6 SEQUENCE AND SERIES –ARITHMETIC AND GEOMETRIC PROGRESSIONS

TRY YOURSELF - 1

1. For what value of k is the sequence $2k+4, 3k-7, k+12$ an arithmetic sequence.

(a) $k=10.$ (b) $k=9.$ (c) $k=8.$ (d) None of these

Sol. If a, b, c are in A.P., then $2b=a+c.$

$\therefore \quad 2(3-7)=2k+4+k+12$

$\Rightarrow \quad 6k-14=3k+16$

$\Rightarrow \quad 3k=30$

$\Rightarrow \quad k=10.$

$\therefore$ (a) is correct

2. Find arithmetic mean between 7 and 15.

(a) 8 (b) 11 (c) 5 (d) None of these

Sol. Here $a=7, b=15$

The arithmetic mean between a and b is $\frac{a+b}{2}$

$\therefore$ The required arithmetic mean = $\frac{7+15}{2}=11$

$\therefore$ (b) is correct

3. Insert 4 arithmetic means between 4 and 29.

(a) 9, 14, 19 and 29 (b) 9, 14, 20 and 25

(c) 9, 14, 25 and 35 (d) none of these

Sol. If d is the common difference, then $d=\frac{b-a}{n+1}=\frac{29-4}{5}=5$

The arithmetic means are $4+5, 4+2\times5, 4+3\times5$ and $4+4\times5$ i.e. 9, 14, 19 and 29 are required arithmetic means.

4. The tenth term of an arithmetic progression is 25 and the fifteenth term is 40. Find the first term and common difference and then find the fifth term.

(a) 5 (b) 15 (c) 10 (d) none of these

Sol. It is given that $t_{10}=25, t_{15}=40$, where t_n denotes the nth term. By using arithmetic progression, $t_n=a+(n-1)d$, where $a=$ first term and $d=$ common difference. It is given that

$25=a+9d$ ....(1)

$40=a+14d$ .....(2)

From (1) and (2), we get

$5d=15$ or $d=3$

$d=3\Rightarrow a=-2.$ Hence $t_n=-2+(n-1).3$

Fifth term $=t_5=-2+4\times3=10.$

∴ (c) is correct

5. The third term of an Arithmetic progression is 7 and its seventh term is 2 more than thrice of its third term. Find the first term, common difference and the sum of first 20 terms of the progression.

(a) 180 (b) 740 (c) 190 (d) none of these

Sol. Let the A.P. be $a, a+d, a+2d, ..., a+(n-1)d, ...;$ a being first term, and d the common difference

According to the question,

$t_3=7$ i.e, $a=(3-1)d=7$ or, $a+2d=7$ .....(i)

$t_7=2+3t_3$

and

i.e., $a+6d=2+3\times7$ [using (i)] or, $a+6d=23$ ...(ii)

(ii)....(i) gives, $4d=16$ i.e., $d=4$. Substituting this value of d, in (i), we find $a=-1$.

Also, sum to 20 terms,

$S_{20}=20/2\{2\times(-1)+(20-1)\times4\}$

$=10(-2+76)$ $=10\times74=740$

∴ (b) is correct

6. Find the increasing arithmetic progression, the sum of whose first three term is 27 and the sum of their squares is 275.

(a) 5, 9, 13. (b) 5, 10, 13. (c) 5, 10, 14. (d) none of these

Sol. Let the first three terms of the programmes be $a-d,\ a,\ a+d$.

By the description of the problem.

$(a-d)+a+(a+d)=27$ .....(i)

and $(a-d)^2+a^2+(a+d)^2=275$ .....(ii)

From $(i),\ 3a=27$ i.e., $a=9$ .....(iii)

From (ii), $3a^2+2d^2=275$

or, $2d^2=275-3\times 81$ [Using (iii)]

or, $2d^2=275-243=32$ i.e., $d=\pm 4$

Using $a=9$ and $d=4$, we get the required increasing arithmetic progression $9-4, 9, 9+4$ i.e., 5, 9, 13.

∴ (a) is correct

7. Divide 20 into 4 parts which are in arithmetic progression such that the product of the first and fourth is to be the product of the second and third in the ratio 2:3.

(a) 2, 4, 6, 8. (b) 8, 4, 6, 8.

(c) 2, 4, 5, 8. (d) none of these

Sol. Let four parts in A.P. be $x-3d,\ x-d,\ x+d,\ x+3d$.

Since their sum is 20, $x-3d+x-d+x+d+x+3d=20$ i.e., $x=5$.

Product of 1st and 4th parts is x^2-9d^2, and

Product of 2nd and 3rd parts is x^2-d^2.

By the given condition, $\frac{x^2-9d^2}{x^2-d^2}=\frac{2}{3}$

Or, $3x^2-27d^2=2x^2-2d^2$

Substituting $x=5$, we get

$75-27d^2=50-2d^2$

$d^2=1$ i.e., $d=\pm 1$

∴ (a) is correct

8. Give the correct answer with reasons for the following question:

If x, y, z be the p^{th}, q^{th}, r^{th} terms respectively of an Arithmetic progression, then $x(q-r)+y(r-p)+z(p-q)$ is equal to

(a) 0 (b) xyz (c) pqr (d) $p+q+r$

Sol. If a be the first term and d the common difference of the A.P. then

$x = a+(p-1)d, y = a+(q-1)d, z = a+(r-1)d$

$\therefore x(q-r)+y(r-p)+z(p-q)$

$=\{a+(p-1)d\}(q-r)+\{a+(q-1)d\}(r-p)+\{a+(r-1)d\}\{p-q\}$

$= a(q-r+r-p+p-q)+d\{(p-1)(q-r)+(q-1)\ (r-p)+(r-1)(p-q)\}$

$= a\times 0+d\times 0=0$

$\therefore$ (a) is correct

9. Find the sum of all numbers between 100 and 1,000 which are divisible by 13.

(a) 37,654 (b) 35,674 (c) 37,674 (d) none of these

Sol. The numbers divisible by 13 form an arithmetic series. The series starts at 104 and ends at 988.

The last term is $a+(n-1)d$. Here $a=104,\ d=13,$

$\therefore \quad 988=104+(n-1)\ 3\times n=69$

Sum of these numbers is given by

$$S=\frac{n}{2}\{2a+(n-1)d\}$$

$$=\frac{69}{2}\ \{208\ +68\times 13\}=\frac{69}{2}\times 1092=37,674$$

$\therefore$ (c) is correct

10. The sum of a certain number of terms in arithmetic progression is 5500. The first and the last terms are 100 and 1000. Find the number of terms.

(a) 10 (b) 12 (c) 16 (d) none of these

Sol. Let the number of terms be n, S the sum, a the first term, and l the last term of the progression.

It is given that = $S=5,000;\ a=100;\ l=1,000.$

We know $\quad S=\frac{n}{2}(a+l)$

$\therefore \quad 5,500=\frac{n}{2}\ (100+1,000)$

or, $\quad 11,000=n\times 1100$ i.e., $n\ =\ 10$

Therefore, the required number of terms is 10.

$\therefore$ (a) is correct

11. The sum of the first n terms of an A.P. is $3n^2-2n+1$. The common difference is

(a) -4 (b) 4 (c) -5 (d) 5

Sol. The sum of the first n terms is = $3n^2 - 2n + 1$

Putting $n = 1$, we get the first term = $3.1^2 - 2.1 + 1 = 2$;

Putting $n = 2$, we get the sum of the two terms = $3.2^2 - 2.2 + 1 = 9$;

Second term is therefore given by $= 9 - 2 = 7$

and common difference = second term - first term $= 7 - 2 = 5$;

$\therefore$ (d) is correct

12. The sum of the digits of a three digit number is 12. The digits are in arithmetic progression. If the digits are reversed, then the number is diminished by 396. Find the number.

(a) 121 (b) 124 (c) 642 (d) none of these

Sol. Let the digits be $x,\ y,\ z.$ The number is $100x + 10y + z.$ From the question

$$x + y + z = 12 \quad ...(1)$$

$$2y = x + z \quad ...(2)$$

and $(100x + 10y + z) - (100z + 10y + x) = 396$

or $99(x - z) = 396$

$\therefore$ $x - z = 4$ ...(3)

From (1) and (2),

$x + z + \frac{1}{2}(x + 2) = 12$

Or, $x + z = 8$ ...(4)

From (3) and (4), $x = 6$

z = 2

From (2). $y = 4$

Therefore the number is 642.

$\therefore$ (c) is correct

13. A piece of equipment cost a certain factory ₹ 6,00,000. If it depreciates in value 15% in the first year, $13\frac{1}{2}\%$ the next year, 12% the third year and so on, what will be its value at the end of 10 years, all percentages applying to the original cost?

(a) ₹105,000 (b) ₹102,000

(c) ₹115,000 (d) none of these

Sol. Cost of piece of equipment = ₹ 6,00,000

Its depreciation in 1st year = $\frac{6,00,000 \times 15}{100}$ = ₹ 90,000

Its depreciation in next year = $6,000 \times \frac{27}{2}$ = ₹ 81,000

Its depreciation in third year = 6,000 × 12 = ₹ 72,000… and so on

Depreciation: 90,000, 81,000, 72,000 ...

It is in A.P. with common difference $d = -9,000$

∴Total depreciation in 10 years

$= \frac{10}{2}\{2 \times = 90,000 + (10-1)\ (-9,000)\} = 5(180,000 - 81,000)$

$= 5 \times 99,000 = ₹\ 4,95,000$

∴Required value at the end of 10 years = $6,00,000 - 4,95,000 = ₹105,000.$

∴ (a) is correct

14. A house with a present value of ₹1,00,000 has to be depreciated over a period of 25 years. 4% of the present value is deducted each year. Find an equation to express the depreciated value as an arithmetic sequence and find the value after 10 years.

(a) ₹60,000 (b) ₹55,000

(c) ₹15,000 (d) none of these

Sol. $d = -\ (4\%\ of\ ₹1,00,000) = \left(1,00,000 \times \frac{4}{100}\right) = -4,000.$

Hence $t_n = 1,00,000 - (n-1)\ 4,000,$ where t_n denotes depreciated value of the machine at the beginning of nth year. The value of machine after 10 years is

$t_n = 1,00,000 - 10 \times 4,000 = ₹60,000.$

∴ (a) is correct

15. The Cricket Control Board of India decides to raise a cricketer's beneficiary fund of ₹ 5 crores. A start is made with ₹ 10 lacs and every year an additional worth ₹ 3 lakhs is made. In how many years will the fund reach the desired value? What should be the last year's contribution to make up the desired fund?

(a) 15,00,000 (b) 35,00,000 (c) 45,00,000 (d) none of these

Sol. a = 10 lakhs; d = 3 lakhs s = 500 lakhs

$$500,00,000 = \frac{n}{2}\left(2 \times 10,00,000 + (n-1)3,00,000\right]$$

Or $1000 = n[20 + 3n - 3]$

Or $3n^2 + 17n - 1,000 = 0$

$$n = \frac{-17 \pm \sqrt{17^2 + 4 \times 3 \times 1000}}{2 \times 3}$$

= 15.64 neglecting the negative value.

Thus, the fund will be raised in 16 years. Last years contribution is equal to

$$= 500,00,000 - \frac{15}{2}\left[2 \times 10,00,000 + (15-1)\ 3,00,000\right]$$

$$= 500,00,000 - \frac{15}{2}\ (20,00,000 + \ 42,00,000)$$

$$= 35,00,000.$$

∴ (b) is correct

16. A pile of bricks is stacked so that there are 24 bricks in the bottom layer and each successive layer contains one brick less. The top layer contains 6 bricks. How many bricks are there in the whole pile.

(a) 285 (b) 100 (c) 125 (d) none of these

Sol. This is a problem in Arithmetic progression. The first term and the last term is given. We have to find the whole sum. Obviously, here $d = -1,\ a = 24,\ I - 6$.To find n, we proceed as

$24 + (n-1)(-1) = 6$

⇒ $(n-1) = 24 - 6 = 18$

$n = 19.$

Therefore, since $S_n = \frac{n}{2}(a+1) = \frac{19}{2}(24+6) = 15 \times 19 = 285.$

∴ (a) is correct

17. The annual salary increment of a monthly salaries person is in the arithmetic progression. Itis known that he drew monthly salary of ₹ 10,000 in the year 2003 and ₹ 14,000 in the year 2019. He started his service in the year 1990 and shall attain the edge of superannuation in the year 2029. Calculate the monthly salary with which he started the job in the year 1990 and also the monthly salary in the year of his super annuation.

(a) ₹13,500 (b) ₹16,500 (c) ₹16,100 (d) none of these

Sol. Let the starting monthly salary of the person be a and annual increment be d .

Then, Salary in 2003 $= a+(14-1)d = 10,000 \Rightarrow a+13d = 10,000$

Salary in 2019 $= a+(30-1)d = 14,000 \Rightarrow a+29d = 14,000$

$\therefore 16\,d = 4,000 \Rightarrow d = 250$ and $a = 10,000 - 13\times 250 = 6,750$

Monthly salary of the person in 1990 = 6,750

Salary in the year of superannuation (2029)

= 6,750+ (40 - 1)250

= 6,750 + 9,750 = ₹ 16,500

∴ (b) is correct

18. A man repays a loan of ₹ 3,250 by paying ₹ 20 in the first month and then increases the payment by ₹ 15 every month. How many months approximately will it take to clear his loan?

(a) 20 (b) 11 (c) 15 (d) none of these

Sol. Payment in the first month = ₹ 20

Payment in the second month = ₹ (20 + 15) = ₹ 35

Payment in the third month = ₹ (35 + 15) = 50

Thus the repayments from an A .P., with first term $a = 20$ common difference $d = 15$; the sum being $S_n = 3,250$. We are to find n, the no. of terms.

Since, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, we may write

$$3,250 = \frac{n}{2}\left[2\times 20+(n-1)15\right]$$

or, $6,500 = n\left[40+(n-1)15\right]$

$= 15n^2 + 25n$

Transposing.

$$15n^2 + 25n - 6,500 = 0,$$

Or, $3n^2 + 5n - 1,300 = 0$ [Dividing throughout by 5]

$$\therefore \quad n = \frac{-5 \pm \sqrt{5^2 + 4\times 3\times 1,300}}{2\times 3}$$

$$= \frac{-5 \pm \sqrt{25 + 15,\ 600}}{6}$$

$$= \frac{-5 \pm 125}{6} = \frac{120}{6} \text{ or } \frac{-130}{6}$$

Neglecting the negative value, $n = \frac{120}{6} = 20.$

Hence, the required answer = 20 months 1 year 8 months.

∴ (a) is correct

19. A man borrows ₹ 1,00,000 from a firm and under the agreement he agrees to pay ₹ 5,000 at the end of each 6 months together with an interest 1% on the opening balance of each period. Find the total interest which he pays on clearing the loan.

(a) 10,500 (b) 10,100

(c) 10,200 (d) none of these

Sol. Interest paid after first period = $1,00,000 \times 0.01 = 1.000$

Interest paid after second period = $(1,00,000 - 5,000)0.01 = 950$

Interest paid after third period and so on = $(1,00,000 - 2 \times 5,000)0.01 = 900$

The stream of interest payments from an decreasing A.P. with 1,000 as first term - 50 is common difference and $\frac{1,00,000}{5,000}$ = 20th number of terms.

The total interest paid is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{20}{2}\left[2 \times 1000 + (20-1)(-50)\right]$$

= 10 [2000 - 950] = 10, 500

∴ (a) is correct

20. A person agrees to pay-off a debt of ₹ 36,000 in 40 annual instalments, which form an A.P. When 30 instalments are paid he dies leaving one-third of the debt unpaid. Find the value of first instalment.

(a) 100 (b) 110 (c) 510 (d) none of these

Sol. Let the value of 1st instalment be a. The no. of instalments being 40, we may write

$S_{40} =$ ₹$36,000$

By the given condition, $S_{30} = 36,000 - \frac{1}{3} \times 36,000 = 24,000$

Since $S_n = \frac{n}{2}[2a + (n-1)d$, we may write

$$S_{40} = \frac{40}{2}(2a + 39d) = 36,000$$

or $2a + 39d = 1800$ ...(i)

Also $S_{30} = \frac{30}{2}(2a + 29d) = 2400$

or $2a + 29d = 1,600$ ...(ii)

Subtracting (ii) from (i),

$10d = 200$ i.e. $d = 20$

Putting the value of d in (ii)

$a = \frac{1600 - 29 \times 20}{2} = 510$ = Required value of first instalment.

∴ (c) is correct

21. On 1st January every year, a person buys NSCs. (National Savings Certificates) of value exceeding that of his last year's purchase by ₹ 100. After 10 years, he finds that the total value of the certificates held by him, is ₹ 54,500. Find the value of the certificates purchased by him.

(i) in the first year

(ii) in the eighth year

(a) 2,700 (b) 5,700 (c) 1,700 (d) none of these

Sol. The investment made by the person form a arithmetic progression with common difference as 100 and sum of first 10 terms as 54,500.

$$S_n = \frac{n}{2} = \left[2a + (n-1)d\right]$$

$$54,500 = \frac{10}{2}\left[2a + (10-1)100\right]$$

or $$\frac{54,500}{5} = 2a + 900$$

$$a = \frac{10,900 - 900}{2} = 5,000$$

In first year he has purchased NSCs worth ₹ 5,000.

$t_8 = 5,000 + (8-1)100 = 5,700$

In eight year he has purchased NSCs worth ₹ 5,700.

∴ (b) is correct

22. Find the 7th term of the A.P. 8, 5, 2, -1, -4,

(a) 11 (b) -10 (c) 12 (d) none of these

Sol. Here a = 8, d = 5 – 8 = -3

Now t_7 = 8 + (7 - 1) d

= 8 + (7 - 1) (- 3)

= 8 + 6 (- 3)

= 8 – 18

= - 10

∴ (b) is correct

23. Which term of the AP $\frac{3}{\sqrt{7}}, \frac{4}{\sqrt{7}}, \frac{5}{\sqrt{7}}$ … … … …is $\frac{17}{\sqrt{7}}$?

(a) $\frac{17}{\sqrt{7}}$ (b) $\frac{18}{\sqrt{7}}$ (c) $\frac{22}{\sqrt{7}}$ (d) none of these

Sol. $a = \frac{3}{\sqrt{7}}, d = \frac{4}{\sqrt{7}} - \frac{3}{\sqrt{7}} = \frac{1}{\sqrt{7}}, t_n = \frac{17}{\sqrt{7}}$

We may write

$\frac{17}{\sqrt{7}} = \frac{3}{\sqrt{7}} + (n-1) \times \frac{1}{\sqrt{7}}$

or, 17 = 3 + (n - 1)

or, n = 17 - 2 = 15

Hence, 15^{th} term of the A.P. is $\frac{17}{\sqrt{7}}$.

∴ (a) is correct

24. If 5^{th} and 12^{th} terms of an A.P. are 14 and 35 respectively, find the A.P.

(a) 2, 5, 8, 11, 14, (b) 2, 5, 8, 11, 15,

(c) 2, 5, 7, 11, 14, (d) none of these

Sol. Let a be the first term & d be the common difference of A.P.

$t_5 = a + 4d = 14$

$t_{12} = a + 11d = 35$

On solving the above two equations,

7d = 21 = i.e., d = 3

and a = 14 - (4 × 3) = 14 - 12 = 2

Hence, the required A.P. is 2, 5, 8, 11, 14,

∴ (a) is correct

25. Divide 69 into three parts which are in A.P. and are such that the product of the first two parts is 483.

(a) 21, 24, 25 (b) 21, 23, 25.

(c) 12, 16, 18 (d) none of these

Sol. Given that the three parts are in A.P., let the three parts which are in A.P. be a - d, a, a + d…..

Thus a - d + a + a + d = 69

or 3a = 69

or a = 23

So the three parts are 23 - d, 23, 23 + d

Since the product of first two parts is 483, therefore, we have

23 (23 - d) = 483

or 23 - d = 483 / 23 = 21

or d = 23 - 21 = 2

Hence, the three parts which are in A.P. are

23 - 2 = 21, 23, 23 + 2 = 25

Hence the three parts are 21, 23, 25.

∴ (b) is correct

26. Find the arithmetic mean between 4 and 10.

(a) 7 (b) 9 (c) 12 (d) none of these

Sol. We know that the A.M. of a & b is = (a + b)/2

Hence, The A. M between 4 &10 = (4 + 10)/2 = 7

∴ (a) is correct

27. Insert 4 arithmetic means between 4 and 324.

(a) 68,132,196,260 (b) 58,132,196,160

(c) 28,132,196,220 (d) none of these

Sol. Here a=4, d = ? n = 2 + 4 = 6, t_n = 324

Now $t_n = a + (n - 1)d$

or 324= 4 + (6 - 1) d

or 320= 5d i.e., = i.e., d = 320 / 5 = 64

So the 1^{st} AM = 4 + 64 = 68

2^{nd} AM = 68 + 64 = 132

3^{rd} AM = 132 + 64 = 196

4^{th} AM = 196 + 64 = 260

∴ (a) is correct

TRY YOURSELF - 2

1. For what values of k is the sequence $\{k-4,\ k-2,\ k+\ 2\}$ a geometric progression.

(a) $k=1$ (b) $k=6$ (c) $k=5$ (d) none of these

Sol. Since in a geometric progression, square of the middle term equals the product of the preceding and succeeding terms, we should have,

$$(k-2)^2=(k-4)(k+2)$$

$\Rightarrow$ $k^2-4k+4=k^2-2k-8$

$\Rightarrow$ $2k-12$

or, $k=6$

Therefore, for $k=6$, the terms $\{k-4,k-2,k+2\}=\{2,\ 4,\ 8\}$ follows a geometric progression.

$\therefore$ (b) is correct

2. Find the G.P. whose 4th term is 8 and 8th term is $\frac{128}{625}$.

(a) 125, 50, 20, 8, $\frac{16}{5}$ (b) $125,\ 50,\ 20,\ 8,\frac{2}{5}$

(c) $125,\ 50,\ 20,\ 8,\frac{11}{5}$ (d) none of these

Sol. If a is the first term and r is the common ratio of a G.P., then

$8=t_n=ar^3$ and $\frac{128}{625}=t_8=ar^7$.

$\Rightarrow$ $\frac{ar^3}{ar^3}=\frac{128}{625}\times\frac{1}{8}$ $\Rightarrow$ $r^4=\frac{16}{625}$

$\Rightarrow$ $r^4=\left(\pm\frac{2}{5}\right)^4$ $\Rightarrow$ $r=\pm\frac{2}{5}$

When $r=\frac{2}{5},a\left(\frac{2}{5}\right)^3=8$ $\Rightarrow$ $a=\frac{8\times125}{8}=125$

When $r=-\frac{2}{5},a\left(-\frac{2}{5}\right)^3=8$ $\Rightarrow$ $a=\frac{8\times125}{-8}=-125$

$\therefore$required G.P. is either 125, 50, 20, 8, $\frac{16}{5}$

or -125, 50 - 20, 8, $\frac{-16}{5}$, ….

$\therefore$ (a) is correct

3. Find the geometric mean between 3 and 27.

(a) 7 (b) 12 (c) 9 (d) none of these

Sol . Here $a = 3, b = 27.$

The geometric mean between a and b is $\sqrt{ab}$.

$\therefore$The geometric mean between 3 and 27 is $\sqrt{3\times 27}=9$.

$\therefore$ (c) is correct

4. Insert 3 geometric means between $\frac{1}{9}$ and 9.

(a) $-\frac{1}{3},1,-3$ (b) $\frac{1}{3},2,+3$ (c) $-\frac{1}{3},2,-4$ (d) none of these

Sol. If' n geometric means are to be inserted between a and b , then common ratio r is given by $r=\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

Here, $r=1/9, b=9, n=3$

$$\therefore r=\left(\frac{9}{\frac{1}{9}}\right)^{\frac{1}{4}}=(81)^{\frac{1}{4}}=(\pm 3)^{\frac{1}{4}}$$

$$r=\pm 3.$$

Therefore required geometric means are

$$\frac{1}{9}\times 3,\frac{1}{9}\times 3^2,\frac{1}{9}\times 3^3$$

or $\frac{1}{9}\times(-3),\frac{1}{9}\times(-3)^2,\frac{1}{9}\times(-3)^3$

i.e. either $\frac{1}{3},1,3$ or $-\frac{1}{3},1,-3$.

$\therefore$ (a & b) is correct

5. Prove that the A.M. of two positive numbers is greater or equal to their geometric mean.

(a) $\frac{a+b}{1}\geq\sqrt{ab}$ (b) $\frac{a+b}{2}\geq\sqrt{ab}$

(c) $\frac{a+b}{2}\leq\sqrt{ab}$ (d) none of these

Sol. Let the two numbers be a and b

Arithmetic mean = $\frac{a+b}{2}$, Geometric mean = $\sqrt{ab}$

To prove $\frac{a+b}{2} \geq \sqrt{ab}$

i.e. $a+b \geq 2\sqrt{ab}$

i.e. $a+b-2\sqrt{ab} \geq 0$

i.e. $\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$

Now the expression $\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$ is true (square of a number is always positive).

Hence $\frac{a+b}{2} \geq \sqrt{ab}$ is also true.

∴ (b) is correct

6. 57 Find three numbers in G.P. whose sum is $\frac{57}{2}$ and whose product is 729.

(a) 6, 9, $\frac{20}{2}$ (b) 6, 9, $\frac{15}{2}$ (c) 6, 9, $\frac{27}{2}$ (d) none of these

Sol. Let the three numbers be $\frac{a}{r}, a, ar.$

Given, $\frac{a}{r}.\ a.\ ar = 729$

or , $a^3 = 729 = 27^2 = \left(3^3\right)^2 = \left(3^2\right)$ $i.e.,\ a = 9$

It is also given that $\frac{a}{r} + a + ar = \frac{57}{2}$

or, $a\left(\frac{1}{r}+1+r\right) = \frac{57}{2}$

or. $1+r+r^2 = \frac{57}{2\times 9}r = \frac{19r}{6}$

or, $6r^2 - 13r + 6 = 0$

or, $(3r-2)(2r-3) = 0$ or, $r = 2/3, 3/2$

Therefore, the required numbers are either $\frac{27}{2}$,9, 6, or, 6, 9, $\frac{27}{2}$.

∴ (c) is correct

7. Give the correct answer with reasons for the following questions:

If $a^{1/x} = b^{1/y} = c^{1/z}$ and a, b, c are in G.P., then x, y, z are in

(a) A.P. (b) G.P (c) M.P (d) none of these

Sol. If $a^{1/x} = b^{1/y} = c^{1/z} = k$ then $a = k^x, b = k^y;\ c = k^z$

Since a, b, c are in G.P., $b^2 = ac$

i.e., $= k^{2y} = k^x.k^z$

or, $= k^{2y} = k^{x+z}$

or. $2y = x + z$ and so x, y, z are in A.P.

∴ (a) is correct

8. If a, b, c, d are in geometric progression, show that: $(b-c)^2 + (c-a)^2 + (d-b)^2 = ?$

(a) $(a+d)^2$ (b) $(a-d)^2$ (c) $(a-d)^3$ (d) none of these

Sol. Let r be the common ratio. Since a, b, c, d are in G.P., $b = ar,\ c = ar^2,\ d = ar^3$.

Now, $(b-c)^2 + (c-a)^2 + (d-b)^2$

$$= (ar - ar^2)^2 + (ar^2 - a)^2 + (ar^3 - ar)^2$$

$$= a^2\left[(r - r^2)^2 + (r^2 - 1)^2 + (r^3 - r)^2\right]$$

$$= a^2\left[r^6 - 2r^3 + 1\right]$$

$$= a^2(r^3 - 1)^2 \quad ...(1)$$

Also $(a-d)^2 = (a - ar^3)^2 = a^2(1 - r^3)^2$ ...(2)

From (1) and (2), we can observe that

$(b-c)^2 + (c-a)^2 + (d-b)^2 = (a-d)^2$.

Which is to be proved.

∴ (b) is correct

9. If S be the sum, P the product and R the sum of the reciprocals of first n terms in a geometric progression, prove that $P^2R^n = ?$

(a) P^4 (b) P^0 (c) S^n (d) none of these

Sol. Let the n terms in G.P. be $a, ar, ar^2, ..., ar^{n-1}$

Then $S = \dfrac{a(1 - r^n)}{1 - r}$

$P = (a)(ar)(ar^2)....(ar^{n-1})$

$= a^n r^{1+2+3+...+(n-1)} = a^n r^{n(n-1)/2}$

$P^2 = a^{2n} r^{n(n-1)}$

$R = \dfrac{1}{a} + \dfrac{1}{ar} + \dfrac{1}{ar^2} ... + \dfrac{1}{ar^{n-1}}$

$$=\frac{\frac{1}{a}\left[\left(\frac{1}{r}\right)^n-1\right]}{\frac{1}{r}-1}=\frac{r\left(1-r^n\right)}{a\left(1-r\right)r^n}=\frac{1-r^n}{ar^{n-1}\left(1-r\right)}$$

$$\frac{S}{R}=\frac{a\left(1-r^n\right)}{\left(1-r\right)}\times\frac{ar^{n-1}\left(1-r\right)}{\left(1-r^n\right)}=a^2r^{n-1}$$

Hence, $$\frac{S^n}{R^n}=\left(a^2r^{n-1}\right)^n=a^{2n}r^{n(n-1)}=P^2$$

Thus, we have $P^2R^n=S^n$

∴ (c) is correct

10. Find the missing numbers and on using suitable formula give the sum of the following:
$1+3+9+*+81+243+*+2187.$

(a) 3280 (b) 3192 (c) 3320 (d) none of these

Sol. Given $1+3+9*+81+243+*+2187$, we may write the sum,

$S=1+3+3^2+*+3^4+3^5+*+3^7$

∴Number of terms 8, and the series is in G.P. with Common ratio 3.

