# Central tendency

**Central tendency:** A centeral value (Number) which represent all the observation of a series (Population or sample)

*** Characteristic of An Ideal Central tendency**

→ Easy Calculation
→ Easy to understand
→ Should be based on all observation
→ It should be properly defined
→ Mathematical properties
→ It should be least affected by extreme values

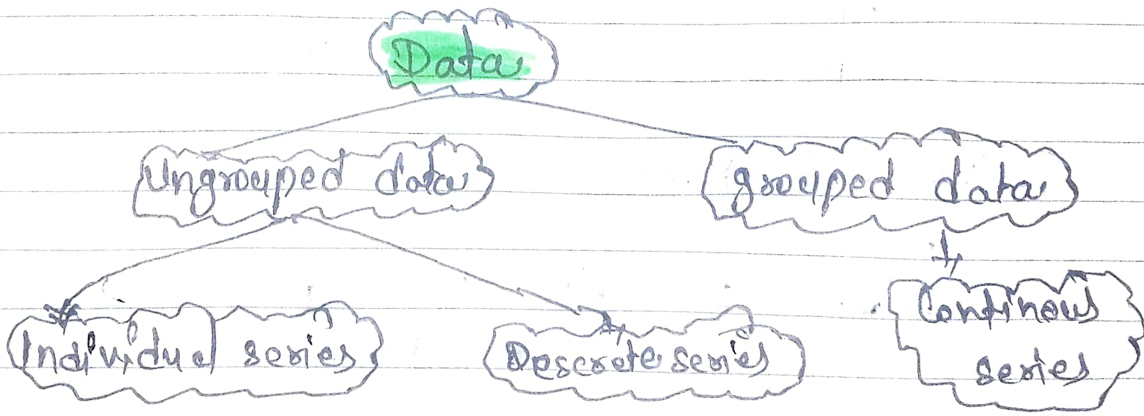
*** Arithmetic Mean**

↳ ise mean (Average) bhi kehate hai'

→ Denoted by $\bar{x}$ or $\mu$
for sample ↳ for population

$$\boxed{AM(\bar{x}) = \frac{\text{Sum of all observation}}{\text{Total no of observation}}}$$

$$\boxed{\bar{X} = \frac{\Sigma x_i}{N}}$$



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| Un grouped → 2 type | | | Grouped data → 1 type | |
|---|---|---|---|---|
| Individual Series | Descrete series | | Continous Series | |
| $x_i$ | $x_i$ | $f_i$ | Class interval | $f_i$ |
| x | xx | xx | 0 - 2 | 6 |
| x | xx | xx | 2 - 4 | 8 |
| x | xx | xx | 4 - 6 | 10 |
| x | xx | xx | 6 - 8 | 12 |
| | | | 8 - 10 | 14 |

Arithmetics Mean ($\bar{x}$) → method

Direct method

$$\bar{x} = \frac{\Sigma f_i x_i}{N}$$

OR

$$\bar{x} = \frac{\Sigma x_i}{N}$$

Short cut method

Assume mean

$$D_i = x_i - A$$

$$\bar{x} = A + \frac{\Sigma f_i d_i}{n}$$

Step deviation

$$u_i = \frac{x_i - A}{h}$$

$$\bar{x} = A + \left[\frac{\Sigma f_i u_i}{n}\right] \times h$$

Properties of Combined Mean

Group A

$n_1$

$\bar{x}_1$

Group B

$n_2$

$\bar{x}_2$

Combined Mean

$$(\bar{x}_{12}) = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

## Weighted Arithmetic mean

$$WM = \frac{\Sigma w_i x_i}{\Sigma w_i}$$

## Change of origin

When a fix number k is added or subtracted from each observation.

