FORMULA SHEET

CH-1 Ratio, Proportion, Logs & Indices

1) $\frac{a}{b}$ → Antecedent
   b → consequent.
2) Fourth Proportion → a, b, c → $\frac{b \times c}{a}$
3) Third Proportion → a, b → = $\frac{b^2}{a}$
4) Mean Proportion → a, b → = $\sqrt{a \times b}$
5) Inverendo → $\frac{a}{b} = \frac{c}{d}$ → $\frac{b}{a} = \frac{d}{c}$
6) Alternendo → $\frac{a}{c} = \frac{b}{d}$
7) Componendo → $\frac{a+b}{b} = \frac{c+d}{d}$
8) Dividendo → $\frac{a-b}{b} = \frac{c-d}{d}$

$\frac{a}{b} = \frac{c}{d}$

9) Comp. & Dividendo $\frac{a+b}{a-b} = \frac{c+d}{c-d}$
10) Addendo $\frac{a+c}{b+d}$
11) Subtrahendo $\frac{a-c}{b-d}$

12) $a^m \times a^n \Rightarrow a^{m+n}$
13) $\frac{a^m}{a^n} \Rightarrow a^{m-n}$
14) $(a^m)^n \Rightarrow a^{m \times n}$
15) $a^x = a^y \Rightarrow x = y$

16) $x^a = y^a \Rightarrow x = y$
17) $a^0 = 1$
18) $(ab)^m = a^m \times b^m$
19) $\sqrt[m]{a} = a^{\frac{1}{m}}$

20) $B^P = N \Rightarrow \log_B N = P$
21) $a^{\frac{1}{6}} = \sqrt{3} \Rightarrow a = (\sqrt{3})^6$
22) $\log(mn) \Rightarrow \log m + \log n$
23) $\log \frac{m}{n} \Rightarrow \log m - \log n$
24) $\log m^n \Rightarrow n \log m$
25) $\log_a 1 \Rightarrow 0$ (with any Base)
26) $\log_a 10 \Rightarrow 1$ (with any Base)
27) $\log_a a \Rightarrow \frac{1}{\log_a b}$

28) $\log_b a \Rightarrow \frac{\log a}{\log b}$
29) $\log_{m^n} a^b \Rightarrow \frac{b}{n} \log_m a$
30) $\log_a a \Rightarrow 1$
31) $\log_b a \times \log_a b \Rightarrow 1$
32) $\log_b a \times \log_c b \Rightarrow \log_c a$
33) $\log_2 8 \Rightarrow ?$ 3 (दूसरा power कौन सा 8 आएगा)
34) $\log_2 n \Rightarrow 8$ (when Single Log Seen Convert it into indices)
35) $\log \frac{ab}{cd} \Rightarrow \log a + \log b - \log c - \log d$

Calculation Tricks

① Trick NO. ①

Variable $= (\text{Base})^{\frac{1}{3}} + (\text{Base})^{\frac{-1}{3}}$

→ $(\text{var.})^3 + 3(\text{var.}) \Rightarrow \text{Base} - \frac{1}{\text{Base}}$

→ $(\text{var.})^3 - 3(\text{var.}) \Rightarrow \text{Base} + \frac{1}{\text{Base}}$

② Trick NO. ②

$\log_{2\sqrt{2}} 512$

- Press $2\sqrt{}$ × 2
- Press =
- Press ×

• Press = Times "Till 512 comes" "Then Add 1" → So Ans :- is 6

③ Trick NO. ③

~~HOW TO FIND LOG OF ANY NO~~

$\log_{7} 343$

- Press 7 (Base)
- Press ×
- Press = (Till 343 come)
- Press 0 + 1 Ans :- 3.

④ Trick NO. ④

If X Trick fail Then we use This Trick

$\log_{0.1} 100$

- Press 0.1 (Base)
- Press ÷
- Press = (Till 100 come)
- Press Ans in − Ans → 2

HOW TO FIND LOG OF ANY NUMBER.

→ Press "NO."
→ Press "√ 19 Times"
→ Press "−1"
→ Press "×227695"
→ Press "="

Eg: log 10
→ Press 10
→ Press √ 19 Times
→ Press −1
→ Press ×227695 → Ans 1

CH-7 Sets, Relation & Function

① No. of Subset $= 2^n$

② No. of Proper Subset $= 2^n - 1$

③ $(A \cup B)' = A' \cap B'$
④ $(A \cap B)' = A' \cup B'$ } De-Morgons Law

⑤ $(A')' = A$

⑥ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
or
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

⑦ $n(A \cup B)' = n(U) - n(A \cup B)$

⑧ "And" ⇒ ∩ & "Or" ⇒ ∪

⑨ Equivalent Set ⇒ Cardinal no. is same
& Equal Set ⇒ Every element of "A" is a element of "B".