$\therefore t_4$, the 4th term $=3^3=27, t_7$, the 7th term = 3^6 = 729

$\left[\therefore t_n=ar^{n1}\right]$

∴Required missing numbers are 27, 729; and

the sum, $S=\frac{a\left(r^n-1\right)}{r-1}=\frac{1\left(3^8-1\right)}{3-1}=\frac{6561-1}{2}=\frac{6560}{2}=3280.$

∴ (a) is correct

11. A man borrows ₹ 8,000 the simple interest rate of 2.76% per annum. It is decided that the principal and the interest are to be paid in 10 monthly instalments. If each instalments is double of the preceding instalment, find the value of the first and the last instalment.

(a) ₹ 4296 (b) ₹ 4096
(c) ₹ 4196 (d) none of these

Sol. Interest to be paid = $\frac{2.76\times 10\times 8,000}{100\times 2}$ =₹184

Total amount to be paid in 10 monthly instalments 8000+ 184 = ₹ 8184

Let a be the first instalment. Since instalments are in G.P. with common ratio 2, we have

$$8184=\frac{a\left(2^{10}-1\right)}{2-1}$$

Hence, $a=\frac{8184}{1023}$ =₹ 8 (First instalment)

The last instalment = $8\times2^9=8\times512=$₹4096.

∴ (b) is correct

12. A person decides to save ₹ 2.00 in January, ₹ 4.00 in February, ₹ 8.00 in March, ₹ 16.00 in April and so on up to the end of year. Determine:

(i) How much amount he will save during the whole year?

(ii) What will he save in the month of October?

(a) ₹ 1024 (b) ₹ 2056

(c) ₹ 4552 (d) none of these

Sol. According to question, person saves

₹ 2, ₹ 4, ₹ 8 ... in respective months

Clearly, it is G.P., where first term is 2 and common ratio is also 2.

(i) Total saving up to end of year $=\dfrac{2\left(2^{12}-1\right)}{2-1}=2\left(4096-1\right)=$₹ $8{,}190$

(ii) Saving in October 10th term, of G.P. = $2\times2^{10-1}=$₹$1{,}024$.

∴ (a) is correct

13. Three numbers are in arithmetic progression and their sum is 21. If 1, 5, 15 be added to them respectively, they form a geometric progression. Find the numbers.

(a) $d=-16;\ 23,$ 7 and -9 (b) $d=-16;\ 23,$ 7 and -8

(c) $d=-16;\ 23,$ 7 and -10 (d) none of these

Sol. Let the three numbers in Arithmetic Progression be $a-d,\ a, a+d$. Then by the given condition,

$$(a-d)+a+(a+d)=21$$

$$3a=21\times a=7$$

or

By adding 1, 5, 15 the numbers become,

$8-d,\ 12,\ 22+d$

Since, it is given that these are in geometric progression, we have

$$12^2-(8-d)(22+d)$$

or, $144=176-22d+8d-d^2$

or, $d^2+14d-32=0$

or, $(d+16)(d-2)=0$

or, $d=-16$ or 2.

Hence the numbers are,

When $d=2;\ 5,\ 7$ and 9; or when $d=-16;\ 23,$ 7 and -9.

∴ (a) is correct

14. If a, b, c are in arithmetic progression and x, y, z are in geometric progression, prove that $x^{b-c}, y^{c-a}, z^{a-b} = ?$

(a) 1 (b) 5

(c) 4 (d) none of these

Sol. If a, b, c are in A.P. then $b = \frac{a+c}{2}$

and x, y, z are in G.P. then $y^2 = xz$

L.H.S. $= x^{\frac{a+c}{2}-c} (xz)^{\frac{c-a}{2}} z^{a-\frac{a+c}{2}}$

$= x^{\frac{a+c}{2}-c} x^{\frac{c-a}{2}} z^{a-\frac{a+c}{2}}$

$= x^{\frac{a-c}{2}+\frac{c-a}{2}} z^{\frac{c-a}{2}} z^{a-\frac{a+c}{2}}$

$= x^0 z^0 = 1 =$ R.H.S.

$\therefore$ (a) is correct

15. Give the correct answer for the following with reasons:

If $\frac{a^{n+1}+b^{n-1}}{a^n+b^n}$ is arithmetic mean and a and b, $a \neq b$, then n is equal to

(a) 1 (b) -1 (c) $\frac{1}{2}$ (d) 0

Sol. Given $\frac{a^{n+1}+b^{n-1}}{a^n+b^n} = \frac{a+b}{2}$

or, $2\left(a^{n+1}+b^{n+1}\right) = a^{n+1}+ab^n+ba^n+b^{n+1}$

or, $a^{n+1}+b^{n+1} = ab^n+ba^n$

or, $a^n(a-b) = b^n(a-b)$

or, $a^n = b^n \ (\because a \neq b)$

or, $\left(\frac{a}{b}\right)^n = 1 = \left(\frac{a}{b}\right)^0 \quad \therefore n = 0$

$\therefore$ (d) is correct

16. If the sum of an infinite Geometric progression is 20 and the sum of the squares of its terms is 100, find the Progression.

(a) $8, \frac{24}{5}, \frac{72}{25}, \ldots$ (b) $5, \frac{24}{5}, \frac{72}{25}, \ldots$

(c) $2, \frac{24}{5}, \frac{72}{25}, \ldots$ (d) none of these

Sol. Let G.P. be $a, ar, ar^2, \ldots$

Given, $20 = a + ar + ar^2 + \ldots \infty$, clearly, $r < 1$.

and $100 = a^2 + a^2r^2 + a^2r^4 + \ldots \infty$

$\Rightarrow$ $20 = \frac{a}{1-r}$ and $100 = \frac{a^2}{1-r^2}$

i.e., $400 = \frac{a^2}{(1-r)^2}$ (1) and $100 = \frac{a^2}{1-r^2}$ .....(2)

Dividing (1) by (2), $4 = \frac{a^2}{(1-r)^2} \times \frac{1-r^2}{a^2} = \frac{1+r}{1-r}$

or, $4(1-r) = 1 + r \Rightarrow 5r = 3$ i.e., $r = \frac{3}{5}$.

Consequently, $20 = \frac{a}{1-\frac{3}{5}}$ or, $5a = 40$ i.e., $a = 8$.

Hence, required progression is $8, \frac{24}{5}, \frac{72}{25}, \ldots$

$\therefore$ (a) is correct

17. By expressing as an infinite geometric series find the value of 0.2175.

(a) $\frac{159}{1650}$ (b) $\frac{359}{1650}$ (c) $\frac{259}{1650}$ (d) none of these

Sol. $0.2175 = 0.21757575\ldots = 0.21 + .0075 + 000075 + .00000075 + \ldots$

$= 0.21 + \frac{75}{10^4} + \frac{75}{10^6} + \frac{75}{10^8} + \ldots = 0.21 + \frac{75}{10^4}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + \ldots\right)$

$= 0.21 + \frac{75}{10^4}\left(\frac{1}{1-\frac{1}{10^2}}\right) = 0.21 + \frac{75}{10^4} \times \frac{100}{99} = \frac{21}{100} + \frac{3}{4} \times \frac{1}{99} = \frac{21}{100} + \frac{1}{132}$

$= \frac{693+25}{3300} = \frac{718}{3300} = \frac{359}{1650}$.

$\therefore$ (b) is correct

18. Find the sum of the series. $3 + 33 + 333 + \ldots +$ to n terms

(a) $\frac{10}{27}\left(10^n - 1\right) + \frac{n}{3}$ (b) $\frac{10}{27}\left(10^n - 1\right) = \frac{n}{3}$

(c) $\frac{10}{27}\left(10^n - 1\right) - \frac{n}{3}$ (d) none of these

Sol. Sun to n terms $=3+33+333+...+$ to n terms

$$=\frac{1}{3}\{9+99+999+\text{... to } n \text{ terms}\}$$

$$=\frac{1}{3}\{(10-1)+(10^2-1)+(10^3-1)+...(10^n-1)\}$$

$$=\frac{1}{3}\{10+10^2+...+10^n-n)$$

$$=\frac{1}{3}\left\{\frac{10(10^n-1)}{10-1}-n\right\}$$

$$=\frac{10}{27}(10^n-1)-\frac{n}{3}$$

$\therefore$ (c) is correct

19. Find the sum n to terms the series $.8+.88+.888+....$

(a) $\frac{1}{9}\left[n-\frac{1}{9\times10^n}(10^n-1)\right]$ (b) $\frac{8}{9}\left[n-\frac{1}{9\times10^n}(10^n-1)\right]$

(c) $\frac{8}{9}\left[n+\frac{1}{9\times10^n}(10^n-1)\right]$ (d) none of these

Sol. Let S_n be the sum of the first n terms. Then

$$S_n=.8+.88+.888+....+\text{....to } n \text{ terms}$$

$$=8(.1+.11+.111+\text{....to } n \text{ terms})$$

$$=\frac{8}{9}(.9+.99+999+\text{....to } n \text{ terms})$$

$$=\frac{8}{9}[(1-.1)+(1-.01)+(1-.001)+\text{....to } n \text{ terms}]$$

$$=\frac{8}{9}\left[(1-10^{-1})+(1-10^{-2})+...+(1-10)^{-n}\right]$$

$$=\frac{8}{9}\left[n-(10^{-1}+10^{-2}+....+10^{-n})\right]$$

$$=\frac{8}{9}\left[n-10^{-1}\left(\frac{1-(10^{-1})^n}{1-\frac{1}{10}}\right)\right]$$

$$=\frac{8}{9}\left[n-\frac{1}{10}\times\frac{10}{9}\left(\frac{(10^n-1)}{10^n}\right)\right]$$

$$=\frac{8}{9}\left[n-\frac{1}{9\times10^n}(10^n-1)\right]$$

$\therefore$ (b) is correct

20. Sum to n terms $6+27+128+629+...$

(a) $\frac{5}{4}\left(5^{n}-1\right)-\frac{n\left(n+1\right)}{2}$ (b) $\frac{5}{4}\left(5^{n}-1\right)=\frac{n\left(n+1\right)}{2}$

(c) $\frac{5}{4}\left(5^{n}-1\right)+\frac{n\left(n+1\right)}{2}$ (d) none of these

Sol. The given series may be written as $(5+1)+\left(5^{2}+2\right)+\left(5^{3}+3\right)+\left(5^{4}+4\right)+.....$ to n terms Required sum = $\left(5+5^{2}+5^{3}+5^{4}+....\text{ to terms}\right)+(1+2+3+....\text{ to } n \text{ terms})$

$$=\frac{5\left(5^{n}-1\right)}{5-1}+\frac{n\left(n+1\right)}{2}$$

$$=\frac{5}{4}\left(5^{n}-1\right)+\frac{n\left(n+1\right)}{2}$$

∴ (c) is correct

21. Sum to first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....$ is:

(a) $2^{n}-n-1$ (b) $n+2^{n}-1$ (c) $2^{n}-1$ (d) none of these

Sol. $t_{1}=\frac{1}{2}=1-\frac{1}{2}$

$t_{2}=\frac{3}{2}=1-\frac{1}{4}=1-\frac{1}{2^{2}}$

$t_{3}=\frac{7}{8}=1-\frac{1}{8}=1-\frac{1}{2^{3}}$

$t_{n}=1-\frac{1}{2^{n}}$

Thus $t_{1}+t_{2}+...t_{n}=(1+1+1+....\text{ to } n \text{ terms})-\left(\frac{1}{2}+\frac{1}{2^{2}}+\frac{1}{2^{3}}+....+\frac{1}{2^{n}}\right)$

$$=n-\frac{1}{2}\left\{\frac{1-\left(\frac{1}{2}\right)^{n}}{1-\frac{1}{2}}\right\}=n-\left\{1-\left(\frac{1}{2}\right)^{n}\right\}$$

$$=n+2^{-n}-1.$$

∴ (b) is correct

22. If a, ar, ar^2, ar^3, ... be in G.P. Find the common ratio.

(a) r (b) d (c) n (d) none of these

Sol. 1^{st} term = a, 2^{nd} term = ar

Ratio of any term to its preceding term = ar/a = r = common ratio.

∴ (a) is correct

23. Which term of the progression 1, 2, 4, 8, ... is 256?

(a) 256 (b) 119 (c) 155 (d) none of these

Sol. a = 1, r = 2/1 = 2, n = ? t_n = 256

$t_n = ar^{n-1}$

or $256 = 1 \times 2^{n-1}$ i.e., $2^8 = 2^{n-1}$ or, n - 1 = 8 i.e., n = 9

Thus 9^{th} term of the G. P. is 256

∴ (a) is correct

24. Insert 3 geometric means between 1/9 and 9.

(a) 1/3,1, 3 (b) -1/3,1,-3

(c) +1/3,-1, 3 (d) none of these

Sol. 1/9, –, –, –, 9

a = 1/9, r = ?, n = 2 + 3 = 5, t_n = 9

we know $t_n = ar^{n-1}$

or $1/9 \times r^{5-1} = 9$

or $r^4 = 81 = 3^4$ => r = 3

Thus 1^{st} G.M = 1/9 × 3 = 1/3

2^{nd} G.M = 1/3 × 3 = 1

3^{rd} G. M = 1 × 3 = 3

∴ (a) is correct

25. Find the G.P where 4^{th} term is 8 and 8^{th} term is 128/625

(a) 125, 50, 20, 8, 16/5, (b) 125, 50, 20, 2, 16/5,

(c) 125, 50, 20, 5, 16/5, (d) none of these

Sol. Let a be the 1^{st} term and r be the common ratio.

By the question $t_4 = 8$ and $t_8 = 128/625$

So $ar^3 = 8$ and $ar^7 = 128 / 625$

Therefore $ar^7 / ar^3 = \frac{128}{625 / 8}$ => $r^4 = 16 / 625 = (\pm 2/5)^4$ => r = 2/5 and -2 /5

Now $ar^3 = 8$ => a × $(2/5)^3$ = 8 => a = 125

Thus the G. P is

125, 50, 20, 8, 16/5,

When r = -2/5 , a = -125 and the G.P is -125, 50, -20, 8, -16/5 ,

Finally, the G.P. is 125, 50, 20, 8, 16/5,

or, -125, 50, -20, 8, -16/5,

∴ (a) is correct

26. Find the sum of 1 + 2 + 4 + 8+ ...to 8 terms.

(a) 125 (b) 255 (c) 455 (d) none of these

Sol. Here a = 1, r = 2/1 = 2, n = 8

Let S = 1 + 2 + 4 + 8 + ….. to 8 terms

$= 1 (2^8 - 1) / (2 - 1) = 2^8 - 1 = 255$

∴ (b) is correct

27. Find the sum to n terms of 6 + 27 + 128 + 629 +......

(a) $\{5 (5^n - 1) /4\} + \{n (n + 1) / 2\}$ (b) $\{6 (5^n - 1) /4\} + \{n (n + 1) / 2\}$

(c) $\{8 (5^n - 1) /4\} + \{n (n + 1) / 2\}$ (d) none of these

Sol. Required Sum $= (5 + 1) + (5^2 + 2) + (5^3 + 3) + (5^4 + 4) + \dots$ to n terms

$= (5 + 5^2 + 5^3 + \dots + 5^n) + (1 + 2 + 3 + .. + n \text{ terms})$

$= \{5 (5^n - 1) / (5 - 1)\} + \{n (n + 1)/2\}$

$= \{5 (5^n - 1) /4\} + \{n (n + 1) / 2\}$

∴ (a) is correct

28. Find the sum to n terms of the series 3 + 33 + 333 + ……

(a) $\frac{5}{27}(10^{n+1} - 9n - 10)$ (b) $\frac{1}{27}(10^{n+1} - 9n - 10)$

(c) $\frac{11}{27}(10^{n+1} - 9n - 10)$ (d) none of these

Sol. Let S denote the required sum.

i.e. S = 3 + 33 + 333 + …………… to n terms

= 3 (1 + 11 + 111 +…………….. to n terms)

$= \frac{3}{9}(9 + 99 + 999 + \dots$ to n terms)

$= \frac{3}{9}\{(10- 1) + (10^2- 1) + (10^3- 1) + \dots + (10^{n - 1})\}$

$= \frac{3}{9}\{(10 + 10^2 + 10^3 + \dots + 10^n) -n\}$

$= \frac{3}{9}\{10 (1 + 10 + 10^2 + \dots + 10^{n - 1}) - n\}$

$= \frac{3}{9}[\{10 (10^n - 1) / (10 - 1)\} - n]$

$= \frac{3}{81}(10^{n+1} - 10 - 9n)$

$= \frac{1}{27}(10^{n+1} - 9n - 10)$

∴ (b) is correct

29. Find the sum of n terms of the series 0.7 + 0.77 + 0.777 + ... to n terms

(a) $\frac{5}{81}\{9n - 1 + 10^{-n}\}$ (b) $\frac{7}{81}\{9n - 1 + 10^{-n}\}$

(c) $\frac{27}{81}\{9n - 1 + 10^{-n}\}$ (d) none of these

Sol. Let S denote the required sum.

i.e. S = 0.7 + 0.77 + 0.777 + to n terms

= 7 (0.1 + 0.11 + 0.111 + ... to n terms)

= $\frac{7}{9}$ (0.9 + 0.99 + 0.999 + ... to n terms)

= $\frac{7}{9}$ {(1 - 1/10) + (1 - $1/10^2$) + (1 - $1/10^3$) + ... + (1 - 1/ 10^n)}

= $\frac{7}{9}$ (n - $\frac{1}{10}$ (1 + 1/10 + $1/10^2$+.... + $1/10^{n-1}$)}

So S = $\frac{7}{9}$ {n - $\frac{1}{10}$ (1 - $1/10^n$)/(1 - 1/10)}

= $\frac{7}{9}$ {n - (1 - 10^{-n}) / 9)}

= $\frac{7}{81}$ {9n - 1 + 10^{-n}}

∴ (b) is correct

30. Evaluate 0.2175 using the sum of an infinite geometric series.

(a) 359/1650 (b) 265/1650

(c) 444/1650 (d) none of these

Sol. 0.2175 = 0.2175757575 ....

0.2175 = 0.21 + 0.0075 + 0.000075 +....

= 0.21 + 75 (1 + $1/10^2$ + $1/10^4$ + ...) / 10^4

= 0.21 + 75 {1 / (1- $1/10^2$} / 10^4

= 0.21 + ($75/10^4$) × 10^2 /99

= 21/100 + (¾) × (1/99)

= 21/100 + 1/132

= (693 + 25)/3300 = 718/3300 = 359/1650

∴ (a) is correct

31. Find three numbers in G. P whose sum is 19 and product is 216.

(a) 9, 6, 4 (b) 9, 5, 4 (c) 9, 2, 4 (d) none of these

Sol. Let the 3 numbers be a/r, a, ar.

According to the question a/r × a × ar = 216

or $a^3 = 6^3$ => a = 6

So the numbers are 6/r, 6, 6r

Again 6/r + 6 + 6r = 19

or 6/r + 6r = 13

or $6 + 6r^2 = 13r$

or $6r^2 - 13r + 6 = 0$

or $6r^2 - 4r - 9r + 6 = 0$

or 2r(3r - 2) - 3 (3r - 2) = 2

or (3r - 2) (2r - 3) = 0 or, r = 2/3, 3/2

So the numbers are

$6/(2/3), 6, 6 \times (2/3) = 9, 6, 4$

or $6/(3/2), 6, 6 \times (3/2) = 4, 6, 9$

$\therefore$ (a) is correct

HOME WORK-1

1. Divide 144 into three parts which are in AP. and such that the largest is twice the smallest, the smallest of three numbers will be :

(a) 48 (b) 36 (c) 13 (d) 32

[June 2010]

Sol. Let t_1 = a and cd = d

∴ a+a+d+ a + 2d=144

or 3a + 3d = 144

or 3(a + d) = 144

or $a+d=\frac{144}{3}=48$

∴ a+d=48 = _______ (1)

∴ Largest = 2x Smallest

∴ a+2d=2a

2d = a

d = a/2

From (1) a

$a+\frac{a}{2}=48$

Or $\frac{3}{2}a=48 \therefore a=48\times\frac{2}{3}$

$\therefore a=32$

∴ (d) is correct

Tricks:- GBC

2. If the sum of n terms of an A.P. is $2n^2 + n$. What is the difference between its 10th term & 1st term

(a) 207 (b) 36 (c) 90 (d) 63

[June 2011]

Sol. $S_n = 2n^2+n$

$\therefore t_1 = s_1 = 2\times1^2+1=3$

$s_2 = 2\times2^2+2=10$

$\therefore d = s_2 - 2s_1 = 10-2\times3=4$

$\therefore t_{10}-t_1 = a+9d-a=9d=9\times4$

= 36

∴ (b) is correct

3. Geometric mean of P, p^2 , p^3 ...., p^n will be

(a) $p^{(n+1)}$ (b) $p^{\left(\frac{1+n}{2}\right)}$ (c) $p\frac{n(n+1)}{2}$ (d) None of the above

[June 2011]

Sol. $GM = \left(p.p^2.p^3............p^n\right)^{1/n}$

$= \left(p^{1+2+3+............+n}\right)^{1/n}$

$= \left[p\frac{n(n+1)}{2}\right] = p^{(n+1/2)}$

Tricks :- Put n=3

$GM = \left(p.p^2, p^3\right)^{1/3} = p^2$

For (a) $GM = p^{3+1} \neq p^2$

(b) $GM = p\frac{1+3}{2} = p^2$

∴ (b) is correct.

4. Find the number whose arithmetic mean is 12.5 and geometric mean is 10.

(a) 20 and 5 (b) 10 and 5 (c) 5 and 4 (d) None of these

[Dec. 2011]

Sol. Tricks:- Go by choices

For (a) $AM = \frac{20+5}{2} = 12.5$

and $GM = \sqrt{20 \times 5} = 10$

∴ 20 & 5 satisfy both given condition in qts.

∴ (a) is correct.

5. If sum 3 arithmetic mean between "a" and 22 is 42, then "a"=__________

(a) 14 (b) 11 (c) 10 (d) 6

[Dec. 2011]

Sol. Tricks:- It A_1; A_2 ;A_3;................;An are “n” AMS

$A_1 + A_2 + A_3 + ... + A_n = n\left(\frac{a+b}{2}\right)$

$= n.(\text{AM of a and b})$

$\therefore 3\left(\frac{a+22}{2}\right) = 42 \therefore a = 6$

∴ (d) is correct.

6. If each month ₹100 increases in any sum then find out the total sum after 10 months, if the sum of first month is ₹2,000.

(a) ₹24,500 (b) ₹24,000 (c) ₹50,000 (d) ₹60,000

[Dec. 2011]

Sol. Sum = $\frac{10}{2}\left[2\times 2000+(10-1).100\right]$

= ₹ 24,500.

∴ (a) is correct.

7. 8th term of an A.P is 15, then sum of its 15 terms is

(a) 15 (b) 0 (c) 225 (d) 225/2

[June 2012]

Sol. $t_8 = a + 7d = 15$

$S_{15} = \frac{15}{2}\left[2a+(15-1)d\right] = \frac{15}{2}\times 2(a+7d)$

$= 15\times 15 = 225$

∴ (c) is correct.

8. Find the sum of the infinite terms $2, \frac{4}{y}, \frac{8}{y^2}, \frac{16}{y^3}$.............; If $y > 2$

(a) $\frac{2y}{y-2}$ (b) $\frac{4y}{y-2}$ (c) $\frac{3y}{y-2}$ (d) None of these

[June 2012]

Sol. $s = \frac{a}{1-r} = \frac{2}{1-\frac{2}{y}} = \frac{2y}{y-2}$

∴ (a) is correct.

9. The 4th term of an A.P. is three times the first and the 7th term exceeds twice the third term by 1. Find the first term 'a' and common difference'd'.

(a) a = 3,d=2 (b) a=4,d=3 (c) a=5,d=4 (d) a=6,d=5

[June 2012]

Sol. $t_4 = 3t_1 \Rightarrow a + 3d = 3a \therefore 2a = 3d; a = \frac{3d}{2}$

$\therefore t_7 = 2t_3 + 1$

or a + 6d = 2(a + 2d) +1

or a + 6d=2a + 4d + 1

or 2d-a=1

or $2d - \frac{3}{2}d = 1 \Rightarrow \frac{d}{2} = 1 \therefore d = 2$

and $a=\frac{3}{2}\times 2=3$

Tricks :- Go by choices

$\therefore$ (a) is correct.

10. In an A.P., if common difference is 2, Sum of n terms is 49, 7th term is 13 then n =__

(a) 0 (b) 5 (c) 7 (d) 13

[Dec. 2012]

Sol. $t_7=a+6\times 2=13\therefore a=1$

$s_n=\frac{n}{2}\left[2\times 1+(n-1).2\right]=49$

or $\frac{n}{2}.2\left[1+(n-1).2\right]=49$

or $n^2=49\therefore n=7$

$\therefore$ (c) is correct.

11. The first term of a GP. When second term is 2 and sum of in term is 8 will be

(a) 6 (b) 3 (c) 4 (d) 1

[Dec. 2012]

Sol. $t_2=ar=2\Rightarrow r=\frac{2}{a}$

$S_\infty=\frac{a}{1-r}=8$

Or $a=8(1-r)$

or $a=8\left(1-\frac{2}{a}\right)$

or $a^2=8(a-2)$

or $a^2-8a+16=0$

or $(a-4)^2=0\Rightarrow a=4$

Tricks :- Go by choices

For (c) $4r=2\therefore r=\frac{1}{2}$

$S=\frac{9}{1-r}=\frac{4}{1-1/2}=8$ (Which is correct)

$\therefore$ (c) is correct.

12. If the sum of n terms of an A.P be $2n^2+5n$, then its 'nth' term is

(a) 4n-2 (b) 3n-4 (c) 4n+3 (d) 3n+4

[Dec. 2012]

Sol. $\therefore S_n = 2n^2 + 5n$

$\therefore S_1 = t_1 = 2\times1^2 + 5\times1 = 7 = a$

$d = S_2 - 2S_1$

$= 2\times2^2 + 5\times2 - 2\times7 = 4$

$t_n = a + (n-1)d = 7 + (n-1)4 = 4n + 3$

Tricks :- Go by choices

For (a) $S_1 = t_1 = 4\times1 - 2 = 2 \neq 7$

(c) $t_1 = 4\times1 + 3 = 7$

$t_2 = 4\times2 + 3 = 11$

$s_2 = t_1 + t_2 = 7 + 11 = 18$

and $S_2 = 2\times2^2 + 5\times2 = 18$

$\therefore$ (c) Satisfies it

$\therefore$ (c) is correct.

