Chapter 3

SEQUENCE & SERIES

CA VINOD REDDY

1. **Terms a, b, c, d, e, f, g are said to be in**

AP, If

$b-a = c-b = d-c = e-d = f-e = g-f$

$= (\text{succ. term} - \text{prec. term})$

GP, If

$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \frac{e}{d} = \frac{f}{e} = \frac{g}{f}$

$= \text{succ. term} / \text{preceding term}$

HP, If

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}, \frac{1}{e}, \frac{1}{f}, \frac{1}{g}$ are in AP

2.

Progression	AP/GP/HP/None of these
8, 16, 32, 64, 128	G.P. as $\frac{16}{8} = \frac{32}{16} = \frac{64}{32} = \frac{128}{64} = 2 = r$
80, 70, 60, 50, 40	A.P. as $70-80 = 60-70 = 50-60 = 40-50 = -10 = d$
2, 8, 32, 128	G.P. as $\frac{8}{2} = \frac{32}{8} = \frac{128}{32} = 4 = r =$ common ratio
0.50, 0.25, 0.1666666, 0.125	$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$ are in H.P. as 2, 4, 6, 8 are in A.P.
$\frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{18}$	None of these
100, 97, 94, 91	A.P. as $97-100 = 94-97 = 91-94 = -3 =$ common Diff $= d$
4, 6, 9, 13.50	G.P. as $\frac{6}{4} = \frac{9}{6} = \frac{13.50}{9} = 1.50 = r$
10,80,150,220	A.P. as $80-10 = 150-80 = 220-150 = 70 = d$
10,0,-10,-20,-30	A.P. $0-10 = -10-0 = -20-(-10) = -30-(-20) = -10 = d =$ common diff

3.

For	t_n	S_n
AP	$a + (n-1)d$	$\frac{n}{2}\left[2a + (n-1)d\right]$ OR $\frac{n}{2}(t_1 + t_n)$
GP	$a \cdot (r)^{n-1}$	$\left[a(r^n - 1)/(r-1)\right]$ when $r > 1$ $\left[a(1-r^n)/(1-r)\right]$ when $r < 1$

4. 80, 87, 94, 101, ………. Find t_{30}, t_{80}, t_{125}, S_{45}, S_{100}, S_{125}

➡ In this A.P. $a = 80$, $d = 7$

$t_{30} = a + 29d = 80 + (29 \times 7) = 283$

$t_{80} = a + 79d = 80 + (79 \times 7) = 633$

$t_{125} = a + 124d = 80 + (124 \times 7) = 948$

$S_{45} = \frac{45}{2}\left[160 + (44 \times 7)\right] = 10{,}530$

S_{100}

$= \frac{100}{2}\left[160 + 99 \times 7\right]$

$= 42650$

S_{125}

$= \frac{125}{2}\left[160 + 124 \times 7\right]$

$= 64{,}250$

5. 5, 10, 20, 40, ………. Find t_{12}, t_{10}, S_{16}, S_{22}

➡ In this G.P. $a = 5$, $r = 2$

$t_{12} = a \cdot r^{11} = 5 \times (2)^{11} = 10{,}240$

$t_{10} = a \cdot r^{9} = 5 \times (2)^{9} = 2{,}560$

$S_{16} = \frac{5\,(2^{16} - 1)}{(2-1)} = 3{,}27{,}675$

$S_{22} = \frac{5\,(2^{22} - 1)}{(2-1)} = 20971515$

Find S_∞ for GP

when $r > 1$: $S_\infty = \infty$

when $1 > r > 0$: $S_\infty = \frac{a}{(1-r)}$

6. 1. Sum of infinite terms of G.P. where r > 1 = ∞ = infinity

$2 + 6 + 18 + 54 + \ldots\ldots\ldots \infty$ terms $= \infty$

2. Sum of infinite terms of G.P. where 0 < r < 1 =

$54 + 18 + 6 + 2 + \ldots\ldots\ldots \infty$ terms $= \frac{a(1 - r^{\infty})}{(1-r)}$

$= \frac{a(1-0)}{(1-r)}$

$\boxed{= \frac{a}{(1-r)}}$

7. 10 + 20 + 40 + 80 + …………. ∞ terms = ?

➡ Here $r > 1$ $\therefore$ $S_\infty = \infty$ = infinite

8. 200 + 100 + 50 + 25 + …………. ∞ terms = ?

