, 2, 3, 4, 5}. The above two sets are:

a) Equal b) Equivalent

c) Disjoint d) Both b) and c) above

14. The number of elements in the power of set of a set containing 'p' elements is:

a) 2^{p-1} b) 2^p c) 2^{p+1} d) 2^{p+1}

15. The number of non - empty subsets of the set {8, 9, 10, 11, 15} is :

a) 32 b) 31 c) 30 d) 33

16. If the set A has m elements and set B has n elements then the number of elements in A x B is:

a) m + n b) mn c) m − n d) m^n

17. Two finite sets have p and q number of elements. The total number of subsets of the first set is eight times the total number of subsets of the second set. Find the value of p − q.

a) 2 b) 3 c) 4 d) None of the above

18. For A and B two given non-empty sets consisting of m and n elements respectively, the total number of subsets of A X B are:

a) 2^m b) 2^{mn} c) $2^m + 2^n$ d) None of the above

19. In a class of 65 students, 35 students have taken Mathematics, 40 have taken Statistics. Find the no. of students who have taken both. Find the no. of students who have taken Mathematics but not Statistics. (Assume that every student has to take atleast one of the two subjects.)

(a) 10, 25 (b) 10, 10 (c) 10, 20 (d) 10, 30

20. In a class of 50 students, 20 students play football and 16 students play hockey. It is found that 10 students play both the games. Use algebra of sets to find out the number of students who play neither.

(a) 26 (b) 25 (c) 24 (d) 20

21. In a class test of 45 students, 23 students passed in paper first, 15 passed in paper first but did not pass in paper second. Using set theory, find the no. of students who passed in both the papers and who passed in paper second but did not pass in paper first. (Assume that each student passed at least one of the two paper.)

(a) 8, 22 (b) 8, 20 (c) 10, 8 (d) None

22. In a statistical investigation of 1003 families of Calcutta it was found that 63 families had neither a radio nor a T.V. 794 families had a radio and 187 a television. How many families in that group had both a radio and a T.V.

(a) 41 (b) 42 (c) 40 (d) None

23. In a City, there are three daily newspaper published X, Y, Z. 65% of the people of the city read X, 54% read Y, 45% read Z, 38% read X and Y, 32% read Y and Z, 28% read X and Z. 12% do not read any of the three papers. If 10,00,000 person live in the city. Find the number of persons who read all the three newspaper.

(a) 220000 (b) 230000 (c) 120000 (d) 200000

(For Q No.24-Q27)

Out of 1600 students in a college, 390 played Kho-Kho, 450 played Kabaddi, and 500 played cricket; 90 played both Kho-Kho and Kabaddi; 125 played Kabaddi and Cricket, and 155 played Kho-Kho and Cricket; 50 played all the three games.

24. How many students did not play any game?

a) 400 b) 500 c) 450 d) None of the above

25. How many played only Kho-Kho?

a) 295 b) 195 c) 95 d) 1000

26. How many played only one game?

a) 1030 b) 930 c) 750 d) 730

27. How many played only two games?

a) 220 b) 320 c) 120 d) None of the above

Refer to the data below and answer the questions that follow. (For Q No.28-Q32)

Kimaya colony has a population of 2800 members.

Number of member listening only English music = 650

Number of member listening only Hindi music = 550

Number of member listening only Bengali music = 450

Number of member listening all three types of music = 100

Number of member listening Hindi as well as English music = 200

Number of member listening Hindi as well as Bengali music = 400

Number of member listening Bengali as well as English music = 300

28. Find the number of members listening Bengali music?

a) 950 b) 1050 c) 650 d) 550

29. Find number of members listening none of the music?

a) 450 b) 2650 c) 2550 d) 550

30. Find the number of members listening only one type of music

a) 450 b) 1100 c) 1600 d) 1650

31. Find number of members listening to at least two types of music

a) 600 b) 400 c) 700 d) 500

32. The ratio of members listening Hindi to that of Bengali music is :

a) 2 : 1 b) 1 : 1 c) 1 : 2 d) 3 : 2

33. A shop has only red, green and blue carpets. 60% of the carpets have red colour, 30% have green colour and 50% have blue colour. If no carpet has all the three colours, what percentage of the carpets have only one colour?

a) 40% b) 60% c) 70% d) None of the above

34. For two non-empty sets A and B containing m and n elements respectively, the total number of relations from A to B will be :

a) 2^{m+n} b) 2^{m} c) 2^{mn} d) $2^{m} + 2^{n}$

35. If A = {a, b, c, d} and B= {p, q, r, s} then which of the following are relations from A to B?

a) R1 = {(a, p), (b, r),(c, s)}
b) R2 = {(q, b), (c, s),(d, r)}
c) R3 = {(a, p), (b, r),(c, r)(s, q)}
d) R4 = {(a, p), (b, s),(s, b)(q, a)}

36. If A = {1, 3, 5, 7} and B = {2, 4, 6, 8, 10} and R = {(1, 8), (3, 6), (5, 2), (1, 4)} be a relation from A to B, then Dom(R) = ?

a) {1, 5} b) { 1, 3, 5} c) (3, 5} d) None of the above

37. In the above question, what is the Range (R)?

a) {1, 3, 5} b) { 8, 6, 2, 4} c) (2, 4, 6} d) None of the above

38. Let A = {1, 2} and B = {3, 4}. The total number of relations from A into B is:

a) 8 b) 16 c) 32 d) 4

39. Let A = {x, y}. The number of all relations on A are:

a) 4 b) 8 c) 16 d) 32

40. What can be said about the relation R ={(a, b),(b, c), (c, a)} defined on set A = {a, b, c}?

a) Reflexive, Symmetric, Transitive
b) Non Reflexive, Symmetric, Transitive
c) Non-Reflexive, Non-Symmetric, Non Transitive
d) None of the above

41. What can be said about the relation R = {(a, a), (a, b), (a, c),(b, b), (b, c), (c, a), (c, b), (c, c)} defined on Set A = {a, b, c}?
    a) Reflexive, Symmetric, Transitive
    b) Non Reflexive, Symmetric, Transitive
    c) Reflexive, Symmetric, Non Transitive
    d) Reflexive, Non-Symmetric, Non Transitive

42. What can be said about the relation R = {(a, b), (b, a),(a, c), (c, a)} defined on Set A = {a, b, c}?
    a) Reflexive, Symmetric, Transitive
    b) Non Reflexive, Symmetric, Non-Transitive
    c) Reflexive, Symmetric, Non Transitive
    d) Reflexive, Non-Symmetric, Non Transitive

43. Let A = {1, 2, 3} and R = {(1, 2), (1, 1,), (2, 3)} be a relation on A. What minimum number of ordered pairs may be added to R so that it may become a transitive relation on A.
    a) (3, 1) b) (1, 3) c) (2, 2) d) None of the above

44. The Range of the relation R where R = {(x, x + 5) : x $\in$ {0, 1, 2, 3, 4, 5} is:
    a) {0, 1, 2, 3, 4, 5} b) {1, 5, 6, 7, 8}
    c) {5, 6, 7, 8, 9} d) {5, 6, 7, 8, 9, 10}

45. A = { 1, 2, 3) R = {(1, 2), (2, 2,), (3, 1), (3, 4)} Find
    a) D(R) b) R(R) c) R^{-1} d) $D(R^{-1})$ e) $R(R^{-1})$

46. Find in each case the type of relation:
    A = {1, 2, 3}
    R_1 = {(1,1), (2,2), (3,3,), (1, 2)}
    R_2 = {(1,1), (2,2), (1,2,), (2, 1)}
    R_3 = {(1,1), (2,2), (3,3,), (1, 2), (2,1), (2, 3)(3,2)}
    R_4 = {(1,1), (2,3), (3,2,)}

47. Find in each case the Type of Relation.

i. "Is smaller than" over the set of eggs in a box is

(a) T (b) S (c) R (d) E

ii. "Is equal to " over the set of all rational numbers is

(a) T (b) S (c) R (d) E

iii. "Is perpendicular to" over the set of straight lines in a given plane is

(a) R (b) S (c) T (d) E

iv. "Is the reciprocal of" ... over the set of non-zero real numbers is

(a) S (b) R (c) T (d) none of these

v. "is the square of " over a set of real numbers is

(a) R (b) S (c) T (d) none of these

FUNCTIONS

1. If A = {x, y, z}, B = {p, q, r, s} which of the relation on A to B are function.

(a) {(x, p), (x, q), (y, r), (z, s)}
(b) {(x, s), (y, s), (z, s)}
(c) {(y, p), (y, q), (y, r), (z, s)
(d) {(x, p), (y, r), (x, s)}

2. {(x, y) |x + y = 5} where x, y $\in$ R is

(a) not a function
(b) a composite function
(c) one-one mapping
(d) none of these

3. {(x, y), y = x^2} where x, y $\in$ R is

(a) not a function
(b) a function
(c) inverse mapping
(d) none of these

4. {(x, y) | x < y} where x, y $\in$ R is

(a) not a function
(b) a function
(c) one-one mapping
(d) none of these

5. The range of {(3, 0), (2, 0), (1, 0), (0, 0)] is

(a) {0, 0}
(b) {0}
(c) {0, 0, 0, 0}
(d) none of these

6. The domain and range of {(x, y) : y = x^2} where x, y $\in$ R is

(a) (reals, natural numbers)
(b) (reals, positive reals including zero)
(c) (reals, reals)
(d) none of these

7. Let the domain of x be the set [1]. Which of the following functions are equal to 1

(a) f(x) = x^2, g(x) = x
(b) f(x) = x, g(x) = 1 − x
(c) f(x) = x^2 + x + 2, g(x) = $(x + 1)^2$
(d) none of these

8. If g(x) = (x − 1)/x, $g\left(-\frac{1}{2}\right)$ is

(a) 1
(b) 2
(c) 3/2
(d) 3

9. If f(x) = 1/1 − x and g(x) = (x − 1)/x, than fog(x) is

(a) x (b) 1/x

(c) −x (d) none of these

10. If f(x) = 1/1 − x and g(x) = (x − 1)/x, then gof (x) is

(a) x − 1 (b) x

(c) 1/x (d) none of these

11. The function $f(x) = 2^x$ is

(a) one-one mapping (b) one-many

(c) many-one (d) none of these

12. The range of the function $f(x) = \log_{10}(1 + x)$ for the domain of real values of x when $0 \leq x \leq 9$ is

(a) [0, 1] (b) [0,2]

(c) [0, 9] (d) none of these

13. The inverse function f^{-1} of f(x) = 2x is

(a) 1/2x (b) $\frac{x}{2}$

(c) 1/x (d) none of these

14. If f(x) = x + 3, $g(x) = x^2$, then f(x) . g(x) is

(a) $(x + 3)^2$ (b) $x^2 + 3$

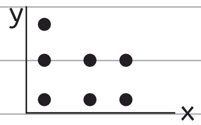
(c) $x^3 + 3x^2$ (d) none of these

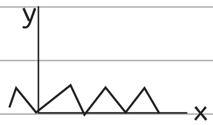
15. The inverse h^{-1} when $h(x) = \log_{10} x$ is

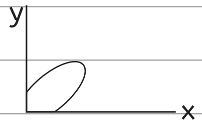
(a) $\log_{10} x$ (b) 10^x

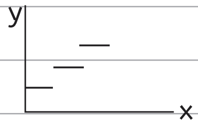
(c) $\log_{10} (1/x)$ (d) none of these

16. Which of the diagram is graph of a function

(a) 

(b) 

(c) 

(d) 

17. If $f(x) = 1/1 - x$, then $f^{-1}(x)$ is

(a) $1 - x$ (b) $(x - 1)/x$

(c) $x/(x - 1)$ (d) none of these

HOMEWORK SECTION

1. Out of 20 members in a family, 11 like to tea and 14 like coffee. Assume that each one likes at least one of the two drinks. Find how many like both coffee a n d tea:

(a) 2 (b) 3
(c) 4 (d) 5

2. In a town of 20,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspaper, then the number of families which buy A only is:

(a) 6,600 (b) 6,300
(c) 5,600 (d) 600

3. If f(x) = $\frac{2+x}{2-x}$, then f^{-1} (x):

(a) $\frac{2(x-1)}{x+1}$ (b) $\frac{2(x+1)}{x-1}$
(c) $\frac{x+1}{x-1}$ (d) $\frac{x-1}{x+1}$

4. If f (x) = 2x + h then f(x +h) – 2f (x)

(a) h – 2x (b) 2x - h
(c) 2x + h (d) None of these

5. If f (x) = $\frac{x}{\sqrt{1+x^2}}$ and g (x) = $\frac{x}{\sqrt{1-x^2}}$ Find fog?

(a) x (b) $\frac{1}{x}$
(c) $\frac{x}{\sqrt{1-x^2}}$ (d) $x\sqrt{1-x^2}$

6. If A = (1, 2, 3, 4, 5), B = (2, 4) and C (1, 3, 5) then (A - C) x B is

(a) {(2, 2), (2, 4), (4, 2), (4, 4), (5, 2), (5, 4)}
(b) {(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4)}
(c) {(2, 2), (4, 2), (4, 4), (4, 5)}
(d) {(2, 2), (2, 4), (4, 2), (4, 4)}

7. The range of the function f: N → N; $f(x) = (-1)^{x-1}$, is

(a) [0, -1] (b) [1, -1]

(c) [1, 0] (d) {1, 0, -1}

8. The number of elements in range of constant function is

(a) One (b) Zero

(c) Infinite (d) Indetermined

9. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then $f\left(\frac{2x}{1+x^2}\right)$ is equal to:

(a) f (x) (b) 2f (x)

(c) 3f (x) (d) -f (x)

10. Of the 200 candidates who were interviewed for a position at call centre, 100 had a two- wheeler, 70 had a credit card and 140 had a mobile phone, 40 of them had both a two- wheeler and a credit card, 30 had both a credit card and mobile phone, 60 had both a two-wheeler and mobile phone, and 10 had all three. How many candidates had none of the three?

(a) 0 (b) 20

(c) 10 (d) 18

11. If $f(x) = \left(\frac{x^2 - 25}{x - 5}\right)$ then $f(5)$ is

(a) 0 (b) 1

(c) 10 (d) Not defined

12. Let A = {1, 2, 3} and B = {6, 4, 7}. Then, the relation R = {(2, 4), (3, 6)} will be:

(a) Function from A to B (b) Function from B to A

(c) Both A and B (d) Not a function

13. If $f(x) = \left(\frac{x}{x-1}\right)$ then $\left(\frac{f(x/y)}{f(y/x)}\right)$ = ______________.

(a) x/y (b) y/x

(c) - x/y (d) - y/x

14. If N be the set of all natural number and E be the set of all even natural numbers then the function $f : N \rightarrow E$, such that $f(x) = 2x$ for all $X \in N$ is

(a) one-one onto (b) one-one into

(c) many-one onto (d) constant

15. In a class of 80 students, 35% students can play only cricket, 45% students can play only table tennis and the remaining students can play both the games. In all how many students can play cricket?

(a) 55 (b) 44

(c) 36 (d) 28

16. The domain (D) and range (R) of the function $f(x) = 2 - |x + 1|$ is

(a) D = Real numbers, R = $(2, \infty)$

(b) D = Integers, R = $(0, 2)$

(c) D = Integers, R = $(-\infty, \infty)$

(d) D = Real numbers, R = $(-\infty, 2]$

17. The number of subsets of the set formed by the word ALLAHABAD is:

(a) 128 (b) 16

(c) 32 (d) 64

18. If $f(x) = \frac{x-1}{x}$ and $g(x) = \frac{1}{1-x}$ then (fog) (x) is equal to:

(a) $x - 1$ (b) x

(c) $1 - x$ (d) $-x$

19. If $f(x) = \frac{x+1}{x+2}$, then $f\left\{f\left(\frac{1}{x}\right)\right\}$ = ______________.

(a) $\frac{2x+3}{3x+5}$ (b) $\frac{2x+5}{3x+2}$

(c) $\frac{3x+2}{5x+3}$ (d) $\frac{5x+2}{2x+3}$

20. In a class of 35 students, 24 like to play cricket and 16 like to play football. Also each student likes to play at least one of the two games. How many students like to play both cricket and football?

(a) 5 (b) 11

(c) 19 (d) 8

21. If A = {1, 2, 3, 4, 5, 6, 7} and B = {2, 4, 6, 8}. Cardinal number of A − B is:

(a) 4 (b) 3

(c) 9 (d) 7

22. If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}

B = {1, 3, 4, 5, 7, 8}; C = {2, 6, 8} then find $(A - B) \cup C$

(a) {2, 6} (b) {2, 6, 8}

(c) {2, 6, 8, 9} (d) None

23. A = {1 2 3 4 .... 10} a relation on A, R = {(x, y)/ x + y = 10, $x \in A$, $y \in A$, $x \ge y$} then domain of R^{-1} is.

(a) {1, 2, 3, 4, 5} (b) {0, 3, 5, 7, 9}

(c) {1, 2, 4, 5, 6, 7} (d) None

24. If $f(x) = x^2$ and $g(x) = \sqrt{x}$ then

(a) gof(3) = 3 (b) gof(-3) = 9

(c) gof(9) = 3 (d) gof(-9) = 3

25. If A = {a, b, c, d}; B = {p, q, r, s} which of the following relation is a function from A to B

(a) R_1 = {(a, p), (b, q), (c, s)} (b) R_2 = {(p, a), (b, r), (d, s)}

(c) R_3 = {(b, p), (c, s), (b, r)} (d) R_4 = {(a, p), (b, r), (c, q), (d, s)}

26. If A = {x,y,z}, B = {a,b,c,d}, then which of the following relation from the set A to set B is a function ?

(a) {(x,a), (x,b), (y,c), (z,d)} (b) {(x,a), (y,b), (z,d)}

(c) {(x,c), (z,b), (z,c)} (d) {(a,z), (b,y), (c,z)}

27. Let A be the set of squares of natural numbers and let $x \in A$, $y \in A$ then

(a) $x + y \in A$ (b) $x - y \in A$

(c) $\frac{X}{Y} \in A$ (d) $xy \in A$

28. If four members a, b, c, d of a decision making body are in a meeting to pass a resolution where rule of majority prevails list the winning conditions. Given that a, b, c, d own 50%, 20%, 15%, 15% shares each. List of winning condition is

(a) {a, b} {a, c} {a, d} {a, b, c} {a, b, d} {a, b, c, d} (b) {b, c, d}

(c) {b, c} {b, d} {c, d} {a, c, d} {b, c, d} {a} {b} {c} {d} ∅ (d) None

29. As per question No.(28) with same order of options (a) (b) (c) and (d) list the blocking conditions.

(a) {a, b} {a, c} {a, d} {a, b, c} {a, b, d} {a, b, c, d} (b) {b, c, d}

(c) {b, c} {b, d} {c, d} {a, c, d} {b, c, d} {a} {b} {c} {d} ∅ (d) None

30. As per question No.(28) with same order of options (a) (b) (c) and (d) list the losing conditions.

(a) {a, b} {a, c} {a, d} {a, b, c} {a, b, d} {a, b, c, d} (b) {b, c, d}

(c) {b, c} {b, d} {c, d} {a, c, d} {b, c, d} {a} {b} {c} {d} ∅ (d) None

31. Consider the following data: –

	Skilled & Direct Worker	Unskilled & Direct Worker	Skilled & Indirect Worker	Unskilled & Indirect Worker
Short Term	6	8	10	20
Medium Term	7	10	16	9
Long Term	3	2	8	0

If S, M, L, T, I denote short, medium, long terms, skilled and indirect workers respectively find the number of workers in set M.

(a) 42 (b) 8 (c) 10 (d) 43

32. Consider the problem No. (31) and find the number of workers in set L ∩ I.

(a) 42 (b) 8 (c) 10 (d) 43

33. Consider the problem No. (31) and find the number of workers in set S ∩ T ∩ I.

(a) 42 (b) 8 (c) 10 (d) 43

34. Consider the problem No. (31) and find the number of workers in set (M∪L) ∩ (T ∪ I).

(a) 42 (b) 8 (c) 10 (d) 43

35. Consider the problem No. (31) and find the number of workers in set $S' \cup (S' \cap I)'$

(a) 42 (b) 44 (c) 43 (d) 99

36. Consider the problem No. (31). Find out which set of the pair has more workers as its members. Pair is $(S \cup M)$ or L: –

(a) $(S \cup M) > L$ (b) $(S \cup M) < L$

(c) $(S \cup M) = L$ (d) None

37. Consider the problem No. (36). Find out which set of the pair has more workers as its members. Pair is $(I \cap T)'$ or $S - (I \cap S')$: –

(a) $(I \cap T)' > [S - (I \cap S')]$ (b) $(I \cap T); < [S - (I \cap S')]$

(c) $(I \cap T)' = [S - (I \cap S')]$ (d) None

38. Out of 1000 students, 658 failed in the aggregate, 166 in the aggregate and in group-I, 434 in aggregate and in group-II, 372 in group-I, 590 in group-II and 126 in both the groups.

Find out how many failed in all the three.

(a) 106 (b) 224 (c) 206 (d) 464

39. As per question No.(38) how many failed in the aggregate but not in group-II?

(a) 106 (b) 224 (c) 206 (d) 464

40. As per question No.(38) how many failed in group-I but not in the aggregate?

(a) 106 (b) 224 (c) 206 (d) 464

41. As per question No.(38) how many failed in group-II but not in group-I?

(a) 106 (b) 224 (c) 206 (d) 464

42. As per question No.(38) how many failed in aggregate or group-II but not in group-I?

(a) 206 (b) 464 (c) 628 (d) 164

43. As per question No.(38) how many failed in aggregate but not in group-I and group-II?

(a) 206 (b) 464 (c) 628 (d) 164

HOMEWORK SOLUTIONS

1. Out of 20 members in a family, 11 like to take tea, 14 like coffee Assume that each one likes atleast one of the two drinks. Find how many like both coffee and tea.

$n(T \cup C) = 20; n(T) = 11; n(c) = 14$

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$20 = 11 + 14 - n(T \cap C)$

$\therefore\ n(T \cap C) = 25 - 20 = 5$

2. As per given data,

$n(A \cup B \cup C) = 20000$ $\quad n(A \cap B) = 1000$

$n(A) = 40\% (20000) = 8000$ $\quad n(B \cap C) = 600$

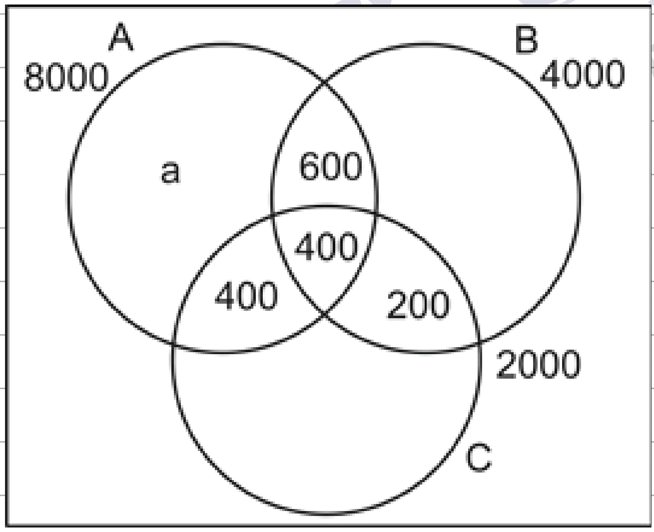
$n(B) = 20\% (20000) = 4000$ $\quad n(C \cap A) = 800$

$n(C) = 10\% (2000) - 2000$ $\quad n(A \cap B \cap C) = 400$

find the no. of families only in A.

$8000 - (600 + 400 + 400)$

$= a = 6600.$



3. $f(x) = \dfrac{2+x}{2-x}$ $f^{-1}(x) = ?$

Let, $y = f(x) = \dfrac{2+x}{2-x}$

$\Rightarrow 2y - xy = 2 + x$

$\Rightarrow 2y - 2 - x + xy \Rightarrow \dfrac{2y-2}{y+1} = x$

$\therefore x = f^{-1}(y) = \dfrac{2(y-1)}{y+1}$

and $f^{-1}(x) = \dfrac{2(x-1)}{x+1}$

4. $f(x) = 2x + h$ find $f(x + h) - 2f(x)$?

$f(x + h) = 2(x + h) + h = 2x + 2h + h = 2x + 3h \ldots (1)$

$2f(x) = 2(2x + h) = 4x + 2h \ldots (2)$

$\therefore f(x + h) - 2f(x) = (2x + 3h) - (4x + 2h)$

$= 2x - 4x + 3h - 2h$

$= h - 2x$

5. $f(x) = \dfrac{x}{\sqrt{1+x^2}}$; $g(x) = \dfrac{x}{\sqrt{1-x^2}}$ find fog

$$fog = f[g(x)] = f\left[\frac{x}{\sqrt{1-x^2}}\right]$$

$$= \frac{\left(\frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{1+\left(\frac{x}{\sqrt{1-x^2}}\right)^2}} = \frac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{1+\frac{x^2}{1-x^2}}} = \frac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{\frac{1-x^2+x^2}{1-x^2}}}$$

$$\therefore fog = \frac{x}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1} = x$$

6. $A = \{1, 2, 3, 4, 5\}$; $B = \{2, 4\}$; $C = \{1, 3, 5\}$

$(A - C) \times B = ?$

$A - C = \{2, 4\}$; $x \in A$ and $x \notin C$

$(A - C) \times B = \{2, 4\} \times \{2, 4\}$

$= \{(2, 2)\ (2, 4)\ (4, 2)\ (4, 4)\}$

7. $f(x) = (-1)^{x-1}$ Range = ? Given $f : N \to M$

Put, x = 1 $\quad f(1) = (-1)^{1-1} = (-1)^{0} = 1$

x = 2 $\quad f(2) = (-1)^{2-1} = (-1)^{1} = -1$

x = 3 $\quad f(3) = (-1)^{3-1} = (-1)^{2} = 1$

x = 4 $\quad f(4) = (-1)^{4-1} = (-1)^{3} = -1$

$\therefore$ **Range = {1, –1}**

8. y is a constant function and $\therefore$ the constant can be any number (y = k)

Hence, Infinite.

9. $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then $f\left(\frac{2x}{1+x^2}\right)$

$$f\left[\frac{2x}{1+x^2}\right] = \log\left[\frac{1+\left(\frac{2x}{1+x^2}\right)}{1-\left(\frac{2x}{1+x^2}\right)}\right]$$

$$= \log\left[\frac{1+x^2+2x}{1+x^2-2x}\right] = \log\frac{(1+x)^2}{(1-x)^2}$$

$$= 2\log\left[\frac{1+x}{1-x}\right] = 2f(x)$$

10. Total no. of candidates = 200

$n(A) = 100$ = who had 2-wheeler

$n(B) = 70$ = who had credit card

$n(C) = 140$ = who had mobile phones

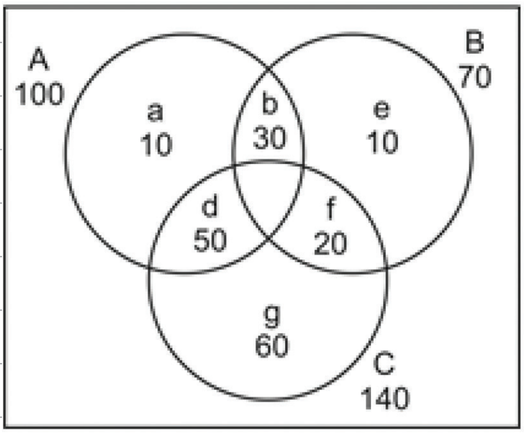
$n(A \cap B) = 40$; $n(B \cap C) = 30$; $n(C \cap A) = 60$

$n(A \cap B \cap C) = 10$

How many had none?

Using Venn – diagram

U=200



Given: b + c = 40 ; c = 10

$\therefore$ b = 30

c + f = 30 $\therefore$ f = 20

c + d = 60 $\therefore$ d = 50

a = [100 − (30 + 10 + 50)] = 10

e = [70 − (30 + 10 + 20)] = 10

g = [140 − (50 + 10 + 20)] = 60

Candidates who has none $= (A \cup B \cup C)'$

$= \text{Total} - (A \cup B \cup C)$

$= 200 - [a + b + c + d + e + f + g]$

$= 200 - 190 = 10$

11. $f(x) = \dfrac{x^2 - 25}{x - 5}$ f(5) = ?

$f(5) = \dfrac{5^2 - 25}{5 - 5} = \dfrac{0}{0}$ = Undefined or Not defined.

12. A = {1, 2, 3} ; B = {6, 4, 7}

Given : R = {(2, 4)(3, 6)}

We observe that, ordered pairs in R has domain values from A and range values from B

$\therefore R : A \rightarrow B$

R is a function from A to B.

13. $f(x) = \dfrac{x}{x-1}$; $\dfrac{f\left(\dfrac{x}{y}\right)}{f\left(\dfrac{y}{x}\right)} = ?$

$$f\left(\frac{x}{y}\right) = \frac{(x/y)}{\left(\frac{x}{y} - 1\right)} = \frac{(x/y)}{\left(\frac{x-y}{y}\right)} = \frac{x}{x-y} \quad \ldots (1)$$

$$f\left(\frac{y}{x}\right) = \frac{(y/x)}{\left(\frac{y}{x} - 1\right)} = \frac{(y/x)}{\left(\frac{y-x}{x}\right)} = \frac{y}{y-x} \quad \ldots (2)$$

$$\frac{(1)}{(2)} = \frac{x}{(x-y)} \times \frac{-(y-x)}{y} = \frac{-x}{y}$$

14. Given: $N = \{1, 2, \ldots \infty\}$

$E = \{2, 4, 6, \ldots \infty\}$

$f : N \to E$

$f(1) = 2 \times 1 = 2$

$f(2) = 2 \times 2 = 4$

$f(3) = 3 \times 2 = 6$

$f(4) = 4 \times 2 = 8$

Here, R = E

$\therefore f(x_1) = f(x_2)$

Hence, f is injective [one-one] and onto.

$\because$ Range = E

15. Total number of students = 80.

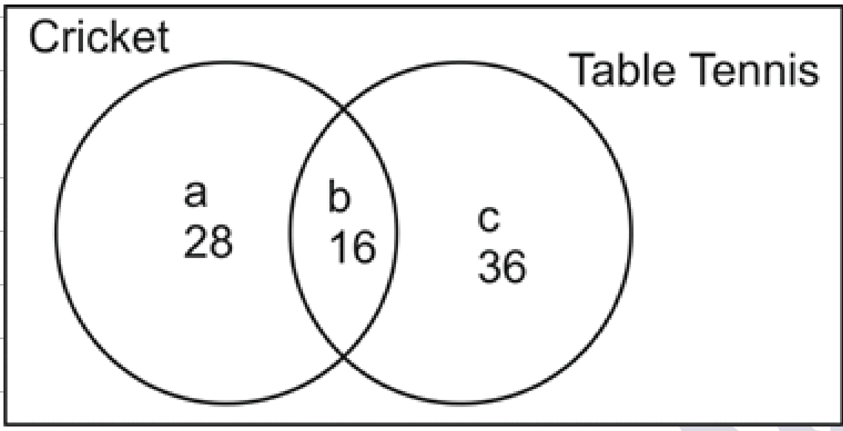
No. of students playing

Cricket only = 35% (80) = 28

No. of students playing

Table Tennis only = 45% (80) = 36

No. of students who play both games = 20% (80) = 16



$\therefore$ Total no. of students playing cricket = a + b

= 28 + 16 = 44

16. Given: D = Domain; R = Range

$f(x) = 2 - |x + 1|$

We know that, $y = f(x) = 2 - [x + 1]$

$\therefore$ D = Real number and

R = $(-\infty, 2]$ Real nos.

17. (c) Number of subsets of the set formed by the word "ALLAHABAD".

Let, the set be A = {A, L, H, B, D}

$n(A) = 5 \;\therefore$ Subsets $= 2^n = 2^5 = 32$

18. $f(x) = \dfrac{x-1}{x}$; $g(x) = \dfrac{1}{1-x}$ fog (x)

$$f[g(x)] = f\left[\frac{1}{1-x}\right] = \frac{\left(\frac{1}{1-x}\right)-1}{\left(\frac{1}{1-x}\right)}$$

$$= \frac{1-(1-x)}{1} = 1 - 1 + x = x$$

19. $f(x) = \dfrac{x+1}{x+2}$ find $f\left[f\left(\dfrac{1}{x}\right)\right]$

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}+1}{\frac{1}{x}+2} = \frac{1+x}{1+2x}$$

$$\text{Now, } f\left[f\left(\frac{1}{x}\right)\right] = \frac{\left(\frac{1+x}{1+2x}\right)+1}{\left(\frac{1+x}{1+2x}\right)+2} = \frac{1+x+1+2x}{1+x+2+4x}$$

$$\therefore \text{ fof}\left(\frac{1}{x}\right) = f\left[f\left(\frac{1}{x}\right)\right] = \frac{2+3x}{3+5x}$$

20. $n(n \cap F)$ = ? when $n(C \cup F) = 35$

$n(C) = 24$; $n(F) = 16$

where ; C = Cricket and F = foot ball

21. A = {1, 2, 3, 4, 5, 6, 7} ; B = {2, 4, 6, 8}

$n(A - B)$ = ?

$A - B = \{1, 3, 5, 7\}$ $\therefore n(A - B) = 4$

22. A = {1, 2, 3, . . . 9}

B = {1, 3, 4, 5, 7, 8} ; C = {2, 6, 8} $\{A - B) \cup C$ = ?

$A - B = \{2, 6, 9\}$ $x \in A$ and $x \notin B$

$(A - B) \cup C = \{2, 6, 8, 9\}$

23. $A = \{1, 2, 3, . . . 10\}$

$R = \{x, y/x + y = 10, \ x \in A \text{ and } y \in A, \ x \geq y\}$

then domain of R^{-1} = ?

$R = \{(5, 5) (6, 4) (7, 3) (8, 2), (9, 1)\}$

$R^{-1} = \{(5, 5) (4, 6) (3, 7) (2, 8) (1, 9)\}$

$\therefore$ Domain $[R^{-1}] = \{5, 4, 3, 2, 1\}$

24. $f(x) = x^2$ and $g(x) = \sqrt{x}$ then,

$fog (x) = f[g(x)] = f[\sqrt{x}] = (\sqrt{x})^2 = x$

$gof (x) = g[f(x)] = g[x^2] = x$

$\therefore$ Option is (a) gof (3) = 3

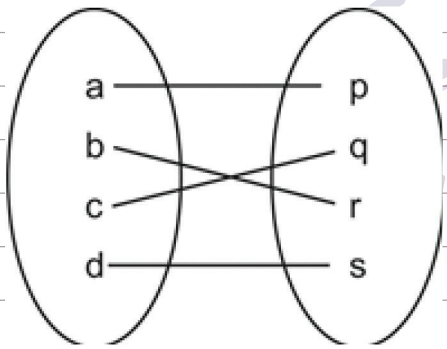
25. $A = \{a, b, c, d\}$

$B = \{p, q, r, s\}$

function from A to B = ?

$f : A \rightarrow B$ is $R_4 = \{(a, p) (b, r) (c, q) (d, s)\}$

is an Injective function (one-one)



$n(C \cup F) = n(C) + n(F) - n(C \cap F)$

$35 = 24 + 16 - x$

$35 = 40 - x$

$\therefore x = 5$ who play both cricket & foot ball.

26. $A = \{x, y, z\}$; $B = \{a, b, c, d\}$ $f : A \rightarrow B$

$A \times B = \{(x, a) (x , b) (x, c) (x, d) (y, a) (y, b) (y, c) (y, d) (z, a) (z, b) (z, c) (z, d)\}$

Then, $\{(x, a) (y, b), (z, d)\}$ is a function.

27. A = Set of squares of natural nos. $x \in A$ and $y \in A$

$A = \{1^2, 2^2, 3^2, \ldots \infty\} = \{1, 4, 9, \ldots\}$

$x \in A$ and $x \in B$ then $xy \in A$, satisfies.

28. If four members a, b, c, d of a decision making body are in a meeting to pass a resolution where rule of majority prevails Given:

a = 50% share

b = 20% share

c = 15% share

d = 15% share

List the winning conditions

In this case, as per the share owned by the 4 members, winning condition is possible only when shares are > 50%

∴ 'a' with any other member or all the members gives us winning condition **more than 50%**

29. **List the blocking condition**

In this case, either if has to be only 'a' or all 3 members 'b, c, d' to give us a blocking condition of **exactly 50%**

30. **List the losing conditions**

Losing condition is possible only when

Share are **< 50%.**

Either it has to be less than 'a' alone or

Less than all 3 b, c, d

31.

	(T)		T & I	I
	Skilled & Direct workers	**Unskilled & Direct**	**Skilled & indirect**	**Unskilled & indirect**
(S) Short	6	8	10	20
(M) Medium	7	10	16	9
(L) Long	3	2	8	0

Set M = 7 + 10 + 16 + 9 = 42

32. Set $L \cap I = 8$ [3^{rd} R 3^{rd} C]

33. Set $S \cap T \cap I$ [1^{St} R 3^{rd} C] = 10

34. $M \cup L = \begin{pmatrix} 7+10+16+19 \\ +3+2+8+0 \end{pmatrix} \cap (T \cup I) \begin{pmatrix} 6 & 10 & 20 \\ 7 & 16 & 9 \\ 3 & 8 & 0 \end{pmatrix} = 43$

35. $S' \cup (S' \cap I)'$

$S' \cap I = \begin{pmatrix} 16 & 9 \\ 8 & 0 \end{pmatrix}$: $(S' \cap I)'$ = Entire table except $\begin{pmatrix} 16 & 9 \\ 8 & 0 \end{pmatrix}$

$S' \cup (S' \cap I)'$ = Entire table = 99

36. $S \cup M = 1^{st}$ 2 Row : $(S \cup M)^1$ = Last Row

$\therefore (S \cup M)' = L$

37. $I \cap T = 3^{rd}$ Column, $(I \cap T)'$ = 1^{st}, 2^{nd}, & 4^{th} column

6 + 8 + 20 + 7 + 10 + 9 + 3 + 2 + 0 = 65

$I \cap S = \begin{pmatrix} 16 & 9 \\ 8 & 0 \end{pmatrix}$; $S - (I \cap S) = 1^{st}$ Row

$6+8+10+20=44$

$\therefore (I \cap T)' > [S - (I \cap S)]$

38. **Using the formula,**

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

As per the given data: there are 3 sets

AGGREGATE, GROUP – 1, GROUP – 2

There are 1000 students, whose results are tabulated

Given: $n(A) = 658$; $n(A \cap G_1) = 166$

$n(G_1) = 372$; $n(A \cap G_2) = 126$

$n(G_2) = 590$; $n(A \cap G_3) = 434$

Now, $1000 = 658 + 372 + 590 - 166 - 126 - 434 + x$

$1000 = 894 + x$

$\therefore x = 106$

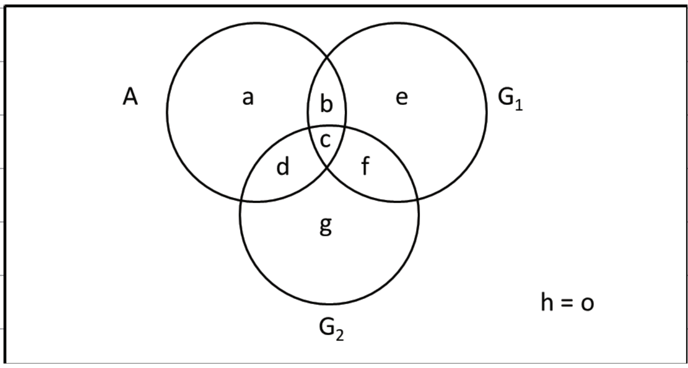
x = No. students who failed in all 3 = 106

Note:-

In the given data, there are no students who have passed in aggregate, G_1, G_2

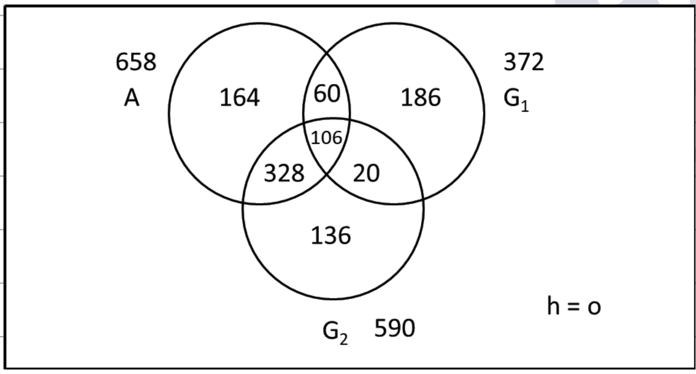
$(A \cup G_1 \cup G_2)'$

U = 1000



Using the given data

U = 1000



b + c = 166 $\rightarrow$ b + 106 = 166 ; $\boxed{b = 60}$

106 + f = 126 ; $\boxed{f = 20}$

106 + d = 434 ; $\boxed{d = 328}$

c + d = 434

c + f = 126

Now, 658 − (328 + 106 + 60) = $\boxed{a = 164}$

372 − (60 + 106 + 20) = $\boxed{e = 186}$

590 − (328 + 106 + 20) = $\boxed{g = 136}$

39. Aggregate but not group 2 $\rightarrow$ 164 + 60 = 224

40. Group – 1 but not aggregate $\rightarrow$ 186 + 20 = 206

41. Group – 2 but not in group 1 $\rightarrow$ 328 + 136 = 464

42. Aggregate OR Group – 2 but not group – 1

= 164 + 328 + 136 = 628

43. Aggregate but not in G_1 or G_2 [only aggregate]

= 164

SELF ASSESSMENT TEST 12
SET THEORY
17 Question, 17 Marks

1. The number of non-empty subsets of a set consisting of 10 elements is:
   a) 1023 b) 1024 c) 2^9 d) $2^{10} - 2$

2. Two finite sets with p, and q as elements. The total number of subsets of the first set is 64 more than the total number of subsets of the second. The values of p and q are:
   a) 5, 7 b) 7, 6 c) 8, 7 d) 9, 7

3. If A and B be two sets containing 3 and 6 elements respectively, then the minimum and maximum number of elements of (A U B) respectively is:
   a) 6, 9 b) 9, 6 c) 3, 6 d) 6, 3

4. In a group of 80 people, 45 drink tea but not coffee and 52 drink tea. Then how many drink coffee but not tea?
   a) 28 b) 27 c) 35 d) 52

5. The number of elements in P[P(P(Φ))] is:
   a) 0 b) 2 c) 5 d) 4

6. Which of the following is a singleton set?
   a) $\{x : |x| = 5, x \in N\}$ b) $\{x : |x| = 6, x \in Z\}$
   c) $\{x : x^2 + 2x + 1 = 0, x \in N\}$ d) $\{x : x^2 = 7, x \in N\}$

7. In a survey, it is found that 80% children like A and 75% of the children like B. If X% like both A and B, then:
   a) $X = 55$ b) $X < 55$ c) $X \geq 55$ d) $55 \leq X \leq 75$

8. In a school, 21 play basketball, 26 play hockey and 29 play football. If 14 play hockey and basketball, 12 play football and basketball, 15 play hockey and football and 8 play all the three games, then the number of players who play only football is:
   a) 10 b) 20 c) 21 d) 43

9. In a college of 10,000 students, it was found that 40% students opt for electronics, 20% students opt for communications, 10% students opt for computers, 5% students opt for both electronics and communication, 3% students opt for communication and computers and 4% opt for electronics and computers, and 2% of the students opt for all the three. Find the number of students who opts only electronics.

a) 3100 b) 3300 c) 290 d) 1400

10. In a college of 10,000 students, it was found that 40% students opt for electronics, 20% students opt for communications, 10% students opt for computers, 5% students opt for both electronics and communication, 3% students opt for communication and computers and 4% opt for electronics and computers, and 2% of the students opt for all the three. Find the number of students who don't opt for any of electronics, computers and communication.

a) 4000 b) 3300 c) 4200 d) 5000

11. In a college of 10,000 students, it was found that 40% students opt for electronics, 20% students opt for communications, 10% students opt for computers, 5% students opt for both electronics and communication, 3% students opt for communication and computers and 4% opt for electronics and computers, and 2% of the students opt for all the three. Find the number of students who opt for only communication.

a) 2000 b) 1400 c) 2500 d) 1800

12. In a survey conducted among certain people, 60 take tea and 30 take only coffee. If 10 people take both, then how many people were involved in the survey? [All the people take at least one of them]

a) 80 b) 90 c) 100 d) 110

(For Q. No. 13 – 17)

During quality control checking of a sample of 1000 TV sets it was found that 100 sets had defective picture tubes, 75 sets had a defective sound system, 80 sets had defective remote control, 20 sets had a defective picture and remote controls, 30 sets had defective picture tube and a sound system, 15 sets had defective sound system and defective remote control system and 5 sets had all the three defects.