13. In an A.P. if $s_n = 3n^2 - n$ and its common difference is 6 then first term is_______

(a) 2 (b) 3 (c) 4 (d) 6

[June 2013]

Sol. $S_n = 3n^2 - n$

$S_1 = 3\times1^2 - 1 = 2 = t_1$

$\therefore$ 1st term =2

14. In an A. P if the sum of 4th & 12th term is 8 then sum of first 15 term is______

(a) 60 (b) 120 (c) 110 (d) 150

[June 2013]

Sol. Given, $t_4 + t_{12} = 8$

or $a + 3d + 1 + 11d = 8$

or $2a + 14d = 8$

$\therefore s_{15} = \frac{15}{2}[2a + (15-1)d]$

$= \frac{15}{2}\times8 = 60$

$\therefore$ (a) is correct

15. There are 'n' AMs between 7 & 71 and 5th AM is 27 then 'n' =

(a) 15 (b) 16 (c) 17 (d) 18

[June 2013]

Sol. c.d $=\frac{b-a}{n+1}$ (Tricks)

$=\frac{71-7}{n+1}=\frac{64}{n+1}$

$A_5=a+5d$ (Tricks)

$=7+5\times\frac{64}{n+1}=27$

or $\frac{5\times 64}{n+1}=20$

or 20n+20= 320

or 20n=300 ∴n=15

∴ (a) is correct

16. An AP has 13 terms whose sum is 143. The third term is 5, then first term is

(a) 4 (b) 7 (c) 9 (d) 2

[Dec. 2013]

Sol. $\therefore t_3=a+2d=5$ ________ (1)

$\therefore 2d=5-a$

$s_{13}=\frac{13}{2}\left[2a+(13-1)d\right]=143$

or $2a+12d=\frac{143\times 2}{13}=22$

or $a+6d=11$

or $a+3\times 2d=11$

or $a+3(5-a)=11$

or a+15-3a=11

or $4=2a\therefore a=2$

Tricks :- Go by choices

[Solve mentally by calculator]

∴ (d) is correct

17. GM of a,b,c,d is 3 then GM of $\frac{1}{a},\frac{1}{b},\frac{1}{c},\frac{1}{d}$ is

(a) $\frac{1}{3}$ (b) 3 (c) $\frac{1}{81}$ (d) 81

[Dec. 2013]

Sol. $G = 3(abcd)^{1/4}$ ____________(1)

New GM $= \left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}\right)^{1/4} = \frac{1}{3}$

Tricks :- GM of a,b,c,d = 3

GM of their Reciprocals = $\frac{1}{3}$

∴ (a) is correct

18. The value of $1^3+2^3+3^3+\ldots\ldots.. +m^3$ is equal to

(a) $\left[\frac{m(m+1)}{2}\right]^3$ (b) $\frac{m(m+1)(2m+1)}{6}$

(c) $\left[\frac{m(m+1)}{2}\right]^2$ (d) None

[June 2014]

Sol. Formula = $\left\{\frac{m(m+1)}{2}\right\}^2$

∴ (c) is correct

19. The sum of the infinite GP $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\ldots\ldots.\infty$ is equal to

(a) 1.95 (b) 1.5 (c) 1.75 (d) None

[June 2014]

Sol. $S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2} = 1.5$

∴ (b) is correct

20. The sum of m terms of the series is 1+11+111+..... is equal to

(a) $\frac{1}{81}\left[10^{m+1} - 9m - 10\right]$ (b) $\frac{1}{2}\left[10^{m+1} - 9m - 10\right]$

(c) $\left[10^{m+1} - 9m - 10\right]$ (d) None of these

[June 2014, June 2015]

Sol. Tricks :- Go by choices

For (a) put m = 1; we get

$s=\frac{1}{81}\left[10^{1+1}-9\times1-10\right]=1=$ 1st term

Put $m-2;S=\frac{1}{81}\left[19^{2+1}-9\times2-10\right]=12$

= 1 + 11 = Sum of 1st 2 terms

∴ (a) is correct

21. If the sum of first 'n' terms of an A.P is $6n^2+6n$, then the fourth term of the series:

(a) 120 (b) 72 (c) 48 (d) 24

[Dec. 2014]

Sol. S_n=Sum of 1st n terms of as AP.

$=6n^2+6n$

$a=t_1=s_1=6\times1^2+6\times1=12$

$s_2=6\times2^2+6\times2=36$

$c,d=d=s_2-2s_1=36-2\times12=12$

$\therefore\ t_4=a+(4-1)d=12+3\times12=48$

∴ (c) is correct

22. If $S_n=n^2p$ and $S_m=m^2p;(m\neq n)$ is the sum of A.P., then S_p = ___________

(a) p^2 (b) p^3 (c) $2p^3$ (d) p^4

[Dec. 2014]

Sol. $\therefore s_n=n^2p$

$s_m=m^2p$

$\therefore s_p=p^2.p=p^3$

∴ (b) is correct

23. If x, y, z are the terms in GP then the terms $x^2+y^2,\ xy+yz,\ y^2+z^2$ are in:

(a) A.P (b) GP (c) H.P (d) None of these

[Dec. 2014]

Sol. $\therefore x;\ y;\ z$ are in G.P

Tricks:- Let x = 1; y=2; z=4 make a GP

$x^2+y^2=1^2+2^2=5$

$xy+yz=1\times2+2\times4=10$

$y^2+z^2=2^2+4^2=20$

$\therefore x^2+y^2;\ xy+yz;y^2+z^2=$

5,10,20………… clearly are in G.P.

$\therefore$ (b) is correct

24. The sum of n terms of an AP is $3n^2 + 5n$, which last term is 164.
(a) 25 (b) 27 (c) 29 (d) 31
[Dec. 2015]

Sol. $S_n = 3n^2 + 5n$

$a = t_1 = S_1 = 3\times1^2 + 5\times1 = 8$

$S_2 = 3\times2^2 + 5\times2 = 22$

$d = S_2 - 2S_1 = 22 - 2\times8 = 6$

$n = \frac{t_n - a}{d} + 1 = \frac{164-8}{6} + 1 = 27$

$\therefore$ (b) is correct

25. Three No's a,b,c are in A.P find a-b+c
(a) a (b) -b (c) b (d) c
[Dec. 2015]

Sol. let a = 1;b=2; c = 3 makes an A.P.

$\therefore a - b + c = 1 - 2 + 3 = 2 = b.$

$\therefore$ (c) is correct

26. Find the numbers whose GM is 5 and AM is 7.5:
(a) 12 and 13 (b) 13.09 and 1.91 (c) 14 and 11 (d) 17 and 19
[Dec. 2015]

Sol. Tricks: Go by Choices

$GM = \sqrt{13.09\times1.51} = 5.$(approx.)

$AM = \frac{13.09 + 1.91}{2} = 7.5$

$\therefore$ (b) is correct

27. If $\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}$ in Arithmetic Progression then a^2, b^2, c^2 are in ________.
(a) Arithmetic Progression (b) Geometric Progression
(c) Both A.P & GP (d) None of these
[June 2016]

Sol. Tricks:- a^2, b^2, c^2 are in AP.

a=1,b=5,c=7 Make it in AP

let

$\therefore \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ in AP

$\frac{1}{5+7}, \frac{1}{7+1}, \frac{1}{1+5}$

$\left[\frac{1}{12}, \frac{1}{8}, \frac{1}{6}\right] \times 24$

2, 3, 4 is also in AP.

$\therefore$ Our assumption is correct.

$\therefore$ (b) is correct

28. $2.353535\ldots\ldots\ldots\ldots = 2.\dot{3}\dot{5}$

(a) $\frac{233}{99}$ (b) $\frac{234}{99}$ (c) $\frac{232}{99}$ (d) None

[Dec. 2016]

Sol. Tricks: Go by choices [use calculator]

Divide 233 by 99 we get 2.3535...

$\therefore$ (a) is correct

29. The number of terms of the series needed for the sum of the series 50 +45+40+ …………. becomes zero

(a) 22 (b) 21 (c) 20 (d) None

[Dec. 2016]

Sol. Tricks: Go by choices

Let (b) is correct.

$$S_{21} = \frac{21}{2}\left[2 \times 50 + (21-1) \times (-5)\right]$$

$= 0$

$\therefore$ (b) is correct.

30. A person received the salary for the 1st year is ₹5,00,000 per year and he received an increment of ₹15,000 per year then the sum of the salary he takes in 10 years

(a) ₹ 56,75,000 (b) ₹ 72,75,000 (c) ₹ 63,75,000 (d) None of these

[Dec. 2016]

Sol. $S_{10} = \frac{10}{2}\left[2 \times 5,00,000 + (10-1) \times 15000\right]$

= ₹56,75,000.

∴ (a) is correct.

31. Find the sum of all natural numbers between 100 and 1000 which are divisible by 11 is:

(a) 44,550 (b) 66,770 (c) 55,440 (d) 33,440

[Dec. 2017]

Sol. Series

S= 110+121+132 +………..+990

$n = \frac{1-a}{d} + 1 = \frac{990-110}{11} + 1 = 81$

$S = \frac{n}{2}(a+1) = \frac{81}{2}(110+990) = 44.550$

∴ (a) is correct.

32. If pth, qth, rth terms of a GP. be a, b, c respectively, then (q-r) log a + (r-p) log b+(p-q) log c=

(a) 0 (b) 1 (c) 2 (d) None

[June 2018]

Sol. Tricks: It is in cyclic order.

∴ (a) is correct.

33. If a, b, c, d are in GP then $(b-c)^2 + (c+a)^2 + (d-b)^2$ = ?

(a) $(a-b)^2$ (b) $(a-d)^2$ (c) $(c-d)^2$ (d) 0

[June 2018]

Sol. a, b, c, d →in GP

let a=1; b=2; c=4; d=8 in

∴ $(b-c)^2 + (c-a)^2 + (d-b)^2$

= $(2-4)^2 + (4-1)^2 + (8-2)^2$

= 4+9+36= 49 = 7^2

GBC

For (b) $(a-d)^2 = (1-8)^2 = 7^2$ =4

∴ (b) is correct

34. If the n th term of a series, $a_n = 3^n - 2^n$ then S_n =?

(a) $\frac{3}{2}(3^n - 1) + 1(n+1)$ (b) $\frac{3}{2}(3^n + 1) - 1(n+1)$

(c) $\frac{3}{2}(3^n - 1) - n(n+1)$ (d) $\frac{3}{2}(3^n + 1) - 1(n-1)$

[June 2018]

Sol. $\because a_n = 3^n - 2^n$

$a_1 = 3^1 - 2^1 = 1$

$a_2 = 3^2 - 2^2 = 5$

$s_2 = a_1 + a_2 = 1 + 5 = 6$

Tricks: Go by choices (GBC)

for (c) let

$s_n = \frac{3}{2}\left(3^n - 1\right) - n\left(n+1\right)$

$s_1 = \frac{3}{2}\left(3^1 - 1\right) - 1\left(1+1\right) = \frac{3}{2}2 - 2 = 1 = a_1$ (True)

Now $s_n = \frac{3}{2}\left(3^2 - 1\right) - 2\left(2+1\right)$

$= \frac{3}{2} \times 8 - 6 = 12 - 6 = 6 = a_1 + a_2$ (True)

$\therefore$ (c) is correct

35. A person pays Rs. 975 in monthly instalments, each instalment is less than former by Rs. 5. The amount of 1st instalment is ₹100. In what time will be entire amount be paid?

(a) 26 months (b) 15 months (c) Both (a) & (b) (d) 18 months

[May 2018]

Sol. Tricks:- Go by choices (GBC)

Series

$S = 100 + 95\,90 + \ldots\ldots$ to n months (let)

= 975.

1st check for n = 15 months

$S = \frac{15}{2}\left[2 \times 100 + (15-1) - (-5)\right]$

If loan is paid off in n = 15 months, then no need of other instalments.

$\therefore$ (b) is correct.

36. If the sum of n terms of an AP is (3n-n) and its common difference is 6, then its first term is:

(a) 3 (b) 2 (c) 4 (d) 1

[May 2018]

Sol. $S_n = 3n^2 - n$

Tricks:-

$\therefore t_1 = S_1 = 3 \times 1^2 - 1 = 2$

= sum of 1st 1 term.

∴ (b) is correct.

37. Insert two arithmetic means between 68 and 260.

(a) 132, 196 (b) 130, 194 (c) 70, 258 (d) None

[May 2018]

Sol. Tricks:-

Go by choices

(a) 68; 132; 196; 260 are in AP.

Hence; 132; 196 are A.Ms. b/w 68 and 260.

∴ (a) is correct.

38. If the P[th] term of an A.P. is 'q' and the q[th] term is 'p', then its r[th] term is

(a) p+q+r (b) p+q-r (c) p-q-r (d) p + q

[Nov. 2018]

Sol. Tricks:-

$$c.d = \frac{q-p}{p-q} = \frac{(p-q)}{p-q} = -1$$

$$\therefore t_r = t_p + (r-p)d$$

$$= q + (r-p).(-1)$$

$$= q + p - r$$

39. The 3[rd] term of a GP. is $\frac{2}{3}$ and the 6[th] term is $\frac{2}{81}$, then the 1[st] term is

(a) 2 (b) 6 (c) 9 (d) $\frac{1}{3}$

[Nov. 2018]

Sol. $t_3 = ar^2 = \frac{2}{3}; t_6 = ar^5 = \frac{2}{81}$

or $ar^2.r^3 = \frac{2}{81}$

or $\frac{2}{3}r^3 = \frac{2}{81} \Rightarrow r^3 = \left(\frac{1}{3}\right)^3$

$\therefore ar^2 = \frac{2}{3}$

or $a.\left(\frac{1}{3}\right)^2 = \frac{2}{3}$

or $a = 6$

∴ (b) is correct.

40. The sum of the series -8, -6, -4,...n terms is 52. The number of terms n is

(a) 10 (b) 11 (c) 13 (d) 12

Sol. Series $S = -8 - 6 - 4..........$ to n terms

first term-$= -8; c.d = d = 2$

Tricks :- Go by choices (Use calculator)

$\therefore$ option (c)

$S_{13} = \frac{13}{2}\left[2 \times (-8) + (13-1) \times 2\right] = 52$

41. The value of K, for which the terms 7K + 3, 4K - 5, 2K + 10 are in A.P., is

(a) -13 (b) -23 (c) 13 (d) 23

[Nov. 2018]

Sol. Formula 2A=a+b

$\therefore 2(4k-5) = 7k + 3 + 2k + 10$

Or $8k - 10 = 9k + 13$

Or $k = -23$

$\therefore$ (b) is correct.

42. The ratio of sum of n terms of the two AP's is (n+1) then the ratio of their m^{th} terms is

(a) (m + 1): 2m (b) (m+1): (m - 1)

(c) (2m-1): (m + 1) (d) m: (m-1)

[June 2019]

Sol. Given that

$\frac{S_n^1}{S_n^{11}} = \frac{n+1}{n-1}$

Tricks:-

To find the ratio of r^{th} term;

put n = 2r – 1

$\therefore$Put n = 2m - 1

Ratio of m^{th} term

$= \frac{2m-1+1}{2m-1-1} = \frac{2m}{2m-2}$

$= \frac{2m}{(2m-2)} = \frac{m}{m-1}$

$\therefore$ (d) is correct.

43. In a G.P. if the fourth term is '3' then the product of first seven terms is

(a) 3^5 (b) 3^7 (c) 3^6 (d) 3^8

[June 2019]

Sol. Tricks:-

Product of 1st (2r-1) terms of a

G.P $= (t_r)^{2r-1}$

$\therefore t_4 = 3$

So; Product of 1st 2x4-1=7 terms

$= (t_r)^{2\times4-1} = 3^7$

$\therefore$ (b) is correct.

Details:-

$\therefore$ Product of 1st 7 terms

$= a.ar.ar^2.ar^3 \ldots\ldots\ldots ar^6$

$= a^7.r^{1+2+3+\ldots\ldots+6}$

$= a^7.r^{\frac{6}{2}(6+1)} = a^7.r^{21}$

$= (ar^3)^7 = 3^7$

44. If 2+6+10+14+18+…..+ x = 882 then the value of x

(a) 78 (b) 80 (c) 82 (d) 86

[June 2019]

Sol. $S = 2 + 6 + 10 + 14 \ldots\ldots + x$ (to n terms) = 882

$\therefore \frac{n}{2}[2 + x] = 882$ (1)

Where x = Last term

Last term = $x = 2 + (n-1)\times 4$

$x = 4n - 2$

or $4n = x + 2$

or $n = \frac{x+2}{4}$

From (1); we get

$\frac{(x+2)}{4\times2}(x+2) = 882$

or $(x+2)^2 = 8\times882 = 84^2$

$\therefore x + 2 = 84 \Rightarrow x = 82$

Tricks:-

Let $t_n = x$

or $2 + (n-1) - 4 = x$

or $4n - 2 = x$

or $n \frac{x+2}{4}$

For GBC

(c)) If $x = 82 \Rightarrow n = \frac{82+2}{4} = 21$

$\therefore S = \frac{n}{2}(a+1) = \frac{21}{2}(2+x)$

$= \frac{21}{2}(2+82) = 882$

$\therefore$ (c) is correct.

45. If $y = 1 + x + x^2 + ...\infty$ then x =

(a) $\frac{y-1}{y}$ (b) $\frac{y+1}{y}$ (c) $\frac{y}{y+1}$ (d) $\frac{y}{y-1}$

[June 2019]

Sol. $y = 1 + x + x^2 + ...\infty$ are in G.P

$\therefore y = \frac{1}{1-x}$ Where $c.r = x$

or $1 - x = \frac{1}{y}$

or $x = 1 - \frac{1}{y} = \frac{y-1}{y}$

$x = 1 - \frac{1}{y} = \frac{y-1}{y} . \left[\therefore S_\infty = \frac{a}{1-r}\right]$

$\therefore$ (a) is correct.

46. In the series 25, 5, 1, ... 1/3125 which term is 1/3125?

(a) 8th term (b) 9th term

(c) 15th term (d) None of these

[Dec. 2019]

Sol. Let

$t_n = \frac{1}{3125}$.

$\therefore 25.\left(\frac{1}{5}\right)^{n-1} = \frac{1}{5^5}$

or $5^2.\frac{1}{5^{n-1}} = \frac{1}{5^5}$

or $5^{n-1} = 5^7 \Rightarrow n - 1 = 7$

$\therefore n = 8$

$\therefore$ (a) is correct.

47. The sum of five terms of AP is 75 find the 3rd term is.

(a) 20 (b) 30 (c) 15 (d) None of these

[Dec. 2019]

Sol. $t_3 = a+(3-1)d = a+2d$.

$S_5 = \frac{5}{2}[2a+(5-1)d] = 75$

or $\frac{5}{2}.2[a+2d] = 75$

or $a+2d = \frac{75}{5} = 15$

So, $t_3 = 15$.

∴ (c) is correct.

48. (c+a-b)/b, (a+b-c)/c, (b+c-a)/a are in AP then a,b,c are in

(a) AP (b) GP (c) HP (d) None of these

[Dec. 2019]

Sol. Adding 2 to each term; we get

$\frac{c+a-b}{b}+2; \frac{c+b-c}{c}+2; \frac{b+c-a}{a}+2$

are also in AP

$\Rightarrow \frac{a+b+c}{b}; \frac{a+b+c}{c}; \frac{a+b+c}{a}$

are in AP

Dividing all terms by (a+b+c); we get

$\frac{1}{b}, \frac{1}{c}; \frac{1}{a}$ are also in AP.

$\Rightarrow$ b; c; a are in HP.

OR a, c; b are in HP.

but a; b; c are not in HP.

∴ (d) is correct.

49. The 20th term of arithmetic progression whose 6^{th} term is 38 and 10^{th} term is 66 is.............

(a) 136 (b) 118 (c) 178 (d) 210

[Dec. 2020]

Sol. Tricks

Common difference

$$= d = \frac{t_{10} - t_6}{10 - 6}$$

$$= \frac{66 - 38}{4} = 7$$

Tricks

$$t_{20} = a + 19d$$

$$= \left[a + (6-1)d\right] + 14d$$

= 38 +14 × 7

= 136

∴ (a) is correct.

50. Three numbers in G.P with their sum is 130 and their product is 27,000 are ……..

(a) 90, 30, 10 (b) 10,30,90 (c) (a) & (b) Both (d) 10,20,30

[Dec. 2020]

Sol. Tricks: GBC (Go by choices)

* (a) & (b) both follow G.P.
* sum of terms = 90+30+10=130 (also follows)
* Their product = 90 × 30 × 10 = 27000/-

Which is also satisfied

∴ option (c) is correct.

51. Divide 69 into 3 parts which are in A.P and are such that the product of first two parts is 460

(a) 20, 23, 26 (b) 21, 23, 25 (c) 19, 23, 27 (d) 22, 23, 24

[Dec. 2020]

Sol. Tricks : GBC (Go by choices)

* All options are in A.P.
* Only in option (a)

Product of 1st two terms

= 20 x 23=460 (True)

∴ (a) is correct

52. The nth terms of the series 3+7+13+21+31+………is

(a) 4n-1 (b) n^2+2n (c) n^2+n+1 (d) n^3+2

[Jan. 2021]

Sol. ricks

In such type of Questions always find answer by GBC (Go by choices).

For $n=1 \Rightarrow t_1 = 3$

for $n=2 \Rightarrow t_2 = 7$

and $n=3 \Rightarrow t_3 = 13$

Putting n = 1 in all options, we get $t_1 = 3$

So, Here, we cannot decide any option.

Now putting n=2 in all options we get in

(a) $t_2 = 4 \times 2 - 1 = 7 = t_2$ (True)

(b) $t_2 = 2^2 + 2 \times 2 = 8 \neq t_2$ (False)

(c) $t_2 = 2^2 + 2 + 1 = 7 = t_2$ (True)

(d) $t_2 = 2^3 + 2 = 10 \neq t_2$ (False)

Hence, we conclude that option (a) or (c) should be answer. (Both same)

So check for n = 3 in (a) & (c); we get

(a) $t_3 = 4 \times 3 - 1 - 11 \neq 13 = t_3$ (False)

(c) $t_3 = 3^2 + 3 + 1 = 13 = t_3$ (True)

$\therefore$ (c) should be correct.

53. In a geometric progression the 3rd and 6th terms are respectively 1 and -1/8. The first term (a) and common ratio are respectively.

(a) 4 and $\frac{1}{2}$ (b) 4 and $\frac{-1}{4}$ (c) 4 and $\frac{-1}{2}$ (d) 4 and $\frac{1}{4}$

[Jan. 2021]

Sol. Tricks GBC [Go by choices]

From (a) $t_3 =, ar^{3-1} = 4\left(\frac{1}{2}\right)^2 = 1$ (True)

and $t_6 =, a\,r^{6-1} = \left(\frac{1}{2}\right)^5 = \frac{1}{8} \neq \frac{1}{8}$ (False)

So (a) is False

$(b)\, t_3 = a\,r^2 = 4\left(-\frac{1}{4}\right)^2 = 4\frac{1}{16} = \frac{1}{4} \neq 1$

(It is also False)

$(c)\, t_3 = a\,r^2 = 4\left(-\frac{1}{2}\right)^2 = 4\frac{1}{4} = 1$ (True)

$t_6 =, a\,r^5 = 4\left(-\frac{1}{2}\right)^5 = 4\left(-\frac{1}{32}\right) = -\frac{1}{8}$ (True)

$\therefore$ (c) is correct

54. The number of terms of the series: 5+7+9+... must be taken so that the sum may be 480

(a) 20 (b) 10 (c) 15 (d) 25

[July 2021]

Sol. Let $S = 5+7+9+...$ to "n" terms = 480

Tricks: Go by choices (GBC)

For (a) at n = 20

$$S = \frac{20}{2}\left[2\times 5+(20-1).2\right]$$

$=10(10+38=480)$ (True)

$\therefore$ (a) is correct

55. If the sum of 'n' terms of an AP (Arithmetic Progression) is $2n^2$, the fifth term is____

(a) 20 (b) 50 (c) 18 (d) 25

[July 2021]

Sol. Qt$_{5=}$S$_5$ ' S$_4$ [i.e. sum of 1st 5 terms sum of 1st 4 terms]

$= 2 \times 5^2 - 2 \times 4^2$

= 50-32=18

$\therefore$ (c) is correct

56. The sum of square of any real positive quantities and its reciprocal is never less that

(a) 1 (b) 2 (c) 3 (d) 4

[July 2021]

Sol. Let a positive no. = x

From question,

Two nos. are x^2 & $\frac{1}{x^2}$

Its Arithmetic mean

$$= A = \frac{x^2+\frac{1}{x^2}}{2}$$

and Its Geometric mean

$$G = \sqrt{x^2\,\frac{1}{x^2}} = \sqrt{1} = 1$$

We know that

$A \geq G$

or $\frac{x^2+\frac{1}{x^2}}{2} \geq 1$

or $x^2 + \frac{1}{x^2} \geq 2$

Minimum value of $x^2 + \frac{1}{x^2}$ is 2

$\therefore$ (b) is correct

57. The sum of series 7+14+21+..... to 17th term is:

(a) 1071 (b) 971 (c) 1171 (d) 1271

[Dec. 2021]

Sol. S=7+14+21... to 17 terms

= 7 [1+2+3.... to 17 terms]

$= 7.\frac{17(17+1)}{2} = 1071$

[$\therefore$ 1+2+3+..... to *n* terms]

$= \frac{n(n+1)}{2}$

$\therefore$ (a) is correct

58. The sum of first n terms of an AP is $3n^2 + 5n$. The series is:

(a) 8, 14, 20, 26.......(b) 8, 22, 42, 68,... (c) 22, 68, 114,… (d) 8, 14, 28, 44,…

[Dec. 2021]

Sol. $\therefore$ $S_n = 3n^2 + 5n$

$S_1 = 3 \times 1^2 + 5 \times 1 = 8$

$S_2 = 3 \text{ x } 2^2 + 5 \times 2 = 22$

$S_3 = 3 \text{ x } 3^2 + 5 \times 3 = 42$

GBC

(A)

S_1 = 8 (True)

S_2=8+14=22 (True)

S_3=8+14+20= 42 (True)

$\therefore$ (a) is correct.

Details

$a=t_1, =S_1=3 \times 1^2+5 \times 1 = 8$

$S_2=3 \times 2^2+5 \text{ x } 2 = 22$

$c.d=d=S_2-2S_1,=22-2\times8=6$

$t_n=a+(n-1)d$

=8+(n-1).6=8+6n-6=6n+2

$t_1=6\times1+2=8$

$t_2=6\times2+2=14$

$t_3 = 6\times3+2=20$

$\therefore$ (a) is correct

59. The largest value of n for which $\frac{1}{2}+\frac{1}{2^2}+....\frac{1}{2^n}<0.998$ is ________.

(a) 9 (b) 6 (c) 7 (d) 8

[Dec. 2021]

Sol . $S=\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+....+\frac{1}{2^n}<0.998$

$$S=\frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}=\frac{\frac{1}{2}\left[1-\frac{1}{2^n}\right]}{\frac{1}{2}}<0.998$$

$=1-\frac{1}{2^n}<0.998$

or $1-0.998<\frac{1}{2^n}$

or $0.002<2^{-n}$

Calculator Trick

Press 2 ÷ button = button 9 times

= 0.00195

(Makes True)

$\therefore$ n = 9

$\therefore$ (a) is correct

60. If the nth term of the arithmetic progression 9, 7, 5... is same as the nth term of the arithmetic progression 15, 12, 9 ..., then n will be

(a) 7 (b) 9 (c) 15 (d) 11

[June 2022]

Sol. t_n of 1^{st} AP = t_n of 2^{nd} AP

$\therefore$9+ (n-1)(-2) = 15+ (n-1) (-3)

or; 9-2n+2 = 15-3n+3

or 3n-2n=18-11

or n = 7

$\therefore$ (a) is correct

61. In a geometric progression, the second term is 12 and the sixth term is 192. Find the 11^{th} term.

(a) 3,072 (b) 1,536 (c) 12,288 (d) 6,144

[June 2022]

Sol. Given

$t_2 = ar = 12$........ (1)

$t_6 = ar^5 = 192$......(2)

Eqn. (2)÷(1); we get

$$\frac{t_6}{t_2} = \frac{ar^5}{ar} = \frac{192}{12}$$

Or; $r^4 = 16 = 2^4$

$\therefore r = 2$

Now $t_{11} = ar^{11-1} = ar^{10}$

$= ar^5.r^5$

$= 192 \times 2^5$ (From (2))

$= 6144$

$\therefore$ (d) is correct

62. The first and last terms of an arithmetic progression are 5 and 905. Sum of the terms is 45,955. The number of terms is

(a) 99 (b) 100 (c) 101 (d) 102

[June 2022]

Sol. Let No. of terms = n.

$$S_n = \frac{n}{2}(a + l) = 45{,}955$$

Where a = 1st term;

l = last term

$$\frac{n}{2}(5 + 905) = 45955$$

or $\frac{n}{2} \times \frac{455}{910} = 45955$ (with 2 and 910 cancelled)

or $455n = 45955$

or; $n = \frac{45955}{455} = 101$

$\therefore$ (c) is correct.

63. The sum of first eight terms of geometric progression is five times the sum of the first four terms. The common ratio is

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 4 (d) 2

[June 2022]

Sol. Given

Sum of 1st 8 terms

= 5 (sum of 1st 4 terms)

Or $\frac{a\left(r^8-1\right)}{r-1}=\frac{5.a\left(r^4-1\right)}{r-1}$

or; $r^8-1 = 5(r^4-1)$

or; $(r^4)^2-1^2=5(r^4-1)$

or; $(r^4-1).(r^4+1)=5(r^4-1)$

or; $r^4+1=5$

or; $r^4=5-1=4=2^2$

or, $(r^2)^2=2^2$

or $r^2=2 \therefore r=\sqrt{2}$

$\therefore$ (a) is correct.

64. If *p*th term of an AP is *q* and its *q*th term is *p*, then what will be the value of (*p*+*q*)th term?

(a) 0 (b) 1 (c) *p*+*q*-1 (d) 2(*p*+*q*-1)

[Dec. 2022]

Sol. Detail:

SEQUENCE & SERIES

Let t_1, = a and common difference = *d*

$\therefore c\times d=d=\frac{t_p-t_q}{p-q}=\frac{q-p}{-(q-p)}$

=-1

Tricks:

$t_{p+q}=t_p+(p+q-p)d$

$=q+(q)(-1)$

$=q-q=0$

$\therefore$ (d) is correct.

65. In a G.P, 5th term is 27 and 8th term is 729. Find its 11th term.

(a) 729 (b) 6,561 (c) 2,187 (d) 19,683

[Dec.2022]

Sol. Let $t_1 = a$ and $c \times r = r$

$\therefore \frac{t_8}{t_5} = \frac{ar^7}{ar^4} = \frac{729}{27}$

Or; $r^3 = 27 = 3^3$

$\therefore r = 3$

$\therefore t_{11} = t_8 \times r^3 = 729 \times 3^3 = 729 \times 27$

$= 19,683.$

$\therefore$ (d) is correct.

