Referencer for Quick Revision

Foundation Course Paper-3: Business Mathematics and Logical Reasoning & Statistics

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CHAPTER 4 : TIME VALUE OF MONEY

At the foundation level with regards to Business Mathematics the topic Time Value of money is very important for students not only to acquire professional knowledge but also for examination point of view. Here in this capsule an attempt is made for solving and understanding the concepts of time value of money with the help of following questions with solutions.

Problems on Simple, Compound and Effective rate of interest

Simple interest = P.T.I., Where P = Principal, T = Time, I = Rate of Interest

1. A sum of money amount to ₹ 6,200 in 2 years and ₹ 7,400 in 3 years. The principal and rate of interest is

Solution: A sum of money in 2 years = P + P.2.I = ₹ 6200
A sum of money in 3 years = P + P.3.I = ₹ 7400
Interest in 1 year = ₹ 1200; Interest in 2 years = ₹ 2400
Amount = ₹ 6200, P= Principal = 6200-2400= ₹ 3800.

$2400 = 3800 \times 2 \times \frac{I}{100}$, I = rate of interest = 31.58% .

2. A sum of money doubles itself in 10 years. The number of years it would triple itself is

Solution: $2P = P + \frac{PTR}{100}$, $P = \frac{PTR}{100}$; T = 10 then R = 10%

$3P = P + \frac{PTR}{100}$ then $2P = \frac{PTR}{100}$ and R= 10%

Time (T) = 20 years.

Amount =P $(1+i)^n$
Compound rate of interest = P(1+i)^n-P= A-P
where P = principle i= interest n =conversion period

3. The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is

Solution: 1.4P = P(1+0.02)n
$(1.02)^n$=1.4, n = 17 years (app)

Depreciation (A)= P$(1-i)^n$
Where, A = Scrap Value, P = Original Cost, I =Depreciated at the rate, n = Number of years

4. A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was ₹ 1,00,000 and its ultimate scrap value was ₹ 30,000. The effective life of the machine is

Solution: Here A = Scrap Value = 30,000 and
P = Original Cost = ₹ 1,00,000
30000 = 100000 $(1-0.2)^n$
3/10 = 0.3 = $(0.8)^n$, n = 5.4 years .

5. The useful life of a machine is estimated to be 10 years and cost ₹ 10,000. Rate of depreciation is 10% p.a. The scrap value at the end of its life is

Solution: Here A = Scrap Value =?,
P= Original cost = 10,000 n= 10 , I = 10%
A = 100000 $(1-0.1)^{10}$ = 10000$(0.9)^{10}$
A = ₹ 3486.78

Effective rate of interest = $(1+i)^n$-1

6. The effective rate of interest corresponding a nominal rate of 7% p.a convertible quarterly is

Solution: Effective rate of interest = $(1+i)^n$-1, here n= 4,
i= 0.07/4 = 0.0175
= $(1+0.0175)^4$-1=1.07186-1=7.19%

The difference between simple and compound interest for 2 years = P.i^2, where P =Principal, i= interest

7. The difference between the S.I and the C.I on ₹ 2,400 for 2 years at 5% p.a is

Solution: The difference between simple and compound interest for 2 years =2400$(0.05)^2$= ₹ 6.

The difference between simple and compound interest for 3 years = 3P.i^2 + P.i^3, where P = Principal i= Interest

8. The difference between the S.I and the C.I on a certain money invested for 3 years at 6% p.a is ₹ 110.16 the principle is

Solution: The differences between simple and compound interest for 3 years
= 110.16 =P($3i^2+i^3$)= P($3\times0.06^2+0.06^3$)
110.16 = P(0.0108+0.000216) =P(0.011016)

$P= \frac{110.16}{0.011016}$ = ₹ 10,000

9. The annual birth and death rates per 1,000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is

Solution: Here given, birth rate per 1,000 = 39.4 and death rates per 1,000 = 19.4
difference = 20 % per 1000 population

growth rate = $\frac{20}{1000} \times 100$=2%

Future population double
P =1000, A = 2000, r = 2%
2000= 1000$(1+0.02)^n$
$(1.02)^n$ = 2
Number of years = n= 35

10. What annual rate of interest compounded annually doubles an investment in 7 years?
(Given that $2^{1/7}$= 1.104090)

Solution: If the principal be P, A_n = 2 p
Since A_n = P$(1+ i)^n$
2P = P (1+i)^7
$2^{1/7}$ = (1 + i)
1.104090 = 1 + i
I = 0.10409, Required rate of interest = 10.41% per annum

11. Vidya deposited ₹ 60000 in a bank for two years with the interest rate of 5.5% p.a. How much interest she would earn? what will be the final value of investment?

Solution: Required interest amount is given by,

I = P × it = ₹ 60,000 × $\frac{5.5}{100}$ ×2 = ₹ 6,600

The amount value of investment is given by, A = P + I = ₹ (60,000 + 6600) = ₹ 66,600

12. Rajiv invested ₹ 75,000 in a bank at the rate of 8% p.a. simple interest rate. He received ₹ 135,000 after the end of term. Find out the period for which sum was invested by Rajiv.

Solution: We know A = P + Pit = P (1+it)

i.e. 135000 = 75000 (1 + $\frac{8}{100}$ × t)

135000/75000 = $\frac{100+8t}{100}$

1.8×100 – 100=8t
80 = 8 t
t (Time) = 10 years

13. Which is a better investment, 3.6% per year compounded monthly or 3.2% per year simple interest? Given that (1+0.003)^12 =1.0366.

Effective rate of interest $E = (1 + i)^n - i$

Solution: $i = 3.6/12 = 0.3\% = 0.003$, $n = 12$

$E = (1 + i)^n - 1$

$= (1 + 0.003)^{12} - 1 = 1.0366 - 1 = 0.0366$ or $= 3.66\%$

Effective rate of interest (E) 3.66% is more than the simple interest so the Effective rate of interest (E) is better investment than the simple interest 3.2% per year.

14. The C.I on ₹ 4,000 for 6 months at 12% p.a payable quarterly is

Solution: Here P = ₹ 4000, n = 6/3= 2 , r= 0.12/4= 0.03

Compound Interest = $[P(1+i)^n - P] = [4000(1+0.03)^2 - 4000]$ =₹ 243.60

Annuity *applications*

F = Future value = C.F. $(1 + i)^n$ Where C.F = Cash flow
i= rate of interest, n = time period

15. Ravi invest ₹ 5000 in a two-year investment that pays you 12% per annum. Calculate the future value of the investment

Solution: We know, F= Future value = C.F.$(1 + i)^n$, Where C.F= Cash flow =₹ 5000, i = rate of interest=0.12,n = time period=2

F= ₹ $5000(1+0.12)^2$=₹ 5000×1.2544 =₹ 6272.

Annuity regular means :First payment at the end of the period.
Future value of the annuity regular =A(n,i)= A. $\left[\frac{(1+i)^n - 1}{i}\right]$

Annuity regular: In annuity regular first payment/receipt takes place at the end of first period.

16. Find the future value of an annuity of ₹ 5000 is made annually for 7 years at interest rate of 14% compounded annually. [Given that $(1.14)^7$= 2.5023]

Solution: Here annual payment A=₹ 5000, n=7, i=14% = 0.14

Future value of the annuity=A(7,0.14)=5000. $\left[\frac{(1+0.14)^7 - 1}{(0.14)}\right]$ = $5000\left[\frac{(2.5023 - 1)}{0.14}\right]$ = ₹ 53653.57

Future value of Annuity due or Annuity Immediate: When the first receipt or payment at the beginning of the annuity) it is called annuity due or annuity immediate.

17. ₹ 2000 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10th payment? Given that $(1.005)^{10}$ = 1.0511

Solution: Here A= ₹ 2000, n= 10, i= 6% per annum= 6/12 % per month = 0.005

Future value of annuity after 10 months is given by

$$A(n,i) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$A(10, 0.005) = 2000\left[\frac{(1+0.005)^{10} - 1}{0.005}\right] = 2000\left[\frac{(1.0511) - 1}{0.005}\right]$$

=2000×10.22 = ₹ 20440

Future value of the annuity regular or annuity due = $A\left[\frac{(1+i)^n - 1}{i}\right] \times (1 + i)$

18.Swati invests ₹ 20,000 every year starting from today for next 10 years. Suppose interest rate 8% per annum compounded annually. Calculate future value of the annuity. Given that $(1 + 0.08)^{10}$ = 2.158925)

Solution: Calculate future value as though it were an ordinary annuity.
Future value of the annuity as if it were an ordinary annuity

$$= ₹\ 20000\left[\frac{(1+0.08)^{10} - 1}{0.08}\right]$$

=₹ 20000 × 14.486563 =₹ 289731.25

Multiply the result by (1 + i) =₹ 289731.25 × (1+0.08)

=₹ 312909.76

19. What is the present value of ₹ 100 to be received after two years compounded annually at 10%.

Solution: Here A_n = ₹ 100, i=10% = 0.1 , n= 2

Required present value= $\frac{A_n}{(1+i)^n} = \frac{100}{(1+0.1)^2} = \frac{100}{(1.21)}$ =82.64

Thus ₹ 82.64 shall grow to ₹ 100 after 2 years at 10% compounded annually.

20. Find the present value of ₹ 10000 to be required after 5 years if the interest rate be 9%. Given that $(1.09)^5$=1.5386.

Solution: Here i= 0.09 , n =5, An=10000

Required present value = $\frac{A_n}{(1+i)^n} = \frac{10000}{(1+0.09)^5} = \frac{10000}{(1.5386)}$=₹ 6499.42

Present Value of Annuity regular= A= P(n, i) =A. $\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$, P(n,i) = $\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

21.Soni borrows ₹ 5,00,000 to buy a car. If he pays equal instalments for 10 years and 10% interest on outstanding balance, what will be the equal annual instalment? Given [P(10,0.10) =6.14457]

Solution: We know, A = $\frac{V}{P(n,i)}$ Here V = ₹ 500000 , n = 10, I = 10% p.a.= 0.10

Annual Instalment= $\frac{V}{P(n,i)}$= ₹ $\frac{5,00,000}{P(10,0.10)}$= ₹ $\frac{5,00,000}{6,14457}$

= ₹ 8,1372.66

22. If ₹ 10,000 is paid every year for ten years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually? Given [P(10,0.14) =5.21611]

Solution: V = A.P(n,i) Here A = ₹ 10000 , n=10, i=0.14

V = 10000×P(10, 0.14)

= 10000×5.21611=₹ 52161.10

Therefore, the loan amount is ₹ 52161.10

23. Ram bought a Scooter costing ₹73000 by making a down payment of ₹ 3000 and agreeing to make equal annual payment for four years. How much would be each payment if the interest on unpaid amount be 14% compounded annually? Given [P(4 ,0.14) =2.91371]

Solution: In the present case we have present value of the annuity i.e. ₹ 70000 (73000-3000) and we have to calculate equal annual payment over the period of four years.

We know that, V = A. P (n , i) Here n = 4 and I = 0.14

$$A = \frac{V}{P(n,i)} = \frac{70000}{P(4,0.14)} = \frac{70000}{2.91371}$$

Therefore, each payment = ₹ 24024.35

24. Suppose your Father decides to gift you ₹ 20,000 every year starting from today for the next six years. You deposit this amount in a bank as and when you receive and get 10% per annum interest rate compounded annually. What is the present value of this annuity?

Solution: For calculating value of the annuity immediate following steps will be followed. Present value of the annuity as if it were a regular annuity for one year less i.e. for five years

= ₹ 20,000 × P(5, 0.10)

= ₹ 20, 000× 3.79079 = ₹ 75815.80

Add initial cash deposit to the value, ₹ (75815.80+20,000) = ₹ 95815.80

Sinking Fund: Interest is computed at end of every period with specified interest rate.

25.How much amount is required to be invested every year so as to accumulate ₹ 5,00,000 at the end of 10 years if interest is compounded annually at 10%? Given A. (10, 0.1)= 15.9374248

Solution: Here A = 500000, n = 10, $A(n,i) = \left[\frac{(1+i)^n - 1}{i}\right] = \left[\frac{(1+01)^{10} - 1}{0.1}\right] = 15.9374248$

since A = P. A. (n ,i)

500000 =P A. (10, 0.1) = P × 15.9374248

$P = \left[\frac{500000}{15.9374248}\right] =$ ₹ 31372.70

26. ABC Ltd. wants to lease out an asset costing ₹ 360000 for a five year period. It has fixed a rental of Rs.105000 per annum payable annually starting from the end of first year. Suppose rate of interest is 14% per annum compounded annually on which money can be invested by the company. Is this agreement favorable to the company?

Solution: First, we have to compute the present value of the annuity of ₹ 105000 for five years at the interest rate of 14% p.a. compounded annually.

The present value V of the annuity is given by

V =A.P (n ,i) = 105000 × P (5, 0.14)

= 105000 ×3.43308 = ₹ 360473.40

which is greater than the initial cost of the asset and consequently leasing is favourable to the lessor.

27.A company is considering proposal of purchasing a machine either by making full payment of ₹ 4000 or by leasing it for four years at an annual rate of ₹ 1250. Which course of action is preferable if the company can borrow money at 14% compounded annually?

Solution: The present value V of annuity is given by

V =A.P (n ,i) = 1250 × P (4,0.14)

= 1250× 2.91371 = ₹ 3642.11

which is less than the purchase price, and consequently leasing is preferable.

28.A machine can be purchased for ₹ 50000. Machine will contribute ₹ 12000 per year for the next five years. Assume borrowing cost is 10% per annum compounded annually. Determine whether machine should be purchased or not.

Solution: The present value of annual contribution

V = A.P. (n ,i)

= 12000 P(5,0.10) = 12000 × 3.79079

= ₹ 45489.48

which is less than the initial cost of the machine. Therefore, machine must not be purchased.

29.A machine with useful life of seven years costs ₹ 10000 while another machine with useful life of five years costs ₹ 8000. The first machine saves labour expenses of ₹ 1900 annually and the second one saves labour expenses of ₹ 2200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded per annum.

Solution: The present value of annual cost savings for the first machine

= ₹ 1900.P (7, 0.10)

= ₹ 1900 × 4.86842 = ₹ 9250

cost of machine being Rs.10000 it costs more by ₹ 750 than it saves in terms of labour cost.

The present value of annual cost savings of the second machine = ₹ 2200.P (5,0.10)= ₹ 2200 x 3.79079 = ₹ 8339.74

Cost of the second machine being ₹ 8000, effective savings in labour cost is ₹ 339.74. Hence the second machine is preferable.

30.An investor intends purchasing a three year ₹ 1000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

Solution: Present value of the bond $= \frac{100}{(1+0.14)^1} + \frac{100}{(1+0.14)^2} + \frac{100}{(1+0.14)^3} + \frac{1000}{(1+0.14)^3}$

= 100 × 0.87719 + 100 × 0.769467 + 100 × 0.674 972

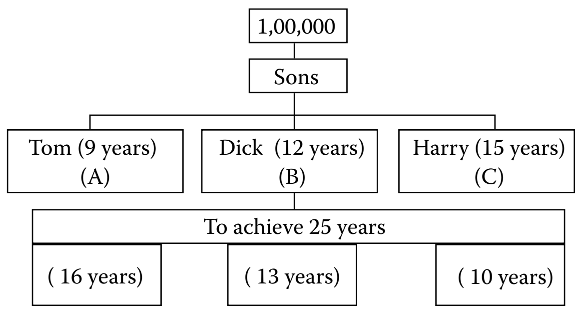
+ 1000 × 0.674972

= 87.719+ 76.947+ 67.497+ 674.972 = 907.125

Thus the purchase value of the bond is ₹ 907.125

31. Johnson left ₹ 1,00,000 with the direction that it should be divided in such a way that his minor sons Tom, Dick and Harry aged 9, 12 and 15 years should each receive equally after attaining the age 25 years. The rate of interest being 3.5%, how much each son receives after getting 25 years old?

Solution: Given problem can be explained as



$$A\left(1+\frac{3.5}{100}\right)^{16} = B\left(1+\frac{3.5}{100}\right)^{13} = +C\left(1+\frac{3.5}{100}\right)^{10}$$

$A(1.035)^{16} = B(1.035)^{13} = C(1.035)^{10}$

$A(1.035)^{6} = B(1.035)^{3} = C$------(I)

$A : B : C = 1 : (1.035)^3 : (1.035)^6$

A + B + C= 100000

$x + x(1.035)^3 + x(1.035)^6 = 100000$

$x[1+(1.035)^3+(1.035)^6] = 10. 0000$

x(3.337973) = 1,00,000 then x = ₹ 29958.30 (A' s share)

B' s share = 29958.30 (1.035)3= ₹ 33215.30

C' s share = 29958.30 (1.035)6= ₹ 36826.40

32. A machine costs ₹ 5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of ₹ 25000. what amount should be set aside every year if the sinking fund investments accumulate at 3.5% compound interest p.a.?

Solution: Cost of new machine = $5{,}20{,}000 \times \frac{125}{100} =$ ₹ 6,50, 000, Scrap value = ₹ 25,000

For new machine = 650000-25000= ₹ 6,25, 000.

Here = ₹ 6,25, 000, n= 25 , i=3.5%=0.035

$$6{,}25{,}000 = P\left[\frac{(1+i)^n - 1}{i}\right] = P\left[\frac{(1+0.035)^{25} - 1}{0.035}\right]$$

$6{,}25{,}000 = P[38.95]$ then $P = \frac{625000}{38.95} =$ ₹ 16046.27

Foundation Paper 3: Logical Reasoning Questions with explanations

At the Foundation level, students are expected to inculcate/evolve logical thinking and reasoning skills to further develop their analytical skills. This section attempts to capture basic techniques in sequential thinking as the underlying concept to solve problems. Here are a few Logical Reasoning Questions with explanations to get you psyched!

Chapter : 9 Number Series, Coding and decoding and Odd man out series

These questions deal in which series or letters in some orders and follows certain pattern throughout.