→ If $\bar{x}$ is the mean of same observation k is added to each observation then new mean $\bar{x} + k$

(ex) $\bar{x} = 15$

$y_i = x_i + 25$

$y_i = 15 + 25$

$= 40$

## Change of scale

When each item is multiplied or divided by same scale 'k'

# If mean of some observation is $\bar{x}$ & each element is multiplied by k then new mean $= k\bar{x}$

(ex) $\bar{x} = 10$

$y_i = 6 x_i$

$y_i = 6 \times 10$

$= 60$

## Properties of AM

If each element is constant k then
$AM = k$

$\Sigma(x_i - \bar{x}) = 0$
$\Sigma(\bar{x} - x_i) = 0$

# Sum of Deviation from arithmetic mean is alway zero $\Sigma(x_i - \bar{x}) = 0$

Sum the square of Deviation is minimum only when Deviation are taken from Arithmetic mean
i.e $\Sigma(x_i - A)^2$ is minimum when
$A$ = Arithmetic mean ~~when~~ $(\bar{x})$

## Median

→ A positional average
→ It represent 50%.
→ A number which divide entire series in two equal parts

# Individual series / Descrete series

Total no of observation $(n)$

Odd
median = $\left(\frac{n+1}{2}\right)^{th}$ term

Even
median = $\frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$

# Medion in continous series

find $n/2$
Locate $n/2$ in cf
select median class

cf = cumulative frequency preceding to median class
L = Lower limit of median class
f = frequency median class
h = class lenth

$$\boxed{Med = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h}$$

# Sum of absolute observation is minimum when deviation are taken from median
$\Sigma |x_i - m|$ is minimum
or
$\Sigma |x_i - A|$ is minimum when A = median

## Quartiles

$Q_1, Q_2, Q_3$

* Individual series, Discrete series

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ term

$Q_3 = \left(3\left(\frac{n+1}{4}\right)\right)^{th}$ term

* Continous series

Locate $n/4$ & $3n/4$ in cf & select $Q_1$ class & $Q_3$

$$Q_1 = l + \left(\frac{n/4 - cf}{f}\right) \times h$$

$$Q_3 = l + \left(\frac{3n/4 - cf}{f}\right) \times h$$

## Decile

Divide entire series in 10 parts

Total Deciles = 9

[$D_1, D_2, D_3$ ...... $D_9$]