66. How many number between 74 and 25,556 are divisible by 5?

(a) 5090 (b) 5097 (c) 5095 (d) 5075

[June 2023]

Sol. Series

S=75+80+85+.....+25,555

Total No. of Nos. divisible by 5

$= \frac{1-a}{d} + 1 = \frac{25,555-75}{5} + 1 = 5097$

Where a 1st term

1 =last term

$\therefore$ (b) is correct.

67. If 9th and 19th term of an Arithmetic Progression are 35 and 75, respectively, then its 20th term is:

(a) 78 (b) 79 (c) 80 (d) 81

[June 2023]

Sol. $= d = \frac{A_{19} - A_9}{19 - 9} = \frac{75 - 35}{10}$

$= \frac{40}{10} = 4$

$t_{20} = t_{19} + cd = 75 + 4 = 79$

$\therefore$ (b) is correct.

68. If 4th, 7th and 10th terms of a Geometric Progression are *p*, *q* and *r*, respectively then:

(a) $p^2 = q^2 + 2$ (b) $p^2 = qr$ (c) $q^2 = pr$ (d) $pqr + pq + 1 = 0$

Sol. Let $t_1 = a$ and $c.r = x$

$t_4 = ax^3 = p$

$t_7 = ax^6 = q$

$t_{10} = ax^9 = r$

Clearly; $q^2 = pr$

$\left(ax^6\right)^2 = ax^3 \times ax^9$

$\Rightarrow a^2X^{12} = a^2 \times X^{3+9}$

$= a^2X^{12}$

(True)

$\therefore$ (b) is correct.

HOME WORK-2

1. If a, b, c are in A.P. as well as in G.P. then -
   (a) They are also in H.P. (Harmonic Progression) (b) Their reciprocals are in A.P.
   (c) Both (a) and (b) are true (d) Both (a) and (b) are false

Sol.

a, b, c are in A.P.

$\therefore a+c = 2b$

$\Rightarrow b = (a+c)/2$ ---(i)

a, b, c are in G.P.

$\therefore b^2 = ac$ ---(ii)

Reciprocals are 1/a, 1/b, 1/c

$$\frac{1}{a}+\frac{1}{c}=\frac{a+c}{ac}$$

from (i) and (ii)

$$\frac{1}{a}+\frac{1}{c}=\frac{2b}{b^2}=\frac{2}{b}$$

$\therefore$ 1/a, 1/b, 1/c are in A.P.

$\therefore$ a, b, c are also H.P.

$\therefore$ Answer : (c)

2. If a, b, c be respectively p^{th}, q^{th} and r^{th} terms of an A.P. the value of a(q - r) + b(r - p)+ c(p - q) is__________.
   (a) 0 (b) 1 (c) -1 (d) None

Sol.

ap =a . aq = b, ar = c

Let 1^{st} term be A and difference be 'd

$\therefore$ A + (p-a)d = a ---(1)
aq= b

A + (a-1)d = b ----(2)

ar = c

A + (r - 1)d = c ---(3)

Replacing value of a,b and c

a(q-r) + b(r-p) + c(p-q)

= [A + pd - d](q-r) + (A + qd -d) (r - p)+ (A +rd -d) (p-q)

= A[q - r + r - p + p - q] -d[p - r +r - p + p - q] + d[p(q - r)+

q(r - p) + r(p - q)]

= 0 - 0 + d(pq -pr + qr - pq + pr - qr)

= 0

∴ Answer : (a)

3. If the p^{th} term of an A.P. is q and the q^{th} term is p the value of the r^{th} term is __________.

(a) p - q - r (b) p + q - r (c) p + q + r (d) None

Sol.

ap = q, aq = p ar = ?

Let a be 1st term and d is common difference

ap = q

a + (p - 1) d = q ---(i)

aq = p

a + (q - 1)d = p ---(ii)

(i) - (ii)

⇒ (p - q)d = q - p

$$d = \frac{-(p-q)}{(p-q)} = -1$$

Substituting value of 'd' we get

a + (p - 1) (-1) = q

a = q + p - 1

Now ar = a + (r - 1)d

ar = (p + q -1) +(r - 1) (-1)

ar = p + q -1 - r +1

ar = p + q - r

∴ Answer : (b)

4. If the p^{th} term of an A.P. is q and the q^{th} term is p the value of the $(p + q)^{th}$ term is __________.

(a) 0 (b) 1 (c) -1 (d) None

Sol.

ap = q, aq = p ar = ?

Let a be 1st term and d is common difference

ap = q

a + (p - 1) d = q ---(i)

aq = p

a + (q - 1)d = p ---(ii)

(i) - (ii)

$\Rightarrow$ (p - q)d = q - p

$$d = \frac{-(p-q)}{(p-q)} = -1$$

Substituting value of 'd' we get

a + (p - 1) (-1) = q

a = q + p - 1

ap + q = a + (p + q - 1)d

ap + q = p + q - 1 + (p + q - 1)(-1)

ap + q = p + q - 1 - p - q + 1

ap + q = 0

$\therefore$ Answer : (a)

5. The sum of first **n** natural number is .

(a) (n/2)(n + 1) (b) (n/6)(n + 1)(2n + 1)

(c) $[(n/2)(n + 1)]^2$ (d) None

Sol.

By formula , sum of n natural number = (n/2)(n+1)

Answer : (a)

6. The sum of square of first **n** natural number is __________.

(a) (n/2)(n + 1) (b) (n/6)(n + 1)(2n + 1)

(c) $[(n/2)(n + 1)]^2$ (d) None

Sol.

Sum of squares of n natural numbers

$= 1^2 + 2^2 + 3^2 + ...n^2$

= (n/6)(n+1)(2n+1) by formula

Answer : (b)

7. The sum of cubes of first n natural number is __________.

(a) (n/2)(n + 1) (b) (n/6)(n + 1)(2n + 1)

(c) $[(n/2)(n + 1)]^2$ (d) None

Sol.

Sum of cubes of n natural numbers

$= 1^3 + 2^3 + 3^3 + ...n^3$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \left[\frac{n(n+1)}{2}\right]^2$$

∴ Answer: (c)

8. The sum of a series in A.P. is 72 the first term is 17 and the common difference -2. The number of terms is __________.

(a) 6 (b) 12 (c) 6 or 12 (d) None

Sol.

$S_n = 72, a = 17, d = -2$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$72 = \frac{n}{2}[2(17)+(n-1)(-2)]$$

$$144 = n(34-2n+2)$$

$$144 = 36n - 2n^2$$

$$2n^2 - 36n + 144 = 0$$

$$n^2 - 18n + 72 = 0$$

$$(n-12)(n-6) = 0$$

$$n = 12, n = 6$$

∴ Answer: (c)

9. Find the sum to **n** terms of (1-1/n) + (1-2/n) + (1-3/n) +

(a) ½(n-1) (b) ½(n+1) (c) (n-1) (d) (n+1)

Sol.

$$\left(1+\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+....$$

$$(1+1+....n\ term)-\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}....n\ term\right)$$

$$= n-\frac{1}{n}(1+2+3....n\ terms)$$

$$= n-\frac{1}{n}\cdot\frac{n(n+1)}{2}$$

$$= \frac{2n-n-1}{2}$$

$$= \frac{n-1}{2}$$

∴ Answer: (a)

10. If S_n the sum of first n terms in a series is given by $2n^2 + 3n$ the series is in __________.

(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

$S_n = 2n^2 + 3n$

$S_1 = 2 + 3 = 5$

$S_2 = 8 + 6 = 14$

$S_3 = 18 + 9 = 27$

$a_2 = S_2 - S_1 = 14 - 5 = 9$

$a_3 = S_3 - S_2 = 27 - 14 = 13$

5, 9, 13, ...

Difference is same so it is A.P.

Answer : (a)

11. The sum of all natural numbers between 200 and 400 which are divisible by 7 is __________.

(a) 7,730 (b) 8,729 (c) 7,729 (d) 8,730

Sol.

Numbers are between 200 and 400 and divisible by 7

are 203, 210, ---- 399

$a = 203$, $d = 7$ and $a_n = 399$

$a_n = a + (n-1)d$

$399 = 203 + (n-1)7$

$n-1 = 196/7 = 28$

$n = 29$

$S_n = (n/2)(a + d)$

$S_n = (29/2)(203 + 399)$

$S_n = 29 \times 301$

$S_n = 8729$

$\therefore$ Answer : (b)

12. The sum of natural numbers upto 200 excluding those divisible by 5 is .

(a) 20,100 (b) 4,100 (c) 16,000 (d) None

Sol.

natural numbers upto 200 divisible by 5 are

5, 10, 15, ... 200

$a = 5,\ d = 5\ l = 100$

$a_n = a + (n-1)d$

$200 = 5 + (n-1)5$

$5n = 200$

$n = 40$

$S_{n_5} = (n/2)(a + l)$

$= (40/2)(5+200) = 4100$

1, 2, 3, ... 200

$$sum = \left[\frac{n(n+1)}{2}\right]$$

$$sum = \left[\frac{200(201)}{2}\right]$$

$sum = 20100$

Sum excluding divisible by 5 = $S_n - S_{n_5}$

$= 20100 - 4100 = 16000$

$\therefore$ Answer: (c)

13. If a, b, c be the sums of p, q, r terms respectively of an A.P. the value of (a/p) (q - r) + (b/q) (r - p) + (c/r) (p - q) is __________.

(a) 0 (b) 1 (c) -1 (d) None

Sol.

$S_p = a,\ S_q = b,\ S_r = c$

Let 1st term be A and difference be 'd'

$S_p = a$

$$\frac{p}{2}\left[2A + (p-1)d\right] = a$$

$$A + \left(\frac{p-1}{2}\right)d = \frac{a}{p} - - - (i)$$

$S_q = b$

Answer: (a)

14. If S_1, S_2, S_3 be the respectively the sum of terms of n, 2n, 3n an A.P. the value of $S_3 \div (S_2 - S_1)$ is given by __________.

(a) 1 (b) 2 (c) 3 (d) None

Sol.

S_1 = Sum of n term of A.P.

S_2 = Sum of 2n term of A.P.

S_3 = Sum of 3n term of A.P.

$$S_1 = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_2 = \frac{2n}{2}\left[2a+(2n-1)d\right]$$

$$S_3 = \frac{3n}{2}\left[2a+(3n-1)d\right]$$

$$S_2 - S_1 = \frac{2n}{2}\left[2a+(2n-1)d\right]-\frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{n}{2}\left[4a+4nd-2d-2a-nd+d\right]$$

$$= \frac{n}{2}\left[2a+3nd-d\right]$$

$$= \frac{n}{2}\left[2a+(3n-1)d\right]$$

$$\frac{S_3}{S_2-S_1} = \frac{\frac{3n}{2}\left[2a+(3n-1)d\right]}{\frac{n}{2}\left[2a+(3n-1)d\right]}$$

$$\frac{S_3}{S_2-S_1} = 3$$

∴ Answer: (c)

15. The sum of **n** terms of two A.P.s are in the ratio of (7n-5)/(5n+17). Then the __________ term of the two series are equal.

(a) 12 (b) 6 (c) 3 (d) None

Sol.

Let there are two A.P. with 1st term a and difference "d"

and second A.P. with first term A and difference D

$$\frac{S_n}{S_N} = \frac{7n-5}{5n+17}$$

$$\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2A+(n-1)D\right]} = \frac{7n-5}{5n+17}$$

$$\frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D}=\frac{7n-5}{5n+17}$$

Terms are equal so there ratio is 1

$$1=\frac{7n-5}{5n+17}$$

5n + 17 = 7n - 5

22 = 2n

n = 11

Replacing n=11 we get

$$\frac{a+5d}{A+5D}=\frac{72}{72}$$

it is 6^{th} term Answer : (b)

16. Find three numbers in A.P. whose sum is 6 and the product is -24

(a) -2, 2, 6 (b) -1, 1, 3 (c) 1, 3, 5 (d) 1, 4, 7

Sol.

Let the three numbers of A.P. be a-d, a and a+c

a-d+a+a+d = 6

3a = 6 ∴ a = 2

Product = -24

(a-d)a(a+d) = -24

$a(a^2 - d^2) = -24$

$2(4 - d^2) = -24$

$4 - d^2 = -12$

$d^2 = 16 \therefore d = \pm 4$

a = 2 and d = 4 then numbers are

-2, 2, 6

Answer : (a)

17. Find three numbers in A.P. whose sum is 6 and the sum of whose square is 44.

(a) -2, 2, 6 (b) -1, 1, 3 (c) 1, 3, 5 (d) 1, 4, 7

Sol.

Let three numbers of A.P. be a-d, a and a+d

Sum = 6

a-d+a+a+d = 6 ∴ a = 2

Sum of squares = 144

$(a-d)^2 + a^2 + (a+d)^2 = 44$

$(2-d)^2 + 2^2 + (2+d)^2 = 44$

$4 - 4d + d^2 + 4 + 4 + 4d + d^2 = 44$

$2d^2 = 32$ ∴ $d = \pm 4$

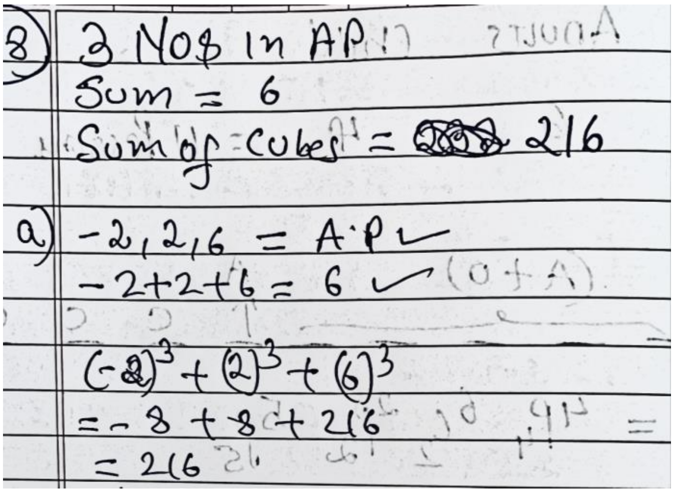
So numbers are -2, 2, 6

∴ Answer: (a)

18 Find three numbers in A.P. whose sum is 6 and the sum of their cubes is 216.

(a) -2, 2, 6 (b) -1, 1, 3 (c) 1, 3, 5 (d) 1, 4, 7

Sol.



8) 3 Nos in A.P.

Sum = 6

Sum of cubes = 216

a) -2, 2, 6 = A.P ✓

-2+2+6 = 6 ✓

$(-2)^3 + (2)^3 + (6)^3$

= -8 + 8 + 216

= 216

19. Divide 12.50 into five parts in A.P. such that the first part and the last part are in the ratio of 2:3

(a) 2, 2.25, 2.5, 2.75, 3 (b) -2, -2.25, -2.5, -2.75, -3

(c) 4, 4.5, 5, 5.5, 6 (d) -4, -4.5, -5, -5.5, -6

Sol.

Let five parts be a-2d, a-d, a, a+d, a+2d

∴ Sum of five parts = 12.5

∴ a-2d+a-d+a+a+d+a+2d = 12.5

5a = 12.5 ∴ a = 2.5

Ratio of first and last term = 2 : 3

$$\frac{a-2d}{a+2d}=\frac{2}{3}$$

3a - 6d = 2a +4d

10d = a 10d = 2.5 ∴ d = 0.25

Terms are 2, 2.25, 2.5, 2.75, 3

∴ Answer : (a)

20. If **a,** b, **c** are in A.P. then the value of $(a^3+4b^3+c^3)/[b(a^2+c^2)]$ is

(a) 1 (b) 2 (c) 3 (d) None

Sol.

a,b,c are in A.P

$$\frac{a+c}{2}=b$$

$$\frac{a^3+4b^3+c^3}{b\left(a^2+c^2\right)}$$

$$=\frac{a^3+4\left(\frac{a+c}{2}\right)^3+c^3}{\left(\frac{a+c}{2}\right)\left[a^2+c^2\right]}$$

$$=\frac{a^3+\frac{a^3+c^3+3a^2c+3ac^2}{2}+c^3}{\left(\frac{a+c}{2}\right)\left(a^2+c^2\right)}$$

$$=\frac{\frac{1}{2}\left[2a^3+a^3+c^3+3a^2c+3ac^2+2c^3\right]}{\left(\frac{a+c}{2}\right)\left(a^2+c^2\right)}$$

$$=\frac{3a^2+3a^2c+3ac^2+3c^3}{(a+c)\left(a^2+c^2\right)}$$

$$=\frac{3(a+c)\left(a^2+c^2\right)}{(a+c)\left(a^2+c^2\right)}$$

=3

∴ Answer: (c)

21. If a, b, c are in A.P. then the value of $(a^2+4ac+c^2)/(ab+bc+ca)$ is

(a) 1 (b) 2 (c) 3 (d) None

Sol.

a, b,c are in A.P.

$\therefore \frac{a+c}{2}=b$

$\frac{a^2+4ac+c^2}{ab+bc+ca}$

$=\frac{a^2+4ac+c^2}{b(a+c)+ca}$

$=\frac{a^2+4ac+c^2}{\left(\frac{a+c}{2}\right)(a+c)+ca}$

$=\frac{2(a^2+4ac+c^2)}{a^2+2ac+c^2+2ac}$

$=\frac{2(a^2+4ac+c^2)}{(a^2+4ac+c^2)}=2$

∴ Answer: (b)

22. If **a,** b, **c** are in A.P. then (a/bc) (b + c), (b/ca) (c + a), (c/ab) (a + b) are in __________.

(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

a, b, c are in A.P.

$\frac{a+c}{2}=b$

Now

$\frac{a}{bc}(b+c)+\frac{c}{ab}(a+b)$

$=\frac{a^2(b+c)+c^2(a+b)}{abc}$

$=\frac{a^2b+a^2c+ac^2+bc^2}{abc}$

$=\frac{b(a^2+c^2)+ac(a+c)}{abc}$

$=\frac{b(a^2+c^2)+ac(2b)}{abc}$

$=\frac{a^2+c^2+2ac}{abc}$

$=\frac{1}{ac}(a+c)^2$

$= \frac{(a+c)(a+c)}{ac}$

$= \frac{2b(a+c)}{ac}$

$= 2\left[\frac{b(a+c)}{ac}\right]$

$\therefore \frac{a}{bc}(b+c), \frac{b}{ac}(a+c), \frac{c}{ab}(a+b)$

are in A.P.

∴ Answer: (a)

23. If a, b, c are in A.P. then $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in __________.

(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

a, b, c are in AP

$\therefore a+c = 2b$

Now $a^2(b + c) + c^2(a+b)$

$= a^2b + a^2c + ac^2 + b^2c$

$=b(a^2 + c^2) + ac(a+c)$

$=b(a^2+c^2) + ac(2b)$

$=b[A^2 + c^2+2ac]$

$= b(a+c)^2$

$=b(a+c)(a+c)$

$=b(2b)(a+c)$

$=2b^2(a+c)$

$=2[b^2(a+c)]$

$\Rightarrow a^2(b+c), b^2(a+c), c^2(a+b)$ given are in

A.P.

Answer : (a)

24. If $(b + c)^{-1}$, $(c + a)^{-1}$, $(a + b)^{-1}$ are in A.P. then a^2, b^2, c^2 are in __________.

(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

$(b + c)^{-1}$, $(c + a)^{-1}$, $(a + b)^{-1}$ are in A.P.

$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$

$\frac{b+c-c-a}{(c+a)(b+c)} = \frac{c+a-a-b}{(a+b)(c+a)}$

$$\frac{b-a}{(c+a)(b+c)}=\frac{c-b}{(a+b)(c+a)}$$

$$a^2-b^2=b^2-c^2$$

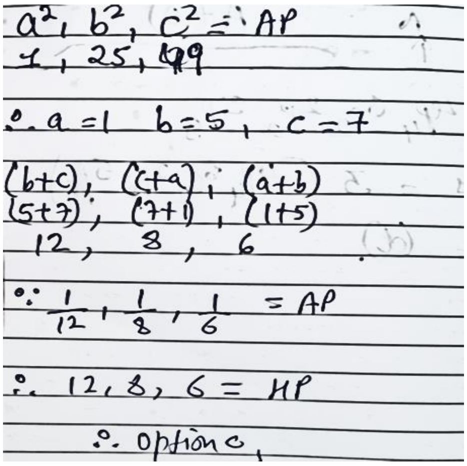
$$2b^2=a^2+c^2$$

a^2 , b^2 , c^2 are in A.P.

$\therefore$ Answer : (a)

25. If a^2, b^2, c^2 are in A.P. then (b + c), (c + a), (a + b) are in __________.
(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.



$a^2, b^2, c^2 = AP$

$1, 25, 49$

$\therefore a=1 \quad b=5, \quad c=7$

$(b+c), (c+a), (a+b)$

$(5+7), (7+1), (1+5)$

$12, 8, 6$

$\because \frac{1}{12}, \frac{1}{8}, \frac{1}{6} = AP$

$\therefore 12, 8, 6 = HP$

$\therefore$ option c.

26. If a^2, b^2, c^2 are in A.P. then a/(b + c), b/(c + a), c/(a + b) are in __________.
(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

$$a^2+c^2=2b^2$$

$$\therefore \frac{a}{b+c}+\frac{c}{a+b}=\frac{a(a+b)+c(b+c)}{(b+c)(a+b)}$$

$$\frac{a}{b+c}+\frac{c}{a+b}=\frac{a^2+ab+bc+c^2}{(b+c)(a+b)}$$

$$\frac{a}{b+c}+\frac{c}{a+b}=\frac{2b^2+ab+bc}{ab+b^2+ac+bc}$$

$$\frac{a}{b+c}+\frac{c}{a+b}=\frac{b(2b+a+c)}{ab+\frac{a^2+c^2}{2}+ac+bc}$$

$$\frac{a}{b+c}+\frac{c}{a+b}=\frac{2b(2b+a+c)}{2ab+a^2+c^2+2ac+2bc}$$

$$\frac{a}{b+c}+\frac{c}{a+b}=\frac{2b(2b+a+c)}{2ab+(a+c)^2+2bc}$$

$$\frac{a}{b+c}+\frac{c}{a+b}=\frac{2b(2b+a+c)}{2b(a+c)+(a+c)^2}$$

$$\frac{a}{b+c}+\frac{c}{a+b}=2\frac{b}{a+c}$$

$$\frac{a}{b+c},\frac{b}{a+c},\frac{c}{a+b} \text{ are in A.P}$$

Answer : (a)

27. If (b + c - a)/a, (c + a - b)/b, (a + b - c)/c are in A.P. then a, b, c are in __________.
(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

(b + c – a)/a, (c + a – b)/b, (a + b – c)/c

are in A.P.

$$\frac{b+c-a}{a}+\frac{a+b-c}{c}=\frac{2(c+a-b)}{b}$$

LCM = abc

bc(b+c-a) + ab(a+b+c) = 2ac(c+a-b)

$b^2C+BC^2-bac+a^2b+ab^2-abc=$

$2ac^2+2a^2c-2abc$

$b^2c+bc^2+a^2b+ab^2=2ac^2+2c^2c$

$a^2b-a^2c+ab^2-ac^2 = a^2c -bc^2+ac^2-b^2c$

$a^2(b-c) + a(b-c)(b+c) = c^2(a-b) +c(a-b)$

(a+b)

a(b-c)[a+b+a] = c(a-b)[c+a+b]

a(b-c) = c(a-b)

ab-ac = ac -bc

ab+bc = 2ac

Divide by abc

1/c + 1/a = 2/b

⇒ a, b, c are in H.P

Answer : (c)

28. If $(b - c)^2$, $(c - a)^2$, $(a - b)^2$ are in A.P. then (b - c), (c - a), (a - b) are in .

(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

$(b-c)^2, (c-a)^2, (a-b)^2 = AP$

$\therefore (c-a)^2 - (b-c)^2 = (a-b)^2 - (c-a)^2$

$c^2 - 2ac + a^2 - b^2 + 2bc - c^2 = a^2 - 2ab + b^2 - c^2 + 2ac - a^2$

$\therefore a^2 - 2b^2 + c^2 + 2ab + 2bc - 4ac = 0$

(c) HP

$\frac{1}{(b-c)}, \frac{1}{(c-a)}, \frac{1}{(a-b)} = AP$

$2\left(\frac{1}{c-a}\right) = \frac{1}{(b-c)} + \frac{1}{(a-b)}$

$\frac{2}{(c-a)} = \frac{a-b+b-c}{(b-c)(a-b)}$

$2[ab - ca - b^2 + bc] = (a-c)(c-a)$

$2ab - 2ac - 2b^2 + 2bc = ac - a^2 - c^2 + ac$

$a^2 - 2b^2 + c^2 + 2ab + 2bc - 4ac = 0$

29. If a b c are in A.P. then (b + c), (c + a), (a + b) are in __________.

(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

a, b, c are in A.P.

$\therefore$ a+c = 2b

Now b+c+a+b = 2b+a+c

b+c+a+b = (a+c) + (a+c)

b+c+a+b = = 2(a+c)

(b + c), (c + a), (a + b) are in A.P.

$\therefore$ Answer : (a)

30. Find the number which should be added to the sum of any number of terms of the A.P. 3, 5, 7, 9, 11 ……….resulting in a perfect square.

(a) -1 (b) 0 (c) 1 (d) None

Sol.

3, 5, 7, 9, 11 …

a = 3, d = 2

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[6 + 2n - 2]$$

$$S_n = \frac{n}{2}(2n + 4)$$

$$S_n = n^2 + 2n$$

To get perfect (n^2+2n+1)

1 is to be added

Answer : (c)

31. The sum of **n** terms of an A.P. is $2n^2 + 3n$. Find the n^{th} term.

(a) 4n + 1 (b) 4n - 1 (c) 2n + 1 (d) 2n – 1

Sol.

$S_n = 2n^2 + 3n$

$a_n = S_n - S_{n-1}$

$a_n = (2n^2 + 3n) - [2(n-1)^2 + 3(n - 1)]$

$a_n = (2n^2 + 3n) - [2n^2 -4n +2 +3n -3]$

$a_n = 2n^2 +3n - 2n^2 + n + 1$

$a_n = = 4n +1$

Answer : (a)

32. The p^{th} term of an A.P. is 1/q and the q^{th} term is 1 /p. The sum of the pq^{th} term is __________.

(a) $\frac{1}{2}(pq+1)$ (b) $\frac{1}{2}(pq-1)$ (c) pq+1 (d) pq-1

Sol.

$a_p = 1/q$ and $a_q = 1/p$

$S_{pq.} = ?$

$a_p = 1/q$

$a + (p-1)d = 1/q$ ---(1)

$a_q = 1/p$

$a (q-1)d = 1/p$ ---(2)

(1) - (2)

$$\Rightarrow (p-q)d = \frac{1}{q} - \frac{1}{p}$$

$$(p-q)d = \frac{p-q}{qp}$$

$$d = \frac{1}{qp}$$

Replace "d" in equation (1)

$$a + (p-1)\frac{1}{qp} = \frac{1}{q}$$

$$a + \frac{1}{q} - \frac{1}{qp} = \frac{1}{q}$$

$$a = \frac{1}{qp}$$

$$S_{pq} = \frac{pq}{2}\left[2a + (pq-1)d\right]$$

$$S_{pq} = \frac{pq}{2}\left[\frac{2}{pq} + (pq-1)\frac{1}{pq}\right]$$

$$S_{pq} = \frac{pq}{2}\left(\frac{pq+1}{pq}\right) = \frac{pq+1}{2}$$

∴ Answer : (a)

33. The sum of p terms of an A.P. is q and the sum of q terms is p. The sum of p + q terms is

(a) - (p + q) (b) p + q (c) $(p-q)^2$ (d) p^2-q^2

Sol.

$S_p = q$, $S_q = P$, $S_{p+q} = ?$

$$S_p = \frac{p}{2}[2a+(p-1)d] = q---(1)$$

$$S_q = \frac{q}{2}[2a+(q-1)d] = p---(2)$$

$$(1)-(2)$$

$$a(p-q)+\frac{p}{2}(p-1)d-\frac{q}{2}(q-1)d = q-p$$

$$a(p-q)+\frac{d}{2}[p^2-p-q^2+q] = q-p$$

$$a(p-q)+\frac{d}{2}[(p+q)(p-q)-(p-q)] = q-p$$

$$(p-q)\left[a+\frac{d}{2}(p-q-1)\right] = -(p-q)$$

$$2a+d(p+q-1) = -2$$

$$S_{p+q} = \frac{p+q}{2}[2a+(p+q-1)d]$$

$$S_{p+q} = \frac{p+q}{2}(-2) = -(p+q)$$

34. If S_1, S_2 S_3 be the sums of **n** terms of three A.P.s the first term of each being unity and the respective common differences 1, 2, 3 then $(S_1 + S_3) / S_2$ is __________.

(a) 1 (b) 2 (c) -1 (d) None

Sol.