I. Find missing term of the series

(1) 101, 102, 106,115,131, 176, ?

(a)212 (b) 220 (c) 211 (d) 235

Explanation: Answer: (a)

The pattern of the series by adding $+1^2, +2^2, +3^2, +4^2, + 5^2, +6^2$,

So missing term is 176+ 6^2= 212.

(2) 3, 10, 29, 66, 127, ?

(a) 164 (b) 187 (c) 216 (d) 218

Explanation: Answer (d)

The pattern of the series is 1^3+2 , 2^3+2, 3^3+2 , $4^3+ 2$, $5^3+ 2$, $6^3+ 2$

So missing number is, 6^3+2= 216 + 2 = 218.

(3) 8, 13, 21, 32, 46, 63, 83 , ?

(a) 104 (b) 106 (c) 108 (d) 110

Explanation: Answer(b)

The pattern of the series is +5, +8, +11, +14, +17, +20, +23

So missing number is 83 + 23 = 106

4) 3, 4, 4, 6, 12, 15, 45, ?, 196

(a) 42 (b) 49 (c) 43 (d) 40

Explanation: Answer (b)

The pattern of the series is 3+1 ,4×1, 4+2, 6×2, 12+3, 15× 3, 45 + 4 , 49 × 4,

So missing term is = 45 + 4= 49

(5) 10, 12, 22, 34, 56, 90, ?

(a) 146 (b) 147 (c) 136 (d) 156

Explanation: Answer (a)

Each term in the series, except the first two terms, is the sum of preceding two terms

The right answer 56+90 = 146

(6) 4, 9, 19, 39, ?, 159, 319

(a) 40 (b) 41 (c) 78 (d) 79

Explanation: Answer (d)

Each number of the series is one more than the twice the preceding number

Therefore, missing term = 39×2+1= 79

(7) 7,15,29, 59, 117 ,?

(a) 238 (b) 235 (c) 120 (d) 155

Explanation: Answer (b)

The pattern is 7 ×2+1, 15 × 2-1, 29 × 2 + 1, 59 × 2 - 1, 117 × 2 + 1

So missing term is = 117 × 2 + 1 = 235

II. Find missing term of the letter series

(8) DBA, IDE, NFI, SHO, ?

(a) XJU (b) XYU(c) XUV (d) XUY

Explanation: Correct Option: Answer (a)

The first letter of the series is

D → +5 I → +5 N → +5 S → +5 X

The second letter of the series B →+2 D →+2 F →+2 H →+2 J

The third letters of the series are pattern

A – E – I – O – U › Vowels

So missing letter series is XJU

(9) cccaa_ bb_ cc_ aa_bb_

(a) abcab (b) babda (c) badna (d) bdanb

Explanation: Answer (a)

The pattern of the series is ccc, aaa, bbb, ccc, aaa follows.

(10) m _ n v _ n _a n _ a_ m a _

(a)aamvnn (b) aanvmm (c) vamaal (d) vanmak

Explanation: Answer (a)

The series man and van repeated

(11) In a certain language 'BISLERI' is written as 'CHTKFQJ' and 'AQUA' is written as 'BPVZ'. How is 'COMPUTER' written in the same Code?

(a) DNNOVSFQ (b) DNNVOSFX (c) DNNOVSXF

(d) DNONVSFX

Explanation: Answer (A)

B •+1 c → I •-1 H → S •+1 T → L •-1 K → E •+1 F → R •-1 Q → I •+1 J

A •+1 B → Q •-1 P → U •+1 V → A •-1 Z

C •+1 D → O •-1 N → M •+1 N → P •-1 o → U •+1 V → T •-1 S → E •+1 F → R •-1 Q

So "COMPUTER' is coded as 'DNNOVSFQ

(12) If 'BROTHER, is coded as 2456784. 'SISTER' is coded as 919684 , what is the code for 'ROBBERS'?

(a) 4562684 (b) 9245784 (c) 4522849 (d) 4652684

Explanation: Answer (c)

B •2 → R •4 → O •5 → T •6 → H •7 → E •8 → R •4

S •9 → I •1 → S •9 → T •6 → E •8 → R •4

R •4 → O •5 → B •2 → B •2 → E •8 → R •4 → S •9

Find odd one of the following series

(13) (a) 144 (b) 169 (c) 288 (d) 324

Explanation: Answer (c), All others are perfect square numbers except 288

(14) (a) 73 (b) 53 (c) 87 (d) 23

Explanation: Answer (c), Except 87 all others are prime numbers

(15) (a) 4867 (b) 5555(c) 6243 (d) 6157

Explanation: Answer (d)

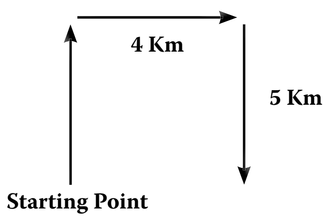
4867= 4+8+6+7 = 25, which is divisible by 5
5555= 5+5+5+5 = 20, which is divisible by 5
6243= 6+4+2+3 = 15, which is divisible by 5
6157= 6+1+5+7 =19, which is not divisible by 5

Chapter 10. Direction tests: In this test,the questions consist of a sort of direction puzzle. A successive follow-up of direction is formulated and the student is required to ascertain the final direction. The test is meant to judge the candidate's ability to trace and follow correctly and sense the direction correctly.

(1) A man walks 6 km North, turns right and walks 4 km, again turns right and walks 5 km, in which direction is he facing now?

(a) South (b) North (c) East (d) West

Explanation:

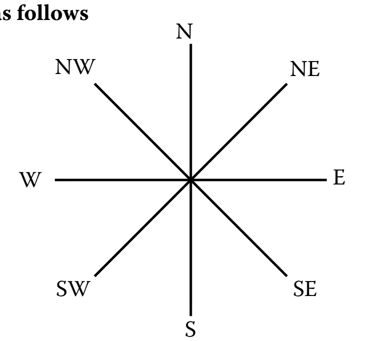


Answer: (a) South

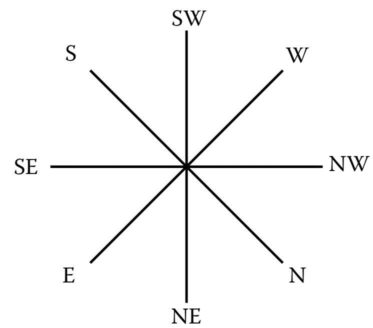
(2) If South-East becomes North, North–East becomes West and all the rest of the directions are changed in same manner, what will be direction of the East?

(a) North-West (b) South (c) South-East (d) South-West

Explanation: According to question the direction of the diagram as follows



After changing the directions



Now from the above diagrams North-West will be the direction for East.

Foundation Paper 3: Logical Reasoning Questions with explanations

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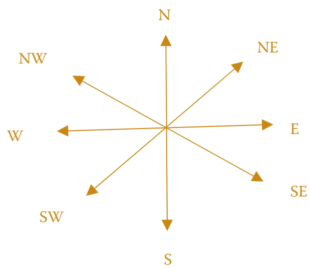
CHAPTER 10: DIRECTION SENSE TESTS

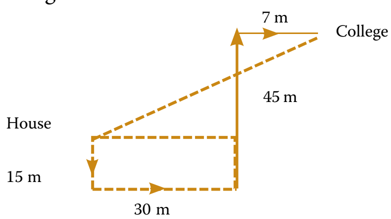


1. Gopal goes 15 m south from his house, turns left and walks 30 m , again turns left and walks 45 m , then turns right and walks 7 m to reach the college. In which direction is the college from his house?

   (a) North-East (b) West (c) East (d) North

   Explanation: Answer (a)

   According to the information stated in the question, direction diagram can be drawn as follows.

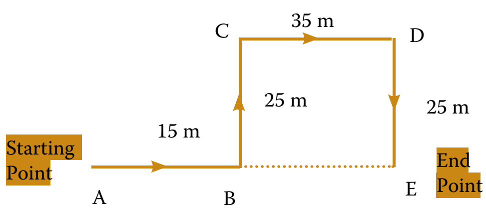


   So it's clear from the diagram that college is North -East direction from Gopal's house

2. Ram start moving from a point, facing in East direction. After walking 15 m, he turned to his left and walked 25m, before turning to his right. Then, he walked a distance of 35 m, then turned to his right and stop after walking further a distance of 25 m. Find how far Ram is from his starting point.

   (a) 20 m (b) 50 m (c) 15 m (d) 25 m

   Explanation: Answer (b), the direction map of Ram's walk can be drawn as,

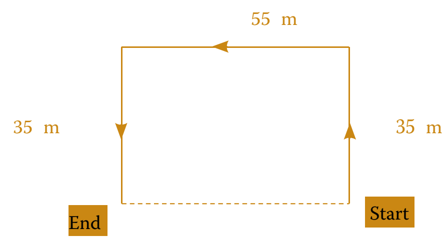


   The distance between the starting point and end point is AB + BE = 15 + 35 = 50 m.

3. Facing towards North, Ravi walks 35 m. He then turns left and walks 55 m. He again turns left and walks 35 m. How far is from original position and towards which direction.

   (a) 30 m, North (b) 20 m, East (c) 55 m, West (d) 20 m, South

   Explanation: Answer (c)



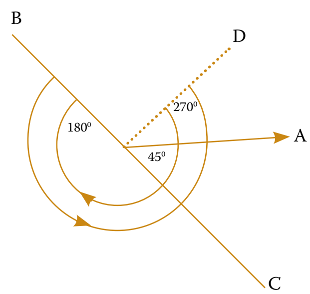
   From the figure it is clear that, Ravi is 55 m away in West direction from his original position.

4. A man is facing towards East and turns through 45° clock wise again 180° clock wise and then turns through 270° anti-clock wise. In which direction is he facing now?

   (a) West (b) North- East (c) South (d) South-West

   Explanation: Answer (b)

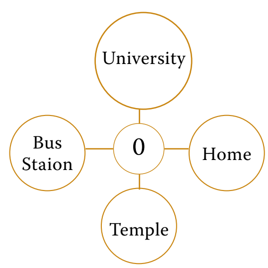
   As shown in figure , the man initially faces in the direction of OA. On moving 45° clockwise, the man faces the direction OB . On further moving 180° clockwise, he faces in the direction of OC. Finally on moving 270° anti-clock wise, he faces the direction OD, which is North-East.

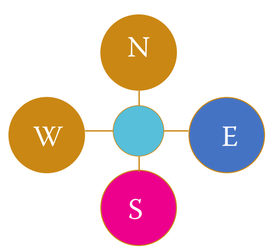


5. Kamal wants to go to university which is situated in a direction opposite to that of a temple. He starts from his house, which is in the East and comes at a four-ways place. His left side road goes to the temple and straight in front is the Bus Station. In which direction is university is located?

   (a) North (b) North- East (c) South (d) East

   Explanation: Answer (c)





   Kamal comes from East towards West. He reached O (four-way place). Now university will not in front or left. It will be towards the right, so it will be north direction.

CHAPTER 11. SEATING ARRANGEMENTS

The process of making group of people to sit as per a prefixed manner is called seating arrangement these questions, some conditions are given on the basis of which students are required to arrange objects, either in a row or on in circular order.

1. Six Children A, B, C, D, E and F are sitting in a row facing towards North. C is sitting between A and E, D is not at the end. B is sitting immediate right of E , F is not at the right of end, but D is sitting 3rd left of E . Which of the following is right of D.

   (a) A (b) F (c) E (d) C

   Explanation: Answer (a)

   According to the question A, B , C, D, E and F are sitting as follows.

F	D	A	C	E	B

   Clearly A is sitting to the right of D.

2. Read the following information carefully and then answer the questions (i), (ii) and (iii).

   Six friends A, B, C, D, E and F are sitting on a bench, facing towards North.

   I. A is sitting next to B.
   II. C is sitting left to D.
   III. D is not sitting with E.
   IV. E is on the left end of the bench.
   V. C is third position from right.
   VI. A is on the right side of B and to the right side of E.
   VII. A and C are sitting together.
   VIII. F is sitting Right of D.

   (i) At what position A is sitting?
   (a) Between B and C (b) Between D and C
   (c) Between E and D (d) Between C and E
   (ii) What is position of B?
   (a) Second from right (b) Centre
   (c) Extreme left (d) Second from left
   (iii) What is position of D?
   (a) Extreme from left (b) Extreme right
   (c) Third from left (d) Second from right.

   Explanation: Arrangement according to the question is as follows.

Left	E	B	A	C	D	F	Right

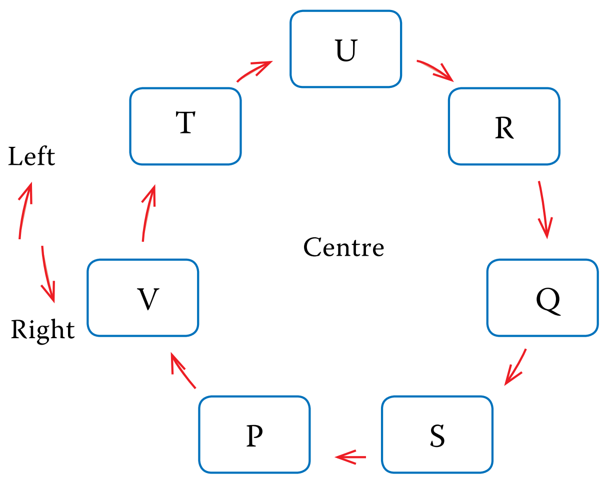
   2(i)Answer (a), A is sitting between B and C
   2(ii) Answer (d), B is sitting second from left.
   2 (iii) Answer (d), D is second from right

3. Read the following information carefully to answer the questions (i) and (ii)

I. P, Q, R, S, T, U, and V are sitting along a circle facing the centre
II. P is between V and S
III. R, who is second to the right of S , is between Q and U
IV. Q is not neighbour of T

3 (i) Which of the following statement is a correct statement?
(a) V is between T and P (b) S is second to left of V
(c) R is third to the left of P (d) P is to the immediate right of S
(ii) What is the position of P ?
(a) P is immediate left of S (b) to the immediate left of V
(c) 2nd to the left of R (d) 2nd to the right of Q

Explanation: Following seating arrangement is formed from the given information.



(i) Answer (a), based on diagram V is sitting T and P .
(ii) Answer (a), based on diagram P is immediate left of S.

4. Read the following information carefully to answer the questions given below:

   Seven boys A, B, C, D, E, F and G are standing in a line
   I . G is between A and D
   II. F and A have one boy between them
   III. D and C have two boys between them.
   IV E is immediate right of F.
   V. C and B have three boys between them
   (i) Who is second from right?
   (a) C (b) G (c) E (d) F
   (ii) Who is standing in the centre?
   (a) A (b) D (c) C (d) G

   Explanation: Arrangements according to the question.

Left	B	D	G	A	C	F	E	Right

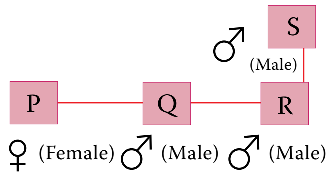
   (i) Answer (d), Clearly F is second from right.
   (ii) Answer (a), Clearly A is standing in the centre.

FOUNDATION: PAPER 3 LOGICAL REASONING CHAPTER 12. BLOOD RELATIONS

Blood relations of a group of persons are given in jumbled form. In these tests, the questions which are asked depend on relation.

1. P is the sister of Q, Q is the brother of R, R is son of S. How S is related to P?

   (a) Father
   (b) Daughter
   (c) Son
   (d) Uncle

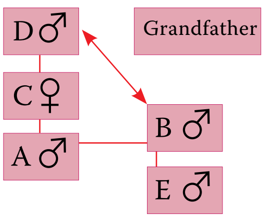
   

   Explanation:
   Answer: (a)

   Based on the diagram Q and R brothers and P is their sister. Therefore, S is the father P.

2. A and B are brothers, C is A's mother, D is C's father, E is B's son. How is B is related to D.

   (a) Son (b) Grandfather
   (c) Grandson
   (d) Great Grandfather

   Explanation: Answer(c)
   D is father of C.
   C is mother of A and B.
   Therefore, B is Grandson of D.

3. A man showed a boy next to him and said – "he is the son of my wife's sister-in- law, but I am the only child of my parents ". How is my son is related to him?

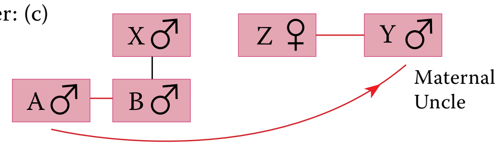
   (a) Nephew (b) Cousin (c) Brother (d) Uncle

   Explanation: Answer (b). The boy is the son of man's brother-in -law. Therefore, man's son is the cousin of that boy.

4. A and B are brothers. X is the father of B, Z is the only sister of Y and Y is maternal uncle of A, what is Z is related to X?

   (a) Sister (b) Brother (c) Wife (d) Mother

   Answer: (c)

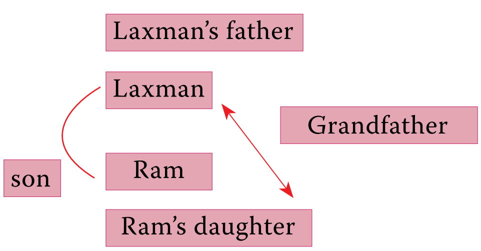
   

   Based on the diagram A and B are bothers.
   Y is bother of Z, therefore z is a female and Z is wife of X.

5. Introducing Ram to guests, Laxman said, "His father is the only son of my father". How is Ram's daughter is related Laxman?

   (a) Nephew (b) Grandson (c) Grandfather (d) Son

   Explanation Answer: (c)

   Only son of Laxman's father is Laxman himself. Therefore, Ram' s father is Laxman.

   Therefore, Laxman is Grandfather of Ram's daughter.

6. Pointing to a man in photograph, a woman said "His brother's father is the only son of my grandfather". Then How is women related to the man's son in the photograph?