| $D_1$ | $D_2$ | $D_3$ | ... | $D_9$ |
|---|---|---|---|---|
| ↓ | ↓ | ↓ | | ↓ |
| 10% | 20% | 30% | | 90% |

* Individual series / Descrete

$$D_1 = \left(\frac{n+1}{10}\right)^{th}$$

$$D_8 = 8\left(\frac{n+1}{10}\right)^{th}$$

$$D_3 = 3\left(\frac{n+1}{10}\right)^{th}$$

* Continous series

for $D_1$

Locate $n/10$ in cf & select $D_1$ class

$$D_1 = l + \left(\frac{n/10 - cf}{f}\right) \times h$$

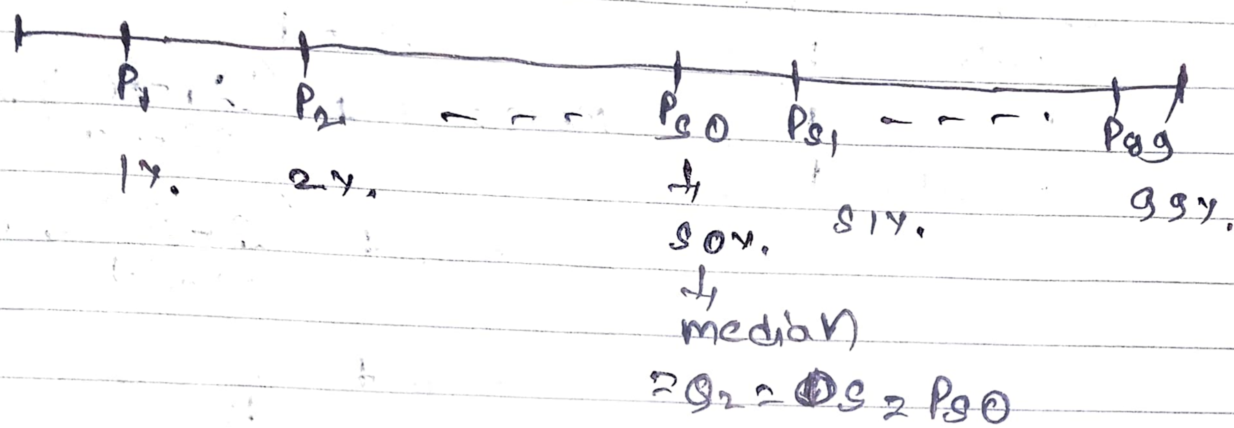
for $D_6$

Locate $6n/10$ in cf & select $D_6$ class

$$D_6 = l + \left(\frac{6n/10 - cf}{f}\right) \times h$$

# Percentile

Divide entire series in 100 parts
total percentile = 99
$(P_1, P_2, P_3 \ldots\ldots P_{99})$




## Individual series / Individual series

$P_1 = (n+1/100)^{th}$ $\qquad P_3 = 3(n+1/100)^{th}$

## Continuous series

for P1

→ Locate n/100 in cf

→ $P_1 = l + \left(\frac{n/100 - cf}{f}\right) \times h$

for $P_{26}$

→ Locate 26n/100 in cf

$P_{26} = l + \left(\frac{26n/100 - cf}{f}\right) \times h$

# Mode

Observation with highest frequency

$$\boxed{\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h}$$

Ex

| Marks | $f_1$ |
|---|---|
| 0-2 | 4 |
| 2-4 | 3 |
| 4-6 | 3 $f_0$ |
| $\boxed{6-8}$ | $\boxed{8}$ $f_1$ |
| 8-10 | 1 $f_2$ |
| 10-12 | 1 |
| 12-14 | 1 |

→ Higher number choice

$h \to 2$

$\underbrace{6 - 8}_{h}$

$l$

$h = 2$

$l = 6$

given that

$l = 6 \quad h = 2 \quad f_0 = 3 \quad f_1 = 8$

$f_2 = 1$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 6 + \left(\frac{8-3}{2\times 8 - 3 - 1}\right) \times 2$$

$$= 6 + \frac{5}{\cancel{12}_{6}} \times \cancel{2}$$

$$= 6 + 0.833$$

$$= 6.83 \text{ Ans}$$

## Relation b/w Mean, Median & Mode

# $\boxed{3 \text{ Median} = \text{Mode} + 2 \text{ Mean}}$

Ex Mean = 20, Mode = 15 Median = ?

$3 \text{ median} = 15 + 2(20)$

$= 15 + 40$

$3 \text{ Median} = 55$

$\text{Med} = \frac{55}{3} = 18.83$

# Difference b/w Mode & Mean

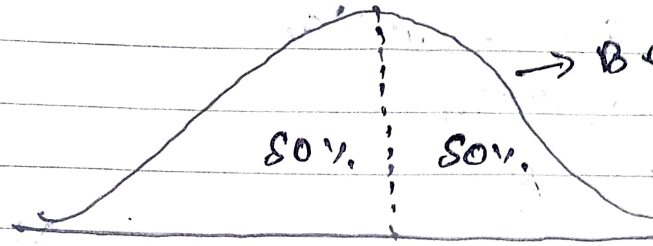
= 3 (Difference b/w median & mean)