➡ Here $r = 0.50 < 1$ $S_\infty = \frac{a}{1-r} = \frac{200}{1-0.50} = 400$

9. For AP $t_5 = 80$, $t_{15} = 580$. Find a, d, t_{80}, t_{100}, S_{80}

$a + 14d = 580$

$a + 4d = 80$

$-\quad -\quad\quad -$

$10d = 500$

$\boxed{d = 50}$

$a + 14(50) = 580$

$\boxed{a = -120}$

$t_{80} = -120 + (79 \times 50)$

$= 3830$

$t_{100} = a + 99d$

$= -120 + (99 \times 50)$

$= 4830$

S_{80}

$= \frac{80}{2}\left[-240 + (79 \times 50)\right]$

$= 1{,}48{,}400$

10. For AP $t_3 = 15$, $S_3 = 30$

Find first term, common difference, S_{40}, S_{100}, t_{30}

$S_3 = \frac{3}{2}\left[t_1 + t_3\right]$

$30 = \frac{3}{2}\left[t_1 + 15\right]$

$\boxed{t_1 = a = 5}$

$t_3 = a + 2d = 15$

$5 + 2d = 15$

$\boxed{d = 5}$

$S_{40} = \frac{40}{2}\left[10 + 39 \times 5\right]$

$= 4100$

$S_{100} = \frac{100}{2}\left[10 + 99 \times 5\right]$

$= 25250$

$t_{30} = a + 29d$

$= 5 + 29 \times 5$

$= 150$

11. For AP t_n = (3n+5). Find S_n

➡ $t_n = 3n+5$

$t_1 = 3(1)+5 = 8$

$t_2 = 3(2)+5 = 11$

$d = 3$

$S_n = \frac{n}{2}(t_1 + t_n)$

$= \frac{n}{2}(8 + 3n+5)$

$= \frac{n}{2}(3n+13) = \left(\frac{3n^2+13n}{2}\right)$

12. For AP t_n = ?, if S_n = ($8n^2$ - 3n) t_n = ?

➡ $S_n = 8n^2 - 3n$

$S_1 = 8(1)^2 - 3(1) = 5$

$S_2 = 8(2)^2 - 3(2) = 26$

$a = 5, \quad t_2 = 21, \quad d = 16$

$t_n = 5 + (n-1)16$

$= 5 + 16n - 16$

$= 16n - 11$

13. For AP - Please Remember

- **1. If S_m=S_n, then S_{m+n} = zero**
- **2. If t_m=n, and t_n=m, then t_{m+n} = zero**
- **3. If m x t_m= n x t_n, then t_{m+n} = zero**

① If $S_{80} = S_{210}$ then $S_{290} = 0$

② If $t_{10} = 200$ & $t_{200} = 10$ then $t_{210} = 0$

③ If $3 \times t_3 = 16 \times t_{16}$ then $t_{19} = 0$

14. **For 2 observations x,y**

AM = $\left(\frac{x+y}{2}\right)$

GM = $\pm\sqrt{xy}$

HM = $\frac{2}{\frac{1}{x}+\frac{1}{y}}$ $= \left(\frac{2xy}{x+y}\right)$

15. For 2 observations relation between AM, GM, HM is

➡ $GM^2 = AM \times HM$ (i.e. GM is GM of AM & HM)

For any no. of observations relation between AM, GM, HM is

➡ $AM \geq GM \geq HM$

16. For 2 observations if GM = 10 and AM = 12, HM = ?

$GM^2 = AM \times HM$

$10^2 = 12 \times HM$

$HM = 8.333333 = 8\frac{1}{3} = \frac{25}{3}$

17. Insert 2 A.means between -200 and 1600

$-200, \; 400, \; 1000, \; 1600$

$a = -200$

$t_4 = a + 3d = 1600$

$-200 + 3d = 1600 \quad \therefore d = 600$

If a, b, c, d are in A.P. then b, c are 2 A.means betn a & d

18. Insert 3 A.means between 5000 and 8520.

$5000, \; 5880, \; 6760, \; 7640, \; 8520$

$t_5 = a + 4d = 8520$

$5000 + 4d = 8520 \quad d = 880$

19. Insert one A.means between 100 and 250.

$\frac{100 + 250}{2} = 175$

20. Insert 5 G.means between 500 and 8,000.

$500, \; \boxed{793.74}, \; \boxed{1260.05}, \; \boxed{2000}, \; \boxed{3174.97}, \; \boxed{5040.21}, \; 8000$

$500 \, r \times r \times r = 2000$

$r^3 = 4 \quad \therefore r = 1.5874837467$

21. a. Sum of first 'n' natural numbers = $n(n+1)/2$

b. Sum of first 'n' odd numbers = n^2

c. Sum of squares of first 'n' natural numbers = $n(n+1)(2n+1)/6$

d. Sum of cubes of first 'n' natural numbers = $\left[n(n+1)/2\right]^2$

e. Sum of first 'n' even numbers = $n(n+1) = n^2+n$

22. $19^2 + 20^2 + 21^2 + 22^2 + \ldots\ldots\ldots\ldots + 105^2$

$= (1^2+2^2+ \cdots + 105^2) - (1^2+2^2+ \cdots + 18^2)$

$= \left(\frac{105 \times 106 \times 211}{6}\right) - \left(\frac{18 \times 19 \times 37}{6}\right)$

$= 3,89,296/-$

23. $28^3 + 29^3 + 30^3 + \ldots\ldots\ldots\ldots + 62^3$

$(1^3+2^3+3^3+ \cdots + 62^3) - (1^3+2^3+ \cdots + 27^3)$

$= \left(\frac{62 \times 63}{2}\right)^2 - \left(\frac{27 \times 28}{2}\right)^2$

