[CA] CA-Foundation

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CHAPTER-WISE MARKS DISTRIBUTION

S. No.	Chapter	2018		2019		2020	2021			2022		2023	Average
		J	D	J	D	D	Jan	July	D	J	D	J	
1	Ratio & Proportion	1	2	1	1	3	3	3	3	2	1	-	1.81
2	Indices	1	1	2	2	1	1	1	4	2	1	3	1.72
3	Logarithm	2	1	2	2	2	1	1	2	2	2	2	1.72
4	Linear Equation	2	-	1	1	0	0	-	1	2	3	2	1.09
5	Quadratic Equation	3	2	1	3	4	3	4	2	1	2	2	2.45
6	Inequalities	1	1	3	1	1	1	1	2	1	1	2	1.36
7	Mathematics of Finance - Simple Interest	1	3	4	5	1	3	2	1	3	1	1	2.27
8	Mathematics of Finance - Compound Interest	2	11	3	7	6	7	5	4	3	8	5	5.54
9	Mathematics of Finance - Annuity	3	-	3	1	4	4	7	1	8	5	8	4.00
10	Permutations and Combinations	2	4	4	4	4	6	4	4	7	4	4	4.27
11	Sequence & Series	4	4	4	4	3	3	3	3	4	2	3	3.36
12	Sets, Function and Relation	3	4	5	2	4	3	4	3	3	3	5	3.54
13	*Calculus (Limit & Continuity)												
14	Differential Calculus	1	2	2	3	2	1	3	3	1	3	3	2.18
15	Integration	3	2	3	3	3	1	1	1	2	3	1	2.09
16	Number Series, Coding & Decoding	3	5	4	5	4	5	5	5	5	6	5	4.72
17	Direction Tests	6	5	4	6	4	3	4	8	5	5	4	4.90
18	Seating Arrangement	5	3	4	2	5	4	4	3	4	2	4	3.63
19	Blood Relations	4	4	4	4	3	4	4	5	5	7	7	4.63
20	Description of Data	4	7	5	4	7	10	8	5	8	4	5	6.09
21	Central Tendency	5	7	5	7	11	4	4	10	6	8	8	6.81
22	Measures of Dispersion	2	4	8	8	1	5	7	1	5	8	6	5.00
23	Probability	7	6	5	3	4	6	6	4	6	7	5	5.36

S. No.	Chapter	2018		2019		2020	2021			2022		2023	Average
		J	D	J	D	D	Jan	July	D	J	D	J	
24	Probability (Theoretical) Distribution	6	5	4	7	7	4	5	8	5	4	6	5.54
25	*Sampling Theory of Estimation												
26	Correlation	8	1	4	3	3	2	1	1	4	2	2	2.81
27	Regression Analysis	5	5	2	2	-	3	4	3	1	3	3	2.81
28	Index Numbers	8	3	4	3	3	3	4	5	5	5	4	4.27

Note : J: June; **D :** December

***Chapter 13 & Chapter 25 are newly added Chapters in Syllabus.**

CHAPTER-WISE COMPARISON WITH STUDY MATERIAL

No.	Name of Chapter	Study Material Chapter
1	Ratio & Proportion	1
2	Indices	1
3	Logarithm	1
4	Linear Equation	2
5	Quadratic Equation	2
6	Inequalities	3
7	Mathematics of Finance - Simple Interest	4
8	Mathematics of Finance - Compound Interest	4
9	Mathematics of Finance - Annuity	4
10	Permutations and Combinations	5
11	Sequence & Series	6
12	Sets, Function and Relation	7
13	Calculus (Limit & Continuity)	7
14	Differential Calculus	8
15	Integration	8
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17	Direction Tests	10
18	Seating Arrangement	11
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23	Probability	15
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Contents

1 CHAPTER

RATIO & PROPORTION

RATIO

The comparison of two or more things of same kind is called RATIO. If x and y are two values of same kind (in same units), then the ratio of x to y is written as x : y and read as x is to y.

In $\frac{x}{y}$

- Numerator "x" is called **1st term or Antecedent** and
- Denominator "y" is called **2nd term or Consequent.**
- Antecedent and Consequent must be of **same units**
- Ratio has no unit.

Remarks:

1. Normally a ratio is expressed in simplest form. As. 10 : 16 = 5 : 8.
2. The order of the terms in a ratio must be maintained. As 3 : 4 is not same as 4: 3.
3. Ratio exists only with quantities having same unit (kind).
4. (*i*) If x > y, then the ratio x : y is called of **greater inequality.**
   (*ii*) If x < y, then the ratio x : y is called of **lesser inequality.**
   (*iii*) If x = y, then the ratio a : b is called ratio of Equal **Equality**.
5. (*i*) **Duplicate ratio** of a : b is $a^2 : b^2$
   (*ii*) **Triplicate ratio** of a : b is $a^3 : b^3$
   (*iii*) **Sub-Duplicate ratio** of a : b is $\sqrt{a} : \sqrt{b} = a^{1/2} : b^{1/2}$
   (*iv*) **Sub-Triplicate ratio** of a : b is $\sqrt[3]{a} : \sqrt[3]{b} = a^{1/3} : b^{1/3}$

6. **Inverse ratio** of x : y is y : x.
7. (*i*) ***Commensurable :*** If the terms of the ratio are **integers, the ratio** is called commensurable. ***As.*** 3 : 2

   (*ii*) ***Incommensurable :*** If the terms of the ratio are not **integers, the ratio** is called Incommensurable.

   As. $\sqrt{3} : \sqrt{2}$ cannot be expressed in terms of integers. So, it is **Incommensurable.**
8. **Compound/Combined Ratio = Product of all ratios.**

PROPORTION

An equality of two ratios is called **Proportion.** Four quantities a, b, c, d are said to be in proportion if a : b = c : d.

It is also written as

a : b : : c : d

Here a, b, c, d are called 1st; 2nd; 3rd and 4th term of proportionals respectively

If, $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc.$

$\therefore$ Product of extreme terms = Product of middle terms.

This rule is called **Cross – Product Rule.**

In a : b : : c : d. proportion, unit of a and b should be same and that of c and d should also be same.

As. ₹ 6 : ₹ 8 = 12 toffees : 16 toffees are in proportion.

Let a, b and c are of same kind (in same units).

If $\frac{a}{b} = \frac{b}{c} \Rightarrow a : b : : b : c$

i.e. a, b, c are in proportion.

Then, this proportion is called **continuous proportion.**

Here **a** is **1st proportion** c is called **3rd proportion** and **b** is called **mean proportion.**

$\therefore\ b = \sqrt{ac}$ = GM of a & c.

Properties of Proportion

1. **Cross - Product**

   If a : b : : c : d.

   $\Rightarrow \frac{a}{b} = \frac{c}{d}$ $\therefore$ ad = bc.
2. **Invertendo**

   If a : b : : c : d.; Then its inverse

   b : a : : d : c also becomes in proportion.

   i.e. If $\frac{a}{b} = \frac{c}{d}$ Then, $\frac{b}{a} = \frac{d}{c}$.
3. **Componendo**

   If a : b : : c : d.

   Then a + b : b : : c + d : d.

   Proof : $\frac{a}{b} + 1 = \frac{c}{d} + 1 \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$.
4. **Dividendo**

   If a : b = c : d.

   Then a - b : b = c - d : d.

   Proof : $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} - 1 = \frac{c}{d} - 1$

   or $\frac{a-b}{b} = \frac{c-d}{d}$
5. **Componendo and Dividendo**

   If a : b : : c : d. ; Dividing (3) by (4)

   Then, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.
6. **Alternendo**

   If a : b : : c : d.

   Then a : c : : b : d.

   i.e. ratio of alternate terms are also in proportion.
7. **Addendo**

   If a : b = c : d = e : f = ...

Then each ratio = $\frac{\text{Sum of antecedents of all ratios}}{\text{Sum of consequents of all ratios}}$

$\therefore \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots\ldots\ldots=\frac{a+c+e+\ldots\ldots}{b+d+f+\ldots\ldots}$

8. Subtrahendo

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots\ldots\ldots$

Then each ratio

$=\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots\ldots\ldots=\frac{a-c-e-\ldots\ldots}{b-d-f-\ldots\ldots}$

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. What must be added to each term of the ratio 49 : 68 so that it becomes 3 : 4?

(*a*) 3 (*b*) 5

(*c*) 8 (*d*) 9

[June 2010]

Solution : Detail Method

Let x is added to each term

Then $\frac{49+x}{68+x}=\frac{3}{4}$

or 196 + 4x = 204 + 3x

or 4x - 3x = 204 - 196

or x = 8

:- (*c*) is Correct

Tricks : Go by Choices

1st Find $\frac{3}{4}=0.75$ (By Calculator)

Then For (*a*) $\frac{49+3}{68+3}\neq 0.75$

(*b*) $\frac{49+5}{68+5}\neq 0.75$

(*c*) $\frac{49+8}{68+8}=\frac{57}{76}=0.75$

$\therefore$ (*c*) is Correct

Q.2. The students of two classes are in the ratios 5 : 7, if 10 students left from each class, the remaining students are in the ratio of 4 : 6, then the number of students in each class was :

(*a*) 30, 40 (*b*) 25, 24

(*c*) 40, 60 (*d*) 50, 70

[June 2010]

Solution : Tricks : Go by choices:

(*a*) ; (*b*) and (*c*) are not in the ratio 5 : 7

$\therefore$ (*d*) is Correct.

Q.3. If A : B = 2:5, then (10A + 3B) : (5A+2B) is equal to

(*a*) 7:4 (*b*) 7:3

(*c*) 6:5 (*d*) 7:9

[Dec. 2010]

Solution : It A : B = 2 : 5 Then

$\frac{10A+3B}{5A+2B}=\frac{10\times2+3\times5}{5\times2+2\times5}=\frac{35}{20}=\frac{7}{4}$

= 7 : 4

(*a*) is correct

Q.4. In a film shooting, A and B received money in a certain ratio and B and C also received the money in the same ratio. If A gets ₹1,60,000 and C gets ₹2,50,000. Find the amount received by B ?

(*a*) ₹2,00,000 (*b*) ₹2,50,000

(*c*) ₹1,00,000 (*d*) ₹1,50,000

[June 2011]

Solution : Detail Method

A : B = B : C

So, $B^2 = AC$;

So, B

$=\sqrt{AC}=\sqrt{1,60,000\times2,50,000}$

$=400\times 500 = 2,00,000$

Q.5. The ratio compounded of 4:5 and sub-duplicate of " a" : 9 is 8:15. Then value of "a" is

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

[Dec. 2011]

Solution : (*c*) $\frac{4}{5}\times\sqrt{\frac{a}{9}}=\frac{8}{15}$

or $\frac{4}{5}\times\frac{\sqrt{a}}{3}=\frac{8}{15}$

$\therefore \sqrt{a}=2\Rightarrow a=4$

$\therefore$ (*c*) is correct

Q.6. If X varies inversely as square of Y and given that Y =2 for X = 1, then the value of X for Y = 6 will be

(*a*) 3 (*b*) 9

(*c*) 1/3 (*d*) 6

[Dec. 2011]

Solution : (*d*) is correct

$x\propto\frac{1}{y^2}\Rightarrow x=K.\frac{1}{y^2}\Rightarrow x=\frac{k}{y^2}$; where

k = proportional constant

When x = 1 Then y = 2

$\therefore 1=\frac{k}{2^2}\Rightarrow k=4\ \therefore x=\frac{4}{y^2}$

When y = 6, Then $x=\frac{4}{6^2}=\frac{1}{9}$

$\therefore x=\frac{1}{9}$

Q.7. Which of the numbers are not in proportion ?

(*a*) 6,8,5,7 (*b*) 7,14,6,12

(*c*) 18,27,12,18 (*d*) 8,6,12,9

[June 2012]

Solution : (*a*) Go by choices

For (*a*) $\frac{6}{8}=\frac{3}{4}\neq\frac{5}{7}$

$\therefore$ (*a*) is not in proportion

Q.8. Find two numbers such that mean proportional between them is 18 and third proportional between them is 144

(*a*) 9 ; 36 (*b*) 8 ; 32

(*c*) 7 ; 28 (*d*) 6 ; 14

[Dec. 2012]

Solution : (*a*) is correct

Tricks : Go by choices

For (*a*) Mean Proportional of 9 and 36

$= \sqrt{9\times36} = 18$

It satisfies 1st condition.

If 144 is its 3rd condition.

$36^2 = 9\times144$

It also satisfies the 2nd Condition.

Q.9. Triplicate ratio of 4 : 5 is

(*a*) 125 : 64 (*b*) 16 : 25

(*c*) 64 : 125 (*d*) 120 : 46

[June 2013]

Solution : (*c*) Triplicate ratio of 4:5

$= 4^3 : 5^3 = 64 : 125$

Q.10. The mean proportion between 24 and 54 is _____

(*a*) 33 (*b*) 34

(*c*) 35 (*d*) 36

[June 2013]

Solution : (*d*) Mean - Proportion

$= \sqrt{24\times54} = 36$

Q.11. The ratio of numbers is 1 : 2 : 3 and sum of their squares is 504 then the numbers are

(*a*) 6,12,18 (*b*) 3,6,9

(*c*) 4,8,12 (*d*) 5,10,15

[Dec. 2013]

Solution : (*a*) is correct

Tricks : Go by choices

6:12:18 = 1:2:3 (True)

and $6^2 + 12^2 + 18^2 = 504$ (True)

Q.12. If P is 25% less than Q and R is 20% higher than Q the Ratio of R and P

(*a*) 5:8 (*b*) 8:5

(*c*) 5:3 (*d*) 3:5

[Dec. 2013]

Solution : (*b*) is correct

Let Q = 100, So, P = 100-025 = 75

& R = 100 + 20 = 120

$\frac{R}{P} = \frac{120}{75} = \frac{8}{5}$

Q.13. A person has assets worth ₹1,48,200. He wish to divide it amongst his wife, son and daughter in the ratio 3 : 2 : 1 respectively. From this assets the share of his son will be

(*a*) ₹ 74,100 (*b*) ₹ 37,050

(*c*) ₹ 49,400 (*d*) ₹ 24,700

[June 2014]

Solution : (*c*) is correct

Share of son $= \frac{2}{3+2+1}\times1,48,200$

= ₹49,400

Q.14. If x : y = 2 : 3 then (5x+2y) : (3x - y) =

(*a*) 19 : 3 (*b*) 16 : 3

(*c*) 7 : 2 (*d*) 7 : 3

[June 2014]

Solution : (*b*) is correct

$\frac{5x+2y}{3x-y} = \frac{5\times2+2\times3}{3\times2-3} = \frac{16}{3}$

Q.15. The first, second and third month salaries of a person are in the ratio 2:4:5. The difference between the product of the salaries of first 2 months & last 2 months is ₹4,80,00,000. Find the salary of the second month

(*a*) ₹ 4,000 (*b*) ₹ 6,000

(*c*) ₹ 12,000 (*d*) ₹ 8,000

[Dec. 2014]

Solution : (*d*) is correct

Let x is common in the ratio.

$1^{st}, 2^{nd}$ *and* 3^{rd} month salaries of a person = 2x ; 4x ; 5x

:- From Qts.

$4x\times5x - 2x\times4x = 4,80,00,000.$

$or, 12x^2 = 4,80,00,000.$

$or, x^2 = 4000000$

$x = 2000.$

$\therefore 2^{nd}$ month salary $= 4x = 4\times2000$

= ₹ 8000

Q.16. $15(2p^2 - q^2) = 7pq$, where p, q are positive then p : q

(*a*) 5:6 (*b*) 5:7

(*c*) 3:5 (*d*) 3:7

[June 2015]

Solution : (*a*) is correct

$15(2p^2 - q^2) = 7pq$

Tricks : Go by choices

For (a) put p = 5; q = 6 we get

$15\left[2\times5^2 - 6^2\right] = 3\times5\times6$

or $15\times14 = 210$

or $210 = 210$

Q.17. If one type of rice of cost ₹13.84 is mixed with another type of rice of cost ₹15.54, the mixture is sold at ₹17.60 with a profit of 14.6% on selling price then in which proportion the two types of rice mixed ?

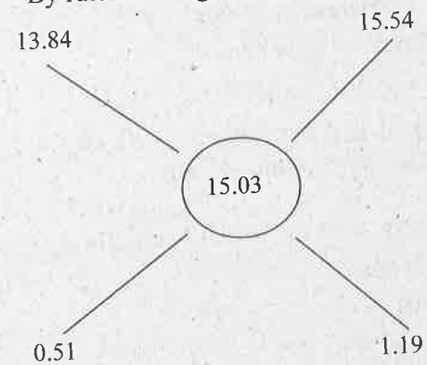
(*a*) 3:7 (*b*) 5:7

(*c*) 7:9 (*d*) 9:1

[June 2015]

Solution : Cost of mixture per kg = 17.60 - 14.6% = 15.0304 = 15.03 (approx.)

By rules of Alligation



51:119 = 3:7

Go by choices

(*a*) is correct (approx)

Q.18. Find the ratio of third proportional of 12 ; 30 and mean proportional of 9 ; 25 :

(*a*) 7 : 2 (*b*) 5 : 1

(*c*) 9 : 4 (*d*) None of these

[Dec. 2015]

Solution : 3rd proportional $= \frac{30^2}{12} = 75$

Mean Proportional $= \sqrt{9\times25} = 15$

Ratio $= \frac{75}{15} = 5:1$

(*b*) is correct

Q.19. What must be added to each of the numbers 10, 18, 22, 38 to make them proportional:

(a) 5 (b) 2
(c) 3 (d) 9

[Dec. 2015]

Solution : (b) is correct

let x be added.

$$\therefore \frac{10+x}{18+x}=\frac{22+x}{38+x}$$

Tricks: Go by choices.

$\therefore x=2$ satisfies it.

Q.20. x, y, z together starts a business, if x invests 3 times as much as y invests and y invests two third of what z invests, then the ratio of capitals of x, y, z is _____

(a) 3 : 9 : 2 (b) 6 : 3 : 2
(c) 3 : 6 : 2 (d) 6 : 2 : 3

[June 2016]

Solution : (d)

Detail Method

$$x=3y \Rightarrow \frac{x}{y}=\frac{3}{1} \Rightarrow x:y=3:1$$

and $y=\frac{2}{3}z \Rightarrow y:z=2:3$

$x:y = 3:1$] × 2
$y:z = 2:3$
$\Rightarrow x:y = 6:2$
$y:z = 2:3$
$x:y:z = 6:2:3$

(d) is correct

Tricks: Go by choices

$6=3\times2$ and $2=3\times\frac{2}{3}$

Q.21. A bag contains 23 number of coins in the form of 1 rupee, 2 rupee and 5 rupee coins. The total sum of the coins is ₹43. The ratio between 1 rupee and 2 rupees coins is 3 : 2. Then the number of 1 rupee coins is

(a) 12 (b) 8
(c) 10 (d) 16

[Dec. 2016]

Solution : (a)

Tricks : Go by choices

Let option (a) is correct.

Let x is common in the ratio.

So, ₹1 coins = 3x = 12 ; So, x = 4

No. of ₹ 2 coins = 2 × 4 = 8

Hence No. of coins of ₹ 5 coins = 23-12 - 8 = 3

Total money = 12 × 1 + 8 × 2 + 3 × 5 = ₹ 43 Satisfied.

So (a) is correct

Detail Method:

Let x is common in the ratio.

$\therefore$ No. of ₹ 1 coins & ₹ 2 coins are $3x$; $2x$

$\therefore$ No. of ₹ 5 coins = $23-3x-2x = 23-5x$

Total Sum

$= 3x\times1+2x\times2+(23-5x)\times5=43$

$7x-25x+115=43$

or $115-43=18x$

or $72=18x$

or $x=4$

$\therefore$ No. of ₹ 1 coins

$= 3x$

$= 3\times4$

$= 12$

Q.22. If a : b = 2 : 3, b : c = 4 : 5, c : d = 6 : 7 then a : d is _____

(a) 24 : 35 (b) 8 : 15
(c) 16 : 35 (d) 7 : 15

[June 2017]

Solution : Option (c) is correct.

Multiply all ratios.

$$\frac{a}{b}\cdot\frac{b}{c}\cdot\frac{c}{d}=\frac{a}{d}$$

$$=\frac{2}{3}\times\frac{4}{5}\times\frac{6}{7}=\frac{16}{35}$$

Q.23. The ratio of the number of five rupee coins to number of ten rupee coins is 8: 15. If the total value of five rupee coins is 360, then the no. of ten rupee coins is _____

(a) 72 (b) 60
(c) 150 (d) 135

[Dec. 2017]

Solution : Option (d) is correct.

Total No. of ₹5 coins = 360/5 = 72

Let x is common in the ratio.

So, ₹5 coins = 8x = 72 ; So, x = 9

No. of ₹ 10 coins = 15×9 = 135

Q.24. If $\frac{1}{2},\frac{1}{3},\frac{1}{5},\frac{1}{x}$ are in proportion then x =

(a) $\frac{15}{2}$ (b) $\frac{3}{15}$
(c) $\frac{2}{15}$ (d) $\frac{1}{15}$

[Dec. 2017]

Solution : Option (a) is correct.

Product of middle two terms

= Product of extremes

So, $\frac{1}{2x}=\frac{1}{15}$; x = 15/2

Q.25. If (a + b) : (b + c) : (c + a) = 7 : 8 : 9 and a + b + c = 18 then a : b : c =

(a) 5 : 4 : 3 (b) 3 : 4 : 5
(c) 4 : 3 : 5 (d) 4 : 5 : 3

[June 2018]

Solution : (c) is correct.

Tricks: Go by choices.

(c) Let a : b : c = 4 : 3 : 5

It is in ratio. So, it should must **satisfy given ratio** (a + b) : (b + c) : (c + a) = 7 : 8 : 9 *i.e.* (4 + 3) : (3 + 5) : (5 + 4) = 7 : 8 : 9 (True) Avoid 2nd condition.

In detail method it will take too much time.

Q.26. If p : q is the sub-duplicate ratio of $p-x^2 : q-x^2$, then x^2 is :

(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$
(c) $\frac{qp}{p-q}$ (d) None

[May 2018]

Solution : Detail Method:

$$\frac{\sqrt{p-x^2}}{\sqrt{q-x^2}}=\frac{p}{q}$$

Squaring on both side; we get

$$\frac{p-x^2}{q-x^2}=\frac{p^2}{q^2}$$

or $pq^2 - q^2x^2 = p^2q - p^2x^2$

or $p^2x^2 - q^2x^2 = p^2q - pq^2$

or $x^2(p^2 - q^2) = Pq(P - q)$

or x^2 (p + q)(p - q) = pq (p - q)

or $x^2 = \frac{pq}{p+q}$

$\therefore$ (*d*) is correct

Tricks : Go by choices.

Q.27. The mean proportional between 24 and 54 is :

(*a*) 33 (*b*) 34

(*c*) 35 (*d*) 36

Solution : Formula

Mean Proportion of a & b $= \sqrt{ab}$

(*d*) $= \sqrt{24 \times 54} = 36$

[May 2018]

Q.28. $\frac{3x-2}{5x+6}$ is the duplicate ratio of $\frac{2}{3}$ then find the value of x :

(*a*) 6 (*b*) 2

(*c*) 5 (*d*) 9

[Nov. 2018]

Solution : (*a*)

Given $\frac{3x-2}{5x+6} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

Tricks : Go by choices

for option (*a*) putting x = 6 in LHS; we get

$\frac{3\times2-2}{5\times2+6} = \frac{4}{9}$ (R.H.S)

$\therefore$ (*a*) is correct.

Q.29. If x : y : z = 7 : 4 : 11 then $\frac{x+y+z}{z}$ is:

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

[Nov. 2018]

Solution : (*a*)

$\frac{x+y+z}{z} = \frac{7+4+11}{11} = 2$

Q.30. If the ratio of two numbers is 7 : 11. If 7 is added to each number then the new ratio will be 2 : 3 then the numbers are.

(*a*) 49, 77 (*b*) 42, 45

(*c*) 43, 42 (*d*) 39, 40

[June 2019]

Solution : Tricks:- GBC (Go by Choices)

(*a*) $\begin{bmatrix} 49 \div 7 = 7 \\ 77 \div 11 = 7 \end{bmatrix}$ both must be equal.

Here it is correct.

Now:

$\frac{49+7}{77+7} = \frac{56}{84} = \frac{2}{3}$

Divide 56 by numerator (2) and 84 by Denominator (3) we get same value "28"

Note:- No need to solve ; only check.

By Calculator.

Q.31. The two numbers are in ratio 3 : 4. The difference between their squares is 28. Find the greater number.

(*a*) 12 (*b*) 8

(*c*) 16 (*d*) 10

[Nov. 2019]

Solution : (*b*)

Let x is common in the ratio.

So, Numbers are 3*x* & 4*x*.

From Qts.

$(4x)^2 - (3x)^2 = 28$

or; $16x^2 - 9x^2 = 28$

or $7x^2 = 28 \Rightarrow x^2 = 4$

$\therefore x = 2$

$\therefore$ Greater No. = $4x = 4 \times 2 = 8$

Q.32. The ratio of No. of boys and the No. of girls in a school is found to be 15 : 32. How many boys and equal no. of girls should be added to bring the ratio to 2/3 ?

(*a*) 20 (*b*) 19

(*c*) 23 (*d*) 27

[Dec. 2020]

Solution : Tricks GBC

(*b*) $\frac{15+19}{32+19} = \frac{34}{51} = \frac{2}{3}$ (True)

$\therefore$ (*b*) is correct.

Detail Method

Let *x* is added.

$\therefore \frac{15+x}{32+x} = \frac{2}{3}$

or $45 + 3x = 64 + 2x$

or $3x - 2x = 64 - 45$

or $x = 19$

(*b*) is correct.

Q.33. If a : b = 9:4 then =

$\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = ?$

(*a*) 2/3 (*b*) 3/2

(*c*) 6/13 (*d*) 13/6

[Dec. 2020]

Solution : $\because$ a : b = 9 : 4

$\therefore \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = \sqrt{\frac{9}{4}} + \sqrt{\frac{4}{9}}$

$= \frac{3}{2} + \frac{2}{3} = \frac{9+4}{6} = \frac{13}{6}$

$\therefore$ (*d*) is correct.

Q.34. If a : b = 3 : 7 then 3a + 2b : 4a + 5b = ?

(*a*) 27 : 43 (*b*) 23 : 47

(*c*) 24 : 51 (*d*) 29 : 53

[Dec. 2020]

Solution : Putting $a = 3$; $b = 7$

We get

$\frac{3a+2b}{4a+5b} = \frac{3\times3+2\times7}{4\times3+5\times7}$

$= \frac{23}{47}$

$\therefore$ (*b*) is correct

Q.35. In a certain business A and B received profit in a certain ratio; B and C received profits in the same ratio. If A gets ₹ 1600 and C gets ₹ 2500 then how much does B get?

(*a*) ₹ 2,000 (*b*) ₹ 2,500

(*c*) ₹ 1,000 (*d*) ₹ 1,500

[Jan. 2021]

Solution : (*a*) is given

$$\frac{A}{B}=\frac{B}{C}$$

or, $B^2 = AC$

or $B = \sqrt{AC}$

$= \sqrt{1600 \times 2500}$

$= 40 \times 50$

$= ₹\ 2000$

Q.36. The ratio of two quantities is 15 : 17. If the consequent of its inverse ratio is 15, then the antecedent is;

(*a*) 15 (*b*) $\sqrt{15}$

(*c*) 17 (*d*) 14

[Jan. 2021]

Solution : (*c*) is correct

After inversing the ratio 15 : 17

We get $\frac{17}{15}$ i.e. 15 as consequent

Clearly its Antecedent = 17

Q.37. The salaries of A, B and C are in the ratio 2 : 3 : 5. If increments of 15%, 10% and 20% are allowed respectively to their salary, then what will be the new ratio of their salaries?

(*a*) 3 : 3 : 10

(*b*) 10 : 11 : 20

(*c*) 23 : 33 : 60

(*d*) Cannot be determined

[Jan. 2021 & July 2021]

Solution : (*c*) is correct

Using Calculator.

New ratio

$= A : B : C$

$= (2 + 15\%) : (3 + 10\%) : (5 + 20\%)$

$= 2.3 : 3.3 : 6.0$

[**Note:** On calculator type 2 + 15% button; we get 2.3. Similarly do for rest terms]

Multiplying each term by 10; we get

$= 23 : 33 : 60$

Q.38. A vessel contained a solution of acid and water in which water was 64%. Four litres of the solution were taken out of the vessel and the same quantity of water was added. If the resulting solution contains 30% acid, the quantity (in litres) of the solution, in the beginning in the vessel, was

(*a*) 12 (*b*) 36

(*c*) 24 (*d*) 27

[July 2021]

Solution: (*c*) is correct

Let quantity of mixture = x

Acid = $x \times (100 - 64)\% = 0{\cdot}36x$

Quantity of Acid in 4 litre mixture

$= 4 \times 36\% = 1{\cdot}44$

After reduction of 4 litres of mixture; remaining Acid = $0{\cdot}36x - 1{\cdot}44$

After adding 4 litre water, Total quantity

$= x - 4 + 4 = x$ litre

$\therefore$ Percentage of Acid

$$= \frac{0.36x - 1.44}{x} \times 100 = 30$$

or,

$0{\cdot}36x - 1{\cdot}44 = 0{\cdot}3x$

or $0{\cdot}36x - 0{\cdot}30x = 1{\cdot}44$

or $0{\cdot}06x = 1{\cdot}44$

or $x = \frac{1.44}{0.06} = 24$ litres

Q.39. If A : B = 5 : 3, B : C = 6 : 7 and C : D = 14 : 9, then the value of A: B : C : D

(*a*) 20 : 14 : 12 : 9

(*b*) 20 : 9 : 12 : 14

(*c*) 20 : 9 : 14 : 12

(*d*) 20 : 12 : 14 : 9

[July 2021]

Solution : (*d*) is correct

Tricks : Go by Choices (GBC)

Checking for all ratios, we get

option (a) 20 : 14 : 12 : 9

$$\frac{20}{14} \neq \frac{5}{3} = \frac{A}{B}$$

So (*a*) False

Doing in this way for rest options

At the end

For option (*d*) 20 : 12 : 14 : 9

A : B : C : D

$\frac{20}{12} = \frac{5}{3} = \frac{A}{B}$ (True)

$\frac{12}{14} = \frac{6}{7} = \frac{B}{C}$ (Also True)

$\frac{14}{9} = \frac{C}{D}$ (True)

Hence (*d*) is correct

Q.40. Incomes of R and S are in the ratio 7 : 9 and their expenditures are in the ratio 4 : 5. Their total expenditure is equal to income of R. What is the ratio of their savings?

(*a*) 23 : 36 (*b*) 28 : 41

(*c*) 31 : 43 (*d*) 35 : 46

[Dec. 2021]

Solution : (*d*)

Let x is common in the ratio of incomes

$$\therefore \frac{R}{S} = \frac{7x}{9x}$$

and y is common in the ratio of expenditure $= \frac{4y}{5y}$

From Question

Total expenditure = Income of R

$\Rightarrow 4y + 5y = 7x$

or $9y = 7x \Rightarrow y = \frac{7}{9}x$

Ratio of saving $= \frac{7x - 4y}{9x - 5y}$

$$= \frac{7x - 4 \times \frac{7}{9}x}{9x - 5 \times \frac{7}{9}x} = \frac{\left(\frac{63 - 28}{9}\right)x}{\left(\frac{81 - 35}{9}\right)x}$$

$= \frac{35}{46}$

Q.41. A bag has 105 coins containing some 50 paise, and 25 paise coins. The ratio of the number of these coins is 4 : 3. The total value (in ₹) in the bag is

(*a*) 43.25 (*b*) 41.25

(*c*) 39.25 (*d*) 35.25

[Dec. 2021]

Solution : (*b*)

Total Numbers of 50 paise coins

$= \frac{105}{4+3} \times 4 = 60$

and 25 paise coins $= \frac{105}{4+3} \times 3 = 45$

Total Value $= 60 \times 0.50 + 45 \times 0.25$

$= 30 + 11.25$

$= ₹ 41.25$

Q.42. In a department, the number of males and females are in the ratio 3 : 2. If two males and 5 females join department, then the ratio becomes 1 : 1, initially the number of female in the department is

(*a*) 9 (*b*) 6

(*c*) 3 (*d*) 8

[Dec. 2021]

Solution : (*b*)

Let x is common in the ratio

$\therefore \frac{\text{Males}}{\text{Females}} = \frac{3x}{2x}$

From Questions

Now $\frac{3x+2}{2x+5} = \frac{1}{1} \Rightarrow 3x+2 = 2x+5$

or $x = 5 - 2 = 3$

$\therefore$ Initially No. of Females $= 2x = 2 \times 3 = 6$

Q.43. A bag contains 25 paise, 10 paise and 5 paise coins in the ratio 3:2:1. The total value is ₹ 40, the number of 5 paise coins is

(*a*) 45 (*b*) 48

(*c*) 40 (*d*) 20

[June 2022]

Solution : Tricks:

Common Factor $= \frac{\text{Total Value}}{\text{Total Value of ratio terms}}$

$= \frac{40}{3(0.25)+2(0.10)+1(0.05)} = 40$

Calculator Tricks

Type 3×0.25 = button press

2×0.10 = button press

1×0.05 = button press

Then Press GT button then M+ button then type 40 ÷ button

Then MRC button. We get 40

$\therefore$ No. of 5 paise coins $= 1 \times 40 = 40$

(*c*) is correct

Q.44. If x:y = 4:6 and 2: x = 1: 2 the y = ____

(*a*) 4 (*b*) 6

(*c*) $\frac{1}{3}$ (*d*) $\frac{3}{2}$

[June 2022]

Solution :

$\because \frac{2}{x} = \frac{1}{2} \Rightarrow x = 4$

$\because \frac{x}{y} = \frac{4}{6}$

or $\frac{\cancel{4}}{y} = \frac{\cancel{4}}{6} \therefore y = 6$

(*b*) is correct

Q.45. A sum of money is to be distributed among A, B, C, D in the proportion of 5:2:4:3. If C gets ₹ 1000 more than D, what is B's share?

(*a*) 2000 (*b*) 1500

(*c*) 2500 (*d*) 1000

[Dec. 2022]

Solution :

Let x is common in the ratio

$\therefore$ A : B : C : D $= 5x : 2x : 4x : 3x$

Given

$C - D = 4x - 3x = 1000$

So, $x = 1000$

$\therefore$ B's share $= 2x = 2 \times 1000$

$= ₹ 2000$

$\therefore$ (*a*) is correct

2 CHAPTER

INDICES

If a number x is multiplied 5 times written as.

x. x. x. x. x. = x^5.

Here "x" is called BASE and 5 is called Power or INDEX.

Some Related Formulae.

1. $a^m = a \times a \times a \times$ ..... to m times.
2. $a^o = 1$ where $a \neq 0; \infty$
3. $a^{-1} = \frac{1}{a}$.
4. $a^{-m} = \frac{1}{a^m}$
5. (*i*) $a^m \times a^n = a^{m+n}$
   (*ii*) $a^m \times a^n \times a^r \times$ ..... $= a^{m+n+r+}$ .....
6. (*i*) $\frac{a^m}{a^n} = a^{m-n}$.
   (*ii*) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$
7. (*i*) $\left(a^m\right)^n = a^{mn}$
   (*ii*) $a^{m^n} \neq a^{mn}$
8. (*i*) If $a^m = b^m$ Then a = b
   (*ii*) If $a^m = a^n$ Then m = n
9. (*i*) $\sqrt[m]{a^n} = a^{\frac{n}{m}}$

(*ii*) $\sqrt{a} = a^{\frac{1}{2}}$

(*iii*) $\sqrt[3]{a} = = a^{\frac{1}{3}}$

10. (*i*) If $a^m = k \Rightarrow a = k^{1/m}$

(*ii*) If $a^m = k^n \Rightarrow a = k^{n/m}$

(*iii*) If $a^{1/m} = k \Rightarrow a = k^m$

(*iv*) If $a^{1/m} = k^n \Rightarrow a = k^{mn}$

11. (*i*) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(*ii*) $(ab....)^m = a^m . b^m$ .....

12. (*i*) $\sqrt[m]{ab.....} = \sqrt[m]{a} \cdot \sqrt[m]{b}$ .....

(*ii*) $\sqrt{ab} = \sqrt{a} . \sqrt{b}$.

13. $\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$

14. If $a^b = b^a$ Then

Either (*i*) a = b

or (*ii*) If a = 2

Then b = 4

or (*iii*) If a = 4

Then b = 2

15. If a > 1 and x < y

Then $a^x < a^y$

CALCULATOR TRICKS

1. For a^{+m}

Steps (*i*) Type a

(*ii*) Press x button.

(*iii*) Continue pressing "=" button (Power - 1 = m - 1) times.

Example

$4^{+5} = 1024$

Type 4 Then Press × button Then "=" button 4 times; we get the result.

2. For a^{-m}

Steps (*i*) Type "a"

(*ii*) Press ÷ button.

(*iii*) Continue pressing "=" button (Power = m) times

Example

$2^{-4} = 0.0625$

Type base 2 then press ÷ button then continue pressing = button 4 times.

We get the required result.

3. For $(1.05)^{60}$

1st Method:

Type base 1.05

Then press "×" button 60 - 1 = 59 times.

IInd Method:

$(1.05)^{60} = [(1.05)^{12}]^5$

1st work for 1.05^{12}

Then "×" button and work for power 5.

Work As. :

Type 1.05 then press × button

Then press = button 12-1 = 11 times

Then press × button and continue

Pressing = button 5 - 1 = 4 times.

We get the required result.

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. If $2^x - 2^{x-1} = 4$, then the value of x^x is :

(*a*) 2 (*b*) 1 (*c*) 64 (*d*) 27

[Feb. 2008, June 2010]

Solution : $2^x - 2^{x-1} = 4$

or $2^{x-1}(2-1) = 4$

or $2^{x-1} \times 1 = 2^2$

or $2^{x-1} = 2^2$

:- x - 1 = 2

:- x = 3 $\therefore x^x = 3^3 = 27$

:- (*d*) is Correct

Tricks : Go by choices

For (*d*) $27 = 3^3 = x^x$

x = 3

Put x = 3 in $2^x - 2^{x-1} = 4$

It satisfies it

(*d*) is correct

Q.2. If $x = y^a$, $y = z^b$ and $z = x^c$ then abc is :

(*a*) 2 (*b*) 1 (*c*) 3 (*d*) 4

[June 2008]

Solution : $x = y^a = (z^b)^a = z^{ab}$

$\Rightarrow x = (x^c)^{ab} = x^{abc}$

$\therefore x' = x^{abc}$

$\therefore$ abc = 1

:- (*b*) is correct

Q.3. If $x = 3^{1/3} + 3^{-1/3}$ then find value of $3x^3 - 9x$

(*a*) 3 (*b*) 9 (*c*) 12 (*d*) 10

[June 2009]

Solution : Detail Method

It $x = 3^{1/3} + 3^{-1/3}$ _____(I)

Cubing on both sides; we get $x^3 = (3^{1/3})^3 + (3^{-1/3})^3 + 3.3^{1/3}.3^{-1/3}(3^{1/3} + 3^{-1/3})$

$= 3 + 3^{-1} + 3 \times 1 \times x$ (From (I)

or $x^3 = 3 + \frac{1}{3} + 3x$

or $x^3 - 3x = \frac{9+1}{3}$

or $3x^2 - 9x = 10$

$\therefore$ (*d*) is correct

Q.4. Find the value of: $\left[1-\left\{1-\left(1-x^2\right)^{-1}\right\}^{-1}\right]^{-\frac{1}{2}}$ is

(*a*) $1/x$ (*b*) x (*c*) 1 (*d*) none of these

[June 2009]

Solution : $\left[1-\{1-(1-x^2)^{-1}\}^{-1}\right]^{-\frac{1}{2}} \left[1-\left\{1-\frac{1}{1-x^2}\right\}^{-1}\right]^{-1/2}$

$= \left[1-\left\{\frac{1-x^2-1}{1-x^2}\right\}^{-1}\right]^{-1/2}$

$= \left[1-\left\{\frac{-x^2}{1-x^2}\right\}^{-1}\right]^{-1/2}$

$= \left[1-\frac{1-x^2}{-x^2}\right]^{-1/2} = \left[1+\frac{1-x^2}{x^2}\right]^{-\frac{1}{2}}$

$= \left[\frac{x^2+1-x^2}{x^2}\right]^{-1/2}$

$= \left(\frac{1}{x^2}\right)^{-1/2} = (x^{-2})^{-\frac{1}{2}} = x$

$\therefore$ (*b*) is correct

Q.5. $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

(*a*) 1/2 (*b*) 3/2 (*c*) 2/3 (*d*) 1/3

[Dec. 2009]

Solution : $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

$= \frac{2^n(2^0 + 2^{-1})}{2^n(2^1 - 2^0)} = \frac{1+\frac{1}{2}}{2-1} = \frac{3}{2}$

$\therefore$ (*b*) is correct

Tricks : Put n = 1

Q.6. If $2^x \times 3^y \times 5^z = 360$. Then what is the value of x, y, z. ?

(*a*) 3,2,1 (*b*) 1,2,3 (*c*) 2,3,1 (*d*) 1,3,2

[Dec. 2009]

Solution : If $2^x \times 3^y \times 5 = 360$

$\therefore 2^x \times 3^y \times 5 = 2^3 \times 3^2 \times 5^1$

Comparing it; we get

x = 3; y = 2 ; Z = 1 ;

$\therefore$ (*a*) is correct

Tricks : Go by choices.

For (*a*) x = 3; y = 2; Z = 1

LHS = $2^3 \times 3^2 \times 5^1 = 360$ = RHS.

$\therefore$ (*a*) is correct

Q.7. The recurring decimal 2.7777 ..... can be expressed as

(*a*) 24/9 (*b*) 22/9 (*c*) 26/9 (*d*) 25/9

[Dec. 2010]

Solution : Tricks : Go by choices.

By calculator

(*a*) $\frac{24}{9} = 2.666..... \neq 2.777.....$

(*b*) $\frac{22}{9} = 2.444..... \neq 2.777.....$

(*c*) $\frac{26}{9} = 2.888..... \neq 2.77.....$

(*d*) $\frac{25}{9} = 2.777.....$

$\therefore$ (*d*) is correct

Q.8. The value of $\frac{(3^{n+1}+3^n)}{(3^{n+3}-3^{n+1})}$ is equal to

(*a*) 1/5 (*b*) 1/6 (*c*) 1/4 (*d*) 1/9

[June 2012]

Solution : (*b*) **Tricks**

Put n = 0

$\frac{3+3^0}{3^3-3} = \frac{3+1}{27-3} = \frac{4}{24} = \frac{1}{6}$

Detail Method

$\frac{3^{n+1}+3^n}{3^{n+3}-3^{n+1}} = \frac{3^n(3+1)}{3^n(3^3-3)}$

$= \frac{4}{24} = \frac{1}{6}$

Q.9. Find the value of X, if $x.(x)^{1/3} = (x^{1/3})^x$

(*a*) 3 (*b*) 4 (*c*) 2 (*d*) 6

[Dec. 2012]

Solution : (*b*) is correct

$x.x^{\frac{1}{3}} = x^{\frac{x}{3}}$

or $x^{1+\frac{1}{3}} = x^{x/3}$ $\therefore 1+\frac{1}{3} = \frac{x}{3}$

or $\frac{4}{3} = \frac{x}{3} \therefore x = 4$

(*b*) is correct

Q.10. If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$; then find the value of $\left[\frac{a+b+c}{3}\right]^3 =$

(*a*) 9abc (*b*) $\frac{1}{9abc}$ (*c*) abc (*d*) $\frac{1}{abc}$

[Dec. 2013, June 2023]

Solution : (*c*) is correct

Detail

Let $x = \sqrt[3]{a}$; $y = \sqrt[3]{b}$; $z = \sqrt[3]{c}$

$\therefore$ $x^3 = a$; $y^3 = b$; $z^3 = c$

$\therefore$ $x + y = 2 = 0$

Formula

$x^3 + y^3 + z^3 - 3xyz = 0$

If $x + y + z = 0$

$x^3 + y^3 + z^3 = 3xyz$

or : $a + b + c = 3 \cdot \sqrt[3]{a} \cdot \sqrt[3]{b} \cdot \sqrt[3]{c}$

or; $\frac{a+b+c}{3} = \sqrt[3]{abc}$

Cubing on both sides; we get

$\left(\frac{a+b+c}{3}\right)^3 = \left(\sqrt[3]{abc}\right)^3 = abc$

$\therefore$ (*c*) is correct

Tricks : Let a = -1; b = -1 and c = 8, because $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = \sqrt[3]{-1} + \sqrt[3]{-1} + \sqrt[3]{8}$

$= -1 - 1 + 2 = 0$ (R.H.S) $\therefore \left[\frac{a+b+c}{3}\right]^3 = \left[\frac{-1-1+8}{3}\right]^3 = (2)^3 = 8$

$= (-1).(-1).(8) = abc$

$\therefore$ (*c*) is correct

Q.11. The value of

$$\left(\frac{y^a}{y^b}\right)^{a^2+ab+b^2} \left(\frac{y^b}{y^c}\right)^{b^2+bc+c^2} \left(\frac{y^c}{y^a}\right)^{c^2+ca+a^2} =$$

(*a*) y (*b*) -1 (*c*) 1 (*d*) None

[June 2014]

Solution : (*c*) is correct

Tricks : It is in Cyclic order

Q.12. If $p^x = q, q^y = r, r^z = p^6$, then the value of xyz is .....

(*a*) 0 (*b*) 1 (*c*) 3 (*d*) 6

[June 2015]

Solution : $q^y = r \Rightarrow (p^x)^y = r \Rightarrow r = p^{xy}$

Now $r^z = p^6 \Rightarrow (p^{xy})^z = p^6 \Rightarrow p^{xyz} = p^6$

$\therefore$ xyz = 6

Q.13. The value of $\frac{x^2-(y-z)^2}{(x+z)^2-y^2} + \frac{y^2-(x-z)^2}{(x+y)^2-z^2} + \frac{z^2-(x-y)^2}{(y+z)^2-x^2} =$

(*a*) 0 (*b*) 1 (*c*) -1 (*d*) ∞

[June 2016]

Solution : (*b*) is correct

$$\frac{(x+y-z)(x-y+z)}{(x+y+z)(x-y+z)} + \frac{(y+x-z)(y-x+z)}{(x+y+z)(x+y-z)} + \frac{(x-y+z)(z-x+y)}{(x+y+z)(y+z-x)}$$

$$= \frac{x+y-z+y-x+z+x-y+z}{x+y+z} = \frac{x+y+z}{x+y+z} = 1$$

Tricks : Cyclic order

Q.14. If $3^x = 5^y = (75)^Z$ then

(*a*) $\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$ (*b*) $\frac{2}{x} + \frac{1}{y} = \frac{1}{z}$ (*c*) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ (*d*) None

[June 2016]

Solution :

$3^x = 5^y = (75)^Z$ .....(1)

$3^1 \times 5^2 = 75^1$ .....(2)

Tricks : Power of (2) ÷ power of (1) and put + sign at the place of "×".

We get, $\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$

So, (*a*) is correct.

Detail:

Let $3^x = 5^y = 75^z = k$

$\therefore 3 = k^{1/x}; 5 = k^{1/y}; 75 = k^{1/z}$

$\therefore 75 = 3 \times 5^2$

or; $k^{1/z} = k^{1/x}(k^{1/y})^2 = k^{1/x}.k^{2/y}$

or; $1/z = 1/x + 2/y$

(*a*) is correct

Q.15. If $abc = 2$, then the value of $\frac{1}{1+a+2b^{-1}} + \frac{1}{1+\frac{b}{2}+c^{-1}} + \frac{1}{1+a^{-1}+c} =$

(*a*) 1 (*b*) 2 (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{4}$

[June 2016]

Solution : Tricks

"Put a = 1, b = 2 & c = 1. So that abc = 2" in the given question. We get

$$\frac{1}{1+1+\frac{2}{2}}+\frac{1}{1+\frac{2}{2}+1^{-1}}+\frac{1}{1+1^{-1}+1}=1$$

Option (*a*) is correct.

Q.16. If $a=\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}, b=\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ then the value of $\frac{1}{a^2}+\frac{1}{b^2}$ is

(*a*) 486 (*b*) 484 (*c*) 482 (*d*) 500

[June 2017]

Solution : $\frac{1}{a}+\frac{1}{b}=$

$$\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}+\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}=\frac{\left(\sqrt{6}-\sqrt{5}\right)^2+\left(\sqrt{6}+\sqrt{5}\right)^2}{\left(\sqrt{6}+\sqrt{5}\right)\left(\sqrt{6}-\sqrt{5}\right)}$$

$$=\frac{6+5-2\sqrt{6}.\sqrt{5}+6+5+2.\sqrt{6}.\sqrt{5}}{6-5}$$

$$=\frac{22}{1}=22$$

$$\& \quad \frac{1}{a}.\frac{1}{b}=1$$

$$\frac{1}{a^2}+\frac{1}{b^2}=\left(\frac{1}{a}+\frac{1}{b}\right)^2-2\left(\frac{1}{a}.\frac{1}{b}\right)=22^2-2=482$$

Option (*a*) is correct.

Tricks : By Calculator $\left(\textbf{Calculate } \frac{1}{a^2}+\frac{1}{b^2}\right)$

Q.17. If $u^{5x}=v^{5y}=w^{5z}$ and $u^2=vw$ then $xy+zx-2yz=$ _____.

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) None of these

[Dec. 2017]

Solution : (*a*) $u^{5x}=v^{5y}=w^{5z} \Rightarrow u^x=v^y=w^z$

Tricks :

$$\because \ u^2=vw; \ \therefore \ \frac{2}{x}=\frac{1}{y}+\frac{1}{z}=\frac{y+z}{yz}$$

or ; $xy + zx = 2yz$

or; $xy + zx - 2yz = 0$

Detail

Let $4^x = V^y = W^z = k$

$\therefore \quad 4 = k^{1/x}; \ V = k^{1/y}; \ W = k^{1/z}$

$\therefore \quad 4^2 = VW$

$(k^{1/x})^2 = k^{1/y}.\ k^{1/z}$

or $\quad k^{2/x} = k^{1/y+1/z}$

$$\therefore \quad \frac{2}{x}=\frac{1}{y}+\frac{1}{z}$$

$xy + zx - 2yz = 0$

Q.18. $\left(x^{\sum_{n=1}^{\infty} ap^{n-1}}\right)^{1-p}\left(x^{\sum_{n=1}^{\infty} bq^{n-1}}\right)^{1-q}\left(x^{\sum_{n=1}^{\infty} cr^{n-1}}\right)^{1-r}$

(*a*) $x^{-(ap+bq+cr)}$ (*b*) x^{a+b+c} (*c*) $x^{(ap+bq+cr)}$ (*d*) x^{abc}

[June 2018]

Solution : (*b*)

$$\sum_{n=1}^{\infty} ap^{n-1}=a+ap+ap^2+\ldots\ldots=\frac{a}{1-p} \text{ [In G.P.]}$$

$$\left(x^{\sum_{n=1}^{\infty} ap^{n-1}}\right)^{1-p}=\left(x^{\frac{a}{1-p}}\right)^{1-p}=x^a$$

Similarly doing as above ; We get

$$\left(x^{\sum_{n=1}^{\infty} ap^{n-1}}\right)^{1-p}\left(x^{\sum_{n=1}^{\infty} bq^{n-1}}\right)^{1-q}\left(x^{\sum_{n=1}^{\infty} cr^{n-1}}\right)^{1-r}$$

$= x^a . x^b . x^c$

$= x^{a+b+c}$

Option (*b*) is correct.

Q.19. $\dfrac{2^n+2^{n-1}}{2^{n+1}-2^n}$

(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{2}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{1}{3}$

[May 2018]

Solution : (*b*)

Tricks :-

Put minimum power = n - 1 = 0 or n = 1 in the question.

$$\therefore \frac{2^n+2^{n-1}}{2^{n+1}-2^n}=\frac{2^1+2^{1-1}}{2^{1+1}-2^1}=\frac{2+1}{4-2}=\frac{3}{2}$$

Q.20. $\dfrac{2^{m+1}x3^{2m-n+3}x5^{n+m+4}x6^{2n+m}}{6^{2m+n}x10^{n+1}x15^{m+3}}$

(*a*) 3^{2m-2n} (*b*) 3^{2n-2m} (*c*) 1 (*d*) None

[Nov. 2018]

Solution : Tricks

Put m = n = 0 in this equation. $\dfrac{2^{m+1}x3^{2m-n+3}x5^{n+m+4}x6^{2n+m}}{6^{2m+n}x10^{n+1}x15^{m+3}}=1$

Q.21. If $2^{x^2}=3^{y^2}=12^{z^2}$ then

(*a*) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{z^2}$ (*b*) $\frac{1}{x^2}+\frac{2}{y^2}=\frac{1}{z^2}$

(*c*) $\frac{2}{x^2}+\frac{1}{y^2}=\frac{1}{z^2}$ (*d*) None

[June 2019]

Solution :

$\because 2^{x^2}=3^{y^2}=12^{z^2}$ ..... (1) (Given)

Tricks:-

Factorize 12 in terms of 2 & 3. We get

$2^2\times3^1=12^1$ ..... (2)

Always write as power of base of (2) ÷ Power on same base of 1 ; put "+" Sign at the place of "×" Sign.

So;

$$\frac{2}{x^2}+\frac{1}{y^2}=\frac{1}{z^2}$$

So (c) is correct.

Details:-

Let $2^{x^2}=3^{y^2}=12^{z^2}=K^1$.

So ; $2=K^{\frac{1}{x^2}}$; $3=K^{\frac{1}{y^2}}$; $12=K^{\frac{1}{z^2}}$

Now

$12=2^2\times3^1$

or $K^{\frac{1}{z^2}}=\left(K^{\frac{1}{x^2}}\right)^2\times K^{\frac{1}{y^2}}$

or $K^{\frac{1}{z^2}}=K^{\frac{2}{x^2}}\cdot K^{\frac{1}{y^2}}$

or $K^{\frac{1}{z^2}}=K^{\left(\frac{2}{x^2}+\frac{1}{y^2}\right)}$

$\therefore \frac{1}{z^2}=\frac{2}{x^2}+\frac{1}{y^2}$

(*c*) is correct

Q.22. The value of $\left[\dfrac{9^{n+1/4}\cdot\sqrt{3.3^n}}{3.\sqrt{3^{-n}}}\right]^{\frac{1}{n}}$ **is ....**

(*a*) 1 (*b*) 3 (*c*) 9 (*d*) 27

[Dec. 2019]

Solution : (*d*)

Tricks:

Putting n = 1 in the Question.

We get

$$\left[\frac{9^{1+\frac{1}{4}}\cdot\sqrt{3\times3^1}}{3\cdot\sqrt{3^{-1}}}\right]^{\frac{1}{1}}$$

$$= \left[\frac{(3^2)^{3/4} \cdot \cancel{3}}{\cancel{3} \cdot 3^{-1/2}}\right]$$

$$= \frac{3^{\frac{5}{2}}}{3^{-1/2}} = 3^{\left(\frac{5}{2}+\frac{1}{2}\right)} = 3^{3}$$

$$= 3^3 = 27.$$

Q.23. If $x = \sqrt{3} + \frac{1}{\sqrt{3}}$ then $\left(x - \frac{\sqrt{126}}{\sqrt{42}}\right)\left(x - \frac{1}{x - \frac{2\sqrt{3}}{3}}\right) =$

(*a*) $\frac{5}{6}$ (*b*) $\frac{6}{5}$ (*c*) $\frac{2}{3}$ (*d*) $-\frac{3}{5}$

[Dec. 2019]

Solution : (*a*)

$\because\ x = \sqrt{3} + \frac{1}{\sqrt{3}}$

Then

$$\left(x - \frac{\sqrt{126}}{\sqrt{42}}\right)\left(x - \frac{1}{x - \frac{2\sqrt{3}}{3}}\right)$$

$$= \left(x - \sqrt{\frac{126}{42}}\right)\left(x - \frac{1}{\sqrt{3} + \frac{1}{\sqrt{3}} - \frac{2\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}}\right)$$

$$= \left(x - \sqrt{3}\right)\left(x - \frac{1}{\sqrt{3} + \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}}}\right)$$

$$= \left(\sqrt{3} + \frac{1}{\sqrt{3}} - \sqrt{3}\right)\left(x - \frac{1}{\sqrt{3} - \frac{1}{\sqrt{3}}}\right)$$

$$= \frac{1}{\sqrt{3}}\left(x - \frac{\sqrt{3}}{3-1}\right)$$

$$= \frac{1}{\sqrt{3}}\left(\sqrt{3} + \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{2}\right)$$

$$= 1 + \frac{1}{3} - \frac{1}{2} = \frac{6+2-3}{6} = \frac{5}{6}$$

Q.24. Find the value of 'a' from the following

$$\left(\sqrt{9}\right)^{-5} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$$

(*a*) 13 (*b*) 11 (*c*) 15 (*d*) 17

[Dec. 2020]

Solution : (*d*)

$$\left(\sqrt{9}\right)^{-5} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$$

or $3^{-5} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$

or $3^{\frac{-10}{2}} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$

or $\left(\sqrt{3}\right)^{-10} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$

or $\left(\sqrt{3}\right)^{-10-7} = \left(\sqrt{3}\right)^{-a}$

$\therefore -a = -17 \Rightarrow a = 17$

(*d*) is correct.

Q.25. Find the value of $\frac{3t^{-1}}{t^{-1/3}}$

(*a*) $\frac{3}{t^{2/3}}$ (*b*) $\frac{3}{t^{3/2}}$ (*c*) $\frac{3}{t^{1/3}}$ (*d*) $\frac{3}{t^2}$

[Jan. 2021]

Solution : (*a*) is correct

$$3.t^{-1+\frac{1}{3}} = 3t^{-\frac{2}{3}} = 3.\frac{1}{t^{2/3}} = \frac{3}{t^{2/3}}$$

Q.26. If xy + yz + zx = -1, then the value of $\left(\frac{xy}{1+xy}+\frac{z+y}{1+zy}+\frac{x+z}{1+zx}\right)$ is

(*a*) xyz (*b*) $\frac{-1+y}{yz}$ (*c*) $\frac{1}{xyz}$ (*d*) $\frac{1}{x+y+z}$

[July 2021]

Solution : (*c*)

$$= \frac{x+y}{1+xy} + \frac{z+y}{1+zy} + \frac{x+z}{1+zx}$$

$$= \frac{z(x+y)}{z(1+xy)} + \frac{x(z+y)}{x(1+zy)} + \frac{y(x+z)}{y(1+zx)}$$

$$= \frac{zx+yz}{z(1+xy)} + \frac{zx+xy}{x(1+yz)} + \frac{xy+yz}{y(1+zx)}$$

$$= \frac{-1+xy}{z(1+xy)} + \frac{-1-yz}{x(1+yz)} + \frac{-1-zx}{y(1+zx)}$$

$$= \frac{-(1+xy)}{z(1+xy)} + \frac{-(1+yz)}{x(1+yz)} + \frac{-(1+zx)}{y(1+zx)}$$

$$= \frac{-1}{z} + \frac{-1}{x} + \frac{-1}{y}$$

$$= -\left[\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right] = -\left[\frac{yz+zx+xy}{xyz}\right]$$

$$= -\left(\frac{-1}{xyz}\right)$$

$$= \frac{1}{xyz}$$

(*c*) is correct

Q.27. Let $a = \left(\sqrt{5}+\sqrt{3}\right)/\left(\sqrt{5}-\sqrt{3}\right)$ and $b = \left(\sqrt{5}-\sqrt{3}\right)/\left(\sqrt{5}+\sqrt{3}\right)$. What is the Value of $a^2 + b^2$?

(*a*) 64 (*b*) 62 (*c*) 60 (*d*) 254

[Dec. 2021]

Solution : (*b*)

$$a = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-3} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \text{(Rationalising)}$$

$$\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2} = \frac{5+3+2.\sqrt{5}.\sqrt{3}}{5-3}$$

$$\frac{8+2.\sqrt{15}}{2}$$

$$= 4+\sqrt{15}$$

Similarly

$$b = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 4-\sqrt{15}$$

$$a + b = 4+\sqrt{15} + 4 - \sqrt{15} = 8$$

$$ab = (4+\sqrt{15})(4-\sqrt{15}) = 4^2 - \left(\sqrt{15}\right)^2 = 1b - 15 = 1$$

$\therefore a^2 + b^2 = (a+b)^2 - 2ab$

$= 8^2 - 2 \times 1 = 62$

(*b*) is correct

Q.28. The value of $\frac{6^{n+4}+3^{n+3}\times 2^{n+3}}{5\times 6^n+6^n}$ is :

(*a*) 232 (*b*) 242 (*c*) 252 (*d*) 262

[Dec. 2021]

Solution : (*c*)

$$= \frac{6^{n+4}+3^{n+3}\times 2^{n+3}}{5\times 6^n+6^n}$$

$$= \frac{6^n.6^4+3^n.3^3\times 2^n.2^3}{6^n(5+1)}$$

$$= \frac{6^n.6^4+(3\times 2)^n.3^3.2^3}{6^n.6}$$

$$= \frac{\cancel{6^n}\left[6^4+3^3\times 2^3\right]}{\cancel{6^n}.6} = \frac{1296+27\times 8}{6}$$

$= 252$

Tricks :-

Put $n = 0$

$$\frac{6^4+3^3\times 2^3}{5+1}$$

$$\frac{1296+27\times 8}{6}$$

$= 252$

Q.29. If $\left(\frac{3a}{2b}\right)^{2x-4} = \left(\frac{2b}{3a}\right)^{2x-4}$, for some a and b, then the value of x is

(*a*) 8 (*b*) 6 (*c*) 4 (*d*) 2

Solution : (*d*)

Given

$$\left(\frac{3a}{2b}\right)^{2x-4} = \left(\frac{2b}{3a}\right)^{2x-4}$$

$$\Rightarrow \left(\frac{3a}{2b}\right)^{2x-4} = \left(\frac{3a}{2b}\right)^{-2x+4}$$

$\Rightarrow 2x - 4 = -2x + 4$

or $2x + 2x = 4 + 4$

or $4x = 8 \Rightarrow x = 2$

Q.30. The value of $\left(1 - \sqrt[3]{0.027}\left(\frac{5}{6}\right)\left(\frac{1}{2}\right)^2\right)$ is :

(*a*) 11/16 (*b*) 13/16 (*c*) 15/16 (*d*) 1

[Dec. 2021]

Solution : (*c*) is correct

$$1 - \left\{(0.3)^3\right\}^{\frac{1}{3}} \frac{5}{6} \cdot \frac{1}{4}$$

$$= 1 - (0.3)\frac{5}{6} \cdot \frac{1}{4}$$

$$= 1 - \frac{1}{16} = \frac{15}{16}$$

Q.31. Find the value of z from $(\sqrt{9})^{-8} \times (\sqrt{3})^{-5} = 3^z$

(*a*) $\frac{2}{21}$ (*b*) $\frac{-21}{2}$ (*c*) $\frac{21}{2}$ (*d*) $\frac{-2}{21}$

[June 2022]

Solution : $(\sqrt{9})^{-8} \times (\sqrt{3})^{-5} = 3^z$

or; $3^z = 3^{-8} \times (3^{1/2})^{-5}$

$= 3^{-8} \times 3^{-5/2}$

$= 3^{-8-5/2} = 3^{\frac{-16-5}{2}}$

or; $3^z = 3^{\frac{-21}{2}}$

$\therefore z = \frac{-21}{2}$

(*b*) is correct

Q.32. Find the value of $\frac{3t^{-1}}{t^{-1/3}}$

(*a*) $\frac{3}{t^{\frac{2}{3}}}$ (*b*) $\frac{3}{t^{\frac{3}{2}}}$ (*c*) $\frac{3}{t^{\frac{1}{3}}}$ (*d*) $\frac{3}{t^2}$

[June 2022]

Solution : $\frac{3t^{-1}}{t^{-1/3}} = 3t^{-1+\frac{1}{3}} = 3t^{-2/3}$

$= \frac{3}{t^{-2/3}}$

(*a*) is correct

Q.33. By simplifying $\left(2a^3b^4\right)^6 \Big/ \left(\left(4a^3b\right)^2 \times \left(a^2b^2\right)\right)$, the answer will be

(*a*) $4a^3 b^x$ (*b*) $4a^3 b^{2x}$ (*c*) $4a^{28} bw^{22}$ (*d*) $4a^{10}.b^{20}$

[Dec. 2022]

Solution : $\frac{\left(2a^3b^4\right)^6}{\left(4a^3b\right)^2\left(a^2b^2\right)} = \frac{64 \times a^{18} \times b^{24}}{16a^6 \times b^2 \times a^2 \times b^2}$

$= 4\, a^{10} \times b^{20}$

(*d*) is correct

Q.34. If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$ then the value of $\left(\frac{a+b+c}{3}\right)^3$ is equal to:

(*a*) *abc* (*b*) *9abc* (*c*) 1/(*abc*) (*d*) (1/9)*bac*

[June 2023]

Solution : $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$

or; $\sqrt[3]{a} + \sqrt[3]{b} = -\sqrt[3]{c}$ ——(1)

Cubing on both sides; we get

$(a^{1/3} + b^{1/3})^3 = (-c^{1/3})^3$

or; $(a^{1/3})^3 + (b^{1/3})^3 + 3a^{1/3} \cdot b^{1/3} (a^{1/3} + b^{1/3}) = - (c^{1/3})^3$

or; $a + b + 3(ab)^{1/3} (-c^{1/3}) = - c$

or; $a + b + c = 3(abc)^{1/3}$

or; $\frac{a+b+c}{3} = (abc)^{1/3}$

Cubing again on both sides; we get

$\left(\frac{a+b+c}{3}\right)^3 = abc$

Trick

$\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$

Put $\sqrt[3]{a} = 1$; $\sqrt[3]{b} = 1$; then $\sqrt[3]{c} = -2$

$\therefore a = 1$; $b = 1$ and $c = -8$

Now $\left(\frac{a+b+c}{3}\right)^3 = \left(\frac{1+1-8}{3}\right)^3 = -8$

Then Go by choices (GBC)

(*a*) $abc = 1.1.(-8) = -8$ (True)

$\therefore$ (*a*) is correct.

Q.35. If $x = y^a$, $y = z^b$, $z = x^c$, then the value of *abc* is:

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

[June 2023]

Solution :

$\therefore x = y^a = (z^b)^a = z^{ab}$

$\therefore z = x^c$

or $z^1 = (z^{ab})^c = z^{abc}$

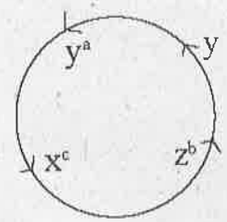
$\therefore abc = 1.$

Tricks

Product of powers = Power on last y.

$\therefore$ abc = 1

$\therefore$ (*a*) is correct.



3 CHAPTER LOGARITHM

If $a^b = c$ Where $a \neq 1$ and a; c > o (positive)

Then b is said to be the logarithm of the number c to the base "a" and expressed as

$Log_a c = b$; Where $a \neq 1$.

Types of Logarithm

(*i*) ***Natural Logarithm:***

The Logarithm of a number to base "e" is called Natural Logarithm.

i.e. $Log_e x$

where x = a number

e = 2.7183

(*ii*) ***Common Logarithm:***

Logarithm of a number to the base 10 is called common Logarithm.

i.e. $Log_{10} x$

where x = A number

Note: If base is not given then in arithmetical or commercial work; base is always taken as 10.

Remember Some Formulae

1. If $a^b = c \Leftrightarrow Log_a c = b$; Where $a \neq 1$.
2. $a^{x \log_a b} = b^x$
3. $\log_a a = 1$

4. $\log_a 1 = 0$

5. $\log_b a = \dfrac{1}{\log_a b} \Rightarrow \log_b a . \log_a b = 1$

6. (*i*) $\log_b a = \log_b x \log_x a$

$= \log_x a \log_b x$

(*ii*) $\log_b a = \log_x a . \log_y x . \log_z y .... \log_b k$

$\log_b a = \log_b x . \log_x y . \log_y z .... \log_k a$

7. (*i*) $\log_b a = \dfrac{\log_x a}{\log_x b}$

(*ii*) $\log_b a = \dfrac{\log_b x}{\log_a x}$

8. If $\log_b a = x$

Then (*i*) $\log_{\frac{1}{b}} a = -x$

(*ii*) $\log_b \dfrac{1}{a} = -x$

(*iii*) $\log_{\frac{1}{b}} \dfrac{1}{a} = +x$

9. (*i*) $\log_a (mn) = \log_a m + \log_a n$

(*ii*) $\log_a (mnr....) = \log_a m + \log_a n + \log_a r$ +.....

10. $\log_a \left(\dfrac{m}{n}\right) = \log_a m - \log_a n$

11. (*i*) $\log a^b (m^n) = \dfrac{n}{b} \log_a m.$

(*ii*) $\log_{a^{(m^n)}} = n \log_a m.$

(*iii*) $\log_{a^b} m = \dfrac{1}{b} \log_a m$

12. (*i*) If $\log_a m = \log_b m \Rightarrow a = b.$

(*ii*) If $\log_a m = \log_a n \Rightarrow m = n.$

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. If $\log_a b + \log_a c = 0$ then

(*a*) b = c (*b*) b = -c

(*c*) b + c = 1 (*d*) b and c are reciprocals.

[June 2010]

Solution : $\log_a b + \log_a c = 0$

$or \log_a (bc) = \log_a 1$

$\therefore$ bc = 1

$\therefore b = \dfrac{1}{c}$

$\therefore$ (*d*) is correct

Q.2. The value of $2\log x + 2\log x^2 + 2\log x^3 + ... + 2\log x^n$ will be :-

(*a*) $\dfrac{n(n+1)\log x}{2}$ (*b*) n(n+1) log *x*

(*c*) $n^2 \log x$ (*d*) None of these

[Dec. 2010]

Solution : Detail Method

$2\log x + 2\log x^2 + 2\log x^3 + ----+2\log x^n$

$= 2\ \log x + 2.2\log x + 2.3\log x + -------+ 2.n.\log x$

$= 2\ \log x. [1 + 2 + 3 + ----------+n]$

$= 2\ \log x. \dfrac{n(n+1)}{2} = n(n+1)\log x$

= (*b*) is correct

Tricks :- Put n = 2 in options directly.

This should be equal to sum of 1st 2 terms = $2\log x + 2.2\log x = 6\log x$

Which gives option (*b*)

$\therefore$ (*b*) is correct.

Q.3. Solve: $\dfrac{\log_x 10 - 3}{2} + \dfrac{11 - \log_x 10}{3} = 2$

(*a*) 10^{-1} (*b*) 10^2 (*c*) 10 (*d*) 10^3

[Dec. 2010, June 2011]

Solution : $\frac{\log_x 10-3}{2}+\frac{11-\log_x 10}{3}=2$

Tricks:- Go by choices [Do Mentally]

For $(a)\ x=10^{-1}$

$$\text{L.H.S}=\frac{\log_{(10^{-1})}10-3}{2}+\frac{11-\log_{10^{-1}}10}{3}$$

$$=\frac{-1-3}{2}+\frac{11-(-1)}{3}$$

$= -2 + 4 = 2 = (\text{R.H.S})$

$\therefore$ (a) is correct

Q.4. If n = m! where ('m' is a positive integer > 2) then the value of: $\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+.....+\frac{1}{\log_m n}$ **is**

(a) 1 (b) 0 (c) -1 (d) 2

[June 2011]

Solution : Given n = m !

$$\therefore \frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+.....+\frac{1}{\log_m n}$$

$$=\log_n 2+\log_n 3+\log_n 4+.....+\log_n m$$

$$=\log_n(2.\ 3.\ 4.....\text{m})$$

$$=\log_n(1.\ 2.\ 3.\ 4.....\text{m})=\log_{(m!)}(m!)=1$$

$\therefore$ (a) is correct

Q.5. If $\log_2$ x + $\log_4$ x = 6, then the value of x is

(a) 16 (b) 32 (c) 64 (d) 128

[Dec. 2011]

Solution : (a) is correct

Tricks: Go by choices for (a) if x = 16

$\text{L.H.S}=\log_2 16+\log_4 16=4+2=6(R.H.S)$

$\therefore$ (a) is correct

Detail Method $\log_2 x+\log_4 x=6$

or $\log_2 x+\log_{2^2} x=6$

or $\log_2 x+\frac{1}{2}\log_2 x=6$

or $\left(1+\frac{1}{2}\right)\log_2 x=6$

or $\log_2 x=\frac{6\times 2}{3}=4\ \ \therefore x=2^4=16$

Q.6. If $\log_x$Y= 100 and $\log_2$ x = 10, then the value of 'Y' is :

(a) 2^{10} (b) 2^{100}

(c) $2^{1,000}$ (d) $2^{10,000}$

[June 2012]

Solution : (c) $\log_2 x=10\ \ \therefore x=2^{10}$

Now $\log_x y=100\ \ \therefore y=x^{100}$

$\therefore y=(2^{10})^{100}=2^{1000}$

$\therefore$ (c) is correct

Q.7. Which of the following is true.

If $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=\frac{1}{abc}$

(a) log (ab + bc +ca) = abc

(b) $\log\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=abc$

(c) log (abc) = 0

(d) log (a+ b + c) = 0

[Dec. 2012]

Solution : (d) is correct

$\therefore \frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=\frac{1}{abc}$

Multiplying both sides by abc;

$\frac{abc}{ab}+\frac{abc}{bc}+\frac{abc}{ca}=\frac{abc}{abc}$

or ; c + a + b = 1

or ; a + b + c = 1

Taking log on both sides ; we get

log (a + b + c) = log1 = 0

Q.8. *If* $(\log_{\sqrt{x}} 2)^2=\log_x 2$ **then x =**

(a) 16 (b) 32

(c) 8 (d) 4

[June 2013]

Solution : (a) is correct

$(\log_{\sqrt{x}} 2)^2=\log_x 2$

or $(\log_{x^{1/2}} 2)^2=\log_x 2$

or $\left(\frac{1}{\frac{1}{2}}\log_x 2\right)^2=\log_x 2$

or $4(\log_x 2)^2-\log_x 2=0$

or $\log_x 2[4\log_x 2-1]=0$

If $\log_x 2=0$ (Invalid)

$\therefore 4\log_x 2-1=0$

or $4\log_x 2=1$

or $\log_x 2=\frac{1}{4}$

or $x^{1/4}=2\Rightarrow x=2^4=16$

Tricks :- Go by choices

For (a) LHS

$$=(\log_{\sqrt{16}} 2)^2=(\log_4 2)^2$$

$$=\left(\frac{1}{2}\log_2 2\right)^2=\frac{1}{4}$$

$$\text{RHS}=\log_{16} 2=\log_{2^4} 2=\frac{1}{4}\log_2 2$$

$$=\frac{1}{4}$$

$\therefore$ (a) is correct

Note :- Never write; check mentally.

Q.9. Find Value of
$\left[\log_y x.\log_z y.\log_x z\right]^3 =$

(*a*) 0 (*b*) -1
(*c*) 1 (*d*) 3

[Dec. 2013]

Solution : (*c*) is correct

$[\log_y x.\log_z y.\log_x z]^3$

$= [\log_x x]^3 = [1]^3 = 1$

Q.10. Find the value of $Log_4 9.Log_3 2 =$

(*a*) 3 (*b*) 9
(*c*) 2 (*d*) 1

[Dec. 2013]

Solution : (*d*) is correct $Log_4 9.Log_3 2 =$

$\log_{(2^2)}(3^2).\log_3 2$

$= \frac{2}{2}\log_2 3.\log_3 2$

$= 1 \times 1 = 1$

Q.11. If $X = \log_{24} 12; y = \log_{36} 24; z = \log_{48} 36$ **then xyz + 1= ?**

(*a*) 2xy (*b*) 2zx (*c*) 2yz (*d*) 2

[June 2014]

Solution : (*c*) is correct

$xyz+1 = \log_{24} 12.\log_{36} 24.\log_{48} 36+1$

$= \log_{48} 12+\log_{48} 48$

$= \log_{48}(12\times 48) = \log_{48}(12\times 2)^2$

$= 2\log_{48} 24 = 2\log_{36} 24.\log_{48} 36$

$= 2yz$

Q.12. If $x^2+y^2 = 7xy$ **then** $\log\frac{1}{3}(x+y) =$

(*a*) logx + logy (*b*) $\frac{1}{2}(\log x+\log y)$ (*c*) $\frac{1}{3}(\log x+\log y)$ (*d*) $\frac{1}{3}(\log x.\log y)$

[June 2014]

Solution : (*b*) is correct

$\log\frac{1}{3}(x+y) = \frac{1}{2}2\log\left\{\frac{1}{3}(x+y)\right\}$

$= \frac{1}{2}\log\left\{\frac{1}{3}(x+y)\right\}^2 = \frac{1}{2}\log\left(\frac{x^2+y^2+2xy}{9}\right)$

$= \frac{1}{2}\log\left(\frac{7xy+2xy}{9}\right) = \frac{1}{2}\log(xy) = \frac{1}{2}(\log x+\log y)$

Q.13. If log x = a-b; log y = a + b then $\log\left(\frac{10x}{y^2}\right)$

(*a*) 1-a-3b (*b*) a-1+3b (*c*) a+3b-1 (*d*) 1-b+3a

[Dec. 2014]

Solution : (*a*) is correct

$\therefore$ log x = a+b ; log y = a-b.

$\log\left(\frac{10x}{y^2}\right) = \log_{10} 10 + \log x - \log y^2$

$= 1 + a + b - 2\log y = 1 + a + b - 2(a-b)$

$= 1 + a + b - 2a + 2b = 1 - a + 3b$

Q.14. If $x = 1+\log_p qr, y = 1+\log_q rp$ **and** $z = 1+\log_r pq$ **; then the value of**

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z} =$ _____

(*a*) 0 (*b*) 1 (*c*) -1 (*d*) 3

[Dec. 2014]

Solution : (*b*) is correct

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$

$= \frac{1}{1+\log_p qr}+\frac{1}{1+\log_q rp}+\frac{1}{1+\log_r pq}$

$= \frac{1}{\log_p p+\log_p qr}+\frac{1}{\log_q q+\log_q rp}+\frac{1}{\log_r r+\log_r pq}$

$= \frac{1}{\log_p pqr}+\frac{1}{\log_q pqr}+\frac{1}{\log_r pqr}$

$= \log_{pqr} p+\log_{pqr} q+\log_{pqr} r$

$= \log_{pqr} pqr = 1$

Tricks:- Cyclic order

So, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

Q.15. If $\log x = m+n$; $\log y = m-n$ then $\log\left(\frac{10x}{y^2}\right)=$

(a) 1-m+3n (b) m-1+3n

(c) m+3n+1 (d) None

[June 2015]

Solution : (a)

If $\log x = m+n$; $\log y = m-n$

Then $\log\left(\frac{10x}{y^2}\right)$

$= \log 10 + \log x - \log y^2$

$= 1 + \log x - 2\log y$

$= 1 + (m+n) - 2(m-n)$

$= 1 + m + n - 2m + 2n$

$= 1 - m + 3n$

$\therefore$ (a) is correct.

Q.16. $\log_3 5. \times \log_5 4 \times \log_2 3$:

(a) 2 (b) 5

(c) -2 (d) None of these

[Dec. 2015]

Solution : (a) is correct

$\log_3 5.\log_5 4.\log_2 3$

$= \log_3 4.\log_2 3 = \log_2 4 = 2$

Q.17. The integral part of a logarithm is called _____, and the decimal part of a logarithm is called _____.

(a) Mantissa, Characteristic

(b) Characteristic, Mantissa

(c) Whole, Decimal

(d) None of these

[June 2016]

Solution : (b) is correct.

Q.18. The value of

$\frac{1}{\log_3 60}+\frac{1}{\log_4 60}+\frac{1}{\log_5 60}=$ _____

(a) 0 (b) 1

(c) 5 (d) 60

[June 2016]

Solution : (b) is correct.

$\log_{60} 3+\log_{60} 4+\log_{60} 5$

$= \log_{60}(3\times4\times5) = \log_{60} 60 = 1$

Q.19. If $\log_4(X^2+x)-\log_4(x+1)=2$ then the value of x is

(a) 2 (b) 3

(c) 16 (d) 8

[June 2016]

Solution : (c) is correct.

$\log_4 \frac{(x^2+x)}{(x+1)}=2$

or $\log_4\left\{\frac{x(x+1)}{(x+1)}\right\}=2$

or $\log_4 x = 2 \Rightarrow x = 4^2 = 16$

Q.20. Given log 2 = 0.3010 and log 3 = 0.4771 then the value of log 24

(a) 1.3081 (b) 1.1038

(c) 1.3801 (d) 1.8301

[Dec. 2016]

Solution : (c) is correct.

Calculator Tricks:

Type 24 then $\sqrt{}$ button 19 times - 1× 227695 = button. We will get the required value of log 24.

Q.21. $\log(1^3+2^3+3^3+.....+n^3)=$ _____

(a) $2\log n+2\log(n+1)-2\log 2$

(b) $\log n+2\log(n+1)-2\log 2$

(c) $2\log n+\log(n+1)-2\log 2$

(d) None

[June 2017]

Solution :

$\log(1^3+2^3+3^3+.....+n^3)$

$= \log\left(\frac{n(n+1)}{2}\right)^2 = 2\log\frac{n(n+1)}{2}$

$= 2\left[\log n+\log(n+1)-\log 2\right]$

$= 2\log n+2\log(n+1)-2\log 2$

So, (a) is correct.

Tricks :- Go by Choices

Q.22. If $\log_3[\log_4(\log_2 x)]=0$ then X =

(a) 4 (b) 8

(c) 16 (d) 32

[Dec. 2017]

Solution : (c)

Tricks :- GBC

for option (c) $\log_3\left[\log_4(\log_2 x)\right]$

$= \log_3\left[\log_4(\log_2 16)\right]$

$= \log_3(\log_4 4) = \log_3 1 = 0$

$\therefore$ (c) is correct.

Q.23. If

$\log\left(\frac{x-y}{2}\right)=\frac{1}{2}(\log x+\log y)$ then

$x^2+y^2=$ _____

(a) $6xy$ (b) $2xy$

(c) $3x^2y^2$ (d) $4x^2y^2$

[Dec. 2017]

Solution : $2\log\left(\frac{x-y}{2}\right)=\log xy$

or ; $\log\left(\frac{x-y}{2}\right)^2=\log(xy)$

or ; $\frac{x^2+y^2-2xy}{4}=xy$

or ; $x^2+y^2-2xy=4xy$

or ; $x^2+y^2=6xy$

$\therefore$ (a) is correct.

Q.24. If $\log_x\left(\sqrt[3]{2}\right)=\frac{1}{15}$ then x =

(a) 2 (b) 8

(c) 16 (d) 32

[June 2018]

Solution : (d) $\log_x\left(\sqrt[3]{2}\right)=\frac{1}{15}$

or $x^{1/15}=\sqrt[3]{2}=2^{1/3}$

or $x=\left(2^{1/3}\right)^{15}=2^5=32$.

Q.25. The value of the expression :

$a^{\log_a b.\log_b c.\log_c d.\log_d t}$

(a) t

(b) abcdt

(c) (a + b + c + d + t)

(d) None

[May 2018]

Solution : $a^{\log_a b.\log_b c.\log_c d.\log_d t}$

$= a^{\log_a t} = a^{1.\log_a t} = t^1 = t$

[Formula $a^{x\log_a b} = b^x$]

Q.26. $\log_2 \log_2 \log_2 16 = ?$

(*a*) 0 (*b*) 3 (*c*) 1 (*d*) 2

[Nov. 2018]

Solution : (*c*) $\log_2 \log_2 \log_2 16 = \log_2 \log_2 4 = \log_2 2 = 1$

Q.27. The value of

$\log_5(1+\frac{1}{5}) + \log_5(1+\frac{1}{6}) + ... + \log_5(1+\frac{1}{624})$

(*a*) 2 (*b*) 3 (*c*) 5 (*d*) 0

[June 2019]

Solution :

$\log_5(1+\frac{1}{5}) + \log_5(1+\frac{1}{6}) + ... + \log_5(1+\frac{1}{624})$

$= \log_5\left(\frac{6}{5}\right) + \log_5\left(\frac{7}{6}\right) + ... + \log_5\left(\frac{625}{624}\right)$

$= \log_5\left(\frac{6}{5}.\frac{7}{6}.....\frac{625}{624}\right)$

$= \log_5\left(\frac{625}{5}\right) = \log_5(125)$

$= \log_5 5^3 = 3.\log_5 5$ **[Calculator Tricks]**

$= 3 \times 1 = 3.$

Q.28. $\log_{2\sqrt{2}}(512) : \log_{3\sqrt{2}} 324 =$

(*a*) 128 : 61 (*b*) 2 : 3

(*c*) 3 : 2 (*d*) None

[June 2019]

Solution :

Calculator Tricks :-

$\log_{2\sqrt{2}} 512 = 5+1$

Type $2 \times 2\sqrt{}$ button = button.

Then press × button then continue pressing = button until to get 512

Here = button has been pressed 5 times. So log value

= (No. of = button pressings + 1)

Similarly

For $\log_{3\sqrt{2}} 324$

Type $3 \times 2\sqrt{}$ button = button then × = button 3 times ; we get

$\log_{3\sqrt{2}} 324$ value = 3 + 1 = 4

So; $\log_{2\sqrt{2}} 512 : \log_{3\sqrt{2}} 324$

= 6 : 4 = 3 : 2

Q.29. $\log_{0.01}(10,000) = x$; Find the value of *x*?

(*a*) 1 (*b*) -2

(*c*) -4 (*d*) 2

[Dec. 2019]

Solution : (*b*)

Calculator Tricks

Type 0.01 then press ÷ button

then press = button 2 times; we get 10,000.

Hence ; $\log_{0.01} 10,000 = -2$

Q.30. $\log xy^2 - \log y = \log(x+y)$

Find the value of y in term of x

(*a*) *x*-1 (*b*) $\frac{x}{x+1}$

(*c*) $\frac{x}{x-1}$ (*d*) $x+1$

[Dec. 2019]

Solution : (*c*)

$\log xy^2 - \log y = \log(x+y)$

or; $\log\left(\frac{xy^2}{y}\right) = \log(x+y)$

or; $\log xy = \log(x+y)$

or; $xy = x+y$

or; $xy - y = x$

or; $y(x-1) = x$

or; $y = \frac{x}{x-1}$

∴ (*c*) is correct

Q.31. If $\log_a(\sqrt{3}) = \frac{1}{6}$ find the value of 'a'

(*a*) 81 (*b*) 9

(*c*) 27 (*d*) 3

[Dec. 2020]

Solution : (*c*)

$\log_a \sqrt{3} = \frac{1}{6}$

or $a^{\frac{1}{6}} = \sqrt{3}$

or a = $\left(\sqrt{3}\right)^6 = \left(3^{1/2}\right)^6 = 3^3 = 27$

(*c*) is correct.

Q.32. Log 9 + log 5 is expressed as

(*a*) log (9/5) (*b*) log 4

(*c*) log (5/9) (*d*) log 45

[Dec. 2020]

Solution : log 9 + log 5

= log (9 × 5) = log45

Q.33. If $\log_a(ab) = x$, then $\log_b(ab)$ is

(a) $1/x$ (b) $\frac{x}{t+x}$

(c) $\frac{x}{x-1}$ (d) None of these

[Jan. 2021]

Solution : (c) is correct

$$\log_a a + \log_a b = x$$

or $$1 + \log_a b = x$$

or $$\log_a b = x - 1$$

$\therefore$ $$\log_b ab = \log_b a + \log_b b$$

$$= \frac{1}{\log_a b} + 1$$

$$= \frac{1}{x-1} + 1 = \frac{1+x-1}{x-1}$$

$$= \frac{x}{x-1}$$

Q.34. If $\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = 25/6$ then the value of x is

(a) 64 (b) 4 (c) 16 (d) 2

[July 2021]

Solution : $$\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$$

$\Rightarrow$ $$\log_{2^2} x^1 + \log_{2^4} x^1 + \log_{2^6} x^1 + \log_{2^8} x^1 = \frac{25}{6}$$

$\Rightarrow$ $$\frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x + \frac{1}{6}\log_2 x + \frac{1}{8}\log_2 x = \frac{25}{6}$$

or $$\log_2 x \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right] = \frac{25}{6}$$

or, $$\log_2 x \cdot \left[\frac{12+6+4+3}{24}\right] = \frac{25}{6}$$

or $$\log_2 x \times \frac{\cancel{25}}{\cancel{24}_4} = \frac{\cancel{25}}{\cancel{6}}$$

or $$\log_2 x = 4$$

or $$x = 2^4 = 16$$

(c) is correct

Q.35. If $\log_{10} 3 = x$ and $\log_{10} 4 = y$, then the value of $\log_{10} 120$ can be expressed as

(a) $x - y + 1$ (b) $x + y + 1$

(c) $x + y - 1$ (d) $2x + y - 1$

[Dec. 2021]

Solution : (b)

$\log_{10} 120 = \log_{10}(10 \times 3 \times 4)$

$= \log_{10} 10 + \log_{10} 3 + \log_{10} 4$

$= 1 + x + y$

Q.36. Find the value of $\log(x^6)$ if $\log(x) + 2\log(x^2) + 3\log(x^3) = 14$.

(a) 3 (b) 4

(c) 5 (d) 6

[Dec. 2022]

Solution : (d)

$\log x + 2.2 \log x + 3.3 \log x = 14$

or $14 \log x = 14 \Rightarrow \log x = 1$

So $\log x^6 = 6 \log x = 6 \times 1 = 6$

Q.37. If $\log a^{\sqrt{3}} = \frac{1}{6}$, find the value of a.

(a) 9 (b) 81

(c) 27 (d) 3

[June 2022]

Solution : $\log a^{\sqrt{3}} = \frac{1}{6}$

or; $a^{\frac{1}{6}} = \sqrt{3} = 3^{1/2}$

or; $a = (3^{1/2})^6 = 3^3 = 27$

(c) is correct

Q.38. $\log \frac{p^2}{qr} + \log \frac{q^2}{pr} + \log \frac{r^2}{pq} =$

(a) pqr (b) $\frac{1}{pqr}$

(c) 1 (d) 0

[June 2022]

Solution : $\log \frac{p^2}{qr} + \log \frac{q^2}{Pr} + \log \frac{r^2}{Pq}$

$= \log\left(\frac{p^2}{qr} \cdot \frac{q^2}{pr} \cdot \frac{r^2}{pq}\right)$

$= \log(1) = 0$

(d) is correct

Q.39. If $\log_{10} 2 = y$ and $\log_{10} 3 = x$, then the value of $\log_{10} 15$ is :

(a) $x - y + 1$ (b) $x + y + 1$

(c) $x - y - 1$ (d) $y - x + 1$

[Dec. 2022]

Solution : $\because x = \log_{10} 2$; $y = \log_{10} 3$

$\therefore \log_{10} 15 = \log_{10}\left(\frac{30}{2}\right)$

$= \log_{10}\left(\frac{3 \times 10}{2}\right)$

$= \log_{10} 3 + \log_{10} 10 - \log_{10} 2$

$= y + 1 - x = y - x + 1.$

$\therefore$ (d) is correct

Q.40. $\log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$ equals to:

(a) 3 (b) 2

(c) 1 (d) 0

[Dec. 2022]

Solution : $\log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$

$= \log_3 9 = \log_3 3^2$

$= 2 \log_3 3 = 2 \times 1 = 2$

(b) is correct

Q.41. The value of $[\log_{10}(5\log_{10} 100)]^2$ is:

(*a*) 1 (*b*) 2

(*c*) 10 (*d*) 25

[June 2023]

Solution : $[\log_{10}(5.\log_{10} 100)]^2$

$= [\log_{10}(5\times2)]^2$

$= (\log_{10}10)^2 = 1^2 = 1$

(*a*) is correct.

Q.42. Given that $\log_{10}x = m + n - 1$ and $\log_{10} y = m - n$, the value of $\log_{10}(100x/y^2)$ is expressed in terms of *m* and *n* as:

(*a*) $1 - m + 3n$

(*b*) $m - 1 + 3n$

(*c*) $m + 3n + 1$

(*d*) $m^2 - n^2$

[June 2023]

Solution : $\log_{10}\left(\frac{100x}{y^2}\right)$

$= \log_{10}100 + \log_{10}x - \log_{10}y^2$

$= 2 + \log_{10}x - 2\log_{10}y$

$= 2 + m + n - 1 - 2(m - n)$

$= 2 + m + n - 1 - 2m + 2n$

$= 1 - m + 3n$

$\therefore$ (*a*) is correct.

4 CHAPTER LINEAR EQUATION

The mathematical statement of equality is called **Equation**.

Linear Equation

The equation having highest power on the variable one is called linear equation. This is also called Equation of 1 degree.

Example

$7x - 5 = 9$.

Simultaneous Equation

Two or more linear equations having two or more variables is called simultaneous equation.

Example

$3x + 2y = 10$ &

$2x + 3y = 2$ are jointly called Simultaneous Equation.

Quadratic Equation (Equation of degree 2):

The equation of degree 2 is called Quadratic Equation or polynomial of degree 2.

Example

$3x^2 + 5x + 6 = 0$ is a Quadratic Equation.

Cubic Equation:

The equation of degree 3 is called Cubic Equation.

Example

$4x^3 + 3x^2 + 5x - 17 = 0$ is a Cubic Equation.

Simple Equation

The Equation with one unknown variable x in the form $ax + b = 0$ is called Simple Equation. Where $a \neq 0$.

Illustrative Examples

Example 1

The Equation -x + 1 = 5-2x will be satisfied for x equal to :

(*a*) 2 (*b*) 4 (*c*) 1 (*d*) None of these

Solution : (*b*) is correct.

$\because$ -x + 1 = 5 - 2x.

or -x + 2x = 5 - 1.

or x = 4

Tricks

Do not solve such types of equation Go by choices. It takes minimum time.

Option (b)

Put x = 4 in the equation mentally and see

LHS $-4+1=-3$

RHS $= 7-2\times4=-3$

$\therefore$ LHS = RHS.

So option (*b*) is correct because it satisfies the equation.

Example 2

The value of y of fraction $\frac{x}{y}$ exceeds with x by 5 and if 3 be added to both the fractions becomes $\frac{3}{4}$. Find the fraction.

(*a*) $\frac{12}{17}$ (*b*) $\frac{13}{17}$ (*c*) $-\frac{1}{3}$ (*d*) None

Solution : Option (*a*) is correct.

$\therefore$ y = x + 5.

Fraction $= \frac{x}{x+5}$

From question $= \frac{x+3}{x+5+3}=\frac{3}{4}$

or $\frac{x+3}{x+8}=\frac{3}{4}$

or 4x + 12 = 3x + 24

or 4x -3x = 24 - 12

or x = 12.

$\therefore$ Fraction $= \frac{x}{x+5}=\frac{12}{12+5}=\frac{12}{17}$.

$\therefore$ Option (*a*) is correct.

Tricks

Go by choices.

Let we see option (*a*) $\frac{12}{17}$

Clearly y = 17 is 5 more than x = 12.

$\therefore$ Option (*a*) satisfies the 1st condition of the question.

If $\frac{12+3}{17+3}=\frac{15}{20}=\frac{3}{4}$

$\therefore$ Option (*a*) also satisfy 2nd condition of the equation.

Example 3

In a two digits number ; the digit in the ten's place is twice the digit in the unit's place. If 18 be subtracted from the number the digits are reversed. Find the number.

(*a*) 63 (*b*) 21 (*c*) 42 (*d*) None

Solution: (*c*) is correct.

Let x be the unit place digit.

So, Ten's place digit = 2x.

$\therefore$ Number $= 10\times2x + x = 21x$.

By question.

21x - 18 = 10x + 2x

or 21x - 12x = 18

or 9x = 18

$\therefore$ x = 2.

So, the required No. $= 21x = 21\times2 = 42$

Tricks

Go by choices.

Option (*a*); (*b*) and (*c*) all satisfy 1st condition of the question.

i.e. in (*a*), (*b*), (*c*) Ten's place digit is twice of unit place digit.

Let we see for 2nd condition.

(*a*) $63 - 18 = 45 \neq 36$ (Digits not reserved)

(*b*) $21 - 18 = 3 \neq 12$ (Digits not reserved)

(*c*) $42 - 18 = 24$ (Digits reserved)

So option (*c*) also satisfies 2nd condition of the question.