# Mode − Mean = 3 [Median − Mean]

# Mean − Mode = 3 (Mean − median).

# Symethical distribution

→ Mean = medion = mode




→ Bell shape curve

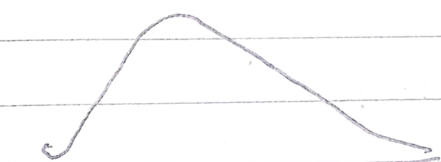
mean = mode = Median

# Non symethical distribution

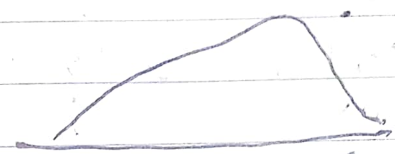
→ Mean ≠ median ≠ mode

Mean < Median < Mode

mean > Median > mode



unequal



unequal

## Gemetric Mean

$n^{th}$ Root of the product of 'n' observation

$GM = (x_1 \times x_2 \times x_3 \times \ldots\ldots \times x_n)^{1/n}$

$(x^{1/n})$ → Calculator trick

→ Calculator trick Ratio & Proportion mai Likha hai

→ $\sqrt{\ }$ 12 time, −1, ÷n, ×=, ×= ...... 12 time

# If all observation are same (Let say k) then GM = k

ex = GM of 2, 2, 2, 2

$GM = (2\times2\times2\times2)^{1/4}$

$= (2)^{4\times 1/4}$

$= 2$ Ans

# $\log G = \log(x_1 \times x_2 \times x_3 - - - \times x_n)^{1/n}$

$= \frac{1}{n}\log(x_1 \times x_2 \times - - - \times x_n)$ $\boxed{\log x^n = n\log x}$

$\log G = \dfrac{\log x_1 + \log x_2 + \log x_3 - - + \log x_n}{n}$ $\boxed{\log(xy) = \log x + \log y}$

# $\boxed{\log G = \dfrac{\sum^{n} \log x_i}{n}}$ Properties

# GM of $(xy)$ = GM of $x$ × GM of $y$

# GM of $x/y$ = $\dfrac{\text{GM of } x}{\text{GM of } y}$

# Combind GM = $(G_1^{n_1} \times G_2^{n_2})^{1/n_1+n_2}$

# $\sqrt{a}$ => ~~domined~~ defined

# $\sqrt{-a}$ = not defined

# Harmonic mean

Donte hote hai (HM)

→ 'HM' is the Reciprocal of the average of Reciprocal of 'n' item

$$HM = \left(\frac{\frac{1}{x_1} + \frac{1}{x_2} + \cdots \frac{1}{x_n}}{n}\right)$$

$$\boxed{HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots \frac{1}{x_n}}}$$

Individual series.

example: HM of 2, 3 & 5

$$HM = \frac{3}{\frac{1}{2} + \frac{1}{3} + \frac{1}{5}} = \frac{3}{0.5 + 0.33 + 0.20}$$

$$\Rightarrow \frac{3}{1.0333} = 2.90 \text{ Ans}$$

## Discrete Series

$$\boxed{HM = \frac{n}{\Sigma(f_i/x_i)}}$$

Ex.

| $x_i$ | $f_i$ | $f_i/x_i$ |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 5 | 2.50 |
| 3 | 6 | 2 |
| 4 | 2 | 0.50 |
| | $\Sigma f = 15$ | $\Sigma = 7$ |

$$HM = \frac{n}{\Sigma(f_i/x_i)}$$

$$= \frac{15}{7}$$

$$= 2.14 \text{ Ans}$$

# If all observation are same (Let say k)
then HM = K

(ex) HM of : 2, 2, 2, 2

$$= \frac{4}{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}}$$

$$= \frac{4}{2} = 2$$

# Combined HM = $\frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$

# If all item are same, AM = HM = GM
ex = 2, 2, 2, 2
AM = 2
HM = 2
GM = 2

# If all observation are different
ex : 2 8

$AM = \frac{2+8}{2} = 5$

$GM = \sqrt{2 \times 8} = 4$

$HM = \frac{2}{\frac{1}{2}+\frac{1}{8}} = \frac{2}{0.625} = 3.2$

AM > GM > HM

# for only two number a & b
$AM \times HM = GM^2$

# weighted AM $= \frac{\Sigma w_i x_i}{\Sigma w_i}$

# weighted HM $= \frac{\Sigma w_i}{\Sigma\left(\frac{w_i}{x_i}\right)}$

# weighted GM $= A.L\left[\frac{\Sigma w_i (\log x_i)}{\Sigma w_i}\right]$

# $1+2+3+ \dots + n = \frac{n(n+1)}{2}$

# $1^2+2^2+3^2+ \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

# AM of first n natural no. $= \frac{n+1}{2}$

example:

AM of first 5 natural no.

soln: $\Rightarrow \frac{5+1}{2} = \frac{6}{2} = 3$ Ans

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