$= 3671325$

24. $1 + 3 + 5 + 7 + \ldots\ldots\ldots\ldots + 989 = ?$

$989 = 1 + (n-1)2$

$n = 495$

S_{495} = sum of first 495 odd numbers $= 495^2$

$= 245025$

25. $4484 + 4488 + 4492 + \ldots\ldots\ldots\ldots + 16880 = ?$

In this A.P. $a = 4484$, $d = 4$, $t_n = 16880$

$16880 = 4484 + (n-1)4$

$n = 3100$

$S_{3100} = \frac{3100}{2}(4484 + 16880)$

$= 3,31,14,200$

26. n[th] term of sequence 1, 3, 5, 7, ... is

→ In this A.P. $a = 1$, $d = 2$

$t_n = a + (n-1)d$

$= 1 + (n-1)2 = 1 + 2n - 2 = 2n - 1$

27. $\sum_{i=4}^{i=7} \sqrt{2i-1} =$

→ $\sqrt{7} + \sqrt{9} + \sqrt{11} + \sqrt{13}$

$= 12.5679273768$

28. If $S_n = 2n^2 + 8n$, first 3 terms of AP are :

→ $S_n = 2n^2 + 8n$

$S_1 = 2(1)^2 + 8(1) = 10$

$S_2 = 2(2)^2 + 8(2) = 24$

$a = 10$, $t_2 = 14$, $d = 4$

First 3 terms are

10, 14, 18

29. For AP $t_1 = -4$, $t_n = 146$, $S_n = 7171$. The number of terms is :

→ $S_n = \frac{n}{2}(t_1 + t_n)$

$7171 = \frac{n}{2}(-4 + 146)$

$\therefore 7171 = \frac{n}{2} \times 142$

$n = 101$

30. $3\frac{1}{2} + 7 + 10\frac{1}{2} + 14 + \ldots\ldots$ **Find S_{17}**

→ In this A.P. $a = 3.50$, $d = 3.50$

$S_{17} = \frac{17}{2}\left[7 + (16 \times 3.50)\right] = 535.50$

31. 4 A.means between -2 & 23 are

$-2, 3, 8, 13, 18, 23$

$a = -2$

$t_6 = 23 = a + 5d = -2 + 5d$

$d = 5$

32. Find x such that 8x + 4, 6x - 2, 2x + 7, are in A.P

$t_2 - t_1 = t_3 - t_2$

$6x - 2 - 8x - 4 = 2x + 7 - 6x + 2$

$-2x - 6 = -4x + 9$

$2x = 15 \quad \therefore x = 7.50$

33. Find k such that (10k+8), (18k-19), (22k-81) are in A.P.

$t_2 - t_1 = t_3 - t_2$

$18k - 19 - 10k - 8 = 22k - 81 - 18k + 19$

$8k - 27 = 4k - 62$

$4k = -35$

$k = -35/4 = -8.75$

34. 4 A.means between -20 and 880 are

$-20, 160, 340, 520, 700, 880$

$a = -20, \; t_6 = 880$

$a + 5d = 880$

$-20 + 5d = 880$

$\boxed{d = 180}$

35. Insert 3 G.means between $\frac{1}{9}$ and 9.

➡ $\frac{1}{9}, \frac{1}{3}, 1, 3, 9$

$a = \frac{1}{9}$

$t_5 = 9 = a \cdot r^4$

$9 = \frac{1}{9} \times r^4$

$\therefore r^4 = 81$

$r = 3$

36. 3 + 33 + 333 + ... n terms = ?

➡ $3 + 33 + 333 + 3333 + \text{--------} \; n \text{ terms}$

$= 3(1 + 11 + 111 + \text{------} \; n \text{ terms})$

$= \frac{3}{9}(9 + 99 + 999 + \text{----} \; n \text{ terms})$

$= \frac{3}{9}\left[(10-1) + (100-1) + (1000-1) + \text{------} \; n \text{ terms}\right]$

$= \frac{3}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$

37. 6, 12, 24, 48, ... Find t_{10}, S_{12}

➡ In this G.P. $a = 6$, $r = 2$

$t_{10} = a \times r^9 = 6 \times 2^9 = 3072$

$S_{12} = \frac{a(r^{12} - 1)}{r - 1} = \frac{6(2^{12} - 1)}{2 - 1} = 24570$

38. For GP $t_2 = 24$, $t_5 = 81$ then find common ratio.

➡ $a \cdot r = 24$

$a \cdot r^4 = 81$

$a \cdot r \cdot r^3 = 81$

$24 \, r^3 = 81$

$r^3 = 81/24$

$r^3 = \frac{27}{8} = \left(\frac{3}{2}\right)^3$

$r = 3/2$

39. Sum of first 20 terms of G.P. is equal to 244 times of sum of first 10 terms of G.P. then common ratio = ?

$$S_{20} = 244 \times S_{10}$$

$$\frac{\cancel{a}(r^{20}-1)}{(r-\cancel{1})} = \frac{244 \times \cancel{a} \times (r^{10}-1)}{(r-\cancel{1})}$$

$$(r^{10}\cancel{-1})(r^{10}+1) = 244 \times (r^{10}\cancel{-1})$$

$$r^{10} = 243 = 3^5 = (\sqrt{3}^2)^5 = \sqrt{3}^{10}$$

$$\therefore \; r = \sqrt{3}$$

40. 1 + 2 + 4 + 8 + ... = 8191.

How many terms are there in the above G.P.?

$a = 1$, $S_n = 8191$, $r = 2$

$$8191 = \frac{1(2^n - 1)}{(2-1)}$$

$$8191 = 2^n - 1$$

$\therefore \; 2^n = 8192 = 2^{13}$

$n = 13$

$\therefore$ There are 13 terms in above GP

41. 4 G.Means between 4 and 972 are :

4, 12, 36, 108, 324, 972

$a = 4$

$t_6 = 972$

$a \cdot r^5 = 972$

$\therefore \; 4 \times r^5 = 972$

$r^5 = 243 = 3^5$

$$\boxed{r = 3}$$

42. For G.P., Find $t_4 = x$, $t_{10} = y$, $t_{16} = z$ then $y^2 = xz$. True / False

$a \cdot r^3 = x$

$a \cdot r^9 = y$

$a \cdot r^{15} = z$

$y^2 = (a \cdot r^9)^2 = a^2 \cdot r^{18}$

$xz = a \cdot r^3 \cdot a \cdot r^{15} = a^2 \cdot r^{18}$

$\therefore \; y^2 = xz$ is True.

43. Find sum of all odd numbers divisible by 9 between 5,000 and 15,000.

→ ① $5004 + 5013 + 5022 + \dots + 14994 = ?$

$t_n = 5004 + (n-1)9 = 14994$

$n = 1111$

$S_{1111} = \frac{1111}{2}(5004 + 14994) = 11108889$

② $5013 + 5031 + 5049 + \dots + 14985 = ?$

$14985 = 5013 + (n-1)18 \quad \therefore n = 555 \quad \therefore S_{555} = \frac{555}{2}(5013 + 14985)$

$= 55,49,445$

44. Find sum of all numbers divisible by 7 between 800 and 8000.

→ $805 + 812 + 819 + \dots + 7994$

$\therefore 7994 = 805 + (n-1)7$

$n = 1028$

$S_{1028} = \frac{1028}{2}(805 + 7994) = 45,22,686$

45. $1.03 + 1.03^2 + 1.03^3 + \dots$ Find S_{11}

→ In this G.P. $a = 1.03$, $r = 1.03$

$S_{11} = \left[\frac{1.03(1.03^{11} - 1)}{1.03 - 1}\right] = 13.1920295602$

46. The n^{th} element of the sequence -1, 2, -4, 8 .... is

a. ~~$(-1)^n \times 2^{n-1}$~~ b. 2^{n-1} c. 2^n d. None of these

In this G.P. $a = -1$, $r = -2$

$t_n = a \times (r)^{n-1}$

$= -1 \times (-2)^{n-1} = -1 \times \frac{(-2)^n}{(-2)^1} = \frac{1}{2} \times (-2)^n = \frac{1}{2} \times (-1 \times 2)^n$