13. Number of TV sets having at least one defect is:

a) 195 b) 185 c) 175 d) 165

14. Find the number of TV sets having no defects?

a) 705 b) 805 c) 605 d) 300

15. Find the number of TV sets having only defective picture tubes.

a) 45 b) 55 c) 65 d) 75

16. Find the number of TV sets having defective sound system only.

a) 45 b) 55 c) 35 d) 25

17. Find the number of TV sets having defective remote control only.

a) 50 b) 60 c) 70 d) 80

EXPLANATORY ANSWERS

1. Number of non empty subsets = $2^{10} - 1 = 1024 - 1 = 1023$. Option A

2. $2^p = 2^q + 64$.
   Only option placement shall help.
   $2^7 = 128 = 2^6 + 64$. Option B

3. Minimum (A U B) = Max of A or B = 6
   Maximum (A U B) = n(A) + n(B) = 9
   Option A

4. People drinking both tea and coffee = 52 − 45 = 7
   People drinking only coffee and not tea = 80 − 52 = 28; Option A

5. $P(\Phi) = \{\Phi\}$
   $P\{P(\Phi)\} = \{\Phi, \{\Phi\}\}$
   $P[P(P(\Phi))] = \{\Phi, \{\Phi\}, \{\{\Phi\}\}, \{\Phi,\{\Phi\}\}\}$
   Thus $n\{P[P(P(\Phi))]\} = 4$; Option D
6. Set a = {5}
   Set b = {-6, 6}, is not a singleton set
   Set c = $(x + 1)^2 = 0$, is not a singleton set
   Set d = $x^2 = 7$, is not a singleton set; Option A

7. Maximum X = 75
   For Minimum: 100 = 80 + 75 − X; X = 55
   X lies between 55 and 75, both inclusive; Option D

8. Students playing only football = 29 − 12 − 15 + 8 = 10. Option A

9. Only electronics = 40% - 5% - 4% + 2% = 33% of 10000 = 3300. Option B

10. Students opted for any course = 40% + 20% + 10% - 5% - 3% - 4% + 2% = 60%
    Students not opting for any course = 40% of 10000 = 4000. Option A

11. Only communications = 20% - 5% - 3% + 2% = 14% of 10000 = 1400. Option B

12. People involved = 60 + (30+ 10) − 10 = 90. Option B

13. At least one defect = 100 + 75 +80 − 20 − 30 − 15 + 5 = 195. Option A

14. No defects = 1000 − 195 = 805 (Refer explanation of Q. No. 21). Option B

15. Only defective picture tubes = 100 − 20 − 30 + 5 = 55. Option B

16. Only defective sound system = 75 − 30 − 15 + 5 = 35. Option C

17. Only defective remote control = 80 − 20 − 15 + 5 = 50. Option A

SELF ASSESSMENT TEST 13
RELATIONS AND FUNCTIONS
16 Question, 16 Marks

1. If the ordered pairs (x − 1, y + 3) and (2, x + 4) are equal, what is the value of x and y?

   a) 3, 3   b) 4, 3
   c) 3, 4   d) Can't be determined

2. If A = {x | x ∈ W, x < 3}, B = {x | x ∈ N, 2 ≤ x < 4} and C = {3, 4}, find the set (A U B) X C.

   a) (A X C) ∩ (B X C)   b) (A X C) U (B X C)
   c) (A X B) U (A X C)   d) (A X C) ∩ (A X B)

3. Let A = {1, 2, 3}, B = {1, 2, 3, 4} and R = {(x, y) : (x, y) ∈ A X B, y = x + 1}. Which of the following represents R?

   a) R = {(1, 2), (2, 3), (3, 4), (4, 5)}   b) R = {(1, 2), (2, 3), (3, 4)}
   c) R = {(2, 1), (3, 2), (3, 4)}   d) R = {(1, 2), (3, 2), (4, 3)}

4. Let R = {(x, y) : x, y ∈ W, 2x + y = 8}. Find the domain of R.

   a) {1, 2, 3, 4}   b) {0, 1, 2, 3, 4}
   c) {-1, 1, 2, 3, 5}   d) None of the above

5. Let R = {(x, y) : x, y ∈ W, 2x + y = 8}. Find the range of R.

   a) {0, 2, 4, 6, 8}   b) {2, 4, 6, 8}
   c) {10, 8, 4, 2, 0}   d) None of the above

6. If A and B are finite sets such that n(A) = m and n(B) = k, find the number of relations from A to B.

   a) 2^{m+k} b) $2^m.2^k$
   c) 2^{mk} d) $2^m - 2^k$

7. Let A = {2, 4, 6, 8} and B = {0, 6, 8, 9, 10}. Find the elements of S = (A ∩ B) X (A − B), corresponding to the relation "is a multiple of".

   a) {(6, 2), (8, 2)}   b) {(2, 6), (2, 8), (4, 8)}
   c) {(6, 2), (8, 2), (8, 4)}   d) None of the above

8. If $f(x + 1) = 3x + 5$, find $f(2x)$.

a) $6x + 5$ b) $6x + 10$

c) $6x + 2$ d) $3x + 10$

9. If $f(x) = \frac{x^2 - 3x + 1}{x - 1}$, find the value of $f(-2) + f(1/3)$.

a) $-32/6$ b) $-23/6$

c) $32/6$ d) $23/6$

10. A function 'f' is defined by $f(x) = x^2 - 3$, $x \in R$, and $-2 \leq x < 2$. Find f(3).

a) 6 b) -6

c) 12 d) Does not exist.

11. A function 'f' is defined by $f(x) = x^2 - 3$, $x \in R$, and $-2 \leq x < 2$. Find x so that $f(x) = 6$.

a) 3 b) -3

c) Both a) and b) above d) None of the above

12. A function f is defined on the set of integers as follows:

$f(x) = x + 1$ if $0 \leq x < 2$

$= 2x - 1$ if $2 \leq x < 4$

$= 3x - 10$ if $4 \leq x < 6$

Which of the following represents f in the roaster form?

a) {(0, 1), (2, 2), (2, 3), (3, 3), (4, 4), (5, 5)}

b) {(0, 1), (2, 2), (2, 3), (3, 3), (4, 2), (5, 5)}

c) {(0, 1), (2, 2), (2, 3), (3, 5), (4, 2), (5, 5)}

d) {(0, 1), (1, 2), (2, 3), (3, 5), (4, 2), (5, 5)}

13. A function f is defined on the set of integers as follows:

$f(x) = x + 1$ if $0 \leq x < 2$

$= 2x - 1$ if $2 \leq x < 4$

$= 3x - 10$ if $4 \leq x < 6$

Which of the following represents range of f?

a) {1, 2, 3, 4, 5}

b) {1, 2, 3, 5}

c) {0, 1, 2, 3, 4, 5}

d) {0, 1, 2, 3, 5}

14. A function f on the set R of real numbers is defined as:

$f(x) = 2x + 1$ for $0 \le x < 2$

$= x - 2$ for $2 \le x \le 5$

What type of function is f?

a) One-One function

b) Many-One function

c) Both a) and b) above

d) Neither a) nor b) above

15. A function f on the set R of real numbers is defined as:

$f(x) = 2x + 1$ for $0 \le x < 2$

$= x - 2$ for $2 \le x \le 5$

Find the value of x for which $f(x) = \frac{1}{2}$.

a) 5/2 b) 2/5

c) $-\frac{1}{4}$ d) None of the above

16. R is a set of all real numbers. If $f : R \to R$ is defined by $f(x) = 2x + 1$ and $g : R \to R$ by $g(x) = 1 - 2x$ for all $x \in R$, which of the following is TRUE?

a) $fog(1) < gof(1)$ b) $fog(1) > gof(1)$

c) Neither a) nor b) d) Both a) and b)

EXPLANATORY ANSWERS

1. $(x - 1) = 2; x = 3$

   $(y + 3) = (x + 4) = 7; y = 4$

   $(x, y) = (3, 4)$. Option C

2. $A = \{0, 1, 2\}$

   $B = \{2, 3\}$

   $C = \{3, 4\}$

   $A \cup B = \{0, 1, 2, 3\}$

   $(A \cup B) \times C = \{(0, 3), (1, 3), (2, 3), (3, 3), (0, 4), (1, 4), (2, 4), (3, 4)\}$

   $(A \times C) = \{(0, 3), (1, 3), (2, 3), (0, 4), (1, 4), (2, 4)\}$

   $(B \times C) = \{(2, 3), (2, 4), (3, 3), (3, 4)\}$

   $(A \times C) \cup (B \times C) = \{(0, 3), (1, 3), (2, 3), (3, 3), (0, 4), (1, 4), (2, 4), (3, 4)\}$

   $(A \cup B) \times C = (A \times C) \cup (B \times C)$

   Option B

3. $A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)\}$

   R have pairs where $y = x + 1$

   $R = \{(1, 2), (2, 3), (3, 4)\}$

   Option B

4. $R = \{(0, 8), (1, 6), (2, 4), (3, 2), (4, 0)\}$

   Domain of $R = \{0, 1, 2, 3, 4\}$

   Option B

5. $R = \{(0, 8), (1, 6), (2, 4), (3, 2), (4, 0)\}$

   Range of $R = \{8, 6, 4, 2, 0\}$

   Option A

6. Number of relations from A to B = Subsets of (Number of elements in the set A X B)

   $= 2^{mk}$

   Option C

7. A = {2, 4, 6, 8}

B = {0, 6, 8, 9, 10}

$(A \cap B) = \{6, 8\}$

$(A - B) = \{2, 4\}$

$S = (A \cap B) \times (A - B) = \{(6, 2), (8, 2), (6, 4), (8, 4)\}$

Of these the relation "is a multiple of" is followed by (6, 2), (8, 2), (8, 4)

Thus R = {(6, 2), (8, 2), (8, 4)}, Option C

8. $f(x + 1) = 3x + 5$

$f(y) = 3(y - 1) + 5 = 3y + 2$

$f(2x) = 3(2x) + 2 = 6x + 2$

Option C

9. $$f(x)=\frac{x^2-3x+1}{x-1}$$

$$f(-2)=\frac{(-2)^2-3(-2)+1}{(-2)-1}=\frac{4+6+1}{-2-1}=\frac{11}{-3}$$

$$f\left(\frac{1}{3}\right)=\frac{\frac{1}{9}-3.\frac{1}{3}+1}{\frac{1}{3}-1}=\frac{\frac{1}{9}}{\frac{-2}{3}}=\frac{-1}{6}$$

f(-2) + f(1/3) = -11/3 – 1/6 = -69/18 = -23/6

Option B

10. Domain of x : $-2 \le x < 2$. 3 does not belong to the domain of f. Thus f(3) does not exist.

Option D

11. $-2 \le x < 2 \rightarrow 0 \le x^2 \le 4 \rightarrow -3 \le (x^2 - 3) \le 1$

Range of f = $\{y : y \in R \text{ and } -3 \le y \le 1\}$

Now, 6 does not belong to the range of f, there is no x in domain of f so that f(x)= 6.

Option D

12. $f(x) = x + 1$ if $0 \le x < 2 \rightarrow (0, 1), (1, 2)$

$= 2x - 1$ if $2 \le x < 4 \rightarrow (2, 3), (3, 5)$

$= 3x - 10$ if $4 \le x < 6 \rightarrow (4, 2), (5, 5)$

Thus function f in roaster form is {(0,1), (1, 2), (2, 3), (3, 5), (4, 2), (5, 5)}

Option D

13. Thus function f in roaster form is {(0,1), (1, 2), (2, 3), (3, 5), (4, 2), (5, 5)}

Range of f = {1, 2, 3, 5}

Option B

14. The function f is many-one because $f(1/2) = 2(1/2) + 1 = 2$ and $f(4) = 4 - 2 = 2$, therefore the elements ½ and 4 of the domain have the same image 2.

Option B

15. We need to find the range of f

$0 \le x < 2 \rightarrow 0 \le 2x < 4 \rightarrow 1 \le (2x + 1) < 4 \rightarrow 1 \le y < 5$

$2 \le x \le 5 \rightarrow 0 \le (x - 2) \le 3 \rightarrow 0 \le y \le 3$

On combining both we get, Range of f: $0 \le y < 5$

Now $y = ½$, lie in the range $0 \le y \le 3$

Thus, $f(x) = ½ = (x - 2)$. Thus, $x = 2 + ½ = 5/2$

Option A

16. $f(x) = 2x + 1$

$g(x) = 1 - 2x$

$fog(x) = f(g(x)) = 2g + 1 = 2(1 - 2x) + 1 = 3 - 4x$

$fog(1) = 3 - 4 = -1$

$gof(x) = g(f(x)) = 1 - 2f = 1 - 2(2x + 1) = -4x - 1$

$gof(1) = -4 - 1 = -5$

Thus, $fog(1) > gof(1)$

Option B

7B LIMITS & CONTINUITY

Limits

Type I

$\underset{x\to a}{Lt}\ f(x) = f(a)$ $\quad\quad$ $\underset{x\to a}{Lt}\dfrac{f(x)}{g(x)} = \dfrac{f(a)}{g(a)}; if\ g(a) \neq 0$

Type II

$\underset{x\to a}{Lt}\dfrac{f(x)}{g(x)}\ \&\ g(a) = 0$, then cancel the common terms from numerator and denominator using algebraic treatments.

The reduced form would be: $\underset{x\to a}{Lt}\dfrac{f(x)}{g(x)} = \underset{x\to a}{Lt}\dfrac{p(x)}{q(x)} = \dfrac{p(a)}{q(a)}$

Type III

$\underset{x\to\infty}{Lt}\dfrac{f(x)}{g(x)}$, Divide numerator and denominator by the highest power of x, and then put 1/x = 0.

Type IV(Standard Limits)

- $\underset{x\to 0}{Lt}\dfrac{e^x - 1}{x} = 1 \quad \underset{x\to 0}{Lt}\dfrac{e^{mx} - 1}{x} = m \quad \underset{x\to 0}{Lt}\dfrac{e^{mx} - 1}{mx} = 1$
- $\underset{x\to 0}{Lt}\dfrac{a^x - 1}{x} = \log_e a \quad \underset{x\to 0}{Lt}\dfrac{a^{mx} - 1}{x} = m.\log_e a \quad \underset{x\to 0}{Lt}\dfrac{a^{mx} - 1}{mx} = \log_e a$
- $\underset{x\to 0}{Lt}\dfrac{\log(1+x)}{x} = 1 \quad \underset{x\to 0}{Lt}\dfrac{\log(1+mx)}{x} = m \quad \underset{x\to 0}{Lt}\dfrac{\log(1+mx)}{mx} = 1$
- $\underset{x\to a}{Lt}\dfrac{x^n - a^n}{x - a} = n.a^{n-1} \quad\quad \underset{x\to a}{Lt}\dfrac{x^n - a^n}{x^m - a^m} = \dfrac{n.a^{n-1}}{m.a^{m-1}} = \dfrac{n}{m}.a^{n-m}$
- $\underset{x\to\infty}{Lt}\left(1 + \dfrac{1}{x}\right)^x = e \quad \underset{x\to\infty}{Lt}\left(1 + \dfrac{a}{x}\right)^x = e^a$
- $\underset{x\to 0}{Lt}(1+x)^{\frac{1}{x}} = e;\ \underset{x\to\infty}{Lt}(1+x)^{\frac{a}{x}} = e^a;\ \underset{x\to 0}{Lt}(1+ax)^{\frac{1}{x}} = e^a$

CLASSWORK SECTION

Type I

1. $\underset{x \to 2}{Lt} \dfrac{x^2 - x + 3}{x + 4}$

a) 3/6 b) 3/5 c) 1/5 d) 5/6

2. $\underset{x \to 1}{Lt} \dfrac{4x^5 + 9x + 7}{3x^6 + x^3 + 1}$

a) 4 b) 5 c) 6 d) 2

Type II

3. $\underset{x \to 2}{Lt} \dfrac{x^2 + x - 6}{x^2 - x - 2}$

a) 6/3 b) 5 c) 5/3 d) 3/5

4. $\underset{x \to 9}{Lt} \dfrac{\sqrt{x} - 3}{x - 9}$

a) 1/6 b) 1/5 c) 1/9 d) 1/7

5. $\underset{x \to 0}{Lt} \dfrac{\sqrt{2+3x} - \sqrt{2-5x}}{4x}$

a) $\dfrac{1}{2\sqrt{2}}$ b) $\dfrac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) None of the above

6. $\underset{x \to 0}{Lt} \dfrac{1 - \sqrt{1 - x^2}}{x^2}$

a) $\dfrac{1}{3}$ b) $\dfrac{2}{5}$ c) $\dfrac{1}{2}$ d) None of the above

7. $\underset{x \to 1}{Lt} \frac{x^2 - 3x + 2}{x^3 - 4x + 3}$

a) 0 b) 1 c) 2 d) 4

8. $\underset{x \to 1}{Lt} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

a) 0 b) 1 c) 2 d) 4

Type – III – Limits, When the variable tends to Infinity

9. $\underset{x \to \infty}{Lt} \frac{x^2 + 3x + 2}{x^3 + x - 4}$

a) 0 b) 1 c) 2 d) 4

10. $\underset{x \to \infty}{Lt} \frac{1 + \sqrt{x}}{1 - \sqrt{x}}$

a) 0 b) - 1 c) 1 d) 2

11. $\underset{x \to \infty}{Lt} \frac{(1 + 2x^2)(3 - x^4)}{(1 + x^2)(5 + x^4)}$

a) -2 b) 2 c) 1 d) -1

12. $\underset{x \to \infty}{Lt} \frac{1 + 2 + 3 + \ldots + x}{x^2}$

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) None of the above

13. $\underset{x \to \infty}{Lt} \frac{1^2 + 2^2 + 3^2 + \ldots + x^2}{x^3}$

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{2}{5}$

14. $\underset{x \to \infty}{Lt} \frac{1^3 + 2^3 + 3^3 + \ldots + x^3}{x^4}$

a) $\frac{1}{3}$ b) $-\frac{1}{4}$ c) $\frac{1}{4}$ d) None of the above

15. $\underset{n\to\infty}{Lt}\ 1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\ldots t_n$

a) 1 b) 2 c) 3 d) 5

Type – IV - Definition

For each of the following functions (from Q No. 20 to25),evaluate the following limit:

$\underset{h\to 0}{Lt}\ \frac{f(x+h)-f(x)}{h}$

16. $f(x) = \frac{1}{x}$

a) $\frac{1}{x}$ b) $\frac{1}{x^2}$ c) $-\frac{1}{x^2}$ d) None of the above

17. $f(x) = \frac{1}{x^2}$

a) $\frac{1}{x^3}$ b) $\frac{2}{x^3}$ c) $-\frac{1}{x^3}$ d) $\frac{-2}{x^3}$

18. $f(x) = \sqrt{x}$

a) $\frac{1}{2x}$ b) $\frac{1}{2\sqrt{x}}$ c) $\frac{1}{\sqrt{x}}$ d) None of the above

19. $f(x) = \frac{1}{\sqrt{x}}$

a) $\frac{1}{2\sqrt{x}}$ b) $\frac{-1}{2\sqrt{x}}$ c) $\frac{-1}{2x\sqrt{x}}$ d) None of the above

20. $f(x) = ax^2 + bx + c$

a) ax b) 2ax c) 2ax + b d) None of the above

21. f(x) = c (c = constant)

a) 0 b) 1 c) c d) none of the above

Type – V – Standard Limits

22. $\underset{x\to 0}{Lt}\ \frac{7^{11x}-1}{x}$

a) $11.\log_7 e$ b) $7.\log_e 7$ c) $11.\log_e 7$ d) $11.\log_e 11$

23. $\lim\limits_{x \to 0} \dfrac{e^{4x} - 1}{x}$

a) 0   b) 4   c) 8   d) – 4

24. $\lim\limits_{x \to 0} \dfrac{2^{-x} - 1}{x}$

a) - $\log_e 2$ b) log 3   c) $\log_e 2$ d) none of the above

25. $\lim\limits_{x \to 0} \dfrac{e^{-2x} - 1}{x}$

a) 1   b) 2   c) -1   d) -2

26. $\lim\limits_{x \to 0} \dfrac{e^{\alpha x} - e^{\beta x}}{x}$

a) $\alpha + \beta$ b) $\alpha . \beta$ c) $\alpha - \beta$ d) None of the above

27. $\lim\limits_{x \to 0} \dfrac{3^{x} - 2^{x}}{x}$

a) $\log_e(3.2)$ b) $\log_e\left(\dfrac{2}{3}\right)$ c) $\log_e\left(\dfrac{3}{2}\right)$ d) None of the above

28. $\lim\limits_{x \to 0} \dfrac{e^{\alpha x} + e^{\beta x} - 2}{x}$

a) $\alpha + \beta$ b) $\alpha . \beta$ c) $\alpha - \beta$ d) None of the above

29. $\lim\limits_{x \to 0} \dfrac{e^{5x} - e^{3x} - e^{2x} + 1}{x^2}$

a) 3   b) 2   c) 6   d) -6

30. $\lim\limits_{x \to 0} \dfrac{6^{x} - 3^{x} - 2^{x} + 1}{x^2}$

a) $\log_e 3 + \log_e 2$ b) $\log_e 3 - \log_e 2$

c) $\log_e 6$ d) $\log_e 3 . \log_e 2$

31. $\lim\limits_{h \to 0} \dfrac{e^{(x+h)^2} - e^{x^2}}{h}$

a) e^{x^2} b) xe^{x^2} c) $2xe^{x}$ d) $2\,xe^{x^2}$

32. $\underset{h \to 0}{Lt} \frac{\log(x+h)-\log x}{h}$

a) x b) 1 c) $\frac{1}{x}$ d) None of the above

33. $\underset{h \to 0}{Lt} \frac{(x+h)^n - x^n}{h}$

i) n. x^{n-1} j) n.a^{n-1} k) a^{n-1} l) $n^2.a^{n-1}$

34. $\underset{x \to \infty}{Lt} \left(1+\frac{a}{x}\right)^x$

a) e b) e^a c) e^3 d) e^{4a}

35. $\underset{x \to \infty}{Lt} \left(\frac{x+6}{x+1}\right)^{(x+6)}$

a. e b. e^2 c. e^3 d. e^5

36. $\underset{x \to 0}{Lt} \{1+ax\}^{\frac{1}{x}}$

a. e b. e^{2a} c. e^{3a} d. e^a

37. $\underset{x \to 2}{Lt} \frac{ax^2-b}{x-2} = 4$, find a & b.

a. 1, 2 b. 1, 3 c. 1, 1 d. 1, 4

38. $\underset{x \to 1}{Lt} \frac{ax^2+bx-2}{x-1} = 3$, find a & b.

a. 1, 1 b. 1, 2 c. 1, 3 d. 1, 4

CONCEPT OF CONTINUITY OF A FUNCTION

A function f (x) is said to be Continuous at a particular point, x =a , if it satisfy the following conditions:

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

Left hand = Right hand =Functional
Limit (LHL) Limit (RHL) Value

Note1: Equality of RHL and LHL is treated as a condition for existence of limit i.e, limit of a function will exist if LHL=RHL

Note2: For Continuity, equality of the functional value at that point is also necessary.

Note3: For all Continuous functions, limit must exist, but existence of limit, is not a sufficient condition for continuity of a function.

Note4: Sum, difference , product and quotient of all continuous functions are always continuous.

Note5: All polynomials are continuous.

Note6: If a given function is of the form $\frac{f(x)}{g(x)}$, where both f (x) and g(x) are polynomials in x, it will be everywhere continuous except at the points at which it is undefined i.e; points of discontinuity of such functions are the points where g(x) =0.

Example: In each of the following cases, discuss continuity of the functions at x=5

i) $f(x) = \frac{x^2 - 25}{x - 5}$

Solution: LHL= $\lim_{x \to 5^-} \frac{x^2 - 25}{x - 5} = \lim_{x \to 5^-} \frac{2x}{1} = 2 \times 5 = 10$

RHL= $\lim_{x \to 5^+} \frac{x^2 - 25}{x - 5} = \lim_{x \to 5^+} \frac{2x}{1} = 2 \times 5 = 10$

$$f(5)=\frac{25-25}{5-5}=\frac{0}{0}(undifined)$$

since,LHL = RHL $\neq$ f (5), f (x)is discontinuous atx = 5,although the limit has existed.

ii) $f(x)=\frac{x^2-25}{x-5}, when \quad x \neq 5$

=10, when x=5

Solution: LHL=10=RHL taken from(i)

Given , f (5) =10 since, LHL=RHL= f (5), f (x) is continuous at x = 5

iii) $f(x)=\frac{x^2-25}{x-5}, when \quad x \neq 5$

=2, when x = 5

Solution: LHL=RHL=10 taken from(ii)

Given , f (5) = 2 since, LHL=RHL$\neq$ f (5), f (x) is discontinuous at x = 5

Example 2: Find the points of discontinuity of the function, $f(x)=\frac{(x^2-3x+2)}{(x^2-5x+6)}$

Solution: The given function will be continuous at all points, except at the points at which it is undefined i.e the points at which its denominator is 0. $(x^2-5x+6)=0$

Points of discontinuity are 2 and 3

$\Rightarrow(x-2)(x-3)=0$

$\Rightarrow x=2,3$

WORKING CODES for Q. No. 1 to 18

Mark C : if function is continuous at the given point

Mark D : if function is discontinuous at the given point

Mark X : if nothing can be said about the continuity of the function at the given point

Mark Y : if function is neither continuous nor discontinuous at the given point

CLASSWORK SECTION

39. f(x) = $\frac{x^2-9}{x-3}$, Check continuity at x = 3

a) C b) D c) X d) Y

40. f(x) = $\begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & \text{when } x = 3 \end{cases}$; Check continuity at x=3

a) X b) Y c) D d) C

41. f(x) = $\begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 1, & \text{when } x = 3 \end{cases}$; Check continuity at x = 3

a) C b) X c) D d) Y

42. f(x) = $\begin{cases} x+1, x \geq 1 \\ 2x+1, x < 1 \end{cases}$, Check continuity x = 1

a) C b) X c) D d) Y

43. f(x) = $\left.\begin{matrix} x^2, x > 2 \\ 4, x = 2 \\ 2x, x < 2 \end{matrix}\right\}$ Check continuity at x = 2

a) C b) D c) X d) Y

44. f(x) = $\begin{cases} 4x+3, x \neq 4 \\ 3x+7, x = 4 \end{cases}$, Check continuity at x = 4

a) X b) Y c) C d) D

45. f(x) = $\left.\begin{matrix} x^2, 0 < x < 1 \\ x, 1 \leq x < 2 \\ \frac{1}{4}x^3, 2 \leq x < 3 \end{matrix}\right\}$ Check continuity at x = 2

a) C b) X c) D d) Y

46. f(x) = $\left.\begin{array}{l} 3+2x, -\frac{2}{3} \le x < 0 \\ 3-2x, 0 \le x < \frac{3}{2} \\ -3-2x,\ x \ \ge \frac{3}{2} \end{array}\right\}$ Check continuity at x = 0

a) D b) Y c) C d) X

47. f(x) = $\left.\begin{array}{l} 3+2x, -\frac{2}{3} \le x < 0 \\ 3-2x, 0 \le x < \frac{3}{2} \\ -3-2x,\ x \ \ge \frac{3}{2} \end{array}\right\}$ Check continuity at x = $\frac{3}{2}$

a) C b) D c) X d) Y

48. f(x) = |x|, Check continuity at x = 0

a) D b) C c) X d) Y

49. $f(x) = \left.\begin{array}{l} \frac{|x-3|}{x-3}, \quad x \ne 3 \\ =1, \ \text{when } x = 3 \end{array}\right\}$, Check continuity at x = 3

a) X b) Y c) D d) C

(for Q. No. 14 and15)

Find the points of discontinuity of the following functions:

50. f(x) = $\frac{2x^2-6x+5}{12x^2+x-20}$

a) $-\frac{4}{3}$ b) $\frac{5}{4}$ c) $\frac{4}{3}$ d) Both of a) and b) above

51. f(x) = $\frac{3x^2-4x}{x^3+x^2-x-1}$

a) ± 1 b) 1 c) -1 d) None of the above

52. Given f(x) $\left\{\begin{array}{l} = x+1, \quad x \le 1 \\ = 4-ax, x > 1 \end{array}\right.$, for what value of a, will f(x) be continuous at x = 1?

a) 2 b) 1 c) 3 d) 4

53. Given f(x) $\begin{cases} = x+1, & x \le 1 \\ = 3-ax^2, & x > 1 \end{cases}$, for what value of a, will f(x) be continuous at x = 1?

a. 2 b. 1 c. 8 d. 6

54. Given f(x) = $\dfrac{2x^2-8}{x-2}$ is undefined at x = 2. What value must be assigned to f(2), if f(x is to be continuous at x = 2 ?

a. 6 b. 2 c. 8 d. 1

BASIC CONCEPTS OF DIFFERENTIAL AND INTEGRAL CALCULUS

8 DIFFERENTIAL CALCULUS

THEORY

Let y = f(x) be a continuous function. Then, the value of y depends upon the value of x and it changes with a change in the value of x. We use the word increment to denote a small change, i.e., increase or decrease in the values of x and y.

Let Δy be an increment in y corresponding to an increment Δx in x.

Then, $\frac{dy}{dx} = \underset{h\to 0}{Lt}\frac{f(x+h)-f(x)}{h}$. This limit, if it exists finitely, is called the derivative or differential coefficient of y = f(x) with respect to x and is denoted by $\frac{dy}{dx}$ or $f^{'}(x)$ or y_1. The process of finding the derivative is known as differentiation.

Standard Derivatives

$\frac{d}{dx}x^n = n.x^{n-1}$	$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}x = 1$	$\frac{d}{dx}\frac{1}{x^n} = -\frac{n}{x^{n+1}}$
$\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$	$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$	$\frac{d}{dx}\frac{1}{\sqrt{x}} = -\frac{1}{2x\sqrt{x}}$	$\frac{d}{dx}e^x = e^x$
$\frac{d}{dx}e^{mx} = m.e^{mx}$	$\frac{d}{dx}a^x = .a^x.\log_e a$	$\frac{d}{dx}a^{mx} = m.a^{mx}.\log_e a$	$\frac{d}{dx}\log_e x = \frac{1}{x}$

Product and Quotient Rule

$$\frac{d}{dx}u.v = u.\frac{d}{dx}v + v.\frac{d}{dx}u \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$$

Parametric Functions

Sometimes x and y are given as function of another variable t. Then t is called a parameter. Let x = f(t) and y = g(t), then:

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$

Implicit Functions

When the variables x and y are not explicitly or clearly defined in terms of each other ,the function takes an implicit form. We differentiate both sides of the equation term wise, keeping in mind that $\frac{d}{dx}2y = 2.\frac{dy}{dx}$ & $\frac{d}{dt}m^2 = 2m.\frac{dm}{dt}$ and so on.

Function of a Function – Chain Rule

If $y = f(t)$ and $t = g(x)$, then $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$, and the rule can be further extended.

Logarithmic Differentiation – Log Rule

When the given function is a power of some expression or a product of expressions, we take logarithm on both sides and differentiate the implicit functions so obtained.

If $y = f(x)^{g(x)}$, then; $\log y = g(x).\log f(x)$......Then proceed.

Slope – Applied Differentiation

For $y = f(x)$, slope at any point (x_1, y_1) is given by $\frac{dy}{dx}_{at\ x_1, y_1}$

Higher Order Derivatives

Let y = f(x) be a differentiable function of x whose second and higher order derivatives exists. The first, second, third, ... and the nth derivatives of this function are denoted by;

dy/dx, d^2y/dx^2, d^3y/dx^3, ..., d^ny/dx^n or y_1, y_2, y_3, ... , y_n or $f'(x)$, $f''(x)$, ...

Integral Calculus

THEORY

Fundamental Integrals

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\int \frac{dx}{x^n} = \frac{1}{(1-n).x^{n-1}} + C$	$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$	$\int dx = x + C$
$\int \frac{dx}{x} = \log\lvert x\rvert + C$	$\int e^x dx = e^x + C$	$\int e^{mx} dx = \frac{e^{mx}}{m} + C$	$\int a^x dx = \frac{a^x}{\log_e a} + C$
$\int a^{mx} dx = \frac{a^{mx}}{m \log_e a} + C$			

Integration by Parts

$$\int u.v\ dx = u.\int v dx - \int\left\{\frac{du}{dx}\int v dx\right\}dx$$

Standard Integrals

- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$, **Given** $(\lvert x\rvert > \lvert a\rvert)$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$, **Given** $(\lvert x\rvert > \lvert a\rvert)$
- $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log\left|(x + \sqrt{x^2 \pm a^2})\right| + C$
- $\int \sqrt{x^2 + a^2}\, dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2+a^2}\right| + C$
- $\int \sqrt{x^2 - a^2}\, dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + C$

Definite Integrals:- Important Properties

$\int_a^b f(x)\,dx = \int_a^b f(z)\,dz$

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \quad (a<c<b)$

$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$\int_{-a}^{+a} f(x)\,dx = 0$, if f(x) is an odd function.

$\int_{-a}^{+a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f(x) is an even function.

$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$\int_a^b f(x)\,dx = \phi(b) - \phi(a)$

$\int_a^b x\,dx = \left.\frac{x^2}{2}\right]_a^b = \frac{b^2}{2} - \frac{a^2}{2} = \frac{b^2-a^2}{2}$

APPLICATION OF DERIVATIVE & INTEGRATION IN COMMERCE AND ECONOMICS

1. COST FUNCTION = C(x) = Fixed Cost + Variable Cost

$$= a + bx$$

2. AVERAGE COST = AC = $\dfrac{\text{TOTAL COST}}{\text{QUANTITY}} = \dfrac{C(x)}{x}$

3. MARGINAL COST = MC = $\dfrac{d}{dx}$ (TOTAL COST) = $\dfrac{d}{dx}(C(x))$

4. REVENUE FUNCTION = R (x) = Quantity x Price

   R (x) = x.p

5. AVERAGE REVENUE = AR = $\dfrac{\text{TOTAL REVENUE}}{\text{QUANTITY}} = \dfrac{x.p}{x} = p$

6. MARGINAL REVENUE = MR = $\dfrac{d}{dx}$ (TOTAL REVENUE) = $\dfrac{d}{dx}(R\,(x))$

7. PROFIT FUNCTION = P(x) = R(x) − C(x)

8. AVERAGE PROFIT = $\dfrac{P(x)}{x} = \dfrac{\text{TOTAL PROFIT}}{\text{QUANTITY}}$

9. MARGINAL PROFIT = MP = $\dfrac{d}{dx}(R(x) - C(x)) = \dfrac{d}{dx}(P\,(x))$

10. MAXIMUM PROFIT:

    Level of output at which profit is maximum can be obtained through the following :

    $$MR = MC$$

11. Consumption Function. The consumption function expresses a relationship between the total income (I) and the total national consumption (C). It is denoted by C = f(I).

    Marginal Propensity to Consume. It is the rate of change of the consumption with respect to income.

    ∴ Marginal Propensity to Consume = MPC = $\dfrac{dC}{dI}$.

    Marginal Propensity to Save

    Let S denote the saving, then Saving S = (Total income − Total consumption) = I − C

    ∴ Marginal Propensity to Save = MPS = $\dfrac{dS}{dI}$.

    It indicates how fast saving changes with respect to income.

12. At Equilibrium, Qd = Qs

On solving the demand and supply equation, we obtain the equilibrium Price and Quantity.

13. Total Cost $= \int$ Marginal Cost dx $= \int MC \; dx$

$= C(x) + k$

where k = fixed cost

i). $AC = \int MAC\, dx$

ii). $VC = \int MVC\, dx$

14. Total Cost for 'n' units $= \int_0^n MC \; dx$

15. Total Revenue $= \int$ Marginal Revenue dx

$= \int MR\, dx = R(x)$

16. Total Revenue for 'n' units $= \int_0^n MR \; dx$

MAXIMA AND MINIMA (EXTREME VALUE)

Given : y = f (x)

Steps for finding Maxima and Minima of a function.

1. Find $\frac{dy}{dx}$

2. Equate $\frac{dy}{dx} = 0$ to obtain the value/values of x

3. Find $\frac{d^2y}{dx^2}$ and put therein the values of x obtained from Step 2, and observe the result:

   (i) if $\frac{d^2y}{dx^2} < 0$, then the function attains its **Maximum Value**, at that point and the maximum value of the function can be obtained by putting the value in the original function.

   (ii) If $\frac{d^2y}{dx^2} > 0$, the function attains the **Minimum Value**, at that point and the minimum value of the function can be obtained by putting the value in the original function.

   (iii) If on putting the value of 'x' $\frac{d^2y}{dx^2} = 0$, but $\frac{d^3y}{dx^3} \neq 0$, then the function will have a Point of Inflexion, at a point.

   In other words, at Point of Inflexion, the curve changes its Curvature.