$\therefore$ (*c*) is correct.

Example 4

For a certain commodity the demand equation giving "y" units for a price "p" in rupees per unit is y = 100 (10 - p). The supply equation giving the supply z units for a price "p" in rupees per unit is z = 75(p-3). The market price is such at which demand equals supply. Find the market price and quantity that will be brought and sold.

(*a*) ₹ 7; 300 units (*b*) ₹ 8; 400 units

(*c*) ₹ 5; 200 units (*d*) None

Solution :

(*a*) is correct.

Demand = Supply (given)

100 (10 - P) = 75 (P - 3)

or 40 - 4p = 3p - 9

or - 4p - 3p = -40 - 9

or - 7p = - 49 or p = 7

$\therefore$ y = 100 (10 - p) =100 (10 - 7) = 300 units

z = 75 (p - 3) = 75 (7 - 3) = 300 units.

$\therefore$ Price = ₹ 7 per unit.

Quantity = 300 units

$\therefore$ (*a*) is correct.

Tricks

Go by Choices

Option (*a*) satisfies given both conditions.

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. For all $\lambda \in R$, the line $(2+\lambda)x + (3-\lambda)y + 5 = 0$ passing through a fixed point, then the fixed point is_____.

(*a*) (1,1) (*b*) (-1,-1)

(*c*) (1,-1) (*d*) (-1,1)

[June 2011]

Solution : (*b*)

Tricks : Go by choices

For option (*b*) Point (-1 ; -1) satisfies the eqn.

LHS = $= (2+\lambda)x + (3-\lambda)y + 5$

or $(2+\lambda)(-1) + (3-\lambda)(-1) + 5$

$= -2 - \lambda - 3 + \lambda + 5 = 0 = RHS.$

$\therefore$ (*b*) is correct

Q.2. If kx - 4 = (k -1).x which of the following is true

(*a*) x = -5 (*b*) x = -4

(*c*) x = -3 (*d*) x = 4

[Dec. 2013]

Solution : (*d*) is correct

kx - 4 = (k - 1) x

or kx - 4 = kx - x

or -4 = -x $\therefore$ x = 4

Q.3. If the equations kx + 2y = 5, 3x + y = 1 has no solution then the value of k is

(*a*) 5 (*b*) 2/3

(*c*) 6 (*d*) 3/2

[Dec. 2013]

Solution : (*c*) is correct

kx + 2y = 5

3x + y = 1

They have no soln. (given)

$\frac{k}{3} = \frac{2}{1} \neq \frac{5}{1}$; $\Rightarrow -k = 6$

Q.4. The equation

$$x + 5y = 33; \frac{x+y}{x-y} = \frac{13}{3}$$

has the solution (x,y) as:

(*a*) (4,8) (*b*) (8,5)

(*c*) (4,16) (*d*) (16,4)

[Dec. 2014]

Solution : (*b*) is correct

Tricks: Go by choices

For LHS = $x + 5y = 8 + 5 \times 5 = 33$

and $\frac{x+y}{x-y} + \frac{8+5}{8-5} = \frac{13}{3}$

Clearly (*b*) satisfies both eqns.

Q.5. The age of a person is 8 years more than thrice the age of the sum of his two grandsons who were twins. After 8 years his age will be 10 years more than twice the sum of the ages of his grandsons. Then the age of the person when the twins were born is_

(*a*) 86 yrs (*b*) 73 yrs

(*c*) 68 yrs (*d*) 63 yrs

[June 2015]

Solution : (*b*) Let age of 1st grandson = x

$\therefore$ Person's Age = P = 3(x+x) + 8

P = 6x + 8

After 8 years

$P + 8 = 2[x + 8 + x + 8] + 10$

$= 2(2x + 16) + 10$

or $6x + 8 + 8 = 4x + 32 + 10$

or $2x = 42 - 16 = 26$

$\therefore x = 13$

$\therefore$ Age of person when grandsons were born

$= 6x + 8 - x$

$= 6\times13+8-13 = 73$

(*b*) is correct

Q.6. In a school number of students in each section is 36. If 12 new students are added, then the number of sections are increased by 4 and the number of students in each section becomes 30. The original number of section at first is

(*a*) 6 (*b*) 10

(*c*) 14 (*d*) 18

[June 2015]

Solution : (*d*) ; Let original No. of sections = x

Total students = 36x

Again Qts.

$36x + 12 = (x+4).30$

or $36x + 12 = 30x + 120$

or $6x = 108 \Rightarrow x = 18$

Tricks: Go by choices

Q.7. A person on a tour has ₹9600 for his expenses. But the tour was extended for another 16 days, so he has to cut down his daily expenses by ₹ 20. The original duration of the tour had been ?

(*a*) 48 days (*b*) 64 days

(*c*) 80 days (*d*) 96 days

[June 2015]

Solution : (*c*) ; Let No. of tour days = x

$\therefore$ Expense per day $= \frac{9600}{x}$

Now Expense per day $= \frac{9600}{x+16}$

From $\frac{9600}{x} - \frac{9600}{x+16} = 20$

Tricks Go by choices

From here we get

For (*c*) LHS

$\frac{9600}{80} - \frac{9600}{80+16} = 20$ RHS.

$\therefore$ (*c*) is correct

Do by Calculator

Q.8. If $2^{x+y} = 2^{2x-y} = \sqrt{8}$ then the respective values of x and y are _____

(*a*) $1, \frac{1}{2}$ (*b*) $\frac{1}{2}, 1$

(*c*) $\frac{1}{2}, \frac{1}{2}$ (*d*) None of these

[June 2016]

Solution : (*a*) is Correct.

$2^{x+y} = 2^{2x-y} = \sqrt{8} = \sqrt[2]{2^3} = 2^{3/2}$

$\therefore x + y = \frac{3}{2}$ (1)

$2x - y = \frac{3}{2}$ (2)

Tricks: Then Go by Choices

(*a*) satisfies (1) & (2) both.

Q.9. Let E_1, E_2 are two linear equations in two variables x and y .(0, 1) is a solution for both the equations E_1 & E_2 (2, -1) is a solution of equation E_1 only and (-2, -1) is a solution of equation E_2 only, then E_1, E_2 are _____.

(*a*) x = 0, y = 1;

(*b*) 2x - y = -1, 4x + y = 1

(*c*) x + y = 1, x - y = -1

(*d*) x + 2y = 2, x + y = 1

[June 2016]

Solution : (*c*) is correct.

Tricks: Go by Choices

(0 ; 1) satisfies E_1 & E_2 both

(2, -1) satisfies 1st Eqn. 2 - 1 = 1 (True)

But (-2 ; -1) also satisfies E_2

i.e. -2 -(-1) = -1 (True)

Q.10. Particular company produces some articles on a day. The cost of production per article is ₹2 more than thrice the number of articles and the total cost of production is ₹800 on a day then the number of articles is :

(*a*) 16 (*b*) 14

(*c*) 18 (*d*) 15

[Dec 2016]

Solution : (*a*) is correct.

Tricks : Go by choices

Let (*A*) is correct.

So, cost per unit = 800/16 = ₹ 50

It is 2 more than 3 times of 16 (as given in Qts.)

Q.11. The sides of equilateral triangle are shortened by 3 units, 4 units, 5 units respectively then a right angle triangle is formed. The side of the equilateral triangle was

(*a*) 5 (*b*) 6

(*c*) 8 (*d*) 10

[June 2017]

Solution : Tricks : Go by Choices

For option (*c*)

1st side of right angled Δ = 8 - 3 = 5

2nd side = 8 - 4 = 4

and 3rd side = 8 - 5 = 3

Here ; 5 ; 4 and 3 are making a right angled triangle.

So, $5^2 = 4^2 + 3^2$

Hence, option (*c*) is correct.

Q.12. If $\frac{3}{x+y} + \frac{2}{x-y} = -1$ and $\frac{1}{x+y} - \frac{1}{x-y} = \frac{4}{3}$ then (x, y) is

(*a*) (2, 1) (*b*) (1, 2)

(*c*) (-1, 2) (*d*) (-2, 1)

[June 2017]

Solution : Tricks: Go by Choices

Option (*B*)

$\frac{3}{1+2} + \frac{2}{1-2} = 1 - 2 = -1$ (True)

$\frac{1}{1+2} - \frac{1}{1-2} = \frac{1}{3} + 1 = \frac{4}{3}$ (True)

So ; option (*B*) is correct.

Q.13. The line $3x + 2y = 6$ intersects the line $3x - y = 12$ in _____ quadrant:

(*a*) 1st (*b*) 2nd

(*c*) 3rd (*d*) 4th

[Dec. 2017]

Solution : (*d*), Eqn. (1) - Eqn. (2) ; we get

$$3x+2y=6$$
$$3x-y=12$$
$$-\quad +\quad -$$

$$3y=-6 \Rightarrow y=-2$$

From (1) ;

$$3x = 6-2y=6-2(-2)=10$$

$$\therefore x=\frac{10}{3}$$

∴ Co-ordinate of the point of intersection

$$=(x\ ;\ y)=\left(\frac{10}{3}\ ;-2\right)$$

It is in 4th Quadrant.

Q.14. If $2^{x+y}=2^{2x-y}=\sqrt{8}$, then the respective values of x and y .....

(*a*) $1, \frac{1}{2}$ (*b*) $\frac{1}{2}, 1$

(*c*) $\frac{1}{2}, \frac{1}{2}$ (*d*) None

[May 2018]

Solution : (*a*)

$$2^{x+y}=2^{2x-y}=\sqrt{8}=\left(2^3\right)^{1/2}=2^{3/2}$$

Tricks:- Go by Choices (GBC)

(*a*) $2^{1+\frac{1}{2}}=2^{\frac{3}{2}}=\text{RHS}$ and

$2^{2\times1-\frac{1}{2}}=2^{3/2}=\text{RHS}$

∴ (*a*) is correct

[Note :- Try to do it mentally]

Q.15. If $\frac{3}{x+y}+\frac{2}{x-y}=-1$;

$\frac{1}{x+y}-\frac{1}{x-y}=\frac{4}{3}$; then (x; y) is -

(*a*) 2 ; 1 (*b*) 1 ; 2

(*c*) -1; 2 (*d*) -2 ;1

[May 2018]

Solution : (*b*) is correct

Tricks :- Go by choices.

(*a*) $\frac{3}{2+1}+\frac{2}{2-1}=1+2=3\neq-1$

So(a) is False.

(*b*) $\frac{3}{1+2}+\frac{2}{1-(2)}=1+\frac{2}{-1}=-1$ (True)

and $\frac{1}{1+2}-\frac{1}{1-2}=\frac{1}{3}-(-1)=\frac{1}{3}+1=\frac{4}{3}$ (True)

Q.16. If the sides of an equilateral triangle are shortened by 3 units , 4 units and 5 units respectively and a right triangle is formed then the sides of an equilateral triangle is

(*a*) 6 units (*b*) 7 units

(*c*) 8 units (*d*) 10 units

[May 2018]

Solution : (*c*) is correct.

Tricks:- Go by Choices.

Check for option (*a*).

1st Side = 6 - 3 = 3

2nd Side = 6 - 4 = 2

3rd Side = 6 - 5 = 1

But $1^2+2^2\neq3^2$

So (*a*) is False.

(*c*) 1st Side = 8 - 3 = 5

2nd Side = 8 - 4 = 4

3rd Side = 8 - 5 = 3

But $3^2+4^2=5^2$ (True)

(Pythagoras Formula)

∴ (*c*) is correct.

Q.17. A number consists of two digits such that the digit in one's place is thrice the digit at ten's place. If 36 be added then the digits are reversed. Find the number .....

(*a*) 62 (*b*) 26

(*c*) 39 (*d*) None

[June 2019]

Solution : (*b*)

Tricks :- Go by choices

(*a*) $62\rightarrow2\neq3\times6$ (False)

and $62+36=98\neq26$ (False)

(*b*) 26 Clearly $6=3\times2$ (True)

and 26 + 36 = 62 (Orders of digits reversed)

So; (*b*) is correct.

Q.18. Find the roots of equation is $4^x,8^y=128$ and $3^x/27^y=1/3$

(*a*) 2, 1 (*b*) -2, 1

(*c*) 2, -1 (*d*) 1, 2

[Dec. 2019]

Solution : (*a*)

Trick: GBC

for option (a) x = 2 ; y = 1

$$4^2\times8^1=16\times8=128=\text{RHS}(\text{True})$$

and

$$\frac{3^2}{27^1}=\frac{9}{27}=\frac{1}{3}=\text{RHS}(\text{Also True})$$

Hence ; option (a) is correct.

Q.19. The cost of 2 oranges and 3 apples is ₹ 28. If the cost of an apple is doubled then the cost of 3 oranges and 5 apples is ₹ 75 The original cost of 7 oranges and 4 apples (in ₹) is

(*a*) 59 (*b*) 47

(*c*) 71 (*d*) 63

[July 2021]

Solution : (*a*) is correct

Let cost of 1 orange = x

and cost of 1 apples = y

∴ $2x + 3y = 28$ ...(1)

$3x + 5(2y) = 75$

or $3x + 10y = 75$ ...(2)

Solving (1) & (2); we get

$[2x + 3y = 28] \times 3$

$[3x + 10y = 75] \times 2$

$-\quad -\quad -$

Subtracting; or, $9y - 20y = 84 - 150$

or; $-11y = -66$

or; $y = 6$

From (1); $2x + 3 \times 6 = 28$

or $x = 5$

So $7x + 4y = 7 \times 5 + 4 \times 6 = ₹\ 59.$

Q.20. In a multiple choice question paper consisting of 100 questions of 1 mark each, a candidate get 60% marks. If the candidate attempted all questions and there was a penalty of 0.25

marks for wrong answer, the difference between number of right answers and wrong answers is :

(*a*) 32 (*b*) 36
(*c*) 40 (*d*) 38

[Dec. 2021]

Solution : (*b*)

Let No. of right answers = x

$\therefore$ No. of Wrong answers = $100 - x$

Marks obtained = 60

or $x \times 1 - (100 - x) \times 0.25 = 60$

or; $x - 25 + 0.25x = 60$

or; $1.25x = 85$

or $x = \dfrac{85}{1.25} = 68$

No. of correct Questions = 68.

No. of incorrect questions
= 100 – 68 = 32

Difference between correct and incorrect questions = 68 – 32
= 36

Q.21. Solve the following pair of Linear equations for x and y: $\left(\frac{b}{a}\right)x + \left(\frac{a}{b}\right)y = a^2 + b^2$

(*a*) $x = \frac{a}{b}, y = \frac{b}{a}$

(*b*) x = 3ab, y = -ab

(*c*) x = -ab, y = 3ab

(*d*) x = ab, y = ab

[June 2022]

Solution : Tricks : GBC (Go by choices)

For (d) x= ab; y = ab

LHS

$\left(\frac{b}{a}\right)x + \left(\frac{a}{b}\right)y$

$= \frac{b}{\cancel{a}} \times \cancel{a}b + \frac{a}{\cancel{b}} \times \cancel{a}b$

$= b^2 + a^2$ = RHS.

$\therefore$ (*d*) is correct

Q.22. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and third length is to be twice as the shortest. What is the possible length for the shortest piece?

(*a*) 22 (*b*) 20
(*c*) 15 (*d*) 18

[June 2022]

Solution : Tricks

GBC

Let (*a*) is correct

Smallest length = 22

Second length = 22 + 3 =25

3rd length = 2 × smallest

= 2 × 22 = 44

Here Total Length

= 22 + 25 + 44 = 91 (True)

$\therefore$ (*a*) is correct

Q.23. If the cost of 3 bags and 4 pens is ₹ 257 whereas the cost of 4 bags and 3 pens is ₹ 324, then the cost of one bag is:

(*a*) 8 (*b*) 24
(*c*) 32 (*d*) 75

[Dec. 2022]

Solution : Let cost of 1 bag = x

and cost of 1 Pen = y

So,

$3x + 4y = 257$ ..... (1)

and $4x + 3y = 324$ ..... (2)

Eqn. (1) + (2); We get

$7x + 7y = 581$

Or $7(x + y) = 581$

$\therefore x + y = 83$ ..... (3)

Now

Eqn. (2) – (1); We get

$x - y = 324 - 257$

or $x - y = 67$ ..... (4)

Now Eqn. (3) + (4); We get

$x + y = 83$
$x - y = 67$
$2x = 150$
or $x = 75$

$\therefore$ Cost of 1 bag = ₹ 75

(*d*) is correct

Q.24. A group of 400 soldiers posted at border area had a provision for 31 days. After 28 days 280 soldiers from this group were called back. Find the number of days for which the remaining ration will be sufficient?

(*a*) 3 (*b*) 6
(*c*) 8 (*d*) 10

[Dec. 2022]

Solution : No. of soldiers after 28 days
= 400 – 280 = 120

Remaining No. of days = 31 – 28
= 3

$\because$ 400 soldiers ration will last 3 days.

$\therefore$ 1 soldier ration will last 3 × 400 days

$\therefore$ 120 soldiers ration will last $\frac{3 \times 400}{120} =$ 10 days

(*d*) is correct

Q.25. The solution of the following system of linear equations: $2x - 5y + 4 = 0$ and $2x + y - 8 = 0$ will be

(*a*) (2, –3) (*b*) (1, –3)
(*c*) (3, 2) (*d*) (–2, 2)

[Dec. 2022]

Solution : Given eqns. are

$2x - 5y + 4 = 0$ .....(1)
$2x + y - 8 = 0$ .....(2)

Subtracting $-6y + 12 = 0$
or $6y = 12$ $\therefore y = 12$

$\therefore y = 2$

From Eqn. (2)

$2x + 2 - 8 = 0$

or $2x = 6$

$\therefore x = 3$

$\therefore$ Soln. is (3, 2)

(*c*) is correct

Tricks : Go by choices

Q.26. The largest side of a triangle is 3 times the shortest side and third side is 4 cm shorter than largest side. If the

perimeter of the triangle is at least 59 cm, what is the length of shortest side?

(*a*) Less than 7 cm

(*b*) Greater than or equal to 7 cm

(*c*) Less than 9 cm

(*d*) Greater than or equal to 9 cm

[June 2023]

Solution : Let smallest side = x

$\therefore$ Largest side = $3x$

and 3rd side = $3x - 4$

Perimeter = Sum of sides ≥ 59

or; $x + 3x + 3x - 4 \geq 59$

or; $7x \geq 59 + 4$

or; $7x \geq 63$

$\therefore x \geq 9$

(*d*) is correct.

Q.27. The age of a man is four times the sum of the ages of his two sons and after 10 years, his age will be double the sum of their ages. The present age of the man must be:

(*a*) 56 Years

(*b*) 45 Years

(*c*) 60 Years

(*d*) 64 Years

[June 2023]

Solution : Let ages of his two sons are x and y.

Father's age = z (let)

From Qts.

$z = 4(x + y)$

$\therefore x + y = \frac{z}{4}$

After 10 yrs

$z + 10 = 2(x + 10 + y + 10)$

$= 2(x + y + 20)$

$= 2\left(\frac{z}{4} + 20\right)$

or; $z + 10 = \frac{z}{2} + 40$

or $z - \frac{z}{2} = 40 - 10$

or; $\frac{z}{2} = 30 \therefore z = 60$ yrs

(*c*) is correct.

Tricks:– GBC (Go by choices)

(*c*) Sum of ages two son = $\frac{60}{4} = 15$

After 10 yrs $\Rightarrow$ Age of Father

= 60 + 10

= 70 yrs

and Sum of ages of two sons

= 15 + 10 + 10 = 35 yrs [10 yrs. increase for 1st and again 10 yrs. increase for 2nd son]

Clearly Father's Age = 2×35

= Twice of sum of son's age.

(True).

5 CHAPTER QUADRATIC EQUATION

$ax^2 + bx + c = 0$; where $a \neq 0$; a,b,c, are constants form equation is called Quadratic Equation or **Second degree equation.**

I. If $b = 0$ Then $ax^2 + c = 0$ is called **PURE Quadratic** Equation.

II. If $b \neq 0$ Then the equation. $ax^2 + bx + c = 0$ where $a \neq 0$ is called an **AFFECTED Quadratic** Equation.

Roots

The value of the variable "x" which satisfies the given equation is called its **Solution** or roots of the Quadratic Equation.

Discriminant

For Quad. Eqn. $ax^2 + bx + c = 0$.

Discriminant D $= b^2 - 4ac$.

Example

For Eqn. $3x^2 + 7x + 2 = 0$.

$a = 3; b = 7; c = 2$

Discriminant $D = b^2 - 4a\,c$

$= 7^2 - 4 \cdot 3 \cdot 2 = 49 - 24 = 25$.

III. Roots of Quad. Eqn. $ax^2 + bx + c = 0$

are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

[Remember this formula, No need to prove it.]

IV. If α and β are roots of a Quadratic Equation $ax^2 + bx + c = 0$

Then $\alpha + \beta = -\frac{b}{a}$

$\therefore$ Sum of roots $= -\frac{Co\text{-}efficient\ of\ x}{Co\text{-}efficient\ of\ x^2}$

$\alpha\ \beta = \frac{c}{a}$

$\therefore$ Product of roots $= \frac{\text{Constant terms}}{\text{Co-efficient of } x^2}$

V. If α and β are roots of a Quadratic Eqn. Then the eqn. is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2$ - (sum of roots) x + Product of roots = 0.

VI. Nature of Roots

Nature of roots of a Quad. Eqn. depends upon **Discriminant D = b^2 - 4ac.**

(A) If D > 0, Roots Real & Unequal

(*i*) D a perfect square then roots are Rational & unequal

As. $\frac{2}{3}; -\frac{2}{3}$.

(*ii*) D not a perfect Square.

Then roots are irrational & unequal and Conjugate ***As.*** $2+\sqrt{3}; 2-\sqrt{3}$.

(B) If D = 0 , Then Roots are Real & equal.

Each root $= -\frac{b}{2a}$

(C) If D < 0, Then Roots are imaginary.

VII. If one root of a quadratic Eqn. is irrational then its other root is its irrational conjugate.

Example

If one root = $3 + \sqrt{5}$

Then other root = $3 - \sqrt{5}$

(irrational conjugate)

[To find conjugate change the sign of irrational part.]

***Note*: [Tricks]**

(i) If one root is reciprocal to the other Then c = a

(ii) If one root is equal to the other but opposite in sign. Then b = 0.

Cubic Equations

1. ***Meaning of Cubic Equation***

The equation having form.

$ax^3 + bx^2 + cx + d = 0, a \neq 0,$

Where a, b, c, d are real numbers , is called a **cubic equation.**

2. **Relation Between Roots and Coefficients**

If α, γ are the roots of the cubic equation $ax^3 + bx^3 + cx + d = 0, a \neq 0$, then

(*i*) $\alpha+\beta+\gamma = \frac{-b}{a}$

(*ii*) $\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a}$

(*iii*) $\alpha\beta\gamma = \frac{-d}{a}$

3. ***The Cubic equation having roots α,β,γ is***

$x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha+\beta+\gamma\alpha)x - \alpha\beta = 0$

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. Roots of the equation $3x^2 - 14x - k = 0$ will be reciprocal of each other if:

(*a*) k= -3 (*b*) k=0

(*c*) k = 3 (*d*) k =14

[June 2010]

Solution : (*a*) is correct.

Let one root = α & Another root = $\frac{1}{\alpha}$ (given)

$\because$ Product of roots = c/a

$\therefore \alpha.\frac{1}{\alpha} = \frac{-k}{3}$

$\therefore 1 = \frac{-k}{3}$ So, k = 3

$\therefore$ (*a*) is correct

Tricks: If one root is reciprocal to other then

$a = c$

$\therefore 3 = -k$

$\therefore k = -3$

Q.2. Positive value of 'k' for which the roots at equation $12x^2 + kx + 5 = 0$ are in ratio 3 : 2, is

(a) 5/12 (b) 12/5

(c) $\frac{5\sqrt{10}}{2}$ (d) $5\sqrt{10}$

[Dec. 2010]

Solution : Let α is common in the ratio.

$\therefore$ Roots are 3α and 2α

Sum of roots $= -\frac{b}{a}$

$\therefore 3\alpha+2\alpha=5\alpha=-\frac{k}{12}$

So, $\alpha=-\frac{k}{60}$

Product of roots $= 3\alpha\times2\alpha=\frac{c}{a}$

$\therefore 6\alpha^2=\frac{5}{12}$

$\Rightarrow 6.\left(-\frac{k}{60}\right)^2=\frac{5}{12}$, So, $k^2=250$

$\therefore k=5\sqrt{10}$

$\therefore$ (*d*) is correct

Q.3. If one root of the equation $x^2-3x+k=0$ is 2, then value of k will be

(a) -10 (b) 0

(c) 2 (d) 10

[Dec. 2010]

Solution : (*c*) is correct

2 is a root of given eqn.

$\therefore 2^2-3\times2+k=0$

or -2 + k = 0

$\therefore$ k = 2

$\therefore$ (*c*) is correct

Q.4. It roots of equation $x^2+x+r=0$ are 'α' and 'β' and $\alpha^3+\beta^3=-6$. Find the value of 'r'?

(a) $\frac{-5}{3}$ (b) $\frac{7}{3}$

(c) $\frac{-4}{3}$ (d) 1

[June 2011]

Solution : (*a*) is correct.

$\alpha+\beta=\frac{-1}{1}=-1$ & $\alpha\beta=\frac{r}{1}=r$

$\therefore \alpha^3+\beta^3=-6$

or $(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=-6$

or $(-1)^3-3r(-1)=-6$

or -1 + 3r = -6; or 3r = -5

$\therefore$ r = -5/3

$\therefore$ (*a*) is correct

Q.5. If one root of the Equation $px^2+qx+r=0$ is r then other root of the Equation will be

(a) 1/q (b) 1/r

(c) 1/p (d) $\frac{1}{p+q}$

[Dec. 2011]

Solution : (*c*) Let α is another root.

$\therefore r\alpha=\frac{r}{p}\therefore\alpha=\frac{1}{p}$

Q.6. If the ratio of the root of the Equation $4x^2-6x+p=0$ is 1: 2 then the value of p is

(a) 1 (b) 2

(c) -2 (d) -1

[Dec. 2011]

Solution : (*b*) Let α is common in the ratio

$\therefore \alpha+2\alpha=\frac{-(-6)}{4}\Rightarrow\alpha=\frac{1}{2}$

$\therefore \alpha.2\alpha=\frac{p}{4}\therefore p=8\alpha^2=8.\frac{1}{4}=2$

Q.7. If p & q are the root of the Equation x^2 - bx + c = 0, then what is the Equation whose roots are (pq + p + q) and (pq - p - q) ?

(a) $x^2-2cx+c^2-b^2=0$

(b) $x^2-2cx+c^2+b^2=0$

(c) $8cx^2-2(a+c)x+c^2=0$

(d) $x^2+2bx-(c^2-b^2)=0$

[Dec. 2011]

Solution : (*a*) Eqn is

$x^2-(p+q)x+pq=0$

$\therefore$ b = p + q ; C = pq

New roots are

pq + (p + q) = c + b

& pq - (p + q) = c - b

$\therefore$ Eqn is

$x^2-(c+b+c-b)x+(c+b)(c-b)=0$

or $x^2-2cx+c^2-b^2=0$

Q.8. If arithmetic mean between roots of a quadratic equation is 8 and the geometric mean between them is 5, the equation is -

(a) $x^2-16x-25=0$

(b) $x^2-16x+25=0$

(c) $x^2-16x+5=0$

(d) None of these

[June 2012]

Solution : (*b*) Let α and β are two roots.

$(\alpha+\beta)/2=8$ and $\sqrt{\alpha b}=5$

$\Rightarrow \alpha+\beta=16$ & $\alpha\beta=25$

Eqn. is $x^2-16x+25=0$

Q.9. The minimum value of the function $x^2-6x+10$ is _____

(a) 1 (b) 2

(c) 3 (d) 10

[June 2012]

Solution : (*a*) coeff. of $x^2=1>0$; function is minimum (Formula)

$\therefore$ Minimum value $=\frac{4ac-b^2}{4a}$

$=\frac{4.1.10-(-b)^2}{4\times1}=\frac{4}{4}=1$

Q.10. If one of the roots of the equation x^2+px+a is $\sqrt{3}+2$, then the value of 'p' and 'a' is

(a) -4,-1 (b) 4,-1

(c) -4,1 (d) 4,1

[June 2012]

Solution : (*c*) Roots are

$2+\sqrt{3}$ *and* $2-\sqrt{3}$

[$\therefore$ If one root is an irrational No. then its other root is its irrational conjugate]

(conjugate of $2+\sqrt{3}$)

$\therefore$ Eqn is

x^2 - (Sum of roots) x + product of roots = 0

$x^2 - 4x + (4 - 3) = 0$

$x^2 + px + a = 0 \therefore P = -4 ; a = 1$

Q.11. Roots of equation $2x^2+3x+7=0$ are α and β. The value of $\alpha\beta^{-1}+\beta\alpha^{-1}$ is

(a) 2 (b) 3/7

(c) 7/2 (d) -19/14

[Dec. 2012]

Solution : (d) $\alpha+\beta=-\frac{3}{2};\alpha\beta=7/2$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{\frac{9}{4}-2.\frac{7}{2}}{\frac{7}{2}}=\frac{9-28}{4}\times\frac{2}{7}=\frac{-19}{14}$$

Q.12. The quadratic equation $x^2-2kx+16=0$ will have equal roots when the value of 'k' is -

(a) ±1 (b) ±2

(c) ±3 (d) ±4

[Dec. 2012]

Solution : (d) Let roots are $\alpha;\alpha$

$$\therefore \alpha+\alpha=\frac{-(-2k)}{1}\Rightarrow\alpha=k$$

$$\therefore \alpha.\alpha=k.k=16\Rightarrow k^2=16\Rightarrow k=\pm4$$

Q.13. If α,β are roots of $x^2+7x+11=0$ then the equation whose roots as $(\alpha+\beta)^2$ & $(\alpha-\beta)^2$ is___

(a) $x^2-54x+245=0$

(b) $x^2-14x+49=0$

(c) $x^2-24x+144=0$

(d) $x^2-50x+49=0$

[June 2013]

Solution : (a) is correct

$$\alpha+B=-\frac{b}{a}=-\frac{7}{1}=-7$$

$$\alpha\beta=\frac{c}{a}=\frac{11}{1}=11$$

$$\therefore(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$$

$$=(-7)^2-4\times11=5$$

Required eqn. is

$$x^2-\{(\alpha+\beta)^2+(\alpha-\beta)^2\}x+(\alpha+\beta)^2(\alpha-\beta)^2=0$$

or $x^2-(49+5)x+49\times5=0$

or $x^2-54x+245=0$

$\therefore$ (a) is correct

Q.14. If b^2-4ac is perfect square but not equal to zero then the roots of the equation $ax^2+bx+c=0$ are

(a) Real and equal

(b) Real irrational and equal

(c) Real rational and unequal

(d) Imaginary

[Dec. 2013]

Solution : (c) $b^2-4ac>0$ & perfect square

$\therefore$ Roots are real rational and unequal

Q.15. Divide 80 into two parts so that their products is maximum then the numbers are

(a) 15,65 (b) 25,55

(c) 35,45 (d) 40,40

[Dec. 2013]

Solution : (d) is correct

Let 1st part = x

$\therefore$ 2nd part = 80 -x

Let $y = x(80 - x) = -x^2+80x$

Here co. eff. of $x^2<0$

$\therefore$ y is maximum at

$$x=\frac{-b}{2a}=-\frac{80}{2(-1)}=40$$

$\therefore$ Numbers are (40 ; 40)

Tricks : Go by choices.

Q.16. The roots of equation $y^3+y^2-y-1=0$ are

(a) 1,1,-1 (b) -1,-1,1

(c) 1,1,1 (d) None

[June 2014]

Solution : (b) is correct

$y^3+y^2-y-1=0$

or $y^2(y+1)-1(y+1)=0$

or $(y+1)(y^2-1)=0$

or (y + 1) (y + 1) (y - 1) = 0

$\therefore$ y = -1 ; -1; 1.

Tricks : Go by choices.

Q.17. If α,β are the roots of the quadratic equation $2x^2-4x=1$ then the value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=$

(a) -11 (b) 22

(c) -22 (d) 11

[June 2015]

Solution : $2x^2-4x-1=0$

Let α and β are its roots

$$\alpha+\beta=\frac{-b}{a}=\frac{-(-4)}{2}=2$$

$$\alpha\beta=\frac{c}{a}=\frac{-1}{2}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{2^3-3.(-\frac{1}{2}).(2)}{(-\frac{1}{2})}=\frac{8+3}{-\frac{1}{2}}$$

$$=11\times\left(\frac{-2}{1}\right)=-22$$ So, (c) is correct

Q.18. If α,β be the roots of a quadratic equation if $\alpha+\beta=-2,\alpha\beta=-3$ Find quadratic equation:

(a) $x^2+2x-7=0$

(b) $x^2+2x-3=0$

(c) $x^2-2x-3=0$

(d) $x^2-2x+7=0$

[Dec. 2015]

Solution : (b) is correct

Quadratic Eqn. is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-(-2)x+(-3)=0$

$\therefore\ x^2+2x-3=0$

Q.19. Value of k for which roots are equal of given equation $4x^2-12x+k=0$:

(a) 144 (b) 9

(c) 5 (d) None of these

[Dec. 2015]

Solution : (b) is correct

$4x^2-12x+k=0$

$D=b^2-4.ac=0$

$=(-12)^2=4\times4.k$

Or, $144=16k \therefore k=9$

Q.20. If difference between the roots of the equation $x^2-kx+8=0$ is 4 then the value of K is

(a) 0 (b) ±4

(c) $\pm8\sqrt{3}$ (d) $\pm4\sqrt{3}$

[June 2016]

Solution : (d) is correct.

let $\alpha;\beta$ are roots of $x^2-kx+8=0$

$\therefore \alpha+\beta=-b/a=-\frac{(-k)}{1}=k$ &

$\alpha.\beta=c/a=8/1=8$

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=4^2$

$\Rightarrow k^2-4\times8=16$

or $k^2=48\Rightarrow k=\pm\sqrt{16\times3}$

$\Rightarrow k=\pm4\sqrt{3}$

(d) is correct.

Q.21. If α, β be the roots of $x^2+x+5=0$ then $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=$ ____

(a) $\frac{16}{5}$ (b) 2

(c) 3 (d) $\frac{14}{5}$

[June 2017, May 2018]

Solution : $\alpha+\beta=-\frac{b}{a}=-\frac{1}{1}=-1$

$\alpha\beta=\frac{c}{a}=\frac{5}{1}=5$

$\therefore \frac{\alpha}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$

$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$=\frac{(-1)^3-3.5.(-1)}{5}$

$=\frac{-1+15}{5}=\frac{14}{5}$

$\therefore$ (d) is correct.

Q.22. If the sum of two numbers is 13 and the sum of their squares is 85 then the numbers are :

(a) 6, 7 (b) 4, 9

(c) 10, 3 (d) 5, 8

[Dec. 2017]

Solution : (a)

Tricks : GBC (Go by choices)

6 + 7 = 13 (True)

& $6^2+7^2=36+49=85$ (True)

Q.23. The difference between the roots of the equation $x^2-7x-9=0$ is _____:

(a) 7 (b) $\sqrt{85}$

(c) 9 (d) $2\sqrt{85}$

[Dec. 2017]

Solution : (b)

Let α and β are roots.

$\therefore \alpha+\beta=-\frac{b}{a}=-\frac{-7}{1}=7$

$\alpha\beta=\frac{c}{a}=\frac{-9}{1}=-9$.

$\therefore (\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=7^2-4(-9)=85$

$\therefore \alpha-\beta=\sqrt{85}$

Q.24. The roots of the cubic equation $x^3+7x^2-21x-27=0$ is ____ :

(a) -1, 3, 9 (b) 1, -3, 9

(c) -1, 3, -9 (d) -1, -3, 9

[Dec. 2017]

Solution : (c)

Tricks: - Go by choices.

Q.25. If the roots of the equation $kx^2-3x-1=0$ are the reciprocal of the roots of the equation $x^2+3x-4=0$ then K =

(a) 4 (b) -4

(c) 3 (d) -3

[June 2018]

Solution :

Tricks : Eqn. having roots the reciprocal of the roots of $ax^2+bx+c=0$ is $cx^2+bx+a=0$ *i.e.* 1st and last term interchanges.

Eqn. is

$-4x^2+3x+1=0$

Multiplying it by (-1); we get

$4x^2-3x-1=0$

Comparing it with

$Kx^2-3x-1=0$

$\therefore K=3$

(a) is correct

Q.26. If the roots of the equation $x^3-15x^2+kx-45=0$ are in A.P., find value of k:

(a) 56 (b) 59

(c) -56 (d) -59

[June 2018]

Solution : $\because$ Roots are in A.P.

Let roots are $a-d\ ;\ a\ ;\ a+d$

So, $(a-d)+a+(a+d)=15$

or ; $3a=15$

or ; $a=5$

And Product of roots

$(a-d).a.(a+d)=45$

or $(5-d).5.(5+d)=45$

or $25-d^2=9$

or ; $d^2=25-9=16$

or ; $d=\sqrt{16}=4$

Hence ; roots are

$a-d\ ;\ a\ ;\ a+d=5-4\ ;\ 5\ ;\ 5+4$

$=1\ ;\ 5\ ;\ 9$.

The value of K

= Sum of product of two roots in a order

$=(1\times5)+(5\times9)+(9\times1)$

$=5+45+9=59$

$\therefore$ (b) is correct.

Tricks : If α ; β and γ are the roots of a cubic Eqn.

So ; Cubic Eqn. is

$x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta\gamma=0$

Given Eqn. is

$x^3-15x^2+k\ .\ x-45=0$

Comparing it,

$\alpha+\beta+\gamma=15\Rightarrow1+5+9=15$

& $\alpha\beta\gamma=45\Rightarrow1\times5\times9=45$

[Apply Hit and Trial method which can satisfy both]

Hence ; we can say ;

$\alpha=1\,;\beta=5\,;\gamma=9\,;$

$\therefore\ k=\alpha\beta+\beta\gamma+\gamma\alpha=1\times5+5\times9+9\times1=59$

Q.27. If $\alpha+\beta=-2$ and $\alpha\beta=-3$ where α and β are the roots of the equation, which is

(*a*) $x^2-2x-3=0$

(*b*) $x^2+2x-3=0$

(*c*) $x^2+2x+3=0$

(*d*) $x^2-2x+3=0$

[May 2018]

Solution : (*b*)

Quadratic Eqn. having roots α and β is

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

or ; $x^2-(-2)x+(-3)=0$

or ; $x^2+2x-3=0$

(*b*) is correct.

Q.28. When two roots of quadratic equation are $\alpha,\dfrac{1}{\alpha}$ then what will be the quadratic equation:

(*a*) $\alpha x^2-(\alpha^2+1)x+\alpha=0$

(*b*) $\alpha x^2-\alpha x^2+1=0$

(*c*) $\alpha x^2-(\alpha^2+1)x+1=0$

(*d*) None of these

[Nov. 2018]

Solution : (*a*)

Tricks : For (*a*) $\alpha.\dfrac{1}{\alpha}=1=\dfrac{c}{a}=\dfrac{\alpha}{\alpha}=1$ (True)

Q.29. Let α and β be the roots of $x^2+7x+12=0$. Then the value of $\left(\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}\right)$ will be

(*a*) $\dfrac{49}{144}+\dfrac{144}{49}$

(*b*) $\dfrac{7}{12}+\dfrac{12}{7}$

(*c*) $-\dfrac{91}{12}$

(*d*) None of the above

[Nov. 2018]

Solution : (*c*)

$x^2+7x+12=0$

or $x^2+4x+3x+12=0$

or $x(x+4)+3(x+4)=0$

or $(x+4)(x+3)=0$

$\therefore\ x=-3\,;-4$

let $\alpha=-3\,;\beta=-4$

$\therefore\ \dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}=\dfrac{9}{-4}+\dfrac{16}{-3}$

$=-\left[\dfrac{9}{4}+\dfrac{16}{3}\right]=-\dfrac{91}{12}$

IInd Method

$\alpha+\beta=\dfrac{-7}{1}=-7$

$\alpha\beta=\dfrac{c}{a}=\dfrac{12}{1}=12$

$\therefore\ \dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}=\dfrac{\alpha^3+\beta^3}{\alpha\beta}=\dfrac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$=\dfrac{(-7)^3-3\times12(-7)}{12}=-\dfrac{91}{12}$

Q.30. Find the condition that one root is double the of $ax^2+bx+c=0$

(*a*) $2b^2=3ac$ (*b*) $b^2=3ac$

(*c*) $2b^2=9.ac$ (*d*) None

[June 2019]

Solution : (*c*)

Tricks:-

Let 1st root = 1

Then 2nd root = 2

The Eqn. is

$x^2-(1+2)x+1\times2=0$

or $x^2-3x+2=0$

Comparing it with

$ax^2+bx+c=0$

we get ;

$a=1\,;\ b=-3\,;\ c=2$

Go by choices (GBC)

(*a*) $2b^2=3ac$

$2.(-3)^2=3.1.2$ (False)

(c) $2b^2=9.ac$

$\therefore\ 2.(-3)^2=9.1.2$

$\Rightarrow$ 18 = 18 (True)

Hence, Option (*c*) is (true)

Q.31. If $x=\dfrac{1}{5+2\sqrt{6}}$ then the value of the expression $x^2-10x+1$ is

(*a*) 0 (*b*) 10 (*c*) $26-12\sqrt{2}$ (*d*) $\sqrt{15}+\sqrt{3}$

[Dec. 2019]

Solution : (*a*)

$\because\ x=\dfrac{1}{5+2\sqrt{6}}=\dfrac{1}{5+2\sqrt{3}}\times\dfrac{5-2\sqrt{6}}{5-2\sqrt{6}}$ [rationalising]

$=\dfrac{5-2\sqrt{6}}{25-24}=5-2\sqrt{6}$

If one root $=5-2\sqrt{6}$

Then other root $=5+2\sqrt{6}$ [Irrational Conjugate of first]

$\therefore$ The Quadratic Eqn. is ——

x^2 - (sum of roots) x + product of roots = 0

So, $x^2-\left(5+2\sqrt{6}+5-2\sqrt{6}\right)x+\left(5+2\sqrt{6}\right)\cdot\left(5-2\sqrt{6}\right)=0$

or $x^2-10x+1=0$

(*a*) is correct.

Q.32. The three roots of equation is $x^3+9x^2-x-9=0$

(*a*) 1, -1, -9 (*b*) 1, -1, 9

(*c*) 1, 1, 9 (*d*) -1, -1, -9

[Dec. 2019]

Solution : (*a*) is correct.

Tricks: GBC

* Co-efficient of $x^3=1$ (Should be)

For (a)

* Sum of roots = Co-efficient of x^2 but opposite in sign.

$\Rightarrow 1+(-1)+(-9)=-9$

Which should be equal to the multiple of x^2 but its sign should be opposite.

i.e. +9 of the question (True)

** Product of roots = The constant term but opposite sign.

Here, Product of roots

$=1.(-1).(-9)=+9$

Which is equal to constant term "-9" but opposite in sign.

Hence ; option (a) satisfies these conditions. No. other condition is required.

So; option (a) is correct.

Detail: (*a*)

$x^3+9x^2-x-9=0$

or $x^2(x+9)-1(x+9)=0$

or $(x+9)(x^2-1)=0$

If $x+9=0\Rightarrow x=-9$

and if $x^2-1=0\Rightarrow x^2=+1$

$\therefore\ x=\pm1$

Hence ; roots $=-1\ ;\ 1\ ;\ -9$.

Q.33. Find the value of K so that x = 2 is a root of the equation $3x^2$ - 2kx + 5 = 0

(*a*) 17/4 (*b*) 4/17

(*c*) -17/4 (*d*) -4/17

[Dec. 2019]

Solution : (*a*)

$\because\ x=2$ is a root.

$\therefore\ 3.2^2-2.K.2+5=0$

or, $-4K=-17$

or, $K=\frac{17}{4}$

Q.34. If $2x^2$ - (a + 6)2x + 12a = 0 then roots are

(*a*) 4 & a^2 (*b*) 6 & a

(*c*) 3 & 2a (*d*) 6 & 3a

[Dec. 2020]

Solution : Tricks : GBC

* Product of roots $=\frac{c}{a}=\frac{12a}{2}$

$=6a$.

It is satisfied by option (*b*) & (*c*)

* sum of roots $=\frac{-b}{a}=-\frac{-(a+6)2}{2}$

$=a+6$

It is satisfied by option (*b*) only.

$\therefore$ option (*b*) is correct.

Q.35. Solving equation $3g^2$ - 14g + 16 = 0, we get roots as

(*a*) 0 (*b*) ± 5

(*c*) 8 and 2/3 (*d*) 2 and 8/3

[Dec. 2020]

Solution : Tricks : GBC

* Product of roots $=\frac{c}{a}=\frac{16}{3}$.

Options (*c*) & (*d*) satisfy it.

* sum of roots $=\frac{-b}{a}=-\frac{-14}{3}=\frac{14}{3}$

Only option (*d*) satisfies it.

because $2+\frac{8}{3}=\frac{6+8}{3}=\frac{14}{3}$ (True)

$\therefore$ (*d*) is correct.

Q.36. Solving equations m + $\sqrt{m}$ = 6/25 the value of 'm' works out to

(*a*) 2/25 (*b*) 1/25

(*c*) 3/25 (*d*) 1

[Dec. 2020]

Solution : Tricks GBC

In $m+\sqrt{m}=\frac{6}{25}$

* For $\sqrt{m}$; we should select that option, of which roots can be obtained.

So option (*b*) & (*d*) are suitable

but for (*d*) $m+\sqrt{m}$

$=1+\sqrt{1}=1+1=2\neq\frac{6}{25}$

$\therefore$ (*b*) should be correct.

For (*b*)

$m+\sqrt{m}=\frac{1}{25}+\sqrt{\frac{1}{25}}$

$=\frac{1}{25}+\frac{1}{5}=\frac{1+5}{25}$

$=\frac{6}{25}$ RHS (True)

$\therefore$ (*b*) is correct.

Q.37. The rational root of the equation $0=2p^3-p^2-4p+2$ is

(*a*) - 2 (*b*) 2

(*c*) ½ (*d*) -1/2

[Dec. 2020]

Solution :

$2p^3-p^2-4p+2=0$

or ; $p^2(2p-1)-2(2p^2-1)=0$

or $(2p-1)(p^2-2)=0$

If $2p-1=0\Rightarrow p=\frac{1}{2}$ * [Rational]

If $p^2-2=0\Rightarrow p^2=2$

$\therefore\ p=\pm\sqrt{2}$ [Irrational]

Clearly option (*c*) is correct.

Tricks : GBC

Q.38. The value of P for which the difference between the root of equation $x^2 + px + 8 = 0$ is 2 is

(*a*) ± 2 (*b*) ± 4

(*c*) ± 6 (*d*) ± 8

[Jan. 2021]

Solution : (*c*) is correct

Let α and β are

roots of Eqn.

$$x^2 + px + 8 = 0$$

$$\therefore \quad \alpha + \beta = -\frac{b}{a} = -\frac{p}{1} = -p$$

and $$\alpha\beta = \frac{c}{a} = \frac{8}{1} = 8$$

Given $\alpha - \beta = 2$

Formula

$$(\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$$

$$\Rightarrow \quad p^2 = 2^2 + 4 \times 8$$

$$= 36$$

$$\therefore \quad p = \pm\sqrt{36} = \pm 6$$

Q.39. If the quadratic equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root then p + q = ?

(*a*) 0 (*b*) 1

(*c*) -1 (*d*) 2

[Jan. 2021]

Solution : (*c*) is correct

Let α be a common root.

$$\therefore \quad \alpha^2 + p\alpha + q = 0 \qquad (1)$$

$$\alpha^2 + q\alpha + p = 0 \qquad (2)$$

Eqn. (1)—Eqn (2); we get

$$p\alpha - q\alpha + q - p = 0$$

or $$\alpha(p - q) = p - q$$

or $$\alpha = 1$$

$\Rightarrow$ Putting $\alpha = 1$ in (1);

We get

$1^2 + p \times 1 + q = 0$

$\therefore p + q = -1$

Q.40. The harmonic mean of the roots of the equation $(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$ is

(*a*) 2 (*b*) 4

(*c*) 6 (*d*) 8

[Jan. 2021]

Solution : (*b*) is correct Let α and β are roots quadratic

Equation. $(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$

Comparing it with standard form $ax^2 + bx + c = 0$

$\therefore a = 5 + \sqrt{2}$, $b = -(4 + \sqrt{5})$ and $c = 8 + 2\sqrt{5}$

We have to find H.M. of α & β

$$= \frac{2\alpha\beta}{\alpha + \beta} = \frac{2\frac{c}{a}}{-\frac{b}{a}}$$

$$= -\frac{2c}{b} = -\frac{2(8 + 2\sqrt{5})}{-(4 + \sqrt{5})}$$

$$= \frac{2.2(4 + \sqrt{5})}{(4 + \sqrt{5})} = 4$$

Q.41. If α and β are the roots of the equation $2x^2 + 5x + k = 0$, and $4(\alpha^2 + \beta^2 + \alpha\beta) = 23$, then which of the following is true?

(*a*) $k^2 + 3k - 2 = 0$

(*b*) $k^2 - 2k + 3 = 0$

(*c*) $k^2 - 2k - 3 = 0$

(*d*) $k^2 - 3k + 2 = 0$

[July 2021]

Solution: (*d*) is correct

$$\therefore \quad 2x^2 + 5x + k = 0$$

$$\therefore \quad \alpha + \beta = \frac{-5}{2} \text{ and } \alpha\beta = \frac{K}{2}$$

[Let α & β are roots]

$$\because \quad 4(\alpha^2 + \beta^2 + \alpha\beta) = 23$$

or $4[(\alpha + \beta)^2 - 2\alpha\beta + \alpha\beta] = 23$

or $$4\left[\left(\frac{-5}{2}\right)^2 - \alpha\beta\right] = 23$$

or $$4\left[\frac{25}{4} - \frac{K}{2}\right] = 23$$

or $25 - 2k = 23$

or $2k = 2 \Rightarrow k = 1$

Go by choices (GBC)

Putting k = 1 in option (d). It satisfies.

$\therefore$ (*d*) is correct

Q.42. The value of 'K' is ....., if 2 is a root of the following cubic equation: $x^3 - (k + 1)x + k = 0$

(*a*) 2 (*b*) 6

(*c*) 1 (*d*) 4

[July 2021]

Solution : (*b*) is correct

Given Equation is

$$x^3 - (K + 1)x + K = 0$$

$\because x = 2$ is its roots

So, $$2^3 - (K + 1)2 + K = 0$$

or $$8 - 2K - 2 + K = 0$$

or $$-K + 6 = 0$$

or $$K = 6$$

Q.43. If one root is half of the other of a quadratic equation and the difference in roots is *a* then the equation is

(*a*) $x^2 + ax + 2a^2 = 0$

(*b*) $x^2 - 3ax - 2a^2 = 0$

(*c*) $x^2 - 3ax + 2a^2 = 0$

(*d*) $x^2 + 3ax - 2a^2 = 0$

[Dec. 2021]

Solution: (*c*)

Let 1st root = α ; $\therefore$ 2nd root = $\alpha/2$

From question

$$\alpha - \frac{\alpha}{2} = a \Rightarrow \frac{\alpha}{2} = a \Rightarrow \alpha = 2a$$

$\therefore$ 1st root = $\alpha = 2a$.

2nd root = $\frac{\alpha}{2} = \frac{2a}{2} = a$.

The equation is

$x^2 - (2a + a)x + 2a \times a = 0$

$\Rightarrow x^2 - 3ax + 2a^2 = 0$

Q.44. If the square of a number exceeds twice of the number by 15, then number that satisfies the condition is

(a) -5 (b) 3

(c) 5 (d) 15

[Dec. 2021]

Solution: (c) is Correct.

Tricks GBC C $5^2 = 2 \times 5 + 15$ (True)

Detail

Let No. = x;

Given

$\therefore x^2 = 2x + 15$

or $x^2 - 2x - 15 = 0$

or $x^2 - 5x + 3x - 15 = 0$

or $x(x-5) + 3(x-5) = 0$

or $(x-5)(x+3) = 0$

If $x - 5 = 0 \Rightarrow x = 5$

If $x + 3 = 0 \Rightarrow x = -3$ (Invalid)

$\therefore$ No. = 5; (C) is True

Q.45. If one root of $5z^2 + 13z + y = 0$ be reciprocal of the other then the value of y is

(a) $\frac{1}{5}$ (b) $\frac{-1}{5}$

(c) 5 (d) -5

[June 2022]

Solution: Tricks

If one root of $ax^2 + bx + c = 0$

is reciprocal of other root

Then a = c

Here one root of $5z^2 + 13z + y = 0$

is reciprocal of other

So 5 = y

(c) is correct.

Q.46. If the roots of the equation $x^3 - px + q = 0$ are in the ratio 2:3, then

(a) $p^2 = 25q$ (b) $p^2 = 6q$

(c) $6p^2 = 5q$ (d) $6p^2 = 25q$

[Dec. 2022]

Solution: *Given* Eqn.

$1.x^2 - px + q = 0$

Let α is common in the ratio 2 : 3.

$\therefore$ Roots = 2α and 3α

Now Sum of roots $= 2\alpha + 3\alpha = -\frac{b}{a}$

$$\Rightarrow 5\alpha = -\frac{-p}{1} = p$$

$$\text{or } \alpha = \frac{P}{5}$$

And product of roots

$$= 2\alpha \times 3\alpha = 6\alpha^2 = \frac{c}{a}$$

$$\Rightarrow 6 \times \left(\frac{p}{5}\right)^2 = \frac{q}{1}$$

$$\text{or } \frac{6p^2}{25} = q$$

$\therefore 6p^2 = 25q$

(d) is correct

Tricks

Let roots = 2; 3

Q. Eqn. is $x^2 - (2+3)x + 2 \times 3 = 0$

or; $x^2 - 5x + 6 = 0$

Comparing it with

$x^2 - px + q = 0$

$\therefore p = 5; q = 6$

GBC

(d) $6p^2 = 25q$

$6 \times 5^2 = 25 \times 6$

$150 = 150$ (True)

$\therefore$ (d) is correct

Q.47. What will be the value of k, if the roots of the equation $(k-4)x^2 - 2kx + (k+5) = 0$ are equal?

(a) 18 (b) 20

(c) 19 (d) 21

[Dec. 2022]

Solution:

Quad. Eqn. is

$(k-4)x^2 - 2kx + (k+5) = 0$

$\because$ Roots equal

$\therefore D = b^2 - 4ac = 0$

$\Rightarrow b^2 = 4ac$

Or; $(-2k)^2 = 4 \times (k-4) \times (k+5)$

Or; $4k^2 = 4(k^2 + k - 20)$

Or; $k^2 = k^2 + k - 20$

$\therefore k = 20$ Ans.

$\therefore$ (b) is correct

Q.48. If α and β are roots of the quadratic equation $x^2 - 2x - 3 = 0$, then the equation whose roots are $\alpha + \beta$ and $\alpha - \beta$ is:

(a) $x^2 - 6x - 8 = 0$

(b) $x^2 - 6x + 8 = 0$

(c) $x^2 + 6x + 8 = 0$

(d) $x^2 + 6x - 8 = 0$

[June 2023]

Solution: $1 \times x^2 - 2x - 3 = 0$

Let α and β are its roots.

$$\therefore \alpha + \beta = -\frac{b}{a} = -\frac{-2}{1} = 2$$

$$\alpha\beta = \frac{c}{a} = \frac{-3}{1} = -3$$

$$\therefore (\alpha - \beta)^2 = (\alpha - \beta)^2 - 4\alpha\beta$$

$$= 2^2 - 4 \times (-3)$$

$$= 4 + 12$$

$$= 16$$

$$\therefore \alpha - \beta = \sqrt{16} = 4$$

Given roots are:

$\alpha + \beta = 2$ and $\alpha - \beta = 4$

$\therefore$ Eqn. is

x^2 – (Sum of roots) x + Product of roots = 0

$\therefore x^2 - (2+4)x + 2 \times 4 = 0$

$\therefore x^2 - 6x + 8 = 0$

(b) is correct.

Q.49. If α and β are roots of the equation $x^2 - (n^2+1)x + \frac{1}{2}(n^4 + n^2 + 1) = 0$, then the value of $\alpha^2 + \beta^2$ is:

(a) $2n$

(b) n^2

(c) $2n^2$

(d) n^3

[June 2023]

Solution:

$$\alpha + \beta = \frac{-b}{a} = -\frac{-(n^2+1)}{1} = n^2 + 1$$

$$\alpha\beta = \frac{c}{a} = -\frac{\frac{1}{2}(n^4+n^2+1)}{1}$$

$$= \frac{1}{2}(n^4 + n^2 + 1)$$

$\therefore \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (n^2 + 1)^2 - 2\frac{1}{2}(n^4 + n^2 + 1)$

$n^4 + 2n^2 \times 1 + 1^2 - (n^4 + n^2 + 1)$

$n^4 + 2n^2 \times 1 - n^4 - n^2 - 1$

$= n^2$

(*b*) is correct.

6 CHAPTER INEQUALITIES

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. The solution of the inequality $\frac{(5-2x)}{3} \le \frac{x}{6} - 5$ is

(*a*) $x \ge 8$ (*b*) $x \le 8$

(*c*) x=8 (*d*) none of these

[June 2010]

Solution : (*a*) is correct.

$$\frac{5-2x}{3} \le \frac{x-5}{6}$$

or $\frac{5-2x}{3} \le \frac{x-30}{6}$

or $5-2x \le \frac{x-30}{2}$

or $10-4x \le x-30$

or $10+30 \le x+4x$

or $5x \ge 40$

or $x \ge 8$

$\therefore$ option (*a*) is correct

Q.2. Solution space of inequalities $2x+y \le 10$ and $x-y \le 5$:

***(i)* includes the origin.**

***(ii)* includes the point (4,3) which one is correct?**

(*a*) Only (*i*)

(*b*) Only (*ii*)

(*c*) Both (*i*) and (*ii*)

(*d*) none of the above

[June 2011]

Solution : (*a*) is correct

Tricks : Go by choices

(0, 0) satisfies both ineqns. but (4 ; 3) does not satisfy 1st

$\therefore$ (*a*) is correct

Q.3. On the average, experienced person does 5 units work while a fresh one 3 units work daily but the employer have to maintain the output of atleast 30 units of work per day.

The situation can be expressed as.

(*a*) $5x+3y \le 30$ (*b*) $5x+3y \ge 30$

(*c*) 5x+3y=30 (*d*) None of these

[Dec. 2011 & 12]

Solution : (*b*) Let No. of experienced persons = x and No. of Freshers = y

$\therefore 5x+3y \ge 30$

Q.4. Find the range of real of x satisfying the inequalities 3x-2 > 7 and 4x-13 > 15

(*a*) x > 3 (*b*) x > 7

(*c*) x < 7 (*d*) x < 3

[June 2012]

Solution : (*b*) is correct.

$3x-2>7 \Rightarrow 3x>9 \therefore x>3$ _____(1)

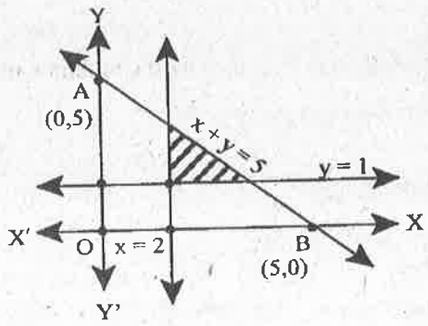
$4x>15+13 \Rightarrow 4x>28$:- x >7 ____(2)

Clearly From (1) and (2) ;

x > 7 satisfies both

$\therefore$ (*b*) is correct.

Q.5. The shaded region represents :



(*a*) x + y ≤ 5, x ≥ 2, y ≤ 1

(*b*) x + y ≤ 5, x ≥ 2, y ≥ 1

(*c*) x + y ≥ 5, x ≥ 2, y ≥ 1

(*d*) None of these

[Feb. 2008]

Solution : Tricks: Go by choices.

option (*b*)

Q.6. The union forbids the employer to employ less than 2 experienced person (x) to each fresh person (y). This situation can be expressed as

(*a*) $x \le y/2$ (*b*) $y \le x/2$

(*c*) $y \ge x/2$ (*d*) None

[June 2013]

Solution : (*b*) is correct

No. of Fresh persons for x Experienced person $= \frac{x}{2}$

$\therefore \frac{x}{2} \ge y$ (given) $\therefore y \le \frac{x}{2}$.

Q.7. The solution of the inequality 8x + 6 < 12x + 14 is

(*a*) (-2, 2) (*b*) (-2, 0)

(*c*) $(2, \infty)$ (*d*) $(-2, \infty)$

[Dec. 2013]

Solution : (*d*) is correct

8x + 6 < 12x + 14

or - 8 < 4x

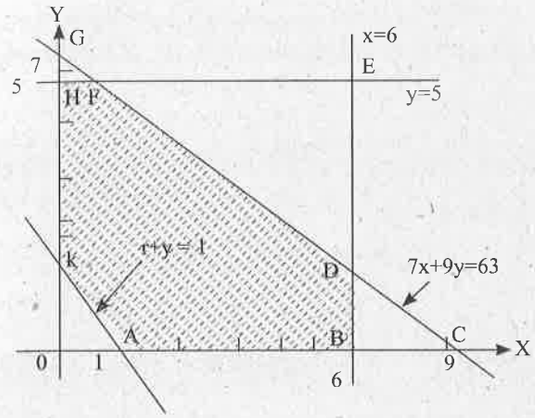
or -2 < x

x > -2

$\therefore$ Soln. is $(-2; \infty)$

Q.8. The graph of linear inequalities

$7x + 9y \le 63$; $x + y \ge 1$; $0 \le x \le 6$ *and* $0 \le y \le 6$ has been given below



(*a*) BCDB and DEFD (*b*) Unbounded

(*c*) HFGH (*d*) ABDFHKA

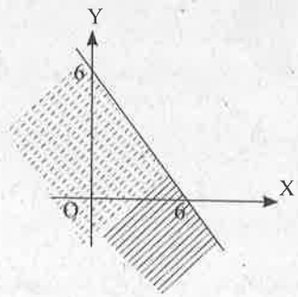
[June 2014]

Solution : (*d*) Clearly common region is ABDFHKA.

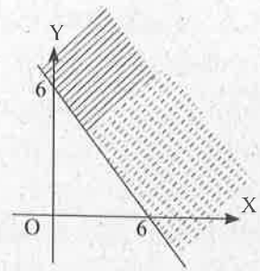
Q.9. Which of the following graph represents the in equality x + y ≤ 6 is

[Dec. 2014]

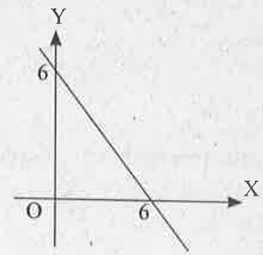
(*a*)



(*b*)

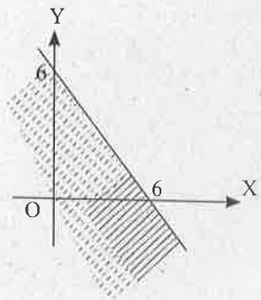


(*c*)

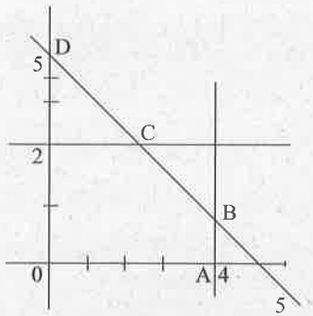


(*d*) None of these

Solution : (*a*) is correct. The graphical representation of $x + y \le 6$ is as follows :



Q.10. The graph of linear inequalities $x + y \geq 5; x + y \leq 5; 0 \leq x \leq 4$ and $0 \leq y \geq 2$ is given below:



The common region of the inequalities will be:

(*a*) OABCEO

(*b*) ECDE

(*c*) Line Segment DC

(*d*) Line Segment BC

[Dec. 2014]

Solution : (*c*)

Q.11. The common region represented by the inequalities $2x + y \geq 8, x+y \geq 12, 3x+2y \leq 34$ is

(*a*) Unbounded

(*b*) In feasible

(*c*) Feasible and bounded

(*d*) Feasible and unbounded

[June 2015]

Solution : (*c*) is correct.

$2x + y = 8 \Rightarrow$

x	0	4
y	8	0

$x + y = 12 \Rightarrow$

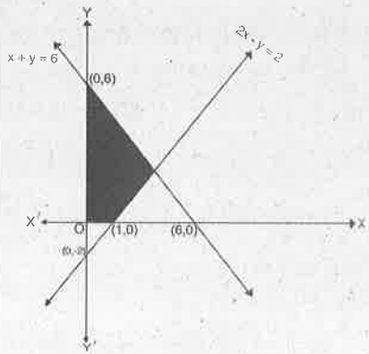
x	6	5
y	6	7

and $3x + 2y = 34$;

x	10	8
y	2	5

clearly It is Feasible and bounded.

Q.12. By lines $x + y = 6$, $2x - y = 2$, the common region shown is the diagram refers to:



(*a*) $x + y \geq 6, 2x - y \leq 2, x \geq 0, y \geq 0$

(*b*) $x + y \leq 6, 2x - y \leq 2, x \geq 0, y \geq 0$

(*c*) $x + y \leq 6, 2x - y \geq 2, x \geq 0, y \geq 0$

(*d*) None of these

[Dec. 2015]

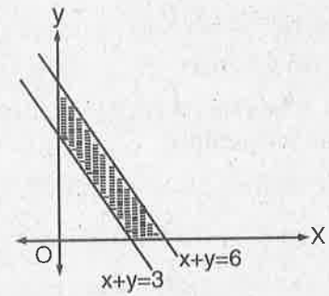
Solution : (*b*) is correct

Tricks: Go by choices

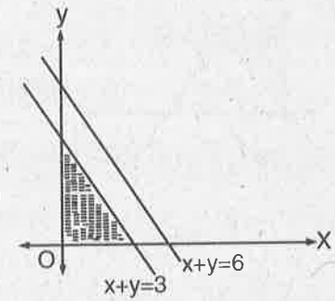
A point (1,1) (let) satisfies all inequations of (*b*).

Q.13. The common region of $x + y \leq 6; x + y \geq 3$, is for shown by shaded region

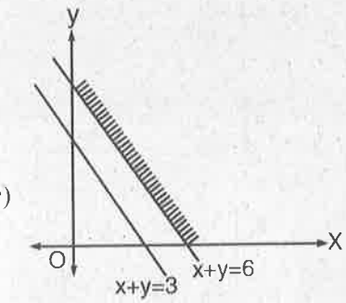
(*a*)



(*b*)



(*c*)



(*d*) None of these

[June 2016]

Solution : (*a*) is correct.

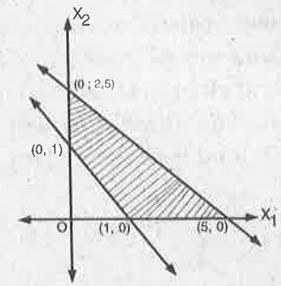
Tricks : Go by choices.

Clearly a point of the common region of option (*a*) satisfy all given constraints

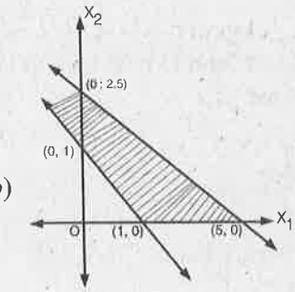
$x + y \leq 6$ & $x + y \geq 3$.

Q.14. The inequalities $x_1 + 2x_2 \leq 5; x_1 + x_2 \geq 1; x_1 \geq 0; x_2 \geq 0$ represents the region.

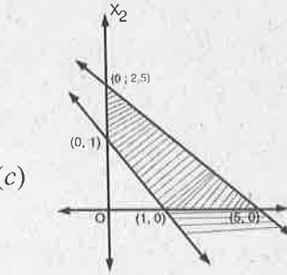
(*a*)



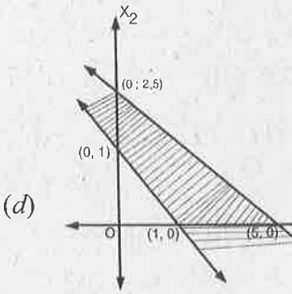
(*b*)



(*c*)



(*d*)



[Dec. 2016]

Solution : (a) is correct

Tricks : Go by choices.

Q.15. A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is atleast 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D. The vitamin content per kg. of each food is shown below:

	A	B	C	D
Food I :	2	1	1	2
Food II :	1	1	2	3

Assuming x kgs of food I is to be mixed with y kgs of food II the situation can be expressed as

(a) $2x+y\le 9$; $x+y\le 7$;
$x+2y\le 10$;
$2x+3y\le 12$; $x>0, y>0$

(b) $2x+y\ge 30$; $x+y\le 7$;
$x+2y\ge 10$;
$x+3y\ge 12$; $x\ge 0$; $y>0$

(c) $2x+y\ge 9$; $x+y\le 7$;
$x+y\le 10$;
$x+3y\ge 12$; $x\ge 0, y\ge 0$

(d) $2x+y\ge 9$; $x+y\ge 7$;
$x+2y\ge 10$;
$2x+3y\ge 12$; $x\ge 0$; $y\ge 0$

[June 2017]

Solution : Atleast $\rightarrow$ Minimum

So, use $\ge$ Sign here.

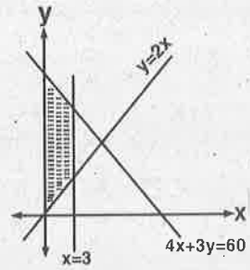
Constraints are :

$2x+y\ge 9$;
$x+y\ge 7$
$x+2y\ge 1$
$2x+3y\ge 12$
$\therefore$ (d) is correct.

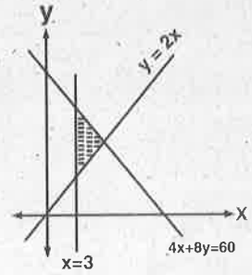
Q.16. The shaped region represented by the inequalities

$4x+3y\le 60,\ y\ge 2x,\ x\ge 3,\ x\ge 0,\ y\ge 0$

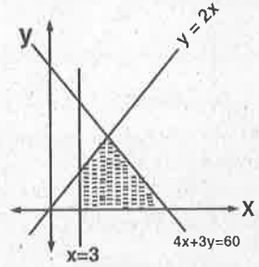
(a)



(b)



(c)



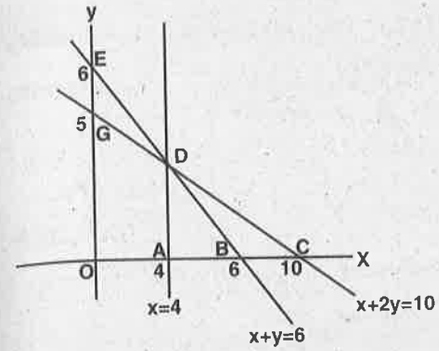
(d) None

[June 2017]

Solution : Tricks : Go by choices

Option (b) is correct.

Q.17. In the following diagram, the region represented by the inequalities $x+2y\le 10, x+y\le 6. x\le 4$ & $x\ge 0, y\ge 0$ is :



(a) OADGO (b) ADC
(c) ACD (d) DEG

[June 2018]

Solution : (a)

Tricks : Go by choices

Q.18. The linear relationship between two variables in an inequality

(a) $ax+by\le c$

(b) $ax\,.\,by\le c$

(c) $axy+by\le c$

(d) $ax+bxy\le c$

[May 2018]

Solution : (a)

Standard form of Linear Eqn. is

$ax+by=c$.

So; $ax+by\le c$ is a Linear Ineqn.

Q.19. On Solving the Inequalities $5x+y\le 100, x+y\le 60, x\ge 0, y\ge 0$, we get the following situation:

(a) $(0,0),(20,0),(10,50)$ & $(0,60)$
(b) $(0,0),(60,0),(10,50)$ & $(0,60)$
(c) $(0,0),(20,0),(0,100)$ & $(10,50)$
(d) None of these ***[Nov. 2018]***

Solution : (a)

Tricks : Go by choices

Q.20. An employer recruits experienced (x) and fresh workmen (y) under the condition that he cannot employ more than 11 people. x and y can be related by the inequality

(a) $x+y\ne 11$;
(b) $x+y\le 11, x\ge 0, y\ge 0$
(c) $x+y\ge 11, x\ge 0, y\ge 0$
(d) None of these

[June 2019]

Solution : (b)

Clearly $x+y\le 11$.

and $x\ ;\ y\ge 0$.

Q.21. The solution set of the inequations $x+2>0$ and $2x-6>0$ is

(a) $(-2,\infty)$; (b) $(3,\infty)$
(c) $(-\infty,-2)$ (d) $(-\infty,-3)$

[June 2019]

Solution : $\because\ x+2>0\Rightarrow x>-2$

$\Rightarrow x=\{-1\ ;\ 0,\ 1,\ 2,\ 3,\ 4,\ .....\}$ (1)

and $2x-6>0\Rightarrow\ x>3$

$\Rightarrow x=\{4\ ;\ 5\ ;\ 6\ ;\ 7\ ;\ .....\}$ (2)

From (1) and (2) ; we get $x=\{4, 5, 6, ...\}$ satisfies both conditions.

$\therefore$ Solution Set $=(3\ ;\ \infty)$

Q.22. The solutions of the set of inequations $2x+y \geq 12, 5x+8y \geq 74, x+6y \geq 24, x \geq 0, y \geq 0$ are

(*a*) (24,. 0), $\left(\frac{126}{11}, \frac{23}{11}\right)$, (2, 8) (0, 12)

(*b*) (0, 24), (2, 8), (0, 12), $\left(\frac{126}{11}, \frac{23}{11}\right)$

(*c*) (8, 4), (2, 8), (0, 12), (0, 24)

(*d*) (8, 4), (0, 0) (0, 6) (2, 0)

[Nov. 2019]

Solution : Tricks : GBC

* In option (*a*) & (*b*) ; all points are common except (0 ; 24).

putting $x = 0$; $y = 24$ in all eqns.; No Eqn. is satisfied by (0 ; 24).

Hence option (*b*) is not the answer.

* In option (*a*) & (*c*) ; point (8 ; 4) does not satisfy any eqn. ;

So option (*c*) is not the answer.

* In option (*a*) & (*d*) ; No point is common. Point (8 ; 4) is also present in option (*d*) which does not satisfy any eqn.

Hence ; option (*b*) ; (*c*) and (*d*) are eliminated.

So ; option (*a*) is correct.

Q.23. The solution of the inequality $\frac{(5-2x)}{3} \leq \frac{x}{6} - 5$ is

(*a*) $x \geq 8$ (*b*) $x \leq 8$

(*c*) $x = 8$ (*d*) None of these

[Dec. 2020]

Solution : (*a*) is correct.

$$\frac{5-2x}{3} \leq \frac{x-5}{6}$$

or $\frac{5-2x}{3} \leq \frac{x-30}{6}$

or $5-2x \leq \frac{x-30}{2}$

or $10-4x \leq x-30$

or $10+30 \leq x+4x$

or $5x \geq 40$

or $x \geq 8$

$\therefore$ option (*a*) is correct

Q.24. The common region in the graph of the inequalities $x+y \leq 4$, $x-y \leq 4$, $x \geq 2$ is

(*a*) Equilateral triangle (*b*) Isosceles triangle

(*c*) Quadrilateral (*d*) Square

[Jan. 2021]

Solution : For $x+y \leq 4$

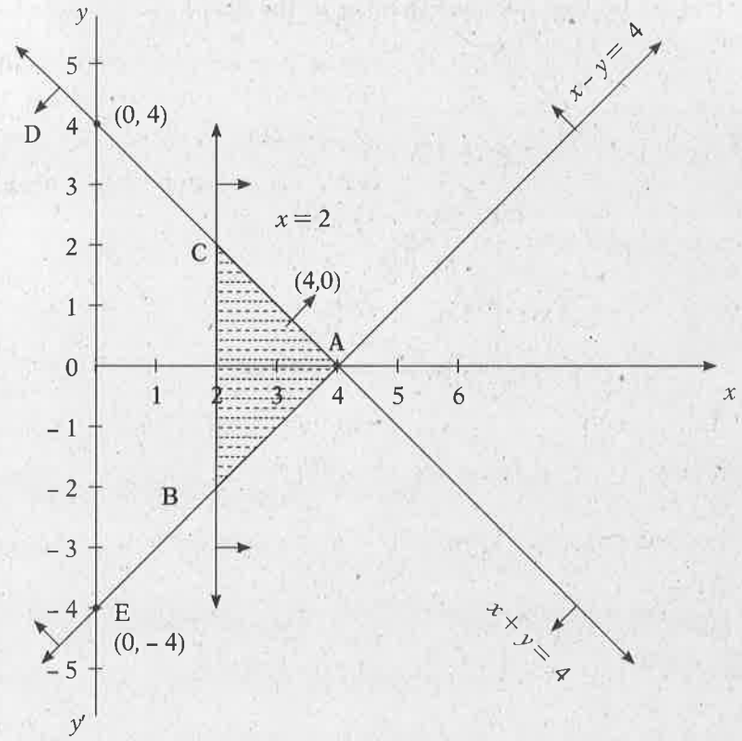
x	0	4
y	4	0

For $x-y \leq 4$

x	0	4
y	–4	0

Clearly ABC is common region. Here AB = AC

$\therefore$ Isosceles Triangle (*b*) correct



Q.25. If $y = 4+9 \sin 5x$ then which holds good?

(*a*) $-5 \leq y \leq 13$ (*b*) $-4 \leq y \leq 8$ (*c*) $0 < y < 1$ (*d*) $-5 < y < 5$

[July 2021]

Solution : (*a*) is correct

We know the value of sin θ always his between –1 and + 1

So, Minimum value of sin 5x = – 1

and Maximum value of sin 5x = + 1

$\therefore$ Minimum value of $y = 4 + 9 \sin 5x$

$= 4 + 9(-1) = -5$

And Maximum value of $4 + 9 \sin 5x$

$= 4 + 9 \times 1 = 13$

$\therefore -5 \leq y \leq 13$

Solution: Let No. of bags manufactured are x_1 & x_2 of Grade I and Grade II respectively.

$\therefore$ Total hrs taken

$= 4x_1 + 10x_2$

Available Time = 180 hrs.

i.e. Maximum Time

$\therefore 4x_1 + 10x_2 \leq 180$

$\therefore$ (*d*) is correct.

CHAPTER 7

SIMPLE INTEREST

SIMPLE INTEREST

When we borrow money from a bank or co-operative society or an individual institution, for a period of time, we pay some extra money in addition to the money borrowed, to the lender for using his money. **This extra money paid is called Interest (I),** the amount borrowed is called **Principal (P)** and the time period for which money is borrowed is called **Time (t)**. The money with interest paid back to the lender is called of **Amount (A or S)**. In other words

Amount = Principal + Interest

A = P + I

Simple Interest

The interest computed on the principal only (*i.e.* not on interest earned) for the entire period of borrowing is called **SIMPLE INTEREST**.

Type - I

Some Important Formulae.

(*i*) $I = \dfrac{P.r.t}{100}$ [when r in %]

(*ii*) I = p.r.t. [when r in decimal form]

(*iii*) $r = \dfrac{I \times 100}{pt}$

(*iv*) $t = \dfrac{I \times 100}{pr}$

(*v*) $P = \dfrac{I \times 100}{rt}$

(*vi*) A = P + I

(*vii*) I = A - P

(*viii*) $A = P\left(1+\frac{rt}{100}\right)$

Where

A = Accumulated amount
[Final value of investment]

P = Principal. [Initial value of an investment]

r = Rate of interest

t = time (years.)

I = Amount if interest

Illustrative examples

Example 1

How much interest will be earned on ₹ 2000 at 6% simple interest for 2 years.

Solution: $I=\frac{Prt}{100}=\frac{2000\times6\times2}{100}=₹\ 240$

Example 2

Sarita deposited ₹ 50,000 in a bank for 2 years with the interest rate of 5.5% p.a. what will be the final value of Investment ?

Solution: $A = P\left(1+\frac{rt}{100}\right)$

$= 50{,}000\left[1+\frac{5.5\times2}{100}\right] = ₹55{,}500$

Calculator Tricks:-

rate for 2 years = 2 × 5.5 = 11%

Amount = 50000 + 11% button = ₹55000

Example 3

Find the rate of interest if the amount owed after 6 months is ₹1050 borrowed amount being ₹ 1000.

(*a*) 5% (*b*) 10% (*c*) 15% (*d*) None

Solution :

SI = ₹ (1050 - 1000) = ₹ 50.

$r = \frac{I\times100}{pt}=\frac{50\times100\times12}{1000\times6} = 10\%$

Tricks: Go by choices

For (*b*) A = 1000 + 1000 × rate of interest of 6 months

$= 1000 + 1000\times\frac{10}{2}(\%) = ₹1050$

∴ option (*b*) is correct

Calculator Tricks:- GBC

For option (b)

$r=\frac{10}{2}=5\%$

∴ A = 1000 + 5% button (press) = 1050 (True)

Example 4

Kapil deposited some amount in a bank for $7\frac{1}{2}$ years at the rate of 6% p.a. simple interest.

Kapil received ₹ 1,01,500 at the end of the term. Compute initial deposit of Kapil.

(*a*) ₹ 70,000 (*b*) ₹ 60,000 (*c*) ₹ 80,000 (*d*) None

Solution: Tricks : Go by Choices

For option (*a*)

= 70,000 + [6 × 7.5] %

= ₹ 1,01,500 (True)

∴ Option (*a*) is correct.

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. ₹ 8, 000 becomes ₹ 10,000 in two years at simple interest. The amount that will become ₹ 6,875 in 3 years at the same rate of interest is :

(*a*) ₹ 4,850 (*b*) ₹ 5,000

(*c*) ₹ 5,500 (*d*) ₹ 5,275

[Nov. 2006]

Solution : Tricks

(*b*) S.I./year = $\frac{10000-8000}{2}=₹1000$

$r=\frac{1000\times100}{8000}=12.5\%$

$P=\frac{Amt}{Amt\ at\ ₹1}=\frac{6875}{1+0.125\times3}=₹5000$

∴ (*b*) is correct.

Q.2. The rate of simple: interest on a sum of money is 6% p.a. for first 3 years, 8% p.a. for the next five years and 10% p.a. for the period beyond 8 years. If the simple interest accrued by the sum for a period for 10 years is ₹ 1,560. The sum is :

(*a*) ₹ 1,500 (*b*) ₹ 2,000

(*c*) ₹ 3,000 (*d*) ₹ 5,000

[Feb. 2007]

Solution : (*b*) is correct.

Single S.I For 1 yrs

$= (6\times3+8\times5+10\times2)\% = 78\%$

Tricks:

$$P = \frac{Total\ S.I}{S.I\ on\ Rs1} = \frac{1560}{0.78} = ₹\ 2000$$

(*b*) is correct

Q.3. A sum of money doubles itself in 10 years. The number of years it would treble itself is :

(*a*) 25 years (*b*) 15 years

(*c*) 20 years (*d*) None

[Feb. 2007]

Solution : (*c*) is correct.

Tricks :

$$\frac{t_2}{t_1} = \frac{x_2 - 1}{x_1 - 1}$$

or $\frac{t_2}{10} = \frac{3-1}{2-1}$ or; $t_2 = 20$ yrs.

(*c*) is correct

Q.4. A certain sum of money amounts to ₹ 6,300 in two years and ₹7,875 in three years nine months at simple interest. Find the rate of interest per annum :

(*a*) 20% (*b*) 18%

(*c*) 15% (*d*) 10%

[May 2007]

Solution : (*a*) is correct.

Tricks:

S.I. Per year =

$$\frac{\text{Amount for 3.75 yrs} - \text{Amount for 2yrs}}{(3.75-2)\text{ yrs}}$$

$= ₹\ 900$

$P = 6300 - 2 \times 900 = ₹\ 4500$

$$r = \frac{900\times100}{4500\times1} = 20\%$$

Q.5. A person borrows ₹ 5,000 for 2 years at 4% p.a. simple interest. He immediately lends to another person at $6\frac{1}{4}\%$ p.a. for 2 years. Find his gain in the transaction per year:

(*a*) ₹ 112.50 (*b*) ₹ 125

(*c*) ₹ 225 (*d*) ₹ 167.50

[Nov. 2007]

Solution : (*a*) % Gain = $6\frac{1}{4}\% - 4 = 2.25\%$

S.I for 2 yrs. $= 5000\times2.25\%\times2 = ₹\ 225$

S.I per yr. $= 5000\times2.25\% = ₹\ 112.50$

$\therefore$ (*a*) is correct

Q.6. Two equal sums of money were lent at simple interest at 11% p.a. for $3\frac{1}{2}$ years and $4\frac{1}{2}$ years respectively. If the difference in interests for two periods was ₹ 412.50, then each sum is:

(*a*) ₹ 3,250 (*b*) ₹ 3,500

(*c*) ₹ 3,750 (*d*) ₹ 4,350

[Feb. 2008]

Solution : (*c*) **Tricks** : Difference in interest is due to time

$\therefore$ rate of interest for the whole duration $= (11\times4.5 - 11\times3.5) = 11\%$

$$\therefore P = \frac{\text{Total S.I}}{\text{Interest on ₹1}} = \frac{412.50}{0.11} = ₹3750$$

(*c*) is correct

Q.7. In how much time would the simple interest on a certain sum be 0.125 times the principal at 10% per annum ?

(*a*) $1\frac{1}{4}$ years (*b*) $1\frac{3}{4}$ years

(*c*) $2\frac{1}{4}$ years (*d*) $2\frac{3}{4}$ years

[June 2008]

Solution : (*a*) is correct

Tricks : $t = \frac{I/P}{r\%} = \frac{0.125}{0.10} = 1.25\,yrs$

Detail: $S.I = \frac{p.r.t}{100}$

or; $0.125P = \frac{P.10\times t}{100}$

or $t = 0.125\times10 = 1.25\,yrs$

$\therefore$ (*a*) is correct

Q.8. Find the numbers of years in which a sum doubles itself at the rate of 8% per annum.

(*a*) $11\frac{1}{2}$ (*b*) $12\frac{1}{2}$

(*c*) $9\frac{1}{2}$ (*d*) $13\frac{1}{2}$

[Dec. 2008]

Solution : (*b*) is correct

Tricks : $t = \frac{(x-1)\times100}{r}$

$= \frac{(2-1)\times100}{8} = 12.5\,yrs$

Q.9. The time by which a sum of money is 8 times of itself if it double itself in 15 years.

(*a*) 42 years (*b*) 43 years

(*c*) 45 years (*d*) 46 years

[June 2009]

Solution : (*c*) is correct

It is Compound Interest Qts.

Tricks : $2^{t_2} = 8^{15}$

or $2^{t_2} = (2^3)^{15}$: $t_2 = 45\,yrs$

Q.10. What is the rate of simple interest if a sum of money amount ₹ 2,784 in 4 years and ₹2,688 in 3 years ?

(*a*) 1% p.a. (*b*) 4% p.a.

(*c*) 5% p.a. (*d*) 8% p.a.

[June 2009]

Solution : (*b*) is correct

$$\text{S.I. pa} = \frac{\text{Difference in S.I}}{\text{Difference in time}}$$

$$= \frac{SI_2 - SI_1}{t_2 - t_1} = \frac{2784-2688}{4-3} = ₹\ 96$$

Principal $= ₹\,(2688 - 3\times96) = ₹\ 2400$

$$r = \frac{I\times100}{p\times t} = \frac{96\times100}{2400\times1} = 4\%$$

Q.11. If a simple interest on a sum of money at 6% p.a. for 7 years is equal to twice of simple interest on another sum for 9 years at 5% p.a. The ratio will be:

(*a*) 2:15 (*b*) 7:15

(*c*) 15:7 (*d*) 1:7

[June 2011]

Solution : (*c*) is correct

$$P_1 \cdot \frac{6.7}{100} = 2 \times \frac{P_2 \cdot 9.5}{100}$$

$$\text{or } \frac{P_1}{P_2} = 2 \times \frac{9 \times 5}{6 \times 7} = \frac{15}{7} \Rightarrow \frac{P_1}{P_2} = \frac{15}{7}$$

Tricks:- GBC

Q.12. By mistake a clerk, calculated the simple interest on principal for 5 months at 6.5% p.a. instead of 6 months at 5.5% p.a. If the error in calculation was ₹ 25.40. The original sum of principal was_____.

(*a*) ₹ 60,690 (*b*) ₹ 60,960

(*c*) ₹ 90,660 (*d*) ₹ 90,690

Solution : (*b*) is correct

$$P = \frac{25.40}{\frac{5.5}{100} \times \frac{6}{12} - \frac{6.5}{100} \times \frac{5}{12}}$$

$$= \frac{25.40 \times 1200}{5.5 \times 6 - 6.5 \times 5} = ₹\ 60,960$$

Q.13. If the Simple Interest on ₹1,400 for 3 years is less than the simple interest on ₹ 1,800 for the same period by ₹ 80, then the rate of interest is:

(*a*) 5.67% (*b*) 6.67%

(*c*) 7.20% (*d*) 5.00%

[Dec. 2011]

Solution : (*b*) is correct.

Tricks : $r = \frac{80 \times 100}{(1800 - 1400) \times 3}$

$= 6.67\%$

Q.14. The S.I. on a sum of money is $\frac{4}{9}$ of the principal and the No. of years is equal to the rate of interest per annum. Find the rate of interest per annum?

(*a*) 5% (*b*) 20/3%

(*c*) 22/7% (*d*) 6%

[June 2012]

Solution : (*b*)

$$\text{S.I.} = \frac{p.r.r}{100} \Rightarrow \frac{4}{9} p. = p. \left(\frac{r}{10}\right)^2$$

$$\therefore \frac{r}{10} = \frac{2}{3} \therefore r = \frac{20}{3}\%$$

Q.15. Simple interest on ₹2,000 for 5 months at 16% p.a. is_____.

(*a*) ₹133.33 (*b*) ₹133.26

(*c*) ₹134.00 (*d*) ₹132.09

[June 2012, Dec. 2012]

Solution : (*a*)

$$\text{S.I.} = 2000 \times \frac{5}{12} \times \frac{16}{100} = ₹\ 133.33$$

Q.16. How much investment is required to yield an Annual income of ₹420 at 7% p.a. Simple interest.

(*a*) ₹ 6,000 (*b*) ₹ 6,420

(*c*) ₹ 5,580 (*d*) ₹ 5,000

[Dec. 2012]

Solution : (*a*) $P = \frac{420 \times 100}{7 \times 1} = ₹\ 6000$

Calculator Tricks II GBC :

P = 420 ÷ 7% button = ₹ 6000

Q.17. Mr. X invests ₹ 90,500 in post office at 7.5% p.a. simple interest. While calculating the rate was wrongly taken as 5.7% p.a. The difference in amounts at maturity is ₹ 9,774. Find the period for which the sum was invested.

(*a*) 7 years (*b*) 5.8 years

(*c*) 6 years (*d*) 8 years

[Dec. 2012]

Solution : (*c*) **Tricks**

$$t = \frac{9774 \times 100}{90{,}500 \times (7.5 - 5.7)} = 6 \text{ yrs}$$

Q.18. If the sum of money when compounded annually become ₹1140 in 2 years and ₹1710 in 3 years at rate of interest

(*a*) 30% (*b*) 40%

(*c*) 50% (*d*) 60%

[June 2013]

Solution : (*c*) Interest in 3rd

yr = ₹ 1710 - ₹1140

= ₹ 570

Tricks Note : For 3rd yr ; it will be like S.I

$$r = \therefore \frac{I \times 100}{P.t} = \frac{570 \times 100}{1140 \times 1} = 50\%$$

Tricks II Go by choices.

For (*c*) A = 1140 + 50% (Calculator)

= ₹ 1710

$\therefore$ (*c*) is correct

Q.19. In what time will a sum of money double itself at 6.25% p.a. at simple interest

(*a*) 5 yrs (*b*) 8 year

(*c*) 12 yrs (*d*) 16 yrs

[Dec. 2013]

Solution : (*d*) is correct.

Tricks :

$$t = \frac{(2-1) \times 100}{6.25} = 16 \text{ years}$$

Q.20. What principal will amount to ₹370 in 6 years at 8% p.a. at simple interest

(*a*) ₹ 210 (*b*) ₹ 250

(*c*) ₹310 (*d*) ₹ 350

[Dec. 2013]

Solution : (*b*) is correct

Tricks : $P = \frac{370}{1 + 6 \times 0.08} = ₹\ 250$

Calculator Tricks :- GBC

(b) Amt = 250 + (6 × 8)% button press = 370

Q.21. If a sum triples in 15 yrs at Simple rate of interest then the rate of interest per annum will be

(*a*) 13.0% (*b*) 13.3%

(*c*) 13.5% (*d*) 18%

[June 2014]

Solution : (*b*) is correct

Tricks $r = \frac{(3-1) \times 100}{1 \times 15} = 13.3\%$

Calculator Tricks :- GBC

(b) r = 15 × 13.333% = 200%

A = 1 + 200% (button) = 3

$\therefore$ (*b*) is correct

Q.22. A certain sum of money was invested at simple rate of interest for three years. If it was invested at 7% higher, the interest have been ₹ 882 more, then sum has been invested at that rate was

(a) ₹ 12,600 (b) ₹ 6,800
(c) ₹ 4,200 (d) ₹ 2,800

[Dec. 2014]

Solution : (c) is correct

S.I = ₹882 for r = 7%

t = 3 years.

$$P = \frac{I \times 100}{rt} = \frac{882 \times 100}{7 \times 3} = ₹\,4200$$

Calculator Tricks :- GBC

Q.23. A sum of money will be doubled itself in 8 years at S.I. In how many years the sum will be tripled itself ?

(a) 20 years (b) 12 years
(c) 16 years (d) None

[June 2015]

Solution : (c) is correct.

Tricks : $\frac{t_2}{8} = \frac{3-1}{2-1}$

$t_2 = 16$ yrs.

Q.24. A sum of 44,000 is divided into 3 parts such that the corresponding interest earned after 2 years, 3 years and 6 years may be equal at the rate of simple interest are 6% p.a. 8% p.a. & 6% p.a., respectively. Then the smallest part of the sum will be.

(a) ₹ 4,000 (b) ₹ 8,000
(c) ₹ 10,000 (d) ₹ 12,000

[June 2015]

Solution : (b) is correct.

Tricks: $P_1 : P_2 : P_3 = \frac{1}{r_1 t_1} : \frac{1}{r_2 t_2} : \frac{1}{r_3 t_3}$

$= \frac{1}{2\times6} : \frac{1}{8\times3} : \frac{1}{6\times6}$

$= \left[\frac{1}{12} : \frac{1}{24} : \frac{1}{36}\right] \times 72$ LCM of denominators

= 6:3:2

So, Smallest principal

$= \frac{44000}{6+3+2} \times 2 = ₹8000$

Q.25. No. of years a sum of money becomes 4 times itself at 12% p.a. at simple interest:

(a) 20 (b) 21
(c) 25 (d) 30

[Dec. 2015]

Solution : (c) is correct

Tricks: $t = \frac{(4-1)\times100}{1\times12} = 25\,yrs.$

Q.26. If a person lends ₹ 6,000 for 4 years and ₹ 8,000 for 3 years at S.I. The total interest earned is ₹ 2400 then the rate of interest is.....

(a) 5% (b) 6%
(c) 7% (d) 8%

[Dec. 2016]

Solution : (a) is correct.

Tricks: Go by choices.

For (a) ;

Total SI = 6000×4×5% + 8000×3 ×5%

= ₹2400

So, (a) is correct.

Q.27. In simple interest, a certain sum becomes ₹ 97,920 in 3 years, and ₹ 1,15,200 in 5 years, then the rate of interest is:

(a) 10% (b) 11.2%
(c) 12% (d) 13.6%

[June 2018]

Solution : (c)

Tricks :

$$\text{S.I p.a.} = \frac{1,15,200 - 97,920}{5-3}$$

= ₹8640

Principal = 97,920 - 3 yrs interest = 97,920 - 3 × 8640 = ₹72,000

$$r = \frac{8640\times100}{72000} = 12\%$$

Calculator Tricks :

Amounts = 72000 + (12×3 = 36) % button = ₹ 97,920 (True)

So, option (C) is correct.

Q.28. A person borrows ₹ 5,000 for 2 years at 4% per annual simple interest. He immediately lends to another person at $6\frac{1}{4}\%$. Per annual for 2 years find his gain in the transaction .