   (a) Daughter (b) Mother (c) Aunt (d) Sister

   Explanation: Answer: (c) Only son of woman's grandfather means father of that woman.

   Father of women is the father of man's brother and hence father of that man.

   Therefore, the women is sister of the man and aunt to his son in photograph.

7. Read the following information carefully and answer the questions that follow

   I: 'A + B' means A is the son of B.
   II. 'A - B' means A is the wife of B.
   III. 'A × B' means A is the sister of B.
   IV. 'A ÷ B' means A is the mother of B.
   V. 'A $ B' means A is the bother of B.

   (i) What does P+ R - Q means
   (a) Q is the father of P (b) Q is the son P
   (c) Q is the uncle of P (d) Q is the bother of P.

   (ii) What does P × R ÷ Q meaning
   (a) P is the brother of R (b) P is the father of Q
   (c) P is the aunt of Q (d) P is the nephew of Q

   (iii) What does P $ R+ Q mean?
   (a) P is the aunt of Q (b) P is the son of Q
   (c) P is the niece of Q (d) P is the sister of Q

   (iv) What does P $ R ÷ Q mean?
   (a) P is the aunt of Q (b) P is the sister of Q
   (c) Q is the niece of P (d) P is the uncle of Q

   Explanation:

   (i) Option (a), P+ R - Q, means P is the son of R, R is wife of Q, So Q is the father of P.

   (ii) Option (c), P× R ÷ Q, means P is the sister of R, R is the mother of Q, So P is the aunt of Q.

   (iii) Option (b), P $ R + Q, means P is the brother of R, R is the son of Q, So P is the son of Q.

   (iv) Option (d), P $ R ÷ Q means P is the brother of R, R is the mother of Q. So P is uncle of Q.

8. On the basis of this information, you have to select the option which shows that A is the grandfather of T.

   I. 'S x T' means that S is the mother of T,
   II. 'S + T' means that S is the father of T,
   III. 'S÷T' means that S is the brother of T.

   (a) A + S + B ÷T
   (b) A x B + C ÷ T
   (c) A + C ÷T
   (d) A + B ÷C x T

   Explanation:
   Option (a) represents that A is the grandfather of T

   (i) B ÷T=> B is the brother of T.

   (ii) S + B=> S is the father of B, hence S will be father of T [from information (i)).

   (iii) A + S => A is the father of S, hence A will be grandfather of B and hence A is the grandfather of T.

CHAPTER.13 SYLLOGISM

Syllogism is a 'Greek' word that means inference or deduction. As such inferences are based on logic, then these inferences are called logical deduction. These deductions are based on propositions (premise).In this section we are going to understand the few basic important questions .These deductions are based on propositions (premise).

'Syllogism' checks basic aptitude and ability of a candidate to derive inferences from given statements using step by step methods of solving problems.

Directions: In each group of questions below are two or more statements followed by two or more conclusions You have to take the given statements to be true even if they seem to be variance from commonly known facts. Read the conclusion and then decide which of the conclusions logically follows from given statements, disregarding commonly known facts.

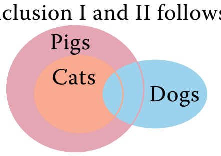
1.

Statements:
I. Some dogs are cats.
II. All cats are pigs .

Conclusions:
I. Some cats are dogs.
II. some dogs are pigs.

Given answer:
(a) If only conclusion I follows
(b) If only conclusion II follows
(c) If either conclusion I or II follows
(d) If neither conclusion I nor II follows
(e) If both I and II follow

Answer: (e), Conclusion I and II follows



Some dogs are cats(I-Type), All cats Are pigs (A-type)

I + A = I type conclusion = Some dogs are pigs = Conclusion II

Again, some dogs are cats → (Conversion Some cats are dogs = Conclusion I

Clearly both the conclusion I and II follows.

2.

Statements:
I. Some cats are dogs.
II. Some dogs are stones.

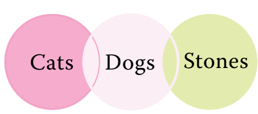
Conclusions:
I. No cat is stone.
II. All dogs are stones.
III. Some stones are cats.
IV. No dog is cat.

Given answer:
(a) only conclusion I and III follow
(b) only conclusion II and III follow.
(c) only I , III and IV follow
(d) none follows

Answer: (d)

Explanation: Both the premises are particular Affirmative (I-type).

No conclusion follows from the two particular premises. Conclusions I and III from complementary pair. Therefore, either conclusion I or III follows.



3.

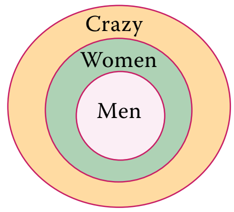
Statements:
I. All men are women.
II. All women are crazy.

Conclusions:
I. All men are crazy.
II. All the crazy are men.
III. Some of the crazy are men.
IV. Some of the crazy are women.

Answers
(a) None of the conclusions follows
(b) All the conclusions follow
(c) Only I, III, and IV follow
(d) Only II and III follow

Answer: (c), Only, I, III, and IV follow.

Explanation: Venn diagram:



Both the premises are universal Affirmative (A- type)

All men are women ↔All women are crazy.

A + A → A- type of conclusion.

"All men are crazy".

This is conclusion I.

Conclusion III is the converse of it.

Conclusion IV is the converse of Statement II.

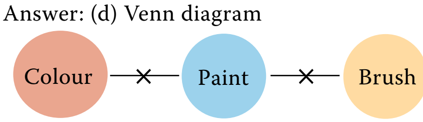
4.

Statements:
I. No colour is a paint.
II. No paint is a brush.

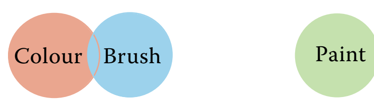
Conclusions:
I. No colour is a brush.
II. All brushres are colours.

Given answer:
(a) If only conclusion I follows.
(b) If only conclusion II follows.
(c) If either conclusion I or II follows.
(d) If neither conclusion I nor II follows.
(e) both conclusion I and II follows.

Answer: (d) Venn diagram



Conclusions: I No colour is a paint (×) . II. All brushes are colours (×) Possible diagram as follows



5.

Statements:
I. All soldiers are men.
II. Some citizens are soldiers.

Conclusions:
I. Some citizens are men.
II. All soldiers are citizens.

Answers
(a) Only I follows.
(b) Neither I nor II follows
(c) Only II follows
(d) Only I and II follow

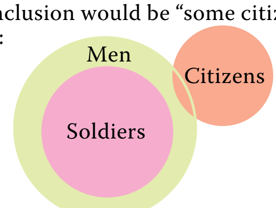
Answer: (a) We can align the premises by changing their orders. Some citizens are soldiers.

All soldiers are men. We know that

I + A → I Type conclusion.

Hence our conclusion would be "some citizens would be men.

Venn diagram:



CAPSULE: FOUNDATION PAPER 3: BUSINESS MATHEMATICS LOGICAL REASONING AND STATISTICS: CHAPTER 15 UNIT-I: MEASURES OF CENTRAL TENDENCY

At the foundation level with regards to Paper 3 Statistics part of the topic Measures of Central Tendency is very important for students not only to acquire professional knowledge but also for examination point of view. Here in this capsule an attempt is made for solving and understanding the concepts of Measures of Central Tendency with the help of following questions with solutions.

Definition of Central Tendency: Central tendency defined as the tendency of a given set of observations to cluster around a single central or middle value and the single value that represents the given set of observations is described as a measure of central tendency or, location, or average.

Following are the different measures of central tendency:

- Arithmetic Mean (AM)
- Median (Me)
- Mode (Mo)
- Geometric Mean (GM)
- Harmonic Mean (HM)

Criteria for an Idea l Measure of Central Tendency

- It should be properly and unambiguously defined.
- It should be easy to comprehend.
- It should be simple to compute.
- It should be based on all the observations.
- It should have certain desirable mathematical properties.
- It should be least affected by the presence of extreme observations.

Arithmetic Mean: defined as the sum of all the observations divided by the number of observations. Thus, if a variable x assumes n values x_1, x_2, x_3,..........x_n, then the AM of x, to be denoted by,

is given by, $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$, $\bar{X} = \frac{X_1 + X_2 + X_3 + \dots\dots\dots + X_n}{n}$

In case of a simple frequency distribution relating to an attribute, we have $\bar{X} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

$\bar{X} = \frac{\sum_{i=1}^{n} f_i x_i}{N}$ assuming the observation xi occurs fi times, i=1,2,3,……..n and N=≤fi.

In case of grouped frequency distribution also we may use formula with xi as the mid value of the i-th class interval, on the assumption that all the values belonging to the i-th class interval are equal to xi.

If classification is uniform, we consider the following formula for the computation of AM from grouped frequency the distribution:

$$\bar{x} = A + \frac{\sum f_i d_i}{N} \times C$$

Where, $d_i = \frac{x_i - A}{C}$ A = Assumed Mean C = Class Length

- **Properties of AM**
- **If all the observations assumed by a variable are constants, say k, then the AM is also k.**
- The algebraic sum of deviations of a set of observations from their AM is zero
- i.e. for unclassified data, ∑ (xi -) = 0 and for grouped frequency distribution, $\sum(f_i(xi -) = 0$
- AM is affected due to a change of origin and/or scale which implies that if the original variable x is changed to another variable y by effecting a change of origin, say a, and scale say b, of x i.e. $\bar{y} = a + b\bar{x}$, then the AM of y is given by
- If there are two groups containing n1 and n2 observations and $\bar{x}_1$ and $\bar{x}_2$ as the respective arithmetic means, then combined AM is given by $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

Question 1: Following are the daily wages in rupees of a sample of 10 workers: 58, 62, 48, 53, 70, 52, 60, 84, 75, 100. Compute the mean wage.

Solution: Let x denote the daily wage in rupees.

Applying the mean wage is given by,

$$\bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{(58 + 62 + 48 + 53 + 70 + 52 + 60 + 84 + 75 + 100)}{10} = \frac{₹662}{10} = ₹66.2$$

Question 2: Compute the mean weight of a group of B. Com students of Sri Ram College from the following data:

Weight in kgs	44-48	49-53	54-58	59-63	64-68	69-73
No. of students	3	4	5	7	9	12

Solution: Computation of mean weight of 40 B. Com students Applying formula, we get the average weight as

$$\bar{x} = \frac{\sum f_i x_i}{N} \frac{2495}{40} \text{ kgs.=62.38 kgs.}$$

Weight in kgs. (1)	No. of Student (f_i) (2)	Mid-Value (x_i) (3)	$f_i x_i$ (4) = (2) x (3)
44 – 48	3	46	138
49 – 53	4	51	204
54 – 58	5	56	280
59 – 63	7	61	427
64 – 68	9	66	594
69 – 73	12	71	852
Total	40	–	2495

Question 3: Find the AM for the following distribution:

Class Interval	5-14	15-24	25-34	35-44	45-54	55-64
Frequency	10	18	32	26	14	10

Solution: Any mid value can be taken as A. However, usually A is taken as the middle most mid-value for an odd number of class intervals and any one of the two middle most mid-values for an even number of class intervals. The class length is taken as C.

The required AM is given by

$$\bar{x} = A + \frac{\sum f_i d_i}{N} \times C = 39.5 + \frac{-64}{110} \times 10 = 39.5 - 5.82 = 33.68$$

Table: Computation of AM

Class Interval	Frequency (f_i)	Mid-Value (x_i)	$d_i = \frac{x_i - A}{C}$	$f_i d_i$
(1)	(2)	(3)	(4)	(5) = (2) x (4)
5-14	10	9.5	– 3	– 30
15-24	18	19,5	– 2	– 36
25-34	32	29.5	– 1	– 32
35-44	26	39.5(A)	0	0
45-54	14	49.5	1	14
55-64	10	59.5	2	20
Total	110	-	-	-64

Question 4: Given that the mean height of a group of students is 67.45 inches. Find the missing frequencies for the following incomplete distribution of height of 100 students.

Height in inches	60-62	63-65	66-68	69-71	72-74
No. of students	5	18	-	-	8

Solution: Let x denote the height and f_3 and f_4 as the two missing frequencies

Table: Estimation of missing frequencies

Class Interval	Frequency	Mid-Value (x_i)	$d_i = \frac{x_i - A}{C}$ $= \frac{x_i - 67}{3}$	$F_i d_i$ (f_i)
(1)	(2)	(3)	(4)	(5) = (2) x (4)
60-62	5	61	-2	-10
63 – 65	18	64	– 1	– 18
66 – 68	f3	67 (A)	0	0
69 – 71	f4	70	1	f4
72 – 74	8	73	2	16
Total	$31+ f_3+ f_4$	–	–	$– 12+f_4$

As given, we have

$31+ f_3+ f_4= 100$, $f_3+ f_4= 69$................(1)

$\bar{x} = 67.45$

and $A + \frac{\sum f_i d_i}{N} \times C = 67.45 = 67 + \frac{(12 + f_4)}{100} \times 3 = 67.45$

$(-12 + f_4) \times 3 = (67.45- 67) \times 100$

$-12 + f_4 = 15$, $f_4 = 27$

On substituting 27 for f_4 in (1), we get ,$f_3 + 27 = 69, f_3 = 42$, Thus, the missing frequencies would be 42 and 27.

Question 5: The mean salary for a group of 40 female workers is ₹5,400 per month and that for a group of 60 male workers is ₹7,800 per month. What is the combined mean salary?

Solution: As given n_1=40, n_2=60, $\bar{x}_1$=₹5,400 and $\bar{x}_2$=₹7,800 hence, the combined mean salary per month is

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{40 \times ₹5,400 + 60 \times ₹7,800}{40+60} = ₹6,840.$$

Question 6: The mean weight of 150 students (boys and girls) in a class is 60 kg. The mean weight of boy student is 70 kg and that of girl student is 55 kg . Find number of boys and girls in that class.

Solution: Le t the number of boy students be n1 and girl students be n_2 , as given $n_1+n_2 = 150$,

Then $n_2 = 150-n_1$, also $\bar{x} = 60$, $\bar{x}_1 = 70$, $\bar{x}_2 = 55$

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}, 60 = \frac{n_1 \times 70 + (150 - n_1) \times 55}{150}$$

$$60 = \frac{70n_1 + 8250 - 55n_1}{150} = \frac{15n_1 + 8250}{150}$$

$$= 9000 = 15n_1 + 8250$$

$$15n_1 = 750, n_1 = 50$$

$$n_2 = 150 - n_1 = 150 - 50 = 100$$

Therefore, number of boys (n_1)= 50.

Number of girls (n_2)= 100

Question 7: The average salary of a group of unskilled workers is Rs. 10,000 and that of a group of skilled workers is Rs. 15000.If the combined salary is Rs.12000, then what is the percentage of skilled workers

Solution: Let x be unskilled and y be skilled

10000x+15000y= 12000(x+y) = 12000x+12000y

2000x=3000y then 2x=3y

skilled workers is 2x/3

total workers x+2x/3=5x/3

percentage of skilled=2x/3 divided by 5x/3=40%

Question 8: The average age of a group of 10 students was 20 years. The average age increased by two years when the two new students joined in the group. What is the average age of two new students joined who joined in the group?

Solution: Average age of 10 students = 20 years, then sum of ages of 10 students = 200 years

If the two boys are included, then total number of students = 10+2 = 12

And average increased by two years = 20 +2 =22

The average age of 12 students = 22, then sum of ages of 12 students = 22×12= 264

The Sum of ages of two boys = 264-200= 64

Average age of boys = 64/2 = 32

Median

- Partioned Values
- As compared to AM, median is a positional average which means that the value of the median is dependent upon the position of the given set of observations for which the median is wanted. Median, for a given set of observations, may be defined as the middle-most value when the observations are arranged either in an ascending order or a descending order of magnitude.

Question 9: The median of the data 13, 8, 11, 6, 4, 15, 2, 18, 20 is

Solution: Arranging the data in a ascending order, we get 2, 4, 6, 8, 11, 13, 15, 18, 20

Here n= 9, which is odd number of observations.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{item} = \left(\frac{9+1}{2}\right)^{th} \text{item} = 5^{th} \text{item} = 11$$

Question 10: What is the median for the observations 5, 8, 6, 9, 11 and 4

Solution: We write in ascending order 4, 5, 6, 8, 9 and 11

Here n = 6. So Median = Average of 3^{rd} and 4^{th} term = $\frac{6+8}{2} = 7$

In case of a grouped frequency distribution, we find median from the cumulative frequency distribution of the variableunder consideration.

$$M = l_1 + \left(\frac{\frac{N}{2} - N_l}{N_u - N_l}\right) \times C$$

Where,

l_1 = lower class boundary of the median class i.e. the class containing median.

N = total frequency.

N_l = less than cumulative frequency corresponding to l_1.(Pre median class)

N_u = less than cumulative frequency corresponding to l_2. (Post median class)

l_2 being the upper class boundary of the median class.

$C = l_2 – l_1$ = length of the median class.

Question 11: What is the Median for the following data?

Marks	5-14	15-24	25-34	35-44	45-5	50-59
No.of Stu dents	10	18	32	26	14	10

Solution: First, we find the cumulative frequency distribution which is exhibited in the table

Marks	Frequency (f)	Marks	Less than Cumulative Frequency (CF)
5-14	10	14.5	10
15-24	18	24.5	28(N_l)
25-34	32	34.5	60(N_u)
35-44	26	44.5	86
45-54	14	54.5	100
55-64	10	64.5	110
Total (N)	110		

We find from table N/2 = 55

$\frac{N}{2} = \frac{110}{2}$ = 55 lies between the two cumulative frequencies 28 and 60 i.e. 28 < 55 < 60.

Thus, we have $N_l = 28$, $N_u = 60$, $l_1 = 24.5$ and $l_2 = 34.5$.

Hence C = 34.5 – 24.5 =10.

Substituting these values formula, we get,

$$M = 24.5 + \frac{55-28}{60-28} \times 10 = 24.5 + 8.44 = 32.94$$

Question 12: Find the missing frequency from the following data, given that the median mark is 23.