⑩ Power Set ⇒ Set of All Subsets

⑪ Cartesian Product :- A = {1, 2, 3}

(Basic Formula)

① $(a+b)^2 = a^2 + 2ab + b^2$

② $(a-b)^2 = a^2 - 2ab + b^2$

③ $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

④ $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

⑤ $(a^3+b^3) = (a+b)(a^2 - ab + b^2)$

⑥ $(a^3-b^3) = (a-b)(a^2 + ab + b^2)$

⑦ $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

if $(a+b+c) = 0$

$\therefore a^3 + b^3 + c^3 - 3abc = 0$

$a^3 + b^3 + c^3 = 3abc$

⑧ Sum of 1st n natural Nos.

$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

⑨ Mean of 1st n Natural

(12) $B \subset A$ (13) $A \Delta B = (A-B) \cup (B-A)$

Subset Superset Symmetric Difference

Every equal Set is equivalent Set, but each equivalent Set is not a equal Set.

Every Set is a Subset of itself

Empty Set is a Subset of every set.

Collection of all Subsets is called Power Set.

(14) $n(A-B) = n(A) - n(A \cap B)$

(15) $n(B-A) = n(B) - n(A \cap B)$

(16) $n(A \cup B)' = n(U) - n(A \cup B)$

Equilateral $\Delta \subset$ Right Isoceles $\Delta \subset$ Isoceles $\subset$ Scalen Δ

$gof(x) = g(f(x))$

Every universal relation is Reflexive

Every identity relation is reflexive but it converse is not true.

(17) Types of Relations

(i) Symmetric Rel^n $\rightarrow (a,b) \in R$ & $(b,a) \in R$.

(ii) Transitive Rel^n $\rightarrow (a,b) \in R$ & $(b,c) \in R \therefore \Rightarrow (a,c) \in R$.

(iii) Equivalence Rel^n $\rightarrow$ If a R is Symmetric, Reflexive, & Transitive then it is equivalence.

Is equal to = equivalence

Is parallel to = equivalence

CH-14 Measures of Central Tendency and Distribution

Five Basic Measures of Central Tendency

(1) Arithmatic Mean :- $\bar{x} = \frac{\text{Sum of observation}}{\text{No. of observation}} = \frac{x_1 + x_2 \cdots x_n}{n}$.

(a) Direct Method

$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

(b) Shortcut Method

$\bar{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i}$ $[u_i = x_i - A]$, A = Assumed Mean.

* when all observations are constant (same) then Arithmatic mean is constant.
* Sum of deviation from mean is 0.
* combined mean = $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$
* A.M. is least affected by the presence of extreme observations.
* A.M. is based on all observations
* Sum of square of deviation from mean is Minimum
* Mean of 1st n natural no. = $\frac{n+1}{2}$
* Mean of 1st n Odd no. = n
* Mean of 1st n Even no. = n+1.

② Median :- central Value of the given data.

when n = Odd: $Me = \left(\frac{n+1}{2}\right)^{th}$

when n = Even: $Me = \left(\frac{n}{2}\right)^{th} + \left(\frac{n+1}{2}\right)^{th}$

when Continuous Series: $Me = L + \frac{(n/2 - C)}{f} i$

Relation Between Mean, Median & Mode.

→ 3 Median = Mode + 2 Mean.

→ Mean - Mode = 3 [Mean - Median]

* Presence of extreme observation doesn't affect median.
* For a given Set of observation the Sum of absolute deviation is minimum when deviation is taken from Median i.e. $\Sigma|x_i - \text{Median}| = \text{Minimum}$

③ Mode :- Highest frequency number is called Mode.

Eg :- 1, 2, 2, 2, 3, 4, 6 , Mode = 2.

when series is continuous.

$$\text{Mode} = L + \frac{(f_m - f_1)}{(2f_m - f_1 - f_2)} \times i.$$

L = Lower Limit
f_m = frequency of Modal class
i = width of the class
f_1 = Preceding class frequency
f_2 = Succeeding frequency.

④ Geometric Mean :- Let $x_1, x_2, x_3 \dots x_n$.

GM → $(x_1 \times x_2 \times x_3 \dots \times x_n)^{1/n}$ Ans.

GM. when frequency is given

$$GM = [x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \dots x_n^{f_n}]^{1/\Sigma f_i}$$

★ when all the observations are constant so GM. is also constant.

GM when percentage is given.

100%, 200% & 400%.

Find Average rate of return

$GM. = [100 \times 200 \times 400]^{1/3}$

$= [8000000]^{1/3}$

$= 200.$

⑤ Harmonic Mean :-

Let $x_1, x_2, x_3 \dots x_n$. are non-zero observations

$$H.M. = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} \dots + \frac{1}{x_n}}$$

⇒ In case of grouped frequency distribution

$$H.M. = \frac{\Sigma f_i}{\Sigma (f_i / x_i)}$$

★ when all the observation are constant then HM. is also constant.

★ combined $H.M. = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$.