(a) ₹ 112.50 (b) ₹ 225
(c) ₹ 125 (d) ₹ 107.50

[May 2018]

Solution : (b)

Interest Gain = $\left(6\frac{1}{4} - 4\right)$

$= 2\frac{1}{4} = 2.25\%$

So, Interest Gain

$= \frac{5000\times2\times2.25}{100} =$ ₹ 225.

Q.29. A certain money doubles itself in 10 years when deposited on simple interest. It would triple itself in

(a) 30 years (b) 20 years
(c) 25 years (d) 15 years

[Nov. 2018]

Solution : (b)

Tricks : See Simple Interest (Quicker BMLRS)

$$\frac{t_2}{t_1} = \frac{x_2 - 1}{x_1 - 1}$$

$$\Rightarrow \frac{t_2}{10} = \frac{3-1}{2-1} \Rightarrow t_2 = 20\,yrs.$$

Q.30. A certain sum of money Q was deposited for 5 year and 4 months at 4.5% simple interest and amounted to ₹ 248, then the value of Q is

(a) ₹ 240 (b) ₹ 200
(c) ₹ 220 (d) ₹ 210

[Nov. 2018]

Solution : (b)

t = 5 yrs 4 months = $5 + \frac{4}{12} = \frac{16}{3}\,yrs$

$$A = Q\left(1 + \frac{rt}{100}\right)$$

or $248 = Q\left[1 + \frac{4.5}{100} \times \frac{16}{3}\right]$

$\therefore Q = \frac{248\times300}{372} = \text{Rs.}200$

Tricks : GBC

Rates for 5 yrs 4 Months = 5 × 4.5 %+ one third of 4.5% = 24%

Note : 4 months means one third of one year, so rate for 4 months = one third of one year interest rate.

(*b*) Amounts = 200 + 24% = 248 (True)

So, (*b*) is correct.

Q.31. The certain sum of money became ₹ 692 in 2 yrs. and ₹ 800 in 5 yrs. then the principle amount is

(*a*) ₹ 520 (*b*) ₹ 620

(*c*) ₹ 720 (*d*) ₹ 820

[June 2019]

Solution : (*b*)

Tricks:- If a certain sum of money becomes A_1 in t_1 years and A_2 in t_2 years then

$$\text{S.I. per annum} = \frac{A_2 - A_1}{t_2 - t_1}$$

$$\therefore \text{S.I. p.a} = \frac{800 - 692}{5 - 2}$$

$= ₹ 36.$

$\therefore$ Principal = A - Interest

= 692 - Interest of 2 yrs.

$= 692 - 2 \times 36 = ₹ 620.$

(b) is correct.

Q.32. A sum of money amount to ₹ 6,200 in 2 years and ₹ 7,400 in 3 years as per S.I. then the principal is

(*a*) ₹ 3,000 (*b*) ₹ 3,500

(*c*) ₹ 3,800 (*d*) None

[June 2019]

Solution : (*c*)

Tricks:- S.I. p.a $= \frac{7400 - 6200}{3 - 2}$

$= ₹ 1200.$

$\therefore$ Principal $= 6200 - 2 \times 1200$

$= ₹ 3800.$

Q.33. P = ₹ 5,000; R = 15%; T = $4\frac{1}{2}$

using $I = \frac{PTR}{100}$ then I will be $I = \frac{Prt}{100}$

(*a*) ₹ 3,375 (*b*) ₹ 3,300

(*c*) ₹ 3,735 (*d*) None

[June 2019]

Solution : (*a*)

$$I = \frac{5000 \times 15 \times 4.5}{100} = 3375$$

[Use Calculator ; Never Write]

Q.34. In simple interest if the principal is ₹ 2,000 and the rate and time are the roots of the equation $x^2 - 11x - 30 = 0$ then simple interest is

(*a*) ₹ 500 (*b*) ₹ 600

(*c*) ₹ 700 (*d*) ₹ 800

[June 2019]

Solution : (*b*)

$\because x^2 - 11x + 30 = 0$

or $x^2 - 5x - 6x + 30 = 0$

or $x(x-5) - 6(x-5) = 0$

or $(x-5)(x-6) = 0$

$\therefore x = 5 ; 6$

If r = 5% then t = 6 yrs.

$$\therefore \text{S.I} = \frac{Prt}{100} = \frac{2000 \times 5 \times 6}{100}$$

$= ₹ 600.$

(*b*) is correct.

Q.35. If the interest of a money is equal to its one by nine, the rate of interest and time are equal then find rate of interest is.

(*a*) $3\frac{1}{3}\%$ (*b*) 4½%

(*c*) 3% (*d*) 3.5%

[Dec. 2019]

Solution : (*a*)

Let Principal = ₹ 1

$$\text{S.I} = \frac{1}{9}$$

Given ; r = t

$$\therefore \text{S.I} = \frac{P.r.t}{100}$$

$$\text{or } \frac{1}{9} = 1.\frac{r.r.}{100}$$

$$\text{or ; } r^2 = \frac{100}{9} \Rightarrow r = \frac{10}{3} = 3\frac{1}{3}\%$$

Q.36. 1/7 of a money is deposited at 4% per annum, 1/2 of a money deposited at 5% per annum and the remaining at the rate of 6%, then total interest gained ₹ 730 find deposit amount is

(*a*) ₹ 14000 (*b*) ₹ 215500

(*c*) ₹ 212800 (*d*) ₹ 214500

[Dec. 2019]

Solution : (*a*)

Calculator Tricks : GBC

For (*a*) S.I. =

$\left(\frac{1}{7} \times 14000 = 2000\right) \times 4\%$ button *i.e.* type 2000 then press multiply button then type 4 then percentage button [never press any other button]

then $\left(\frac{1}{2} \times 14000 = 7000\right) \times 5\%$ button *i.e* type 7000 then press multiply button then type 5 then percentage button and $(14000 - 2000 - 7000) \times 6\%$ button *i. e.* type 5000 then press multiply button then type 6 then percentage button

then Press GT button ; we get ₹ 730.

$\therefore$ (*a*) is correct.

Q.37. Ram deposited ₹ 12000 in a bank at 10% per annum and remaining amount deposit in other bank at 20% per annum, if he received interest according to 14% per annum find the Ram's amount.

(*a*) ₹ 20000 (*b*) ₹ 22000

(*c*) ₹ 30000 (*d*) ₹ 25000

[Dec. 2019]

Solution : (*a*)

Tricks : GBC

For (*a*) S.I on ₹ 20,000

$= 20,000 \times 14\%$

$= ₹ 2800.$

S.I. on ₹ 12,000 = 12000 × 10%

$= ₹ 1200.$

Rest principal = 20000 - 12000

$= ₹ 8000.$

S.I. on rest money = 8000× 20%

= 1600.

Sum of these S.I.

= 1200 + 1600 = 2800 (True)

$\therefore$ (*a*) is correct.

Q.38. If the difference between interest received by two persons A and B on the same sum of ₹1500 for 3 years is ₹ 18. Then what is the difference between the two rates of interest.

(*a*) 1% (*b*) 2.5%

(*c*) 3% (*d*) 0.4%

[Dec. 2019]

Solution : (*d*)

Principal same ; time same but interest differ by ₹ 18.

It means rate differs.

$$\therefore \quad r = \frac{I \times 100}{p \times r}$$

Here ; r = difference between rates.

$$\therefore \quad r = \frac{18 \times 100}{1500 \times 3} = 0.4\%$$

Detail : Let their rates are r_1 and r_2

$$\therefore \quad \frac{1500 \times r_1 \cdot 3}{100} - \frac{1500 \times r_2 \cdot 3}{100} = 18$$

$$\text{or} \quad \frac{\overset{15}{\cancel{1500}} \times 3}{\cancel{100}}(r_1 - r_2) = 18$$

$$\text{or} \quad r_1 - r_2 = \frac{18}{15 \times 3} = 0.4\%$$

Q.39. If the compound interest on a certain sum for 2 years at 3% p.a. is ₹ 1015. What would be the simple interest on the sum at the same rate and same time is

(*a*) 1005 (*b*) 1010

(*c*) 1000 (*d*) 1003

[Dec. 2019]

Solution : (*c*)

Tricks : GBC

For (*c*) S.I. for 2 yrs.

= ₹ 1000.

$\therefore$ S.I for 1 yr. = ₹ 500. *i.e.* $(1000 \div 2)$

C.I for 1st yr. = S.I of 1st yr. = ₹ 500.

C.I for 2nd yr. = S.I for 1st yr + Interest on this S.I

$$= 500 + \frac{500 \times 3 \times 1}{100} = \text{Rs. } 515.$$

$\therefore$ Total C.I = 500 + 515 = ₹ 1015 [True]

Calculator Tricks for C.I

C.I. = $500 + (500 + 5\%\,\text{button})$

= ₹ 1015 **(True)**

Q.40. What sum of money will produce ₹ 42,800 as an interest in 3 years and 3 months at 2.5% p.a simple interest ?

(*a*) ₹ 3,78,000 (*b*) ₹ 5,26,769

(*c*) ₹ 4,22,000 (*d*) ₹ 2,24,000

[Dec. 2020]

Solution : time = t = 3 years & 3 months

$$= 3 + \frac{3}{12} = 3.25 \text{ years}$$

Formula

$$P = \frac{1 \times 100}{t \times r} = \frac{42{,}800 \times 100}{(3.25) \times (2.5)}$$

= ₹ 5,26,769.23 = ₹ 5,26,769/-

(*b*) is correct.

Calculator Tricks

P = 42,800 ÷ 2.5% button ÷ (3.25)

= ₹ 5,26,769

Q.41. A certain sum amounted to ₹ 575 at 5% in a time in which ₹ 750 amounted to ₹ 840 at 4%. If the rate of interest is simple, find the sum—

(*a*) 525 (*b*) 550

(*c*) 515 (*d*) 500

[Jan. 2021]

Solution : (*d*) is correct S.I. = A – P = 840 – 750 = ₹ 90

$$t = \frac{I \times 100}{p.r.} = \frac{90 \times 100}{750 \times 4} = 3 \text{ years}$$

TRICKS

Then use GBC (Go by choices) with calculator.

[$\therefore$ 1 year interest rate = 5%

$\therefore$ 3 years interest rate = 3 × 5 = 15%]

(*a*) A = 525 + (3 × 5)% = 603.75 ≠ 575

(*a*) wrong

Amount ₹ 603.75 is more than 575

So, principal must be less than ₹ 525.

For option (*c*)

A = 515 + (3 × 5)% = 592.25 ≠ 575

So option (*d*)

A = 500 + 15% = 575 (True)

(*d*) is correct.

Q.42. A man invested one-third of his capital at 7% one fourth at 8% and the remainder at 10%. If the annual income is ₹ 561. The capital is -

(*a*) ₹ 4,400 (*b*) ₹ 5,500

(*c*) ₹ 6,600 (*d*) ₹ 5,800

[Jan. 2021]

Solution : (*c*) is correct

Tricks

Remainder

$$= 1 - \frac{1}{3} - \frac{1}{4} = \frac{12 - 4 - 3}{12} = \frac{5}{12}$$

Single rate of interest

$$= \left(7 \times \frac{1}{3} + 8 \times \frac{1}{4} + 10 \times \frac{5}{12}\right)\%$$

[Do by using calculator]

= 8.5%

Annual income = Annual interest = ₹ 561

Capital = Interest ÷ rate of interest

= 561 ÷ 8.5% (button)

= ₹ 6600

$\therefore$ (*c*) is correct

Q.43. Certain sum amounts to ₹ 15748 in 3 years at simple interest at r% p.a. The same sum amounts to ₹ 16,510 at (r+2)% p.a. simple interest in the same time. What is the value of r?

(*a*) 10% (*b*) 8%

(*c*) 12% (*d*) 6%

[July 2021]

Solution : (*b*) is correct

Due to increase in interest by 2% in 3 years

Interest increase

= 16510 – 15748

= ₹ 762

$$\text{So, } P = \frac{I \times 100}{r.t.} = \frac{762 \times 100}{2 \times 3} = ₹\ 12{,}700$$

Then apply tricks GBC

(*a*) A = 12700 + (3 × 10% = 30%) button ≠ 15748

So (*a*) Wrong

(*b*) A = 12700 + (3 × 8% = 24%) button = ₹ 15748 (True)

So (*b*) correct

Q.44. Two equal amounts of money are deposited in two banks each at 15% p.a. S.I. fix 3.5 year in the bank and fix 5 years respectively. The difference between the interest amount from the bank is ₹ 144. Find the sum

(*a*) ₹ 620 (*b*) ₹ 640

(*c*) ₹ 820 (*d*) ₹ 840

[Jan. 2021]

Solution : (*b*) is correct

Let Each deposit = P

Tricks

Rate of interest for 3.5 years

= 3.5 × 15 = 52.5% pa S.I.

Rate of interest for 5 years = 5 × 15 = 75% pa S.I.

Difference in rate of interest

= 75 – 52.5% = 22.5% pa S.I.

∴ P = Interest difference ÷ Rate of interest difference

= 144 ÷ 22.5% (button) = ₹ 640.

Detail each

Let Principal money = P

From Question

$$\frac{P\cdot 15\times 5}{100}-\frac{P\cdot 15\times 3.5}{100}=144$$

or $\frac{P}{100}$ [75 - 52.5] = 144

or $P=\frac{144\times 100}{22.5}$ = ₹ 640

(*b*) is correct

Q.45. A sum of money in simple interest doubles itself in 7 years. How many years will it take to triple itself?

(*a*) End of 12 years

(*b*) End of 14 years

(*c*) End of 18 years

(*d*) End of 16 years ***[Dec. 2021]***

Solution : (*b*)

Tricks

A certain sum of money becomes x_1 times in t_1 years and x_2 times in t_2 years at same rate of S.I. The relationship is

$$\frac{t_2}{t_1}=\frac{x_2-1}{x_1-1}$$

$$\therefore \frac{t_2}{7}=\frac{3-1}{2-1}=t_2=7\times 2=14 \text{ years.}$$

Q.46. Simple interest on a sum of money is amount to ₹ 59,000 in 3 years and ₹ 62,000 in 4 years at same rate of interest. What are the principal amount and rate of interest?

(*a*) ₹ 50,000, 6%

(*b*) ₹ 45,000, 5.5%

(*c*) ₹ 55,000, 5%

(*d*) ₹ 52,000, 7% ***[Dec. 2021]***

Solution : (*a*)

Tricks

$$\text{S.I. Pa}=\frac{62000-59000}{4-3}=₹\ 3{,}000/-$$

Principal

P = 59000 -3 × SI Pa

= 59000-3 × 3000 = ₹ 50,000/-

Rate of Interest

$$=r=\frac{I\times 100}{P.t}=\frac{3000\times 100}{50{,}000\times 1}$$

= 6%

Q.47. An amount is lent at R% simple interest for R years and the simple interest amount was one fourth of the principal amount. Then it is _____

(*a*) 5 (*b*) 6

(*c*) 5½ (*d*) 6½

[Dec. 2021]

Solution : (*a*)

Let Principal = P

$$\text{Interest}=\frac{1}{4}P$$

$$\therefore \text{rate}=\frac{I\times 100}{\cdot P.t}$$

$$\text{or } R=\frac{\frac{1}{4}P\times \cancel{100^{25}}}{P\times R}$$

or; $R^2 = 25 \Rightarrow R = 5\%$

Q.48. An investor is saving to pay off an obligation of ₹ 15,250 which will be due in seven years, if the investor is earning 7.5% simple interest rate per annum, he must deposit ₹ _____ to meet the obligation.

(*a*) 8,000 (*b*) 9,000

(*c*) 10,000 (*d*) 11,000

[June 2022]

Solution : FV = ₹ 15,250

S.I rate for 1 yr = 7.5%

∴ S.I for 7 yrs = 7 × 7.5

= 52.5 %

Tricks

$$PV=P=\frac{A}{1+\frac{rt}{100}}=\frac{15250}{1+\frac{52.5}{100}}$$

$$=\frac{15250}{1.525}$$

= ₹ 10,000

∴ (*c*) is correct

Q.49. The annual rate of simple interest is 12.5%. In how many years does the principal double?

(*a*) 11 years (*b*) 9 years

(*c*) 8 years (*d*) 7 years

[June 2022]

Solution : Tricks

$$t=\frac{(m-1)\ 100}{r}$$

$$\frac{(2-1)\times 100}{12.5}=8 \text{ yrs.}$$

Calculator Tricks

(2–1) ÷ 12.5% button = 8 yrs

(*c*) is correct.

Q.50. A farmer borrowed ₹ 3600 at the rate of 15% simple interest per Annum. At the end of 4 years. He cleared this account by paying ₹ 4000 and a cow. The cost of the cow is :

(*a*) ₹ 1000 (*b*) ₹ 1200

(*c*) ₹ 1550 (*d*) ₹ 1760

[Dec. 2022]

Solution: *Given :*

Let *P* = Principle = ₹ 3600

r = rate of interest = 15%

t = time = 4 years

Simple Interest

$$= \frac{p \times r \times t}{100} = \frac{3600 \times 15 \times 4}{100}$$

$= ₹\,2160$

Total amount payable after 4 years

$= 3600 + 2160 = ₹\,5760$

Which is equal to ₹ 4000 + cost of 1 cow

$\therefore$ Cost of 1 cow = 5760 – 4000

$= ₹\,1760$

$\therefore$ (*d*) is correct.

Q.51. Mr. Ram invested a total of ₹ 1,00,000 in two different banks for a fixed period. The first bank yields an interest of 9% per annum and second, 11% per annum. If the total interest at the end of one year is 9.75% per annum, then the amount invested in these banks are respectively:

(*a*) ₹ 52,500, ₹ 47,500

(*b*) ₹ 62,500, ₹ 37,500

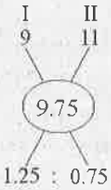
(*c*) ₹ 57,500, ₹ 42,500

(*d*) ₹ 67,500, ₹ 32,500

Solution :

Trick-I

Investment ratio =



125 : 75 ÷ 25

5 : 3

$\therefore$ Investment in 1st bank

$$= \frac{1,00,000}{5+3} \times 5 = 62,500$$

$$\text{in 2nd bank} = \frac{1,00,000}{5+3} \times 3 = 37,500$$

(*b*) is correct.

Trick-II

GBC

Total interest = 1,00,000 × 9.75%

$= ₹\,9750$

(*b*) Total Interest (by Calculator)

62,500 × 9% =

37,500 × 11% = (GT button)

Press = ₹ 9,750 (True)

$\therefore$ (*b*) is correct.

8 CHAPTER COMPOUND INTEREST

Definition :- Compound interest (or compounding interest) is interest calculated on the initial principal and which also includes all of the accumulated interest of previous periods of a deposit or loan, the computed interest is called **Compound Interest.**

Conversion Period:- The period at the end of which the interest is computed is called **Conversion period.**

Description	Conversion period	No. of Conversion periods in 1 year = m	Rate of interest of a conversion period $i = \frac{r}{100m}$
10% compounded yearly	1 year	m = 1	i = 10/100
10% compounded half-yearly	6 months	m = 2	i = 10/200
10% compounded Quarterly	3 months	m = 4	i = 10/400
10% compounded monthly	1 months	m = 12	i = 10/1200

Formula Compound Amount = $A = P\left(1+\frac{r}{100m}\right)^{mt}$

$= P\,(1 + i)^n$

Where $i = \frac{r}{100m}$ & n = mt

Compound Interest = $P\left[(1+i)^n - 1\right]$

Type - I (To find Amount & Compound Interest)

Working Rule:-

(*i*) If rate of interest compounded yearly then divide r by 100 *i.e.* $i = \frac{r}{100}$.

(*ii*) If rate of interest compounded $\frac{1}{2}$ yearly Then divide r by 200 *i.e.* $i = \frac{r}{200}$.

(*iii*) If rate of interest compounded $\frac{1}{4}$ yearly then $i = \frac{r}{400}$

(*iv*) If rate of interest compounded monthly then $i = \frac{r}{1200}$ and so on.

CALCULATOR TRICKS

Step-I Type r then push ÷ button then type 100 if r is yrly.; 200 if r is half yearly and so on.

Step-II Press + 1.

Step-III Press × button

Step-IV Continue pressing "=" button (n-1 = power - 1) times.

Step -V Press × button then type principal value then push = button.

We will get the result of amount on calculator screen.

Step -VI Press - button then type principal value, we will get the value of compound Interest on the screen.

Ex.1 ₹ 2000 is invested at annual rate of interest of 10%. What is the amount after 2 years if compounding is done.

(*a*) Annually (*b*) Semi-annually

(*c*) Quarterly (*d*) Monthly.

Solution : (*a*) Given that P = ₹ 2000; r = 10% yrly.; t = 2 years.

$$i = \frac{r}{100} = \frac{10}{100} = 0.10$$

$$n = mt = 1 \times 2 = 2$$

[∵ m = No. of conversion periods in a year = 1]

$$\therefore A = P(1+i)^n$$

$$= 2000\left(1+\frac{10}{100}\right)^{1\times 2} = 2000(1.10)^2$$

= ₹2420.

Calculator Trick

Work As

Type 10 ÷ 100 + 1 × button then press = button (2-1=1) time then press × button type 2000 then press = button. (This is the required result).

(*b*) $i = \frac{10}{200}; m = 2$

$n = mt = 2 \times 2 = 4$

$$A = 2000\left[1+\frac{10}{200}\right]^{2\times 2} = 2000(1+0.05)^4$$

$= 2000 \times 1.21550625 = Rs.\ 2431.0125 = Rs.\ 2431.$

Calculator Trick

Type 10 ÷ 200 then + 1 then × button then press = button (4 - 1 = 3) times then type × 2000 = button (we get the required result)

[**This Trick** will take minimum 5 seconds and saves lots of time]

(*c*) $n = mt = 4 \times 2 = 8$

$$\therefore A = P(1+i)^n = 2000\left(1+\frac{10}{400}\right)^{4\times 2}$$

$$= 2000 \times [1+0.025]^8 = 2000 \times 1.2184$$

= ₹ 2436.81

Calculator Trick

Type 10 ÷ 400 + 1 × Then press = button (8-1 = 7) times then × 2000 = button (we get the result)

(*d*) $i = \frac{10}{1200}; m = 12; n = mt = 12 \times 2 = 24$

Calculator Trick

Type 10 ÷ 1200 + 1 × button then press = button (24-1 = 23) times then press × 2000 = button (we get the result) = ₹ 2440.78.

Ex. 2 Determine the compound amount and compound interest on ₹1000 at 6% compounded semi-annually for 6 years. Given that $(1+i)^n = 1.42576$ for $i = 3\%$ and n = 12.

(*a*) ₹ 1425.76; ₹425.76 (*b*) ₹1420.76 ; ₹420.76

(*c*) ₹1525.76 ; ₹525.76 (*d*) None

Solution : (*a*) is correct.

Given that P = ₹ 1000; t = 6 years

r = 6 % compounded half -yearly

$$\therefore m = 2 \; ; \; i = \frac{6}{200} = 0.03$$

$$n = mt = 2 \times 6 = 12$$

$$\therefore A = 1000(1+0.03)^{12} = 1000 \times 1.42576$$

= ₹ 1425.76

C.I = A - P = 1425.76 - 1000 = ₹ 425.76

[Note:- If $(1 + i)^n$ value is given in the question then use that given value]

Calculator Trick [If values not given]

Type $6 \div 200 + 1 \times$ Then press = button (12 - 1 = 11) time then $\times$ 1000 = button ; we will get the required result = ₹1425.76 = Amount. Press -1000 = button we will get compound interest value.

EFFECTIVE RATE OF INTEREST

Type - II

The equivalent annual rate of interest compounded annually if interest is compounded more than once in a year is called EFFECTIVE RATE of INTEREST. It is denoted by E or r_e.

Formula

$$r_e = E = \left[\left(1+\frac{r}{100m}\right)^m - 1\right] \times 100 = (1+i)^m - 1$$

where r = Nominal rate of interest, m = No. of conversion periods in a year.

Calculator Trick

Type r ÷ 100m then push $\times$ button then push = button (m - 1) times then - 1 then push $\times$ 100 = we get r_e in %.

Ex- Find effective rate of interest of the nominal rate of interest 6% compounded quarterly.

Solution : r = 6% compounded quarterly.

m = 4

$$\therefore r_e = \left[\left(1+\frac{6}{400}\right)^4 - 1\right] \times 100 = 6.136355062 \quad \% = 6.14\,\%$$

Calculator Trick

Type 6 ÷ 400 + 1 $\times$ button then press = button (4 - 1 = 3) times then - 1 then $\times$ 100 then press = button we get the required result.

Ex- Find effective rate of interest of 12% compounded monthly.

Solution : r = 12% compounded monthly, m = 12

$$\therefore r_e = \left[\left(1+\frac{12}{1200}\right)^{12} - 1\right] \times 100\% = 12.6825\%$$

Type 12 ÷ 1200 + 1 $\times$ button then press = button 11 times then - 1 then $\times$ 100 = button. [we get the required result]

Type-III (To find Present Value)

$$\because \; A = P(1+i)^n$$

$$or \; P = \frac{A}{(1+i)^n}$$

$$or \; P = A(1+i)^{-n}$$

Calculator Trick

Step-I Type i = r ÷ 100 m + 1

Step-II Press ÷ button

Step-III Press = buttons n times *i.e.* (power times)

Step-IV Press $\times$ button then type A value then press = button (we get the required result)

Ex- A certain sum invested at 4% per annum compounded semi annually amounts to ₹78030 at the end of one year. Find the sum.

(*a*) ₹75000 (*b*) ₹70,000 (*c*) ₹72000 (*d*) None

Solution : option (*a*) is correct

A = ₹78030; r = 4% compounded semi - annually

m = 2, t = 1 yr.

n = mt = 2× 1 = 2

$$P = A(1+i)^{-n} = 78030\left(1+\frac{4}{200}\right)^{-2}$$

=74999.999....= ₹ 75000

Calculator Trick

Type 4 ÷ 200 + 1 then press ÷ button then press = button (power = 2) times then press × button then type A value = 78030 then press = button we get the required result.

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. The difference between the simple and compound interest on a certain sum for 3 year at 5% p.a. is ₹ 228.75. The compound interest on the sum for 2 years at 5% p.a. is :

(*a*) ₹ 3,175 (*b*) ₹ 3,075

(*c*) ₹ 3,275 (*d*) ₹ 2,975

[Nov. 2006]

Solution : Tricks

$$P = \frac{Difference \times (100)^3}{r^2(300+r)}$$

[For 3 years only]

$$= \frac{228.75\times(100)^3}{5\times5(300+5)} = ₹\ 30{,}000$$

A = 30000 + 5 % + 5% buttons = ₹ 33075

∴ C.I = A - P = ₹ 33075 - 30000

= ₹ 3,075

∴ (*b*) is Correct

Q.2. In what time will ₹ 3,90,625 amount to ₹ 4,56,976 at 8% per annum, when the interest is compounded semi-annually ? [Given: $(1.04)^4 = 1.16986$]

(*a*) 2 years (*b*) 4 years

(*c*) 5 years (*d*) 7 years

[Feb. 2007]

Solution : (*a*)

$$A = P\left(1+\frac{r}{100m}\right)^{mt}$$

$$\frac{4,56,976}{3,90,625} = \left(1+\frac{8}{200}\right)^{2t}$$

or $1.16985856 = (1.04)^{2t}$

or $1.16966 = (1.04)^{2t}$

or $(1.04)^4 = (1.04)^{2t}$

∴ 2t = 4 ∴ t = 2 years

∴ (a) is correct

Q.3. How long will ₹12,000 take to amount to ₹14,000 at 5% p.a. converted quarterly? [Given: $(1.0125)^{12.4} = 1.1666$]

(*a*) 3 years (*b*) 3.1 years

(*c*) 13.5 years (*d*) 12.4 years.

[May 2007]

Solution : (*b*) $\frac{A}{p} = \left(1+\frac{5}{400}\right)^{4t}$

or $\frac{14000}{12000} = (1.0125)^{4t}$

or 1.16666..... $(1.0125)^{4t}$

or $(1.0125)^{12.4} = (1.0125)^{4t}$ [Note :- Always use values given in question]

or 4t = 12.4 ∴ t = 3.1yrs.

∴ (*b*) is correct

Q.4. If ₹ 1,000 be invested at interest rate of 5% and the interest be added to the principal every 10 years, then the number of years in which it will amount to ₹ 2,000 is :

(*a*) $16\frac{2}{3}$ years (*b*) $\frac{1}{10}$ years

(*c*) 16 years (*d*) $6\frac{2}{3}$ years

[Aug. 2007]

Solution : Given P = ₹ 1000; m = $\frac{1}{10}$;

$$n = mt = \frac{1}{10}\times t = 0.1t$$

r = 5% p.a.

$$A = P\left(1+\frac{5}{0.1\times100}\right)^{0.1t}$$

$$\frac{2000}{1000} = (1.50)^{0.1t}$$

or $2 = (1.5)^{0.1t}$

or $0.1t = \frac{\log 2}{\log(1.5)}$

or 0.1t = 1.709 or $t = \frac{1.709}{0.1}$

$= 17.09 = 16\frac{2}{3}$, (a) is correct

Q.5. The annual birth and death rates per 1000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is :

(*a*) 35 years (*b*) 30 years

(*c*) 25 years (*d*) None of these

[Aug. 2007]

Solution : (*a*) is correct

	Per 1000	Per 100
Birth rate	39.4	3.94
Death rate	19.4	1.94

Population increase 2.00%

Population increase = 2.%

$$\frac{A}{P} = 2 = (1+0.02)^t$$

Calculator Trick

Type 1.02 Then push × button then continue pressing = button until to get 2. No. of pressings of = button is 34 times. So, t = 35 years (approx.)

Q.6. The effective rate equivalent to nominal rate of 6% compounded monthly is:

(*a*) 6.05 (*b*) 6.16

(*c*) 6.26 (*d*) 6.07

[Aug. 2007]

Solution : (*b*)

$$r_e = \left[\left(1+\frac{r}{100m}\right)^m - 1\right]\times100$$

$$= \left[\left(1+\frac{6}{1200}\right)^{12} - 1\right]\times100 = 6.16\%$$

option (b) is correct.

Q.7. A person deposited ₹ 5,000 in a bank. The deposit was left to accumulate at 6% compounded quarterly for the first five years and at 8% compounded semiannually for the next eight years. The compound amount at the end of 13 years is :

(*a*) ₹ 12621.50 (*b*) ₹ 12613.10

(*c*) ₹ 13613.10 (*d*) None

[Nov. 2007]

Solution :

Calculator Tricks :-

$$A = 5000\left(1+\frac{6}{400}\right)^{5\times4}\left(1+\frac{8}{200}\right)^{8\times2}$$

₹12613.17 = ₹ 12610.00 (approx)

∴ (*b*) is correct.

Q.8. Anshul's father wishes to have ₹ 75,000 in a bank account when his first college expenses begin. How much amount his father should deposit now at 6.5% compounded annually if Anshul is to start college in 8 years hence from now?

(*a*) ₹ 45,320 (*b*) ₹ 46,360

(*c*) ₹ 55,360 (*d*) ₹ 48,360.

[Feb. 2008]

Solution : (*a*)

Calculator Tricks

$P = A(1+i)^{-n}$

$$= 75000\left(1+\frac{6}{100}\right)^{-8}$$

Calculator Tricks:

Type 6 ÷ 100 + 1 ÷ button

then press = button 8 times

× 75000 = button

we get ₹ 45,317 ≅ 45320

(*a*) is correct.

Q.9. The difference between compound interest and simple interest on a certain sum for 2 years @ 10% p.a. is ₹ 10. Find the sum:

(*a*) ₹ 1,010 (*b*) ₹ 1,095

(*c*) ₹ 1,000 (*d*) ₹ 990

[June 2008]

Solution : (*c*) **Tricks**

$$P = \frac{\text{Actual (C.I - S.I.)}}{\text{(C.I - S.I.) at Re.1}}$$

$$P = \frac{10}{[(1.10)^2-1]-0.2} = ₹\,1000$$

∴ (*c*) is correct

Calculator Tricks :- For 2 years. P = 10 ÷ 10% ÷ 10% button = ₹ 1000.

Q.10. A machine worth ₹ 4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹ 2,00,000 :

(*a*) 5 years 6 months

(*b*) 5 years 7 months

(*c*) 5 years 5 months

(*d*) None

[June 2008]

Solution : (*a*) is correct

Tricks :

$$t = \frac{\log(2,00,000/4,90,740)}{\log(1-15/100)}$$

= 5.5 years (approx.)

= 5 yrs. 6 months

Note:

Calculator Tricks : Type 0.85 $\sqrt{\text{button}}$ 19 times –1 × 227695 = button then press M+ then type 200,000 ÷ 4,90,740 = button then $\sqrt{\text{button}}$ 19 times –1 × 227695 = button then ÷ MRC button = button we get answer.

Q.11. If the difference between simple interest and compound interest is ₹ 11 at the rate of 10% for two years, then find the sum:

(*a*) ₹ 1,200 (*b*) ₹ 1,100

(*c*) ₹ 1,000 (*d*) None of these

[Dec. 2008]

Solution : (*b*) is correct

Tricks $P = \frac{Difference\times(100)^2}{(rate)^2}$

$$= \frac{11\times(100)^2}{(10)^2} = ₹\,1100$$

Calculator Tricks :- P = 11 ÷ 10% ÷ 10% button = ₹ 1100

Q.12. In how many years, a sum will become double at 5% p.a. compound interest.

(*a*) 14.0 years (*b*) 14.1 years

(*c*) 14.2 years (*d*) 14.3 years

[June 2009]

Solution : (*c*) is correct

Tricks $t = \frac{Log(A/P)}{me\log(1+r/100m)}$

$$= \frac{\log 2}{\log(1.05)} = 14.2 yrs.\,(approx)$$

Q.13. A sum amount to ₹ 1331 at a principal of ₹1,000 at 10 % compounded annually. Find the time.

(*a*) 3.31 years (*b*) 4 years

(*c*) 3 years (*d*) 2 years

[June 2009; Dec. 2009]

Solution : (*c*) is correct

Tricks :- Go by choices

For (*c*) ; $A = 1000\left(1+\frac{10}{100}\right)^3 = ₹\,1331.$

So; t = 3 yrs.

Calculator Tricks :- GBC

(*c*) A = 1000 + 10% + 10% + 10% button = ₹ 1,331

Q.14. The compound interest for a certain sum @ 5% p.a. for first years is ₹25. The S-I for the same money @ 5% p.a. for 2 years will be.

(*a*) ₹40 (*b*) ₹50

(*c*) ₹60 (*d*) ₹70

[Dec. 2009]

Solution : (*b*) is correct

Tricks :-

S.I For 1st yrs. = C . I for 1st yrs. = ₹25

S.I For 2 yrs. For same 'p' = 2 × 25 = ₹ 50

Q.15. At what % rate of compound interest corresponding (C.I) will a sum of money became 16 times in four years, if interest is being calculated compounding annually:

(*a*) r = 100% (*b*) r = 10%

(*c*) r = 200% (*d*) r = 20%

[June 2010]

Solution : (*a*) is correct

Tricks :- Go by choices

For (*a*) Let P = 1; $A = 1\left(1+\frac{100}{100}\right)^4 =$ $(2)^4 = 16$

∴ (*a*) is correct

Q.16. If the simple interest on a sum of money at 12% p.a. for two years is ₹ 3,600. The compound interest on the same sum for two years at the same rate is:

(*a*) ₹ 3,816 (*b*) ₹ 3,806
(*c*) ₹ 3,861 (*d*) ₹ 3,860

[June 2010]

Solution : (*a*) is correct

$$P = \frac{3600\times100}{12\times2} = ₹\ 15000$$

$$\therefore c.i = 15000\left(1+\frac{12}{100}\right)^2 - 15000$$

$$= ₹\ 3816.$$

Tricks:-

CI for 1st yr. = SI for 1st year = 3600 ÷ 2 = ₹ 1800

CI for 2nd year = 1800 + 1800 × 12% = ₹ 2016

∴ C.I for 2 years = 1800 + 2016 = ₹ 3816.

Q.17. The effective annual rate of interest corresponding to nominal rate 6% p.a. payable half yearly is

(*a*) 6.06% (*b*) 6.07%
(*c*) 6.08% (*d*) 6.09%

[Dec. 2010]

Solution : (*d*) is correct

$$r_e = \left[\left(1+\frac{6}{200}\right)^2 - 1\right]\times100 = 6.09\%$$

Q.18. The cost of Machinery, is ₹1,25,000/- If its useful life is estimated to be 20 years and the rate of depreciation of its cost is 10% p.a., then the scrap value of the Machinery is (given that $(0.9)^{20} = 0.1215$)

(*a*) 15,187 (*b*) 15,400
(*c*) 15,300 (*d*) 15,250

[Dec. 2010]

Solution : (*a*) is correct

$$S \text{ (Scrap Value)} = P\left(1-\frac{d}{100}\right)^t$$

where P = Principal;

d = rate of depreciation

$$\therefore S = 1{,}25{,}000\left(1-\frac{10}{100}\right)^{20}$$

$$= ₹15{,}187.50$$

Q.19. Mr. X invests 'P' amount at Simple Interest rate 10% and Mr. Y invests 'Q' amount at Compound Interest rate 5% compounded annually. At the end of two years both get the same amount of interest, then the relation between two amounts P and Q is given by:

(*a*) $p = \frac{41Q}{80}$ (*b*) $p = \frac{41Q}{40}$

(*c*) $p = \frac{41Q}{100}$ (*d*) $p = \frac{41Q}{200}$

[Dec. 2010]

Solution : (*a*) is correct

$$S.I = \frac{P.10\times2}{100} = \frac{P}{5}$$

$$C.I = Q\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$$

$$= 0.1025.Q$$

From Question

S.I = C.I

$$\frac{P}{5} = 0.1025Q$$

or $P = 5\times0.1025Q = 0.5125Q$

$$\therefore P = \frac{5125}{10000}Q = \frac{205Q}{400} = \frac{41Q}{80}$$

$$\therefore P = \frac{41Q}{80}$$

Tricks :- GBC

Q.20. If the difference of S.I and C.I is ₹ 72 at 12% for 2 years. Calculate the amount.

(*a*) 8,000 (*b*) 6,000
(*c*) 5,000 (*d*) 7,750

[June 2011]

Solution : (*c*) is correct

Tricks: $P = \frac{(C.I - S.I)\times(100)^2}{r^2}$

$$= \frac{72\times100\times100}{12\times12} = ₹\ 5000$$

Calculator Tricks :- P = 72 ÷ 12% ÷ 12% = 5000

Q.21. Nominal rate of interest is 9.9% p.a. If interest is Compounded monthly, What will be the effective rate of interest

(Given $\left(\frac{4033}{4000}\right)^{12} = 1.1036$ (approx))?

(*a*) 10.36% (*b*) 9.36%
(*c*) 11.36% (*d*) 9.9%

[Dec. 2011, June 2012]

Solution : (*a*) is correct.

Tricks:-

$$r_e = \left[\left(1+\frac{9.9}{1200}\right)^{12} - 1\right]\times100$$

= 10.36 %

Q.22. The difference between CI and SI on a certain sum of money for 2 years at 4% per annum is ₹1. The sum is

(*a*) 625 (*b*) 630
(*c*) 640 (*d*) 635

[June 2013]

Solution : (*a*) is correct

Tricks :- For 2 yrs

$$\text{Sum of Money} = \frac{Diff.(100)^2}{r^2}$$

$$= \frac{1\times(100)^2}{4^2} = ₹\ 625$$

Calculator Tricks:- P = 1 ÷ 4% ÷ 4% button = ₹ 625.

Q.23. If the sum of money when compounded annually become 1140 in 2 years and 1710 in 3 years at rate of interest

(*a*) 30% (*b*) 40%
(*c*) 50% (*d*) 60%

[June 2013]

Solution : (*c*) is correct.

Interest in 3rd yr. = ₹ 1710 - ₹1140 = ₹570

Tricks Note :- For 3rd yr ; it will be like S.I

$$r = \therefore \frac{1\times100}{Pt} = \frac{570\times100}{1140\times1} = 50\%$$

Tricks II Go by choices.

For (*c*) A = 1140 + 50% (Calculator)

= ₹ 1710

∴ (*c*) is correct

Q.24. The difference between and C.I & S.I at 7% p.a. for 2 years is ₹29.4 then principal is

(a) ₹5,000 (b) ₹5,5000
(c) ₹6,000 (d) ₹6,500

[Dec. 2013]

Solution : (c) is correct

Tricks $P = \frac{Difference \times (100)^2}{r^2}$

$= \frac{29.4 \times (100)^2}{(7)^2} = ₹\ 6000.$

Calculator Tricks :- P = 29.4 ÷ 7% ÷ 7% button = ₹ 6000

Q.25. The Partners A & B together lent ₹3903 at 4% p.a. interest compounded annually. After a spam of 7 years, A gets the same amount as B gets after 9 years. The share of A in the sum of ₹3903 would have been

(a) ₹1875 (b) ₹2280
(c) ₹2028 (d) ₹2820

[June 2014]

Solution : (c) is correct

$$A\left(1+\frac{4}{100}\right)^7 = B\left(1+\frac{4}{100}\right)^9$$

$$\text{or } \frac{A}{B} = \left(1+\frac{4}{100}\right)^2 = \left(\frac{26}{25}\right)^2$$

$$= \frac{676}{625}$$

A : B = 676 : 625

$$A = \frac{676}{676+625} \times 3903 = ₹\ 2028$$

Tricks:- GBC

Q.26. A certain sum of money double itself in 4 years at C.I. In how many years it will become 32 times to itself

(a) 15 years (b) 24 years
(c) 20 years (d) None

[Dec. 2014]

Solution : (c) is correct

Tricks:- $2^{t_2} = 32^4$

$= 2^{t_2} = (2^5)^4 = 2^{20}$

$= t_2 = 20$ yrs.

Q.27. On a certain sum rate of interest @ 10% p.a., S.I = ₹ 90 Term = 2 year, Find Compound interest for the same :

(a) ₹ 544.5 (b) ₹ 94.5
(c) ₹ 450 (d) ₹18

[Dec. 2015]

Solution : (b) is correct

$$\text{S.I. } p.a = \frac{90}{2} = ₹45$$

Tricks: Compound interest

= 45+(45+10%) = ₹ 94.5

Q.28. If an amount is kept at simple interest, it earns ₹ 600 in first 2 years but when kept at Compound interest it earns at interest of ₹ 660 for the same period; then the rate of interest and principle amount respectively are

(a) 20%; ₹1200 (b) 10%; ₹1200
(c) 20%; ₹1500 (d) 10%; ₹1500

[June 2016]

Solution : (c)

Tricks:- Go by choices

(c) $\text{S.I.} = \frac{1500 \times 2 \times 20}{100} = ₹\ 600$ (True)

$$\text{C.I} = 1500\left[\left(1+\frac{20}{100}\right)^2 - 1\right] = ₹\ 660 \text{ (also True)}$$

(c) is correct

Q.29. Mr. X bought an electronic item for ₹1000. What would be the future value of the same item after two years, if the value is compounded semi-annually at the rate of 22% per annum ?

(a) ₹1488.40 (b) ₹1518.07
(c) ₹2008.07 (d) ₹2200.00

[June 2016]

Solution : (b) is correct

$$FV = P\ (1+i)^n$$

$$= 1000\left(1+\frac{22}{200}\right)^{2\times 2}$$

= ₹ 1518.07 (approx.)

Q.30. The difference between the simple interest and compound interest on a certain sum of money invested for 2 years at 5% p.a. is ₹30. Then the sum =

(a) 10,000 (b) 12,000
(c) 13,000 (d) None

[Dec. 2016]

Solution : (b)

Calculator Tricks:

P = 30 ÷ 5% ÷ 5% button = ₹12,000

Q.31. A sum of money amounts to ₹7803 for one year at the rate of 4% compounded semi-annually then the sum invested is

(a) 7,000 (b) 7,500
(c) 7,750 (d) 8,000

[Dec. 2016]

Solution : (b)

$$P = 7803\left[1+\frac{4}{200}\right]^{-2}$$

Calculator Tricks:

P = (4 ÷ 200 +1) ÷ = button 2 times × 7803 = button

= ₹ 7500

Tricks : (b) (GBC) → A = 7500 + 2% + 2% button = 7803.

Q.32. The difference between simple and compound interest on a sum of ₹ 10000 for 4 years at the rate of interest 10% per annum is _____

(a) 650 (b) 640
(c) 641 (d) 600

[June 2017]

Solution : C.I – S.I

$$= \left[10,000\left(1+\frac{10}{100}\right)^4 - 10,000\right] - \left[\frac{10,000 \times 10 \times 4}{100}\right]$$

= 4641 - 4000 = ₹ 641.

option (c) is correct. [Note :- Do by Calculator]

Q.33. If the compound interest on a sum for two year at the rate 5% p.a. is ₹512.50, then the principal is _____:

(a) 4,000 (b) 3,000
(c) 5,000 (d) None of these

[Dec. 2017]

Solution : (c)

Tricks:- GBC

Amount = 5000 + 5% + 5% button

= 5512.50.

C.I = 5512.50 - 5000 = ₹ 512.50.

Q.34. Find effective rate of interest corresponding to the nominal rate of interest 7% compounded monthly is _____ :

(*a*) 7.26 % (*b*) 7.22 %

(*c*) 7.02 % (*d*) 7.20 %

[Dec. 2017]

Solution : (*b*)

$$r_e = \left[\left(1+\frac{7}{1200}\right)^{12} - 1\right] \times 100\%$$

$= 7.229\% = 7.22\%$.

Q.35. In compound interest, if the amount is 9 times to its principle in two years then the rate of interest is ?

(*a*) 300% (*b*) 200%

(*c*) 150% (*d*) 100%

[June 2018]

Solution : (*b*) Given,

$$A = P\left(1+\frac{r}{100}\right)^t$$

$$\text{or ; } 9P = P\left(1+\frac{r}{100}\right)^2$$

$$\text{or ; } 9 = \left(1+\frac{r}{100}\right)^2$$

$$\text{or ; } 3^2 = \left(1+\frac{r}{100}\right)^2 \Rightarrow 3 = 1+\frac{r}{100}$$

$$\Rightarrow 2 = \frac{r}{100} \Rightarrow r = 200\%$$

Tricks : - 1 + 200 % + 200 % = 9

So, (*b*) is correct.

Q.36. If difference between Compound Interest and Simple Interest for 3 years is ₹ 912 at the rate 4 % p.a., the principal is

(*a*) ₹1,87,500 (*b*) ₹1,87,000

(*c*) ₹1,87,550 (*d*) ₹1,85,700

[June 2018]

Solution : (*a*)

Tricks :-

$P = 912 \div 4\% \div 4\% \div (300 + 4)\%$

= ₹ 1,87,500

Q.37. If Rs. 1,000 be invested at interest at interest rate of 5% and the interest be added to the principal every 10 years, than the number of years in which it will amount to Rs. 2,000 is :

(*a*) $16\frac{2}{3}$ years (*b*) $6\frac{1}{4}$

(*c*) 16 years (*d*) $6\frac{2}{3}$

[May 2018]

Solution : (*a*)

$\because$ Interest is added to the principal every 10 years. So, within 10 years ; simple interest will apply.

So, Amount after 10 yrs.

$$= 1000 + 1000 \times \frac{10 \times 5}{100}$$

= Rs. 1500.

Total amount = Rs. 2000

Extra Interest needed = 2000 - 1500

= Rs. 500.

$$\text{Time} = \frac{500 \times 100}{1500 \times 5} = \frac{20}{3}$$

$$= 6\frac{2}{3} \text{ yrs.}$$

So; Total time $= 10 + 6\frac{2}{3}$

$$= 16\frac{2}{3} \text{ yrs.}$$

Q.38. If an amount is kept at S.I. it earns an interest of Rs. 600 in first two years but when kept at compound interest it earns an interest of Rs. 660 for the same period, then the rate of interest and principal amount respectively are :

(*a*) 20%., Rs. 1,200

(*b*) 20%, Rs. 1,500

(*c*) 10%, Rs. 1,200

(*d*) 10%., Rs. 1,500

[May 2018]

Solution : (*b*)

Tricks:- Go by choices (GBC)

$$\text{(a) S.I} = \frac{1200 \times 2 \times 20}{100} = 480 \neq 600$$

So; (a) is false.

$$\text{(b) S.I} = \frac{1500 \times 2 \times 20}{100} = \text{Rs. } 600$$

C.I = $(-1500 + 20\% + 20\%)$ (button)

= 660.

So; (*b*) is True.

Q.39. If ₹10,000 is invested at 8% per year compound quarterly, then the value of the investment after 2 years is [Given $(1 + 0.2)^8 = 1.171659$]

(*a*) ₹ 10,716.59 (*b*) ₹ 11,716.59

(*c*) ₹ 117.1659 (*d*) None of these

[Nov. 2018]

Solution : (*b*)

$$FV = 100000\left(1+\frac{8}{400}\right)^{2\times4}$$

= ₹ 11716.59

Q.40. A bank pays 10% rate of interest, interest being calculated half yearly. A sum of ₹ 400 is deposited in the bank. The amount at the end of 1 years will be

(*a*) ₹ 439 (*b*) ₹ 440

(*c*) ₹ 442 (*d*) ₹ 441

[Nov. 2018]

Solution : (*d*)

$$FV = 400\left(1+\frac{10}{200}\right)^2 = 441$$

Calculator Tricks :-

FV = 400 + 5 % + 5 % = 441

Q.41. A men deposited ₹ 8,000 in a bank for 3 years at 5% per annum compound interest, after 3 years he will get

(*a*) ₹ 9,000 (*b*) ₹ 8,800

(*c*) ₹ 9,200 (*d*) ₹ 9,261

[Nov. 2018]

Solution : (*d*)

$$FV = 8000\left(1+\frac{5}{100}\right)^3 = ₹\ 9261.$$

Calculator Tricks :

FV = 8000 + 5 % + 5 % + 5 % buttons = 9261

Q.42. If in two years time a principal of ₹ 100 amounts to ₹ 121 when the interest at the rate of *r* % is compounded annually, then the value of *r* will be

(*a*) 14 (*b*) 10.5

(*c*) 15 (*d*) 10

[Nov. 2018]

Solution : (*d*)

Details :-

$$121 = 100\left(1+\frac{r}{100}\right)^2 \Rightarrow \frac{121}{100} = \left(1+\frac{r}{100}\right)^2$$

$$\text{or } \left(\frac{11}{10}\right)^2 = \left(1+\frac{r}{100}\right)^2 \Rightarrow 1+\frac{r}{100} = \frac{11}{10}$$

$$\text{or } \frac{r}{100} = \frac{11}{10} - 1 = \frac{1}{10}$$

$$\therefore r = 10\%$$

I Tricks :- GBC

$$\text{for FV} = 100\left(1+\frac{10}{100}\right)^2 = 121$$

(True)

II Calculator Tricks :-

FV = 100 + 10 % + 10 % buttons = 121

Q.43. The effective rate of interest for one year deposit corresponding to a nominal 7% rate of interest per annum convertible quarterly is

(*a*) 7% (*b*) 7.4%

(*c*) 7.5% (*d*) 7.18%

[Nov. 2018]

Solution : (*d*)

$$r_e = \left[\left(1+\frac{7}{400}\right)^4 - 1\right] \times 100 = 7.18\%$$

Q.44. How much will ₹ 25,000 amount to in 2 years at compound interest if the rates for the successive years are 4% and 5% per year

(*a*) ₹ 27,000 (*b*) ₹ 27,300

(*c*) ₹ 27,500 (*d*) ₹ 27,900

[Nov. 2018]

Solution : (*b*)

$$\text{FV} = 25000\left(1+\frac{4}{100}\right) \times \left(1+\frac{5}{100}\right)$$

= ₹ 27,300/-

Calculator Tricks:- 25000 + 4% + 5% buttons

= ₹27,300/-

Q.45. ₹ 8,000/- at 10% per annum interest compounded half yearly will become at the end of one year

(*a*) ₹ 8,800 (*b*) ₹ 8,900

(*c*) ₹ 8820 (*d*) ₹ 9,600

[Nov. 2018]

Solution : (*c*)

$$\text{FV} = 8000\left(1+\frac{10}{200}\right)^2 = ₹\ 8{,}820$$

Calculator Tricks :-

FV = 8000 + 5% + 5% buttons = 8820

Q.46. The value of furniture depreciates by 10% a year, if the present value of the furniture in an office is ₹ 21870, calculate the value of furniture 3 years ago

(*a*) ₹ 30,000 (*b*) ₹ 40,000

(*c*) ₹ 35,000 (*d*) ₹ 50,000

[Nov. 2018]

Solution : (*a*)

Calculator Tricks :- GBC

(*a*) 30000 - 10% - 10% - 10% button = 21870.

Details Method

$$21870 = \text{P}\left(1+\frac{10}{100}\right)^3$$

$$\therefore \text{P} = \frac{21870}{(0.9)^3} = ₹\ 30{,}000$$

Q.47. If compound interest on a sum for 2 years at 4% per annum is ₹ 102, then the simple interest on the same period at the same rate will be

(*a*) ₹ 90 (*b*) ₹ 100

(*c*) ₹ 101 (*d*) ₹ 93

[Nov. 2018]

Solution : (*b*)

$$\text{Details :- C.I} = \text{P}\left(1+\frac{4}{100}\right)^2 - \text{P} = 102$$

$$\text{or P}\left[(1.04)^2 - 1\right] = 102$$

$$\text{or P} \times 0.0816 = 102$$

$$\text{or P} = \frac{102}{0.0816} = 1250$$

$$\therefore \text{S.I.} = \frac{\text{p.r.t}}{100} = \frac{1250 \times 4 \times 2}{100} = ₹\ 100$$

Tricks :- Go by choices

For option (b)

S.I. for 2 years = ₹ 100

∴ S.I. for 1 years = ₹ 50

S.I. of 1st yr. = C.I. of 1st yr. = ₹ 50

C.I. for 2nd yr. = 50 + 4% = ₹ 52

Total C.I. for 2 yrs = 50 + 52 = ₹ 102 (True)

∴ Option (*b*) is correct

Q.48. If the difference between the compound interest compounded annually and simple interest on a certain amount at 10% per annum for two years is ₹ 372, then the principal amount is

(*a*) ₹ 37,000

(*b*) ₹ 37,200

(*c*) ₹ 37,500

(*d*) None of the above

[Nov. 2018]

Solution : (*b*)

Tricks :-

P = 372 ÷ 10% ÷ 10% = ₹ 37,200

Q.49. What is the net present value of piece of property which would be valued at ₹ 2 lakh at the end of 2 years? (Annual rate of increase = 5%)

(*a*) ₹ 2.00 lakh

(*b*) ₹ 1.81 lakh

(*c*) 2.01 lakh

(*d*) None of the above

[Nov. 2018]

Solution : (*b*)

$$NPV = 2\left(1+\frac{5}{100}\right)^{-2} = 1.81 \text{ lakh}$$

(approx)

Q.50. A sum was invested for 3 years as per C.I. and the rate of interest for first year is 9%, 2nd year is 6% and 3rd year is 3% p.a. respectively. Find the sum if the amount in three years is ₹ 550 ?

(*a*) ₹ 250 (*b*) ₹ 300

(*c*) ₹ 462.16 (*d*) ₹ 350

[June 2019]

Solution :

Tricks :- GBC

(*c*) 462.16 on Calculator, do as Type 462.16 + 9% + 6% + 3% button we get 550. So (*c*) is correct.

Q.51. The effective rate of interest does not depend upon

(*a*) Amount of Principal

(*b*) Amount of Interest

(*c*) Number of Conversion Periods

(*d*) None of these

[June 2019]

Solution : (*a*) is correct

Q.52. If p.i^2 = 96 , and R = 8% compounded annually then P =

(*a*) ₹ 14,000 (*b*) ₹ 15,000

(*c*) ₹ 16,000 (*d*) ₹ 17,000

[June 2019]

Solution : (*b*)

Tricks :- Given , p.$i^2 = 96$

Means interest of two periods (yrs. here) is 96.

So; GBC (Calculator Tricks)

(*a*) $I = 14000 \times 8\% \times 8\%$

$= 89.6 \neq 96$

So; (*a*) is False.

(*b*) Type $15000 \times 8\% \times 8\%$ button

We get 96.

So, (*b*) is correct.

Tricks II

$P = 96 \div 8\% \div 8\%$ buttons.

= ₹ 15,000.

Q.53. The present value of a scooter is ₹ 7290. The rate of depreciation is 10%. What was its value 3 years ago?

(*a*) 10,000 (*b*) 10010

(*c*) 9990 (*d*) 12000

[Dec. 2019]

Solution : (*a*)

Calculator Tricks : GBC

after 3 years;

PV. = 7290.

For option (*a*), Type 10,000 - 10% button - 10% button -10% button ;

we get ₹ 7290.

∴ (*a*) is correct.

Q.54. The difference between compound interest, compounded semi annually and simple interest on ₹ 400 at 10% p.a. for one year.

(*a*) ₹ 1 (*b*) ₹ 28

(*c*) ₹ 35 (*d*) ₹ 40

[Dec. 2019]

Solution : (*a*)

In C.I. 10% p.a compounded semi-annually ⇒ (10/2) = 5% interest in 6 months.

There are 2 periods in 1 yr.

∴ FV = 400 + 5% + 5% button = 441

∴ C.I = 441 - 400 = 41.

S.I = $400 \times 10\% = 40$ for 1 yr.

∴ C.I - S.I = 41 - 40

= ₹ 1.

[**Another Trick:** Difference = $400 \times 5\% \times 5\%$ = ₹ 1]

Q.55. In how much time the S.I. on a certain sum becomes 0.125 times to its principle at 10% p.a. is

(*a*) 1.00 yrs (*b*) 1.25 yrs

(*c*) 1.50 yrs (*d*) 2.00 yrs

[Dec. 2019]

Solution : (*b*)

Let principal = ₹ 1

∴ S.I = 0.125

$$\therefore t = \frac{I \times 100}{p.r} = \frac{0.125 \times 100}{1 \times 10}$$

= 1.25 yrs

Q.56. In what time will a sum ₹ 800 amounts to ₹ 882 at 5% p.a. compounded annually

(*a*) 1 yrs (*b*) 2 yrs

(*c*) 3 yrs (*d*) 4 yrs

[Dec. 2019]

Solution : (*b*)

Tricks : GBC

for (*b*) FV = Amounts = 800 + 5% + 5% button = ₹ 882

∴ (*b*) is correct.

Q.57. Find the effective rate of interest if an amount of ₹ 30,000 deposited in a bank. For 1 year at the rate of 10% p.a. compounded semi annually.

(*a*) 10.05% (*b*) 10.10%

(*c*) 10.20% (*d*) 10.25%

[Dec. 2019]

Solution : (*d*)

Here, No need of Principals value.

Formula

∴ Effective rate of interest

$$= \left[(1+1)^m - 1\right] \times 100$$

$$r_e = \left[\left(1+\frac{10}{200}\right)^2 - 1\right] \times 100$$

Calculator Tricks

Type $10 \div 200 + 1 \times$ button = button 1 time -1×100 = button

We get 10.25%

∴ r_e = 10.25%

Q.58. The present population of a town is 25,000. If it grows at the rate of 4%, 5%, 8% during 1st year, 2nd year, 3rd year respectively. Then find the population after 3 years.

(*a*) 29,484 (*b*) 29,844

(*c*) 29,448 (*d*) 28,944

[Dec. 2019]

Solution : (*a*)

Tricks : Population after 3 yrs

= 25000 + 4% + 5% + 8% buttons

= 29484.

Q.59. An amount ₹ 35000 with the rate of interest is 7% per annum, it is compounded on a monthly basis, then tell the effective rate of interest.

(*a*) 7.22% (*b*) 7.64%

(*c*) 7.0% (*d*) 7.5%

[Dec. 2019]

Solution : (*a*)

$$r_e = \left[\left(1+\frac{7}{1200}\right)^{12} - 1\right] \times 100$$

Calculator Tricks

Type 7 ÷ 1200 + 1 × button = button 11 times -1 × 100 = button

We get 7.229% = 7.22% (approx.)

Q.60. On what sum will the compound interest at 5% p.a for 2 years compounded annually be ₹ 3,280

(*a*) ₹ 16,000 (*b*) ₹ 32,000

(*c*) ₹ 48,000 (*d*) ₹ 64,000

[Dec. 2020]

Solution : (*b*)? Formula

C.I. = P[(1 + *i*)^n - 1]

$$3280 = P\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$$

= P[1.1025 - 1]

3280 = P(0.1025)

$$\therefore P = \frac{3280}{0.1085} = 32000$$

Calculator Tricks :- GBC

(*a*) CI = 16000 + 5% + 5% - 16000

= ₹ 1640

≠ 3280

∴ (*a*) is incorrect.

(*b*) C.I. = 32000 + 5% + 5% button – 32000

= ₹ 3280 (Correct)

∴ (*b*) is correct.

Note : No Need to write anything else

Only on Calculator in seconds.

Q.61. An amount P becomes ₹ 5,100.5 and ₹ 5,203 after second and fourth years respectively, at r% of interest per annum compounded annually. Thus, values of P and r are

(*a*) ₹ 5,000 and 1 (*b*) ₹ 4,000 and 1.5

(*c*) ₹ 6,000 and 2 (*d*) ₹ 5,500 and 3

[Dec. 2020]

Solution : Tricks GBC (Calculator Tricks)

(*a*) FV after 2 yrs = 5000 + 1% + 1% button (Press) = ₹ 5100.5 (True)

FV after 4 yrs = 5000 + 1% + 1% + 1% + 1% button = ₹ 5203 (True)

∴ (*a*) is correct.

Q.62. The useful life of a machine whose cost is ₹ 10,000 is 10 years. If it depreciates at 10% p.a. then the scrap value of the machine is.

(*a*) 3486.70 (*b*) 3158.30

(*c*) 3500 (*d*) 7033

[Dec. 2019]

Solution : (*a*)

Scrap Value

$$= 10,000\left(1-\frac{10}{100}\right)^{10} = 10,000(0.9)^{10}$$

Calculator Tricks:

Type 0.9 × = button 9 times × button 10,000 = button, we get ₹ 3486.78

Q.63. A certain sum invested at 4% per annum compounded semi-annually amounts to ₹ 1,20,000 at the end of one year. Find the sum

(*a*) 1,10,120 (*b*) 1,15,340

(*c*) 1,12,812 (*d*) 1,13,113

[Dec. 2020]

Solution : Tricks GBC

Calculator Tricks Interest for 1st 6 months = 2%.

(*a*) FV = 1,10,120 + 2% + 2%

≠ 1,20,000

(*a*) is incorrect.

(*b*) FV = ₹ 1,15,340 + 2% + 2% button

= ₹ 1,19,999.736 = ₹ 120,000

(*b*) is correct.

II. **Calculator Tricks**

P = A (1 + *i*)$^{-n}$

$$= 1,20,000\left(1+\frac{4}{200}\right)^{-2}$$

on Calculator → Type 4 ÷ 200 + 1 ÷ button

then Press = button 2 times

then press "×" button then type 1,20,000 = button = ₹ 1,15,340

Q.64. The ratio of principal and the compound interest value for three years (compounded annually) is 216 : 127. The rate of interest is

(*a*) 0.1567 (*b*) 0.1777

(*c*) 0.1666 (*d*) 0.1588

[Dec. 2020]

Solution : Tricks GBC

Use calculator [No need to write anything else]

(*a*) Let r = 0.1567 = 15.67%

$$C.I = 216\left(1+\frac{15.67}{100}\right)^3 - 216 \neq 127$$

(*b*) $$C.I = 216\left(1+\frac{17.77}{100}\right)^3 - 216$$

≠ 127

(*c*) *r* = 0.1666 = 16.66%

$$\therefore C.I = 216\left(1+\frac{16.66}{100}\right)^3 - 216.$$

Cal. 16.66 ÷ 100 + 1 × = 2 times – 216

= 126.9412 ≅ ₹ 127.

∴ (c) is correct.

Detail

$$\frac{P}{P\left[\left(1+\frac{r}{100}\right)^3 - 1\right]} = \frac{216}{127}$$

$$\text{or } \left[\left(1+\frac{r}{100}\right)^3 - 1\right] = \frac{127}{216}$$

$$\text{or } \left[\left(1+\frac{r}{100}\right)^3\right] = \frac{127}{216} + 1$$

$$= \frac{343}{216} = \left(\frac{7}{6}\right)^3$$

$$\therefore 1 + \frac{r}{100} = \frac{7}{6}$$

$$\therefore \frac{r}{100} = \frac{7}{6} - 1 = \frac{1}{6}$$

$$\therefore r = \frac{100}{6} = 16.66\%$$

$r = 0.1666$

(c) is correct.

Q.65. Find the present value of ₹ 1,00,000 be required after 5 years if the rate of interest is 9% given that $(1.09)^5 = 1.5386$

(a) 78,995.98 (b) 64,994.20

(c) 88,992.43 (d) 93,902.12

[Dec. 2020]

Solution : Calculator Tricks

$PV = A(1 + i)^{-n}$

$$= 100{,}000\left(1 + \frac{9}{100}\right)^{-5}$$

On Calculator Type 9 ÷ 100 + 1 ÷ = 5 times × 100000

= buttons = ₹ 64,994.20

$$\text{OR } PV = \frac{A}{(1+1)^n}$$

$$= \frac{1{,}00{,}000}{\left(1 + \frac{9}{100}\right)^5} = \frac{1{,}00{,}000}{(1.09)^5}$$

$$= \frac{1{,}00{,}000}{1.5386} = 64{,}994.15 \text{ [Given } (1.09)^5$$

$= 1.5386$]

$\therefore$ (b) is correct.

Q.66. An amount is lent at a nominal rate of 4.5% per annum compounded quarterly. What would be the gain in rupees over when compounded annually.

(a) 0.56 (b) 0.45

(c) 0.076 (d) 0.85

[Dec. 2020]

Solution : Let P = ₹ 1

Effective rate

$$= \left[1\left(1 + \frac{4.5}{400}\right)^4 - 1\right] \times 100$$

= 4.576 % (approx) yearly.

If 4.5% compounded yearly

Then Gain % = 4.576 – 4.5% = 0.076%

(c) is correct.

Q.67. Find the amount of compound interest, if an amount of ₹ 50,000 is deposited in a bank for one year at the rate of 8% per annum compounded semiannually

(a) 3080 (b) 4080

(c) 5456 (d) 7856

[Jan. 2021]

Solution : Tricks (b) is correct.

Principal = ₹ 50,000

Semi annually rate of interest

$$r = \frac{8}{2} = 4\%$$

Amount = 50,000 + 4% + 4% (button)

(By using calculator)

= ₹ 54,080

C.I. = A – P = 54080 – 50000

= ₹ 4080

(b) is correct

Q.68. The population of a town increase by 2% of the population at the beginning of the year. The number of year by which the total increases in population would be 40% is :

(a) 7 years

(b) 10 years

(c) 17 years

(d) 19 years (approx)

[Jan. 2021]

Solution : (c) is correct

% Increase in population = r = 2%

Let after t years population = 1 + 40% = 1.40

In population increase case, always use compound Interest. Formula.

$\therefore A = P(1 + 1)^n$

$$\text{or } 1.40 = 1\left(1 + \frac{2}{100}\right)^{t \times 1}$$

$= 1.40 = (1{\cdot}02)^t$

Calculator

Type 1·02 × = button (Press)

until to get 1.40

After pressing = button 16 times, we get 1.40 approx,

$\therefore$ t = 16 + 1 = 17 years

Q.69. The simple on sum at 4% p.a. for 2 years is ₹ 80. Find the CI on the same sum for the same period.

(a) ₹ 81.6 (b) ₹ 80.3

(c) ₹ 83.2 (d) ₹ 82.3

[Jan. 2021]

Solution : Tricks (a) is correct

S.I. for 2 year = ₹ 80

$$\therefore \text{S.I. for each year} = \frac{80}{2} = ₹\,40$$

S.I. of 1st year = C.I. of 1st year = ₹ 40

C.I. for 2nd years = Interest on Principal + Interest on interest earned in 1st year.

= S.I. + Interest on S.I.

= 40 + 40 × 4% = ₹ 41.6

[Calculator 40 + 4% = 41.6]

$\therefore$ Total Compound Interest = 40 + 41.6 = ₹ 81.6

i.e. C.I. = 40 + 4% + 40 = ₹ 81.6

Q.70. Which is a better investment 9% p.a. compounded quarterly or 9.1% p.a. simple interest?

(a) 9% compounded

(b) 9.1% S.I.

(c) Both are same

(d) Cannot be said

[Jan. 2021]

Solution : (a) is correct

$r_e = [(1 + i)^m - 1] \times 100\%$

$$= \left[\left(1 + \frac{9}{400}\right)^4 - 1\right] \times 100$$

Where m = No. of conversion periods in 1 year = 4

Calculator

[9 ÷ 400 + 1 × = button 3 times – 1]

× 100 (button) = 9.308%

= 9.31% p.a.

Clearly 9.31% is more than 9.1%

9% compounded quarterly is better.

Q.71. The effective rate of interest corresponding to a nominal rate of 7% p.a. compounded quarterly is

(a) 7.5% (b) 7.6%

(c) 7.7% (d) 7.185%

[Jan. 2021]

Solution : (d) is correct

$$r_e = \left[\left(1+\frac{7}{400}\right)^4 - 1\right] \times 100$$

Calculator

7 ÷ 400 + 1 × = button 3 times –1 × 100 = button

We get 7.185% approx

Q.72. A sum of money is lent at C.I. Rate 20% p.a. 2 years. It would fetch ₹ 482 more if the interest is compounded half yearly. The sum is :

(a) ₹ 19,800 (b) ₹ 19,900

(c) ₹ 20,000 (d) ₹ 20,100

[Jan. 2021]

Solution : Tricks (c) is correct

Rate of interest difference in 2 years

$$= \left(1+\frac{20}{200}\right)^{2\times2} - \left(1+\frac{20}{100}\right)^2$$

$$= (1{\cdot}1)^4 - (1{\cdot}2)^2 = 0.0241 = 2.41\%$$

[Calculator → 1·1 × = button 3 times M + button 1.2 × button = button 1 time M – button (press)

Then press MRC button we get 0.0241]

P = Difference ÷ rate of interest difference

= 482 ÷ 2.41% button, we get

= ₹ 20,000

Q.73. What 'I' denote the actual rate of interest in decimal, and n denote the number of conversion periods, the formula for computing the effective rate of interest E is given by.

(a) $(1+i)^n$ (b) $(1+i)^n - 1$

(c) $1-(1+i)^n$ (d) $(1+i)^{-n}$

[Jan. 2021]

Solution : (b) is correct

$$r_e = E = (1+i)^m - 1$$

Here, $r_e = E = (1+i)^n - 1$

Q.74. A sum of ₹ 7500 amounts to ₹ 9075 at 10% p.a., interest being compounded yearly in a certain time. The simple interest (in ₹) on the same sum for the same time and the same rate is

(a) 1000 (b) 1250

(c) 1800 (d) 1500

[July 2021]

Solution : (d) is correct

Calculator Tricks

Type 7500 + 10% + 10% = ₹ 9075 (We get) (True)

Hence t = 2 years

$$\text{S.I.} = \frac{\text{P.r.t.}}{100} = \frac{7500\times10\times2}{100} = ₹\ 1500$$

or; on calculator S.I. = 7500 × 20% = 1500

Q.75. If the desired future value after 5 years with 18% interest rate is ₹ 1,50,000, then the present value (in ₹) is (Given that $(1.18)^5 = 2.2877$)

(a) 63,712 (b) 65,568

(c) 53,712 (d) 41,712

[July 2021]

Solution : (b) is correct

$$PV = P = A(1+i)^{-n}$$

$$= 150{,}000\left(1+\frac{18}{100}\right)^{-5} = ₹\ 65{,}568$$

Calculator Tricks

Type 2.2877 (given in question)

Then ÷ = button then × button type

150,000 Press = button

We get ₹ 65568

Q.76. What is the compound interest (in ₹) on a sum of ₹ 12,600 for 1½ years at 20% per annum if the interest is compounded half yearly? (Nearest to a Rupee)

(a) 4271 (b) 4171

(c) 4711 (d) 4117

[July 2021]

Solution : (b) is correct

$$\text{C.I.} = P[(1+i)^n - 1]$$

$$= 12{,}600\left[\left(1+\frac{20}{200}\right)^{2\times\frac{3}{2}} - 1\right]$$

= ₹ 4171

Calculator Tricks

Type 20 ÷ 200 + 1 × = button 2 times

–1 × 12,600 (Type) = button

We get ₹ 4171

Q.77. A sum of ₹ x amounts to ₹ 27,900 in 3 years and to ₹ 41,850 in 6 years at a certain rate per cent per annum, when the interest is compounded yearly. The value of x is

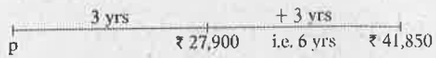
(a) 16080 (b) 18600

(c) 18060 (d) 16800

[July 2021]

Solution : (b) is correct

Tricks :



Let = Principal = P

It means in 3 years it becomes

$$\frac{41{,}850}{27{,}900} = 1{\cdot}5 \text{ times}$$

Means in 3 years principal becomes 1.5 times

Calculator Tricks

∴ P = 27900 ÷ 1.5 = button

= ₹ 18600

Hence (b) is correct

Q.78. The effective rate of return for 24% per annum convertible monthly is given as

(a) 24% (b) 26.82%

(c) 18% (d) 24.24%

[July 2021]

Solution : (b) is correct

Effective rate of return

$$= \left[\left(1+\frac{24}{1200}\right)^{12} - 1\right] \times 100 = 26{\cdot}82\%$$

Calculator Tricks

Type 24 ÷ 1200 + 1 × = button 11 times then –1 × 100 = button. We get 26·82%

Q.79. What is the difference (in ₹) between the simple interest and the compound interest on a sum of ₹ 8,000

for $2\frac{2}{5}$ years at the rate of 10% p.a., when the interest is compounded yearly?

(a) 135.75 (b) 129.50
(c) 151.75 (d) 147.20

[July 2021]

Solution : (d) is correct

$$\text{S.I.} = \frac{\text{Prt}}{100} = \frac{800 \times 1 \times \frac{12}{5}}{100}$$

$= ₹ 1920$

For compound interest

Rate of interest for $\frac{2}{5}$ years

$$= \frac{2}{5} \times 10 = 4\%$$

Tricks

Hence

C.I. = 8000 + 10% + 10% + 4% button – 8000 (Type)

= 2067·20

So, Difference between C.I. & S.I.

= 2067.20 – 1920 = ₹ 147·20

Q.80. S deposits an amount in bank which gives 10% compound interest, compounded annually for 5 years. What is effective rate of simple interest?

(a) 12.21 (b) 11.11
(c) 13.21 (d) 12.81

[Dec. 2021]

Solution : (a)

Given

r = 10% Compounded yearly

t = Time = 5 years

Let P = ₹ 100

∴ Compound Interest

$$= P\left[\left(1+\frac{r}{100}\right)^t - 1\right]$$

$$= 100\left[\left(1+\frac{10}{100}\right)^5 - 1\right] = ₹ 61.051$$

Let R be @ single rate of interest which gives ₹ 61.051 in 5 years

∴ S.I = C.I.

$$\therefore \frac{\text{P.R.T}}{100} = 61.051$$

$$\text{or } \frac{100 \times R \times 5}{100} = 61.051.$$

$$\text{or } R = \frac{61.051}{5} = 12.2102\%$$

$= 12.21\%$

Q.81. Cost of a laptop is ₹ 1,10,000 and its value depreciate 12% annually its life is 6 years its scrap value.....times its cost

(a) 0.44 (b) 0.42
(c) 0.45 (d) 0.48

[Dec. 2021]

Solution : (c)

Scrap Value

$$S = P\left(1-\frac{d}{100}\right)^t$$

$$\text{or } \frac{S}{P} = \left(1-\frac{12}{100}\right)^6$$

$= (0.88)^6 = 0.4644$

S = 0.46

$P \Rightarrow S = 0.46P$

Calculator

Type 0.88 × = 5 times

Q.82. If the compound interest earned at i% p.a. in n years is to be earned at s% simple interest rate for n years, the s =

(a) i (b) $i\frac{1}{n}$

(c) $\frac{(1+i)^n - 1}{n}$ (d) $\frac{1-(1+i)^n}{n}$

[Dec. 2021]

Solution : (c)

Compound Interest = Simple Interest

$\therefore p\,[(1+i)^n - 1] = p.n.s$

[Where S in decimal form]

$$\text{or } S = \frac{(1+i)^n - 1}{n}$$

Q.83. A company needs ₹ 10,000 in five years to replace as equipment. How much (in ₹) must be invested now at the interest rate of 8% p.a. is order to provide for the equipment ?

(a) ₹ 6,606 (b) ₹ 6,806
(c) ₹ 10,500 (d) ₹ 11,500

[Dec. 2021]

Solution : (b)

$PV = A(1+i)^{-n}$

$$= 10000\left(1+\frac{8}{100}\right)^{-5}$$

$= 1000(1.08)^{-5} = ₹ 6806$

Calculator Tricks → Type 8 ÷ 100 + 1 ÷ = button

5 times × 10000 = button = **₹ 6805.8** = ₹ 6806

Q.84. It needs to pay ₹ 5,00,000 after 10 years. He invested a sum in a scheme at 9% rate of interest compounded half-yearly. How much amount (in ₹) he invested? ($1.045^{19} = 2.41171$)

(a) 3.97.321 (b) 2.70.321
(c) 2.97.321 (d) 3.40.321

[Dec. 2021]

Solution : (c)

Given

FV = A = ₹ 5,00,000

t = 10 years

r = 9% compounded half yearly m = 2;

n = mt = 2×10 = 20

$$PV = A\left(1+\frac{r}{100\,m}\right)^{-n}$$

$$= 5,00,000\left(1+\frac{9}{200}\right)^{-20}$$

$$= \frac{500,000}{(1.045)^{20}} = \frac{5,00,000}{2.41171}$$

$= ₹ 2,07,321$

Q.85. A sum of money is put at 20% compound interest rate p.a. At which year the aggregated amount just exceeds the double of the original sum?

(a) 5 (b) 6
(c) 4 (d) 3

[Dec. 2021]

Solution : (c)

I. Calculator Tricks

1 + 20% + 20% + 20% + 20% button

= 2.0736 > 2

∴ t = 4 yrs.

2nd

Method

$\frac{A}{P}=\left(1+\frac{20}{100}\right)^t$

$=(1.20)^t$

On Calculator

Press 1.20 × = button 3 times

It becomes greater than 2.

So t = 3+1 = 4 years.

Q.86. An investment is earning compound interest, ₹ 100 invested in the year 2 accumulated to ₹ 105 by year 4. If ₹ 500 invested in the year 5, will become ₹ ..... by year 10.

(*a*) 364.80 (*b*) 564.80

(*c*) 464.80 (*d*) 664.80

[June 2022]

Solution : 1st Condition P = ₹ 100; A = ₹ 105; n = 2; r = ?

Now $\frac{A}{P}=(1+i)^n$

or $\frac{105}{100}=(1+i)^2$

$\Rightarrow (1+i)^2=1.05$ .....(1)

Secondly ∴ P = ₹ 500/-; A = ?

t = 5 yrs; n = 5

∴ $A=P(1+i)^n$

$=500(1+i)^5$

$=500(1+i)^4(1+i)$

$=500\{(1+i)^2\}^2.\sqrt{(1+i)^2}$

$=500(1.05)^2.\sqrt{1.05}$

= ₹ 564.86

∴ (*a*) is correct

Q.87. There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest of ₹ 12,000 after 3 years at the same rate?

(*a*) ₹ 3,972 (*b*) ₹ 2,160

(*c*) ₹ 3,120 (*d*) ₹ 3,742

[June 2022]

Solution :

Let P = ₹ 100

A = ₹ 160

S.I = A – P = 160 – 100 = ₹ 60

$r=\frac{I\times 100}{P.t}=\frac{60\times 100}{100\times 6}=10\%$

Now

As CI A = 12,000 + 10% + 10% + 10% button

= ₹ 15,972

C.I = A–P = ₹ 15,972–12,000

= ₹ 3,972

∴ (*a*) is correct

Q.88. The present value of ₹ 2,000, after 8 years at the rate of 6% per annum, is_____. ($1.06^8=1.59385$)

(*a*) ₹ 1,054 (*b*) ₹ 1,254

(*c*) ₹ 3,054 (*d*) ₹ 2,054

[June 2022]

Solution : $PV=A(1+i)^{-n}$

$=2000\left(1+\frac{6}{100}\right)^{-8}$

Calculator Tricks

Type 6 ÷ 100 + 1 ÷ = button 8 times × 2000 = button

∴ PV = ₹ 1254 (Approx)

(*b*) is correct.

Q.89. A machine worth ₹ 4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹ 2,00,750.

(*a*) 5 years 5 months

(*b*) 5 years 6 months

(*c*) 5 years 7 months

(*d*) 5 years 8 months

[Dec. 2022]

Solution:

Tricks :

$t=\frac{\log\left(\frac{S}{P}\right)}{\log\left(1-\frac{d}{100}\right)}$

$=\left[\frac{\log\left(\frac{2,00,750}{4,90,740}\right)}{\log\left(1-\frac{15}{100}\right)}\right]$

$=\frac{\log(0.409076)}{\log(0.85)}=5.4998=5.5$ yrs

= 5 yrs 6 months

Calculator Tricks

I. Type 0.85 press $\sqrt{}$ button 19 times –1 × 2,27,695 = button then press M+ button.

II. Type 2,00,750 ÷ 4,90,740 = button

Then press $\sqrt{}$ button 19 times

–1 × 2,27,695 = button

III. Then press ÷ button then press MRC = button

[It is time value in decimal form]

IV. We get 5.4998 yrs

= 5.5 yrs

Means 5 years and 6 months

(*b*) is correct.

Q.90. If ₹ 64 Amount to ₹ 83.20 in 2 years, what will ₹ 86 Amount to in 4 years at the same. Rate per cent per annum?

(*a*) ₹ 127.60 (*b*) ₹ 147.60

(*c*) ₹ 145.34 (*d*) ₹ 117.60

[Dec. 2022]

Solution : ₹ 64 Amounts to ₹ 83.20 in 2 yrs.

It means ₹ 64 becomes 83.20 ÷ 64

= 1.3 times in 2 years.

Hence, ₹ 86 will become 86 × 1.3 (times) in 2 yrs

and ₹ 86 × 1.3 will become (86 × 1.3) × 1.3 (times)

= ₹ 145.34 in next 2 yrs.

So ₹ 86 will become ₹ 145.34 in 4 years.

(*c*) is correct

Q.91. The effective annual rate of interest corresponding to a normal rate of 6% per annum payable half yearly is :

(*a*) 6.06% (*b*) 6.07%

(*c*) 6.08% (*d*) 6.09%

[Dec. 2022]

Solution: $r_e = \left[(1+i)^m - 1\right] \times 100\%$

$= \left[\left(1+\frac{6}{200}\right)^2 - 1\right] \times 100\% = 6.09\%$

[Note : Calculator :– Type 6 ÷ 200 + 1 × = button 1 time –1 × 100 = button]

(We get result)

∴ (*d*) is correct.

Q.92. 10 years ago the Earning Per Share (EPS) of ABC Ltd. was ₹ 5 share. Its EPS for this year is ₹ 22. Compute at what rate, EPS of the company grow annually?

(*a*) 15.97% (*b*) 16.77%

(*c*) 18.64% (*d*) 14.79%

[Dec. 2022]

Solution: ∵ EPS growing annually

So compound interest applies

Let C.I *pa* = *r*

₹ 5 10 years ₹22 Now

Go by choices

(*a*) *r* = 15.97% (let)

$\therefore Fv = 5\left(1+\frac{15.97}{100}\right)^{10}$

= 22.00019 .....

= 22 (True)

[Note : Calculator:-

Type 15.97 ÷ 100 + 1 × = button 9 times × 5 = button.

We get 22]

∴ (*a*) is correct.

Q.93. Mr. Prakash invested money in two schemes 'A' and 'B' offering compound interest at the rate of 8% and 9% per annum respectively. If the total amount of interest accrued through these two schemes together in two years was ₹ 4818.30 and total amount invested was ₹ 27,000. What was the amount invested in Scheme 'A'?

(*a*) ₹ 12,000 (*b*) ₹ 12,500

(*c*) ₹ 13,000 (*d*) ₹ 13,500

[Dec. 2022]

Solution:

₹27,000/- (Invested)
Schemes

	A	B
	8%	9%
Time	2 years	2 years

Go by choices

(*a*) Let $P_1 = 12000$ $P_2 = 27,000 - 12,000$

$= ₹ 15,000$

Calculator : CI | = 12,000 + 8% + 8% – 12,000 = M + button |

Then | 15,000 + 9% + 9% – 15,000 = M + button |

Then press MRC button

We get ₹ 4818.30 (True)

∴ (*a*) is correct

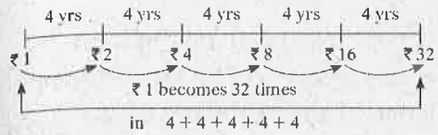
Q.94. A sum of money invested of compound interest doubles itself in four years. In how many years it becomes 32 times of itself at the same rate of compound interest.

(*a*) 12 years (*b*) 16 years

(*c*) 20 years (*d*) 24 years

[Dec. 2022]

Solution: Tricks :



$= 4 \times 5 = 20$ years

∴ (*c*) is correct

Q.95. The difference between compound interest and simple interest on an amount of ₹ 15,000 for 2 years is ₹ 96. What is the rate of interest per annum?

(*a*) 9% (*b*) 8%

(*c*) 11% (*d*) 10%

[Dec. 2022]

Solution: *Given,*

P = ₹ 15000

r = 9; *t* = 2 yrs

C.I – S.I = ₹ 96

Tricks

P = D ÷ *r*% ÷ *r*% button

GBC (*a*) 96 ÷ 9% ÷ 9% button

≠ 15000 (False)

So (*a*) is wrong.

(*b*) For 8%

P = 96 ÷ 8% ÷ 8% button (Press)

= ₹ 15000 (True)

∴ (*b*) is correct

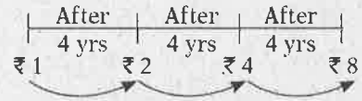
Q.96. A sum of money doubles itself in 4 years at certain compound interest rate. In how many years this sum will become 8 times at the same compound interest rate?

(*a*) 12 years (*b*) 14 years

(*c*) 16 years (*d*) 18 years

[Dec. 2022]

Solution : Tricks



∴ ₹ 1 becomes ₹ 8 in 4 + 4 + 4 = 12 yrs.

(*a*) is correct

Q.97. The Nominal rate of interest is 10% per annum. The interest is compounded quarterly. The effective rate of interest per annum will be:

(*a*) 10 % (*b*) 10.10 %

(*c*) 10.25 % (*d*) 10.38 %

[June 2023]

Solution : Here r = 10% Quarterly ∴ m = 4

$r_e = \left[(1+i)^m - 1\right] \times 100\%$

$= \left[\left(1+\frac{10}{400}\right)^4 - 1\right] \times 100\% = 10.38\%$

Calculator

10 ÷ 400 + 1 × = button 3 times

–1 × 100% (button)

= 10.38%

(*d*) is correct.

Q.98. The difference between compound interest and simple interest on a certain sum of money invested for 3 years at 6% per annum is ₹ 110.16. The principle is:

(*a*) ₹ 3,000 (*b*) ₹ 3,700

(*c*) ₹ 12,000 (*d*) ₹ 10,000

[June 2023]

Solution : Calculator Tricks

P = D÷ r%÷ r% ÷ (300 + r)%

= 110.16÷ 6%÷ 6%÷ (300 + 6)% button. (Press)

= 10,000/-

(*d*) is correct.

Q.99. A machine depreciates at 10% of its value at the beginning of a year. The cost and scrap value realized at the time of sale being ₹ 23.240 and ₹ 9,000 respectively. Approximately, for how many years the machine is put to use?

(*a*) 7 (*b*) 8

(*c*) 9 (*d*) 10

[June 2023]

Solution :

$$\because S = P\left(1-\frac{d}{100}\right)^t$$

Where P = Present Value = Cost

S = Scrap Value

d = Rate of depreciation

t = Time

$$9000 = 23240\left(1-\frac{10}{100}\right)^t$$

$$\text{or } (0.9)^t = \frac{9000}{23240}$$

or $(0.9)^t = 0.387...$

By Calculator

Press 0.9 x = button 8 times

So t = 8 + 1 = 9 years.

∴ (*c*) is correct.

Q.100. The population of a town increases every year by 2% of the population at the beginning of that year. The approximate number of years, by which the total increase of population will be 40%, is _____(Given $1.02^8 = 1.17166$)

(*a*) 15 (*b*) 17

(*c*) 19 (*d*) 20

[June 2023]

Solution : Let Present population P = 100

Let after "t" yrs; population

A = 100 + 40 = 140

$$\therefore \frac{A}{O} = \left(1-\frac{r}{100}\right)^t$$

$$\frac{140}{100} = 1.40 = \left(1+\frac{2}{100}\right)^t$$

or $1.40 = (1.02)^t$.

Calculator Tricks

Press 1.02 x = button 16 times

We get 1.4000 Approx.

∴ t = 16 + 1 = 17 yrs.

Q.101. The Compound interest on ₹ 15,625 for 9 months at 16% per annum compounded quarterly is:

(*a*) ₹ 1,851 (*b*) ₹ 1,941

(*c*) ₹ 1,951 (*d*) ₹ 1,961

[June 2023]

Solution :

P = ₹ 15,625

$$t = 9 \text{ months} = \frac{9}{12} \text{ year}$$

$$n = mt = 4 \times \frac{9}{12} = 3$$

$$r = 1.6\% \ \frac{1}{4} \text{ yearly}$$

∴ C.I. = P[(1 + i)^n – 1]

$$= 15{,}625\left[\left(1+\frac{16}{400}\right)^3 - 1\right]$$

Calculator Tricks

Press 16 ÷ 400 + 1 x = button

2 times – 1 x 15625 = button; we get ₹ 1951

∴ (*c*) is correct.

9

CHAPTER

ANNUITY

Definition:

A sequence of payments, generally equal in size, made at equal intervals of times is called an **annuity.**

Monthly Rent ; premiums of LIC; deposit into a recurring account in a bank; equal monthly payments got by a retired government servant as pension and loan instalments to houses or automobiles etc.

Some terms related with annuities

Periodic Payment: The size of each payment of an annuity is called the periodic payment of the annuity.

Annual Rent: The sum of all payments of an annuity made in one year is called its annual rent.

Payment Period/Interval : The duration between two successive payments of an annuity is called the payment period (or payment interval) of the annuity

Term: The total duration from the beginning time of the first payment period to the end of the last payment period is called the **term** of the annuity.

Amount of an Annuity: The total Value of all the payments at the maturity time of an annuity is called the amount (or future value) of the annuity.

Present Value of an Annuity: Sum of the present values of all the payments of an annuity is called the present value or capital value of the annuity.

TYPES OF ANNUITIES

Ordinary Annuity: If the payments of an annuity are made at the end of payment interval is called An Ordinary annuity or Regular annuity.

Annuity Due: If the payments of an annuity are made at the beginning of payment interval is called An Annuity Due or Annuity Immediate.

Perpetuity: A perpetuity is an annuity whose payments continue forever.

Note. In what is to follow, it is understood that the payment interval coincides with the interest period unless statement to the contrary is made.

ORDINARY ANNUITY OR ANNUITY REGULAR

Definition: Payments of an annuity are made at the end of payment interval.

Type-I

(TO Find Amount)

$$S = A\left[\frac{(1+i)^n - 1}{r}\right] \times 100m.$$

Where S = Amount of an Annuity

A= Value of each instalment

r = rate of interest

m = No. of conversion periods in a year

n = m.t = No. of instalments made in t yrs.

$i = \frac{r}{100m}$ = Rate of interest of one conversion Period

Calculator Trick

Step - I Find $(1 + i)^n$ by calculator *i.e.* Type r ÷ 100 m + 1 Then push × button then push = button (n - 1) times.

Step - II Then - 1

Step - III ÷ r× 100m

Step - IV Then× A push = button (We get the required value of Amount)

Ex-1. Find the future value of an annuity of ₹500 is made annually for 7 years at interest rate of 14% compounded annually. [Given that $(1.14)^7 = 2.5023$]

(*a*) ₹5365.25 (*b*) ₹5265.25 (*c*) ₹5465.25 (*d*) none

Solution : option (*a*) is correct

Calculator Trick

$$S = A\left[\frac{(1+i)^n - 1}{r}\right] \times 100m. = ₹\ 5365.25$$

$$= 500\left[\frac{\left(1+\frac{14}{100}\right)^7 - 1}{14}\right] \times 100 = ₹\ 5365.25$$

Step - I Find $\left(\frac{14}{100}+1\right)^7$ *As* Type 14 ÷ 100 + 1× Push = button 6 times.

Step - II Type - 1 ÷ 14 then ×100 (Because it is annually)

Step - III Then× 500 = (we get the result)

Ex-2. ₹200 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10th payment? Given that $(1.005)^{10} = 1.0511$

(*a*) ₹2544 (*b*) ₹2144 (*c*) ₹2544 (*d*) None

Solution : (*a*) is correct.

Here A = 200 ; r = 6% compounded monthly

n = 10 = No. of payments.

$$FV = S = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 200\left[\frac{\left(1+\frac{6}{1200}\right)^{10} - 1}{6}\right] \times 1200 = ₹2045.60$$

Calculator Trick

Step- I Type 6 ÷1200 + 1 Then push ×button then push = button 9 times.

Step-II Type - 1 Then ÷ 6× 1200

Step-III Then Type× 200 = buttons we get the required amount.

Note: If $(1 + i)^n$ value is given in the question then use given value in the question otherwise answer may vary.

Type - II

To find the **Value of EACH INSTALMENT**

Ex: If a bank pays 6% interest compounded quarterly what equal deposit have to be made at the end of the each quarter for 3 years if you want to have ₹1500 at the end of 3 years?

(*a*) ₹117.86 (*b*) ₹115.01 (*c*) ₹150.50 (*d*) None of these

Solution : (*b*) is correct

$$FV = S = A\left[\frac{(1+i)^n - 1}{r}\right] \times 100m$$

$$1500 = A\left[\frac{\left(1+\frac{6}{400}\right)^{12} - 1}{6}\right] \times 400$$

A = ₹150.01

Calculator Trick

Step-I Type 6 ÷ 400 + 1 Then push × button then push = buttons 11 times

Step-II Then push -1 ÷ 6 × 400 buttons

Step-III Then push M+ button to save the typed value.

Step-IV Then type 1500 then ÷ button then push "MRC" button 2 times then push = button.

[we get the required result]

Type-III

(To find Present Value for Ordinary Annuity)

$$\text{PV} = \text{Present value} = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Calculator Trick

Step-I Type (1 + i) value then push ÷ button

Step-II Then push = buttons "n" times

Step-III Push GT button

Step-IV Then type × A (value) then push = button

we get the required result.

Ex- Find the present value of an annuity which pays 200 at the end of each 3 months for 10 years assuming money to be worth 5% converted quarterly?

(*a*) ₹3473.86 (*b*) ₹3108.60 (*c*) ₹6265.38 (*d*) None of these

Solution : option (*c*) is correct

Here A = 200 ; m = 4 ; r = 5% 1/4 yrly.

t = 10 years $\Rightarrow$ n = mt = 4 × 10 = 40 year PV=?

Calculator Trick

Step-I Type 5 ÷ 400 + 1 then push ÷ button

Step-II Then push = buttons 40 times

Step-III Then Push GT button

Step-IV Then type × 200 = buttons

[We get the resulting value]

Type-IV

(To find instalment value if PV is given).

Ex- Mr. A borrows 5,00,000 to buy a house.

If he pays equal instalments for 20 years and 10% interest on outstanding balance what will be the equal annual instalment?

(*a*) ₹58239.84 (*b*) ₹58729.84 (*c*) ₹68729.84 (*d*) None of these

Solution : (*b*) is correct

Here PV = ₹5,00,000 ; r = 10% yrly.

t = 20 years ∴ n = 20; A = ?

$$5,00,000 = A\left[\frac{1-\left(1+\frac{10}{100}\right)^{-20}}{0.10}\right] = ₹\,58729.84$$

Calculator Trick

Step-I Type 10 ÷ 100 + 1 then push ÷ button

Step-II Push = buttons 20 times

Step-III Then Push GT button

Step-IV Then M+ buttons to save the result.