Marks	0-10	10-20	20-30	30-40	40-50
No.of Students	5	8	?	6	3

Solution: Let us denote the missing frequency by f_3. Following table shows the relevant computation.

Table (Estimation of Missing frequency)

Marks	Less than cumulative frequency
0	0
10	5
20 (l_1)	13(N_l)
30(l_2)	13+f_3(N_u)
40	19+f_3
50	22+f_3

Going through the mark column, we find that 20<23<30. Hence l_1=20, l_2 =30 and accordingly Nl=13, Nu=13+f_3. Also the total frequency i.e. N is 22+f_3. Thus,

$$M = l_1 + \left(\frac{\frac{N}{2} - N_l}{N_u - N_l}\right) \times C$$

$$23 = 20 + \frac{\left(\frac{22+f_3}{2}\right) - 13}{(13+f_3) - 13} \times 10$$

$$3 = + \frac{22 + f_3 - 26}{f_3} \times 5 \ , \ 3f_3 = 5f_3 - 20 \ , \ f_3 = 20$$

f_3 = 10 ,So, the missing frequency is 10.

Properties of median: We cannot treat median mathematically; the way we can do with arithmetic mean. We consider below two important features of median.

(i) If x and y are two variables, to be related by y=a+bx for any two constants a and b, then the median of y is given by

$y_{me} = a + bx_{me}$

For example, if the relationship between x and y is given by 2x – 5y = 10 and if x_{me} i.e. the median of x is known to be 16.

Then 2x – 5y = 10

⇨ y = –2 + 0.40x

⇨ $y_{me} = -2 + 0.40\ x_{me}$

⇨ $y_{me} = -2 + 0.40 \times 16$

⇨ $y_{me} = 4.40$.

(ii) For a set of observations, the sum of absolute deviations is minimum when the deviations are taken from the median. This property states that $\Sigma|x_i - A|$ is minimum if we choose A as the median.

PARTITION VALUES OR QUARTILES : These may be defined as values dividing a given set of observations into a number of equal parts. When we want to divide the given set of observations into two equal parts, we consider median. Similarly, quartiles are values dividing a given set of observations into four equal parts. So there are three quartiles – first quartile or lower quartile denoted by Q_1, second quartile or median to be denoted by Q_2 or Me and third quartile or upper quartile denoted by Q_3. First quartile is the value for which one fourth of the observations are less than or equal to Q_1 and the remaining three – fourths observations are more than or equal to Q_1. In a similar manner, we may define Q_2 and Q_3.

Deciles : Deciles are the values dividing a given set of observation into ten equal parts. Thus, there are nine deciles to be denoted by D_1, D_2, D_3,.....D_9. D_1 is the value for which one-tenth of the given observations are less than or equal to D_1 and the remaining nine-tenth observations are greater than or equal to D_1 when the observations are arranged in an ascending order of magnitude.

percentiles or centiles Percentiles divide a given set of observations into 100 equal parts. The points of sub-divisions being P_1, P_2,..........P_{99}. P_1 is the value for which one hundredth of the observations are less than or equal to P_1 and the remaining ninety-nine hundredths observations are greater than or equal to P_1 once the observations are arranged in an ascending order of magnitude.

For unclassified data, the p[th] quartile is given by the (n+1)p[th] value, where n denotes the total number of observations. p = 1/4, 2/4, 3/4 for Q_1, Q_2 and Q_3 respectively. p=1/10, 2/10,............9/10.

For D_1, D_2,......,D_9 respectively and l astly p=1/100, 2/100,....,99/100 for P_1, P_2, P_3....P_{99} respectively.

In case of a grouped frequency distribution, we consider the following formula for the computation of quartiles.

$$Q = l_1 + \left(\frac{N_p - N_l}{N_u - N_l}\right) \times C$$

The symbols, except p, have their usual interpretation which we have already discussed while computing median and just like the unclassified data, we assign different values to p depending on the quartile.

Another way to find quartiles for a grouped frequency distribution is to draw the ogive (less than type) for the given distribution. In order to find a particular quartile, we draw a line parallel to the horizontal axis through the point Np. We draw perpendicular from the point of intersection of this parallel line and the ogive. The x-value of this perpendicular line gives us the value of the quartile.

Question 13: Following are the wages of the labourers: ₹82, ₹56, ₹90, ₹50, ₹120, ₹75, ₹75, ₹80, ₹130, ₹65. Find Q_1, D_6 and P_{82}.

Solution: Arranging the wages in an ascending order, we get ₹50, ₹56, ₹65, ₹75, ₹75, ₹80, ₹82, ₹90, ₹120, ₹130.

Hence, we have

$Q_1 = \frac{(n+1)}{4}$th value $= \frac{(10+1)}{4}$th value = 2.75th value

= 2nd value + 0.75 × difference between the third and the 2nd values.
= ₹[56 + 0.75 × (65 – 56)] = ₹62.75

D_6 = (10 + 1) × $\frac{6}{10}$ th value = 6.60th value

= 6th value + 0.60 × difference between the 7th and the 6th values.
= ₹(80 + 0.60 × 2) = ₹81.20

P_{82} = (10+1) x $\frac{82}{100}$ th value = 9.02th value

= 9th value + 0.02 × difference between the 10th and the 9th values
= ₹(120 + 0.02 ×10) = ₹120.20

Question 14: Compute the Third Quartile and 65th percentile for the following data

Profits '000 Rs	Less than 10	10-19	20-29	30-39	40-49	50-59
No. of firms	5	18	38	20	9	2

Solution:

Profits'000(Rs.)	Frequency (f)	Cumulative Frequency (CF)
Less than 9.5	5	5
9.5-19.5	18	23
19.5-29.5	38	61
29.5-39.5	20	81
39.5-49.5	9	90
49.5-59.5	2	92
110		

Q_3 = Third Quartile = $\frac{3N}{4} = \frac{3\times 92}{4} = 69$ Q_3 lies 29.5 and 39.5

$$Q_3 = 29.5 + \left(\frac{69-61}{20}\right)\times 10 = 33.5$$

Q_3 = ₹33,500

For 65th percentile = $P_{65} = \frac{iN}{100} = \frac{65\times 92}{100} = 59.8 = 65^{th}$

percentile lies in the class 19.5-29.5, here l= 19.5, c = 23, f = 38 and C = 10

$$P_{65} = 19.5 + \frac{(59.8-23)}{38}\times 10$$

$$P_{65} = 29.184$$

= ₹29,184

Question 15: Compute mode for the distribution for the following distribution

Class Interval	350-369	370-389	390-409	410-429	430-449	450-469
Frequency	15	27 (f-1)	31(fo)	19 (f1)	13	6

Solution: Going through the frequency column, we note that the highest frequency i.e., fo = 31 and f_{-1}= 27, f_1 = 19, LCB = 389.50, C = 409.5-389.50= 20

$$\text{Mode} = 389.5 + \frac{(31-27)}{2\times 31-(27+19)}\times 20$$

$$\text{Mode} = 389.5 + \frac{4}{16}\times 20 = 389.5 + 5 = 394.5$$

Question 16: For a moderately skewed distribution of marks in statistics for a group of 200 students, the mean mark and median mark were found to be 55.60 and 52.40. What is the modal mark?

Solution: Since in this case, mean = 55.60 and median = 52.40, applying, we get the modal mark as,
Mode = 3 × Median – 2 × Mean = 3 × 52.40 – 2 × 55.60 = 46.

Question 17: If x and y related by x-y-10=0 and mode of x is known to be 23, then the mode of y is :

Solution: Mode of x = 23 , x-y-10= 0 then y = x-10 , Mode of y = 23-10 = 13

Geometric Mean: For a given set of n positive observations, the geometric mean is defined as the n-th root of the product of the observations. Thus if a variable x assumes n values $x_1, x_2, x_3, \ldots\ldots$, x_n, all the values being positive, then the GM of x is given by $G = (x_1 \times x_2 \times x_3 \ldots\ldots \times x_n)^{1/n}$

For a grouped frequency distribution, the GM is given by
$G = (x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \ldots\ldots \times x_n^{f_n})^{1/N}$, Where $N = \Sigma f_i$

In connection with GM, we may note the following properties:

(i) Logarithm of G for a set of observations is the AM of the logarithm of the observations

(ii) if all the observations assumed by a variable are constants, say K > 0, then the GM of the observations is also K.

(iii) GM of the product of two variables is the product of their GM's i.e. if z = xy, then
GM of z = (GM of x) × (GM of y)

(iv) GM of the ratio of two variables is the ratio of the GM's of the two variables i.e. if z = x/y then

$$\text{GM of } z = \frac{\text{GM of } x}{\text{GM of } y}$$

Question 18: Find the GM of 8, 24 and 40.
Solution: As given x_1=8, x_2=24, x_3=40 and n=3.
Applying, we have $G = (8\times 24\times 40)^{1/3} = 8\sqrt[3]{15}$

Question 19: If GM of x is 10, and GM of y is 15, then GM of xy
Solution: According to the GM of XY= GM of x * GM of y
= 10×15 = 150

Harmonic Mean: For a given set of non-zero observations, harmonic mean is defined as the reciprocal of the AM of the reciprocals of the observation. So, if a variable x assumes n non-zero values $x_1, x_2, x_3, \ldots\ldots, x_n$, then the HM of x is given by

$$H = \frac{n}{\Sigma(1/x_i)}$$

For a grouped frequency distribution, we have $H = \frac{N}{\Sigma\left[\frac{f_i}{x_i}\right]}$

Properties of HM

(i) If all the observations taken by a variable are constants, say k, then the HM of the observations is also k.

(ii) If there are two groups with n1 and n2 observations and H1 and H2 as respective HM's than the combined HM is given by = $\frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$

Question 20: A man travels at a speed of 20 km/ hr and then returns at a speed of 30 km/hr. His average of the whole journey is

Solution: Harmonic Mean is the method which is preferred for the computation of average speed

$$HM = \frac{2ab}{a+b} = \frac{2 \times 20 \times 30}{20+30} = 24\text{km/hr}$$

Question 21: Find the HM for 4, 6 and 10.

Solution: Applying formula, we have

$$H = \frac{3}{\frac{1}{4}+\frac{1}{6}+\frac{1}{10}} = \frac{3}{0.25+0.17+0.10} = 5.77$$

Question 22: An aeroplane flies from A to B at the rate of 500 km/ hr and comes back B to A at the rate of 700 km / hr . The average speed of the aeroplane is;

Solution: Required average speed of the aeroplane =

$$\frac{2}{\left(\frac{1}{500}+\frac{1}{700}\right)} = \frac{2 \times 3500}{7+5} = 583.33\text{km / hr}$$

Question 23: Find the HM for the following data:

x	2	4	8	16
f	2	3	3	2

Solution: Using formula, we get $H = \frac{10}{\frac{2}{2}+\frac{3}{4}+\frac{3}{8}+\frac{2}{16}} = 4.44$

Relation between AM, GM, and HM → For any set of positive observations, we have the following inequality: AM ≥ GM ≥HM → The equality sign occurs, as we have already seen, when all the observations are equal.

Question 24: compute AM, GM, and HM for the numbers 6, 8, 12, 36.

Solution: In accordance with the definition, we have

$$AM = \frac{6+8+12+36}{4} = 15.50$$

$$GM = (6 \times 8 \times 12 \times 36)^{1/4}$$
$$= (2^8 \times 3^4)^{1/4} = 12$$

$$HM = \frac{4}{\frac{1}{6}+\frac{1}{8}+\frac{1}{12}+\frac{1}{36}} = 9.93$$

The computed values of AM, GM, and HM establish AM ≥ GM ≥HM

Question 25: If there are two groups with 75 and 65 as harmonic means and containing 15 and 13 observations, then the combined HM is given by

Solution: Combined HM is given by $= \frac{n_1+n_2}{\left(\frac{n_1}{H_1}+\frac{n_2}{H_2}\right)} = \frac{15+13}{\left(\frac{15}{75}+\frac{13}{65}\right)} = 70$

Weighted average

When the observations under consideration have a hierarchical order of importance, we take recourse to computing weighted average, which could be either weighted AM or weighted GM or weighted HM.

$$\text{Weighted AM} = \frac{\sum w_i x_i}{\sum w_i}$$

$$\text{Weighted GM} = \text{Ante log}\left(\frac{\sum w_i \log x_i}{\sum w_i}\right)$$

$$\text{Weighted HM} = \frac{\sum w_i}{\sum\left(\frac{w_i}{x_i}\right)}$$

FOUNDATION: PAPER 3 - BUSINESS MATHEMATICS LOGICAL REASONING AND STATISTICS

At the foundation level with regards to Paper 3 Statistics part of the topic Measures of Dispersion is very important for students not only to acquire professional knowledge but also for examination point of view. Here in this capsule an attempt is made for solving and understanding the concepts of Measures of Dispersion with the help of following questions with solutions

CHAPTER 15 UNIT-II: MEASURES OF DISPERSION

The second important characteristic of a distribution is given by dispersion. Two distributions may be identical in respect of its first important characteristic i.e. central tendency and yet they may differ on account of scatterness. The following figure shows a number of distributions having identical measure of central tendency and yet varying measure of scatterness. Obviously, distribution is having the maximum amount of dispersion.

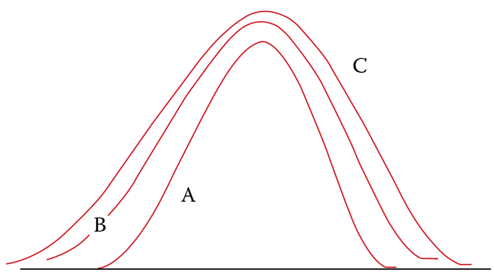


Figure: Showing distributions with identical measure of central tendency and varying amount of dispersion.

Dispersion

for a given set of observations may be defined as the amount of deviation of the observations, usually, from an appropriate measure of central tendency. Measures of dispersion may be broadly classified into the following:

Absolute measures of dispersion

(i) Range
(ii) Mean Deviation
(iii) Standard Deviation
(iv) Quartile Deviation

Relative measures of dispersion

(i) Coefficient of range.
(ii) Coefficient of Mean Deviation
(iii) Coefficient of Variation
(iv) Coefficient of Quartile Deviation

Distinction between the absolute and relative measures of dispersion :

- Absolute measures are dependent on the unit of the variable under consideration whereas the relative measures of dispersion are unit free.
- For comparing two or more distributions, relative measures and not absolute measures of dispersion are considered.
- Compared to absolute measures of dispersion, relative measures of dispersion are difficult to compute and comprehend.

Characteristics for an ideal measure of dispersion

An ideal measure of dispersion should be properly defined, easy to comprehend, simple to compute, based on all the observations, unaffected by sampling fluctuations and amenable to some desirable mathematical treatment

Range :
- For a given set of observations, range may be defined as the difference between the largest and smallest of observations.
- Thus if L and S denote the largest and smallest observations respectively thenRange = L – S

Coefficient of Range = $\frac{L-S}{L+S}\times 100$

For a grouped frequency distribution: Range is defined as the difference between the two extreme class boundaries. The corresponding relative measure of dispersion is given by the ratio of the difference between the two extreme class boundaries to the total of these class boundaries, expressed as a percentage.

Range remains unaffected due to a change of origin but affected in the same ratio due to a change in scale i.e., if for any two constants a and b, two variables x and y are related by y = a + bx, Then the range of y is given by
$R_y = |b| \times R_x$

Example 1: Following are the wages of 8 workers expressed in rupees: 80, 65, 90, 60, 75, 70, 72, 85. Find the range and find its coefficient.

Solution: The largest and the smallest wages are L = ₹90 and S= ₹60
Thus range = ₹90 – ₹60 = ₹30

Coefficient of range = $\frac{90-60}{90+60}\times 100 = 20$

Example 2: What is the range and its coefficient for the following distribution of weights?

Weight in kgs	10-19	20-29	30-39	40-49	50-59
No. of Students	11	25	16	7	3

Solution: The lowest class boundary is 9.50 kgs. and the highest-class boundary is 59.50 kgs. Thus, we have
Range = 59.50 kgs. – 9.50 kgs. = 50 kgs.

Also, coefficient of range = $\frac{59.50-9.50}{59.50+9.50}\times 100 = \frac{50}{69}\times 100 = 72.46$

Example 3: If the relationship between x and y is given by 2x+3y=10 and the range of x is ₹15, what would be the range of y?

Solution: Since $2x+3y=10$

Therefore, $y = \frac{10}{3} - \frac{2}{3}x$, Applying the range of y is given by

$R_y = |b| \times R_x$ = 2/3 × ₹15 = ₹10.

Mean Deviation : Since range is based on only two observations, it is not regarded as an ideal measure of dispersion. A better measure of dispersion is provided by mean deviation which, unlike range, is based on all the observations. For a given set of observation, mean deviation is defined as the arithmetic mean of the absolute deviations of the observations from an appropriate measure of central tendency. Hence if a variable x assumes n values x_1, x_2, x_3...x_n, then the mean deviation of x about an average A is given by

$MD_A = \frac{1}{n}\sum|x_i - A|$ For a grouped frequency distribution, mean deviation about A is given by $MD_A = \frac{1}{n}\sum|x_i - A|.f_i$

Where x_i and f_i denote the mid value and frequency of the i^{th} class interval and $N = \sum f_i$

In most cases we take A as mean or median and accordingly, we get mean deviation about mean or mean deviation about median or mode.

A relative measure of dispersion applying mean deviation is given by

$$\text{Coefficient of Mean Deviation} = \frac{\text{Mean deviation about A}}{A} \times 100$$

Mean deviation takes its minimum value when the deviations are taken from the median. Also mean deviation remains unchanged due to a change of origin but changes in the same ratio due to a change in scale i.e. if y = a + bx, a and b being constants,

then MD of y = |b| × MD

Example 4: What is the mean deviation about mean for the following numbers?

50,60,50,50,60,60,60,50,50,50,60,60,60,50.