CLASS WORK

DIFFERENTIATION

1. Differential coefficient of $5x^2 + 3x + 2$ at $x = 0$ is
   (a) 10 (b) 3 (c) −3 (d) 2

2. If $y = a^x + x^a + a^a$, then $\frac{dy}{dx} =$
   (a) $xa^{x-1} + ax^{a-1} + aa^{a-1}$ (b) $a^x \log a + ax^{a-1}$
   (c) $a^x \log a + ax^{a-1} + aa^{a-1}$ (d) none

3. If $f(x) = \frac{x^3+1}{x^3-1}$, then $f^1(x)=$
   (a) $6x^2 (x^3 - 1)^{-2}$ (b) $-6x^2(x^3 - 1)^{-2}$
   (c) $x^2(x^3 - 1)^{-2}$ (d) $-x^2(x^3 - 1)^{-2}$

4. If $f(x) = x^2 - 6x + 5$, then $f'(2) - f'(5) =$
   (a) $-3f'(2)$ (b) $3f'(2)$
   (c) $2f'(2)$ (d) $4f'(2)$

5. If $f(x) = x^2e^x$, then $f'(x)$ is
   (a) $xe^x(x^2 + 2)$ (b) $xe^x (x + 1)$
   (c) $x e^x (x + 2)$ (d) non

6. $y = x(x - 2) (x - 3)$, then $\frac{dy}{dx} =$
   (a) $3x^2 - 10x + 6$ (b) $3x^2 - 10x - 6$
   (c) $3x^2 + 10x + 6$ (d) $3x^2 + 10x - 6$

7. $f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then $f'(x)$ at $x = 4$ is
   (a) $\frac{17}{16}$ (b) $\frac{1}{16}$
   (c) $\frac{15}{16}$ (d) $-\frac{1}{16}$

8. $f(x) = a^x x^k$, then $f'(x)$

(a) $f(x)(a - \log a)$ (b) $f(x)(a + \log a)$

(c) $f(x)\left(\frac{k}{x} - \log a\right)$ (d) $f(x)\left(\frac{k}{x} + \log a\right)$

9. $f(x) = \frac{e^x - 1}{e^x + 1}$, then $f'(1) =$

(a) $\frac{2e^x}{(e^x+1)^2}$ (b) $\frac{-2e^x}{(e^x+1)^2}$

(c) $\frac{e^x}{(e^x+1)^2}$ (d) $\frac{-e^x}{(e^x+1)^2}$

10. If $y = \log(\log x)$, then $dy/dx =$

(a) $\frac{x}{\log x}$ (b) $\frac{1}{\log x^x}$

(c) $\frac{1}{x \log x}$ (d) both (b) and (c)

11. $f(x) = {}^xC_2$, then $f'(1) =$

(a) 1 (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) $\frac{1}{6}$

12. Derivative of $\sqrt{x^2 + \sqrt{x}}$

(a) $\frac{1}{2\sqrt{x^2+\sqrt{x}}}$ (b) $2x + \frac{1}{2\sqrt{x}}$

(c) $\frac{1}{2\sqrt{x^2+\sqrt{x}}}\left(2x + \frac{1}{2\sqrt{x}}\right)$ (d) none

13. Derivative of e^{x^2+5x+6}

(a) $(2x+5)e^{x^2+5x+6}$ (b) e^{x^2+5x+6}

(c) $(x+5)e^{x^2+5x+6}$ (d) none

14. $y = \sqrt{x^2+a^2}$, then $y\dfrac{dy}{dx} =$

(a) x (b) 2x (c) y (d) 2y

15. $\dfrac{d}{dx}(3^{\log_3 x})$

(a) 1 (b) 0 (c) $\dfrac{1}{2}$ (d) $2^x \log_2 x$

16. $y = \log(x + \sqrt{x^2+5})$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{1}{\sqrt{x^2+5}}$ (b) $\dfrac{-1}{\sqrt{x^2+5}}$

(c) $\dfrac{x}{\sqrt{x^2+5}}$ (d) none

17. Find $\dfrac{dy}{dx}$, if $y = 7^{x^2+x}$

(a) $7^{x^2+x}\log 7$ (b) $7^{x^2+x}\log 7(x+1)$

(c) $7^{x^2+x}\log 7(2x+1)$ (d) $2(x+1)7^{x^2+x}\log 7$

18. $x^2 + xy + y^2 = 0$, then $\dfrac{dy}{dx} =$

(a) $-\left(\dfrac{2x+y}{x+2y}\right)$ (b) $-\left(\dfrac{2x-y}{x+2y}\right)$

(c) $-\left(\dfrac{x+2y}{2x+y}\right)$ (d) $\dfrac{2x+y}{x+2y}$

19. $ax^2 + 2hxy + by^2 = 0$, then $\frac{dy}{dx}$ =

(a) $-(ax + hy)(hx + by)^{-1}$ (b) $-(ax + hy)(hx + by)$

(c) $\frac{ax + by}{bx + by}$ (d) none

20. The slope of tangent to the curve $x^2 + xy + y^2 = 0$ at (2,1).

(a) $-\frac{5}{4}$ (b) $\frac{5}{3}$ (c) $\frac{3}{5}$ (d) $-\frac{3}{5}$

21. $x^y = e^x$, then $\frac{dy}{dx}$ is

(a) $\frac{\log x - 1}{\log x}$ (b) $\frac{\log x - 1}{(\log x)^2}$

(c) $\frac{\log x + 1}{\log x}$ (d) $\frac{\log x + 1}{(\log x)}$

22. $x^3 y^4 = (x + y)^7$, then $\frac{dy}{dx}$ =

(a) $-\frac{y}{x}$ (b) $\frac{y}{x}$

(c) 0 (d) 1

23. $y = x^{x^{x \ldots \infty}}$, then $\frac{dy}{dx}$ =

(a) $\frac{y^2}{x(1 + y \log x)}$ (b) $\frac{y^2}{x(1 - y \log x)}$

(c) $\frac{y}{x(1 + y \log x)}$ (d) $\frac{y}{x(1 - y \log x)}$

24. $y = \sqrt{x}^{\sqrt{x} \ldots \infty}$, then $\frac{dy}{dx}$ =

(a) $\frac{y^2}{x(2 + y \log x)}$ (b) $\frac{y}{x(2 - y \log x)}$

(c) $\frac{y^2}{x(1 - y \log x)}$ (d) none

25. Find $\frac{dy}{dx}$, if $x = at^3$; $y = \frac{a}{t^3}$.

(a) $\frac{1}{t^5}$ (b) $-\frac{1}{t^5}$

(c) $\frac{1}{t^6}$ (d) $-\frac{1}{t^6}$

26. $y = 2at$; $x = at^2$, then $\frac{dy}{dx}$ at $t = 1$

(a) 1 (b) 0 (c) 2 (d) a

27. $e^{xy} + \log xy = 0$, then $\frac{dy}{dx} =$

(a) $\frac{y}{x}$ (b) $-\frac{y}{x}$ (c) $\frac{y^2}{x^2}$ (d) $-\frac{y^2}{x^2}$

28. $y = x^x$, then $\frac{dy}{dx} =$

(a) $x^x (\log ex)$ (b) $1 + \log x$

(c) $x^x (1 - \log x)$ (d) x^x

29. $y = x^x$, then $\frac{d^2y}{dx^2} =$

(a) $\frac{dy}{dx}(1+\log x) + y\frac{d}{dx}(1+\log x)$ (b) $\frac{dy}{dx}(1+\log x) + \frac{d}{dx}(1+\log x)$

(c) $\frac{dy}{dx}(1+\log x) - y\frac{d}{dx}(1+\log x)$ (d) $\frac{dy}{dx}(1+\log x) - \frac{d}{dx}(1+\log x)$

30. $y = x^{\log x}$, then $\frac{\frac{dy}{dx}}{y} =$

(a) $2x^{-1} \log x$ (b) $2x \log x$

(c) $x^{\log x}\left(\frac{2\log x}{x}\right)$ (d) none

31. $y = (3x+1)^{\frac{1}{4}}(4x+1)^{\frac{1}{5}}(5x+1)^{\frac{1}{6}}$, then $\dfrac{\frac{dy}{dx}}{y} =$

(a) $\frac{3}{4}\left(\frac{1}{3x+1}\right)+\frac{4}{5}\left(\frac{1}{4x+1}\right)+\frac{5}{6}\left(\frac{1}{5x+1}\right)$

(b) $y\left[\frac{3}{4}\left(\frac{1}{3x+1}\right)+\frac{4}{5}\left(\frac{1}{4x+1}\right)+\frac{5}{6}\left(\frac{1}{5x+1}\right)\right]$

(c) $(x-3)^{-1}+\frac{1}{3}(x-4)^{-1}+\frac{1}{4}(x-5)^{-1}$

(d) none

32. $y = e^{k\log x} + e^{x\log k}$, then $\frac{dy}{dx} =$

(a) $x^k + k^x$ (b) $kx^{k-1} + k^x \log k$

(c) $kx^{k-1} + xk^{x-1}$ (d) none

33. The slope of the tangent to the curve $y = \sqrt{9 - x^2}$ at the point where ordinate is equal to abscissa is

(a) 1 (b) −1 (c) 0 (d) none

34. The slope of tangent to the curve $y = x^2 - x$ at the point where the line $y = 6$ meets the curve in the first quadrant is

(a) 4 (b) 5 (c) 6 (d) 2

35. If $y = \frac{\sqrt{x}}{\sqrt{a}} + \frac{\sqrt{a}}{\sqrt{x}}$, then $2xy\frac{dy}{dx} - \frac{x}{a} + \frac{a}{x} =$

(a) 1 (b) 0 (c) 2 (d) −1

36. If $y = \log\left(x + \sqrt{x^2 - a^2}\right)$, then $(x^2 - a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} =$

(a) 1 (b) −1 (c) 0 (d) none

37. If $y = \left(x + \sqrt{x^2 - 4}\right)^m$, then $(x^2 - 4)\left(\frac{dy}{dx}\right)^2 - m^2y^2 =$

(a) 0 (b) 1 (c) 2 (d) none

PAST EXAM QUESTIONS

38. The derivative of $x^2 \log x$ is

(a) $1 + 2 \log x$ (b) $2 \log x$

(c) $x(1 + 2 \log x)$ (d) none of these

39. If $x^y = y^x$, then $\frac{dy}{dx}$ gives:

(a) $\frac{x(x \log y - y)}{y(y \log x - x)}$ (b) $\frac{x(y \log x - x)}{y(x \log y - y)}$

(c) $\frac{y(x \log y - y)}{x(y \log x - x)}$ (d) none of these

40. If $x^3 - 2x^2y^2 + 5x + y = 5$, then $\frac{dy}{dx}$ at $x = 1$ and $y = 1$ is

(a) 4/3 (b) −5/4

(c) 4/5 (d) −4/3

41. If $y = \left(x + \sqrt{x^2 + m^2}\right)^n$ then $\frac{dy}{dx} =$

(a) $\frac{ny}{\sqrt{x^2 + m^2}}$ (b) ny

(c) $-\frac{ny}{\sqrt{x^2 + m^2}}$ (d) none

42. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$ then $\frac{dy}{dx} - y$ is equal to

(a) 1 (b) −1

(c) 0 (d) none

43. Differentiate e^{x^x} :

(a) $(1 + \log x)$ (b) $x^x (1 + \log x)$

(c) $e^{x^x}(1 + \log x)x^x$ (d) $e^{x^x}(1 + \log x)$

44. If $x^2 + y^2 = 4$, then

(a) $y\frac{d^2y}{dx^2} - \left(2\frac{dy}{dx}\right)^2 + 1 = 0$ (b) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0$

(c) $y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 - 1 = 0$ (d) $y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 1 = 0$

45. For the functions $y = x^3 - 3x$, the value of $\frac{d^2y}{dx^2}$ at which $\frac{dy}{dx}$ is zero, is

(a) ±1 (b) ±3

(c) ±6 (d) none of these

46. If $x^y = e^{x-y}$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{2\log x}{(1+\log x)^2}$ (b) $\frac{\log x}{(1+\log x)}$

(c) $\frac{\log x}{(1+\log x)^2}$ (d) none of the above

47. If $y = \log x^x$ then $\frac{dy}{dx}$ is equal to:

(a) $\log ex$ (b) $\log \frac{e}{x}$

(c) $\log \frac{x}{e}$ (d) 1

48. $xy = 1$ then $y^2 + \frac{dy}{dx} = ?$

(a) 1 (b) 0

(c) 2 (d) none of the above

INTEGRATION

1. $\int x\sqrt{x}\ dx$

(a) $\frac{2}{5}x^{5/2} + c$ (b) $\frac{2}{5}x^{2/5} + c$

(c) $\frac{2}{3}x^{3/2} + c$ (d) $\frac{5}{2}x^{5/2} + c$

2. $\int \log(x^2)\,dx$

(a) $x(\log x - 1) + c$ (b) $2x(\log x - 1) + c$

(c) $2(\log x - 1) + c$ (d) $2x \log x - x + c$

3. $\int (e^{2x} + e^{-3x})dx$

(a) $\frac{e^{2x}}{2} + \frac{e^{-3x}}{3} + c$ (b) $\frac{e^{2x}}{2} - \frac{e^{-3x}}{3} + c$

(c) $\frac{e^{2x}}{2} - \frac{e^{+3x}}{3}$ (d) none

4. $\int \left(x + \frac{1}{x}\right)^2 dx$

(a) $\frac{x^3}{3} + 2x - \frac{1}{x} + c$ (b) $\frac{x^3}{3} + 2x + \frac{1}{x} + c$

(c) $\frac{x^3}{3} + x - \frac{1}{x} + c$ (d) $\frac{x^3}{3} - 2x + \frac{1}{x} + c$

5. $\int (1 - 4x)(1 + x)\ dx$

(a) $x - \frac{3x^2}{2} - \frac{4x^3}{3} + c$ (b) $x - \frac{x^2}{2} - \frac{x^3}{3} + c$

(c) $x + \frac{3x^2}{2} - \frac{4x^3}{3} + c$ (d) $x - \frac{3x^2}{2} + \frac{4x^3}{3} + c$

6. $\int x^x(1 + \log x)\,dx$

(a) $x^x + c$ (b) $x^x(\log x) + c$

(c) $x \log x + c$ (d) done

7. $\int \frac{dx}{\sqrt{x+2}}$

(a) $\sqrt{x+2} + c$ (b) $2\sqrt{x+2} + c$

(c) $\frac{2}{\sqrt{x+2}} + c$ (d) none

8. $\int x^3 e^{2x}\, dx$

(a) $\frac{x^3 e^{2x}}{2} + \frac{3}{4}(x^2 e^{2x}) + \frac{3}{4}(xe^{2x}) + \frac{3}{8}e^{2x} + c$

(b) $\frac{x^3 e^{2x}}{2} - \frac{3}{4}(x^2 e^{2x}) + \frac{3}{4}(xe^{2x}) - \frac{3}{8}e^{2x} + c$

(c) $\frac{x^3 e^{2x}}{2} + \frac{3}{4}(x^2 e^{2x}) - \frac{3}{4}(xe^{2x}) - \frac{3}{8}(e^{2x}) + c$

(d) none

9. $\int x^2 2^x\, dx$

(a) $x^2 \frac{2^x}{\log 2} - \frac{x\,2^x}{(\log 2)^2} + \frac{2\,2^x}{(\log 2)^3} + c$

(b) $\frac{x^2\,2x}{\log 2} + \frac{2x\,2^x}{(\log 2)^2} + \frac{2^x}{(\log 2)^3} + c$

(c) $\frac{x^2\,2^x}{\log 2} + \frac{2x\,2^x}{(\log 2)^2} + \frac{2^x}{(\log 2)^3} + c$

(d) none

10. $\int x^n \log x\, dx$

(a) $\frac{x^{n+1}}{(n+1)^2}[(n+1)\log x + 1] + c$ (b) $\frac{x^{n+1}}{(n+1)^2}[(n+1)\log x - 1] + c$

(c) $\frac{x^{n+1}}{n+1}[\log x - 1] + c$ (d) none

11. $\int \log x \, dx$

(a) $x \log x - x^2 + c$ (b) $x \log x + c$

(c) $x \log x - x + c$ (d) none

12. $\int (\log x)^2 \, dx$

(a) $x[(\log x)^2 - 2 \log x + 2] + c$ (b) $x[(\log x)^2 + 2 \log x - 2] + c$

(c) $x^2 (\log x)^2 - 2x \log x + 2x + c$ (d) none

13. $\int \frac{3x+2}{(x-2)(x-3)} dx$

(a) $11 \log (x - 3) - 8 \log |x - 2| + c$ (b) $8 \log |x - 3| - 11 \log |x - 2| + c$

(c) $-11 \log |x - 3| + 8 \log |x - 2| + c$ (d) $- 8 \log |x - 3| + 11 \log |x - 2| + c$

14. $\int \frac{1}{(x-1)(x-2)} dx$

(a) $\log \left|\frac{x-2}{x-1}\right| + c$ (b) $\log \left|\frac{x-1}{x-2}\right| + c$

(c) $\log \left|\frac{x-2}{x-1}\right|$ (d) $\log \left|\frac{x-1}{x-2}\right|$

15. $\int \frac{1}{(x-1)(x-2)(x-3)} dx$

(a) $\frac{1}{2} \log |x-3| - \log |x-2| + \frac{1}{2} \log |x-1| + c$

(b) $-\frac{1}{2} \log |x-3| + \log |x-2| - \frac{1}{2} \log |x-1| + c$

(c) $\log |x - 3| + \log |x - 2| - \log |x - 1| + c$

(d) none

16. $\int \frac{1}{(x-1)(x-3)^2} dx$

(a) $\frac{1}{4} \log |x-1| - \frac{1}{4} \log |x-3| - \frac{1}{2(x-3)} + c$

(b) $\frac{1}{4} \log |x-1| + \frac{1}{4} \log |x-3| - \frac{1}{2}\left(\frac{1}{x-3}\right) + c$

(c) $\frac{1}{4}\log|x-1| - \frac{1}{4}\log|x-3| - \frac{1}{2}\left(\frac{1}{x-3}\right) + c$

(d) $\frac{1}{4}\log|x-1| + \frac{1}{4}\log|x-3| + \frac{1}{2}\left(\frac{1}{x-3}\right) + c$

17. $\int (x^2+1)^9 2x\,dx$

(a) $(x^2+1)^{10} + c$ (b) $\frac{(x^2+1)^{10}}{10} + c$

(c) $\frac{(x^2+1)^9}{10} + c$ (d) none

18. $\int \frac{7x^2}{(x^3+2)^3}\,dx$

(a) $\frac{7}{6}(x^3+2)^2 + c$ (b) $-\frac{7}{6}(x^3+2)^2 + c$

(c) $-\frac{7}{6}\left[\frac{1}{(x^3+2)^2}\right] + c$ (d) $\frac{7}{6}\left[\frac{1}{(x^3+2)^2}\right] + c$

19. $\int \frac{1}{x(\log x)^2}\,dx$

(a) $\frac{1}{\log x} + c$ (b) $-\frac{1}{\log x} + c$

(c) $\frac{1}{x\log x} + c$ (d) $-\frac{-1}{x\log x} + c$

20. $\int e^x(x^3+5x^2+4x)\,dx$

(a) $e^x(x^3+x^2) + c$ (b) $e^x(x^3+2x^2) + c$

(c) $e^x(x^3+3x^2) + c$ (d) $e^x(x^3+4x^2) + c$

21. $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$

(a) $e^x(x)^{-1} + c$ (b) $e^x(x) + c$

(c) $e^x\left(\frac{1}{x^2}\right) + c$ (d) none

22. $\int e^x (x \log x + 1)(x)^{-1}\, dx$

(a) $e^x\left(\frac{1}{\log x}\right) + c$ (b) $e^x (\log x) + c$

(c) $e^x (x \log x) + c$ (d) none

23. $\int e^x\left(\frac{2-x}{(1-x)^2}\right) dx$

(a) $e^x\left(\frac{1}{1-x}\right) + c$ (b) $e^x\left(\frac{-1}{1-x}\right) + c$

(c) $e^x\left(\frac{1}{2-x}\right) + c$ (d) none

24. $\int \frac{1}{x^2-9}\, dx$

(a) $\frac{1}{3}\log\left|\frac{x-3}{x+3}\right| + c$ (b) $\frac{1}{6}\log\left|\frac{x-3}{x+3}\right| + c$

(c) $-\frac{1}{6}\log\left|\frac{x-3}{x+3}\right| + c$ (d) $-\frac{1}{3}\log\left|\frac{x-3}{x+3}\right| + c$

25. $\int \sqrt{x^2+4}\, dx$

(a) $\frac{x}{2}\sqrt{x^2+4} + c$

(b) $\frac{x}{2}\sqrt{x^2+4} + 8\log|x+\sqrt{x^2+4}| + c$

(c) $\frac{x}{2}\sqrt{x^2+4} + 2\log|x+\sqrt{x^2+4}| + c$

(d) $\frac{x}{2}\sqrt{x^2+4} - 2\log|x+\sqrt{x^2+4}| + c$

26. $\int \frac{1}{\sqrt{x^2+9}}\, dx$

(a) $\log|x+\sqrt{x^2+9}| + c$ (b) $\log|x-\sqrt{x^2+9}| + c$

(c) $\sqrt{x^2+9} + c$ (d) $2\sqrt{x^2+9} + c$

27. $\int_{0}^{2} 6x^2 \, dx$

(a) 8 (b) 16
(c) 4 (d) 2

28. $\int_{4}^{5} f(x)\,dx - \int_{4}^{5} f(9-x)\,dx$

(a) 0 (b) 1
(c) −1 (d) none

29. $\int_{0}^{a} [f(x) + f(-x)]dx =$

(a) 0 [for an odd function] (b) $2\int_{0}^{a} f(x)\,dx$ (for an even function)
(c) $\int_{-a}^{a} f(x)\,dx$ (d) all of the above

30. $\int_{1}^{4} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{5-x}} dx =$

(a) 4 (b) 1
(c) $\frac{3}{2}$ (d) $\frac{5}{2}$

31. $\int_{-3}^{3} (x^3 + x)dx$

(a) 0 (b) 3
(c) −3 (d) 1

32. Equation of the curve which passes through the point (1, 0) and $F^1(x) = 2x - 1$

(a) $y = x^2 - x - 1$ (b) $y = x^2 - x - 2$
(c) $y = x^2 - x$ (d) none

33. $\int \frac{\log(\log x)}{x}\,dx$

(a) $\log x\,[\log(\log x) - 1] + c$ (b) $\log x\,[\log x - 1] + c$

(c) $\log x\,[\log(\log x)] + c$ (d) none

34. $\int e^{x^3} x^2\,dx$

(a) $\frac{1}{3}e^{x} + c$ (b) $\frac{1}{2}e^{x^3} + c$

(c) $\frac{1}{3}e^{x^3} + c$ (d) $e^{x^3} + c$

35. $\int (\log x)^3\,dx$

(a) $x[(\log x)^3 - 3(\log x)^2 + 6(\log x) - 6] + c$

(b) $(\log x)^3 - 3(\log x)^2 + 6(\log x) - 6 + c$

(c) $x^3(\log x)^3 - x^2(\log x)^2 + 6(\log x) - 6 + c$

(d) none

PAST EXAM QUESTIONS

36. The value of $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}\,dx$ is

(a) 0 (b) 3 (c) 2 (d) 1

37. $\int_1^2 \frac{2x}{1+x^2}\,dx$:

(a) $\log_e \frac{5}{2}$
(b) $\log_e 5 - \log_e 2 + 1$
(c) $\log_e \frac{2}{5}$
(d) none of these

38. The value of $\int_1^e \frac{(1+\log x)}{x}\,dx$ is : [Given log e = 1]

(a) 1/2 (b) 3/2 (c) 1 (d) 5/2

39. Find $\int \frac{x^3}{(x^2+1)^3}\,dx$:

(a) $\frac{1}{4}\left[\frac{2x^2+1}{(x^2+1)^2}\right] + c$
(b) $-\frac{1}{4}\left[\frac{2x^2+1}{(x^2+1)^2}\right] + c$
(c) $\frac{1}{2}\left[\frac{2x^2+1}{(x^2+1)^2}\right] + c$
(d) $-\frac{1}{2}\left[\frac{2x^2+1}{(x^2+1)^2}\right] + c$

40. The value of $\int_0^1 \frac{dx}{(1+x)(2+x)}$ is :

(a) $\log \frac{3}{4}$
(b) $\log \frac{4}{3}$
(c) log 12
(d) none

41. $\int \frac{1}{x(x^5+1)} dx$

(a) $\log\left(\frac{x^5}{x^5-1}\right) + c$

(b) $\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right) + c$

(c) $\frac{1}{3}\log\left(\frac{x^5}{x^5+1}\right) + c$

(d) $\frac{1}{3}\log\left(\frac{x^5+1}{x^5}\right) + c$

42. Find the value of $\int_{-3}^{3} x\sqrt{8-x^2}\, dx$

(a) 1 (b) −1 (c) 0 (d) none of these

43. $\int_0^1 \left(\frac{1-x}{1+x}\right) dx$

(a) 2 log 2 − 1

(b) 4 log 2 − 1

(c) 2 log 2

(d) none of these

44. Solve: $\int \frac{(\log x^x)^2}{x^3} . dx$

(a) $\frac{3}{2}(\log x)^3 + c$

(b) $\frac{1}{3}(\log x)^3 + c$

(c) $\frac{1}{6}(\log x)^3 + c$

(d) $\frac{3}{7}(\log x)^3 + c$

45. $\int 2^{3x}.3^{2x}.5^x dx =$ ______

(a) $\frac{2^{3x}.3^{2x}.5^x}{\log(270)} + c$

(b) $\frac{2^{3x}.3^{2x}.5^x}{\log(360)} + c$

(c) $\frac{2^{3x}.3^{2x}.5^x}{\log(180)} + c$

(d) $\frac{2^{3x}.3^{2x}.5^x}{\log(90)} + c$

PRACTICAL PROBLEMS ON APPLICATION OF DIFFERENTIAL & INTEGRATION CALCULUS

Q1. Find the maximum and minimum values of

$$f(x) = \frac{1}{2}x^4 - x^2 + 1.$$

Q.2. The total cost C(x) of a firm is $C(x) = 0.0005x^3 - 0.02x^2 - 30x + 5000$, where x is the output. Determine

(i) average of (AC)

(ii) slope of AC

(iii) Marginal cost (MC)

(iv) slope of MC

(v) value of x, for which MVC = AVC, where VC denotes the variable cost

Q.3. The total revenue received from the sale of a x units or product is given by:

$R(x) = 200 + \frac{x^2}{5}$. Find

(i) the average revenue;

(ii) the marginal revenue;

(iii) the marginal revenue when x = 25

(iv) the actual revenue from the sale of twenty sixth unit.

Q.4. If the consumption is given by $C = 71 + 15\sqrt{I}$, where I is the income. When I = 25

(a) determine the marginal propensity to consume;

(b) marginal propensity to save.

Q.5. The manufacturing cost of an article involves a fixed overhead of ₹ 100 per day. ₹ 0.5 for material and $(x^2/100)$ per day for labour and machinery to produce x articles. How many articles should be produced per day to minimize the cost per article.

Q.6. A company charges ₹ 550 for a transistor set on orders of 50 or less sets. The charge is reduced by ₹ 5 per set for each set ordered in excess of 50. Find the largest size order company should allow so as to receive maximum revenue.

Q.7. A manufacturer can sell x items per day at a price p rupees each, where $p = 125 - \frac{5}{3}x$. The cost of production for x items is $500 + 13x + 0.2x^2$.

(i) Find how much he should produce to have a maximum profit, assuming all items produced are sold.

(ii) What is the maximum profit?

Q.8. A seller makes an offer of selling certain articles that can be described by the equation $x = 25 - 2y$ where x is price per unit and denotes the no. of units. The cost price of the article is ₹ 10 per unit. The maximum quantity that can be offered in single deal to avoid loss is

Q.9. Determine the cost of producing 200 cars, if the marginal cost (in rupees per unit) is given by $MC(x) = \frac{15}{2}x^2 - 4x + 8000$

Q.10. The marginal cost of production (in rupees) is $3 + \frac{x}{3000} + e^{-0.03x}$, where x denotes the number of units. Find the cost of producing 100 units. It is given that $e^{-3} = 0.05$ (approx.) [CA (Foundation), Dec. 1993]

Q.11. The marginal cost function of manufacturing X shoes is $6 + 10x - 6x^2$. The total cost of producing a pair of shoes is ₹ 12. Find the total and average cost function.

Q.12. If the marginal revenue function for output x is given by $Rm = \frac{6}{(x+2)^2} + 5$, find the total revenue function and the demand function.

HOMEWORK SECTION (DIFFERENTIATION)

1. The gradient of the curve $y = 2x^3 - 3x^2 - 12x + 8$ at $x = 0$ is

   (a) -12 (b) 12

   (c) 0 (d) none of these

2. The gradient of the curve $y = 2x^3 - 5x^2 - 3x$ at $x = 0$ is

   (a) 3 (b) -3

   (c) $1/3$ (d) none of these

3. The derivative of $y = \sqrt{x+1}$ is

   (a) $1/\sqrt{x+1}$ (b) $-1/\sqrt{x+1}$

   (c) $1/2\sqrt{x+1}$ (d) none of these

4. If $f(x) = e^{ax^2+bx+c}$, then $f'(x)$ is

   (a) e^{ax^2+bx+c} (b) $e^{ax^2+bx+c}(2ax+b)$

   (c) $2ax + b$ (d) none of these

5. If $f(x) = \dfrac{x^2+1}{x^2-1}$, then $f'(x)$ is

   (a) $-4x/(x^2-1)^2$ (b) 4x/(x2 − 1)2

   (c) $x/(x^2-1)^2$ (d) none of these

6. If $y = x(x-1)(x-2)$, then $\dfrac{dy}{dx}$ is

   (a) $3x^2 - 6x + 2$ (b) $-6x + 2$

   (c) $3x^2 + 2$ (d) none of these

7. The gradient of the curve $y - xy + 2px + 3qy = 0$ at the point $(3, 2)$ is $-\dfrac{2}{3}$. The values of p and q are

   (a) $(1/2, 1/2)$ (b) $(2, 2)$

   (c) $(-1/2, -1/2)$ (d) $(1/2, 1/6)$

8. The curve $y^2 = ux^2 + v$ passes through the point P(2, 3) and $\frac{dy}{dx} = 4$ at P. The values of u and v are

(a) (u = 2, v = 7) (b) (u = 2, v = −7)

(c) (u = −2, v = −7) (d) (0, −1)

9. The gradient of the curve y + px + qy = 0 at (1, 1) is 1/2. The values of p and q are

(a) (−1, 1) (b) (2, −1)

(c) (1, 2) (d) (0, −1)

10. If xy = 1 then $y^2 + dy/dx$ is equal to

(a) 1 (b) 0

(c) −1 (d) none of these

11. The derivative of the function $\sqrt{x + \sqrt{x}}$ is

(a) $\frac{1}{2\sqrt{x+\sqrt{x}}}$ (b) $1 + \frac{1}{2\sqrt{x}}$

(c) $\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)$ (d) none of these

12. Given $e^{-xy} - 4xy = 0$, $\frac{dy}{dx}$ can be proved to be

(a) −y/x (b) y/x

(c) x/y (d) none of these

13. If $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$, $\frac{dy}{dx}$ can be expressed as

(a) $\frac{x}{y}$ (b) $\frac{x}{\sqrt{x^2 - a^2}}$

(c) $\frac{1}{\sqrt{\frac{x^2}{a^2} - 1}}$ (d) none of these

14. If log (x/y) = x + y, $\frac{dy}{dx}$ is

(a) $\frac{y(1+x)}{x(1+y)}$ (b) $\frac{y}{x}$

(c) $\frac{1-x}{1+y}$ (d) none of these

15. If f(x, y) = $x^3 + y^3 - 3axy = 0$, $\frac{dy}{dx}$ can be found out as

(a) $\frac{ay-x^2}{y^2+ax}$ (b) $\frac{ay-x^2}{y^2-ax}$

(c) $\frac{ay+x^2}{y^2+ax}$ (d) none of these

16. Given $x = at^2$, $y = 2at$; $\frac{dy}{dx}$ is calculated as

(a) t (b) −1/t

(c) 1/t (d) none of these

17. Given $x = 2t + 5$, $y = t^2 - 2$; $\frac{dy}{dx}$ is calculated as

(a) t (b) −1/t

(c) 1/t (d) none of these

18. If $y = \frac{1}{\sqrt{x}}$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2x\sqrt{x}}$ (b) $\frac{-1}{x\sqrt{x}}$

(c) $-\frac{1}{2x\sqrt{x}}$ (d) none of these

19. If $x = 3t^2 - 1$, $y = t^3 - t$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{3t^2-1}{6t}$ (b) $3t^2 - 1$

(c) $\frac{3t-1}{6t}$ (d) none of these

20. If $x^y \cdot y^x = M$, where M is constant then $\frac{dy}{dx}$ is equal to

(a) $-\frac{y}{x}$ (b) $\frac{-y(y + x\log y)}{x(x\log x + x)}$

(c) $\frac{y + x\log y}{y\log x + x}$ (d) none of these

21. Given $x = t + t^{-1}$ and $y = t - t^{-1}$ the value of $\frac{dy}{dx}$ at $t = 2$ is

(a) 3/5 (b) −3/5

(c) 5/3 (d) none of these

22. If $x^3 - 2x^2 y^2 + 5x + y - 5 = 0$ then $\frac{dy}{dx}$ at $x = 1, y = 1$ is equal to

(a) 4/3 (b) −4/3

(c) ¾ (d) none of these

23. The derivative of $\frac{3-5x}{3+5x}$ is

(a) $30/(3 + 5x)^2$ (b) $1/(3 + 5x)^2$

(c) $-30/(3 + 5x)^2$ (d) none of these

24. Let $y = \sqrt{2x} + 3^{2x}$ then $\frac{dy}{dx}$ is equal to

(a) $(1/\sqrt{2x}) + 2.3^{2x}\log_e 3$ (b) $1/\sqrt{2x}$

(c) $2.3^{2x}\log_e 3$ (d) none of these

25. The derivative of e^{3x^2-6x+2} is

(a) $30(1-5x)^5$ (b) $(1-5x)^5$

(c) $6(x-1)e^{3x^2-6x+2}$ (d) none of these

26. If $y = \dfrac{e^x + 1}{e^x - 1}$ then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{-2e^x}{(e^x - 1)^2}$ (b) $\dfrac{2e^x}{(e^x - 1)^2}$

(c) $\dfrac{-2}{(e^x \quad 1)^2}$ (d) none of these

27. If $x = at^2$, $y = 2at$ then $\left[\dfrac{dy}{dx}\right]_{t=2}$ is equal to

(a) 1/2 (b) −2

(c) −1/2 (d) none of these

28. Let $f(x) = \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)^2$ then $f'(2)$ is equal to

(a) 3/4 (b) 1/2

(c) 0 (d) none of these

29. If $f(x) = x^2 - 6x + 8$ then $f'(5) - f'(8)$ is equal to

(a) $f'(2)$ (b) $3f'(2)$

(c) $2f'(2)$ (d) none of these

30. If $y = \left(x + \sqrt{x^2 + m^2}\right)^n$ then dy/dx is equal to

(a) ny (b) $ny / \sqrt{x^2 + m^2}$

(c) $-ny / \sqrt{x^2 + m^2}$ (d) none of these

31. If $y = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$ then $\dfrac{dy}{dx} - y$ is proved to be

(a) 1 (b) −1

(c) 0 (d) none of these

32. If $f(x) = x^k$ and $f'(1) = 10$ the value of k is

(a) 10 (b) −10

(c) 1/10 (d) none of these

33. If $y = \sqrt{x^2 + m^2}$, then $y\,y_1$ (where $y_1 = dy/dx$) is equal to

(a) $-x$ (b) x
(c) $1/x$ (d) none of these

34. The derivative of $(x^2 - 1)/x$ is

(a) $1 + 1/x^2$ (b) $1 - 1/x^2$
(c) $1/x^2$ (d) none of these

35. The differential coefficients of $(x^2 + 1)/x$ is

(a) $1 + 1/x^2$ (b) $1 - 1/x^2$
(c) $1/x^2$ (d) none of these

36. If $y = e^{\sqrt{2x}}$, then $\frac{dy}{dx}$ is equal to ______

(a) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ (b) $e^{\sqrt{2x}}$
(c) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ (d) none of these

37. If $y = \sqrt{x}^{\sqrt{x}\ldots\infty}$, then $\frac{dy}{dx}$ is equal to ____

(a) $\frac{y^2}{2 - y\log x}$ (b) $\frac{y^2}{x(2 - y\log x)}$
(c) $\frac{y^2}{\log x}$ (d) none of these

38. If $x = (1 - t^2)/(1 + t^2)$; $y = 2t/(1 + t^2)$, then dy/dx at $t = 1$ is _____

(a) $1/2$ (b) 1
(c) 0 (d) none of these

HOMEWORK SOLUTION (DIFFERENTIATION)

1. $y = 2x^3 - 3x^2 - 12x + 8$ at $x = 0$

$\text{gradient} = m = \left(\frac{dy}{dx}\right)_{x=0} \Rightarrow 6x^2 - 6x - 12$

$\therefore\ m = 6(0)^2 - 6(0) - 12$

$\therefore\ m = -12$

2. $y = 2x^3 - 5x^2 - 3x$ at $x = 0$

$\frac{dy}{dx} = 6x^2 - 10x - 3$

$\therefore\ m = \text{gradient} = 6(0)^2 - 10(0) - 3$

$\therefore\ m = -3$

3. **y =** $\sqrt{x+1}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}}$

4. $f(x) = e^{ax^2+bx+c}$. $f'(x) = ?$

$f'(x) = e^{ax^2+bx+c}.\frac{d}{dx}(ax^2 + bx + c)$

$\therefore\ f'(x) = e^{ax^2+bx+c}.(2ax + b)$

5. $f(x) = \frac{x^2+1}{x^2-1}$; $f'(x) = ?$

Using quotient rule

$f'(x) = \frac{1}{(x^2-1)^2}[(x^2 - 1)(2x) - (x^2 + 1)(2x)]$

$= \frac{2x}{(x^2-1)^2}[x^2 - 1 - x^2 - 1] = \frac{-4x}{(x^2-1)^2}$

6. $y = x(x-1)(x-2)$, then $\frac{dy}{dx} = ?$

$y = (x^2 - x)(x-2)$

$y = x^3 - x^2 - 2x^2 + 2x = x^3 - 3x^2 + 2x$

$\therefore \frac{dy}{dx} = 3x^2 - 6x + 2$

7. $y - xy + 2px + 3qy = 0$ point (3, 2)

$2 - 6 + 6p + 6q = 0 \Rightarrow 6p + 6q = 4$

Using plug-in option, put $p = \frac{1}{2}$ and $q = \frac{1}{6}$

$6\left(\frac{1}{2}\right) + 6\left(\frac{1}{6}\right) = 3 + 1 = 4$

8. $y^2 = ux^3 + v$ passes through (2, 3), then u, v = ?

$9 = 8u + v$

Using plugin option, put $u = 2$ and $v = -7$

$9 = 8(2) - 7$ LHS = RHS

9. $y + px + qy = 0$ at (1, 1)

$(1) + p + q = 0$ putting $p = 0$ and $q = -1$

$1 + 0 - 1 = 0$

LHS = RHS

10. $xy = 1$, then $y^2 + \frac{dy}{dx} = ?$

$y = \frac{1}{x} \Rightarrow$'s $y^2 = \left(\frac{1}{x}\right)^2 = \frac{1}{x^2}$

$\frac{dy}{dx} = \frac{-1}{x^2}$

$\therefore y^2 + \frac{dy}{dx} = \frac{1}{x^2} - \frac{1}{x^2} = 0$

11. $y = \sqrt{x + \sqrt{x}}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x\sqrt{x}}} \times \frac{d}{dx}(x + \sqrt{x})$$

$$= \frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right)$$

12. $e^{-xy} - 4xy = 0$, then $\frac{dy}{dx} = ?$

$e^{-xy} = 4xy$

Applying log on both sides, we get,

$-xy \log e = \log 4 + \log x + \log y$

$xy + \log 4 + \log x + \log y = 0$

Differentiating using implicit function formula,

$$\frac{dy}{dx} = -\frac{\left[y(1) + \frac{1}{x}\right]}{\left[x(1) + \frac{1}{y}\right]} = -\frac{\left[\frac{xy+1}{x}\right]}{\left[\frac{xy+1}{y}\right]} = -\frac{y}{x}$$

13. $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1 \Rightarrow x^2 - y^2 = a^2 \Rightarrow x^2 - y^2 - a^2 = 0$

$$\frac{dy}{dx} = -\frac{[2x]}{[2y]} = \frac{-x}{y}$$

14. $\log\left(\frac{x}{y}\right) = x + y$, then $\frac{dy}{dx} = ?$

$\log x - \log y - x - y = 0$

$$\frac{dy}{dx} = \frac{-\left[\frac{1}{x} - 1\right]}{\left[-\frac{1}{y} - 1\right]} = \frac{-\left[\frac{1-x}{x}\right]}{-\left[\frac{1+y}{y}\right]} = \frac{(1-x)y}{(x)(1+y)}$$

15. $f(x, y) = x^3 + y^3 - 3axy = 0$, then $\frac{dy}{dx} = ?$

$$\frac{dy}{dx} = \frac{-[3x^2 - 3ay(1)]}{[3y^2 - 3ax(1)]} = \frac{-3(x^2 - ay)}{3(y^2 - ax)} = \frac{ay - x^2}{y^2 - ax}$$

16. $x = at^2$; $y = 2at$; $\frac{dy}{dx} = ?$

$$\frac{dx}{dt} = 2at \rightarrow (1) ; \frac{dy}{dt} = 2a \rightarrow (2)$$

$$\frac{(2)}{(1)} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{2a}{2at} = \frac{1}{t}$$

17. $x = 2t + 5$; $y = t^2 - 2$, Then $\frac{dy}{dx} = ?$

$$\frac{dx}{dt} = 2 \ldots (1) ; \frac{dy}{dt} = 2t \ldots (2)$$

$$\frac{(2)}{(1)} = \frac{dy}{dx} = \frac{2t}{2} = t$$

18. $y = \frac{1}{\sqrt{x}} = x^{-1/2}$, then $\frac{dy}{dx} = ?$

$$\frac{dy}{dx} = \frac{-1}{2}x^{-1/2-1} = \frac{-1}{2}x^{-3/2} = \frac{-1}{2x\sqrt{x}}$$

Note: $x^{3/2} = x \,.\, x^{1/2} = x\sqrt{x}$

19. $x = 3t^2 - 1$; $y = t^3 - t$, then $\frac{dy}{dx} = ?$

$$\frac{dx}{dt} = 6t \ldots (1) ; \frac{dy}{dt} = 3t^2 - 1 \rightarrow (2)$$

$$\frac{(2)}{(1)} = \frac{dy}{dx} = \frac{3t^2 - 1}{6t}$$

20. $x^y \,.\, y^x = M$; M is a constant; $\frac{dy}{dx} = ?$

Taking long on both rider, we get,

$y \log x + x \log y = \log M$

$\Rightarrow y \log x + x \log y - \log M = 0$

$$\frac{dy}{dx} = \frac{-\left[y\left(\frac{1}{x}\right) + \log y(1)\right]}{\left[\log x(1) + x\left(\frac{1}{y}\right)\right]} = \frac{-\left[\frac{y + x\log y}{x}\right]}{\left[\frac{y\log x + x}{y}\right]}$$

$$\therefore \frac{dy}{dx} = \frac{-y(y + x\log y)}{x(y\log x + x)}$$

21. $x = t + \frac{1}{t}$; $y = t - \frac{1}{t}$, then $\frac{dy}{dx}$ at $t = 2$

$$\frac{dx}{dt} = 1 - \frac{1}{t^2} = \frac{t^2 - 1}{t^2} \quad \ldots (1)$$

$$\frac{dy}{dx} = 1 + \frac{1}{t^2} = \frac{t^2 + 1}{t^2} \quad \ldots (2)$$

$$\frac{(2)}{(1)} = \frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1}$$

$$\left(\frac{dy}{dx}\right)_{t=2} = \frac{2^2 + 1}{2^2 - 1} = \frac{5}{3}$$

22. $x^3 - 2x^2y^2 + 5x + y - 5$, then $\frac{dy}{dx}$ at $= 1, y = 1$

$$\frac{dy}{dx} = \frac{-[3x^2 - 2y^2(2x) + 5]}{[-2x^2(2y) + 1]}$$

$$\left(\frac{dy}{dx}\right)_{(1,1)} = \frac{-[3 - 4 + 5]}{[-4 + 1]} = \frac{-4}{-3} = \frac{4}{3}$$

23. $y = \frac{3 - 5x}{3 + 5x}$ Using quotient rule, we get,

$$\frac{dy}{dx} = \frac{1}{(3 + 5x)^2}[(3 + 5x)(-5) - (3 - 5x)(5)]$$

$$= \frac{1}{(3 + 5x)^2}[-15 - 25x - 15 + 25x] = \frac{-30}{(3 + 5x)^2}$$

24. $y = \sqrt{2x} + 3^{2x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{2x}} \cdot \frac{dy}{dx}(2x) + 3^{2x} \log 3 \times \frac{d}{dx}(2x)$$

$$= \frac{2}{2\sqrt{2x}} + 3^{2x}.\log 3(2)$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{2x}} + 2.3^{2x} \log_e 3$$

25. $y = e^{3x^2 - bx + 2}$

$$\frac{dy}{dx} = e^{3x^2 - 6x + 2} \times \frac{d}{dx}(3x^2 - 6x + 2)$$

$$\frac{dy}{dx} = (6x - 6)e^{3x^2 - 6x + 2} = 6(x - 1)e^{3x^2 - 6x + 2}$$

26. $y = \dfrac{e^x + 1}{e^2 - 1}$

Using Quotient Rule, we get

$$\frac{dy}{dx} = \frac{1}{(e^x - 1)^2}[(e^x - 1)(e^x) - (e^x + 1)(e^x)]$$

$$= \frac{e^x}{(e^x - 1)^2}[e^x - 1 - e^x - 1] = \frac{-2e^x}{(e^x - 1)^2}$$

27. $x = at^2$; $y = at$; $\dfrac{dy}{dx}$ at $t = 2$

$$\frac{dx}{dt} = 2at \ . . . (1) ; \ \frac{dy}{dt} = 2a \ . . . (2)$$

$$\frac{(2)}{(1)} \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} \frac{2a}{2at} = \frac{1}{t} \therefore \left(\frac{dy}{dx}\right)_{t=2} = \frac{1}{2}$$

28. $f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ find $f'(2)$ $f(x) = x + \frac{1}{x} + 2$