Step-V Type 5,00,000 then push ÷ button then- MRC button 2 time and then = button.

(We get the required result)

Annuity Immediate/Due

Definition: An annuity due is an annuity the first payment of which is made at the beginning of the first payment interval

Type-V

(To find Amount)

$$\text{FV} = \text{Amount } S = A\left[\left\{\frac{(1+i)^{n+1}-1}{r}\right\}\times 100m - 1\right]$$

Calculator Trick (work as ordinary annuity)

Step-I Type r ÷ 100 m + 1 then push × button

Step-II Push = buttons n + 1 - 1 = n times then push - 1 button then push ÷ button then push r value then push × 100m value buttons.

Step-III Push - 1 button then × button and then type A value & then push = button (we get the required result)

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. Mr. X Invests ₹ 10,000 every year starting from today for next 10 years suppose: interest rate is 8% per annum compounded annually. Calculate future value of the annuity:

(Given that $(1 + 0.08)^{10} = 2.15892500$]

(*a*) ₹ 156454.88 (*b*) ₹ 144865.625

(*c*) ₹ 156554.88 (*d*) None of these

[Nov. 2006]

Solution : (*a*) It is Annuity Due Question

$$A = FV = R\left[\left\{\frac{(1+i)^{n+1}-1}{r}\right\}\times 100m - 1\right]$$

$$= 10{,}000\left[\frac{(1+0.08)^{10+1}-1}{8}\times 100 - 1\right]$$

= ₹ 1,56,454.88.

(*a*) is correct

Q.2. The present value of an annuity of ₹ 3,000 for 15 years at 4.5% p.a. C.I. is:

[Given that $(1.045)^{15}$,= 1.935282]

(*a*) ₹ 23,809.67 (*b*) ₹ 32,218.67

(*c*) ₹ 32,908.67 (*d*) None of these

[Nov. 2006]

Solution : $\text{PV} = \text{R}\left[\frac{1-(1+1)^{-n}}{i}\right]$

$$= 3000\left[\frac{1-(1.045)^{-15}}{0.045}\right]$$

Tricks = ₹ 32,218.67

Q.3. A machine can be purchased for ₹ 50,000. Machine will contribute ₹ 12,000 per year for the next five years. Assume borrowing cost is 10% per annum. Determine whether machine should be purchased or not:

(*a*) Should be purchased

(*b*) Should not be purchased

(*c*) Can't say about purchase

(*d*) None of the above

[Feb. 2007]

Solution : (*b*) $\text{PV} = \text{R}\left[\frac{1-(1+i)^{-n}}{i}\right]$

$$\text{PV} = 12000\left[\frac{1-(1.10)^{-5}}{0.10}\right]$$

= ₹ 45,489.44

But it costs ₹ 50,000

: It should not be purchased

:- (*b*) is correct

Q.4. How much amount is required to be invested every year so as to accumulate ₹ 3,00,000 at the end of 10 years, if interest is compounded annually at 10 % ?

[Give $(1.1)^{10} = 2.5937$]

(*a*) ₹ 18,823.65 (*b*) ₹18,828.65

(*c*) ₹ 18,832.65 (*d*) ₹18,882.65

[Feb. 2007]

Solution : (*a*) $\text{FV} = \text{R}\left[\frac{(1+i)^{n}-1}{r}\times 100m\right]$

$$3{,}00{,}000 = \text{R}\left[\frac{(1+\frac{10}{100})^{10}-1}{10}\times 100\right]$$

$$\text{R} = \frac{3{,}00{,}000}{\left[\frac{(1.1)^{10}-1}{10}\times 100\right]} = ₹\,18{,}823.65$$

(*a*) is correct

Q.5. A company is considering proposal of purchasing a machine either by making full payment of ₹ 4,000 or by leasing it for four years at an annual rate of ₹ 1,250. Which course of action is preferable, if the company can borrow money at 14% compounded annually ? [Given: (1.14) = 1.68896]

(*a*) Leasing is preferable

(*b*) Should be purchased

(*c*) No difference

(*d*) None of these

[May 2007]

Solution : (*a*) ₹ 4000 = Present value

$$\text{PV} = R\left[\frac{1-(1\_i)^{-n}}{r}\times 100m\right]$$

$$= 1250\left[\frac{1-(1+\frac{14}{100})^{-4}}{14}\times 100\right] = ₹\,3642.14$$

It is less than real cost price.

∴ Leasing is better

∴ (*a*) is correct

Q.6. Vipul purchases a car for ₹ 5,50,000. He gets a loan of ₹ 5,00,000 at 15% p.a. from a Bank and balance ₹ 50,000 he pays at the time of purchase. He has to pay the whole amount of loan in 12 equal monthly instalments with interest starting from the end of the first month. The money he has to pay at the end of every month is :

[Given $(1.0125)^{12} = 1.16075452$]

(*a*) ₹ 45,130.43 (*b*) ₹ 45,230.43

(*c*) ₹ 45,330.43 (*d*) None of these

[May 2007]

Solution : Loan Value = ₹ 5,00,000 = PV

R = Instalment value = ?

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$5,00,000 = R\left[\frac{1-\left(1+\frac{15}{1200}\right)^{-12}}{i}\right]$$

R = 45,130.43.

Q.7. A company establishes a sinking fund to provide for the payment of ₹ 2,00,000 debt maturing in 20 years. Contributions to the fund are to be made at the end of every year. Find the amount of each annual deposit if interest is 5% per annum :

(*a*) ₹ 6,142 (*b*) ₹ 6,049

(*c*) ₹ 6,052 (*d*) 6,159

[Aug. 2007]

Solution : A = ₹ 200,000

$$200,000 = R\left[\frac{\left(1+5/100\right)^{20}-1}{5}\times 100\right]$$

$$\text{or } R = \frac{2,00,000\times 5}{[(1.05)^{20}-1)]\times 100}$$

= ₹ 6049 (Approx)

Q.8. Raja aged 40 wishes his wife Rani to have ₹ 40 lakhs at his death. If his expectation of life is another 30 years and he starts making equal annual investments commencing now at 3% compound interest p.a. How much should he invest annually?

(*a*) ₹ 84,077 (*b*) ₹ 81,628

(*c*) ₹ 84,449 (*d*) ₹ 84

[Nov. 2007]

Solution : (*b*) is correct.

R = value of instalment

$$= \frac{40,00,000}{\left[\left\{\frac{(1+0.03)^{30+1}-1}{0.03}\right\}-1\right]}$$

= ₹ 81,628.19

Calculator Trick

Type 1.03 × = button 30 times – 1 ÷ 0.03 – 1 = M + button (Press) Then type 40,00,000 ÷ MRC button = button we get answer.

Q.9. A company may obtain a machine either by leasing it for 5 years (useful life) at an annual rent of ₹ 2,000 or by purchasing the machine for ₹ 8,100. If the company can borrow money at 18% per annum, which alternative is preferable?

(*a*) Leasing (*b*) Purchasing

(*c*) Can't say (*d*) None of these

[Feb. 2008]

Solution : (*a*) PV = ₹8100

It is an ordinary Annuity

$$PV = 2000\left[\frac{1-\left(1+\frac{8}{100}\right)^{-5}}{18}\times 100\right]$$

= ₹6254.34

It is less than ₹8100.

∴ (*a*) is correct

Q.10. A sinking fund is created for redeeming debentures worth ₹ 5 lacs at the end of 25 years. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a.?

(*a*) 12,006 (*b*) 12,040

(*c*) 12,039 (*d*) 12,035

[June 2008]

Solution : (*a*) is correct

Tricks : $₹\ 5,00,000 = R\left[\frac{(1.04)^{25}-1}{0.04}\right]$

∴ R = 12006.00 approx

Q.11. Future value of an ordinary annuity :

(*a*) $A(n,i) = A\left[\frac{(1+i)^n-1}{i}\right]$

(*b*) $A(n,i) = A\left[\frac{(1+i)^n+1}{i}\right]$

(*c*) $A(n,i) = A\left[\frac{1-(1+i)^n}{i}\right]$

(*d*) $A(n,i) = A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$

[Dec. 2008]

Solution : (*a*) is correct.

It is Formulae.

Q.12. Paul borrowers ₹20,000 on condition to repay it with compound interest at 5% p.a. in annual instalment of ₹2,000 each. Find the number of years in which the debt would be paid off.

(*a*) 10 years (*b*) 12 years

(*c*) 14 years (*d*) 15 years

[June 2009]

Solution : (*d*) is correct

$$20,000 = 2000\left[\frac{1-\left(1+\frac{5}{100}\right)^{-t}}{5}\times 100\right]$$

$$\text{or } 10 = \left[\frac{1-(1.05)^{-t}}{5}\right]\times 100$$

$$\text{or } \frac{10\times 5}{100} = 1-(1.05)^{-t}$$

$$\text{or } 0.5 - 1 = (1.05)^{-t}$$

$$\text{or } 0.5-1 = -(1.05)^{-t}$$

$$\text{or } (1.05)^{t} = \frac{1}{0.5} = 2$$

$$\text{or } t = \frac{\log 2}{\log(1.05)} = 15 \text{ yrs. approx.}$$

Tricks : Go by choices

Q.13. Find the present value of an annuity of ₹1,000 payable at the end of each year for 10 years. If rate of interest is 6% compounding per annum. (given $(1.06)^{-10} = 0.5584$):

(*a*) ₹7,360 (*b*) ₹ 8,360

(*c*) ₹ 12,000 (*d*) None of these

[June 2010]

Solution : (*a*) is correct

$$PV = 1000\left[\frac{1-(1.06)^{-10}}{0.06}\right]$$

= ₹7360

(*a*) is correct.

Q.14. The future value of an annuity of ₹ 5,000 is made annually for 8 years at interest rate of 9% compounded annually

[Given that $(1.09)^8 = 1.99256$ _____]

(*a*) ₹ 55,142.22 (*b*) ₹ 65,142.22

(*c*) ₹ 65,532.22 (*d*) ₹57,425.22

[Dec. 2010]

Solution : (*a*) is correct

$$FV = 5000\left[\frac{(1.09)^8 - 1}{0.09}\right] = ₹55,142.22$$

∴ (*a*) is correct

Q.15. How much amount is required to be invested every year as to accumulate ₹6,00,000 at the end of 10th year, if interest is compounded annually at 10% rate of interest?

(*a*) ₹37,467 (*b*) ₹37,476

(*c*) ₹37,647 (*d*) ₹37,674

[June 2014]

Solution : (*c*) is correct

Let amount invested annually = R

$$R = \frac{6,00,000}{\left[\frac{\left(1+\frac{0.1}{100}\right)^{10}-1}{10}\times 100\right]}$$

= ₹ 37,647 (approx)

Q.16. The future value of an annuity of ₹1,000 made annually for 5 years at the rate of interest 14% compound annually is

(*a*) ₹5610 (*b*) ₹6610

(*c*) ₹6160 (*d*) ₹5160

[Dec. 2014]

Solution : (*b*)

$$FV = 1000\left[\frac{\left(1+\frac{14}{100}\right)^{5}-1}{14}\times 100\right]$$

= ₹ 6610.104 = ₹ 6610.

Q.17. Suppose your mom decides to gift you ₹10,000 every year starting from today for the next sixteen years. You deposit this amount in a bank as and when you receive and get 8.5% per annum interest rate compounded annually. What is the present value of this money: [Given that P (15, 0.085) = 8.304236]

(*a*) 83,042 (*b*) 90,100

(*c*) 93,042 (*d*) 10,100

[Dec. 2015]

Solution : (*c*) is correct

PV = 10,000

$$\left[\frac{1-\left(1+\frac{8.5}{100}\right)^{(-16-1)}}{8.5}\times 100+1\right]$$

$= 10,000\ (8.304236+1)$

= ₹ 93,042

Q.18. The future value of an annuity of ₹1500 made annually for 5 years at an interest rate of 10% compounded annually is ___

[Given that $(1.1)^5 = 1.61051$]

(*a*) 9517.56 (*b*) 9157.65

(*c*) 9715.56 (*d*) 9175.65

[June 2017]

Solution :

$$FV = 1500\left[\frac{\left(1+\frac{10}{100}\right)^{5}-1}{10}\times 100\right]$$

Use Calculator tricks

= ₹ 9157.65

option (*b*) is correct.

Q.19. What sum should be invested at the end of every year so as to accumulate an amount of ₹ 796870 at the end of 10 years at the rate of interest 10% compounded annually, [given that A(10 ; 0.1) = 15.9374]

(*a*) 40,000 (*b*) 4,50,000

(*c*) 4,80,000 (*d*) 50,000

[June 2017]

Solution : Calculator Tricks:

$$R = \frac{796870}{\left[\frac{\left(1+\frac{10}{100}\right)^{10}-1}{10}\times 100\right]}$$

= ₹ 50,000

option (*d*) is correct.

Q.20. A person invests ₹2,000 at the end of each month @ of interest 6% compounding monthly, find the amount of annuity after the 10th payment is :

(*a*) ₹20,456 (*b*) ₹20,156

(*c*) ₹20,256 (*d*) ₹20,356

[June 2018]

Solution : (*a*)

$$FV = 2000\left[\frac{\left(1+\frac{6}{1200}\right)^{10}-1}{6}\times 1200\right]$$

= ₹ 20, 456

Type 6 ÷ 1200 + 1 then press × button then = button 9 times - 1 ÷ 6 × 1200 × 2000 = button ; we will get the required result.

Q.21. Determine the present value of perpetuity of ₹ 50,000 per month @ Rate of interest 12% p.a. is _____

(*a*) ₹ 45,00,000

(*b*) ₹ 50,00,000

(*c*) ₹ 55,00,000

(*d*) ₹ 60,00,000

[June 2019]

Solution : (*b*)

$$i = \frac{12}{1200} = 0.01$$

Formula

$$PV = \frac{R}{i} = \frac{50,000}{0.01}$$

= ₹ 50,00,000

(*b*) is correct.

Q.22. A person wants to lease out a machine costing ₹ 5,00,000 for a 10 year period. It has fixed a rental of

₹ 51,272 per annum payable annually starting from the end of first year. Suppose rate of interest is 10% per annum, compounded annually on which money can be invested. To whom this agreement is favourable?

(*a*) Favour for lessee

(*b*) Favour for lessor

(*c*) Not for both

(*d*) Can't be determined

[June 2019]

Solution : (*a*)

Cost = ₹ 5,00,000.

So; GST = PV of Instalments made

$$= PV = 51{,}272\left[\frac{1-\left(1+\frac{10}{100}\right)^{-10}}{i}\right]$$

Calculator Tricks

Type $10 \div 100 + 1 \div =$ button 10 times then press GT button then $\times 51{,}272$ = button = ₹ 3,15,044.25.

Which is less than ₹ 5,00,000.

So, Leasing is preferable.

(*a*) is correct.

Q.23. Let a person invest a fixed sum at the end of each month in an account paying interest 12% per year compounded monthly. It the future value of this annuity after the 12th payment is Rs. 55,000 then the amount invested every month is?

(*a*) ₹ 4,837 (*b*) ₹ 4,637

(*c*) ₹ 4,337 (*d*) ₹ 3337

[June 2019]

Solution : (*c*)

Calculator Tricks

Value of each instalments

$$= R = \frac{FV}{FV \text{ for } ₹\ 1}$$

$$= \frac{55{,}000}{\left[\frac{\left(1+\frac{12}{1200}\right)^{12}-1}{12}\times 1200\right]}$$

= ₹ 4337

* Type $12 \div 1200 + 1 \times = 11$ times

$-1 \div 12 \times 1200 =$ button.

Then press (m+) button.

* Type 55000 ÷ button

then press MRC button

then = button.

We get ₹ 4337.

Q.24. Find the future value of annuity of ₹ 500 is made annually for 7 years interest rate of 14% compound at annually. Given that $(1.14)^7 = 2.5023$

(*a*) ₹ 15635.35 (*b*) ₹ 10,730.71

(*c*) ₹ 16535.35 (*d*) ₹ 16355.35

[Dec. 2019]

Solution : (*b*)

$$FV = 500\left[\frac{\left(1+\frac{14}{100}\right)^{7}-1}{7}\times 100\right]$$

$$= 500\times\left[\frac{2.5023-1}{7}\times 100\right]$$

= ₹ 10,730.71

Q.25. Determine the present value of perpetuity ₹ 10 per month for infinite period at an effective rate of interest of 14% p.a.?

(*a*) ₹ 657 (*b*) ₹ 757

(*c*) ₹ 857 (*d*) ₹ 957

[Dec. 2020]

Solution : $i = \left[\frac{14}{1200}\right]$

$$\therefore PV = \left[\frac{R}{i}\right] = \frac{10}{\frac{14}{1200}} = \frac{10}{14}\times 1200$$

= ₹ 857.14 = ₹ 857.

(*c*) is correct.

Q.26. Which of the following statement is true?

(*a*) F.V of ordinary annuity < F.V of annuity due

(*b*) F.V of ordinary annuity > F.V of annuity due

(*c*) P.V of ordinary annuity > P.V of annuity due

(*d*) None of these

[Dec. 2020]

Solution : (*a*) is correct.

Q.27. Suppose you deposit ₹ 900 per month into an account that pays 14.8% interest compounded monthly. How much money will you get after 9 months?

(*a*) ₹ 8,511 (*b*) ₹ 9,000

(*c*) ₹ 9,200 (*d*) ₹ 1,000

[Dec. 2020]

Solution :

$$FV = R\left[\frac{(1+i)^n-1}{r}\times 100m\right]$$

$$= 900 = \left[\frac{\left(1+\frac{14.8}{1200}\right)^{9}-1}{14.8}\times 1200\right]$$

= ₹ 8511.31 = ₹ 8511

Calculator Trick

$14.8 \div 1200 + 1 \times =$ button 8 times -1

$\div 14.8 \times 1200 \times 900 =$ button. We get FV ₹ 8511.

(*a*) is correct.

Q.28. ₹ 2,500 is paid every year for 10 years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually?

(*a*) 13,040.27 (*b*) 15,847.90

(*c*) 14,674.21 (*d*) 16,345.11

[Dec. 2020]

Solution : Calculator Tricks

Loan amount

$$= PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 2500\left[\frac{1-\left(1+\frac{4}{100}\right)^{-10}}{i}\right]$$

Calculator Tricks

Type 14 ÷ 100 + 1 ÷ = button 10 times (Press)

Then press GT button. then × button.

Type 2500 then = button. (Press)

We get PV = ₹ 13,040.28

(*a*) is correct.

Q.29. Assuming that the discount rate is 7% p.a. how much would pay to receive ₹ 200 growing at 5% annually for ever?

(*a*) ₹ 2,500 (*b*) ₹ 5,000

(*c*) ₹ 7,500 (*d*) ₹ 10,000

[Jan. 2021]

Solution : (*d*) is correct

$$\text{Discount rate} = i = 7\% = \frac{7}{100} = 0.07$$

Growing rate = g = 5% = 0.05

R = Value of each payment received = ₹ 200

$$\therefore \text{PVA} = \frac{R}{i-g} = \frac{200}{0.07-0.05}$$

= ₹ 10,000

Q.30. ₹ 800 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annually after 10th payment?

(*a*) ₹ 4,444 (*b*) ₹ 8,756

(*c*) ₹ 3,491 (*d*) ₹ 8,182

[Jan. 2021]

Solution: (*d*) is correct.

Monthly Instalment = A = ₹ 800

rate of interest = r = 6% p.a. compounded monthly

n = No. of Payments = 10

$$\therefore \text{FV} = A_{(n,i)} = A\left[\frac{(1+r)^n-1}{r}\times 100m\right]$$

$$= 800\left[\frac{\left(1+\frac{6}{1200}\right)^{10}-1}{6}\times 1200\right]$$

= ₹ 8182

[Calculator Tricks 6 ÷ 1200 + 1 × = 9 times –1 ÷ 6 × 1200 × 800 = button; we get ₹ 8182]

Q.31. The present value of an Annuity immediate is the same as

(*a*) Annuity regular for (n - 1) year plus the initial receipt in the beginning of the period

(*b*) Annuity regular for (n - 1) years

(*c*) Annuity regular for (n + 1) years

(*d*) Annuity regular for (n + 1) years plus the initial receipt in the beginning of the period

[Jan. 2021]

Solution : (*a*) is correct

$$\therefore \text{PV} = R\left[\frac{1-(1+i)^{-(n-1)}}{i}+1\right]$$

$$= R\left[\frac{1-(1+i)^{-(n-1)}}{i}\right]+R$$

= PV of Annuity Regular + Value of 1st instalment

(*a*) is correct

Q.32. Find the future value of annuity of ₹ 1,000 made annually for 7 year at interest rate of 14% compounded annually (Given that $1.14^7 = 2.5023$)

(*a*) ₹ 10,730.7 (*b*) ₹ 5,365.35

(*c*) ₹ 8,756 (*d*) ₹ 9892.34

[Jan. 2021]

Solution : (*a*) is correct

$$\text{FV} = A\left[\frac{(1+1)^n-1}{r}\times 100m\right]$$

Where m = No. of conversion periods in 1 year = 1

n = No. of payments made = mt

= n = 1 × 7 = 7

r = 14% yearly

A = Value of each instalment

= ₹ 1000

$$\text{FV} = A_{(n,i)} = 1000\left[\frac{\left(1+\frac{14}{100}\right)^7-1}{14}\times 100\right]$$

Calculator Tricks

Type 14 ÷ 100 × = button 6 times –1 ÷ 14 × 100 × 1000 = button

We get 10,730.5 which is nearest to option (*a*) ₹ 10,730.70

∴ (*a*) is correct

Q.33. A loan of ₹ 1,02,000 is to be paid back in two equal annual instalments. If the rate of interest is 4% p.a., compounded annually, then the total interest charged (in ₹) under this instalment plan is

(*a*) 6160 (*b*) 8120

(*c*) 5980 (*d*) 7560

[July 2021]

Solution : (*a*) is correct

Value of one Instalment

$$= \frac{\text{PV}}{\left[\frac{1+(1+i)^{-(n-1)}}{i}+1\right]}$$

$$= \frac{1,02,000}{\left[\frac{1-(1\cdot04)^{-1}}{i}+1\right]}$$

Calculator Tricks

Type 1·04 ÷ = 1 time press GT button + 1 = button then press M + button

Then type 1,02,000 ÷ MRC button then = button.

We get

R = ₹ 52,000

1st year, C.I. = ₹ 1,02,000 × 4% = ₹ 4080

2nd year, C.I. = ₹ 52000 × 4% = ₹ 2080

Total compound Interest = ₹ 6160

∴ (*a*) is correct

Q.34. If the nominal rate of growth is 17% and inflation is 9% for the five years. Let P be the Gross Domestic Product (GDP) amount at the present

year then the projected real GDP after 6 years is

(*a*) 1.587 P (*b*) 1.921 P

(*c*) 1.403 P (*d*) 2.51 P

[July 2021]

Solution : Real rate of return = Nominal rate of return – Inflation = 17 - 9 = 8%

Expected Real GDP after 6 years

$$= P\left(1+\frac{8}{100}\right)^6$$

$$= 1{\cdot}587\ P$$

Where P = Gross Domestic Product (GDP) Amount

(*a*) is correct

Q.35. If a person bought a house by paying ₹ 45,00,000 down payment and ₹ 80,000 at the end of each year till the perpetuity, assuming the rate of interest as 16%, the present value of house (in ₹) is given as

(*a*) 47,00,000 (*b*) 45,00,000

(*c*) 57,80,000 (*d*) 50,00,000

[July 2021]

Solution : (*d*) is correct

Present value of House

$$= 45,00,000 + \frac{R}{i}$$

$$= 45,00,000 + \frac{80000}{0{\cdot}16}$$

Here $i = \frac{r}{100m} = \frac{16}{100\times1} = 0{\cdot}16$

= 45,00,000 + 5,00,000

= 50,00,000

Q.36. Let the operating profit of a manufacturer for five years is given as:

Year	1	2	3	4	5	6
Operating profit (in lakh ₹)	90	100	106.4	107.14	120.24	157.35

Then the operating profit of Compound Annual Growth Rate (CAGR) for year 6 with respect to year 2 is given at

(*a*) 9% (*b*) 12% (*c*) 11% (*d*) 13%

[July 2021]

Solution : (*b*) is correct

Compound Annual Growth

Rate (CAGR) for year 6 with respect to year 2

$$= \left[\left(\frac{V_6}{V_2}\right)^{\frac{1}{6-2}} - 1\right] \times 100\%$$

$$= \left[\left(\frac{157{\cdot}35}{100}\right)^{\frac{1}{4}} - 1\right] \times 100\%$$

$$= \sqrt[4]{1.5735} - 1 \times 100\%$$

$$= 11{\cdot}999\% = 12\%$$

Calculator Tricks

Type 1·5735 press $\sqrt{\text{button}}$ two times –1 × 100 = button

We get 12%

Q.37. If discount rate is 14% per annum, then how much a company has to pay to receive ₹ 280 growing at 9% annually forever.

(*a*) ₹ 5,600 (*b*) ₹ 2,800

(*c*) ₹ 1,400 (*d*) ₹ 4,200

[July 2021]

Solution : (*a*) is correct

$$PVA = \frac{R}{i-g} = \frac{280}{0{\cdot}14-0{\cdot}03}$$

= ₹ 5600

Q.38. If the cost of capital be 12% per annum, then the net present value (in nearest ₹) from the given cash flow is given as

Year	0	1	2	3
Operating profit (in thousands ₹)	(100)	60	40	50

(*a*) 31048 (*b*) 34185 (*c*) 51048 (*d*) 24187

Solution : (*d*) is correct

Year	**Operating profit 1**	**PVIF @ 12% 2**	**Discounted Cash Flows = 1 ₹ 2**
0	(100,000)	$\left(1+\frac{12}{100}\right)^0 = 1$	(1,00,000)
1	60,000	$\left(1+\frac{12}{100}\right)^{-1} = 0{\cdot}893$	53,580
2	40,000	$\left(1+\frac{12}{100}\right)^{-2} = 0{\cdot}797$	31,880
3	50,000	0·712	35,600
Net present Value (NPA)			₹ 21,060

Its nearest value in option is (*d*)

So (*d*) is correct

Formula
PVIF = $A(1+i)^{-n} = (1+i)^{-n}$
When A = 1

Q.39. The future value of annuity of ₹ 2,000 for 5 years at 5% compounded annually is given (in nearest ₹) as

(*a*) 51051 (*b*) 21021

(*c*) 15624 (*d*) 61254

[July 2021]

Solution : (*c*) is correct

$$FV = R\left[\frac{(1+i)^n-1}{r}\times 100m\right]$$

$$= 2000\left[\frac{(1+\frac{5}{100})^5-1}{5}\times 100\right] = ₹\ 11051{\cdot}26$$

Calculator Tricks

Type 5 ÷ 100 + 1 × = 4 times –1

÷ 5 × 100 × 2000 = button

We get ₹ 11,051.26

Which is nearest to smallest value in option (c)

Q.40. Mr. X wants to accumulate ₹ 50,00,000 at the end of 10 years. Then how much amount is required to be invested every year if interest is compounded annually at 10% (Given that P(10,0.10) = 15.9374298)

(*a*) ₹ 3,13,726.87

(*b*) ₹ 4,13,726.87

(*c*) ₹ 3,53,726.87

(*d*) ₹ 4,53,726.87 ***[Dec. 2021]***

Solution : (*a*)

$$R = \frac{50,00,000}{\left[\frac{\left(1+\frac{10}{100}\right)^{10}-1}{10}\times 100\right]}$$

[Calculator Tricks: Type 10 ÷ 100 + 1 × = button 9 Times – 1 ÷ 10 × 100 (M+) button (Press) Then type 50,00,000 ÷ button then MRC button = button]. We get the Ans.

= ₹ 3,13,726.87

Q.41. The present value of an annuity of ₹ 25,000 to be received after 10 years at 6% per annum compounded annually is ₹ _____. (1.06^5 = 1.33823)

(*a*) ₹ 15,960 (*b*) ₹ 13,960

(*c*) ₹ 11,960 (*d*) ₹ 17,960

[Dec. 2021]

Solution : (*b*)

Note : - Rectification : Delete word "an Annuity" from the question,

Then Answer will match.

Formula

$$PV = A\left(1+\frac{r}{100}\right)^{-t}$$

$$= 25,000\left(1+\frac{6}{100}\right)^{-10} = ₹\ 13,960$$

On Calculator.

Type 6 ÷ 100 + 1 ÷ = button 10 times × 25,000 = button

we get 13,959.8 = ₹ 13,960.

Q.42. ₹ 800 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10th payment? Given that $1.005^{10} = 1.0511$

(*a*) ₹ 4,444 (*b*) ₹ 8,766

(*c*) ₹ 3,491 (*d*) ₹ 8,176

[June 2022]

Solution : Given that

R = ₹ 800 = Value of one instalment

r = 6% Compounded Monthly

n = 10

$$\therefore FV = R\left[\frac{(1+i)^n-1}{r}\times 100m\right]$$

$$= 800\left[\frac{\left(1+\frac{6}{1200}\right)^{10}-1}{6}\times 1200\right]$$

$$= 800\left[\frac{1.0511-1}{6}\times 1200\right]$$

= ₹ 8176/-

(*d*) is correct

Q.43. Lokesh deposits ₹ 3,000 at the start of each quarter in his savings account. If the account earns interest 5.75% per annum compounded quarterly, how much money (in ₹) will he have at the end of 4 years? (1.014375^{16} = 1.25696)

(*a*) ₹ 53,624.4 (*b*) ₹ 58,353.6

(*c*) ₹ 68,353.6 (*d*) ₹ 63,624.4

[June 2022]

Solution : Given

R = ₹ 3000/- = value of one instalment

r = 5.75% compounded quarterly

t= 4 years; n = mt = 4×4=16

$$FV = 3000\left[\frac{\left(1+\frac{5.75}{400}\right)^{16}-1}{5.75}\times 400\right]$$

$$= 3000\left[\frac{1.25696-1}{5.75}\times 400\right]$$

= ₹ 53,626.43 ≅ ₹ 53,624.40

(*a*) is correct

Q.44. Find the future value of annuity of ₹ 1,000 made annually for 7 years at interest rate 14% compounded annually. Given that $(1.14)^7$ = 2.5023

(*a*) ₹ 10,730.71 (*b*) ₹ 5,365.35

(*c*) ₹ 8,756 (*d*) ₹ 9,892.34

[June 2022]

Solution : $FV = R\left[\frac{(1+i)^n-1}{i}\right]$

$$=1000\left[\frac{\left(1+\frac{14}{100}\right)^{7}-1}{14}\times 100\right]$$

Where

R = ₹ 1000; $i = \frac{14}{100}$; n = m t = 1× 7 = 7

$$=1000\left[\left(\frac{2.5023-1}{14}\right)\times 100\right]$$

= ₹ 10,730.71

∴ (*a*) is correct

Q.45. Assuming that the discount rate is 7% p.a. How much would you pay to receive ₹ 200. Growing at 5% annually forever?

(*a*) ₹ 2,500 (*b*) ₹ 5,000

(*c*) ₹ 7,500 (*d*) ₹ 10,000

[June 2022]

Solution : Given

R = ₹ 200; r = 7% yearly

∴ i = 0.07

g = 5%, = 0.05

$$\therefore \text{Present Value} = PV = \frac{R}{i-g}$$

$$= \frac{200}{0.07-0.05} = \frac{200}{0.02}$$

= ₹ 10,000

∴ (*d*) is correct

Q.46. ₹ 2,500 is paid every year for 10 years to pay off a loan. What is the loan amount if interest rate be 14% p.a. compounded annually?

(*a*) ₹ 15,847.90 (*b*) ₹ 13,040.27

(*c*) ₹ 14,674.21 (*d*) ₹ 16,345.11

[June 2022]

Solution : Given that

Each instalment = R = ₹ 2500

time = 10 yrs

r = 14% compounded yearly

m = 1

∴ n = mt = 1 × 10 = 10

$$\text{Loan} = PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 2500\left[\frac{1-\left(1+\frac{14}{100}\right)^{-10}}{i}\right]$$

Calculator Tricks

Type 14 ÷ 100 + 1÷ = button 10 times then press GT button then × button

Type 2500 = button.

∴ Loan = ₹ 13,040.27

∴ (*b*) is correct

Q.47. Raj made an investment of ₹ 15,000 in a scheme and at the time of maturity the amount was ₹ 25,000. If Compound Annual Growth Rate (CAGR) for this investment is 8.88%. Calculate the approximate number of years for which he has invested the amount.

(*a*) 6 (*b*) 7.7

(*c*) 5.5 (*d*) 7

[June 2022]

Solution : Given

Vn = ₹ 25000

Vo = ₹ 15000

CAGR = r = 8.88%

Formula

$$\left[\left(\frac{Vn}{Vo}\right)^{\frac{1}{n-0}}-1\right]\times 100 = CAGR$$

$$\text{or}\left[\left(\frac{25000}{15000}\right)^{\frac{1}{n}}-1\right]\times 100 = 8.88$$

$$\text{or}\left(\frac{25000}{15000}\right)^{1/n}-1 = \frac{8.88}{100}$$

$$\text{or}\left(\frac{25000}{15000}\right)^{1/n} = 1+0.0888$$

$$\text{or}\,(1.0888)^n = \frac{25000}{15000} = 1.666....=1.67$$

GBC

@ n = 6

∴ $(1.0888)^6$ ⇒ By calculator

1.0888 × = 5 times

= 1.666605 = 1.67 Approx. (True)

∴ t = n = 6 (True)

(*a*) is correct

Q.48. Madhu takes a loan of ₹ 50,000 from XYZ Bank. The rate of interest is 10% per annum. The first instalment will be paid at the end of year 5. Determine the amount (in ₹) of equal instalments, if Madhu wishes to repay the amount in five instalments.

(*a*) ₹ 19,510 (*b*) ₹ 19,430

(*c*) ₹ 19,310 (*d*) ₹ 19,630

[June 2022]

Solution :

R...5 instalment

R = Value of one instalment.

It is a Question of Deferred Annuity

1st Payment made at the end of 5 years.

So Deferred Period = m-1 = 5-1 = 4

No. of instalments = n = 5

$$i = \frac{r}{100\,m} = \frac{10}{100} = 0.10$$

Loan = PV = ₹ 50, 000

Formula

PV for Deferred Annuity

$$= \frac{R\left[1-(1+i)^{-n}\right]}{i(1+i)^{m-1}}$$

$$\Rightarrow 50{,}000 = R\,\frac{\left[1-(1+0.1)^{-5}\right]}{(0.10)(1+0.1)^4}$$

$$= R\left[\frac{1-(1.1)^{-5}}{0.1}\right](1.1)^{-4}$$

[On Calculator 1÷ =Type 1.1 ÷ = button 5 times than press GT button then on + button

Then 1.1÷ = button 4 times × MRC button = button]

∴ 50,000 = R [2.589158]

$$\therefore R = \frac{50{,}000}{2.589158}$$

= ₹ 19311/-

∴ (*c*) is correct.

Q.49. Ramesh invests ₹ 20,000 per year in a stock index fund, which earns 9% per year, for the next ten years. What would be the closest value of the accumulated value of the investment upon payment of the last instalment? ($1.09^{10} = 2.36736$)

(*a*) ₹ 3,88,764.968

(*b*) ₹ 3,03,858.594

(c) ₹ 2,68,728.484

(d) ₹ 4,08,718.364

[June 2022]

Solution : Given

R = Volume of one instalment = ₹ 20,000/-

r = 9% yrly ; t =10 yrs

n = 1 × 10 = 10

$$\therefore FV = 20,000\left[\frac{\left(1+\frac{9}{100}\right)^{10}-1}{0.09}\right]$$

$$= 20000\left[\frac{(1.09)^{10}-1}{0.09}\right]$$

$$= 20000\left[\frac{2.36736-1}{0.09}\right]$$

= 3,03,857.77

≅ 3,03,858.594 (approx.)

∴ (b) is correct

Q.50. A company creates a sinking fund of ₹ 2,00,000 in a bank account for 15 years bank offers interest rate 6% per annum the yearly payment to be paid by company is approximately ↔ (if need, use: $1.06^{14} = 2.209$)

(a) ₹ 8,945 (b) ₹ 8,145

(c) ₹ 9,345 (d) ₹ 9,645

[Dec. 2022]

Solution : Given that

FV = ₹ 2,00,000

T = 15 yrs; r = 6% yearly

n = mt = 1× 15 = 15

∴ Value of one instalment

$$= R = \frac{FV}{\left[\frac{(1+i)^n-1}{r}\times 100\ m\right]}$$

$$= \frac{2,00,000}{\left[\frac{\left(1+\frac{6}{100}\right)^{15}-1}{6}\times 100\right]}$$

$$= \frac{2,00,000}{\frac{(1.06)^{14}\ (1.06)-1}{6}\times 100}$$

[Note:- We have to use given value $(1.06)^{14} = 2.209$]

$$= \frac{200,000}{\frac{(2.209\times 1.06-1)}{6}\times 100}$$

= ₹ 8944.94 = ₹ 8945/-

∴ (a) is correct

Q.51. How much amount is required to be invested every year so as to accumulate ₹ 5,00,000 at the end of 12 years if interest is compounded annually at 10%? (Where A (12, 0.1) = 21.384284).

(a) ₹ 23381.65 (b) ₹ 24385.85

(c) ₹ 26381.65 (d) ₹ 28362.75

[Dec. 2022]

Solution: FV = ₹ 5,00,000

R. A(n, i) = FV

Where

R = value of 1 instalment (yearly)

∴ R. A(12, 0.10) = 5,00,000

$$\text{or } R = \frac{5,00,000}{A(12,0.10)}$$

$$= \frac{5,00,000}{21.38165}$$

= ₹ 23,381.65

∴ (a) is correct.

Q.52. Raju invests ₹ 20,000 every year in a deposit scheme starting from today for next 12 years. Assuming that interest rate on this deposit is 7% per annum compounded annually. What will be the future value of this annuity? Given that $(1 + 0.07)^{12} = 2.25219159$.

(a) ₹ 5,40,526 (b) ₹ 3,82,813

(c) ₹ 6,43,483 (d) ₹ 3,57,769

[Dec. 2022]

Solution: *Given :*

Value of 1 instalment = R = ₹ 20,000 (yearly)

Starting from today means annuity due

r = 7% compounded yearly; t = 12 yrs

$$FV = (1+i)\times R\left[\frac{(1+i)^n-1}{i}\right]$$

$$= \left(1+\frac{7}{100}\right)\times 20,000\left[\frac{\left(1+\frac{7}{100}\right)^{12}-1}{7}\times 100\right]$$

$$= (1.07)\times 20,000\left[\frac{2.25219159-1}{7}\times 100\right]$$

= ₹ 3,82,813

∴ (b) is correct

Q.53. Mr. A invested ₹ 10,000 every year for next 3 year at the interest rate of 8 per cent per annum compounded annually. What is future value of the annuity?

(a) 32,644 (b) 32,464

(c) 34,4264 (d) 36,442

[Dec. 2022]

Solution: *Given :*

Value of 1 instalment = R = ₹ 10,000

Time = t = 3 years

r = 8% p.a. compounded yearly

∴ m = 1

n = mt = 3 ×1 = 3

FV = ?

$$FV = R\left[\frac{(1+i)^n-1}{r}\times 100\ m\right]$$

$$= 10,000\left[\frac{\left(1+\frac{8}{100}\right)^{3}-1}{8}\times 100\right]$$

= ₹ 32,464

∴ (b) is correct

Q.54. ₹ 5,000 is invested every month end in an account paying interest @ 12% per annum compounded monthly. What is the future value of this annuity just after making 11th payment? (Given that $(1.01)^{11} = 1.1156$)

(a) ₹ 57,800 (b) ₹ 56,100

(c) ₹ 56,800 (d) ₹ 57,100

[Dec. 2022]

Solution: *Given*

R = ₹ 5000;

r = 12% p.a. compounded monthly

n = 11 (ordinary Annuity)

$$\therefore FV = R\left[\frac{(1+i)^n - 1}{r} \times 100m\right]$$

$$= 5000\left[\frac{\left(1+\frac{12}{1200}\right)^{11} - 1}{12} \times 1200\right]$$

$$= 5000\left[\frac{1.1156 - 1}{12} \times 1200\right]$$

$= ₹\ 57,800$

(a) is correct

Q.55. Sinking fund factor is the reciprocal of :

(a) Present value interest factor of a single cash flow

(b) Present value interest factor of an annuity

(c) Future value interest factor of an annuity

(d) Future value interest factor of a single cash flow

[Dec. 2022]

Solution: (c) FV = A.A (n ; i) = A = $\frac{Fv}{A\,(n; i)}$

Where A = Value of one instalment

A[n; i] = Annuity factor

(a) is correct

Q.56. Suppose you have decided to make a Systematic Investment Plan (SIP) in a mutual fund with ₹ 1,00,000 every year from today for next 10 years where you get return at the rate of 10% per annum compounded annually. What is the future value of this annuity? Given $1.1^{10} = 2.59374$

(a) ₹ 17,35,114

(b) ₹ 17,53,411

(c) ₹ 17,35,411

(d) ₹ 17,53,114

[June 2023]

Solution: Given

R = ₹ 1,00,000/- Today ⇒ Annuity Due

r = 10% pa. yearly; n = 10

$$FV = (1 + i) \times A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\left(1+\frac{1}{100}\right) \times 1{,}00{,}0000\left[\frac{\left(1+\frac{10}{100}\right)^{10} - 1}{0.10}\right]$$

$$= (1.10) \times (1{,}00{,}000) \times \left[\frac{2.59374 - 1}{0.10}\right]$$

$= ₹\ 17,53,114$

(d) is correct.

Q.57. Ms. Paul invested ₹ 1,00,000 in a mutual fund scheme in January 2018. After one year in January, 2019, she got a dividend amounting to ₹ 10,000 for first year. ₹ 12,000 for second year, ₹ 16,000 for third year, ₹ 18,000 for fourth year and ₹ 21,000 for fifth year in January 2023. What is Compounded Annual Growth Rate (CAGR) of dividend return? Given $1.2038^4 = 2.1$.

(a) 20.38%

(b) 18.59%

(c) 16.36%

(d) 15.89%

[June 2023]

Solution: Given

Investment = ₹ 1,00,000/- (in 2018)

Years	2019	2020	2021	2022	2023
	10,000	12,000	16,000	18,000	21,000

Formula

$$CAGR = \left[\left(\frac{V_n}{V_o}\right)^{\frac{1}{t_n - t_o}} - 1\right] \times 100$$

$$= \left[\left(\frac{21{,}000}{10{,}000}\right)^{\frac{1}{2023-2019}} - 1\right] \times 100$$

$$= \left[(2.1)^{\frac{1}{4}} - 1\right] \times 100 \qquad \left[\because (1.2038)^4 = 2.1,\ \therefore (2.1)^{1/4} = 1.2038\right]$$

$= [1.2038 - 1] \times 100 = 20.38\%$

$\therefore$ (a) is correct.

Q.58. A company want to replace its existing tool room machine at the end of 10 years, the expected cost of machine would be ₹ 10,00,000. If management of the company creates a sinking fund, how much provision needs to be made out of revenue each year which can earn at the rate of 10% compounded annually? Given A(10,0.10) = 15.937425

(a) ₹ 74,625

(b) ₹ 72,514

(c) ₹ 62,745

(d) ₹ 67,245

[June 2023]

Solution: Value of 1st instalment

$$= \frac{FV}{A(n; i)} = \frac{10{,}00{,}000}{A(10; 0.10)}$$

$$\frac{10{,}00{,}000}{15.937425} = ₹\ 62,745.39$$

$= ₹\ 62,745$

(c) is correct.

Q.59. A car is available for ₹ 4,98,200 cash payment or ₹ 60,000 cash down payment followed by three equal annual instalments. If the rate of interest charged is 14% per annum compounded yearly, then total interest charged in the instalment plan is (Given P(3,0.14) = 2.32163):

(*a*) ₹ 1,46,314

(*b*) ₹ 1,46,137

(*c*) ₹ 1,28,040

(*d*) ₹ 1,58,040

[June 2023]

Solution:

Cost of Car = ₹ 4,98,200

Cash Down = ₹ 60,000

Rest Amount for instalments = ₹ 4,38,200 = PV

Value of each Instalment

$$= \frac{PV}{PV \text{ for } ₹\ 1}$$

$$= \frac{4,38,200}{P(3,0.14)}$$

$$= \frac{4,38,200}{2.32163} \quad ₹\ 188,746.65$$

Total Value of 3 Instalments

= 188,746.65 × 3

= ₹ 5,66,240.

Total Interest = 5,66,240 - 4,38,200

= ₹ 1,28,040

(*c*) is correct.

Q.60. Govinda's mother decides to gift him ₹ 50,000 every year starting from today for the next five years. Govinda deposits this amount in a bank as and when he receives and gets 10% per annum interest rate, compounded annually. What is the present value of this annuity? Given P(4,0.10) = 3.16987.

(*a*) ₹ 2,80,493.5

(*b*) ₹ 2,08,493.5

(*c*) ₹ 2,08,943.5

(*d*) ₹ 2,58,493.5

[June 2023]

Solution: Given

Value of one instalment

= R = ₹ 50,000/- yearly

t = 5 years; r = 10% yearly

n = 5 × 1 = 5

(Annuity Due Qts.)

$$PV = R\left[\frac{1-(1+i)^{-(n-1)}}{i}+1\right]$$

$$= 50,000\left[\frac{1-\left(1+\frac{10}{100}\right)^{-(5-1)}}{i}+1\right]$$

= 50,000[P(4, 0.10) + 1]

= 50,000[3.16987 + 1]

= 2,08,493.5

∴ (*b*) is correct.

Q.61. If the discount rate is 10% per annum, how much amount would you pay to receive ₹ 2,500 growing at 8%, annually forever?

(*a*) ₹ 1,25,000

(*b*) ₹ 2,50,000

(*c*) ₹ 1,50,000

(*d*) ₹ 2,00,000

[June 2023]

Solution:

Given

R = ₹ 2,500/-

$$PVA = \frac{R}{i-g} = \frac{2500}{\frac{10}{100}-\frac{8}{100}}$$

$$= \frac{2500}{\frac{2}{100}} = 2500 \times \frac{100}{2}$$

= ₹ 1,25,000

∴ (*a*) is correct.

Q.62. Mr. Sharad got his retirement benefits amounting to ₹ 50,00,000. He want to receive a fixed monthly sum of amount for his rest of life, starting after one month and thereafter he want to pass on the same to future generation. He expects to earn an interest of 9% compounded annually. Determine how much perpetuity amount he will receive every month?

(*a*) ₹ 39,500

(*b*) ₹ 38,500

(*c*) ₹ 37,500

(*d*) ₹ 36,600

[June 2023]

Solution:

Given PVA = ₹ 50,00,000

Value of 1st instalment = R (let)

i = 9% monthly,

$$\therefore PVA = \frac{R}{i} \Rightarrow R = PVA \times i$$

$$\therefore \quad R = 50,00,000 \times \frac{9}{1200}$$

= ₹ 37,500

∴ (*c*) is correct.

Tricks

P ∪ A = R ÷ (r – g)% button

= 2500 ÷ 2% button

= ₹ 1,25,000

Q.63. Jonny wants to have ₹ 2,00,000 in his saving account after three year. The rate of interest offered by bank is 8% per annum compounded annually. How much should he invest today to achieve his target amount?

(*a*) ₹ 1,47,489.10

(*b*) ₹ 1,58,766.44

(*c*) ₹ 1,71,035.59

(*d*) ₹ 1,84,417.96

[June 2023]

Solution: Given

FV = ₹ 2,00,000

r = 8% yearly; t = 3 yrs; n = 3

Annuity Due Qts.;

Value of one instalment = R = ?

$$R = \frac{FV}{\left[\frac{(1+i)^{n+1}-I}{i}\right]}$$

$$= \frac{2,00,000}{\left[\frac{\left(1+\frac{8}{100}\right)^{3+1}-1}{0.08}-1\right]}$$

$$= \frac{2,00,000}{\left[\frac{(1.80)^4 - 1}{0.08} - 1\right]} = ₹\ 57043.24$$

Calculator Tricks

Type 1.08 x = button 3 times – 1

÷ 0.08 – 1 = button then press (M^+) button. After this

Type 2,00,000 ÷ MRC button 2 times (press) then press = button. We get R Value

Now 1st investment value

$$= PV = 57043.24 \left[\frac{1-\left(1+\frac{8}{100}\right)^{-(3-1)}}{0.08} + 1\right]$$

$= ₹\ 1,58,766.44$ approx.

$\therefore$ (*b*) is correct.

10 CHAPTER PERMUTATIONS AND COMBINATIONS

Permutations and combinations :

There are various ways in which objects from a set may be selected, generally without replacement, to form subsets. This selection of subsets is called a **Permutation** when the order of selection is a factor and this selection of subsets is called a **Combination** when order is not a factor.

The concepts of and differences between permutations and combinations can be illustrated by examination of all the different ways in which a pair of objects can be selected from five distinguishable objects such as the letters A, B and C. If both the letters selected and the order of selection are considered, then the following 6 outcomes are possible they are AB ; AC ; BC; BA; CA ; CB.

Each of these 6 different possible selections is called a **permutation**. In particular, they are called the **permutations of three objects taken two at a time,** and the number of such permutations possible is denoted by the symbol ${}^3\mathbf{P}_2$, read "3 permute 2." In general, if there are n objects available from which to select, and permutations (P) are to be formed using r of the objects at a time, the number of different permutations possible is denoted by the symbol ${}^nP_r$ & formulated as

$${}^n p_r = \frac{n!}{(n-r)!}$$

$${}^3 p_2 = \frac{3!}{(3-2)!} = 3! = 3.2.1 = 6$$

For **Combinations,** r objects are selected from a set of n objects to produce subsets without ordering. Contrasting the previous permutation example with the corresponding combination, the AB and BA subsets are no longer distinct selections; by eliminating such cases there remain only 3 different possible subsets—AB, AC, BC.

The number of such subsets is denoted by ${}^nC_r$, read "n choose r" & is formulated as ${}^nC_r = \frac{n!}{r!(n-r)!}$

Fundamental Rule of Counting

(*i*) The SUM Rule/Addition Rule

Ex- 7 buses

CP ⟶ Laxmi Nagar

5 Autoes

There are 7 buses and 5 Autoes to go from CP to Laxmi Nagar . In how many ways a person can reach Laxmi Nagar from CP by buses or autoes. We have two options/ choices of vehicles Bus or Auto. There are 7 options to reach by bus or 5 options to reach by auto.

∴ Total ways to reach Laxmi Nagar from CP

= 7 + 5 = 12 ways.

Rule: If there are two alternative jobs, one can be done by "p" ways and another by "q" ways, either of these two jobs can be done by (p + q) ways.

(*ii*) Multiplication (Product) Rule:

If one things is done by "p" different ways and when it has been done a second thing can be done by "q" different ways then total number of ways of doing both things simultaneously

= p × q. ways

Ex-1 Suppose, there are 7 different buses to reach Jaipur from Delhi. In how many ways a person can go to Jaipur by a bus and can come back by different bus. There are 7 options to go to Jaipur by bus but 6 options to come back by a different bus.

∴ Total No. of ways to reach Jaipur and come back

= 7 × 6 = 42 ways

Ex-2 There are 5 roads to reach LUCKNOW from Delhi; 4 roads to reach Gorakhpur from Lucknow and 6 roads for Patna from Gorakhpur. In how many different ways can a person reach Patna from Delhi.

Solution : Total No. of ways = 5 × 4 × 6 = 120

Note:

(*i*) In case of either or ; or ; one of them ; atleast one ; or options then use addition rule

(*ii*) In case of AND ; then use product rule.

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. Six points are on a circle. The number of quadrilaterals that can be formed are:

(*a*) 30 (*b*) 360

(*c*) 15 (*d*) None

[June 2010]

Solution : (*c*) is correct

No. of Quadrilaterals

${}^6C_4 = \frac{6!}{4!.2!} = \frac{6.5.4!}{4!.2!} = 15$

Q.2. The number of ways of arranging 6 boys and 4 girls in a row so that all 4 girls are together is:

(*a*) 6!.4! (*b*) 2(7!.4!)

(*c*) 7!.4! (*d*) 2.(6!.4!)

[June 2010]

Solution : (*c*) is correct

No. of arrangements of 6 boys and 4 girls so that all 4 girls are together

= (6 +1)! .4!

= 7!. 4! = 120960.

Q.3. How many numbers not exceeding 1000 can be made from the digits 1,2, 3, 4, 5, 6, 7, 8, 9 if repetition is not allowed.

(*a*) 364 (*b*) 585

(*c*) 728 (*d*) 819

[June 2010]

Solution : (*b*) is correct

Given digits = 1, 2, 3, ...;9

Numbers less than 1000 will be of 1 digit 2 digits and of 3 digits.

∴ Total No. of Numbers = ${}^9P_1 + {}^9P_2 + {}^9P_3$

= 9 + 9 × 8 + 9 × 8 × 7 = 585

Q.4. A garden having 6 tall trees in a row. In how many ways 5 children stand, one in a gap between the trees in order to pose for a photograph?

(*a*) 24 (*b*) 120

(*c*) 720 (*d*) 30

[Dec. 2010]

Solution : (*b*) is correct

Treas

1 * 2 * 3 * 4 * 5 * 6

Clearly there will be 5 positions for children

= ${}^5P_5 = 120$

Q.5. ${}^{15}C_3 + {}^{15}C_3$ **is equal to:**

(*a*) 16_{c_3} (*b*) $30_{c_{16}}$

(*c*) $15_{c_{16}}$ (*d*) $15_{c_{15}}$

[Dec. 2010]

Solution : (*a*) is correct

${}^{15}C_3 + {}^{15}C_2 = {}^{16}C_3$

Q.6. How many ways a team of 11 players can be made out of 15 players if one particular player is not to be selected in the team.

(*a*) 364 (*b*) 728

(*c*) 1,001 (*d*) 1,234

[Dec. 2010]

Solution : No. of ways to make a 11 - member teams

$= {}^{15-1}C_{11} = {}^{14}C_{11} = \frac{14!}{11!3!}$

$= 364$

Q.7. Find the number of arrangements of 5 things taken out of 12 things, in which one particular thing must always be included.

(*a*) 39,000 (*b*) 37,600

(*c*) 39,600 (*d*) 36,000

[June 2011]

Solution : (*c*) is correct

No. of arrangements of 5 things

$= {}^{12-1}C_{5-1}.5! = {}^{11}C_4.5!$

$= \frac{11!}{4!.7!} \times 120 = 330 \times 120 = 39600$

Q.8. In how many ways 3 prizes out of 5 can be distributed amongst 3 brothers Equally ?

(*a*) 10 (*b*) 45

(*c*) 60 (*d*) 120

[Dec. 2011]

Solution : (*c*) is correct

No. of ways $= {}^5C_1.{}^4C_1.{}^3C_1 = 5.4.3 = 60$

Q.9. There are 12 question are to be answered to be Yes or No. How many ways can these be answered ?

(*a*) 1024 (*b*) 2048

(*c*) 4096 (*d*) None

[Dec. 2011]

Solution : (*c*) No. of ways $= 2^{12} = 4096$

Q.10. The letters of the word VIOLENT are arranged so that the vowels occupy even place only. The number of permutations is

(*a*) 144 (*b*) 120

(*c*) 24 (*d*) 72

[June 2012]

Solution : (*a*) Vowels = I , O, E

Consonants = V, L, N, T

No. of perms. of vowels

$= {}^3P_3 = 3! = 6$

$\therefore$ Total no. of words

$= 4! \times 3! = 24 \times 6 = 144$

Q.11. If ${}^nP_4 = 20({}^np_2)$ then the value of 'n' is_____

(*a*) -2 (*b*) 7

(*c*) -2 and 7 both (*d*) None of these

[June 2012]

Solution : (*b*) Note : n is always positive Go by choices.

Q.12. A man has 3 sons and 6 schools within his reach, in how many ways, he can send them to school, if no two of his sons are to read in the same school ?

(*a*) ${}^6p_2$ (*b*) ${}^6p_3$

(*c*) 6^3 (*d*) 3^6

[Dec. 2012]

Solution : (*b*) No. of ways $= {}^6P_3$

Q.13. How many permutations can be formed from the letters of the word "DRAUGHT", if both vowels may not be separated?

(*a*) 720 (*b*) 1,440

(*c*) 140 (*d*) 1,000

[Dec. 2012]

Solution : (*b*) Total Perms. of DRAUGHT

$= 6!.2! = 720 \times 2 = 1440.$

Q.14. If ${}^{13}C_6 + 2{}^{13}C_5 + {}^{13}C_4 = {}^{15}C_x$ then, x = _____

(*a*) 6 (*b*) 7

(*c*) 8 (*d*) 9

[Dec. 2012]

Solution : (*a*)

${}^{13}C_6 + {}^{13}C_5 + {}^{13}C_5 + {}^{13}C_4 = {}^{15}C_x$

or ${}^{14}C_6 + {}^{14}C_5 = {}^{15}C_x$

or ${}^{15}C_6 = {}^{15}C_x \therefore x = 6$

Q.15. The total number of shake hands in a group of 10 persons to each other are_____

(*a*) 45 (*b*) 54

(*c*) 90 (*d*) 10

[June 2013]

Solution : (*a*) Total No. of hand shakes

$= {}^{10}c_2 = 45$

Q.16. A regular polygon has 44 diagonals then the No. of sides are_____

(*a*) 8 (*b*) 9

(*c*) 10 (*d*) 11

[June 2013]

Solution : (*d*) No. of Diagonals in a polygon of "n" sides $= {}^nc_2 - n = 44$

Tricks : Go by choices.

For.- (*a*) ${}^8c_2 - 8 = 28 - 8 \neq 44$

(*b*) ${}^9c_2 - 9 = 36 - 9 \neq 44$

(*c*) ${}^{10}c_2 - 10 = 45 - 10 \neq 44$

(*d*) ${}^{11}c_2 - 11 = 55 - 11 = 44$

$\therefore$ (*d*) is correct

Q.17. In how many ways the word "ARTICLE" can be arranged in a row so that vowels occupy even places?

(*a*) 132 (*b*) 144

(*c*) 72 (*d*) 160

[June 2013]

Solution : (*b*) In word ARTICLE

Vowels = A,I,E

Positions For vowels = 2, 4, 6

$\therefore$ Vowels can be arranged in 3! = 6 ways

Rest letters can be arranged in 4! = 24 ways

$\therefore$ Total No. of such arrangements

$= 6 \times 24 = 144$

Q.18. How many different words can be formed with the letters of the word "LIBERTY"

(*a*) 4050 (*b*) 5040

(*c*) 5400 (*d*) 4500

[Dec. 2013]

Solution : (*b*) is correct

LIBERTY

No. of words = 7! = 5040

Q.19. In how ways can a family consist of 3 children have different birthday in a leap year

(*a*) $366 \times 365 \times 364$

(*b*) ${}^{366}c_6$

(c) $^{365}c_3$

(d) $^{366}c_3 - 3$

[Dec. 2013]

Solution : (a) is correct

1 Leap year = 366

No. of ways = $^{366}c_1 . ^{365}c_1 . ^{364}c_1$

$= 366 \times 365 \times 364$

Q.20. If $^{15}c_{3r} = {}^{15}c_{r+3}$ then r =

(a) 2 (b) 3

(c) 4 (d) 5

[Dec. 2013]

Solution : (b) is correct

$^{15}c_{3r} = {}^{15}c_{r+3}$

Either $3r = r + 3$

$2r = 3$; So, $r = \frac{3}{2}$ (In fraction; so invalid)

Or $3r + r + 3 = 15$

or $4r = 12$; So, $r = 3$

Q.21. If 6 times the No. of permutations of n items taken 3 at a times is equal to 7 times the No. of permutations of (n-1) items taken 3 at a time then the value of n will be

(a) 7 (b) 9

(c) 13 (d) 21

[June 2014]

Solution : (d) is correct

$6.{}^nP_3 = 7.{}^{(n-1)}P_3$ (given)

or; $6.\frac{n!}{(n-3)!} = 7.\frac{(n-1)!}{(n-1-3)!}$

or; $\frac{6.n.(n-1)!}{(n-3)(n-4)!} = \frac{7(n-1)!}{(n-4)!}$

or $\frac{6n}{n-3} = 7$

or 6n = 7n - 21

or n = 21

Tricks : By Calculator (Go by choices)

Q.22. If $^{1000}c_{98} = {}^{999}c_{97} + {}^{x}c_{901}$ then the value of x will be

(a) 999 (b) 998

(c) 997 (d) None

[June 2014]

Solution : (a) is correct

Tricks : Go by choices

For (a) RHS = $^{999}C_{97} + {}^{999}C_{901}$

$= {}^{999}C_{97} + {}^{999}C_{98}$ $\left[\because {}^nC_r = {}^nC_{n-r}\right]$

$= {}^{1000}C_{98}(L.H.S)$

$\left[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\right]$

Q.23. $^6p_r = 360$ then find r

(a) 4 (b) 5

(c) 6 (d) None

[Dec. 2014]

Solution : (a) is correct

Tricks: Go by choices:

$^6p_r = 360$

LHS at r = 4

$= {}^6p_4 = 6.5.4.3 = 360$. R.H.S.

Q.24. If 5 books of English 4 books of Tamil and 3 books of Hindi are to be arranged in a single row so that books of same language come together

(a) 1,80,630 (b) 1,60,830

(c) 1,03,680 (d) 1,30,680

[Dec. 2014]

Solution : (c) is correct

Total No. of ways so that same language

books remain together

$= 5!.4!.3!.3!.$

$= 120 \times 24 \times 6 \times 6 = 1,03,680$

Q.25. 5 Boys and 4 girls are to be seated in row. If the girls occupy even places then the No. of such arrangements

(a) 288 (b) 2808

(c) 2008 (d) 2880

[Dec. 2014]

Solution : (d) is correct

Total No. of students = 9

For girls positions may be 2,4,6,8,

Total no. of arrangements of girls

$^4p_4 = 4! = 24$

For boys = 5 ! = 120

Total ways = $5!.4! = 120 \times 24 = 2880$

Q.26. A person has 10 friends of which 6 of them are relatives. He wishes to invite 5 persons so that 3 of them are relatives. In how many ways he can invites?

(a) 450 (b) 600

(c) 120 (d) 810

[June 2015]

Solution : Friends = 10

Relatives = 6 ; So, Rest = 4 friends

Selection of 5 in which 3 are relatives

$= {}^6c_3 . {}^4c_2 = 20 \times 6 = 120$

(c) is correct

Q.27. A student has 3 books on computer, 3 books on Economics, 5 on Commerce. If these books are to be arranged subject wise then these can be placed on a shelf in the__ number, of ways.

(a) 25.290 (b) 25,920

(c) 4,230 (d) 4,320

[June 2015]

Solution : Total ways = $3! \times 3! \times 5! \times 3!$

$= 6 \times 6 \times 120 \times 6$

$= 25920$

Q.28. The number of 4 digit numbers that can be formed from seven digits 1, 2, 3, 5, 7, 8, 9 such that no digit being repeated in any number, which are greater than 3000 are

(a) 120 (b) 480

(c) 600 (d) 840

[June 2015]

Solution :

5	6	5	4

At 1st place 3,5,7,8,9 these 5 digits are suitable.

So, permutation of 1st place = 5

One of them will be used at that place.

Remaining 6 digits are suitable for next place.

Similarly doing as above,

Total ways = 5.6.5.4 = 600

(c) is correct.

Q.29. A question paper consist 10 questions, 6 in math and 4 in stats. Find out number of ways to solve question paper if at least one question is to be attempted from each section.

(*a*) 1024 (*b*) 950

(*c*) 945 (*d*) 1022

[Dec. 2015]

Solution : (*c*) is correct

No. of ways to attempt at least one from each $= (2^6 - 1)(2^4 - 1) = 945.$

Q.30. There are 6 gents and 4 ladies. A committee of 5 is to be formed if it include at least two ladies.

(*a*) 64 (*b*) 162 (*c*) 102 (*d*) 186

[Dec. 2015]

Solution : (*d*) is correct

	Gents	Ladies		Combs
	(6)	(4)		
A Committee	(*i*) 3	2	=	$6c_3.4c_2 = 120$
of 5	(*ii*) 2	3	=	$6c_2.4c_3 = 60$
	(*iii*) 1	4	=	$6c_1.4c_4 = 6$
	Total	= 186		

Q.31. ${}^nP_r = 720$ **and** ${}^nC_r = 120$ **, Find r ?**

(*a*) 6 (*b*) 4

(*c*) 3 (*d*) 2

[Dec. 2015]

Solution : (*c*) is correct

$${}^nP_r = {}^nC_r \; r! = 720$$

$$120.r! = 720$$

$$r! = 6 = 3! \quad \therefore r = 3$$

Q.32. There are 10 students in a class, including 3 girls. The number of ways to arrange them in a row, when any two girls out of them never come together

(*a*) $8_{p_3} \times 7!$ (*b*) $3_{p_3} \times 7!$

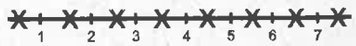
(*c*) $8_{p_3} \times 10!$ (*d*) None

[June 2016]

Solution : (*a*)

Rest Students = 7

Total no. of permutations of rest 7 stds. = 7 !



Total no. of places for girls = 7 + 1 = 8

Total ways to arrange girls = ${}^8P_3$

Total No. of required ways = ${}^8P_3 \cdot (7!)$

Q.33. In how many ways can a selection of 6 out of 4 teachers and 8 students be done so as to include atleast two teachers ?

(*a*) 220 (*b*) 672 (*c*) 896 (*d*) 968

[June 2016]

Solution : (*b*)

		Teachers	Stds.	Combs.
		4	8	
Selection of	(*i*)	2	4	$= {}^4C_2 \cdot {}^8C_4 = 420$
6 Persons =	(*ii*)	3	3	$= {}^4C_3 \cdot {}^8C_3 = 224$
	(*iii*)	4	2	$= {}^4C_4 \cdot {}^8C_2 = 28$ Total = 672

Q.34. The maximum number of points of inter section of 10 circles will be

(*a*) 2 (*b*) 20

(*c*) 90 (*d*) 180

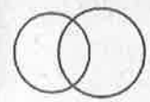
[June 2016]

Solution : (*c*)

To intersect 2 circles out of 10 are required

$= {}^{10}C_2 \times 2 = 45 \times 2 = 90$ Nos. of ways

(Each circle will cut at 2 points)



Q.35. How many numbers between 1,000 and 10,000 can be formed with the digits 1, 2, 3, 4, 5, 6

(*a*) 720 (*b*) 360

(*c*) 120 (*d*) 60

[Dec. 2016]

Solution : (*b*) is correct.

Total no. of numbers = ${}^6p_4 = 6.5.4.3 = 360$

Q.36. If ${}^{n+1}C_{r+1} : {}^nC_r : {}^{n-1}C_{r-1} = 8:3:1$ **; then find the value of *n*.**

(*a*) 14 (*b*) 15

(*c*) 16 (*d*) 17

[Dec. 2016]

Solution : (*b*) is correct.

Formula, ${}^nC_r : {}^{n-1}C_{r-1} = \frac{n}{r} = \frac{3}{1}$;

So, n = 3r

& ${}^{n+1}C_{r+1} : {}^nC_r = \frac{n+1}{r+1} = \frac{8}{3}$;

So, $3n + 3 = 8r + 8$

or, $3 \times 3r + 3 = 8r + 8$

or, $r = 8 - 3 = 5$

Hence, $n = 3r = 3 \times 5 = 15$

Q.37. In how many ways 4 members can occupy 9 vacant seats in a row

(*a*) 3204 (*b*) 3024

(*c*) 4^9 (*d*) 9^4

[Dec. 2016]

Solution : (*b*) is correct.

Total ways = ${}^{9}p_{4} = 9.8.7.6 = 3024$

Q.38. The number of arrangements that can be formed from the letters of the word "ALLAHABAD"

(*a*) 7560 (*b*) 3780

(*c*) 30240 (*d*) 15320

[June 2017]

Solution : $\dfrac{9!}{(4!).(2!)}$

$= \dfrac{9.8.7.6.5(4!)}{(4!).2.1.} = 7560$

$\therefore$ Option (*a*) is correct

Q.39. If ${}^{10}c_{3} + 2.{}^{10}c_{4} + {}^{10}c_{5} = {}^{n}c_{5}$ then the value of n = _____

(*a*) 10 (*b*) 11 (*c*) 12 (*d*) 13

[June 2017]

Solution : ${}^{10}C_{3} + 2.\ {}^{10}C_{4} + {}^{10}C_{5} = {}^{n}C_{5}$

or ; ${}^{10}C_{3} + {}^{10}C_{4} + {}^{10}C_{4} + {}^{10}C_{5} = {}^{n}C_{5}$

or ; ${}^{11}C_{4} + {}^{11}C_{5} = {}^{n}C_{5}$

or ; ${}^{12}C_{5} = {}^{n}C_{5} \Rightarrow n = 12$

$\therefore$ Option (*c*) is correct

Q.40. The number of parallelograms that can be formed by a set of 6 parallel lines intersected by the another set of 4 parallel lines is _____

(*a*) 360 (*b*) 90 (*c*) 180 (*d*) 45

[June 2017]

Solution : No. of parallelograms = ${}^{6}C_{2} \cdot {}^{4}C_{2}$

$= 15 \times 6 = 90$

$\therefore$ Option (*b*) is correct

Q.41. If ${}^{n}p_{13} : {}^{(n+1)}p_{12} = 3:4$ then 'n' is _____:

(*a*) 13 (*b*) 15 (*c*) 18 (*d*) 31

[Dec. 2017]

Solution : (*b*) is correct

Tricks : Go by choices (Test from beginning)

OR $\dfrac{{}^{n}P_{13}}{{}^{n+1}P_{12}} = \dfrac{3}{4}$

or ; $\dfrac{n!}{(n-13)!} \div \dfrac{(n+1)!}{(n-11)!} = \dfrac{3}{4}$

or ; $\dfrac{n!}{(n-13)!} \times \dfrac{(n+11)!}{(n-1)!} = \dfrac{3}{4}$

or ; $\dfrac{n!}{(n-13)!} \cdot \dfrac{(n+11)(n-12).(n-13)!}{(n-1).n!} = \dfrac{3}{4}$

or ; $\dfrac{(n-11)(n-12)}{(n-1)} = \dfrac{3}{4}$

Tricks : GBC

for (b) ; $\text{LHS} = \dfrac{(15-11)(15-12)}{(15+1)} = \dfrac{4 \times 3}{16}$

$= \dfrac{3}{4}(\text{RHS}).$

Q.42. In how many ways that 3 commerce books, 3 computer books and 5 economics books be arranged along a row, so that books of same subjects are come together is _____ :

(*a*) 29,950 (*b*) 25,940 (*c*) 25,920 (*d*) None of these

[Dec. 2017]

Solution : (*c*)

From Qts. ; Total no. of ways

$= (3!) \cdot (3!) \cdot (5!) \cdot (3!)$

$= 6 \times 6 \times 120 \times 6$

$= 25{,}920.$

Q.43. If ${}^{12}C_{3} + 2.{}^{12}C_{4} + {}^{12}C_{5} = {}^{14}C_{x}$,The value of x

(*a*) 3 or 5 (*b*) 5 or 9 (*c*) 7 or 1 (*d*) 9 or 12

[June 2018]

Solution : (*b*)

${}^{12}C_{3} + 2\ {}^{12}C_{4} + {}^{12}C_{5} = {}^{14}C_{x}$

$\Rightarrow\ {}^{12}C_{3} + {}^{12}C_{4} + {}^{12}C_{4} + {}^{12}C_{5} = {}^{14}C_{x}$

$\Rightarrow \quad {}^{13}C_4 + {}^{13}C_5 = {}^{14}C_x$

$\Rightarrow \quad {}^{14}C_5 = {}^{14}C_x \Rightarrow x = 5$

and ${}^{14}C_5 = {}^{14}C_{14-5} = {}^{14}C_x \Rightarrow x = 14 - 5 = 9$

$\therefore \; x = 5$ or 9.

Q.44. The number of ways in which a man can invite one or more of his 7 friends to dinner is

(*a*) 64 (*b*) 128

(*c*) 127 (*d*) 63

[June 2018]

Solution : (*c*)

No. of ways to invite at least one friend

$= 2^7 - 1 = 128 - 1 = 127.$

Q.45. The number of words from the letters of the word BHARAT, in which B and H will never come together, is

(*a*) 120 (*b*) 360

(*c*) 240 (*d*) None

[Nov. 2018]

Solution : (*c*)

Use Gap rule.

* A * R * A * T *

Total words $= \frac{4!}{2!} \times {}^5p_2$

$= \frac{24}{2} \times 5 \times 4 = 240$

Q.46. The value of *N* in $\frac{1}{7!} + \frac{1}{8!} = \frac{N}{9!}$ is

(*a*) 81 (*b*) 64

(*c*) 78 (*d*) 89

[Nov. 2018]

Solution : (*a*)

$\frac{1}{7!} + \frac{1}{8!} = \frac{N}{9!}$

or ; $\frac{1}{7!} + \frac{1}{8 \times 7!} = \frac{N}{9 \times 8 \times 7!}$

or ; $\frac{1}{7!}\left(1 + \frac{1}{8} = \right) = \frac{N}{9 \times 8 \times 7!}$

$\Rightarrow \frac{9}{8} = \frac{N}{9 \times 8}$

$\Rightarrow$ N=81.

Q.47. If ${}^nP_r = 720$ and ${}^nC_r = 120$ then *r* is

(*a*) 4 (*b*) 5

(*c*) 3 (*d*) 6

[Nov. 2018]

Solution : (*c*)

$\because \; {}^np_r = {}^nC_r \cdot r!$

or ; $720 = 120 \cdot (r!)$

or $6 = r! \Rightarrow r! = 3!$

$\therefore \; r = 3$

Q.48. A bag contains 4 red, 3 black and 2 white balls. In how many ways 3 balls can be drawn from this bag so that they include at least one black ball ?

(*a*) 46 (*b*) 64

(*c*) 86 (*d*) None

[Nov. 2018]

Solution : (*b*)

		Black balls	Rest balls	
Selection		3	4 + 2 = 6	Combs.
of 3 balls	(*i*)	1	2	$= {}^3C_1 \cdot {}^6C_2 = 45$
	(*ii*)	2	1	$= {}^3C_2 \cdot {}^6C_1 = 18$
	(*iii*)	3	0	$= {}^3C_3 \cdot {}^6C_0 = 1$

Total ways = 64

Q.49. If ${}^{11}C_x = {}^{11}C_{2x-4}$ and x ≠ 4 than the value of ${}^7C_x =$

(*a*) 20 (*b*) 21

(*c*) 22 (*d*) 23

[June 2019]

Solution : (*b*)

Formula ; ${}^nC_r = {}^nC_k \Rightarrow r = k$

or $r + k = n.$

$\therefore \; {}^{11}C_x = {}^{11}C_{2x-4}$

$\Rightarrow \quad x = 2x - 4 \Rightarrow \; x = 4$

but $\quad x \neq 4$

Then, ${}^{11}C_x = {}^{11}C_{2x-4}$

$\therefore \; x + 2x - 4 = 11$

or; $3x = 15 \Rightarrow \; x = 5$

$\therefore \; {}^7C_x = {}^7C_5 = \frac{7!}{5! \cdot 2!} = \frac{7 \cdot 6 \cdot 5!}{5! \cdot 2 \cdot 1}$

$= 21$

(*b*) is correct.

Q.50. Which of the following is a correct statement

(*a*) ${}^nP_n = {}^nP_{n-1}$

(*b*) ${}^nP_n = {}^{2n}P_{n-2}$

(*c*) $P_n = {}^{3n}P_{n-3}$

(*d*) ${}^nP_n = {}^{n(n+1)}P_{n-1}$

[June 2019]

Solution : (*a*)

Tricks:-Put n = 2 and then GBC.

(*a*) ${}^nP_n = {}^nP_{n-1} \Rightarrow {}^2P_2 = {}^2P_1$

$\Rightarrow 2 + 1 = 2$ (True)

(*b*) ${}^nP_n = {}^{2n}P_{n-2} \Rightarrow {}^2P_2 = {}^4P_0$

$\Rightarrow \quad 2 \times 1 \neq 1$

$\left(\because {}^4P_0 = 1\right)$

(*c*) Put n = 3

${}^3P_3 = {}^9P_0$

$\Rightarrow 3 \cdot 2 \cdot 1 \neq 1$ (False)

(d) Put n = 2

${}^2P_2 = 2 \cdot (2 + 1)$

$\Rightarrow \quad {}^2P_2 = {}^{2 \cdot (2+1)}P_{2-1}$

$\Rightarrow \quad 2 \times 1 = {}^6P_1$

$\Rightarrow \quad 2 = 6$ (False)

(*a*) is correct.

Q.51. If these are 40 guests in a party. If each guest takes a shake hand with all the remaining guests. Then the total number of hands shake is _____ :

(a) 780 (b) 840
(c) 1,560 (d) 1,600

[June 2019]

Solution : (a)

No. of guests = 40

To shake hands 2 persons are required.

∴ Total No. of hand shakes

$$= {}^{40}C_2 = \frac{401}{(2!)(38!)}$$

$$= \frac{40.39.38!}{2\times1\times38!} = 780.$$

Q. 52. In how many ways that the crew of an eight oared be arranged so that if 3 of crew can row only on a stoke side and 2 row on the other side is _____ :

(a) 1728 (b) 256
(c) 164 (d) 126

[June 2019]

Solution : (a)

3 this side || 2 this side

Total No. of rest persons

= 8 - 3 - 2 = 3

∴ Total No. of ways =

$$= \left[{}^3C_1 \cdot (4!)\right] \cdot \left[{}^2C_2 \cdot (4!)\right]$$

$$= (3\times24)\times(1\times24)$$

$$= 1728$$

Q.53. How many numbers divisible by 5 of 6 digit can be made from the digit 2, 3, 4, 5, 6, 7

(a) 120 (b) 600
(c) 240 (d) none

[Dec. 2019]

Solution : (a)

Given digits are 2, 3, 4, 5, 6, 7 for being divisible by 5 means 5 should be at unit place.

∴ Permutation of 5 at unit place = 1

Remaining digits are 2, 3, 4, 6, 7.

They can be arranged in 5! = 120 ways

∴ Total No. of Nos. divisible by 5 =

$120\times1=120$

Q.54. 5 boys and 3 girls are to be seated together such that no two girls are together

(a) 14,400 (b) 2400
(c) 720 (d) None of these

[Dec. 2019]

Solution : (a)

Use Gap Rule

Arrangement of 5 boys = 5! = 120.

$*B_1\ *B_2\ *B_3\ *B_4\ *B_5\ *$

After arranging boys; we get 6 places for girls can be arranged in

$${}^6P_3 = 6.5.4 = 120 \text{ ways}$$

∴ Total Arrangements =

$120\times120=14400$

Q.55. Out of 6 Boys & 4 girls, find the number of ways for selecting 5 members committee in which there is exactly two girls ?

(a) 120 (b) 1440
(c) 720 (d) 71

[Dec. 2019]

Solution : (a)

Total members in 1 committee = 5.

In this committee, girls should be 2

∴ Boys should be 3

∴ Total no. of ways to select 3 boys and 2 girls

$= {}^6C_3 \cdot {}^4C_2 = 20\times6 = 120$

Q.56. If $^np_5 : {}^np_3$ is 2:1 than value of n is

(a) 2 (b) -5
(c) -2 (d) 5

[Dec. 2015]

Solution : (d)

Tricks: GBC

In ${}^nP_r$; n & r are always positive.

So options (b) & (c) are wrong.

In ${}^nP_r$; $n \geq r$

So; in ${}^nP_5 \Rightarrow n \geq 5$

∴ (a) is rejected.

Hence (d) is correct.

Q.57. If $^np_4 = 20\,{}^np_2$ where P denotes the number of permutations n =.....

(a) 4 (b) 2
(c) 5 (d) 7

[Dec. 2020]

Solution : ${}^nP_4 = 20.\ {}^nP_2$

Tricks : GBC

(c) For $n = 5 \Rightarrow {}^5P_4 = 20.\ {}^5P_2$

$= 5.4.3.2 \neq 20 \times 5 \times 4$

(c) is incorrect.

(d) For $n = 7$

$\therefore {}^nP_4 = 20.\ {}^nP_2$

$\Rightarrow {}^7P_4 = 20.\ {}^7P_2$

$\Rightarrow 7.6.5.4 = 20 \times 7 \times 6$ (Correct)

Detail.

${}^nP_4 = 20.\ {}^nP_2$

$$\Rightarrow \frac{\lfloor n}{\lfloor n-4} = 20.\ \frac{\lfloor n}{\lfloor n-2}$$

$$\Rightarrow \lfloor n-2 = 20\ \lfloor n-4$$

$\Rightarrow (n-2)(n-3)\ \lfloor n-4 = 20\ \lfloor n-4$

$\therefore (n-2)(n-3) = 20$

Then **GBC** (d) (7 - 2) (7 - 3)

$= 5 \times 4 = 20$ (RHS)

∴ (d) is correct.

Q.58. A fruit basket contains 7 apples, 6 bananas and 4 mangoes. How many selections of 3 fruits can be made so that all 3 are apples?

(a) 120 ways (b) 35 ways
(c) 168 ways (d) 70 ways

[Dec. 2020]

Solution : No. of selections of 3 apples

$$= {}^7C_3 = \frac{\lfloor 7}{\lfloor 3\ \lfloor 7-3}$$

$$= \frac{7.6.5\ (4!)}{(3!)\ (4!)}$$

$$= \frac{7.6.5}{3\times2\times1} = 35$$

(b) is correct.

Q.59. Out of 7 boys and 4 girls a team of a debate club of 5 is to be chosen. The number of teams such that each team includes at least one girl is.....

(*a*) 429 (*b*) 439 (*c*) 419 (*d*) 441

[Dec. 2020]

Solution :

	7 Boys	4 Girls		Combinations
Team of 5 (*i*)	4	1	=	$^7c_4.^4c_1 = \frac{7.6.5.4}{\lfloor 4} \times 4 = 140$
(*ii*)	3	2	=	$^7c_3.^4c_2 = \frac{7.6.5}{\lfloor 3}\ 6 = 210$
(*iii*)	2	3	=	$^7c_2.^4c_3 = \frac{7\times6}{\lfloor 2} \times 4 = 84$
(*iv*)	1	4	=	$^7c_1.^4c_4 = 7 \times 1 = 7$
				Total = 441

$\therefore$ (*d*) is correct.

Q.60. From a group of 8 men and 4 women, 4 persons are to be selected to form a committee so that at least 2 women are there on the committee. In how many ways can it be done?

(*a*) 201 (*b*) 168 (*c*) 202 (*d*) 220

[Dec. 2020]

Solution :

	8 Men	4 Women		Combinations
Selection of 4 (*i*)	2	2	=	$^8c_2.^4c_2 = \frac{8\times7}{\lfloor 2} \times \frac{4\times3}{\lfloor 2} = 168$
(*ii*)	1	3	=	$^8c_1.^4c_3 = 8 \times 4 = 32$
(*iii*)	0	4	=	$^8c_0.^4c_4 = 1 \times 1 = 1$
				Total ways = 201

$\therefore$ (*a*) is correct.

Q.61. Eight Chairs are numbered from 1 to 8. Two women and three men are to be seated by allowing one Chair for each. First, the women choose the chairs from the chairs numbered 1 to 4 and then men select the chairs from the remaining.

The number of possible arrangement is

(*a*) 120 (*b*) 288

(*c*) 32 (*d*) 1440

[Jan. 2021]

Solution : (*d*) is correct

Total No. of possible arrangements

= (Arrange of women) (Arrangement of men)

$= {}^4P_2\ {}^{(8-2)}P_3$

$= (4 \times 3) \times (6 \times 5 \times 4)$

$= 12 \times 120 = 1440$

$\therefore$ (*d*) is correct

Q.62. 'n' locks and 'n' corresponding keys are available but the actual combination is not known. The maximum number of trails that are needed to assigns the keys to the corresponding locks is

(*a*) $(n-1)C_2$ (*b*) $(n+1)C_2$

(*c*) $\sum_{k=2}^{n} k$ (*d*) $\sum_{k=2}^{n} (k-1)$

[Jan. 2021]

Solution : (*c*) is correct

Total No. of locks = n and Total no. of keys = n

1st Step: For 1st key, only one key will match with any one lock. It means in 'n' trails, (n – 1) trial will fail and nth trial will match.

2nd Step : Now available locks = n – 1 and available keys = n – 1

Similarly as above, in (n – 1) trials, (n – 2) trials will fail and $(n-1)^{th}$ trial will match with the lock.

3rd Step : Similarly doing as above Maximum No. of trials

$= n + (n-1) + (n-2) + \ldots + 3 + 2$

It can be written as $= \sum_{k=2}^{n} k$

Q.63. There are ten flights operating between city A and city B. The number of ways in which a person can travel from city A to city B and return by different flight is

(*a*) 90 (*b*) 95

(*c*) 8C (*d*) 78

[Jan. 2021; June 2022]

Solution : (*a*) is correct

There are 10 options to reach city B from city A.

$\therefore$ Permutations of going = 10

The person has to return by different flight: So, there are 9 options of returning.

Total ways of doing so = 10 × 9 = 90.

Q.64. How many odd numbers of four digit can be formed with digit 0, 1, 2, 3, 4, 7 and 8?

(*a*) 150 (*b*) 300

(*c*) 120 (*d*) 210

[Jan. 2021]

Solution : (*b*) is correct

Given digits are

0, 1, 2, 3, 4, 7 and 8

We have to make 4 - digits No.

So, make Four boxes as

Th	H	T	U
5	5	4	3

$5\times5\times4\times3 = 300$

There are 3 options for unit place.

They are 1, 3, 7

So, Permutation of unit place = 3

Out of 7 digits, 1 digit will be used at unit place.

Total No. of remaining digits = 7 – 1 = 6 in which zero (0) is included

But at thousand place, zero (0) cannot be used. So No. of no. of options for thousand place = 5

Similarly, No. of options for hundred places = 7 – 1 –1 = 5

and for Tens place = 7 – 1 –1 – 1 = 4 options

Hence, Total no. of 4- digit odd Nos.

= Product of permutations of each box.

$5\times5\times4\times3 = 300$

∴ (*b*) is correct

Q.65. In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd numbered positions

(*a*) 32 (*b*) 36

(*c*) 48 (*d*) 60

[Jan. 2021; Dec. 2021]

Solution : (*b*) is correct

In word "DETAIL"

No. of letters = 6

No. of vowels (E, A, I) = 3

Places

1	2	3	4	5	6
V		V		V	

Suitable places for vowels = 1, 3, 5

So Permutation of vowels = $^3P_3$

$= 3\times2\times1 = 6$

Permutations of consonants

$= (6-3)! = 3!$

$= 3\times2\times1 = 6$

Total no. of permutations (i.e. such words) = 6 × 6 = 36

∴ (*b*) is correct

Q.66. $^nC_p + 2\,^nC_{P-1} + {}^nC_{P-2}$**?**

(*a*) $^{n+}C_P$ (*b*) $^{n+2}C_P$

(*c*) $^{n+1}C_{P+1}$ (*d*) $^{n+2}C_{P-1}$

[Jan. 2021]

Solution : $^nC_p + 2\cdot{}^nC_{p-1} + {}^nC_{p-2}$

$= {}^nC_p + {}^nC_{p-1} + {}^nC_{p-1} + {}^nC_{p-2}$

$= ({}^nC_p + {}^nC_{p-1}) + ({}^nC_{p-1} + {}^nC_{p-2})$

$= n + {}^1C_p + {}^{n+1}C_{p-1}$

(∵ Formula $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$= {}^{n+2}C_p$

∴ (*b*) is correct

Q.67. A business houses wishes to simultaneously elevate two of its six branch heads. In how many ways these elevation can take place?

(*a*) 12 (*b*) 3

(*c*) 6 (*d*) 15

[Jan. 2021]

Solution : (*b*) is correct

Total no. of branches = 6

No. of ways to elevate 2 out of 6 branches

$= {}^6C_2 = \frac{\lfloor 6}{\lfloor 2 \lfloor 4} = \frac{6\cdot5\cdot\lfloor 4}{\lfloor 2 \lfloor 4} = 15$

∴ (*b*) is correct.

Q.68. If $^np_6 = 20\,{}^np_4$ **then the value of n is given by**

(*a*) n = 5 (*b*) n = 3

(*c*) n = 9 (*d*) n = 8

[July 2021]

Solution : (*c*) is correct

$^nP_6 = 20\,{}^nP_4$

or $\frac{n!}{(n-6)!} = 20\,\frac{n!}{(n-4)!}$

$\Rightarrow (n-4)! = 20\,(n-6)!$

or $(n-4)(n-5)(n-6)! = 5\times4\,(n-6)!$

or $(n-4)(n-5) = 5\times4$

Equating on both sides; we get

Largest factor of LHS = Largest factor of RHS

$\Rightarrow n-4 = 5$

$\therefore n = 9$

Tricks : Go by choices (GBC)

(*c*) is correct

Q.69. How many numbers of seven digit numbers which can be formed from the digits 3, 4, 5, 6, 7, 8, 9 no digits being repeated are not divisible by 5?

(*a*) 4320 (*b*) 4690

(*c*) 3900 (*d*) 3890

[July 2021]

Solution : (*a*) is correct

Total no. of 7 digit numbers not divisible by 5 = Total Nos. — Nos. divisible by 5

$= \lfloor 7 - \lfloor 6 \times 1$

where 1 is the permutation of unit place

$= 5040 - 720\times1$

$= 4320$

Q.70. A person can go from place 'A' to 'B' by 11 different modes of transport but is allowed to return back to "A" by any mode other than the one earlier. The number of different ways, the entire journey can be complete is

(*a*) 110 (*b*) 10^{10}

(*c*) 9^5 (*d*) 10^9

[July 2021]

Solution : (*a*) is correct

There are 11 choices to go from A to B and 10 choices to return from B to A.

So, Total No. of ways to complete the journey = 11 × 10 = 110

Q.71. The number of ways 5 boys and 5 girls can be seated at a round table, so no two boys are adjacent is

(*a*) 2550 (*b*) 2880

(*c*) 625 (*d*) 2476

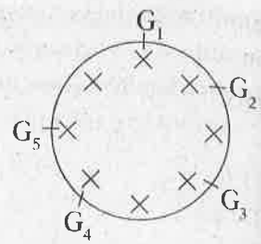
[July 2021]

Solution : (*b*) is correct

Using Gap rule

Permutation of Girls

$= \lfloor 5-1 = \lfloor 4 = 24$



There are 5 places suitable for boys.

They can be arranged in ${}^5P_5 = \lfloor 5 = 120$ ways

Hence, Total no. of arrangements

$= 24 \times 120 = 2880$

Q.72. The number of four letter words can be formed using the letters of the word DICTIONARY is

(*a*) 5040 (*b*) 720 (*c*) 90 (*d*) 30240

[Dec. 2021]

Solution : (*a*)

Total No. of words

$= {}^{10}P_4 = 10.9.8.7 = 5040$

Q.73. The number of words that can be formed using the letters of "PETROL" such that the words do not have "P" in the first position, is

(*a*) 720 (*b*) 120 (*c*) 600 (*d*) 540

[Dec. 2021]

Solution : (*c*)

Total No. of Words = | 5 | 5 | 4 | 3 | 2 | 1 |

E, T, R, O, L (Eligible letters for 1st place).

$= 5.5.4.3.2.1 = 600$

OR

Total words = $\lfloor 6 - \boxed{1 \mid \lfloor 5}$

P(Fix)

$= 720 - 1 \times 120$

$= 600$

Q.74. If ${}^nP_2 = 12$, then the value of n is

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 6

[Dec. 2021]

Solution : (*c*)

${}^nP_2 = 12$

GBC (C) ${}^4P_2 = 4 \times 3 = 12$ (True)

$\therefore n = 4$

Detail

${}^nP_2 = n(n-1) = 4 \times 3$

Comparing; we get

$n = 4$

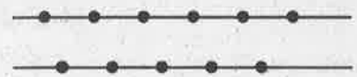
Q.75. Six points are marked on a straight line and five points are marked on another line which is parallel to the first line, how many straight lines, including the first two, can be formed with these points?

(*a*) 28 (*b*) 30

(*c*) 32 (*d*) 34

[June 2022]

Solution :



Total No. of Points = 6 + 5 = 11

Total No. of Lines =

${}^{11}C_2 - {}^6C_2 - {}^5C_2 + 2$

$= \frac{(11)!}{(2)!(11-2)!} - \frac{6!}{2!(6-2)!} - \frac{5!}{2!(5-2)!} + 2$

$= \frac{11.^5\cancel{10}.(9)!}{\cancel{2}\times 1.(9)!} - \frac{^3\cancel{6}.5.(4)!}{\cancel{2}\times 1.(4)!} - \frac{5.^2\cancel{4}\,(3)!}{\cancel{2}\times 1\times 3!} + 2$

$= 55 - 15 - 10 + 2$

$= 32$

(*c*) is correct.

Q.76. How many 4 letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

(*a*) 7,020 (*b*) 5,040

(*c*) 1,480 (*d*) 2,520

[June 2022]

Solution : LOGARITHMS

Total Letters = 10

Total No. of 4 Letters word

$= {}^{10}P_4 = 10.9.8.7$

$= 5040$

(*b*) is correct.

Q.77. Out of 7 boys and 4 girls, a team of a debate club of 5 is to be chosen. The number of teams such that each team includes at least one girl is

(*a*) 4439 (*b*) 429 (*c*) 419 (*d*) 441

[June 2022]

Solution :

		7 Boys	4 girls	Combinations
Selection of 5	(*i*)	4	1=	${}^7C_4 . {}^4C_1 = 35 \times 4 = 140$
	(*ii*)	3	2=	${}^7C_3 . {}^4C_2 = 35 \times 6 = 210$

(iii)	2	3=	$^7C_2 . {}^4C_3 = 21 \times 4 = 84$
(iv)	1	4=	$^7C_1 . {}^4C_4 = 7 \times 1 = 7$
			Total = 441

∴ (*d*) is correct.

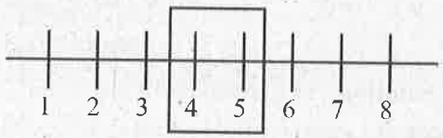
Q.78. The total number of sitting arrangements of 8 members of a board on a row with the president and the vice-president occupying central places is

(*a*) 7! × 2! (*b*) 6! × 2!

(*c*) 6! (*d*) 7!

[June 2022]

Solution :



Central Place

Arrangement of rest 6 persons = 6!

Arrangement of 2 persons

President and Vice-President at Central Place =2!

Total Ways = (6!). (2!)

∴ (*b*) is correct.

Q.79. The solution for $\frac{n!}{10} = {}^{n-1}P_{n-3}$ then n =

(*a*) 5 (*b*) 6

(*c*) 7 (*d*) 8

[June 2022]

Solution :

$\frac{n!}{10} = {}^{n-1}P_{n-3}$

or; $\frac{n!}{10} = \frac{(n-1)!}{(n-1-n+3)!}$

or; $\frac{n.(n-1)!}{10} = \frac{(n-1)!}{2!}$

or; $\frac{n}{\cancel{10}_5} = \frac{1}{\cancel{2}}$ ∴ n =5

∴ (*a*) is correct.

Q.80. A multiple - choice test contains five questions and each has four possible options. How many different answer keys are possible?

(*a*) 20 (*b*) 120

(*c*) 256 (*d*) 1024

[June 2022]

Solution : There are 4 options in each question.

Total No. of ways to answer 1st question = 4

Similarly Total No. of ways to answer 2nd question = 4

Similarly Total No. of ways to answer 3rd question = 4

Similarly Total No. of ways to answer 4th question = 4

Similarly Total No. of ways to answer 5th question = 4

∴ Total ways to answer 5 questions

= 4. 4. 4. 4. 4. = 1024

(*d*) is correct.

Q.81. There are 20 points in a plane area. How many triangles can be formed by these points if 5 points are collinear?

(*a*) 550 (*b*) 560

(*c*) 1130 (*d*) 1140

[Dec. 2022]

Solution : No. of Triangles made

$= {}^{20}C_3 - {}^5C_3 = \frac{20!}{(3!)(17!)} - \frac{5!}{(3!)(2!)}$

$= \frac{20 \times 19 \times 18 \times (17!)}{3 \times 2 \times 1 \times (17!)} - \frac{5 \times 4(3!)}{(3!) \times 2!}$

= 1140 – 10 = 1130

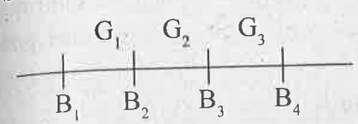
(*c*) is correct

Q.82. The number of ways 4 boys and 3 girls can be seated in a row so that they are alternate is:

(*a*) 12 (*b*) 288 (*c*) 144 (*d*) 256

[Dec. 2022]

Solution : 4 boys and 3 girls are to be arranged in = $\underline{|4} \times \underline{|3} = 24 \times 6 = 144$ ways



(Use GAP rule)

(*c*) is correct.