Solution: The mean is given by

$$\bar{X} = \frac{50+60+50+50+60+60+60+50+50+50+60+60+60+50}{14} = \frac{770}{14} = 55$$

x_i	50	60	50	50	60	60	60	50	50	50	60	60	60	50	Total
$\lvert x_i-\bar{x}\rvert$	5	5	5	5	5	5	5	5	5	5	5	5	5	5	70

Thus, mean deviation about mean is given by $\frac{\sum|x_i-\bar{x}|}{n} = \frac{70}{14} = 5$

Example. 5 : The coefficient of Mean Deviation about the first 9 natural numbers ?

Solution: The Mean of first 9 natural numbers = $\frac{n+1}{2} = \frac{9+1}{2} = 5$

coefficient of Mean Deviation about the first 9 natural numbers =

$$\frac{\text{Mean deviation about A}}{A} \times 100 = \frac{\frac{20}{9}}{5} = \frac{4}{9} \times 100 = \frac{400}{9}$$

Example. 6 : . The mean deviation about the mode for the following observations 4/11, 6/11, 8/11, 9/11, ,12/11, 8/11 is

Solution: For the 4/11, 6/11, 8/11, 9/11, 12/11, 8/11 Mode is 8/11

$$\text{Mean deviation from Mode} = \frac{\sum|x_i\text{-Mode}|}{n}$$

$$= \frac{\left|\frac{4}{11}-\frac{8}{11}\right| + \left|\frac{6}{11}-\frac{8}{11}\right| + \left|\frac{8}{11}-\frac{8}{11}\right| + \left|\frac{9}{11}-\frac{8}{11}\right| + \left|\frac{12}{11}-\frac{8}{11}\right| + \left|\frac{8}{11}-\frac{8}{11}\right|}{6}$$

$$= \frac{\frac{4}{11}+\frac{2}{11}+0+\frac{1}{11}+\frac{4}{11}+0}{6} = \frac{\frac{11}{11}}{6} = \frac{1}{6}$$

Example 7: Find mean deviations about median and the corresponding coefficient for the following profits ('000₹) of a firm during a week. 82, 56, 75, 70, 52, 80, 68.

Solution: The profits in thousand rupees is denoted by x. Arranging the values of x in an ascending order, we get 52, 56, 68, 70, 75, 80, 82. Therefore, Median = $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{7+1}{2}\right)^{th}$ item = 4^{th} item = 70, thus,

Median profit = ₹70,000.

Computation of Mean deviation about median

x_i	52	56	68	70	75	80	82	Total
$\lvert x_i\text{-Me}\rvert$	18	14	2	0	5	10	12	61

Thus mean deviation about median $\frac{\sum|x_i - \text{Median}|}{n} = ₹\frac{61}{7} \times 1000$ = ₹8714.29

$$\text{Coefficient of mean deviation} = \frac{\text{MD about median}}{\text{Median}} \times 100$$

$$= \frac{8714.29}{70000} \times 100 = 12.45$$

Example 8: Compute the mean deviation about the arithmetic mean for the following data:

Variable (x)	5	10	15	20	25	30
Frequency (f)	3	4	6	5	3	2

Solution: We are to apply formula as these data refer to a grouped frequency distribution the AM is given by

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{5\times3+10\times4+15\times6+20\times5+25\times3+30\times2}{3+4+6+5+3+2} = 16.52$$

$$\text{Mean deviation from Mean} = \frac{\sum f_i.|x_i - \bar{x}|}{n} = \frac{139.56}{23} = 6.07$$

Coefficient of MD about its AM = $\frac{\text{MD about AM}}{\text{AM}} \times 100 = \frac{6.07}{16.52} \times 100 =$ 36.73

Example 9: The mean and SD for a, b and 2 are 3 and $\frac{2}{\sqrt{3}}$ respectively, The value of ab would be

Solution: Here the mean a, b and 2 ($\bar{x}$) = 3 , $\bar{x} = \frac{a+b+2}{3}$, 9 = a+b+2

Then a +b = 9-2= 7

$$\text{Standard deviation} = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$$

$$\frac{2}{\sqrt{3}} = \sqrt{\frac{\sum x^2}{n} - (3)^2}$$

$$\Rightarrow \frac{4}{3} = \frac{\sum x^2}{3} - (3)^2$$

$$\frac{4}{3} = \frac{\sum x^2}{3} - 9$$

$$\sum x^2 = 27 + 4 = 31 \Rightarrow \sum x^2 = 31$$

$$a^2 + b^2 + 2^2 = 31$$

$$a^2 + b^2 = 31 - 4 = 27$$

$$(a+b)^2 - 2ab = a^2 + b^2$$

$(7)^2 - 2ab = 27$, 2ab = 49-27

2ab = 22

ab = 11

Example 10: If x and y are related as 4x+3y+11 = 0 and mean deviation of x is 5.40, what is the mean deviation of y?

Solution: Since 4x + 3y + 11 = 0

Therefore, $y = \left(\frac{-11}{3}\right) + \left(\frac{-4}{3}\right)x$

Hence MD of y= |b| × MD of x $= \frac{4}{3} \times 5.40 = 7.20$

Standard Deviation: Although mean deviation is an improvement over range so far as a measure of dispersion is concerned, mean deviation is difficult to compute and further more, it cannot be treated mathematically. The best measure of dispersion is, usually, standard deviation which does not possess the demerits of range and mean deviation.

Standard deviation for a given set of observations is defined as the root mean square deviation when the deviations are taken from the AM of the observations. If a variable x assumes n values x_1, x_2, x_3 ....x_n then its standard deviation(s) is given by

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

For a grouped frequency distribution, the standard deviation is given by

$$S = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{N}}$$

can be simplified to the following forms for unclassified data

$$S = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2}$$

$$= \sqrt{\frac{\sum f_i x_i^2}{N} - \bar{x}^2}$$ for a grouped frequency distribution.

Variance: The square of standard deviation, known as variance

Variance = $s^2 = \sqrt{\frac{\sum x_i - \bar{x}^2}{n}}$ for unclassified data

$= \sqrt{\frac{\sum f_i (x_i - \bar{x}^2)}{N}}$ for a grouped frequency distribution

Coefficient of variation (CV) $= \frac{SD}{AM} \times 100$ (A relative measure of dispersion using standard deviation is given by Coefficient of Variation (CV) which is defined as the ratio of standard deviation to the corresponding arithmetic mean, expressed as a percentage.)

Example 11: Find the standard deviation and the coefficient of variation for the following numbers: 5, 8, 9, 2, 6

Solution: We present the computation in the following table: **Computation of standard deviation**

x_i	5	8	9	2	6	$\Sigma x_i = 30$
x_i^2	25	64	81	4	36	$\Sigma x_i^2 = 210$

Applying, we get the standard deviation as

$$= \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2} = \sqrt{\frac{210}{5} - \left(\frac{30}{5}\right)^2} \quad \left(\text{since } \bar{x} = \frac{\sum x_i}{n}\right)$$

$= \sqrt{42-36} = \sqrt{6} = 2.45$

The coefficient of variation is $CV = 100 \times \frac{SD}{AM} = 100 \times \frac{2.45}{6} = 40.83$

We consider the following formula for computing standard deviation from grouped frequency distribution with a view to saving time and computational labour:

$$S = \sqrt{\frac{\sum f d_i^2}{N} - \left(\frac{\sum f d_i}{N}\right)^2} \times C, \text{ Where } d_i = \frac{x_i - A}{C}$$

Properties of standard deviation

1. If all the observations assumed by a variable are constant i.e. equal, then the SD is zero. This means that if all the values taken by a variable x is k, say , then s = 0. This result applies to range as well as mean deviation.

2. SD remains unaffected due to a change of origin but is affected in the same ratio due to a change of scale i.e., if there are two variables x and y related as y = a+bx for any two constants a and b, then SD of y is given by $s_y = |b|\, s_x$

3. If there are two groups containing n_1 and n_2 observations, 1 and 2 as respective AM's, s_1 and s_2 as respective SD's, then the combined SD is given by

$$s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$ where, $d_1 = \bar{x}_1 - \bar{x}$, $d_2 = \bar{x}_2 - \bar{x}$ and

$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$ = combined AM

This result can be extended to more than 2 groups,we have

$s = \sqrt{\frac{\sum n_i s_i^2 + \sum n_i d_i^2}{\sum n_i}}$ With $d = x_i - \bar{x}$ and $\bar{x} = \frac{\sum n_i \bar{x}_i}{\sum n_i}$

Where $\bar{x}_1 = \bar{x}_2$ is reduced to $s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}}$

4. For any two numbers a and b, standard deviation is given by $\frac{|a-b|}{2}$

5.SD of first n natural numbers is SD $= \frac{\sqrt{n^2 - 1}}{12}$

Example 12: If the S.D. of x is 3, what is the variance of (5 – 2x)?

Solution: If y = a + bx, then $\sigma_y = |b|\, \sigma_x$

Let y = 5 – 2x

$\therefore\ \sigma_y = |-2|\, \sigma x$

= 2 × 3 = 6

∴ Variance (5 – 2x) = $(2)^2 \times 9 = 36$

Example 13: The coefficient of variation of the following numbers 53, 52, 61, 60 64, is

Solution: $\bar{x} = \frac{(53+52+61+60+64)}{5} = 58$

$$\therefore\ \sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

$$\therefore\ \sigma = \sqrt{\frac{(-5)^2 + (-6)^2 + 3^2 + 2^2 + 6^2}{5}} = 4.69.$$

Coefficient of variation $= \frac{\text{S.D.}}{\text{A.M.}} \times 100$

$= \frac{4.69}{58} \times 100 = 8.09.$

Example 14: What is the standard deviation of 5,5,9,9,9,10,5,10,10?

Solution: Mean = $\frac{3\times5+3\times9+3\times10}{9} = \frac{72}{9} = 8.$

Standard Deviation = $\sqrt{\frac{\sum f\left(x_i-\bar{x}\right)^2}{n}} = \sqrt{\frac{3(9)+3(1)+3(4)}{9}} = \sqrt{42/9} = 2.16.$

Example 15: If x and y are related by 2x + 3y + 4 = 0 and S.D. of x is 6, then S.D. of y is :

Solution: $y = \frac{-2}{3}x - \frac{4}{3};$

$\sigma_y = |-2|\ 3|\ \sigma_x = (2/3)\ 6 = 4.$

Example 16: If x and y are related by y = 2x + 5 and the S.D. and A.M. of x are known to be 5 and 10 respectively, then the coefficient of variation of y is:

Solution: Y = 2x+5

$\sigma_y = |-2|\ \sigma_x = 2 \times 5 = 10.$

Also $\bar{y} = 2\bar{x} + 5 = 20+5 = 25$

Coefficient of variation of y = $\frac{\sigma_y}{\bar{y}}\times100 = \frac{10}{25}\times100 = 40.$

Example 17: If the mean and S.D. of x are a and b respectively, then the S.D. of $\frac{x-a}{b}$ is

Solution: Let $y = \frac{(x-a)}{b} = \frac{1}{b}.x - \frac{a}{b}$

$\sigma_y = \left|\frac{1}{b}\right|\ \sigma_x$

$\frac{1}{b}b = 1.$

Example 18: If x and y are related by 3y = 7x – 9 and the S.D. of y is 7, then what is he variance of x?

Solution: 3y = 7x – 9

$X = \frac{3}{7}y + 9$

Also $\sigma_x = \left|\frac{3}{7}\right|\ \sigma_y = \frac{3}{7}\times7 = 3.$

$\therefore$ Variance : $\sigma_x^2 = 3^2 = 9$

Example 19: Which of the following companies A and B is more consistent so far as the payment of dividend is concerned ?

Dividend paid by A	5	9	6	12	15	10	8	10
Dividend paid by B:	4	8	7	15	18	9	6	6

Solution: Here $\sum x_A = 75$

$\therefore \bar{x}_A = 75/\ 8 = 9.375$

$\sum x_A^2 = 775$

$\sigma_A^2 = \frac{\sum x_A^2}{N} - \left(\frac{\sum x_A}{N}\right)^2$

$= \frac{775}{8} - \left(\frac{75}{8}\right)^2 = 9$

$\sigma_A = 3.$

$C.V_A = \frac{\sigma_A}{\bar{x}}\times100 = \frac{3}{9.375}\times100 = 32.$

Also $\Sigma x_B = 73,\ \therefore\ \bar{x}_B = \frac{73}{8} = 9.125$

$\Sigma x_B^2 = 831,$

$\sigma_B^2 = \frac{831}{8} - \left(\frac{73}{8}\right)^2 = 20.61$

$\therefore\ \sigma = \sqrt{20.61} = 4.54$

$C.V_B = \frac{4.54}{9.125}\times100 = 49.75$

C.VA < C.VB

Company A is more consistent

Example 20: Find the SD of the following distribution:

Weight(kgs)	50-52	52-54	54-56	56-58	58-60
No.of Students	17	35	28	15	5

Solution:

Weight	No .of Students	Mid – Value	$d_i = x_i - 55$	$f_i d_i$	$f_i d_i^2$
50-52	17	51	-2	-34	68
52-54	35	53	-1	-35	35
54-56	28	55	0	0	0
56-58	15	57	1	15	15
58-60	5	59	2	10	20
Total	100			-44	138

Applying, we get the SD of weight as

$= \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}\ xC = \sqrt{\frac{138}{100} - \frac{(-44)^2}{100}} \times 2\text{ kgs.} = \sqrt{1.38 - 0.1936} \times 2\text{ kgs.}$

=2.18 kgs

Example 21: The mean and variance of the 10 observations are found to be 17 and 33 respectively. Later it is found that one observation (i.e.26) is inaccurate and is removed. What is mean and standard deviation of remaining?

Solution: Mean of 10 observations = 17 then Total of the observations = 17 ×10 = 170

Total of the 9 observations = 170-26 = 144

Changed Mean = 144/9 = 16

Variance (σ^2) = 33

$\frac{\sum x^2}{n} - (17)^2 = 33 \Rightarrow \frac{\sum x^2}{10} = 33 + 289 = 322$

$\frac{\sum x^2}{10} = 322$

$\sum x^2 = 3220 - (26)^2 = 3220 - 676 = 2544$

Changed Variance = $\frac{\text{Changed} \sum x^2}{n} - (\text{Changed } \bar{x})^2$

$= \frac{2544}{9} - (16)^2 = 26.67$

SD of remaining observations = $\sqrt{26.67} = 5.16$

Example 22: If AM and coefficient of variation of x are 10 and 40 respectively, what is the variance of (15–2x)?

Solution: let y = 15 – 2x; AM of x= 10

Then applying formula, we get,

$s_y = 2 \times s_x$

As given cv_x = coefficient of variation of x = 40 and= 10

Thus $cv_x = \frac{S_x}{\bar{x}}\times100 \Rightarrow 40 = \frac{S_x}{10}\times100$

$\Rightarrow s_x = 4$

Then, S_y = 2 x 4 = 8 Therefore, variance of (15-2x) = S_y^2 = 64

Example 23: Compute the SD of 9, 5, 8, 6, 2. Without any more computation, obtain the SD of

Sample I	-1	-5	-2	-4	-8
Sample II	90	50	80	60	20
Sample III	23	15	21	17	9

Solution:

x_i	9	5	8	6	2	30
x_i^2	81	25	64	36	4	210

The SD of the original set of observations is given by

$$s = \sqrt{\frac{210}{5} - \left(\frac{30}{5}\right)^2} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} = \sqrt{42-36} = \sqrt{6} = 2.45$$

If we denote the original observations by x and the observations of sample I by y, then we have

$y = -10 + x$

$y = (-10) + (1)\,x$

$\therefore S_y = |1| \times S_x = 1 \times 2.45 = 2.45$

In case of sample II, x and y are related as

$Y = 10x = 0 + (10)x$

$\therefore S_y = |10| \times S_x$

$= 10 \times 2.45 = 24.50$

And lastly, $y = (5) + (2)x$

$\Rightarrow S_y = 2 \times 2.45 = 4.90$

Example 24: For a group of 60 boy students, the mean and SD of stats. marks are 45 and 2 respectively. The same figures for a group of 40 girl students are 55 and 3 respectively. What is the mean and SD of marks if the two groups are pooled together?

Solution: As given $n_1 = 60$, $\bar{x}_1 = 45$, $s_1 = 2$ $n_2 = 40$, $\bar{x}_1 = \bar{x}_2 = 55$, $s_2 = 3$

Thus the combined mean is given by

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{60 \times 45 + 40 \times 55}{60 + 40} = 49$$

Thus $d_1 = \bar{x}_1 = \bar{x} = 45 - 49 = -4$

$d_2 = \bar{x}_2 - \bar{x} = 55 - 49 = 6$

Applying formula, we get the combined SD as

$$s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

$$s = \sqrt{\frac{60 \times 2^2 + 40 \times 3^2 + 60 \times (-4)^2 + 40 \times 6^2}{60 + 40}}$$

$= \sqrt{30}$

$= 5.48$

Example 25: The mean and standard deviation of the salaries of the two factories are provided below:

Factory	No. of Employees	Mean Salary	SD of Salary
A	30	₹4800	₹10
B	20	₹5000	₹12

(i) Find the combined mean salary and standard deviation of salary.

(ii) Examine which factory has more consistent structure so far as satisfying its employees are concerned.

Solution: Here we are given

$n_1 = 30$, $\bar{x}_1 = ₹4800$, $s_1 = ₹10$,

$n_2 = 20$, $\bar{x}_2 = ₹5000$, $s_2 = ₹12$

(i) Combined mean = $\frac{30 \times ₹4800 + 20 \times 5000}{30 + 20}$ ₹4880

$d_1 = \bar{x}_1 = \bar{x} = ₹4{,}800 - ₹4880 = -₹80$

$d_2 = \bar{x}_2 - \bar{x} = ₹5{,}000 - ₹4880 = ₹120$

hence, the combined SD in rupees is given by

$$s = \sqrt{\frac{30 \times 10^2 + 20 \times 12^2 + 30 \times (-80)^2 + 20 \times 120^2}{30 + 20}} = \sqrt{9717.60} = 98.58$$

thus the combined mean salary and the combined standard deviation of salary are ₹4880 and ₹98.58 respectively.