Three A.P. have a = 1

Common difference $d_1 = 1$, $d_2 = 2$ and $d_3 = 3$

S_1 = Sum of n terms of first A.P.

$$S_1 = \frac{n}{2}[2a+(n-1)d_1]$$

$$S_1 = \frac{n}{2}[2+(n-1)1]$$

$$S_1 = \frac{n}{2}[n+1]$$

$$S_1 = \frac{n}{2}[n+1]$$

S_2 = Sum of n terms of second A.P.

$$S_2 = \frac{n}{2}\left[2a + (n-1)d_2\right]$$

$$S_2 = \frac{n}{2}\left[2 + (n-1)\,2\right]$$

$$S_2 = \frac{n}{2}\cdot 2n = n^2$$

S_3 = Sum of n terms of third A.P.

$$S_3 = \frac{n}{2}\left[2a + (n-1)d_3\right]$$

$$S_3 = \frac{n}{2}\left[2 + (n-1)3\right]$$

$$S_3 = \frac{n}{2}\left[3n-1\right]$$

$$Now\ \frac{S_1 + S_3}{S_2}$$

$$\frac{S_1 + S_3}{S_2} = \frac{\frac{n}{2}(n+1) + \frac{n}{2}(3n-1)}{n^2}$$

$$\frac{S_1 + S_3}{S_2} = \frac{\frac{n}{2}(n+1+3n-1)}{n^2}$$

$$\frac{S_1 + S_3}{S_2} = \frac{1}{2}\cdot\frac{4n}{n} = 2$$

Answer : (b)

35. The sum of all natural numbers between 500 and 1000, which are divisible by 13, is __________.

(a) 28,400 (b) 28,405 (c) 28,410 (d) None

Sol.

Number between 500 and 1000 divisible by 13 are 507, 520,…..988

a = 507, d = 13, and l = 988

l = a + (n-1)d

988 = 507 + (n-1)13

n-1 = 481/13

n-1 = 37

n = 38

$$S_n = \frac{n}{2}\left[a + l\right]$$

$$S_n = \frac{38}{2}\left[507 + 988\right]$$

S_n = 19 (1495)

$S_n = 28405$

Answer : (b)

36. The sum of all natural numbers between 100 and 300, which are divisible by 4, is __________.

(a) 10,200 (b) 30,000 (c) 8,200 (d) 2,200

Sol.

Number from 100 and 300 divisible by 4 are

100, 104, 108 300

a = 104, d = 4, l = 296

l = a + (n-1)d

300 = 100 + (n-1)4

n = 51

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{51}{2}(100+300)$$

Answer : (a)

37. The sum of all natural numbers from 100 to 300 excluding those, which are divisible by 4, is

(a) 10,200 (b) 30,000 (c) 8,200 (d) 2,200

Sol.

Number from 100 and 300 divisible by 4 are

100, 104, 108 300

a = 104, d = 4, l = 296

l = a + (n-1)d

300 = 100 + (n-1)4

n = 51

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{51}{2}(100+300)$$

Sum of the all numbers between 100 and 300

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$S_n = \frac{201}{2}[400]$

$S_n = 40200$

Sum of number NOT divisible by 4

=40200 - 10,200 = 30,000

Answer : (b)

38. The sum of all natural numbers from 100 to 300, which are divisible by 5, is .

(a) 10,200 (b) 30,000 (c) 8,200 (d) 2,200

Sol.

From 100 to 300 divisible by 5 not are 100, 105, 110….300

a = 100, d = 5, l = 300

l = a + (n-1)d

300 = 100 + (n-1)5

n = 41

$S_n = \frac{n}{2}(a+l)$

$S_n = \frac{41}{2}(100+300)$

$S_n = 8200$

Answer : (c)

39. The sum of all natural numbers from 100 to 300, which are divisible by 4 and 5, is .

(a) 10,200 (b) 30,000 (c) 8,200 (d) 2,200

Sol.

From 100 and 300 divisible by 4 and 5 are 100, 120, 140….300

120, 140, ...300

a = 100, d = 20, l = 300

l = a + (n-1)d

300 = 100 + (n-1)20

n = 11

S_n = (n/2) (a+l)

S_n = (11/2) (100 + 300) = 2200

Answer : (d)

40. The sum of all natural numbers from 100 to 300, which are divisible by 4 or 5, is .

(a) 10,200 (b) 8,200 (c) 2,200 (d) 16,200

Sol.

Sum of numbers from 100 to 300 divisible by 4 or 5.

First we will find sum of number divisible by 4, then sum of number divisible by 5 and sum of numbers divisible by 4 and 5

sum of all natural numbers between 100 and 300, which are divisible by 4

Number from 100 and 300 divisible by 4 are

100, 104, 108 300

a = 104, d = 4, l = 296

l = a + (n-1)d

300 = 100 + (n-1)4

n = 51

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{51}{2}(100+300)$$

sum of all natural numbers from 100 to 300, which are divisible by 5 From 100 to 300 divisible by 5 not are 100, 105, 110, ... 300

a = 100, d = 5, l = 300

l = a + (n-1)d

300 = 100 + (n-1)5

n = 41

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{41}{2}(100+300)$$

$$S_n = 8200$$

The sum of all natural numbers from 100 to 300, which are divisible by 4 and 5,

From 100 and 300, divisible by 4 and 5 are , 100, 120, 140, ...300

a = 100, d = 20, l = 300

l = a + (n-1)d

300 = 100 + (n-1)20

n = 11

S_n = (n/2) (a+l)

S_n = (11/2) (100 + 300) = 2200

Sum of number divisible by 4 or 5

= Sum of number divisible by 4 + Sum of number divisible by 5 - Sum of number divisible by 4 and 5

= 10,200 + 8200 -2200 = 16,200

Answer : (d)

41. If the n terms of two A.P.s are in the ratio (3n+4) : (n+4) the ratio of the fourth term is ___________.

(a) 2 (b) 3 (c) 4 (d) None

Sol.

$$\frac{a_n}{A_n} = \frac{3n+4}{n+4}$$

$$\frac{a_4}{A_4} = \frac{3(4)+4}{4+4}$$

$$\frac{a_4}{A_4} = \frac{16}{8} = 2$$

42. If a, b, c, d are in A.P. then

(a) $a^2-3b^2+3c^2-d^2=0$ (b) $a^2+3b^2+3c^2+d^2=0$ (c) $a^2+3b^2+3c^2-d^2=0$ (d) None

Sol.

a, b, c and d are in A.P.

b-a = k; c-b=k; d-c=k

∴ b = a+k; c = a+2k; d = a +3k

$a^2 - 3b^2 + 3c^2 - d^2$

$= a^2 - 3(a+k)^2 + 3(a+2k)^2 - (a+3k)^2$

$= a^2 - 3a^2 - 6ak - 3k^2 + 3a^2 + 12ak + 12k^2 - a^2 - 6ak + 9k^2$

= 0

Answer : (a)

43. If a, b, c, d, e are in A.P. then

(a) a - b - d + e = 0 (b) a - 2c + e = 0 (c) b - 2c + d = 0 (d) all the above

Sol.

a, b, c, d, e are in A.P.

b = a+k, c = a +2k , d = a+3k, e = a+4k

a+e = a + a + 4k

a+e = 2a + 4k

a+e = (a+k)+(a+3k)

a+e = b+d

∴ a – b – d + e = 0 Option a is true

b+d = a+k +a+3k = 2a+4k

b+d = 2(a+2k) = 2c option b is true

∴ b – 2c + d = 0 Option c is true

∴ All the above is answer

Answer : (d)

44. The three numbers in A.P. whose sum is 18 and product is 192 are __________.

(a) 4, 6, 8 (b) -4, -6, -8 (c) 8, 6, 4 (d) both (a) & (c)

Sol.

Let numbers be a-d,a+d

Sum = 18

a-d+a+a+d= 18

3a=18

a = 6

Product = 192

a(a-d)(a+d) = 192

$6(36-d^2) = 192$

$36 - d^2 = 32$

$d = \pm 2$

If d = 2, numbers are 4, 6, 8

If d = -2, numbers are 8, 6, 4

Answer : (d)

45. The three numbers in A.P., whose sum is 27 and the sum of their squares is 341, are __________.

(a) 2, 9, 16 (b) 16, 9, 2 (c) both (a) and (b) (d) -2, -9, -16

Sol.

Let numbers be a-d, a , a+d

Sum = 27

$a-d+a+a+d = 27$

$3a = 27$

$a = 9$

Sum of the squares are 341

$(a-d)^2+a^2+(a+d)^2 = 341$

$(9-d)^2+9^2+(9+d)^2 = 341$

$81-18d+d^2+81+81+18d+d^2=341$

$2d^2=341-243=98$

$d^2=49$

$d=\pm7$

If = 7 numbers are 2,9,16

If d = -7 numbers are 16, 9,2

Answer : (c)

46. The four numbers in A.P., whose sum is 24 and their product is 945, are __________.

(a) 3, 5, 7, 9 (b) 2, 4, 6, 8 (c) 5, 9, 13, 17 (d) None

Sol.

Let numbers be a-3d, a-d, a+d, a+3d

Sum = 24

$a-3d+a-d+a+d+a+3d = 24$

$4a=24$

$a=6$

Product = 945

$(a-3d)(a-d)(a+d)(a+3d) = 945$

$(a^2-9d^2)(a^2-d^2)=945$

$(36-9d^2)(36-d^2) = 945$

$1296 - 360d^2+9d^4=945$

$9d^4 -360d^2+351 = 0$

$9d^4 -360d^2+351 = 0$

$d^4 -40d^2+39 = 0$

$d^2 = 39$ and $d = 1$

If d = 1 numbers are

$a-3d = 6-3=3$

$a-d = 6-1=5$

$a+d = 6+1=7$

$a+3d = 6+3 = 9$

Numbers are 3, 5, 7, 9

Answer : (a)

47. The four numbers in A.P., whose sum is 20 and the sum of their squares is 120, are __________.

(a) 3, 5, 7, 9 (b) 2, 4, 6, 8 (c) 5, 9, 13, 17 (d) None

Sol.

Let numbers be a-3d, a-d, a+d, a+3d

Sum = 20

$a-3d+a-d+a+d+a+3d = 20$

$4a = 20 \therefore a = 5$

Sum of squres = 120

$\Rightarrow (a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2=120$

$\Rightarrow (5-3d)^2+(5-d)^2+(5+d)^2+(5+3d)^2=120$

$\therefore 25-30d+9d^2+25-$

$10d+d^2+25+10d+d^2+25+30d+9d^2 = 120$

$\Rightarrow 20d^2 = 120 - 100 = 20$

$d = 1$

So $a-3d = 5 - 3 = 2$

$a-d = 5-1 = 4$

$a+d = 5+1 = 6$

$a+3d = 5+3 = 8$

Answer : (b)

48. The four numbers in A.P. with the sum of second and third being 22 and the product of the first and fourth beinf 85 are __________.

(a) 3, 5, 7, 9 (b) 2, 4, 6, 8 (c) 5, 9, 13, 17 (d) None

Sol.

Let numbers be a-3d, a-d, a+d and a+3d

$a_2+a_3 = 22$

$a = 11$

$a_1a_4 = 85$

$(a-3d)(a+3d) = 85$

$a^2 - 9d^2 = 85$

$121 - 9d^2 = 85$

$9d^2 = 36$

d = 2

Numbers are a-3d = 11 - 6 = 5

a - d = 11 - 2 = 9

a +d = 11+2 = 13

a+3d = 11 + 6 = 17

Numbers are 5, 9, 13, 17

Answer : (c)

49. The five numbers in A.P. with their sum 25 and the sum of their squares 135 are __________.

(a) 3, 4, 5, 6, 7 (b) 3, 3.5, 4, 4.5, 5

(c) -3, -4, -5, -6, -7 (d) -3, -3.5, -4, -4.5, -5

Sol.

Let five terms of A.P. be a-2d, a-d, a, a+d , a+2d

Sum = 25

∴ a-2d+ a-d + a + a+d + a+2d = 25

5a = 25

a = 5

Sum of their squares = 135

∴ $(a-2d)^2 + (a-d)^2 + a^2 + (a+d)^2 + (a+2d)^2 = 135$

$a^2 - 4ad+4d^2+a^2-$

$2ad+d^2+a^2+a^2+2ad+d^2+a^2+4ad+d^2 = 135$

$5a^2+10d^2= 135$

$a^2+2d^2 = 27$

∴ $25+2d^2 = 27$

$2d^2 = 2$

∴ d = ± 1

If d = 1, a-2d = 3, a-d = 4., a+d = 6, a+2d = 7

Numbers are 3, 4, 5, 6, 7

If d = -1 a-2d = 7, a-d = 6, a+d = 4, a+2d = 3

So numbers are 7,6,5, 4, 3

∴ numbers rae 3,4,5,6,7

Answer : (a)

50. The five numbers in A.P. with the sum 20 and product of the first and last 15 are __________.

(a) 3, 4, 5, 6, 7 (b) 3, 3.5, 4, 4.5, 5

(c) -3, -4, -5, -6, -7 (d) -3, -3.5, -4, -4.5, -5

Sol.

Let numbers be

a - 2d, a-d, a, a+d, a+2d

Sum =20

a - 2d+ a-d+a+a+d+a+2d = 20

5a = 20

a = 4

Product of (a-2d) and (a+2d) is 15

$(a-2d)\times(a+2d)=15$

$a^2 - 4d^2 = 15$

Substituting the value of a in above equation, we

get $16 - 4d^2 = 15$

$-4d^2 = 15-16$

$d^2 = 1/4$

d = 1/2 or 0.5

Therefore numbers are 3, 3.5, 4, 4.5 and 5

Answer : (b)

51. The sum of **n** terms of 2, 4, 6, 8 ……….is

(a) n(n + 1) (b) (n/2)(n + 1) (c) n(n - 1) (d) (n/2)(n - 1)

Sol.

2, 4, 6, 8……

a=2, d=2

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_n = \frac{n}{2}[4+(n-1)2]$$

$$S_n = \frac{n}{2}[4+2n-2]$$

$$S_n = \frac{n}{2}(2n+2)$$

$$S_n = n(n+1)$$

Answer : (a)

52. The sum of n terms of a+b, 2a, 3a-b, ...is
(a) n(a-b)+2b (b) n(a+b) (c) both the above (d) None

Sol.

a+b, 2a, 3a–b, ...

$a_1 = a+b$

$d = 2a-a-b = a-b$

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[2(a+b) + (n-1)(a-b)\right]$$

$$S_n = \frac{n}{2}\left[2a + 2b + (n-1)(a-b) - a + b\right]$$

$$S_n = \frac{n}{2}\left[a + 3b + n(a-b)\right]$$

Answer : (d)

53. The sum of **n** terms of $(x + y)^2$, $(x^2 + y^2)$, $(x - y)^2$, ...is
(a) $(x + y)^2 - 2(n - 1)xy$ (b) $n(x + y)^2 - n(n - 1)xy$
(c) both the above (d) None

Sol.

$(x + y)^2$, $(x^2 + y^2)$, $(x - y)^2$

$a = (x+y)^2$ and $d = x^2 + y^2 - (x - y)^2 = -2xy$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[2(x+y)^2 + (n-1)(-2xy)\right]$$

$$S_n = n\left[(x+y)^2 + (n-1)(-xy)\right]$$

$$S_n = n(x+y)^2 - n(n-1)(xy)$$

Answer : (b)

54. The sum of **n** terms of (1/n)(n - 1), (1/n) (n - 2), (1/n) (n - 3) ....is
(a) 0 (b) (1/2)(n - 1) (c) (1/2)(n + 1) (d) None

Sol.

$$\frac{n-1}{n}, \frac{n-2}{n}, \frac{n-3}{n}$$

$$a = \frac{n-1}{n}$$

$$d = \frac{n-2}{n} - \frac{n-1}{n} = \frac{-1}{n}$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[2\left(\frac{n-1}{n}\right) + (n-1)\left(\frac{-1}{n}\right)\right]$$

$$S_n = \frac{n}{2} \cdot \frac{1}{n}\left[2n - 2 - n + 1\right]$$

$$S_n = \frac{n-1}{2} = \frac{1}{2}(n-1)$$

Answer : (b)

55. The sum of **n** terms of 1.4, 3.7, 5.10 Is
(a) $(n/2)(4n^2+5n - 1)$ (b) $n(4n^2+5n - 1)$ (c) $(n/2)(4n^2-5n - 1)$ (d) None

Sol.

1.4, 3.7, 5.10…….

First take all 1st digit

1, 3, 5, ... is AP with a =- 1 and d = 2

$a_i = a + (i-1)d$

$a_i = 1 + (i-1)2 = 2i - 1$

Now will take 2_{nd} digit

4, 7, 10, ... it is A.P

a = 4 and d = 3

$a_i = a + (i-1)d$

$a_i = 4 + (i-1)3 = 3i + 1$

So i^{th} term of series is

$a_i = (2i-1)(3i+1)$

$a_i = 6i^2 - i - 1$

$$S_n = \sum a_i$$

$$S_n = \sum_{i=1}^{n}\left(6i^2 - i - 1\right)$$

$$S_n = 6\sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} i - \sum_{i=1}^{n} 1$$

$$S_n = \frac{6n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} - n$$

$$S_n = \frac{n}{2}\left[2(n+1)(2n+1) - (n+1) - 2\right]$$

$$S_n = \frac{n}{2}\left[4n^2 + 6n + 2 - n - 1 - 2\right]$$

$$S_n = \frac{n}{2}\left[4n^2 + 5n - 1\right]$$

Answer : (a)

56. The sum of **n** terms of $1^2, 3^2, 5^2, 7^2$, ...is

(a) $(n/3)(4n^2-1)$ (b) $(n/2)(4n^2-1)$ (c) $(n/3)(4n^2+1)$ (d) None

Sol.

$1^2, 3^2, 5^2, 7^2$,......

1, 3, 5, 7 .... is AP with a = 1 and d = 2

$a_i = a +)i-1)d$

$a_i = 1 +)i-1) 2 = 2i - 1$

$1^2, 3^2, 5^2, 7^2 .... = (2i-1)^2$

$1^2, 3^2, 5^2, 7^2 .... = 4i^2 - 4i + 1$

$$S_n = \sum_{i=1}^{n} a_i$$

$$S_n = \sum_{i=1}^{n}\left(4i^2 - 4i + 1\right)$$

$$S_n = 4\sum_{i=1}^{n} i^2 - 4\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$$

$$S_n = \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$$

$$S_n = \frac{n}{3}\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$$

$$S_n = \frac{n}{3}\left[4n^2 + 6n + 2 - 6n - 6 + 3\right]$$

$$S_n = \frac{n}{3}\left(4n^2 - 1\right)$$

Answer : (a)

57. The sum of n terms of 1, (1 + 2), (1 + 2 + 3) ....is
    (a) (n/3)(n + 1)(n - 2) (b) (n/3)(n + 1)(n + 2)
    (c) n(n + 1)(n + 2) (d) None

Sol.

Answer : (d)

58. The sum of **n** terms of the series $1^2/1+(1^2+2^2)/2+(1^2+2^2+3^2)/3+$ ....is
    (a) $(n/36)(4n^2 +15n+17)$ (b) $(n/12)(4n^2+15n+17)$
    (c) $(n/12)(4n^2 +15n+17)$ (d) None

Sol.

$$\frac{1^2}{1}+\frac{1^2+2^2}{2}+\frac{1^2+2^2+3^2}{3}....$$

a_i of Numerator

1^2 , 1^2+2^2, $1^2+2^2+3^2$ ....

$a_i = 1^2+2^2+3^2+ \ldots + i^2$

$$a_i = \sum i^2$$

$$a_i = \frac{i(i+1)(2i+1)}{6}$$

For denominator 1, 2, 3, ...

$a_i = i$

$$i^{th} term = \frac{i(i+1)(2i+1)}{6i}$$

$$i^{th} term = \frac{2i^2+3i+1}{6}$$

$$S_n = \sum a_i$$

$$S_n = \sum_{i=1}^{n} \frac{2i^2+3i+1}{6}$$

$$S_n = \frac{2}{6}\sum_{i=1}^{n} i^2 + \frac{3}{6}\sum_{i=1}^{n} i + \frac{1}{6}\sum_{i=1}^{n} 1$$

$$S_n = \frac{2}{6}\frac{n(n+1)(2n+1)}{6} + \frac{3}{6}\frac{n(n+1)}{2} + \frac{1}{6}n$$

$$S_n = \frac{n}{36}\left[2n(n+1)(2n+1) + 9(n+1) + 6\right]$$

$$S_n = \frac{n}{36}\left[4n^2 + 6n + 2 + 9n + 9 + 6\right]$$

$$S_n = \frac{n}{36}\left(4n^2 + 15n + 17\right)$$

Answer : (a)

59. The sum of n terms of the series 2.4.6 + 4.6.8 + 6.8.10 + ...is

(a) $2n(n^3+6n^2+11n+6)$ (b) $2n(n^3-6n^2+11n-6)$

(c) n(n3+6n2+11n+6) (d) n(n3+6n2+11n-6)

Sol.

2.4.6 + 4.6.8 + 6.8.10 + ...

First all 1^{st} digit of series

2, 4, 6, ... It is AP a=4 and d=2

$a_i = a + (i-1)d = 2 + (i-1)2 = 2i$

Now 2^{nd} digit

4, 6, 8, ... It is AP a=4, d = 2

$a_i = a + (i-1)d = 4 + (i-1)2 = 2i+2$

3^{rd} digits

6, 8, 10, ... It is AP a = 6, d = 2

$a_i = a + (i-1)d = 6 + (i-1)2 = 2i+4$

So i^{th} is $a_i = 2i(2i+2)(2i+4)$

$a_i = 8i(i+1)(i+2)$

$a_i = 8i(i^2+3i+2)$

$a_i = 8i^3 + 24i^2 + 16i$

$$S_n = \sum a_i$$

$$S_n = \sum\left(8i^3 + 24i^2 + 16i\right)$$

$$S_n = 8\sum_{i=1}^{n} i^3 + 24\sum_{i=1}^{n} i^2 + 16\sum_{i=1}^{n} i$$

$$= \frac{8 \cdot n^2(n+1)^2}{4} + \frac{24 \cdot n(n+1)(2n+1)}{6} + \frac{16 \cdot n(n+1)}{2}$$
$$= 2n(n+1)[n(n+1)+2(2n+1)+4]$$
$$= 2n(n+1)(n^2+n+4n+2+4)$$
$$= 2n(n+1)(n^2+5n+6)$$
$$= 2n(n^3+6n^2+11n+6)$$

Answer : (a)

60. The sum of **n** terms of the series $1.3^2+4, 4^2+7.5^2+10.6^2+$ ....is

(a) $(n/12)(n + 1)(9n^2+49n+44)-8n$ (b) $(n/12)(n + 1)(9n^2+49n+44)+8n$

(c) $(n/6)(2n + 1)(9n^2+49n+44)-8n$ (d) None

Sol.

$1.3^2 + 4.4^2 + 7.5^2 + 10.6^2 +$........

First digits are 1, 4, 7, 10, ... It is AP

a = 1 and d = 3

$a_i = a + (i-1)d = 1 + (i-1)3 = 3i-2$

Now Second degits are $3^2, 4^2, 5^2, 6^2$ .....

Now 3, 4, 5, 6 ... are in AP

a = 3 and d=1

$a_i = a + (i-1)d = 3+(i-1)1 = i+2$

$\therefore i^{th}$ term is $(i+2)^2$

i^{th} term of series is

$a_i = (3i-2)(i+2)^2$

$a_i = (3i-2)(i^2+4i+4)$

$a_i = 3i^3+12i^2+12i-2i^2-8i-8$

$a_i = 3i^3+10i^2+4i-8$

$$S_n = \sum a_i$$
$$S_n = \sum_{i=1}^{n}(3i^3+10i^2+4i-8)$$
$$S_n = 3\sum_{i=1}^{n} i^3 + 10\sum_{i=1}^{n} i^2 + 4\sum_{i=1}^{n} i - 8\sum_{i=1}^{n} 1$$
$$= \frac{3n^2(n+1)^2}{4} + \frac{10n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} - 8n$$

$$=\frac{n(n+1)}{12}\left[9n(n+1)+20(2n+1)+12\right]-8n$$

$$=\frac{n(n+1)}{12}\left[9n^2+9n+40n+20+24\right]-8n$$

$$=\frac{n(n+1)}{12}\left(9n^2+49n+44\right)-8n$$

Answer : (a)

61. The sum of **n** terms of the series 4 + 6 + 9 + 13 ....is

(a) $(n/6)(n^2+3n + 20)$ (b) $(n/6)(n + 1)(n + 2)$

(c) $(n/3)(n + 1)(n + 2)$ (d) None

Sol.

S_n = 4 + 6 + 9 + 13 ……+ a_n ----(1)

S_n = +4 + 6 + 9 + 13 ……+ a_n ----(2)

(1) - (2)

0 = 4 + (2+3+4+....(n-1) terms) - a_n

a_n = 4 + (2+3+4+....(n-1) terms)

2, 3, 4, ... is A.P. with a = 2 and d = 1

$$a_n=4+\sum_{i=1}^{n-1}(i+1)$$

$$a_n=4+\sum_{i=1}^{n-1}i+\sum_{i=1}^{n-1}1$$

$$a_n=4+\frac{(n-1)n}{2}+(n-1)$$

$$a_n=\frac{8+n^2-n+2n-2}{2}$$

$$a_n=\frac{n^2+n+6}{2}$$

$S_n = \sum a_n$

$$S_n=\sum\frac{n^2+n+6}{2}$$

$$=\frac{1}{2}\sum n^2+\frac{1}{2}\sum n+3\sum 1$$

$$=\frac{1}{2}\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{4}+3n$$

$$=\frac{n}{12}\left[(n+1)(2n+1)+3(n+1)+36\right]$$

$$=\frac{n}{12}\left(2n^2+6n+40\right)$$

$$= \frac{n}{12} \cdot 2\left(n^2 + 3n + 20\right)$$

$$= \frac{n}{6}\left(n^2 + 3n + 20\right)$$

Answer : (a)

62. The sum to **n** terms of the series 11, 23, 59, 167 ....is

(a) $3^{n+1}+5n - 3$ (b) $3^{n+1}+5n + 3$ (c) $3^n+5n - 3$ (d) None

Sol.

11, 23, 59,167……..

S_n = 11 + 23 + 59 + 167+ ……+a_n ---(1)

S_n = + 11 + 23 + 59 + 167+ ……+a_n ---(2)

(1) - (2)

0 = 11 + (12 +36 + 108 + ...) - a_n

∴ a_n

= 11 + (12 +36 + 108 + ...(n-1) terms)

12, 36, 108 .... are in G.P

a = 12 and r = 3

$$12+36+18+\ldots = \frac{12\left(3^{n-1}-1\right)}{3-1}$$

$$= 6\left(3^{n-1}-1\right) = \left(2\cdot 3^n - 6\right)$$

$$a_n = 11 + 2\cdot 3^n - 6$$

$$a_n = 5 + 2\left(3^n\right)$$

$$S_n = \sum a_n$$

$$S_n = \sum\left(5 + 2\cdot 3^n\right)$$

$$S_n = \sum 5 + 2\sum 3^n$$

$$S_n = 5n + 2\left[3 + 3^2 + 3^3 + \ldots 3^n\right]$$

3, 3^2, 3^3 ... in G.P.

a = 3, r = 3 > 1

$$3 + 3^2 + 3^3 + \ldots + 3^n = \frac{3\left(3^n-1\right)}{3-1} = \frac{3\left(3^n-1\right)}{2}$$

$$S_n = 5n + 2\frac{3\left(3^n-1\right)}{2}$$

$$S_n = 5n + 3^{n+1} - 3$$

$$S_n = 3^{n+1} + 5n - 3$$

Answer : (a)

63. The sum of n terms of the series 1/(4.9)+1/(9.14)+1/(14.19)+1/(19.24)+ ....is
(a) $(n/4)(5n+4)^{-1}$ (b) (n/4)(5n+4) (c) $(n/4)(5n-4)^{-1}$ (d) None

Sol.

$$\frac{1}{4\cdot 9}+\frac{1}{9\cdot 14}+\frac{1}{14\cdot 19}+\frac{1}{19\cdot 24}+..$$

1^{st} digit of denominator are
4, 9, 14, 19, ... It is AP

a = 4 and d=5

$a_i = a + (i-1)d$

$a_i = 4 + (i-1)5$

$a_i = 5i - 1$

2^{nd} diigit of fenominator are

9, 14, 19, 24, ... It is AP

a = 9 and d= 5

$a_i = a + (i-1)d$

$a_i = 9 + (i-1)5 = 5i + 4$

i^{th} term of series

$$a_i = \frac{1}{(5i-1)(5i+4)}$$

$$S_n = \sum a_i$$

$$S_n = \sum_{i=1}^{n} \frac{1}{(5i-1)(5i+4)}$$

$$S_n = \frac{1}{5}\sum_{i=1}^{n} \frac{(5i+4)-(5i-1)}{(5i-1)(5i+4)}$$

$$= \frac{1}{5}\left[\sum_{i=1}^{n}\frac{1}{5i-1} - \sum_{i=1}^{n}\frac{1}{5i+4}\right]$$

$$= \frac{1}{5}\left[\left(\frac{1}{4}+\frac{1}{9}+\frac{1}{14}+...+\frac{1}{5n-1}\right)-\left(\frac{1}{9}+\frac{1}{14}+...+\frac{1}{5n+4}\right)\right]$$

$$= \frac{1}{5}\left[\frac{1}{4}-\frac{1}{5n+4}\right]$$

$$= \frac{1}{5}\left[\frac{5n+4-4}{4(5n+4)}\right]$$

$$= \frac{n}{4(5n+4)}$$

Answer : (a)

64. The sum of **n** terms of the series 1 + 3 + 5 + ...Is

(a) n^2 (b) $2n^2$ (c) $n^2/2$ (d) None

Sol.

