3 LINEAR INEQUALITIES

Try Your Self - 1

1. An employer recruits experienced (x) and fresh workmen (y) for his firm under the condition that he cannot employ more than 9 people. x and y can be related by the inequality

(a) $x + y \neq 9$ (b) $x + y \leq 9$

(c) $x + y \geq 9$ (d) none of the above

Sol. An employer recruits experienced (x) and fresh workmen (y) for his firm under the condition that he cannot employ more than 9 people, it means he has to recruit less than or equal to 9.

$\therefore x + y \leq 9$.

Answer: (b)

2. On the average experienced person does 5 units of work while a fresh one 3 units on work daily but the employer has to maintain an output of at least 30 units of work per day. This situation can be expressed as

(a) $5x + 3y \leq 30$ (b) $5x + 3y > 30$

(c) $5x + 3y \geq 30$ (d) None of these

Sol. On the average experienced person does 5 units of work while a fresh one 3 units of work daily but the employer has to maintain an output of at least 30 units of work per day.

$\therefore 5x+3y \geq 30, x \geq 0, y \geq 0$

Answer: (c)

3. A firm makes two types of products. Type A and type B. The profit on product A is Rs 20 each and that on product B is Rs 30 each. Both types are processed on three machines M1, M2 and M3. The time required in hours by each product and total time available in hours per week on each machine are as follows:

Machine	Product A	Product B	Available Time
MI	3	3	36
M2	5	2	50
M2	2	6	60

The constraints can be formulated taking x_1 = number of units of A and x2 = number of unit of B as

(a) $x_1 + x_2 < 12$
$5x_1 + 2x_2 < 50$
$2x_1 + 6x_2 < 60$
$x_1 \geq 0, x_2 \geq 0$

(b) $3x_1 + 3x_2 \geq 36$
$5x_1 + 2x_2 \leq 50$
$2x_1 + 6x_2 \geq 60$
$x_1 \geq 0, x_2 \geq 0$

(c) $3x_1 + 3x_2 \leq 36$
$5x_1 + 2x_2 \leq 50$
$2x_1 + 6x_2 \leq 60$
$x_1 \geq 0$
$x_2 \geq 0$

(d) none of the above

Sol.

Machine	Product A	Product B	Available Time
MI	3	3	36
M2	5	2	50
M2	2	6	60

$3x_1 + 3x_2 \leq 36$
$5x_1 + 2x_2 \leq 50$
$2x_1 + 6x_2 \leq 60$
$x_1 \geq 0$
$x_2 \geq 0$

$\therefore$ Option (c) is correct

4. A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is at least 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D. The vitamin content per kg. of each food is shown below:

	A	B	C	D
Food I:	2	1	1	2
Food II:	1	1	2	3

Assuming x units of food I is to be mixed with y units of food II the situation can be expressed as

(a) $2x + y \le 9$, $x + y \le 7$, $x + 2y \le 10$, $2x + 3y \le 12$, $x > 0, y > 0$

(b) $2x + y \ge 30$, $x + y \le 7$, $x + 2y \ge 10$, $x + 3y \ge 12$

(c) $2x + y \ge 9$, $x + y \le 7$, $x + y \le 10$, $x + 3y \ge 12$

(d) $2x + y \ge 9$, $x + y \ge 7$, $x + 2y \ge 10$, $2x + 3y \ge 12$, $x \ge 0, y \ge 0$

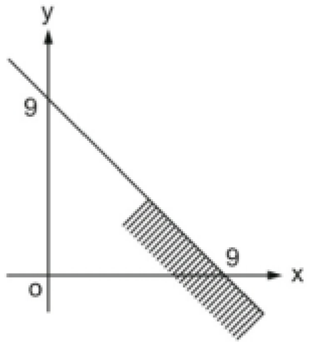
Sol. According the data given we get

$2x + y \ge 9$, $x + y \ge 7$, $x + 2y \ge 10$ and $2x + 3y \ge 12$, $x \ge 0$ and $y \ge 0$

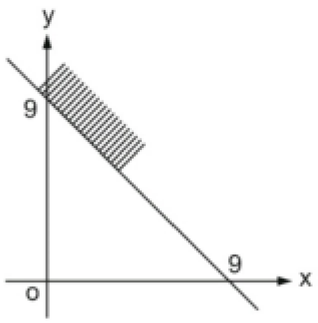
the correct the option is d.