Q.83. If $^nP_r = 3024$ and $^nC_r = 126$, then find n and *r*

(*a*) 9, 4 (*b*) 10, 3

(*c*) 12, 4 (*d*) 11, 4

[Dec. 2022]

Solution : ∵ $^nP_r = {}^nC_r \times \underline{|r}$ (Formula)

or; $3024 = 126 \times \underline{|r}$

or $\underline{|r} = 24 = \underline{|4}$

∴ $r = 4$

Then Go by choices

(*a*) $^9P_4 = 9 \times 8 \times 7 \times 6 = 3024$ (True)

∴ (*a*) is correct.

Q.84. How many 3 digit odd numbers can be formed using the digits 5, 6, 7, 8, 9, if the digits can be repeated?

(*a*) 55 (*b*) 75

(*c*) 65 (*d*) 85

[Dec. 2022]

Solution: Given digits are 5, 6, 7, 8, 9

No. of 3-digit odd Nos. = 5×5×3 = 75

5	5	3

↑ 5, 7, 9

[Note : Only 5, 7, 9 digits are suitable for unit place. So, permutation of unit place = 3. Permutation of Rest Places = 5]

Total No. of numbers = 5×5×3 = 75

Q.85. A committee of 3 women and 4 men is to be formed out of 8 women and 7 men. Mrs. Kajal refuses to serve in a committee in which Mr. Yash is a member. The number of such committees can be:

(*a*) 1530 (*b*) 1500 (*c*) 1520 (*d*) 1540

[June 2023]

Solution:

		Mrs. Kajal	Mr. Yash	Rest Men	Rest Women	Comb.
Committee		1	1	6	7	
of 4 men	(*i*)	0	0	4	3	$= {}^1C_0 . {}^1C_0 . {}^6C_4 . {}^7C_3 = 15 \times 35$
& 3 women	(*ii*)	1	0	4	2	$= {}^1C_1 . {}^1C_0 . {}^6C_4 . {}^7C_2 = 15 \times 21$
or	(*iii*)	0	1	3	3	$= {}^1C_0 . {}^1C_1 . {}^6C_3 . {}^7C_3 = 20 \times 35$

Total ways $= 15 \times 35 + 15 \times 21 + 20 \times 35$

$= 525 + 315 + 700 = 1540$

(*d*) is correct.

Q.86. If ${}^6P_{2r} = 12 \times {}^6P_r$, then *r* is equal to:

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

[June 2023]

Solution:

${}^6P_{2r} = 12 \times {}^6P_r$

or; $\frac{\lfloor 6}{\lfloor 6-2r} = 12 \times \frac{\lfloor 6}{\lfloor 6-r}$

$\frac{\lfloor 6-r}{\lfloor 6-2r} = 12 = \frac{4 \times 3 \times \lfloor 2}{\lfloor 2}$

$= \frac{\lfloor 4}{\lfloor 2}$

Comparing it; we get

$\lfloor 6-r = \lfloor 4 \Rightarrow 6-r = 4$

$\therefore r = 2$

OR $\lfloor 6-2r = \lfloor 2 \Rightarrow 6-2r = 2$

or $2r = 4$

$r = 2$

Tricks GBC

(*b*) LHS $= {}^6p_{2\times2} = {}^6p_4$

$= 6 \times 5 \times 4 \times 3$

$= 360$

RHS $= 12 \times {}^6p_2$

$= 12 \times 6 \times 5 = 360$

$= LHS$

$\therefore$ (*b*) is correct.

Q.87. In how many different ways can the letters of the word "SOFTWARE" be arranged so that the vowels always come together?

(*a*) 720

(*b*) 144

(*c*) 2880

(*d*) 4320

[June 2023]

Solution: Word = SOFTWARE

Vowels = O, A, E

Total No. of words as per Qts.

$= \boxed{O, A, E}, S, F, T, W, R$

[Vowels will be taken as one unit]

$= \lfloor 6 \times \lfloor 3 = 720 \times 6$

$= 4320$

Note:— All vowels are put in a box.

$\therefore$ (*d*) is correct.

Q.88. In the next world cup of cricket, there will be 12 teams divided equally into two equal groups. Team of each group will play a match against other teams of the group. From each group, 3 top teams will qualify for next round. In this round, each team will play against each other. Four top teams of this round will qualify for semi-finals and play against each other and then two top teams will go to final, where they play the best of three matches. How much minimum number of matches in the next world cup will be?

(*a*) 54 (*b*) 53

(*c*) 38 (*d*) 43

[June 2023]

Solution: Total teams = 12

They are divided into 2 groups.

Each group has 6 teams.

Each team of each group will play with every team.

So in 1st round

Total no. of matches played

$= 6_{C_2} + 6_{C_2} = 15 + 15 = 30$

For second round 6 teams are selected. Each team of this round will play with other.

So total matches played in this second round

$= 6_{C_2} = 15$

In semi-final round top 4 teams are selected. Total matches played in this round

$= 4_{C_2} = 6$

For final round, top 3 teams are selected.

In this round, if a team win 1st and 2nd match than there is no need of 3rd match. Hence in this round, minimum no. of matches played = 2.

$\therefore$ Minimum total no. of matches played $= (15 + 15) + 15 + 6 + 2 = 53$

$\therefore$ (*b*) is correct.

11 CHAPTER SEQUENCE & SERIES

PREVIOUS YEAR EXAM QUESTIONS (MEMORY BASED)

Q.1. If a_1, a_2, a_3 represent first, second and third term of an A.P respectively, the first term is 2 and $(a_1 + a_2)a_3$ is minimum, then the common difference is equal to

(*a*) 5/2 (*b*) -5/2

(*c*) 2/5 (*d*) -2/5

[June 2010]

Solution : (*b*) Given $a_1 = 2$

Let c.d = x

$\therefore a_2 = 2 + x$ *and* $a_3 = 2 + 2x$

The Function is $(a_1 + a_2)a_3$

$= (2 + 2 + x)(2 + 2x)$

$= (4 + x)(2 + 2x)$

$= 8 + 8x + 2x + 2x^2$

$= 2x^2 + 10x + 8$ (a Quadratic Function)

Formula ;

$ax^2 + bx + c$ is minimum when

$a > 0$ *i.e.* + ve at $x = -b/2a$

$= \frac{-10}{2\times 2} = \frac{-5}{2}$

$\therefore$ (*b*) is correct

Q.2. Divide 144 into three parts which are in AP. and such that the largest is twice the smallest, the smallest of three numbers will be :

(*a*) 48 (*b*) 36

(*c*) 13 (*d*) 32

[June 2010]

Solution : (*d*) Let t_1 = a and cd = d

$\therefore$ a + a + d + a + 2d = 144

or 3a + 3d = 144

or 3(a + d) = 144

or $a + d = \frac{144}{3} = 48$

$\therefore$ a + d = 48 _____(1)

$\therefore$ Largest = 2× Smallest

$\therefore$ a + 2d = 2a

2d = a

d = a/2

From (1)

$a + \frac{a}{2} = 48$

or $\frac{3}{2}a = 48 \therefore a = 48 \times \frac{2}{3}$

$\therefore$ a = 32

$\therefore$ (*d*) is correct

Tricks:- GBC

Q.3. Sum of series $\frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + ...\infty$

(*a*) 15/36 (*b*) 35/36 (*c*) 35/16 (*d*) 15/16

[June 2010]

Solution : (*c*) $S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + ... \text{ to } \infty$ _____ (1)

$\frac{1}{5}S = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + ... \text{ to } \infty$ _____ (2)

(1) - (2); we get

$S - \frac{1}{5}S = 1 + \frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + ... \text{ to } \infty$

$\frac{4}{5}S = 1 + \frac{\frac{3}{5}}{1 - 1/5} = 1 + \frac{3}{5} \times \frac{5}{4}$

$\frac{4}{5}S = 1 + \frac{3}{4}$

$\frac{4}{5}S = \frac{7}{4}; \quad \therefore S = \frac{35}{16}$

(*c*) is correct

Q.4. If G be geometric mean between a & b, then the value of $\frac{1}{G^2 - a^2} + \frac{1}{G^2 - b^2}$ **is equal to**

(*a*) G^2 (*b*) $3G^2$ (*c*) $1/G^2$ (*d*) $2/G^2$

[Dec. 2010]

Solution : (*c*) **Tricks** :- 1, 2, 4 are in GP

$\therefore$ 2 is the GM of 1 & 4

$\therefore$ a = 1; b = 4 and G = 2

$\therefore \frac{1}{G^2 - a^2} + \frac{1}{G^2 - b^2} = \frac{1}{4-1} + \frac{1}{4-16} = \frac{1}{3} - \frac{1}{12}$

$= \frac{4-1}{12} = \frac{3}{12} = \frac{1}{4}$

Then Go by choices

For (*a*) $G^2 = 2^2 = 4 \neq \frac{1}{4}$

(*c*) $\frac{1}{G^2} = \frac{1}{2^2} = \frac{1}{4}$

$\therefore$ (*c*) is correct

Q.5. If the sum of n terms of an A.P. is $2n^2 + n$. What is the difference between its 10th term & 1st term

(*a*) 207 (*b*) 36 (*c*) 90 (*d*) 63

[June 2011]

Solution : (*b*) $S_n = 2n^2 + n$

$\therefore t_1 = s_1 = 2 \times 1^2 + 1 = 3$

$s_2 = 2 \times 2^2 + 2 = 10$

$\therefore d = s_2 - 2s_1 = 10 - 2 \times 3 = 4$

$\therefore t_{10} - t_1 = a + 9d - a = 9d = 9 \times 4$

$= 36 \quad \therefore$ (*b*) is correct

Q.6. Find the product of $243.243^{\frac{1}{6}}.243^{\frac{1}{36}}.....$ **to** ∞

(*a*) 1024 (*b*) 27 (*c*) 729 (*d*) 246

[June 2011]

Solution : (*c*) is correct $P = (243).(243)^{\frac{1}{6}}.(243)^{\frac{1}{36}}.....\text{to } \infty$

$= 243^{\frac{1}{1-1/6}} = 243^{6/5} = (3^5)^{6/5}$

$= 3^6 = 729$

Q.7. Insert two arithmetic means between 68 and 260

(*a*) 132, 196 (*b*) 130, 194 (*c*) 70, 258 (*d*) None

[June 2011]

Solution : (*a*) Trick I

$A_1 + A_2 + A_3 + ... + A_n = n \times \text{AM of a \& b}$

$= n\left(\frac{a+b}{2}\right)$

$132 + 196 = 2\left(\frac{68+260}{2}\right)$

$328 = 328$ (True)

Trick II

Go by choices

For (*a*) 68, 132, 196, 260 marks an AP

$\therefore$ (*a*) is correct

Q.8. Geometric mean of p, p^2, p^3....., p^n will be

(*a*) p^{n+1}

(*b*) $p^{\left(\frac{1+n}{2}\right)}$

(*c*) $p^{\frac{n(n+1)}{2}}$

(*d*) None of the above

[June 2011]

Solution : (*b*)

$GM = \left(p.p^2.p^3.....p^n\right)^{1/n}$

$= \left(p^{1+2+3+.....+n}\right)^{1/n}$

$= \left[p^{\frac{n(n+1)}{2}}\right] = p^{(n+1/2)}$

Tricks :- Put n = 3

$Gm = \left(p.p^2, p^3\right)^{1/3} = p^2$

For (*a*) $GM = p^{3+1} \neq p^2$

(*b*) $GM = p^{\frac{1+3}{2}} = p^2$

$\therefore$ (*b*) is correct.

Q.9. Find the number whose arithmetic mean is 12.5 and geometric mean is 10.

(*a*) 20 and 5 (*b*) 10 and 5

(*c*) 5 and 4 (*d*) None of these

[Dec. 2011]

Solution : (*a*) is correct

Tricks :- Go by choices

For (*a*) $AM = \frac{20+5}{2} = 12.5$

and $GM = \sqrt{20\times5} = 10$

$\therefore$ 20 & 5 satisfy both given condition in qts.

$\therefore$ (*a*) is correct.

Q.10. If sum 3 arithmetic mean between "a" and 22 is 42, then "a"=

(*a*) 14 (*b*) 11

(*c*) 10 (*d*) 6

[Dec. 2011]

Solution : (*d*) is correct

Tricks :- It $A_1; A_2; A_3;$.....; An are "n" AMS between a and b

$A_1 + A_2 + A_3 + ... + A_n = n\left(\frac{a+b}{2}\right)$

= n.(AM of a and b)

$\therefore 3\left(\frac{a+22}{2}\right) = 42 \therefore a = 6$

Q.11. If each month ₹100 increases in any sum then find out the total sum after 10 months, if the sum of first month is ₹2,000.

(*a*) ₹24,500 (*b*) ₹24,000

(*c*) ₹50,000 (*d*) ₹60,000

[Dec. 2011]

Solution : (*a*) Sum =

$\frac{10}{2}[2\times2000 + (10-1).100]$

= ₹24,500.

Q.12. If 5th term of a G.P. is $\sqrt[3]{3}$, then the product of first nine terms is

(*a*) 8 (*b*) 27

(*c*) 243 (*d*) 9

[Dec. 2011]

Solution : (*b*) ; $t_5 = a.r^4 = \sqrt[3]{3} = 3^{1/3}$

Product of 1st 9 terms

$= a.ar.ar^2.....ar^8$

$= a^9.r^{1+2+3+.....+8}$

$= a^9.r^{\frac{8}{2}(1+8)} = a^9.r^{36}$

$= (ar^4)^9 = (3^{y_3})^9 = 3^3 = 27$

Q.13. The sum of the third and ninth term of an A.P. is 8. Find the sum of the first 11 terms of the progression.

(*a*) 44 (*b*) 22

(*c*) 19 (*d*) 11

[Dec. 2011]

Solution : (*a*) is correct

$\because t_3 + t_9 = 8$

a + 2d + a + 8d

or 2a + 10d = 8

$\therefore S_{11} = \frac{11}{2}[2a + (11-1)d]$

$= \frac{11}{2}[2a + 10d] = \frac{11}{2}\times8 = 44$

Q.14. 8th term of an A.P is 15, then sum of its 15 terms is

(*a*) 15 (*b*) 0

(*c*) 225 (*d*) 225/2

[June 2012]

Solution : (*c*) is correct

$t_8 = a + 7d = 15$

$S_{15} = \frac{15}{2}[2a + (15-1)d] = \frac{15}{2}\times2(a+7d)$

$= 15 \times 15 = 225$

Q.15. Find the sum of the infinite terms

2, $\frac{4}{y}, \frac{8}{y^2}, \frac{16}{y^3}$.....; If y>2

(*a*) $\frac{2y}{y-2}$ (*b*) $\frac{4y}{y-2}$

(*c*) $\frac{3y}{y-2}$ (*d*) None of these

[June 2012]

Solution : (*a*) is correct

$S = \frac{a}{1-r} = \frac{2}{1-\frac{2}{y}} = \frac{2y}{y-2}$

Q.16. The 4th term of an A.P. is three times the first and the 7th term exceeds twice the third term by 1. Find the first term 'a' and common difference 'd'.

(a) a=3,d=2 (b) a=4,d=3

(c) a=5,d=4 (d) a=6,d=5

[June 2012]

Solution : (a) is correct

$t_4 = 3t_1 \Rightarrow a+3d = 3a \therefore 2a = 3d;\ a = \frac{3d}{2}$

$\because t_7 = 2t_3 + 1$

or a + 6d = 2(a + 2d) +1

or a + 6d = 2a + 4d + 1

or 2d - a = 1

or $2d - \frac{3}{2}d = 1 \Rightarrow \frac{d}{2} = 1 \therefore d = 2$

and $a = \frac{3}{2} \times 2 = 3$

Tricks :- Go by choices.

Q.17. In an A.P., if common difference is 2, Sum of n terms is 49, 7th term is 13 then n = _____

(a) 0 (b) 5

(c) 7 (d) 13

[Dec. 2012]

Solution : (c) is correct.

$t_7 = a + 6\times 2 = 13 \therefore a = 1$

$s_n = \frac{n}{2}[2\times 1 + (n-1).2] = 49$

or $\frac{n}{2}.2[1+n-1] = 49$

or $n^2 = 49 \therefore n = 7$

Q.18. The first term of a G.P. where second term is 2 and sum of infinite term is 8 will be

(a) 6 (b) 3

(c) 4 (d) 1

[Dec. 2012]

Solution : (c) is correct

$t_2 = ar = 2 \Rightarrow r = \frac{2}{a}$

$S_\infty = \frac{a}{1-r} = 8$

or a = 8 (1 - r)

or $a = 8\left(1 - \frac{2}{a}\right)$

or $a^2 = 8(a-2)$

or $a^2 - 8a + 16 = 0$

or $(a-4)^2 = 0 \Rightarrow a = 4$

Tricks :- Go by choices

For (c) $4r = 2 \therefore r = \frac{1}{2}$

$S = \frac{9}{1-r} = \frac{4}{1-1/2} = 8$ (Which is correct)

$\therefore$ (c) is correct

Q.19. If the sum of n terms of an A.P be $2n^2 + 5n$, then its 'n^{th}' term is

(a) 4n-2 (b) 3n-4

(c) 4n+3 (d) 3n+4

[Dec. 2012]

Solution : (c) is correct

$\therefore S_n = 2n^2 + 5n$

$\therefore S_1 = t_1 = 2\times 1^2 + 5\times 1 = 7 = a$

$d = S_2 - 2S_1$

$= 2\times 2^2 + 5\times 2 - 2\times 7 = 4$

$t_n = a + (n-1)d = 7 + (n-1)4 = 4n + 3$

Tricks :- Go by choices

For (a) $S_1 = t_1 = 4\times 1 - 2 = 2 \neq 7$

(c) $t_1 = 4\times 1 + 3 = 7$

$t_2 = 4\times 2 + 3 = 11$

$s_2 = t_1 + t_2 = 7 + 11 = 18$

and $S_2 = 2\times 2^2 + 5\times 2 = 18$

$\therefore$ (c) Satisfies it

$\therefore$ (c) is correct.

Q.20. In an A.P. if $s_n = 3n^2 - n$ and its common difference is 6 then first term is_____

(a) 2 (b) 3

(c) 4 (d) 6

[June 2013]

Solution : (a) is correct.

$s_n = 3n^2 - n$

$s_1 = 3\times 1^2 - 1 = 2 = t_1$

$\therefore$ 1st term =2

Q.21. In an A.P if the sum of 4th & 12th term is 8 then sum of first 15 term is_____

(a) 60 (b) 120

(c) 110 (d) 150

[June 2013]

Solution : (a) is correct.

Given, $t_4 + t_{12} = 8$

or a + 3d + 1 + 11d = 8

or 2a + 14d = 8

$\therefore s_{15} = \frac{15}{2}[2a + (15-1)d]$

$= \frac{15}{2} \times 8 = 60$

$\therefore$ (a) is correct

Q.22. There are 'n' AMs between 7 & 71 and 5th AM is 27 then 'n' = _____

(a) 15 (b) 16

(c) 17 (d) 18

[June 2013]

Solution : (a) is correct.

c.d = $\frac{b-a}{n+1}$ **(Tricks)**

$= \frac{71-7}{n+1} = \frac{64}{n+1}$

$A_5 = a + 5d$ (Tricks)

$= 7 + 5\times \frac{64}{n+1} = 27$

or $\frac{5\times 64}{n+1} = 20$

or 20n + 20 = 320

or 20n = 300 $\therefore$ n =15

$\therefore$ (a) is correct

Q.23. In a G.P the 6th term is 729 and the common ratio is 3 then is_____

(a) 2 (b) 3

(c) 4 (d) 7

[June 2013]

Solution : (b) is correct .

$t_1 = a; cr = r = 3$

$t_6 = 729$

or $a.r^5 = 729$

or $a\times 3^5 = 3^6$

$\therefore$ a = 3

Q.24. An AP has 13 terms whose sum is 143. The third term is 5, then first term is

(a) 4 (b) 7

(c) 9 (d) 2

[Dec. 2013]

Solution : (d) is correct

$\therefore t_3 = a+2d = 5$ _____(1)

$\therefore$ 2d = 5 - a

$s_{13} = \frac{13}{2}[2a+(13-1)d] = 143$

or $2a + 12d = \frac{143\times 2}{13} = 22$

or a + 6d = 11

or $a + 3\times 2d = 11$

or a +3 (5 - a) =11

or a + 15 - 3a = 11

or 4 = 2a $\therefore$ a = 2

Tricks :- Go by choices

[Solve mentally by calculator]

Q.25. G.M of a, b, c, d is 3 then G.M of $\frac{1}{a},\frac{1}{b},\frac{1}{c},\frac{1}{d}$ is

(a) $\frac{1}{3}$ (b) 3

(c) $\frac{1}{81}$ (d) 81

[Dec. 2013]

Solution : (a) is correct

$G = 3 = (abcd)^{1/4}$ _____(1)

New GM $= \left(\frac{1}{a}.\frac{1}{b}.\frac{1}{c}.\frac{1}{d}\right)^{1/4} = \frac{1}{3}$

Tricks :- GM of a,b,c,d = 3

GM of their Reciprocals = $\frac{1}{3}$

Q.26. The value of

$1^3+2^3+3^3+.....+m^3$ is equal to

(a) $\left[\frac{m(m+1)}{2}\right]^3$

(b) $\frac{m(m+1)(2m+1)}{6}$

(c) $\left[\frac{m(m+1)}{2}\right]^2$

(d) None

[June 2014]

Solution : (c) is correct

Formula $= \left\{\frac{m(m+1)}{2}\right\}^2$

Q.27. The sum of the infinite GP $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+.....\infty$ is equal to

(a) 1.95 (b) 1.5

(c) 1.75 (d) None

[June 2014]

Solution : (b) is correct

$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2} = 1.5$

Q.28. The sum of m terms of the series is 1 + 11 + 111+..... is equal to

(a) $\frac{1}{81}\left[10^{m+1}-9m-10\right]$

(b) $\frac{1}{2}\left[10^{m+1}-9m-10\right]$

(c) $[10^{m+1}-9m-10]$

(d) None of these

[June 2014, June 2015]

Solution : (a) is correct

Tricks :- Go by choices

For (a) put m = 1 ; we get

$s = \frac{1}{81}\left[10^{1+1}-9\times 1-10\right] = 1 = 1$st term

Put m=2; $S = \frac{1}{81}\left[19^{2+1}-9\times 2-10\right] = 12$

=1 +11 = Sum of 1st 2 terms

So; (a) is correct.

Q.29. If the sum of first 'n' terms of an A.P is $6n^2+6n$, then the fourth term of the series:

(a) 120 (b) 72

(c) 48 (d) 24

[Dec. 2014]

Solution : (c) is correct

S_n = Sum of 1st n terms of as AP.

$= 6n^2 + 6n$

$a = t_1 = s_1 = 6\times 1^2 + 6\times 1 = 12$

$s_2 = 6\times 2^2 + 6\times 2 = 36$

$c, d = d = s_2 - 2s_1 = 36 - 2\times 12 = 12$

$\therefore$

$t_4 = a+(4-1)d = 12+3\times 12 = 48$

Q.30. If $S_n = n^2p$ and $S_m = m^2p$; $(m \neq n)$ is the sum of A.P., then S_p = _____

(a) p^2 (b) p^3

(c) $2p^3$ (d) p^4

[Dec. 2014]

Solution : (b) is correct

$\because s_n = n^2 p$

$s_m = m^2 p$

$\therefore s_p = p^2.p = p^3$

Q.31. If x, y, z are the terms in G.P then the terms $x^2+y^2, xy+yz, y^2+z^2$ are in:

(a) A.P (b) G.P

(c) H.P (d) None of these

[Dec. 2014]

Solution : (b) is correct

$\because x; y; z$ are in G.P

Tricks:- Let x = 1 ; y = 2; z = 4 make a G.P

$\therefore x^2+y^2 = 1^2+2^2 = 5$

$xy+yz = 1\times 2+2\times 4 = 10$

$y^2+z^2 = 2^2+4^2 = 20$

$\therefore x^2+y^2; xy+yz; y^2+z^2 =$

5,10,20..... clearly are in G.P.

Q.32. Let S be the sum, P be the product and R be the sum of reciprocals of n terms of a G.P then P^2R^n =

(a) S^{2n} (b) S^{-n}

(c) S^n (d) S^{-2n}

[June 2015]

Solution : Let n = 3

Let S = 1+ 2 + 4 ----- a GP. = 7

Tricks:- $P = 1\times 2\times 4 = 8$

$R = \frac{1}{1}+\frac{1}{2}+\frac{1}{4} = \frac{4+2+1}{4} = \frac{7}{4}$

Let n = 3

$$\therefore P^2R^n = P^2.R^3 = 8^2 \times \left(\frac{7}{4}\right)^3$$

$$= 64 \times \frac{349}{64} = 7^3$$

$$= s^3$$

$$= s^n$$

$\therefore$ (*c*) is correct.

Q.33. The sum of n terms of an AP is $3n^2 + 5n$, which last term is 164.

(*a*) 25 (*b*) 27

(*c*) 29 (*d*) 31

[Dec. 2015]

Solution : (*b*) is correct

$$S_n = 3n^2 + 5n$$

$$a = t_1 = S_1 = 3\times1^2 + 5\times1 = 8$$

$$S_2 = 3\times2^2 + 5\times2 = 22$$

$$d = S_2 - 2S_1 = 22 - 2\times8 = 6$$

$$n = \frac{t_n - a}{d} + 1 = \frac{164-8}{6} + 1 = 27$$

Q.34. Three No's a,b,c are in A.P find a - b + c

(*a*) a (*b*) -b

(*c*) b (*d*) c

[Dec. 2015]

Solution : (*c*) is correct

let $a = 1; b = 2; c = 3$ makes an A.P.

$\therefore a - b + c = 1 - 2 + 3 = 2 = b.$

Q.35. Find the numbers whose GM is 5 and AM is 7.5:

(*a*) 12 and 13

(*b*) 13.09 and 1.91

(*c*) 14 and 11

(*d*) 17 and 19

[Dec. 2015]

Solution : (*b*) is correct

Tricks: Go by Choices

$$GM = \sqrt{13.09\times1.51} = 5. \text{ (approx.)}$$

$$AM = \frac{13.09+1.91}{2} = 7.5$$

Q.36. If $\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}$ are in Arithmetic Progression then a^2, b^2, c^2 are in _____.

(*a*) Arithmetic Progression

(*b*) Geometric Progression

(*c*) Both A.P & G.P

(*d*) None of these

[June 2016]

Solution : (*a*)

Tricks:- a^2, b^2, c^2 are in AP.

$a = 1, b = 5, c = 7$ Make it in AP.

let

$$\therefore \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b} \text{ in AP}$$

$$\frac{1}{5+7}, \frac{1}{7+1}, \frac{1}{1+5}$$

$$\left[\frac{1}{12}, \frac{1}{8}, \frac{1}{6}\right] \times 24$$

2, 3, 4 is also in AP.

$\therefore$ Our assumption is correct.

Q.37. The sum of n terms of the series

If $\log(x) + \log\left(\frac{x^2}{y}\right) + \log\left(\frac{x^3}{y^2}\right) + ...$ *is*

(*a*) $\frac{n}{2}\left[2n\log\left(\frac{x}{y}\right) + \log xy\right]$

(*b*) $\frac{n}{2}\left[n\log xy + \log\left(\frac{x}{y}\right)\right]$

(*c*) $\frac{n}{2}\left[n\log\left(\frac{x}{y}\right) - \log xy\right]$

(*d*) $\frac{n}{2}\left[n\log\left(\frac{x}{y}\right) + \log xy\right]$

[June 2016]

Solution : (*d*) is correct.

Tricks : Go by Choices

put $n = 1; 2$ Then

(*d*) Should be equal to first term *i.e.* $\log x$

$\therefore$ option (*d*)

$$= \frac{1}{2}\left[1.\log\frac{x}{y} + \log xy\right]$$

$$= \frac{1}{2}\left[\log\left(\frac{x}{y}xy\right)\right]$$

$$= \frac{1}{2}\log x^2 = \frac{1}{2}2\log x = \log x$$

if $n = 2$

then option (*d*) should be equal to

$$\log x + \log\frac{x^2}{y} = \log\frac{x^3}{y}$$

Option (*d*)

Q.38. A G.P (Geometric Progression) consists of 2n terms. If the sum of the terms occupying the odd places is S_1 and that of the terms in even places is S_2, The common ratio of the progression is

(*a*) n (*b*) $2S_1$

(*c*) $\frac{S_2}{S_1}$ (*d*) $\frac{S_1}{S_2}$

[June 2016]

Solution : (*c*) **Tricks:-**

let $S = \underline{1} + 2 + \underline{4} + 8 + \underline{16} + ...$ to $2n$ terms

$S_1 = 1 + 4 + 16 + ...$ to n terms

$S_2 = 2 + 8 + 32 + ...$ to n terms

for $n = 1$

$S_1 = 1;\ S_2 = 2$

$$\text{c.r.} = \frac{S_2}{S_1} = \frac{2}{1} = 2 \text{ (True)}$$

for $n = 2$

$$\text{c.r.} = \frac{2+8}{1+4} = 2 \text{ (Also True)}$$

$\therefore$ (*c*) is correct.

Q.39. 2.353535 ..... = $2.3\dot{5}$

(*a*) $\frac{233}{99}$ (*b*) $\frac{234}{99}$

(*c*) $\frac{232}{99}$ (*d*) None

[Dec. 2016]

Solution : (*a*) is correct.

Tricks : Go by choices [use calculator]

Divide 233 by 99 we get 2.3535...

Q.40. The number of terms of the series needed for the sum of the series 50 + 45 + 40 + ... becomes zero

(*a*) 22 (*b*) 21

(*c*) 20 (*d*) None

[Dec. 2016]

Solution : (b) is correct.

Tricks : Go by choices

Let (b) is correct.

$$S_{21} = \frac{21}{2}[2\times 50 + (21-1)\times(-5)]$$

$$= 0$$

So, (b) is correct.

Q.41. A person received the salary for the 1st year is ₹5,00,000 per year and he received an increment of ₹15,000 per year then the sum of the salary he taken in 10 years

(*a*) ₹56,75,000 (*b*) ₹72,75,000 (*c*) ₹63,75,000 (*d*) None of these

[Dec. 2016]

Solution : (*a*) is correct.

$$S_{10} = \frac{10}{2}[2\times 5,00,000 + (10-1)\times 15000]$$

$$= ₹\ 56,75,000.$$

Q.42. If a, -3, b, 5, c are in A.P then the value of c is

(*a*) -7 (*b*) 1 (*c*) 9 (*d*) 13

[June 2017]

Solution : a, -3 ; b ; 5, are in AP.

So; $b = \frac{-3+5}{2}$ [$\because$ b is A.M. of -3 & 5]

$b = 1$

Similarly ;

$5 = \frac{b+c}{2} \Rightarrow b + c = 10$

or $c = 10 - b = 10 - 1 = 9$

$\therefore$ (*c*) is correct.

Q.43. The sum of n terms of the series 1+(1+3)+(1+3+5)+.....

(*a*) $\frac{n(n+1)(2n+1)}{6}$ (*b*) $\frac{n(n+1)(2n+1)}{3}$

(*c*) $\frac{n(n+1)(n+2)}{6}$ (*d*) None

[June 2017]

Solution : Tricks :- Go by Choices

For (*a*) put $n = 1 \Rightarrow$ It should be equal

$$\frac{1(1+1)(2\times 1+1)}{6} = 1 \text{ (True)}$$

If $n = 2$ Then it should be equal to sum of 1st 2 term

So ; $\frac{2(2+1)(2\times 2+1)}{6} = 1 + (1+3)$

or $\frac{30}{6} = 5$ (True)

$\therefore$ Option (*a*) is correct.

Q.44. The sum of first 20 terms of a G.P is 1025 times the sum of first 10 terms then the common ratio is _____

(*a*) 2 (*b*) $2\sqrt{2}$ (*c*) $\frac{1}{2}$ (*d*) $\sqrt{2}$

[June 2017]

Solution : Given

$S_{20} = 1025 . S_{10}$

$\frac{a(r^{20}-1)}{r-1} = 1025 . \frac{a(r^{10}-1)}{r-1}$

or ; $r^{20} - 1 = 1025 (r^{10} - 1)$

or ; $(r^{10})^2 - 1^2 = 1025 (r^{10} - 1)$

or ; $(r^{10} + 1)(r^{10} - 1) = 1025 (r^{10} - 1)$

or ; $r^{10} + 1 = 1025$

or ; $r = 2$

$\therefore$ Option (*a*) is correct.

Q.45. Find the sum of all natural numbers between 100 and 1000 which are divisible by 11 is :

(*a*) 44,550 (*b*) 66,770 (*c*) 55,440 (*d*) 33,440

[Dec. 2017]

Solution : (*a*)

Series

$S = 110 + 121 + 132 + ... + 990$

$$n=\frac{l-a}{d}+1=\frac{990-110}{11}+1=81$$

$$S=\frac{n}{2}(a+l)=\frac{81}{2}(110+990)=44,550$$

Q.46. If pth, qth, rth terms of a G.P. be a, b, c respectively, then (q - r) log a + (r - p) log b + (p - q) log c =

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) None

[June 2018]

Solution : (*a*)

Tricks :- It is in cyclic order.

Q.47. If a, b, c, d are in GP then $(b-c)^2 + (c-a)^2 + (d-b)^2$ = ?

(*a*) $(a-b)^2$ (*b*) $(a-d)^2$

(*c*) $(c-d)^2$ (*d*) 0

[June 2018]

Solution : (*b*)

a, b, c, d $\rightarrow$ in GP

let a = 1 ; b = 2 ; c = 4 ; d = 8 in G.P.

$\therefore (b-c)^2+(c-a)^2+(d-b)^2$

$=(2-4)^2+(4-1)^2+(8-2)^2$

$=4+9+36=49=7^2$

GBC

For (*b*) $(a-d)^2=(1-8)^2=7^2=49.$

$\therefore$ (*b*) is correct.

Q.48. If the n th term of a series, $a_n = 3^n - 2^n$, then S_n = ?

(*a*) $\frac{3}{2}(3^n-1)+1(n+1)$ (*b*) $\frac{3}{2}(3^n+1)-1(n+1)$

(*c*) $\frac{3}{2}(3^n-1)-n(n+1)$ (*d*) $\frac{3}{2}(3^n+1)-1(n-1)$

[June 2018]

Solution : (*c*)

$\because a_n=3^n-2^n$

$a_1=3^1-2^1=1$

$a_2=3^2-2^2=5$

$s_2=a_1+a_2=1+5=6$

Tricks :- Go by choices (GBC)

for (*c*) let

$$s_n=\frac{3}{2}(3^n-1)-n(n+1)$$

$$s_1=\frac{3}{2}(3^1-1)-1(1+1)=\frac{3}{2}2-2=1=a_1 \text{ (True)}$$

Now $s_2=\frac{3}{2}(3^2-1)-2(2+1)$

$=\frac{3}{2}\times 8-6=12-6=6=a_1+a_2$ (True)

$\therefore$ (*c*) is correct.

Q.49. The sum to m terms of the series 1 + 11 + 111 + ... upto m terms, is equal to :

(*a*) $\frac{1}{81}(10^{m+1}-9m-10)$ (*b*) $\frac{1}{27}(10^{m+1}-9m-10)$

(*c*) $10^{m+1}-9m-10$ (*d*) None of these

[May 2018]

Solution : (a)

Tricks:-GBC

If m = 1 $\Rightarrow$ S_1 = Sum of 1st 1 term = 1

If m = 2 $\Rightarrow$ S_2 = Sum of 1st 2 term

= 1 + 11 = 12

Note:- Check only for m = 1 and m = 2

(a) If m = 1

$$S_1=\frac{1}{81}(10^{1+1}-9\times 1-10)=\frac{1}{81}(100-9-10)=1$$

(a) If m = 2

$$S_2=\frac{1}{81}\left[10^{2+1}-9\times 2-10\right]=12$$

So; Option (a) is true for m = 1 & m = 2.

So; (a) is correct.

Q.50. A person pays Rs. 975 in monthly instalments, each instalment is less than former by Rs. 5. The amount of Ist instalment is ₹ 100. In what time will be entire amount be paid?

(*a*) 26 months

(*b*) 15 months

(*c*) Both (*a*) & (*b*)

(*d*) 18 months

[May 2018]

Solution : (*b*)

Tricks:- Go by choices (GBC)

Series

S = 100 + 95 90 +.....to n months (let)

= 975.

1st check for n = 15 months

$$S=\frac{15}{2}\left[2\times100+(15-1)\cdot(-5)\right]$$

If loan is paid off in n = 15 months, then no need of other instalments. So (b) is correct.

Q.51. If the sum of n terms of an AP is $(3n^2-n)$ and its common difference is 6, then its first term is :

(*a*) 3 (*b*) 2

(*c*) 4 (*d*) 1

[May 2018]

Solution : (b)

$$S_n = 3n^2 - n$$

Tricks:-

$$\because\ t_1 = S_1 = 3\times1^2 - 1 = 2$$

= sum of 1st 1 term.

Q.52. Insert two arithmetic means between 68 and 260.

(*a*) 132, 196 (*b*) 130, 194

(*c*) 70, 258 (*d*) None

[May 2018]

Solution : (a)

Tricks:-

Go by choices

(a) 68; 132 ; 196; 260 are in AP.

Hence ; 132 ; 196 are

A.Ms. b/w 68 and 260.

Hence (a) is correct.

Q.53. If the P^{th} term of an A.P. is 'q' and the q^{th} term is 'p', then its r^{th} term is

(*a*) p + q + r (*b*) p + q - r

(*c*) p - q - r (*d*) p + q

[Nov. 2018]

Solution : (b)

Tricks:- $\text{c.d} = \frac{q-p}{p-q} = \frac{(p-q)}{p-q} = -1$

$$\therefore\ t_r = t_p + (r-p)d$$

$$= q + (r-p)\cdot(-1)$$

$$= q + p - r$$

Q.54. The 3^{rd} term of a G.P. is $\frac{2}{3}$ and the 6^{th} term is $\frac{2}{81}$, then the 1^{st} term is

(*a*) 2 (*b*) 6

(*c*) 9 (*d*) $\frac{1}{3}$

[Nov. 2018]

Solution : (*b*)

$$t_3 = ar^2 = \frac{2}{3};\ t_6 = ar^5 = \frac{2}{81}$$

$$\text{or } ar^2\cdot r^3 = \frac{2}{81}$$

$$\text{or } \frac{2}{3}r^3 = \frac{2}{81} \Rightarrow r^3 = \left(\frac{1}{3}\right)^3$$

$$\because\ ar^2 = \frac{2}{3}$$

$$\text{or } a\cdot\left(\frac{1}{3}\right)^2 = \frac{2}{3}$$

or a = 6

Q.55. The sum of the series –8, –6, –4,...n terms is 52. The number of terms n is

(*a*) 10 (*b*) 11

(*c*) 13 (*d*) 12

Solution : (*c*)

Series S = - 8 - 6 - 4 ..... to n terms

first term = -8 ; c.d = d = 2

Tricks :- Go by choices (Use calculator)

option (*c*)

$$S_{13} = \frac{13}{2}\left[2\times(-8)+(13-1)\times2\right] = 52$$

Q.56. The value of K, for which the terms 7K + 3, 4K - 5, 2K + 10 are in A.P., is

(*a*) –13 (*b*) –23

(*c*) 13 (*d*) 23

[Nov. 2018]

Solution : (*b*)

Formula 2A = a + b

$\therefore$ 2(4k - 5) = 7k + 3 + 2k + 10

or 8k - 10 = 9k + 13

or k = -23

Q.57. The ratio of sum of n terms of the two AP's is (n+1) then the ratio of their m^{th} terms is

(*a*) (m + 1) : 2m

(*b*) (m + 1) : (m - 1)

(*c*) (2m - 1) : (m + 1)

(*d*) m : (m - 1)

[June 2019]

Solution : Given that

$$\frac{S_n^{I}}{S_n^{II}} = \frac{n+1}{n-1}$$

Tricks:-

To find the ratio of r^{th} term ;

put n = 2r - 1

$\therefore$ Put n = 2m - 1

Ratio of m^{th} term

$$= \frac{2m-1+1}{2m-1-1} = \frac{2m}{2m-2}$$

$$= \frac{2m}{(2m-2)} = \frac{m}{m-1}$$

(d) is correct.

Q.58. In a G.P. if the fourth term is '3' then the product of first seven terms is

(*a*) 3^5 (*b*) 3^7

(*c*) 3^6 (*d*) 3^8

[June 2019]

Solution :(*b*)

Tricks:-

Product of 1st (2r-1) terms of a G.P = $(t_r)^{2r-1}$

$\therefore t_4 = 3$

So; Product of 1st $2\times4-1=7$ terms

$= (t_r)^{2\times4-1} = 3^7$

(b) is correct.

Details:-

$\because$ Product of 1st 7 terms

$= a.ar.ar^2.ar^3.....ar^6$

$= a^7 . r^{1+2+3+.....+6}$

$= a^7 . r^{\frac{6}{2}(6+1)} = a^7 . r^{21}$

$= (ar^3)^7 = 3^7$

Q.59. If 2 + 6 + 10 + 14 + 18 + ... + x = 882 then the value of x

(a) 78 (b) 80

(c) 82 (d) 86

[June 2019]

Solution : (c)

S = 2 + 6 + 10 + 14 ..... + x(to n terms) = 882

$\therefore \frac{n}{2}[2+x] = 882$ (1)

Where x = Last term

Last term $= x = 2+(n-1)\times4$

$x = 4n-2$

or $4n = x+2$

or $n = \frac{x+2}{4}$

$\therefore$ From (1) ; we get

$\frac{(x+2)}{4\times2}(x+2) = 882$

or $(x+2)^2 = 8\times882 = 84^2$

$\therefore x+2 = 84 \Rightarrow x = 82$.

Tricks:-

Let $t_n = x$

or $2+(n-1)\cdot4 = x$

or $4n-2 = x$

or $n = \frac{x+2}{4}$

For GBC

(c) If $x = 82 \Rightarrow n = \frac{82+2}{4} = 21$

$\therefore S = \frac{n}{2}(a+l) = \frac{21}{2}(2+x)$

$= \frac{21}{2}(2+82) = 882$

$\therefore$ (c) is correct.

Q.60. If $y = 1+x+x^2+.....\infty$ then x =

(a) $\frac{y-1}{y}$ (b) $\frac{y+1}{y}$

(c) $\frac{y}{y+1}$ (d) $\frac{y}{y-1}$

[June 2019]

Solution : (a)

$y = 1+x+x^2+..... \infty$ are in G.P.

$\therefore y = \frac{1}{1-x}$ Where c.r = x

or $1-x = \frac{1}{y}$

or $x = 1-\frac{1}{y} = \frac{y-1}{y}$. $\left[\because S_\infty = \frac{a}{1-r}\right]$

Q.61. In the series 25, 5, 1, ... 1/3125 which term is 1/3125?

(a) 8th term (b) 9th term

(c) 15th term (d) None of these

[Dec. 2019]

Solution : (a)

Let $t_n = \frac{1}{3125}$.

$\therefore 25\cdot\left(\frac{1}{5}\right)^{n-1} = \frac{1}{5^5}$

or $5^2\cdot\frac{1}{5^{n-1}} = \frac{1}{5^5}$

or $5^{n-1} = 5^7 \Rightarrow n-1 = 7$

$\therefore n = 8$

Q.62. The sum of five terms of AP is 75 find the 3rd term is.

(a) 20 (b) 30

(c) 15 (d) None of these

[Dec. 2019]

Solution : (c)

$t_3 = a+(3-1)d = a+2d.$

$S_5 = \frac{5}{2}[2a+(5-1)d] = 75$

or $\frac{5}{2}\cdot2[a+2d] = 75$

or $a+2d = \frac{75}{5} = 15$

So, $t_3 = 15$.

Q.63. (c+a-b)/b, (a+b-c)/c, (b+c-a)/a are in AP then a,b,c are in

(a) AP (b) GP

(c) HP (d) None of these

[Dec. 2019]

Solution : (d)

Adding 2 to each term; we get

$\frac{c+a-b}{b}+2; \frac{a+b-c}{c}+2; \frac{b+c-a}{a}+2$

are also in AP

$\Rightarrow \frac{a+b+c}{b}; \frac{a+b+c}{c}; \frac{a+b+c}{a}$

are in AP.

Dividing all terms by (a + b + c) ; we get

$\frac{1}{b}, \frac{1}{c}; \frac{1}{a}$ are also in AP.

$\Rightarrow$ b ; c ; a are in HP.

OR a, c ; b are in HP.

but a ; b ; c are not in HP.

Q.64. The sum of series 1/2+1/3²+1/2³+1/3⁴ + ... up to infinity is

(a) 25/24 (b) 19/24

(c) 1/12 (d) None of these

[Dec. 2019]

Solution : (b)

$S = \frac{1}{2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}+.....$ to ∞

$$= \left[\frac{1}{2}+\frac{1}{2^3}+\ldots\ldots\text{to }\infty\right]+\left[\frac{1}{3^2}+\frac{1}{3^4}+\ldots\ldots\text{to }\infty\right]$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2^2}}+\frac{\frac{1}{3^2}}{1-\frac{1}{3^2}}$$

$$= \frac{1/2}{3/4}+\frac{1/9}{8/9}$$

$$= \frac{1}{2}\times\frac{4}{3}+\frac{1}{8} = \frac{2}{3}+\frac{1}{8}$$

$$= \frac{16+3}{24} = \frac{19}{24}$$

Q.65. The 20th term of arithmetic progression whose 6^{th} term is 38 and 10^{th} term is 66 is.....

(*a*) 136 (*b*) 118

(*c*) 178 (*d*) 210

[Dec. 2020]

Solution : Tricks

Common difference

$$= d = \frac{t_{10}-t_6}{10-6}$$

$$= \frac{66-38}{4} = 7$$

Tricks

$t_{20} = a + 19d$

$= [a + (6 - 1)\, d] + 14d$

$= 38 + 14 \times 7$

$= 136$

$\therefore$ (*a*) is correct.

Q.66. Three numbers in G.P with their sum is 130 and their product is 27,000 are.....

(*a*) 90, 30, 10

(*b*) 10,30,90

(*c*) (*a*) & (*b*) Both

(*d*) 10,20,30

[Dec. 2020]

Solution : (*c*)

Tricks : GBC (Go by choices)

* (*a*) & (*b*) both follow G.P.
* sum of terms = 90 + 30 +10 = 130 (also follows)
* Their product = 90 × 30 × 10 = 27000/-

Which is also satisfied

$\therefore$ option (*c*) is correct.

Q.67. Divide 69 into 3 parts which are in A.P and are such that the product of first two parts is 460

(*a*) 20, 23, 26 (*b*) 21, 23, 25

(*c*) 19, 23, 27 (*d*) 22, 23, 24

[Dec. 2020]

Solution : Tricks : GBC (Go by choices)

* All options are in A.P.
* Only in option (*a*)

Product of 1st two terms

$= 20 \times 23 = 460$ (True)

$\therefore$ (*a*) is correct.

Q.68. The nth terms of the series 3 + 7 + 13 + 21 + 31 +.....is

(*a*) 4n - 1 (*b*) $n^2 + 2n$

(*c*) $n^2 + n + 1$ (*d*) $n^3 + 2$

[Jan. 2021]

Solution : Tricks

In such type of Questions always find answer by GBC (Go by choices).

For $n = 1 \Rightarrow t_1 = 3$

for $n = 2 \Rightarrow t_2 = 7$

and $n = 3 \Rightarrow t_3 = 13$

Putting n = 1 in all options, we get $t_1 = 3$

So, Here, we cannot decide any option.

Now putting n = 2 in all options we get in

(*a*) $t_2 = 4 \times 2 - 1 = 7 = t_2$ (True)

(*b*) $t_2 = 2^2 + 2 \times 2 = 8 \neq t_2$ (False)

(*c*) $t_2 = 2^2 + 2 + 1 = 7 = t_2$ (True)

(*d*) $t_2 = 2^3 + 2 = 10 \neq t_2$ (False)

Hence, we conclude that option (*a*) or (*c*) should be answer. (Both same)

So check for n = 3 in (a) & (c); we get

(*a*) $t_3 = 4 \times 3 - 1 = 11 \neq 13 = t_3$ (False)

(*c*) $t_3 = 3^2 + 3 + 1 = 13 = t_3$ (True)

Hence (*c*) should be correct.

Q.69. The number of integers from 1 to 100 which are neither divisible by 3 nor by 5 nor by 7 is

(*a*) 67 (*b*) 55 (*c*) 45 (*d*) 33

[Jan. 2021]

Solution : (*c*) is correct.

Let U = Set of integers from 1 to 100 = 1, 2, 3, ———; 100

$\therefore$ n (U) = 100

Let A = Integers divisible by 3

= {3, 6, 9, ...; 99} in A.P.

Let

B = Integers divisible by 5.

= {5, 10, 15, ...; 100} in A.P.

and C = Integers divisible by 7

= {7, 14, 21, ...; 98} in A.P.

We have to find No. of integers from 1 to 100 which are neither divisible by 3, nor divisible by 5, nor divisible by 7

$= n(A \cup B \cup C)' = n(U) - N(A \cup B \cup C)$

$= 100 - [n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)] = ?$ (1)

$\therefore n(A) = \frac{1-a}{d} + 1 = \frac{99-3}{3} + 1 = 33$

$n(B) = \frac{100-5}{5} + 1 = 20$

$n(C) = \frac{98-7}{7} + 1 = 14$

Now $(A \cap B)$ = Integers divisible by 3 & 5 both i.e., divisible by 15.

$= \{15, 30, 45, \ldots; 90\}$

$n(A \cap B) = \frac{90-15}{15} + 1 = 6$

$(B \cap C)$ = Integers divisible by 5 and 7 i.e. divisible by LCM of 5 & 7 = 35

$= \{35, 70\}$

$n(B \cap C) = 2$

$(C \cap A)$ = Integers divisible by 3 and 7 i.e. divisible by 21.

$= \{21, 42, 63, 84\}$

$n(C \cap A) = 4$

And $(A \cap B \cap C)$ = Integers divisible by 3, 5 and 7 all i.e. LCM of 3, 5 & 7 = 105

$= \{\}$

$n(A \cap B \cap C) = 0$

$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 33 + 20 + 14 - 6 - 2 - 4 + 0$

$= 55$

$\therefore$ From (1)

$n(A \cup B \cup C)' = 100 - n(A \cup B \cup C)$

$= 100 - 55 = 45.$

$\therefore$ (*c*) is correct.

Q.70. In a geometric progression the 3rd and 6th terms are respectively 1 and -1/8. The first term (*a*) and common ratio are respectively.

(*a*) 4 and $\frac{1}{2}$ (*b*) 4 and $\frac{-1}{4}$ (*c*) 4 and $\frac{-1}{2}$ (*d*) 4 and $\frac{1}{4}$

[Jan. 2021]

Solution : (*c*) is correct.

Tricks GBC [Go by choices]

From (*a*) $t_3 = a r^{3-1} = 4\left(\frac{1}{2}\right)^2 = 1$ (True)

and $t_6 = a r^{6-1} = \left(\frac{1}{2}\right)^5 = \frac{1}{8} \neq -\frac{1}{8}$ (False)

So (*a*) is False

(*b*) $t_3 = a r^2 = 4\left(-\frac{1}{4}\right)^2 = 4\frac{1}{16} = \frac{1}{4} \neq 1$

(It is also False)

(*c*) $t_3 = a r^2 = 4\left(-\frac{1}{2}\right)^2 = 4\frac{1}{4} = 1$ (True)

$t_6 = a r^5 = 4\left(-\frac{1}{2}\right)^5 = 4\left(-\frac{1}{32}\right) = -\frac{1}{8}$ (True)

$\therefore$ (*c*) is correct

Q.71. The number of terms of the series: 5 + 7 + 9 + ... must be taken so that the sum may be 480

(*a*) 20 (*b*) 10

(*c*) 15 (*d*) 25

[July 2021]

Solution : (*a*) is correct

Let S = 5 + 7 + 9 + ... to "n" terms

= 480

Tricks : Go by choices (GBC)

For (*a*) at n = 20

$S = \frac{20}{2}[2 \times 5 + (20-1)\cdot 2]$

$= 10(10 + 38) = 480$ (True)

Hence, (*a*) is correct

Q.72. If the sum of 'n' terms of an AP (Arithmetic Progression) is $2n^2$, the fifth term is _____

(*a*) 20 (*b*) 50

(*c*) 18 (*d*) 25

[July 2021]

Solution : (*c*) is correct

Q $t_5 = S_5 - S_4$ [i.e. sum of 1st 5 terms – sum of 1st 4 terms]

$= 2 \times 5^2 - 2 \times 4^2$

$= 50 - 32 = 18$

Q.73. The sum of three numbers in a geometric progression is 28. When 7,2 and 1 are subtracted from the first, second and the third numbers respectively, then the resulting numbers are in arithmetic progression. What is the sum of squares of the original three numbers?

(*a*) 510 (*b*) 456 (*c*) 400 (*d*) 336

[July 2021]

Solution : (*d*) is correct

Let first term = a and common ratio = r

3 Numbers are a, ar, ar^2

$\because$ $a + ar + ar^2 = 28$ (1)

From question $a - 7$; $ar - 2$ and $ar^2 - 1$ are in AP

$\therefore$ $2(ar - 2) = (a - 7) + (ar^2 - 1)$

or; $2ar - 4 = a + ar^2 - 8$

Adding "ar" on both sides; we get

$\Rightarrow$ $2ar - 4 + ar = a + ar + ar^2 - 8$

or $3ar - 4 = 28 - 8$

(From (*i*))

or; $3ar = 20 + 4 = 24$

or $ar = \frac{24}{3} = 8$ (2)

From (1); $a + ar + ar^2 = 28$

or ; $a + ar + ar.\, r = 28$

or $a + 8 + 8r = 28$

or ; $a = 20 - 8r$

From (2)

$ar = 8$

or $(20 - 8r)\, r = 8$

or $4(5 - 2r)\, r = 8$

or $5r - 2r^2 = 2$

or $2r^2 - 5r + 2 = 0$

$2r^2 - 4r - r + 2 = 0$

or $2r\,(r - 2) - 1\,(r - 2) = 0$

or $(r - 2)\,(2r - 1) = 0$

$\therefore$ $r = 2; \frac{1}{2}$

From (2) $ar = 8$

or $a \cdot 2 = 8 \Rightarrow a = 4$

Sum of squares of these numbers

$= a^2 + a^2 r^2 + a^2 r^4$

$= 4^2 + 4^2 \times 2^2 + 4^2 \times 2^4$

$= 16 + 64 + 256 = 336$

Q.74. The sum of square of any real positive quantities and its reciprocal is never less that

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

[July 2021]

Solution : (*b*) is correct

Let a positive no. = x

From question,

Two nos. are x^2 & $\frac{1}{x^2}$

Its Arithmetic mean

$= A = \frac{x^2 + \frac{1}{x^2}}{2}$

and Its Geometric mean

$G = \sqrt{x^2 \frac{1}{x^2}} = \sqrt{1} = 1$

We know that

$A \geq G$

or $\frac{x^2 + \frac{1}{x^2}}{2} \geq 1$

or $x^2 + \frac{1}{x^2} \geq 2$

$\therefore$ Minimum value of $x^2 + \frac{1}{x^2}$ is 2

Q.75. If the sum and products of three numbers in G.P. are 7 and 8 respectively, then 4th term of the series is

(*a*) 6 (*b*) 4

(*c*) 8 (*d*) 16

[Dec. 2021]

Solution : (*c*) If the sum and product of three numbers in G.P. are 7 and 8 respectively, then 4th term of the series is

(*a*) 6 (*b*) 4

(*c*) 8 (*d*) 16

Tricks

Product of 3 terms in G.P. = 8-1.2.4

[Note - Find 3 Factors of 8 such that the factors in G.P.]

So Nos. are = 1, 2, 4

And their sum = 7

Here 1 + 2 + 4 = 7 (True)

$\therefore T_4 = 4 \times CR$

$= 4 \times 2$

$= 8$

Q.76. The sum of series 7+14+21+..... to 17th term is:

(*a*) 1071 (*b*) 971

(*c*) 1171 (*d*) 1271

[Dec. 2021]

Solution : (*a*)

S = 7 + 14 + 21... to 17 terms

= 7 [1 + 2 + 3 .... to 17 terms]

$= 7.\frac{17(17+1)}{2} = 1071$

[$\because$ 1 + 2 + 3 + ... to *n* terms]

$= \frac{n(n+1)}{2}$

Q.77. The sum of first n terms of an AP is $3n^2 + 5n$. The series is:

(*a*) 8, 14, 20, 26.....

(*b*) 8, 22, 42, 68,.....

(*c*) 22, 68, 114,.....

(*d*) 8, 14, 28, 44,.....

[Dec. 2021]

Solution : (*a*)

$\because\ S_n = 3n^2 + 5n$

$S_1 = 3 \times 1^2 + 5 \times 1 = 8$

$S_2 = 3 \times 2^2 + 5 \times 2 = 22$

$S_3 = 3 \times 3^2 + 5 \times 3 = 42$

GBC

(A)

$S_1 = 8$ (True)

$S_2 = 8 + 14 = 22$ (True)

$S_3 = 8 + 14 + 20 = 42$ (True)

$\therefore$ **a** is correct.

Details

$a = t_1 = S_1 = 3 \times 1^2 + 5 \times 1 = 8$

$S_2 = 3 \times 2^2 + 5 \times 2 = 22$

$c.d = d = S_2 - 2S_1 = 22 - 2 \times 8 = 6$

$t_n = a + (n-1)\,d$

$= 8 + (n-1).6 = 8 + 6n - 6 = 6n + 2$

$\therefore\ t_1 = 6 \times 1 + 2 = 8$

$t_2 = 6 \times 2 + 2 = 14$

$t_3 = 6 \times 3 + 2 = 20$

So (a) is Correct.

Q.78. The largest value of n for which $\frac{1}{2} + \frac{1}{2^2} + \ldots \frac{1}{2^n} < 0.998$ is_____.

(*a*) 9 (*b*) 6

(*c*) 7 (*d*) 8

[Dec. 2021]

Solution : (*a*)

$$S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^n} < 0.998$$

$$S = \frac{\frac{1}{2}\left[1 - (1/2)^n\right]}{1 - 1/2} = \frac{\frac{1}{2}\left(1 - \frac{1}{2^n}\right)}{1/2} < 0.998$$

$$= 1 - \frac{1}{2^n} < 0.998$$

or $1 - 0.998 < \frac{1}{2^n}$

or $0.002 < 2^{-n}$

Calculator Trick

Press 2 ÷ button = button 9 times

= 0.00195

(Makes True)

$\therefore\ n = 9$

Q.79. If the nth term of the arithmetic progression 9, 7, 5 ... is same as the nth term of the arithmetic progression 15, 12, 9 ..., then n will be

(*a*) 7 (*b*) 9

(*c*) 15 (*d*) 11

[June 2022]

Solution : t_n of 1st AP = t_n of 2nd AP

$\therefore\ 9 + (n-1)(-2) = 15 + (n-1)(-3)$

or; $9 - 2n + 2 = 15 - 3n + 3$

or $3n - 2n = 18 - 11$

or $n = 7$

$\therefore$ (a) is correct

Q.80. In a geometric progression, the second term is 12 and the sixth term is 192. Find the 11th term.

(*a*) 3,072 (*b*) 1,536

(*c*) 12,288 (*d*) 6,144

[June 2022]

Solution : Given

$t_2 = ar = 12$.....(1)

$t_6 = ar^5 = 192$.....(2)

Eqn. (2) ÷ (1); we get

$$\frac{t_6}{t_2} = \frac{ar^5}{ar} = \frac{192}{12}$$

or; $r^4 = 16 = 2^4$

$\therefore\ r = 2$

Now $t_{11} = ar^{11-1} = ar^{10}$

$= ar^5.r^5$

$= 192 \times 2^5$ (From (2))

$= 6144$

$\therefore$ (d) is correct

Q.81. The first and last terms of an arithmetic progression are 5 and 905. Sum of the terms is 45,955. The number of terms is

(*a*) 99 (*b*) 100

(*c*) 101 (*d*) 102

[June 2022]

Solution : Let No. of terms = n.

$$S_n = \frac{n}{2}(a + l) = 45{,}955$$

Where a = 1st term;

l = last term.

$$\frac{n}{2}(5 + 905) = 45955$$

or $\frac{n}{\cancel{2}} \times \frac{455}{\cancel{910}} = 45955$

or $455n = 45955$

or; $n = \frac{45955}{455} = 101$

(*c*) is correct.

Q.82. The sum of first eight terms of geometric progression is five times the sum of the first four terms. The common ratio is

(*a*) $\sqrt{2}$ (*b*) $\sqrt{3}$

(*c*) 4 (*d*) 2

[June 2022]

Solution : Given

Sum of 1st 8 terms

= 5 (sum of 1st 4 terms)

$$\text{or; } \frac{a\left(r^8 - 1\right)}{r - 1} = \frac{5.a\left(r^4 - 1\right)}{r - 1}$$

or; $r^8 - 1 = 5\,(r^4 - 1)$

or; $(r^4)^2 - 1^2 = 5(r^4 - 1)$

or; $(r^4 - 1).\,(r^4 + 1) = 5\,(r^4 - 1)$

or; $r^4 + 1 = 5$

or; $r^4 = 5 - 1 = 4 = 2^2$

or, $(r^2)^2 = 2^2$

or $r^2 = 2\ \therefore r = \sqrt{2}$

(*a*) is correct.

Q.83. If *p*th term of an AP is q and its *q*th term is *p*, then what will be the value of (*p* + *q*)th term?

(*a*) 0 (*b*) 1

(*c*) $p + q - 1$ (*d*) $2(p + q - 1)$

[Dec. 2022]

Solution: *Detail :*

Let $t_1 = a$ and common difference $= d$

$$\therefore c \times d = d = \frac{t_p - t_q}{p - q} = \frac{q - p}{-(q - p)}$$

$= -1$

Tricks :

$t_{p+q} = t_p + (p + q - p)d$

$= q + (q)(-1)$

$= q - q = 0$

$\therefore$ (*a*) is correct.

Q.84. In a G P, 5th term is 27 and 8th term is 729. Find its 11th term.

(*a*) 729 (*b*) 6,561

(*c*) 2,187 (*d*) 19,683

[Dec. 2022]

Solution: Let $t_1 = a$ and $c \times r = r$

$$\therefore \frac{t_8}{t_5} = \frac{ar^7}{ar^4} = \frac{729}{27}$$

or; $r^3 = 27 = 3^3$

$\therefore r = 3$

$\therefore t_{11} = t_8 \times r^3 = 729 \times 3^3 = 729 \times 27$

$= 19{,}683.$

$\therefore$ (*d*) is correct.

Q.85. How many number between 74 and 25,556 are divisible by 5?

(*a*) 5090 (*b*) 5097

(*c*) 5095 (*d*) 5075

[June 2023]

Solution: Series

S = 75 + 80 + 85 + ... + 25,555

Total No. of Nos. divisible by 5

$$= \frac{l - a}{d} + 1 = \frac{25{,}555 - 75}{5} + 1 = 5097$$

Where a = 1st term

l = last term

(*b*) is correct.

Q.86. If 9th and 19th term of an Arithmetic Progression are 35 and 75, respectively, then its 20th term is:

(*a*) 78 (*b*) 79

(*c*) 80 (*d*) 81

[June 2023]

Solution: Tricks

Common difference

$$= d = \frac{A_{19} - A_9}{19 - 9} = \frac{75 - 35}{10}$$

$$= \frac{40}{10} = 4$$

$t_{20} = t_{19} + cd = 75 + 4 = 79$

(*b*) is correct.

Q.87. If 4th, 7th and 10th terms of a Geometric Progression are *p*, *q* and *r*, respectively then:

(*a*) $p^2 = q^2 + r^2$

(*b*) $p^2 = qr$

(*c*) $q^2 = pr$

(*d*) $pqr + pq + 1 = 0$

Solution: Let $t_1 = a$ and $c.r = x$

$t_4 = ax^3 = p$

$t_7 = ax^6 = q$

$t_{10} = ax^9 = r$

Clearly ; $q^2 = pr$

$(ax^6)^2 = ax^3 \times ax^9$

$\Rightarrow a^2x^{12} = a^2 \times x^{3+9}$

$= a^2 x^{12}$

(True)

$\therefore$ (*b*) is correct.

12 CHAPTER

SETS, FUNCTION AND RELATION

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. If $A=\{X : X^2-3X+2=0\}$

$B=\{X : X^2-4X+12=0\}$,

Then B-A is equal to

(*a*) $\{-6\}$ (*b*) $\{1\}$

(*c*) $\{1,2\}$ (*d*) $\{2,-6\}$

[June 2010]

Solution : (*a*) $\because\ x^2-3x+2=0$

or $x^2-2x-x+2=0$

or x(x - 2) -1 (x - 2) = 0

or (x- 2) (x - 1) = 0

$\because$ x = 1;2

$\because$ A = {1,2}

And x^2 + 4x - 12 = 0

or x^2 + 6x - 2x - 12 = 0

or x(x +6) -2 (x + 6) = 0

or (x + 6) (x - 2) = 0

$\therefore$ x = -6 ; 2

$\therefore$ B = {-6 ; 2}

$\therefore$ B - A = {-6 ; 2} - {1 ; 2}

= {-6}

$\therefore$ (*a*) is correct

Q.2. If F: A $\rightarrow$ R is a real valued function defined by f(x) = $\frac{1}{x}$ then

(*a*) R (*b*) $R-\{1\}$

(*c*) $R-\{0\}$ (*d*) R-N

[June 2010]

Solution : (*c*)

f(x) = $\frac{1}{x}$ is defined at all x $\in$R except x = 0

$\therefore$ A = R - { 0}

$\therefore$ (*c*) is correct

Q.3. In the set N of all natural numbers the relation R defined by a R b "if and only if, a divide b", then the relation R is:

(*a*) Partial order relation

(*b*) Equivalence relation

(*c*) Symmetric relation

(*d*) None of these

[June 2010]

Solution : (*a*) It is transitive relation. *i.e.* partial order relation

Q.4. For any two sets A and B, $A \cap (A' \cup B) =$ _____, where A' represent the compliment of the set A

(a) $A \cap B$ (b) $A \cup B$

(c) $A \cup B$ (d) None of these

[Dec. 2010]

Solution : Tricks : Take an example and then decide the answer

Let U = {0, 1, 2, 3, 4, 5}

A = {0, 1, 2, 3}

B = {2, 3, 4, 5}

$\therefore A^1 = \cup - A$

$= \{4;5\}$

$A^1 \cup B = \{4,5\} \cup \{2,3,4,5\}$

$= \{2,3,4,5\}$

$\therefore A \cap (A^1 \cup B)$

$= \{0,1,2,3\} \cap \{2,3,4,5\}$

$= \{2,3\}$

$= A \cap B$

$\therefore$ (a) is correct

IInd method = $(A \cap A') \cup (A \cap B) =$ { } $\cup (A \cap B) = A \cap B$

Q.5. If $f : R \to R$, $f(x) = x + 1$,

$g : R \to R$ $g(x) = x^2 + 1$ then fog(-2) equals to

(a) 6 (b) 5

(c) -2 (d) None

[Dec. 2010]

Solution : (a) $\because$ f(x) = x + 1

$g(x) = x^2 + 1. \Rightarrow g(-2) = (-2)^2 + 1 = 5$

fog (-2) = f{g(-2)} = f(5)

= 5 + 1 = 6

$\therefore$ (a) is correct

Q.6. If $A \subset B$, then following is true

(a) $A \cap B = B$ (b) $A \cup B = B$

(c) $A \cap B = A'$ (d) $A \cap B$

[Dec. 2010]

Solution : (b)

Q.7. If $f(x-1) = x^2 - 4x + 8$, then $f(x+1) =$

(a) $x^2 + 8$ (b) $x^2 + 7$

(c) $x^2 + 4$ (d) $x^2 - 4x$

[Dec. 2010]

Solution : (c); $f(x - 1) = x^2 - 4x + 8$

$= (x-1+1)^2 - 4(x - 1 + 1) + 8$

$\therefore f(x + 1)$

$= (x+1+1)^2 - 4(x + 1 + 1) + 8$

$= (x+2)^2 - 4(x + 2) + 8$

$= x^2 + 4x + 4 - 4x - 8 + 8$

$= x^2 + 4$

$\therefore$ (c) is correct.

Q.8. There are 40 students, 30 of them passed in English, 25 of them passed in Maths and 15 of them passed in both. Assuming that every Student has passed atleast in one subject. How many student's passed in English only but not in Maths.

(a) 15 (b) 20

(c) 10 (d) 25

[June 2011]

Solution : (a) Total students = 40

n(E) = 30 ; n(M) = 25

$(E \cap M) = 15$

No. of stds. passed in English only

n(E) - n$(E \cap M)$

= 30 - 15 = 15

$\therefore$ (a) is correct

Q.9. If $A = \{\pm 2, \pm 3\}$, $B = \{1,4,9\}$ AND $F = \{(2,4),(-2,4),(3,9),(-3,4)\}$ then 'F' is defined as:

(a) One to one function from A into B

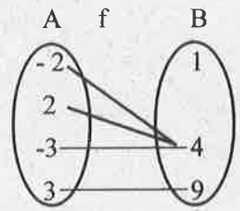
(b) One to one function from A onto B

(c) Many to one function from A onto B.

(d) Many to one function from A into 8.

[June 2011]

Solution : (c)



$\therefore$ (c) is correct

Q.10. If f(x) $= \frac{x}{\sqrt{1+x^2}}$ and $g(x) = \frac{x}{\sqrt{1-x^2}}$ Find fog ?

(a) x (b) $\frac{1}{x}$

(c) $\frac{x}{\sqrt{1-x^2}}$ (d) $x\sqrt{1-x^2}$

[June 2011]

Solution : (a) $fog = f\{g(x)\} = f\left(\frac{x}{\sqrt{1-x^2}}\right)$

$$= \frac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{1+\frac{x^2}{1-x^2}}}$$

$$= \frac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{\frac{1-x^2+x^2}{1-x^2}}}$$

$$= \frac{x}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{\sqrt{1}} = x$$

$\therefore$ (a) is correct

Q.11. f(x) = 3+x, for $-3 < x < 0$ and 3-2x for 0<x<3, then value of f(2) will be

(a) -1 (b) 1

(c) 3 (d) 5

[Dec. 2011]

Solution: (a) is correct

$\because$ f(x) = 3 + 2x ; when $-3 < x < 0$

= 3 - 2x ; when $0 < x < 3$

f(x = 2) = 3 - 2× 2 = -1

$\therefore$ 2 lies in 2nd condition

Q.12. If A = (1,2,3,4,5), B = (2,4) and C = (1,3,5) then (A -C) × B is

(a) {(2,2), (2,4), (4,2), (4,4), (5,2), (5,4)}

(b) {(1,2), (1,4), (3,2), (3,4), (5,2), (5,4)}

(c) {(2,2), (4,2), (4,4), (4,5)}

(d) {(2,2), (2,4), (4,2), (4,4)}

[Dec. 2011]

Solution : (d) is correct

(A -C)× B = {2,4}× {2, 4} = {(2, 2), (2, 4) ; (4 ;2) ; (4 ; 4)}

Q.13. For any two sets A and B the set $(A \cup B')'$ is Equal to (where' denotes compliment of the set)

(a) B-A (b) A-B

(c) A'-B' (d) B'-A'

[Dec. 2011]

Solution: (a) is correct

By De-Morgan's formula

$(A \cup B')'$

$= A' \cap (B')'$

$= A' \cap B$

$= B - A \cap B$

$= B - A$

Q.14. The number of proper sub-set of the set $\{3,4,5,6,7\}$ is

(a) 32 (b) 31

(c) 30 (d) 25

[June 2012]

Solution : (b) No. of proper sub-sets $= 2^n - 1$

$= 2^5 - 1 = 31$

Q.15. On the set of lines, being perpendicular is a _____

(a) Reflexive (b) Symmetric

(c) Transitive (d) None of these

[June 2012]

Solution : (b) is correct

It is symmetric relation Because it x is perpendicular to y

Then y is also perpendicular to x

Q.16. The range of the function f : N → N; $f(x) = (-1)^{x-1}$, is

(a) $\{0,-1\}$ (b) $\{1,-1\}$

(c) $\{1,0\}$ (d) $\{1,0,-1\}$

[June 2012]

Solution : (b) is correct

$\therefore f(x) = (-1)^{x-1}$

If x = odd No.

f(x) = 1

It x = 0; even No.

f(x) = -1

∴ Range = {1 ; -1}

Domain = {any real No.}

Q.17. For a group of 200 persons, 100 are interested in music, 70 in photography and 40 in swimming, Further more 40 are interested in both music and photography, 30 in both music and swimming, 20 in photography and swimming and 10 in all the three. How many are interested in photography but not in music and swimming ?

(a) 30 (b) 15 (c) 25 (d) 20

[Dec. 2012]

Solution : (d) is correct

Let A = No. of persons interested in Music

B = No. of persons interested in photography

C = No. of persons interested in Swimming

n(A) = 100; n(B) = 70 ; n(C) = 40;

$n(A \cap B) = 40; n(A \cap C) = 30;$

$n(B \cap C) = 20; n(A \cap B \cap C) = 10.$

$\therefore n(B \cap A' \cap C') = n(B) - n(B \cap A) - n(B \cap C) + n(A \cap B \cap C)$

$= 70 - 40 - 20 + 10$

$= 20$

Q.18. If $f : R \to R$ is a function, defined by f(x) = 10x-7, if $g(x) = f^{-1}(x)$, then the value of g(x) is equal to

(a) $\frac{1}{10x-7}$ (b) $\frac{1}{10x+7}$

(c) $\frac{x+7}{10}$ (d) $\frac{x-7}{10}$

[Dec. 2012]

Solution : (c) is correct

Let y = f(x) = 10x - 7

or 10x = y + 7

$\therefore x = \frac{y+7}{10}$

$\therefore f^{-1}(x) = \frac{x+7}{10}$

$\therefore g(x) = \frac{x+7}{10}$

Q.19. The No. of elements in range of constant function is

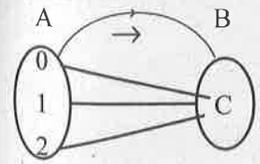
(a) One (b) Zero

(c) Infinite (d) None

[Dec. 2012]

Solution : (a) is correct

Let f(x) = c (where c = constant)



Domain = {x/x∈R}

Range = {c}

Q.20. If f(x) = x+2, $g(x) = 7^x$ then go f(x)=

(a) $7^x.x + 2.7^x$ (b) 7^{x+2}

(c) $(7^x)+2$ (d) none

[June 2013]

Solution : f(x) = x+2 ; $g(x) = 7^x$

$g\ of\ (x) = g\{f(x)\} = g(x+2) = 7^{x+2}$

∴ (b) is correct

Q.21. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then $f\left(\frac{2x}{1+x^2}\right)$

(a) f(x) (b) 2 f(x)

(c) 3 f(x) (d) -f(x)

[June 2013]

Solution :

$$(b)\quad f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$

$$= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$

$$= \log\frac{(1+x)^2}{(1-x)^2} = \log\left(\frac{1+x}{1-x}\right)^2$$

$$= 2\log\left(\frac{1+x}{1-x}\right) = 2f(x)$$

∴ (b) is correct

Q.22. If A = {1,2,3} then the relation R={(1, 1), (2, 3), (2, 2), (3, 3), (1, 2)} on A is:

(*a*) Reflexive (*b*) Symmetric
(*c*) Transitive (*d*) Equivalence

[June 2013]

Solution : (*a*) is correct.

Reflexive Relation.

xRx ; (x ; x) ∈ R

Here (1, 1), (2, 2), (3, 3)∈R

So; It is Reflexive

Q.23. Of the 200 candidates who were interviewed for a position at call center, 100 had a two wheeler, 70 had a credit card and 140 had a mobile phone 40 of them had both a two wheeler and a credit card, 30 had both a credit card and mobile phone, 60 had both a two wheeler and a mobile phone and 10 had all the three. How many candidates had none of them ?

(*a*) 0 (*b*) 20
(*c*) 10 (*d*) 18

[Dec. 2013]

Solution : (*c*) is correct

Let n(A) = No. of Candidates having two wheeler

n(B) = No. of candidates having credit cards

n(C) = No. of candidates having mobile phone.

Given

n(A) = 100 ; n(B) = 70 ; n(c) = 140

$n(A\cap B)=40;\ n(B\cap C)=30;\ n(C\cap A)=60\ n(A\cap B\cap C)=10.$

$\therefore n(A\cup B\cup C)=100+70+140-40-30-60+10=190$

No. of candidates having none = 200 -190 = 10

Q.24. If $f(x)=\frac{x^2-25}{x-5}$ then f(5)

(*a*) 0 (*b*) 1
(*c*) 10 (*d*) Undefined

[Dec. 2013]

Solution : (*d*) is correct

$$f(5)=\frac{x^2-25}{x-5}=\frac{5^2-25}{5-5}=\frac{0}{0}$$

∴ Undefined

Q.25. $f(x)=(a-x^n)^{\frac{1}{n}}$, a > 0 and n is positive integer then f [f(x)]=

(*a*) x (*b*) a
(*c*) $x^{1/n}$ (*d*) $a^{1/n}$

[Dec. 2013]

Solution : (*a*) is correct

$$f\{f(x)\}=f\left\{(a-n^n)^{1/n}\right\}$$

$$=\left[a-\left\{(a-x^n)^{\frac{1}{n}}\right\}^n\right]^{\frac{1}{n}}$$

$$=\left[a-(a-x^n)\right]^{\frac{1}{n}}=\left[x^n\right]^{\frac{1}{n}}=x.$$

Q.26. In a class of 50 students 35 opted for Maths, 37 opted for commerce. The number of such student who opted for both maths and commerce is

(*a*) 13 (*b*) 15
(*c*) 22 (*d*) 28

[June 2014]

Solution : (*c*) is correct

n(M) = No. of students opted for Maths = 35

n(C) = No. of Student opted for Commerce = 37

So; $(M\cup C)=50$

$n(M\cap C)=35+37-50=22$

Q.27. The range of the relation {(1, 0) (2, 0) (3, 0) (4, 0) (0, 0)} is

(*a*) {1, 2, 3, 4, 0}
(*b*) {0}
(*c*) {1, 2, 3, 4}
(*d*) None

[June 2014]

Solution : (*b*) is correct

Range = {0}

Q.28. If A = {1, 2, 3} and B = {4, 6, 7} then the relation R = {(2, 4) (3, 6)} is

(*a*) A function
(*b*) A function from A to B
(*c*) Both (*a*) and (*b*)
(*d*) Not a function

[June 2014]

Solution : (*d*) is correct.

Note:- 1 has no image

Q.29. A = (2, 3), B=(4, 5), C=(5, 6) then $A\times(B\cap C)$

(*a*) [(5, 2), (5, 3)] (*b*) [(2, 5), (3, 5)]
(*c*) [(2, 4), (5, 3)] (*d*) [(3, 5), (2, 6)]

[Dec. 2014]

Solution : (*b*) is correct

$B\cap C=\{4,5\}\cap\{5,6\}=\{5\}$

$\therefore A\times(B\cap C)=\{2,3\}\times\{5\}$
$=\{(2,5);(3,5)\}$

Q.30. If a relation S = ((1,1), (2,2), (1,2), (2,1)) is symmetric and

(*a*) Reflexive but not transitive
(*b*) Reflexive as well as transitive
(*c*) Transitive but not reflexive
(*d*) Neither transitive nor reflexive

[Dec. 2014]

Solution : If S = {1,2,3} then

Then relation {(1,1); (2,2); (1,2); (2,1)} is symmetric and transitive but not Reflexive.

(*b*) is correct

Q.31. If

$f(x)=\frac{x}{x-1}$, then $\frac{f(x/y)}{f(y/x)}=$ ——

(*a*) x/y (*b*) y/x
(*c*) -x/y (*d*) -y/x

[Dec. 2014]

Solution : $f(x)=\frac{x}{x-1}$

$$f\left(\frac{x}{y}\right)=\frac{\frac{x}{y}}{\frac{x}{y}-1}=\frac{\frac{x}{y}}{\frac{x-y}{y}}=\frac{x}{x-y}$$

$$f\left(\frac{y}{x}\right)=\frac{\frac{y}{x}}{\frac{y}{x}-1}=\frac{\frac{y}{x}}{\frac{y-x}{x}}=\frac{y}{-(x-y)}$$

$$\therefore -\frac{f\left(\frac{x}{y}\right)}{f\left(\frac{y}{x}\right)}=\frac{\frac{x}{x-y}}{\frac{y}{-(x-y)}}=-\frac{x}{y}$$

Q.32. Let N be the set of all Natural number ; E be the set of all even natural numbers then the function

f : N → E defined as f(x) =2x ; ∨ x ∈ N is :

(*a*) One-one into

(*b*) One-one onto

(*c*) Many-one into

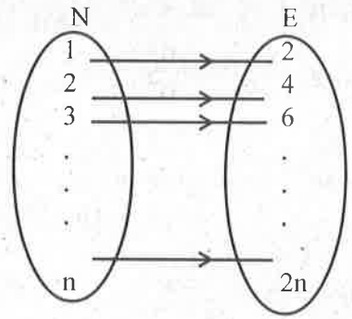
(*d*) Many-one onto

[Dec. 2014]

Solution : (*b*) is correct

N = {1,2,3,.....;n}

E = {2,4,6,.....;2n}



Clearly it is one-one onto mapping

Q.33. Which of these is a function from A → B; A= {x,y,z}; B={a,b,c,d}

(*a*) {(x,a) (x,b) (y,c)}

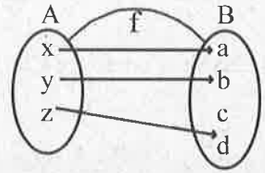
(*b*) {(x,a) (x,b) (y,c) (z,d)}

(*c*) {(x,a) (y,b) (z,d)}

(*d*) {(a,x) (b,z) (c,y)}

[Dec. 2015]

Solution : (*c*) is correct.



Q.34. f(x) = 2x+2, g(x) = x^2, fog(4) = ?

(*a*) 100 (*b*) 10

(*c*) 34 (*d*) None of these

[Dec. 2015]

Solution : (*c*) is correct

$fog(x) = f\{g(x)\}$

$= f(x^2) = 2.x^2+2$

$fog(4) = 2\times4^2+2 = 34$

Q. 35. In a class of 80 students, 35% play only cricket, 45% only Tennis. How many play Cricket?

(*a*) 86 (*b*) 54

(*c*) 36 (*d*) 44

[Dec. 2015]

Solution : (*d*) is correct

Given $n(C-T)=n(C)-n(C\cap T)=35\%$

$n(T-C)=n(T)-n(C\cap T)=45\%$

$n(CUT)=n(C)+n(T)-n(C\cap T)=100$

or; $35+n(C\cap T)+45+n(C\cap T)-n(C\cap T)=100$

or $80+n(C\cap T)=100$

$n(C\cap T)=20\%$

$\therefore n(C)=35+n(C\cap T)=35+20=55\%$

$=80\times55\%=44$

Q.36. If set $A=\left\{x:\frac{x}{2}\in Z, 0\le x\le 10\right\}$

$B=\{x : x \text{ is one digit prime number}\}$ and

$C=\left\{x:\frac{x}{3}\in N, x\le 12\right\}$ **then**

$A\cap(B\cap C)=$

(*a*) ϕ (*b*) Set A

(*c*) Set B (*d*) Set C

[June 2016]

Solution : (a)

$A=\{2,4,6,8,10\}$

$B=\{2,3,5,7\}$

$C=\{3,6,9,12\}$

$A\cap(B\cap C)$

$=A\cap(B\cap C)=\varphi$

No Common element in all 3 sets.

Q.37. The domain (D) and range (R) of the function

$f(x)=2-|x+1|$ is

(*a*) D = Real numbers, R = $(2,\infty)$

(*b*) D = Integers, R = (0, 2)

(*c*) D = Integers, R = $(-\infty,\infty)$

(*d*) D = Real numbers, R = $(-\infty, 2)$

[June 2016]

Solution : (*d*) is correct.

let $y=f(x)=2-|x+1|$

For any real values of x ; $f(x)$ is defined.

∴ Domain = D Real numbers

Minimum value of $|x+1|$ is Zero

∴ Maximum value of Range

$=2-0=2$

∴ Range $=-\infty<y\le 2$

$=(-\infty;2]$

Q.38. Let A be the set of the squares of natural numbers and $x\in A, y\in A$. Then _____.

(*a*) $x+y\in A$ (*b*) $x-y\in A$

(*c*) $\frac{x}{y}\in A$ (*d*) $xy\in A$

[June 2016]

Solution : (*d*) is correct.

$A = \{x / x$ is the squares of natural Nos.$\}$

$= \{1, 4, 9, 16, 25, ...\}$

Tricks: then Go by Choices

let $x = 1; y = 4 \in A$.

$\therefore\ x + y = 1 + 4 = 5 \notin A$.

$x - y = 1 - 4 = -3 \notin A$.

$\frac{x}{y} = \frac{1}{4} \notin A$.

But $xy = 1 \times 4 = 4 \in A$.

$\therefore$ (*d*) is correct.

Q.39. The number of sub-sets formed from the letters of the word "ALLAHABAD".

(*a*) 128 (*b*) 16

(*c*) 32 (*d*) None

[June 2016]

Solution : (c) is correct.

Let X = {Letters of word ALLAHABAD}

= { A, L, H, B, D }

No. of sub-sets = $2^5 = 32$

Q.40. If $f(x) = 100\,x$ then $f^{-1}(x) =$

(*a*) $\frac{x}{100}$ (*b*) $\frac{1}{100x}$

(*c*) $\frac{1}{100}$ (*d*) None of these

[June 2016]

Solution : (a) is correct.

Let y = f (x) = 100x

$x = \frac{y}{100}$; So, $f^{-1}(x) = \frac{x}{100}$

Q.41. $f : R \to R$ is defined by $f(x) = 2^x$ then f is

(*a*) One - one and onto

(*b*) One - one and into

(*c*) Many to one

(*d*) One to many

[June 2016]

Solution : (*b*) is correct.

Q.42. In a class, 80 students speak Hindi, 60 students speak English and 40 students speak both Hindi and English then the number of students in the class is _____.

(*a*) 100 (*b*) 120

(*c*) 140 (*d*) 180

[June 2017]

Solution : Let H = Students speak Hindi

E = Students speak English

Given

n(H) = 80 ; n (E) = 60

and $n(H \cap E) = 40$

$n(H \cup E) = n(H) + n(E) - n(H \cap E)$

= 80 + 60 - 40 = 100.

$\therefore$ Option (a) is correct

Q.43. If $f(x) = \frac{x-1}{x}$ and $g(x) = \frac{1}{1-x}$ then fog(x)= $\cup$

(*a*) x-1 (*b*) x

(*c*) 1-x (*d*) -x

[June 2017]

Solution : $fog(x) = f\{g(x)\} =$

$\frac{g(x) - 1}{g(x)}$

$= \frac{\frac{1}{1-x} - 1}{\frac{1}{1-x}} = \left(\frac{1 - 1 + x}{1 - x}\right) \times \frac{(1-x)}{1}$

$= x$

$\therefore$ Option (b) is correct

Q.44. The Range of the function f is defined by $f(x) = \frac{x}{x^2+2}$ is

(*a*) $\left\{x : \frac{-1}{2} < x < \frac{1}{2}\right\}$

(*b*) $\left\{x : \frac{-1}{2} \le x < \frac{1}{2}\right\}$

(*c*) $\left\{x : \frac{-1}{2} \le x \le \frac{1}{2}\right\}$

(*d*) $\left\{x : x > \frac{-1}{2}\ or\ x < \frac{-1}{2}\right\}$

[June 2017]

Solution : Let $y = \frac{x}{x^2+2} = f(x)$

or ; $yx^2 + 2y = x$

or $yx^2 - x + 2y = 0$

It is a quadratic equation in terms of x.

$\therefore$ Discriminant = D = $b^2 - 4ac$

$= (-1)^2 - 4.y.2y = 1 - 8y^2$

To be Real solutions ;

$D \ge 0 \Rightarrow 1 - 8y^2 \ge 0$

or $1 \ge 8y^2 \Rightarrow 8y^2 \le 1$

or ; $y^2 \le \frac{1}{8}$

or $\frac{-1}{2\sqrt{2}} \le y \le \frac{1}{2\sqrt{2}}$

$\left[\because If\ y^2 = \frac{1}{8} \Rightarrow y = \pm\frac{1}{2\sqrt{2}}\right]$

$\therefore$ (c) is correct.

Q.45. In a class of 35 students, 16 students play football and 24 students play cricket. Assume that each one play atleast one game, then number of students who play both the games is _____ :

(*a*) 5 (*b*) 11

(*c*) 12 (*d*) 17

[June 2017]

Solution : $n(F \cap C) = n(F) + n(C) - n(F \cup C)$

= 16 + 24 - 35 = 5

option (*a*) is correct.

Q.46. If $f(x) = \frac{x+1}{x+2}$ = then

$f[f\left(\frac{1}{x}\right)] =$ _____ :

(*a*) $\frac{2x+3}{3x+5}$ (*b*) $\frac{2x+5}{3x+2}$

(*c*) $\frac{3x+2}{5x+3}$ (*d*) $\frac{5x+2}{2x+3}$

[Dec. 2017]

Solution : (*c*)

$f\left(\frac{1}{x}\right) = \frac{1/x + 1}{1/x + 2} = \frac{1+x}{x} \times \frac{x}{1+2x}$

$= \frac{1+x}{1+2x}$

Now ; $f\left[f\left(\frac{1}{x}\right)\right] = f\left(\frac{1+x}{1+2x}\right)$

$$= \frac{\frac{1+x}{1+2x}+1}{\frac{(1+x)}{1+2x}+2} = \frac{\frac{3x+2}{1+2x}}{\frac{5x+3}{1+2x}}$$

$$= \frac{3x+2}{5x+3}.$$

Q.47. If A = $\{\phi, \{\phi\}\}$ then the Power Set of A is

(*a*) $\{\phi\}, \{0\}$

(*b*) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

(*c*) A

(*d*) $\{A\}; \{\phi\}$

[June 2018]

Solution : (*b*)

$A = \{\phi; \{\phi; \}\}$

$\therefore P(A) = \{\{\ \}\}; \{\phi\}; \{\{\phi\}\}; \{\phi; \{\phi\}\}$

$= \{\phi; \{\phi\}; \{\{\phi\}; A\}\}$

Q.48. If A = { x / x =3^n - 2n - 1 , where n$\in$ N },

B = { x/x = 4(n - 1), where n$\in$ N }. Then

(*a*) $A \subset B$ (*b*) $B \subset A$

(*c*) $A = B$ (*d*) None

[June 2018]

Solution : (*a*)

Putting n = 1, 2, 3,..... ; we get

$A = \{x / x = 3^n - 2n - 1\}$

= {0 ; 4 ; 20 ;..... }

$B = \{x / x = 4(n-1)\}$

= {0 ; 4 ; 8 ; 12 ; 16 ; 20 ;..... }

Clearly ; $A \subset B$

Q.49. The range of the function $\frac{x^6}{x^{12}+1}$ is:

(*a*) $(0, \infty)$

(*b*) $\left[0, \frac{1}{2}\right]$

(*c*) $(-\infty, 0) \cup [2, \infty)$

(*d*) $\left(0, \frac{1}{2}\right)$

[June 2018]

Solution : (*b*)

Let $y = \frac{x^6}{x^{12}+1}$

or $yx^{12} + y = x^6$

let $z = x^6$;

$\therefore yz^2 + y = z \Rightarrow yz^2 - z + y = 0$

It is a Quadratic Eqn. in terms of Z. for real solns.

$D = b^2 - 4ac = (-1)^2 - 4.y.y$

$= 1 - 4y^2$

$D \geq 0$

or ; $1 - 4y^2 \geq 0 \Rightarrow 1 \geq 4y^2$

or ; $4y^2 \leq 1$

or $y^2 \leq \frac{1}{4}$ If $y^2 = \frac{1}{4} \Rightarrow y \pm \frac{1}{2}$

$\therefore -\frac{1}{2} \leq y \leq \frac{1}{2}$

From Qts. $\left[0, \frac{1}{2}\right]$ [$\because$ y is always positive.]

Q.50. Let N be the set of all natural numbers; E be the set of all even natural numbers then the function;

(*a*) One-one-into

(*b*) Many-one-into

(*c*) One-one-onto

(*d*) Many-one-onto

[May 2018]

Solution : (*c*)

$\because$ N = set of Natural Numbers

= {1, 2, 3, ...}

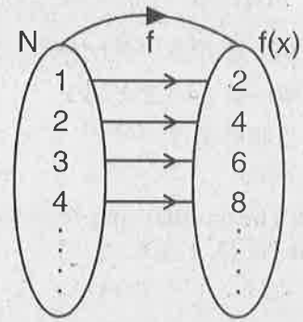
$f\{x\} = 2x$; $\forall X \in N$

So; $f(1) = 2 \times 1 = 2$

$f(2) = 2 \times 2 = 4$

$f(3) = 2 \times 3 = 6$

So;



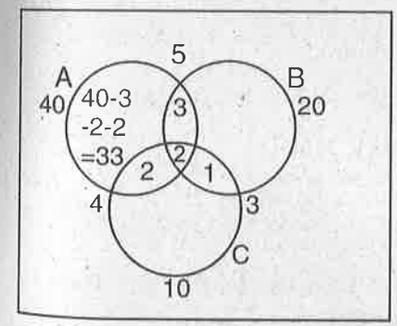
Clearly ; It is one-one and onto mapping.

Q.51. In a town of 20,000 families it was found that 40% families buy newspaper. A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C if 2% families buy all the three newspapers, then the number of families which buy A only is :

(*a*) 6600 (*b*) 6300 (*c*) 5600 (*d*) 600

[May 2018]

Solution : (*a*)



Given that

$n(A) = 40\%$; $n(B) = 20\%$

$n(C) = 10\%$; $n(A \cap B) = 5\%$

$n(B \cap C) = 3\%$; $n(C \cap A) = 4\%$

$n(A \cap B \cap C) = 2\%$

$\therefore n(A \cap \overline{B} \cap \overline{C}) =$ Only A

$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$= 40 - 5 - 4 + 2 = 33\%$

$= 20000 \times 33\%$

$= 6600.$

Q.52. The numbers of proper sub-sets of the set {3, 4, 5, 6, 7} is :

(*a*) 32 (*b*) 31

(*c*) 30 (*d*) 25

[May 2018]

Solution : (*b*)

Formula

No. of proper sub-sets = $2^n - 1$

$= 2^5 - 1 = 31.$

Q.53. A is {1,2,3,4} and B is {1,4,9,16,25} if a function f is defined from set A to B where $f(x) = x^2$ then the range of f is:

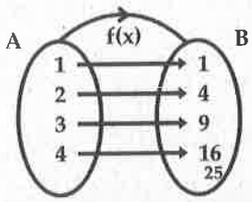
(*a*) {1,2,3,4} (*b*) {1,4,9,16}

(*c*) {1,4,9,16,25} (*d*) None of these

[Nov. 2018]

Solution : (*b*)

$\because f(x) = x^2$



$\therefore$ Range = {1, 4, 9, 16}

Q.54. If A = {1, 2} and B = {3, 4}. Determine the number of relations from A and B :

(*a*) 3 (*b*) 16

(*c*) 5 (*d*) 6

[Nov. 2018]

Solution : (*b*)

No. of Relations $= 2^{n(A \times B)}$

$= 2^{(2 \times 2)} = 16.$

Q.55. If A = {1,2,3,4,5,6,7} and B = {2, 4,6,8}. Cardinal number of A - B is:

(*a*) 4 (*b*) 3

(*c*) 9 (*d*) 7

[Nov. 2018]

Solution :

$A - B = \{1, 2, 3, 4, 5, 6, 7\} - \{2, 4, 6, 8\}$

$= \{1, 3, 5, 7\}$

$\therefore n(A - B) = 4$

(*a*) is correct.