$f'(x) = 1 - \frac{1}{x^2}$ $\therefore f'(2) = 1 - \frac{1}{4} = \frac{3}{4}$

29. $f(x) = x^2 - 6x + 8$ $f'(5) - f'(8) = ?$

$f'(x) = 2x - 6$

$f'(5) = 10 - 6 = 4$; $f'(8) = 16 - 6 = 10$

$\Rightarrow$'s $f'(5) - f'(8) = 4 - 10 = -6$

Now, $f'(2) = 2(2) - 6 = -2$

$\therefore 3f'(2) = 3(-2) = -6$

30. $y = \left(x + \sqrt{x^2 + m^2}\right)^m$ . . . (1) $\frac{dy}{dx} = ?$

$$\frac{dy}{dx} = m\left(x + \sqrt{x^2 + m^2}\right)^{m-1}\left[1 + \frac{1(2x)}{2\sqrt{x^2 + m^2}}\right]$$

$$= m\left(x + \sqrt{x^2 + m^2}\right)^{m-1}\left[\frac{\sqrt{x^2 + m^2} + x}{\sqrt{x^2 + m^2}}\right]$$

$$\frac{dy}{dx} = \frac{m[x + \sqrt{x^2 + m^2}]^{m-1+1}}{\sqrt{x^2 + m^2}} = \frac{my}{\sqrt{x^2 + m^2}} \text{ from (1)}$$

31. $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...,$ then $\frac{dy}{dx} - y = ?$

$$\frac{dy}{dx} = 1 + \frac{1}{2!}(2x) + \frac{1}{3!}(3x^2) + ...$$

$$\frac{dy}{dx} = 1 + \frac{x}{1} + \frac{x^2}{2!} + ... = y$$

$$\therefore \frac{dy}{dx} - y = 0$$

32. $f(x) = x^k$; $f'(1) = 10$; $k = ?$

$f'(x) = k \cdot x^{k-1}$

$f'(1) = k \cdot (1)^{k-1} = 10$

$\therefore k = 10$

33. $y = \sqrt{x^2 + m^2}$, then $yy_1 = ?$

$$\frac{dy}{dx} = y = \frac{1}{2\sqrt{x^2+m^2}}(2x) = \frac{x}{y}$$

$\therefore yy_1 = x$

34. $y = \frac{x^2+1}{x} = x + \frac{1}{x}$

$$\frac{dy}{dx} = 1 - \frac{1}{x^2}$$

35. $y = \frac{x^2-1}{x} = x - \frac{1}{x}$

$$\frac{dy}{dx} = 1 + \frac{1}{x^2}$$

36. $y = e^{\sqrt{2x}}$; $\frac{dy}{dx} = ?$

$$\frac{dy}{dx} = e^{\sqrt{2x}} \times \frac{d}{dx}(\sqrt{2x}) \times \frac{d}{dx}(2x)$$

$$\frac{dy}{dx} = e^{\sqrt{2x}} . \frac{1}{2\sqrt{2x}}(2) = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

37. $y = \sqrt{x}^{\sqrt{x}\ldots\infty}$ ... (1)

$y = (\sqrt{x})^y$ from (1)

Taking log on both sides, we get,

$\log y = \log x^{y/2} = \frac{y}{2}\log x$

$2\log y = y\log x \Rightarrow 2\log y - y\log x = 0$

$$\frac{dy}{dx} = \frac{-\left[-\frac{y}{x}\right]}{\left[\frac{2}{y} - \log x(1)\right]} = \frac{\frac{y}{x}}{\frac{2 - y\log x}{y}} = \frac{y^2}{x(2 - y\log x)}$$

38. $x = \dfrac{1-t^2}{1+t^2}$; $y = \dfrac{2t}{1+t^2}$ find $\dfrac{dy}{dx}$ at t = 1

$$\frac{dx}{dt} = \frac{1}{(1+t^2)^2}[(1+t^2)(-2t) - (1-t^2)(2t)]$$

$$= \frac{-2t}{(1+t^2)^2}[1 + t^2 + 1 - t^2] = \frac{-4t}{(1+t^2)^2} \quad \ldots (1)$$

$$\frac{dy}{dt} = \frac{1}{(1+t^2)^2}[(1+t^2)(2) - (2t)(2t)]$$

$$= \frac{1}{(1+t^2)^2}\left[2 + 2t^2 - 4t^2\right] = \frac{2-2t^2}{(1+t^2)^2} \rightarrow (2)$$

$$\frac{(2)}{(1)} = \frac{dy}{dx} = \frac{\frac{2-2t^2}{\cancel{(1+t^2)^2}}}{\frac{-4t}{\cancel{(1+t^2)^2}}} = \frac{2-2t^2}{-4t}$$

$$\left(\frac{dy}{dx}\right)_{t=1} = \frac{2-2(1)^2}{-4(1)} = \frac{0}{-4} = 0$$

HOMEWORK (INTEGRATION)

1. Evaluate $\int 5x^2\,dx$

(a) $5/3x^3 + k$ (b) $\frac{5x^3}{3} + k$

(c) $5x^3$ (d) none of these

2. Integration of $3 - 2x - x^4$ will become

(a) $-x^2 - x^5/5$ (b) $3x - x^2 - \frac{x^3}{5} + k$

(c) $3x - x^2 + \frac{x^5}{5} + k$ (d) none of these

3. Given $f(x) = 4x^3 - 3x^2 - 2x + 5$ and $\int f(x)\,dx$ is

(a) $x^4 + x^3 - x^2 + 5x$ (b) $x^4 + x^3 - x^2 + 5x + k$

(c) $12x^2 + 6x - 2x^2$ (d) none of these

4. Evaluate: $\int (x^2 - 1)\,dx$

(a) $x^5/5 - 2/3x^3 + x + k$ (b) $\frac{x^3}{3} - x + k$

(c) $2x$ (d) none of these

5. $\int (1-3x)(1+x)\,dx$ is equal to

(a) $x - x^2 - x^3$ (b) $x^3 - x^2 + x$

(c) $x - x^2 - x^3 + k$ (d) none of these

6. $\int \left[\sqrt{x} - \frac{1}{\sqrt{x}}\right] dx$ is equal to

(a) $\frac{2}{3}x^{3/2} - 2x^{1/2} + k$ (b) $\frac{2}{3}\sqrt{x} - 2\sqrt{x} + k$

(c) $\frac{1}{2\sqrt{x}} + \frac{1}{2x\sqrt{x}} + k$ (d) none of these

7. The integral of $px^3 + qx^2 + rx + w/x$ is equal to

(a) $px^2 + qx + r + k$ (b) $px^3 / 3 + qx^2 / 2 + rx$

(c) $3px + 2q - w/x^2$ (d) none of these

8. Use method of substitution to integrate the function $f(x) = (4x + 5)^6$ and the answer is

(a) $1/28(4x + 5)^7 + k$ (b) $(4x + 5)^7/7 + k$

(c) $(4x + 5)^7/7$ (d) none of these

9. Use method of substitution to evaluate $\int x(x^2 + 4)^5\, dx$ and the answer is

(a) $(x^2 + 4)^6 + k$ (b) $1/12(x^2 + 4)^6 + k$

(c) $(x^2 + 4)^6 / + k$ (d) none of these

10. Integrate $(x + a)^n$ and the result will be

(a) $\dfrac{(x+a)^{n+1}}{n+1} + k$ (b) $\dfrac{(x+a)^{n+1}}{n+1}$

(c) $(x + a)^{n+1}$ (d) none of these

11. $\int 8x^2 / (x^3 + 2)^3 dx$ is equal to

(a) $-4/3(x^3 + 2)^2 + k$ (b) $-\dfrac{4}{3(x^3+2)^2} + k$

(c) $\dfrac{4}{3(x^3+2)^2} + k$ (d) none of these

12. Using method of partial fraction the integration of f(x) when $f(x) = \dfrac{1}{x^2 - a^2}$ and the answer is

(a) $\log x - \dfrac{a}{x+a} + k$ (b) $\log(x - a) - \log(x + a) + k$

(c) $\dfrac{1}{2a}\log\left(\dfrac{x-a}{x+a}\right) + k$ (d) none of these

13. Use integration by parts to evaluate $\int x^2 e^{3x}\, dx$

(a) $x^2 e^{3x}/3 - 2x e^{3x}/9 + 2/27 e^{3x} + k$ (b) $x^2 e^{3x} - 2x e^{3x} + 2e^{3x} + k$

(c) $e^{3x} / 3 - x e^{3x} / 9 + 2e^{3x} + k$ (d) none of these

14. $\int \log x \, dx$ is equal to
(a) $x \log x + k$ (b) $x \log x - x^2 + k$
(c) $x \log x + k$ (d) none of these

15. $\int xe^x \, dx$ is
(a) $(x - 1)e^x + k$ (b) $(x - 1)e^x$
(c) $xe^x + k$ (d) none of these

16. Evaluate $\int_0^1 (2x^2 - x^3) \, dx$ and the value is
(a) $4/3 + k$ (b) $5/12$
(c) $-4/3$ (d) none of these

17. Evaluate $\int_2^4 (3x - 2)^2 \, dx$ and the value is
(a) 104 (b) 100
(c) 10 (d) none of these

18. $\int x^x (1 + \log x) \, dx$ is equal to
(a) $x \log x + k$ (b) $e^{x^2} + k$
(c) $\frac{x^2}{2} + k$ (d) $x^x + c$

19. If $f(x) = \sqrt{1 + x^2}$ then $\int f(x) \, dx$ is
(a) $\frac{2}{3} x(1 + x^2)^{3/2} + k$ (b) $\frac{x}{2}\sqrt{1 + x^2} + \frac{1}{2}\log(x + \sqrt{x^2 + 1}) + k$
(c) $\frac{2}{3} x(1 + x^2)^{3/2} + k$ (d) none of these

20. $\int (e^x + e^{-x})^2 (e^x - e^{-x}) \, dx$ is
(a) $\frac{1}{3}(e^x + e^x)^3 + k$ (b) $\frac{1}{2}(e^x - e^{-x})^2 + k$
(c) $e^x + k$ (d) none of these

21. $\int_0^a [f(x) + f(-x)]\ dx$ is equal to

(a) $\int_0^a 2f(x)\ dx$ (b) $\int_{-a}^a f(x)\ dx$

(c) 0 (d) $\int_{-a}^a -f(-x)\ dx$

22. $\int xe^x / (x + 1)^2\ dx$ is equal to

(a) $e^x / (x + 1) + k$ (b) $e^x / x + k$

(c) $e^x + k$ (d) none of these

23. $\int (x^4 + 3 / x)\ dx$ is equal to

(a) $x^5/5 + 3 \log |x|$ (b) $1/5x^5 + 3 \log |x| + k$

(c) $1/5\ x^5 + k$ (d) none of these

24. Evaluate the integral $\int (1 - x)^3 / x\ dx$ and the answer is equal to

(a) $\log |x| - 3x + 3/2x^2 + k$ (b) $\log x - 2 + 3x^2 + k$

(c) $\log x + 3x^2 + k$ (d) none of these

25. The equation of the curve in the form y = f(x) if the curve passes through the point (1, 0) and $f'(x) = 2x - 1$ is

(a) $y = x^2 - x$ (b) $x = y^2 - y$

(c) $y = x^2$ (d) none of these

26. Evaluate $\int_1^4 (2x + 5)\ dx$ and the value is

(a) 3 (b) 10

(c) 30 (d) none of these

27. $\int_1^2 \frac{2x}{1 + x^2}\ dx$ is equal to

(a) $\log_e (5/2)$ (b) $\log_e 5 - \log_e 2 + k$

(c) $\log_e (2/5)$ (d) none of these

28. $\int_0^4 \sqrt{3x+4}\, dx$ is equal to

(a) 9/112 (b) 112/9
(c) 11/9 (d) none of these

29. $\int_0^2 \frac{x+2}{x+1} dx$ is

(a) $2 + \log_e 2$ (b) $2 + \log_e 3$
(c) $\log_e 3$ (d) none of these

30. Evaluate $\int_1^{e^2} \frac{dx}{x(1+\log x)^2}$ and the value is

(a) 3/2 (b) 1/3
(c) 26/3 (d) 1/2 (loge 5)

31. $\int_0^4 \frac{(x+1)(x+4)}{\sqrt{x}} dx$ is equal to

(a) $51\frac{1}{5}$ (b) 48/5 (c) 48 (d) $55\frac{7}{15}$

32. The equation of the curve which passes through the point (1, 3) and has the slope $4x - 3$ at any point (x, y) is

(a) $y = 2x^3 - 3x + 4$ (b) $y = 2x^2 - 3x + 4$
(c) $x = 2y^2 - 3y + 4$ (d) none of these

33. The value of $\int_2^3 f(5-x)dx - \int_2^3 f(x)dx$ is

(a) 1 (b) 0 (c) −1 (d) none of these

34. $\int \frac{e^x(x\log x+1)}{x} dx$ is equal to

(a) $e^x \log x + k$ (b) $e^x + k$
(c) $\log x + k$ (d) none of these

35. $\int \log x^2\, dx$ is equal to

(a) $x(\log x - 1) + k$ (b) $2x(\log x - 1) + k$
(c) $2(\log x - 1) + k$ (d) none of these

36. $\int_1^2 x \log x \, dx$ is equal to

(a) 2 log 2 (b) −3/4
(c) 2 log 2 − 3/4 (d) none of these

37. $\int_0^2 3x^2 \, dx$ is

(a) 7 (b) −8 (c) 8 (d) none of these

38. Evaluate $\int \frac{(2-x)e^x}{(1-x)^2} dx$ and the value is

(a) $\frac{e^x}{1-x} + k$ (b) $e^x + k$
(c) $\frac{1}{1-x} + k$ (d) none of these

39. Using integration by parts $\int x^3 \log x \, dx$
(a) $x^4/16 + k$ (b) $x^4/16(4 \log x - 1) + k$
(c) $4 \log x - 1 + k$ (d) none of these

40. Evaluate $\int \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) dx$ and the value is

(a) $\log e \, |e^x + e^{-x}|$ (b) $\log^e |e^x + e^{-x}| + k$
(c) $\log_e |e^x - e^{-x}| + k$ (d) none of these

41. By the method of partial fraction $\int \frac{3x}{(x^2 - x - 2)} dx$ is

(a) $2 \log_e |x-2| + \log_e |x+1| + k$ (b) $2 \log_e |x-2| - \log_e |x+1| + k$
(c) $\log_e |x-2| + \log_e |x+1| + k$ (d) none of these

42. If f′(x) = x − 1, the equation of a curve y = f(x) passing through the point (1, 0) is given by
(a) $y = x^2 - 2x + 1$ (b) $y = x^2 / 2 - x + 1$
(c) $y = x^2 / 2 - x + 1/2$ (d) none of these

HOMEWORK SOLUTION (INTEGRATION)

1. $I = \int 5x^2 = \frac{5x^3}{3} + k$

2. $I = \int (3 - 2x - x^4)\ dx$

$= 3x - \frac{2x^2}{x} - \frac{x^5}{5} = 3x - x^2 - \frac{x^5}{5} + k$

3. $f(x) = 4x^3 + 3x^2 - 2x + 5$

$\int f(x) = \int (4x^3 + 3x^2 - 2x + 5)\ dx$

$= \frac{4x^4}{4} + \frac{3x^3}{3} - \frac{2x^2}{2} + 5x$

$= x^4 + x^3 - x^2 + 5x + k$

4. $\int (x^2 - 1)\ dx = \frac{x^3}{3} - x + k$

5. $\int (1 - 3x)(1 + x)\ dx = \int (1 - 2x - 3x^2)\ dx$

$I = x - \frac{2x^2}{2} - \frac{3x^3}{3} + k$

$\therefore I = x - x^2 - x^3 + k$

6. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx = \int (x^{1/2} - x^{-1/2})\ dx$

$I = \frac{x^{1/2+1}}{\frac{1}{2}+1} - \frac{x^{-1/2+1}}{-\frac{1}{2}+1} = \frac{x^{3/2}}{\frac{3}{2}} - \frac{x^{1/2}}{\frac{1}{2}}$

$I = \frac{2}{3}x^{3/2} - 2x^{1/2} + k$

7. $\int\left(px^3 + qx^2 + rx + \frac{w}{x}\right)dx$

$$I = \frac{px^4}{4} + \frac{qx^3}{3} + \frac{rx^2}{2} + w\log x + k$$

8. Let, $I = \int (4x+5)^5\ dx$

$$= \frac{(5x+5)^7}{7} \cdot \frac{1}{4} = \frac{(4x+5)^7}{28} + k$$

9. Let, $I = \int x(x^4+4)^5 dx$

Let, $x^2 + 4 = t$

$2xdx = dt$

Now, $I = \int t^5 \frac{dt}{2}$

$= \frac{1}{2} \cdot \frac{t^6}{6}$

$I = \frac{1}{12}(x^2+4)^6 + k$

10. $I = \int (x+a)^n dx$

$$I = \frac{(x+a)^{n+1}}{n+1} + k$$

11. Let, $I = \int \frac{8x^2}{(x^2+2)^3} dx$

Let, $x^3 + 2 = t$

$3x^2 = \frac{dt}{dx}$

$x^3\,dx = \frac{dt}{3}$

$= 8\int \frac{x^2}{(x^3+2)^3} dx$

$= \frac{8}{3}\int \frac{dt}{t^3} = \frac{8}{3}\int t^{-3}\, dt$

$= \frac{8}{3}\int \frac{dt}{t^3} = \frac{8}{3}\int t^{-3}\ dt$

$I = \frac{8}{3}\frac{t^{-3+1}}{(-3+1)} = \frac{-8}{3\times 2} \cdot t^{-2}$

$I = -\frac{4}{3}\frac{1}{t^2} = \frac{-4}{3}\frac{1}{(x^3+2)^2} + c$

12. Given: $f(x) = \frac{1}{x^2 - a^2}$

Using the standard integral formula

$$\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + k$$

13. Let $I = \int x^2 . e^{3x}\, dx$

Using integration by parts, we get

$$I = x^2 . \frac{e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{27} + k$$

14. Let, $I = \int \log x\, dx$

Using the formula : $\int x^n \log x\, dx = \frac{x^{n+1}}{(n+1)^2}[(n+1)\log x - 1]$

$\therefore I = \int (1) \log x\, dx$ Here, n = 0

$$= \frac{x^1}{1}[1\log x - 1] = x\log x - x + k$$

15. Let, $I = \int xe^x\, dx$

$$= xe^x - e^x = e^x(x-1) + k$$

16. $I = \int_0^1 (2x^2 - x^3)\, dx$

$$= \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{2}{3}(x^3)_0^1 - \frac{1}{4}(x^4)_0^1$$

$$I = \frac{2}{3}(1-0) - \frac{1}{4}(1-0)$$

$$I = \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}$$

17. $I = \int_2^4 (3x-2)^2\, dx$

$$= \left[\frac{(3x-2)^3}{3} . \frac{1}{3}\right]_2^4 = \frac{1}{9}\left[(3x-2)^3\right]_2^4$$

$$I = \frac{1}{9}[(12-2)^3 - (6-2)^3] = \frac{1}{9}[1000-64] = \frac{936}{9} = 104$$

18.
$I = \int x^x (1 + \log x)\ dx$

Let, $x^x = t$

differentiating using logarithmic differentiation,

$$\frac{dt}{dx} = x^x \left[\frac{x}{x} \frac{d}{dx}(x) + \log x \cdot \frac{d}{dx}(x) \right]$$

$= x^x [1(1) + \log x(1)]$

$dt = x^x (1 + \log x)\ dx$

$\therefore I = \int dt = t = x^x + k$

19. $I = \int \sqrt{1 + x^2}\ dx$

Using std integral, $\int \sqrt{a^2 + x^2}\ dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2} \log x + \sqrt{a^2 + x^2}$

$$\therefore \int \sqrt{1 + x^2}\ dx = \frac{x}{2}\sqrt{1 + x^2} + \frac{1}{2} \log |\ x + \sqrt{1 + x^2}\ | + k$$

20. $\int (e^x + e^{-x})^2 (e^x - e^x) dx$

Let, $t = e^x + e^{-x}$

$$\frac{dt}{dx} = e^x - e^{-x} \Rightarrow dt = (e^x - e^{-x})\ dx$$

$$\therefore I = \int t^2 . dt = \frac{t^3}{3} = \frac{(e^x + e^{-x})^3}{3} + k$$

21. $\int_0^a [f(x) + f(-x)]\ dx$

We know the property of definite integral

$\int_{-a}^{a} f(x)\ dx = 0$ if $f(x)$ is odd

$\int_{-a}^{a} f(x)\ dx = \int_0^a f(x)$ if $f(x)$ is even

$f(-x) = -f(x) \Rightarrow f(x)$ is odd

$f(x) = f(x) \Rightarrow f(x)$ is even

and $\therefore \int_0^a [f(x) + f(-x)]\ dx = \int_{-a}^{a} f(x)\ dx$

22. Let, $I = \int \frac{xe^x}{(x+1)^2}\,dx$

$= \int e^x \left[\frac{x+1-1}{(x+1)^2}\right] dx$

$= \int e^x \left[\frac{(x+1)}{(x+1)^2} - \frac{1}{(x+1)^2}\right] dx$

$= \int e^x \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right] dx = e^x.\left(\frac{1}{x+1}\right) + k$

[Using $\int e^x[f(x) + f'(x)]\, dx = e^x.\, f(x) + c$

23. $I = \int \frac{(1-x)^3}{x} dx \qquad (a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$= \int \frac{(1 - x^3 + 3x^2 - 3x)}{x}\, dx = \int \left(\frac{1}{x} - x^2 + 3x - 3\right) dx$

$I = \log x - \frac{x^3}{3} + \frac{3x^2}{2} - 3x + k$

24. $I = \int \left(x^4 + \frac{3}{x}\right) dx = \frac{x^5}{5} + 3\log|x| + k$

25. $y = f(x)$. $f'(x) = 2x - 1$ passes through $(1, 0)$

$I = \int f'(x)\, dx = f(x) = y$

$= \int (2x-1)\, dx = \left[\frac{2x^2}{x} - 1\right] + c$

$\therefore y = x^2 - x + c$

Putting $x = 1$ and $y = 0$

$0 = 1 - 1 + c \Rightarrow c = 0$

$\therefore f(x) \Rightarrow y = x^2 = x$

26. $I = \int_1^4 (2x+5)\, dx = \left(\frac{2x^2}{2} + 5x\right)_1^4$

$= (x^2)_1^4 + 5(x)_1^4 = (16-1) + 5(4-1)$

$I = 15 + 15 = 30$

27. $I = \int_1^2 \frac{2x}{1+x^2}\, dx$

Let, $t = 1 + x^2$

$\frac{dt}{dx} = 2x \Rightarrow dt = 2x\, dx$

$I = \int \frac{dt}{t} = \log t = \log(1 + x^2)$

$I = \log(1+x^2)\Big]_1^2 = \log[1 + 2^2] - \log[1 + 1^2]$

$I = \log\left(\frac{5}{2}\right)$

28. $I = \int_0^4 \sqrt{3x+4}\, dx = \int_0^4 (3x+4)^{1/2}\, dx$

$= \left[\frac{(3x+4)^{2/2}}{3/2} \cdot \frac{1}{3}\right]_0^4$

$= I = \frac{2}{9}\left[(3x+4)^{3/2}\right]_0^4 = \frac{2}{9}[16^{3/2} - 4^{3/2}]$

$\therefore I = \frac{2}{9}[16\sqrt{16} - 4\sqrt{4}] = \frac{2}{9}[64 - 8]$

$\therefore I = \frac{112}{9}$

29. Let, $I = \int_0^2 \frac{x+2}{x+1}\, dx$

$= \int_0^2 \frac{x+1+1}{x+1}\, dx = \int_0^2 \left(\frac{x+1}{x+1} + \frac{1}{x+1}\right) dx$

$I = \int_0^2 1\, dx + \int_0^2 \frac{1}{x+1}\, dx$

$= [x + \log(x+1)]_0^2$

$= [(2 - 0) + \log(2 + 1) - \log(1)]$

$I = 2 + \log_e 3$

30. $I = \int_1^2 \frac{dx}{x(1+\log x)^2}$

Let $1 + \log x = t$

$\frac{1}{x}\,dx = dt$

$\therefore 2 = \int -\frac{dt}{t^2} = \int t^2\ dt = \frac{-1}{t}$

$I = \left[\frac{-1}{1+\log x}\right]_1^{e^2} = -\left[\frac{1}{1+\log e^2} - \frac{1}{1+\log 1}\right]$

$= -\left[\frac{1}{1+2} - \frac{1}{1+0}\right] = -\left[\frac{1}{3} - 1\right]$

$I = \frac{2}{3}$

31. $\int_0^4 \frac{(x+1)(x+4)}{\sqrt{x}}\ dx$

$I = \int_0^4 \frac{(x^2+5x+4)}{\sqrt{x}}\ dx$

$= \int_0^4 (x^{3/2} + 5.x^{1/2} + 4x^{-1/2})\ dx$

$= \left[\frac{x^{5/2}}{5/2} + \frac{5x^{3/2}}{3/2} + \frac{4x^{1/2}}{1/2}\right]_0^4$

$= \frac{2}{5}[x^{5/2}]_0^4 + \frac{10}{3}[x^{3/2}]_0^4 + 8[x^{1/2}]_0^4$

$= \frac{2}{5}[4^2\sqrt{a} - 0] + \frac{10}{3}[4\sqrt{4} - 0] + 8[\sqrt{4} - 0]$

$= \frac{2}{5}[16 \times 2] + \frac{10}{3}[4 \times 2] + 8(2)$

$I = \frac{64}{5} + \frac{80}{3} + 16 = \frac{832}{15} = 55\frac{7}{15}$

32. Given: Slope = m = 4x – 3 point (1,3)

$$\left(\frac{dy}{dx}\right) = 4x - 3$$

$$I = \int \frac{dy}{dx} = \int (4x-3)dx = \frac{4x^2}{2} - 3x + k$$

$\therefore\ y = 2x^2 - 3x + k$

Now, $3 = 2 - 3 + k$ $\therefore\ y = 2x^2 - 3x + 4$

33. $$I = \int_2^3 f(5-x)\,dx - \int_2^3 f(x)\,dx$$

Using the property of definite integral

$$\int_a^b f(x)\ dx = \int_a^b f(a+b-x)\,dx$$

$$\text{Now, } I = \int_2^3 f(5-x)\ dx - \int_2^3 f(3+2-x)\ dx$$

$$= \int_2^3 f(5-x)\ dx - \int_2^3 f(5-x)\ dx = 0$$

34. $$I = \int e^x \left[\frac{x\log x + 1}{x}\right] dx$$

$$= \int e^x \left[\log x + \frac{1}{x}\right] dx = e^x \log x + c$$

35. $$I = \int \log x^2\ dx = 2\int \log\ x\,dx$$

$= 2\,[x \log x - x] = 2x\,(\log x - 1) + k$

$[\int \log\ x\,dx = x \log x - x]$

36. $I = \int_1^2 (x \log x)\ dx$

$$\int x^n \log x\ dx = \frac{x^{n+1}}{(n+1)^2}[(n+1)\log x - 1]$$

$$\therefore\ I = \int_1^2 x \log x\ dx = \left[\frac{x^2}{4}[2\log x - 1]\right]_1^2$$

$$= \left(\frac{2x^2}{4}\log x\right)_1^2 - \left(\frac{x^2}{4}\right)_1^2$$

$$= \left(\frac{4}{2}\log 2 - \frac{1}{2}\log 1\right) - \left(\frac{4}{4} - \frac{1}{4}\right)$$

$$= (2\log 2) - \left(\frac{3}{4}\right) \quad \therefore\ I = 2\log 2 - \frac{3}{4}.$$

37. $I = \int_0^2 3x^2\ dx - \left(\frac{3.x^3}{3}\right)_0^2 = 8 - 0 = 8$

38. $I = \int \frac{(2-x)e^x}{(1-x)^2}\ dx$

$$= \int e^x\left[\frac{2-x}{(1-x)^2}\right] dx = \int e^x\left[\frac{1-x+1}{(1-x)^2}\right] dx$$

$$= \int e^x\left[\frac{1-x}{(1-x)^2} + \frac{1}{(1-x)^2}\right] dx$$

$$I = e^x\left[\frac{1}{1-x}\right] + k$$

Using, $\int e^x[f(x) + f'(x)]\ dx = e^x f(x) + c$

39.
$$I = \int x^3 \log x \, dx$$
$$= \frac{x^4}{4^2}[4\log x - 1] = \frac{x^4}{16}[4\log x - 1] + k$$

40. $$I = \int \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) dx$$

Let, $t = e^x + e^{-x}$ $\frac{dt}{dx} = e^x - e^{-x}$ $\Rightarrow dt = (e^x - e^{-x})\,dx$

$$I = \int \frac{dt}{t} = \log t = \log(e^x + e^{-x}) + k$$

41. $$I = \int \frac{3x}{x^2 - x - 2} dx$$

Put, $x = 1$ $f(x) = \frac{3x}{x^2 - x - 2}$ $f(1) = \frac{3}{x - 1 - 2} = \frac{-3}{2} \longrightarrow (1)$

$2\log|x - 2| + \log|x + 1| + k$

$\Rightarrow \frac{2}{x-2} + \frac{1}{x+1}$ put $x = 1$

$$\Rightarrow \frac{2}{1-2} + \frac{1}{1+1} = \frac{2}{-1} + \frac{1}{2} = \frac{4-1}{-2} = \frac{-3}{2} \longrightarrow (2)$$

$(1) = (2)$

42. $f'(x) = x - 1$ point $(1, 0)$
$$y = \int f'(x)\,dx = \int (x - 1)\,dx$$

$$y = \left[\frac{x^2}{2} - x\right] + k$$

$$0 = \frac{1}{2} - 1 + k \Rightarrow k = \frac{1}{2}$$

$$\therefore y = \frac{x^2}{2} - x + \frac{1}{2}$$

SELF ASSESSMENT TEST 14
DIFFERENTIAL CALCULUS
18 Question, 18 Marks

1. Find dy/dx of the function: $y = 5x^4 + 3x^3 - 7x^2 + x - 8$
   a) $20x^3 + 9x^2 - 14x + 1$
   b) $20x^3 + 9x - 14x^2 + 1$
   c) $2x^3 + 19x^2 - 14x + 1$
   d) None of the above

2. Find dy/dx of the function: $y = \sqrt{2x} + 3^{2x}$
   a) $\sqrt{2}\sqrt{x} + 3^{2x}\log_e 3$
   b) $\frac{1}{\sqrt{2x}} + 2.3^{2x}\log_e 3$
   c) $\sqrt{x} + 3^{2x}\log_e 3$
   d) None of the above

3. If $y = 2x^3 + 3x^2 - 36x + 7$, find the value of "x" for which dy/dx = 0.
   a) 2
   b) – 3
   c) 3
   d) Both a) and c) above

4. Find dy/dx of the function: $y = \frac{x^2-1}{x^2+1}$
   a) $2x\left(1+x^2\right)^2$
   b) $2x\left(1+x^2\right)^{-2}$
   c) $4x\left(1+x^2\right)^{-2}$
   d) $4x\left(1+x^{-2}\right)^{-2}$

5. Find dy/dx of the function: $y = \frac{\log x}{2^x}$
   a) $2^x\left(x^{-1} - \log x \log 2\right)\left(2^x\right)^2$
   b) $2^x\left(x - \log x \log 2\right)\left(2^x\right)^{-2}$
   c) $2^x\left(x^{-1} - \log x \log 2\right)\left(2^x\right)^{-2}$
   d) None of the above

6. If $y = \frac{x^2}{4x+1}$, find "x" such that y_1 is non-existent.
   a) ¼
   b) ½
   c) – ¼
   d) – ½

7. Find dy/dx of the function: $x = t^2$, $y = 2 - t$, at $t = 1$

a) $-\frac{1}{2}$ b) $\frac{1}{2}$

c) $\frac{1}{4}$ d) 1

8. Find dy/dx of the function: $x = \frac{3at}{1+t^3}, y = \frac{3at^2}{1+t^3}$

a) $\frac{2t - t^4}{1-2t^3}$ b) $\frac{1-2t^3}{1-t^2}$

c) $\frac{2t-t^4}{1-t^2}$ d) None of the above

9. Find dy/dx of the function: $\sqrt{x} + \sqrt{y} = \sqrt{a}$

a) $\sqrt{\frac{x}{y}}$ b) $-\sqrt{\frac{x}{y}}$ c) $\sqrt{\frac{y}{x}}$ d) $-\sqrt{\frac{y}{x}}$

10. Find dy/dx of the function: $y = \log\sqrt{1-x^2}$

a) $\frac{x}{(x^2-)1}$ b) $\frac{x}{(1-x)^2}$

c) $\frac{x}{(x^2+)1}$ d) None of the above

11. Find dy/dx of the function : $y = \log\log\log\log x$

a) $\frac{1}{\log\log\log x}\cdot\frac{1}{\log\log x}\cdot\frac{1}{\log x}\cdot\frac{1}{x}$ b) $\frac{1}{\log\log x}\cdot\frac{1}{\log x}\cdot\frac{1}{x}$

c) $\frac{1}{\log\log x}\cdot\frac{1}{\log\log x}\cdot\frac{1}{\log x}\cdot\frac{1}{x}$ d) $\frac{1}{\log\log\log x}\cdot\frac{1}{\log\log x}\cdot\frac{1}{\log x}$

12. Find dy/dx of the function : $y = x^x$

a) $(1+\log x)$ b) x^x

c) $x^2.\log x$ d) $x^x.(1+\log x)$

13. If $y = \frac{x-2}{x+2}$, Find the value of $2x.\frac{dy}{dx}$

a) $1 + y$ b) $1 - y$

c) $1 - y^2$ d) $1 + y^2$

14. Find the derivative of x^4 with respect to $\sqrt{x+1}$.

a) $\sqrt{x+1}$ b) $x^3\sqrt{x+1}$

c) $8x^3\sqrt{x+1}$ d) $8x^2\sqrt{x+1}$

15. If y = $e^{4x} + e^{-4x}$, find the value of $\frac{d^2y}{dx^2}$.

a) 8y b) 4y

c) 16y d) 20y

16. Find the maximum value of the function y = $2/3\ x^3 - 6x^2 + 20x - 5$.

a) 6 b) 10

c) − 6 d) None of the above

17. The total cost function C(x) for producing x units of an article per day is given by: C(x) = Rs. $(400 + x^2 - 16x)$. Find the level of output at which the average cost function is minimum.

a) 16 b) 20 c) 24 d) None of the above

18. The total cost C(x) of a firm is C(x) = $1500 + 30x + x^2$, where x is the output. What is the marginal cost when 20 units are produced?

a) 20 b) 30 c) 70 d) 40

EXPLANATORY ANSWERS

1. $y = 5x^4 + 3x^3 - 7x^2 + x - 8$

$\frac{dy}{dx} = 5.4x^3 + 3.3x^2 - 7.2x + 1 = 20x^3 + 9x^2 - 14x + 1$

Option A

2. $y = \sqrt{2x} + 3^{2x}$

$\frac{dy}{dx} = \sqrt{2}.\frac{1}{2\sqrt{x}} + 2.3^{2x}.\log_e 3 = \frac{1}{\sqrt{2x}} + 2.3^{2x}.\log_e 3$

Option B

3. $y = 2x^3 + 3x^2 - 36x + 7$

$\frac{dy}{dx} = 6x^2 + 6x - 36 = 0$

$\therefore\ x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

x = 2 or – 3. Option D

4. $y = \frac{x^2 - 1}{x^2 + 1}$

$$\frac{dy}{dx} = \frac{\left(x^2+1\right).\frac{d}{dx}\left(x^2-1\right) - \left(x^2-1\right).\frac{d}{dx}\left(x^2+1\right)}{\left(x^2+1\right)^2} = \frac{\left(x^2+1\right)2x - \left(x^2-1\right)2x}{\left(x^2+1\right)^2} = \frac{4x}{\left(x^2+1\right)^2}$$

Option C

5. $y = \frac{\log x}{2^x}$

$$\frac{dy}{dx} = \frac{2^x.\frac{d}{dx}(\log x) - \log x.\frac{d}{dx}2^x}{\left(2^x\right)^2} = \frac{2^x.\frac{1}{x} - \log x.2^x.\log 2}{\left(2^x\right)^2} = \frac{2^x\left(x^{-1} - \log x\right)\log 2}{\left(2^x\right)^2}$$

Option C

6. $y = \frac{x^2}{4x+1}$

$$\frac{dy}{dx} = \frac{(4x+1).2x - x^2(4)}{(4x+1)^2} = \frac{4x^2 + 2x}{(4x+1)^2}$$

dy/dx is not-existent, when $(4x + 1)^2 = 0$, or $x = -1/4$

Option C

7. dx/dt = 2t, dy/dt = -1

dy/dx = (dy/dt).(dt/dx) = -1(1/2t) = -1/2t

dy/dx at t = 1, = -1/2; Option A

8. $x=\frac{3at}{1+t^3};\frac{dx}{dt}=\frac{3a\left(1+t^3\right)-3at\left(3t^2\right)}{\left(1+t^3\right)^2}=\frac{3a\left(1-2t^3\right)}{\left(1+t^3\right)^2}$

$y=\frac{3at^2}{1+t^3};\frac{dy}{dt}=\frac{6at\left(1+t^3\right)-3at^2\left(3t^2\right)}{\left(1+t^3\right)^2}=\frac{3at\left(2-t^3\right)}{\left(1+t^3\right)^2}$

$\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}=\frac{3at\left(2-t^3\right)}{\left(1+t^3\right)^2}.\frac{\left(1+t^3\right)^2}{3a\left(1-2t^3\right)}=\frac{2t-t^4}{1-2t^3}$

Option A

9. $\sqrt{x}+\sqrt{y}=\sqrt{a}$

$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}.\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-1}{2\sqrt{x}}\Big/\frac{1}{2\sqrt{y}}=-\frac{\sqrt{y}}{\sqrt{x}}=-\sqrt{\frac{y}{x}}$

Option D

10. $y=\log\sqrt{1-x^2}$

$y=\frac{1}{2}\log\left(1-x^2\right)$

$\frac{dy}{dx}=\frac{1}{2}.\frac{-2x}{\left(1-x^2\right)}=\frac{-x}{\left(1-x^2\right)}=\frac{x}{\left(x^2-1\right)}$

Option A

11. $y=\log\log\log\log x$

$\frac{dy}{dx}=\frac{1}{\log\log\log x}.\frac{1}{\log\log x}.\frac{1}{\log x}.\frac{1}{x}$

Option A

12. $y=x^x$

$\log y=x\log x$

$\frac{1}{y}.\frac{dy}{dx}=1.\log x+x.\frac{1}{x}=\log x+1$

$\frac{dy}{dx}=y(1+\log x)=x^x\left(1+\log x\right)$

Option D

13. $y=\frac{x-2}{x+2}$

$\frac{dy}{dx}=\frac{(x+2).1-(x-2).1}{(x+2)^2}=\frac{4}{(x+2)^2}$

$2x.\frac{dy}{dx}=\frac{8x}{(x+2)^2}=\frac{(x+2)^2-(x-2)^2}{(x+2)^2}=1-\left(\frac{x-2}{x+2}\right)^2=1-y^2$

Option C

14. $A=x^4;B=\sqrt{x+1}$

$\frac{dA}{dx}=4x^3;\frac{dB}{dx}=\frac{1}{2\sqrt{x+1}}$

$\frac{dA}{dB}=\frac{dA}{dx}.\frac{dx}{dB}=4x^3.2\sqrt{x+1}=8x^3\sqrt{x+1}$

Option C

15. $y = e^{4x} + e^{-4x}$

$dy/dx = 4.e^{4x} - 4.e^{-4x}$

$d^2y/dx^2 = 16.e^{4x} + 16.e^{-4x} = 16y$;

Option C

16. $y=\frac{2}{3}x^3-6x^2+20x-5$

$y=\frac{2}{3}x^3-6x^2+20x-5$

$\frac{dy}{dx}=2x^2-12x+20=0$

$x^2-6x+10=0;\therefore\ x=\frac{6\pm\sqrt{36-40}}{2}=3\pm i$

No real value of x exists for which $dy/dx = 0$. Given function has neither maximum nor minimum. Option D

17. $C(x) = 400 + x^2 - 16x$

$AC(x) = C(x)/x = 400/x + x - 16$

$AC'(x) = -400/x^2 + 1 = 0$, we get $x = 20, -20$

$AC''(x) = 800/x^3$, $AC''(20) > 0$

At $x = 20$, $AC(x)$ is minimum Option B

18. $C(x) = 1500 + 30x + x^2$

$MC(x) = dC/dx = 30 + 2x$

$MC(20) = 30 + 40 = 70$ Option C

SELF ASSESSMENT TEST 15
INTEGRAL CALCULUS
19 Question, 19 Marks

1. Evaluate: $\int \frac{dx}{9x^2-4}$

a) $\log\left|\frac{3x+2}{3x-2}\right|+c$

b) $\frac{1}{12}\log\left|\frac{3x+2}{3x-2}\right|+c$

c) $\frac{1}{12}\log\left|\frac{3x-2}{3x+2}\right|+c$

d) $12\log\left|\frac{3x+2}{3x-2}\right|+c$

2. Evaluate: $\int \frac{dx}{\sqrt{16x^2+25}}$

a) $\log\left|x+\sqrt{16x^2+25}\right|+c$

b) $\frac{1}{4}\log\left|x+\sqrt{x^2+\left(\frac{5}{4}\right)^2}\right|+c$

c) $\frac{1}{4}\log\left|x+\sqrt{16x^2+25}\right|+c$

d) $\log\left|x+\sqrt{x^2+\left(\frac{5}{4}\right)^2}\right|+c$

3. Evaluate: $\int \frac{dx}{2x^2+x-1}$

a) $\frac{1}{3}\log\left|\frac{2x-1}{2(x+1)}\right|+c$

b) $\frac{1}{3}\log\left|\frac{2x-1}{(x+1)}\right|+c$

c) $\frac{1}{3}\log\left|\frac{x-1}{2(x+1)}\right|+c$

d) $\frac{1}{3}\log\left|\frac{x-1}{x+1}\right|+c$

4. Evaluate: $\int \frac{dx}{3+2x-x^2}$

a) $\frac{1}{4}\log\left|\frac{x+1}{x+3}\right|+c$

b) $\frac{1}{4}\log\left|\frac{x+1}{x-3}\right|+c$

c) $\frac{1}{4}\log\left|\frac{x+1}{3-x}\right|+c$

d) $\frac{1}{2}\log\left|\frac{x+1}{3-x}\right|+c$

5. Evaluate : $\int \frac{dx}{x\left(x^5+1\right)}$

a) $\frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right|+c$

b) $\frac{1}{4}\log\left|\frac{x^5}{x^4+1}\right|+c$

c) $\frac{1}{4}\log\left|\frac{x^4}{x^4+1}\right|+c$

d) $\frac{5}{4}\log\left|\frac{x^5}{x^4+1}\right|+c$

6. Integrate : $\int 2^{(x+3)} dx$

a) $\frac{2^x}{\log 2} + C$ b) $\frac{2^3}{\log 2} + C$

c) $\frac{2^{x-3}}{\log 2} + C$ d) $\frac{2^{(x+3)}}{\log 2} + C$

7. Integrate : $\int \frac{\log x}{x^2} dx$

a) $-\log x - \frac{1}{x} + c$ b) $-x\log x - \frac{1}{x} + c$

c) $-\frac{1}{x}\log x - \frac{1}{x} + c$ d) $-x^2 \log x - \frac{1}{x^2} + c$

8. Integrate: $\int e^x \left\{ \frac{1}{x^2} - \frac{2}{x^3} \right\} dx$

a) $e^x + C$ b) $\frac{e^x}{x^2} + C$

c) $-\frac{e^x}{x^2} + C$ d) $\frac{e^x}{x} + C$

9. Integrate : $\int \frac{x^2 + 5x + 2}{x + 2} dx$

a) $\frac{x^2}{2} - 4\log(x+2) + C$ b) $\frac{x^2}{2} + 3x - 4\log(x+2) + C$

c) $\frac{x^2}{2} - 3x + 4\log(x+2) + C$ d) $\frac{x^2}{2} + x + 4\log(x+2) + C$

10. Integrate : $\int \frac{dx}{x(x^3+1)}$

a) $\frac{1}{3}\log \frac{x^3}{x^3+1} + c$ b) $\log \frac{x^3}{x^3+1} + c$

c) $\frac{1}{2}\log \frac{x^2}{x^3+1} + c$ d) $\frac{2}{3}\log \frac{x^2}{x^3+1} + c$

11. The value of the marginal revenue function (given in thousands of rupees) for a particular commodity is $R(x) = 4 + e^{-0.03x}$, where x denotes the number of units sold. What is the total revenue from the sale of 100 units of the commodity?