★ $G^2 = A \times H.$

★ $AM \geq GM \geq HM$

[$AM > GM > HM$] quantity unequal

[$AM = GM = HM$] quantity equal.

weighted Average :

1) weighted A.M. ⇒ $\frac{\Sigma w_i x_i}{\Sigma w_i}$

Partition Values.

① Quartiles → $Q_1 = \left(\frac{n+1}{4}\right)^{th}$, $Q_2 = \left(2\left(\frac{n+1}{4}\right)\right)^{th}$

② Deciles → $D_1 = \left(\frac{n+1}{10}\right)^{th}$, $D_2 = \left(2\left(\frac{n+1}{10}\right)\right)^{th}$

③ Percentiles → $P_1 = \left(\frac{n+1}{100}\right)^{th}$, $P_2 = \left(2\left(\frac{n+1}{100}\right)\right)^{th}$.

★ Q_2, D_5 and P_{50} are Medians.

Dispersion.

Absolute Measure of Dispersion	Relative Measure of Dispersion
① Range. Range = Largest − Smallest $= L - S$	① Coefficient of Range $= \frac{L-S}{L+S} \times 100$
② Mean Deviation $\frac{\Sigma \lvert x_i - \bar{x} \rvert}{n}$ ⇒ $\frac{\Sigma f_i (x_i - \bar{x})}{\Sigma f_i}$	② Coefficient of Mean Deviation $= \frac{\text{Mean Deviation about Mean}}{\text{Median}} \times 100$
③ Quartile Deviation / Semi-Interquartile Range (SIQR) $= \frac{Q_3 - Q_1}{2}$ S.I.Q.R. $= Q_3 - Q_1$.	③ Coefficient of Quartile Deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$
④ Standard Deviation $\sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$ ⇒ $\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{n}}$	④ Coefficient of Variation S.D. $= \frac{\text{S.D.}}{\text{Mean}}$ $= \frac{\sigma}{\bar{x}}$

⑤ Variance

$$\sigma^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}$$

⑤ Coefficient of Variance

$$= \frac{\sigma}{\bar{x}} \times 100$$

CH-6 Sequence and Series

$a_n = a + (n-1)d$ | $a_n = l - (n-1)d$

★ n^{th} Term of an A.P. is always Linear in n and its common difference is coefficient of n.

If $m.a_m = n.a_n$, Then $a(m+n) = 0$

★ Sum of the term equidistant from "beginning" and "end" is always equal to Sum of 1st & last Term

Ex:- 5, 10, 15 ..... 90, 95, 100

$(5+100) = 105$

★ ① $p^{th} \longrightarrow q$, $q^{th} \longrightarrow p$, Then n^{th} Term $= (p+q-n)$

② $x^{th} \longrightarrow y$, $y^{th} \longrightarrow x$, Then n^{th} Term $= (x+y-n)$

★ $m^{th} \longrightarrow \frac{1}{n}$ & $n^{th} \longrightarrow \frac{1}{m}$, Then $(mn)^{th} = 1$.

★ Sum of n terms of an A.P.

$S_n = \frac{n}{2}[2a + (n-1)d]$

or

$= \frac{n}{2}[a + l]$

★ Sum of n terms of an G.P.

$S_n = a\frac{(r^n - 1)}{r - 1}$, $r > 1$

$= a\frac{(1 - r^n)}{1 - r}$, $r < 1$

★ Sum of n terms of an A.P. is always quadratic in n.

★ Sum of n terms of an A.P. is always 2 the coefficient of n^2 i.e. $an^2 + bn$.

If its C.D. is

* Insertion of A.M.

$d = \frac{b-a}{n+1}$, n = no. of mean

* Insertion of G.M.

$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

* For what value of n $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. of a & b

Ans:- $\boxed{n=0}$

* If $S_n = n^2p$ & $S_m = m^2p$.

then. $S_p = p^2.p = p^3$

* n^{th} term. or general term or last term.

$a_n = a_1.r^{n-1}$.

* G.P. from end. $= l\left(\frac{1}{r}\right)^{n-1}$

* If a, b, c are in A.P. $= \boxed{2b = a+c}$

* If a, b, c are in G.P. $= \boxed{b^2 = ac}$

* Important point.

(1) $9 + 99 + 999 + \ldots\ldots$

$\Rightarrow 10\left[\frac{(10^n-1)}{9} - n\right]$

(2) $0.7 + 0.77 + 0.777 + \ldots\ldots$

$\Rightarrow \frac{0.7}{9}\left[10\left(\frac{10^n-1}{9}\right) - n\right] \Rightarrow \frac{7}{81}\left[9n - 1 + 10^{-n}\right]$

(3) $11 + 103 + 1005 + \ldots\ldots$

$(10+1) + (10^2+1) + (10^3+1) \ldots\ldots$

$\Rightarrow \frac{10(10^n-1)}{} + n^2$.

* Sum of an Infinite G.P.