(ii) In order to find the more consistent structure, we compare the coefficients of variation of the two factories.

Letting $CV_A = 100 \times \frac{S_A}{\bar{x}_A}$ and $CV_B = 100 \times \frac{S_B}{\bar{x}_B}$

We would say factory A is more consistent

if $CV_A < CV_B$. Otherwise factory B would be more consistent.

Now $CV_A = 100 \times \frac{S_1}{\bar{x}_1} = \frac{100 \times 10}{4800} = 0.21$

and $CV_B = 100 \times \frac{S_2}{\bar{x}_2} = \frac{100 \times 12}{5000} = 0.24$

Thus we conclude that factory A has more consistent structure.

Quartile Deviation: Another measure of dispersion is provided by quartile deviation or semi - inter –quartile range which is given by

$$Q_d = \frac{Q_3 - Q_1}{2}$$

A relative measure of dispersion using quartiles is given by coefficient of quartile deviation which is

$$\text{Coefficient of quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

Merits

1) Quartile deviation provides the best measure of dispersion for open-end classification.
2) It is also less affected due to sampling fluctuations.
3) Like other measures of dispersion, quartile deviation remains unaffected due to a change of origin but is affected in the same ratio due to change in scale.

Example 26: The quartiles of a variable are 45, 52, and 65 respectively. Its quartile deviation is:

Solution: Quartile Deviation = $\frac{Q_3 - Q_1}{2} = \frac{65 - 45}{2} = 10$

Example 27: If x and y are related as 3x + 4y = 20 and the quartile deviation of x is 12, then the quartile deviation of y is

Solution: If $y = ax + b$

Q.D of y = a × (Q.D. of x)

$3x + 4y = 20$

then $y = \frac{-3}{4}x + 5$

Q.D. of y = (3/4) (Q.D. of x)

= |(-3/4)| 12 = 9.

Example 28: Following are the marks of the 10 students : 56, 48, 65, 35, 42, 75, 82, 60, 55, 50. Find quartile deviation and also its coefficient.

Solution: After arranging the marks in an ascending order of magnitude, we get 35, 42, 48, 50, 55, 56, 60, 65, 75, 82

First quartile observation $Q_1 = \frac{(n+1)}{4}$th observation$=\frac{(10+1)}{4}$th observation

= 2.75^{th} observation

= 2^{nd} observation + 0.75 × difference between the third and the 2^{nd} observation.

= 42 + 0.75 × (48 – 42)

= 46.50

Third quartile (Q_3) = $\frac{3(n+1)}{4}$th observation

= 8.25^{th} observation

= 65 + 0.25 × 10

= 67.50

Thus applying, we get the quartile deviation as

$$\frac{Q_3 - Q_1}{2} = \frac{67.50 - 46.50}{2} = 10.50$$

Also, using the coefficient of quartile deviation = $\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

$$= \frac{67.50 - 46.50}{67.50 + 46.50} = 18.42$$

Example 29: If the quartile deviation of x is 6 and 3x + 6y = 20, what is the quartile deviation of y?

Solution: 3x + 6y = 20

$$y = \left(\frac{20}{6}\right) + \left(\frac{-3}{6}\right)x$$

Therefore, quartile deviation of y = $\frac{|-3|}{6}$ X quartile deviation of X

$$= \frac{1}{2} \times 6 = 3$$

Example 30: Find an appropriate measures of dispersion from the following data:

Daily wages (₹)	upto 20	20-40	40-60	60-80	80-100
No. of workers	5	11	14	7	3

Solution: Since this is an open-end classification, the appropriate measure of dispersion would be quartile deviation as quartile deviation does not taken into account the first twenty five percent and the last twenty five per cent of the observations.

Here a denotes the first Class Boundary

Table Computation of Quartile

Daily wages in ₹ (Class boundary)	No. of workers (less than cumulative frequency)
a	0
20	5
40	16
60	30
80	37
100	40

$$Q_1 = \left[20 + \frac{10-5}{16-5} \times 20\right] = ₹29.09$$

$$Q_3 = \left[40 + \frac{30-16}{30-16} \times 20\right] = 60$$

Thus quartile deviation of wages is given by = $\frac{Q_3 - Q_1}{2} = \frac{₹60 - ₹29.09}{2}$

= ₹15.46

Example 31: The mean and variance of 5 observations are 4.80 and 6.16 respectively. If three of the observations are 2, 3 and 6, what are the remaining observations?

Solution: Let the remaining two observations be a and b, then as given

$$\frac{2+3+6+a+b}{5}$$

⇨ 11+a+b =24 ⇨ a+b =13 .... (1)

and $\frac{2^2 + a^2 + b^2 + 3^2 + 6^2}{5} - (4.80)^2$ ⇨ $\frac{49^2 + a^2 + b^2}{5} - 23.04 = 6.16$

⇨ 49 + a^2 + b^2 =146

⇨ a^2 + b^2 =97 .... (2)

From (1), we get a = 13 – b .... (3)

Eliminating a from (2) and (3), we get

$(13 - b)^2 + b^2 = 97$ ⇨ $169 - 26b + 2b^2 = 97$

⇨ $b^2 - 13b + 36 = 0$

⇨ (b–4)(b–9) =0

⇨ b = 4 or 9

From (3), a= 9 or 4

Thus the remaining observations are 4 and 9.

Example 32: If Standard deviation of x is σ , then standard deviation of $\frac{ax+b}{c}$, where a, b and c are constants, will be , then SD of y will be

Solution: SD of X = σ.

Let $y = \frac{ax+b}{c} = \frac{ax}{c} + \frac{b}{c}$

$y = \frac{b}{c} + \frac{ax}{c}$

SD of y = $\left|\frac{a}{c}\right|$SD of x = $\left|\frac{a}{c}\right|.\sigma$

Example 33: Find at the variance given arithmetic mean = $\frac{(8+4)}{2}$

Solution: Here Largest Value (L) = 8

Smallest Value (S) = 4

Range = Largest Value – Smallest Value = 8-4 = 4

We know that SD = $\frac{\text{Range}}{2} = \frac{4}{2} = 2$

Variance = $(SD)^2$= (2) =4

Example 34: If Mean and coefficient of variation of the marks of 10 students is 20 and 80, respectively. What will be the variance of them?

Solution: Given No. of observations (N) = 10

Mean ($\bar{x}$) = 20

CV = 80

$$CV = \frac{SD}{AM} \times 100$$

$$80 = \frac{SD}{20} \times 100$$

$$SD = \frac{80 \times 20}{100} = 16$$

Variance = $(SD)^2$= $(16)^2$= 256

Example 35: If arithmetic mean and coefficient of variation x are 10 and 40 respectively then variance of 15- $\frac{3x}{2}$ will be

Solution: Given Mean of x = 10 , Coefficient of Variation of (x) = 40

$$C\,V \text{ of } X = \frac{\text{SD of X}}{\text{Mean of X}} \times 100$$

400 = SD of x × 100

SD of x = $\frac{400}{100} = 4$

Now $y = 15 - \frac{3x}{2}$

$2y = -30+3x \quad \therefore 2y= 3x-30$

$\therefore y = \frac{3x}{2} - \frac{30}{2} \quad \therefore y = \frac{3x}{2} -15$

$\therefore 3x-2y-30 = 0$

S.D of y = $|b|$ S.D of X

S.D of y = $\left|\frac{3}{2}\right| \times 4 = 6$

Variance of $y = (6)^2 = 36$

Example 36: Coefficient of Quartile deviation is ¼ then $\frac{Q_3}{Q_1}$ is

Solution: Coefficient of QD = $\frac{1}{4}$

$\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{1}{4}$

$4Q_3 - 4Q_1 = Q_3 + Q_1$

$\frac{Q_3}{Q_1} = \frac{5}{3}$

Example 37: SD from numbers 1, 4, 5, 7, 8 is 2.45 . If 10 is added to each then SD will be

Solution: We know a change in origin of SD have no change in SD .
So , New SD = Original Sd when 10 will be added,
So , SD will not change .

Example 38: A student computes the AM and SD for a set of 100 observations as 50 and 5 respectively. Later on, she discovers that she has made a mistake in taking one observation as 60 instead of 50. What would be the correct mean and SD if

i) The wrong observation is left out?

ii) The wrong observation is replaced by the correct observation?

Solution: As given, n = 100, $\bar{x}$ = 50, S = 5
Wrong observation = 60(x), correct observation = 50(V)

$$\bar{x} = \frac{\sum x_i}{n}$$

$$\therefore \sum x_i = n\bar{x} = 100 \times 50 = 5000$$

$$\text{and } s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$\therefore \sum x_i^2 = n\ (\bar{x}^2 + s^2) = 100(50^2 + 5^2) = 252500$$

i) Sum of the 99 observations = 5000 – 60 = 4940
AM after leaving the wrong observation = 4940/99 = 49.90
Sum of squares of the observation after leaving the wrong observation
= 252500 – 60^2 = 248900
Variance of the 99 observations = 248900/99 – $(49.90)^2$
= 2514.14 – 2490.01
= 24.13
∴ SD of 99 observations = 4.91

ii) Sum of the 100 observations after replacing the wrong observation by the correct observation = 5000 – 60 + 50 =4990

AM = $\frac{4990}{100}$ = 49.90

Corrected sum of squares = 252500 + 50^2 – 60^2 = 251400

Corrected SD = $\sqrt{\frac{251400}{100} - (49.90)^2}$

= $\sqrt{23.94}$ = 4.90

Example 39: Compute coefficient of variation from the following data:

Age	under 10	under 20	under 30	under 40	under 50	under 60
No. of persons Dying	10	18	30	45	60	80

Solution: Given in this problem is less than cumulative frequency distribution. We need first convert it to a frequency distribution and then compute the coefficient of variation.

Table : Computation of coefficient of variation

Class Interval Age in years	No. of persons dying	Mid-value	$d_i = \frac{x_i - 25}{10}$	$f_i d_i$	$f_i d_i^2$
	(f_i)	(x_i)	10		
0-10	10	5	–2	–20	40
10-20	18–10= 8	15	–1	–8	8
20-30	30–18=12	25	0	0	0
30-40	45–30=15	35	1	15	15
40-50	60–45=15	45	2	30	60
50-60	80–60=20	55	3	60	180
Total	80	–	–	77	303

The AM is given by:

$= \bar{x} = A + \frac{\sum f_i d_i}{N} \times C$

$= \left(25 + \frac{77}{80} \times 10\right)$ years

= 34.63 years

The standard deviation is

$s = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times C$

$= \sqrt{\frac{303}{80} - \left(\frac{77}{80}\right)^2} \times 10$ years

$= \sqrt{3.79 - 0.93} \times 10$ years

= 16.91 years

Thus the coefficient of variation is given by

$CV = \frac{S}{\bar{X}} \times 100$

$= \frac{16.91}{34.63} \times 100 = 48.83$

Comparison between different measures of dispersion

We may now have a review of the different measures of dispersion on the basis of their relative merits and demerits.

1. Standard deviation, like AM, is the best measure of dispersion. It is rigidly defined, based on all the observations, not too difficult to compute, not much affected by sampling fluctuations and moreover it has some desirable mathematical properties. All these merits of standard deviation make SD as the most widely and commonly used measure of dispersion.
2. Range is the quickest to compute and as such, has its application in statistical quality control. However, range is based on only two observations and affected too much by the presence of extreme observation(s).
3. Mean deviation is rigidly defined, based on all the observations, and not much affected by sampling fluctuations. However, mean deviation is difficult to comprehend and its computation is also time consuming and laborious. Furthermore, unlike SD, mean deviation does not possess mathematical properties.
4. Quartile deviation is also rigidly defined, easy to compute and not much affected by sampling fluctuations. The presence of extreme observations has no impact on quartile deviation since quartile deviation is based on the central fifty-percent of the observations. However, quartile deviation is not based on all the observations and it has no desirable mathematical properties. Nevertheless, quartile deviation is the best measure of dispersion for open-end classifications.

CA FOUNDATION - PAPER 3 - BUSINESS MATHEMATICS, LOGICAL REASONING AND STATISTICS

At the Foundation level the concept of Probability is used in accounting and finance to understand the likelihood of occurrence or non-occurrence of a variable. It helps in developing financial forecasting in which you need to develop expertise at an advanced stage of chartered accountancy course. Here in this capsule an attempt is made for solving and understanding the concepts of probability.

Chapter 16 : Probability

The terms 'Probably' 'in all likelihood', 'chance', 'odds in favour', 'odds against' are too familiar nowadays and they have their origin in a branch of Mathematics, known as Probability. In recent time, probability has developed itself into a full-fledged subject and become an integral part of statistics.

Random Experiment: An experiment is defined to be random if the results of the experiment depend on chance only. For example if a coin is tossed, then we get two outcomes—Head (H) and Tail (T). It is impossible to say in advance whether a Head or a Tail would turn up when we toss the coin once. Thus, tossing a coin is an example of a random experiment. Similarly, rolling a dice (or any number of dice), drawing items from a box containing both defective and non—defective items, drawing cards from a pack of well shuffled fifty—two cards etc. are all random experiments.

Events: The results or outcomes of a random experiment are known as events. Sometimes events may be combination of outcomes. The events are of two types:
(i) Simple or Elementary,
(ii) Composite or Compound.
An event is known to be simple if it cannot be decomposed into further events. Tossing a coin once provides us two simple events namely Head and Tail. On the other hand, a composite event is one that can be decomposed into two or more events. Getting a head when a coin is tossed twice is an example of composite event as it can be split into the events HT and TH which are both elementary events.

Mutually Exclusive Events or Incompatible Events: A set of events A_1, A_2, A_3, ... is known to be mutually exclusive if not more than one of them can occur simultaneously. Thus, occurrence of one such event implies the non-occurrence of the other events of the set. Once a coin is tossed, we get two mutually exclusive events Head and Tail.

Exhaustive Events: The events A_1, A_2, A_3, ... are known to form an exhaustive set if one of these events must necessarily occur. As an example, the two events Head and Tail, when a coin is tossed once, are exhaustive as no other event except these two can occur.

Equally Likely Events or Mutually Symmetric Events or Equi-Probable Events: The events of a random experiment are known to be equally likely when all necessary evidence are taken into account, no event is expected to occur more frequently as compared to the other events of the set of events. The two events Head and Tail when a coin is tossed is an example of a pair of equally likely events because there is no reason to assume that Head (or Tail) would occur more frequently as compared to Tail (or Head).

CLASSICAL DEFINITION OF PROBABILITY OR A PRIORI DEFINITION

Let us consider a random experiment that result in n finite elementary events, which are assumed to be equally likely. We next assume that out of these n events, $n_A(\leq n)$ events are favourable to an event A. Then the probability of occurrence of the event A is defined as the ratio of the number of events favourable to A to the total number of events. Denoting this by P(A), we have

$$P(A) = \frac{n_A}{n} = \frac{\text{Number of equally likely events favourable to A}}{\text{Total Number of equally likely events}}$$

However, if instead of considering all elementary events, we focus our attention to only those composite events, which are mutually exclusive, exhaustive and equally likely and if $m(\leq n)$ denotes such events and is furthermore $m_A(\leq n_A)$ denotes the no. of mutually exclusive, exhaustive and equally likely events favourable to A, then we have

$$P(A) = \frac{m_A}{m} = \frac{\text{"Number of mutually exclusive,exhaustive and equally likely events favourable to A"}}{\text{"Total Number of mutually exclusive,exhaustive and equally likely events"}}$$

PROBABILITY AND EXPECTED VALUE BY MATHEMATICAL EXPECTATION

For this definition of probability, we are indebted to Bernoulli and Laplace. This definition is also termed as a priori definition because probability of the event A is defined based on prior knowledge.

This classical definition of probability has the following demerits or limitations:
(i) It is applicable only when the total no. of events is finite.
(ii) It can be used only when the events are equally likely or equi-probable. This assumption is made well before the experiment is performed.
(iii) This definition has only a limited field of application like coin tossing, dice throwing, drawing cards etc. where the possible events are known well in advance. In the field of uncertainty or where no prior knowledge is provided, this definition is inapplicable.

In connection with classical definition of probability, we may note the following points:
(a) The probability of an event lies between 0 and 1, both inclusive.
i.e. $0 \leq P(A) \leq 1$
When P(A) = 0, A is known to be an impossible event and when P(A) = 1, A is known to be a sure event.
(b) Non-occurrence of event A is denoted by A′ or A^C or and it is known as complimentary event of A. The event A along with its complimentary A′ forms a set of mutually exclusive and exhaustive events.
i.e. $P(A)+P(A')=1$, $P(A')=1-P(A) = 1 - \frac{m_A}{m} = \frac{m - m_A}{m}$

Statistical definition of Probability: Owing to the limitations of the classical definition of probability, there are cases when we consider the statistical definition of probability based on the concept of relative frequency. This definition of probability was first developed by the British mathematicians in connection with the survival probability of a group of people.

Let us consider a random experiment repeated a very good number of times, say n, under an identical set of conditions. We next assume that an event A occurs F_A times. Then the limiting value of the ratio of F_A to n as n tends to infinity is defined as the probability of A.

i.e. $P(A) = \lim_{n \to \infty} \frac{F_A}{n}$

This statistical definition is applicable if the above limit exists and tends to a finite value.

Two events A and B are mutually exclusive if P (A ∩ B) = 0 or more precisely

$P(A \cup B) = P(A) + P(B)$

Similarly, three events A, B and C are mutually exclusive if.

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

Two events A and B are exhaustive if.

$P(A \cup B) = 1$

Similarly, three events A, B and C are exhaustive if.