$= \frac{1}{2} \times -1^n \times 2^n = (-1)^n \times 2^{n-1}$

47. $\sum_{i=4}^{7} \sqrt{2i-1}$ can be written as :

~~a.~~ $\sqrt{7}+\sqrt{9}+\sqrt{11}+\sqrt{13}$ b. $2\sqrt{7}+2\sqrt{9}+2\sqrt{11}+2\sqrt{13}$

c. $\sqrt{7+9+11+13}$ d. None of these

Question

$$\sum_{x=1}^{x=7} x^2+3 = ?$$

$= 1^2+3 + 2^2+3 + 3^2+3 + 4^2+3 + \cdots + (7^2+3)$

$= (1^2+2^2+\cdots+7^2) + 21$

$= \frac{7\times8\times15}{6} + 21 = 161$

48. Which term of AP -1, -3, -5, ...is -39

a. 21st   ~~b.~~ 20th   c. 19th   d. None of these

In this A.P. $a=-1$, $t_n=-39$, $d=-2$

$-39 = -1 + (n-1)(-2)$

$-38 = (n-1)(-2)$

$19 = n-1$

$\therefore n = 20$

$\therefore$ 20th term is -39

49. For AP $t_m = n$, $t_n = m$ then t_r = ?

a. m+n+r   b. m+n-2r   c. (m+n+r)/2   d. m+n-r

For A.P. If $t_m=n$, $t_n=m$ then $t_r = m+n-r$

For AP If $t_m=n$, $t_n=m$ then $t_{m+n} = m+n-(m+n)=0$

50. $10 + 9\frac{2}{3} + 9\frac{1}{3} + 9 + 8\frac{2}{3} + \ldots\ldots$ Find S_{30}

a. 155   b. 551   c. 1010   d. 305

$\Rightarrow \frac{30}{3}, \frac{29}{3}, \frac{28}{3}, \frac{27}{3}, \frac{26}{3}, \ldots\ldots$

$a=10$, $d=-\frac{1}{3}$

$S_{30} = \frac{30}{2}\left[20 + (29)\left(-\frac{1}{3}\right)\right] = \frac{30}{2}\left(\frac{60}{3} - \frac{29}{3}\right) = 15\times\frac{31}{3} = 155$

51. 2 A.means between terms -6 and 14 are

a. 2/3, 1/3 ~~b.~~ 2/3, 22/3 c. -2/3, -22/3 d. None of these

$-6, \; 0.666666, \; 7.333333, \; 14$

$a = -6$

$14 = t_4 = -6 + 3d \quad \therefore 3d = 20$

$d = 6.666666$

52. The number of numbers between 74 and 25,556 divisible by 5 are:

a. 5090 b. 5097 c. 5095 d. None of these

$75, 80, 85, \text{-------}, 25555$

$25555 = 75 + (n-1)5$

$\therefore n = 5097$

53. The 4 arithmetic means between -2 and 23 are :

a. 3, 13, 8, 18 b. 18, 3, 8, 13 ~~c.~~ 3, 8, 13, 18 d. None of these

$-2, \; 3, \; 8, \; 13, \; 18, \; 23$

$a = -2$

$t_6 = a + 5d = 23 \quad \therefore -2 + 5d = 23 \quad \therefore d = 5$

54. t_1 = -4 and t_n = 146, S_n = 7171. Find n

~~a.~~ 101 b. 100 c. 99 d. None of these

$7171 = \frac{n}{2}(-4 + 146) \qquad S_n = \frac{n}{2}(t_1 + t_n)$

$7171 = n \times 71$

$n = 101$

55. x^2, x, 1 .... t_{31} = ?

a. x^{28} b. $1/x$ c. $1/x^{28}$ d. $1/x^{35}$

In this G.P. $a = x^2$, $r = 1/x$

$t_{31} = a \cdot r^{30} = x^2 \times \left(\frac{1}{x}\right)^{30} = x^2 \times \frac{1^{30}}{x^{30}} = \frac{1}{x^{28}}$

56. For G.P. $t_2 = 24$, $t_5 = 81$. The series is,

a. 16, 36, 24, 54...... b. 24, 36, 53....... c. 16, 24, 36, 54....... d. None of these

$t_2 = ar = 24$

$t_5 = a \cdot r^4 = 81$

$a \cdot r \cdot r^3 = 81$

$24 \times r^3 = 81$

$\therefore r^3 = \frac{81}{24} = \frac{27}{8}$

$\therefore r = \frac{3}{2}$

$ar = 24$

$a \times \frac{3}{2} = 24$

$a = 16$

$\therefore$ Series is : 16, 24, 36, 54, 81, -------

57. The sum of 3 numbers in G.P. is 39 and their product is 729. The numbers are :

a. 3, 9, 27 b. 27, 9, 3 c. Both (a) & (b) d. None of these

$\frac{a}{r}, a, ar$ $\qquad \frac{9}{r}, 9, 9r$

$\frac{a}{\cancel{r}} \times a \times a\cancel{r} = 729$

$a^3 = 729 \quad \therefore a = 9$

58. In a G.P, product of first 3 terms is 27/8. The middle term is

a. 2/3 b. 3/2 c. 9/8 d. None of these

$\frac{a}{r}, a, ar$

$\frac{a}{r} \times a \times ar = \frac{27}{8}$

$a^3 = \frac{27}{8} = \left(\frac{3}{2}\right)^3 \quad \therefore a = \frac{3}{2}$

59. If you have 1 paise today, 2 paise next day, 4 paise succeeding day and so on. Total saving in 2 weeks will be :

a. ₹ 163 b. ₹ 183 ~~c.~~ ₹ 163.83 d. None of these

$1 + 2 + 4 + 8 + \cdots\cdots\cdots$ $S_{14} = \frac{1(2^{14}-1)}{2-1}$

$= 16383$ paise

$= ₹\ 163.83$

60. Sum of first 20 terms of G.P. is 244 times of sum of its first ten terms. The common ratio is :

~~a.~~ $\sqrt{3}$ b. 3 c. $1/\sqrt{3}$ d. None of these

Refer Q.No. 39

61. The sum of the series 1 + 2 + 4 + 8 + ... + n terms is

~~a.~~ 2^n-1 b. $2n - 1$ c. $(1/2^n) - 1$ d. None of these

In this G.P.