[Given: e-3 = 0.05]

a) Rs. 430,000 b) Rs. 341,670 c) Rs. 340,000 d) Rs. 431,670

12. Evaluate : $\int_0^1 \frac{dx}{(1+x)(2+x)}$

a) $\log\frac{3}{4}$

b) $\frac{1}{2}.\log\frac{3}{4}$

c) $\log\frac{4}{3}$

d) $\frac{1}{3}\log\frac{4}{3}$

13. If the marginal revenue of a firm is given by: MR = 9 − $4x^2$, what can be maximum revenue possible?

a) 9

b) 5

c) 13

d) None of the above

14. Calculate the cost of producing certain type of 10 articles, if the marginal cost (in Rs. Per unit) is C(x) = $0.3x^2 - 2.4x + 30$.

a) Rs. 200

b) Rs. 420

c) Rs. 280

d) Rs. 180

15. Evaluate: $\int_0^{\log 2} \frac{e^x}{1+e^x}dx$

a) $\log\left|\frac{2}{3}\right|$

b) $\log\left|\frac{3}{2}\right|$

c) $\log\left|\frac{5}{3}\right|$

d) $\log\left|\frac{7}{5}\right|$

16. Evaluate : $\int_1^{e^2} \frac{dx}{x(1+\log x)^2}$

a) 1 / 2

b) 2 / 3

c) 3 / 4

d) None of the above

17. Integrate : $\int \frac{(x-1)dx}{(x-3)(x+2)}$

a) $\frac{2}{5}\log(x-3)+\frac{3}{5}\log(x+2)+c$

b) $\frac{3}{5}\log(x-3)+\frac{2}{5}\log(x+2)+c$

c) $\frac{2}{5}\log(x-3)-\frac{3}{5}\log(x+2)+c$

d) $\frac{3}{5}\log(x-3)-\frac{2}{5}\log(x+2)+c$

18. Integrate : $\int \frac{(x-1)(x-5)}{(x-2)(x-4)}dx$

a) $x+\frac{3}{2}\log\left|\frac{x+2}{x+4}\right|+c$

b) $x+\frac{3}{2}\log\left|\frac{x-2}{x-4}\right|+c$

c) $x+\frac{2}{3}\log\left|\frac{x-2}{x-4}\right|+c$

d) $x+\frac{2}{3}\log\left|\frac{2-x}{x-4}\right|+c$

19. Integrate : $\int \frac{5x^2+1}{(x+1)^2(2x-1)}dx$

a) $2\log|x+1|+\frac{2}{x+1}+\log|2x-1|+c$

b) $\log|x+1|+\frac{2}{x+1}+\frac{1}{2}\log|2x-1|+c$

c) $2\log|x+1|+\frac{1}{x+1}+\frac{1}{2}\log|2x-1|+c$

d) $2\log|x+1|+\frac{2}{x+1}+\frac{1}{2}\log|2x-1|+c$

EXPLANATORY ANSWERS

1. $\int \frac{dx}{9x^2-4}$

$$= \frac{1}{9}\int \frac{dx}{x^2-(2/3)^2} = \frac{1}{9}.\frac{1}{2x\frac{2}{}}.Log\left|\frac{x-\frac{2}{3}}{x+\frac{2}{}}\right| + c = \frac{1}{12}Log\left|\frac{3x-2}{3x+2}\right| + c$$

Option C

2. $\int \frac{dx}{\sqrt{16x^2+25}}$

$$= \frac{1}{4}\int \frac{dx}{\sqrt{x^2+\left(\frac{5}{4}\right)^2}} = \frac{1}{4}\log\left|x+\sqrt{x^2+\left(\frac{5}{4}\right)^2}\right| + c$$

Option B

3. $\int \frac{dx}{2x^2+x-1}$

$$= \frac{1}{2}\int \frac{dx}{x^2+\frac{x}{2}-\frac{1}{2}} = \frac{1}{2}\int \frac{dx}{x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-\frac{1}{2}} = \frac{1}{2}\int \frac{dx}{\left(x+\frac{1}{4}\right)^2-\left(\frac{3}{4}\right)^2}$$

$$= \frac{1}{2}.\frac{1}{2\left(\frac{3}{4}\right)}\log\left|\frac{x+\frac{1}{4}-\frac{3}{4}}{x+\frac{1}{4}+\frac{3}{4}}\right| + c = \frac{1}{3}\log\left|\frac{x-\frac{1}{2}}{x+1}\right| + c = \frac{1}{3}\log\left|\frac{2x-1}{2(x+1)}\right| + c$$

Option A

4. $\int \frac{dx}{3+2x-x^2}$

$$= \int \frac{dx}{-\left(x^2-2x-3\right)} = \int \frac{dx}{-\left[x^2-2x+1-1-3\right]} = \int \frac{dx}{-\left[(x-1)^2-2^2\right]}$$

$$= \int \frac{dx}{2^2-(x-1)^2} = \frac{1}{2(2)}\log\left|\frac{2+(x-1)}{2-(x-1)}\right| + c = \frac{1}{4}\log\left|\frac{x+1}{3-x}\right| + c$$

Option C

5. $\int \frac{dx}{x\left(x^5+1\right)}$

$= \int \frac{x^4}{x^5\left(x^5+1\right)} dx$

Let $(x^5+1) = t$; Then, $d(x^5+1) = dt \Rightarrow 5x^4 dx = dt \Rightarrow dx = \frac{dt}{5x^4}$

$\therefore \int \frac{x^4}{x^5\left(x^5+1\right)} dx = \frac{1}{5}\int \frac{dt}{t.x^5} = \frac{1}{5}\int \frac{dt}{t(t-1)} = \frac{1}{5}\int \frac{dt}{t^2-t}$

$= \frac{1}{5}\int \frac{dt}{t^2-t+\frac{1}{4}-\frac{1}{4}} = \frac{1}{5}\int \frac{dt}{\left(t-\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2} = \frac{1}{5}.\frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{t-\frac{1}{2}-\frac{1}{2}}{t-\frac{1}{2}+\frac{1}{2}}\right| + c$

$= \frac{1}{5}\log\left|\frac{t-1}{t}\right| + c = \frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right| + c$

Option A

6. $\int 2^{(x+3)} dx$

$= 2^3 \int 2^x dx = 2^3.\frac{2^x}{\log_e 2} + c = \frac{2^{(x+3)}}{\log 2} + c$

Option D

7. $\int \frac{\log x}{x^2} dx$

$= \log x \int \frac{dx}{x^2} - \int \frac{1}{x}.\left(\frac{-1}{x}\right) dx = -\frac{1}{x}\log x - \frac{1}{x} + c$

Option C

8. $\int e^x \left\{\frac{1}{x^2} - \frac{2}{x^3}\right\} dx$

$= \int \frac{e^x}{x^2} dx - \int \frac{2e^x}{x^3} dx$

$= \frac{1}{x^2}.e^x - \int \frac{-2}{x^3} e^x dx - \int \frac{2e^x}{x^3} dx = \frac{e^x}{x^2} + c$

Option B

9. $\int \frac{x^2+5x+2}{x+2}dx$

$$= \int \frac{x(x+2)+3(x+2)-4}{x+2}dx$$

$$= \int xdx + 3\int dx - 4\int \frac{dx}{x+2} = \frac{x^2}{2} + 3x - 4\log(x+2) + c$$

Option B

10. $\int \frac{dx}{x\left(x^3+1\right)}$

Let $u = x^3$, so that $du = 3x^2dx \rightarrow x^2dx = du/3$

$$\int \frac{dx}{x\left(x^3+1\right)} = \int \frac{x^2dx}{x^3\left(x^3+1\right)} = \frac{1}{3}\int \frac{du}{u(u+1)} = \frac{1}{3}\left[\int \frac{du}{u} - \int \frac{du}{u+1}\right]$$

$$= \frac{1}{3}[\log u - \log(u+1)] + c = \frac{1}{3}\log\left(\frac{u}{u+1}\right) + c = \frac{1}{3}\log\frac{x^3}{x^3+1} + c$$

Option A

11. Total Revenue:

$$= \int_0^{100} \left(4+e^{-0.03x}\right) dx = [4x]_0^{100} + \left[\frac{e^{-0.03x}}{-0.03}\right]_0^{100} = (400-0) - \frac{1}{0.03}\left(e^{-3}-e^{0}\right)$$

$$= 400 - \frac{100}{3}(0.05-1) = 400 + \frac{95}{3} = 431.67 = 431{,}670$$

Option D

12. $\int_0^1 \frac{dx}{(1+x)(2+x)}$

$$= \int_0^1 \left[\frac{1}{1+x} - \frac{1}{2+x}\right]dx = \left[\log\frac{1+x}{2+x}\right]_0^1 = \log\frac{2}{3} - \log\frac{1}{2} = \log\frac{4}{3}$$

Option C

13. MR = dR/dx = $9 - 4x^2$

$$R = \int \frac{dR}{dx}.dx = \int\left(9-4x^2\right)dx = 9x - \frac{4x^3}{3} + c$$

We know, R(x) = 0, when x = 0. Thus, c = 0

$$\therefore R = 9x - \frac{4x^3}{3}$$

Revenue is maximum, when dR/dx = 0 and $d^2R/dx^2 < 0$

dR/dx = 9 – 4x2 = 0, x = 3/2

$d^2R/dx^2 = -8x < 0$ at x = 3/2

Thus, maximum revenue = R(3/2) = $9(1.5) - 4(1.5)^3/3 = 9$

Option A

14. MC(x) = $0.3x^2 - 2.4x + 30$

$$TC(x) = \int_0^{10}\left(0.3x^2 - 2.4x + 30\right)dx = \left[0.3\frac{x^3}{3} - 2.4\frac{x^2}{2} + 30x\right]_0^{10} = 100 - 120 + 300 = 280$$

Option C

15. $\int_0^{\log 2} \frac{e^x}{1+e^x}dx$

Let $(1 + e^x) = z$; As x = 0, z = 2 and as x = log2, z = 3. And, $e^x dx = dz$

$$\therefore \int_0^{\log 2}\frac{e^x}{1+e^x}dx = \int_2^3 \frac{dz}{z} = \left[\log|z|\right]_2^3 = \log 3 - \log 2 = \log\left|\frac{3}{2}\right|$$

Option B

16. $\int_1^{e^2} \frac{dx}{x(1+\log x)^2}$

Let (1 + logx) = z. When x = 1, z = 1 & when x = e^2, z = 3. And dx = x.dz

$$\therefore \int_1^{e^2}\frac{dx}{x(1+\log x)^2} = \int_1^3 \frac{dz}{z^2} = \left[\frac{-1}{z}\right]_1^3 = \frac{-1}{3} + 1 = \frac{2}{3}$$

Option B

17. $\int \frac{(x-1)dx}{(x-3)(x+2)}$

$Let \int \frac{(x-1)dx}{(x-3)(x+2)} = \int \frac{A}{x-3}dx + \int \frac{B}{x+2}dx$

On solving, we get: A = 2/5 and B = 3/5

$\therefore \int \frac{(x-1)dx}{(x-3)(x+2)} = \frac{2}{5}\int \frac{dx}{x-3} + \frac{3}{5}\int \frac{dx}{x+2} = \frac{2}{5}\log(x-3) + \frac{3}{5}\log(x+2) + c$

Option A

18. $\int \frac{(x-1)(x-5)}{(x-2)(x-4)}dx$

Let $\int \frac{(x-1)(x-5)}{(x-2)(x-4)}dx = \int \frac{x^2-6x+5}{(x-2)(x-4)}dx = \int \left[1 + \frac{A}{x-2} + \frac{B}{x-4}\right]dx$

On Solving: $(x^2 - 6x + 5) = (x - 2)(x - 4) + A(x - 4) + B(x - 2)$, we get: A = 3/2 & B = -3/2

$\therefore \int \frac{(x-1)(x-5)}{(x-2)(x-4)}dx = \int dx + \frac{3}{2}\int \frac{dx}{x-2} - \frac{3}{2}\int \frac{dx}{x-4} = x + \frac{3}{2}\log(x-2) - \frac{3}{2}\log(x-4) + c$

Option B

19. $\int \frac{5x^2+1}{(x+1)^2(2x-1)}dx$

Let $\int \frac{5x^2+1}{(x+1)^2(2x-1)}dx = \int \left[\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{2x-1}\right]dx$

On Solving: $(5x^2 + 1) = A(x + 1)(2x - 1) + B(2x - 1) + C(x + 1)^2$, we get: A = 2, B = - 2; C = 1

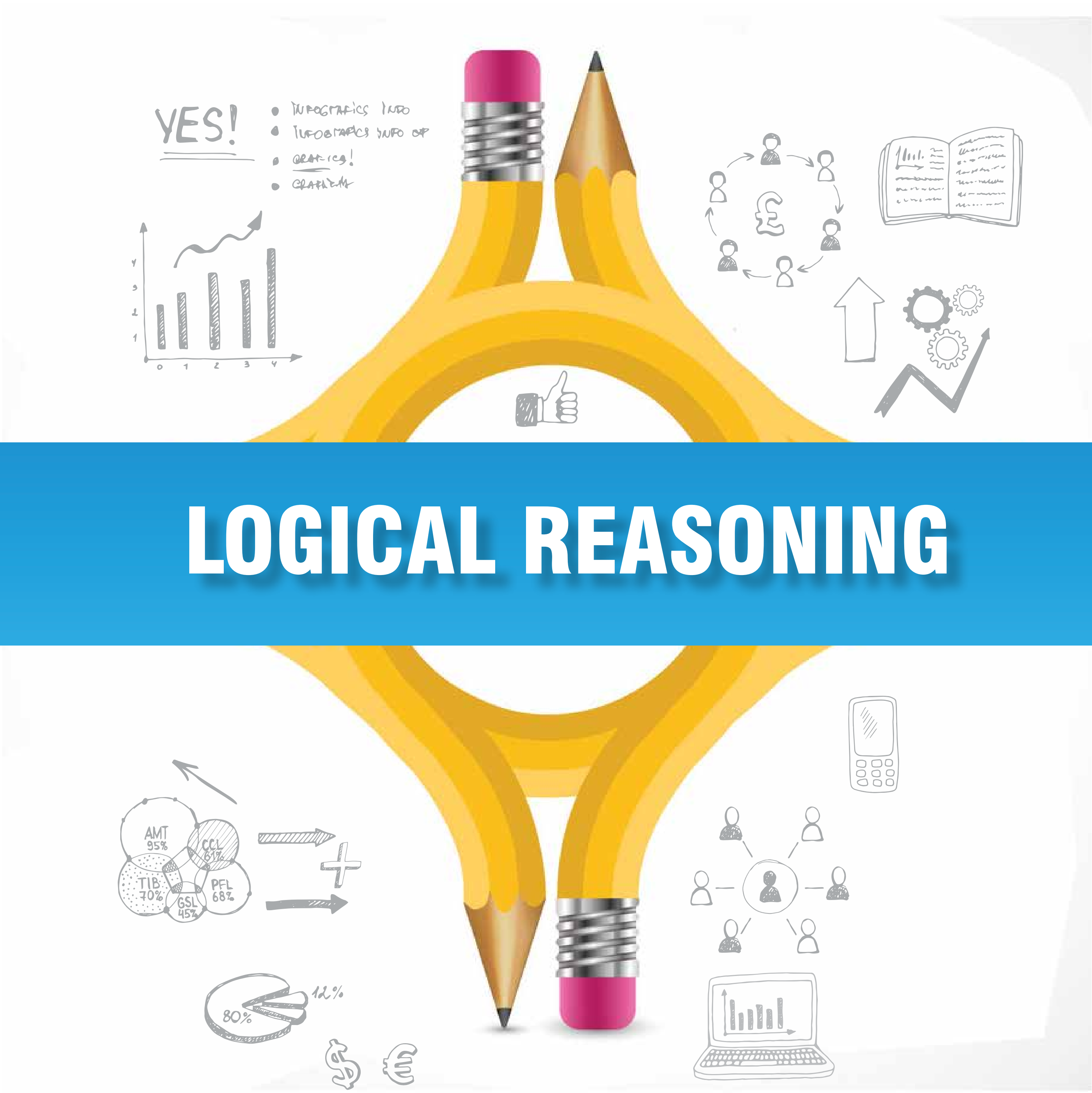
$\therefore \int \frac{5x^2+1}{(x+1)^2(2x-1)}dx = 2\int \frac{dx}{x+1} - 2\int \frac{dx}{(x+1)^2} + \int \frac{dx}{2x-1}$

$= 2\log|x+1| - 2\frac{(x+1)^{-1}}{-1} + \frac{1}{2}\log|2x-1| + c$

$= 2\log|x+1| + \frac{2}{x+1} + \frac{1}{2}\log|2x-1| + c$

Option D

LOGICAL REASONING



1 NUMBER SERIES, CODING AND DECODING AND ODD MAN OUT

- Series is a sequential order of numbers, letters or both arranged in some specific rules.
- These Rules can be based on mathematical operations, place of letters in alphabetical order etc.

Different types of Series

1. Number Series
2. Letter Series
3. Alpha-Numeric Series
4. Continuous pattern Series

NUMBER SERIES

Number series is a logical sequence of more than one elements made of arithmetical digits.

Some Types of number series:

1. Same numbers addition or subtraction series.
2. Increasing order addition or subtraction series.
3. Same number multiplication or division series
4. Increasing order multiplication or division series
5. Same number multiplication and addition or subtraction series

6. Same number multiplication and addition or subtraction in increasing order series
7. Increasing order multiplication and same number addition or subtraction series.
8. Increasing order multiplication and increasing order addition or subtraction series
9. Multiplication and division series.
10. Square series
11. Cube series
12. Square addition series
13. Prime number series
14. Digital operation of number series
15. Mixed combination series

CLASS WORK SECTION

In the following series replace the question (?) with the suitable option.

1. 6,9,12, 15, 18,?
   a) 21 b)20 c) 19 d) 22

2. 7, 12, 19, 28, 39,?
   a) 52 b) 50 c) 51 d) 48

3. 36, 28, 24, 22,?
   a) 18 b) 19 c) 21 d) 22

4. 7, 11, 13, 17, 19, 23, 25, 29.? (Nov-2018)
   a) 30 b) 31 c) 32 d) 33

5. 7, 9, 13, 21, 37,?
   a) 58 b) 63 c) 69 d) 72

6. 27, 32, 30, 35, 33, ?
   a) 28 b) 31 c) 36 d) 38

7. 71, 59, 48, 38, 29, ?
   a) 18 b) 21 c) 20 d) 12

8. 2, 5, 10, ?, 26
   a) 19 b) 21 c) 17 d) 25

9. 1, 6, 15, ? , 45, 66, 91
   a) 25 b) 26 c) 27 d) 28

10. 97, 86, 99, 88, 101, ?, ?
    a) 88,99 b) 90, 103 c) 121, 108, d) 114, 103

11. 107, 97, 82, 62, ?

a) 52 b) 42 c) 47 d) 37

12. 2, 6, 12, ? , 30

a) 18 b) 24 c) 20 d) 26

13. 24, 60, 120, 210, ?

a) 300 b) 336 c) 420 d) 525

14. 8, 16, 28, 44, ?

a) 60 b) 64 c) 62 d) 66

15. 1, 4, 9, 25, ?

a) 49 b) 60 c) 30 d) 36

16. 198, 194, 185, 169, ?

a) 92 b) 136 c) 144 d) 112

17. 1, 2, 5, 26,_______

a) 620 b) 650 c) 677 d) 687

18. 7, 23, 47, 119, 167, ? (J-2019)

a) 211 b) 223 c) 287 d) 319

19. 0, 6, 24, 60, 120, 210, ?

a) 290 b) 240 c) 336 d) 504

20. 3, 15, 35, 63, ?, 143

a) 120 b) 110 c) 99 d) 91

21. 17, 36, 74, 150, ? , 606

a) 250 b) 303 c) 300 d) 302

22. 2, 1, 4, 3, 6, 5, 8, ?

a) 9 b) 10 c) 7 d) 8

23. 3, 128, 6, 64, 9, ?, 12, 16, 15, 8
    a) 32   b) 12   c) 108   d) 72

24. 6, 13, 38, ?, 532, 2675
    a) 129   b) 123   c) 172   d) 164

25. 45, 46, 70,141, ? , 1061.5
    a) 353   b) 353.5   c) 352.5   d) 352

ODD MAN OUT

Find the odd out of the following

1. Fish, Starfish, Crocodile, Hen
    a) Fish   b) Hen   c) Starfish   d) Crocodile

2. 15, 21, 63,81, 69 (N-2018)
    a) 15   b) 21   c) 63   d) 81

3. 4, 12, 44, 176, 890 (J-2019)
    a) 4   b) 12   c) 44   d) 176

4. 3, 12, 36, 144, 431, 1728, 5184
    a) 144   b) 431   c) 36   d) 5184

5. 56, 72, 90, 110, 132, 150
    a) 72   b) 90   c) 110   d) 150

6. 9, 14,19,25, 32, 40
    a) 14   b) 25   c) 32   d) 9

7. 20, 40, 200, 400, 2000, 4000, 8000
    a) 200   b) 40   c) 8000   d) 4000

8. 4, 5, 12, 38, 160 805, 4836
    a) 12,   b) 160   c) 38   d) 805

9. 7, 4, 5, 9, 20, 51, 160.5
   a) 4   b) 51   c) 9   d) 20

10. 1788, 892, 444, 220, 112, 52, 24
   a) 52   b) 112   c) 220   d) 444

LETTER SERIES, ALPHA NUMERIC AND CONTINUOUS PATTERN SERIES

Letter series is a sequence of letters taken from English alphabet and such sequence follows a certain logical pattern

1. B, D, F, I, L, P, ?
   a) U   b) R   c) S   d) T

2. W, U, S, P, M, I, ?
   a) E   b) A   c) H   d) F

3. ACE, BDF, CEG, ?
   a) DFE   b) DEF   c) DFH   d) DEH

4. PMK, MPK, MKP, KMP, ?
   a) PMK   b) KMP   c) MPK   d) KPM

5. PBA, QDC, RFE, ?
   a) SHG   b) OAB   c) TJI   d) ULK

6. XWA, VTC, SPF, OKJ, ?
   a) JDN   b) JEO   c) LPN   d) JDP

7. OTE, PUF, QVG, RWH, ?
   a) SYJ   b) TXI   c) SXJ   d) SXI

8. BFG, HLM, NRS, ?
   a) TWX   b) RVW   c) TYZ   d) TXY

9. P3, M8, ?, G24, D35
   a) K15   b) J13   c) I13   d) J15

10. A1, C3, F6, J10, O15,?

a) U21 b) V21 c) T20 d) U20

11. Which of the following is odd one: (J-2019)

a) CEHL b) KMPT c) OQTX d) NPSV

12. ac_ga_eg_ce_

a) dbag b) ecag c) deag d) ebdg

13. ba_b_aab_a_b

a) abaa b) abba c) baab d) babb

14. _sr_tr_srs_r_srst_

a) ttssrr b) tsrtsr c) strtrs d) tstttr

15. ab_cabb_caa_bccab_cab_cc

a) abbcc b) baabc c) bcbbb d) bcaac

CODING AND DECODING

Coding-Decoding is process of transmitting an information from one place to other using some suitable codes, so that it might reach to other person safely.

Different Types of coding and decoding:

1. Coding based on Rearrangement of Letters
2. Coding based on replacement of letters
3. Opposite letter coding
4. Coding of Letters by their Left and Right Letters
5. Number coding
6. Symbol coding based on Similarity
7. Coding by substitution or word replacement
8. Fictitious Language Coding
9. Coding by Comparison

CLASS WORK SECTION

1. HONEY is coded as JQPGA, which word is code as VCTIGVU? (N-2018)
   a) CARPETS b) TRAPETS c) TARGETS d) UMBRELU

2. In a certain language, MADRAS is coded as NBESBT, How is BOMBAY coded in that language? (J-19)
   a) CPNCBX b) CPNCBZ c) CPOCBZ d) CQOCBZ

3. In a certain code language, COMPUTRONE is written as PMOCTUENOR. How is ADVANTAGES written in that same code?
   a) ADVANSEGAS b) ADTANSEAG c) AVDANTAGES d) AVDATNSEGA

4. In a certain code language EARTH is written as JVTCG, then how will GLOBE be written in that language?
   a) GDQNI b) GDQIN c) GDNIQ d) GDIQN

5. In a certain language FAME is written as LGGY then how will LION be coded in that language?
   a) RHIO b) ROIH c) RHOI d) RIOH

6. In a code language APPLE is written as PQQRS, RIS is written as ABC and MANGO is written as TPXYZ. How will ROSE written in that language?
   a) ABCS b) ACBS c) AZSC d) AZCS

7. If in a certain code language SIMILAR is written as IZORNRH, how will NATURAL be written in that language?
   a) OZIFGZM b) OZIFGMZ c) OZIFZMG d) OZIFMZG

8. If in a certain code language GO is written as FHNP, then how will SUN be written in that language?
   a) RTTOMV b) RTTOVM c) RTTVOM d) RTTVMO

9. If in a certain code RAMAN is coded as 18113114, then how will KAPILA be coded in that language?
   a) 111196112 b) 111169112 c) 111169121 d) 116119121

10. In a certain code RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code? (M-2018)
   a) 318826 b) 318286 c) 618826 d) 338816

11. If PLAY is coded as 8123 and RHYME is coded as 49367. What will be code of MALE? (N-18)
   a) 6217 b) 6198 c) 6395 d) 6285

12. If LOSE is coded as 1357 and GAIN coded as 2468, what do figure 82146 for? (M-2018)
   a) NGLAI b) NGLIA c) GNLIA d) GNLA

13. In a certain code language, DOME is written as 8943 and MEAL is written as 4321. What group of letters can be formed for the code 38249?
   a) EOADM b) MEDOA c) EMDAO d) EDAMO

14. In a certain language 'DEW' written as 1625529 'GET' is written as 4925400, then how will TWO be written in that language?
   a) 400529522 b) 400529225 c) 400925225 d) 400225925

15. If A = 1 and LOT = 47 then MAT = ?
   a) 40 b) 66 c) 34 d) 51

16. If P = 16 and PUT = 6720 then PICK?
   a) 4137 b) 4590 c) 4032 d) 4752

17. In certain code language STAR is written as 5 $ * 2, TORE is written as $ 3 2 @ then how will OATS be written in that language?
   a) 3 * 5 $ b) 3 * $ 5 c) 3 $ * 5 d) 3 5 * $

18. If GREEN is called BLACK, BLACK is called BLUE, BLUE is called RED, RED is called WHITE and WHITE is called ORANGE, then what is the colour of BLOOD?

a) RED b) BLACK c) GREEN d) WHITE

19. If LION is called FISH, FISH is called PARROT, PARROT is called RAT, RAT is called CAT, CAT is called TIGER, then which of the following is a bird?

a) FISH b) PARROT c) RAT d) TIGER

Directions (Q. no. 20 to 23)

In a certain code language 461 means 'where are you', 169 means 'you are good' and 8652 means 'flowers are not bad'.

20. What is the code for 'not'?

a) 6 b) 8 c) 2 d) 8 or 5 or 2

21. What is the code for 'good'?

a) 4 b) 9 c) 6 d) 6 or 1

22. How will 'where not are good flowers' be written in coded language?

a) 68954 b) 46598 c) 45698 d) Data inadequate

23. How will 'are you where' be written in the code?

a) 614 b) 163 c) 618 d) 168

24. In a certain code 256 means 'you are good' 637 means 'we are bad' and 358 mens 'good and bad'. Which of the following represents 'and' in that code? (M-2018)

a) 2 b) 5 c) 8 d) 3

25. If in a certain code language 'Sachin is great' is written as 'ga ma ra'. 'is he poor' is written as 'ta saga', then find the code for 'is'.

a) ta b) ma c) ga d) ra

HOMEWORK SECTION

1. 6, 11, 21, 36, 56 ?
   (a) 42 (b) 51 (c) 81 (d) 91

2. 10 , 100, 200, 310 ?
   (a) 400 (b) 410 (c) 420 (d) 430

3. 11, 13, 17, 19, 23, 25 ?
   (a) 33 (b) 27 (c) 29 (d) 49

4. 6, 12, 21, 33 ?
   (a) 33 (b) 38 (c) 40 (d) 48

5. 2, 5, 9, 14, ? , 27
   (a) 20 (b) 16 (c) 18 (d) 24

6. 6, 11, 21, ? , 56, 81
   (a) 42 (b) 36 (c) 91 (d) 51

7. 10, 18, 28, 40, 54, ?, 88
   (a) 70 (b) 86 (c) 87 (d) 98

8. 120, 99, ?, 63, 48, 35
   (a) 80 (b) 36 (c) 45 (d) 40

9. 22, 24, 28, 36, ? , 84
   (a) 44 (b) 52 (c) 38 (d) 54

10. 4832, 5840, 6848, 7856 ?
    (a) 8864 (b) 8815 (c) 8846 (d) 8887

11. 10, 100, 200, 310, 430 ?
    (a) 560 (b) 540 (c) 550 (d) 590

12. 28, 33, 31, 36, 34 ?
(a) 38 (b) 39 (c) 40 (d) 42

13. 120, 80, 40, 45, ?, 5
(a) 15 (b) 20 (c) 25 (d) 30

14. 2, 15, 41, 80, 132 ?
(a) 184 (b) 144 (c) 186 (d) 197

15. 6, 17, 39, ?, 116
(a) 72 b) 75 (c) 85 (d) 80

16. 1, 4, 10, 22, ?, 94
(a) 46 (b) 48 (c) 49 (d) 47

17. 4, 9, 25, 49, ? , 169, 289, 361
(a) 120 (b) 121 (c) 122 (d) 164

18. 4, 12, 36, ? , 324
(a) 107 (b) 109 (c) 108 (d) 110

19. 1, 1, 4, 8 , 9, ? , 16, 64
(a) 27 (b) 28 (c) 32 (d) 40

20. 5760, 960, 192, ? 16, 8
(a) 47 (b) 48 (c) 52 (d) 50

21. 1, 2, 6, 7, 21, 22, 66, ? , 201
(a) 69 (b) 68 (c) 67 (d) 69

22. 48, 24, 96 , ? 192
(a) 48 (b) 47 (c) 44 (d) 54

23. 165, 195, 255, 285, ?, 435
(a) 345 (b) 390 (c) 335 (d) 395

24. 2, 3, 3, 5, 10, 13, 39, ?, 172, 177
   (a) 42   (b) 44   (c) 43   (d) 40

25. 7, 26, 63, 124, 215, ?, 511
   (a) 342   (b) 343   (c) 441   (d) 421

26. 3, 7, 15, 31, ? 127
   (a) 62   (b) 63   (c) 64   (d) 65

27. 8, 28, 116, 584, ?
   (a) 1752   (b) 3502   (c) 3504   (d) 3508

28. 6, 13, 28, 59, ?
   (a) 122   (b) 114   (c) 113   (d) 112

29. 2, 7, 27, 107, 427, ?
   (a) 1707   (b) 4027   (c) 4207   (d) 1207

30. 5, 2, 7, 9, 16, 25, 41?
   (a) 65   (b) 66   (c) 67   (d) 68

31. In a certain language, MADRAS is coded NBESBT, how DELHI is coded in that code?
   (a) EMMJI   (b) EFMIJ   (c) EMFIJ   (d) JIFEM

32. If RAMAN is written as 12325 and DINESH as 675489 how HAMAM is written?
   (a) 92323   (b) 92233   (c) 93233   (d) 93292

33. If RED is coded as 6720 then GREEN would be coded as
   (a) 9207716   (b) 167129   (c) 1677209   (d) 1672091

34. If A = 1, FAT = 27, FAITH = ?
   (a) 44   (b) 45   (c) 46   (d) 36

35. If BROTHER is coded as 2456784, SISTER is coded as 919684, what is the code for BORBEES?
   (a) 2542889   (b) 2542898   (c) 2454889   (d) 2524889

36. If DELHI is coded 73541 and CALCUTTA as 82589662, How can CALICUT be coded?
(a) 5279431 (b) 5978213 (c) 8251896 (d) 8543962

37. If CLOCK is coded 34235 and TIME is 8679, what will be code of MOTEL?
(a) 72894 (b) 77684 (c) 72964 (d) 27894

38. If PALE is coded as 2134 and EARTH is coded as 41590, how is PERAL coded?
(a) 29530 (b) 24153 (c) 25430 (d) 254313

39. If LOSE is coded as 1357 and GAIN is coded as 2468, what do figure 82146 stands for?
(a) NGLAI (b) NGLIA (c) GNLIA (d) GNLIA

40. If MEKLF is coded as 91782 and LLLJK as 88867, how can IHJED is coded?
(a) 97854 (b) 64512 (c) 54610 (d) 75632

41. If in a certain code language NAME is written as 4258 then what is coded as MEAN ?
(a) 2458 (b) 5842 (c) 8524 (d) 5824

42. If GOLD is written as IQNF, how WIND can be written in that code?
(a) YKPF (b) VHCM (c) XJOE (d) DNIW

43. If ROSE is written as TQUG, how BISCUIT can be written in that code?
(a) DKUEWKV (b) CJTDVJU
(c) DKVEWKV (d) DKUEWKY

LETTER: C Z N V R S W F D CODE
DIGIT: 8 6 4 7 2 9 3 5 1 (Q. no. 44 to 46)
In each of the following questions find out the correctly coded alternative from amongst the given four alternatives (a), (b), (c), (d).

44. ZDRCVF
(a) 612875 (b) 619875 (c) 612845 (d) 612835

45. WNCSZV

(a) 348267 (b) 318267 (c) 348957 (d) 348967

46. RDNFVS

(a) 21679 (b) 216549 (c) 214579 (d) 218579

47. If DELHI is coded as CCIDD, how would you encode BOMBAY?

(a) AJMTVT (b) AMJXVS (c) MJXVSU (d) WXYZAX

48. In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code?

(a) 318826 (b) 318286 (c) 618826 (d) 338816

49. If PALAM could be given the code number 43, what code number can be given to SANTACRUZ?

(a) 123 (b) 85 (c) 120 (d) 125

Directions: The number in each question below is to be codified in the following code:

Digit	7	2	1	5	3	9	8	6	4
Letter	W	L	M	S	I	N	D	J	B

50. 184632

(a) MDJBSI (b) MDJBIL (c) MDJBWL (d) MDBJIL

51. In a certain code '256' means 'you are good', '637' means 'we are bad' and '358' means 'good and bad'. Which of the following represents 'and' in that code?

(a) 2 (b) 5 (c) 8 (d) 3

Directions: Find odd man out of the following (52 - 61):

52. 3, 5, 7, 15, 17, 19

(a) 15 (b) 17 (c) 19 (d) 7

53. 10, 14, 16, 18, 23, 24, 26

(a) 26 (b) 23 (c) 24 (d) 18

54. 1, 4, 9, 16, 24, 25, 36

(a) 9 (b) 24 (c) 25 (d) 36

55. 41, 43, 47, 53, 61, 71, 73, 75

(a) 75 (b) 73 (c) 71 (d) 53

56. 16, 25, 36, 73, 144, 196, 225

(a) 36 (b) 73 (c) 196 (d) 225

57. 1, 4, 9, 16, 19, 36, 49

(a) 19 (b) 9 (c) 49 (d) 16

58. 1, 5, 14, 30, 49, 55, 91

(a) 49 (b) 30 (c) 55 (d) 91

59. 835, 734, 642, 751, 853, 981, 532

(a) 751 (b) 853 (c) 981 (d) 532

60. 4, 5, 7, 10, 14, 18, 25, 32

(a) 7 (b) 14 (c) 18 (d) 33

61. 52, 51, 48, 43, 34, 27, 16

(a) 27 (b) 34 (c) 43 (d) 48

HOMEWORK SOLUTIONS

1. 6 , 11, 21, 36, 56, ...

5 10 15 20 25

∴ 56 + 25 =81
option C

2. 10 , 100, 200, 310, ...

90 100 110 120

∴ 310 + 120 = 430
option D

3. 11 ,13, 17, 19, 23, 25, ...

2 4 2 4 2 4

∴ 25 + 4 = 29
option C

4. 6 , 12, 21, 33, ...

6 9 12 15

∴ 33 + 15 = 48
option D

5. 2 , 5, 9, 14, ... , 27

3 4 5 6 7

∴ 14 + 6 = 20
option A

6. 6 , 11, 21, ... , 56, 81

5 10 15 20 25

∴ 21 + 15 = 36
option B

7. 10 , 18, 28, 40, 54, ….. 88

8 10 12 14 16 18

∴ 54 + 16 = 70
option A

8. 120 , 99, ….. , 63, 48, 35

21 19 17 15 13

∴ 99 - 19 = 80
option A

9. 22 , 24, 28, 36, ….. , 84,

2 4 8 16 32

∴ 36 + 16 = 52
option B

10. 4832 , 5840, 6848, 7856, …..

1008 1008 1008 1008

∴ 7856 + 1088 = 8864
option A

11. 10 , 100, 200, 310, 430, …..

90 100 110 120 130

∴ 430 + 130 = 560
option A

12. 28 , 33, 31, 36, 34, …..

+5 -2 +5 -2 +5

∴ 34 + 5 = 39
option B

13. 120 , 80 , 40 , 45 , ……….., 5

option C

$\frac{120+40}{2}$, $\frac{45+5}{2} = 25$

14. 2 , 15, 41, 80, 132, …..

13 26 39 52 65

∴ 132 + 65 = 197

option D

15. 6 , 17, 39, ….., 116

11 22 33 44

∴ 39 + 33 = 72

option A

16. 1 , 4, 10, 22, ….., 34

3 6 12 24 48

∴ 22 + 24 = 46

option A

17. 4 , 9, 25, 49, ….., 169, 289, 361 / $11^2 = 121$

2^2 3^2 5^2 7^2 11^2 13^2 17^2 19^2 option B

Square of Prime Numbers

18. 4, 12, 36, ….., 324,

X3 X3 X3 X3

∴ 36 x 3 = 108

option C

19. 1 , 1, 4, 8, 9, ….., 16, 64

1^2 1^3 2^2 2^3 3^2 (3^3) 4^2 4^3

$(3^3) = 27$

option A

20. 5760 , 960 , 192 , -------, 16 , 8

5760 = 6 960 = 5 $^{16}/_{8} = 2$

5760/960 = 6 960/192=5 192/4 = 48 Option B

21. 1, 2, 6 , 7 , 21 , 22 , 66 , ----- , 201

5 15 45 66 + 1 = 67

OR 22 + 45 = 67

Option C

22. $\overset{x2}{\frown}$ $\overset{x2}{\frown}$

48 , 24 , 96 , ------, 192

x2 24 x 2 = 48 Option A

23. 165 , 195 , 255 , 285, -------- , 435 15 x 29

↓ ↓ ↓ ↓ ↓

15x11 15x13 15x17 15x19 15x23 15x23=345

15 x Prime No. Option A

24. 2 , 3 , 3 , 5 , 10 , 13 , 39 , ... , 172 , 177

1 2 3 4 5 39 + 4 = 43

Option C

25. 7 , 26 , 63 , 124 , 215 , ..., 511

⇓ ⇓ ⇓ ⇓ ⇓ ⇓

2^3-1 3^3-1 4^3-1 5^3-1 6^3-1 7^3-1 8^3-1

7^3-1 = 342

Option A

26. 3 , 7 , 15 , 31 , ... , 127

⇓ ⇓ ⇓ ⇓

2(3)+1 2(7)+1 2(15)+1 2(31)+1 = 63 Option B

(2n+1 ⇒ Potion)

27. 8 , 28 , 116 , 584 , ...

⇓ ⇓ ⇓ ⇓

3n+4 4n+4 5n+4 6n+4

Option D

28. 6 , 13 , 28 , 59 , ...

$\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$

2n+1 2n+2 2n+3 2n+4

Option A

29. 2 , 7 , 27 , 107 , 427 , ...

5 20 80 320 1280

x4 x4 x4 x4

427 + 1280

= 1707

Option A

30. 5 , 2 , 7 , 9 , 16 , 25 , 41, ...

$\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$

5+2 2+7 7+9 9+16 16+25 25+41

= 66

Option B

31. Word $\Rightarrow$ M A D R A S D E L H I

+1 $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ +1 $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$

Code $\Rightarrow$ N B E S B T E F M I J

Option B

32. R A M A N D I N E S H

$\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$

1 2 3 2 5 6 7 5 4 8 9

H A M A M

$\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$

9 2 3 2 3

Option A

33. R = 18 + 2 = 20 | G = 7 + 2 = 9
E = 5 + 2 = 7 | R = 18 + 2 = 20
D = 4 + 2 = 6 | E = 5 + 2 = 7
E = 5 + 2 = 7
6720 | N = 14 + 2 = 16
1677209
Option C

34. A = 1
F A T = 27
F A I T H
6 + 1 + 9 + 20 + 8 = 44 Option A

35.

B R O T H E R	S I S T E R
2 4 5 6 7 8 4	9 1 9 6 8 4

Question Error
B O R B E E S
⇓ ⇓ ⇓ ⇓ ⇓ ⇓ ⇓
2 5 4 2 8 8 9
Option A

36.