∴ (d) is correct

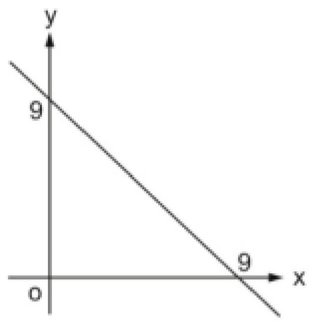
5. The graph to express the inequality $x + y \le 9$ is



(a)



(b)



(c)

(d) None of these

Sol. To plot the graph of $x + \le 9$, first plot the graph of $x + y = 9$.

It will be a straight line passing through (0,9) and (9,0).

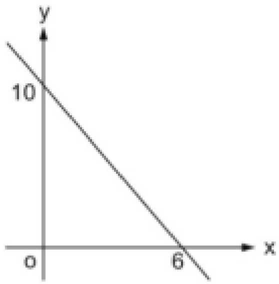
The graph of inequality is the region satisfying the inequality.

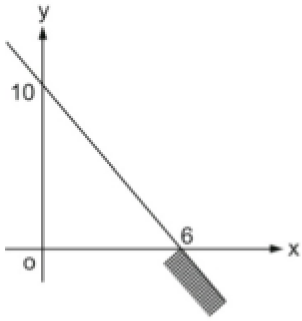
So it is on side of a line.

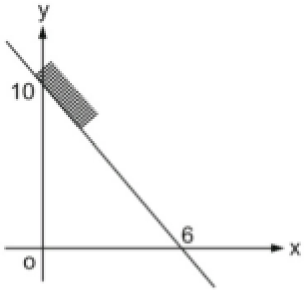
Now we will check any one point of one side, if it satisfy the inequality then the shading is done on that side or on the other side. Let us check (0,0). 0+0=0 < 9, so the shading has to be done towards the origin.

∴ (a) is correct

6. The graph to express the inequality $5x + 3y \geq 30$ is

(a) 

(b) 

(c) 

(d) None of these

Sol. To plot the graph of $5x + 3y \geq 30$, first plote the graph of $5x + 3y = 30$.

It will be a straight line passing through (0, 10) and (6, 0).

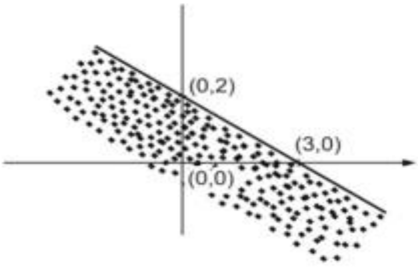
The graph of inequality is the region satisfying the inequality.

So it is one side of a line.

Now we will check any one point of one side, if it satisfy the inequality then the shading is done on that side or on the other side. Let us check (0,0). 0+0=0 <30, So the origin does not satisfy the inequality. so the shading has to be done opposite to the origin.

$\therefore$ (c) is correct

7. The following region is represented by



(a) $2x + 3y \leq 6$ (b) $2x + 3y \geq 6$ (c) $2x + 3y = 6$ (d) $2x + 4y \leq 6$

Sol. (a) $2x+3y\le 6$

(0 , 2) & (3 , 0)