Q.56. Identify the function from the following:

(*a*) {(1,1), (1,2), (1,3)}

(*b*) {(1,1), (2,1), (2,3)}

(*c*) {(1,2), (2,2), (3,2), (4,2)}

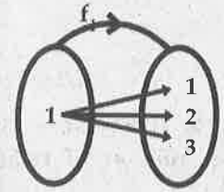
(*d*) None of these

[Nov. 2018]

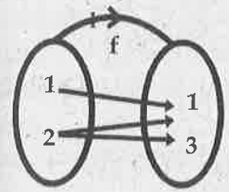
Solution : (*c*)

Go by choices

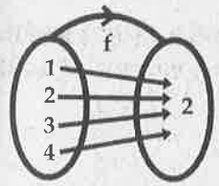
(*a*)



(*b*)



(*c*)



Q.57. If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}; B = {1, 3, 4, 5, 7, 8}; C = {2, 6, 8}

Then find $(A - B) \cup C$

(*a*) {2, 6} (*b*) {2, 6, 8}

(*c*) {2, 6, 8, 9} (*d*) None

[June 2019]

Solution : (*c*)

$A - B = A - (A \cap B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$\{1, 3, 4, 5, 7, 8\}$

$= \{2, 6, 9\}$

$\therefore (A - B) \cup C = \{2;6;9\} \cup \{2;6;8\}$

$= \{2 ; 6 ; 8 ; 9\}$

(*c*) is correct.

Q.58. A = {1, 2, 3, 4 ..... 10} a relation on A, R = {(x, y)/x + y = 10, x ∈ A, Y ∈ A, x ≥ Y} then domain of R^{-1} is

(*a*) {1, 2, 3, 4, 5}

(*b*) {0, 3, 5, 7, 9}

(*c*) {1, 2, 4, 5, 6, 7}

(*d*) None

[June 2019]

Solution : (*a*)

Given ; A = {1, 2, 3, ... ; 10}

R = {x ; y) / x + y = 10 ; $X \in A$; $Y \in A$; $x \geq y$}

$\Rightarrow$ R = (5 ; 5) ; (6 ; 4) ; (7 ; 3) ; (8 ; 2) ; (9 ; 1)

$\Rightarrow$ R^{-1} = (5 ; 5) ; (4 ; 6) ; (3 ; 7) ; (2 ; 8) ; (1 ; 9)

$\therefore$ Domain of R^{-1} = (5 ; 4 ; 3 ; 2 ; 1)

Q.59. The no. of sub-sets of the set {3, 4, 5} is :

(*a*) 4 (*b*) 8

(*c*) 16 (*d*) 32

[June 2019]

Solution : (*b*)

No. of sub-sets = $2^n = 2^3 = 8.$

Q.60. If $f(x) = x^2$ and $g(x) = \sqrt{x}$ then

(*a*) go f(3) = 3

(*b*) go f(-3) = 9

(*c*) go f(9) = 3

(*d*) go f(-9) = 3

[June 2019]

Solution : (*a*)

$\because f(x)=x^2;\ g(x)=\sqrt{x}$

$\therefore go\ f(x)=g\{f(x)\}$

$\sqrt{f(x)}=\sqrt{x^2}$

$=x$

$\therefore go\ f(3)=3$

$\therefore$ (a) is correct.

Q.61. If A = {a, b, c, d}; B = {p, q, r, s} which of the following relation is a function from A to B

(*a*) R_1 = {(a, p), (b, q), (c, s)}

(*b*) R_2 = {(p, a), (b, r), (d, s)}

(*c*) R_3 = {(b, p), (c, s), (b, r)}

(*d*) R_4 = {(a, p), (b, r), (c, q), (d, s)}

[June 2019]

Solution : (*d*)

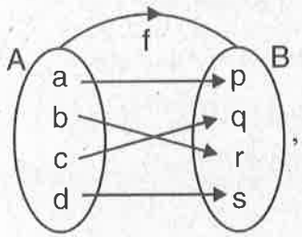
GBC

(*a*) All have one and only one solution of A in B except d i.e. d has no solution in B

$\Rightarrow$ Clearly R_1 is not Function.

Similarly (*b*) & (*c*) are not functions or mappings.

(*d*)



$\therefore R_4$ is a function or mapping from A to B.

Q.62. $f(x)=f(x-1)+f(x-2)$ **if** $f(0)=0, f(1)=1, x=2,3,4,.....$ **then what is** $f(7)$

(*a*) 8 (*b*) 13

(*c*) 3 (*d*) 5

[Dec. 2019]

Solution : (*b*)

$f(2)=f(2-1)+f(2-2)$

$=f(1)+f(0)=1+0=1$

$f(3)=f(3-1)+f(3-2)$

$=f(2)+f(1)=1+1=2$

$f(4)=f(4-1)+f(4-2)=f(3)+f(2)$

$=2+1=3$

$f(5)=f(4)+f(3)=3+2=5$

$f(6)=f(5)+f(4)=5+3=8$

$f(7)=f(6)+f(5)=8+5=13.$

Q.63. $f(x)=2x^3+1$ **then what is** $f^{-1}(x)$ **options**

(*a*) $\frac{1}{2}(x-1)^{1/3}$

(*b*) $\left(\frac{x-1}{2}\right)^{1/3}$

(*c*) $\left(\frac{x-1}{2}\right)^{1/2}$

(*d*) None of these

[Dec. 2019]

Solution : (*b*)

Let $y=f(x)=2x^3+1$

or ; $2x^3=y-1$

or $x^3=\frac{y-1}{2}$

or $x=\left(\frac{y-1}{2}\right)^{1/3}$

$\therefore f^{-1}(x)=\left(\frac{x-1}{2}\right)^{1/3}$

[Replacing y by *x*]

Q.64. Two finite sets respectively have x and y number of elements. The total number of subsets of the first is 56 more than the total no. of subsets of the second. The values of x, y are respectively.....

(*a*) 4 and 2 (*b*) 6 and 3

(*c*) 2 and 4 (*d*) 3 and 6

[Dec. 2020]

Solution : (*b*) **Tricks :- Go By Choices (GBC)**

Total No. of Subsets = 2^n

(*a*) $2^4 - 2^2 = 16 - 4 = 12 \neq 56$

$\therefore$ (*a*) is in correct.

(*b*) $2^{n(x)} - 2^{n(y)} = 56$

$\Rightarrow 2^6 - 2^3 = 64 - 8 = 56$

(*b*) is correct

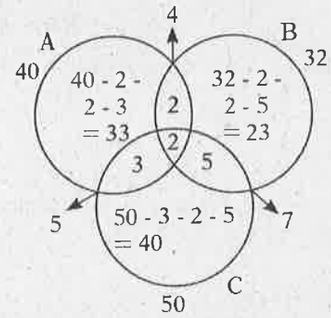
Q.65. The number of items in the set A is 40 in the Set B is 32; in the Set C is 50; in both A and B is 4; in both A and C is 5; in both B and C is 7; in all the set is 2. How many are in only one set?

(*a*) 65 (*b*) 110

(*c*) 96 (*d*) 84

[Dec. 2020]

Solution : Solution (*c*).



Given

$n(A) = 40$

$n(B) = 32$

$n(C) = 50$

$n(A \cap B) = 4$

$n(A \cap C) = 5$

$n(B \cap C) = 7$

and $n(A \cap B \cap C) = 2$

No. of items in only one set.

= only A + only B + only C

= (40 - 2 - 2 - 3) + (32 - 2 - 2 - 5)

+ (50 - 3 - 2 - 5)

= 43 + 23+ 40 = 96

Q.66. The set of cubes of natural numbers is

(*a*) Null set

(*b*) Finite set

(*c*) Infinite set

(*d*) A finite set of three numbers

[Dec. 2020]

Solution : (*c*) is correct.

Q.67. The inverse function $f'(y)$ of $f(y)$ = 3y is ....

(*a*) 1/3y (*b*) y/3

(*c*) -3y (*d*) 1/y

[Dec. 2020]

Solution : (*b*) Let $x = f(y) = 3y$

$$\therefore y = \frac{x}{3}$$

$$\therefore f^{1}(y) = \frac{y}{3}$$ ($\because$ Replace *x* by *y*)

(*b*) is correct.

Q.68. The set of cubes of natural number is

(*a*) Null set (*b*) A finite set

(*c*) An infinite set (*d*) Singleton Set

[Jan. 2021]

Solution : Let A = Set of cubes of natural Numbers

$= \{1^3, 2^3, 3^3 ...\}$

$n(A) = \infty$

$\therefore$ (*c*) is correct

Q.69. In the set of all straight lines on a plane which of the following is Not 'TRUE'?

(*a*) Parallel to an equivalence relation

(*b*) Perpendicular to is a symmetric relation

(*c*) Perpendicular to is an equivalence relation

(*d*) Parallel to a reflexive relation

[Jan. 2021]

Solution : (*c*) is correct

Q.70. Let F : R R be defined by

$$f(x) = \begin{cases} 2x & \text{for } x > 3 \\ x^2 & \text{for } 1 < x \le 3 \\ 3x & \text{for } x \le 1 \end{cases}$$

The value of f(-1) + f(2) + f(4) is

(*a*) 9 (*b*) 14

(*c*) 5 (*d*) 6

[Jan. 2021]

Solution : (*c*) is correct

Let R = being a relation of perpendicular Reflexive Relation

Let x be a straight line on a plane.

x R x i.e. x is perpendicular to itself is not true.

Hence R is not Equivalence.

(*c*) is correct.

Q.71. Let U be the universal set, A and B are the subsets of U. If n(U) = 650, n(A) = 310, n(A∩B). = 95 and n(B) = 190, then $n(\bar{A} \cap \bar{B})$ is equal to

($\bar{A}$ and $\bar{B}$ are the complement of A and B, respectively)

(*a*) 400 (*b*) 200

(*c*) 300 (*d*) 245

[July 2021]

Solution :

$\because \eta(\bar{A} \cap \bar{B}) = \eta(A \cup B)'$

$= \eta(\cup) - \eta(A \cup B) = ?$ (1)

Now

$\eta(A \cup B) = \eta(A) + \eta(B) - \eta(A \cap B)$

$= 310 + 190 - 95 = 405$

From (1)

$\eta(\bar{A} \cap \bar{B}) = \eta(\cup) - \eta(A \cup B)$

$= 650 - 405 = 245$

(*d*) is correct.

Q.72. The range of the function f defined by $f(x) = \sqrt{16 - x^2}$ is

(*a*) [-4, 0] (*b*) [-4,4]

(*c*) [0, 4] (*d*) (-4, 4)

[July 2021]

Solution : (*c*) is correct

Given $f(x) = \sqrt{16 - x^2}$

For Domain $(16 - x^2)$ should not be negative.

i.e. $16 - x^2 \ge 0$

or $16 \ge x^2$

Hence $x \le 4$ or $x \ge -4$

$\Rightarrow -4 \le x \le 4$

$\because$ Domain = [- 4, 4]

For Range : *f*(x) is maximum

if x = $0 \Rightarrow f(x) = 4$

and f(x) is minimum at $x = \pm 4$

i.e. $f(x) = 0$.

Hence Range = $0 \le f(x) \le 4$

= [0, 4]

Q.73. Let A = R - {3} and B = R - {1}. Let f : A → B defined by $f(x) = \dfrac{x-2}{x-3}$

What is the value of $f^{-1}\left(\dfrac{1}{2}\right)$?

(*a*) 2/3 (*b*) 3/4

(*c*) 1 (*d*) -1

[July 2021]

Solution : (*c*) is correct

Net $y = f(x) = \dfrac{x-2}{x-3}$

or $xy - 3y = x - 2$

or $xy - x = 3y - 2$

or $x(y - 1) = 3y - 2$

or $x = \dfrac{3y-2}{y-1}$

$\therefore f^{1}(x) = \dfrac{3x-2}{x-1}$

Now $= f^{-1}\left(\dfrac{1}{2}\right) \dfrac{3x\frac{1}{2}-2}{\frac{1}{2}-1} = \dfrac{-\frac{1}{2}}{-\frac{1}{2}}$

= 1

Q.74. If f(x) = x²-1 and g(x) = |2x + 3|, then fog(3) - g of(-3) =

(*a*) 71 (*b*) 61

(*c*) 41 (*d*) 51

[July 2021]

Solution : (*b*) is correct

$g(3) = |2 \times 3 + 3| = 9$

$f(-3) = (-3)^2 - 1 = 8$

$\therefore$ fog(3) – g of(–3)

$= f\{g(3)\} - g\{f(-3)\}$

$= f(9) - g(8)$

$= (9^2 - 1) - |2 \times 8 + 3|$

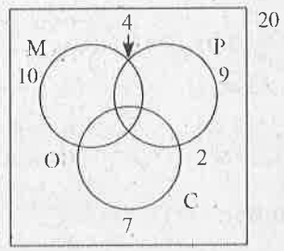
$= 80 - 19 = 61$

Q.75. Out of a group of 20 teachers in school, 10 teach Mathematics, 9 teach Physics and 7 teach Chemistry, 4 teach Mathematics and Physics but none teach both Mathematics and Chemistry. How many teach Chemistry and Physics, how may teach only Physics?

(*a*) 2, 3 (*b*) 3, 2 (*c*) 4, 6 (*d*) 6, 4

[Dec. 2021]

Solution : (*a*)



Given $n(M \cup P \cup C) = 20$;

$n(m) = 10$; $n(P) = 9$

$n(c) = 7$; $n(M \cap P) = 4$

$n(M \cap C) = 0 \Rightarrow n(M \cap P \cap C)$

$\therefore n(M \cup P \cup C) = n(M) + n(P) + n(C)$

$-n(M \cap P) - n(M \cap C) - n(P \cap C)$

$+ n(M \cap P \cap C)$

or $20 = 10 + 9 + 7 - 4 - 0 - x(\text{let}) + 0$

or; $x = 22 - 20$

$= 2$

$\therefore n(P \cap C) = 2$

only physics $= n(P) - n(P \cap C)$

$-n(M \cap P)$

$= 9 - 4 - 2 = 3$

$\therefore$ (*a*) is correct.

Q.76. If a is related to b if and only if the difference in a and b is an even integer. This relation is

(*a*) symmetric, reflexive but not transitive

(*b*) symmetric, transitive but not reflexive

(*c*) transitive, reflexive but not symmetric

(*d*) equivalence relation

[Dec. 2021]

Solution : (*d*)

(*i*) $\because a - a = 0$ and 0 is an even integer.

$\Rightarrow (a, a) \in R$ (True)

$\therefore$ R is reflexive

(*ii*) If $a - b$ = even integer

$\Rightarrow b - a$ = Also even integer

It means

If $(a - b) \in R \Rightarrow (b - a) \in R$.

$\therefore$ The relation is symmetric

(*iii*) Let $a = 1, b = 3, c = 5$.

$(1, 3) \in R$; $(3, 5), \in R$

$\Rightarrow$ (1–3) even integer

(3–5) also even integer

Then (1–5) also even integer

i.e. If $(a, b) \in R$, $(b, c) \in R$

$\Rightarrow (a, c) \in R$

$\therefore$ R is Transitive relation.

Since R is Reflexive, Symmetric and Transitive. So it is an Equivalence relation.

Q.77. If $u(x) = \dfrac{1}{1-x}$, then $u^{-1}(x)$ is :

(*a*) $\dfrac{1}{x-1}$ (*b*) $1 - x$

(*c*) $1 - \dfrac{1}{x}$ (*d*) $\dfrac{1}{x} - 1$

[Dec. 2021]

Solution : (*c*)

Net $y = u(x) = \dfrac{1}{1-x}$

or $\dfrac{1}{y} = 1 - x$

or $x = 1 - \dfrac{1}{y}$

$\therefore u^{-1}(x) = 1 - \dfrac{1}{x}$

Q.78. If $f(p) = \dfrac{1}{1-p}$, then f^{-1} is

(*a*) 1-p (*b*) $\dfrac{p-1}{p}$

(*c*) $\dfrac{p}{p-1}$ (*d*) $\dfrac{1}{p}$

[June 2022]

Solution : Let $y = f(p) = \dfrac{1}{1-p}$

$\therefore 1 - p = \dfrac{1}{y}$

or $p = 1 - \dfrac{1}{y} = \dfrac{1-1}{y}$

So $f^{-1}(p) = \dfrac{P-1}{P}$

(replace y by p).

(b) is correct.

Q.79. Let R = {(3, 3) (6, 6), (9, 9) ,(12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}. Then relation is

(*a*) an equivalence relation

(*b*) reflexive and transitive only

(*c*) reflexive only

(*d*) reflexive and symmetric only

[June 2022]

Solution : Given that

A = {3, 6, 9,12}

A relation R = {(3,3); (6,6); (9,9); (12,12); (6,12); (3,9); (3,12); (3,6)}

Reflexive Relation :

If x R x $\Rightarrow$ (x; x) $\notin$ R

Then R is Reflexive

Here (3, 3); (6, 6); (9, 9), (12, 12) $\notin$ R

So R is Reflexive

Symmetric Relation

$\because$ if (x; y) $\in$ R then (y, x) $\in$ R

Here (3, 9) $\in$ R but (9, 3) $\notin$ R

So R is not Symmetric

Transitive Relation::

if (x; y) $\in$ R; (y, z) $\in$ R

Then (x; z) $\in$ R

Here $(3, 6) \in R$; $(6, 12) \in R$

$\Rightarrow (3, 12) \in R$ (True)

$\therefore$ R is Reflexive and Transitive

(*b*) is correct

Q.80. Two finite sets have m and n elements. The total number of sub-sets of the first set is 56 more than the total number of sub-sets of the second set. The values of m and n are

(*a*) 6, 3 (*b*) 7, 6

(*c*) 5, 1 (*d*) 8, 7

[June 2022]

Solution : Formula

Let A has n elements

$\therefore$ Total No. of sub-sets $= 2^n$

Now Difference between No. of sub-sets of both sets

$= 2^m - 2^n$

Go by choices

Let (a) is correct

So, $2^m - 2^n = 2^6 - 2^3$

$= 64 - 8 = 56$ (True)

$\therefore$ (*a*) is correct

Q.81. If A = {1,2,3,4,5,7,8,9} and B = {2, 4, 6, 7, 9} then how many proper subset of $A \cap B$ can be created.

(*a*) 16 (*b*) 15

(*c*) 32 (*d*) 31

[Dec. 2022]

Solution: A = {1, 2, 3, 4, 5, 7, 8, 9}

B = {2, 4, 6, 7, 9}

$A \cap B = \{2, 4, 7, 9\} \Rightarrow n(A \cap B) = 4$

No. of proper subsets of $(A \cap B)$

$= 2^n - 1$

$= 2^4 - 1 = 15$

(*b*) is correct.

Q.82. Let A = {1, 2, 3} and consider the relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}, Then R is:

(*a*) Symmetric and transitive

(*b*) Reflexive but not transitive

(*c*) Reflexive but not symmetric

(*d*) Neither symmetric nor transitive

[Dec. 2022]

Solution: *Given :*

A = {1, 2, 3}

Relation = R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

Reflexive : $(x, x) \in R$ (True)

Symmetric : If $(x, y) \in R \Rightarrow (y, x) \in R$

$\therefore$ If $(1, 2) \in R$ but $(2, 1) \notin R$

Not symmetric.

(*c*) is correct.

Q.83. The number of subjects of the set {0, 1, 2, 3} is:

(*a*) 2 (*b*) 4

(*c*) 8 (*d*) 16

[Dec. 2022]

Solution: Let A = {0, 1, 2, 3} be a set.

No. of Subsets = n(Power set of A)

$= nP(A) = 2^n = 2^4 = 16$

(*d*) is correct

Q.84. Given the relation *R* = {(1, 2), (2, 3)} on the set *A* = {1, 2, 3}, the minimum number of ordered pairs which when added to R make it equivalence relation is:

(*a*) 5 (*b*) 7

(*c*) 6 (*d*) 8

[June 2023]

Solution:

$A \times A = \{1, 2, 3\} \times \{1, 2, 3\}$

$= \{(1, 1)(1, 2)(1, 3)(2, 1), (2, 2), (2, 3), (3, 1)(3, 2), (3, 3)\}$

Minimum No. of ordered pairs added to R to make it equivalence

$= n(A \times A) - n(R)$

$= 9 - 2 = 7$

(*b*) is correct.

Q.85. A survey shows that 74% of the Canadian like grapes, whereas 68% like bananas. What percentage of the Canadians like both grapes and bananas, if everybody like either of two?

(*a*) 32% (*b*) 26%

(*c*) 6% (*d*) 42%

[June 2023]

Solution:

Let n (A) = 74% = Grapes

n (B) = 68% = Banana

and $n(A \cup B) = 100\%$

$\therefore n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 74 + 68 - 100 = 42\%$

$\therefore$ (*d*) is correct.

Q.86. If *R* be a relation defined, on the set of Natural numbers as "$x\ Ry \Leftrightarrow (x - y)$ is divisible by 5" $\forall\ x, y \in N$, then the relation *R* is:

(*a*) Equivalence

(*b*) Anti-symmetric

(*c*) Symmetric but not transitive

(*d*) Symmetric but not reflexive

[June 2023]

Solution: Given

$xRy \Rightarrow (x - y)$ is divisible by 5

Let $x - y = 5m$

where m = An integer

Reflexive Relation:—

$xRx \Rightarrow x - x \div 5m$

$= 0 \times 5$

Hence it is Reflexive.

Symmetric Relation:—

Let $xRy \Rightarrow x - y$ is divisible by 5.

Let $x - y = 5m$

where m is an integer.

$\therefore y - x = -(x - y)$

$= -5m = 5(-m)$

Here $-m$ is also an integer.

Hence $y - x$ is also divisible by 5.

$\therefore xRy$ is also Symmetric Relation.

Transitive Relation:—

Let $xRy \Rightarrow x - y = 5m$

and $yRz \Rightarrow y - z = 5n$

i.e. $y - z$ is also divisible by 5.

So, $x - z = x - y + y - z$

$= 5m + 5n$

$= 5 (m + n)$

Here m + n is also an integer.

$\therefore$ x - z is also divisible by 5

$\therefore$ xRy is also Transitive.

Hence $x - y$ divisible by 5 is an equivalence relation.

$\therefore$ (a) is correct.

Q.87. If $A = (a, b, c)$, $B = (b, c, d)$ and $C = (a, d, c)$, then $(A - B) \times (B \cap C)$ is equal to:

(a) $\{(a, d), (c, d)\}$

(b) $\{(a, c), (a, d)\}$

(c) $\{(c, a), (d, a)\}$

(d) $\{(a, c), (a, d), (b, d)\}$

[June 2023]

Solution:

$A - B = \{a, b, c\} - \{b, c, d\}$

$= \{a\}$

Now $B \cap C = \{b, c, d\} \cap \{a, d, c\}$

$= \{c, d\}$

$\therefore (A - B) \times (B \cap C)$

$= \{a\} \times \{c, d\}$

$= \{(a, c)\}; \{a, d\}$

$\therefore$ (b) is correct.

Q.88. If $f(x)$: $N \to R$ is a function defined as $f(x) = 4x + 3$, $\forall\, x \in N$, then $f^{-1}(x)$ is:

(a) $4 + \frac{x+3}{4}$

(b) $\frac{x+3}{4}$

(c) $\frac{x-3}{4}$

(d) $\frac{3x+4}{4}$

[June 2023]

Solution:

Let $f(x) = 4x + 3 = y$

or $4x = y - 3$

or $x = \frac{y-3}{4}$

Replacing y by x. We get

$\therefore$ $f^{-1}(x) = \frac{x-3}{4}$

$\therefore$ (c) is correct.

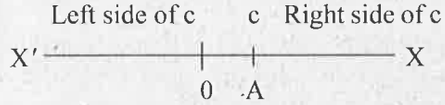
13 CHAPTER CALCULUS

LIMITS AND CONTINUITY

Limits

Meaning of x $\to$ c

1. It is known that all real numbers can be plotted on a straight line and every point on it, represents a real number. Let c (zero or non-zero) be any real number. Let it corresponds to the point A on the line as shown below:



We can move to the point A, along the line (X′ OX), from the left side of A as well as from the right side of A.

2. To be definite, let c = 1. Consider two sequences of number:

(i) 0.9, 0.99, 0.999, 0.9999,....

(ii) 1,1, 1.01, 1.001, 1.0001,...

As one can easily see, the difference of 1 from these numbers is becoming smaller and smaller as we move nearer to 1; we say that these numbers are approaching (or tending to) 1 and we write this in symbols as $x \to 1$.

3. It may further be noted that the numerical difference between x and 1 i.e., $|x, -1|$ can be made less than any pre-assigned positive number, howsoever small, when we keep moving along these sequences of numbers.

4. Notation

If we reach to c through the right side, we shall write; $x \to c^+$ (or $x \to c + 0$) and if we reach to c through the left side, we shall write: $x \to c^-$ (or $x \to c - 0$).

LIMIT OF A FUNCTION

1. Before defining this concept, let us consider two examples. Let f be any function with domain and codomain = R.

Example 1: Let f(x) = x + 2. Let us consider: $x \to 1$, i.e., x approaches 1, i.e., x takes values nearer and nearer to 1 from either side. We prepare the following table:

x	$f(x) = x + 2$	$\lvert f(x) - 3 \rvert$
0.9	2.9	0.1

0.99	2.99	0.01
0.999	2.999	0.001
-	-	-
1.0001	3.0001	0.0001
1.001	3.001	0.001

Looking at the above table. we find that as x goes on taking values nearer and nearer to 1, the function. f(x), takes values nearer and nearer to 3. Thus, as x $\to$ 1 (from the left or from the right), f(x) $\to$ 3. This means, we can bring f(x) as near to 3 as we please by taking x sufficiently near to 1, i.e., the difference, |f(x) - 3| can be made as small as we please, by taking x sufficiently nearer to 1. In such a case, we say that 3 is the limit of f(x) as x tends to 1.

Example 2:

Let $f(x)=\dfrac{x^2-9}{x-3}(x\neq 3)$. Let x approach 3 and we prepare the following table:

x	f(x)	|f(x) - 6|
2.9	5.9	0.1
2.99	5.99	0.01
2.999	5.999	0.001
2.9999	5.9999	0.0001
-	-	-
-	-	-
3.0001	6.0001	0.0001
3.001	6.001	0.001
3.01	6.01	0.01
3.1	6.1	0.1

We observe from the table that as x aproaches 3, from the left or from the right, f(x) approaches 6, i.e., the difference, |f(x) - 6| can be made as small as we please, by taking x sufficiently close to 3.

Hence, 6 is the limit of f(x) as x tends to 3.

These illustrations lead us to define the concept of limit as follows:

2. Definition

If there exists a finite real number l such that the difference, $|f(x) - l|$ can be made as small as we please, by taking x sufficiently close to c (but not equal to c), then ***l is said to be the limit of f(x) as x tends to c.*** we shall express this in symbols as:

$$\lim_{x\to c} f(x)=l$$

or $f(x)\to l$ as $x\to c$

More precisely: Let f(x) be any function defined in a neighbourhood of c, not necessarily at c. Then f(x) is said to tend to a limit l (a finite real number) as x $\to$ c, if for every number $\in >0$, there exists a number $\delta>0$ (depending upon $\in$)such that $|f(x) - l| < \in$ whenever $0<|x - c|<\delta$.

3. ONE SIDE LIMITS

4. When $x\to c$ from the left side (i.e., through values of x less than c) and f(x) is very close to l_1: then we say that l_1 is **the Left Hand Limit (L.H.L.)** of $f(x)$ and we shall write it as:

$$\lim_{x\to c-} f(x)=l_1.$$

5. When x $\to$ c from the right side (i.e., through values of x greater than c) and f(x) is very close to l2: then we say that

l2 is the right hand limit (R.H.L.) of f(x) and we shall write it as:

$$\lim_{x\to c+} f(x)=l_2.$$

Note: If the left hand and the right hand limits of f(x) as x tends to c, both exit and are equal then we say that limit of f(x) as $x\to c$ exists and $\lim_{x\to c-} f(x)=\lim_{x\to c+} f(x)=\lim_{x\to c} f(x)$: otherwise we say that $\lim_{x\to c} f(x)$ does not exist.

SOME RESULTS ON LIMITS

The calculation of limits is based on the following results:

1. $\lim_{x\to c}(f(x)\pm g(x))=\lim_{x\to c} f(x)\pm\lim_{x\to c} g(x).$
2. $\lim_{x\to c}[kf(x)]=k\lim_{x\to c} f(x)$ where k is a scalar.
3. $\lim_{x\to c}(f(x).g(x))=\lim_{x\to c} f(x).\lim_{x\to c} g(x).$
4. $\lim_{x\to c}\dfrac{f(x)}{g(x)}=\dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c}}(provided\lim_{x\to c} g(x)\neq 0).$

SOME IMPORTANT LIMITS

1 $\lim_{x\to 0}\dfrac{(e^x-1)}{x}=1$

2 $\lim_{x\to 0}\dfrac{a^x-1}{x}=\log_e a(a>0)$

3 $\lim_{x\to 0}\dfrac{\log(1+x)}{x}=1$

4 $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x=e$

5 $\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$

6 $\lim_{x\to\infty}\left(1+\dfrac{a}{x}\right)^x=e^a$

7 $\lim_{x\to a}\dfrac{x^n-a^n}{x-a}=na^{n-1}$

8 $\lim_{x\to 0}\dfrac{(1+x)^n-1}{x}=n$

9 $e^x\to\infty, as\ x\to\infty$

10 $e^{-x}\to 0, as\ x\to\infty$

11 $\lim_{x\to 0}\dfrac{1}{x}$ does not exist.

COMPUTATION OF LIMITS

Suppose we have to find $\lim_{x\to a}\frac{f(x)}{g(x)}$.

Direct Substitution Method :-

If f. g are polynomial, Logarithmic or Exponential function such that $g(a) \neq 0$.

Then provided f(a) and g(a) are finite.

Type-I

ILLUSTRATION 1

Evaluate $\lim_{x\to 2}\frac{x^2+x+2}{x^3+1}$

Solution

$$\lim_{x\to 2}\frac{x^2+x+2}{x^3+1}=\frac{2^2+2+2}{2^3+1}=\frac{8}{9}.$$

ILLUSTRATION 2

Evaluate

(*i*) $\lim_{x\to 4}\sqrt[3]{5x+7}$

(*ii*) $\lim_{x\to 1}\frac{x^2-3x+2}{x^2+5}$.

Solution

(*i*) $\lim_{x\to 4}(5x+7)^{1/3}=(5\times 4+7)^{1/3}=(27)^{1/3}=3.$

(*ii*) $\lim_{x\to 1}\frac{x^2-3x+2}{x^2+5}=\frac{1^2-3\times 1+2}{1^2+5}=0.$

Indeterminate Form

After putting the limit value in the function; if we get the result $\frac{0}{0};\frac{\infty}{\infty};\infty-\infty;1^{\infty}$ etc. form then these forms are called indeterminate form.

Type-II L' Hospetal Rule.

Tricks:-

If the Limits are in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form then

I we use L' Hospetal Rule. **working steps** 1st we put x = given limiting value and find the form.

II If the form is $\frac{0}{0}$ then we differentiate numerator and denominator separately.

III After defferentiation ; we again put the limit value then find the result.

If the result is not equal to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ then this result is the answer.

IV If we again get the result $\frac{0}{0}$ form then we again differentiate numerator and denominator separately and then put the limit value we continue this process untill not to get $\frac{0}{0}$ form. The found result is the answer.

Ex-1 $\lim_{x\to 1}\frac{x^3-1}{x^2-1}$ is equal to

(*a*) $\frac{3}{2}$ (*b*) $-\frac{3}{2}$ (*c*) $\frac{4}{5}$ (*d*) None

Soln.:- (*a*) is correct.

Tricks:- **L' Hospetal Rule**

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1}\left[Form\ \frac{1^3-1}{1^2-1}=\frac{0}{0}\right]$$

Differentiate numerator and denominator with respect to x. ; we get

$$\lim_{x\to 1}\frac{3x^2-0}{2x-0}=\lim_{x\to 1}\frac{3x}{2}=\frac{3\times 1}{2}=\frac{3}{2}$$

Ex-2 $\lim_{x\to 2}\frac{2x^2-7x+6}{5x^2-11x+2}$ is equal to

(*a*) 1/9 (*b*) 9 (*c*) -1/9 (*d*) None

Soln.:- (*a*) is correct

$$\lim_{x\to 2}\frac{2x^2-7x+6}{5x^2-11x+2}\ (\text{Form}\ \frac{0}{0})$$

Using L' Hospetal Rule; we get

$$\lim_{x\to 2}\frac{2\times 2x-7\times 1+0}{5\times 2x-11\times 1+0}=\lim_{x\to 2}\frac{4x-7}{10x-11}$$

$$=\frac{4\times 2-7}{10\times 2-11}=\frac{1}{9}$$

Ex-3 $\lim_{x\to 1}\frac{x^3-5x^2+2x+2}{x^3+2x^2-6x+3}$ is equal to

(*a*) 5 (*b*) -5 (*c*) $\frac{1}{5}$ (*d*) None

Soln.:- (*b*) is correct.

$$\lim_{x\to 1}\frac{x^3-5x^2+2x+2}{x^3+2x^2-6x+3}\ (\text{Form}\ \frac{0}{0})$$

Using L' Hospetal Rule ; we get

$$\lim_{x\to 1}\frac{3x^2-10x+2}{3x^2+4x-6}=\frac{3\times 1^2-10\times 1+2}{3\times 1^2+4\times 1-6}=\frac{3-10+2}{3+4-6}$$

$$=\frac{-5}{1}=-5 \text{ Ans}$$

Type - III (Rationalisation Method)

1st Rationalise then put value.

Ex-1 $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ is equal to

(*a*) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 2 (*d*) None

Soln.:- (*b*) is correct

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\quad\left(form\frac{0}{0}\right)$$

$$=\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x}\right)^2-1^2}{x\left(\sqrt{1+x}+1\right)}=\lim_{x\to 0}\frac{1+x-1}{x\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{x}{x\left(\sqrt{1+x}-1\right)}$$

$$=\frac{1}{\sqrt{1+0}+1}=\frac{1}{1+1}=\frac{1}{2}\quad\text{Ans}$$

Ex-2 $\lim_{x\to 0}\frac{\sqrt{1+2x^2}-\sqrt{1-2x^2}}{x^2}$ is equal to

(*a*) 2 (*b*) -2 (*c*) 2 (*d*) None

Soln.:- (*a*) is correct.

$$\lim_{x\to 0}\frac{\sqrt{1+2x^2}-\sqrt{1-2x^2}}{x^2}\times\frac{\sqrt{1+2x^2}+\sqrt{1-2x^2}}{\sqrt{1+2x^2}+\sqrt{1-2x^2}}$$

$$=\lim_{x\to 0}\frac{(1+2x^2)-(1-2x^2)}{x^2\left(\sqrt{1+2x^2}+\sqrt{1-2x^2}\right)}$$

$$=\lim_{x\to 0}\frac{1+2x^2-1+2x^2}{x^2\left(\sqrt{1+2x^2}+\sqrt{1-2x^2}\right)}$$

$$=\lim_{x\to 0}\frac{4x^2}{x^2\left(\sqrt{1+2x^2}+\sqrt{1-2x^2}\right)}$$

$$=\lim_{x\to 0}\frac{4}{x^2\left(\sqrt{1+2x^2}+\sqrt{1-2x^2}\right)}$$

$$=\frac{4}{\sqrt{1+0}+\sqrt{1-0}}=\frac{4}{2}=2$$

Type - IV From $\frac{\infty}{\infty}$

Working Rule

(*i*) First of all simplify the given erpression.

(*ii*) Then divide each term of numerator and denominator by x^m

Where x = In dependent variable.

m = maximum power of x.

(*iii*) Then put expression of the form $\frac{a}{x^k}=o$; where a = constant ; a = a positive number

When $x\to\infty$ *or* $x\to-\infty;\frac{c}{x^k}\to o;k>o$

Ex-1 $\lim_{x\to\infty}\frac{2x^2+7x+5}{4x^2+3x+1}$ is equal to l where l is

(*a*) $-\frac{1}{2}$ (*b*) $\frac{1}{2}$ (*c*) 2 (*d*) None

Soln.:- (*b*) is correct.

$$\lim_{x\to\infty}\frac{2x^2+7x+5}{4x^2+3x+1}\quad\left[Form\frac{\infty}{\infty}\right]$$

$$= \lim_{x \to \infty} \frac{\frac{2x^2}{x^2} + \frac{7x}{x^2} + \frac{5}{x^2}}{\frac{4x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}}$$

[Dividing each term of numerator and denominator by x^2]

$$= \lim_{x \to \infty} \frac{2 + \frac{7}{x} + \frac{5}{x^2}}{4 + \frac{3}{x} + \frac{1}{x^2}} = \frac{2 + \frac{7}{\infty} + \frac{5}{\infty}}{4 + \frac{3}{\infty} + \frac{1}{\infty}}$$

$$= \frac{2+0+0}{4+0+0} = \frac{2}{4} = \frac{1}{2} \quad Ans.$$

Ex-2 $\lim_{n \to \infty} \left(\frac{1^2}{n^3} + \frac{2^2}{n^3} + \frac{3^2}{n^3} + \ldots\ldots + \frac{n^2}{n^3} \right)$ is

(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{4}$ (*c*) $-\frac{1}{2}$ (*d*) None

Soln.:- (*a*) is correct.

$$\lim_{n \to \infty} \left[\frac{1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2}{n^3} \right]$$

$$= \lim_{n \to \infty} \frac{\sum n^2}{n^3} = \lim_{n \to \infty} \frac{n(n+1)(2n+1)}{6n^3}$$

$$= \lim_{n \to \infty} \frac{2n^2 + 3n + 1}{6n^2} \quad \left[Form \frac{\infty}{\infty} \right]$$

$$= \lim_{n \to \infty} \frac{2 + \frac{3}{n} + \frac{1}{n^2}}{6} = \frac{2 + \frac{3}{\infty} + \frac{1}{\infty}}{6}$$

[Dividing each term of numerator and denominator by n^2]

$$= \frac{2+0+0}{6} = \frac{1}{3} \quad Ans.$$

Ex-3 Find $\lim_{n \to \infty} (2^n - 2)(2^n + 1)^{-1}$

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) None

Soln.:- (*c*) is correct.

$$\lim_{n \to \infty} \frac{2^n - 2}{2^n + 1} \quad \left[Form \frac{\infty}{\infty} \right] = \lim_{n \to} \frac{2^n \left[1 - \frac{2}{2^n} \right]}{2^n \left[1 + \frac{1}{2^n} \right]}$$

$$= \lim_{n \to \infty} \frac{1 - \frac{2}{2^n}}{1 + \frac{1}{2^n}} = \frac{1-0}{1+0} = 1.$$

Type - V Form $[\infty - \infty]$

(*i*) First of all simplify the given expression.

(*ii*) Then rationalise the expression.

(*iii*) Then take $x^{\text{max. power}}$ common and simplify it.

(*iv*) Then put the lim. value as $x \to \infty$ *or* $x \to -\infty$ and find the answer.

Ex-1 $\lim_{x \to \infty} \left(\sqrt{x+1} - \sqrt{x} \right)$ is

(*a*) 0 (*b*) 1 (*c*) -1 (*d*) None

Soln. (*a*) is correct.

$$\lim_{x \to \infty} \left(\sqrt{x+1} - \sqrt{x} \right)$$

$$= \lim_{x \to \infty} \frac{\sqrt{x+1} - \sqrt{x}}{1} \times \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x+1} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{x+1-x}{\sqrt{x+1} + \sqrt{x}} = \lim_{x \to \infty} \frac{1}{\sqrt{x+1} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{\infty+1} + \sqrt{\infty}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0$$

Ex-2 $\lim_{x\to\infty}\left(\sqrt{x^2+7x}-x\right)$ is

(*a*) 5/3 (*b*) 7/2 (*c*) -7/2 (*d*) None

Soln.:- (*b*) is correct $\lim_{x\to\infty}\frac{\left(\sqrt{x^2+7x}-x\right)}{1}\times\frac{\sqrt{x^2+7x}+x}{\sqrt{x^2+7x}+x}$

$$=\lim_{x\to\infty}\frac{x^2+7x-x^2}{\sqrt{x^2+7x}+x}=\lim_{x\to\infty}\frac{7x}{\sqrt{x^2\left(1+\frac{7}{x}\right)}+x}$$

$$=\lim_{x\to\infty}\frac{7x}{x\sqrt{\left(1+\frac{7}{x}\right)}+x}=\lim_{x\to\infty}\frac{7x}{x\left[\sqrt{1+\frac{7}{x}}+1\right]}$$

$$=\frac{7}{\sqrt{1+0}+1}=\frac{7}{2}$$

Type - VI Form $[1^\infty]$

Formula

(*i*) $\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$ (*ii*) $\lim_{x\to 0}(1+mn)^{\frac{1}{mx}}=e$

(*iii*) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=e$ (*iv*) $\lim_{x\to\infty}\left(1+\frac{m}{x}\right)^{\frac{x}{m}}=e$

Ex-1 $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{7x}$ *is*

(*a*) e^7 (*b*) 1 (*c*) 0 (*d*) None

Soln.:- (*a*) is correct.

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^7=(e)^7=e^7$$

Ex-2 Find $\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^{x}$.

(*a*) 0 (*b*) 1 (*c*) - 1 (*d*) None

Soln.:- (*b*) is correct.

$$\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^{x}=\lim_{x\to\infty}\left[\left(1+\frac{1}{x^2}\right)^{x^2}\right]^{\frac{1}{x}}$$

$$=e^{\frac{1}{\infty}}=e^0=1$$

Ex-3 The value of the limit when x tends to zero of the expression $(1+n)^{\frac{1}{n}}$ is

(*a*) e (*b*) 0 (*c*) 1 (*d*) -1

Soln.:- (*a*) is correct.

$$\lim_{n-\infty}(1+n)^{\frac{1}{n}}=e.$$

Type - VII Form $\left[\frac{0}{0}\right]$

Formula $\lim_{x\to 0}\frac{e^{3x}-1}{x}=1$

Ex-1 $\lim_{x\to 0}\frac{e^{3x}-1}{x}$ is

(*a*) 0 (*b*) 3 (*c*) 1 (*d*) None

Soln.:- (*b*) is correct.

Tricks

Use L' Hospetal Rule.

$$\lim_{x\to 0}\frac{e^{3x}-1}{x}\left[\text{Form }\frac{0}{0}\right]$$

Differentiating numerator and denominater with respect to x ; we get

$$\lim_{x\to 0}\frac{3e^{3x}-0}{1}=3\times e^{0}=3\times 1=3$$

Ex-2 $\lim\limits_{x\to\infty}\dfrac{e^{x}+1}{e^{x}+2}$ is evaluated to be

(*a*) 0 (*b*) -1 (*c*) 1 (*d*) None

Soln.:- (*c*) is correct.

$$\lim_{x\to\infty}\frac{e^{x}+1}{e^{x}+2}\quad\left[\text{Form}\frac{\infty}{\infty}\right]$$

Using L' Hospetal rule ; we get

$$\lim_{x\to\infty}\frac{e^{x}+0}{e^{x}+0}$$

[Differentiating numerator and denominator with respect to x]

$$=\lim_{x\to\infty}1=1.$$

Type - VIII

Formula.

(*i*) $\lim\limits_{x\to 0}\dfrac{\log_e(1+x)}{x}=1$ (*ii*) $\lim\limits_{x\to 0}\dfrac{x}{\log_e(1+x)}=1$

Ex-1 $\lim\limits_{x\to 0}\dfrac{\log(1+3x)}{x}$ *is*

(*a*) 1 (*b*) 3 (*c*) -3 (*d*) None

Soln.:- (*b*) is correct

$$\lim_{x\to 0}\frac{\log_e(1+3x)}{x}\lim_{x\to 0}\frac{\log(1+3x)}{3x}\times 3=1\times 3=3$$

Ex-2 $\lim\limits_{x\to 0}\dfrac{\log(1-x)}{x}$ is eual to

(*a*) 1 (*b*) -1

(*c*) cannot be determined (*d*) none

Soln.:- (*b*) is correct

$$\lim_{x\to 0}\frac{\log(1-x)}{x}=\lim_{x\to 0}\frac{\log[1+(-x)]}{-x}\times(-1)$$

$$=1\times(-1)=-1$$

Ex-3 $\lim\limits_{x\to 0}\dfrac{\log(1+px)}{e^{3x}-1}$ is eual to

(*a*) $\dfrac{p}{3}$ (*b*) p (*c*) $\dfrac{1}{3}$ (*d*) None

Soln.:- (*a*) is correct

$$\lim_{x\to 0}\frac{\left(\dfrac{\log(1+px)}{px}\right)\times px}{\left(\dfrac{e^{3x}-1}{3x}\right)\times 3x}=\frac{\lim\limits_{x\to 0}\dfrac{\log(1+px)}{px}}{\lim\limits_{x\to 0}\dfrac{e^{3x}-1}{3x}}$$

$$\times\frac{p}{3}=\frac{1\times p}{1\times 3}=\frac{p}{3}$$

Ex-4 The value of $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{\log(1+x)}$ is

(*a*) 0.5 (*b*) 2 (c) -0.5 (*d*) None

Soln.:- (*a*) is correct.

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\log(1+x)}=\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\log(1+x)}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$= \lim_{x \to 0} \frac{1+x-1}{\log(1+x).(\sqrt{1+x}+1)}$$

$$= \lim_{x \to 0} \frac{x}{\log(1+x)} \times \frac{1}{\sqrt{1+x}+1}$$

$$= 1 \times \frac{1}{\sqrt{1+0}+1} = 1 \times \frac{1}{1+1} = \frac{1}{2} = 0.5$$

Type - IX

Formula

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$$

Ex-1 $\lim_{x \to 0} \frac{3^x - 1}{x}$ is equal

(*a*) $10^3 . \log_{10} 3$ (*b*) $\log_{10} e$ (*c*) $\log_e 3$ (*d*) None

Soln. (*c*) is correct.

Ex-2 $\lim_{x \to 0} \frac{5^x + 3^x - 2}{x}$ will be equal to

(*a*) $\log_e 15$ (*b*) $-\log_e 15$ (*c*) $\log e$ (*d*) None

Soln.:-

$$\lim_{x \to 0} \frac{5^x - 1 + 3^x - 1}{x} = \lim_{x \to 0} \left[\frac{5^x - 1}{x} + \frac{3^x - 1}{x} \right]$$

$$= \log_e 5 + \log_e 3 = \log_e 15$$

Ex-3 $\lim_{x \to 0} \frac{10^x - 5^x - 2^x + 1}{x^2}$

(*a*) $\log_e 2 + \log_e 5$ (*b*) $\log_e 2 . \log_e 5$

(*c*) $\log_e 10$ (*d*) None

Soln.:- (*b*) is correct

$$\lim_{x \to 0} \frac{10^x - 5^x - 2^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{(2 \times 5)^x - 5^x - 2^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{2^x \times 5^x - 5^x - 2^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x (2^x - 1) - 1(2^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{(2^x - 1)(5^x - 1)}{x \times x}$$

$$= \log_e 2 \times \log_e 5$$

Type - X

(Differentiation by 1st principle method)

Formula

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x) = \frac{df(x)}{dx}$$

Ex-1 If $f(x) = ax^2 + bx + c$ then

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ is eual to

(a) ax + b (*b*) ax + 2b (*c*) 2ax + b (*d*) None

Soln.:- (*c*) is correct

$$\because \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$= \frac{d}{dx}(ax^2 + bx + c) = 2ax + b \times 1 + 0 = 2ax + b.$$

Ex-2 If $f(x) = 5x^3 + 7x + 2$ then

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ is equal to

(*a*) $15x^2 + 7$ (*b*) $5x^2 + 7$

(*c*) $15x^2 + 7x + 2$ (*d*) None

Soln.:- (*a*) is correct.

$$\because \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

$$= \frac{d}{dx}(5x^3 + 7x + 2) = 5 \times 3x^2 + 7 \times 1 + 0 = 15x^2 + 7.$$

Type - XI

(For Two or more than two functions)

Formula.

$\lim_{x \to a} f(x)$ exists if $\lim_{x \to a+} f(x) = \lim_{x \to a} f(x)$

or RHL = LHL

i.e. Right hand limit = Left hand limit

Ex-1 Evaluate limits of the function defined by

$$f(x) = \begin{cases} 1+x^2 \ ; \ if \ 0 \le x \le 1 \\ 2-x \ ; \ if \ x > 1 \end{cases}$$

at x = 1. Then $\lim_{x \to 1} f(x)$ is

(*a*) 2 (*b*) 1 (*c*) does not exist (*d*) None

Soln.:- (*c*) is correct.

$$\text{LHL} = \lim_{x \to 1 \ n<1} \frac{f(x)}{} = \lim_{x \to 1} (1+x^2) = 1+1^2 = 2$$

$$\text{RHL} = \lim_{x \to 1 \ x>1} \frac{f(x)}{} + = \lim_{x \to 1} (2-x) = 2-1 = 1$$

Clearly $\lim_{x \to 1} - f(x) \neq \lim_{x \to 1+} f(x)$

$\therefore \lim_{x \to 1} f(x)$ does not exist.

Ex-2 Find $\lim_{x \to 0} \frac{3x+|x|}{7x-5|x|}$

(*a*) 2 (*b*) $\frac{1}{6}$

(*c*) Limit does not exist. (*d*) None

Soln.:- (*c*) is correct

$$\therefore f(x) = \frac{3x+x}{7x-5x} = 2 \ if \ x > 0$$

$$= \frac{3x-x}{7x+5x} = \frac{1}{6} \ if \ x < 0$$

$$\therefore RHL = \lim_{x \to 0} + f(x) = \lim_{x \to 0} (2) = 2$$

$$LHL = \lim_{x \to 0} - f(x) = \lim_{x \to 0} \frac{1}{6} = \frac{1}{6}$$

Clearly $\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$

$\therefore \lim_{x \to 0} f(x)$ **does not exist**

Ex-3 If $f(x) = \begin{cases} 2x-1; \ where \ x < 2 \\ \frac{3}{2}x; \ where \ x \ge 2 \end{cases}$ then $\lim_{x \to 2} f(x) = ?$

(*a*) 3 (*b*) $\frac{3}{2}$

(*c*) 2 (*d*) Does not exist

Soln.:- (*a*) is correct.

$$LHL = \lim_{n \to 2^-} f(x) = \lim_{x \to 2} (2x-1) = 2\times 2 - 1 = 3$$

$$RHL = \lim_{n \to 2^+} f(x) = \lim_{x \to 2} \left(\frac{3}{2}x\right) = \frac{3}{2}\times 2 = 3$$

$$\therefore \lim_{x \to 2^-} (x) = \lim_{x \to 2^+} f(x) = 3$$

$$\therefore \lim_{x \to 2} f(x) \text{ exists } = 3$$

CONTINUITY

Continuity : - Something which goes on without interruption and without abrupt changes is called **Continuous**.

A function f(x) is said to be continuous at x = a

if and only if

(*i*) f(x) is defined at x = a

(*ii*) $\lim_{x \to a-} f(x) = \lim_{x \to a+} f(x)$

(*iii*) $\lim_{x \to a} f(x) = f(a)$

If *f(x)* is not continuous at x = a. we say that it is discontinuous at x = a.

Notes:

(*i*) The sum, difference and product of two continuous functions is a continuous function. This property holds good for any finite number of functions.

(*ii*) The quotient of two continuous functions is a continuous function provided the denominator is not equal to zero.

(*iii*) Constant functions are continuous every where.

(*iv*) Polynomial functions are continuous every where.

(*v*) A rational function is continuous at all points except at those points where the denominator is zero.

(*vi*) Exponential and logarithmic functions are continuous at every point of their respective domains.

Ex-1 A function g (x) is defined as follows.

g(x) = x when 0 < x < 1

= 2 - x when x ≥ 1.

Is g(x) continuous at x = 1. ?

(*a*) continuous (*b*) Discontinuous

(*c*) undefined at x = 1 (*d*) None

Soln.:- (*a*) is correct

$\because g(x) = x$; when $0 < x < 1$

$= 2 - x$; when $x \geq 1$

$\therefore g(x=1) = 2-1 = 1.$

$$\lim_{x \to 1^+} g(x) = \lim_{x \to 1} (2-x) = 2-1 = 1$$

$$\lim_{x \to 1^-} g(x) = \lim_{x \to 1} (x) = 1.$$

$$\therefore g(x=1) = \lim_{x \to 1^+} g(x) = \lim_{x \to 1^-} g(x) = 1$$

$\therefore g(x)$ is continuous at x = 1

Ex-2 *If* $f(x) = \frac{1}{2} - x$; when $0 < x < \frac{1}{2}$

$= \frac{3}{2} - x$; when $\frac{1}{2} < x < 1$

$= \frac{1}{2}$; when $x = \frac{1}{2}$

Discuss the continuity of *f(x)* at $x = \frac{1}{2}$

(*a*) continuous at $x = \frac{1}{2}$

(*b*) discontinuous and defined at x = 1/2

(*c*) discontinuous and undefined at $x = \frac{1}{2}$

(*d*) None of these

Soln.:- (*b*) is correct

$$f\left(x=\frac{1}{2}\right)=\frac{1}{2}(defined)$$

$$\lim_{x\to\frac{1}{2}^+} f(x)=\lim_{x\to\frac{1}{2}}\left(\frac{3}{2}-x\right)=\frac{3}{2}-\frac{1}{2}=1$$

$$\lim_{x\to\frac{1}{2}^-} f(x)=\lim_{x\to\frac{1}{2}}\left(\frac{1}{2}-x\right)=\frac{1}{2}-\frac{1}{2}=0.$$

$$\therefore\ f\left(x=\frac{1}{2}\right)\neq\lim_{x\to\frac{1}{2}^+} f(x)\neq\lim_{x\to\frac{1}{2}^-} f(x)$$

Hence f(x) is discontinuous at $x=\frac{1}{2}$ but defined at $x=\frac{1}{2}$.

Ex-3 Find the points of discontinuity of the function $f(x)=\frac{x^2+2x+5}{x^2-3x+2}$

(*a*) 1; (*b*) 2;

(c) (*a*) and (*b*) both (d) None of these

Soln.:- (*c*) is correct.

For discontinuity of *f(x)* ; denominator should be equal to zero.

$$\therefore\ x^2-3x+2=0$$

$$or\ \ x^2-2x-x+2=0$$

$$or\ \ x(x-2)-1(x-2)=0$$

$$or\ \ (x-2)(x-1)=0$$

$$\therefore\ x=2\ ;\ 1.$$

$\therefore$ f(x) is discontinuous at x = 1 ; 2

Ex-4 The function $f(x)=\frac{x^2-9}{x-3}$ is undefined at x = 3, what value must be assigned to f(3) if f(x) is to be continuous at x = 3 ?

(*a*) 6 (*b*) -6

(*c*) cannot be decided (*d*) none

Soln.:- (*a*) is correct.

$$\because f(x)=\frac{x^2-9}{x-3}$$

$\because$ *f(x) is undefined at x =* .

$$\therefore x-3\neq 0.$$

$$\therefore\ \lim_{x\to 3} f(x)=\lim_{x\to 3}\frac{(x+3)(x-3)}{(x-3)}$$

$$\lim_{x\to 3}(x+3)=3+3=6$$

$\because$ *f(x) is continuous at* $x=3$

$$\therefore f(3)=\lim_{x\to 3} f(x)=6$$

MODEL EXAM QUESTIONS (FOR PRACTICE)

Choose the most appropriate option (a) (b) (c) or (d)

1. $\lim_{x\to 0} f(x)$ when $f(x)=6$ is

(*a*) 6 (*b*) 0 (*c*) 1/6 (*d*) none of these

2. $\lim_{x\to 2}(3x+2)$ is equal to

(*a*) 6 (*b*) 4 (*c*) 8 (*d*) none of these

3. $\lim_{x\to -2}\frac{x^2-4}{x+2}$

(*a*) 4 (*b*) -4 (*c*) does not exist (*d*) none of these

4. $\lim_{x\to\infty}\left(\frac{3}{x^2}+2\right)$

(*a*) 0 (*b*) 5 (*c*) 2 (*d*) none of these

5. $\lim_{x\to 1}$ log e^x is evaluated to be

(*a*) 0 (*b*) *e* (*c*) 1 (*d*) none of these

6. The value of the limit of f (x) as x $x\to 3$ when $f(x)=e^{x^2+2x+1}$ is

(*a*) e^{15} (*b*) e^{16} (*c*) e^{10} (*d*) none of these

7. $\lim_{x\to 1/2}\left(\frac{8x^3-1}{6x^2-5x+1}\right)$ is equal to

(*a*) 5 (*b*) -6 (*c*) 6 (*d*) none of these

8. $\lim_{x\to 1}\frac{x^3-5x^2+2x+2}{x^3+2x^2-6x+3}$ is equa to

(*a*) 5 (*b*) -5 (*c*) 1/5 (*d*) none of these

9. $\lim_{x\to t}\frac{x^3-t^3}{x^2-t^2}$ evaluated to be

(*a*) -1 (*b*) 1 (*c*) 0 (*d*) none of these

10. $\lim_{x\to 0}\frac{4x^4+5x^3 7x^2+6x}{5x^5+7x^2+x}$ is equal to

(*a*) 6 (*b*) 5 (*c*) -6 (*d*) none of these

11. $\lim_{x\to 2}\frac{(x^2-5x+6)(x^2-3x+2)}{x^3-3x^2+4}$ is equal to

(*a*) 1/3 (*b*) 3 (*c*) -1/3 (*d*) none of these

12. $\lim_{x\to\infty}\frac{\sqrt{3x^4+5x^2+7x+5}}{4x^2}$ is evaluated

(*a*) $\frac{\sqrt{3}}{4}$ (*b*) $\sqrt{3}$ (*c*) -1/4 (*d*) none of these

13. $\lim_{x\to 1}\frac{(1-x^{-1/3})}{(1-x^{-2/3})}$ is equal to

(*a*) - 1/2 (*b*) 1/2 (*c*) 2 (*d*) none of these

14. $\lim_{x\to 4}\frac{(x^2-16)}{(x-4)}$ is evaluated as

(*a*) -3 (*b*) 1/3 (*c*) 3 (*d*) none of these

15. If $f(x)=\frac{(1+x)^6-1}{(1+x)^2-1}$ then $\lim_{x\to 0} f(x)$ exists and is equal to

(*a*) 3 (*b*) -1/3 (*c*) -3 (*d*) none of these

16. $\lim_{x\to 2}\frac{4-x^2}{3-\sqrt{x^2+5}}$ is equal to

(*a*) 6 (*b*) 1/6 (*c*) -6 (*d*) none of these

17. $\lim_{x\to\sqrt{2}}\frac{x^{3/2}-2^{3/4}}{\sqrt{x}-2^{1/4}}$ exists and is equal to a finite value where is

(*a*) -5 (*b*) 1/6 (c) 6 (*d*) none of these

18. $\lim_{x\to 0}\left(\frac{x^2-\sqrt{x}}{\sqrt{x}-1}\right)$ is equal to

(*a*) 3 (*b*) -3 (*c*) 1/3 (*d*) none of these

19. $\lim_{x\to 0}\frac{e^{x^2}-1}{x^2}$ is evaluated to be

(*a*) 1 (*b*) 1/2 (*c*) -1 (*d*) none of these

20. If $\lim_{x\to 2}\frac{e^n-2^n}{x-2}=80$ and n E N then

(*a*) n = 5 (*b*) n = 4 (*c*) n = 0 (*d*) none of these

21. $\lim_{x\to\sqrt{2}}\frac{x^{5/2}-2^{5/4}}{\sqrt{x}-2^{1/4}}$ is equal to

(*a*) 1/10 (*b*) 10 (*c*) 20 (*d*) none of these

22. $\lim_{x\to 1}\left(\frac{1}{x^2+x-2}-\frac{x}{x^3-1}\right)$ is evaluated to be

(*a*) 1/9 (*b*) 9 (*c*) -1/9 (*d*) none of these

23. $\lim_{x\to 0}\frac{\sqrt{1+2x^2}-\sqrt{1-2x^2}}{x^2}$ is equal to

24. $\lim_{x\to p}\frac{\sqrt{x-q}-\sqrt{p-q}}{x^2-p^2}$ $(p>q)$ is evaluated as

(*a*) $\frac{1}{p\sqrt{p-q}}$ (*b*) $\frac{1}{p\sqrt{p-q}}$ (*c*) $\frac{1}{2p\sqrt{p-q}}$ (*d*) none of these

25. $\lim_{x\to 1}\frac{x^2-1}{\sqrt{3x+1}-\sqrt{5x-1}}$ is evaluated to be

(*a*) 4 (*b*) 1/4 (*c*) -4 (*d*) none of these

26. $\lim(\sqrt{x+h}-\sqrt{x})/h$ where $h\to 0$ is equal to

(*a*) 1/2 *x* (*b*) 1/2*x* (*c*) *x* / 2 (*d*) none of these

27. $\lim_{x\to 0}\frac{(e^{2x}-1)}{x}$ is equal to

(*a*) 1/2 (*b*) 2 (*c*) 0 (*d*) none of these

28. $\lim_{x\to 0}\frac{e^{x^2}-1}{x^2}$ is evaluated to be

(*a*) 1 (*b*) 1/2 (*c*) -1 (*d*) none of these

29. $\lim_{x\to\infty}\frac{e^x+1}{e^x+2}$ is evaluated to be

(*a*) 1/2 (*b*) 2 (*c*) 0 (*d*) none of these

30. $\lim_{x\to 0}\frac{(e^x+e^{-x}-2)(x^2-3x+2)}{(x-1)}$ is equal to

(*a*) 1 (*b*) 0 (*c*) -1 (*d*) none of these

31. $\lim_{x\to 0}\frac{(3^x-1)}{x}$ is equal to

(*a*) $10^3 \log_{10} 3$ (*b*) $\log_3 e$ (*c*) $\log_e 3$ (*d*) none of these

32. $\lim_{x\to 0}\frac{5^x+3^x-2}{x}$ will be equal to

(*a*) $\log_e 15$ (*b*) $\log(1/15)$ (*c*) $\log e$ (*d*) none of these

33. $\lim_{x\to 0}\frac{10^x-5^x-2^x+1}{x^2}$ equal to

(*a*) $\log_e 2 + \log_e 5$ (*b*) $\log_e 2 \log_e 5$

(*c*) $\log_e 10$ (*d*) none of these

34. If $f(x)=\frac{e^{\log x}-1}{e^{x-1}-1}$ then $\lim_{x\to 0} f(x)$ exists and is equal to

(*a*) -1 (*b*) 1 (*c*) 0 (*d*) none of these

35. $\lim_{x\to 0}\log\frac{(1+px)}{e^{3x}-1}$ is equal to

(*a*) p/3 (*b*) P (*c*) 1/3 (*d*) none of these

36. The value of $\lim_{x\to 0}\frac{\sqrt{1}+x-1}{\log(1+x)}$

(*a*) 0.5 (*b*) 2 (*c*) -0.5 (*d*) none of these

37. If $f(x) = ax^2 + bx + c$ then $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is equal to

(*a*) $ax + b$ (*b*) $ax + 2b$ (*c*) $ax + b$ (*d*) none of these

38. $\lim_{x\to\infty}\frac{(2x^2+7x+5)}{4x^2+3x+1}$ is equal to 1 where 1 is

(*a*) -1/3 (*b*) 1/2 (*c*) 2 (*d*) none of these

39. $\lim_{x\to\infty}\frac{(x\sqrt{x}-m\sqrt{m})}{1-x^{-2/3}}$ is equal to

(*a*) 1 (*b*) -1 (*c*) 1/2 (*d*) none of these

40. $\lim_{x\to 0}$ 1/x log (1- x /2) is equal to

(*a*) -1 / 2 (*b*) 1 / 2 (*c*) 2 (*d*) none of these

41. A function f(x) is defined as follows: f(x)= x when $x \le 1$ = 1 + x when $x > 1$ = 3/2 when x = 1 Then f(x) is

(*a*) continuous at x = 1/2 (*b*) discontinuous at x = 1

(*c*) undefined at x = 1,2 (*d*) none of these

42. A function f(x) is defined by f(x) = (x - 2) + 1 over all real values of x. Now f(x) is

(*a*) continuous at x = 2 (*b*) discontinuous at x = 2

(*c*) undefined at x = 2 (*d*) none of these

43. Let f(x) = x/ | x |. Now f(x) is

(*a*) continuous at x = 0 (*b*) discontinuous at x = 0

(*c*) defined at x = 0 (*d*) none of these

44. *f*(x) = x - 1 when x > a = - 1/2 when x = 0 = x + 1 when x < a f(x) is

(*a*) continuous at x = 0 (*b*) undefined at x

(*c*) discontinuous at x = 0 (*d*) none of these

45. $f(x) = (x^2 - 1) / (x^3 - 1)$ is undefined at x = 1 the value of f(x) at x = 1, such that it is continuous at x = 1 is

(*a*) 3/2 (*b*) 2/3 (*c*) -3/2 (*d*) none of these

46. f(x) = 2x - | x | is

(*a*) undefined at x = 0 (*b*) discontinuous at x = 0

(*c*) continuous at x = 0 (*d*) none of these

47. $f(x) = \frac{x^2 - 3x + 2}{x - 1}$ $x \neq 1$ becomes continuous at x = 1. Then the value of f(1) is

(*a*) 1 (*b*) -1 (*c*) 0 (*d*) none of these

48. $f(x) = \frac{(x^2 - 3x - 2}{(x + 1)}$ $x \neq -1$ and the function f(x) is continuous at x = - 1. The value of k will be

(*a*) -1 (*b*) 1 (*c*) -4 (*d*) none of these

49. If f(x) = 3 x < 2 = kx² when x ≥ 2 is continuous at x = 2 then the value of k is

(*a*) 3/4 (*b*) 4/3 (*c*) 1/3 (*d*) none of these

ANSWERS

1.	(*a*)	**2.**	(*c*)	**3.**	(*b*)	**4.**	(*c*)	**5.**	(*a*)	**6.**	(*b*)	**7.**	(*c*)
8.	(*b*)	**9.**	(*c*)	**10.**	(*a*)	**11.**	(*c*)	**12.**	(*a*)	**13.**	(*b*)	**14.**	(*a*)
15.	(*c*)	**16.**	(*a*)	**17.**	(*c*)	**18.**	(*d*)	**19.**	(*d*)	**20.**	(*a*)	**21.**	(*b*)
22.	(*c*)	**23.**	(*a*)	**24.**	(*b*)	**25.**	(*c*)	**26.**	(*d*)	**27.**	(*b*)	**28.**	(*d*)
29.	(*c*)	**30.**	(*b*)	**31.**	(*c*)	**32.**	(*a*)	**33.**	(*b*)	**34.**	(*b*)	**35.**	(*a*)
36.	(*a*)	**37.**	(*c*)	**38.**	(*b*)	**39.**	(*d*)	**40.**	(*a*)	**41.**	(*a*)	**42.**	(*a*)
43.	(*b*)	**44.**	(*c*)	**45.**	(*b*)	**46.**	(*c*)	**47.**	(*b*)	**48.**	(*c*)	**49.**	(*a*)

14 CHAPTER DIFFERENTIAL CALCULUS

DIFFERENTIATON

Concept of Differentiation

We are familiar with the expressions like kilometres/hour, price/metre, etc. All such expressions (or functions) represent rates. The rate of change of function of one variable with respect to another on which it depends is called the **derivative of the function.** The process of finding the derivative in terms of a limit involving the increments of the independent and the dependent variables is called **differentiation.**

Let $y = f(x)$ be a function of x. If values of a variable x is changed from one value x_0 to another value x_1, then the difference $x_1 - x_0$ is called the increment of x (which may be positive or negative) and we denote it by Δx (or h). In the similar manner, the increment *of y or f(x)* is denoted by Δy *or* $\Delta f(x)$, *i.e.*, $f(x + \Delta x) - f(x)$.

∴ The rate of change is given by

$$\frac{\Delta y}{\Delta x} = \frac{\text{increment in the value of } y \text{(dependent variable)}}{\text{increment in the value of } x \text{(independe nt variabl e)}}$$

$$= \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

If this ratio tends to a definite finite limit as Δx tends to zero from either side, then this limit is called the differential coefficient (or derivative) of *f(x)* with respect to *x*. Symbolically, the differential coefficient of y with respect to *x* is denoted by

$$\frac{dy}{dx} \text{ or } f'(x) y_1 \text{ or } \frac{d}{dx}[f(x)] \text{ or } Df(x).$$

Thus, $\frac{dy}{dx}$ or $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

The process of finding the differential coefficient is known as **differentiation.**

Remarks 1. For the differential coefficient f'(x) to exist, the right-hand and left-hand different coefficient must both exists and equal, i.e.,

$$\lim_{\Delta x \to 0^+} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2. The differential coefficient of f(x) at any fixed value 'c' of x is given by

$f'(c) = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$, provided this limit exists.

Illustration. Consider the function, $y = f(x) = x^2$.

By definition,

$$\frac{dy}{dx} = \frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x + 0 = 2x.$$

FORMULAE OF DIFFERENTIATION

Let f(x) and g(x) be differentiable function and $\alpha \in R$

1. Sum and Difference Rule

$$\frac{d}{dx}(f(x) \pm g(g)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

2. Scalar Multiple Rule

$$\frac{d}{dx}(\alpha f(x)) = \alpha \frac{d}{dx} f(x)$$

3. $\frac{dc}{dx} = 0$

Where c = Constant

4. $\frac{dx^n}{dx} = nx^{n-1}$

5. $\frac{da^x}{dx} = a^x.\log e^a$

$= a^x.\log a$

Where a = Constant

6. $\frac{de^x}{dx} = e^x$

7. $\frac{d}{dx} \log x = \frac{d}{dx} \log e^x = \frac{1}{x}$

8. $\frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$

9. $\frac{d}{dx} \log a^x = \frac{1}{x \log e^a} = \frac{1}{x \log a}$

10. $\frac{dx^x}{dx} = x^x (1 + \log x)$

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. If $f(x) = {}^xC_3$; then $f'(1) = ?$

(a) $\frac{1}{6}$ (b) $-\frac{1}{6}$

(c) $\frac{5}{6}$ (d) $-\frac{5}{6}$

[Dec. 2010]

Solution : (b) is correct.

Given $f(x) = x_{c_3} = \frac{x!}{3!(x-3)!}$

$= \frac{x(x-1)(x-2).(x-3)!}{6.(x-3)!}$

$\therefore f(x) = \frac{(x^3 - 3x^2 + 2x)}{6}$

Differentiating w.r.t(x) both the sides, we get

$f^1(x) = \frac{1}{6}(3x^2 - 6x + 2)$

$\therefore f^1(1) = \frac{1}{6}(3 \times 1^2 - 6 \times 1 + 2)$

$\therefore f^1(1) = \frac{-1}{6}$

Q.2. $\frac{d}{dx}\left[2^{\log_2 x}\right] =$ _____

(a) 1 (b) 0

(c) 1/2 (d) $2^x.\log_2 x$

[Dec. 2011]

Solution : (a) is correct.

$\frac{d}{dx} 2^{\log_2 x} = \frac{d(x)}{dx} = 1$

Formula ($\because a^{\log_a x} = x$)

Q.3. If $Y = X^x$ then $\frac{d^2y}{dx^2} =$ _____

(a) $z\frac{dY}{dx}(1 + \log x) + Y\frac{d}{dx}(1 + \log x)$

(b) $\frac{dY}{dx}(1 + \log x) + \frac{d}{dx}(1 + \log x)$

(c) $\frac{dY}{dx}(1 + \log x) - Y\frac{d}{dx}(1 + \log x)$

(d) $\frac{dY}{dx}(1 + \log x) - \frac{d}{dx}(1 + \log x)$

[Dec. 2011]

Solution : (*a*) is correct.

If $y = x^x$

taking log on both side

$\log y = \log x^x$

$\log y = x \log x$

Diff w.r.t (x)

$$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{x} + \log x.1$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \log x$$

$$\frac{dy}{dx} = y(1 + \log x)$$

Again differentiating w.r.t 'x'

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\{y(1 + \log x)\}$$

$$\frac{d^2y}{dx^2} = y.\frac{d}{dx}(1 + \log x) + (1 + \log x)\frac{dy}{dx}$$

$$= \frac{dy}{dx}(1 + \log x) + y.\frac{d}{dx}(1 + \log x)$$

Q.4. If x = c t, y = c/t, then $\frac{dy}{dx}$ is equal to :

(*a*) 1/t (*b*) $t.e^t$

(*c*) $-1/t^2$ (*d*) None of these

[June 2012]

Solution : (*c*) is correct.

$$\frac{dx}{dt} = C \times 1 = C$$

$$\frac{dy}{dt} = C \times \frac{d}{dt}\left(\frac{1}{t}\right)$$

$$= C\left(-\frac{1}{t^2}\right) = -\frac{c}{t^2}$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-c/t^2}{C}$$

$$= -\frac{1}{t^2} = -t^{-2}$$

Q.5. If $y = e^{a \log x} + e^{x \log a}$, then $\frac{dy}{dx}$ =

(*a*) $X^a + a^x$

(*b*) a. $X^{a-1} + a^x \log a$

(*c*) $a X^{a-1} + X a^{x-1}$

(*d*) $X^x + a^a$

[June 2012]

Solution : (*b*) is correct.

$y = e^{a \log x} + e^{x \log a}$

$\Rightarrow y = x^a + a^x \quad [\because e^{\log m} = m]$

Diff. w.r.t. x on both side ; we get

$$\frac{dy}{dx} = ax^{a-1} + a^x \log a$$

Q.6. For the functions $y = x^3 - 3x$, the value of $\frac{d^2y}{dx^2}$ at which $\frac{dy}{dx}$ is zero, is

(*a*) ±1 (*b*) ±3

(*c*) ±6 (*d*) None of these

[Dec. 2012]

Solution : (*c*) is correct.

Given $y = x^3 - 3x$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = 3x^2 - 3 \quad \text{_____(1)}$$

$0 = 3(x^2 - 1)$

$x^2 - 1 = 0$

$x^2 = 1$; so $x = \pm 1$

Diff. (1) w.r.t 'x'

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2 - 3) = 6x$$

$$\left(\frac{d^2y}{dx^2}\right)_{(x=\pm 1)} = 6(\pm 1) = \pm 6$$

Q.7. The equation of the tangent to the curve, $x^3 - 2x + 3$, at the point (2,7) is

(*a*) y = 2x-13 (*b*) y = 10x

(*c*) y = 10x -13 (*d*) y = 10

[Dec. 2012]

Solution : (*c*) is correct. Given that

$f(x) = x^3 - 2x + 3$

$i.e. y = x^3 - 2x + 3$

$$\frac{dy}{dx} = 3x^2 - 2$$

$$\left(\frac{dy}{dx}\right)_{(2,7)} = 3(2)^2 - 2$$

$= 12 - 2$

$$\left(\frac{dy}{dx}\right)_{(2,7)} = 10$$

Slope of tangent $m = \left(\frac{dy}{dx}\right)_{(2,7)} = 10$

The equation of tangent at (2,7)

$y - y_1 = m(x - x_1)$

y - 7 = 10 (x - 2)

y - 7 = 10x - 20

y = 10x - 20 + 7

y = 10x - 13 **Tricks** : GBC.

Q.8. If $y = \log\left[\frac{5 - 4x^2}{3 + 5x^2}\right]$, then $\frac{dy}{dx}$ = ____

(*a*) $\frac{8}{4x-5} - \frac{10}{3+5x}$

(*b*) $(4x^2 - 5) - (3 + 5x^2)$

(*c*) $\frac{8x}{4x^2-5} - \frac{10x}{3+5x^2}$

(*d*) 8x-10

[Dec. 2012]

Solution : (*c*) is correct.

Since, $y = \log\left(\frac{5-4x^2}{3+5x^2}\right)$

$y = \log(5 - 4x^2) - \log(3 + 5x^2)$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\log(5 - 4x^2) - \frac{d}{dx}\log(3 + 5x^2)$$

$$= \frac{1}{(5-4x^2)}\frac{d}{dx}(5 - 4x^2) - \frac{1}{(3+5x^2)}\frac{d}{dx}(3 + 5x^2)$$

$$= \frac{1}{(5-4x^2)}(0 - 8x) - \frac{1}{(3+5x^2)}(0 + 10x)$$

$$= \frac{8x}{4x^2-5} - \frac{10x}{3x+5x^2}$$

Q.9. If $y = \log_y x$ then $\frac{dy}{dx} =$

(a) $\frac{1}{x \log y}$ (b) $\frac{1}{x + x \log y}$

(c) $\frac{1}{1 + x \log y}$ (d) $\frac{1}{y + \log x}$

[June 2013]

Solution : (b) Let $y = \log_y x = \frac{\log_e x}{\log_e y}$

or $\log_e x = y \log_e y$ _____(1)

Diff. on both sides w.r.t.y; we get

$\frac{1}{x} \cdot \frac{dx}{dy} = y \cdot \frac{1}{y} + 1 \cdot \log_e y = 1 + \log_e y$

or $\frac{dx}{dy} = x + x \log_e y$

$\therefore \frac{dy}{dx} = \frac{1}{x + x \log_e y}$ $\therefore$ (b) is correct

Q.10. $y = e^t$ & $x = \log t$; then $\frac{dy}{dx} =$

(a) $\frac{1}{t}$ (b) $t.e^t$

(c) $-\frac{t}{t^2}$ (d) none

[June 2013]

Solution : (b) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t}{\frac{1}{t}} = t.e^t$

$\therefore$ (b) is correct.

Q.11. The points on the curve $y = x^3 - x^2 - x + 1$. Where the tangent is parallel to x-axis are:

(a) (1,0) $\left(\frac{-1}{3}, \frac{32}{27}\right)$

(b) (1,0) (1,1)

(c) $\left(\frac{-1}{3}, \frac{21}{37}\right)$ (0,0)

(d) (0,0) (1,0)

[Dec. 2013]

Solution : (a) is correct.

$\because y = x^3 - x^2 - x + 1$

$\frac{dy}{dx} = 3x^2 - 2x - 1$

$\because$ Tangent is parallel to x - axis

$\therefore \frac{dy}{dx} = 0$

$3x^2 - 2x - 1 = 0$

or $3x^2 - 3x + x - 1 = 0$

or 3x(x-1)+1(x-1)=0

or (x-1) (3x+1)=0

$\therefore$ x=1; x = -1/3

At x = 1

$y = 1^3 - 1^2 - 1 + 1 = 0$

$\therefore$ Point is (1;0)

and At x = -1/3

Then $y = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - \left(\frac{-1}{3}\right) + 1$

$= \frac{32}{27}$

Or; Another Point is = $\left(-\frac{1}{3}; \frac{32}{27}\right)$

Tricks : Go by choices

Q.12. A seller makes an offer of selling certain articles that can be described by the equation x = 25 - 2y where x is price per unit and y denotes the No. of units. The cost price of the article is ₹10 per unit. The maximum quantity that can be offered in single deal to avoid loss is

(a) 6 (b) 7

(c) 8 (d) 9

[Dec. 2013]

Solution : (b) is correct.

$\because$ x = 25 - 2y

Total cost = Cost price per unit $\times$ No. of unit sold

= 10y

Total sale = Selling price per unit $\times$ No. of units sold

$= x.y = (25 - 2y)y = 25y - 2y^2$

Profit = sale - Cost

$= 25y - 2y^2 - 10y = 15y - 2y^2$

For No loss; Profit > 0

$15y - 2y^2 > 0$

or 15 -2y>0

or $\frac{15}{2} > y \Rightarrow y < 7.5$

y = No. of units (a whole No.)

$\therefore$ y = 7

$\therefore$ Maximum Quantity sold y = 7

Tricks : Go by choices

Q.13. If $y = ae^{nx} + be^{-nx}$ then $\frac{d^2y}{dx^2}$

(a) $n^2 y$ (b) $-n^2 y$

(c) ny (d) None

[June 2014]

Solution : (a) is correct.

$\therefore y = ae^{nx} + be^{-nx}$

So; $\frac{dy}{dx} = a.e^{nx}.n + b.e^{-nx}(-n)$

$= n\left[ae^{nx} - be^{-nx}\right]$

$\therefore \frac{d^2y}{dx^2} = n\left[a.e^{nx}.n - b.e^{-nx}(-n)\right]$

$= n.n\left[ae^{nx} + be^{-nx}\right]$

$= n^2.y$

Q.14. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots\ldots\ldots + \frac{x^n}{n!} + \ldots\ldots\ldots$, then the value of $\frac{dy}{dx} - y =$ ____

(a) 1 (b) 0

(c) -1 (d) None

Solution : (b) is correct.

$\because y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots + \frac{x^n}{n!} + \ldots\ldots$

$\therefore y = e^x$ _____(1)

$\therefore \frac{dy}{dx} = \frac{de^x}{dx} = e^x = y$ [From (1)]

$$\therefore \frac{dy}{dx} - y = 0$$

Q.15. If $e^{xy} - 4xy = 4$ then $\frac{dy}{dx} =$

(a) $\frac{y}{x}$ (b) $\frac{-y}{x}$ (c) $\frac{x}{y}$ (d) $\frac{-x}{y}$

[June 2015]

Solution : If $e^{xy} - 4xy - 4 = 0$

Tricks : $\frac{dy}{dx} = -\frac{D.C.w.r.t.x \text{ keeping y constant}}{D.C.w.r.t.y \text{ keeping x constant}}$

$$= -\frac{e^{xy}.y - 4y - 0}{e^{xy}.x - 4x - 0}$$

$$= -\frac{(4xy+4)y - 4x}{(4xy+4)x - 4x}$$

$$= -\frac{4xy^2 + 4y - 4y}{4x^2y + 4x - 4x} = -\frac{4xy^2}{4x^2y}$$

$$= -y/x$$

$\therefore$ (b) is correct.

Q.16. If $x^p.y^q = (x+y)^{p+q}$ then $\frac{dy}{dx} =$

(a) $\frac{y}{x}$ (b) $\frac{-y}{x}$ (c) $\frac{p}{q}$ (d) $\frac{-p}{q}$

[June 2015]

Solution : $x^p.y^2 = (x+y)^{p+q}$

Tricks $\frac{dy}{dx} = \frac{y}{x}$

(a) is correct

Q.17. Find slope of tangent of curve y $= \frac{x-1}{x+2}$ at x = 2.

(a) 3/16 (b) 5/17

(c) 9/11 (d) None of the above

[Dec. 2015]

Solution : Slope - m $= \frac{dy}{dx} = \frac{\frac{d}{dx}(x-1)(x+2) - (x-1)\frac{d}{dx}(x+2)}{(x+2)^2}$

$$= \frac{(1-0)(x+2) - (x-1)(1+0)}{(x+2)^2}$$

$$= \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$$

Slope at $(x = 2) = \frac{3}{(2+2)^2} = \frac{3}{16}$

(a) is correct

Q.18. u = $5t^4 + 4t^4 + 2t^4 + 4$ at t=-1 find du/dt

(a) -11 (b) 11

(c) -16 (d) 16

[Dec. 2015]

Solution : (a) is correct.

$u = 5t^4 + 4t^3 + 2t^2 + t + 4$

$$\frac{du}{dt} = 5\times4t^3 + 4\times3t^2 + 2\times2t + 1 + 0$$

$$= 20t^3 + 12t^2 + 4t + 1$$

$$\frac{du}{dt} at\ = -1$$

$$= 20(-1)^3 + 12(-1)^2 + 4(-1) + 1$$

$$= -20 + 12 - 4 + 1 = -11$$

Q.19. Y $= \sqrt{\frac{1-x}{1+x}}$ then $\frac{dy}{dx}$ is equal to _____

(a) $\frac{y}{x^2-1}$ (b) $\frac{y}{1-x^2}$

(c) $\frac{y}{1+x^2}$ (d) $\frac{y}{y^2-1}$

[June 2016]

Solution : (a) is correct.

$$\therefore\ y = \sqrt{\frac{1-x}{1+x}} = \left(\frac{1-x}{1+x}\right)^{1/2}$$

Taking log on both sides; we get

$$\log y = \log\left(\frac{1-x}{1+x}\right)^{1/2} = \frac{1}{2}\log\left(\frac{1-x}{1+x}\right)$$

$$= \frac{1}{2}[\log(1-x) - \log(1+x)]$$

Differentiating *w.r.t. x; we.get*

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1-x}(0-1) - \frac{1}{1+x}(0+1)\right]$$

$$= \frac{1}{2}\left[\frac{1}{x-1} - \frac{1}{x+1}\right]$$

$$= \frac{1}{2}\left(\frac{x+1-x+1}{x^2-1^2}\right)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x^2-1}$$

$$\frac{dy}{dx} = \frac{y}{x^2-1}$$

Q.20. $\frac{d}{dx}\left(\log\left(\sqrt{x-1} + \sqrt{x+1}\right)\right) =$

(a) $\frac{1}{2\sqrt{x^2-1}}$

(b) $\frac{1}{2\sqrt{x^2+1}}$

(c) $\frac{1}{\sqrt{x-1}+\sqrt{x+1}}$

(d) None of these

[Dec. 2016]

Solution : (a) is correct.

Soln. : $\frac{d}{dx}\left\{\log\left(\sqrt{x-1}+\sqrt{x+1}\right)\right\}=\frac{1}{\sqrt{x-1}+\sqrt{x+1}}\times\left[\frac{d\sqrt{x-1}}{dx}+\frac{d\sqrt{x+1}}{dx}\right]$

$=\frac{1}{\sqrt{x-1}+\sqrt{x+1}}\cdot\left[\frac{1}{2\sqrt{x-1}}+\frac{1}{2\sqrt{x+1}}\right]$

$=\frac{1}{\sqrt{x-1}+\sqrt{x+1}}\times\frac{1}{2}\left(\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x-1}.\sqrt{x+1}}\right)$

$=\frac{1}{2\sqrt{(x-1)(x+1)}}=\frac{1}{2\sqrt{x^2-1}}$

$\therefore$ (a) is correct

Q.21. $f(x)=\log_e\left(\frac{x-1}{x+1}\right)$ **and** $f^1(x)=1$ **then the value of** $x=$

(a) 1 (b) 0 (c) $\pm\sqrt{3}$ (d) $\pm\sqrt{2}$

[Dec. 2016]

Solution : (c) is correct.

Soln.

$f(x)=\log_e\left(\frac{x-1}{x+1}\right)=\log_e(x-1)-\log_e(x+1)$

$f^1(x)=\frac{d\log(x-1)}{dx}-\frac{d\log(x+1)}{dx}$

$=\frac{1}{x-1}-\frac{1}{x+1}$

$=\frac{x+1-(x-1)}{(x-1)(x+1)}$

$=\frac{x+1-x+1}{x^2-1}=\frac{2}{x^2-1}\Rightarrow 1=\frac{2}{x^2-1}$

$\Rightarrow x^2-1=2$

$\Rightarrow x^2=3$

$\therefore x=\pm\sqrt{3}$

Q.22. The equation of the curve which passes through the point (1, 2) and has the slope 3x - 4 at any point (x, y) is

(a) $2y=3x^2-8x+9$

(b) $y=6x^2-8x+9$

(c) $y=x^2-8x+9$

(d) $2y=3x^2-8x+c$

[June 2017]

Solution : Tricks: Go by choices for option (a) point (1, 2) satisfies

$2y=3x^2-8x+9$

and its slope is

$2.\frac{dx}{dx}=3\times2x-8\times1+0$

$=6x-8=2(3x-4)$

$\therefore \frac{dx}{dx}=\text{slope}=3x-4(\text{True})$

$\therefore$ Option (a) is correct.

Q.23. If $y=1+\frac{x^1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots\ldots$ **then** $\frac{dy}{dx}=$ _____ :

(a) x (b) y (c) 1 (d) 0

[Dec. 2017]

Solution : $y=1+\frac{1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots$

$\Rightarrow y=e^x$ (Formula)

$$\frac{dy}{dx}=\frac{de^x}{dx}=e^x=y$$

$$\therefore \frac{dy}{dx}=y.$$

(*b*) is correct.

Q.24. If $x=at^2$ and $y=2at$ then $\left(\frac{dy}{dx}\right)_{t=2}=$ _____ :

(*a*) 2 (*b*) 4 (*c*) 1/2 (*d*) $\frac{1}{4}$

[Dec. 2017]

Solution : (*c*) is correct.

$$\frac{dy}{dx}=a\times 2t=2at \Rightarrow \frac{dy}{dx}=2a\times 2=4a \text{ at } t=2$$

$$\frac{dy}{dx}=2a\times 1=2a \Rightarrow \frac{dy}{dx}=2a \text{ . at } t=2$$

$$\therefore \left(\frac{dy}{dx}\right)at\ t=2 \quad =\frac{\left(dy/dt\right)at\ t=2}{\left(dx/dt\right)at\ t=2}$$

$$=\frac{2a}{4a}=\frac{1}{2}$$

Q.25. If $x^y=e^{x-y}$ then

$$\frac{dy}{dx}=\text{\_\_\_\_\_}:$$

(*a*) $\frac{2\log x}{(1+\log x)^2}$

(*b*) $\frac{\log x}{1+\log x}$

(*c*) $\frac{\log x}{(1+\log x)^2}$

(*d*) None of these

[Dec. 2017]

Solution : (*c*)

$\because\ x^y=e^{x-y}$

Taking log on both sides ; we get

$$y\log x=(x-y).\log e=x-y$$

or ; $y+y\log x=x$

or ; $y(1+\log x)=x$

or ; $y=\frac{x}{1+\log x}$

$$\therefore \frac{dy}{dx}=\frac{1.(1+\log x)-x\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$$

$$=\frac{1+\log x-1}{(1+\log x)^2}$$

$$=\frac{\log x}{(1+\log x)^2}$$

Q.26. If $y=\log x^x$ then

$$\frac{dy}{dx}=\text{\_\_\_\_\_}:$$

(*a*) log (ex) (*b*) log (e/x)

(*c*) log (x/e) (*d*) 1

[Dec. 2017]

Solution : (*a*)

$\because\ y=\log x^x=x\log x$

$$\frac{dy}{dx}=1.\log x+x\times\frac{1}{x}$$

$$=1+\log x$$

$$=\log_e e+\log_e x$$

$$=\log_e(ex)$$

$$=\log(ex)$$

Q.27. The cost function for the production of x units of a commodity is given by

$C(x)=2x^3-15x^2+36x+15$

The cost will be minimum when x = ?

[May 2018]

(*a*) 3 (*b*) 2

(*c*) 1 (*d*) 4

Solution : (*a*)

Let

$\because\ c(x)=y=2x^3-15x^2+36x+15$

$$\therefore \frac{dy}{dx}=6x^2-30x+36$$

$$\frac{d^2y}{dx^2}=12x-30$$

If $\frac{dy}{dx}=0\Rightarrow 6x^2-30x+36=0$

or $6(x^2-5x+6)=0$

or ; $x^2-3x-2x+6=0$

or ; $x(x-3)-2(x-3)=0$

or $(x-2)(x-3)=0$

$\therefore\ x=2;3.$

Case-I $\frac{d^2y}{dx^2}at\ x=2$

$=12\times 2-30=-6<0.$

$\therefore\ c(x)$ is maximum at $x=2.$

Case-II $\frac{d^2y}{dx^2}at\ x=3$

$=12\times 3-30=6>0.$

$\therefore\ y=c(x)$ is minimum at $x=3.$

$\therefore$ (*a*) is correct.

Q.28. Let $x=at^3, y=\frac{a}{t^2}$, Then $\frac{dy}{dx}$

(*a*) $\frac{-3a}{t^6}$ (*b*) $\frac{-1}{t^6}$

(*c*) $\frac{1}{3at^2}$ (*d*) None

[Nov. 2018]

Solution : (*d*)

$$\frac{dx}{dt} = a \times 3t^2 = 3at^2$$

$$\frac{dy}{dt} = a \cdot \frac{dt^{-2}}{dt} = a(-2)t^{-3} = -2at^{-3}$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2at^{-3}}{3at^2} = \frac{-2}{3}t^{-5}$$

Q.29. xy = 1 then $y^2 + \frac{dy}{dx} = ?$

(*a*) 1 (*b*) 0 (*c*) 2 (*d*) None

[Nov. 2018]

Solution : (*b*)

$$\because xy = 1 \Rightarrow y = \frac{1}{x} \Rightarrow \frac{dy}{dx} = -\frac{1}{x^2} = -\left(\frac{1}{x}\right)^2$$

$$\Rightarrow \frac{dy}{dx} = -y^2$$

$$\Rightarrow y^2 + \frac{dy}{dx} = 0$$

Q.30. If the given cost function of commodity is given by $C = 150x - 5x^2 + \frac{x^3}{6}$, where C stands for cost and x stands for output, if the average cost is equal to the marginal cost then the output x = _____.

(*a*) 5 (*b*) 10

(*c*) 15 (*d*) 20

[June 2019]

Solution :

$$\because c(x) = 150x - 5x^2 + \frac{x^3}{6}$$

$$AC = \frac{c(x)}{x} = 150 - 5x + \frac{x^2}{6}$$

$$MC = \frac{dc}{dx} = 150 \times 1 - 5 \times 2x + \frac{3x^2}{6}$$

$$= 150 - 10x + \frac{3x^2}{6}$$

$\because$ AC = MC (given)

$$\because 150 - 5x + \frac{x^2}{6} = 150 - 10x + \frac{3x^2}{6}$$

$$\Rightarrow \frac{2x^2}{6} - 10x + 5x = 0$$

$$\text{or } \frac{x^2}{3} - 5x = 0$$

$$\text{or } x\left(\frac{x}{3} - 5\right) = 0$$

$$\text{or } x = 0 ; \frac{x}{3} - 5 = 0$$

$$\text{or } x = 0 ; \frac{x}{3} - 5 = 0$$

$$\text{or } \frac{x}{3} = 5 \quad \therefore x = 15.$$

$\therefore$ (c) is correct.

Q.31. If $2^x - 2^y = 2^{x-y}$ then $\frac{dy}{dx}$ at x = y = 2

(*a*) 1 (*b*) 2 (*c*) 4 (*d*) 5

[June 2019]

Solution : (*a*)

Given $2^x - 2^y - 2^{x-y} = 0$.

Tricks:- It is implicit function.

So;

$$\frac{dy}{dx} = -\frac{\text{D.C of } f(x;y) \text{ w.r.t.x keeping } y \text{ constant}}{\text{D.C of } f(x;y) \text{ w.r.t.y keeping } x \text{ constant}}$$

$$= -\frac{2^x \cdot \log 2 - 0 - 2^{x-y} \cdot \log 2 \cdot (1-0)}{0 - 2^y \cdot \log 2 - 2^{x-y} \cdot \log 2 \cdot (0-1)}$$

$$= \frac{\log 2 \cdot \left[2^x - 2^{x-y}\right]}{\log 2 \cdot \left[-2^y + 2^{x-y}\right]}$$

$$\therefore \frac{dy}{dx} \text{ at } x = y = 2 = -\frac{2^2 - 2^0}{-2^2 + 2^0}$$

$$= -\frac{4-1}{-4+1} = -\frac{3}{-3} = +1$$

(*a*) is correct.