$S_n = \frac{a(r^n-1)}{r-1} = \frac{1 \times 2^n - 1}{2-1} = 2^n - 1$

62. The number of terms to be taken so that 1 + 2 + 4 + 8 + ... will be 8191 is :

a. 10 ~~b.~~ 13 c. 12 d. None of these

Refer Q.No. 40

63. Four Geometric means between 4 and 972 are

~~a.~~ 12,36,108,324 b. 12,24,108,320 c. 10,36,108,320 d. None of these

$\boxed{\text{Refer Q.No. 41}}$

64. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\ldots \infty$ terms = ?

a. 0.75 ~~b.~~ 1.50 c. ∞ d. None of these

In this G.P. $a = 1$, $r = \frac{1}{3}$

As $0 < r < 1$ $S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{1}{2/3} = 3/2$
$= 1.50$

65. If p, q, r are in AP and x, y, z are in GP then $x^{q-r} \times y^{r-p} \times z^{p-q}$ **=?**

a. zero ~~b.~~ 1 c. -1 d. None of these

$p = 2$ $x = 10$
$q = 3$ $y = 20$
$r = 4$ $z = 40$

$x^{q-r} \times y^{r-p} \times z^{p-q}$
$= (10)^{3-4} \times (20)^{4-2} \times (40)^{2-3}$
$= 10^{-1} \times 20^{2} \times 40^{-1} = \frac{1}{10} \times 400 \times \frac{1}{40} = 1$

66. For G.P, $t_4 = x$, $t_{10} = y$, $t_{16} = z$. **Then**

a. $x^2 = y.z$ b. $z^2 = x.y$ ~~c.~~ $y^2 = x.z$ d. None of these

$\boxed{\text{Refer Q.No. 42}}$

67. A person saved ₹ 16,500 in 10 years. In each year after first year he saved ₹ 100 more than he did in preceeding year. The amount of money he saved in first year was

a. ₹ 1,000 b. ₹ 1,500 ~~c.~~ ₹ 1,200 d. None of these

$a, a+100, a+200, \ldots\ldots$ In this A.P. $d = 100$

$S_{10} = 16500 = \frac{10}{2}\left[2a + 9\times 100\right]$

$16500 = 5(2a+900)$

$a = 1200$

68. Sum of first 30 even natural numbers is :

~~a.~~ 930 b. 465 c. 900 d. None of these

$= 30\times 31$

$= 930$

69. t_n for AP is (8n + 3). Find S_n

a. $7n^2+7n$ b. $7n^2+4n$ ~~c.~~ $4n^2+7n$ d. $2n^2+7n$

$t_n = 8n+3$

$t_1 = 8(1)+3 = 11$

$S_n = \frac{n}{2}(t_1 + t_n)$

$= \frac{n}{2}(11 + 8n+3) = \frac{n}{2}(8n+14)$

$= n(4n+7) = 4n^2+7n$

70. $101^3 + 102^3 + 103^3 + \ldots\ldots\ldots\ldots + 123^3 = ?$

a. 3,23,11,450 ~~b.~~ 3,26,53,376 c. 3,15,45,295 d. None

$= (1^3+2^3+3^3+\ldots\ldots+123^3) - (1^3+2^3+\ldots\ldots+100^3)$

$= \left(\frac{123\times 124}{2}\right)^2 - \left(\frac{100\times 101}{2}\right)^2$

$= 3265\ 3376$

71. For A.P t_9 = 40 and t_{40} = 9 then t_{49} = ?

a. 49 b. -98 c. zero d. None of these

If $t_m = n$, $t_n = m$ then $t_{m+n} = 0$

If $m \cdot t_m = n \cdot t_n$ then $t_{m+n} = 0$

If $S_m = S_n$ then $S_{m+n} = 0$

3 short cuts for AP

72. If Loga, Logb, Logc are in AP, then

$$\log b - \log a = \log c - \log b$$

$$\log\left(\frac{b}{a}\right) = \log\left(\frac{c}{b}\right)$$

$$\frac{b}{a} = \frac{c}{b}$$

$\therefore$ a, b, c are in G.P.

73. For 2 positive observations G.M. is G.M of AM & HM

a. True b. False

4 & 9

$$AM = \frac{4+9}{2} = 6.50$$

$$GM = \sqrt{4 \times 9} = 6$$

$$HM = \frac{2 \times 4 \times 9}{4+9} = 5.538461538 46$$

AM GM HM

6.50 6 5.538461538 4 $\Rightarrow$ G.P.

$\therefore$ GM is GM of AM & HM

74. For AP First term = common difference then ratio of m[th] term to n[th] term is -

a. m:n b. n:m c. $m^2:n^2$ d. None

$a = d$

$$\frac{t_m}{t_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{\cancel{d}+md-\cancel{d}}{\cancel{d}+nd-\cancel{d}} = \frac{md}{nd} = m:n$$

75. $a^{1/x} = b^{1/y} = c^{1/z}$ and a, b, c are in G.P, then x, y , z are in

~~a. A.P~~ b. G.P c. Both d. H.P

$a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$

$a^{\frac{1}{x}} = k \quad \therefore a = k^x$

$b = k^y$

$c = k^z$

k^x, k^y, k^z are in G.P.