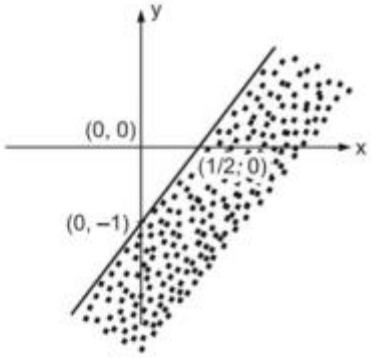
Sub in equation

2 (0) + 3 (2) = 0 + 6 = 6 satisfy equation

2 (3) + 3 (0) = 6 + 0 = 6 satisfy equation

∴ (a) is correct

8. The following region is represented by



(a) $2x+y\ge 1$ (b) $2x-y=1$ (c) $2x-y\ge 1$ (d) $2x-y\le 1$

Sol. By option (c) $2x-y\ge 1$

∵ Shading is in 1^{st} , 4^{th} & 3^{rd} Quadrant

∴ Take point (5,0) on X-axis

& (0 , -5) on Y – axis

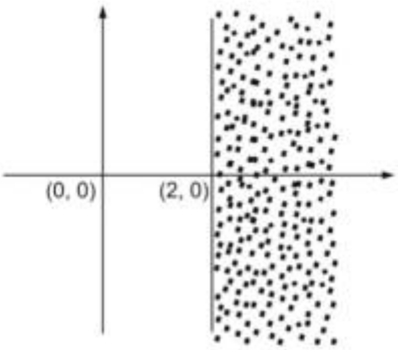
Sub in equation

2(5) – 0 = 10 – 0 = 10 > 1 satisfy

2(0) – (-5) = 0 + 5 = 5 >1 satisfy

∴ (c) is correct

9. The following region is represented by



(a) $x\le 2$ (b) $x=2$ (c) $y\ge 2$ (d) $x\ge 2$

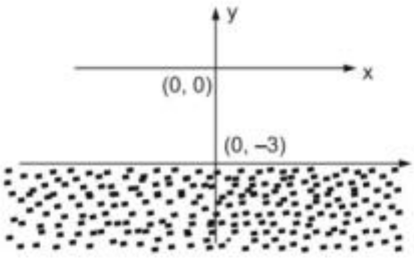
Sol. ∵ shading is greater than 2 on X-axis

And line is parallel to Y-axis

∴ $x \geq 2$

∴ (d) is correct

10. The following region is represented by



(a) $y \leq -3$ (b) $x \leq -3$ (c) $x \geq -3$ (d) $y \geq -3$

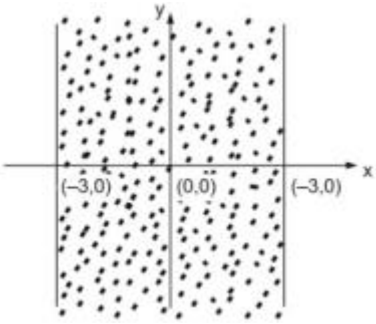
Sol. ∵ shading is below (0 , -3) on Y-axis

And line is parallel to X-axis

∴ $y \leq -3$

∴ (a) is correct

11. The following region is represented by



(a) $|x| = 3$ (b) $|y| \leq 3$ (c) $|y| \geq 3$ (d) $|x| \leq 3$

Sol. $|x| \leq 3$

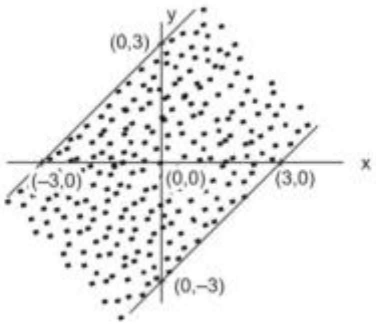
∴ $x \leq \pm 3$

$-x \leq +3 or\ x \geq -3$

$-3 \leq x \leq +3$ satisfies the shading.

∴ (d) is correct

12. The following region is represented by



(a) $|y-x|=3$ (b) $|y-x|\le 3$ (c) $|y-x|\ge 3$ (d) $|x+y|\le 3$

Sol. $|y-x|\le 3$

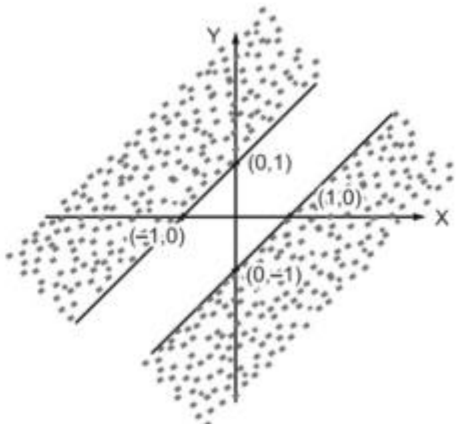
$\therefore \quad y-x \le \pm 3$

$\therefore \quad y-x \le 3$ or $y-x \ge -3$

-3 ≤ $y-x \le 3$ satisfy the shading

∴ (b) is correct

13. The following region is represented by



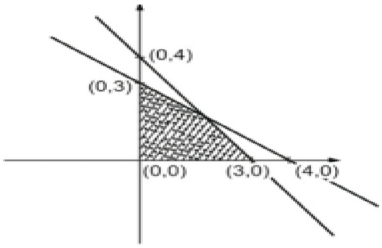
(a) $|x-y|\le 1$ (b) $|x+y|\ge 1$ (c) $|x-y|\ge 1$ (d) $|x+y|\le 1$