$S\infty = a + ar + ar^2 + \ldots\ldots\infty$

$S\infty = \frac{a}{1-r}$

CH-13 Statistical Description of Data

Singular → Scientific Method → P Present, A Analyses, C Collect.

Plural → Data → Quantitative, Qualitative

Collection of Data

- Quantitative (variable)
  - Discrete (whole)
  - Continuous (Decimal)
- Qualitative (Attribute) Eg: Nation, Color & Gender.

Collection of Data

- Observation Method → Most Accurate
- Mailed Questionnaire Method → Maximum Non-responsiveness, Widest Range
- Questionaires by Enumerator method + Sending someone with list of questions
- Interview Method.
  - Personal
  - Telephonic
  - Indirect

Classification of Data

- Chronological or Temporal or Time series Data
- Geographical or Spatial Series Data
- Qualitative (ordinal) Data
- Quantitative (Cardinal) Data

* Some Important Terms

① Range.
Range = L − S

② No. of class Interval.
$= \frac{\text{Range}}{\text{Length of class}}$

③ class Mark / Mid value / Mid Point
$= \frac{UCB + LCB}{2}$

④ Frequency Density.
$= \frac{\text{Frequency}}{\text{Width or class Length}}$

⑤ Relative Frequency.
$= \frac{\text{Frequency}}{\text{Total Frequency}}$
Sum of Relative Frequency is always 1.

⑥ Percentage Frequency.
$= \frac{\text{Frequency}}{\text{Total Frequency}}$
Sum of % frequency is always 100.

⑦ class Boundary
Overlapping classification

⑧ Exclusive Series
upper limit not included.

⑨ Less than Ogive Curve - S-Shaped. (J-Shaped)

⑩ Most Commonly used Bell-Shaped.

⑪ Histogram
Compute → Mode.
Cannot Compute → S.D.

Classification is of 4 kinds.
Logarithm of variable is also known as Ratio Chart.

CH-18 INDEX NUMBERS

TESTS	Laspeyre's	Paasche's	Fisher's	Marshall E.W.	Simple (AM & GM)
Unit Test	✓	✓	✓	✓	X
Circular Test	X	X	X	X	✓
Time Reversal Test	X	X	✓	✓	✓
Factor Reversal Test	X	X	✓	X	X

Simple

Laspeyre
Metho
$= \frac{\Sigma P_1 Q}{\Sigma P_0 Q}$

Dick-E
B.Y.

S
Sp
#
#
Link
→ Link
of

14-15 Marks

6 - Easy
5 - Mod
4 - Hard

Time Value of Money

Value of Money ↑ Acc. to time

- Inflation
- Opportunity Cost
- Risk factor
- Liquidity preference

(3) Simple Interest

(5-6) Compound Interest

(4) Annuity

(2) Various Applications

P = 5 Lakh

Lender (Bank) — 6 Years at 10% (Rate of Interest) — Borrower (Home)

5L + 3L = 8 Lakh (Accumulated Amount)

Interest

Rate of Interest = $\frac{I}{P} \times \frac{100}{T}$

P + I = A

	Simple Interest	Compound Interest
Meaning	• Interest is calculated on initial principal	• Interest is calculated on Accumulated Amt. in previous year.
formula	• $\frac{PRT}{100}$ / $\frac{A_3 - A_2}{\text{Acc. to time}}$ IV	• $P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$ by (A − P)
Accumulated Amt.	• P + S.I $P + \frac{PRT}{100} = P\left(1 + \frac{RT}{100}\right)$	• $P\left(1+\frac{R}{100}\right)^t$ $A = C.I + P = \left[P\left(1+\frac{R}{100}\right)^t - P\right] + P$

Difference

$$C.I - S.I = \left[P\left(1+\frac{R}{100}\right)^t - P - \frac{P \times R \times t}{100}\right] = \left[P\left[\left(1+\frac{R}{100}\right)^t - 1 - \frac{Rt}{100}\right]\right]$$

Conversion Period | C

D Q S M N/y

Acc. to time period, which it is going to be compounded.

④ Quater

② Semi annualy

⑫ Monthly

① Yearly / annualy

Compounding time

$$A = P\left(1+\frac{r}{C\times 100}\right)^{C\times t}$$

$$C.I = P\left[\left(1+\frac{r}{C\times 100}\right)^{C\times t}-1\right]$$

Effective Rate of Interest

Depend on conversion period

$$E.R.I = \left[\left(1+\frac{r}{100\times C}\right)^{C}-1\right]\times 100$$

Annuity

When we Pay & Recieve a fixed Amt. of money over a specific time period

EMI / Py

SIP / Recive

Annuity Regular — at the End of the Month (uses Mostly i)