Q.32. $f(x) = \begin{cases} x, & x < 0 \\ 0, & x = 0 \\ x^2, & x > 0 \end{cases}$ **then find** $\lim_{x \to 0} f(x)$

(*a*) 1 (*b*) 0

(*c*) -2 (*d*) 2

[Dec. 2019]

Solution : (*b*) $\text{LHL} = \lim_{x \to 0^-} f(x)$

$$= \lim_{x \to 0}(x) = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0}(x^2)$$

$$= 0^2 = 0$$

Here, LHL = RHL = 0

$\therefore$ Limit exists

$$\therefore \lim_{x \to 0} f(x) = 0$$

Q.33. Find the value of dy/dx if $y = x^x$

(a) $x^x \log x$ (b) $1+\log x$

(c) $y \log x$ (d) none of these

[Dec. 2019]

Solution : (a)

$\because\ y = x^x$ ——————(1)

Taking log on both sides ; we get

$\log y = \log x^x$

$= x \log x.$

Differentiating on both sides with respect to x ; we get

$$\frac{d \log y}{dx} = \frac{dx}{dx} \cdot \log x + x. \frac{d \log x}{dx}$$

$$\text{or } \frac{1}{y} \cdot \frac{dy}{dx} = 1.\log x + x. \frac{1}{x}$$

$$\text{or } \frac{dy}{dx} = y[\log x + 1]$$

from (1) ; we get

$$\frac{dy}{dx} = x^x (\log_e x + \log_e e)$$

$$= x^x . \log_e (ex) = x^x . \log e^x$$

Q.34. If $f(x) = a(x^2 + x + 1)^2$ and $f^1(-1) = -6$ then the value of a =

(a) 1 (b) 2 (c) 3 (d) 4

[Dec. 2019]

Solution : (c)

$\because\ f(x) = a(x^2 + x + 1)^2$

$f^1(x) = a \cdot 2(x^2 + x + 1)^{2-1} \cdot \frac{d}{dx}(x^2 + x + 1)$; [by chain Rule]

$= 2a(x^2 + x + 1) \cdot (2x + 1)$

$f^1(-1) = 2a[(-1)^2 + (-1) + 1] \cdot [2 \cdot (-1) + 1]$

or; $-6 = 2a[1 - 1 + 1](-2 + 1)$

or; $-6 = 2a \cdot 1 \cdot (-1)$

or $-2a = -6 \Rightarrow a = 3$

Q.35. If $Y = x(x - 1)(x - 2)$ then dy/dx is

(a) -6x (b) $3x^2$-6x+2

(c) 6x+4 (d) $3x^2$-6x

[Dec. 2020]

Solution : $y = x(x - 1)(x - 2)$

$= (x^2 - x)(x - 2)$

$= x^3 - 2x^2 - x^2 + 2x$

$= x^3 - 3x^2 + 2x$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^3 - 3x^2 + 2x)$$

$= 3x^2 - 6x + 2$

(b) is correct.

Q.36. The cost function of a good is $2Q + 6 + \frac{13}{Q}$ where Q is the quantity produced.

The approx. cost at Q = 15 is

(a) 42 (b) 36

(c) 66 (d) 130

[Dec. 2020]

Solution : Cost function $= 2Q + 6 + \frac{13}{Q}$

Cost at Q = 15

$= 2 \times 15 + 6 + \frac{13}{15}$

$= 36.86 = \cong ₹\ 36$

$\therefore$ (b) is correct.

Q.37. The cost function of production is given by $C(x) = \frac{x^3}{2} - 15x^2 + 36x$ where x, denotes the number of items produced.

The level of output for which marginal cost is minimum and the level of output for which the average cost is minimum are given by, respectively.

(a) 10 and 15 (b) 10 and 12

(c) 12 and 15 (d) 15 and 10

[Jan. 2021]

Solution : $C(x) = \frac{x^3}{2} - 15x^2 + 36x$

MC = Marginal Cost $= \frac{dC}{dx}$

$= \frac{1}{2} 3x^2 - 15 \times 2x + 36 \times 1$

$= \frac{3}{2} x^2 - 30x + 36$

Now $\frac{dMC}{dx} = \frac{3}{2} \times 2x - 30 \times 1 + 0$

$= 3x - 30$

And $\frac{d^2MC}{dx^2} = 3 \times 1 - 0 = 3 > 0$ (+ Ve)

For Maximum/Minimum

$\frac{dMC}{dx} = 0 \Rightarrow 3x - 30 = 0 \Rightarrow x = 10$

$\because \frac{d^2MC}{dx^2} > 0$

$\therefore$ MC is minimum at $x = 10$

Now $AC = \frac{C(x)}{x} = \frac{x^2}{2} - 15x + 36$

$\frac{dAC}{dx} = \frac{1}{2} 2x - 15 \times 1 + 0 = x - 15$

For maximum/Minimum

$\frac{dAC}{dx} = 0 \Rightarrow x - 15 = 0$

$\therefore x = 15$

Now $\frac{d^2AC}{dx^2} = 1 - 0 = +1 > 0$ (+ ve)

$\therefore$ AC is minimum at $x = 15$.

$\therefore$ (a) is correct.

Q.38. The cost function $C(x) = 125 + 500x - x^2 + x^3/3$, $0 \le x \le 100$ and the demand function for the items is given by, $p(x) = 1500 - x$, then the marginal profit when 18 items are sold is

(a) 751 (b) 571

(c) 676 (d) 875

[July 2021]

Solution : (c) is correct

Revenue Function $= R(x) = Px = (1500 - x)x$

$= 1500x - x^2$

Profit Function $= P(x) = R(x) - C(x)$

$P(x) = 1500x - x^2 - 125 - 500x + x^2 - \frac{-x^3}{3}$

$= \frac{-x^3}{3} + 1000x - 125$

Marginal Profit = MP = $P'(x) = \frac{-1}{3} \times 3x^2 + 1000 \times 1 - 0$

$= -x^2 + 1000$

MP at $x = 18 = -18^2 + 1000$

$= 676$

(c) is correct

Q.39. If $f(x) = 3e^{x^4}$ then $f'(x) - 4x^3 f(x) + \left(\frac{1}{3}\right) f(0) - f'(0)$ is equal to

(a) 0 (b) e^{x^2}

(c) 1 (d) -1

[July 2021]

Solution : (c) is correct

$\because$ $f(x) = 3e^{x^4}$

$f'(x) = 3.e^{x^4}$

$\frac{dx^4}{dx} = 3e^{x^4}.4x^3$

$= 4x^3.(3e^{x^4})$

$f'(x) = 4x^3.f(x)$

Now $f(0) = 3.e^{0^4} = 3.e^0 = 3 \times 1 = 3$

$f'(0) = 4.0^3.(3.e^{04})$

$= 0$

Hence

$f'(x) - 4x^3 f(x) + \frac{1}{3} f(0) - f'(0)$

$= 4x^3.f(x) - 4x^3 f(x) + \frac{1}{3} \times 3 - 0$

$= 0 + 1 - 0 = 1$

(c) is correct

Q.40. The cost for producing x units is $500 - 20x^2 + x^3/3$. The material cost is minimum at $x =$ _____

(a) 5 (b) 10

(c) 20 (d) 50

[Dec. 2021]

Solution : (c)

$C(x) = 500 - 20x^2 + \frac{x^3}{3}$

$MC = \frac{dc}{dx} = 0 - 20 \times 2x + \frac{3x^2}{3}$

$= -40x + x^2$

Now $\frac{dMC}{dx} = -40 \times 1 + 2x = 2x - 40$

$\frac{d^2MC}{dx^2} = 2 \times 1 - 0 = +2 > 0$

For Maxima/Minima.

$\frac{dMC}{dx} = 0 \Rightarrow 2x - 40 = 0$

$\therefore x = 20$

So MC is minimum at $x = 20$

$\therefore$ (c) is Correct.

Q.41. If $y = \frac{x^4}{e^x}$ then $\frac{dy}{dx}$ is equal to :

(a) $x^3 (4 - x)/(e^x)^2$

(b) $x^3 (4 - x)/e^x$

(c) $x^2 (4 - x)/e^x$

(d) $x^3 (4x - 1)/e^x$ ***[Dec. 2021]***

Solution : (b)

$\because y = \frac{x^4}{e^x}$

$= \frac{dy}{dx} = \frac{\frac{dx^4}{dx}.e^x - x^4.\frac{de^x}{dx}}{\left(e^x\right)^2}$

$= \frac{4x^3.e^x - x^4.e^x}{e^x.e^x}$

$\frac{x^3.(4-x)e^x}{e^x.e^x} = \frac{x^3(4-x)}{e^x}$

$\therefore$ (b) is Correct

Q.42. The speed of a train at a distance x (from the starting point) is given by $3x^2 - 5x + 4$. What is the rate of change (of distance) at $x = 1$?

(a) -1 (b) 0

(c) 1 (d) 2

[Dec. 2021]

Solution : (d)

Rate of Change of distance is called speed.

Given $\frac{dx}{dt} = 3x^2 - 5x + 4$

When $x = 1$

Then Rate of change of distance

$= 3 \times 1^2 - 5 \times 1 + 4$

$= 3 - 5 + 4$

$= 2$

(d) is correct.

Q.43. Which of the following is the differentiation of $e^t.\log_e t$ with respect to 't'?

(a) $e^t(t \log_e t)$

(b) $\frac{e^t(1 + t \log_e t)}{t}$

(c) $\frac{e^t}{t}$

(d) $e^t(1 - \log_e t)$

[June 2022]

Solution : Let $y = e^t \log_e t$

$\therefore \frac{dy}{dt} = \frac{de^t}{dt}.\log_e t + e^t.\frac{d\log_e t}{dt}$

$= e^t.\log_{et} + e^t.\frac{1}{t}$

$= e^t\left[\log_e t + \frac{1}{t}\right]$

$= e^t\left(\frac{t.\log_e t + 1}{t}\right)$

(b) is correct

Q.44. The maxima and minima of the function $y = 2x^3 - 15x^2 + 36x + 10$ occurs respectively at

(a) $x = 2$ and $x = 3$

(b) $x = 1$ and $x = 3$

(c) $x = 3$ and $x = 2$

(d) $x = 3$ and $x = 1$

[Dec. 2022]

Solution: $\because y = 2x^3 - 15x^2 + 36x + 10$

$\frac{dy}{dx} = 2.3x^2 - 15 \times 2x + 36 \times 1 + 0$

$= 6x^2 - 30x + 36$

$\frac{dy^2}{dx^2} = 6 \times 2x - 30 \times 1 + 0$

$= 12x - 30$

If $\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 30x + 36 = 0$

or $6(x^2 - 5x + 6) = 0$

or; $x^2 - 3x - 2x + 6 = 0$

or $x(x-3) - 2(x-3) = 0$

or $(x-3)(x-2) = 0$

$\therefore x = 2; 3$

For Maxima/Minima :

At $x = 2$

$\frac{dy^2}{dx^2} = 12x - 30$

$= 12 \times 2 - 30 = -6$ (–ve)

$\therefore y$ is max. at $x = 2$

Now at $x = 3$

$\frac{dy^2}{dx^2} = 12 \times 3 - 30 = 6$ (+ve)

$\therefore y$ is minima at $x = 3$

$\therefore$ Max. at $x = 2$ and

Minima at $x = 3$

$\therefore$ (*a*) is correct.

Q.45. If $y = x^x$, then dy/dx at $x = 1$ is equal to

(*a*) 0 (*b*) 1

(*c*) –1 (*d*) 2

[Dec. 2022]

Solution : $\because y = x^x = e^{x.\log_e x}$

$\therefore \frac{dy}{dx} = \frac{de^{(x.\log_e x)}}{dx}$

$= e^{x.\log_e x} \times \frac{d(x.\log_e x)}{dx}$

$= x^x \left[\frac{dx}{dx} \times \log_e x + x \times \frac{d \log_e x}{dx}\right]$

$= x^x \left[1 \times \log_e x + x \times \frac{1}{x}\right]$

$= x^x [\log x + 1]$

$\therefore \frac{dy}{dx}(\text{at } x = 1) = 1^1 (\log 1 + 1)$

$= 1 \times (0 + 1) = 1$

$\therefore$ (*b*) is correct

Q.46. If $x^5 + y^5 - 5xy = 0$ then $\frac{dy}{dx}$ is

(*a*) $\frac{y + x^4}{x + y^4}$ (*b*) $\frac{y - x^4}{y^4 - x}$

(*c*) $\frac{x - y^4}{x^4 + y}$ (*d*) $\frac{x + y^4}{x^4 + y}$

[Dec. 2022]

Solution : *Given* Eqn. is

$x^5 + y^5 - 5xy = 0$

Tricks : It is an Implicit function

$$\frac{dy}{dx} = -\frac{\text{D.C of } f(x;y) \text{ w.r.t. } x \text{ keeping } y \text{ constant}}{\text{D.C of } f(x;y) \text{ w.r.t. } y \text{ keeping } x \text{ constant}}$$

[D.C means differential coefficient]

$$\therefore \frac{dy}{dx} = -\frac{\frac{dx^5}{dx} + \frac{dy^5}{dx} - \frac{d}{dx}(5xy)}{\frac{dx^5}{dy} + \frac{dy^5}{dy} - \frac{d}{dy}(5xy)}$$

$$= -\frac{5x^4 + 0 - 5y \times \frac{dx}{dx}}{0 + 5y^4 - 5x \times \frac{dy}{dy}}$$

$$= -\frac{5x^4 - 5y \times 1}{5y^4 - 5x \times 1}$$

$$= -\frac{5(x^4 - y)}{5(y^4 - x)}$$

$$= \frac{x^4 - y}{x - y^4} = \frac{y - x^4}{y^4 - x}$$

(*b*) is correct

Q.47. For a given curve $y = 2 - x^2$, when 'x' increases at the rate of 3 units/s, then the slope of curve will:

(*a*) Increase at 6 units/s

(*b*) Increase at 3 units/s

(*c*) Decrease at 6 units/s

(*d*) Decrease at 3 units/s

[June 2023]

Solution : $\because y = 2 - x^2$

Slope of curve $= \frac{dy}{dx} = 0 - 2x$

at $x = 3$

$= -2 \times 3 = -6$

i.e. decreases at 6 units/sec.

(*c*) is correct.

Q.48. If $y = \frac{x}{x+5}$, then $\frac{dx}{dy}$ is equal to:

(*a*) $\frac{5}{(1-y)^2}$

(*b*) $\frac{5}{(1+y)^2}$

(*c*) $\frac{3}{(1-y)^2}$

(*d*) $\frac{3}{(1+y)^2}$

[June 2023]

Solution : $\because y = \frac{x}{x+5}$

$\therefore xy + 5y = x$

or $xy + 5y - x = 0$

$\Rightarrow x = \frac{5y}{1-y}$

Difference on both sides w.r.t. x; we get

$\frac{dx}{dx} \times y + x \times \frac{dy}{dx} + 5\frac{dy}{dx} - \frac{dx}{dx}$

$= 0$

or $1 \times y + x\frac{dy}{dx} + 5\frac{dy}{dx} - 1 = 0$

or $(x+5)\frac{dy}{dx} = 1 - y$

or $\left(\frac{5y}{1-y} + 5\right)\frac{dy}{dx} = 1 - y$

or; $\left(\frac{5y + 5 - 5x}{1-y}\right)\frac{dy}{dx} = 1 - y$

or; $\frac{dy}{dx} = \frac{(1-y)^2}{5}$

$\therefore \frac{dx}{dy} = \frac{5}{(1-y)^2}$

(*a*) is correct.

Q.49. If $xy = 1$, then $y^2 + dy/dx$ is equal to:

(*a*) 1 (*b*) 0

(*c*) –1 (*d*) 1/2

[June 2023]

Solution : $\because \quad xy = 1$

$$y = \frac{1}{x} \quad \text{.....(1)}$$

$$\frac{dy}{dx} = \frac{d\left(\frac{1}{x}\right)}{dx} = -\frac{1}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{1}{x}\right)^2 = -y^2$$

$$\therefore \frac{dy}{dx} + y^2 = 0$$

$\therefore$ (*b*) is correct.

15 CHAPTER INTEGRATION

FORMULA

1. $\int o\,dx = c$ (Constant)

2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$; Where $n \neq -1$

   Where c = Integration Constant (I.C.)

3. $\int a^x\,dx = \frac{a^x}{\log a} = \frac{a^x}{\log e^a} + c$; Where c = I.C.

4. $\int e^x\,dx = e^x + c$; Where c = I.C.

5. $\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \log e^x + c$; Where c = I.C.

6. $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)^a} + c$; Where c = I.C.

7. $\int a^{bx+c}\,dx = \frac{a^{bx+c}}{b.\log a} + K$; Where K = I.C.

8. $\int e^{ax+b}\,dx = \frac{e^{ax+b}}{a} + c$; Where c = I.C.

9. $\int \frac{1}{a^{x+b}}\,dx = \log e\,(ax+b) + c$; Where c = I.C.

10. $\int_a^b f(x)\,dx = [F(x)]_a^b = f(b) - f(a)$; Where $\int f(x)\,dx = F(x)$

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. $\int \frac{dx}{\sqrt{3x+4}-\sqrt{3x+1}}$ **Equal to**

(*a*) $\frac{2}{27}\left[(3x+4)^{3/2}-(3x+1)^{3/2}\right]+c$ (*b*) $\frac{2}{27}\left[(3x+4)^{3/2}+(3x+1)^{3/2}\right]+c$

(*c*) $\frac{2}{3}\left[(3x+4)^{3/2}-(3x+1)^{3/2}\right]+c$ (*d*) None of these

[June 2010]

Solution : (*b*) is correct.

$$\int \frac{dx}{\sqrt{3x+4}-\sqrt{3x+1}}$$

$$= \int \frac{dx}{\sqrt{3x+4}-\sqrt{3x+1}} \times \frac{\sqrt{3x+4}+\sqrt{3x+1}}{\sqrt{3x+4}+\sqrt{3x+1}}$$

$$= \int \frac{\sqrt{3x+4}+\sqrt{3x+1}dx}{3x+4-3x-1}$$

$$= \frac{1}{3}\int \sqrt{3x+4}dx + \frac{1}{3}\int \sqrt{3x+1}dx$$

$$= \frac{1}{3}\cdot\frac{(3x+4)^{3/2}}{3/2}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{(3x+1)^{3/2}}{3/2}\cdot\frac{1}{3}$$

$$= \frac{2(3x+4)^{3/2}}{27} + \frac{2(3x+1)^{3/2}}{27}$$

$$= \frac{2}{27}\left[(3x+4)^{3/2}+(3x+1)^{3/2}\right]$$

Q.2. $\int_1^2 \frac{xdx}{x^2+2} =$ _____

(*a*) $\log\sqrt{2}$ (*b*) $\log\sqrt{3}$ (*c*) $\log\frac{1}{\sqrt{2}}$ (*d*) $\log\frac{1}{\sqrt{3}}$

[June 2010]

Solution : (*a*) is correct.

$$\int_1^2 \frac{xdx}{x^2+2}$$

Let $x^2+2=t$

→ Differentiating both sides w.r.t. x

$2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$\int_3^6 \frac{dt}{2t}$; when, x = 1, t = 3; x = 2. t = 6

$$= \left[\frac{\log|t|}{2}\right]_3^6$$

$$= \frac{1}{2}[\log 6 - \log 3] = \frac{1}{2}\log\frac{6}{3} = \frac{1}{2}\log 2$$

$= \log\sqrt{2}$

Q.3. $\int \frac{6x+4}{(x-2)(x-3)}dx$ **is equal to**

(*a*) 22log (x-3)-16(x-2) (*b*) 11log(x-3)-8(x-2)

(*c*) 22log (x-3) -16log(x-2) (*d*) 232log (x-3) +16log (x-2)

[Dec. 2010]

Solution : (*c*) is correct

$$\int \frac{6x+4}{(x-2)(x-3)}dx$$

Let, $\frac{6x+4}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$ (Partial Fractions)

$6x + 4 = A(x - 3) + B(x - 2)$

When x = 3 ; B = 22

& When x = 2; A = - 16

$$\int \frac{-16}{x-2}dx + \int \frac{22}{x-3}dx$$

$$= -16 \log(x-2) + 22 \log(x-3) + c$$

$$= 22\log(x-3) - 16 \log(x-2) + c$$

Q.4. $\int \frac{1}{x(1+\log x)^2}dx$ **is equal to**

(*a*) $-\frac{1}{2(1+\log x)^2}+c$ (*b*) $\frac{1}{(1+\log x)}+c$

(*c*) $-\frac{1}{(1+\log x)}+c$ (*d*) None of these

[Dec. 2010]

Solution : (*c*) is correct .

$$\int \frac{1}{x(1+\log x)^2}dx$$

Let 1 + log x = t

$$\frac{1}{x}dx = dt$$

$$\int \frac{dt}{t^2} = -\frac{1}{t}+c$$

$$= -\frac{1}{(1+\log x)}+c$$

Q.5. $\int_{-1}^{1} (e^x - e^{-x})dx$ **= ?**

(*a*) 0 (*b*) 1 (*c*) 12 (*d*) None

[Dec. 2010]

Solution : (*a*) is correct.

Let f(x) = $e^x - e^{-x}$

then f (-x) = - $(e^x - e^{-x}) = -f(x)$

So, the function is a "ODD" function.

so; $\int_{-1}^{1} (e^x - e^{-x})dx = 0$

Q.6. $\int \frac{(\log x^x)^2}{x^3}dx$ **=?**

(*a*) $\frac{3}{2}(\log x)^3 + c$ (*b*) $\frac{1}{3}(\log x)^3 + c$

(*c*) $\frac{1}{6}(\log x)^3 + c$ (*d*) $\frac{3}{7}(\log x)^3 + c$

[Dec. 2010]

Solution : (*b*) is correct.

$$\int \frac{(\log x^x)^2}{x^3}dx = I \text{ (say)}$$

$$\therefore I = \int \frac{(x\log x)^2}{x^3}dx = \int \frac{(\log x)^2}{x}dx$$

Let log x = t

$$\therefore \frac{1}{x}dx = dt$$

$$\therefore I = \int t^2 dt = \frac{t^3}{3}+C = \frac{(\log x)^3}{3}+C = \frac{1}{3}(\log x)^3 + C$$

Q.7. Given $y = \int (e^{a\log x} + e^{x\log a})dx$ **then** $\frac{dy}{dx}$

(*a*) $x^a\ a^x$ (*b*) $x^a + a^x$

(*c*) $ax^{x-1} + a^x \log a$ (*d*) None

[Dec. 2010]

Solution : (*b*) Since, $e^{\log x^a} + e^{\log a^x} = x^a + a^x$ .....(1)

$$\therefore Y = \int (e^{a\log x} + e^{x\log a})dx$$

or Y = $\int (x^a + a^x)dx$ using (1)

So, $\frac{dy}{dx} = x^a + a^x$

Differentiation is just reverse of Integration.

Q.8. If $f'(x)=3x^2-\frac{2}{x^3}, f(1)=0$ *and* $f(x)=$_____.

(*a*) $\frac{x^3}{3}-x^{-2}-2$ (*b*) x^3+x^2+2 (*c*) $x^3+x^{-2}-2$ (*d*) None of these.

[June 2011]

Solution : (*c*) is correct

Given : $f'(x)=3x^2-\frac{2}{x^3}$

Formula, $f(x)=\int f'(x)dx=\int(3x^2-\frac{2}{x^3})dx$

$\therefore f(x)=x^3+x^{-2}+c$ .....(1)

Given $f(1)=0$

$\Rightarrow f(1)=(1)^3+(1)^{-2}+c$

$0=1+1+c$

$\Rightarrow c=-2$

$\therefore$ from (1); $f(x)=x^3+x^{-2}-2$

Q.9. $\int_{-1}^{1}\frac{|x|}{x}dx=$_____

(*a*) -1 (*b*) 0 (*c*) 1 (*d*) 2

[Dec. 2011]

Solution : (*b*) is correct

$\int_{-1}^{1}\frac{|x|}{x}dx$

Let $f(x)=\frac{|x|}{x}$

$f(-x)=\frac{|-x|}{-x}=\frac{|x|}{-x}=\frac{-|x|}{x}$

f(-x) = - f(x)

So, it is an odd function

$\int_{-1}^{1}\frac{|x|}{x}dx=0$

$\because \int_{-a}^{a}f(x)dx=0$; if f(x) is odd function.

$=2\int_{0}^{a}f(x)dx$; if f(x) an Even function

Q.10. $\int\frac{e^x}{(1+x)^3}dx-\int\frac{e^x}{2(1+x)^2}dx=$____

(*a*) 0 (*b*) $\frac{e^x}{2(1+x)^2}+c$ (*c*) $-\frac{e^x}{2(1+x)^2}+C$ (*d*) $\frac{e^x}{(1+x)^2}+C$

[Dec. 2011]

Solution : (*c*) is correct.

$=\int\frac{e^x}{(1+x)^3}dx-\int\frac{e^x}{2(1+x)^2}dx$

$=\int\left\{\frac{e^x}{(1+x)^3}-\frac{e^x}{2(1+x)^2}\right\}dx$

$=\int e^x\left\{\frac{1}{(1+x)^3}-\frac{1}{2(1+x)^2}\right\}dx$

$=\int e^x\left(-\frac{1}{2}\right)\left\{\frac{1}{(1+x)^2}-\frac{2}{(1+x)^3}\right\}dx$

$=\frac{-1}{2}\int e^x\left\{\frac{1}{(1+x)^2}-\frac{2}{(1+x)^3}\right\}dx$

$=\frac{-1}{2}e^x\frac{1}{(1+x)^2}+c$

$[\because [\int e^x\{f(x)+f'(x)\}dx=e^xf(x)+c]$

Q.11. $\int_0^1\frac{dx}{[ax+b(1-x)]^2}=$_____

(*a*) a/b (*b*) b/a (*c*) ab (*d*) 1/ab

[June 2012]

Solution : (*d*) $\int_0^1\frac{dx}{[ax+b(1-x)]^2}=$.....

Let t = ax + b (1 -x)

$= dt = [a + b(-1)]\, dx$

$= dt = (a - b)\, dx$

or $dx = \dfrac{dt}{(a-b)}$

Let $I = \int_0^1 \dfrac{dx}{[ax+b(1-x)]^2}$

$= \int_b^a \dfrac{dt}{(a-b).t^2}$ [if x = 0, t = b & x =1 then t = a]

$= \dfrac{1}{(a-b)} \int_b^a t^{-2} dt$

$= \dfrac{1}{(a-b)} \left[\dfrac{t^{-2+1}}{-2+1}\right]_b^a$

$= -\dfrac{1}{(a-b)} \left[\dfrac{1}{a} - \dfrac{1}{b}\right] = \dfrac{1}{ab}$

Q.12. $\int 2^{3x}.3^{2x}.5^x dx =$ ____

(*a*) $\dfrac{2^{3x}.3^{2x}.5^x}{\log(720)} + c$ (*b*) $\dfrac{2^{3x}.3^{2x}.5^x}{\log(360)} + c$ (*c*) $\dfrac{2^{3x}.3^{2x}.5^x}{\log(180)} + c$ (*d*) $\dfrac{2^{3x}.3^{2x}.5^x}{\log(90)} + c$

[Dec. 2012, June 2013]

Solution : (*b*) is correct

$\int 2^{3x}.3^{2x}.5^x dx$

$= \int 8^x.9^x.5^x dx$

$= \int (8 \ .9 \ .5)^x dx$

$= \int (360)^x dx$

$= \dfrac{(360)^x}{\log 360} + c$

$= \dfrac{2^{3x}.3^{2x}.5^x}{\log 360} + c$

Q.13. $\int_1^2 \dfrac{[\log_e(ex)]^n}{x} dx =$ **(where n** $\neq$ **-1)**

(*a*) $\dfrac{[\log 2e]^{n+1}}{n+1}$ (*b*) $\dfrac{[\log 2e]^{n+1}}{n+1} - \dfrac{\log e}{n+1}$

(*c*) $\dfrac{[\log 2e - \log 2]^{n+1}}{n+1}$ (*d*) None

[June 2013]

Solution : (*b*) $\int_1^2 \dfrac{[\log_e e + \log_e x]^n}{x} dx$

$= \int_1^2 \dfrac{(1+\log x)^n}{x} dx$

$= \left[\dfrac{(1+\log x)^{n+1}}{n+1}\right]_1^2$

$= \dfrac{1}{n+1}\left[(1+\log 2)^{n+1} - (1+\log 1)^{n+1}\right]$

$= \dfrac{1}{n+1}\left[(\log 2e)^{n+1} - \log e\right]$

$= \dfrac{(\log 2e)^{n+1}}{n+1} - \dfrac{\log e}{n+1}$

Q.14. $\int a^{2x} dx =$

(*a*) $\dfrac{a^{2x} \log a}{2}$ (*b*) $\dfrac{2.a^{2x}}{\log a}$ (*c*) $\dfrac{a^{2x}}{2 \log a}$ (*d*) None

[Dec. 2013]

Solution : (*c*) is correct

$\int a^{2x} dx = \dfrac{a^{2x}}{2 \log_e a}$ (Formula)

Q.15. $\int_0^5 \dfrac{x^2}{x^2+(5-x)^2}$ **dx is equal to**

(*a*) 5 (*b*) 5/2 (*c*) 1 (*d*) None

[June 2014]

Solution : (*b*) is correct

Let $I = \int_0^5 \dfrac{x^2}{x^2+(5-x)^2} dx$

$$I=\int_0^5 \frac{(5-x)^2}{(5-x)^2+x^2}dx$$

$$\left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]=0$$

Adding Them ; We get

$$2I=\int_0^5 \frac{x^2+(5-x)^2}{x^2+(5-x)^2}dx$$

$$=\int_0^5 dx=[x]_0^5=5-0=5$$

$$\therefore I=\frac{5}{2}$$

Q.16. $\int_0^2 |1-x|dx=$

(*a*) $\frac{3}{2}$ (*b*) $\frac{1}{2}$ (*c*) 0 (*d*) 1

[June 2014]

Solution : (*d*) is correct

$$\int_0^2 |1-x|\,dx=\int_0^1 (1-x)dx+\int_1^2 -(1-x)dx$$

$$=\left[x-\frac{x^2}{2}\right]_0^1-\left[x-\frac{x^2}{2}\right]_1^2$$

$$=\left[\left(1-\frac{1^2}{2}\right)-(0-0)\right]-\left[\left(2-\frac{2^2}{2}\right)-\left(1-\frac{1^2}{2}\right)\right]$$

$$=\frac{1}{2}-\left(0-\frac{1}{2}\right)=\frac{1}{2}+\frac{1}{2}=1$$

Q.17. $\int_0^{1/2} \frac{1}{\sqrt{3-2x}}dx=$

(*a*) 1 (*b*) $1-\frac{\sqrt{3}}{2}$ (*c*) $\sqrt{3}-\sqrt{2}$ (*d*) $2-\sqrt{3}$

[June 2015]

Solution : $\int_0^{1/2} \frac{1}{\sqrt{3-2x}}dx=\int_0^{1/2}(3-2x)^{-1/2}dx$

$$\left[\frac{(3-2x)^{-\frac{1}{2}+1}}{\left(-\frac{1}{2}+1\right)(-2)}\right]_0^{1/2}$$

$$=\left[\frac{\sqrt{3-2x}}{\frac{1}{2}.(-2)}\right]_0^{1/2}$$

$$=-\left[\sqrt{3-2.\frac{1}{2}}-\sqrt{3-2\times 0}\right]$$

$$=-\left[\sqrt{2}-\sqrt{3}\right]=\sqrt{3}-\sqrt{2}$$

(c) is correct

Q.18. $\int_0^1 xe^{x^2}dx=$

(*a*) 1 (*b*) e-1 (*c*) $\frac{e}{2}-1$ (*d*) $\frac{1}{2}(e-1)$

Solution : $I=\frac{1}{2}\int_0^1 2xe^{x^2}dx$

$$=\frac{1}{2}\int_0^1 e^{x^2}2xdx=\frac{1}{2}\left[e^{x^2}\right]_0^1$$

$$=\frac{1}{2}\left[e^{1^2}-e^{0^2}\right]=\frac{1}{2}(e-1)$$

(*d*) is correct

Q.19. $\int_1^2 \frac{1-x}{1+x}$**:**

(*a*) 2 log 3/2 - 1 (*b*) 2 log 3 + 1 (*c*) ½ log 3/2 - 1 (*d*) 2 log 2 – 1 + k

[Dec. 2015]

Solution : (*a*) is correct $\int_1^2 \frac{1-x}{1+x}dx=\int_1^2 \frac{2-1-x}{1+x}dx=\int_1^2 \frac{2-(1+x)}{1+x}dx$

$$= \int_1^2 \left(\frac{2}{1+x} - 1\right) dx = [2\log(1+x) - x]_1^2$$

$$= 2[\log(1+x)]_1^2 - [x]_1^2$$

$$= 2[\log 3 - \log 2] - [2-1] = 2\log\frac{3}{2} - 1$$

Q.20. $\int \frac{x}{(x^2+1)(x^2+2)} dx$ **is equal to**

(*a*) $\log\left|\frac{x^2+1}{x^2+2}\right| + c$ (*b*) $\frac{1}{2}\log\left|\frac{x^2+1}{x^2+2}\right| + c$

(*c*) $\frac{1}{2}\log\left|\frac{x^2+1}{x^2+1}\right| + c$ (*d*) $-\log\left|\frac{x^2+1}{x^2+2}\right| + c$

[June 2016]

Solution : (*b*) is correct.

$$I = \frac{1}{2}\int \frac{2x \quad dx}{(x^2+1)(x^2+2)}$$

let $t = x^2$

dt $= 2xdx$

$$\therefore \quad I = \frac{1}{2}\int \frac{dt}{(t+1)(t+2)}$$

$$= \frac{1}{2}\int \left(\frac{1}{t+1} - \frac{1}{t+2}\right) dt$$

$$= \frac{1}{2}[\log(t+1) - \log(t+2)] + C$$

$$= \frac{1}{2}\log\frac{t+1}{t+2} + C$$

$$= \frac{1}{2}\log\left(\frac{x^2+1}{x^2+2}\right) + C$$

where C = Integration Constant.

(b) is correct.

Q.21. $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}} dx$ **is equal to**

(*a*) $\frac{3\sqrt{2}}{\log_e 3}$ (*b*) 0 (*c*) $\frac{2}{\log 3}\left(3^{\sqrt{2}} - 1\right)$ (*d*) None

[June 2016]

Solution : (c)

let $I = 2\int \frac{3^{\sqrt{x}}}{2\sqrt{x}} dx$

let $t = \sqrt{x}$

$$dt = \frac{1}{2\sqrt{x}} dx$$

$$I = 2\int 3^t dt = 2 \times \frac{3^t}{\log_e 3} + C$$

$$\therefore \int_0^2 \frac{3^{\sqrt{x}}}{2\sqrt{x}} dx \quad = 2\left[\frac{3^{\sqrt{x}}}{\log_e 3}\right]_0^2$$

$$= 2\left[\frac{3^{\sqrt{2}} - 1}{\log 3}\right] = \frac{2}{\log 3}\left(3^{\sqrt{2}} - 1\right)$$

Q.22. $\int_1^e \frac{e^x (x\log_e x + 1)}{x} dx =$

(*a*) $e-1$ (*b*) e^e (*c*) $e^e - 1$ (*d*) None

[Dec. 2016]

Solution : (*b*) is correct.

$$\int_1^e \frac{e^x (x\log_e x + 1)}{x} dx =$$

$$= \int_1^e e^x \left(\frac{x\log_e x}{x} + \frac{1}{x}\right) dx$$

$$= \int_1^e e^x \left(\log x + \frac{1}{x}\right) dx = \left[e^x \log_e x\right]_1^e$$

$$= = e^e \log_e e - e^1 \log_e 1$$

$$= e^e \times 1 - 0 = e^e$$

Q.23. Evaluate $\int_1^2 \frac{x}{x^2+1}dx$

(a) $\log_e(5/2)$ (b) $\frac{1}{2}\log_e(5/2)$ (c) $\log_{e^5}-\log_{e^2}$ (d) None

[June 2017]

Solution : $I=\int_1^2 \frac{x}{x^2+1}dx=\frac{1}{2}\int_1^2 \frac{2x}{x^2+1}dx$

$=\frac{1}{2}\left[\log\left(x^2+1\right)\right]_1^2=\frac{1}{2}[\log 5-\log 2]$

$=\frac{1}{2}\log 5/2$

$\therefore$ Option (b) is correct.

Q.24. $\int e^x[f(x)+f'(x)]dx=$:

(a) $e^x.f(x)+c$ (b) $e^x.f'(x)+c$

(c) $\frac{f'(x)}{f(x)}+c$ (d) $e^x.\frac{f(x)}{f'(x)}+c$

[Dec. 2017]

Solution : (a)

It is formula

Q.25. $\int x.e^{x^2}.dx=$ _____

(a) $2.e^{x^2}+c$ (b) $\frac{1}{3}e^{x^2}+c$ (c) $\frac{1}{2}e^{x^2}+c$ (d) None of these

[Dec. 2017]

Solution : (c)

$I=\frac{1}{2}\int 2x\,.\,e^{x^2}\,dx=\frac{1}{2}\int e^{x^2}\,.\,2x\,dx$

let $t=x^2$

$dt=2xdx$

$\therefore I=\frac{1}{2}\int e^t dt=\frac{1}{2}e^t+C=\frac{1}{2}e^{x^2}+C$

Where C = Integration constant

Q.26. $\int_0^1 \log\left(\frac{1}{x}-1\right)dx$ **is equal to :**

(a) 0 (b) 0 (c) 2 (d) -1

[June 2018]

Solution : (b)

$I=\int_0^1\left[\log(1-x)-\log x\right]dx$ - (1)

$I=\int_0^1\left[\log\{1-(1-x)\}-\log(1-x)\right]dx$

$\left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$I=\int_0^1\left[\log x-\log(1-x)\right]dx$ - (2)

Eqn. (1) + Eqn. (2) ; we get

$2I=\int_0^1 0.dx=0$

$I=0$

Q.27. If $f(x)=\frac{x}{x+\sqrt{1+x^2}}$ **then** $\int_0^1 f(x)\,dx$:

(a) $\frac{2}{3}\left(\sqrt{2}-1\right)$ (b) $\frac{2}{3}\left(\sqrt{2}-1\right)$ (c) $\frac{2}{3}\left(1-\sqrt{2}\right)$ (d) $\frac{3}{2}\left(1-\sqrt{2}\right)$

[June 2018]

Solution : (a)

$f(x)=\frac{x}{x+\sqrt{1+x^2}}\times\frac{x-\sqrt{1+x^2}}{x-\sqrt{1+x^2}}$

$=\frac{x^2-x\sqrt{1+x^2}}{x^2-1-x^2}=x\sqrt{1+x^2}-x^2$

$\therefore I=\int_0^1 f(x)dx=\int_0^1\left\{\sqrt{1+x^2}.\frac{2x}{2}-x^2\right\}dx$

$$= \left[\frac{1}{2}\frac{(1+x^2)^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{3}\left[(1+x^2)^{3/2} - x^3\right]_0^1$$

$$= \frac{1}{3}\left[\left(2^{3/2}-1\right) - \left(1^{3/2}-0\right)\right] = \frac{1}{3}\left[2\sqrt{2}-1-1\right]$$

$$= \frac{1}{3}\left(2\sqrt{2}-2\right) = \frac{2}{3}\left(\sqrt{2}-1\right)$$

Q.28. $\int_0^5 \frac{x^2}{x^2+(5-x)^2}$ **dx is equal to :**

(*a*) 0 (*b*) 5/2 (*c*) 1 (*d*) -1

Solution : (*b*)

$$I = \int_0^5 \frac{x^2}{x^2+(5-x)^2}dx$$

$$I = \int_0^5 \frac{(5-x)^2}{(5-x)^2+x^2}dx$$

Adding them ; we get

$$2I = \int_0^5 \frac{x^2+(5-x)^2}{x^2+(5-x)^2}dx$$

$$\int_0^5 dx = [x]_0^5 = 5-0 = 5$$

$$\therefore 2I = 5 \Rightarrow I = 5/2$$

Q.29. The value of $\int_1^2 \frac{1-x}{1+x}dx$ **is equal to :**

(*a*) $\log\frac{3}{2}-1$ (*b*) $2\log\frac{3}{2}-1$ (*c*) $\frac{1}{2}\log\frac{3}{2}-x$ (*d*) $\frac{1}{2}\log\frac{2}{3}-1$

[May 2018]

Solution : (*b*)

$$I = \int_1^2 \frac{1-x}{1+x}dx$$

$$= \int_1^2 \frac{2-1-x}{1+x}dx = \int_1^2 \frac{2-(1+x)}{1+x}dx$$

$$= \int_1^2 \left(\frac{2}{1+x} - \frac{1+x}{1+x}\right)dx$$

$$= \left[2\cdot\log(1+x) - x\right]_1^2$$

$$= \left[2\log(1+2)-2\right] - \left[2\log(1+1)-1\right]$$

$$= \left[\log 3 - \log 2\right] - 2 + 1$$

$$= 2\log\frac{3}{2} - 1$$

(*b*) is correct.

Q.30. $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}dx$ **is equal to**

(*a*) $\frac{2\sqrt{2}}{\log_e 3}$ (*b*) 0 (*c*) $\frac{2(3\sqrt{2}-1)}{\log_e 3}$ (*d*) $\frac{3\sqrt{2}}{\sqrt{2}}$

[May 2018]

Solution : (*c*)

$$I = \int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}dx = 2\int_0^2 3^{\sqrt{x}}\frac{1}{2\sqrt{x}}dx$$

let $t = \sqrt{x}$

$$dt = \frac{1}{2\sqrt{x}}dx$$

If $x = 0 \Rightarrow t = \sqrt{0} = 0$

and if $x = 2 \Rightarrow t = \sqrt{2}$

$$\therefore I = 2\int_0^{\sqrt{2}} 3^t \cdot dt = 2\left[\frac{3^t}{\log_e 3}\right]_0^{\sqrt{2}}$$

$$= \frac{2}{\log_e 3}\left[3^t\right]_0^{\sqrt{2}} = \frac{2}{\log 3}\left[3^{\sqrt{2}} - 3^0\right]$$

$$= \frac{2\left(3^{\sqrt{2}}-1\right)}{\log 3}$$

$\therefore$ (c) is correct.

Q.31. The value of $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}\,dx$ is :

(*a*) 0 (*b*) 3 (*c*) 2 (*d*) 1

[May 2018]

Solution : (*d*)

Tricks:-

$$\int_a^b \frac{x^n}{x^n+(a+b-x)^n}dx = \frac{b-a}{2}$$

$$\Rightarrow \int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}dx = \frac{2-0}{2} = 1.$$

Q.32. $\int x\left(x^2+4\right)^5 dx$ is equal to

(*a*) $\frac{1}{12}\left(x^2+4\right)^6+c$ (*b*) $\left(x^2+4\right)^6+c$

(*c*) $\frac{1}{6}\left(x^2+4\right)^6+c$ (*d*) None

[Nov. 2018]

Solution : (*a*)

$$I = \int x\left(x^2+4\right)^5 dx$$

$$= \frac{1}{2}\int \left(x^2+4\right)^5 \cdot 2x\,dx$$

$$= \frac{1}{2}\frac{\left(x^2+4\right)^6}{6}+C$$

$$= \frac{\left(x^2+4\right)^6}{12}+C$$

Q.33. $\int_{-1}^{3}\left(1+3x-x^3\right)dx$ is equal to

(*a*) –3 (*b*) –4 (*c*) 3 (*d*) 4

Solution : (*b*)

$$\int_{-1}^{3}\left(1+3x-x^3\right)dx$$

$$= \left[x+3\frac{x^2}{2}-\frac{x^4}{4}\right]_{-1}^{3}$$

$$= [x]_{-1}^{3}+\frac{3}{2}\left[x^2\right]_{-1}^{3}-\frac{1}{4}\left[x^4\right]_{-1}^{3}$$

$$= 3-(-1)+\frac{3}{2}\left[3^2-(-1)^2\right]-\frac{1}{4}\left[3^4-(-1)^4\right]$$

$$= 3+1+\frac{3}{2}(9-1)-\frac{1}{4}(81-1)$$

$$= 4+12-20 = -4$$

Q.34. $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx =$

(*a*) 1 (*b*) 1/2 (*c*) 2 (*d*) 3/2

[June 2019]

Solution : (*b*)

Tricks:-

$$\int_2^3 \frac{\sqrt{x}}{\sqrt{3+2-x}+\sqrt{x}}dx = \frac{3-2}{2} = \frac{1}{2}.$$

Q.35. $\int e^x\left(x^2+2x\right)dx =$

(*a*) $x^2\cdot e^x+c$ (*b*) $e^x\cdot x+c$

(*c*) $-e^x\cdot x^2+c$ (*d*) $-e^x\cdot x+c$

[June 2019]

Solution : (*a*)

Formula

$$I = \int e^x.\left\{f(x)+f^1(x)\right\}dx$$

$$= e^x.f(x)+c.$$

$$\therefore \int e^x.\left(x^2+2x\right)dx = e^x.x^2+c$$

Q.36. $\int \log\left(a^x\right)dx =$

(a) $\log a\left(\frac{x^2}{2}\right)+c$ (b) $\log a\left(\frac{x}{2}\right)+c$

(c) $x\log a^x - x + c$ (d) $x\log a^x + c$

[June 2019]

Solution : (a)

$$\int \log\left(a^x\right)dx$$

$$= \int x.\log a\,dx$$

$$= \log a.\int x.dx$$

$$= \log a.\left(\frac{x^2}{2}\right)+c$$

Where C = Integration Constant

(a) is correct.

Q.37. Find the value of $\int xe^x dx$

(a) $e^x(x-1)+c$ (b) $e^x(2x-1)+c$

(c) $e^x(x-1)$ (d) None of these

[Dec. 2019]

Solution : (a)

$$I = \int xe^x dx$$

By Parts ; we get

$$I = x\int e^x dx - \int\left\{\frac{dx}{dx}\int e^x dx\right\}dx$$

$$= xe^x - \int 1.e^x dx$$

$$= xe^x - e^x + c$$

$$= e^x.(x-1)+c$$

Q.38. Find the value of $\int (4x+5)^6 dx$ **is equal to**

(a) 1/7(4x+5)^7 + c (b) 1/28(4x+5)^7 + C

(c) 1/4(4x+5)^7 + c (d) None of these

[Dec. 2019]

Solution : (b)

$$\int (4x+5)^6 dx = \frac{(4x+5)^7}{7\times 4}+c$$

$$= \frac{1}{28}(4x+5)^7 + c$$

Where C = Integration constant.

Q.39. $\int_{-1}^{1}\left(2x^2 - x^3\right)dx =$

(a) 14 (b) 104 (c) $\frac{2x^3}{3}-\frac{x^4}{4}$ (d) $\frac{4}{3}$

[Dec. 2019]

Solution : (d)

$$I = \int_{-1}^{1}\left(2x^2 - x^3\right)dx$$

$$= \left[2\cdot\frac{x^3}{3}-\frac{x^4}{4}\right]_{-1}^{+1}$$

$$= \frac{2}{3}\left[x^3\right]_{-1}^{+1} - \frac{1}{4}\left[x^4\right]_{-1}^{+1}$$

$$= \frac{2}{3}\left[1^3-(-1)^3\right]-\frac{1}{4}\left[1^4-(-1)^4\right]$$

$$= \frac{2}{3}(1+1)-\frac{1}{4}(1-1)$$

$$= \frac{4}{3}-0=\frac{4}{3}$$

Q.40. $\int_{-1}^{1}(e^x - e^{-x})dx = ?$

(*a*) 0 (*b*) 1 (*c*) 12 (*d*) None

[Dec. 2020]

Solution : (*a*) is correct.

Let $f(x) = e^x - e^{-x}$

then $f(-x) = -(e^x - e^{-x}) = -f(x)$

So, the function is a "ODD" function.

so; $\int_{-1}^{1}(e^x - e^{-x})dx = 0$

Q.41. $\int_{1}^{2}\frac{1-x}{1+x}dx$

(*a*) 2 log 3/2 - 1 (*b*) 2 log 3 + 1 (*c*) ½ log 3/2 - 1 (*d*) 2 log 2 – 1 + k

[Dec. 2020]

Solution : (*a*) is correct $\int_{1}^{2}\frac{1-x}{1+x}dx = \int_{1}^{2}\frac{2-1-x}{1+x}dx = \int_{1}^{2}\frac{2-(1+x)}{1+x}dx$

$= \int_{1}^{2}\left(\frac{2}{1+x}-1\right)dx = \left[2\log(1+x)-x\right]_1^2$

$= 2\left[\log(1+x)\right]_1^2 - [x]_1^2$

$= 2[\log 3 - \log 2] - [2-1] = 2\log\frac{3}{2} - 1$

Q.42. The value of $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}dx$ **is :**

(*a*) 0 (*b*) 3 (*c*) 2 (*d*) 1

[Dec. 2020]

Solution : (*d*)

Tricks:-

$$\int_a^b \frac{x^n}{x^n + (a+b-x)^n}dx = \frac{b-a}{2}$$

$$\Rightarrow \int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}dx = \frac{2-0}{2} = 1.$$

Q.43. $\int_1^2 e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx =$

(*a*) $e\left(\frac{e}{2}-1\right)$ (*b*) $e(e-1)$ (*c*) a (*d*) $e^2(e-i)$

[Jan. 2021]

Solution : (*a*) is correct

$$I = \int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$$

$$\int e^x [f(x) + f'(x)]dx = e^x f(x) + c$$

$$\therefore \quad I = \int e^x \frac{1}{x} - \frac{1}{x^2}dx$$

$$= e^x \frac{1}{x}$$

Now $$I = \int_1^2 e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx = \left[e^x \frac{1}{x}\right]_1^2$$

$$= \frac{e^2}{2} - \frac{e^1}{1} = e\left(\frac{c}{2} - 1\right)$$

$\therefore$ (*a*) is correct

Q.44. The value of $\int_{-2}^{2} f(x)\,dx$ **, where** $f(x) = 1 + x, \le 0$; $f(x) = 1-2x, x \ge 0$ **is**

(*a*) 20 (*b*) -2 (*c*) -4 (*d*) 0

[July 2021]

Solution : (*b*) is correct

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_0^2 f(x)\,dx$$

$$= \int_{-2}^{0}(1+x)\,dx + \int_0^2 (1-2x)\,dx$$

$$= \left[x + \frac{x^2}{2}\right]_{-2}^{0} + \left[x - 2\frac{x^2}{2}\right]_0^2$$

$$= 0 + 0 - \left[(-2) + \frac{(-2)^2}{2}\right] + [(2 - 22) - 0 - 0]$$

$$= -[-2 + 2] + (-2)$$

$$= -2$$

Q.45. Integrate with respect to x, $1/[x(\log x)^2]$

(*a*) $-1/\log x + k$ (*b*) $1/\log x + k$ (*c*) $\log x$ (*d*) x

[Dec. 2021]

Solution : (*a*)

$$I = \int \frac{1}{x(\log x)^2} dx$$

Let $t = \log x$

$$\Rightarrow dt = \frac{1}{x} dx$$

$$I = \int \frac{1}{(\log x)^2} \frac{1}{x} dx$$

$$= \int \frac{1}{t^2} . dt = \int t^{-2} dt$$

$$= \frac{t^{-2+1}}{-2+1} + K = \frac{t^{-1}}{-1} + K$$

$$= \frac{1}{-t} + K$$

$$= = \frac{-1}{\log x} + K$$

Where

K = Integration constant

Q.46. $\int_0^1 xe^x dx$

(*a*) -1 (*b*) 1 (*c*) e^1 (*d*) e^{-1}

[June 2022]

Solution : $I = \int_0^1 xe^x dx$

By Parts ; we get

$$\int xe^x dx = x\int e^x dx - \int \left\{\frac{dx}{dx}\int e^x dx\right\} dx$$

$$= xe^x - \int 1 . e^x dx$$

$$= xe^x - e^x$$

$$= e^x (x - 1)$$

$$\therefore I = \int_0^1 xe^x dx = \left[e^x (x-1)\right]_0^1$$

$$= e^1 (1-1) - e^0(0 - 1)$$

$$= 0 - 1. (-1) = +1$$

$\therefore$ (*b*) is correct

Q.47. Determine f(x), given that $f'(x) = 12x^2 - 4x$ and $f(-3) = 17$.

(*a*) $f(x) = 4x^3 - 2x^2 + 143$ (*b*) $f(x) = 6x^2 - x^4 + 137$

(*c*) $f(x) = 3x^4 - x^3 - 137$ (*d*) $f(x) = 4x^3 - 2x^2 - 143$

[June 2022]

Solution : $f(x) = \int f^1(x) dx$

$$\int (12x^2 - 4x)\, dx$$

$$= 12. \frac{x^3}{3} - 4.\frac{x^2}{2} + c$$

where c = integration constant.

$\therefore f(x) = 4x^3 - 2x^2 + C$.....(1)

Now $f(-3) = 4(-3)^3 - 2(-3)^2 + c$

or; $17 = -108 - 18 + C$

$\therefore C = 143$

$\therefore$ From (1)

$f(x) = 4x^3 - 2x^2 + 143$

(*a*) is correct

Q.48. Find the area under the curve $f(x) = x^2 + 5x + 2$ with the limits 0 to 1.

(*a*) 3.833 (*b*) 4.388 (*c*) 4.833 (*d*) 3.338

[Dec. 2022]

Solution: Required Area $= \int_0^1 f(x) dx$

$$= \int_0^1 (x^2 + 5x + 2) dx$$

$$= \left[\frac{x^3}{3} + 5 \times \frac{x^2}{2} + 2x\right]_0^1$$

$= \frac{1}{3}\left[x^3\right]_0^1 + \frac{5}{2}\left[x^2\right]_0^1 + 2\left[x\right]_0^1$

$= \frac{1}{3}\left[1^3 - 0\right] + \frac{5}{2}\left[1^2 - 0\right] + 2\left[1 - 0\right]$

$= \frac{1}{3} + \frac{5}{2} + 2 = \frac{2+15+12}{6} = \frac{29}{6} = 4.833$

$\therefore$ (*c*) is correct.

Q.49. $\int (2x-3)^6\, dx$ **is**

(*a*) $\frac{(2x-3)^6}{6}$ (*b*) $\frac{(2x-3)^6}{2}$ (*c*) $\frac{(2x-3)^6}{12}$ (*d*) $\frac{(2x-3)^6}{3}$

[Dec. 2022]

Solution: $\int (2x-3)^5 dx$

$= \frac{(2x-3)^{5+1}}{(5+1)\times 2} + C$

$= \frac{(2x-3)^6}{12} + C$

(*c*) is correct

Q.50. $\int_2^4 \frac{x\,dx}{x^2+1}$ **is**

(*a*) $\frac{1}{2}\log\left(\frac{17}{5}\right)$ (*b*) $2\log\left(\frac{17}{5}\right)$ (*c*) $\frac{1}{2}\log\left(\frac{5}{17}\right)$ (*d*) $2\log\left(\frac{5}{17}\right)$

[Dec. 2022]

Solution: $I = \int_2^4 \frac{x}{x^2+1} dx$

$= \frac{1}{2}\int_2^4 \frac{2x}{x^2+1} dx$

$= \frac{1}{2}\left[\log\left|x^2+1\right|\right]_2^4$

$\left[Tricks: -\int \frac{f^1(x)}{f(x)} dx = \log|f(x)|\right]$

$= \frac{1}{2}\left[\log(4^2+1) - \log(2^2+1)\right]$

$= \frac{1}{2}\log\frac{17}{5}$

So, (*c*) is correct.

Q.51. Evaluate the integral $f \frac{1}{(x-1)(x-2)} dx$

(*a*) $\log\left(\frac{x-2}{x-1}\right) + C$

(*b*) $\log[(x-2)(x-1)] + C$

(*c*) $\log\left(\frac{x-1}{x-2}\right) + C$

(*d*) $\log[(x-2)(x+1)] + C$

[June 2023]

Solution:

$I = \int \frac{1}{(x-1)(x-2)} dx$

$= \int\left(\frac{1}{x-2} - \frac{1}{x-1}\right) dx$

$= \log(x-2) - \log(x-1) + C$

where C = Integration constant

$= \log\left(\frac{x-2}{x-1}\right) + C$

(*a*) is correct.

16 CHAPTER

NUMBER SERIES, CODING & DECODING

Number Series

Arrangement of numbers which follows a particular rule in a certain order is called **Number series.** In the series, some number is wrongly putted OR some number is missing. We need to observe and find the accurate number to the series of numbers.

Examples : 2, 4, 6, 8, ... is a series, because it follows a particular rule/pattern. According to this rule, next number will be 10.

First of all we have to decide that what type of series are given in papers & according to that we have to use shortcut tricks as fast as we can.

Different types of Number Series

Perfect Square Series:

This type of Series are based on square of a number which is in same order and one square number is missing in that given series.

Example 1 : 400, 441, 484, 529, 576 , ?

Ans: $400 = 20^2$; $441 = 21^2$, $484 = 22^2$, $529 = 23^2$,
$576 = 24^2$, **$625 = 25^2$**.

Perfect Cube Series:

This type of Series are based on cube of a number which is in same order and one cube number is missing in that given series.

Example 2 : 1000, 1331, 1728, 2197, ?

Ans : $1000 = 10^3$, $1331 = 11^3$, $1728 = 12^3$, $2197 = 13^3$, **$2744 = 14^3$**

Geometric Series:

This type of series are based on ascending or descending order of numbers and each successive number is obtain by multiplying or dividing the previous number with a fixed number.

Example 3: 45, 405, 3645, 32805 , ?

Ans: $45 \times 9 = 405$, $405 \times 9 = 3645$, $3645 \times 9 = 32805$; **$32805 \times 9 = 295245$**

Difference Series: Here difference between two consecutive terms OR difference between alternate terms are equal OR these difference make an another series.

Example 4 : 7, 12, 17, 22, ...

Ans. Here difference between two consecutive terms is equal i.e. 5 ; **Next term = 22 +5 = 27**

Ratio Series : Here the ratio OR division of two consecutive terms are equal. It is same pattern for alternate terms.

Example 5: 8, 24, 72, 216, ...

Ans. 24 / 8 = 3; 72/24 = 3; 216 /72 = 3 ; **Next term = $216 \times 3 = 648$**

Prime Number : Series of prime numbers.

Example 6 : 2, 5, 11, 19, 29, ...

Ans. It is a series of prime numbers. Here, between two consecutive prime numbers, there is one missing prime number. After 29 next prime number is 31. Leave it. Hence **Next term = 37**

Mixed Series : It is combination of two or more different type of series.

Example 7 : 3, 4, 7, 9, 11, 16, 15, 25, 19, ...

Here 1st, 3rd, 5th, 7th, 9th term are in Arithmetic progression while 4, 9, 16, are squares of consecutive numbers. **Hence Next Term = $5^2 = 25$.**

Arithmetic Progression / Series : Here arrangement of numbers makes an Arithmetic Progression.

Example 8 : 100, 95, 90, 85, ...

It is in AP. Here difference between two consecutive terms is equal. **Hence Next Term = 80.**

Geometric Progression / Series. : Here arrangement of numbers makes a Geometric Progression.

Example 9 : 4, 12, 36, 108, ...

It is in GP. Here ratio between two consecutive terms is equal.

Hence Next Term = $108 \times 3 = 324$.

Two-Tier Arithmetic or Geometric Series : - In this series, difference between two consecutive terms makes an Arithmetic or Geometric progression or series.

Example 10 : 2, 5, 10, 17, 26, ...

Here (5 - 2 = 3) ; (10 - 5 =5) ; (17 - 10 = 7) ; (26 - 17 = 9) i.e. difference of consecutive terms 3; 5; 7; 9; ...are making Arithmetic progression. Hence **next difference = 11.**

& Next term = 26 + 11 = 37

Example 11 : 2, 5, 11, 23, 47, ...

Here (5 - 2 = 3); (11 - 5 = 6) ; (23 - 11=12) ; (47 - 23=24) i.e. difference of consecutive terms 3; 6; 12; 24; ...are making Geometric progression. Hence **next difference = 48.**

& Next term = 47 + 48 = 95

Alphabet Series: -

Questions are also asked relating to positions of English alphabets. This topic is very important from examination point of view. So we need to memorize the positions of the alphabets so that we can handle each and every question based on this topic. If it is not memorised, then write these letters sequence as given below on blank page of the Question paper with their positions from both ends as given below :

Going forward & backward both.

26	**25**	**24**	**23**	**22**	**21**	**20**	**19**	**18**	**17**	**16**	**15**	**14**
A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
13	**12**	**11**	**10**	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**
N	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**

On that basis we will try to understand the logic behind the series or sequence of letters, numbers or alphanumeric. There are some important assumptions given below for solving the letter series questions.

Check some points as (i) whether it is a continuous series or not (ii) Missing terms in the series (iii) Immediate next or before term of the series (iv) Squares or cubes of numbers (v) Mathematical operations like addition, subtraction, multiplication or division (vi) Arithmetic or Geometric series or Harmonic series.

Example 1:

E, J, O, ?

(*a*) T (*b*) S (*c*) V (*d*) None

Solution : First, we write the position of letters of given question. They are 5, 10, 15, in AP

Hence next position = 20. Letter at this position is T.

So, (*a*) is correct.

Example 2 :

C, G, K, O, ?

(*a*) T (*b*) S (*c*) U (*d*) None

Solution : First, we write the position of letters of given question. They are 3, 7, 11, 15, in AP

Hence next position = 19. Letter at this position is S.

So, (*b*) is correct.

Example 3 :

A, D, H, M, S, ?

(*a*) X (*b*) Y (*c*) Z (*d*) None

Solution : First, we write the position of letters of given question. They are 1, 4, 8, 13 , 19

It can be understand as (1 + 3 =4); (4 + 4 = 8) ; (8 + 5 = 13); (13 + 6 = 19).

So , Next term should be 19 + 7 = 26.

Hence, Letter at 26th position is Z.

So, (c) is correct.

Example 4 : AK , FP , KU , ? , UE , ZJ

(*a*) PZ (*b*) KT (*c*) JU (*d*) None

Solution :- (*a*) is correct.

First , we write the position of letters of given series. They are:

(1, 11), (6, 16), (11, 21) , ?, (21, 5), (26, 10).

Clearly we see that the first numbers of each pair making a definite pattern as 1, 6, 11, ?, 21, 26;

Difference between two successive terms = 5. So, 4th term = 11 + 5 = 16. Letter at this position is P.

Similarly, we see that the 2nd numbers of each pair making a definite pattern as 11, 16, 21, ?, 5, 10.

If all alphabets are put on circle to make it as cyclic order, we consider that after Z (26th position), position will start from A i.e. 1st position. Here, try to understand the sequence as (11+ 5 = 16) ;

(16 +5 =21) ; (21 + 5 = 26); (0 + 5 = 5) ; (5 + 5 =10). So 4th term = (21 + 5 = 26). Letter at this position = Z. Hence ordered pair in terms of letter is (PZ).

Hence option (a) is correct.

Letter Series & Continuous Pattern Series

Letter Series type questions usually consists of a series of small letters which follow a specific pattern. However, some letters are missing from the set. The students need to identify the missing letter in the series. These missing letters are then given in proper order as one of the alternatives. Thus, the candidates can know the tips to solve the Letter Series Logical Reasoning Questions by practicing. To understand the tips of the Letter Series, first of all, the candidates must have the knowledge of the alphabetical order, its numerical position, and its opposite word. Some examples are given below :

Example 1. QPO, NML, KJI, _____, EDC

(*a*) HGF (*b*) CAB (*c*) JKL (*d*) GHI

Solution : (*a*) is correct. All terms in reverse order.

Directions : In each of the following letter series, some of the letters are missing which are given in that order as one of the alternatives below it. Choose the correct alternative.

Example 2. __ __ aba __ __ ba __ ab

(*a*) abbba (*b*) abbab (*c*) baabb (*d*) bbaba

Solution : (*b*) : The series is ab/ab/ab/ab/ab/ab. Thus, the pattern 'ab' is repeated.

Odd Man Out : In this type of question, all options follow a particular character except one option. That exception a option is called **Odd Man Out or Odd One Out.**

Example 1 : Find odd man out of the following:

2, 3, 5, 7, 11, 13, 17.

(*a*) 2 (*b*) 11 (*c*) 7 (*d*) 13

Solution : (*a*) is correct.

All are odd & prime number except 2 which is even & prime number.

Example 2 : Find odd man out of the following:

1, 3, 5, 7, 11, 13, 17.

(*a*) 1 (*b*) 11 (*c*) 7 (*d*) 13

Solution : (*a*) is correct.

All are prime number except 1 which is neither prime nor composite.

Example 3 : Find odd man out of the following:

(*a*) 13 (*b*) 17 (*c*) 23 (*d*) 63

Solution :- Each of the numbers except 63, is a prime number. Hence, the answer is (*d*).

Example 4 : Find odd man out of the following:

(*a*) 15 (*b*) 25 (*c*) 37 (*d*) 49

Solution : Clearly, 37 is the only prime number in the group. Hence, the answer is (c).

Example 5 : Find odd man out of the following:

(*a*) 16 (*b*) 36 (*c*) 78 (*d*) 256

Solution : Each of the numbers except 78, is a perfect square. Hence, the answer is (*c*).

Example 6 : Find odd man out of the following:

(*a*) 8 (*b*) 28 (*c*) 343 (*d*) 125

Solution : Each of the numbers except 28, is a perfect cube. Hence, the answer is (*b*).

Example 7 : Find odd man out of the following:

(*a*) 21 (*b*) 36 (*c*) 49 (*d*) 56

Solution : Each of the numbers except 36, is divisible by 7. Hence, the answer is (*b*).

Example 8 : Find odd man out of the following:

(*a*) 295 (*b*) 381 (*c*) 552 (*d*) 729

Solution : Each of the numbers except 552, is an odd number. Hence, the answer is (*c*).

Example 9 : Find odd man out of the following:

1, 4, 9, 16, 19, 36, 49, 64, 81

(*a*) 9 (*b*) 19 (*c*) 49 (*d*) 16

Solution : (*b*) is correct.

All except 19 are perfect square. Square root of 19 does not give a whole number.

Hence, the answer is (*b*).

CODING - DECODING

A CODE is a system of words, letters, figures, or symbols used to represent others, especially for the purposes of secrecy. Therefore, Coding is a method of transmitting

a message between the sender and the receiver without knowing it by a third person. The Test of Coding and Decoding is set up to judge the candidate's ability.

LETTER CODING

In this case, the letters in a word are replaced by certain other letters according to a specific rule to make its code. The candidate is required to detect the pattern or rule and answer the questions according to that rule.

Example 1 : In a certain code, TEACHER is written as VGCEJGT. How is CHILDREN written in that code ?

(*a*) EJKNEGTP (*b*) EGKNEITP (*c*) EJKNFGTO (*d*) EJKNFTGP

(B.T.M. 2006)

Solution : First of all , compare 1st letter of both words i.e. compare V with T. Clearly V is 2 steps ahead with respect of T. Similarly compare remaining letters as above accordingly. We get as

T	E	A	C	H	E	R
+2 ↓	+2 ↓	+2 ↓	+2 ↓	+2 ↓	+2 ↓	+2 ↓
V	G	C	E	J	G	T

Similarly, posting letters which are 2 steps ahead with respect to each letter of the word CHILDREN; we get the required result as given below.

C	H	I	L	D	R	E	N
+2 ↓	+2 ↓	+2 ↓	+2 ↓	+2 ↓	+2 ↓	+2 ↓	+2 ↓
E	J	K	N	F	T	G	P

Hence, the answer is option (d).

Example 2 : If HEALTH is written as GSKZDG, then how will NORTH be written in that code ?

(*a*) OPSUI (*b*) GSQNM (*c*) FRPML (*d*) IUSPO

(M.C.A. 2005)

Solution : Analysing it, we get that : 1st reverse the order of letters of the word HEALTH as

HTLAEH then compare it with the letters of the coded word GSKZDG. We get as

H	T	L	A	E	H
−1 ↓	−1 ↓	−1 ↓	-1 ↓	-1 ↓	-1 ↓
G	S	K	Z	D	G

Similarly doing as above, we get as : Reversing the order of letters in NORTH, we get HTRON.

H	T	R	O	N
−1↓	−1↓	−1↓	−1↓	−1↓
G	S	Q	N	M

Hence , the required code is GSQNM . So, the answer is (b).

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code?

(*a*) 318826 (*b*) 318286 (*c*) 618826 (*d*) 338816

[May 2018]

Solution :

(*a*) R I P P L E L I F E

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

Then 6 1 3 3 8 2 8 1 9 2

P I L L E R

↓ ↓ ↓ ↓ ↓ ↓

3 1 8 8 2 6.

Hence code of **PILLER is 318826.**

Q.2. In a certain code '256' mean 'you are good', '637' means 'we are bad' and '358' means 'good and bad'. Which of the following represents 'and' in that code ?

(*a*) 2 (*b*) 5 (*c*) 8 (*d*) 3

[May 2018]

Solution :

(c)

256 means 'Your are good'

637 means 'We are Bad'

358 means 'Good and Bad'

Here , word " good " is common in 1st & 3rd sentence and 5 is common in 1st & 3rd. So, Code of good is 5 . BAD is common in 2nd & 3rd. Hence Code of 'bad ' is 3. So, Code of "and" is 8

Q.3. If LOSE is coded as 1357 and GAIN is coded as 2468, what do figure 82146 for ?

(*a*) NGLAI (*b*) NGLIA (*c*) GNLIA (*d*) GNLA

[May 2018]

Solution :

(*a*) L O S E G A I N Hence 8 2 1 4 6

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

1 3 5 7 2 4 6 8 N G L A I

Code for 82146 is NGLAI

Q.4. If PLAY is coded as 8123 and RHYME is coded as 49367. What will be code of MALE ?

(*a*) 6217 (*b*) 6198 (*c*) 6395 (*d*) 6285

[Nov. 2018]

Solution :

(a) If P L A Y ↓ ↓ ↓ ↓ 8 1 2 3 and R H Y M E ↓ ↓ ↓ ↓ ↓ 4 9 3 6 7

Hence Code for word "MALE" IS M A L E ↓ ↓ ↓ ↓ 6 2 1 7

Q.5. Find out the next number in the following series 7, 11, 13, 17, 19, 23, 25, 29 ?

(*a*) 30 (*b*) 31 (*c*) 32 (*d*) 33

[Nov. 2018]

Solution :

(*b*) Given series

7, 11, 13, 17, 19, 23, 25, 29, 31

+4= +2= +4= +2= +4= +2= +4= +2=

Q.6. If HONEY is coded as JQPGA, which word is code as VCTIGVU ?

(*a*) CARPETS (*b*) TRAPETS (*c*) TARGETS (*d*) UMBRELU

[Nov. 2018]

Solution : (*c*)

H O N E Y
+2↓ +2↓ +2↓ +2↓ +2↓
J Q P G A

then

V C T I G V U
−2↓ −2↓ −2↓ −2↓ −2↓ −2↓ −2↓
U A R G E T S

Q.7. Find odd man out of the following series 15, 21, 63, 81, 69

(*a*) 15 (*b*) 21 (*c*) 63 (*d*) 81

[Nov. 2018]

Solution :

(*d*) 15, 21, 63, 81, 69

In this series only 81 is a Perfect square.

Q.8. Find odd man out of the following series 7, 9, 13, 17, 19

(*a*) 7 (*b*) 9 (*c*) 19 (*d*) 13

[Nov. 2018]

Solution :

(*b*) 7, 9, 13, 17, 19

Since only 9 is a perfect square and not prime No. So, 9 is the odd man out.

OR, 9 - 7 = 2 | (11) - 7 = 4
13 - 9 = 4 | 13 - 11 = 2
17 - 13 = 4 | 17 - 13 = 4
19 - 17 = 2 | 19 - 17 = 2

Note :- If 11 is used at the place of 9, we get a perfect series.

Q.9. 7, 23, 47, 119, 167

(*a*) 211 (*b*) 223 (*c*) 287 (*d*) 319

[June 2019]

Solution : (*c*)

7, 23, 47, 119, 167, 287

$3^2 - 2 = 7$

$5^2 - 2 = 23$

$7^2 - 2 = 47$

$11^2 - 2 = 119$

$13^2 - 2 = 167$

$17^2 - 2 = 287$

Q.10. Which of the following is odd one 4, 12, 44, 176, 890?

(*a*) 4 (*b*) 12 (*c*) 44 (*d*) 176

[June 2019]

Solution : (*c*)

4, 12, 44, 176, 890,

$4 \times 2 + 4 = 12$

$12 \times 3 + 6 = (42)$

$42 \times 4 + 8 = 176$

$176 \times 5 + 10 = 890$

Q.11. If in a certain language, MADRAS is coded as NBESBT. How is BOMBAY coded in that language ?

(*a*) CPNCBX (*b*) CPNCBZ (*c*) CPOCBZ (*d*) CQOCBZ

[June 2019]

Solution : (*b*)

M A D R A S ⇒ B O M B A Y
+1↓ +1↓ +1↓ +1↓ +1↓ +1↓ +1↓ +1↓ +1↓ +1↓ +1↓ +1↓
N B E S B T C P N C B Z

Q.12. Which of the following is odd one?

(*a*) CEHL (*b*) KMPT (*c*) OQTX (*d*) NPSV

[June 2019]

Solution: (*d*)

(a)	(b)	(c)	(d)
C E H L	K M P T	O Q T X	N P S V
3 5 8 12	11 13 16 20	15 17 20 24	14 16 19 22
+2 +3 +4	+2 +3 +4	+2 +3 +4	+2 +3 +3

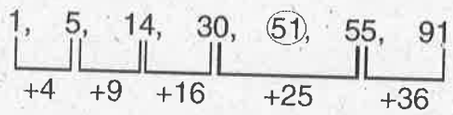
So NPSV is odd one.

Q.13. Find odd one out; 1, 5, 14, 30, 51, 55, 91,?

(*a*) 14 (*b*) 55 (*c*) 51 (*d*) 91

[Dec. 2019]

Solution: (*c*)



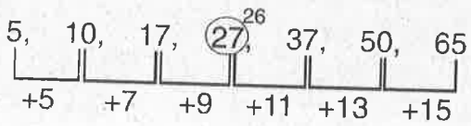
Clearly 51 is odd one out.

Q.14. Find odd one out ; 5, 10, 17, 27, 37, 50, 65 ?

(*a*) 17 (*b*) 27 (*c*) 37 (*d*) 65

[Dec. 2019]

Solution: (*b*)



Clearly (b) 27 is odd one out. 26 should be there.

Q.15. Find the missing figures; 4, 16, 36, 64, 100, ?

(*a*) 92 (*b*) 121 (*c*) 144 (*d*) 169

[Dec. 2019]

Solution: (*c*)

4; 16; 36; 64; 100; ?

$\Rightarrow 2^2; 4^2; 6^2; 8^2; 10^2; 12^2 = 144$

Q.16. If "SYSTEM" be coded 131625; then 'TERMS' may be coded as :—

(*a*) 62251 (*b*) 62451 (*c*) 64951 (*d*) 62415

[Dec. 2019]

Solution: (*b*)

S	Y	S	T	E	M
↓	↓	↓	↓	↓	↓
1	3	1	6	2	5

In word "TERM" "R" is different. Code of R should be different out of 1, 2, 3, 5, 6.

Hence code of R may be 0 ; 4, 7, 8, 9.

That is code for "TERMS" may be 62051 ; 62451 ; 62751 ; 62851 ; 62951.

Clearly (*b*) 62451 is present in the option.

So ; (*b*) is correct.

Q.17. If "MADRAS" written as "NBESBT"; then "DELHI" may be coded as:—

(*a*) EMTIF (*b*) EFMIJ (*c*) JEMFT (*d*) EEMJI

[Dec. 2019]

Solution: (*b*)

	M	A	D	R	A	S		D	E	L	H I
+1	↓	↓	↓	↓	↓	↓	⇒ +1	↓	↓	↓	↓ ↓
	N	B	E	S	B	T		E	F	M	I J

Q.18. Find the missing value in the series 0, 2, 3, 6, 10, 17, 28, ? 75

(*a*) 58 (*b*) 46 (*c*) 48 (*d*) 54

[Dec. 2020]

Solution: 0, 2, 3, 6, 10, 17, 28, ? 75

Rule: sum of two terms + 1 = Next term

(0 + 2) + 1 = 3

(2 + 3) + 1 = 6

(3 + 6) + 1 =10

(6 + 10) + 1 = 17

(10 + 17) + 1 = 28

(17 + 28) + 1 = 46

(28 + 46) + 1 = 75

∴ (*b*) is correct.

Q.19. $\frac{3}{8}, \frac{8}{19}, \frac{18}{41}, ?, \frac{78}{173}$

(*a*) $\frac{38}{85}$ (*b*) $\frac{83}{38}$ (*c*) $\frac{31}{38}$ (*d*) None

[Dec. 2020]

Solution : $\frac{3}{8}, \frac{8}{19}, \frac{18}{41}, ?, \frac{78}{173}$

Series of Numerators.

$3 + 5 = 8$

$8 + 10 = 18$

$\boxed{18 + 20 = 38}$

$38 + 40 = 78$

Series of Denominators.

$8 + 11 = 19$

$19 + 22 = 41$

$\boxed{41 + 44 = 85}$

$85 + 88 = 173$

Hence $? = \frac{38}{85}$

$\therefore$ (*a*) is correct

Q.20. Find Odd man out of the following 6, 9, 12, 18, 21, 26, 30

(*a*) 24 (*b*) 30 (*c*) 26 (*d*) 9

[Dec. 2020]

Solution : In 6, 9, 12, 18, 21, 26, 30

All terms are divisible by 3 except 26.

$\therefore$ 26 is odd man out

(*c*) is correct.

Q.21. If in a certain language HEALTH is coded as IFBMUI then what is the code for NORTH

(*a*) OPSUI (*b*) OPUSI (*c*) OUSPI (*d*) OIPSU

[Dec. 2020]

Solution :

H	E	A	L	T	H		N	O	R	T	H
+1↓	+1↓	+1↓	↓+1	↓+1	↓+1	⇒	+1↓	+1↓	+1↓	↓+1	↓+1
I	F	B	M	U	I		O	P	S	U	I

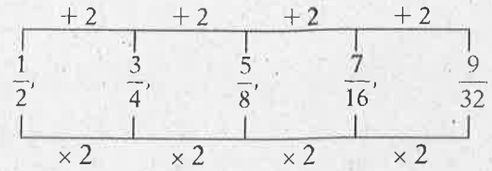
$\therefore$ (*a*) is correct.

Q.22. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16} = ?$

(*a*) 9/32 (*b*) 10/17 (*c*) 11/34 (*d*) 12/35

[Jan. 2021]

Solution : (*a*) is correct



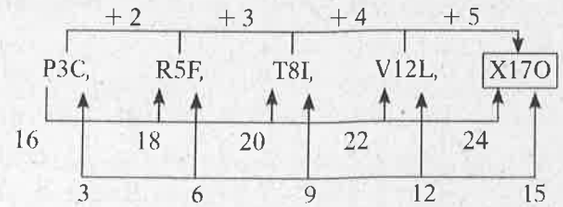
Q.23. Find the missing term

P 3 C; R 5 F; T 8 I; V 12 L; _____?

(*a*) Y17O (*b*) X17M (*c*) X17O (*d*) X16O

[Jan. 2021]

Solution : (*c*) is correct



Q.24. Find out the odd man out in the Sequence 8, 27, 64, 125, 216.

(*a*) 27 (*b*) 64 (*c*) 125 (*d*) 216

[Jan. 2021]

Solution : (*a*) is correct

$8 = 2^3$

$27 = 3^3$

$64 = 4^3$

$125 = 5^3$

$216 = 6^3$

Bases are in A.P. and Powers on bases are equal. Only in 3^3 base and power are equal but in all, base and Power are different.

Q.25. In a certain Code Language BEAT is written as YVZG, then what will be Code for MILD?

(*a*) ONRW (*b*) NOWR (*c*) ONWR (*d*) NROW

[Jan. 2021]

Solution : (*d*)

From left numberings					From left			
2	5	1	20		13	9	12	4
B	E	A	T		M	I	L	D
|	|	|	|	⇒	|	|	|	|
Y	V	Z	G		N	R	O	W
2	5	1	20		13	9	12	4

From right (end) numberings

Clearly option (*d*) is correct.

Q.26. In a certain code RIPPLE is written as 613382, and LIFE is written as 8192. How will RFFLE be written in that code?

(*a*) 618892 (*b*) 689912 (*c*) 619982 (*d*) 629981

[Jan. 2021]

Solution : (*c*) is correct

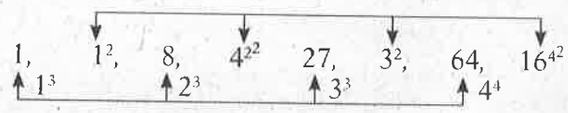
R	I	P	P	L	E		L	I	F	E		R	I	F	F	L	E
|	|	|	|	|	|		|	|	|	|	⇒	|	|	|	|	|	|
6	1	3	3	8	2		8	1	9	2		6	1	9	9	8	2

Q.27. Choose the missing term in the series.... 1, 1, 8, 4, 27, _____ 64, 16

(*a*) 27 (*b*) 11 (*c*) 9 (*d*) 125

[July 2021]

Solution : (*c*) is correct



Hence option (*c*) is correct

Q.28. The wrong term in the series.... 225, 196, 169, 144, 121, 100, 77, 64, is

(*a*) 121 (*b*) 77 (*c*) 100 (*d*) 169

[July 2021]

Solution : (*b*) is correct

225;	196;	169;	144;	121;	100;	77;	64
$= 15^2$	$=14^2$	$= 13^2$	$= 12^2$	$=11^2$	$= 10^2$	$=9^2 = 81$	$=8^2$

Hence "77" is wrong term

Q.29. If DELHI is coded as EFMIJ then JAIPUR is coded as

(*a*) JQVSBK (*b*) QVSKBJ (*c*) BJQVSK (*d*) KBJQVS

[July 2021]

Solution : (*d*) is correct

	D	E	L	H	I			J	A	I	P	U	R
+1	↓	↓	↓	↓	↓	⇒	+1	↓	↓	↓	↓	↓	↓
	E	F	M	I	J			K	B	J	Q	V	S

Q.30. If FRAME is coded as 0618011305 then ARISE is coded as

(*a*) 0118091905 (*b*) 0119091805

(*c*) 0118190905 (*d*) 0118091805

[July 2021]

Solution : (*a*) is correct

F	R	A	M	E		A	R	I	S	E
↓	↓	↓	↓	↓		↓	↓	↓	↓	↓
06	18	01	13	05		01	18	09	19	05

Q.31. If CLOCK is coded as 34235 and TIME as 8679, then MOTEL is coded as

(*a*) 27894 (*b*) 72964 (*c*) 72894 (*d*) 77684

[July 2021]

Solution : (*c*) is correct

C	L	O	C	K		T	I	M	E		M	O	T	E	L
↓	↓	↓	↓	↓		↓	↓	↓	↓	⇒	↓	↓	↓	↓	↓
3	4	2	3	5		8	6	7	9		7	2	8	9	4

Q.32. If MOUSE is coded as 34651 and KEY is coded as 217, then how will YES be coded?

(*a*) 715 (*b*) 517 (*c*) 175 (*d*) 571

[Dec. 2021]

Solution : (*a*)

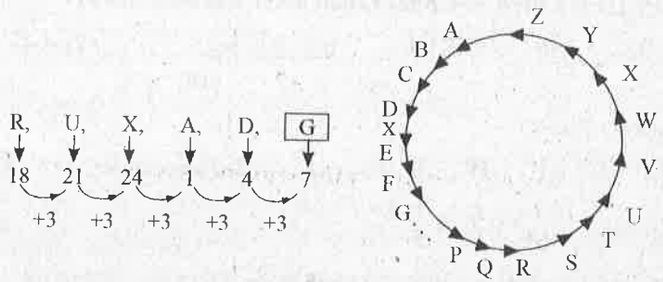
MOUSE	KEY		YES
↓↓↓↓↓	↓↓↓	⇒	↓↓↓
34651	217		715

Q.33. What comes at the last place in R, U, X, A, D, _____?

(*a*) E (*b*) F (*c*) G (*d*) H

[Dec. 2021]

Solution : (*c*)

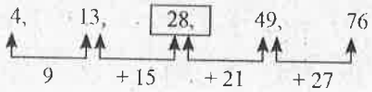


Q.34. The missing term of the series 4, 13, _____, 49, 76 is

(*a*) 26 (*b*) 28 (*c*) 30 (*d*) 32

[Dec. 2021]

Solution : (*b*)



Q.35. Find the odd one from the following :

(*a*) Zebra (*b*) Giraffe (*c*) Horse (*d*) Tiger

[Dec. 2021]

Solution : (*d*)

Tiger is a Meat eating animal But rest are grass eating animal.

Q.36. If in a certain code "THANKS" is written as "SKNTHA", then how is "STUPID" written?

(*a*) DIPUTS (*b*) DISPUT (*c*) DIPUST (*d*) DIPSTU

[Dec. 2021]

Solution : (*d*)

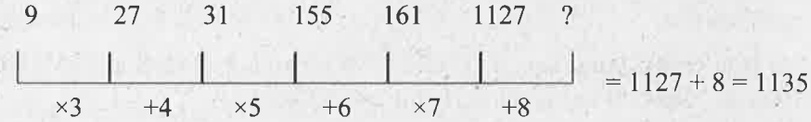
First three letters at the end and last three letters at first place in reverse order.

Q.37. 9,27,31,155,161,1127, ?- Find missing number?

(*a*) 1316 (*b*) 1135 (*c*) 1288 (*d*) 2254

[June 2022]

Solution :



(*b*) is correct.

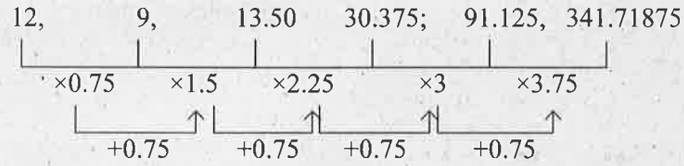
Q.38. What is the missing number in the sequence given below?

12, 9,13.50, 30.375, ?, 341.71875

(*a*) 91.125 (*b*) 89.145 (*c*) 90.475 (*d*) 92.485

[June 2022]

Solution :



∴ (*a*) is correct.

Q.39. 7, 26, 63, 124, 215, 342 ?- Find the missing number.

(*a*) 391 (*b*) 421 (*c*) 481 (*d*) 511

[June 2022]

Solution : 7; 26; 63; 124; 215; 342; 511

2^3-1 3^3-1 4^3-1 5^3-1 6^3-1 7^3-1 8^3-1 =511

(*d*) is correct.

Q.40. In a certain code 'TELEPHONE' is written as ENOHPELET. Then ALIGATOR is written as

(*a*) ROTAGILA (*b*) ROTAGAIL (*c*) ROTAGILE (*d*) TOTAGILA

[June 2022]

Solution : TELEPHONE→ENOHPELET

Reversing its order

ALIGATOR→ROTAGILA

Reversing its order

∴ (*a*) is correct

Q.41. In a coded language, if 'EARTH' is written as 34215 and 'VENUS' is written as 73089. What is the code for "SATURN"?

(*a*) 941012 (*b*) 941820 (*c*) 914281 (*d*) 912418

[June 2022]

Solution :

E	A	R	T	H		V	E	N	U	S
↓	↓	↓	↓	↓		↓	↓	↓	↓	↓
3	4	2	1	5		7	3	0	8	9

S	A	T	U	R	N	(Copying Codes of letters)
↓	↓	↓	↓	↓	↓	
9	4	1	8	2	0	

∴ (*b*) is correct

Q.42. If 'FROZEN' is decoded as 'OFAPSG'. Tick the right option that depicts 'MOLTEN' written in this way?

(*a*) OFPOMN (*b*) OFSMPN (*c*) OFUMPN (*d*) OFUNPN

[Dec. 2022]

Solution :

FROZEN $\xrightarrow{\text{Reversing Order}}$

N	E	Z	O	R	F
↓	↓	↓	↓	↓	↓
O	F	A	P	S	G

∴ MOLTEN $\xrightarrow{\text{Reversing Order}}$

N	E	T	L	O	M
↓	↓	↓	↓	↓	↓
O	F	U	M	P	N

∴ (*c*) is correct.

Q.43. Find the odd man our:

34, 105, 424, 2123, 12756

(*a*) 12756 (*b*) 2123 (*c*) 424 (*d*) 34

[Dec. 2022]

Solution: Only 2123 is divisible by 11.

∴ (*b*) is correct

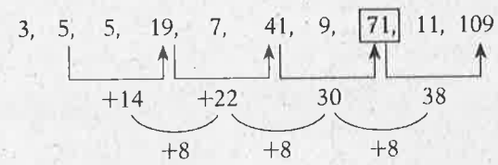
Q.44. Find the missing in the following series?

3, 5, 5, 19, 7, 41, 9, ?, 11, 109

(*a*) 71 (*b*) 61 (*c*) 69 (*d*) 79

[Dec. 2022]

Solution:



i.e. 5 + 14 = 19

19 + 22 = 41

41 + 30 = 71 *Ans.*

71 + 38 = 109

Q.45. In certain code language, if TOUR is written as 1234, CLEAR is written 5678 and SPARE is written as 90847, find the code for CARE?

(*a*) 1247 (*b*) 4847 (*c*) 5247 (*d*) 5847

[Dec. 2022]

Solution:

T	O	U	R		C	L	E	A	R	
↓	↓	↓	↓		↓	↓	↓	↓	↓*	[*Missing in Qts.]
1	2	3	4		5	6	7	8	4	

S	P	A	R	E
↓	↓	↓	↓	↓
9	0	8	4	7

∴ C A R E
↓ ↓ ↓ ↓
5 8 4 7

∴ (*d*) is correct

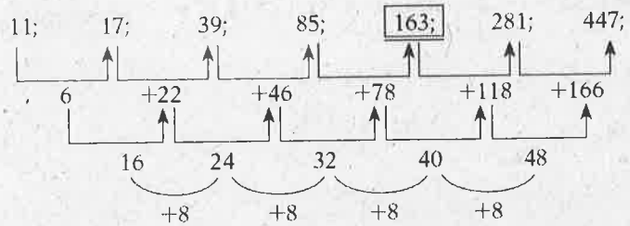
Q.46. Find the next number in the given sequence?

11, 17, 39, 85, ?, 281, 447

(*a*) 133 (*b*) 143 (*c*) 153 (*d*) 163

[Dec. 2022]

Solution:



∴ (*d*) is correct

Q.47. If ROSE is coded as 6821, CHAIR is coded as 73456 and PREACH is coded as 961473, what will be the code for SEARCH?

(*a*) 246173 (*b*) 214673 (*c*) 216473 (*d*) 214763

[Dec. 2022]

Solution:

R O S E C H A I R P R E A C H
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
6 8 2 1 7 3 4 5 6 9 6 1 4 7 3

S E A R C H
↓ ↓ ↓ ↓ ↓ ↓
2 1 4 6 7 3

∴ (*b*) is correct

Q.48. The number in place of question mark in: 7, 26, 63, 124, 215, ?, 511 is:

(*a*) 342 (*b*) 343 (*c*) 441 (*d*) 421

[June 2023]

Solution:

$7 = 2^3 - 1$

$26 = 3^3 - 1$

$63 = 4^3 - 1$

$124 = 5^3 - 1$

$215 = 6^3 - 1$

$\boxed{342 = 7^3 - 1}$

$511 = 8^3 - 1$

(*a*) is correct.

Q.49. In a certain code, MENTION is written as LNEITNO. How is PRESENT written in that code?

(*a*) QFSFTUM (*b*) ONESERP

(*c*) QRESTNO (*d*) OERESTN

[June 2023]

Solution:

MENTION
−1 ↓ ↓ ↓ ↓ ↓ ↓
L NE IT NO

PRESENT
−1 ↓ ↓ ↓ ↓ ↓ ↓
OERESTN

(*d*) is correct.

Q.50. Out of following 41, 43, 47, 53, 61, 71, 83, 95 the odd man out shall be:

(*a*) 95 (*b*) 81 (*c*) 71 (*d*) 53

[June 2023]

Solution:

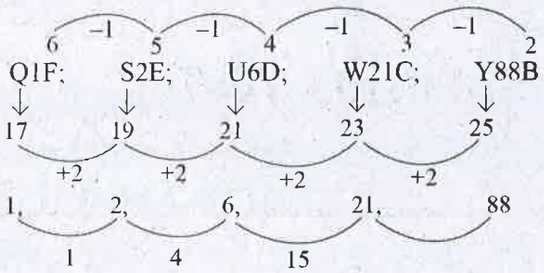
(*a*) All are Prime Numbers Except "95".

Q.51. Find the next number in the series: Q1F, S2E, U6D, W21C,.....?

(*a*) Y66B (*b*) Y44B (*c*) Y88B (*d*) Z66B

[June 2023]

Solution:



$1 \times 1 + 1 = 2$

$2 \times 2 + 2 = 6$

$6 \times 3 + 3 = 21$

$21 \times 4 + 4 = 88$

$\therefore$ (*c*) is correct.

Q.52. Find the odd man out in the following series: 190, 145, 136, 352, 460, 324, 631, 244.

(*a*) 136 (*b*) 244 (*c*) 460 (*d*) 324

[June 2023]

Solution:

190, 145, 136, 352, 460, 324, 631, 244.

No. 324 is odd man out because sum of all digits of each No. is equal to 10 except 324.

$\therefore$ (*d*) is correct.

17 CHAPTER DIRECTION TESTS

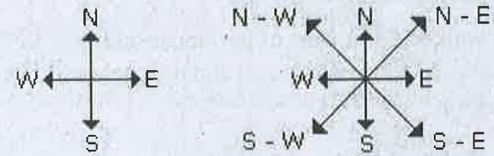
Direction and Distance topic is one of the easiest topic in the reasoning section and we can score full marks with great accuracy if we learn the nuances of this topic.

In this topic, we are going to discuss how to solve direction sense questions along with the shortcut tricks to solve the question within a minute. First make a sketch of the data that is provided to solve the direction test.

Directions questions asked in the exam are based on two principles-

(i) Distance (ii) Direction

There are four main directions – East, West, North & South and four cardinal directions – North-East (N-E), North-West (N-W), South-East (S-E), and South-West (S-W) as shown below:



The first point you need to remember is that each main direction change undergoes a 90° change in direction *e.g.* from North to West / East it will be 90° change. But the change between North and North-East is only 45°.

Important Points To Keep In Mind In Left Right Movement:

1. Always mark the starting point and end-point different from the other points.
2. Always be attentive while taking right and/or left turns.
3. Mark distances with a scale (if your rough diagrams confuse you).
4. A person facing east, on taking left will face towards north and on taking the right turn towards south.

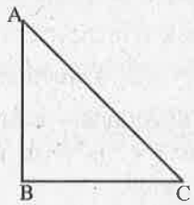
5. A person facing north, on taking left will face towards west and on taking the right turn towards east.
6. A person facing west, on taking left will face towards south and on taking right turn towards north.
7. A person facing south, on taking left will face towards east and on taking the right turn towards west.

   Note: When a question says moved towards left or right side, we assume that the movement is at an angle of 90 degrees.

   Keep in mind that when a person moves to his left side, he will move towards anti-clockwise direction and when a person moves to his right side, he will move towards clockwise direction.
8. One should be aware of basic geometric rule, such as Pythagoras Theorem.

   Pythagoras Theorem $\Rightarrow AC^2 = AB^2 + BC^2$

$\therefore AC = \sqrt{AB^2 + BC^2}$



Now, consider the following examples:

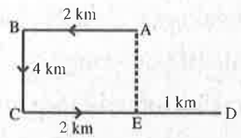
Example 1: Neha walked 2 km west of her house and then turned south covering 4 km. Finally, she moved 3 km towards east and then again 1 km west. How far is she from her initial position?

(*a*) 7 km (*b*) 3 km (*c*) 4 km (*d*) 12 km

Solution :- (*c*) is correct.

Neha starts from her house at A, moves 2 km west upto B, then 4 km to the south upto C, 3 km east upto D and finally 1 km west upto E. Thus, her distance from the initial position A = AE = BC = 4 km.

Hence, the answer is (c).

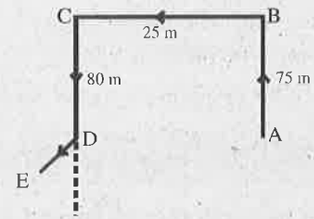


Example 2: Shweta moved a distance of 75 metres towards the north. She then turned to the left and walking for about 25 metres, turned left again and walked 80 metres. Finally, she turned to the right at an angle of 45°. In which direction was she moving finally?

(*a*) South (*b*) South-West (*c*) North-East (*d*) North-West

Solution :- (*b*) is correct.

Shweta started moving from A, moved 75 m upto B, turned left and walked 25 m upto C. She then turned left again and moved 80 m upto D. She turned to the right at an angle of 45° & finally moved in the direction DE i.e., South-West. Hence, the answer is (b).



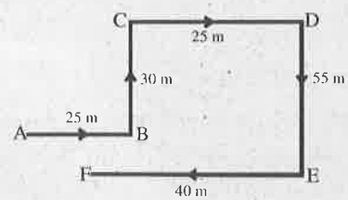
Example 3: Varun faces towards north. Turning to his right, he walks 25 metres. He then turns to his left and walks 30 metres. Next, he moves 25 metres to his right. He then turns to his right again and walks 55 metres. Finally he turns to the right and moves 40 metres. In which direction is he now from his starting point ?

(*a*) South-East (*b*) South-West (*c*) South (*d*) North-West

Solution :- (*a*) is correct.

Varun face is towards north. He turns towards right from north direction & walks 25 m towards east upto B, turns left and moves 30 m upto C, turns right and goes 25 m upto D. At D, he turns to right towards the south and walks 55 m upto E. Next, he again turns to right and walks 40 m upto F, which is his final position. F is to the south-east of A. So, he is to the south-east from his starting point.