Sol. $|x-y|\ge 1$

$\therefore \quad x-y \ge \pm 1$

$\therefore \quad x-y \ge 1$ or $x-y \le -1$ satisfy the shading

14. The following shaded region is the solution set of the linear inequations



(a) $3x+4y \geq 12, y+3x \geq 3, x \geq 0, y \geq 0$ (b) $3x+4y \leq 12, 4x+3y \leq 12, x \geq 0, y \geq 0$

(c) $3x+4y \leq 12, 4x+3y \geq 12, x \geq 0, y \geq 0$ (d) $3x+4y = 12, 4x+3y = 12, x \geq 0, y \geq 0$

Sol. $\because$ shading is towards origin side. And in first quadrant only.

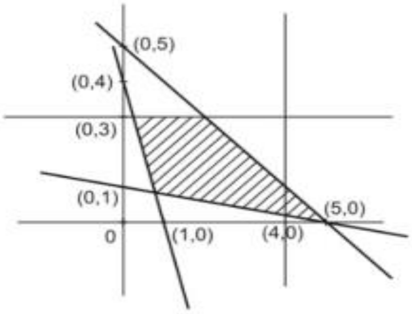
$\therefore$ all inequalities will have " $\leq$"

And $x \geq 0$

$y \geq 0$

$\therefore$ (b) is correct

15. The following shaded region is the solution set of the linear inequations



(a) $x+y \geq 5, 4x+y \geq 4, x+5y \leq 5$

(b) $x \leq 4, y \leq 3, x+y \leq 5, 4x+y \geq 4$

(c) $x+y \leq 5, 4x+y \leq 4, x+5y \geq 5, y \leq 3$

(d) $x+y \leq 5, 4x+y \leq 4, x+5y \leq 5$

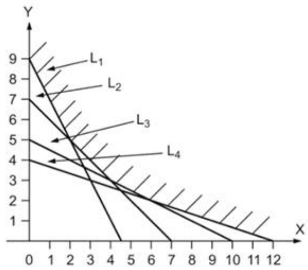
Sol. Line (0 ,5) $\leftrightarrow$ (5,0) $\Rightarrow$ origin side $\therefore$ $\leq$

Line (0,1) (5,0) $\Rightarrow$ Non Origin side $\therefore$ $\geq$

Line (0, 4) (1, 0) $\Rightarrow$ Non Origin side $\therefore$ $\geq$

$\therefore$ (c) is correct

16.



$L_1 : 2x+y=9, L_2 : x+y=7, L_3 : x+2y=10, L_4 : x+3y=12$

The common region (shaded part) indicated on the diagram is expressed by the set of inequalities

(a) $2x+y\le 9$, $x+y\ge 7$, $x+2y\ge 10$, $x+3y\ge 12$

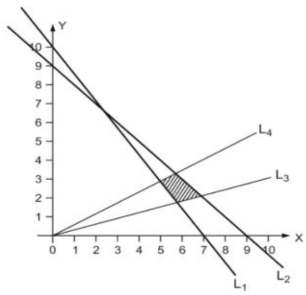
(b) $2x+y\ge 9$, $x+y\le 7$, $x+2y\ge 10$, $x+3y\ge 12$

(c) $2x+y\ge 9$, $x+y\ge 7$, $x+2y\ge 10$, $x+3y\ge 12$, $x\ge 0, y\ge 0$

(d) None of these

Sol. The diagram shows the region which is common of the equations of inequality.
We will select one point from the shaded region and write the equations of inequalities.
Let's the consider the point (6,5) and substitute in equations.
2x + y = 12+5=179, x+y=6+5=11>7
x + 2y = 6+10=16>10, x + 3y = 6+15=21>12.
x = 6 >0, y = 5 >0
So the correct option is c.
∴ (c) is correct

17.