1+3+15+.... It is AP

a= 1 and d=2

$a_i = a + (i-1)d = 1 + (i-1)2 = 2i-1$

$$S_n = \sum_{i=1}^{n}(2i-1)$$

$$S_n = 2\sum_{i=1}^{n} i - \sum_{i=1}^{n} 1$$

$$S_n = \frac{2n(n+1)}{2} - n$$

$S_n = n^2 + n - n = n^2$

Answer : (a)

65. The sum of **n** terms of the series 2 + 6 + 10 + ...is

(a) $2n^2$ (b) n^2 (c) $n^2/2$ (d) $4n^2$

Sol.

2 + 6 + 10 + …….

2(1+3+5+...)

$$S_n = 2\sum_{i=1}^{n}(2i-1)$$

$$S_n = 2\left(2\sum_{i=1}^{n} i - \sum_{i=1}^{n} 1\right)$$

$$S_n = 2\left(\frac{2n(n+1)}{2} - n\right)$$

$$S_n = 2(n^2 + n - n)$$

$$S_n = 2n^2$$

Answer : (a)

66. The sum of **n** terms of the series 1.2 + 2.3 + 3.4 + ... Is

(a) (n/3)(n + 1)(n + 2) (b) (n/2)(n + 1)(n + 2)

(c) (n/3)(n + 1)(n - 2) (d) None

Sol.

1.2 + 2.3 + 3.4 + ...

1^{st} digit are 1, 2, 3, ... It is AP

a = 1 and d = 1

$a_i = a + (i-1)d = 1 + (i-1)1 = i$

2^{nd} digits are 2, 3, 4, ... It is AP a =2 and d=1

$a_i = a + (i-1)d = 2 + (i-1)1 = i + 1$

i^{th} term of series is

$a_i = i(i+1) = i^2 + i$

$$S_n = \sum a_i$$

$$S_n = \sum_{i=1}^{n}(i^2 + i)$$

$$S_n = \sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i$$

$$S_n = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{6}[2n+1+3]$$

$$= \frac{n(n+1)(2n+4)}{6}$$

$$= \frac{n(n+1)(n+2)}{3}$$

Answer : (a)

67. The sum of **n** terms of the series 1.2.3 + 2.3.4 + 3.4.5 + ...is

(a) (n/4)(n + 1)(n + 2)(n + 3) (b) (n/3)(n + 1)(n + 2)(n + 3)

(c) (n/2)(n + 1)(n + 2)(n + 3) (d) None

Sol.

1.2.3 + 2.3.4 + 3.4.5 + ...

1^{st} digit number = 1, 2, 3, ... It is AP

a = 1 and d=1

$a_i = a + (i-1)d = 1 + (i-1)1 = i$

2^{nd} digits are 3, 4, 5 , ... It is AP a=3 and d=1

$a_i = a + (i-1)d = 3 + (i-1)d = i + 2$

$\therefore$ i^{th} term of series

$a_i = i(i+1)(i+2)$

$a_i = i(i^2 + 3i + 2)$

$a_i = i^3 + 3i^2 + 2i$

$S_n = \sum a_n$

$S_n = \sum_{i=1}^{n}(i^3 + 3i^2 + 2i)$

$S_n = \sum_{i=1}^{n} i^3 + 3\sum_{i=1}^{n} i^2 + 2\sum_{i=1}^{n} i$

$= \frac{n^2(n+1)^2}{4} + \frac{3n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$

$= \frac{n(n+1)}{4}[n(n+1) + 2(2n+1) + 4]$

$= \frac{n(n+1)}{4}(n^2 + n + 4n + 2 + 4)$

$= \frac{n(n+1)}{4}(n^2 + 5n + 6)$

$= \frac{n(n+1)(n+2)(n+3)}{4}$

Answer : (a)

68. The sum of **n** terms of the series $1.2+3.2^2+5.2^3+7.2^4+$ ....is
(a) $(n-1)2^{n+2}-2^{n+1}+6$ (b) $(n+1)2^{n+2}-2^{n+1}+6$ (c) $(n-1)2^{n+2}-2^{n+1}-6$ (d) None

Sol.

$1.2 + 3.2^2 + 5.2^3 + 7.2^4 + \ldots\ldots.$

1^{st} digits are 1, 3, 5, 7 ... are AP

a = 1 and d = 2

$a_i = a + (i-1)d$

$a_i = 1 + (i-1)2 = 2i-1$

2^{nd} digits are 2, 2^2, 2^3 .... It is GP

a = 2 and r=2 > 1

$a_i = ar^{i-1} = 2(2)^{i-1} = 2^i$

i^{th} term of sequence is

$a_i = (2i - 1).\ 2^i$

$a_i = (2^{i+1}\ .\ i - 2^i)$

Answer : (d)

69. The sum of **n** terms of the series 1/(3.8)+1/(8.13)+1/(13.18)+ ....is

(a) $(n/3)(5n + 3)^{-1}$ (b) $(n/2)(5n + 3)^{-1}$ (c) $(n/2)(5n - 3)^{-1}$ (d) None

Sol.

$1.2 + 3.2^2 + 5.2^3 + 7.2^4 + ...$

1^{st} digits are 1, 3, 5, 7 ... are AP

a = 1 and d = 2

$a_i = a + (i-1)d$

$a_i = 1 + (i-1)2 = 2i-1$

2^{nd} digits are 2, 2^2, 2^3 .... It is GP

a = 2 and r=2 > 1

$a_i = ar^{i-1} = 2(2)^{i-1} = 2^i$

i^{th} term of sequence is

$a_i = (2i - 1).\ 2^i$

$a_i = (2^{i+1}\ .\ i - 2^i)$

Answer : (d)

70. The sum of n terms of the series 1/1+1/(1+2)+1/(1+2+3)+ ……….is

(a) $2n(n + 1)^{-1}$ (b) n(n+1) (c) $2n(n-1)^{-1}$ (d) None

Sol.

$$\frac{1}{1}+\frac{1}{1+2}+\frac{1}{1+2+3}.$$

TErms in denominator are 1, (1+2), (1+2+3), ...

$a_i = (1+2+3+ ....+i)$

$$a_i = \sum i = \frac{i(i+1)}{2}$$

i^{th} term of series is

$$a_i = \frac{1}{\frac{i(i+1)}{2}} = \frac{2}{i(i+1)}$$

$$S_n = \sum a_i = \sum \frac{2}{i(i+1)}$$

$$= 2\sum \frac{1}{i(i+1)}$$

$$= 2\sum \left[\frac{(i+1)-i}{i(i+1)}\right]$$

$$= 2\left[\sum_{i=1}^{n}\frac{1}{i} - \sum_{i=1}^{n}\frac{1}{i+1}\right]$$

$$= 2\left[\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right)-\left(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n+1}\right)\right]$$

$$= 2\left[1 - \frac{1}{n+1}\right]$$

$$= 2\left[\frac{n+1-1}{n+1}\right]$$

$$= \frac{2n}{n+1} = 2n(n+1)^{-1}$$

Answer : (a)

71. The sum of n terms of the series $2^2+5^2+8^2+$ ……….is

(a) $(n/2)(6n^2+3n-1)$ (b) $(n/2)(6n^2-3n-1)$

(c) $(n/2)(6n^2+3n+1)$ (d) None

Sol.

$2^2 + 5^2 + 8^2 +$.......

Numbers are 2. 5. 8. .... It is AP

a = 2 and d = 3

$a_i = a + (i-1)d = 2 + (i-1)3 = 3i-1$

i^{th} term of series

$a_i = (3i-1)^2 = 9i^2 - 6i + 1$

$2^2 + 5^2 + 8^2 +$.......

Numbers are 2, 5, 8, ... It is AP

a = 2 and d = 3

$a_i = a + (i-1)d = 2 + (i-1)3 = 3i-1$

i^{th} term of series

$a_i = (3i-1)^2 = 9i^2 - 6i + 1$

$$S_n = \sum a_i$$

$$= \sum(9i^2 - 6i + 1)$$

$$= 9\sum_{i=1}^{n} i^2 - 6\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$$

$$= \frac{9n(n+1)(2n+1)}{6} - \frac{6n(n+1)}{2} + n$$

$$= \frac{n}{2}[3(n+1)(2n+1) - 6(n+1) + 2]$$

$$= \frac{n}{2}[3(2n^2+3n+1) - 6n - 6 + 2]$$

$$= \frac{n}{2}(6n^2+3n-1)$$

72. The sum of **n** terms of the series $1^2+3^2+5^2+$ ……….is

(a) $\frac{n}{3}(4n^2-1)$ (b) $n^2(2n^2+l)$ (c) $n(2n-1)$ (d) $n(2n+1)$

Sol.

$1^2 + 3^2 + 5^2 + \ldots\ldots..$

Numbers are 1, 3, 5, ... It is AP

a = 1 and d=2

$a_i = a + (i-1)d = 1 + (i-1)2 = 2i-1$

i^{th} term of series is

$a_i = (2i-1)^2 = 4i^2 - 4i + 1$

$S_n = \sum a_i$

$S_n = \sum(4i^2 - 4i + 1)$

$= 4\sum i^2 - 4\sum i + \sum 1$

$= \frac{4n(n-1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$

$= \frac{n}{3}[2(n+1)(2n+1) - 6(n+1) + 3]$

$= \frac{n}{3}(4n^2 + 6n + 2 - 6n - 6 + 3)$

$= \frac{n}{3}(4n^2 - 1)$

Answer : (a)

73. The sum of **n** terms of the series 1.4 + 3.7 + 5.10 + ……….is

(a) $(n/2)(4n^2+5\ 1)$ (b) $(n/2)(5n^2+4n - 1)$

(c) $(n/2)(4n^2+5n + 1)$ (d) None

Sol.

1.4 + 3.7 + 5.10 + ...

First digit are 1, 3, 5, ... It is AP

a = 1 and d=2

$a_i = a + (i-1)d = 1 + (i-1)2 = (2i-1)$

Second digits are 4, 7, 10.... It is AP

a = 4,d = 3

$a_i = a + (i-1)d = 4 + (i-1)d = 4+(i-1)3 = (3i+1)$

i^{st} term of series

$a_i = (2i-1)(3i+1)$

$a_i = 6i^2 - i - 1$

$S_n = \sum a_i$

$S_n = \sum(6i^2 - i - 1)$ $S_n = 6\sum i^2 - \sum i - \sum 1$

$S_n = \frac{6n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} - n$

$$S_n = \frac{n}{2}\left[2(n+1)(2n+1)-(n+1)-2\right]$$

$$S_n = \frac{n}{2}\left[4n^2+6n+2-n-1+2\right]$$

$$S_n = \frac{n}{2}\left(4n^2\right)$$

Answer : (a)

74. The sum of **n** terms of the series $2.3^2 + 5.4^2 + 8.5^2 + \ldots\ldots\ldots$is

(a) $(n/12)(9n^3 + 62n^2 + 123n + 22)$ (b) $(n/12)(9n^3 - 62n^2 + 123n - 22)$

(c) $(n/6)(9n^3 + 62n^2 + 123n + 22)$ (d) None

Sol.

1^{st} digits are 2, 5, 8, ...It is AP

a =2 and d= 3

$a_i = a + (i-1)d = 2 + (i-1)3 = 3i-1$

2^{nd} digits are 3, 4, 5, ... It is AP

a = 3 and d = 1

$a_i = a + (i-1)d = 3 + (i-1)1 = i+2$

i^{th} term of series

$a_i = (3i-1)(i+2)^2$

$a_i = (3i-1)(i^2 + 4i + 4)$

$a_i = 3i^3 + 12i^2 + 12i - i^2 - 4i - 4$

$a_i = 3i^3 + 11i^2 + 8i - 4$

$S_n = \sum a_i$

$S_n = \sum (3i^3 + 11n^2 + 8i - 4$

$S_n = 3\sum i^3 + 11\sum n^2 + 8\sum i - 4\sum 1$

$$= \frac{3n^2(n+1)^2}{4} + \frac{11n(n+1)(2n+1)}{6} + \frac{8n(n+1)}{2} - 4n$$

$$= \frac{n}{12}\left[9n(n+1)^2 + 22(n+1)(2n+1) + 48(n+1) - 48\right]$$

$$= \frac{n}{12}\left[9n\left(n^2+2n+1\right) + 22\left(2n^2+3n+1\right) + 48n + 48 - 48\right]$$

$$= \frac{n}{12}\left[9n^3 + 18n^2 + 9n + 44n^2 + 66n + 22 + 48n\right]$$

$$= \frac{n}{12}\left(9n^3 + 62n^2 + 123n + 22\right)$$

Answer : (a)

75. The sum of **n** terms of the series 1 + (1 + 3) + (1 + 3 + 5) + ………is

(a) (n/6)(n + 1)(2n + 1) (b) (n/6)(n + 1)(n + 2)

(c) (n/3)(n + 1)(2n + 1) (d) None

Sol.

$1 + (1 + 3) + (1 + 3 + 5) + \ldots\ldots$

$a_i = (1+3+5+\ldots)i$ It is AP

$a = 1$ and $d = 2$

$a_i = \sum[a+(i-1)d]$

$a_i = \sum[1 + (i-1)2]$

$a_i = \sum(2i - 1)$

$a_i = 2\sum i - \sum 1$

$$S_n = \frac{2n(n+1)}{2} - n$$

$$S_n = n(n+1-1) = n^2$$

$$S_n = \sum a_i = \sum n^2$$

$$S_n = \frac{n(n+1)(2n+1)}{6}$$

Answer : (a)

76. The sum of **n** terms of the series $1^2+(1^2+2^2)+(1^2+2^2+3^2)+$ ……….is
(a) $(n/12)(n + 1)^2 (n + 2)$ (b) $(n/12)(n - 1)^2 (n + 2)$
(c) $(n/12)(n^2 - 1)(n + 2)$ (d) None

Sol.

$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots$

i^{th} term

$a_i = 1^2 + 2^2 + 3^2 + \ldots + i^2$

$a_i = \sum i^2$

$$a_i = \frac{i(i+1)(2i+1)}{6}$$

$$S_n = \sum a_i$$

$$S_n = \frac{1}{6}\sum i(i+1)(2i+1)$$

$$S_n = \frac{1}{6}\sum(2i^3 + 3i^2 + i)$$

$$S_n = \frac{2}{6}\sum i^3 + \frac{3}{6}\sum i^2 + \frac{1}{6}\sum i$$

$$= \frac{2}{6}\frac{n^2(n+1)^2}{4} + \frac{3}{6}\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{12}\left[n(n+1) + 2n + 1 + 1\right]$$

$$= \frac{n(n+1)}{12}\left[n^2+n+2n+2\right]$$

$$= \frac{n(n+1)}{12}\left(n^2+3n+2\right)$$

$$= \frac{n(n+1)(n+2)(n+1)}{12}$$

$$= \frac{n(n+1)^2(n+2)}{12}$$

Answer : (a)

77. The sum of **n** terms of the series 1+(1+1/3)+(1+1/3+1/3^2)+ ……….is

(a) (3/2) (1-3^{-n}) (b) (3/2)[n-(1/2)(1-3^{-n})]

(c) Both (d) None

Sol.

$$1+\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{3}+\frac{1}{3^2}\right)+\ldots$$

$$a_i = 1+\frac{1}{3}+\frac{1}{3^2}+\ldots+\frac{1}{3^{i-1}}$$

It is G.P. a = 1 and r = 1/3

$$a_i = \frac{a\left(1-r^n\right)}{1-r}$$

$$a_i = \frac{\left[1-\left(\frac{1}{3}\right)^i\right]}{\frac{2}{3}}$$

$$a_i = \frac{3}{2}\left[1-\left(\frac{1}{3}\right)^i\right]$$

$S_n = \sum a_i$

$$S_n = \sum\frac{3}{2}\left[1-\left(\frac{1}{3}\right)^i\right]$$

$$S_n = \sum\frac{3}{2}-\frac{3}{2}\sum\left(\frac{1}{3}\right)^i$$

It is G.P. a=1/3 aand r = 1/3

$$S_n = \frac{3}{2}n-\frac{3}{2}\left[\frac{a\left(1-r^n\right)}{1-r}\right]$$

$$S_n = \frac{3n}{2} - \frac{9}{4}\cdot\frac{1}{3}\left[1-\left(\frac{1}{3}\right)^n\right]$$

$$S_n = \frac{3n}{2} - \frac{3}{4}\left[1-(3)^{-n}\right]$$

$$S_n = \frac{3}{2}\left[n - \frac{1}{2}\left(1-3^{-3}\right)\right]$$

Answer : (b)

78. The sum of n terms of the series n.1+(n-1).2+(n - 2).3+ ……….is

(a) (n/6)(n + 1)(n + 2) (b) (n/3)(n + 1)(n + 2)

(c) (n/2)(n + 1)(n + 2) (d) None

Sol.

n.1+(n-1).2+(n-2).3+ ...

First numbers are n, n-1, n-2 .... It is AP

a = n and d = -1

$a_i = a + (i-1)d = n + (i-1)(-1)$

$a_i = n + 1 - i$

2nd numbers are 1, 2, 3, ... It is A.P.

a = 1 and d = 1

i^{th} terms of series

$a_i = (n+1-i)i$

$a_i = ni + i - i^2$

$S_n = \sum a_i$

$S_n = \sum (ni + i - i^2)$

$S_n = n\sum i + \sum i - \sum i^2$

$$S_n = \frac{n^2(n+1)}{2} + \frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{6}\left[3n+3-(2n+1)\right]$$

$$= \frac{n(n+1)}{6}(3n+3-2n-1)$$

$$= \frac{n(n+1)}{6}(n+2)$$

$$= \frac{n(n+1)(n+2)}{6}$$

Answer : (a)

79. The sum of n terms of the series 1 + 5 + 12 + 22 + ……….is

(a) $(n^2/2)(n + 1)$ (b) $n (n+1)$ (c) $(n^2/2)(n - 1)$ (d) None

Sol.

$S_n = 1 + 5 + 12 + 22 + \ldots\ldots a_n$ ---(1)

$S_n = 1+ 5 + 12 + 22 + \ldots\ldots a_n$ ---(2)

(1) - (2)

$\Rightarrow 0 = 1 + (4 + 7 + 10 + \ldots .a_{n-1}) - a_n$

$a_n = 1 +(4 +7 + 10 + \ldots .$ (n-1) terms)

4, 7, 10, ... is A.P. with a = 4, d = 3

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$4+7+10+\ldots..(n-1)$$

$$= \frac{n-1}{2}\left[2(4)+(n-1-1)3\right]$$

$$= \frac{n-1}{2}[8+3n-6]$$

$$= \frac{n-1}{2}(3n+2)$$

Use formula

$$a_n = 1+\frac{(n-1)(3n+2)}{2}$$

$$a_n = \frac{2+3n^2+2n-3n-2}{2}$$

$$a_n = \frac{3n^2-n}{2}$$

$$S_n = \sum a_n$$

$$S_n = \sum\left(\frac{3n^2-n}{2}\right)$$

$$S_n = \frac{3}{2}\sum n^2 - \frac{1}{2}\sum n$$

$$S_n = \frac{3}{2}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{1}{2}\cdot\frac{n(n+1)}{2}$$

$$S_n = \frac{n(n+1)}{4}[2n+1-1]$$

$$S_n = \frac{n(n+1)2n}{4}$$

$$S_n = \frac{n^2(n+1)}{2}$$

Answer : ()

80. The sum of **n** terms of the series 4 + 14 + 30 + 52 + 80 + ……….is

(a) $n(n + 1)^2$ (b) $n(n - 1)^2$ (c) $n(n^2 - 1)$ (d) None

Sol.

4 + 14 + 30 + 52 + 80 + ……

S_n = 4 + 14 + 30 + 52 + 80 + .. $+a_n$ ----(1)

S_n = 4 + 14 + 30 + 52 + 80 + .. $+a_{n-1}$ $+a_n$ ---(2)

(1) - (2)

$\Rightarrow 0 = 4 + 10 + 16 + 22 + 28 + \ldots - a_n$

$a_n = 4 + (10+16+22+28+\ldots n-1)$

10+16+22+28+...n-1 It is A.P. a = 10 and d = 6

$$a_n = 4 + \frac{n-1}{2}(6n+8)$$

$$a_n = 4 + (n-1)(3n+4)$$

$$a_n = 4 + 3n^2 + n - 4$$

$$a_n = 3n^2 + n$$

$S_n = \sum a_n = \sum(3n^2+n)$

$$S_n = \frac{3n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}(2n+1+1)$$

$$= \frac{n(n+1)2(n+1)}{2}$$

$$= n(n+1)^2$$

Answer : (a)

81. The sum of **n** terms of the series 3 + 6 + 11 + 20 + 37 + ……….is

(a) 2^{n+1} + (n/2)(n + 1)-2
(b) 2^{n+1} + (n/2)(n + 1)-1
(c) 2^{n+1} + (n/2)(n - 1)-2
(d) None

Sol.

3 + 6 + 11 + 20 + 37 + ……..

=(2+1)+(4+2)+(8+3)+(16+4)+....

=(2+4+8+16+...)+(1+2+3+4...)

2+4+8+16+... is G.P a=2 and r = 2>1

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{2(2^n - 1)}{1}$$

$$S_n = 2^{n+1} - 2$$

1+2+3+... is AP a=1 and d = 1

$$S_n = \frac{n}{2}[2+(n-1)1]$$

$$S_n = \frac{n}{2}[2+n-1]$$

$$S_n = \frac{n}{2}(n+1)$$

$$S_n = 2^{n+1} - 2 + \frac{n}{2}(n+1)$$

$$S_n = 2^{n+1} + \frac{n}{2}(n+1) - 2$$

Answer : (a)

82. The n^{th} terms of the series is 1/(4.7) + 1/(7.10) + 1/(10.13) + ……….is

(a) (1/3)[$(3n + 1)^{-1}$-$(3n+4)^{-1}$]
(b) (1/3)[$(3n-1)^{-1}$-$(3n+4)^{-1}$]
(c) (1/3) [$(3n+1)^{-1}$ - $(3n-4)^{-1}$]
(d) None

Sol.

1^{st} number of denominator = 4, 7, 10, ...

a = 4 and d = 3

a_i = a + (i-1)d = 4 + (i-1)3 = 3i+1

Second number of denominator = 7, 10, 13 ....

It is AP a=7 and d= 3

a_i = a + (i-1)d = 7+(i-1)3 = 3i+4

i^{th} term of series

$$a_i = \frac{1}{(3i+1)(3i+4)}$$

$$a_i = \frac{1}{3}\left[\frac{(3i+4)-(3i+1)}{(3i+1)(3i+4)}\right]$$

$$= \frac{1}{3}\left[\frac{1}{3i+1} - \frac{1}{3i+4}\right]$$

$$= \frac{1}{3}\left[(3i+1)^{-1} - (3i+4)^{-1}\right]$$

$\therefore$ Answer : (a)

83. In question No.(82) the sum of the series upto n is

(a) $(n/4)(3n+4)^{-1}$ (b) $(n/4)(3n-4)^{-1}$ (c) $(n/2)(3n+4)^{-1}$ (d) None

Sol.

$$\frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + \frac{1}{10\cdot 13} + \ldots$$

1st number of denominator = 4, 7, 10, ...

a = 4 and d = 3

$a_i = a + (i-1)d = 4 + (i-1)3 = 3i+1$

Second number of denominator = 7, 10, 13 ....

It is AP a=7 and d= 3

$a_i = a + (i-1)d = 7+(i-1)3 = 3i+4$

i^{th} term of series

$$a_i = \frac{1}{(3i+1)(3i+4)}$$

$$S_n = \sum a_i$$

$$S_n = \sum \frac{1}{(3i+1)(3i+4)}$$

$$S_n = \frac{1}{3}\sum \frac{(3i+4)-(3i+1)}{(3i+1)(3i+4)}$$

$$= \frac{1}{3}\left[\sum_{i=1}^{n} \frac{1}{3i+1} - \sum_{i=1}^{n} \frac{1}{3i+4}\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{4}+\frac{1}{7}+\ldots\frac{1}{3n+1}\right)-\left(\frac{1}{7}+\frac{1}{10}+\ldots\frac{1}{3n+4}\right)\right]$$

$$= \frac{1}{3}\left[\frac{1}{4}+\frac{1}{7}+\ldots\frac{1}{3n+1}-\frac{1}{7}+\frac{1}{10}+\ldots\frac{1}{3n+4}\right]$$

$$= \frac{1}{3}\left[\frac{1}{4}-\frac{1}{3n+4}\right]$$

$$= \frac{1}{3}\left[\frac{3n+4-4}{4(3n+4)}\right]$$

$$= \frac{n}{4(3n+4)}$$

Answer : (a)

84. The sum of **n** terms of the series $1^2/1+(1^2+2^2)/(1+2)+(1^2+2^2+3^2)/(1+2+3)+$ ….is

(a) (n/3)(n + 2) (b) (n/3)(n + 1) (c) (n/3)(n + 3) (d) None

Sol.

$$\frac{1^2}{1}+\frac{1^2+2^2}{1+2}+\frac{1^2+2^2+3^2}{1+2+3}+..$$

$$a_i = \frac{1^2+2^2+3^2+\ldots+i^2}{1+2+3+\ldots+i}$$

$$a_i = \frac{\sum i^2}{\sum i}$$

$$a_i = \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}$$

$$a_i = \frac{1}{3}(2n+1)$$

$$a_i = \frac{2}{3}i+\frac{1}{3}$$

$$S_n = \sum a_i$$

$$S_n = \sum\left(\frac{2}{3}i+\frac{1}{3}\right)$$

$$S_n = \sum a_i$$

$$S_n = \sum\left(\frac{2}{3}i+\frac{1}{3}\right)$$

$$S_n = \frac{2}{3}\sum i+\frac{1}{3}\sum 1$$

$$S_n = \frac{2}{3}\cdot\frac{n(n+1)}{2}+\frac{1}{3}n$$

$$S_n = \frac{n}{3}(n+1+1)$$

$$S_n = \frac{n}{3}(n+2)$$

Answer : (a)

85. The sum of n terms of the series $1^3/1+(1^3+2^3)/2+(1^3+2^3+3^3)/3+ \ldots$ is

(a) (n/48)(n + 1)(n + 2)(3n+5)   (b) (n/24)(n + 1)(n + 2)(3n+5)

(c) (n/48)(n + 1)(n + 2)(5n + 3)   (d) None

Sol.

$$\frac{1^3}{1}+\frac{1^3+2^3}{2}+\frac{1^3+2^3+3^3}{3}+\ldots.$$

$$a_i = \frac{1^3+2^3+3^3+\ldots i^3}{i}$$

$$a_i = \frac{\sum i^3}{i} = \frac{i^2(i+1)^2}{4\cdot i}$$

$$a_i = \frac{1}{4}i\left(i^2+2i+1\right)$$

$$a_i = \frac{1}{4}i^3+\frac{2}{4}i^2+\frac{1}{4}i$$

$$S_n = \sum a_i$$

$$S_n = \sum\left(\frac{1}{4}i^3+\frac{2}{4}i^2+\frac{1}{4}i\right)$$

$$S_n = \frac{1}{4}\sum i^3+\frac{2}{4}\sum i^2+\frac{1}{4}\sum i$$

$$= \frac{n^2(n+1)^2}{4\cdot 4} + \frac{2}{4}\frac{n(n+1)(2n+1)}{6} + \frac{1}{4}\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{48}\left[3n(n+1)+4(2n+1)+6\right]$$

$$S_n = \sum a_i$$

$$S_n = \sum\left(\frac{1}{4}i^3 + \frac{2}{4}i^2 + \frac{1}{4}i\right)$$

$$S_n = \frac{1}{4}\sum i^3 + \frac{2}{4}\sum i^2 + \frac{1}{4}\sum i$$

$$= \frac{n^2(n+1)^2}{4\cdot 4} + \frac{2}{4}\frac{n(n+1)(2n+1)}{6} + \frac{1}{4}\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{48}\left[3n(n+1)+4(2n+1)+6\right]$$

$$= \frac{n(n+1)}{48}\left[3n^2+3n+8n+4+6\right]$$

$$= \frac{n(n+1)}{48}\left[3n^2+11n+10\right]$$

$$= \frac{n(n+1)}{48}\left[3n^2+6n+5n+10\right]$$

$$= \frac{n(n+1)}{48}\left[3n(n+2)+5(n+2)\right]$$

$$= \frac{n(n+1)}{48}(n+2)(3n+5)$$

Answer : (a)

86. The value of n^2 ++2n[1+2+3+ ….. +(n - 1)] is

(a) n^3 (b) n^2 (c) n (d) None

Sol.

n^2 + 2n[1+2+3+ .....+(n-1)]

For 1+2+3+ .....+(n-1) use formula for summation

$$S_n = \frac{n(n+1)}{2}$$

Thus total summation

$$S_{n-1} = n^2 + 2n\left[\frac{(n-1)n}{2}\right]$$

$$S_{n-1} = n^2 + n(n-1)(n)$$

$$S_{n-1} = n^2 + n^3 - n^2$$

$S_{n-1} = n^3$

Answer : (a)

87. 2^{4n} - 1 is divisible by

(a) 15 (b) 4 (c) 6 (d) 64

Sol.