Annuity Due — Intermidiate at the Bigning of Month

	Future Value	Present Value
Single Cash flow	• $C.F\left(1+\frac{r}{100}\right)^{t}$	• $\frac{F.V}{(1+i)^{n}}$
Annuity Regular	• $A\left[\frac{(1+i)^{n}-1}{i}\right]$	• $A\left[\frac{(1+i)^{n}-1}{i\,(1+i)^{n}}\right]$
Annuity Due	• $A\left[\frac{(1+i)^{n}-1}{i}\right]\times(1+i)$	• $A\left[\frac{(1+i)^{n}-1}{i\,(1+i)^{n-1}}\right]$

Single Cash flow – Payment only single time
Future Value – Value of total money in future
Present Value – Value of money in Present

$i = \frac{r}{C \times 100}$ $t = n$

Month	1	2	3	4	..	11	12
Annuity Regular (31 Jan)	Empty	1st Install	2nd Inst	3rd 2nd		Empty	12th Inst.
Annuity Due (1 Jan)							

Annuity Due

Various Application

- Sinking Funds
- Leasing
- Capital Expenditure
- Valuation of Bond (Imp)
- Perpetuity
- Net Present Value (Imp)
- CAGR (Imp.)

1) Sinking Funds :– It is a amt which a person from org put aside for sum specific purpose.

Debenture, Depriciation
Future Value

Formula -> $A\frac{(1+i)^n - 1}{i} = F.V$

Find Annuity

2) Leasing :-

10 lakh Car → For 6Y

→ Interest — 15, 86, 874 — 8% — 1 year

Which one is best

→ Lease — 12 lakh — 2 year — 4 lakh

3) Capital Expenditure :-

It means purchasing an asset today in result of benifit of tomorrows which would flow across the life of asset.

C.E = Outflow - Inflow

4) Valuation of Bond

Amount for Equity

	Money	Bank	Time	Amt
Intrest	20%	15%	3Y	1000

1000 × 20% = 200

$$\frac{M \cdot A}{i} + \frac{M \cdot A}{i^2} + \frac{M \cdot A + P}{i^3}$$

5) Perpetuity

One Amt Recieve for life time

$PVA_\infty = \frac{A}{i}$

R = 15
A = 18000
C = yearly

$= \frac{18000}{0.15}$

$= 80000$

Calculating growing Perpetuity

Growing Because of inflation

$g = \frac{G}{C} \times 100$

$PVA_\infty = \frac{A}{i - g}$

G = 4
A = 1000
C = 1
I = 8

g = 0.04

$PVA_\infty = \frac{1000}{0.08 - 0.04} = \frac{1000 \times 100}{4}$

$= 25000$

⑥ Net Present Value

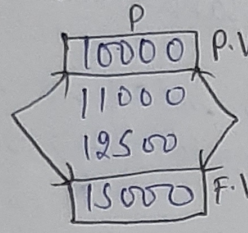
NPV = P.V of Cash inflow − P.V of Cash outflow

+ve (Accept)

−ve (Reject)

⑦ Compound Annualy Growth Rate

CAGR



$$F.V = P.V(1+i)^n$$

$$\frac{F.V}{P.V} = (1+i)^n$$

$$\Rightarrow \frac{F.V}{P.V} - 1 = i^n$$

$$\Rightarrow \left(\frac{F.V}{P.V} - 1\right)^{\frac{1}{n}} = (i^n)^{\frac{1}{n}}$$

CAGR $\Rightarrow$ $$\boxed{i = \left(\frac{F.V}{P.V} - 1\right)^{\frac{1}{n}}}$$

$$i = \left(\frac{10000}{15000} - 1\right)^{\frac{1}{3}}$$

$$i = \left(\frac{95}{150}\right)^{\frac{1}{3}}$$

(4) (i) Sum of 1^{st} n term

$S_n = a + a+d + a+2d _____ + a(n-1)d$

$\boxed{S_n = \frac{n}{2}\{2a + (n-1)d\}}$

(ii) Sum of last(l) term

$a_n = l = a(n-1)d$ - (1)

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_n = \frac{n}{2}(a + a(n-1)d)$

$\boxed{S_n = \frac{n}{2}(a+l)}$ from (1)

(iii) Sum of n term natural no.

$\boxed{\frac{n(n+1)}{2}}$

(iv) Sum of 1^{st} natural odd no.

$S_n = 1+3+5+7+9 _____ + a_n$

$= \frac{n}{2}(2 + (n-1)2)$

$S_n = \frac{n}{2}(2 + 2n - 2)$

$\boxed{S_n = n^2}$

(v) Sum of ~~Cube~~ :-

$S_n = 1^2 + 2^2 + 3^2 + 4^2 _____ + n^2$

$S_n = \frac{n(n+1)}{2} \cdot \frac{(2n+1)}{3}$

$\boxed{S_n = \frac{n(n+1)(2n+1)}{6}}$

(vi) Sum of Cube $\rightarrow S_n = 1^3 + 2^3 + 4^3 ____ + n^3$

$\boxed{S_n = n^2\frac{(n+1)^2}{4}}$ $S_n = \left(\frac{n(n+1)}{2}\right)^2$

(5.) Airthmetic Mean

AP = a, b, c

$b - a = c - b$

$b + b = c + a$

$\boxed{2b = c + a}$

$a = a - d$

$b = a$

$c = a + d$

$\boxed{b = \frac{a+b}{2}}$

$a_n = a + nd$

Sum of Airthmetic Mean

$a - d + a + a + d = n$

$a^3 = n$

3-4

Sets, Relations & functions

Sets → It is a collection of well define Obj. It is universally same.