D E L H I	C A L C U T T A
7 3 5 4 1	8 2 5 8 9 6 6 2

C A L I C U T
⇓ ⇓ ⇓ ⇓ ⇓ ⇓ ⇓
8 2 5 1 8 9 6
Option C

37.

C L O C K	T I M E	M O T E L
⇓ ⇓ ⇓ ⇓ ⇓	⇓ ⇓ ⇓ ⇓	⇓ ⇓ ⇓ ⇓ ⇓
3 4 2 3 5	8 6 7 9	7 2 8 9 4

Option A

38.

P A L E	E A R T H	P E A R L
⇓ ⇓ ⇓ ⇓	⇓ ⇓ ⇓ ⇓ ⇓	⇓ ⇓ ⇓ ⇓ ⇓
2 1 3 4	4 1 5 9 0	2 4 1 5 3

Option B

39.	L O S E	G A I N	8 2 1 4 6
	⇓ ⇓ ⇓ ⇓	⇓ ⇓ ⇓ ⇓	⇓ ⇓ ⇓ ⇓ ⇓
	1 3 5 7	2 4 6 8	N G L A I

Option A

40.	M E K L E	L L L J K	I H J E D
	⇓ ⇓ ⇓ ⇓ ⇓	⇓ ⇓ ⇓ ⇓ ⇓	⇓ ⇓ ⇓ ⇓ ⇓
	9 1 7 8 2	8 8 8 6 7	5 4 6 1 0

Positional Value - 4

Option C

41.	N A M E	M E A N
	⇓ ⇓ ⇓ ⇓	⇓ ⇓ ⇓ ⇓
	4 2 5 8	5 8 2 4

Option C

42.	G O L D	W I N D
	⇓ ⇓ ⇓ ⇓	⇓ ⇓ ⇓ ⇓
	I Q N F	y k P F

Option A

43.	R O S E	B I S C U I T
	⇓ ⇓ ⇓ ⇓	⇓ ⇓ ⇓ ⇓ ⇓ ⇓ ⇓
	T Q U G	D K U E W K V

Option A

44. **Q. 44. to Q. 46.**

LETTER	⇒	C	Z	N	V	R	S	W	F	D
CASE	⇒	8	6	4	7	2	9	3	5	1

44. Z D R C V F

6 1 2 8 7 5 Option A

45. W N C S Z V

3 4 8 9 6 7 Option D

46. R D N F V S

2 1 4 5 7 9 Option C

47. | D E L H I | B O M B A Y

⇓ ⇓ ⇓ ⇓ ⇓ | ⇓ ⇓ ⇓ ⇓ ⇓ ⇓

C C I D D | -1 -2 -3 -4 -5 -6 Option B

A M J X V S

48. R I P P L E L I F E

⇓ ⇓ ⇓ ⇓ ⇓ ⇓ ⇓ ⇓ ⇓ ⇓

6 1 3 3 8 2 8 1 9 2

P I L L E R

⇓ ⇓ ⇓ ⇓ ⇓ ⇓

3 1 8 8 2 6 Option A

49. P A L A M = 16 + 1 + 12 + 1 + 13

= 43

S A N T A C R U Z = 19 + 1 + 14 + 20 + 1 + 3 + 18 + 21 + 26

= 123

Option A

50. 1 8 4 6 3 2

⇓ ⇓ ⇓ ⇓ ⇓ ⇓ From Table Given

M D B J II L

Option D

51. 2 5 6 ⇒ You are Good

6 3 7 ⇒ We are Bad

From observation 6 are

3 5 8 ⇒ Good and Bad

5 ⇒ Good

3 ⇒ Bad

∴ 8 represents 'and' Option C

52. 3, 5, 7, 15, 17, 19

Not Prime No. Option A

53. 10, 14, 16, 18, 23, 24, 26

Not Even No Option B

54. 1, 4, 9, 16, 24, 25, 36

Option B Not Perfect Square

55. Except 75 remaining all are prime number

Option A

56. 16, 25, 36, 73, 144, 196, 225

Not Perfect Square

Option B

57. 1, 4, 9, 16, 19 , 36, 49

Not Perfect Square

Option A

58. 1 5 14 30 49 55 91

Odd Number

4 9 16 25 36 Option A

59. 835, 734, 642, 751, 853, 981, 532

Odd Number

8-3=5 7-3=4 6-4=2 8-5=3 9-8=1 5-3=2 Option A

60. 4 5 7 10 14 18 25 32

Odd Number

1 2 3 4 5 6 7 Option C

61. 52 51 48 43 34 27 16

Odd Number

-1 -3 -5 -7 -9 -11 Option B

SELF ASSESSMENT

1. 10, 7, 12, 9, 14, 11, ________

   a) 16 b) 15 c) 18 d) 17

2. 0, 1, -1, 0, -2, -1, ______

   a) 3 b) -3 c) -2 d) -1

3. 1, 6, 16, 36, ______

   a) 74 b) 76 c)66 d) None of these

4. 3, 4, 7, 16, 43, _____

   a) 120 b) 124 c) 126 d) 130

5. 1000, 100, 200, 20, 40, ______

   a) 4 b) 400 c) 20 d) None of these

6. 2, 7, 17, 37, ______

   a) 66 b) 75 c) 77 d) 80

7. BCB, DED, FGF, HIH, ______

   a) HJH b) JKJ c) KJK d) HKH

8. D2E, H4J, L6O, P8T, ______

   a) T10Y b) U5V c) L7O d) X10Y

9. In a certain code MONKEY is written as XDJMNL. How TIGER can be coded as?

   a) QDFHS b) QDFSH c) QDHFS d) DQFSH

10. If DELHI coded as 73541 and CALCUTTA as 82589662, then CALICUT be coded?

    a) 8251986 b) 2851896 c) 8251896 d) None of these

11. In a system 15789 is coded as EGKPT and 2346 is coded as ALUR. How 23549 can be coded?

a) ALGUT b) ALTUG c) LATUG d) LAUTG

12. In a certain code

"Pit dar na" means "You are good"

"Dar tok pa" means "good and bad"

"Tim na tok" means "they are bad"

Then "they stands for

a) Tim b) dar c) tok d) None of these

13. In a coding

"256" means "you are good"

"637" means "we are bad"

"358" means "good and bad"

Then and coded as

a) 6 b) 8 c) 7 d) 5

14. If LOSE is coded as 1357 and GAIN coded as 2468, what do figure 82146 coded for?

a) NGLAI b) NGLIA c) GNLIA d) GNLA

15. AB, BA, ABC, CBA, ABCD, ______

a) DCBA b) DCAB c) ABDC d) BACD

16. Odd man out 15, 21, 63, 81, 69

a) 15 b) 21 c) 63 d) 81

17. 1, 2, 6, 15, 31, 56, 91

a) 6 b) 31 c) 56 d) 91

18. 22, 33, 66, 77, 121, 279, 594

a) 22 b) 66 c) 121 d) 279

19. 2, 3, 7, 9, 11

a) 2 b) 7 c) 9 d) 11

20. 3, 6, 18, 39, 108, 216

a) 18 b) 216 c) 39 d) 108

ANSWER

1.	A	2.	B
3.	B	4.	B
5.	A	6.	C
7.	B	8.	A
9.	A	10.	C
11.	A	12.	A
13.	B	14.	A
15.	A	16.	D
17.	D	18.	D
19.	C	20.	C

EXPLANATORY SOLUTION

1.
+2 +2
10, 7, 12, 9, 14, 11, 16
+2 +2 +2

Option A

2.
-1 -1
0, 1, -1, 0, -2, -1, -3
-1 -1 -3

Option B

3. 1, 6, 16, 36, 76
5 10 20 40

Option B

4. 3, 4, 7, 16, 43, 124
1 3 9 27 81

Option B

5. 1000, 100, 200, 20, 40, 4
÷ 10 x2 ÷ 10 x2 ÷ 10

Option A

6. 2, 7, 17, 37, 77
5 10 20 40

Option C

7.
+1 +1 +1 +1 +1
BCB, DED, FGF, HIH, JKJ
+2 +2 +2 +2

Option B

8.

	+2		+2		+2		+2	
D2E,		H4J,		L6O,		P8T,		T10Y
	+4		+4		+4		+4	

Option A

9.

M	Q	N	K	E	Y
X	D	J	M	N	L
-1	-1	-1	-1	-1	-1

then

T	I	G	E	R
Q	D	F	H	S

Option A

10.

D	E	L	H	I
↓	↓	↓	↓	↓
7	3	5	4	1

C	A	L	C	U	T	T	A
↓	↓	↓	↓	↓	↓	↓	↓
8	2	5	8	9	6	6	2

C	A	L	I	C	U	T
8	2	5	1	8	9	6

Option C

11.

1	5	7	8	9	2	3	4	6
E	G	K	P	T	A	L	U	R

2	3	5	4	9
↓	↓	↓	↓	↓
A	L	G	U	T

Option A

12. Pit dar $\boxed{\text{Na}}$ → you $\boxed{\text{are}}$ good

Dar tok pa → good and bad

Dar → good

Tim $\boxed{\text{Na}}$ Tok → they $\boxed{\text{are}}$ bad

Na → are

tak → bad

∴ Tim → They

Option A

13. 2 5 6 → you are good

6 3 7 → we are bad

6 → are

3 5 8 → good and bad

3 → bad

5 → good

∴ 8 → and

Option B

14. L O S E
1 3 5 7

G A I N
2 4 6 8

8 2 1 4 6

↓ ↓ ↓ ↓ ↓

N G L A I

Option A

15. AB BA ABC CBA ABCD DCBA

Reverse Reverse Reverse

Option A

16. 15, 21, 63, 81, 69

81 is the perfect square

Option D

17. 1, 2, 6, 15, 31, 56, 91 it should be 92

1 4 9 16 25 36

Option D

18. 22, 33, 66, 77, 121, 279, 594

Other all are multiple of 11

Option D

19. 2, 3, 7, 9, 11

All other are Prime Number

Option C

20.

6 6

3, 6, 18, 39, 108, 216

6 6

∴ Odd man out is number 39

Option C

2 DIRECTION TESTS

Direction is a measurement of position of one thing with respect to another thing or a reference point.

Types :

1. Finding direction only

2. Find the distance only

3. Finding both the distance and direction.

CLASS WORK SECTION

1. A girl is going towards West, then she turned left, then turned 90 degree in clockwise direction. In which direction is she going now?
   a) East b) West c) North d) None of these

2. If south west becomes North, then what will North-East be?
   a) North b) South-East c) South d) East

3. At sunrise, Amit and Deepak are having conversation standing in front of each other. The shadow of Deepak is formed towards the right hand of Amit. What direction is Deepak facing?
   a) North-East b) South c) East d) North

4. One evening before sun set, two friends Raman and Arjun were talking to each other face to face. If Raman's shadow was exactly to his left side, which direction was Arjun facing?
   a) West b) East c) North d) South

5. The time on the watch is 9.15 and the hour hand points towards West. The direction of the minute hand is towards.
   a) North b) South c) East d) West

6. A clock is so place that at 12 noon its minute hand points towards North East. In which direction does its hour hand point at 1.30 pm.
   a) North b) South c) East d) West

7. Samar wants to go college which is situated in a direction opposite to that of a mall. He starts from his house, which is in the east and comes at four-way place. His left side road goes to the mall and straight in front is the railway station. In which direction is the college located?
   a) North b) North-East c) South d) East

8. Ram walks 10m South from his house, turns left and walks 25m, again turns left and walks 40m, then turns right and walks 5m to reach the school. In which direction is the school from his house?

a) North b) South-West c) North-East d) East

9. Rashmi goes towards East from a point P then turns left. She walks some distance and turns to her right. Which direction is she facing now?

a) North b) East c) West d) South

10. A man goes 5 km east, then he turns right and goes 4km, then he turns left and goes 5 km. Which direction is he facing now?

a) North b) East c) West d) South

11. Laxman went 15kms to North then he turned west and covered 10 kms. Then he turned South and covered 5 kms, finally turning to East he covered 10 kms. In which direction he is now moving? (M-2018)

a) East b) West c) North d) South

12. A man is facing East, then he turns left and goes to 10 meter then turns right and goes 5 meter then goes 5meter to the south and from their 5meter to West. In which direction is he from his original place? (M-2018)

a) East b) West c) North d) South

13. Raman starts from his house and goes towards 15m North, then he turns to his right and walks 30 m before taking right turn and moving again upto 30 m to reach temple. In which direction is the temple with respect to Raman's house?

a) North-West b) South c) South-East d) West

14. X walks southwards and turns right then left and then right. In which direction is he moving now? (M-18)

a) South b) North c) West d) South – west

15. You go north, turn right, then right then go to the left. In which direction are you now?

a) South b) East c) West d) North

16. Surbhi is facing east, she turns 100degree in the clockwise direction and then 145 degree in the anti-clockwise direction. Which direction is she facing now?
    a) West b) North-East c) North d) South-West

17. The time in a clock is quarter past twelve. If the hour hand points to the East, which is the direction opposite to the minute hand?
    a) South-West b) South c) West d) North

18. A train runs 120km in West direction, then 30km in South direction and then 80 km in East direction before reaching the station. In which direction is the station from the train's starting point?
    a) South-West b) North-West c) South-East d) South

19. If X stands on his head with his face towards South, to which direction will his left hand point?
    a) East b) West c) South d) North

20. A and B start walking in opposite direction. A covers 3 km and B covers 4 km then A turns right and walks 4 km while B turns left and walks 3 km. How far is A from the starting point?
    a) 10km b) 8 km c) 5 km d) 4 km

21. Shreya started from point P and walked 2m towards West. She, then took a right turn and walked 3m before taking a left turn and walking 5m. She finally took a left turn, walked 3m and stopped at appoint Q. How far is point Q from point P?
    a) 2m b) 6m c) 7m d) 8m

22. Vinod Starts from his house and travels 4km in East direction, after that he turns towards left and moves 4km. Finally, he turns towards left and moves 4km. At what distance and in which direction he finally stands from his starting point?
    a) North, 4km b) North-East 4km
    c) South 12km d) West 4km

23. Two buses start from the opposite points of a main road, 150km apart. The first bus runs for 25km and takes a right turn and then runs for 15km. It then turns left and runs for another 25km and takes the direction back to reach the main road. In the meantime, due to the minor break down the other bus has run only 35km along the main road. What would be the distance between the two buses at this point?

a) 65km b) 80km c) 75km d) 85km

24. A man started to walk East. After moving a certain distance, he turns to his right. After moving some distance, he turns to his right again. After moving a little he turns now to his left, currently he is going in ________ direction.

a) West b) East c) North d) South

25. Raghu is at point A. He walks 3km to the North and then turns to his left. He walks, 4km in this direction. He turns left again and walks 6 km. If he wishes to reach point A again, in which direction should he be walking and what distance will he have to cover?

a) South-East, 5km b) South-East, 4 km

c) North-East, 5 km d) North-East, 4 km.

HOMEWORK SECTION

1. Mohan starts from point A and walks 1 km towards south, turns left and walks 1km. Then he turns left again and walks 1 km. Now he is facing.
   (a) East (b) West (c) North (d) South-west

2. Suresh starts from a point, walks 2 miles towards south, turns right and walks 11/2 miles, turns left and walks ½ miles and then he turns back. What is the direction he is facing now?
   (a) East (b) West (c) South (d) North

3. A man starts from a point, walks 4 miles towards north and turns left and walks 6 miles, turns right and walks for 3 miles and again turns right and walks 4 miles and takes rest for 30 minutes. He gets up and walks straight 2 miles in the same direction and turns right and walks one mile. What is the direction he is facing?
   (a) North (b) South (c) South-east (d) West

4. Arun started from point A and walked 10 km East to point B, then turned to North and walked 3 km to point C and then turned West and walked 12 kms to point D, then again turned South and walked 3 kms to point E. In which direction is he from his starting point?
   (a) East (b) South (c) West (d) North

5. A starts from a point and walks 5 kms north, then turns left and walks 3 kms. Then again turns left and walks 5 km. Point out the direction in which he is going now.
   (a) North (b) South (c) East (d) West

6. A rat run 20 towards East and turns to right runs 10 and turns to right runs 9 and again turns to left runs 5 and then turns to left runs 12 and finally turns to left and rusn 6. Now what direction is the rat facing?
   (a) East (b) North (c) West (d) South

7. A driver left his village and drove North for 20 km, after which he stopped for breakfast. Then he turned left and drove another 30 km, when he stopped for lunch. After some rest, he again turned left and drove 20 kms before stopping for evening tea. Once more he turned left and drove 30 kms to reach the town where he had supper. After evening tea in which direction did he drive?

(a) West (b) East (c) North (d) South

8. A man is facing East, then he turns left and goes 10 m, then turns right and goes 5 m then goes 5 m to the South and from there 5 m to West. In which direction is be from his original place?

(a) East (b) West (c) North (d) South

9. From her home Prerna wishes to go to school. From home she goes towards North and then turns left and then turns right, and finally she turns left and reaches school. In which direction her school is situated with respect to her home?

(a) North-East (b) North-West (c) South-East (d) South-West

10. A child walks 25 feet towards North, turns right and walks 40 feet, turns right again and walks 45 feet. He then turns left and walks 20 feet. He turns left again walks 20 feet. Finally, he turns to his left to walks another 20 feet. In which direction is the child from his starting point?

(a) North (b) South (c) West (d) East

11. Raju facing North and moves 20 km, then he turned to his right and moves 20 km and then he moves 10 km in North-East, then he turned to his right and moves 20 km and then he turned to his right and moves 20 km and again he turned to his left and moves 20 km. Now in which direction Raju is facing?

(a) South-East (b) North-East (c) South-West (d) North-West

12. K is a place which is located 2 km away in the north-west direction from the capital P. R is another place that is located 2 km away in the south-west direction from K. M is another place and that is located 2 km away in the north-west direction from R. T is yet another place that is located 2 km away in the south-west direction from M. In which direction is T located in relation to P?

(a) South-west (b) North-west (c) West (d) North

13. Babu is Rahim's neighbour and his house is 200 meters away in the north-west direction. Joseph is Rahim's neighbour and his house is located 200 meter away in the south-west direction. Gopal is Joseph's neighbour and he stays 200 meters away in the south-east direction. Roy is Gopal's neighbour and his house is located 200 meters away in the north-east direction. Then where is the position of Roys' house in relation to Babu's?

(a) South-east (b) south-west (c) North (d) North-east

14. A tourist drives 10 km towards west and turns to left and takes a drive of another 4 km. He then drives towards east another 4 km and then turns to his right and drives 5 km. Afterwards he turns to his left and travels 6 km. In which direction is he from the starting point?

(a) North (b) East (c) West (d) South

15. A man started walking West. He turned right, then right again and finally turned left. Towards which direction was he walking now?

(a) North (b) South (c) West (d) East

16. One evening, Raja started to walk toward the Sun. After walking a while, he turned to his right and again to his right. After walking a while, he again turned right. In which direction is he facing?

(a) South (b) East (c) West (d) North

17. Five boys A, B, C, D, E, are sitting in a park in a circle. A is facing South-West, D is facing South-East, B and E are right opposite A and D respectively and C is equidistant between D and B. Which direction is C facing?

(a) West (b) South (c) North (d) East

18. If a man on a moped starts from a point and rides 4 km South then turns left and rides 2 km and turn again to the right. Which direction is he moving ?

(a) North (b) West (c) East (d) South

19. A man starts from a point, walk 8 km towards North, turns right and walks 12 km, turns left and walks 7 km turns and walks 20 km towards South, turns right and walks 12 km. In which direction is he from the starting point?
(a) North (b) South (c) West (d) East

20. Daily in the morning the shadow of Gol Gumbaz falls on Bara Kaman and in the evening the shadow of Bara Kaman falls on Gol Gumbaz exactly. So in which direction is Gol Gumbaz to Bara Kaman?
(a) Easter side (b) Western side (c) Northern side (d) Southern side

21. Ashok went 8 km South and turned West and walked 3 km again he turned North and walked 5 kms. He took a final turn to East and walked 3 kms . In which direction was Ashok from the starting point?
(a) East (b) North (c) West (d) South

22. If X stands on his head with his face towards south, to which direction will his left hand point ?
(a) East (b) West (c) North (d) South

23. I drove East for 5 miles then drove North 3 miles, then turned to my left and drove for 2 miles and again turned to my left. Which direction am I going now?
(a) South (b) North (c) West (d) North-west

24. If A stands on his head with his face towards north. In which direction will his left hand point?
(a) North-East (b) North (c) East (d) North-West

25. A car travelling from south covers a distance of 8 km, then turns right and runs another 9 kms and again turns to the right and was stopped. Which direction does it face now?
(a) South (b) North (c) West (d) East

26. A taxi driver commenced his journey from a point and drove 10 km toward north and turned to his left and drove another 5 km. After waiting to meet a friend here, he turned to his right and continued to drive another 10 km. He has covered a distance of 25 km so far, but in which direction would he be now?
    (a) South (b) North (c) East (d) South-east

27. A walks 3 kms northward and then he turns left and goes 2 km. He again turns left and goes 3 km. He turns right and walks straight. In which direction is he walking now?
    (a) East (b) West (c) North (d) South

28. A walks southwards, then turns right, then left and then right. In which direction is he from the starting point?
    (a) Southwest (b) East (c) West (d) North

29. A man starts from a point, walks 15 metres towards East, turns left and walks 10 metres, turns right again and walks. Towards which direction is he now waking?
    (a) North (b) East (c) West (d) South

30. A boy starts walking towards West, he turns right and again he turns right and then turns left at last. Towards which direction is he walking now?
    (a) West (b) North (c) South (d) East

31. I stand with my right hand extended side-ways towards South. Towards which direction will my back be ?
    (a) North (b) West (c) East (d) South

32. If a person moves 4 km towards west, then turns right and moves 3 km and then turns right and moves 6 km, which is the directions in which he is now moving?
    (a) East (b) West (c) North (d) South

33. If Mohan sees the rising sun behind the temple and the setting sun behind the railway station from his house, what is the direction of the temple from the railway station?
    (a) South (b) North (c) East (d) West

34. Laxman went 15 km to North then he turned West and covered 10 kms. Then he turned south and covered 5 kms. Finally turning to East he covered 10 kms. In which direction he is from his house?
(a) East (b) West (c) North (d) South

35. A man starts from a point, walks 4 miles North, turns to his right and walks 2 miles, again turns to his right and walks 2 miles, again turns to his right and walks 2 miles. In which direction would he be now from his starting point?
(a) North (b) South (c) East (d) West

36. I started walking down a road in the morning facing the Sun. After walking for some time I turned to my left. Then I turned to my right. In which direction was I going then ?
(a) East (b) West (c) North (d) South

37. Lakshmi walked 2 furlongs north from her house and took a turn to left and continued to walk another one kilometre and finally she turned left and reached the school. Which direction is she facing now?
(a) West (b) North (c) South (d) North

38. You are going straight, first eastwards, then turn to the right, then right again, then left. In which direction would you be going now?
(a) East (b) West (c) South (d) East

39. If Ahmed travels towards North from his house, then to left, then to South covering equal distances in each direction to reach Sohan's house, in which direction is Ahmed's house now?
(a) East (b) South (c) North (d) West

40. You go North, turn right, then right again and then go to the left. In which direction are you now?
(a) South (b) East (c) West (d) North

41. Roopa starts from a point and walks 15 metre towards west, turns left and walks 12 metre, turns right again and walks. What is the direction she is now facing?
(a) South (b) West (c) East (d) North

42. A man starts his journey facing the sun early morning. He then turns right and walks 2 km. He then walks 3 km after turning right again. Which is the direction he is facing now?
(a) North-East (b) North (c) West (d) South

43. Roy walks 2 km to East, then turns North-West and walks 3 km. Then he turns South and walks 5 km. Then again he turns West and walks 2 km. Finally he turns North and walks 6 km. In which direction, is he from the starting point?
(a) South-West (b) South-East (c) North-West (d) North-East

44. Seeta starts from a point, walks 2 km towards north, turns towards her right and walks 2 km, turns right again and walks. What is the direction she is facing now?
(a) East (b) West (c) South (d) North

45. Shyam was facing East. He walked 5 km forward and then after turning to his right walked 3 km. Again he turned to his right and walked 4 km. After this he turned back. Which direction was he facing at that time?
(a) East (b) West (c) North (d) South

46. Raju is standing facing north. He goes 30 metres ahead and turns left and goes for 15 metres. Now he turns right and goes for 50 metres and finally turns to his right and walks. In which direction is he heading?
(a) North (b) East (c) South (d) West

47. Sanmitra starts from his house and walks 3 km towards north. Then he turns right and walks 2 km and then turns right and walks 5 km, then turns right and walks 2 km and then again turns right and walks 2 km. Which direction is he facing now?
(a) North (b) South (c) West (d) East

48. Raju is Ramu’s neighbour and he stays 100 metres away towards southeast. Venu is Raju’s neighbour and he stays 100 metres away towards southwest. Khader is Venu’s neighbour and he stays 100 metres away towards, north-west. Then where is the position of Khader’s home in relation to Ramu’s?
(a) South-East (b) South-West (c) North-West (d) East

49. Ramesh walked 3 km, towards West and turned to his left and walked 2 km. He, then turned to his right and walked 3 km. Finally, he turned to his right again and walked another 2 km. In which direction is Ramesh from his starting point now?
(a) East (b) West (c) North (d) South

50. Deepa starts walking towards north and after a while she turns to her right. After walking some distance, she turns to her left and walks a distance of 1 km. She then turns to her left again. In which direction she moving now?
(a) North (b) West (c) East (d) South

51. Raman starts walking in the morning facing the Sun. After sometime, he turned to the left later again he turned to his left. At what direction is Raman moving now?
(a) East (b) West (c) South (d) North

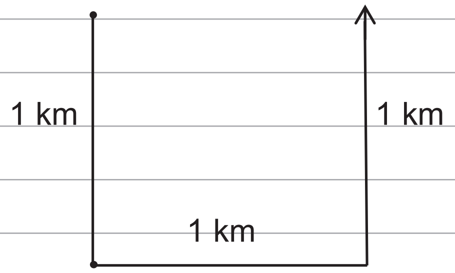
52. A starts walking towards North turns left, again turns left, turns right, again turns right once again turns left. In which direction is A walking now?
(a) East (b) South (c) West (d) South-East

53. X walks southwards and then turns right, then left and then right,. In which direction is he moving now?
(a) South (b) North (c) West (d) South-West

54. A man started to walk East. After moving a distance, he turned to his right. After moving a distance, he turned to his right again. After moving a little he turned in the end to his left. In which direction was he going now.?
(a) North (b) South (c) East (d) West

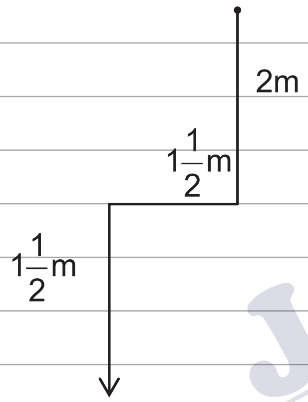
HOMEWORK SOLUTION

Q.1



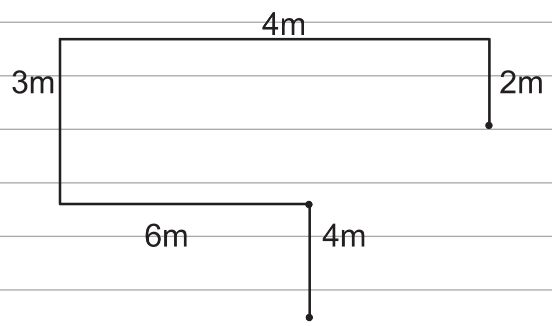
Option C

Q.2



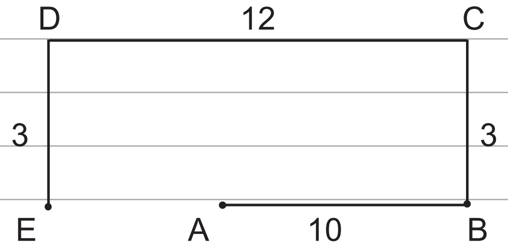
Option D

Q.3



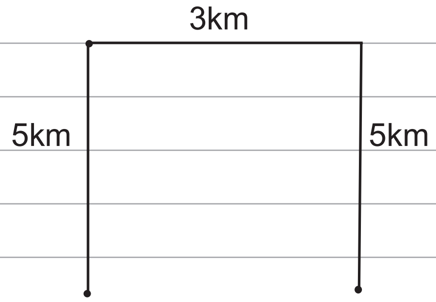
Option B

Q.4



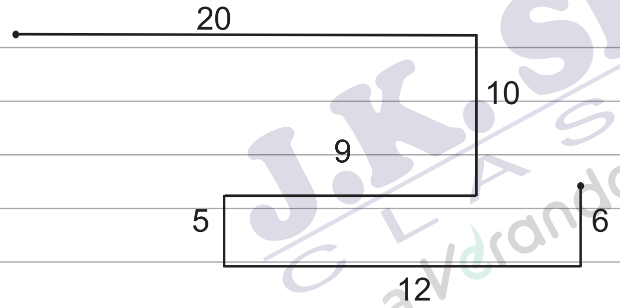
Option C

Q.5



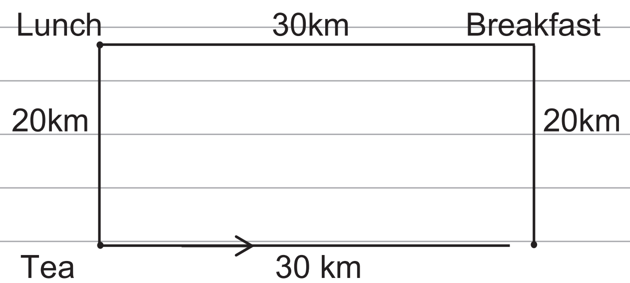
Option B

Q.6



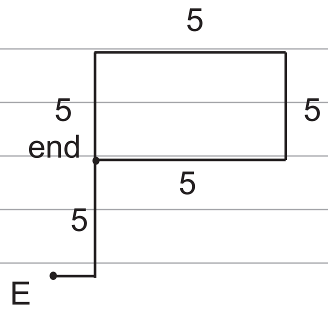
Option B

Q.7



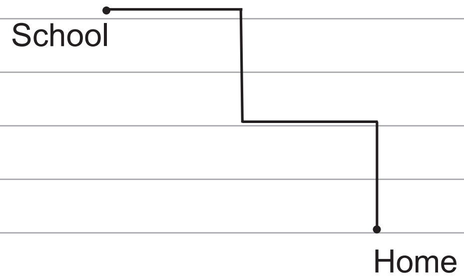
Option B

Q.8



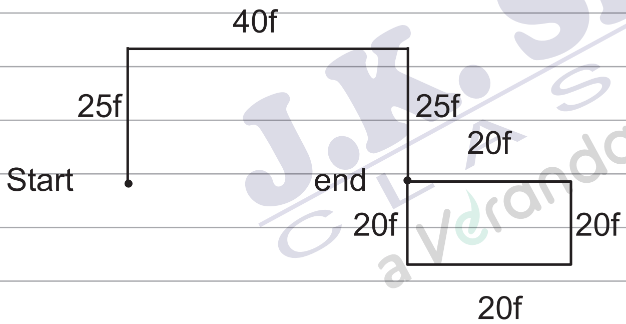
Option C

Q.9



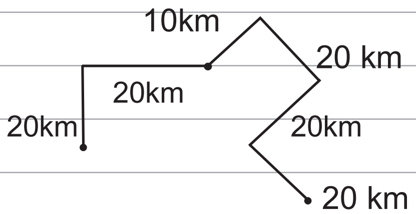
Option B

Q.10



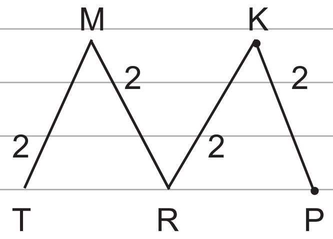
Option D

Q.11



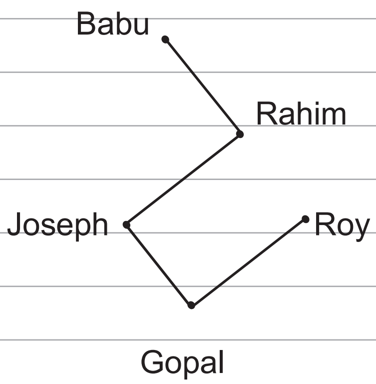
Option A

Q.12



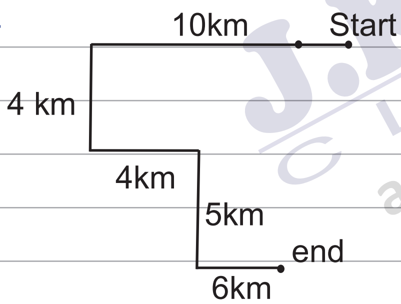
Option C

Q.13



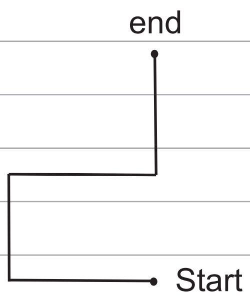
Option A

Q.14



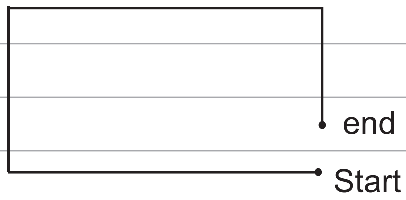
Option D

Q.15



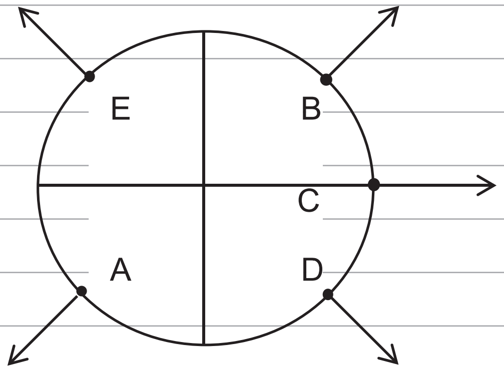
Option A

Q.16



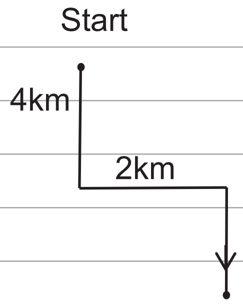
Option A

Q.17



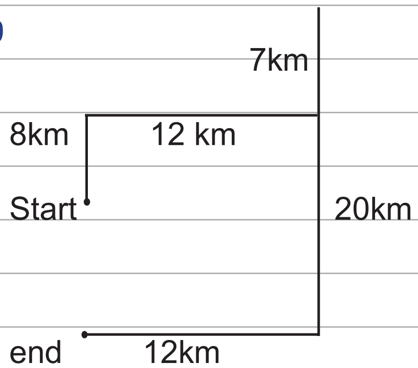
Option D

Q.18



Option D

Q.19

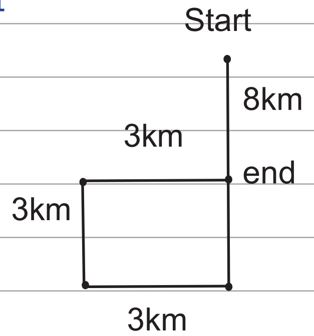


Option B

Q.20

Option A

Q.21



Option D

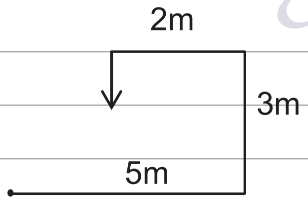
Q.22 If x stands on his head = If x stands on his feet

↓ ↓

Face towards South = Face towards North

Option B

Q.23



Option A

Q.24

If A stands on his head = If A stands on his feet

↓ ↓

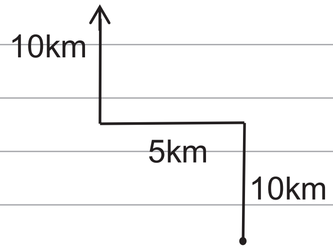
Face towards North = Face towards South

Option C

Q.25

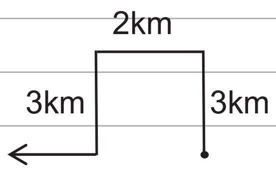
Option A

Q.26



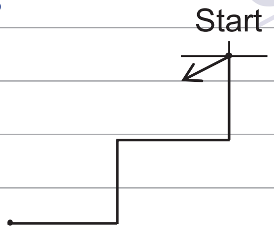
Option B

Q.27



Option B

Q.28



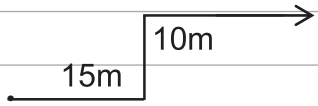
Option A

↓

Correction answer should be

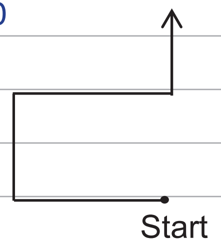
(South West)

Q.29



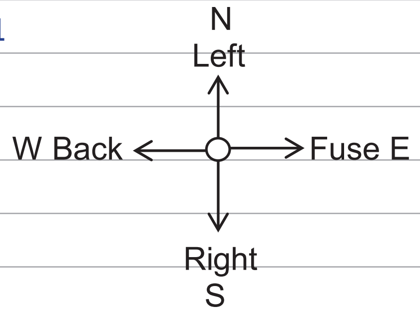
Option B

Q.30



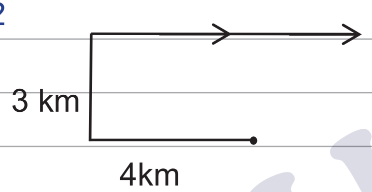
Option B

Q.31



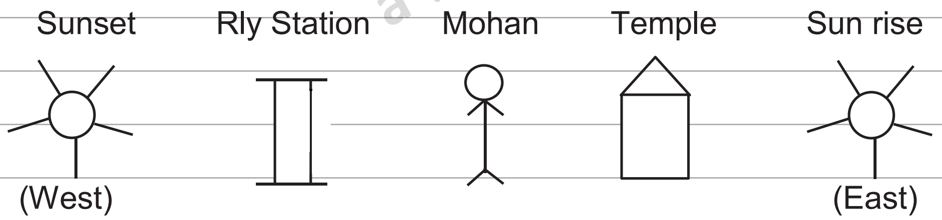
Option B

Q.32



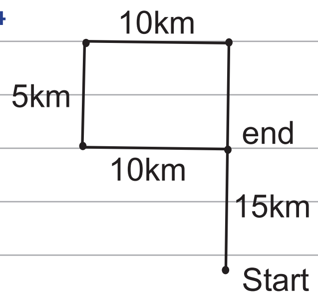
Option A

Q.33



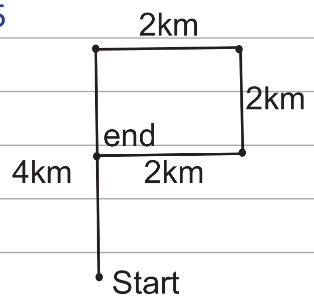
Option C

Q.34



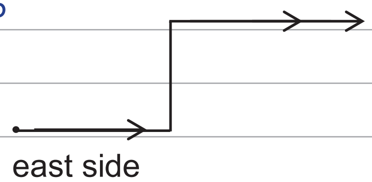
Option C

Q.35



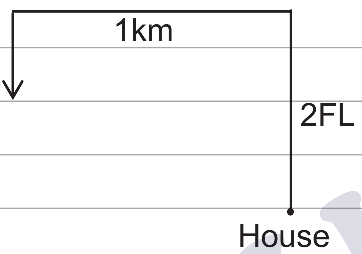
Option A

Q.36



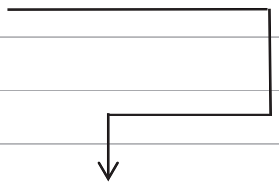
Option A

Q.37



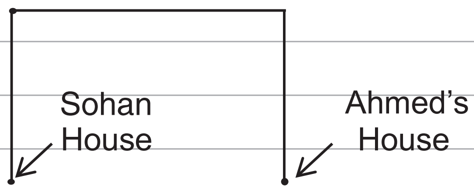
Option C

Q.38



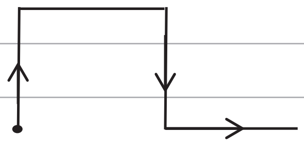
Option C

Q.39



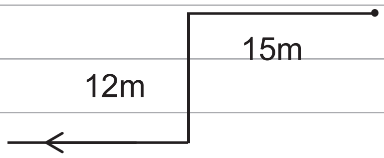
Option A

Q.40



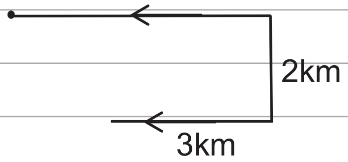
Option B

Q.41



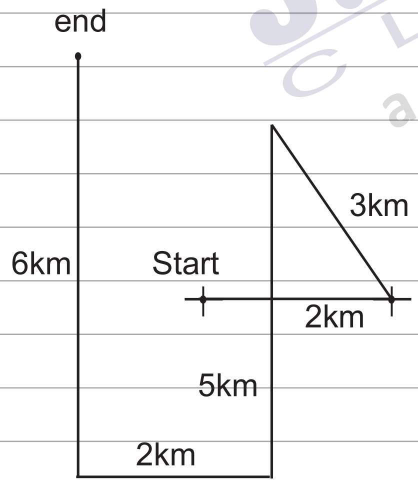
Option B

Q.42



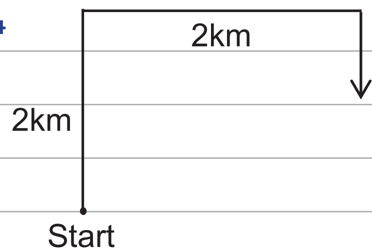
Option C

Q.43



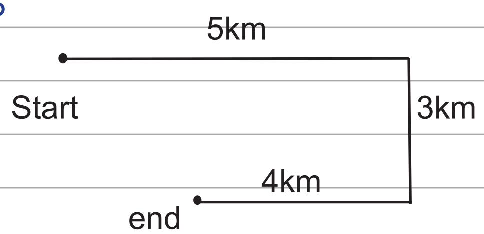
Option C

Q.44



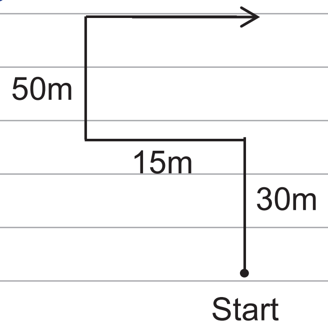
Option C

Q.45



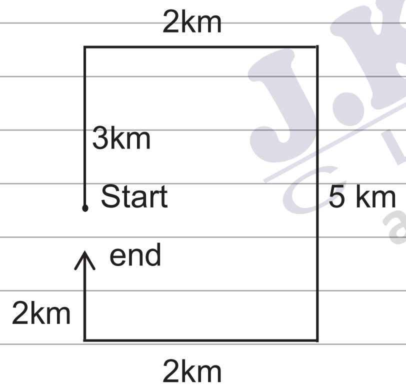
Option A

Q.46



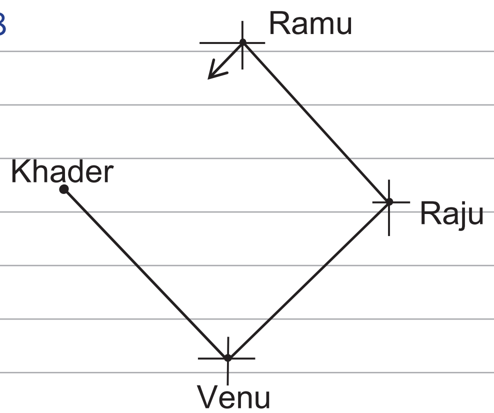
Option B

Q.47



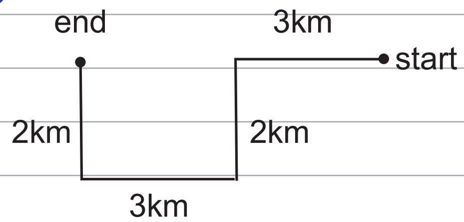
Option A

Q.48



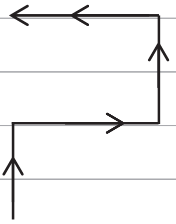
Option B

Q.49



Option B

Q.50

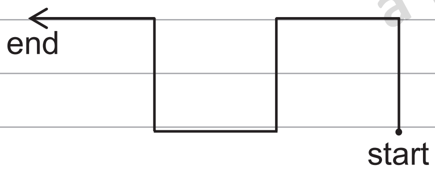


Option B

Q.51

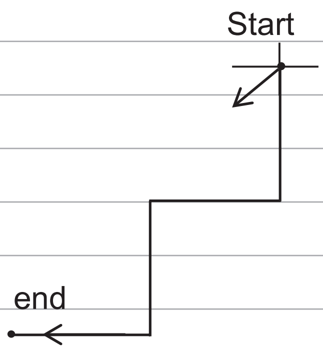
Option B

Q.52



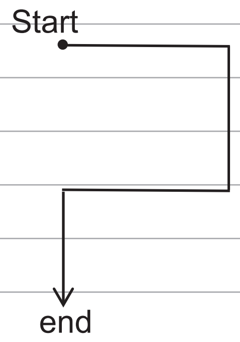
Option C

Q.53



Option C

Q.54



Option B

SELF ASSESSMENT

1. Aditya walked 20 m towards North. Then he turned right and walks 30 m. Then he turns right and walk 35 m. Then he turns left and walk 15 m. finally he turns left and walk 15 m. In which direction and how many meters is he from the starting point?