$n^2 + 2n[1+2+3+ .....+(n-1)]$

For 1+2+3+ .....+(n-1) use formula for summation

$$S_n = \frac{n(n+1)}{2}$$

Thus total summation

$$S_{n-1} = n^2 + 2n\left[\frac{(n-1)n}{2}\right]$$

$$S_{n-1} = n^2 + n(n-1)(n)$$

$$S_{n-1} = n^2 + n^3 - n^2$$

$$S_{n-1} = n^3$$

Answer : (a)

88. 3^n - 2n - 1 is divisible by

(a) 15 (b) 4 (c) 6 (d) 64

Sol.

$3^n - 2n - 1$

$3^n = (1+2)^n$

$= {}^nC_0 + {}^nC_1(2) + {}^nC_2(2)^2 + ... + {}^nC_n2^n$

$= 1+n(2)+{}^nC_2(2)^2 +....+ {}^nC_n2^n$

$3^n - 2n - 1 = 2^2 [{}^nC_2+{}^nC_3(2) + {}^nC_4(2)^2 + ...+{}^nC_n(2)^{n-2}]$

$3^n - 2n - 1 = 4 [{}^nC_2+{}^nC_3(2) + {}^nC_4(2)^2 + ...+{}^nC_n(2)^{n-2}]$

∴ Divisible by 4

Answer : (b)

89. n(n-1) (2n-1) is divisible by

(a) 15 (b) 4 (c) 6 (d) 64

Sol.

n(n-1)(2n-1)

n = 1 , n(n-1)(n-2) = 0

n = 2, n(n-1)(n-2) = 6

n = 3 , n(n-1)(n-2) = 6

It is divisible by 6

Answer : (c)

90. $7^{2n}+16n-1$ is divisible by

(a) 15 (b) 4 (c) 6 (d) 64

Sol.

$7^{2n} + 16n - 1$

$n = 1 \Rightarrow 49+16-1 = 64$

$n = 2 \Rightarrow 2401 + 32-1 = 2432=38\times64$

$\therefore$ It is divisble by 64

Answer : (d)

91. The sum of n terms of the series whose n^{th} term $3n^2 +2n$ is is given by

(a) (n/2)(n + 1)(2n + 3) (b) (n/2)(n + 1)(3n + 2)

(c) (n/2)(n + 1)(3n - 2) (d) (n/2)(n + 1)(2n - 3)

Sol.

$a_n = 3n^2 + 2n$

$S_n = \sum a_n$

$S_n = \sum (3n^2 + 2n)$

$S_n = 3\sum n^2 + 2\sum n$

$$S_n = 3\frac{n(n+1)(2n+1)}{6} + 2\frac{n(n+1)}{2}$$

$$S_n = \frac{n(n+1)}{2}[2n+1+2]$$

$$S_n = \frac{n(n+1)(2n+3)}{2}$$

Answer : (a)

92. The sum of n terms of the series whose n^{th} term $n.2^n$ is is given by

(a) $(n - 1)2^{n+1}+2$ (b) $(n + 1)2^{n+1}+2$ (c) $(n - 1)2^n + 2$ (d) None

Sol. Answer (a)

93. The sum of **n** terms of the series whose n^{th} term $5.3^{n+1}+2n$ is is given by

(a) $(5/2)\ (3^{n+2}-9)+n(n+1)$

(b) $(2/5)\ (3^{n+2}-9)+n(n+1)$

(c) $(5/2)\ (3^{n+2}+9)+n(n+1)$

(d) None

Sol.

$5.3^{n+1}+2n$

$S_n = \sum a_n$

$S_n = \sum(5.3^{n+1}+2n)$

$S_n = 5\sum 3^{n+1} + 2\sum n$

$S_n = 5[3^2 + 3^3 + ... 3^{n+1}] + 2\ [n(n+1)/2]$

$S_n = 5[3^2 + 3^3 + ... 3^{n+1}] + n(n+1)$

$3^2 + 3^3 + ... 3^{n+1}$ is G.P. with

$a = 9,\ r = 3 > 1$

$$3^2 + 3^3 + ...3^{n+1} = \frac{9(3^n - 1)}{3-1}$$

$$3^2 + 3^3 + ...3^{n+1} = \frac{9(3^n - 1)}{2}$$

$$S_n = 5\left[\frac{9}{2}(3^n - 1)\right] + n(n+1)$$

$$S_n = \frac{5}{2}\left[3^2(3^n - 1)\right] + n(n+1)$$

$$S_n = \frac{5}{2}(3^{n+2} - 9) + n(n+1)$$

Answer : (a)

94. If the third term of a G.P. is the square of the first and the fifth term is 64 the series would be __________.

(a) 4 + 8 + 16 + 32 + ...

(b) 4 - 8 + 16 - 32 + ……….

(c) both

(d) None

Sol.

Let "a" be the 1^{st} term and ratio be "r"

$a_3 = a_1^2$

$a_5 = 64$

$\therefore ar^2 = (a)^2$

$r^2 = a$

$a_5 = 64$

$ar^4 = 64$

$r^6 = 3^6$

$\Rightarrow r = 2$

G.P 4+8+16+32+ ....

Answer : (a)

95. Three numbers whose sum is 15 are in A.P. but if they are added by 1, 4, 19 respectively they are in G.P. The numbers are __________.

(a) 2, 5, 8 (b) 26, 5, -16 (c) Both (d) None

Sol.

Let terms be a-d. a. a+d

a-d+a+a+d = 15

3a = 15

a = 5

When 1, 4, 19 is added from G.P.

So, numbers will be a-d+1, a+4, a+d+19 will be in G.P.

$\therefore (a+4)^2 = (a-d+1)(a+d+19)$

Replace a = 5

$(5+4)^2 = (5-d+1)(5+d+19)$

81 = (6-d)(24+d)

$81 = 144+6d-24d-d^2$

$d^2 + 18d - 63 = 0$

(d-21)(d+3) = 0

d = 21 or d = -3

If d= 21, numbers are -16, 5, 26

If d = -3, then numbers are 8, 5, 2

BOTH

$\therefore$ Answer : (c)

96. If a, b, c are the p^{th}, q^{th} and r^{th} terms of a G.P. respectively the value of $a^{q-r}.b^{r-p}.c^{p-q}$ is __________.

(a) 0 (b) 1 (c) -1 (d) None

Sol.

$a_p = a$, $a_q = b$, $a_r = c$

Let 1st term be A and rate be 'R'

$AR^{p-1} = a$, $AR^{q-1} = b$, $AR^{r-1} = c$

Now $a^{q-r}\, b^{r-p}\, c^{p-q}$

$= (AR^{p-1})^{q-r}\, (AR^{q-1})^{r-p}(AR^{r-1})^{p-q}$

$=A^{q-r}\, (R^{(p-1)(q-r)}\, A^{r-p}\, R^{(q-1)(r-p)}\, A^{p-q}R^{(r-1)(p-q)}$

$= A^{q-r+r-p+p-q}\, R^{pq-pr-q+r+pr-pq-r+p+pr-qr-p+q}$

$= A^0\, R^0$

$=1$

∴ Answer : (b)

97. If a, b, c are in A.P. and x, y, z in G.P. then the value of $x^{b-c}.y^{c-a}.z^{a-b}$ is __________.

(a) 0 (b) 1 (c) -1 (d) None

Sol.

a, b, c are in A.P.

∴ $a+c = 2b$

x, y, z in G.P.

$y^2 = xz$

$\Rightarrow y = (xz)^{1/2}$

$= x^{b-c}\, y^{c-a}\, z^{a-b}$

$= x^{b-c}\, (x)^{(c-a)/2} z^{(c-a)/2}\, z^{a-b}$

$$= x^{b-c+\frac{c-a}{2}} \cdot z^{\frac{c-a}{2}+a-b}$$

$$= x^{\frac{2b-2c+c-a}{2}} \cdot z^{\frac{c-a+2a-2b}{2}}$$

$$= x^{\frac{2b-c-a}{2}} \cdot z^{\frac{c+a-2b}{2}}$$

$$= x^0 z^0$$

$$= 1$$

Answer : (b)

98. If a, b, c are in A.P. and x, y, z in G.P. then the value of $(x^b.y^c.z^a)\div(x^c.y^a.z^b)$ is __________.

(a) 0 (b) 1 (c) -1 (d) None

Sol.

a, b, c are in A.P.

$\therefore a+c = 2b$

x, y, z in G.P.

$y^2 = xz$

$\Rightarrow y = (xz)^{1/2}$

$(x^b \cdot y^c \cdot z^a) \div x^c y^a z^b$

$= x^{b-c} y^{c-a} z^{a-b}$

$x^{b-c} (xz)^{(c-a)/2} z^{a-b}$

$= x^{b-c} (x)^{(c-a)/2} z^{(c-a)/2} z^{a-b}$

$$= x^{b-c+\frac{c-a}{2}} \cdot z^{\frac{c-a}{2}+a-b}$$

$$= x^{\frac{2b-2c+c-a}{2}} \cdot z^{\frac{c-a+2a-2b}{2}}$$

$$= x^{\frac{2b-c-a}{2}} \cdot z^{\frac{c+a-2b}{2}}$$

$$= x^0 z^0$$

$$= 1$$

$\therefore$ Answer : (b)

99. The sum of **n** terms of the series 7 + 77 + 777 + …..is

(a) (7/9) [(1/9) (10^{n+1}-10)-n] (b) (9/10) [(1/9) (10^{n+1}-10)-n]

(c) (10/9) [(1/9) (10^{n+1}-10)-n] (d) None

Sol.

7 + 77 + 777 + ……

=7(1+11+111+....)

$$= \frac{7}{9}(9+99+999+\ldots)$$

$$= \frac{7}{9}[(10-1)+(100-1)+(1000-1)+\ldots]$$

$$= \frac{7}{9}[(10+10^2+10^3+\ldots n\,term)-(1+1+1+\ldots n\,term)]$$

$$= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$$

$$= \frac{7}{9}\left[\frac{1}{9}(10^{n+1}-10)-n\right]$$

Answer : (a)

100. The least value of n for which the sum of n terms of the series $1 + 3 + 3^2 +$ ……….is greater than 7000 is __________.

(a) 9 (b) 10 (c) 8 (d) 7

Sol.

$1 + 3 + 3^2 + \ldots\ldots > 7000$

It is G.P. a = 1 and r=3

$S_n > 7000$

$$\frac{(3^n - 1)}{2} > 7000$$

$$3^n - 1 > 14000$$

$$3^n > 14001$$

$$3^9 > 14000$$

$$n \geq 9$$

Least value is n = 9

$\therefore$ Answer : (a)

101. If 'S' be the sum, 'P' the product and 'R' the sum of the reciprocals of n terms in a G.P. then 'P' is the __________ of S^n and R^{-n}.

(a) Arithmetic Mean (b) Geometric Mean

(c) Harmonic Mean (d) None

Sol.

Let series is G.P. with 1st term a and ration r

S is sum of n terms

$$S = \frac{a(r^n - 1)}{r - 1}$$

P is product

$ar.ar^2.ar^3 \ldots\ldots ar^{n-1}$

$= a^{1+1+\ldots n \text{ times}}\ r^{1+2+3+\ldots n-1}$

$= a^n.\ r^{(n-1)n/2}$

R = sum of reciprocals

$$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \ldots + \frac{1}{ar^{n-1}}$$

$$R = \frac{r^{n-1} + r^{n-2} + \ldots + r + 1}{ar^{n-1}}$$

$$R = \frac{1+r+r^2+\ldots+r^{n-1}}{ar^{n-1}}$$

$$R = \frac{(r^n-1)}{r-1} \times \frac{1}{ar^{n-1}}$$

$$R = \frac{(r^n-1)}{ar^{n-1}(r^{n-1})}$$

Now $S^n . R^n$

$$= \left[\frac{a(r^n-1)}{r-1}\right]^n \left[\frac{r^n-1}{(r-1)ar^{n-1}}\right]^{-n}$$

$$= \frac{a^n(r^n-1)^n}{(r-1)^n} \cdot \frac{(r^n-1)^{-n}}{(r-1)^{-n}a^{-n}r^{-n(n-1)}}$$

$$= a^{n+n} \cdot 1 \cdot r^{n(n-1)}$$

$$= a^{2n} \cdot r^{n(n-1)}$$

$$= \left[a^n \cdot r^{\frac{n(n-1)}{2}}\right]^2$$

$$= P^2$$

∴ P ia G.M. of $S^n R^{-n}$

Answer : (b)

102. Sum upto ∞ of the series $8+4\sqrt{2}+4+$ ……….is

(a) $8(2+\sqrt{2})$ (b) $8(2-\sqrt{2})$ (c) $4(2+\sqrt{2})$ (d) $4(2-\sqrt{2})$

Sol.

$8 + 4\sqrt{2} + 4 +$.....

$a = 8$, $r = (4\sqrt{2})/8 = 1/\sqrt{2}$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{8}{1-\frac{1}{\sqrt{2}}}$$

$$S_\infty = \frac{8\sqrt{2}}{\sqrt{2}-1}$$

$$S_\infty = \frac{8\sqrt{2}\left(\sqrt{2}+1\right)}{2-1}$$

$S_\infty = 8\sqrt{2}\,(\sqrt{2} + 1)$

$S_\infty = 8(2+\sqrt{2})$

Answer : (a)

103. Sum upto ∞ of the series $1/2+1/3^2 +1/2^3 +1/3^4 +1/2^5+1/3^6 +$ ……….is

(a) 19/24 (b) 24/19 (c) 5/24 (d) None

Sol.

$$\frac{1}{2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}+\frac{1}{2^5}+\frac{1}{3^6}+\dots$$

$$=\left(\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}+\dots\right)+\left(\frac{1}{3^2}+\frac{1}{3^4}+\dots\right)$$

$$\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}+\dots$$

$$a_1=\frac{1}{2}, r_1=\frac{\frac{1}{2^3}}{\frac{1}{2}}=\frac{1}{4}$$

$$\frac{1}{3^2}+\frac{1}{3^4}+\dots$$

$$a_2=\frac{1}{9}, r_2=\frac{\frac{1}{81}}{\frac{1}{9}}=\frac{1}{9}$$

$$S_\infty=\frac{a_1}{1-r_1}+\frac{a_2}{1-r_2}$$

$$=\frac{\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{9}}{1-\frac{1}{9}}$$

$$=\frac{1}{2}\times\frac{4}{3}+\frac{1}{9}\times\frac{9}{8}$$

$$=\frac{2}{3}+\frac{1}{8}$$

$$=\frac{19}{24}$$

Answer : (a)

104. If $1+a+a^2 + \ldots\infty = x$ and $1+b+b^2 + \ldots\infty = y$ then $1 + ab + a^2b^2 + \ldots\infty = x$ is given by __________.

(a) (xy)/(x+y-1) (b) (xy)/(x-y-1) (c) (xy)/(x+y+1) (d) None

Sol.

$1+a+a^2 + \ldots\ldots\infty$

$$x = \frac{1}{1-a}$$

$x - ax = 1$

$x-1 = ax$

$$a = \frac{x-1}{x}$$

$1+b+b^2 + \ldots\ldots\infty$

a=1 and r = b

$$y = \frac{1}{1-b}$$

$$y - by = 1$$

$$y - 1 = by$$

$$b = \frac{y-1}{y}$$

$1 + ab + a^2b^2 + \ldots$

a = 1 and r=ab

$$S_\infty = \frac{1}{1-ab}$$

$$S_\infty = \frac{1}{1-\left(\frac{x-1}{x}\right)\left(\frac{y-1}{y}\right)}$$

$$S_\infty = \frac{xy}{xy-(xy-x-y+1)}$$

$$S_\infty = \frac{xy}{xy-xy+x+y-1}$$

$$S_\infty = \frac{xy}{x+y-1}$$

Answer : (a)

105. If the sum of three numbers in G.P. is 35 and their product is 1000 the numbers are __________.

(a) 20, 10, 5 (b) 5, 10, 20 (c) both (d) None

Sol.

Let numbers in G.P. be

a/r, a, ar

a/r + a + ar = 35

and a/r×a×ar = 1000

$\therefore a^3$ or $a^3 = 10^3$

$\Rightarrow a = 10$

Now

$$\frac{a}{r}+a+ar=35$$

$$\frac{10}{r}+10+10r=35$$

$$\frac{10}{r}+10r-25=0$$

$10r^2 - 20r - 5r + 10 = 0$

10r(r-2) -5(r-2) = 0

r = 2,and 10r-5=0

r =2 and r = 1/2

IF r = 2, a = 0

a/r = 10/2 = 5

ar = 10(2) = 20

If r=1/2, a = 10

a/r = 20

a=10, ar =5

So numbers are 5, 10 and 20 Answer

is both a and b

$\therefore$ Answer : (c)

106. If the sum of three numbers in G.P. is 21 and the sum of their squares is 189 the numbers are __________.

(a) 3, 6, 12 (b) 12, 6, 3 (c) both (d) None

Sol.

Let number be

a, ar and ar^2

$a+ar+ar^2 = 21$

$\therefore a(1+r+r^2) = 21$

Squaring

$a^2[1+r+r^2]^2 = (21)^2$

$a^2(1+r+r^4+2r+2r^3 + 2r^2) = 441$

$a^2(1+r^2+r^4) + 2a^2r(1+r^2+r) = 441$ ----(1)

Now given sum of their squares is 189

$a^2 + ar^2 + a^2r^4 = 189$

$a^2(1+r^2+r^4 = 189$

----(2)

Substituting (2) in (1)

$189 + 2a^2r(1+r^2+r) = 441$

$2ar[a(1+r+r^2) = 441-189$

$2ar(21) = 252$

$$ar = \frac{252}{2\times 21}$$

$$ar = 6$$

$$a = \frac{6}{r}$$

$a+6+ar^2 = 21$

$a+ar^2 = 15$

$$\frac{6}{r}+\frac{6}{r}r^2 = 15$$

$6+6r^2=15r$

$6r^2-15r+6=0$

$2r^2-5r+2=0$

$2r^2-4r-r+2 = 0$

$2r(r-2) - (r-2) = 0$

r = 2 and r = 1/2

If r =2 and a = 3Number are 3, 6, 12

If r = 1/2, a = 6

a/r = 12,6, 3 ..

∴ 12, 6, 3

Anser is both a and b

Answer : (c)

107. If a, b, c are in G.P. then the value of $a(b^2 + c^2) - c(a^2 + b^2)$ is ________.

(a) 0 (b) 1 (c) -1 (d) None

Sol.

a, b and c are in G.P.

$b^2 = ac$

$a(b^2 + c^2) - c(a^2 + b^2)$

$= a(ac + c^2) - c(a^2 + ac)$

$= ac(a + c) - ac(a + c)$

$= 0$

Answer : (a)

108. If a, b, c, d are in G.P. then the value of $b(ab - cd) - (c + a)(b^2 - c^2)$ is ________

(a) 0 (b) 1 (c) -1 (d) None

Sol.

a, b, c and d are G.P.

$b = ar$, $c = ar^2$, $d = ar^3$

LHS= $b(ab - cd) - (c + a)(b^2 - c^2)$

$= ar(ar^2 - a^2r^5) - (ar^2 + a)(a^2r^2 - a^2r^4)$

$= a^2r^3 - a^3r^6 - a^3r^4 + a^3r^6 - a^3r^2 - a^3r^2 + a^3r^4$

$= 0$

Answer : (a)

109. If a, b, c, d are in G.P. then the value of $(ab + bc + cd)^2 - (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$ is ________.

(a) 0 (b) 1 (c) -1 (d) None

Sol.

a, b, c and d are G.P.

a = a. b = ar and c=ar^2. d = ar^3

LHS

$= (ab + bc + cd)^2 - (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$

$= [a^2r + a^2r^3 + a^2r^5)^2 - (a^2 + a^2r^2 + a^2r^4)$

$(a^2r^2 + a^2r^4 + {}^2r^6)$

$=[a^2r(1+r^2+r^4)]^2 - a^2(1+r^2+r^4)a^2r^2(1+r^2+r^4)$

$= a^4r^2(1+r^2+r^4)^2 - a^4r^2(1+r^2+r^4)^2$

$= 0$

Answer : (a)

110. If a, b, c, d are in G.P. then a+b, b+c, c+d are in

(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

a, b, c and d are in G.P.

a, b = ar , c =ar^2

$$\frac{b+c}{a+b} = \frac{ar^2+ar^3}{a+ar^2}$$

$$\frac{b+c}{a+b} = \frac{ar(1+r)}{a(1+r)} = r$$

$$\frac{c+d}{b+c} = \frac{ar^2+ar^3}{ar+ar^2}$$

$$= \frac{ar^2(1+r)}{ar(1+r)} = r$$

Ratio is ame so

a+b, b+c, c+d are in GP

∴ Answer : (b)

111. If **a,** b, **c** are in G.P. then a^2+b^2, ab+bc, b^2+c^2 are in

(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

a, b, c and d are in G.P.

$\therefore b^2 = ac$

$(a^2 + b^2)\ (b^2+c^2$

$= (a^2+ac)\ (ac+c^2) = (a)(a+c)(c)(a+c)$

$=ac(a+c)^2$

$=b^2(a+c)^2$

$=[\ b(a+c)](a+c)^2$

$= (ab+bc)(a+c)^2$

$\therefore (a^2 +b^2)$, (ab+bc), $(b^2 +c^2)$ are in G.P.

Answer : (b)

112. If a, b, x, y, z are positive numbers such that a, x, b are in A.P. and a, y, b are in G.P. and z=(2ab)/(a+b) then

(a) x, y, z are in G.P. (b) $x \ge y \ge z$

(c) both (d) None

Sol.

a, x, b are in A.P.

$\therefore$ a+b = 2x

a, y, b are in G.P. and z = 2ab/(a+b)

$y^2 = ab$

$$z = \frac{2ab}{a+b}$$

$$z = \frac{2y^2}{2x}$$

$\therefore y^2 = xz$

$\therefore$ x, y, z are in G.P.

Answer : (a)

113. If a, b, c are in G.P. then the value of $(a-b+c)(a+b+c)^2 - (a+b+c)(a^2 + b^2 + c^2)$ is given by

(a) 0 (b) 1 (c) -1 (d) None

Sol.

a,b, and c are in G.P.

$\therefore b^2 = ac$

$(a-b+c)(a+b+c)^2 - (a+b+c)(a^2+b^2+c^2)$

$= (a+b+c)\,[(a-b+c)(a+b+c) - (a^2+b^2+c^2)]$

$= (a+b+c)\,[(a+c)^2 - (b)^2 - {}^2\text{-}B^2\text{-}C^2]$ as $ac = b^2$

$\therefore =(a+b+c)\,(2b^2 - 2b^2]$

$=(a+b+c)(0) = 0$

Answer : (a)

114. If a, b, c are in G.P. then the value of $a(b^2 + c^2) - c(a^2 + b^2)$ is given by

(a) 0 (b) 1 (c) -1 (d) None

Sol.

a, b, c are G.P. $\Rightarrow b^2 = ac$

$a(b^2 + c^2) - c(a^2 + b^2)$

$= a(ac+c^2) - c(a^2 + ac)$

$= ac(a+c) - ac(a+c)$

$= 0$

Answer : (a)

115. If a, b, c are in G.P. then the value of $a^2b^2c^2 (a^{-3}+b^{-3}+c^{-3})-(a^3+b^3+c^3)$ is given by

(a) 0 (b) 1 (c) -1 (d) None

Sol.

a, b, and c are in G.P.

$\therefore b^2 = ac$

$a^2b^2c^2(a^{-3} + b^{-3} + c^{-3}) - (a^3 + b^3 + c^3)$

$$= a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)-a^3-b^3-c^3$$

$$= \frac{b^2c^2}{a}+\frac{a^2c^2}{b}+\frac{a^2b^2}{c}-a^3-b^3-c^3$$

$$= \frac{ac\cdot c^2}{a}+\frac{(b^2)^2}{b}+\frac{a^2\cdot ac}{c}-a^3-b^3-c^3$$

$$= c^3+b^3+a^3-a^3-b^3-c^3$$

$$= 0$$

Answer : (a)

116. If a, b, c, d are in G.P. then $(a-b)^2$, $(b-c)^2$, $(c-d)^2$ are in

(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

a, b, c, and d are in G.P.

$\therefore a= a,\ b = ar,\ c = ar^2,\ d = ar^3$

$(a-b)^2 (c-d)^2 = [a-ar]^2 [ar^2 - ar^3]^2$

$= [a(1-r)]^2 [ar^2(1-r)]^2$

$=a^2(1-r)^2a^2r^4 (1-r)^2$

$= a^4r^4 (1-r)^4$

$(b-c)^2 = [ar-ar^2]^2$

$= [ar(1-r)]^2 = a^2r^2(1-r)^2$

$\therefore [(b-c)^2]^2 = [a^2r^2(1-r)^2]^2$

$= a^4r^4 (1-r)^4$

$= (a-b)^2(c-d)^2$

$\therefore (a-b)^2, (b-c)^2, (c-d)^2$ are in G.P.

Answer : (b)

117. If a, b, c, d are in G.P. then the value of $(b-c)^2 + (c-a)^2 + (d-b)^2 - (a-d)^2$ is given by

(a) 0 (b) 1 (c) -1 (d) None

Sol.

a, b, c, d are in G.P.

$\Rightarrow$ a = a, b = ar, c = ar^2 and d = ar^3

$(b-c)^2 + (c-a)^2 + (d-b)^2 - (a-d)^2$

$= (ar-ar^2)^2 + (ar^2 - a)^2 + (ar^3 - ar)^2 - (a-ar^3)^2$

$= [ar(1-r)]^2 + [a(r^2-1)]^2 + [ar(r^2-1)]^2 - [a(1-r^3)]^2$

$=a^2r^2 (1-r^2) + a^2(r-1)^2 (r+1)^2 + a^2r^2 (r-1)^2 (r+1)^2 - a^2(1-r^3)^2$

$= a^2(1-r^2) [r^2 + (r+1)^2] + a^2[(r-1)^2(r+1)^2 r^2 - (1-r)^2 (1+r+r^2)^2]$

$=a^2(1-r)^2[r^2+r^2+2r+1] + a^2(1-r)^2[r^2(r^2+2r+1) - (1+r+r^2)^2]$

$=a^2(1-r)^2 (2a^2 +2r +1) +a^2(1-r)^2[r^4 +2r^3 + r^2 - 1 -r^2-r^4-2r-2r^3- 2r^2]$

$= a^2 (1-r)^2[2r^2+2r+1-2r^2 - 2r -1]$

$=a^2 (1-r^2) (0)$

$= 0$

Answer : (a)

118. If (a-b), (b-c), (c-a) are in G.P. then the value of $(a+b+c)^2-3(ab+bc+ca)$ is given by

(a) 0 (b) 1 (c) -1 (d) None

Sol.

(a-b), (b-c), (c-a) are in G.P.

$\therefore (b-c)^2 = (a-b) (c-a)$

$\rightarrow b^2 - 2bc +c^2 = ac - a^2 - bc +ab$

$\Rightarrow a^2 + b^2 + c^2 -ab -bc -ca = 0$

$(a+b+c)^2 - 3(ab+bc+ca)$

$= a^2+b^2+c^2 +2ab +2bc +2ca -3ab -3bc -3ca$

$=a^2+b^2+c^2 -ab - bc -ca$

$=0$

Answer : (a)

119. If $a^{1/x}=b^{1/y}=c^{1/z}$ and a, b, c are in G.P. then x, y, z are in

(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

$a^{1/x} = b^{1/y} = c^{1/z} = k$

$\therefore a = k^x, b = k^y, c = k^z$

a, b, c are in G.P.

$\therefore b^2 = ac$

$(k^y)^2 = k^x k^z$

$k^{2y} = k^{x+z}$

$\Rightarrow 2y = x+z$

$\therefore$ x, y, z are A.P.