Representation of sets are :-

1. Roster form
2. Set Builder form.

1. **Roster form** → No matter of order & Doubling in tabular form

$A = \{1, 2, 1, 3, 2, 3, 1, 2\}$

$\boxed{A = \{1, 2, 3\}}$ → Roster form.

2. **Set builder form** → $A = \{1, 2, 3, 4, 5\}$

$A = \{x : x$ ~~is a 1st SN no.~~$\}$

$A = \{x : x \leq N, n \in N, x < 6\}$

Few imp. & used in sets.

1 N = set of all natural no. $\{1, 2, 3, 4, 5, 6 \dots\}$
2 Z = set of all integers $\{-3, -2, -1, 0, 1, 2, 3 \dots\}$
3 Q = Set of all Rational no $(\frac{p}{q})$
4 R = ——— Real No. (any no.)
5 Z^+ = ——— +ve integers $\{0, 1, 2, 3 \dots\}$
6 Q^+ = ——— +ve Rational no. $\{\frac{1}{2}, \frac{2}{5} \dots\}$
7 R^+ = ——— +ve Real no. $\{0, 1 \dots\}$
8 T = ——— irrational no. $\{\sqrt{2}, \sqrt{3}, \sqrt{7} \dots\}$

Some Special Sets :-

Finite	Infinite
→ Countable	Uncontable
→ {1, 2, 3, 4, 5 ---- 100}	{1, 2, 3, 4, 5 ---- .}

(2.) Null / Empty Set (ϕ) → $A = \{\}$ $A = \phi$

(3.) Singleton sets → $A = \{2\}$ $B = \{4\}$

(4.) Disjoint sets → (Never Same)
$A = \{1, 2, 3, 4\}$
$B = \{5, 6, 7, 8\}$
A : B are disjoints

(5.) Equal Set
No. of element are Equal
B : C
$A = \{P, Q, R, S\}$
$B = \{a, b, c, d\}$
$C = \{b, a, d, c\}$

Equavalent Sets.
only one element ~~equal~~

A : C
B : C

?

IM Subset ⊂

$A = \{1, 2, 3\}$
$B = \{1, 2, 3\}$

Main Ques.

Set A will be Subset of Set B
⇓
if every / some element of A is also an element of B

Properties

1.) If $A \subset B$ & $B \subset A$ → $B = A$

2.) Every set is subset of itself $x \in A$, $x \in B$ $(A \subseteq A)$

3.) Null set has zero element
Null set is sub-set of all set
$\phi = A$
$\{\} = \{1, 2, 3\}$

(2) Proper Set ⊂ Super Set

$A \subset B$

$A = \{1, 2, 3\}$
$B = \{1, 2, 3, 4\}$

A will be proper set when $B \in A$

$B \in A$

B is a Super Set $A \subset B$ But $B \in A$

(3) Power Set → It is set of all set of given set

$A = \{1, 2, 3\}$ ~~No~~ no. of element in a set

(i) No. of total subset $= 2^n = 2^3 = 8$

$P(A) = \{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}$

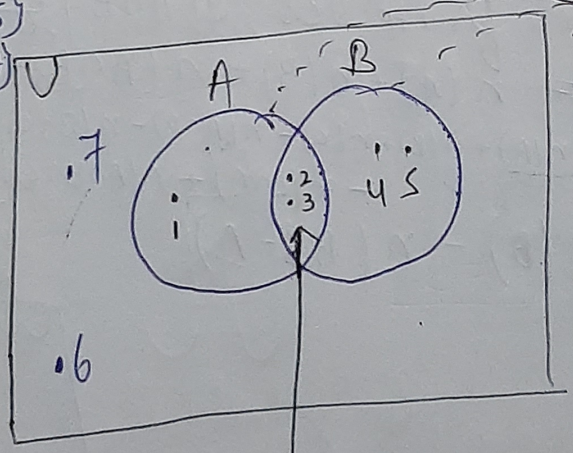
(ii) total no of Proper Set :- $2^n - 1 = 2^3 - 1 = 7$

$P(A) = \{\}, \{1\}, \{2\}, \{3\}, \{2,1\}, \{3,1\}, \{2,3\}$

(4) Universal Set :- donated by U

Venn Diagram

$A = \{1, 2, 3\}$
$B = \{2, 3, 4, 5\}$
$U = \{1, 2, 3, 4, 5, 6, 7\}$



$A \cup B$

Union of Set

$U_{AB} = \{1, 2, 3, 4, 5\}$

$\{x = x \in A, x \in B\}$

$A \cap B$

Intersection of Set

$\{2, 3\}$

Function / Relation → Subset of Cartesian product $R: A \to B$, $R \subseteq A \times B$