$(k^y)^2 = k^x \cdot k^z$

$k^{2y} = k^{x+z}$

$\therefore 2y = x+z$ ...... $\therefore x, y, z$ are in A.P.

76. $x = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\ldots \infty$ terms, $y = 1 + \frac{1}{4} + \frac{1}{4^2} + \ldots\ldots \infty$ terms. Find (x·y).

~~a. 2~~ b. 1 c. 8/9 d. 1/2

$a = 1, r = \frac{1}{3}$

$x = S_\infty = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

$y = \frac{1}{1-\frac{1}{4}}$

$= (1/3/4)$

$= 4/3$

xy

$= \frac{3}{2} \times \frac{4}{3} = 2$

77. For AP if $t_7 : t_{10} = 5:7$, then $t_8 : t_{11} = ?$

a. 13:16 ~~b. 17:23~~ c. 14:17 d. 15:19

$\frac{t_7}{t_{10}} = \frac{5}{7}, \quad \frac{a+6d}{a+9d} = \frac{5}{7}, \quad 7a + 42d = 5a + 45d$

$2a = 3d$

$\frac{t_8}{t_{11}} = \frac{a+7d}{a+10d} = \frac{2a+14d}{2a+20d} = \frac{17d}{23d} = 17:23$

78. If G is GM of a, b then, $\frac{1}{G^2 - a^2} + \frac{1}{G^2 - b^2} = ?$

a. G^2 b. $3G^2$ ~~c. $1/G^2$~~ d. $2/G^2$

$\therefore G^2 = ab$ ---------------------- (1)

$\frac{1}{ab - a^2} + \frac{1}{ab - b^2} = \frac{1 \times b}{ba(b-a)} - \frac{1 \times a}{ab(b-a)} = \frac{\cancel{(b-a)}}{ab\cancel{(b-a)}}$

$= \frac{1}{G^2}$

79. Find the product of $243 \times 243^{1/6} \times 243^{1/36} \times$ ....

a. 1024 b. 27 ~~c.~~ 729 d. 246

$= 243^{1} \times 243^{\frac{1}{6}} \times 243^{\frac{1}{36}} \times 243^{\frac{1}{216}} \times 243^{\frac{1}{1296}} \times \ldots\ldots$

$= 243^{1+\frac{1}{36}+\frac{1}{216}+\frac{1}{1296}+\ldots\ldots} = (243)^{\frac{1}{1-\frac{1}{6}}} = (243)^{6/5}$

$= (3^5)^{6/5} = 3^6 = 729$

80. GM of P, P^2, P^3, P^4, ..., P^n will be

a. P^{n+1} ~~b.~~ $P^{(n+1)/2}$ c. $P^{n(n+1)/2}$ d. None of these

GM of n obs^{n}s = nth root of product of n obs^{n}s

$= (p^1 \times p^2 \times p^3 \times \ldots\ldots \times p^n)^{1/n} = (p^{1+2+3+\ldots+n})^{\frac{1}{n}}$

$= \left(p^{\frac{n(n+1)}{2}}\right)^{1/n} = (p)^{\frac{n+1}{2}}$

81. Find the numbers whose AM is 12.50 and GM is 10 :

~~a.~~ 20,5 b. 10,5 c. 5,4 d. None of these

82. t_5 of GP = $3^{1/3}$ then product of the first 9 terms of GP is :

a. 8 ~~b.~~ 27 c. 243 d. 9

$t_5 = a.r^4 = 3^{1/3}$

product of 1st 9 terms

$= a \times ar \times ar^2 \times \ldots\ldots \times a.r^8$

$= a^9 \times r^{1+2+3+\ldots+8} = a^9.r^{36} = (a.r^4)^9 = (3^{1/3})^9$

$= 27$

83. For AP $t_3 + t_9 = 8$. Find S_{11} for AP

~~a.~~ 44 b. 22 c. 19 d. 11

$a+2d+a+8d = 8$

$2a+10d = 8$

$S_{11} = \frac{11}{2}(t_1+t_{11})$

$= \frac{11}{2}(a+a+10d) = \frac{11}{2} \times 8$

$= 44$

84. t_8 for AP is 15 then S_{15} = ?

a. 15 b. 0 ~~c.~~ 225 d. 225/2

$a+7d=15$

$S_{15}=\frac{15}{2}(2a+14d)$

$=15(a+7d)$

$=15\times 15=225$

85. Find first term of GP if second term is 2 and sum of infinite terms is 8.

a. 6 b. 3 ~~c.~~ 4 d. 1

$a\cdot r=2$

$\therefore\ r=\frac{2}{a}$

$S_\infty=\frac{a}{1-r}=8$

$\frac{a}{1-\frac{2}{a}}=8$

$\frac{a^2}{a-2}=8$

$a^2=8a-16$

$a^2-8a+16=0$

$\therefore(a-4)^2=0$

$a-4=0$

$a=4$

86. If sum of 4th term and 12th term of AP is 8, what is the sum of first 15 terms?

~~a.~~ 60 b. 120 c. 110 d. 150

$a+3d+a+11d=8$

$2a+14d=8$

$S_{15}=\frac{15}{2}[2a+14d]$

$=\frac{15}{2}\times 8$

$=60$

87. In GP, t_6 = 729; r = 3, First term = ?

a. 2 ~~b.~~ 3 c. 4 d. 7

$a\cdot r^5=729$

$a\times 3^5=729$

$\boxed{a=3}$

88. For AP S_{13} = 143, t_3 = 5, find first term.

a. 4 b. 7 c. 9 ~~d.~~ 2

$\frac{13}{2}(2a+12d)=143$

$\therefore\ 2a+12d=22$

$a+6d=11$

$a+2d=5$

$a+6d=11$

$3a+6d=15$

$-2a=-4$

$a=2$

math Comprehensive Revision

$GM \text{ of } x_1, x_2, \ldots, x_n = (x_1 \cdot x_2 \cdot x_3 \cdots x_n)^{1/n}$