Answer : (a)

120. If x = a + a/r + a/r^2 + …..∞, y = b - b/r + b/r^2 - …..∞, and z = c + c/r^2 + c/r^4 + …..∞, then the value of $\frac{xy}{z} - \frac{ab}{c}$ is

(a) 0 (b) 1 (c) -1 (d) None

Sol.

$$x = a + \frac{a}{r} + \frac{a}{r^2} + \ldots\infty$$

$$a = a, r = \frac{1}{r}$$

$$x = S_\infty = \frac{a}{1-r} = \frac{a}{1-\frac{1}{r}}$$

$$x = \frac{ar}{r-1}$$

$$y = b + \frac{b}{r} + \frac{b}{r^2} + \ldots\infty$$

$$a = b, r = -\frac{1}{r}$$

$$y = S_\infty = \frac{a}{1-r}$$

$$y = \frac{b}{1+\frac{1}{r}} = \frac{br}{r+1}$$

$$z = c + \frac{c}{r^2} + \frac{c}{r^4} + \ldots\infty$$

It is G.P.

$$a = c, r = \frac{1}{r^2}$$

$$z = S_\infty = \frac{a}{1-r} = \frac{c}{1-\frac{1}{r^2}}$$

$$z = \frac{cr^2}{r^2-1}$$

$$\frac{xy}{z} - \frac{ab}{c}$$

$$= \frac{\left(\frac{ar}{r-1}\right)\left(\frac{br}{r+1}\right)}{\frac{cr^2}{r^2-1}} - \frac{ab}{c}$$

$$= \frac{abr^2}{(r-1)(r+1)} \times \frac{(r-1)(r+1)}{cr^2} - \frac{ab}{c}$$

$$= \frac{ab}{c} - \frac{ab}{c}$$

$$= 0$$

Answer : (a)

121. If a, b, c are in A.P. a, x, b are in G.P. and b, y, c are in G.P then x^2, b^2, y^2 are in

(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

a, b and c are in A.P.

∴ a+c = 2b

a, x, b in G.P.

∴ $x^2 = ab$

b, y, and c is G.P.

$y^2 = bc$

$x^2 + y^2 = ab + bc$

$= b(a+c)$

$= b(2b) = 2b^2$

x^2, b^2, y^2 are in A.P.

Answer : (a)

122. If a, b-a, c-a are in G.P. and a=b/3=c/5 then a, b, c are in
(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

a, b-a, c-a are in G.P.

$\therefore (b-a)^2 = a(c-a)$

a = b/3 = c/5 = k (say)

a=k, b = 3k, c= 5k

a+c = k+5k = 6k

a+c = 2(3k) = 2b

$\Rightarrow$ a+c = 2b

a, b, c are in AP

Answer : (a)

123. If a, b, (c+1) are in G.P. and a = $(b-c)^2$ then a, b, c are in
(a) A.P. (b) G.P. (c) H.P. (d) None

Sol.

a, b, (c+1) are in G.P. and a = $(b-c)^2$

$\therefore b^2 = a(c+1)$

$b^2 = (b-c)^2 (c+1)$

$b^2 = (b^2 - 2bc + c^2)(c+1)$

$b^2 = b^2c + b^2 - 2bc^2 - 2bc + c^3 + c^2$

$2bc^2 - b^2c + 2bc = c^3 + c^2$

$bc(2c - b + 2) = c^2(c+1)$

$b(2c - b + 2) = c(c+1)$

$2bc - b^2 + 2b = c^2 + c$

$2b - c = c^2 + b^2 - 2bc$

$2b - c = (b-c)^2$

2b - c = a

2b = a+c

$\therefore$ a, b, and c are in A.P.

Answer : (a)

124. If $S_1, S_2, S_3, \ldots\ldots S_n$ are the sums of infinite G.P.s whose first terms are 1, 2, 3 …..n and whose common ratios are 1 /2, 1 /3, …..1 /(n+1) then the value of $S_1 + S_2 + S_3 + \ldots S_n$ is
(a) (n/2) (n + 3) (b) (n/2) (n + 2) (c) (n/2) (n+1) (d) $n^2/2$

Sol.

S_1 = sum of infinite terms of G.P. with a = 1 and r = 1/2

$$S_1 = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 2$$

S_2 = sum of infinite terms of G.P. a = 2, r = 1/3

$$S_2 = \frac{a}{1-r} = \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = 3$$

$$S_3 = \frac{a}{1-r} = \frac{3}{1-\frac{1}{4}} = 4$$

$$S_n = \frac{a}{1-r} = \frac{n}{1-\frac{1}{n+1}} = n+1$$

$S_1 + S_2 + S_3 + \ldots S_n = 2 + 3 + 4 + \ldots (n+1) = [1 + 2 + 3 + \ldots (n+1)] - 1$

$$= \frac{(n+1)(n+2)}{2} - 1$$

$$= \frac{n^2+3n+2-2}{2}$$

$$= \frac{n(n+3)}{2}$$

Answer : (a)

125. The G.P. whose 3rd and 6th terms are 1, -1/8 respectively is

(a) 4, -2, 1 ….. (b) 4, 2, 1 ….. (c) 4, -1, 1 /4 ….. (d) None

Sol.

$a_3 = 1$, $a_6 = -1/8$, It is G.P.

$\therefore ar^2 = 1$ ---(1)

$a_6 = -1/8$

$ar^5 = -1/8$ ---(2)

(2) ÷ (1) ⇒

$$\frac{ar^5}{ar^2} = \frac{-\frac{1}{8}}{1}$$

$$r^3 = -\frac{1}{8}$$

$$r = -\frac{1}{2}$$

$ar^2 = 1$

$a(-1/2)^2 = 1$

$a/4 = 1 \Rightarrow a = 4$

G.P. is 4, -2, 1, -1/2

Answer : (a)

126. In a G.P. if the $(p+q)^{th}$ term is m and the $(p-q)^{th}$ term is n then the p^{th} term is ________.

(a) $(mn)^{1/2}$ (b) mn (c) (m+n) (d) (m-n)

Sol.

It is G.P.

$a_{p+q} = m,\ a_{p-q} = n$

$ar^{p+q-1} = m$ ---(1)

$a_{p-q} = n$

$ar^{p-q-1} = n$ ---(2)

$(2) \div (1) \Rightarrow$

$$\frac{ar^{p-q-1}}{ar^{p+q-1}} = \frac{n}{m}$$

$$r^{-2q} = \frac{n}{m}$$

$$\frac{1}{r^{2q}} = \frac{n}{m}$$

$$r = \left(\frac{m}{n}\right)^{\frac{1}{2q}}$$

$ar^{p+q-1} = m$

$$a\left[\left(\frac{m}{n}\right)^{\frac{1}{2q}}\right]^{p+q-1} = m$$

$$a = \frac{m \times n^{\frac{p+q-1}{2q}}}{m^{\frac{p+q-1}{2q}}} = m^{\frac{q-p+1}{2q}} \cdot n^{\frac{p+q-1}{2q}}$$

$$a_p = ar^{p-1}$$

$$= m^{\frac{q-p+1}{2q}} \cdot n^{\frac{p+q-1}{2q}}\left[\left(\frac{m}{n}\right)^{\frac{1}{2q}}\right]^{p-1}$$

$$= m^{\frac{q-p+1}{2q}} \cdot n^{\frac{p+q-1}{2q}} \cdot \frac{m^{\frac{p-1}{2q}}}{n^{\frac{p-1}{2q}}}$$

$$= m^{\frac{q-p+1+p-1}{2q}} \cdot n^{\frac{p+q-1}{2q}-\frac{p-1}{2q}}$$

$$= m^{\frac{q}{2q}} \cdot n^{\frac{q}{2q}}$$

$$= (mn)^{\frac{1}{2}}$$

Answer : (a)

127. The sum of **n** terms of the series is $1/\sqrt{3}+1+3/\sqrt{3}$ + ...

(a) (1/6) $(3+\sqrt{3})\,(3^{n/2}-1)$ (b) (1/6) $(\sqrt{3}+1)\,(3^{n/2}-1)$

(c) (1/6) $(3+\sqrt{3})\,(3^{n/2}+1)$ (d) None

Sol.

$$\frac{1}{\sqrt{3}}+1+\frac{3}{\sqrt{3}}+\ldots$$

$$It\ is\ G.P.\ a=\frac{1}{\sqrt{3}},\ r=\sqrt{3}>1$$

$$S_n=\frac{a\left(r^n-1\right)}{r-1}$$

$$=\frac{1}{\sqrt{2}}\left[\frac{\left(\sqrt{3}\right)^n-1}{\sqrt{3}-1}\right]$$

$$=\frac{\left(\sqrt{3}\right)^n-1}{3-\sqrt{3}}$$

$$=\frac{\left[\left(\sqrt{3}\right)^n-1\right]}{9-3}$$

$$=\frac{1}{6}\left(3+\sqrt{3}\right)\left(3^{\frac{n}{2}}-1\right)$$

Answer : (a)

128. The sum of **n** terms of the series 5/2-1 + 2/5 - ……….is

(a) (1/14) $(5^n+2^n)/5^{n-2}$ (b) (1/14) $(5^n-2^n)/5^{n-2}$

(c) both (d) None

Sol.

$$\frac{5}{2}, -1, +\frac{5}{2} .... is\ G.P$$

$$a = \frac{5}{2}, r = \frac{-2}{5} < 1$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{\frac{5}{2}\left[1-\left(\frac{-2}{5}\right)^n\right]}{1-\left(\frac{-2}{5}\right)}$$

$$= \frac{5}{2}\left[1-\left(\frac{-2}{5}\right)^n\right] \times \frac{1}{\frac{7}{5}}$$

$$= \frac{25}{14}\left[1-\left(\frac{-2}{5}\right)^n\right]$$

$$= \frac{5^2}{14} \frac{\left(5^n-(-2)^n\right)}{5^n}$$

$$= \frac{1}{14} \times \frac{1}{5^{n-2}}\left[5^n-(-2)^n\right]$$

Answer : (c)

129. The sum of n terms of the series 0.3 + 0.03 + 0.003 + ……….is
(a) $(1/3)(1-1/10^n)$ (b) $(1/3)(1+1/10^n)$ (c) both (d) None

Sol.

0.3 + 0.03 + 0.003 ....

It is G.P. anda r = 0.03/0.3 = 0.1 < 1

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{0.3\left[1-(0.1)^n\right]}{1-0.1}$$

$$= \frac{0.3}{0.9}\left[1-(0.1)^n\right]$$

$$= \frac{1}{3}\left(1-\frac{1}{10^n}\right)$$

Answer : (a)

130. The sum of first eight terms of G.P. is five times the sum of the first four terms. The common ratio is __________.

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) both (d) None

Sol.

It is G.P

$S_8 = 5\ S_4$

$$S_n = \frac{a(r^8-1)}{r-1} = 5\frac{a(r^4-1)}{r-1}$$

$$r^8-1 = 5(r^4-1)$$

$$(r^4-1)(r^4+1) = 5(r^4-1)$$

$$r^4+1 = 5$$

$$r^4 = 4$$

$$r = \pm\sqrt{2}$$

Answer : (c)

131. If the sum of n terms of a G.P. with first term 1 and common ratio 1/2 is 1+127/128, the value of n is __________.

(a) 8 (b) 5 (c) 3 (d) None

Sol.

a = 1 and r =1/2<1

$$S_n = 1+\frac{127}{128}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$1+\frac{127}{128} = \frac{1\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}$$

$$\left(\frac{128+127}{128}\right)\frac{1}{2} = 1-\left(\frac{1}{2}\right)^n$$

$$\frac{255}{256} = 1-\left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^n = 1 - \frac{255}{256}$$

$$\left(\frac{1}{2}\right)^n = \frac{1}{256}$$

$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^8$$

$\Rightarrow n = 8$

Answer : (a)

132. If the sum of n terms of a G.P. with last term 128 and common ratio 2 is 255, the value of n is __________.

(a) 8 (b) 5 (c) 3 (d) None

Sol.

$a_n = 128$, $r = 2$ and $S_n = 255$

$ar^{n-1} = 128$

$a(2)^{n-1} = 128$

$a = 128 / (2^{n-1})$

$S_n = 255$

$$\frac{a\left(r^n - 1\right)}{r - 1} = 255$$

$$\frac{a\left(2^n - 1\right)}{2 - 1} = 255$$

$$\frac{128}{2^{n-1}}\left(2^n - 1\right) = 255$$

$$\frac{2^7}{2^n}\left(2^n - 1\right)2 = 255$$

$$\frac{2^8 2^n}{2^n} - \frac{2^8}{2^n} = 255$$

$$2^8 - 255 = \frac{2^8}{2^n}$$

$$256 - 255 = 2^{8-n}$$

$$1 = 2^{8-n}$$

$$2^0 = 2^{8-n}$$

$$0 = 8 - n$$

$n = 8$

Answer : (a)

133. How many terms of the G.P. 1, 4, 16 .... are to be taken to have their sum 341?

(a) 8 (b) 5 (c) 3 (d) None

Sol.

1, 4, 16, ... are in G.P.

a = 1 and r = 4 > 1, $S_n = 341$

$$\frac{a(r^n - 1)}{r - 1} = 341$$

$$\frac{1(4^n - 1)}{4 - 1} = 341$$

$$4^n - 1 = 1023$$

$$(2^2)^n = 1023 + 1 = 1024$$

$$(2)^{2n} = 2^{10}$$

$$2n = 10$$

$$n = 5$$

Answer : (b)

134. The sum of **n** terms of the series 5 + 55 + 555 + ……….is

(a) (50/81) (10^n - 1)-(5/9)n (b) (50/81) (10^n + 1)-(5/9)n

(c) (50/81) (10^n + 1)+(5/9)n (d) None

Sol.

5 + 55 + 555 + …….

$5[1+11+111+...]$

$$= \frac{5}{9}[9 + 99 + 999 + ...]$$

$$= \frac{5}{9}\left[(10-1) + (10^2 - 1) + (10^3 - 1) + ...\right]$$

$$= \frac{5}{9}\left[(10 + 10^2 + 10^3 + ...) - (1 + 1 + 1 + ...n\ times)\right]$$

$$= \frac{5}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{50}{81}(10^n - 1) - \frac{5n}{9}$$

Answer : (a)

135. The sum of **n** terms of the series 0.5 + 0.55 + 0.555 + ……….is

(a) $(5/9)n-(5/81)(1-10^{-n})$ (b) $(5/9)n+(5/81)(1-10^{-n})$

(c) $(5/9)n+(5/81)(1+10^{-n})$ (d) None

Sol.

0.5 + 0.55 + 0.555 + ……….

0.5 + 0.55 + 0.555 + ……….

$$= \frac{5}{9}(0.9+0.99+0.999+.....)$$

$$= \frac{5}{9}\left[(1-0.1)+(1-0.01)+(1-0.001)+...\right]$$

$$= \frac{5}{9}\left[(1+1+...)-(0.1+0.01+0.001+...)\right]$$

$$= \frac{5}{9}\left[(1+1+...)-\left(0.1+0.1^2+0.1^3+...\right)\right]$$

$$0.1+0.1^2+0.1^3+....$$

$$a = 0.1, r = 0.1 < 1$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{0.1}{0.9}\left[1-(0.1)^n\right]$$

$$S_n = \frac{5}{9}\left[n-\frac{1}{9}\left[1-(0.1)^n\right]\right]$$

$$S_n = \frac{5n}{9}-\frac{5}{81}\left(1-10^{-n}\right)$$

Answer : (a)

136. The sum of **n** terms of the series $1.03+1.03^2+1.03^3$ + ……….is

(a) $(103/3)(1.03^n - 1)$ (b) $(103/3)(1.03^n+1)$

(c) $(103/3)(1.03^{n+1}-1)$ (d) None

Sol.

$1.03+1.03^2+1.03^3$ + ...

It is G.P. a = 1.03, r = 1.03>1

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$= \frac{1.03\left[(1.03)^n-1\right]}{1.03-1}$$

$$= \frac{103}{3}\left[(1.03)^n-1\right]$$

Answer : (a)

137. The sum upto infinity of the series 1/2 + 1/6 + 1/18 + ……….is

(a) 3/4 (b) 1/4 (c) 1/2 (d) None

Sol.

1/2 + 1/6 + 1/18 + …… is G.P. with a=1/2, r = 1/3 < 1

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{3}} = \frac{\frac{1}{2}}{\frac{2}{3}}$$

$$= \frac{1}{2}\cdot\frac{3}{2} = \frac{3}{4}$$

Answer : (a)

138. The sum upto infinity of the series 4 + 0.8 + 0.16 + ……….is

(a) 5 (b) 10 (c) 8 (d) None

Sol.

4 + 0.8 + 0.16 + …… is G.P.

a = 4 and r = 0.2 <1

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{4}{1-0.2}$$

$$= \frac{4}{0.8} = 5$$

Answer : (a)

139. The sum upto infinity of the series $\sqrt{2}$+1/$\sqrt{2}$+1/(2$\sqrt{2}$)+ ……….is

(a) $2\sqrt{2}$ (b) 2 (c) 4 (d) None

Sol.

$$\sqrt{2} + \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} + \ldots.is$$

G.P., a= $\sqrt{2}$ and r = 1/2 < 1

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{\sqrt{2}}{1-\frac{1}{2}}$$

$S_\infty = 2\sqrt{2}$

Answer : (a)

140. The sum upto infinity of the series 2/3 + 5/9 + 2/27 + 5/81 + ……….is

(a) 11/8 (b) 8/11 (c) 3/11 (d) None

Sol.

$$\frac{2}{3}+\frac{5}{9}+\frac{2}{27}+\frac{5}{81}+....$$

$$=\left(\frac{2}{3}+\frac{2}{27}+....\right)+\left(\frac{5}{9}+\frac{5}{81}+...\right)$$

$$=\left(\frac{2}{3}+\frac{2}{27}+....\right)$$

$$a_1=\frac{2}{3}, r_1=\frac{1}{9}$$

$$=\left(\frac{5}{9}+\frac{5}{81}+...\right)$$

$$a_2=\frac{5}{9}, r_2=\frac{1}{9}$$

$$S_\infty=\frac{a_1}{1-r_1}+\frac{a_2}{1-r_2}$$

$$S_\infty=\frac{\frac{2}{3}}{1-\frac{1}{9}}+\frac{\frac{5}{9}}{1-\frac{1}{9}}$$

$$S_\infty=\frac{2}{3}\times\frac{9}{8}+\frac{5}{9}\times\frac{9}{8}$$

$$S_\infty=\frac{6}{8}+\frac{5}{8}=\frac{11}{8}$$

Answer : (a)

141. The sum upto infinity of the series $(\sqrt{2}+1)+1+(\sqrt{2}-1)+$ ……….is

(a) $(1/2)(4+3\sqrt{2})$ (b) $(1/2)(4-3\sqrt{2})$ (c) $4+3\sqrt{2}$ (d) None

Sol.

$$\left(\sqrt{2}+1\right)+1+\left(\sqrt{2}-1\right)+\ldots$$

It is G.P. with

$$a=\sqrt{2}+1, r=\sqrt{2}-1$$

$$S_\infty=\frac{a}{1-r}$$

$$S_\infty=\frac{\sqrt{2}+1}{1-\left(\sqrt{2}-1\right)}$$

$$S_\infty=\frac{\sqrt{2}+1}{2-\sqrt{2}}$$

$$S_\infty=\frac{\left(\sqrt{2}+1\right)\left(2+\sqrt{2}\right)}{4-2}$$

$$S_\infty=\frac{1}{2}\left(2\sqrt{2}+2+2+\sqrt{2}\right)$$

$$S_\infty=\frac{4+3\sqrt{2}}{2}$$

Answer : (a)

142. The sum upto infinity of the series $(1+2^{-2})+(2^{-1}+2^{-4})+(2^{-2}+2^{-6})+$ ……….is

(a) 7/3 (b) 3/7 (c) 4/7 (d) None

Sol.

$(1+2^{-2})+(2^{-1}+2^{-4})+(2^{-2}+2^{-6})+$ ......

$= (1+2^{-1}+2^{-2}+ \ldots\ldots\infty) + (2^{-2}+2^{-4}+2^{-6}+\ldots.)$

$a_1 = 1, r_1 = 1/2; a_2 = 1/4, r_2 = 1/4$

$$S_\infty=\frac{a_1}{1-r_1}+\frac{a_2}{1-r_2}$$

$$S_\infty=\frac{1}{1-\frac{1}{2}}+\frac{\frac{1}{4}}{1-\frac{1}{4}}$$

$$S_\infty=2+\frac{1}{3}=\frac{7}{3}$$

Answer : (a)

143. The sum upto infinity of the series $4/7-5/7^2+4/7^3-5/7^4+$ ……….is

(a) 23/48 (b) 25/48 (c) 1/2 (d) None

Sol.

$$\frac{4}{7}-\frac{5}{7^2}+\frac{4}{7^3}-\frac{5}{7^4}+\ldots\infty$$

$$\left(\frac{4}{7}+\frac{4}{7^3}+\frac{4}{7^5}+\ldots\right)-\left(\frac{5}{7^2}+\frac{5}{7^4}+\ldots\right)$$

$$a_1=\frac{4}{7}, r_1=\frac{1}{7^2}; a_2=\frac{5}{49}, r_2=\frac{1}{7^2}$$

$$S_\infty=\frac{a_1}{1-r_1}-\frac{a_2}{1-r_2}$$

$$S_\infty=\frac{\frac{4}{7}}{1-\frac{1}{49}}-\frac{\frac{5}{49}}{1-\frac{1}{49}}$$

$$S_\infty=\frac{4}{7}\times\frac{49}{48}-\frac{5}{49}\times\frac{49}{49}$$

$$=\frac{49}{48\times7}\left[4-\frac{5}{7}\right]$$

$$=\frac{7}{48}\left(\frac{28-5}{7}\right)$$

$$=\frac{23}{48}$$

Answer : (a)

144. If the sum of infinite terms in a G.P. is 2 and the sum of their squares is 4/3 the series is

(a) 1, 1/2, 1/4 ……… (b) 1, -1/2, 1/4 ……

(c) -1, -1/2, -1/4 .... (d) None

Sol.

Let terms be a, ar, ar^2, ..

$S_\infty = 2$

a/(1-r) = 2

a = 2(1-r)

Square of terms of G.P.

a^2, a^2r^2, a^4r^4 ..... It is G.P. with $a_1 = a^2$ and $r_1 = r^2$

$$S_\infty=\frac{a_1}{1-r_1}$$

$$\frac{4}{3}=\frac{a^2}{1-r^2}$$

$$\frac{4}{3}=\frac{[2(1-r)]^2}{1-r^2}$$

$$\frac{4}{3}=\frac{4(1-r)^2}{(1-r)(1+r)}$$

1+r = 3(1-r)

1+r = 3 - 2r

4r = 2

r = 1/2

Replace r = 1/2

a = 2(1 - 1/2) = 1

So G.P. is 1, 1/2, 1/4, ...

Answer : (a)

145. The infinite G.P. with first term 1/4 and sum 1/3 is

(a) 1/4, 1/16, 1/64 ... (b) 1/4, -1/16, 1/64 ...

(c) 1/4, 1/8, 1/16 .... (d) None

Sol.

a = 1/4 and S_∞ = 1/3

$$S_\infty=\frac{a}{1-r}$$

$$\frac{1}{3}=\frac{\frac{1}{4}}{1-r}$$

$$1-r=\frac{3}{4}$$

$$r=1/4$$

So G.P. is

$$\frac{1}{4},\frac{1}{16},\frac{1}{64}....$$

Answer : (a)

146. If the first term of a G.P. exceeds the second term by 2 and the sum to infinity is 50 the series is __________.

(a) 10, 8, 32/5 ... (b) 10, 8, 5/2 ... (c) 10, 10/3, 10/9 .... (d) None

Sol.

Let G.P. be a, ar, ar^2, ...

$a_1 > a_2$ by 2, $S_\infty = 50$

a > ar by 2

∴ a = ar + 2 a-ar = 2

a(1-r) = 2

$$a = \frac{2}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{\frac{2}{1-r}}{1-r}$$

$$50 = \frac{2}{(1-r)^2}$$

$$(1-r)^2 = \frac{2}{50} = \frac{1}{25}$$

$$1-r = \pm\frac{1}{5}$$

$$r = 1 \mp \frac{1}{5}$$

or

$$r = 1 + \frac{1}{5} = \frac{6}{5}$$

$$r = 1 - \frac{1}{5} = \frac{4}{5}$$

$$a = \frac{2}{1-r} = \frac{2}{1-\frac{4}{5}} = 10$$

Sp G.P. is 10, 8, 32/5, ... ,

Answer : (a)

147. Three numbers in G.P. with their sum 130 and their product 27,000 are __________.

(a) 10, 30, 90 ... (b) 90, 30, 10 ... (c) both (d) None

Sol.

Let terms ofg G.P. be a/r, a, ar

$$\frac{a}{r}+a+ar=130$$

and

$$\frac{a}{r}\times a\times ar=27,000$$

$$a^3=27000$$

$$a=30$$

Now

$$\frac{30}{r}+30+30r=130$$

$$\frac{30}{r}+30r=100$$

and ar = 30(3) = 90

So numbers are 90, 30, 10 or

10, 30, 90

Answer : (c)

148. Three numbers in G.P. with their sum 13/3 and sum of their squares 91/9 are .

(a) 1/3, 1, 3 (b) 3, 1, 1/3 (c) both (d) None

Sol.

Let three terms be a, ar, ar^2

$$a+ar+ar^2=\frac{13}{3}$$

$$a\left(1+r+r^2\right)=\frac{13}{3}---(1)$$

$$Sum\ of\ squares=\frac{91}{9}$$

$$a^2+a^2r^2+a^2r^4=\frac{91}{9}$$

$$a^2\left(1+r^2+r^4\right)=\frac{91}{9}\quad ---(2)$$

Square

$$a^2\left(1+r+r^2\right)^2=\frac{169}{9}$$

$$a^2\left(1+r^2+r^4+2r+2r^3+2r^2\right)=a^2\left(1+r^2+r^4\right)+2a^2r\left(1+r^2+r\right)$$

$$\frac{91}{9}+2ar\left[a\left(1+r^2+r\right)\right]=\frac{169}{9}$$

$$2ar\left(\frac{13}{3}\right)=\frac{169}{9}-\frac{91}{9}$$

$$2ar\left(\frac{13}{3}\right)=\frac{78}{9}$$

$$ar=\frac{78}{9}\times\frac{3}{2\times13}$$

$$ar=1\Rightarrow a=\frac{1}{r}$$

Now

$$\frac{1}{r}+1+\frac{1}{r}\cdot r^2=\frac{13}{3}$$

$$\frac{1}{r}+r=\frac{13}{3}-1$$

$$\frac{1+r^2}{r}=\frac{10}{3}$$

$3r^2 + 3 = 10r$

$3r^2 - 10r + 3 = 0$

$(3r - 1)(r - 3) = 0$

r = 1/3 or r = 3

If r = 1/3 , a = 3

$ar^2 = 1/3$

Number are 3, 1, 1/3

If r = 3, then a = 1/3

Number will be 1/3, 1, 3

Answer : (c)

149. Find five numbers in G.P. such that their product is 32 and the product of the last two is 108.

(a) 2/9, 2/3, 2, 6, 18
(b) 18, 6, 2, 2/3, 2/9
(c) both
(d) None

Sol.

Let the terms of G.P. be

$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

$|product\ of\ 3 = 27$

$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = 32$

$a^5 = 32$

$a = 2$

Product of last two terms = 108

$ar \times ar^2 = 108$

$a^2r^3 = 108$

$2^2r^3 = 108$

$r^3 = 27$

$r = 3$

$\frac{a}{r^2} = \frac{2}{9}, \frac{a}{r} = \frac{2}{3}$

$so\ number\ are\ \frac{2}{9}, \frac{2}{3}, 2, 6, 18$

Answer : (a)

150. If the continued product of three numbers in G.P. is 27 and the sum of their products in pairs is 39 the numbers are __________.

(a) 1, 3, 9 (b) 9, 3, 1 (c) both (d) None

Sol.

Let the three terms G.P. be a/r, a, ar

Product of three term = 27

$(a/r) \times a \times ar = 27$

$a^3 = 27$

$\Rightarrow a = 3$

Sum of product of pairs = 39

$\frac{a}{r} \times a + a \times ar + \frac{a}{r} \times ar = 39$

$\frac{a^2}{r} + a^2r + a^2 = 39$

$\frac{9}{r} + 9r + 9 = 39$

$\frac{9}{r}+9r=30$

$9r^2 - 30r + 9 = 0$

$3r^2 - 10r + 3 = 0$

(3r - 1) (r - 3) = 0

r = 1/3 or r = 3

If r=1/3, a=3

a/r = 2/(1/3) = 9, ar = 1

So numbers are 9, 3, 1

If r=3 and a=3 then a/r = 3/3 = 1, ar =9

So number are 1, 3, 9

Answer : (c)

151. The numbers x, 8, y are in G.P. and the numbers x, y, -8 are in A.P. The values of x, y are __________.

(a) 16, 4 (b) 4, 16 (c) both (d) None

Sol.

x, 7 and y are in G.P.

∴ $xy = 8^2$

xy = 64

x, y and -8 are in A.P.

∴ x + (-8) = 2y

x - 8 = 24

x = 2y + 8

as xy = 64

(2y+8)y = 64

$2y^2 + 4y - 32 = 0$

(y+8) (y-4) = 0

y = -8 or y = 4

If y = -8,

x = 2y + 8

x = -16 +8 = -8

If y = 4 then x +2y+8

x = 8 + 8 = 16

x = 16 anad y = 4

or x = -8 and v = -8

Answer : (a)