→ Ordered Pair

() - Open
[] - closed
[) - semiopen
(] - semi closed

→ Cartesian product of Set

$A = (a, b)$ $B = (p, q)$

$A \times B = \{(a,p), (a,q), (b,p), (b,q)\}$

$B \times A = \{(p,a) (p,b) (q,a) (q,b)\}$

→ Function → $A = (2, 3, 4, 5, 6)$
$B = (4, 9, 16, 25, 36)$

$A \cup A \times B$
$R: A \to B \to \{(2,4), (3,9) (4,16) (5,25)$

Types of Relations :-

1.) Void Relation → $R = \phi$ $A = \{1,2,3)$ $B = (2,5,7)$] Not find Sq. Root
$R = \{\}$

2.) Universal Relation :- $R: A \times B$ $A = (2, -2)$ $B = (4)$
$(2,4) (-2,4)$

3.) Reflexisive Relation :- $R: A \to A$ $A = (1,2,3)$
$\{(1,1) (2,2) (3,3)\}$ (It is equal to itself)

4.) Symetrical Relation :- $a, b \to b, a \to c, a$

$A \to (1,2,3)$
a is parallel to B

$A_1 - \{(1,1) (1,2) (2,1) (1,3)\}$
$R_2 - \{(1,1) (2,2) (3,3)\}$
$R_3 - \{(1,1) (2,1) (1,2) (3,1) (1,3)\}$

5.) Transitive Relations :- $(a,b) (b,c)$
(a,c)
$R: ab = (a,c)$

6.) Equivalence Relation :- Reflex, Symetric, Transitive } Mix

Domain
$R: A \to B$
$R = \{1, 2)$

Range
$R: A \to B$
$R = \{2, 1)$

Co-Domain
$R: A \to B$
$R = 2,$

$A = \{1,2,3)$
$B = \{2,3,5)$
$R = \{(1,2) (1,3) (2,3)\}$

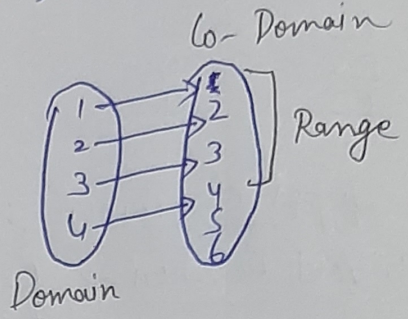
Function

$R \to F$

(i) If 1st component of ordered pair are Unique
(ii) All element of 1st Set are fix in 1st component

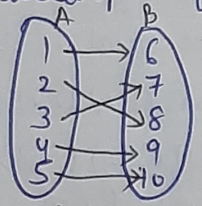
$A = \{1, 2, 3\}$ $B = \{a, b, c\}$
$P_1 = \{(1,a), (2,b), (3,c)\}$

Domain (A-Pair) $= \{1, 2, 3, 4\}$
Co-domain (B-Pair) $= \{2, 1, 3, 4, 5, 6\}$
Range $= \{1, 2, 3, 4\}$
$A \in B$



Various Type of Function :-

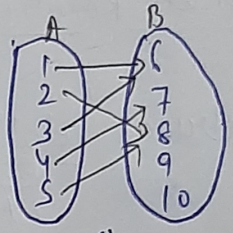
① One to One function
(ordered pair of 2nd Comp. are Repeated)



$f(A \to B)$

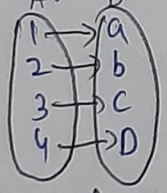
sign ($\to$)

② Many to One function



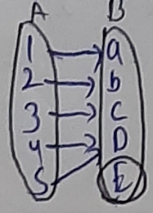
sign ($>, -$)

③ Onto function
(All image have preimage)

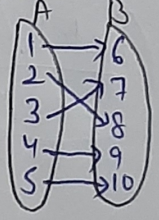


Range = Codomain

④ Onto function
(At least one of Co-domain is not pair)



⑤ Bijective Function
(Both one to one & onto function)



Range = Codomain
$f: A \to B$
$f: B \leftarrow A$

⑥ Constant function
(function is fix)

$R: A \to A$
$f(x) = c$

Unique
t
one

(7) Identity function
(only a real No.)