89. If GM of a, b, c, d is 3 then GM of $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ is

~~a.~~ 1/3 b. 3 c. 81 d. 1/81

$\sqrt[4]{abcd} = 3$

$abcd = 3^4$

$\sqrt[4]{\frac{1}{abcd}} = \left(\frac{1}{3^4}\right)^{1/4} = \frac{1}{3}$

90. Find common difference of AP, if a = 200 and sum of first 6 terms exceeds twice the sum of first 4 terms by 50

a. -10 b. -15 c. -20 ~~d.~~ None of these

$S_6 = 2 \times S_4 + 50$

$3(400 + 5d) = 2 \times 2 \times (400 + 3d) + 50$

$1200 + 15d = 1600 + 12d + 50$

$3d = 450$

$d = 150$

91. 59 + 63 + 67 + 71 + ……… + 107 = ?

a. 972 ~~b.~~ 1079 c. 1083 d. None of these

In this A.P. $a = 59, d = 4$

$107 = 59 + (n-1)4$

$n = 13$

$S_{13} = \frac{13}{2}(59 + 107)$

$= 1079$

92. If one AM 'A' and 2 G. means G_1 & G_2 are inserted between any 2 numbers then $(G_1^3 + G_2^3) =$?

~~a.~~ $2AG_1G_2$ b. $2G_1G_2$ c. $2AG_1$ d. 2A

x, A, y : AP

x, G_1, G_2, y : GP

$A = \frac{x+y}{2}$

$x + y = 2A$

$G_1^2 = x \cdot G_2$

& $G_2^2 = yG_1$

$G_1 \cdot G_1^2 + G_2 \cdot G_2^2$

$= G_1 \cdot x G_2 + G_2 \cdot y \cdot G_1$

$= G_1 \cdot G_2 (x + y)$

$= 2A\, G_1 G_2$

93. If a, b, c are in G.P. a, x, b and b, y, c both are in A.P, then (a/x) + (c/y) = ?

a. 1 b. 0 ~~c.~~ 2 d. None of these

$a = 10 \quad b = 20 \quad c = 40$

$a = 10 \quad x = 15 \quad b = 20$

$b = 20 \quad y = 30 \quad c = 40$

$\left(\frac{a}{x} + \frac{c}{y}\right) = \frac{10}{15} + \frac{40}{30} = \frac{20}{30} + \frac{40}{30} = 2.00$

94. For AP $(t_7 / t_3) = (12/5)$. Find (t_{13}/t_4) =?

a. 8:5 b. 9:4 c. 7:3 ~~d.~~ 10:3

$\frac{t_7}{t_3} = \frac{a+6d}{a+2d} = \frac{12}{5}$

$5a + 30d = 12a + 24d$

$\boxed{6d = 7a}$

$\frac{t_{13}}{t_4} = \frac{a+12d}{a+3d} = \frac{2a+24d}{2a+6d}$

$= \frac{2a + 4\times 7a}{2a + 7a} = \frac{30a}{9a} = \frac{10}{3}$

95. 4^{th} term of AP is equal to 3 times of first term and 7^{th} term exceeds twice of third term by 1. Find first term.

~~a.~~ 3 b. 5 c. 7 d. 9

$t_4 = 3a$

$a + 3d = 3a$

$2a = 3d$

$t_7 = 2\times t_3 + 1$

$a + 6d = 2(a+2d) + 1$

$a + 6d = 2a + 4d + 1$

$-a + 2d = 1$

$-2a + 4d = 2$

$-3d + 4d = 2$

$\boxed{d = 2}$

$\therefore 2a = 3\times 2 = 6$

$a = 3$

96. $t_n = 1/243$. For $3, \sqrt{3}, 1$ .... then n = ?

a. 12 ~~b.~~ 13 c. 14 d. 15

$t_n = \frac{1}{243}$

$a(r)^{n-1} = \frac{1}{243}$

In this G.P. $a = 3$, $r = 1/\sqrt{3}$

$\therefore 3 \times r^{n-1} = \frac{1}{3^5}$

$(3^{-1/2})^{n-1} = \frac{1}{3^6}$

$(3)^{-\frac{1}{2}(n-1)} = 3^{-6}$

$\frac{1}{2}(n-1) = 6$

$n - 1 = 12$

$n = 13$

97. For GP $S_n = 4095$, $r = 2$, $t_n = 2048$. Find n

a. 10 b. 11 ~~c.~~ 12 d. 15

$4095 = \frac{a(2^n - 1)}{2-1}$

$4095 = a(2^n - 1)$

$4095 = a\times 2^n - a$

$a = 4096 - 4095 = 1$

$2048 = a \times (2)^{n-1}$

$2048 = a \times \frac{2^n}{2^1}$

$4096 = a \times 2^n$

$a \times 2^n = 4096$

$1 \times 2^n = 2^{12}$

$\therefore n = 12$

98. Which term of AP 64, 60, 56, 52, ... is zero

a. 18^{th} ~~b.~~ 17^{th} c. 14^{th} d. 15^{th}

In this A.P. $a = 64$, $d = -4$

$t_n = 0 = a + (n-1)d$

$0 = 64 + (n-1)(-4)$

$\therefore (n-1)(-4) = -64$

$(n-1) = 16$

$n = 17$

99. Sum of all 2 digit natural numbers is

a. 4955 b. 4890 c. 3776 ~~d.~~ None of these

$10 + 11 + \cdots\cdots + 99 = ?$

$S_{90} = \frac{90}{2}(10 + 99) = 4905$

100. 1, y, 9 are in A.P, then value of 'y' is

a. 3 b. -3 c. Either (a) or (b) ~~d.~~ None of these

$y = \frac{1+9}{2} = 5$

101. a, b, c are in AP as well as GP, then

~~a.~~ a = b = c b. a ≠ b = c c. a ≠ b ≠ c d. Wrong question.