$L_1: 5x+3y=30, L_2: x+y=9, L_3: y=x/3, L_4: y=x/2$

The common region (shaded part) show in the diagram refers to

(a) $5x+3y\le 30$, $x+y\le 9$, $y\le \frac{1}{5}x$, $y\le x/2$

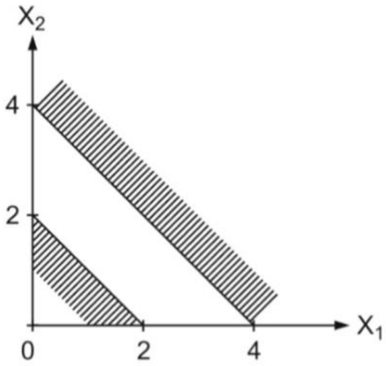
(b) $5x+3y\ge 30$, $x+y\le 9$, $y\ge x/3$, $y\le x/2$, $x\ge 0, y\ge 0$

(c) $5x+3y\ge 30$, $x+y\ge 9$, $y\le x/3$, $y\ge x/2$, $x\ge 0, y\ge 0$

(d) $5x+3y>30$, $x+y<9$, $y\ge 9$, $y\le x/2$, $x\ge 0, y\ge 0$

Sol. The diagram shows the region which is common of the equations of inequality.
We will select one point from the shaded region and write the equations of inequalities.
Let's consider the point (6, 2.5) and substitute in equations.
5x + 3y = 5(6) + 3(2.5) = 30 +7.5 =37.5> 30
x + y = 6+2.5 = 8.5 < 9
x/3=6/3=2<2.5<y
So the answer is none of these.

18. The region indicated by the shading in the graph is expressed by inequalities



(a) $x_1 + x_2 \le 2$, $2x_1 + 2x_2 \ge 8$, $x_1 \ge 0, x_2 \ge 0$

(b) $x_1 + x_2 \le 2$, $x_1 + x_2 \le 4$

(c) $x_1 + x_2 \ge 2$, $2x_1 + 2x_2 \ge 8$

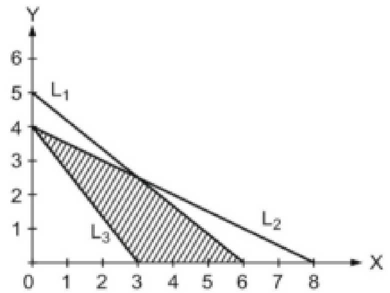
(d) $x_1 + x_2 \le 2$, $2x_1 + 2x_2 > 8$

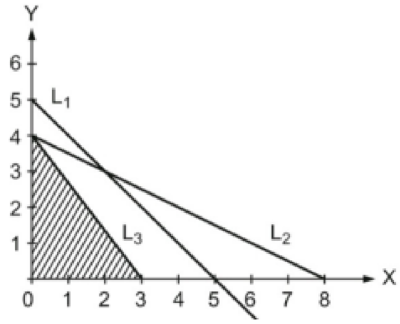
Sol. Select the point from the common region and the check the equations.

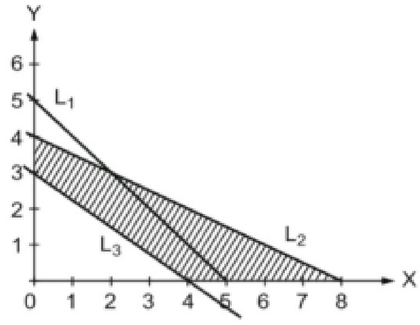
The correct option is a.

$\therefore$ (a) is correct

19. The common region satisfying the set of inequalities $x \ge 0, y \ge 0, L_1 : x + y \le 5, L_2 : x + 2y \le 8$ and $L_3 : 4x + 3y \ge 12$ is indicated by

(a) 

(b) 

(c) 

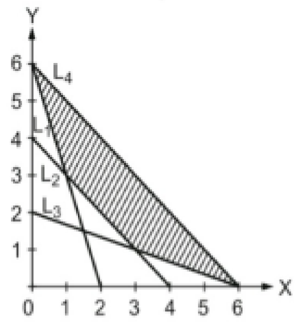
(d) None of these

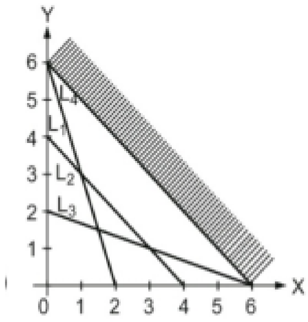
Sol. First plot the graph of equations. Then according the region for inequality, shade the common region.