$f(x) = x$ Input
$f(y) = y$ output
same

$x = y$ same)

(8) Equal function
(Same input = Same Output)

$f(x) = y$
$g(x) = y$
$f(x) = g(x)$

Codomain(f) = (g)
domain(f) = domain(g)
Range(f) = Range(g)

(9) Inverse function
(If it is a bijective function)

$f: A \to B$ $f(x) = y$
$f^{-1} B \to A$ $f^{-1}(y) = x$

(10) Composite function

$f(x) = 2x = y$
$g(x) = 3x^2 = y$

$fog = f(g(x))$
$gof = g(f(x))$

Marks 2-3

Equations

Find the value of x (Root of Eqn)

- Linear
- Simultaneously
- Quadratic
- Cubic

Linear Eq in 1 Variable

$2x + 5 = 0$

Coefficient: 2 (of x) — Variable: x — Constant: 5, 0

Linear eq. in 2 Variable

$3x + 2y = 15$

$2x + 5y = 20$

Mostly use → Eliminate Method

$5(3x + 2y) = 15$

$2x + 5y = 20$

$15x + 10y = 75$

$2x + 10y = 20$

$x = 5$

$y =$

→ Cross Product Method

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

for x: $\frac{x}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$

for y: $\frac{y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$

Quadratic

$ax^2 + bx + c = 0$

$2x^2 - 3x + 2 = 0$

Mostly use → Factorization

$2x^2 - 3x + 2x + 1$

$x(x-3) - 2(x-3)$

$(x-3)(x-2)$

$x = 3$

$x = 2$

For Irrational No → Quadration

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Determinant:

$b^2 - 4ac = 0$ Real, Rational

< 0 imaginary No (N.A)

> 0 Real Distict

Sign	
+ −	−
− +	−
+ +	+
− −	+

Quadratic

$ax^2 + bx + c = 0$

Sum of roots $= \alpha + \beta = \frac{-b}{a}$ (alpha, Beta)

Product of roots $= \alpha\beta = \frac{c}{a}$

$(a+b)^2 = a^2 + b^2 + 2ab$

$a^2 + b^2 = (a+b)^2 - 2ab$

Cubic Eq

$ax^3 + bx^2 + cx + d = 0$

sum of roots $= \alpha + \beta + \gamma = \frac{-b}{a}$ (gama)

Product of Roots $= \alpha\beta\gamma = \frac{-d}{a}$

$ab + bc + ca = \gamma\alpha + \alpha\beta + \beta\gamma = \frac{c}{a}$

$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

Mark |(1-2)|

Linear Inequality

- One Varible
- 2 Variable
- Graphical Sol.

★ Linear inequality

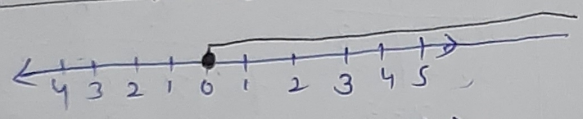
$2x + 5 \leq 0$

Sign	use
$<$ less than	
$>$ greater than	
$\leq$ less then equal to	Max. / at most
$\geq$ greater than equal to	Min / at least

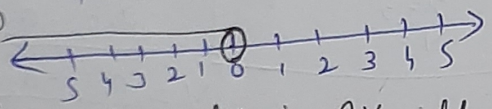
Comparision	use $\times$ & $\div$ Both Side
$x > 5$ (6, 7, 8)	sign change
$x < 5$	
$x \leq 5$	
$x \geq 5$ (5, 6, 78)	

★ Graphical inequality in One Variable

$x \geq 0$



$x \leq 0$

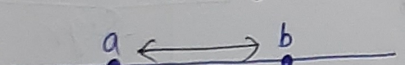


Sign

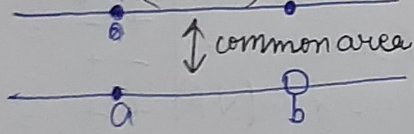
$\geq$	$\leq$	•
$>$	$<$	o

★ Linear inequality in 2 Variable :-

(i)



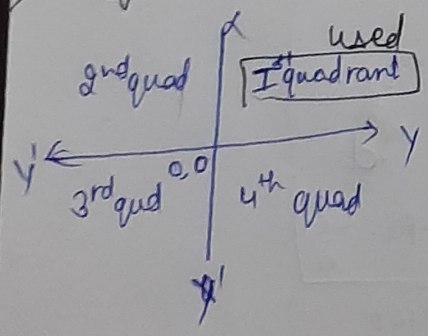
(iii)

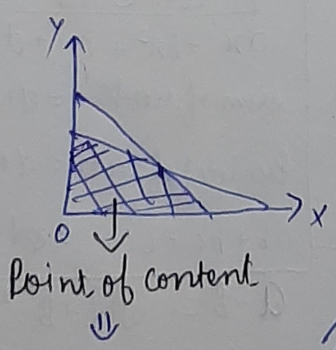


Sign

Interval space	Inequality
$[a, b]$	$a \leq x \leq b$
(a, b)	$a < x < b$
$[a, b)$	$a \leq x < b$
$(a, b]$	$a < x \leq b$

★ Graphical inequality in 2 Variable :-





Point of content

$5x + y \leq 100$

$x + y \leq 60$

x	0	60
y	100	0

$x \geq 0$

$y \geq 0$

case used