   a) 15 m West b) 30 m East c) 30 m West d) 45 m East

2. Jay (Starts from A) walked 5m towards West, then turned left and walked 5m. Again turned left and walked 5 m. again turned left and walked 6 m. How far he is from A.

   a) 1m (b) 2 m c) 3 m d) 4 m

3. Shivam started from his house towards West. After walking a distance of 15 km he turned to the right and walked 10 km. He again turned to the right and walked 5 km. After this he is to turn right at 135° and covered 10 km. In which direction should be go?

   a) South b) S-W c) S-E d) North

4. After walking 6 km, I turned to the right and then walked 2 km. After then I turned to the left and walked 10 km. In the end, I was moving towards the North. From which direction did I start my Journey?

   a) North b) South c) East d) West

5. Reena walked from A to B in the East 10 Feet. The she turned to the right and walked 3 Feet. Again she turned to the right and walked 14 Feet. How far is she from A?

   a) 4 FT b) 5 FT c)12 FT d) 13 FT

6. 'If A xB means A is to the South ofB.A + B means A is to the North of B, A% B means A is to East of B, A - B means A is to the West of B, then in P% Q + R - S, S is in which direction with respect to Q?

   a) S-W b) S-E c) N-E d) N-W

7. Four friends A, B, C, D live in a same locality. The house of B is in the East ofA's house but in the North of C's house. The house of C is in the West of D's house. D's house is in which direction of A's house?

a) S-E b) N-E c) East (d) North

8. Mohan starts from point A and walk 1 km towards South, turns left and walk 1 km. Then he turns left again and walk 1 km. Now he is facing.

a) East b) West c) North d) South

9. Dev walks20m towards North. He then turns left and walks 40 m. He again turns left and walks 20 m. Further, he moved 20 m after turning to the right. How far is he from his original position?

a) 20 m b) 30 m c) 50 m d) 60 m

10. A walks Southwards, then turn right, then left and then right. In which direction is he from the starting point?

a) South West b) West c) East d) North

11. A is located to the West of B, C is located at North in between A and B. D is exactly to the South of B and also in line with B. In which direction of C is D located?

a) South b) S-E c) West d) S-W

12. A man is facing East, then he turns left and goes to 10 meter then turns right and goes 5 meter then goes 5 meter to the south and from their 5 meter to West. In which direction is he from his original place?

a) East b) West c) North d) South

13. I stand with my right-hand extended side-ways towards South. Towards which direction will my back be?

a) North b) West c) East d) South

14. Manu wants to go to the market. He starts from his house towardsNorth reaches at a crossing after 30m. He turns towards East, goes 10m till the second crossing and turns again, moves towards South straight for 30m where marketing complex exits. In which direction is the market from his house?

a) North b) East c) South d) West

15. Madhuri moved a distance of 75 meters toward north. She then turned to the left and walking for about 25m, turned left again and walks 80m. Finally she turned to the right at an angle of 45°. In which direction was she moving finally?

a) South – East   b) South – West
c) North - West   d) North – East

16. a man starts from a point, walk 3 km towards North, turns towards his right and walks 3 km, turns right again and walk. What is the direction nowhe is facing?

a) South   b) North   c) West   d) East

17. A person moves towards North up to n km, then walked towards East upto 10 km, then take right turn towards South and walked upto 10 km, again he moved towards West and reached at start point. What is the value of n.

a) 10 km   b) 20 km   c) 1 km   d) 2 km

18. One morning Sujata started to walk towards the sun. After covering some distance she turned to right then again to the right and after covering some distance she again turn to the right. Now in which direction is she facing.

a) North   b) South   c) N-E   d) S-W

19. Ramu is going towards North from his house. After covering a distance of 6 km he turned towards left and covered a distance of 8 km. What is the shortest distance now from his house?

a) 15 km   b) 10 km   c) 25 km   d) 20 km

20. You go North, turn right, then right again and then go to left. In which direction are you now?

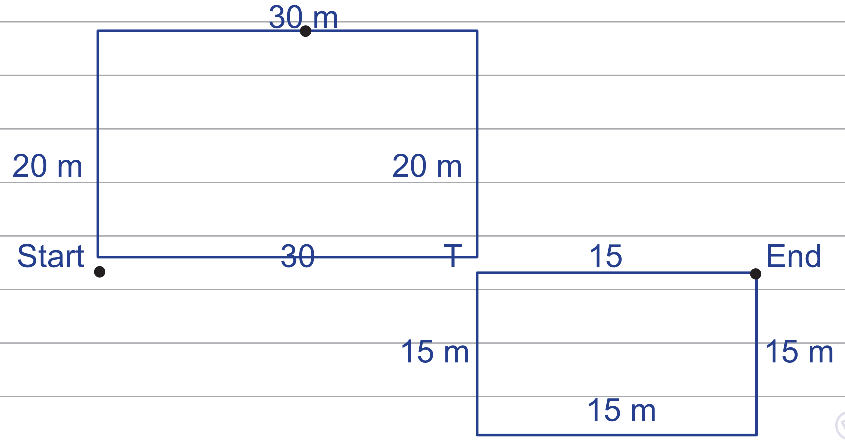
a) South   b) East   c) West   d) North

ANSWER

1.	D	2.	A
3.	B	4.	B
5.	B	6.	B
7.	A	8.	C
9.	D	10.	A
11.	B	12.	C
13.	B	14.	B
15.	C	16.	A
17.	A	18.	A
19.	B	20	B

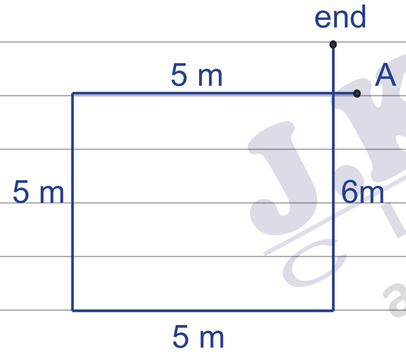
Explanatory solutions

Q.1



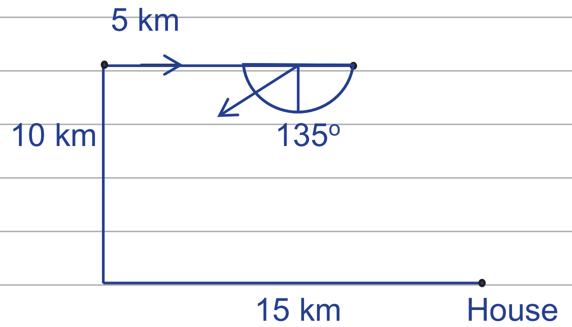
Option D

Q.2



Option A

Q.3

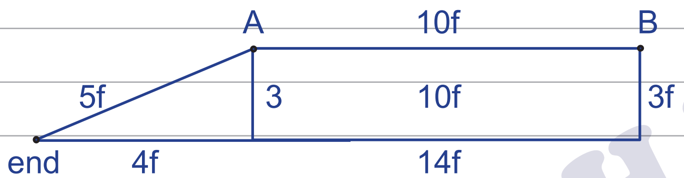


Option B

Q.4

Option B

Q.5



Option B

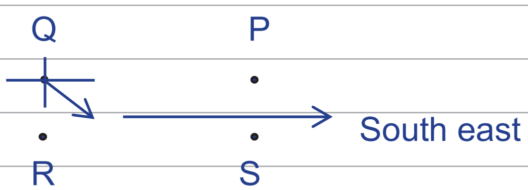
Q.6 A x B → A is to the south of B

A + B → A is to the north of B

A % B → A is to the east of B

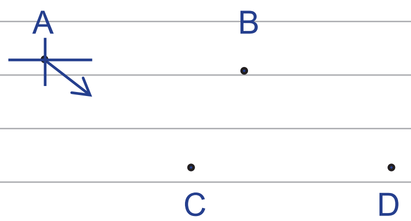
A - B → A is to the west of B

P % Q + R – S



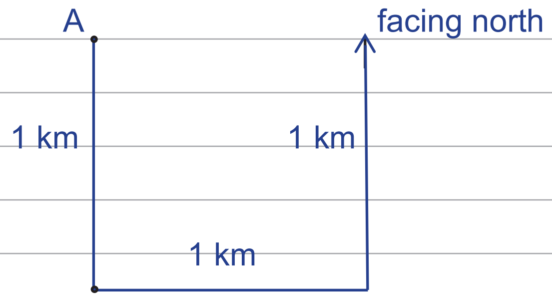
Option B

Q.7



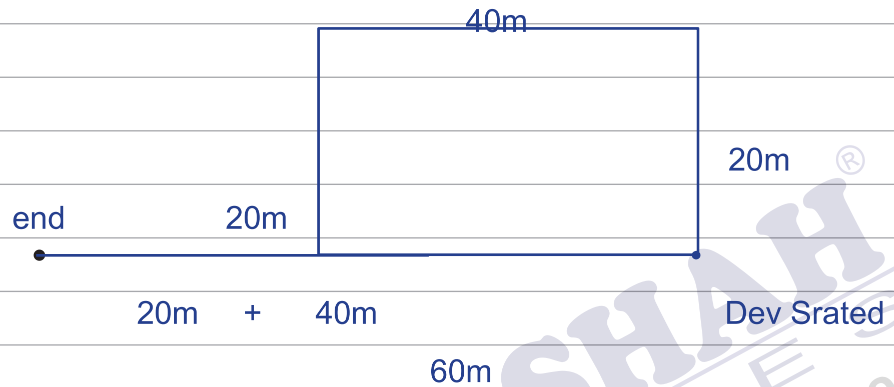
Option A

Q.8



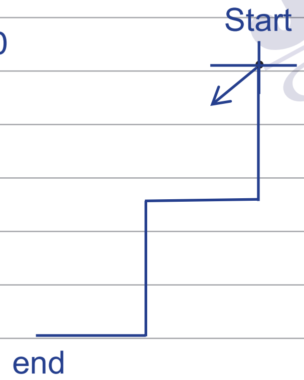
Option C

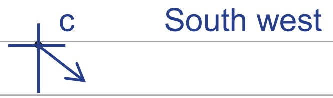
Q.9



Option D

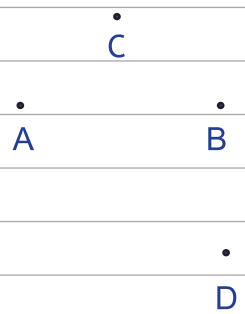
Q.10





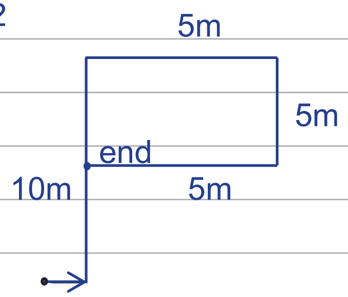
Option D

Q.11



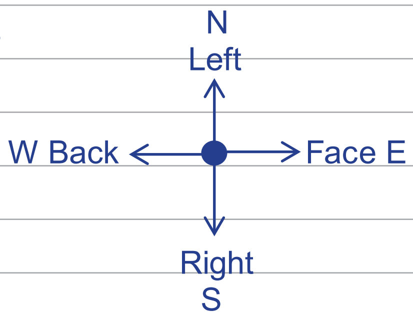
Option B

Q.12



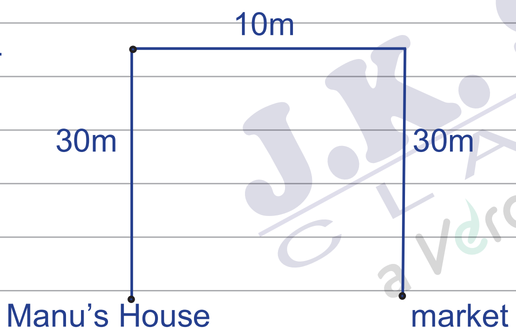
Option C

Q.13



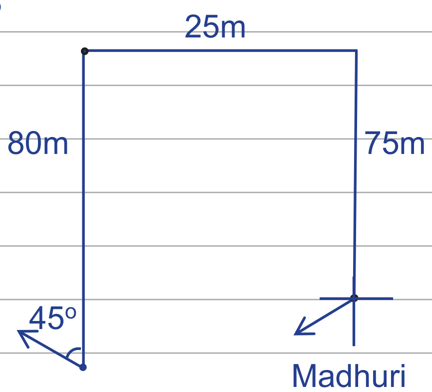
Option B

Q.14



Option B

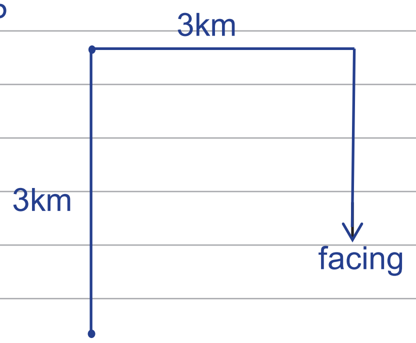
Q.15



Option C

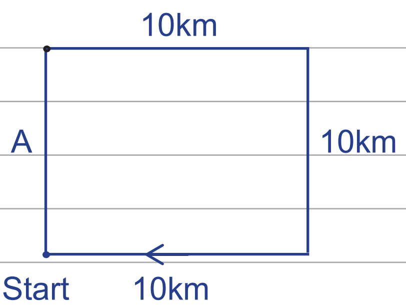
Northwest

Q.16



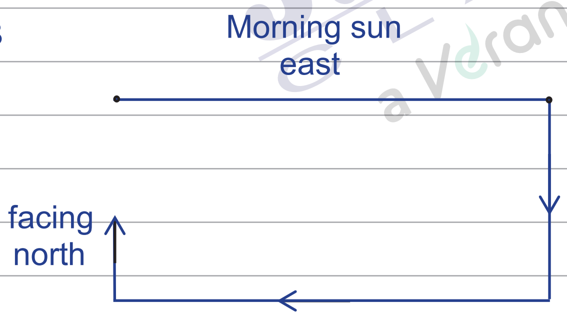
Option A

Q.17



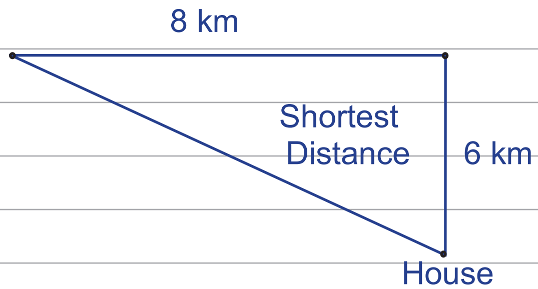
Option A

Q.18



Option A

Q.19



Distance = $\sqrt{(6)^2 + (8)^2}$

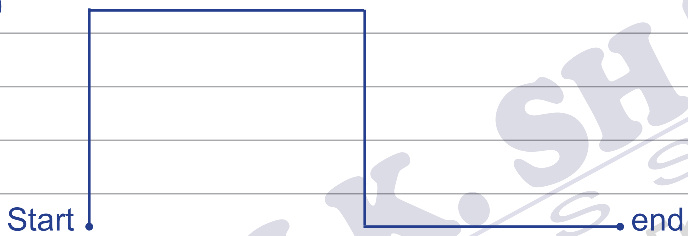
$= \sqrt{36 + 64}$

$= \sqrt{100}$

= 10 km

Option B

Q.20



Option B

3 SEATING ARRANGEMENT

Sitting arrangement questions are based on the sitting sequence pattern, direction, facing outside or inside etc.

Different types of Questions covered.

1. Linear arrangement
2. Circular arrangement
3. Polygonal arrangement

CLASS WORK SECTION

LINEAR ARRANGEMENT

1. Four girls are seated for a photograph Shikha is left of Reena. Manju is to the right of Reena. Reeta is between Reena and Manju. Who is the second left in photograph. (J-2019)

   a) Reena b) Manju c) Reeta d) Shikha

2. 5 friends are sitting on a bench. A is to the left of B but on the right of C. D is to the right of B but on the left of E. Who are at the extremes?

   a) A, B b) A, D c) B, D d) C, E

3. Five students A, B, C, D and E are standing in a row. D is on the right of E, B is on the left of E but on the right of A. D is next to C on his left. The student in middle is (N-2018)

   a) B b) E c) C d) A

4. Five children are sitting in a row. S is sting next to P but not T. K is sitting next to R, who is sitting on the extreme left and T is not sitting next to K. Who is/are adjacent to S? (M-2018)

   a) K and P b) R and P c) only P d) P and T.

5. Five boys are standing in a row facing East, Pavan is to the left of Tavan, Vipin, Chavan. Tavan, Vipin and Chavan are to the left of Nakul. Chavan is between Tavan and Vipin. If Vipin is fourth from the left, then how far is Tavan from the right?

   a) First b) Second c) Third d) Fourth

6. Five senior citizens are living in a multistoried building Mr. Muan lives in a flat above Mr. Ashokan, Mr. Lokesh lives in a flat below Mr. Gaurav, Mr. Ashokan lives in a flat above Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who lives in the topmost flat? (M-2018)

   a) Mr. Lokesh b) Mr. Gaurav c) Mr. Muan d) Mr. Rakesh

7. Five boys A, B, C, D and E are sitting on a stair in the following way

E is above A

D is under B

B is under A

D is between B and C

Who is at the lowest position of the stair?

a) A b) C c) E d) B

8. Six children A, B, C, D E and F are standing in a row.

B is between F and D.

E is between A and C

A does not stand next to F or D.

C does not stand next to D.

F is between which of the following pairs of children? (M-2018)

a) B and E b) B and C c) B and D d) B and A

9. Eight persons A, B, C, D, E, F, G and H are sitting in a line.

E is second right to D.

H sits fourth left to D.

C and F are immediate neighbors, but C is not immediate neighbor of A.

G is not neighbor of E

Only two person sit between A and E.

The persons on left end and right end respectively are

a) G and E b) B and E c) H and E d) G and B

10. There are eight books kept one over the other. Two books are on Organisational Behaviour, two books on TQM, three books on Industrial Relations and one book is on Economics, counting from the top, the second, fifth and sixth books are on Industrial Relations. Two books on Industrial Relations are between two books on TQM. One book of Industrial Relations is between two books on Organisational Behaviour while the book above the book of Economics is book of TQM. Which book is the last book from the top?

a) Economics b) TQM

c) Industrial Relations d) Organisational Behaviour

Directions: (Q. no. 11 to 15)

Read the following information carefully to answer the given questions.

A, B, C, D, E, F, G and H are seated in straight line facing North.

C sits fourth to left of G.

D sits seconds to right of G.

Only two people sit between D and A.

B and F are immediate neighbors of each other.

B is not an immediate neighbour of A.

H is not an immediate neighbour of D.

11. Who amongst the following sits exactly in the middle of the persons who sit fifth from the left and the person who sits sixth from the right?

a) C b) H c) E d) F

12. Who amongst the following sits third to the right of C?

a) B b) F c) A d) E

13. Which of the following represents persons seated at the two extreme ends of the line?

a) C, D b) A, B c) B, G d) D, H

14. What is the position of H with respect to F?

a) Third to the left b) Immediate right

c) Second right d) Fourth to left

15. How many persons are seated between A and E?

a) One b) Two c) Three d) Four

Directions (Q. no. 16 to 20)

Read the following information carefully and answer the following questions based on it.

Ten students – A, B, C, D, E, F, G, H, I and J are sitting in a row facing West.

B and F are not sitting on either of the edges.

G is sitting to the left of D and H is sitting to the right of J

There are four persons between E and A.

I is immediate to the North of B and F immediate is to the South of D.

J is in middle of A and D and G is in middle of E and F

There are two persons between H and C.

16. Who is sitting at the seventh-place counting from left?
    a) H   b) C   c) J   d) Either H or C

17. Who among the following is definitely sitting at one of the ends?
    a) C   b) H   c) E   d) Cannot be determined

18. Who are immediate neighbours of I?
    a) BC   b) BH   c) AH   d) (a) or (b)

19. Who is sitting second left of D?
    a) G   b) F   c) E   d) J

20. If G and A interchange their positions, then who become the immediate neighbours of E?
    a) G and F   b) Only F   c) Only A   d) J and H

Circular Arrangement

1. Four girls A, B, C and D are sitting around a circle facing the centre. B and C are in front of each other, which of the following is definitely true?
    a) A and D are in front of each other
    b) A is not between B and C
    c) D is to the left of C
    d) A is to the left of C

2. Five persons are sitting facing centre of a circle. Pramod is sitting to the right of Rajan. Raju is sitting between Brejesh and Naveen. Raju is to the left of Brejesh and Rajan is to right of Brejesh. Who is sitting to the left of Naveen?
    a) Pramod   b) Raju   c) Brejesh   d) Rajan

3. Six persons are sitting in a circle facing the centre of the circle. Parikh is between Babita and Narendra. Asha is between Chitra and Pankaj. Chitra is to the immediate left of Babita. Who is to the immediate right of Babita?
    a) Parikh   b) Pankaj   c. Narendra   d. Chitra

Directions (Q. no.4 to 6)

Read the following information carefully to answer the question that follow:

Six girls are sitting in a circle.

Sonia is sitting opposite to Radhika.

Poonam is sitting right of Radhika but left of Deepti.

Monika is sitting left of Radhika

Kamini is sitting right of Sonia and left of Monika

Now, Deepti and Kamini, Monika and Radhika mutually exchange their positions.

4. Who will be opposite to Sonia?

a) Radhika b) Monika c) Kamini d) Sonia

5. Who will be sitting left of Kamini?

a) Poonam b) Deepti c) Radhika d) Sonia

6. Who will be sitting left of Deepti?

a) Sonia b) Monika c) Radhika d)Poonam

Direction (Q.no. 7 to 10)

Eight friends A, B, C, D, E, F G and H are sitting around a circle facing the centre but not necessarily in the same order. G sits third to left of D. Only one person sits between D and F. B sits second to right of H. H is not an immediate neighbour of D. C is not an immediate neighbour of D. E is an immediate neighbour of H.

7. What is the position of E with respect to the position of C?

a) Third to the left b) Second to the left

c) Immediate right d) Third to the right

8. Who amongst the following sits exactly between A and G?

a) B b) C c) E d) F

9. Three of the following four are alike in a certain way and thus form a group. Which is the one that does not belong to that group?

a) CG b) AE c) HD d) EC

10. Who amongst the following sits third to the left of F?

a) A b) B c) C d) G

Polygonal Arrangement

1. Four boys and four girls are sitting around a square facing the centre. One person is sitting at each corner and at the midpoint of each side of the square. Madhu is sitting diagonally opposite to Usha who is to the right of Geeta. Ram who is to the left of Geeta is diagonally opposite to Gopi who is to the left of Bose. Position of Suma is not to the right off Madhu but in front of Prema. Who is sitting opposite to Bose?

a) Geeta b)Prema c) Suma d) Madhu

2. Five children A, B, C D and E are sitting along the corners of a pentagonal table facing the centre. B is between E and C. D is to the right of E. Who is to the left of C?

a) B b) A c) D d) C

Directions (Q. no.3 to 6)

Read the following information carefully to answer the question that follow:
Six people A, B, C, D, E and F are sitting on the ground in a hexagonal shape. All the sides of hexagon so formed are of same length. A is not adjacent to B or C. D is not adjacent to C or E. B and C are adjacent. F in the middle of D and C.

3. Which of the following is not a correct neighbour pair?

a) A and F b) D and F c) B and E d) C and F

4. Who is at the same distance from D as E is from D?

a) B b) C c) D d) F

5. Which of the following group has the correct order of arrangement?

a) A,F, B b) F, A, E c) B, C, F d) D, A, B

6. If one neighbour of A is D, who is the other one?

a) B b) C c) E d) F

Directions (Q. no.7 to 10)

Read the following information carefully to answer the question that follow:

Eight friends P, Q, R, S, T, V, W and Y are sitting around a square table. Out of eight, four persons are sitting at the corners of the table and the other four are sitting at the mid-points of each side of the table. Persons at the corners are facing the centre while the persons at the mid-points of side are facing outside. S is third to the right of P. P is facing the centre, Y is not sitting beside P or S. T is third to the right of R. R is not sitting at the mid-point of any side of the table. R is also not beside Y. There is only one person between P and V. Q is not sitting beside V.

7. Which of the following is true regarding Y?
   a) T is not sitting beside Y
   b) Y is sitting at the midpoint of side
   c) R is second to the left of Y
   d) P and V are beside Y

8. Who is forth to the left of V?
   a) Y b) R c) T d) Q

9. What is the position of Q in respect of R?
   a) Immediate right
   b) Second to the left
   c) Third to the left
   d) Third to the right

10. Who is third to the right of W?
   a) R b) S c) Q d) Y

HOMEWORK SECTION

1. Five boys A, B, C, D and E are sitting in a row A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end? (U.P.B.Ed 2013)

   (a) D (b) A (c) E (d) B

2. There are five different houses, A to E, in a row. A is to the right of B and E is to the left of C and right of A, B is to the right of D. Which of the houses is in the middle? IB CA (IO) 2013)

   (a) A (b) B (c) C (d) D

3. Five friends P, Q, R, S and T are sitting in a row facing North. Here, S is between T and Q and Q is to the immediate left of R. P is to the immediate left of T. Who is in the middle? (SSC (Multi Task) 2014)

   (a) S (b) T (c) Q (d) R

4. Six children A, B, C, D, E and F are standing in a row. B is between F and D. E is between A and C. A does not stand next to F or D. C does not stand next to D. F is between which of the following pairs of children? (SSC (FCI) 2012)

   (a) B and E (b) B and C (c) B and D (d) B and A

5. There are eight books kept one over the other. Two books are on Organisation Behaviour, two books on TQM, three books on Industrial Relations and one book is on Economics. Counting from the top, the second, fifth and sixth books are on Industrial Relations. Two books on Industrial Relations are between two books on TQM. One book of Industrial Relations is between two books on Organizational Behaviour while the book above the book of Economics is a book of TQM. Which book is the last book from the top? (MAT 2011)

   (a) Economics (b) TQM
   (c) Industrial Relations (d) Organizational Behaviour

6. Five boys are standing in a row facing East. Pavan is left of Tavan, Vipin and Chavan to the left of Nakul. Chavan is between Tavan and Vipin. Vipin is fourth from the left, then how far is Tavan from the right? (CLAT 2014)

(a) First (b) Second (c) Third (d) Fourth

7. Six persons M, N, O, P, Q and R are sitting in two row with three persons in each row. Both the row are in front of each other. Q is not at the end of any row. P is second the left of R. O is the neighbour of Q and diagonally opposite to P. N is the neighbour of R. Who is in front of N? (UPSC (CSAT) 2011)

(a) R (b) Q (c) P (d) M

8. Six persons A, B, C, D, E and F are sitting in two row, three in each row. (MAT 2011)

(I) E is not at the end of any row

(II) D is second to the left of F

(III) C, the neighbor of E, is sitting diagonally opposite to D

(IV) B is the neighbor of F.

Which of the following are in one of the two rows?

(a) D, B and F (b) C, E and B (c) A, E and F (d) F, B, C

Direction (Q.No.9): Read the following information carefully and answer the question that follows. Five boys A1, A2, A3, A4 and A5 are sitting in a stair in the following way. (RRB (TC/CC) 2010)

I. A5 is above A1

II. A4 is under A2

III. A2 is under A1

IV. A4 is between A2 and A3.

9. Who is at the lowest position of the stair?

(a) A1 (b) A3 (c) A5 (d) A2

10. Five children are sitting in a row. S is sitting next to P but not T. K is sitting next to R, who is sitting on the extreme left and T is not sitting next to K. Who is/are adjacent to S? (NIFT (UG) 2014)

(a) K and P (b) R and P (c) Only P (d) P and T

11. Five senior citizens are living in a multi-storeyed building. Mr. Muan lives in a flat above Mr. Ashokan, Mr. Lokesh in a flat below Mr. Gaurav, Mr. Ashokan lives in a flat above Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who lives in the topmost flat? (MAT 2013).

    (a) Mr. Lokesh (b) Mr. Gaurav (c) Mr. Muan (d) Mr. Rakesh

12. In a gathering seven members are sitting in a row. 'C' is sitting left to 'B' but on the right to 'D'. 'A' is sitting right to 'B', 'F; is sitting right to 'E' but left to 'D'. 'H' is sitting left to 'E'. Find the person sitting in the middle (SSC (10+2) 2013)

    (a) C (b) D (c) E (d) F

Directions (Q. no. 13 to 17): Study the following information carefully to answer the given questions. A to H are seated in straight line facing North. C sits fourth left of G. D sits second to right of G. Only two people sit between D and A. B and F are immediate neighbours of each other. B is not an immediate neighbour of A. H is not neighbour of D. (GIC 2012)

13. Who amongst the following sits exactly in the middle of the persons who sit fifth from the left and the person who sit sixth from the right?

    (a) C (b) H (c) E (d) F

14. Who amongst the following sits third to the right of C?

    (a) B (b) F (c) A (d) E

15. Which of the following represents persons seated at the two extreme ends of the line?

    (a) C, D (b) A, B (c) B, G (d) D, H

16. What is the position of H with respect to F?

    (a) Third to the left (b) Immediate right
    (c) Second to right (d) Fourth to left

17. How many persons are seated between A and E?

    (a) One (b) Two (c) Three (d) Four Directions

(Q. No. 18-22) (MAT 2012)

Study the following information carefully to answer the given questions.

Ten students are A to J are sitting in a row facing west.

I. B and F are not sitting on either of the edges.

II. G is sitting left of D and H is sitting to the right of J.

III. There are four persons between E and A

IV. I is immediate north of B and F is immediate south of D.

V. J is in middle of A and D and G is in middle of E and F.

VI. There are two persons between H and C.

18. Who is sitting at the seventh place counting from left?

(a) H (b) C (c) J (d) Either H or C

19. Who among the following is deﬁnitely sitting at one of the ends?

(a) C (b) H

(c) E (d) Cannot be determined

20. Who are immediate neighbours of I?

(a) BC (b) BH

(c) AH (d) BC or BH

21. Who is sitting second left of D?

(a) G (b) F (c) E (d) J

22. If G and A interchange their positions, then who become the immediate neighbours of E?

(a) G and F (b) Only F (c) Only A (d) J and H

Directions (Q. no. 23 & 24) Read the following information carefully and then answer the questions that follow. A group of singers, facing the audience, are standing in line on the stage as follows.

I. D is to the right to C

II. F is standing beside G.

III. B is immediate left of F

IV. E is immediate left of A

V. C and B have one person between them

VI. A and D have one person between them

23. Who is on the second extreme right?

(a) D (b) F (c) G (d) E

24. If we start counting from the left, on which number is B?

(a) 1st (b) 2nd (c) 3rd (d) 5th

Directions (Q. No. 25- 27): Study the following information carefully to answer the given questions. Eight persons P to W are sitting in front of one another in two rows. Each row has four persons. P is between U and V and facing North. W is opposite to Q, who is to the immediate left of S. R is between T and S and W is to the immediate right of V. (UCO Bank 2011)

25. Who is sitting in front of R?

(a) U (b) Q (c) V (d) P

26. Who is to the immediate right of R?

(a) S (b) U (c) S or Q (d) None of these

27. In which of the following pairs, persons are sitting in front of each other?

(a) SV (b) RV (c) TV (d) UR

28. Four girls A, B, C, D are sitting around a circle facing the centre. B and C infront of each other, which of the following is definitely true? (MAT 2009)

(a) A and D infront of each other (b) A is not between B and C

(c) D is left of C (d) A is left of C

HOMEWORK SOLUTION

1. Arrangement ⟹ C E B A D
   Option C

2. Arrangement ⟹ D B A E C
   Option A

3. Arrangement ⟹ P T S Q R
   Option A

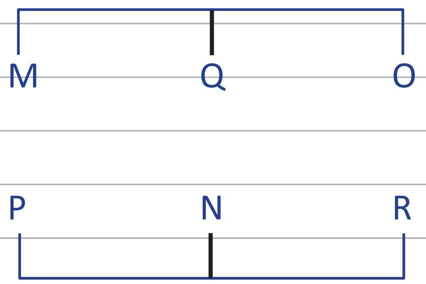
4. Arrangement ⟹ A E C F B D
   Option B

5. 1 ⟶ OB
   2 ⟶ IR
   3 ⟶ OB
   4 ⟶ TQM
   5 ⟶ IR
   6 ⟶ IR
   7 ⟶ TQM
   8 ⟶ ECO

   Option A

6. Arrangement ⟶ Pavan Tavan Chavan Vipul Nakul
   Option D

7.



Option B

8.

A E C

D B F

Option A

9.

Arrangement – A_5, A_1, A_2, A_4, A_3

Option B

10. Arrangement ⟹ R K P S T

Option D

11.

Arrangement – Mr Muan, Mr Ashokan, Mr Gaurav, Mr Lokesh, Mr Rakesh

Option C

12. Arrangement ⟹ H E F D C B A

Option B

13. to 17.

Arrangement ⟹ H C B F A G E D

13. Option D
14. Option C
15. Option D
16. Option A
17. Option A

18. to 22.

Arrangement ⟹ E G F D J A H B I C

18. Option D
19. Option C
20. Option D
21. Option A
22. Option C

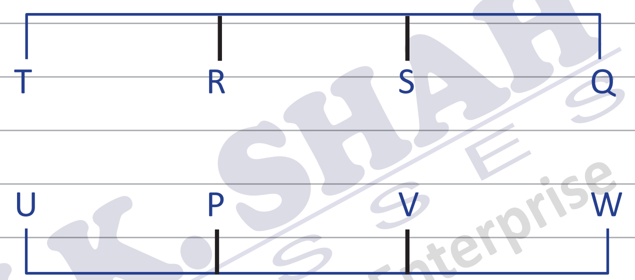
23 to 24

Arrangement ⟹ E A C D B F G

23. Option B
24. Option D

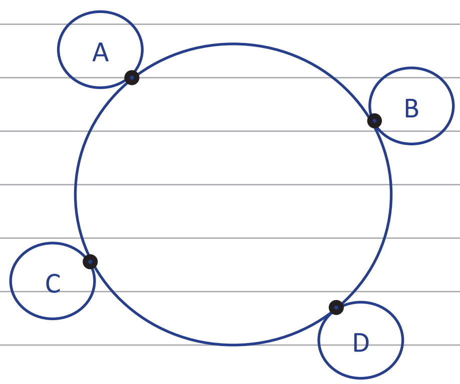
25 to 27

Arrangement



25. Option D
26. Option D
27. Option A

28.



Option A

SELF ASSESSMENT

1. Five senior citizens are living in a multi-storeyard building. Mr Muan lives a flat above Mr. Ashokan, Mr. Lokesh in a flat below Mr. Gaurav, Mr. Ashokan lives in a flat above Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who lives in the top most flat?

   a) Mr. Lokesh b) Mr. Gaurav c) Mr. Muan d) Mr. Rakesh

2. Five children are sitting in a row, S is sitting next to P but not T.K is sitting next to R, who is sitting on the extreme left and T is not sitting next to K. Who is/are adjacent to S.

   a) K+P b) R+P c) Only P d) P and T

3. Five students A, B, C, D and E are standing in a row. D is on the right of E, B is on the left of E but on the right of A. D is next to C on his left. The student in middle is

   a) B b) E c) C d) A

4. In a straight line there are six person sitting in a row? B is between F and D. E is between A and C. A does not stand next to F or D, C does not stand next to D. F is between which of the following person?

   a) B and E b) B and C c) B and D d) B and A

5. Five boys A, B, C, D, E are sitting in a row A is the right of B and E is to the left to B but to the right of C. A is to the left to D. Who is second from the left end?

   a) D b) A c) E d) B

6. Six Persons M N O P Q and R are sitting in two row with three persons in each row. Both the row are in front of each other. Q is not at the end of any row. P is second the left of R. O is the neighbor of Q and diagonally opposite to P. N is the neighbor of R. Who is in front of N?

   a) R b) Q c) P d) M

Direction to solve Q. No. 7 to 15

A, B, C, D and E are five men sitting in a line facing to South-while M, N, O, P and Q are the five ladies sitting in a second line parallel to the first line and are facing to North.

a) B who is just next to the right of D, is opposite to Q.

b) C and N are diagonally opposite to each other.

c) P who is just to the left of O, is opposite to D.

d) M is at one end of the line

e) E is opposite to O

7. Who is sitting between M and P

a) Q b) P c) E d) D

8. Which of the following pair is diagonally opposite to each other?

a) EQ b) BO c) AN d) AM

9. Who is the extreme left of E?

a) D b) A c) E d) O

10. In the original arrangement who is sitting just opposite to N?

a) B b) A c) C d) D

11. How many persons are there between N & P?

a) 1 b) 2 c) 3 d) 4

12. Who among the following sit next to each other ?

a) C and D b) N and P c) M and Q d) D and A

13. Who among the following sit on the immediate right of D?

a) G b) E c) F d) B

14. Who among the following sits third to the right of A?

a) C b) G c) B d) E

15. Which is true with regard to B?

a) B is second to the right of A
b) B is fourth to the left of N.
c) B sits between C and D
d) B sits at the extreme left

Instructions to solve (Q.16 to 20)

I) P, Q, R, S, T, U and V are sitting on a wall and all of them are facing West.
II) S is the immediate left of R.
III) T is at an extreme end and has Q as his neighbour
IV) V is between Q and U.
V) S is sitting third from the north end.

16. Who is the sitting to the left of S?
a) Q b) U c) T d) R e) P

17. Which of the following pairs of people are sitting at the extreme ends?
a) TQ b) PR c) TP d) ST e) VP

18. Name the person who should change places with R such that he gets the fourth place from the South end?
a) P b) S c) Q d) T e) U

19. Immediately between which of the following pairs of people S is sitting?
a) UR b) PQ c) VP d) TU e) RV

20. Which of the conditions given above are not required to find out the place in which P is sitting?
a) I b) II c) IV
d) III e) All required

ANSWER

1.	C	2.	D
3.	B	4.	B
5.	C	6.	B
7.	A	8.	D
9.	B	10.	B
11.	A	12.	C
13.	D	14.	C
15.	C	16.	B
17.	C	18.	E
19.	A	20	E

Explanatory Solution

Q.1

Arrangement $\begin{cases} \text{Mr. Muan} \\ \text{Mr. Ashokan} \\ \text{Mr. Gaurav} \\ \text{Mr. Lokesh} \end{cases}$

Mr. Rakesh — Option C

Q.2 Arrangement → R K P S T — Option D

Q.3 Arrangement → A B $\boxed{E}$ D C — Option B

Q.4 Arrangement → A E C $\boxed{F}$ B D — Option B

Q.5 Arrangement → C $\boxed{E}$ B A D — Option C

Q.6

↓ ↓ ↓
M $\boxed{Q}$ O

P N R
↑ ↑ ↑

Option B

Q.7 to Q.15

North

$\overline{C}$ $\overline{B}$ $\overline{D}$ $\overline{E}$ $\overline{A}$ ↓

$\underline{M}$ $\underline{Q}$ $\underline{P}$ $\underline{O}$ $\underline{N}$ ↑

South

Q.7 → Option A

Q.8 → Option D

Q.9 → Option B

Q.10 → Option B

Q.11 → Option A

Q.12 → Option C

Q.13 → Option D

Q.14 → Option C

Q.15 → Option C

Q.16 to Q.20

Arrangement

			West					
→ South	T	Q	V	U	S	R	P	North
			East					

Q.16 → Option B

Q.17 → Option C

Q.18 → Option E

Q.19 → Option A

Q.20 → Option E

4 BLOOD RELATION

Blood relation between two individuals is defined as a relation between them by the virtue of their birth rather than by their marriage or any other reasons.

Different Types of Blood Relation questions

1. Blood relation based on Conversation
2. Blood relation based on Puzzles
3. Symbolically Coded Blood Relationship

CLASS WORK SECTION

Type 1. Blood relation based on Conversation

1. Pointing towards a woman, Suresh said, "She is the daughter of my father's sister". How is the woman related to Suresh?

   a) Brother b) Cousin c) Uncle d) Sister

2. Vinod introduces Vishal as the son of the only brother of his father's wife. How is Vinod related to Vishal? (M-2018)

   a) Cousin b) Brother c) Son d) Uncle

3. Suresh introduces a man as "he is the son of the woman who is the mother of the husband of my mother". How is Suresh related to the man? (M-2018)

   a) Brother in law b) Son c) Brother d) Father.