102. a,b,c,d,e,f are in AP then (e-c) = ?

a. 2 (c-a) b. 2 (f-d) ~~c.~~ 2(d-c) d. (d-c)

$e - c = 5^{th}$ term $- 3^{rd}$ term

$= 2 \times$ common diff

$= 2 \times (d-c)$

103. The sum of first '2n' terms of AP 2, 5, 8 ...... is equal to sum of first 'n' terms of AP 57, 59, 61, ... then n = ?

a. 10 b. 12 ~~c.~~ 11 d. 13

2, 5, 8 ..... 57, 59, 61 ....

$\frac{\cancel{2}n}{\cancel{2}}\left[4 + (2n-1)3\right] = \frac{\cancel{n}}{2}\left[114 + (n-1)2\right]$

$4 + 6n - 3 = 57 + n - 1$

$6n + 1 = n + 56 \quad \therefore 5n = 55 \quad \therefore n = 11$

104. If $a^x = b^y = c^z$ and x, y, z are in GP then loga, logb, logc are in

a. A.P ~~b.~~ G.P c. Both d. None of these

$a^x = b^y = c^z$

$x \log a = y . \log b = z \log c$

$\frac{y}{x} = \frac{z}{y}$

$\frac{\log a}{\log b} = \frac{\log b}{\log c}$

$(\log b)^2 = \log a \times \log c$

105. (4x+5), (5x+7), (8x-1) are in A.P. then x = ?

~~a.~~ 5 b. 6 c. 7 d. 4

$t_2 - t_1 = t_3 - t_2$

$5x + 7 - 4x - 5 = 8x - 1 - 5x - 7$

$x + 2 = 3x - 8$

$10 = 2x \quad \therefore x = 5$

106. 3 numbers are in G.P. If we double the middle term, we get an A.P. then common ratio of G.P. is equal to

~~a.~~ $2 \pm \sqrt{3}$ b. $3 \pm \sqrt{2}$ c. $3 \pm \sqrt{5}$ d. $5 \pm \sqrt{3}$

$\frac{a}{r}, a, ar$: GP

$\frac{a}{r}, 2a, ar$: AP

$2a - \frac{a}{r} = ar - 2a$

$\cancel{a}\left(2 - \frac{1}{r}\right) = \cancel{a}(r-2)$

$2r - 1 = r^2 - 2r$

$0 = r^2 - 4r + 1$

$r = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2 \times 1} = \frac{4 \pm \sqrt{4 \times 3}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$

$r = 2 \pm \sqrt{3}$

107. $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots = ?$

a. 17/8 b. 9/2 c. 7/2 ~~d.~~ 4

In this G.P. $a = 2$, $r = \frac{1}{2}$ AS $0 < r < 1$

$S_\infty = \frac{a}{1-r} = \frac{2}{1 - \frac{1}{2}} = \frac{2}{\frac{1}{2}} = 4$

108. In AP a,b,c,d,e,f,g,h common difference = k; then in A.P. a,c,e,g common difference. = ?

~~a.~~ 2k b. k^2 c. k d. None of these

58, 61, 64, 67, 70, 73, 76, 79, 82 A.P. with d = 3

58, 64, 70, 76, 82 are also in A.P. with common diff $= 3 \times 2 = 6$

109. In G.P. a,b,c,d,e,f,g,h common ratio = m; then in G.P. a,c,e,g common ratio = ?

a. m b. 2m ~~c.~~ m^2 d. None of these

1, 5, 25, 125, 625, 3125, 15625 are in GP with $r = 5$

1, 25, 625, 15625 are also in G.P. with common ratio $= 5^2$

110. Shall we stop here for the day?

a. Yes ~~b. No~~

111. 8,8,8,8,8 are in

a. A.P b. G.P c. H.P ~~d. All of these~~

112. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ **are in**

a. A.P b. G.P ~~c. H.P~~ d. All of these

as 2,4,6,8,10,12 are in A.P.

113. $\frac{1}{8}, \frac{1}{m}, \frac{1}{18}$ **are in H.P. then m = ?**

a. 1/13 ~~b. 13~~ c. 1/12 d. 144

8, m, 18 must be in A.P.

$m = \frac{8+18}{2} = 13$

114. 3, $\sqrt{m}$, 10 are in G.P.; then m = ?

a. $\sqrt{30}$ ~~b. 30~~ c. 13 d. 13/3

$(\sqrt{m})^2 = 3 \times 10 = m$

115. If a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q are in G.P with r as common ratio; then a,d,g,j,m,p are in GP. with common ratio = ?

a. r b. r^2 ~~c. r^3~~ d. None of these math — Comprehensive Revision

① 3 Short cuts of A.P.

ⓐ If $S_m = S_n$ then $S_{(m+n)} = 0$

ⓑ If $m \times t_m = n \times t_n$ then $t_{(m+n)} = 0$

ⓒ If $t_m = n$ & $t_n = m$ then $t_{(m+n)} = 0$

②

For A P	For G.P.
$t_n = a + (n-1)d$	$t_n = a \times (r)^{n-1}$
$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ OR $\frac{n}{2}(t_1 + t_n)$	$S_n = \frac{a(r^n-1)}{(r-1)}$ when $r > 1$ & $\frac{a(1-r^n)}{(1-r)}$ when $0 < r < 1$
	$S_\infty = \infty$ when $r > 1$
	$S_\infty = \frac{a}{(1-r)}$ when $0 < r < 1$

(3) $1+2+3+4+\text{-------}+m=\dfrac{m(m+1)}{2}$

$1^2+2^2+3^2+\text{------}+k^2=\dfrac{k(k+1)(2k+1)}{6}$

$1+3+5+7+\text{------}+k\text{ terms}=k^2$

$2+4+6+8+\text{------}+x\text{ terms}=x(x+1)$

$1^3+2^3+3^3+\text{------}+y^3=\left[\dfrac{y(y+1)}{2}\right]^2$

Don't chase People chase " DREAMS "

- CA VINOD REDDY -


AND STILL
RISE