$P(A \cup B \cup C) = 1$

Three events A, B and C are equally likely if

$P(A) = P(B) = P(C)$

Axiomatic or modern definition of probability: Then a real valued function p defined on s is known as a probability measure and p(a) is defined as the probability of A if P satisfies the following axioms:

(i) $P(A) \leq 0$ for every $A \subseteq S$ (subset)

(ii) $P(S) = 1$

(iii) For any sequence of mutually exclusive events $A_1, A_2, A_3,..$

$P(A_1 U A_2 U A_3 U....) = P(A_1) + P(A_2) + P(A_3)$

Addition theorems or theorems on total probability: For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B.i.e. P (A∪B) or P(A + B) = P(A) + P(B) or P(A or B) whenever A and B are mutually exclusive

For any three events A, B and C, the probability that at least one of the events occurs is given by

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

(d) For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.

i. e. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

For any three events A, B and C, the probability that at least one of the events occurs is given by

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

(e) Two events A and B are mutually exclusive if

$P(A \cup B) = P(A) + P(B)$

Similarly, three events A, B and C are mutually exclusive if $(A \cup B \cup C) = P(A) + P(B) + P(C)$

(f) $P(A-B) = P(A \cap B') = P(A) - P(A \cap B)$

And $P(B-A) = P(B \cap A') = P(B) - P(A \cap B)$

Some important Results

1.If A and B are two independent events, then the probability of occurrence of both is given by $P(A \cap B) = P(A). P(B)$

2.If A, B and C are three events, then. $P(A \cap B \cap C) = P(A). P(B/A). P(C/A \cap B)$

3.If A and B are two mutually exclusive events of a random experiment, then.

$A \cap B = \phi$, $P(AUB) = P(A) + P(B)$

4. If A and B are associated with a random experiment, then.

$P(AUB) = P(A) + P(B) - P(A \cap B)$

5. If A, B and C are three events connected with random experiment, then

$P(AUBUC) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(g) Compound Probability or Joint Probability

$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}$

(h) For any three events A, B and C, the probability that they occur jointly is given by

$P(A \cap B \cap C) = P(A) \quad P(B/A) \quad P(C/(A \cap B))$ Provided $P(A \cap B) > 0$

(i) $P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

(j) $P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$

(k) $P(A'/B') = \frac{P(A' \cap B')}{P(B')}$

$= \frac{P(A \cup B)'}{P(B')}$ [by De-Morgan's Law $A' \cap B' = (A \cup B)'$]

$= \frac{1 - P(A \cup B)}{1 - P(B)}$

CA FOUNDATION - PAPER 3 - BUSINESS MATHEMATICS, LOGICAL REASONING AND STATISTICS

This capsule is in continuation to the previous edition featured in March 2021. It presents the concepts of Random Variable, Expected value, Variance and Standard Deviation of a random variable. These concepts are extensively applied and widely used in areas such as Finance, Risk Management and Costing. Here an attempt is made to enable the students to understand these concepts of probability calculation with the help of examples and help them attempt diverse questions based on these concepts.

Chapter 16 : Probability - II

(7) **A random variable or stochastic variable** is a function defined on a sample space associated with a random experiment assuming any value from R and assigning a real number to each and every sample point of the random experiment.

(8) **Expected value or Mathematical Expectation or Expectation** of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.

When x is a discrete random variable with probability mass function f(x), then its expected value is given by

$$E(x) = \mu = \sum_x x f(x)$$

and its variance is

$$V(x) = \sigma^2 = E(x^2) - \mu^2$$

$$\text{Where } E(x^2) = \sum_x (x)^2 f(x)$$

For a continuous random variable x defined in [–∞, ∞], its expected value (i.e. mean) and variance are given by

$$E(x) = \int x f(x) dx$$

$$\text{and } \sigma^2 = E(x^2) - \mu^2$$

$$\text{where } E(x^2) = \int x^2 f(x) dx$$

Properties of Expected Values

1. Expectation of a constant is k
   i.e. E(k) = k for any constant k
2. Expectation of sum of two random variables is the sum of their expectations.
   i.e. E(x + y) = E(x) + E(y) for any two random variables x and y.
3. Expectation of the product of a constant and a random variable is the product of the constant and the expectation of the random variable.
   i.e. E(k x) = k.E(x) for any constant k
4. Expectation of the product of two random variables is the product of the expectation of the two random variables, provided the two variables are independent.
   i.e. E(xy) = E(x).E(y)
   Where x and y are independent.

IMPORTANT EXAMPLES:

1. A speaks truth in 60% and B in 75% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?
   Solution:
   The Probability that A speaks the truth and B a lie =
   $$\frac{60}{100} \times \frac{(100-75)}{100} = \frac{60}{100} \times \frac{25}{100} = \frac{3}{20}$$
   The Probability that B speaks the truth and A a lie =
   $$\frac{75}{100} \times \frac{(100-60)}{100} = \frac{75}{100} \times \frac{40}{100} = \frac{3}{10}$$
   $$\therefore \text{ Total Probability} = \frac{3}{20} + \frac{3}{10} = \frac{9}{20}$$
   Hence, the percentage of cases in which they contradict each other = (9/20) × 100 or 45%

2. A Committee of 4 persons is to be appointed from 7 men and 3 women. The probability that the committee contains (i) exactly two women, and (ii) at least one woman is
   Solution:
   Total number of persons = 7+3 = 10. Since 4 out of them can be formed in 10 C_4 ways, where the exhaustive number of cases is 10 C_4 or 210 ways.
   (i) P (exactly 2 women in a committee) of four = 7 C_2X 3C_2/ 210 = 63/210 = 3/10.
   (ii) P (at least one women in committee)
   = 1–p (no women) =1-(7C_4/10C_4)=1-(35/210)=1-1/6=5/6

3. If A and B are two events, such that P(A) = 1/4, P(B) = 1/3 and $P(A \cup B)$ = 1/2; then P(B/A) is equal to
   Solution: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
   $1/2 = 1/4 + 1/3 - P(A \cap B)$
   Or $P(A \cap B) = 1/4+1/3-1/2 = 1/12$
   $$\text{Hence, } P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/12}{1/4} = \frac{1}{3}$$

4. A person applies for a job in two firms, say X and Y. the probability of his being selected in firm X is 0.7 and being rejected in firm Y is 0.5. The probability of at least one of his applications being rejected is 0.6. What is the probability that he will be selected in one of the two firms?

Solution:
Event A; Person is selected in firm X, and
Event B : person is selected in Firm Y .
Then, $P(A) = 0.7$, $P(B^c) = 0.5$ and $P(A^c \cup B^c) = 0.6$
Therefore $P(B) = 1-0.5 = 0.5$
$P(A^c \cup B^c) = P[(A \cap B)^C] = 1-P(A \cap B)$
This implies that $P(A \cap B) = 1- P(A^c \cup B^c) = 1-0.6 = 0.4$
Hence, $P(A \cup B) = 0.7+0.5-0.4 = 0.8$

5. A person is known to hit a target in 5 out of 8 shots, whereas another person is known to hit in 3 out of 5 shots. Find the probability that the target is hit at all when they both try.

Solution: Event A = First person hits the target and
Event B = Another person hits the target.
$P(A) = 5/8$ and $P(B) = 3/5$
$P(A^C) = 1-5/8 = 3/8$ and $P(B^C) = 1-3/5 = 2/5$
Event X = target is hit when they both try i.e.,
When at least one of them hit the target.
$P(X^C)$ = P (the target is not hit at all)
$= P(A^C \cap B^C) = P(A^C) \times P(B^C) = 3/20$
Hence $P(X) = 1-P(X^C) = 1-3/20 = 17/20$

6. The probability that a man will be alive in 25 years is 3/5, and the probability that his wife will be alive in 25 years in 2/3. Find the probability that :
(i) Both will be alive (ii) at least one of the will be alive

Solution:
$P(M) = 3/5$ and $P(W) = 2/3$
$P(M^C) = 1 - 3/5$ and $P(W^C) = 1-2/3 = 1/3$.
The probability that both will be alive
$= P(M) \times P(W) = 3/5 \times 2/3 = 2/5$.
Probability that at least one of them will be alive is given by
$P(M \cup W) = P(M) + P(W) - P(M \cap W)$
$= 3/5+2/3-6/15 = 13/15$.

7. Given the data in Previous Problem find the probability that (i) only wife will be alive, (ii) only man will be alive.

Solution.
(i) Probability that only wife will be alive.
= Probability that wife will be alive but not man
$= P(W) \times P(M^C) = 2/3 \times 2/5 = 4/15$
(ii) Probability that only man will be alive
= Probability that man will be alive but not wife
$= P(M) \times P(W^C) = 3/5 \times 1/3 = 1/5$.

8. A random variable X has the following probability distribution:

Value of X	0	1	2	3
P[X = x]	1/3	1/2	0	1/6

Find $E(\{X - E(X)\}^2]$

Solution :
$E(X) = 0\times1/3+1\times1/2+2\times0+3\times1/6 = 1$
$E(X^2) = 0\times1/3+1\times1/2+4\times0+9\times1/6 = 2$
$E[X - E(X)]^2 = E(X^2) - [E(X)]^2 = 2-1 = 1$

9. Given the data in previous Problem, Find Var (Y), where Y = 2X – 1.

Solution:
$E(Y) = E(2X-1) = 2E(X) - 1 = 1$
$E(Y^2) = E(2x-1)^2 = 2E(X^2) - 4E(X)+1 = 1$
Var. $(Y) = E(Y^2) - [E(Y)]^2 = 1-1 = 0$

10 Daily demand for pen drive is having the following probability distribution. Determine the expected demand and variance of the demand:

Demand	1	2	3	4	5	6
Probability	0.10	0.15	0.20	0.25	0.18	0.12

Solution:
$E(X)=1\times0.10+2\times0.15+3\times0.20+4\times0.25+5\times0.18+6\times0.12 = 3.62$
Variance of the demand.
$E(X^2) = 1\times0.10+4\times0.15+9\times0.20+16\times0.25+25\times0.18+36\times0.12= 15.32$
Var. $(X) = 15.32 - (3.62)^2 = 2.22$

11. An investment consultant predicts the odds against the price of a certain stock going up are 2:1 and odds in favor of the price remaining the same are 1:3. What is the price of stock will go down?
Solution:

P(the prices will go up) $= \frac{1}{2+1} = \frac{1}{3}$

P (the prices will remain the same) $= \frac{1}{1+3} = \frac{1}{4}$

Therefore P (the prices will go down)
= P (the price will neither go up nor remain same)
=1-P (the price will go up or will remain the same)

$$= 1 - \left(\frac{1}{3} + \frac{1}{4}\right) = 1 - \frac{7}{12} = \frac{5}{12}$$

12. A pair of dice is rolled. If the sum of the two dice is 9, find the probability that one of the dice shows 3
Solution:
Let A: Sum of on the two dice is 9. B: one of the dice showed 3.
Total outcomes when two dice are thrown = 6×6 = 36

$P(A) = P\{(6,3), (5,4), (4,5), (3,6)\} = \frac{4}{36}$

$P(A \cap B) = P\{(6,3), (3,6)\} = \frac{2}{36}$

Therefore, required probability = P(B/A) =

$$= \frac{P(A \cap B)}{P(A)} = \frac{2/36}{4/36} = \frac{2}{4} = \frac{1}{2}$$

13. The overall percentage of failures in a certain examination was 30. What is the probability that out of a group of 6 candidates at least four passed the examination?

Solution:

Let passing the examination be a success.

Take n= 6, P= P(a student passes) = $1-\frac{30}{100}=\frac{70}{100}=\frac{7}{10}$

q = P (a student fails) = $\frac{30}{100}=\frac{3}{10}$

Therefore P (at least 4 students pass) = P(4 or 5 or 6)

$$= 6_{C_4}\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right)^2 + 6_{C_5}\left(\frac{7}{10}\right)^5\left(\frac{3}{10}\right)^1 + 6_{C_6}\left(\frac{7}{10}\right)^6\left(\frac{3}{10}\right)^0$$

$$=15\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right)^2+6\left(\frac{7}{10}\right)^5\left(\frac{3}{10}\right)+\left(\frac{7}{10}\right)^6$$

$$=\left(\frac{7}{10}\right)^4\left[\frac{135}{100}+\frac{126}{100}+\frac{49}{100}\right]$$

$$=\frac{31}{10}\left(\frac{7}{10}\right)^4=0.74431.$$

14. What is the probability that a leap year selected at random would contain 53 Saturdays?

Solution:

A normal year has 52 Mondays, 52 Tuesdays, 52 Wednesdays, 52 Thursdays, 52 Fridays, 52 Saturdays and 52 Sundays

52 Saturdays, 52 + 1 day that could be anything depending upon the year under consideration.

- In addition to this, a leap year has an extra day which might be a Monday or Tuesday or Wednesday or Sunday.

Our sample space is S: {Monday-Tuesday, Tuesday-Wednesday, Wednesday-Thursday, Thursday-Friday, Friday-Saturday, Saturday-Sunday, Sunday-Monday} =Number of elements in S = n(S) = 7

set A (say) that comprises of the elements Friday-Saturday and Saturday-Sunday i.e. A : {Friday-Saturday, Saturday-Sunday}

Number of elements in set A = n(A) = 2,

By definition, probability of occurrence of A = n(A)/n(S) = 2/7

Therefore, probability that a leap year has 53 Saturdays is =2/7

15. If two unbiased coin is tossed three times, what is the probability of getting more than one head.

Solution:

One toss can give two (2) possible outcomes - head and tail. So, three tosses can give (2×2×2) = 8 possible outcomes.

2 heads and 1 tail out of 3 tosses can occur in $(3C_2)\times(1C_1) = 3$ ways.

So, the probability = (3/8).

16. If two unbiased are rolled, what is the probability of getting points neither 6 nor 9?

Solution:

Two dice can make 6 in 5 ways: {1,5}, {2,4}, {3,3}, {4,2} and {5,1}.

Two dice can make 9 in 4 ways: {3,6}, {4,5}, {5,4} and {6,3}.

There are 36 possible ways the two dice can fall. Therefore, the probability of 6 or 9 is (5+4)/36 = 1/4.

The probability of not (6 or 9) is therefore 1–1/4 = 3/4.

17. What is probability that 4 children selected at random would have different birthdays

Solution:

There are 365 out of 365 ways to select the birthday of first person. Therefore, the number of ways that we can choose a birthday for second person is 364 out of 365.

The probability that the second child has a different birthdate than the first is 364/365.

The probability that the third child has a different birthday than the first two is 363/365.

The probability that the fourth child has a different birthday than the first three is 362/365.

Since all three of these situations must occur, multiply the three probabilities.

364/365 × 363/365 × 362/365 = 98.364%

18. A box contains 5 white and 7 black balls. Two successive drawn of 3 balls are made (i) with replacement (ii) without replacement. The probability that the first draw would produce white balls and the second draw would produce black balls are respectively.

Solution:

Two successive drawn of 3 balls are made: TOTAL = 12 Balls: (5White balls +7 Black balls)

1st draw white ball and second draw black ball with replacement = $\frac{5_{C_3}}{12_{C_3}}\times\frac{7_{C_3}}{12_{C_3}}=\frac{10}{220}\times\frac{35}{220}=\frac{7}{968}$

1st draw white ball and second draw black ball without replacement = $\frac{5_{C_3}}{12_{C_3}}\times\frac{7_{C_3}}{9_{C_3}}=\frac{10}{220}\times\frac{35}{84}=\frac{5}{264}$

P (both happening) = $\frac{7}{968}$ and $\frac{5}{264}$

19. There are three boxes with the following composition:

Box I: 5 Red + 7 White + 6 Blue balls

Box II: 4 Red + 8 White + 6 Blue balls

Box III: 3 Red + 4 White + 2 Blue balls

If one ball is drawn at random, then what is the probability that they would be of same colour?

Solution:

Either balls would be Red or white or blue

$=P(R_1\cap R_2\cap R_3) + P(W_1\cap W_2\cap W_3) + P(B_1\cap B_2\cap B_3)$

$=P(R_1)\times P(R_2)\times P(R_3)+P(W_1)\times P(W_2)\times P(W_3)+P(B_1)\times P(B_2)\times P(B_3)$

$=\frac{5}{18}\times\frac{4}{18}\times\frac{3}{9}$(Red Balls) $+\frac{7}{18}\times\frac{8}{18}\times\frac{4}{9}$ (White Balls) $+\frac{6}{18}\times\frac{6}{18}\times\frac{2}{9}$ (Black Balls)

$=\frac{89}{729}$

20. A number is selected at random from the first 1000 natural numbers. What is the probability that the number so selected would be a multiple of 7 or 11?

Solution:

First 1000 natural numbers belong to the following set {1,2, 3,…,1000} with cardinality = 1000

Multiples of 7 less than 1000 = Quotient of (1000/7) = 142

Multiples of 11 less than 1000 = Quotient of (1000/11) = 90

As 7 & 11 are both primes so multiples of 7*11 = 77 will be included in both multiples of 7 and multiples of 11

Multiples of 77 less than 1000 = Quotient of (1000/77) = 12
Hence, all-natural numbers below 1000 which are either multiples of 7 or of 11 = 142 + 90–12 = 220
So, Prob (this event) = 220/1000 = 0.22

21. A bag contains 8 red and 5 white balls. Two successive draws of 3 balls are made without replacement. The probability that the first draw will produce 3 white balls and the second 3 red balls is

Solution:
There are total 13 balls out of which 8 are red and 5 are white.
Favourable case of first draw is to get 3 white balls out of 5 white balls.
Probability $P_1 = 5C_3/13C_3 = 5/143$
If this happens then remaining are - 2 white balls and 8 red balls.
Favourable case is to get 3 red balls out of 8 balls.
Probability $P_2 = 8C_3/10C_3 = 7/15$
Both the events are independent of each other, hence total probability is $P_1{}^*P_2 = (5C_3 \times 8C_3)/(13C_3 \times 10C_3) = 5/143 \times 7/15 = 7/429$

22. There are two boxes containing 5 white and 6 blue balls and 3 white and 7 blue balls respectively. If one of the the boxes is selected at random and a ball is drawn from it, then the probability that the ball is blue is

Solution:
First box: No. of white balls = 5, No. of blue balls = 6
Second box: No. of white balls = 3, No. of blue balls = 7
So, total no. of white balls = 8, Total no. of blue balls = 13
So, total no. of balls = 8+13=21
Now probability of getting blue ball: =13/21
Hence the probability of getting blue ball is =13/21

23. A problem in probability was given to three CA students A, B and C whose chances of solving it are 1/3, 1/5 and 1/2 respectively. What is the probability that the problem would be solved?