4. Pointing to a picture, Summit said, she is the mother of my son's wife's daughter. How is lady related to the Summit. (J-2019)

   a) Uncle b) Cousin c) Daughter d) None

5. Mathew told his friend Sham, pointing to a photograph, "Her father is the only son of my mother." The photograph is of whom?

   a) Mathew's niece b) Mathew's mother
   c) Mathew's daughter d) Mathew's sister

6. Introducing Amrita, Raj said "Her mother is the only daughter of my Mother-in Law". How is Raj related to Amrita?

   a) Husband b) Father c) Wife d) Uncle

7. Pointing to a lady Rishi said, "The son of her brother is the brother of my wife". How is this lady related to Rishi?

   a) Mother-in Law b) Mother's sister
   c) Sister of Father-in Law d) None of the above.

8. Pointing towards a boy Veena said "He is son of only son of my grandfather". How is that boy related to Veena?

   a) Uncle b) Brother c) Cousin d) Data inadequate

9. A prisoner introduced a boy who came to visit him to the jailor as "Brothers and sisters I have none, he is my father's son's son". Who is the boy?

   a) Nephew b) Son c) Cousin d) Uncle

10. Pointing towards a girl, Anurag says, "This girl is the daughter of the only child of my father". What is the relation of Anurag's wife with the girl?

    a) Sister b) Aunt c) Daughter d) Mother

Type 2. Blood relation based on Puzzle

Directions (Q. no.1 to 4)

Read the following information carefully to answer the question that follow:

There are six children playing football, namely, P, Q, R, S, T and U.

P and T are brothers,

U is the sister of T.

R is the only son of P's Paternal uncle,

Q and S are the daughters of the only brother of R's father.

1. How is R related to U?

   a) Cousin b) Brother c) Son d) Uncle

2. How many male players are there?

   a) One b) Three c) Four d) Five

3. How many female players are there?

   a) One b) Two c) Three d) Four

4. How is S related to P?

   a) Uncle b) Sister c) Niece d) Cousin

5. Six persons are seen together in a group. They are A, B, C, D, E and F.
B is brother of D, but D is not brother of B,
F is brother of B.
C and A are married together.
F is son of C, but C is not mother of F.
E is brother of A.
The number of female member in the group is (N-2018)
a)1 b) 2 c) 3 d) 4

6. P and Q are brothers R and S are sisters, P's son is R's brother. How is Q related to R? (M-2018)
a) Uncle b) Brother c) Father d) Grandfather

7. P's father is Q's son. M is the paternal uncle of P and N is the brother of Q. How is M related to N?
a) Nephew b) Cousin
c) Data inadequate d) None of these

8. T, S and R are three brothers. T's son Q is married to K and they have one child Rahul blessed to them. M the son of S is married to H and this couple is blessed with a daughter Madhvi. R has daughter N who is married to P. This couple has one daughter Karuna born to them. How is Madhvi related to S?
a) Daughter b) Niece
c) Granddaughter d) None of these

9. Arti and Saurabh are the children of Mr and Mrs Shah. Ritu and Shakti are the children of Mr and Mrs Mehra. Saurabh and Ritu are married to each other and two daughter Mukti and Shruti are born to them. Shakti is married to Rina and two children Subhash and Reshma are born to them. How is Arti related to Shruti?
a) Mother b) Mother-in Law c) Sister d) Aunt

10. In a family, there are seven persons comprising two married couple. T is the only son of M and the grandson of K. M is a widower. M and R are brothers and W is the daughter in law of J, who is the mother of R and the grandmother of D. How is D related to M?
a) Son b) Son in law c) Nephew or Niece d) Brother

Type 3. Symbolically Coded Blood Relation

Directions (Q. no.1 & 2)

Read the following information carefully to answer the question that follow:

'P x Q' means 'P is sister of Q'.

'P + Q' means 'P is mother of Q'

'P − Q' means 'P is father of Q'

'P ÷ Q' means 'P is brother of Q'

1. Which of the following represents 'M is the nephew of R'?

a) M ÷ T − R   b) R ÷ T − M   c) R x T + M x J   d) R ÷ T − M ÷ J

2. Which of the following represents W is the grandfather of H?

a) W + T − H   b) W ÷ T − H   c) W x T + H   d) None of these

3. If P + Q means P is the mother of Q.

P ÷ Q means p is the father of Q.

P − Q means P is the sister of Q.

Then which of the following relationship shows that M is the daughter of R?

(N-2018)

a) R ÷ M + N   b) R + N ÷ M   c) R − M ÷ N   d) None of these

4. If 'A x D' means 'A is sister of D',

'A + D' means 'D is the daughter of A'

'A ÷ D' means 'A is the mother of D,

Then how will N is the aunt of M be denoted?

a) M + L x N   b) M ÷ L + N   c) L x N ÷ M   d) N x L ÷M

5. S x T means that S is the mother of T

S + T means that S is father of T

S − T means that S is the sister of T

On the basis of this information, you have to select the option which shows that A is the grandfather of T?

a) A + S + B − T   b) A x B + C − T

c) A + B − C x T   d) (a) & (c) both

Directions (Q. no.6 to 10)

Read the following information carefully to answer the question that follow:

'A + B' means 'A is the father of B'

'A x B' means 'A is the sister of B'

'A $ B' means 'A is the wife of B'

'A % B' means 'A is the mother of B'

'A ÷ B' means 'A is the son of B.'

6. What should come in place of the question mark, to establish that J is the brother of T in the expression?

   J ÷ P % H ? T % L

   a) x   b) ÷   c) $ d) Either + or x

7. Which among the given expression indicate that M is the daughter of D?

   a) L % R $ D + T x M

   b) L + R $ D + M x T

   c) L % R % D + T ÷ M

   d) L $ D ÷ R % M ÷ T

8. Which among the following options is true, if the expression 'I + T % J x L ÷ K' is definitely true?

   a) L is the daughter of T

   b) K is the son in law of I

   c) I is the grandmother of L

   d) J is the brother of L

9. Which among the following expressions is true, if Y is the son of X is definitely false?

   a) W % L x T x Y ÷ X

   b) W + L x T x Y ÷ X

   c) X + L x T x Y ÷ W

   d) W $ X + L + Y + T

10. What should come in the place of the question mark, to establish that T is the sister in law of Q in the expression

   R % T x P ? Q + V

   a) +   b) %   c) x   d) $

HOMEWORK SECTION

1. A is B's brother. C is A's mother. D is C's father, E is B's son. How is D related to A?
   (a) Son (b) Grandson
   (c) Grandfather (d) Great Grandfather

2. As is B's brother. C is A's father. D is C's sister and E is D's mother. How is B related to E?
   (a) Grand-daughter (b) Great grand daughter
   (c) Grandaunt (d) Daughter

3. A is B's Sister. C is B's Mother. D is C's Father. E is D's Mother. Then how is A related to D?
   (a) Grandmother (b) Grandfather
   (c) Daughter (d) Grand-daughter

4. A is the father of B. C is the daughter of B. D is the brother of B. E is the son of A. What is the relationship between C and E?
   (a) Brother and sister (b) Cousins
   (c) Niece and uncle (d) Uncle and aunt

5. If P is the husband of Q and R is the mother of S and Q. What is R to P?
   (a) Mother (b) Sister
   (c) Aunt (d) Mother-in Law

6. P and Q are brothers. R and S are sister. P's son is S's brother. How is Q related to R?
   (a) Uncle (b) Brother
   (c) Father (d) Grandfather

7. X is the husband of Y. W is the daughter of X. Z is husband of W. N is the daughter of Z. What is the relationship of N to Y?
   (a) Cousin (b) Niece
   (c) Daughter (d) Grand-daughter

8. A reads a book and find the name of the author familiar. The author 'B' is the paternal uncle of C. C is the daughter of A. How is B related to A?
   (a) Brother (b) Sister
   (c) Father (d) Uncle

9. A's mother D is sister of B and D has a daughter C who is 21 years old. How is B related to C?
   (a) Uncle (b) Maternal Uncle
   (c) Niece (d) Son

10. A is B's brother. C is A's mother. D is C's father. F is A's son. How is F related to D?
   (a) Son (b) Grandson
   (c) Grand-grandson (d) Grand-daughter

11. A is B's brother. C is A's mother. D is C's father. E is B's son. How is B related to D?
   (a) Son (b) Grand-daughter
   (c) Grandfather (d) Great grandfather

12. A is B's brother. C is A's mother. D is C's father. F is B's son. How is B related to F's child?
   (a) Aunt (b) Cousin
   (c) Nephew (d) Grandfather

13. A is B's daughter. B is C's mother. D is C's brother. How is D related to A?
   (a) Father (b) Grandfather
   (c) Brother (d) Son

14. A is D's brother. D is B's father. B and C are sisters. How is C related to A?
   (a) Cousin (b) Niece
   (c) Aunt (d) Nephew

15. A is B's brother. C is A's mother, D is C's father. E is B's son. How is D related to E ?
   (a) Grandson (b) Great Grandson
   (c) Great Grandfather (d) Grandfather

16. X and Y are the children of A. A is the father of X but Y is not his son. How is Y related to A?
    (a) Sister (b) Brother
    (c) Son (d) Daughter

17. A is B's brother. C is A's mother. D is C's father. E is B's son. How is E related to A?
    (a) Cousin (b) Nephew
    (c) Uncle (d) Grandson

18. Based on the statements given below, find out who is the uncle of P? (i) K is the bother of J (ii) M is the sister of K (iii) P is the brother of N (iv) N is the daughter of J
    (a) K (b) J (c) N (d) M

19. A and B are sisters. A is mother of D. B has a daughter C who is married to F. G is the husband of A. How is C related to D?
    (a) Cousin (b) Niece
    (c) Aunt (d) Sister-in-law

20. R and S are brothers. X is the sister of Y and X is mother of R. What is Y to S?
    (a) Uncle (b) Brother
    (c) Father (d) Mother

21. A is B's brother. C is A's mother. D is C's father. B is D's grand-daughter. How is B related to C.
    (a) Daughter (b) Cousin
    (c) Niece (d) Grandaunt

22. A is the son of B while B and C are sisters to one another. E is the mother of C. If D is the son of E, which of the following statements is correct?
    (a) D is the maternal uncle of A (b) E is the brother of B
    (c) D is the cousin of A (d) B and D are brothers

23. P is the father of T. T is the daughter of M. M is the daughter of K. What is P to K?
    (a) Father (b) Father-in Law
    (c) Brother (d) Son-in Law

24. A and B are brothers. E is the daughter of F. F is the wife of B. What is the relation of E to A?
    (a) Sister (b) Daughter
    (c) Niece (d) None of these

25. M and F are a married couple. A and B are sisters. A is the sister of F. Who is B to M?
    (a) Sister (b) Sister-in-law
    (c) Niece (d) Daughter

26. If A is the mother of D. B is not the son of C. C is the father of D, D is the sister of B, then how is A related to B?
    (a) Mother (b) Brother
    (c) Step son (d) Sister

27. A and B are brother and sister respectively. C is A's father. D is C's sister and E is D's mother. How is B related to E?
    (a) Grand-daughter (b) Great grand-daughter
    (c) Aunt (d) Daughter

28. Q is the son of P. X is the daughter of Q. R is the aunty (Bua) of X and L is the son of R, then what is L to P?
    (a) Grandson (b) Grand-daughter
    (c) Daughter (d) Nephew

29. P and Q are brothers. R and S are sisters. P's son is S's brother. How is Q related to R?
    (a) Uncle (b) Brother
    (c) Father (d) Grandfather

30. A and B are the young ones of C. If C is the mother of B but A is not the daughter of C, then what is the relationship between C and A?
    (a) Nephew and Aunty (b) Brother and Sister
    (c) Mother and Son (d) Niece and Aunty

31. A is the mother of D and sister of B. B has a daughter C who is married to F. G is the husband of A. How is G related to D?

(a) Uncle (b) Husband
(c) Son (d) Father

32. Pointing towards A, B said "your mother is the younger sister of my mother". How is A related to B?

(a) Uncle (b) Cousin
(c) Nephew (d) Father

33. A is B's wife's husband's brother. C and D are sisters of B. How is A related to C?

(a) Brother (b) Sister-in-law
(c) Wife (d) Sister

34. A and B are brothers. C and D are sisters. A's son is D's brother. How is B related to C?

(a) Father (b) Brother
(c) Uncle (d) Son

35. A is B's sister. C is B's mother. D is C's father. E is D's mother. Then how is A related to D?

(a) Grandmother (b) Grandfather
(c) Daughter (d) Grand-daughter

36. P, Q, R, S, T, U are 6 members of a family in which there are two married couples. T, a teacher is married to a doctor who is mother of R and U. Q the lawyer is married to P. P has one son and one grandson. Of the two married ladies one is a housewife. There is also one student and one male engineer in the family. Which of the following is true about the grand-daughter of the family?

(a) She is a lawyer (b) She is an engineer
(c) She is a student (d) She is a doctor

37. Six members of a family namely A, B, C, D, E and F are travelling together. 'B' is the son of C but C is not the mother of B. A and C are married couple. E is the brother of C. D is the daughter of A. F is the brother of B. How many male members are there in the family?

(a) 3 (b) 2 (c) 4 (d) 1

38. A's mother is sister of B and has a daughter C. How can A be related to B from among the following?

(a) Niece (b) Uncle
(c) Daughter (d) Father

39. Rajiv is the brother of Atul. Sonia is the sister of Sunil. Atul is the son of Sonia. How is Rajiv related to Sonia?

(a) Nephew (b) Son
(c) Brother (d) Father

40. Sita is the niece of Ashok. Ashok's mother is Lakshmi. Kalyani is Lakhshmi's mother. Kalyani's husband is Gopal. Parvathi is the Mother-in Law of Gopal. How is Sita related to Gopal?

(a) Great grandson's daughter (b) Gopal's Sita's father
(c) Sita is Gopal's great grand-daughter (d) Grand niece

41. Seema is the daughter-in-law of Sudhir and sister-in-law of Ramesh. Mohan is the son of Sudhir and only brother of Ramesh. Find the relation between Seema and Mohan.

(a) Sister-in-law (b) Aunt
(c) Cousin (d) Wife

42. Suresh introduces a man as, "He is the son of the woman who is the mother of the husband of my mother". How is Suresh related to the man?

(a) Uncle (b) Son
(c) Cousin (d) Grandson

43. Pointing to a lady in a photograph. Meera said, "Her father's son's wife is my Mother-in Law". How is Meera's husband related to that lady in the photo?

(a) Nephew
(b) Uncle
(c) Son
(d) Father

44. Pointing to a photograph Vikas said, "She is the daughter of my grandfather's only son". How is she related to Vikas in the photograph?

(a) Father
(b) Brother
(c) Sister
(d) Mother

45. Suresh's sister is the wife of Ram. Ram is Rani's brother. Ram's father is Madhur. Sheetal is Ram's grandmother. Rema is Sheetal is daughter-in-law. Rohit is Rani's brother's son. Who is Rohit to Suresh?

(a) Brother-in-law
(b) Son
(c) Brother
(d) Nephew

46. Vinod introduces Vishal as the son of the only brother of his father's wife. How is Vinod related to Vishal?

(a) Cousin
(b) Brother
(c) Son
(d) Uncle

47. Among her children, Ganga's favourites are Ram and Rekha. Rekha is the mother of Sharat, who is loved most by his maternal uncle Mithun. The head of the family is Ram Lal, who is succeeded by his sons Gopal and Mohan. Gopal and Ganga have been married for 35 years and have 3 children. What is the relation between Mithun and Mohan?

(a) Uncle
(b) Son
(c) Brother
(d) No relation

48. Rahul and Robin are brothers. Promod is Rohin's father. Sheela is Pramod's sister. Prema is Promod's niece. Shubha is Sheela's grand-daughter. How is Rahul related to Shubha?

(a) Brother
(b) Cousin
(c) Uncle
(d) Nephew

49. Preeti has a son, named Arun. Ram is Preeti's brother. Neeta too has a daughter named Reema. Neeta is Ram's sister. What is Arun's relationship to Reema?
    (a) Brother (b) Nephew
    (c) Cousin (d) Uncle

50. There are 2 film stars. One is the father of the other's son. What is the relationship of the two with each other?
    (a) Grandfather and Grandson (b) Grandfather and son
    (c) Husband and wife (d) Father and Son

51. Ramu's mother said to Ramu,"My mother has a son whose son is Achyut". How is Achyut relation to Ramu?
    (a) Uncle (b) Cousin
    (c) Brother (d) Nephew

52. Ravi's father has a son Rohit who has an aunt Laxmi who has a husband Rao whose Father-in Law is Mohan. What is the relation of Mohan to Ravi?
    (a) Nephew (b) Grandfather
    (c) Son (d) Uncle

53. Vijay says, "Ananda's mother is the only daughter of my mother". How is Ananda relation to Vijay?
    (a) Brother (b) Father
    (c) Nephew (d) Grandfather

54. Introducing a man, a woman said, "His wife is the only daughter of my mother." How is the woman related with the man?
    (a) Sister-in-law (b) Wife
    (c) Aunt (d) Mother-in Law

55. A prisoner introduced a boy who came to visit him to the jailor as "Brothers and sisters I have none, he is my father's son's son". Who is the boy?
    (a) Nephew (b) Son
    (c) Cousin (d) Uncle

HOMEWORK SOLUTION

1. D (Grand Father) – C (Mother) – A & B (Children) – E (Grand Son)
   D is Grand Father of A, Option C

2. E (Grand Mother) – C (Father) D (Father's Sister) – A (Male) & B (Children)
   B is grand child of E. No mention is there regarding the gender of B. So B can be Grand Son or Grand Daughter of E. In the four options given, only Grand Daughter is given and there is no mention of Grand Son. So we chose Grand Daughter. Option A

   Kindly Note: General answer of these type of missing link question is Can't be determined. If option D is open, i.e., None of the above or Can't be determined, than its advised that you chose Option D – None of the above or Can't be determined.

3. E (Great Grand Mother) –D (Grand Father) – C (Mother) – A (Female) & B (Children)
   A is Grand Daughter of D. Option D

4. A (Father) – E (Male), B & D(Male) (Children) – C (Grand Daughter, Daughter of B)
   C is daughter of B and E is brother of B. C is niece of E, E is uncle of C. Option C

5. R (Mother of Q & S) – P (Husband) + Q (Wife) – S (Brother/Sister of Q)
   R is Mother-in Law of P. Option D

6. P (Father) Q(Brother of P) – P (Son) R (Daughter) S (Daughter)
   Q is brother of R's father. Q is uncle of R. Option A

7. X (Husband) + Y (Wife) – W (Daughter) + Z (Daughter's husband) – N (Grand Daughter)
   N is Grand Daughter of Y.
   Option D

8. A (Father) + B (Uncle of C) – C (Daughter)
   B is uncle of C. B is male. B is brother of A.
   Option A

9. Correct Question is: A's mother is sister of B and she has a daughter C who is 21 years old. How is B related to C?

Solution:
A and C are siblings – B is maternal uncle/aunt of both A and C.
Again in options only maternal uncle is given and all other options are closed. We chose Ma-ternal Uncle. Option B

10. D (Grand Father) – C (Mother) – A (Male) & B (Children) – F (Grand Son, son of A)
F is Grand-grandson of D. Option C

11. D (Grand Father) – C (Mother) – A (Male) & B (Children) – E (Grand Son, son of B)
B is grand child of D. B is either male or female (no clue given). Again as per options given, B is grand-daughter of D. Option B

12. D (Grand Father) – C (Mother) – A (Male) & B (Children) – F (Grand Son, son of B)
B shall be grand-father / mother of F's child. Again in option only grand-father is there. We select Option D

13. B (Father / Mother) – A (Daughter), D (Male) & C (Children).
D is brother of A. Option C

14. A (Male) & D – B & C (Daughters of D).
A is uncle of C. C is niece of A. Option B

15. D (Grandfather) – C (Mother) – A (Male) & B (Children) – E (Son of B).
D is great grandfather of E. Option C

16. A – X & Y (Daughter) (Children)
Y is daughter of A.
Option D

17. D (Grandfather) – C (Mother) – A (Male) & B (Children) – E (Son of B)
A is uncle of B. B is Nephew of A.
Option B

18. M (Female), K (Male) & J – N (Daughter of J) & P (Son of J)

    K is uncle of P.

    Option A

19. A (Female, Mother), G (Husband of A), B (Female, sister of A) – D (Child of A, G) – C (Daughter of B) + F (Husband of C)

    C is cousin of D.

    Option A.

20. X (Mother), Y (Brother / Sister of X) – R + S (Brother) (Son of X)

    Y is maternal uncle / aunt of S.

    Option A

21. D (Grandfather) – C (Mother of A) – A (Son) + B (Daughter of C)

    B is daughter of C

    Option A

22. E (Mother) – D (Brother) + B & C (Sisters) – A (Son of B)

    D is maternal uncle of A

    Option A

23. K (Grandfather / mother, related to M) – P (Father) + M (Mother) – T (Daughter)

    P is Son-in Law of K

    Option D

24. A + B (Brothers), F (wife of B) – E (Daughter of B & F)

    A is uncle of E. E is niece of A.

    Option C

25. M & F (Couple), A & B (Sisters of F)

    If M is male, than B is sister-in-law of M

    If M is female, than B is sister of M's husband, i.e. F, again B is sister-in-law of M

    Option B

26. A (Mother) & C (Father) – D (Female) & B (Female) (Children)

    A is mother of B

Option A

27. E (Grandmother) – C (Father), D (Sister of C) – A (Male) + B (Female) (Children of C)
    B is grand-daughter of E
    Option A

28. P – Q (Son of P), R (Daughter of P) – X (Daughter of Q), L (Son of R)
    L is grandson of P
    Option A

29. P & Q (Brother) – R & S (Daughter of P)
    Q is uncle of R
    Option A

30. C (Mother) – A (Son), B (Children)
    C is mother of A. A is son of C.
    Option C

31. A (Mother, G is husband), B (Brother of A) – D (Child of A & G), C (Daughter of B, F is hus-band)
    G is father of D
    Option D

32. A's mother is the younger sister of B's mother.
    A and B are cousins
    Option B

33. A & B (Brothers), C & D (Sisters)
    A is brother of C
    Option A

34. A & B (Brothers) – C & D (Daughters of A)
    B is uncle of C
    Option C

35. E (Great grandmother) – D (Grandfather) – C (Mother) – A (sister of B, female)
A is grand-daughter of D
Option D

36. Q (Lawyer) + P (Housewife) → T (Teacher, Mother) + S (Doctor, Father) → R & U (Children) (one male Engineer, one female Student)
The grand-daughter of the family is a Student
Option C

37. A (Mother) + C (Father), E (Brother of C) – B (Son), D (Daughter), F (Son)
Male members are = C, E, B, F = 4
Option C

38. Mother, Sister/Brother (B) – C (Daughter), A (Children)
A is either Nephew or Niece of B.
Option A

39. Sonia (Mother), Sunil (Brother of Sonia) – Atul (Son), Rajiv (Son)
Rajiv is son of Sonia
Option B

40. Parvathi (Mother of Kalyani) – Kalyani (Grandmother) + Gopal (Grandfather) – Lakshmi (Mother) – Ashok (Son) – Sita (Niece of Ashok)
Sita is great grand-daughter of Gopal. Option C

41. Sudhir (Father) – Seema (Daughter-in-law), Ramesh (Son), Mohan (Husband of Seema)
Seema is wife of Mohan. Option D

42. A is the son of the woman who is the mother of the husband of Suresh's mother
= A is the son of the woman who is mother of Suresh's father
= A is the son of Suresh's grand-mother
= A is either father or uncle of Suresh
= Suresh is either son or nephew of A
Option B

43. Lady's father's only son's wife is Meera's Mother-in Law.
= Lady's brother's wife is Meera's Mother-in Law
= Lady's sister-in-law is Meera's Mother-in Law
Now, Meera's husband is son of lady's sister-in-law
Thus, Meera's husband is lady's nephew
Option A

44. A is the daughter of Vikas's grandfather's only son
= A is the daughter of Vikas's father
= A is the sister of Vikas. Option C

45. Sheetal (Grandmother of Ram) – Madhur (Father of Ram) + Reema (Mother of Ram) – Suresh, Ram (Husband) + Wife (Suresh's sister), Rani (Ram is Rani's brother) – Rohit (Ram's Son)
Rohit is Ram's Son = Rohit is Suresh's sister's son = Rohit is Nephew of Suresh
Option D

46. Vishal is the son of the only brother of Vinod's father's wife.
= Vishal is the son of only brother of Vinod's mother.
= Vishal is the son of Vinod's maternal uncle.
= Vishal is the cousin of Vinod
Option A

47. Ram Lal (Head of the family) – Gopal & Mohan (Sons), Ganga (Wife of Gopal) – Ram, Rekha (Female), Mithun (Male) (Children of Gopal & Ganga) – Sharat (Rekha is the mother)
Mithun is son of Gopal. Mohan is brother of Gopal.
Mithun is Nephew of Mohan or Mohan is uncle of Mithun
Option A

48. Pramod (Father), Sheela (Sister of Pramod) – Rahul & Robin (Sons of Pramod), Prema (Daugh-ter of Sheela) – Shubha (Daughter of Prema)
Shubha is daughter of Prema. Rahul is cousin of Prema. Rahul is uncle of Shubha.
Option C

49. Preeti (Mother), Ram (Preeti's brother), Neeta (Sister of Ram & Preeti) – Arun (Son of Preeti), Reema (Daughter of Neeta)

Arun and Reema are Cousins.

Option C

50. A is father of B's son, i.e. C. A is father of C. C is B's son too. B is the mother of C. A and B are Husband and Wife.

Option C

51. Ramu's Mother = X

X's mother has a son whose son is Achyut.

Ramu's grandmother has a son whose son is Achyut

Ramu's maternal uncle;s son is Achyut

Ramu and Achyut are cousins

Option B

52. Mohan (Grandfather) – Father, Laxmi (Sister of Father) + Rao (Husband of Lakshmi) – Ravi & Rohit (Son)

Mohan is grandfather of Ravi

Option B

53. Ananda's mother is the only daughter of Vijay's mother

= Ananda's mother is Vijay's sister

= Ananda is Nephew / Niece of Vijay (Vijay is maternal uncle/aunt of Ananda)

Option C

54. A's wife is the only daughter of Lady's mother

= A's wife is the Lady

Lady is the wife of A (the man)

Option B

55. Brothers and Sisters I have none, he is my father's son's son.

= He is MY (my father's son = myself) son

Option B

SELF ASSESSMENT

1. Pointing to a photograph, a man said "His mother husband's sister is my aunt". Then what is relation between the man and the person?

a) Son b) Uncle c) Nephew d) Brother

2. introducing a boy, a girl said, "He is the son of the daughter of the father of my uncle." How is the boy related to the girl?

a) Brother b) Nephew c) Uncle d) Son-in Law

3. Pointing to a photograph. Bajpai said, "He is the son of daughter of father of my brother" How Bajpai is related to the man in the photograph?

a) Nephew b) Brother c) Father d) Maternal uncle

4. Pointing a photograph, X said to his friend Y, "she is the only daughter of the father of my mother", How X is related to the person of photograph?

a) Daughter b) Son c) Nephew d) Cannot be decided

5. Pointing toward a man, a woman said 'His mother is the only daughter of my mother". How is the woman related to the man?

a) Mother b) Grandmother c) Sister d) Daughter

6. Pointing to a photograph. Anjali said, "He is the son of the only son of my grandfather". How is the man in photograph related to Anjali?

a) Brother b) Uncle c) Son (d) Data is inadequate

7. A's son B is married with C whose sister D is married to E the brother of B. How D is related to A?

a) Sister b) Daughter in law
c) Sister in law d) Cousin

8. If M x N means M is the daughter of N, M + N means M is the father of N. M % N means M is the mother of N and M - N means M is the brother of N then P % Q + R - T x K indicates which relation of P to K ?

a) Daughter-in-law   b) Sister-in-law
c) Aunty   d) None of these

Directions (Q. no. 9 to 11): R is the father of P, who is a son- in- law of M and S is the mother of G.S is a sister of K, who is a brother-in-law of P and H is the daughter of T, who is a grandmother of G.

9. How is G related to P?

a) Son   b) Daughter
c) grandson   d) Cannot be determined

10. It M is a female, then how is H related to S?

a) Sister   b) Sister in law
c) Niece   d) Cannot be determined

11. If K married to N, then how is N related to M?

a) Son-in Law   b) Daughter-in-law
c) Mother-in Law   d) Father-in Law

12. Pointing a boy. Ram said ‘He is the only is son of my grandfather’s son’ How is the boy related to Ram?

a) Brother   b) Son   c) Cousin   d) Data inadequate

Q 13 to 15Directions:

P x Q means P is brother of Q.
P ÷ Q means Q is mother of P.
P - Q means P is father of Q.
P + Q means Q is sister of P.

13. Which of the following means M is the daughter of T?

a) M + N ÷ J - T   b) T-JxR + M   c) M - J x T ÷ K   d) M + W x R ÷ T

14. How is K related to R in the expression R ÷ T + K?

a) Daughter b) Sister c) Niece d) None of these

15. Which of the following means D us grandfather of W?

a) D − K x T − W
b) D ÷ K x T ÷ W
c) D − K x T ÷ W
d) D ÷ K x T − W

16. If P + Q means P is the mother of Q.

P ÷ Q means P is the father of Q.

P - Q means P is the sister of Q.

Then which of the following relationship shows that M is the daughter of R?

a) R ÷ M + N b) R + N ÷ M c) R − M ÷ N d) None of the above

17. Pointing in a photograph, Sonia said, "His mother's daughter's is my mother".How is Sonia related to that man?

a) Nephew b) Sister c) Wife d) Niece

18. If A $ B means A is the brother of B. A @ B means A is the wife of B, A # B means A is the daughter of B and A * B means A is the father of B. Which of the following indicates that U is the Father-in Law of P.

a) P @ Q $ T # U * W
b) P @ W $ Q * T # U
c) P @ Q $ W * T # U
d) P @ Q $ T # W * U

19. P is the sister of Q. R is the father of S, who is the brother of Q. R married to T, How is Q related to T?

a) Son
b) Daughter
c) Either son or daughter
d) Data in adequate

20. Pointing a girl, Prasan said, she is the only granddaughter of my wife's grandfather's only child. How is the girl relate to Prasan?

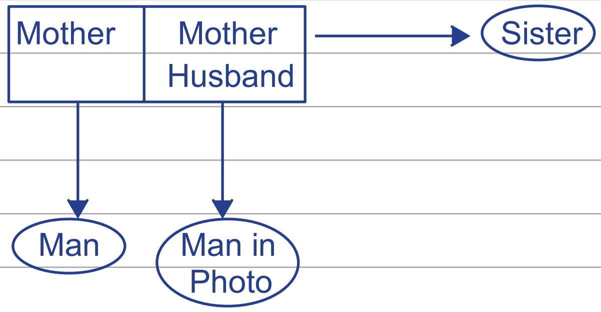
a) Sister
b) Niece
c) Daughter
d) Cannot be determined

ANSWER

1.	D	2.	A
3.	D	4.	B
5.	A	6.	A
7.	B	8.	D
9.	D	10.	B
11.	B	12.	C
13.	B	14.	D
15.	A	16.	A
17.	D	18.	A
19.	C	20	C

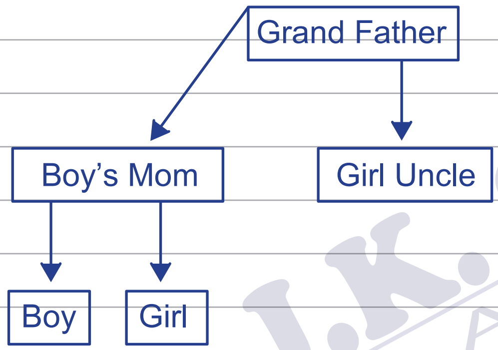
Explanatory Solution

Q.1



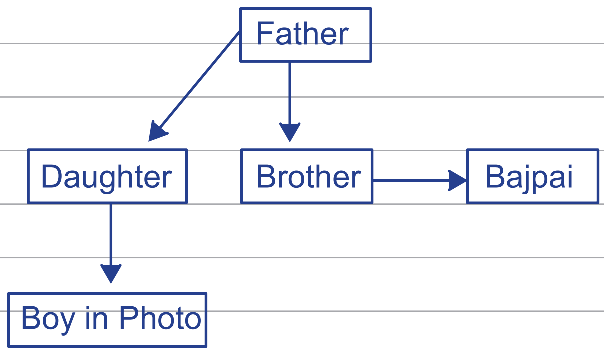
Option D

Q.2



Option A

Q.3



Option D

Q.4

Girl Father
↓
Girl in Photo (=) (x is mom)
↓
x

Option D

Q.5

Woman's | Mom
↓
Man mom (woman)
↓
Man

Option A

Q.6

Grand Father
↓
Boy's Father
↓ ↓
Boy in Photo | Anjali

Option A

Q.7

A
↓
Boy B | C Girl
Boy E | D Girl

Option B

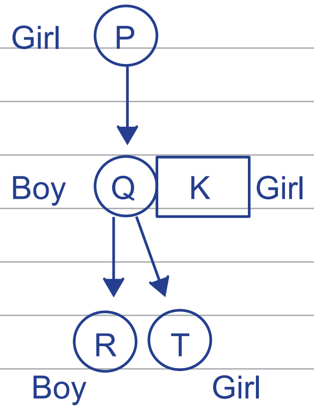
Q.8 M x N → M is daughter of N

M + N → M is father of N

M % N → M is mother of N

M − N → M is brother of N

P % Q + R − T x K

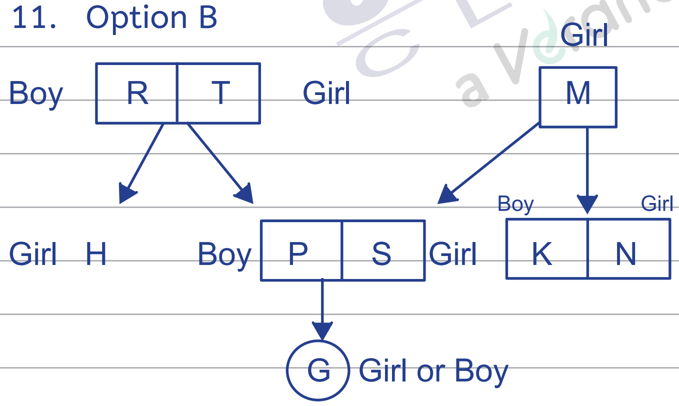


P is Mother-in Law of K

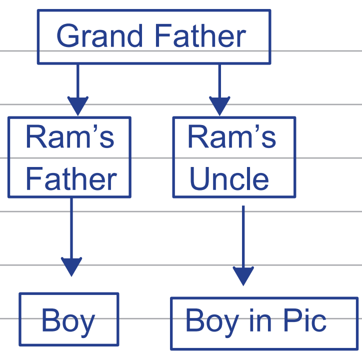
Option D

Q.9 to Q.11

9. Option D
10. Option B
11. Option B



Q.12



Option C

Q.13 to Q.15

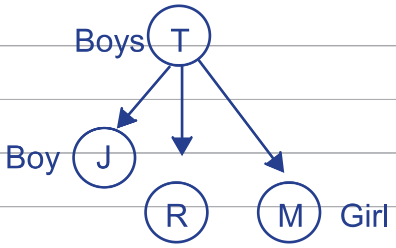
P x Q → P is brother of Q

P ÷ Q → Q is mother of P

P − Q → P is father of Q

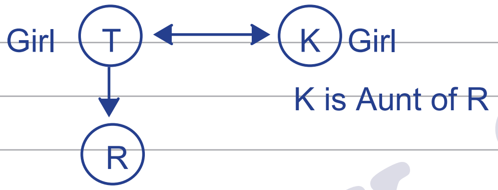
P + Q → Q is sister of P

Q.13



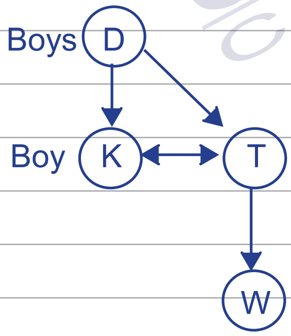
Option B

Q.14



Option D

Q.15



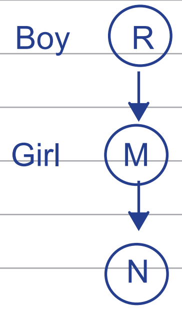
Option A

Q.16

P + Q → P is mother of Q

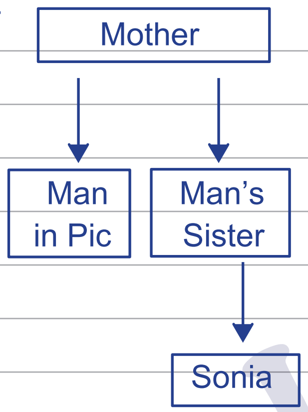
P ÷ Q → P is father of Q

P − Q → P is sister of P



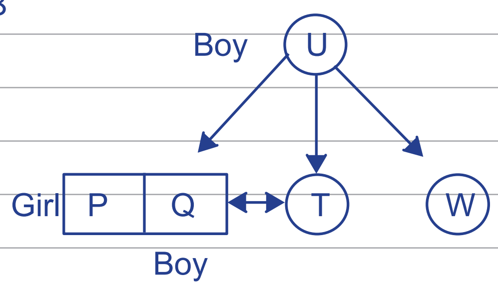
Option A

Q.17



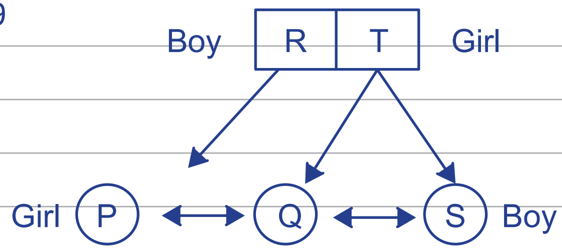
Option D

Q.18



Option A

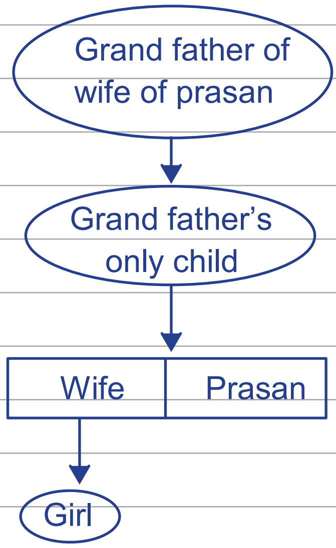
Q.19



Q is either Son or Daughter of T

Option C

Q.20



Option C

Daughter

APPENDIX

Table I - LOGARITHM

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	32	36	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0964	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1208	1239	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523								3	6	9	12	15	19	22	25	28
				1553	1584	1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818								3	6	9	11	14	17	20	23	26
				1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	6	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	3	5	8	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	17	19	21
						2672	2695	2718	2742	2765	2	4	7	9	11	14	16	19	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3023	3054	3075	3096	3116	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3909	3927	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	11	13	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	12	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	10	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	11	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12

32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	631	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6235	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6575	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6037	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

Example:

Log 2 = 0.3010: Log 20 = 1.3010: Log 200 = 2.3010: Log 2,000 = 3.3010 etc.

Log 2 = 0.3010 - 1 – (–) 0.699

Log 0.02 = 0.3010 - 2 – (–) 1.699

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
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51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7166	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7306	7314	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7358	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7452	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7848	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7958	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
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66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
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70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
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74	8692	8698	8704	8710	8716	8722	8727	8733	8738	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	2	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	2	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	2	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	2	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	2	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9450	9455	9460	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4

89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9945	9952	0	1	1	2	2	3	3	4	4
99	9958	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Table II - ANTILOGARITHM

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
100	1000	1002	1005	1007	1009	1012	1014	1016	1018	1021	0	0	1	1	1	1	2	2	2
101	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
102	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
103	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
104	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
105	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
106	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
107	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
108	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
109	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
110	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
111	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
112	1381	1321	1324	1327	1330	1334	1337	1340	1342	1348	0	1	1	1	2	2	2	2	3
113	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
114	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
115	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
116	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
117	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
118	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
119	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
120	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
121	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
122	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
123	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
124	1738	1742	1746	1750	1754	1758	1762	1768	1770	1774	0	1	1	2	2	2	3	3	4
125	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
126	1820	1824	1828	1832	1837	1841	1845	1849	1897	1858	0	1	1	2	2	3	3	3	4
127	1862	1866	1871	1875	1879	1884	1888	1892	1941	1901	0	1	1	2	2	3	3	3	4
128	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
129	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
130	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
131	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
132	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
133	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
134	2188	2193	2198	2203	2206	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
135	2239	2244	2249	2254	2259	2265	2270	2275	2280	2256	1	1	2	2	3	3	4	4	5

136	2291	2286	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
137	2344	2350	2355	2359	2366	2271	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
138	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
139	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
140	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
141	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
142	2630	2636	2642	2649	2655	2661	2667	2673	2679	2624	1	1	2	2	3	4	4	5	6
143	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
144	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
145	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
146	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
147	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
148	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
149	3090	3097	3105	3112	3118	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
150	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
151	3236	3243	3251	3258	3268	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
152	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
153	3388	3396	3404	3412	3420	3428	3436	3442	3451	3459	1	2	2	3	4	5	6	6	7
154	3467	3475	3483	3491	3499	3508	3516	3524	3532	2540	1	2	2	3	4	5	6	6	7
155	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	6	7
156	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
157	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
158	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
159	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	4	5	6	7	8
160	3981	3990	3999	4009	4018	4027	4036	4046	4055	4065	1	2	3	4	5	6	6	7	8
161	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
162	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
163	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
164	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
165	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
166	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
167	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
168	4788	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
169	4898	4909	4920	4932	4943	4955	4986	4977	4989	5000	1	2	3	5	6	7	8	9	10
170	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
171	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
172	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
173	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
174	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
175	5632	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
176	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
177	5858	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
178	6028	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
179	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
180	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
181	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
182	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
183	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
184	6918	6934	6950	6965	6982	6598	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
185	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
186	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
187	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
188	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16

189	7762	7780	7796	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
190	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
191	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
192	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
193	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
194	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
195	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
196	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
197	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
198	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
199	9772	9795	9817	9840	9836	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

Example:

If Log x = 0.301. then x = Antilog 0.301 = 2

If Log x = 1.301. then x = (Antilog 0.301) × 10 = 20

If Log x = 2.301. then x = (Antilog 0.301) × 100 = 200

If Log x = (–) 0.699, then we can write Log x = (– 1 + 0.301) : Thus x = Antilog (0.301) / 10 = 0.2

If Log x = (–) 1.699, then we can write Log x = (– 2 + 0.301) : Thus x = Antilog (0.301) / 100 = 0.02


Good Luck!