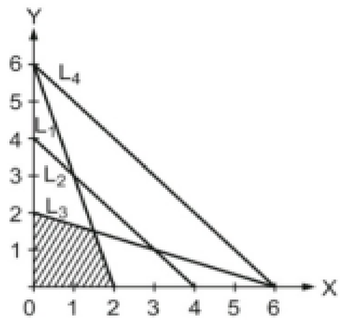
The diagram in option a is the correct diagram, so a is the correct option.

$\therefore$ (a) is correct

20. The common region satisfying the set of inequalities
$L_1 : 3x + y \geq 6, L_2 : x + y \geq 4, L_3 : x + 3y \geq 6, \text{ and } L_4 : x + y \leq 6$ is indicated by

(a)


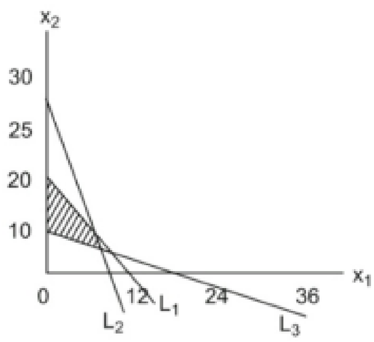
(b)


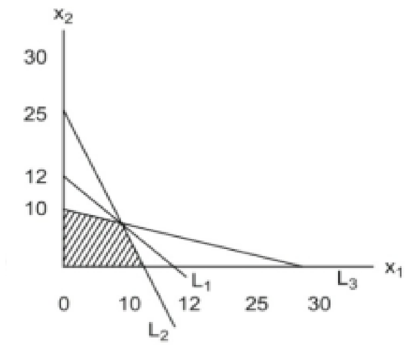
(c)


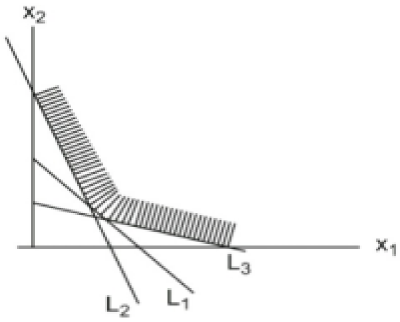
(d) None of these

Sol. we will select the point (0,0) and check the region for each equation and find the common region.
The correct option is a.
∴ (a) is correct

21. The set of inequalities
$L_1 : x_1 + x_{12} \leq 12, L_2 : 5x_1 + 2x_2 \leq 50, L_3 : x_1 + 3x_2 \leq 30, x_1 \geq 0, \text{and} x_2 \geq 0$ is represented by

(a)


(b)


(c) 

(d) None of these

Sol. First plot the graph of equations. Then according the region for inequality shade the common region.

The diagram in option b is the correct diagram. So b is the correct option.

$\therefore$ (b) is correct

22. Two machines (I and II) produce two grades of plywood, grade A and grade B. In one hour of operation machine I produces two units of grade A and one unit of grade B, while machine II, in one hour of operation produces three units of grade A and four units of grade B. The machines are required to meet a production schedule of at least fourteen units of grade A and twelve units of grade B. The constraints can be formulated taking x=number of hours required on machine I and y number of hours required on machine II

(a) $2x + 3y \le 14$, $x + 4y \le 12$, $x \ge 0 \text{ and } y \ge 0$

(b) $2x + 3y \ge 14$, $x + 4y \ge 12$, $x \ge 0 \text{ and } y \ge 0$

(c) $2x + 3y \ge 14$, $x + 4y > 12$, $x \ge 0 \text{ and } y \ge 0$

(d) $2x + 3y \ge 14$, $x + 4y \ge 12$, $x \le 0 \text{ and } y \le 0$

Sol. $\because$ It is atleast fourteen units of grade A & twelve units of grade B,

$2x + 3y \ge 14$

$x + 4y \ge 12$

$x \ge 0 \text{ and } y \ge 0$

$\therefore$ (b) is correct