Solution:
Probability of A solving the problem = 1/3, Probability of A not solving the problem = 1-1/3 = 2/3
Probability of B solving the problem = 1/5, Probability of B not solving the problem = 1-1/5 = 4/5
Probability of C solving the problem = ½, Probability of C not solving the problem = 1-1/2 = ½
Probability of A, B and C not solving the problem = 2/3 x 4/5 x 1/2 = 4/15
Probability of A, B and C solving the problem = 1-4/15 = 11/15

24. There are three persons aged 60, 65 and 70 years old. The survival probabilities for these three persons for another 5 years are 0.7, 0.4 and 0.2 respectively. What is the probability that at least two of them would survive another five years?

Solution:
Probability (At least two alive) = P (two alive) +P (two alive)
=(0.7)(0.4)(1-0.2)+(0.7)(0.2)(1-0.4)+(0.4)(0.2)(1-0.7) +(0.7(0.4)(0.2)
$=0.28\times0.8+0.14\times0.6+0.08\times0.3+0.056=0.224+0.084+0.056=0.388$

25. Tom speaks truth in 30 percent cases and Dick speaks truth in 25 percent cases. What is the probability that they would contradict each other?

Solution:
P(Tom speaks truth)= $P(T_T)$ = 30/100= 3/10;
P(Dick speaks truth)= $P(D_T)$ = 25/100= ¼
$P(T_F)$ = 1-3/10= 7/10; $P(D_F)$ = 1-1/4= ¾
probability that they would contradict each other
= $P(T_T)\times P(D_F) + P(T_F)\times P(D_T)$
= (3/10×3/4) + (7/10×1/4) = 0.40

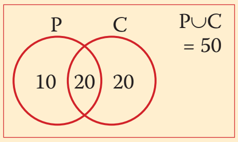
26. There are two urns. The first urn contains 3 red and 5 white balls whereas the second urn contains 4 red and 6 white balls. A ball is taken at random from the first urn and is transferred to the second urn. Now another ball is selected at random from the second urn. The probability that the second ball would be red is

Solution:
the first urn contains 3 red and 5 white balls => total = 8 Balls
second urn contains 4 red and 6 white balls. => Total = 10 Balls
There can be two cases : Ball taken from Urn is Red or White
Case 1 : Red is Taken from Urn:A
Probability of Red = (3/8)
then second urn contains 5 Red & 6 White => total = 11
Probability of Red from Urn:B = (3/8) (5/11) = 15/88
Probability of White from Urn:B = (3/8) (6/11) = 18/88
Case 1 : White is Taken from Urn:A
Probability of White = (5/8)
then second urn contains 4 Red & 7 White => total = 11
Probability of Red from Urn:B = (5/8) (4/11) = 20/88
Probability of White from Urn:B = (5/8) (7/11) = 35/88
probability of that the second ball would be Red = 15/88 + 20/88 = 35/88
probability of that the second ball would be White = 18/88 + 35/88 = 53/88

27. For a group of students, 30%, 40% and 50% failed in Physics, Chemistry and at least one of the two subjects, respectively. If an examinee is selected at random, what is the probability that he passed in Physics if it is known that he failed in Chemistry?

Solution:
Let the total number of students = 100
Number of students failed in physics = 30% of 100 =30
Number of students failed in chemistry = 40% of 100 =40
Number of students failed at least one of the two subjects = 50% of 100 =50
We need to calculate.
P (He passed in Physics but Failed in Chemistry)/P (Failed Chemistry)
$n(P\cup C) = n(p) + n(c) - n(P\cap C) 50 = 30+40- n(P\cap C)$
$n(P\cap C) = 20$
P (He passed in Physics but Failed in Chemistry)/P (failed Chemistry)

$$= P(C-P)/(P(P) = \frac{\frac{20}{100}}{\frac{40}{100}} = \frac{20}{40} = \frac{1}{2}$$



28. A packet of 10 electronic components is known to include 2 defectives. If a sample of 4 components is selected at random from the packet, what is the probability that the sample does not contain more than 1 defective?

Solution:
P (No more than one defective) = P (No defective) + P (One defective)

$$= \frac{^8C_4}{^{10}C_4} + \frac{^2C_1 \times {^8C_3}}{^{10}C_4}$$

$$= \frac{70}{210} + \frac{112}{210}$$

$$= \frac{91}{105} = \frac{13}{15}$$

29. 8 identical balls are placed at random in three bags. What is the probability that the first bag will contain 3 balls?

Solution:
The probability that a ball will be placed in the first bag is 1/3. The probability that exactly 3 of the 8 balls will end up in the first bag can be found by using the binomial distribution:

$$^8C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^5 = \frac{^8C_3 \times 2^5}{3^8} = \frac{56 \times 32}{6561} = 0.2731$$

30. X and Y stand in a line with 6 other people. What is the probability that there are 3 persons between them?

Solution:
There are altogether 8 people $p_1, p_2, \ldots, p_8$ including X & Y and these 8 people can be arranged in 8!= 40320 ways

Now, there should be 3 people between X and Y and these 3 people can be selected out of 6 in C(6,3)=20 =20ways. (6 people because X & Y are excluded from 8).

Now, take (X,*,*,*,Y)(X,*,*,*,Y)as one set of people and together with the remaining 3 people we can think of a total of 4 people which can be arranged in 4!=24 ways.

Again, the 3 people between X & Y can be arranged in 3!=6 ways.

Also, the position of X and Y can also be arranged in 2!=2ways.

So, total arrangements with 3 people between X & Y is 20×24×6× 2=5760

Hence, the required probability is =5760/40320=1/7.

31. Given that P (A) = 1/2, P (B) = 1/3, P (AB) = 1/4, what is P (A'/B')

Solution:

$$P\left(\frac{A'}{B'}\right) = \frac{P(A' \cap B')}{P(B')} = \frac{P(AUB)'}{1-P(B)} = \frac{1-P(AUB)}{1-P(B)} = \frac{1-\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right)}{1-\frac{1}{3}} = \frac{1-\left(\frac{6+4-3}{12}\right)}{\frac{3-1}{3}}$$

$$= \frac{1-\frac{7}{12}}{\frac{2}{3}} = \frac{\frac{5}{12}}{\frac{2}{3}} = \frac{5}{12} \times \frac{3}{2} = \frac{5}{8}$$

32. Four digits 1, 2, 4 and 6 are selected at random to form a four-digit number. What is the probability that the number so formed, would be divisible by 4?

Solution:
From four digits 1,2,4 and 6 last two digits (12,16,24,64) can be selected in (4 ways)

Total possible numbers are divisible by 4 are 4612, 6412, 2416, 4216, 1624, 6124, 1264, 2164 =8

Here are four ways of filling the last two digits. The remaining two places(100's, 1000's digits) can be filled in two ways. Thus there are total 4×2=8 ways

Total possible 4 digit numbers =4! =24

$$\text{Probability} = \frac{8}{4!} = \frac{8}{24} = \frac{1}{3}$$

33. A bag containing 6 white and 4 red balls. Rs 10 is received if he draws white ball and Rs. 20 for red ball. Find the expected amount when the person draws 2 balls.

Solution:
The probability of both being white ball would be $= \frac{^6C_2}{^{10}C_2} = \frac{15}{45}$

The probability of both being red ball would be $= \frac{^4C_2}{^{10}C_2} = \frac{6}{45}$

The probability of one being red ball and another being white ball would be $= \frac{^6C_1 \times {^4c_1}}{^{10}C_2} = \frac{24}{45}$

Hence the expected amount when person draws two balls will be

$$= \frac{15}{45} \times (10+10) + \frac{6}{45}(20+20) + \frac{24}{45}(10+20)$$

= 6.7+5.33+16 = Rs.28

34. If two random variables x and y are related as Y =-3x +4 and Standard Deviation of Y is

Solution:
Given Y =-3x +4

Y = a + bx

$\sigma_y = \sigma(ax+b)$, $\sigma(b)=0$

$\sigma_y = |a|.\sigma_x$

SD of y=$\sigma_y = 3 \times 2 = 6$

35. If 2x+3y+ 4 = 0 and v(x) = 6 then V(y) is

Solution:
Given that 2x+3y+ 4 = 0 and v(x) = 6 then SD of x = √6:V(y) = ?

$$SD_y = \left|-\frac{2}{3}\right|.SD_x = \frac{2}{3}\sqrt{6}$$

$$y^2 = \frac{4}{9}.6 = \frac{8}{3}$$

$$V(y) = \frac{8}{3}$$

36. A pocket of 10 electronic components is known to include 3 defectives. If 4 components are selected from the packet at random, what is the expected value of the number of defective?

Solution:
10 electronic components If 4 components are selected from the packet at random $= \frac{4}{10}$

Expected value of the number of defective $= 3 \times \frac{4}{10} = \frac{12}{10} = 1.2$

37. The Probability there is atleast one error in a account statement prepared by 3 persons A , B and C are 0.2 , 0.3 and 0.1 respectively . If A, B and C prepare 60, 70 and 90 such statements then expected number of correct statements.

Solution:
E(x)=A.(1-P(A))+B.(1-P(B))+C.(1-P(C))=(60×(1-0.2) + (70×(1-0.3) + (90×(1-0.1)

= (60×0.8) +(70×0.70) +(90×0.9) =178

CA FOUNDATION - PAPER 3 - BUSINESS MATHEMATICS, LOGICAL REASONING AND STATISTICS

This capsule on Foundation - Paper 3 - Business Mathematics, Logical Reasoning and Statistics , will enable the students to understand and apply the techniques of developing discrete and continuous probability distributions.

Chapter 17 : Theoretical Distributions

In this chapter we will discuss the probability theory by considering a concept and analogous to the idea of frequency distribution. In frequency distribution where to the total frequency to different class intervals, the total probability (i.e. one) is distributed to different mass points is known as theoretical probability distributions.

- Discrete Random variable.
- Continues Random variable.

Importance of theoretical probability distribution.:

(a) An observed frequency distribution, in many a case, may be regarded as a sample i.e. a representative part of a large, unknown, boundless universe or population and we may be interested to know the form of such a distribution.
For Example: By fitting a theoretical probability distribution.
  - Length of life of the lamps produced by manufacturer up to a reasonable degree of accuracy.
  - The effect of a particular type of missiles, it may be possible for our scientist to suggest the number of such missiles necessary to destroy an army position.
  - By knowing the distribution of smokers, a social activist may warn the people of a locality about the nuisance of active and passive smoking and so on.

(b) Theoretical probability distribution may be profitably employed to make short term projections for the future.

(c) Statistical analysis is possible only on the basis of theoretical probability distribution.

A probability distribution also possesses all the characteristics of an observed distribution. We define population mean, population median, population mode, population standard deviation etc. exactly same way we have done earlier. These characteristics are known as population parameters.

A probability distribution may be either a discrete probability distribution or a Continuous probability distribution depending on the random variable under study.

Two important discrete probability distributions

(a) Binomial Distribution
(b) Poisson distribution.

Important continuous probability distribution

Normal Distribution

Binomial Distribution

One of the most important and frequently used discrete probability distribution is Binomial Distribution. It is derived from a particular type of random experiment known as Bernoulli process named after the famous mathematician Bernoulli. Noting that a 'trial' is an attempt to produce a particular outcome which is neither certain nor impossible, the characteristics of Bernoulli trials are stated below:

(i) Each trial is associated with two mutually exclusive and exhaustive outcomes, the occurrence of one of which is known as a 'success' and as such its non occurrence as a 'failure'. As an example, when a coin is tossed, usually occurrence of a head is known as a success and its non–occurrence i.e. occurrence of a tail is known as a failure.

(ii) The trials are independent.

We may note the following important points in connection with binomial distribution:

(a) As $n > 0$, $p, q \geq 0$, it follows that $f(x) \geq 0$ for every x
Also $\sum f(x) = f(0) + f(1) + f(2) + \ldots + f(n) = 1$

(b) Binomial distribution is known as biparametric distribution as it is characterised by two parameters n and p. This means that if the values of n and p are known, then the distribution is known completely.

(c) The mean of the binomial distribution is given by $\mu = np$

(d) Depending on the values of the two parameters, binomial distribution may be unimodal or bi- modal., the mode of binomial distribution, is given by μ_0=the largest integer contained in (n+1)p
if (n+1)p is a non-integer (n+1)p and (n+1)p - 1 if (n+1)p is an integer

(e) The variance of the binomial distribution is given by
$\sigma^2 = npq$
Since p and q are numerically less than or equal to 1, npq < np variance of a binomial variable is always less than its mean.
Also variance of X attains its maximum value at p = q = 0.5 and this maximum value is n/4.

(f) Additive property of binomial distribution.
If X and Y are two independent variables such that
$X \sim \beta(n_1, P)$
and $Y \sim \beta(n_2, P)$
Then $(X+Y) \sim (n_1 + n_2, P)$

Applications of Binomial Distribution

Binomial distribution is applicable when the trials are independent and each trial has just two outcomes success and failure. It is applied in coin tossing experiments, sampling inspection plan, genetic experiments and so on.

Poisson Distribution

Poisson distribution is a theoretical discrete probability distribution which can describe many processes. Simon Denis Poisson of France introduced this distribution way back in the year 1837.

Poisson Model

Let us think of a random experiment under the following conditions:

I. The probability of finding success in a very small time interval (t, t + dt) is kt, where k (>0) is a constant.

II. The probability of having more than one success in this time interval is very low.

III. The probability of having success in this time interval is independent of t as well as earlier successes.

The above model is known as Poisson Model. The probability of getting x successes in a relatively long time interval T containing m small time intervals t i.e. T = mt. is given by

$$\frac{e^{-kt}.(kt)^x}{x!} \quad \text{for } x = 0, 1, 2, \ldots\ldots\ldots\ldots$$

Taking kT = m, the above form is reduced to

$$\frac{e^{-m}.m^x}{x!} \quad \text{for } x = 0, 1, 2, \ldots\ldots\ldots\ldots$$

Definition of Poisson Distribution

A random variable X is defined to follow Poisson distribution with parameter λ, to be denoted by X ~ P (m) if the probability mass function of x is given by

$$f(x) = P(X = x) = \frac{e^{-m}.m^x}{x!} \text{ for } x = 0, 1, 2, \ldots$$

$$= 0 \quad \text{otherwise}$$

Here e is a transcendental quantity with an approximate value as 2.71828.

Important points in connection with Poisson distribution:

(i) **Since $e^{-m} = 1/e^m > 0$, whatever may be the value of m, m > 0, it follows that f (x)≥ 0 for every x.**
Also it can be established that $\sum_x f(x) = 1$ i.e. f(0) + f(1) + f(2) +....... = 1

(ii) **Poisson distribution is known as a uniparametric distribution as it is characterised by only one parameter m.**

(iii) **The mean of Poisson distribution is given by m i,e μ = m**

(iv) **The variance of Poisson distribution is given by $\sigma^2 = m$**

(v) **Like binomial distribution, Poisson distribution could be also unimodal or bimodal depending upon the value of the parameter m.**
We have μ_0 = The largest integer contained in m if m is a non-integer
= m and m–1 if m is an integer

(vi) **Poisson approximation to Binomial distribution**
If n, the number of independent trials of a binomial distribution, tends to infinity and p, the probability of a success, tends to zero, so that m = np remains finite, then a binomial distribution with parameters n and p can be approximated by a Poisson distribution with parameter m (= np).
In other words when n is rather large and p is rather small so that m = np is moderate then $\beta(n, p) \cong P(m)$

(vii) **Additive property of Poisson distribution**
If X and y are two independent variables following Poisson distribution with parameters m_1 and m_2 respectively, then Z = X + Y also follows Poisson distribution with parameter $(m_1 + m_2)$.
i.e. if $X \sim P(m_1)$
and $Y \sim P(m_2)$
and X and Y are independent, then
$Z = X + Y \sim P(m_1 + m_2)$

Application of Poisson distribution

Poisson distribution is applied when the total number of events is pretty large but the probability of occurrence is very small. Thus we can apply Poisson distribution, rather profitably, for the following cases:

a) The distribution of the no. of printing mistakes per page of a large book.
b) The distribution of the no. of road accidents on a busy road per minute.
c) The distribution of the no. of radio-active elements per minute in a fusion process.
d)

Normal or Gaussian distribution

The two distributions discussed so far, namely binomial and Poisson, are applicable when the random variable is discrete. In case of a continuous random variable like height or weight, it is impossible to distribute the total probability among different mass points because between any two unequal values, there remains an infinite number of values. Thus a continuous random variable is defined in term of its probability density function f (x), provided, of course, such a function really exists, f (x) satisfies the following condition:

$$f(x) \geq 0 \text{ for } x(-\infty,\infty) \text{ and } = \int_{-\infty}^{+\infty} f(x) = 1$$

The most important and universally accepted continuous probability distribution is known as normal distribution. Though many mathematicians like De-Moivre, Laplace etc. contributed towards the development of normal distribution, Karl Gauss was instrumental for deriving normal distribution and as such normal distribution is also referred to as Gaussian Distribution.

A continuous random variable x is defined to follow normal distribution with parameters μ and σ^2, to be denoted by

$$X \sim N(\mu, \sigma^2)$$

If the probability density function of the random variable x is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}.e^{-(\bar{x}-\mu)^2/2\sigma^2} \quad \text{for } -\infty<x<\infty$$

where μ and σ are constants, and > 0