Hence, the answer is (a).

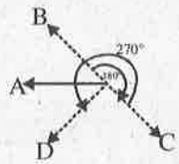


Example 4: Pankaj is facing west. He turns 45° in the clockwise direction and then again another turns with 180° in the same direction i.e. clockwise direction, after that he turns 270° in the anti-clockwise direction. Which direction is he facing now ?

(*a*) North-West (*b*) West (*c*) South-West (*d*) South

Solution :- (*c*) is correct.

Pankaj is facing towards west means initially facing in the direction OA. He turned with 45° in clockwise & faces in the direction OB. After that moving 180° clockwise, he faces in the direction OC. Finally, he turned with 270° anti-clockwise, he faces in the direction OD, which is South-West. Hence, the answer is (c).



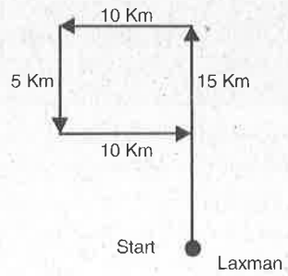
PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

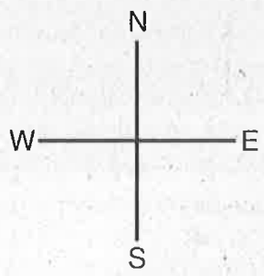
Q.1. Laxman went 15 Kms. to North then he turned West and covered 10 Kms. Then he turned South and covered 5 kms. finally turning to East he covered 10 Kms. In which direction is he moving now ?

(*a*) East (*b*) West (*c*) North (*d*) South

[May 2018]

Solution : (a)





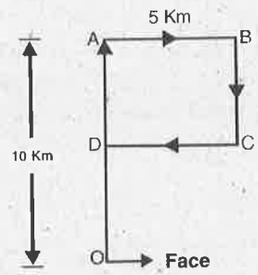
Laxman is moving towards East.

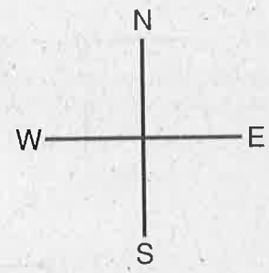
Q.2. A man is facing East, then he turns left and goes to 10 metres then turns right and goes 5 metres then goes 5 metres to the South and from there 5 metres to West. In which direction is he from his original place?

(*a*) East (*b*) West (*c*) North (*d*) South

[May 2018]

Solution : (c)





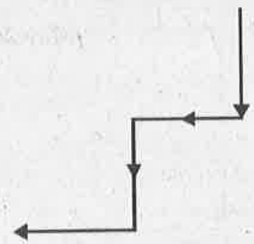
He is in North direction from his original place.

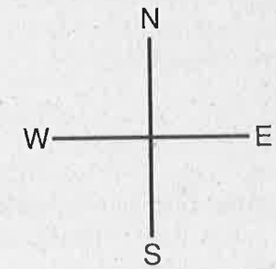
Q.3. X walks southwards and then turns right then left and then right. In which direction is he moving now ?

(*a*) South (*b*) North (*c*) West (*d*) South-west

[May 2018]

Solution : (c)





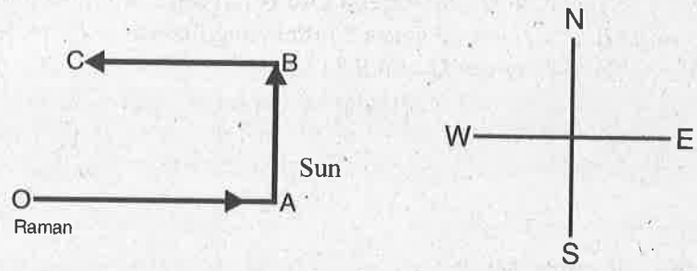
Clearly Mr. X is moving in West direction.

Q.4. Mr. Raman starts walking in the morning facing the Sun. After sometimes, he turned to the left. After sometimes later again he turned to his left. In what direction is he moving now?

(*a*) East (*b*) North (*c*) West (*d*) South

[May 2018]

Solution : (c)



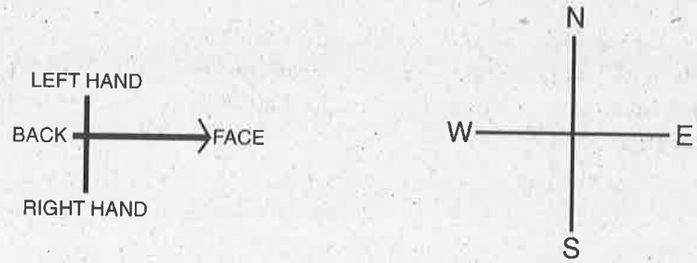
Now Raman is moving towards West.

Q.5. I stand with my right hand extended side-ways towards South. Towards which direction will my back be ?

(*a*) East (*b*) South (*c*) North (*d*) West

[May 2018]

Solution : (d)



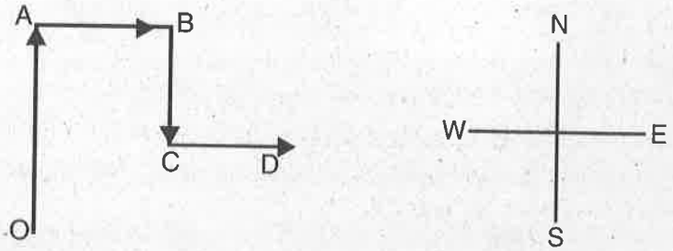
Clearly my right hand extended towards South side - means Left hand towards North. Face towards East & back will be towards West direction.

Q.6. You go towards North, after sometimes turn right, then right then go to the left. In which direction are you now moving ?

(*a*) East (*b*) South (*c*) West (*d*) North

[May 2018]

Solution : (a)



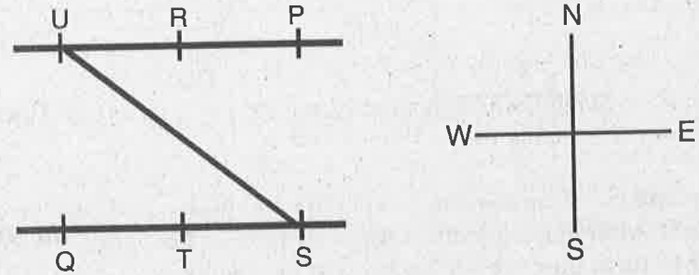
Clearly in East direction.

Q.7. Six flats on a floor in two rows facing North and South are allotted to P, Q, R, S, T and U. If Q gets a North facing flat and is not next to S. S and U get diagonally opposite flat. R next to U gets a South facing flat and T gets a North facing flat. Whose flat is between Q and S ?

(*a*) T (*b*) U (*c*) R (*d*) P

[Nov. 2018]

Solution : (a)



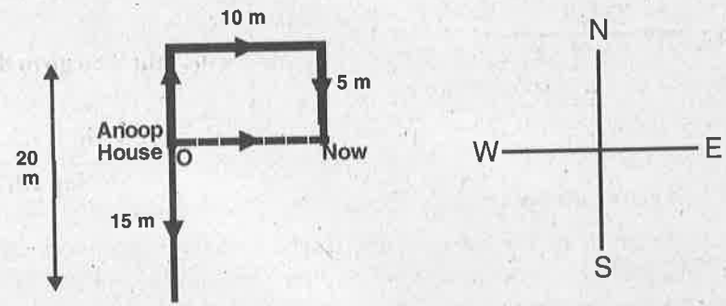
Clearly Flat 'T' is lying between Q & S.

Q.8. Anoop Starts walking towards South. After walking 15 metres he turns towards North. After walking 20 metres he turns towards East and walks 10 metres. He then turns towards South and walks 5 metres. In which direction is he from the original position ?

(*a*) North (*b*) South (*c*) East (*d*) West

[Nov. 2018]

Solution : (c)



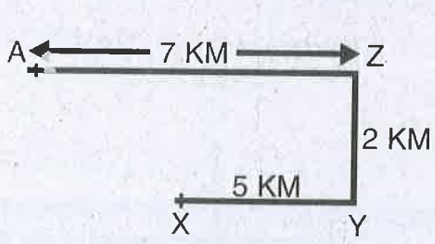
Now Anoop is in 'East direction' with respect to his original Position O.

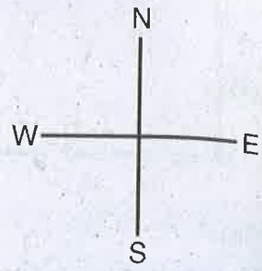
Q.9. Rahim started from point X and walked straight 5 km. East, then turned left and walked straight 2 km. and again turned left and walked straight 7 km. In which direction is he from the point X ?

(*a*) North-East (*b*) South-West (*c*) South-East (*d*) North-West

[Nov. 2018]

Solution : (d)



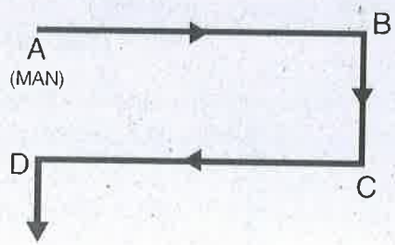


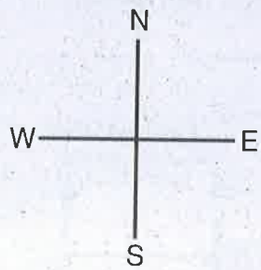
Q.10. A man started walking towards East. After moving a certain distance, he turns to his right. After moving some distance, he turns to his right again. After moving a little he turns now to his left currently, he is going in _____ direction.

(*a*) East (*b*) West (*c*) South (*d*) North

[Nov. 2018]

Solution : (c)





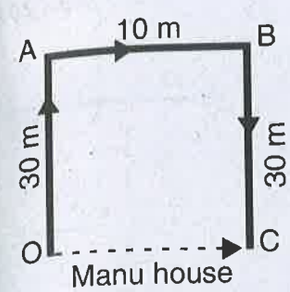
The man is going towards South.

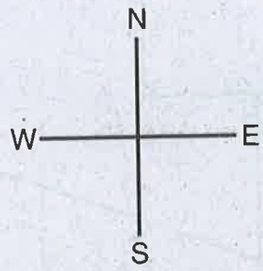
Q.11. Manu wants to go to the market. He starts walking from his house towards North reaches at a crossing after 30 m. He turns towards East, goes 10m till the second crossing and turns again, moves towards South straight for 30 m where marketing complex exists. In which direction is the market from his house ?

(*a*) North (*b*) South (*c*) East (*d*) West

[Nov. 2018]

Solution : (c)





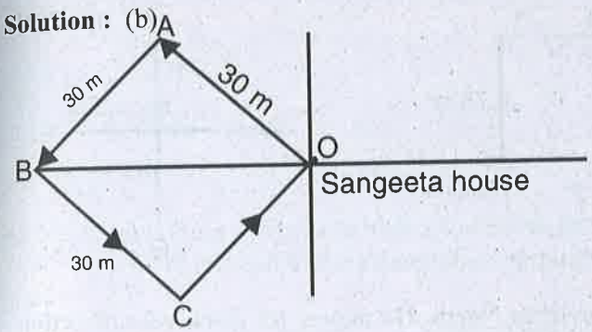
Clearly the Market exists towards East from his house.

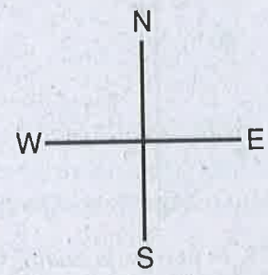
Q.12. Sangeeta leaves from her home. She first walk 30 metres in North-West direction, and then 30m in South West direction, next she walks 30 metres in South-East direction. Finally she turns towards her house. In which direction is she moving?

(*a*) North-West (*b*) North-East (*c*) South-East (*d*) South-West

[June 2019]

Solution : (b)





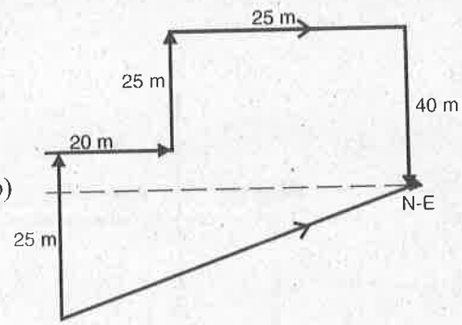
Finally Sangeeta is moving towards North-East direction.

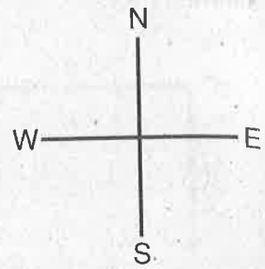
Q.13. When a person faces north and walk 25m and he turn right and walk 20m and again turns left and walk 25m, and turns right 25 m and turns right and walks 40m. In which direction is he now from his starting point?

(*a*) North-West (*b*) North-East (*c*) South-East (*d*) South-West

[June 2019]

Solution : (b)



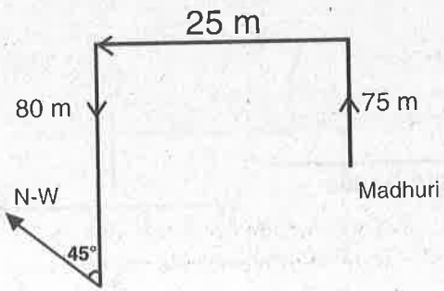


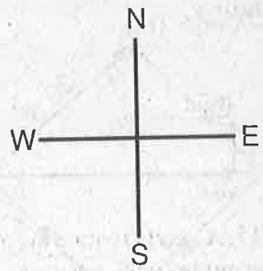
Q.14. Madhuri moved a distance of 75 metres towards North. She then turned to her left and walked about 25m, turned left again and walked 80m. Finally she turned to the right at an angle of 45°. In which direction was she moving finally?

(*a*) South-East (*b*) South-West (*c*) North-West (*d*) North-East

[June 2019]

Solution : (c)



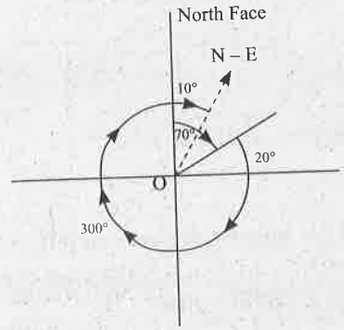


Q.15. A person is facing towards North. He moves 70⁰ clock-wise direction. Again he is moving 300⁰ clock-wise direction. Now, in which direction is he presently facing ?

(*a*) North-West (*b*) South-East (*c*) North-East (*d*) South-West

[June 2019]

Solution : (*c*)

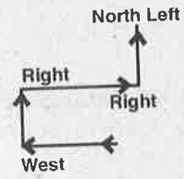


Q.16. A man started walking West. He turned right. Then again right and finally turned left. Towards which direction was he walking now?

(*a*) North (*b*) South (*c*) West (*d*) East

[Dec. 2019]

Solution : (*a*)

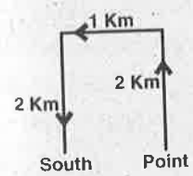


Q. 17. A starts from a point and walk 2 km north, then turns left and walk 1 km, then again turns left and walks 2 km. Point out the direction in which he is going now?

(*a*) East (*b*) West (*c*) North (*d*) South

[Dec. 2019]

Solution : (*d*)

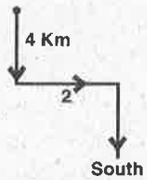


Q. 18. A man is moving on cycle and move 4 km South then turns left and move 2 km and turns again to the right to move to go more. In which direction is he moving?

(*a*) North (*b*) West (*c*) East (*d*) South

[Dec. 2019]

Solution : (*d*)

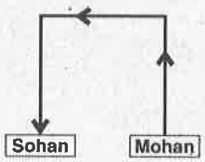


Q. 19. If Mohan travels towards north from his house then turn to left, then to south covering equal distance. In each direction to reach Sohan's house. In which direction Mohan's house is form Sohan's house now?

(*a*) East (*b*) South (*c*) North (*d*) West

[Dec. 2019]

Solution : (*a*)

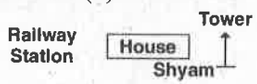


Q.20. If Shyam sees the rising sun behind the tower and setting sun behind the Railway station from his house. What is the direction of tower from the Railway station?

(*a*) South (*b*) North (*c*) West (*d*) East

[Dec. 2019]

Solution : (*d*)



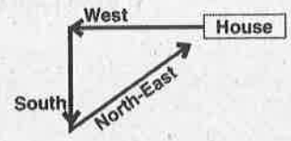
Clearly Tower is in East with respect to Railway Station.

Q.21. A man takes his dog for a walk whose house is facing East. He walks first towards west and then walks towards south. In which direction he has to walk now to reach home?

(*a*) North East (*b*) West (*c*) South (*d*) North West

[Dec. 2019]

Solution : (*a*)

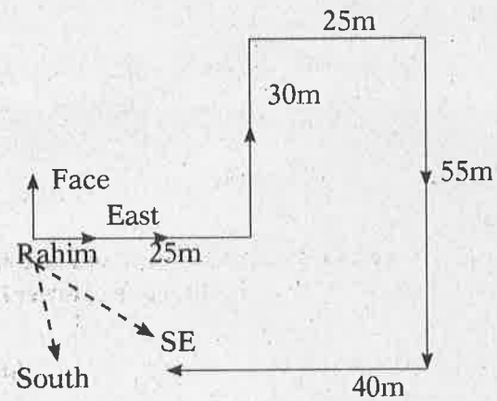


Q.22. Rahim faces towards north turning to his right he walks 25 mtrs. He then turns to his left and walks 30 mtrs. Next he moves 25 mtrs. To his right he then turns to his right again and walks 55 mtrs. Finally he turns to the right and moves 40 mtrs. In which direction is he now from the starting point?

(*a*) South - West (*b*) South (*c*) North - West (*d*) South - East

[Dec. 2020]

Solution :



Clearly now he is in South – East direction from starting point.

(*d*) is correct.

Q.23. A man can walk by having long, medium and short steps. He can cover 60 meters by 100 long steps, 100 meters by 200 medium steps and 80 meters by 200 short steps. He starts walking by 5,000 long steps, then he turns left and walk by taking 6,000 medium steps. He then turns right and walk by taking 2,500 short steps. How far (in meters) is he away from his starting point?

(*a*) 5,000m (*b*) 4,000m (*c*) 6,000m (*d*) 7,000m

[Dec. 2020]

Solution :

(*a*) is correct.

Let L = Long step ; M = Medium step

and S = Short step.

$100L = 60m. \Rightarrow L = 0.6m$

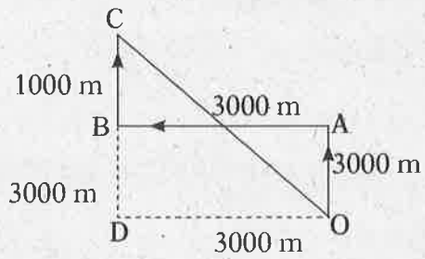
$200M = 100m. \Rightarrow M = \frac{100}{200} = 0.5m.$

$200S = 80m \Rightarrow S = \frac{80}{200} = 0.4m.$

$\therefore$ 5000 Long steps = $5000 \times 0.6 = 3000m.$

6000 Medium steps = $6000 \times 0.5 = 3000m.$

and 2500 short steps = $2500 \times 0.4 = 1000m.$



$\therefore$ AB = OD = 3000*m*

CD = BC + BD

= BC + OA

= 1000 + 3000

= 4000m.

$\therefore$ OC = $\sqrt{OD^2 + CD^2} = \sqrt{3000^2 + 4000^2} = 5000m.$

(*a*) is correct.

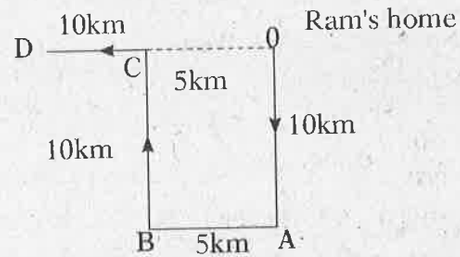
Q.24. One day, Ram left home and cycled 10km southwards, then he turns right and cycled 5km, then he turns right and cycled 10km and then he turns left and cycled 10 km. How many kilometers will he have to cycle to reach his home straight?

(*a*) 15km (*b*) 10km (*c*) 20km (*d*) 25km

[Dec. 2020]

Solution :

(*a*) is correct.



Clearly Ram is at D at the end.

He has to reach his home "O".

Distance covered to reach his home straight = DC + CO

= 10 + 5 = 15 km

$\therefore$ (*a*) is correct.

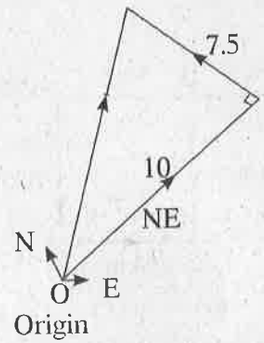
Q.25. You are facing North - east and moved forward 10cms and turned left for 7.5 cm what is your position?

(*a*) North from initial (*b*) South from initial

(*c*) East from initial (*d*) None

[Dec. 2020]

Solution :



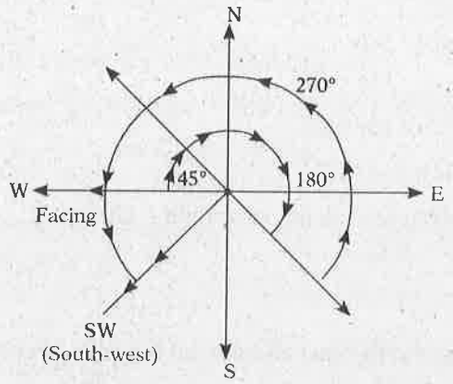
Clearly (d) is correct.

Q.26. A man is facing west. He turns 45° in the clockwise direction and then another 180° in the same direction and then 270° in the anti-clockwise direction. He is the facing now?

(*a*) South-West (*b*) North-West (*c*) West (*d*) South

[Jan. 2021]

Solution : (*a*) is correct

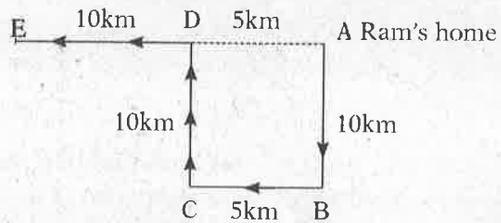


Q.27. One day Ram left home and bi-Cycled 10 km southwards, turned right and travelled 5 km and turned right and went 10 km he turned left and went 10 km how many kilometers he has to cycle to reach his home straight?

(*a*) 10 (*b*) 15 (*c*) 20 (*d*) 25

[Jan. 2021]

Solution : (*b*)



clearly BC = AD = 5km

Distance to bicycle from E to house A = ED + DA

= 10 + 5 = 15km

∴ (*b*) is correct

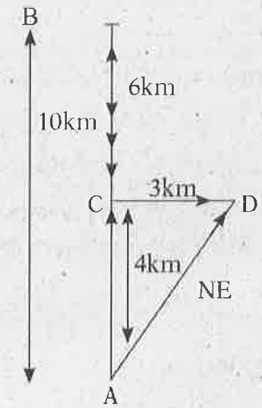
Q.28. Ms. N walks 10km towards North from there she walks 6 km towards South. Then she walks 3 km towards East. How far and in which direction is she with reference to her starting point?

(*a*) 4 km West (*b*) 6 km West

(*c*) 3 km East (*d*) 5 km North-East

[Jan. 2021]

Solution : (*d*) is correct



Here AC = 10 – 6 = 4km

Here ACD is a right-angled triangle

$AD^2 = AC^2 + CD^2$

$= 4^2 + 3^2 = 16 + 9 = 25$

$\therefore AD = \sqrt{25} = 5km$

Direction from A to D is North-East

∴ (*d*) is correct

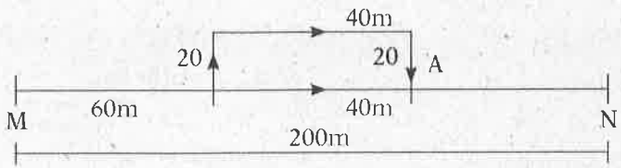
Q.29. A and B start moving towards each other from two places 200 m apart. After walking 60 m, B turns left and goes 20 m, then he turns right and goes 40 m. He then turns right again and comes back to the road on which he had started walking. If A and B walk with the same speed, what is the distance between them now?

(*a*) 80 m (*b*) 70 m (*c*) 40 m (*d*) 60 m

[July 2021]

Solution : (*c*) is correct

Speed of both A & B is equal.



Let distance between M and N = 200m. A starts from M and B from N Walking towards each other.

Distance covered by A = Distance covered by B = 60 + 20 + 40 + 20 = 140m.

B is 140m away from N but A is only 60 + 40 = 100m away from M.

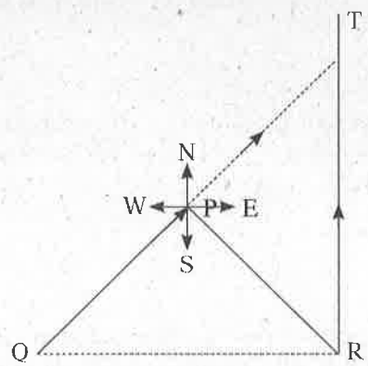
Hence distance between A and B = 140 – 100 = 40m

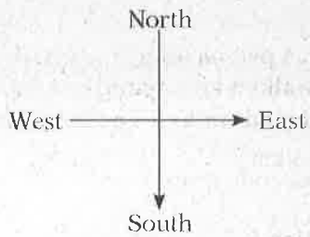
Q.30. There are four towns P, Q, R and T. Q is to the south-west of P, R is to the east of Q and south east of P, and T is to the north of R in line with QP. In which direction of P is T located?

(*a*) North (*b*) North-East (*c*) East (*d*) South-East

[July 2021]

Solution : (*b*) is correct





Clear T is Located in North-East direction with respect to P.

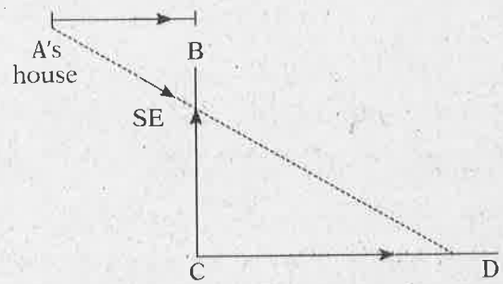
Q.31. Five friends A, B, C, D and E are staying in the same locality. B's house is to the east of A's house and to the north of C's house C's house is to the west of D's house. D's house is in which direction with respect to A's house?

(*a*) North-East (*b*) South-East

(*c*) North-West (*d*) South-West

[July 2021]

Solution : (*b*) is correct

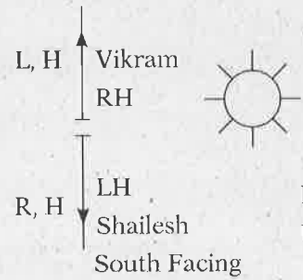


Q.32. One morning, after sunrise, Vikram and Shailesh were standing in a lawn with their backs towards each other. Vikram's shadow fell exactly towards left hand side. Which direction was Shailesh facing?

(*a*) South-West (*b*) West (*c*) South (*d*) East-South

[July 2021]

Solution : (*c*) is correct



LH ⇒ Left hand

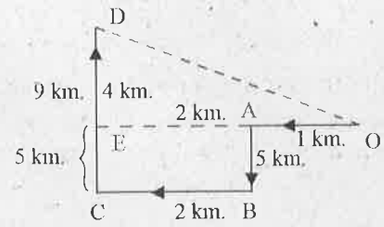
RH ⇒ Right hand

Q.33. A person walks 1 km (kilometer) towards West and then he turns to South and walks 5 km. Again, he turns to West and walks 2 km. After this he turns to North and walks 9 km. How far is he from his starting point?

(*a*) 3 km (*b*) 4 km (*c*) 5 km. (*d*) 7 km.

[Dec. 2021]

Solution : (*c*)



Here,

OE = 1 + 2 = 3 Km.

DE = CD – CE

= 9 – AB

= 9 – 5 = 4 Km.

$OD = \sqrt{ED^2 + OE^2}$

$= \sqrt{4^2 + 3^2} = \sqrt{16+9}$

$= \sqrt{25} = 5Km$.

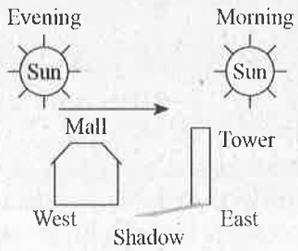
Q.34. Daily in the morning the shadow of a Clock Tower installed on Railway Station falls on high rise Mall and in the evening the shadow of the same Mall falls on the Clock Tower installed on Railway Station exactly. So in which direction is Clock Tower to Mall?

(*a*) Eastern side (*b*) Western side

(*c*) Northern side (*d*) Southern side

[Dec. 2021]

Solution : (*a*)

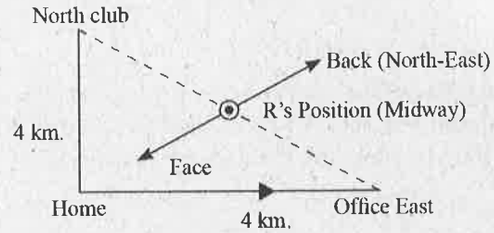


Q.35. R's office is 4 km. in East direction from his home and club is 4 km. in North direction from his home. On midway from office to club R, starts moving towards his home. In which direction is he facing his back?

(*a*) South-East (*b*) North-West (*c*) North-East (*d*) South-West

[Dec. 2021]

Solution : (*c*)

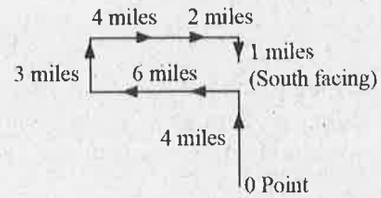


Q.36. A man starts from a point walks 4 miles towards North and turn left and walks 6 miles, turns right and walks for 3 miles and again turns right and walks 4 miles and takes rest for 30 minutes. He gets up and walks straight 2 miles in the same direction and turns right and walks one mile. What is the direction he is facing?

(*a*) North (*b*) South (*c*) South-East (*d*) West

[Dec. 2021]

Solution : (*b*)

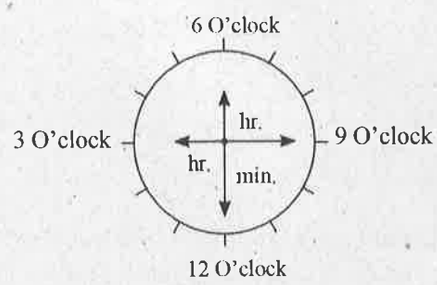


Q.37. The hour hand of a clock is in west direction when time is 3'O clock. What is the direction of minutes hand when time is 6:45?

(*a*) East (*b*) West (*c*) North (*d*) South

[Dec. 2021]

Solution : (*a*)



6.45 hrs means minute hand at point

9 (*i.e.* 9 × 5 = 45 Minutes)

∴ Minute hand in eastern direction

Q.38. If P × Q means P is to the south of Q; P + Q means P is to the north of Q; P % Q means P is to the east of Q; P - Q means P is to the west of Q: then in case of A % B + C - D, D is in which direction with respect to B?

(*a*) North-west (*b*) South-east (*c*) North-east (*d*) South-east

[June 2022]

Solution : A % B + C–D

Clearly D is in South-East with respect to B.

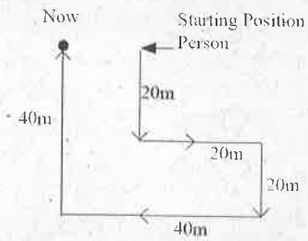
∴ (*b*) is correct

Q.39. I am facing west, turning to the left I go 20 m, then turning to the left I go 20m and turning to the right I go 20m, then again turning to the right I go 40 m and then again I go 40 m to the right. In which direction am I from my original position?

(*a*) North (*b*) West (*c*) South (*d*) East

[June 2022]

Solution :



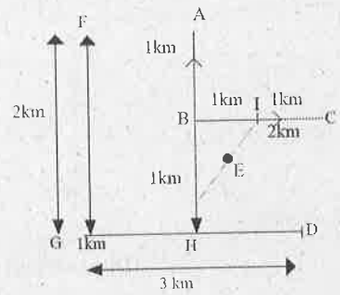
∴ (*b*) is correct

Q.40. A, B, C, D, E, F, G, H, and I are nine poles. C is 2 km east of B. A is 1 km north of B and H is 2 km south of A.G is 1 km west of H while D is 3 km East of G and F is 2 km north of G. I is situated right in the middle of B and C while E is just in the middle of H and I. The Distance between B and I is

(*a*) 1 km (*b*) 1.41 km (*c*) 2 km (*d*) 3 km

[June 2022]

Solution :



∴ (*a*) is correct

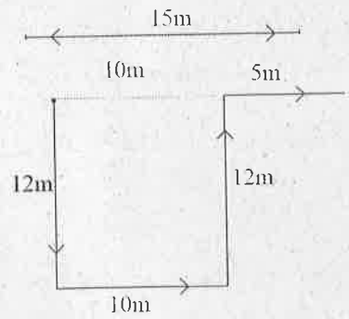
Q.41. Starting from a point, Rani walked 12m South, she turned left and walked 10m, she again turned left and walked 12m, then she turned right and walked 5m. how far is she now and in which direction from the starting point?

(*a*) 27 m towards East (*b*) 5 m towards East

(*c*) 10 m towards West (*d*) 15 m towards East

[June 2022]

Solution :



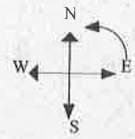
∴ (*d*) is correct

Q.42. Puru was driving his car and at a circle there was direction pole, which was showing all the four correct directions. But due to the wind, it turns in such a manner that now North pointer is showing West. Puru went in the wrong direction thinking that he was travelling East. In what direction he was actually travelling?

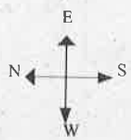
(*a*) West (*b*) East (*c*) North (*d*) South

[June 2022]

Solution :



(Original)



Now
(Due to Wind)

$\therefore$ (*c*) is correct

Q.43. Radha moves towards South-East a distance of 7 km, then she moves towards. West and travels a distance of 14 km. From here she moves towards North. West a distance of 7 km and finally she moves a distance of 4 km towards east. How far is she now from the starting point?

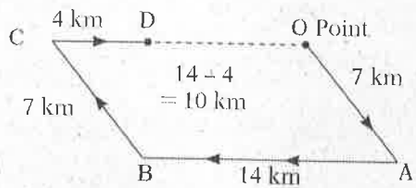
(*a*) 3 km (*b*) 4 km (*c*) 10 km (*d*) 11 km

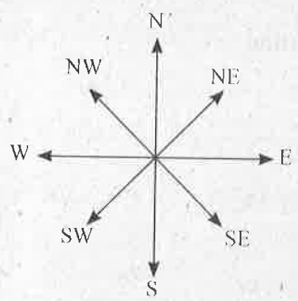
[Dec. 2022]

Solution:

$\therefore$ OD = OC – CD

= 14 – 4 = 10 km





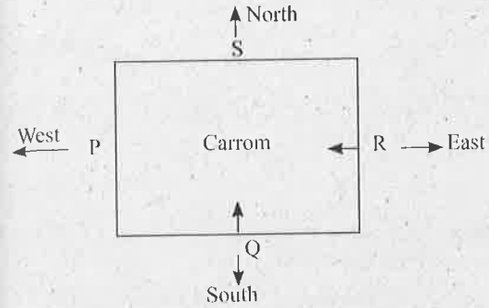
$\therefore$ (*c*) is correct

Q.44. P, Q, R and S are playing a game of carrom P, R and S, Q are partners. 'S' is to the right of 'R'. If 'R' is facing West, then 'Q' is facing which direction?

(*a*) South (*b*) North (*c*) East (*d*) West

[Dec. 2022]

Solution:



Clearly Q is facing North

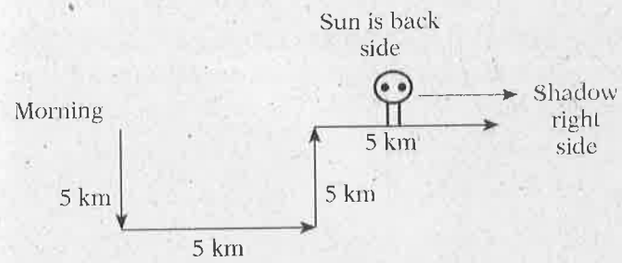
$\therefore$ (*b*) is correct

Q.45. One morning a boy starts walking in a particular direction for 5 km and then takes a left turn and walks another 5 km thereafter he again takes left turn and walks another 5 km and at last he takes right turn walks 5 km. Now he sees his shadow in front of him. What direction he did start initially?

(*a*) South (*b*) North (*c*) West (*d*) East

[Dec. 2022]

Solution :



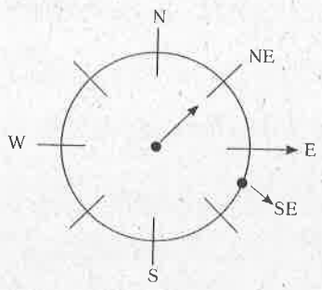
$\therefore$ (*a*) is correct.

Q.46. It is 3'o clock in a watch. If the minute hand points towards the North - East then the hour hand will point towards the

(*a*) South (*b*) South-West (*c*) North-West (*d*) South-East

[Dec. 2022]

Solution :



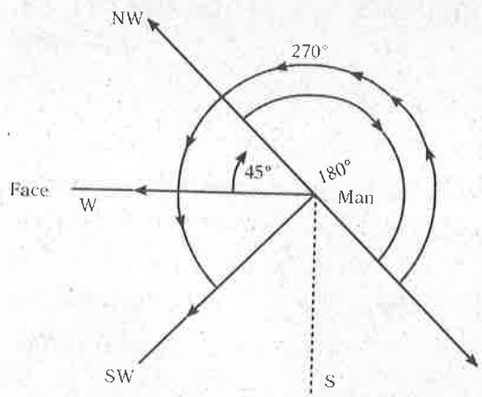
∴ (*d*) is correct

Q.47. A man is facing west. He turns 45 degree in the clock-wise direction and then another 180 degree in the same direction and then 270 degree in the anticlock-wise direction. Find which direction he is facing now?

(*a*) South-East (*b*) West (*c*) South (*d*) South-West

[Dec. 2022]

Solution:



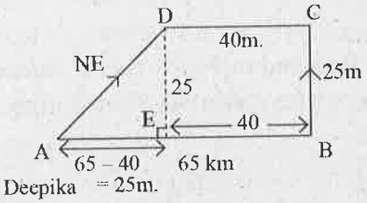
Q.48. Deepika starts walking straight towards ease. After walking 65m, she turns to the left and walks 25m straight. Again she turns to be left and walks a distance of 40m. At what distance and in which direction currently she is from the initial point?

(*a*) 35.35m in North-East (*b*) 35.35m in South-West

(*c*) 25m in North (*d*) 25m in West

[June 2023]

Solution:



AE = 65 – 40 = 25m.

In right angle ΔAED.

$$AD = \sqrt{AE^2 + DE^2}$$

$$= \sqrt{25^2 + 25^2}$$

$$= \sqrt{2 \times 25^2}$$

$$= 25\sqrt{2} = 25 \times 1.414$$

$$= 35.35\text{m}$$

North-East.

(*a*) is correct.

Q.49. Mr. Kartik puts his time piece on the table in such a way that at 6:00 PM, hours hand points to north. In which direction the minute hand will point at 9:15 PM?

(*a*) South-East (*b*) East (*c*) West (*d*) South-West

[June 2023]

Solution:

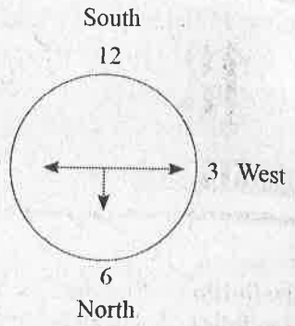
Clearly

Minute hand will show west

∵ North South just opposite.

Hence, East & West will be just opposite.

So (*c*) is correct.



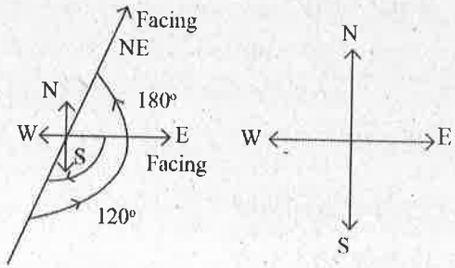
Q.50. Shrikant is facing East and turns 120° in the clockwise direction and then turns 180° in the anticlockwise direction. Which direction is Shrikant facing now?

(*a*) East (*b*) North-East

(*c*) North (*d*) South-West

[June 2023]

Solution:



(*b*) is correct.

18 CHAPTER SEATING ARRANGEMENT

Definition : The process of making an arrangement of a group of persons to sit according to a prefixed (pre-planned) manner is called Seating Arrangement. In seating arrangement Questions, some conditions are given & on the basis of those conditions, students are required to arrange those things.

Pattern or Types of Seating Arrangements:-

(*i*) Linear Arrangement ; (*ii*) Circular Arrangement ; (*iii*) Polygon Arrangement.

Some General instructions to solve seating arrangement questions:

(*i*) Review all informations given in the question.

(*ii*) Recognise 'definite information'; 'comparative information' and 'negative information'.

(*a*) "Person A is sitting on the left end of the bench" is a definite information.

(*b*) "Person B is sitting immediate right to C" is an example of Comparative information.

(*c*) "Person C is not sitting on the immediate right of D" is an example of negative information.

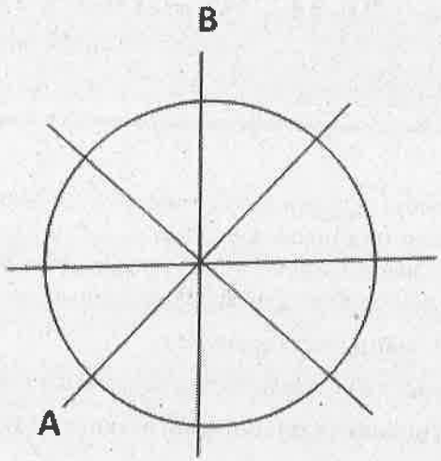
(*iii*) A negative information does not tell us anything definitely but gives an idea to eliminate a possibility.

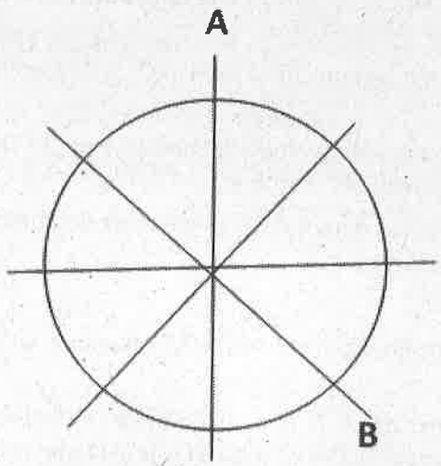
Golden Rules

Rule 1: Always start the arrangement with 100% fixed i.e. confirmed i.e. definite information.

Example: There are 8 persons A, B, C, D, E, F, G, H are sitting around a round table. A is sitting 3rd to the right of B, C is sitting opposite to D who is 3rd left to the person opposite to A

... so on. If the question is like this then students should start with A is sitting 3rd to the right of B. Because this is only the definite or confirmed information.





Note: If nothing is mentioned in the question regarding the direction i.e. they are facing towards centre or facing away from centre then by default we should take it as facing towards centre.

Rule 2: For a closed figure (circle, square, rectangle, octagon etc..,) arrangement, Only persons sit opposite to each other when there are even number of members.

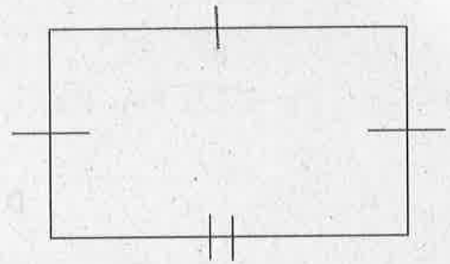


Fig. (i)

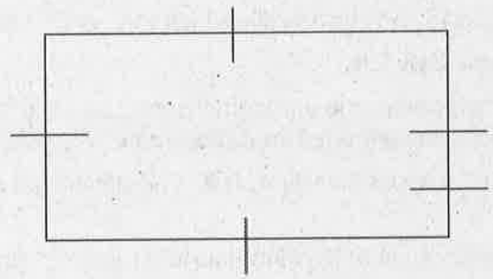
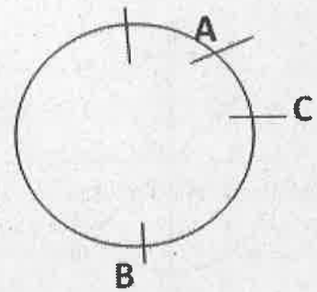


Fig. (ii)

In a given arrangement if four persons are given as in first diagram then two persons can be accommodated exactly opposite to each other but not in case of figure (ii)

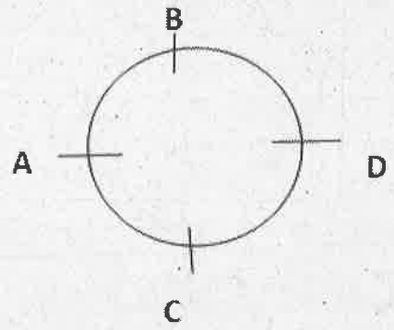
Rule 3: "And" verses "Who"

In a given question if it is given as "In a circular arrangement, there are four persons A, B, C, D sitting in the following way. A is right of B who is left of C"



In this case **Who** refers to B: Thus A is placed right to B and B is placed left to C.

In a circular arrangement, there are four persons A, B, C, D sits in the following way. A is right of B and is left of C



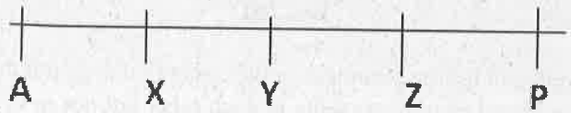
In this case **And** refers to A: Thus A is placed left to C as well as right to B.

Rule 4: Left vs Immediate left

Either in circular arrangement or in linear arrangement unless it is mentioned in the question we cannot assume left or an immediate left.

Example: There are 5 persons namely A, P, X, Y, Z standing in a row. No one sits left to the vowel.

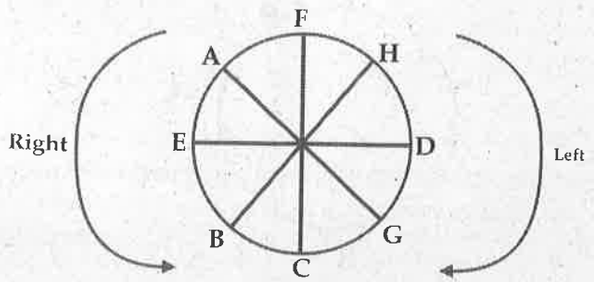
Only 3 persons are to the right of X. X sits immediate left of Y and left to Z. Z does not sit at extreme ends.



In the Above arrangement, X is immediate left to Y, whereas X is left to Z. No matter how many places far from Z but it is left to Z.

Rule 5: Identifying the left and right in a given arrangement

For Circular/closed figure:



* Whenever it is given towards right go for anti-clockwise direction.

* Whenever it is given towards left go for clockwise rotation.

These are the important rules, a student should be aware of while solving seating arrangement problems. **In a competitive exam speed is required, but should not be hurry.**

Example 1 : A, P, R, X, S and Z are sitting in a row. S and Z are in the centre. A and P are at the ends.

R is sitting to the left of A. Who is to the right of P ?

(*a*) A (*b*) X (*c*) S (*d*) Z

Solution : Option (*b*)

The seating arrangement is as follows:

P X S Z R A

Therefore, right of P is X.

Example 2 : A, B, C, D and E are sitting on a bench. A is sitting next to B, C is sitting next to D, D is not sitting with E who is on the left end of the bench. C is on the second position from the right. A is to the right of B and E. A and C are sitting together. In which position A is sitting ?

(*a*) Between B and D (*b*) Between B and C

(*c*) Between E and D (*d*) Between C and E

Solution : Option (*b*)

E B A C D

Therefore, A is sitting in between B and C

Example 3 : There are four children P ,Q , R, S sitting in a row. P occupies seat next to Q but not next to R. If R is not sitting next to S ? Who is occupying seat next to adjacent to S.

(*a*) Q (*b*) P (*c*) P and Q (*d*) None

Solution : (*b*) is correct.

Two possible arrangements can be : R , Q , P , S or S , P , Q , R

Clearly, only P is sitting adjacent to S.

Example 4 : 8 leaders P, Q, R, S, T, U, V and W are sitting on a bench facing towards North.

(*i*) T is fourth to the left of P

(*ii*) S is fourth to the right of W

(*iii*) U and R are not sitting at the ends, but they are neighbours of T and Q respectively.

(*iv*) P is next to the right of W and but left of Q.

Who are sitting at the extreme ends ?

(*a*) T & S (*b*) P & Q

(*c*) U & R (*d*) None

Solution : (*a*) is correct.

(*i*) T is fourth to the left of P $\Rightarrow$ T***P

(*ii*) S is fourth to the right of W $\Rightarrow$ W***S

(*iii*) U and R are neighbours of T and Q respectively $\Rightarrow$ UT or TU & RQ or QR

(*iv*) P is next to the right of W and but left of Q $\Rightarrow$ WPQ

Combining (*i*), (*ii*), (*iv*) & (*iii*) ; we get UT**WPQRS or TU*WPQRS

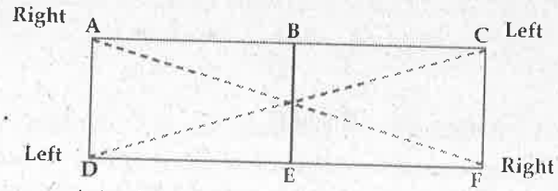
Since, U and R are not sitting at the ends $\Rightarrow$ TU*WPQRS is right .

Rest person is V. So V will sit at rest place. Hence, the arrangement is TUVWPQRS.

Hence, T and S are sitting at the extreme ends.

Two Rows Sequence or Parallel Seating Arrangement

Let A , B , C , D , E , F are 6 students seating in two rows for Group Discussion just opposite to each other.



In the above diagram, it is clear that:

1. A & D are opposite to each other.
2. Also B & E are opposite to each other.
3. C & F are also sitting opposite to each other.
4. A and F are sitting diagonally opposite to each other.
5. C and D are also sitting diagonally opposite to each other.
6. Students A, B, C are sitting Southward while D, E, F Northward.
7. Toward North facing yours right is right side & yours left is left side in the diagram. Toward South facing, it is just opposite as seen in the diagram.

Example 5 : Directions: Read the following informations carefully to answer these questions :

[MAT 2005]

(*i*) Six flats on a floor in two rows facing North and South are allotted to P, Q, R, S, T and U.

(*ii*) Q gets a North facing flat and is not next to S.

(*iii*) S and U get diagonally opposite flats.

(*iv*) R, next to U, gets a South facing flat and T gets a North facing flat.

Q1. The flats of which of the other pairs than SU, are diagonally opposite to each other ?

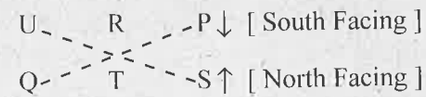
(*a*) QP (*b*) PT (*c*) QR (*d*) TS

Q2. Which of the following combinations gets South facing flats ?

(*a*) UPT (*b*) URP (*c*) QTS (*d*) Data inadequate

Solution :

From Question , the arrangement should be :-



Ans. Q 1. (*a*) is correct.

Q 2. (*b*) is correct.

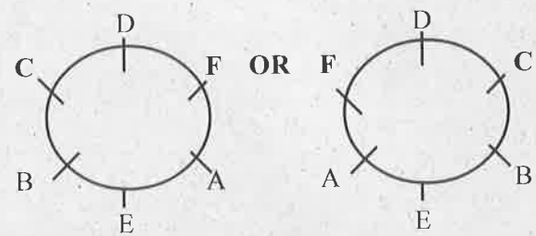
Example 6 : A, B, C, D, E and F are sitting around a round table. A is between E and F, E is opposite to D, and C is not in either of the neighbouring seats of E. Who is opposite to B ?

(*a*) C (*b*) D

(*c*) F (*d*) None of these

[MAT 2003]

Solution : (*c*)



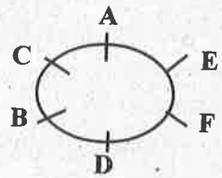
In each arrangement F is opposite to B.

Example 7: Six persons A, B, C, D, E and F are standing in a circle. B is between D and between E and C. F is to the right of D. Who is between A and F ?

(*a*) B (*b*) C (*c*) D (*d*) E

[IITTM 2005]

Solution : (*d*)



PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. Five boys A, B, C, D and E are sitting in a row . A is to the right of B, and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end ?

(*a*) D (*b*) A (*c*) E (*d*) B

[May 2018]

Solution : (c)

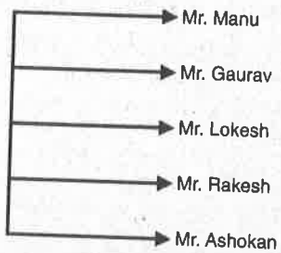
Clearly E is the second from the left end.

Q.2. Five senior citizens are living in a Multi-Storeyed building. Mr. Manu lives in a flat above Mr. Ashokan, Mr. Lokesh in a flat below Mr. Gaurav, Mr. Ashokan lives in a flat below Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who lives in the top most flat ?

(*a*) Mr. Lokesh (*b*) Mr. Gaurav (*c*) Mr. Manu (*d*) Mr. Rakesh

[May 2018]

Solution : (c)



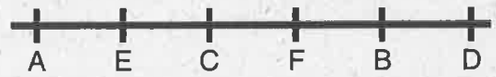
Clearly Mr. Manu lives in the top most flat.

Q.3. Six children A, B, C, D, E and F are standing in a row. B is between F and D. E is between A and C. A does not stand next to F or D. C does not stand next to D. F is between which of the following pairs of children ?

(*a*) B and E (*b*) B and C (*c*) B and D (*d*) B and A

[May 2018]

Solution : (b)



F is between B & C.

Q.4. Five children are sitting in a row. S is sitting next to P but not T. K is sitting next to R who is sitting on the extreme left and T is not sitting next to K. Who is/are adjacent to S.

(*a*) K + P (*b*) R + P (*c*) Only P (*d*) P and T

[May 2018]

Solution : (d)

Q.5. Five boys A, B, C, D and E are sitting in a row. A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end ?

(*a*) D (*b*) A (*c*) E (*d*) B

[May 2018]

Solution :

(c)

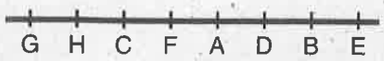
Clearly E is second from the left end.

Q.6. Eight persons A, B, C, D, E, F, G and H are sitting in a line. E sits second right to D. H sits fourth left to D. C and F are immediate neighbours, but C is not immediate neighbour of A. G is not neighbour of E. Only two persons sit between A and E. The persons on left end and right end respectively are:

(*a*) G and E (*b*) B and E (*c*) H and E (*d*) G and B

[Nov. 2018]

Solution : (a); (i) D * E (ii) H ***D (iii) A* CF or FC*A (iv) G * E or E *G (v) A**E or E **A

So, 

Clearly G is on the left end & E on the right end.

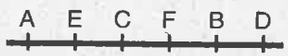
Q.7. Six children A, B, C, D, E and F are sitting in a row. B is between F and D. E between A and C. However, A does not sit next to F or D. C does not sit next to D. Then, F is sitting between ...& ...?

(*a*) B and C (*b*) E and C (*c*) B and D (*d*) None

[Nov. 2018]

Solution : (a)

(i) FBD (ii) AEC

Clearly 

Clearly F is siting between B and C.

Q.8. Five students A, B, C, D and E are standing in a row. D is on the right of E, B is on the left of E but on the right of A. D is next to C on his left. The student in middle is.....

(*a*) B (*b*) E (*c*) C (*d*) A

[Nov. 2018]

Solution : (b)

Clearly 

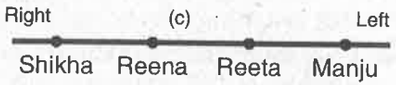
Hence student E is in the middle of these arrangement.

Q.9. Four girls are seated for a photograph. Shikha is left of Reena. Manju is to the right of Reena. Reeta is between Reena and Manju. Who is the second from left in photograph ?

(*a*) Reena (*b*) Manju (*c*) Reeta (*d*) Shikha

[June 2019]

Solution : (c)



(Photograph Sequence)

The second from left in photograph is Reena.

Q.10. In a straight line there are six persons sitting in a row. B is between F and D. E is between A and C. A does not stand next to F or D, C does not stand next to D. F is between which of the following persons ?

(*a*) B and E (*b*) B and C (*c*) B and D (*d*) B and A

[June 2019]

Solution : (b)

(i) FBD or **DBF** (ii) AEC or **CEA**

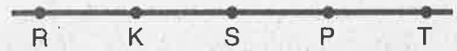
Q.11. 5 Children are sitting in a row. S is sitting next to P but not T. K is sitting next to R. K is not sitting on extreme end. T is not sitting next to K. Who are sitting adjacent S.

(*a*) K & P (*b*) R & P (*c*) Only P (*d*) P & T

[June 2019]

Solution : (a)

(i) T*SP or PS*T (ii) RK (iii) T*K or K*T



Q.12. Five boys A, B, C, D, E are sitting in a row. A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end ?

(*a*) D (*b*) A (*c*) E (*d*) B

[June 2019]

Solution : (c)

(i) CEBAD

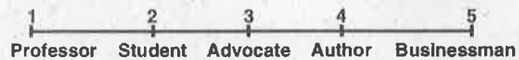
Clearly , E is second from the left end.

Q.13. 5 persons are standing in a line. One of the 2 persons at the extreme ends is a professor and the other a business man. An advocate is standing to the right of student. An author is to the left of the business man. The student is standing between the professor and advocate. Counting from the left, the author is at which place?

(*a*) 2nd (*b*) 3rd (*c*) 4th (*d*) None of these

[Dec. 2019]

Solution : (*c*)

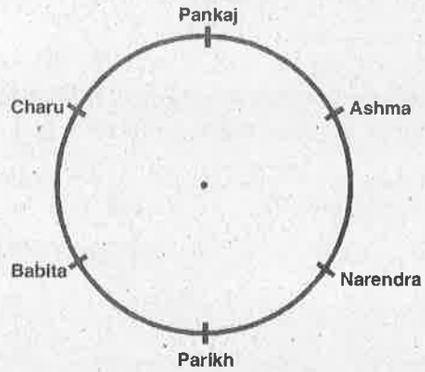


Q.14. Parikh is sitting between Narendra and Babita, Charu is to the left of Babita, Pankaj is sitting between Charu and Ashma , they all sitting around a circle facing the center then who is sitting to the right of Babita ?

(*a*) Parikh (*b*) Ashma (*c*) Charu (*d*) Narendra

[Dec. 2019]

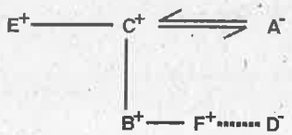
Solution : (*a*)



Q. 15-18. Read the following information carefully to answer the give questions.

Six members of a family namely A, B, C, D, E and F are travelling together. B is the son of C but C is not the mother of B. A and C are married couple. E is the brother of C. D is the daughter of A. F is the brother of B.

Solution :



Q.15. How many male members are there in the family?

(*a*) 3 (*b*) 2 (*c*) 4 (*d*) 1

[Dec. 2019]

Solution : (*c*)

Q.16. How many children does A have

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

[Dec. 2019]

Solution : (c)

Q.17. What is the relation of E to D

(*a*) Uncle (*b*) Brother (*c*) Father (*d*) None of these

[Dec. 2019]

Solution : (*a*)

Q.18. Who is the mother of B?

(*a*) C (*b*) D (*c*) F (*d*) A

[Dec. 2019]

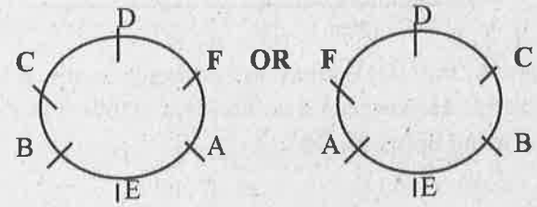
Solution : (*d*)

Q.19. A, B, C, D, E and F are sitting around a round table. A is between E and F, E is opposite to D, and C is not in either of the neighbouring seats of E. Who is opposite to B ?

(*a*) C (*b*) D (*c*) F (*d*) None of these

[Dec. 2020]

Solution : (*c*)



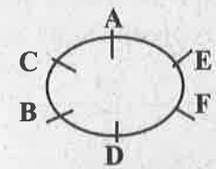
In each arrangement F is opposite to B.

Q.20. Six persons A, B, C, D, E and F are standing in a circle. B is between C and D and A is between E and C. F is to the right of D. Who is between A and F ?

(*a*) B (*b*) C (*c*) D (*d*) E

[Dec. 2020]

Solution : (*d*)

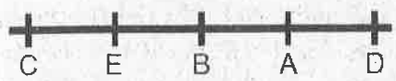


Q.21. Five boys A, B, C, D and E are sitting in a row. A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end ?

(*a*) D (*b*) A (*c*) E (*d*) B

[Dec. 2020]

Solution : (*c*)



Clearly E is second from the left end.

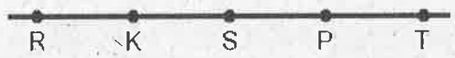
Q.22. 5 Children are sitting in a row. S is sitting next to P but not T. K is sitting next to R. K is not sitting on extreme end. T is not sitting next to K. Who are sitting adjacent S.

(*a*) K & P (*b*) R & P (*c*) Only P (*d*) P & T

[Dec. 2020]

Solution : (*a*)

(i) T*SP or PS*T (ii) RK (iii) T*K or K*T



Q.23. Six children A, B, C, D, E and F are sitting in a row. B is between F and D. E between A and C. However, A does not sit next to F or D. C does not sit next to D. Then, F is sitting between ...& ...?

(*a*) B and C (*b*) E and C (*c*) B and D (*d*) None

[Dec. 2020]

Solution : (*a*)

(i) FBD (ii) AEC

Clearly 

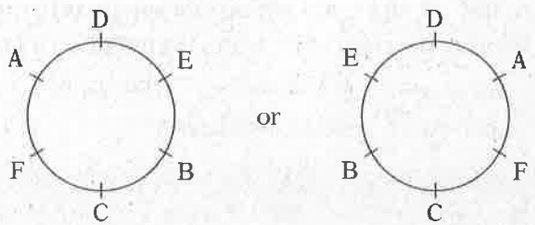
Clearly F is sitting between B and C.

Q.24. A is seated between D and F at a round table. C is seated opposite to D. E is round adjust to D. Who sit opposite to B?

(*a*) A (*b*) D (*c*) C (*d*) F

[Jan. 2021]

Solution :

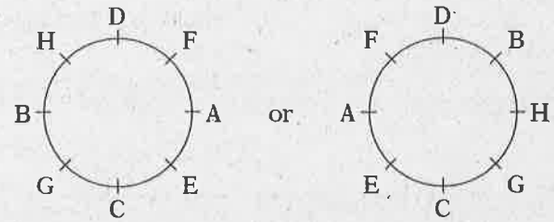


Q.25. Four Indian, A, B, C and D and four Chinese E, F, G and H are sitting in a circle around a table facing the each other in a conference. No two Indians or Chinese are sitting side by side, C who is sitting between G and E is being D, F is between D and A and facing G, H is to the left of B. Who is sitting left of A?

(*a*) E (*b*) F (*c*) G (*d*) H

Clearly A sits opposite to B. ***[Jan. 2021]***

Solution :



(*i*) (*ii*) (Incorrect)

(*ii*) is false because two Indians B & D are siting side by side which is against direction.

So (*i*) is correct.

Hence (*a*) is correct.

Q.26. Five friends A, B, C, D and E are sitting on a bench. A is sitting next to B; C is sitting next to D, D is not sitting with E; E is at the left end of bench. C is on second position from the right; A is on the right side of B who is the right side of E. A and C are sitting together. What is the position of B?

(*a*) Second from right (*b*) Centre

(*c*) Extreme left (*d*) Second from left

[Jan. 2021]

Solution : (*d*) is correct

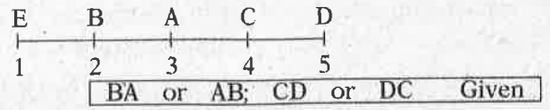
B is second from left.

Q.27. A, B, C, D and E are sitting on a bench. A is sitting next to B, C is sitting next to D, D is not sitting with E who is on the left end of the bench. C is on the second position from the right. A is to the right of B and E. A and C are sitting together. In which position A is sitting between?

(*a*) C and D (*b*) D and E (*c*) B and C (*d*) B and D

[Jan. 2021]

Solution : (*c*) is correct



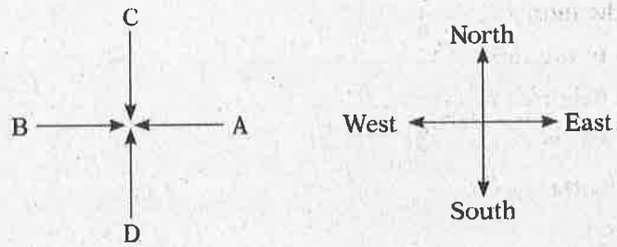
Q.28. A, B, C and D are playing cards, A and B are partners. D faces towards North. If A faces West, then who faces south?

(*a*) C (*b*) B

(*c*) D (*d*) Data is inadequate

[Jan. 2021]

Solution : (*a*) is Correct



Given that A and B are partners. A faces west hence B will face East. D faces North hence rest person 'C' will face South C & D are Partners.

Clearly C is facing South.

(*a*) is correct.

Q.29. Five girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary. Who is sitting immediate right to Reeta?

(*a*) Seema (*b*) Rani (*c*) Bindu (*d*) Mary

[July 2021]

Solution : (*d*) is correct

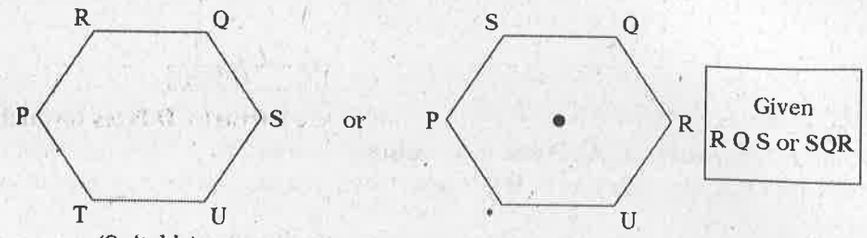
Bindu, Seema, Rani, Reeta, Mary

Q.30. Six friends P, Q, R, S, T and U are sitting around the hexagonal table each at one corner and are facing the centre of the hexagonal. P is second to the left of U. Q is neighbour of R and S. T is second to the left of S. Which one is sitting opposite to S?

(*a*) R (*b*) P (*c*) Q (*d*) T

[July 2021]

Solution : (*b*) is correct



Q.31. A, B, C, D, E, F and G are sitting in a row facing North :

1. **F is to the immediate right of E**
2. **E is 4th to the right of G**
3. **C is the neighbour of B and D**
4. **Person who is third to the left of D is at one of ends.**

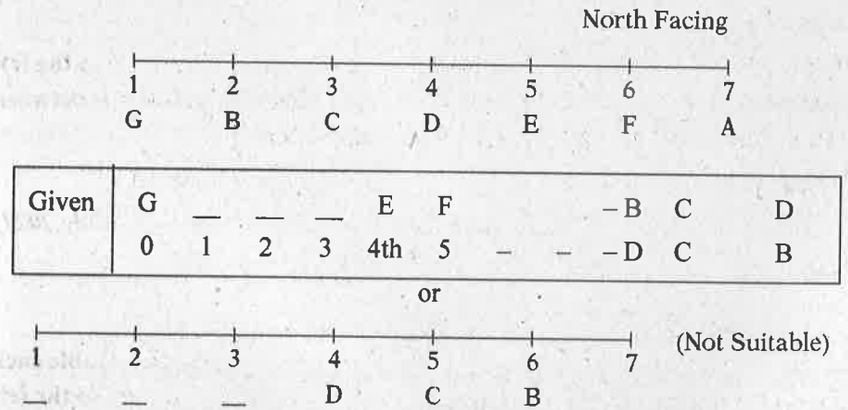
Who are to the right of D?

(*a*) E and F only (*b*) G, B and C

(*c*) E, F and A (*d*) G and B only

[July 2021]

Solution : (*c*) is correct



Q.32. Six children, named as P, Q, R, S, T and U, are sitting in a row, Q is between U and S: T is between P and R; P does not sit next to either U or S; B does not sit next to S. So, U is sitting between the pairs ..... of children.

(*a*) Q and T (*b*) Q and R (*c*) Q and S (*d*) Q and P

[Dec. 2021]

Solution : (*b*)

SQURTP

UQS or SQU
PTR or RTP

Q.33. Five persons A, B, C, D and E are sitting in a row. A sit S left to C and C sit S left to B. E sits rights to B, D sits in between E and B. Who is sitting in the middle?

(*a*) B (*b*) C (*c*) E (*d*) D

[Dec. 2021]

Solution : (*a*)

ACBDE

Q.34. Four Ladies A, B, C and D and four Gentlemen E, F, G and H are sitting in a circle around a table facing each other.

I. No two Ladies or gentlemen are sitting side by side.

II. C, who is sitting between G and E, facing D.

III. F is between D and A and facing G.

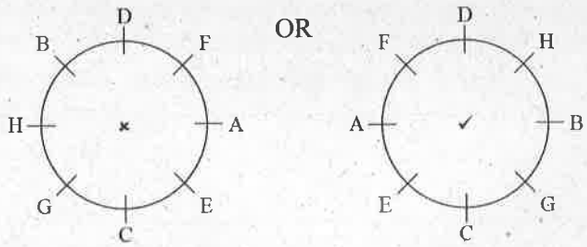
IV. H is to the right of B.

Who is immediate neighbour of B?

(*a*) G and H (*b*) E and F (*c*) E and G (*d*) A and B

[Dec. 2021]

Solution : (*a*)



[Because two ladies B & D are side by side]

Q.35. Persons M, N, O, P, Q, R, S and T are sitting on a compound wall facing North, O sits fourth left of S; P sits second to the right of S; only two people sit between P and M; N and R are immediate neighbours of each other. N is not an immediate neighbour of M; T is not a neighbour of P. How many persons are seated between M and Q?

(*a*) one (*b*) two (*c*) three (*d*) four

[Dec. 2021]

Solution : (*a*)

T O N R M S Q P

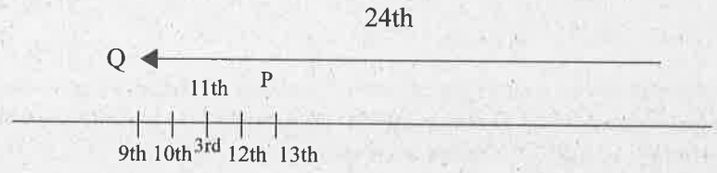
NR or ~~RN~~

Q.36. In a line, P is sitting 13th from left. Q is sitting 24th from the right and 3rd left from P. How many people are sitting in the line?

(*a*) 34 (*b*) 31 (*c*) 32 (*d*) 33

[Dec. 2021]

Solution : (*d*)



Total No. of People = 9 + 24

= 33

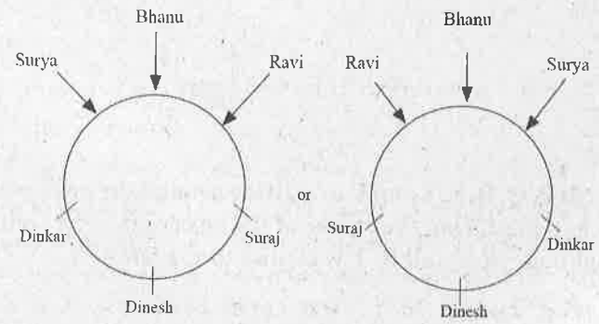
Q.37. Six friends Surya, Bhanu, Dinkar, Ravi, Suraj and Dinesh Are sitting in a circle and are facing the centre of the circle, Dinesh is between Dinkar and Suraj. Bhanu is between Ravi and Surya. Dinkar and Ravi are opposite to each other. Who are the immediate neighbours of Ravi ?

(*a*) Suraj and Dinesh (*b*) Dinkar and Bhanu

(*c*) Surya and Dinesh (*d*) Bhanu and Suraj

[June 2022]

Solution :



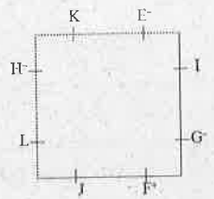
∴ (*d*) is correct

Q.38. Eight persons E, F, G, H, I, J, K and L are seated around a square table, facing table - two on each side. J is between L and F; G is between I and F; H a lady member is second to the left of J; F a male member is seated opposite to E, a lady member. There is a lady member between F and I. Who among the following is to the immediate left of F ?

(*a*) G (*b*) I (*c*) J (*d*) H

[June 2022]

Solution :



(*i*) LJF or FJL (*iii*) H—J

(*ii*) IGF or FGI (*iv*) F
|
E

(*v*) F Lady I or I Lady F

∴(*c*) is correct

Q.39. Five persons are sitting on a bench to be photographed, S is to the left of N and to the right of B. M is to the right of N. R is between N and M. Who is sitting immediate right to R?

(*a*) B (*b*) N (*c*) M (*d*) S

[June 2022]

Solution :

∴(*c*) is correct

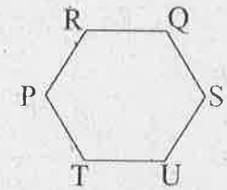
Q.40. Six friends P, Q, R, S, T and U are sitting around the hexagonal table each at one corner and are facing the center of the hexagonal. P is second to the left of U. Q is neighbour of R and S. T is second to the left of S.

Which one is sitting opposite to P?

(*a*) R (*b*) Q (*c*) T (*d*) S

[June 2022]

Solution :



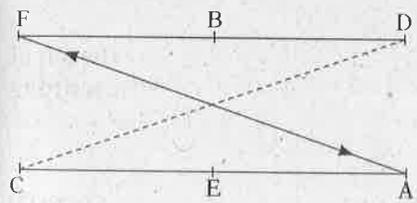
∴(*d*) is correct

Q.41. Six persons A, B, C, D, E and F are sitting in two rows with three persons in each row. Both rows are in front of each other. E is not at the end of the any row and D is second left to the F. C is neighbour of E and diagonally opposite to (*d*) If B is neighbour of F who is in front of C then who is sitting diagonally to F?

(*a*) C (*b*) E (*c*) A (*d*) D

[Dec. 2022]

Solution:



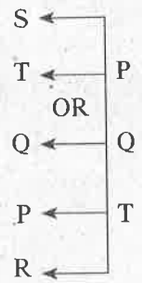
∴ (*c*) is correct

Q.42. P, Q, R, S and T are sitting in a line facing West. P and Q are sitting together. R is sitting at south end and S is sitting at North end. T is neighbour of Q and R. Who is sitting the middle?

(*a*) P (*b*) Q (*c*) R (*d*) S

[Dec. 2022]

Solution: (*b*)



∴ Q is in middle

∴ (*d*) is correct

Q.43. Seven friends A, B, C, D, E, F, & G are watching movie sitting in a row. E is sitting at one extreme end. C is sitting second to E. B is sitting between A & C. G is not sitting at any extreme end. A is not at any extreme end. D is sitting immediate on F, who is sitting in the middle?

(*a*) G (*b*) D (*c*) C (*d*) B

[June 2023]

Solution:

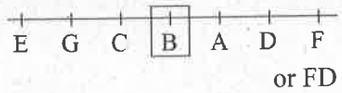
Given

(*i*) DF or FD

(*ii*) ABC – E or

E – CBA

Combining (*i*) & (*ii*)



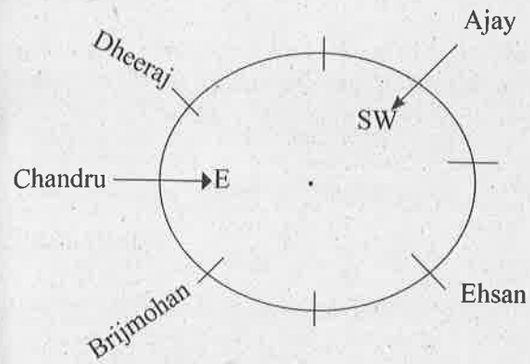
(*d*) is correct.

Q.44. Five boys Ajay, Brijmohan, Chandru, Dheeraj and Ehsan are sitting in a park in a circle facing the centre. Ajay is facing South-West, Dheeraj is facing South-East, Brijmohan and Ehsan are right opposite Ajay and Dheeraj, respectively and Chandru is equidistant between Dheeraj and Brijmohan. Which direction is Chandru facing?

(*a*) West (*b*) South (*c*) North (*d*) East

[June 2023]

Solution:



Clearly, Chandru is facing East.

(*d*) is correct.

Q.45. Pran, Qomal, Ravi, Shalu, Trilok, Urvi, Vasu and Walter are sitting in a row facing North:

(*i*) Pran is fourth to the right of Trilok

(*ii*) Walter is fourth to the left of Shalu

(*iii*) Ravi and Urvi, which are not at the ends, are neighbours of Qomal and Trilok, respectively

(*iv*) Walter is immediate left of Pran and Pran is the neighbour of Qomal

Identify who are sitting at the extreme ends?

(*a*) Pran and Walter (*b*) Trilok and Urvi

(*c*) Trilok and Shalu (*d*) Shalu and Pran

[June 2023]

Solution:

(*i*) Trilok — — — Pran

(*ii*) Walter — — — Shalu

(*iii*) (*a*) Ravi Qomal OR Qomal Ravi

(*b*) Urvi Trilok OR Trilok Urvi

(*iv*) Walter Pran Qomal

Mixing these points; we get

Trilok Urvi Vasu Walter Pran Qomal Ravi Shalu

Clearly Trilok & Shalu are sitting at the extreme ends.

∴ (*c*) is correct.

Q.46. Six friends — A, B, C, D, E and F are sitting around a circular table facing towards the centre of the circle. E is not sitting between B and D. A sits to the left of F and C is fourth to the right of A. D is immediate right of E.

Who sits second to right to F?

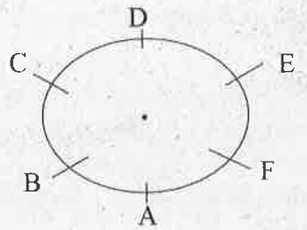
(*a*) C (*b*) A (*c*) D (*d*) B

[June 2023]

Solution:

Clearly D is 2nd to the right of F.

∴ (*c*) is correct.



19 CHAPTER BLOOD RELATIONS

DEFINITION

Those persons who are related to another persons by birth rather than by marriage are called in Blood Relation.

Let's start right from basic and see what are the different blood relations in a family, which we should know to understand the logical reasoning in blood relation questions.

Remember the relations as given below:

(i) Children of Same parents → Siblings
One's Husband or Wife → Spouse
Relatives on Mother's Side → Maternal
Relatives on Father's Side → Paternal

(ii) Father's Father → Paternal Grandfather
Father's Mother → Paternal Grandmother
Mother's Father → Maternal Grandfather
Mother's Mother → Maternal Grandmother
Father's Brother → Paternal Uncle
Father's Sister → Paternal Aunt
Mother's Brother → Maternal Uncle
Mother's Sister → Maternal Aunt
Uncle or Aunt's Child → Cousin
Sibling's Son → Nephew
Sibling's Daughter → Niece
Son or Daughter's Child → Grandchild

(iii) Relations

1. Grandfather's son ⟶ Father or Uncle
2. Grandmother's son ⟶ Father or Uncle
3. Grandfather's only son ⟶ Father
4. Grandmother's only son ⟶ Father
5. Mother's or father's mother ⟶ Grandmother
6. Son's Wife ⟶ Daughter-in-Law
7. Daughter's husband ⟶ Son-in-Law
8. Husband's or wife's sister ⟶ Sister-in-Law
9. Brother's son ⟶ Nephew
10. Brother's daughter ⟶ Niece
11. Uncle or aunt's son or daughter ⟶ Cousin
12. Sister's husband ⟶ Brother-in-Law
13. Brother's wife ⟶ Sister-in-Law
14. Grandson's or grand daughter's daughter ⟶ Great grand Daughter

Family Tree - Generally, the construction of the family tree is considered to be the most appropriate way of solving blood relation questions in Logical reasoning. So we start with representing each by gender by a particular sign. Keep these instructions in mind.

(*i*) Use **" + " sign for male** & **" - " sign for female**. As A is male ⟶ A^+; B is female ⟶ B^-

(*ii*) A is the brother of B ⟶ A^+ ← bro. → B [Here gender of B is not given, So do not show gender of B]

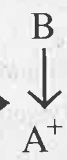
(*iii*) A is the sister of B ⟶ A^- ← Sis. → B [B may be male or female]

(*iv*) A is the wife of B ⟶ $A^- \rightleftharpoons B^+$

In this case we know the gender of both A and B. We are using a double-sided arrow here to show that the two individuals are married to each other.

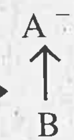
(*v*) We start by plotting the tree on different levels, where each level represents a generation in blood relation questions.

A is the son of B ⟶



[We cannot determine the gender of B since B can be either of the parent, mother or father]

(*vi*) A is the mother of B ⟶



[We cannot determine the gender of B since B can be either of the child, son or daughter]

(*vii*) A is the grandfather of B ⟶

[We cannot determine the gender of B since B can be either of the grandchild, grandson or granddaughter.]

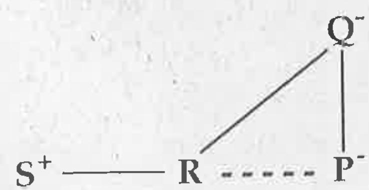
The 2 arrows represent that they are 2 generations away i.e. gap.

Let us try to understand through examples.

Example 1 : P is Q's daughter, Q is R's mother. S is R's brother. How is S related to P ?

(*a*) Father
(*b*) Grandfather
(*c*) Brother
(*d*) Son

Solution : (*c*) is correct.



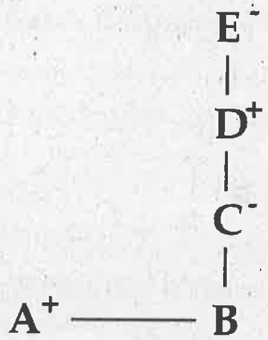
Clearly, S is brother of P.

Example 2 : A is B's brother. C is B's mother. D is C's father. E is D's mother. How is A related to E?

(*a*) Grandmother (*b*) Grand-daughter

(*c*) Great grandson (*d*) Grandson

Solution : : (*c*) is correct.



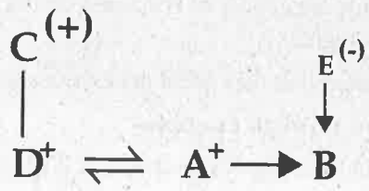
Clearly, E is grandfather of A⟶ A is **Great grandson of E.**

Example 3 : A is B's brother. C is D's father. E is B's mother. A and D are brothers. How is C related to E ?

(*a*) Wife (*b*) Husband

(*c*) Sister-in-law (*d*) Niece

Solution : (*b*) is correct.



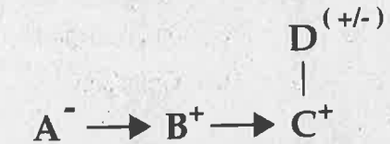
Clearly, E is wife of C ⟶ C is husband of E.

Example 4 : A is the sister of B. B is the brother of C. C is the son of D. How is D related to A?

(*a*) Son (*b*) Mother

(*c*) Daughter (*d*) Uncle

Solution : (*b*) is correct.



Clearly, D is parents of A, B, C. Gender of D is not clear, but in question's options. Mother suits because no other option is there as other part of parent.

Example 5 : (*i*) F is the brother of A.

(*ii*) G is the daughter of A.

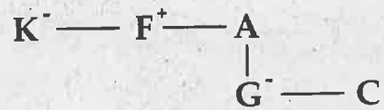
(*iii*) K is the sister of F.

(*iv*) C is the brother of G.

Who is the uncle of G ?

(*a*) K (*b*) F (*c*) A (*d*) C

Solution : (*b*) is correct.



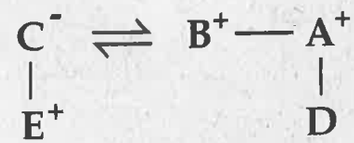
G is A and F is brother of A.

Example 6 : C is wife of B. E is the son of C. A is the brother of B and father of D. What is the relationship of E to D ?

(*a*) Cousin (*b*) Mother

(*c*) Sister (*d*) Brother

Solution : (*a*) is correct .



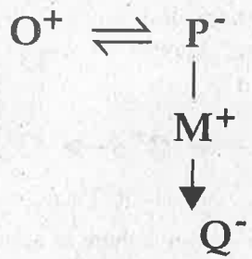
Clearly, E is cousin of D.

Example 7 : M is the son of P. Q is the grand-daughter of O, who is the husband of P. How is M related to O?

(*a*) Father (*b*) Mother

(*c*) Son (*d*) Daughter

Solution : (*c*) is correct .



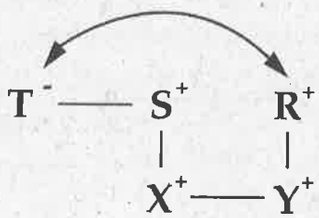
Clearly, M is son of O.

Example 8 : X and Y are brothers. R is the father of Y. S is the brother of T and maternal uncle of X. What is T related to R ?

(*a*) Mother (*b*) Wife

(*c*) Sister (*d*) Brother

Solution : (*b*) is correct.



Clearly, T is wife of R

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

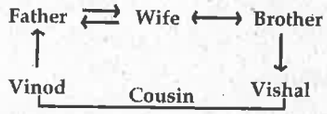
Q.1. Vinod introduce Vishal as the son of the only brother of his father's wife. How is Vinod related to Vishal ?

(*a*) Cousin (*b*) Brother

(*c*) Son (*d*) Uncle

[May 2018]

Solution : (a)



Clearly Vinod is cousin of Vishal.

Q.2. A prisoner introduced a boy who came to visit him to the jailor as "Brothers and sisters, I have none, he is my father's son's son". Who is the boy ?

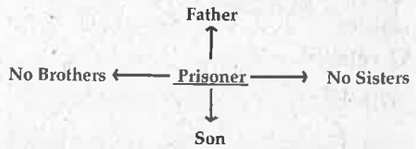
(*a*) Nephew (*b*) Son

(*c*) Cousin (*d*) Uncle

[May 2018]

Solution :

(b) 'Son'



Q.3. Suresh introduce a man as "he is the son of the woman who is the mother of the husband of my mother". How is Suresh is related to the man?

(*a*) Brother-in-law

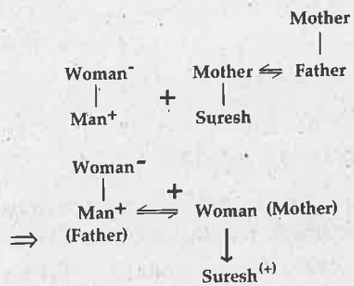
(*b*) Son

(*c*) Brother

(*d*) Nephew

[May 2018]

Solution : (b)



Clearly Suresh is the 'Son' of the man.

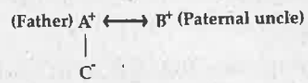
Q.4. A reads a book and find the name of the author familiar. The author 'B' is the paternal uncle of C. C is the daughter of A. How is B related to A?

(*a*) Brother (*b*) Sister

(*c*) Father (*d*) Uncle

[May 2018]

Solution : (a)



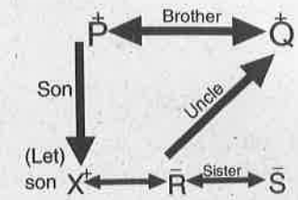
Q.5. P and Q are brothers. R and S are sisters. P's son is R's brother. How is Q related to R?

(*a*) Uncle (*b*) Brother

(*c*) Father (*d*) Grandfather

[May 2018]

Solution : (a)



Clearly 'Q' is the uncle of 'R'

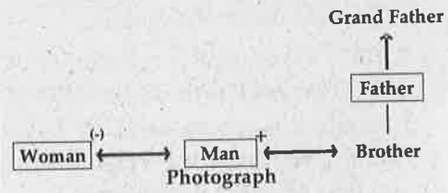
Q.6. Pointing to a man in a photograph, a woman said, "the father of his brother is the only son of my grandfather", how is the woman related to the man in the photograph?

(*a*) Mother (*b*) Aunty

(*c*) Daughter (*d*) Sister

[Nov. 2018]

Solution : (d)



Q.7. Six persons are seen together in a group. They are A, B, C, D, E and F. B is brother of D, but D is not brother of B. F is brother of B, C and A are married together. F is son of C, but C is not mother of F. E is brother of A. The number of female members in the group is.....

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

[Nov. 2018]

Solution :

(b) $C^{+} \rightleftharpoons A^{-} \longleftrightarrow E^{+}$
$F^{+} \longleftrightarrow B^{+} \longleftrightarrow D^{-}$

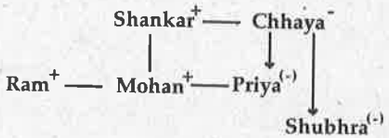
No. of female members = 2

Q.8. Ram and Mohan are brothers, Shankar is Mohan's father. Chhaya is Shankar's sister. Priya is Shankar's niece. Shubhra is Chhaya's grand daughter. Then Ram is Shubhra's.....

(*a*) Brother (*b*) Uncle
(*c*) Cousin (*d*) Nephew

[Nov. 2018]

Solution : (b)



Ram is the uncle of Shubhra.

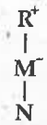
Q.9. If P + Q means P is the mother of Q. P ÷ Q means P is the father of Q. P - Q means P is the sister of Q. Then which of the following relationship shows that M is the daughter of R ?

(*a*) R ÷ M + N
(*b*) R + N ÷ M
(*c*) R - M ÷ N
(*d*) None of the above

[Nov. 2018]

Solution : (a)

Go by choices (GBC)



Clearly, M is the daughter of R.

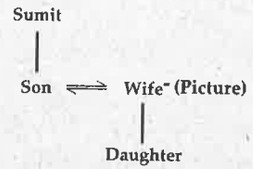
Q.10. Pointing to a picture, Sumit said, she is the mother of my son's wife's daughter. How is lady related to the Sumit ?

(*a*) Uncle (*b*) Cousin
(*c*) Daughter (*d*) None

[June 2019]

Solution :

(d)



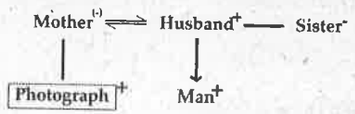
Clearly, the lady (in picture) is the daughter-in-law of Sumit.

Q.11. Pointing to a photograph, a man said "His mother husband's sister is my aunt". Then what is relation between a man and he ?

(*a*) Son (*b*) Uncle
(*c*) Nephew (*d*) Brother

[June 2019]

Solution : (d)



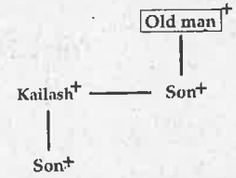
Clearly, the man and he (Photograph) are brothers.

Q.12. Pointing the old man Kailash said "his son is my son's uncle". How is Kailash related to old man?

(*a*) Brother
(*b*) Either son or son-in-law
(*c*) Father
(*d*) Grand Father

[June 2019]

Solution : (b)



Clearly Kailash is either Son or Son-in-law.

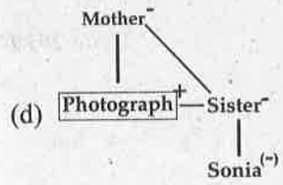
Q.13. Pointing in a photograph, Sonia said, "His mother's only daughter is my mother". How is Sonia relates to that man?

(*a*) Nephew (*b*) Sister
(*c*) Wife (*d*) Niece

[June 2019]

Solution :

(d)



Clearly Sonia is Niece of Photograph.

Direction (Q.14 - Q.17) Read the following information carefully to answer the given questions.

Six members of a family namely A, B, C, D, E and F are travelling together. B is the son of C but C is not the mother of B. A and C are married couple. E is the brother of C. D is the daughter of A. F is the brother of B.

Q.14. How many male members are there in the family ?

(*a*) 3 (*b*) 2
(*c*) 4 (*d*) 1

[Dec. 2019]

Solution : (*c*)

Q.15. How many children does A have

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

[Dec. 2019]

Solution : (*c*)

Q.16. What is the relation of E to D

(*a*) Uncle (*b*) Brother
(*c*) Father (*d*) None of these

[Dec. 2019]

Solution : (*a*)

Q.17. Who is the mother of B?

(*a*) C (*b*) D
(*c*) F (*d*) A

[Dec. 2019]

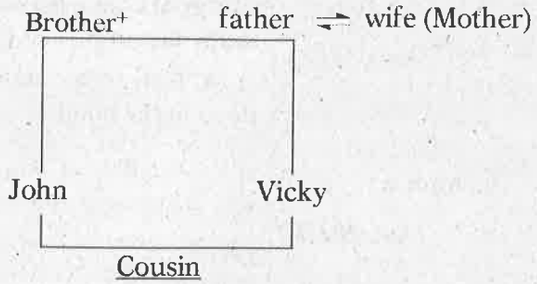
Solution : (*d*)

Q.18. Vicky introduces John as the son of the only brother of his father's wife. How is Vicky related to John?

(*a*) Cousin (*b*) Son
(*c*) Brother (*d*) Uncle

[Dec. 2020]

Solution : (*a*)



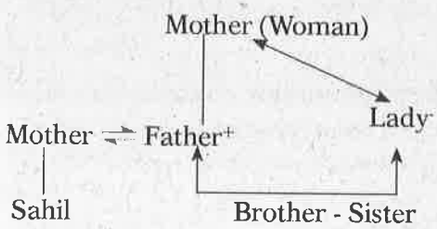
Q.19. Point out to a lady Sahil said she is the daughter of woman. Who is the mother of the husband of my mother. Who is the lady to Sahil?

(*a*) Sister (*b*) Aunt

(*c*) Daughter (*d*) Sister-in-law

[Dec. 2020]

Solution : (*b*)



∴ That Lady is aunt of Sahil.

Q.20. P is the brother of Q and R, S is the mother of R. T is the father of P, Which of the following statement cannot be definitely true?

(*a*) S is the mother of P

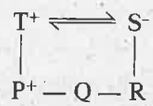
(*b*) P is son of S

(*c*) T is husband of S

(*d*) Q is son of T

[Jan. 2021]

Solution : (*d*) is correct



Here Gender of Q is not known.

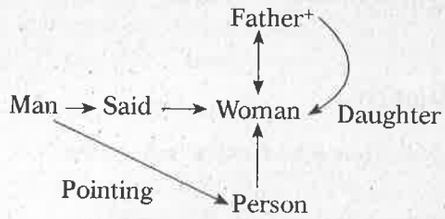
Q.21. Pointing towards a person a man said to a woman. "His mother is the only daughter of your father". How is the woman related to that person?

(*a*) daughter (*b*) mother

(*c*) sister (*d*) wife

[Dec. 2020]

Solution : (*b*)



⇒ Your Father means woman's Father is the only daughter means that woman herself. Clearly that woman is mother of that person.

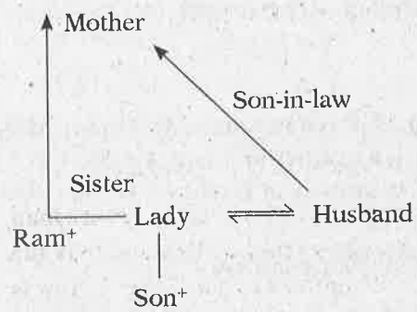
Q.22. Pointing to a lady in a photograph, Ram said "Her son's father is the son in law of my mother". How is Ram related to the lady?

(*a*) Aunt (*b*) Cousin

(*c*) Sister (*d*) Mother

[Jan. 2021]

Solution : (*b*) is correct



Clearly Ram is the brother of that Lady.

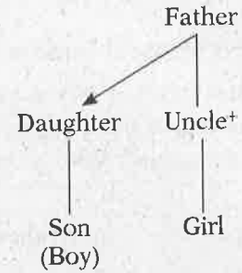
Q.23. A girl Introduced, a boy as the son of the daughter of father of her uncle. The boy is girl's

(*a*) Son (*b*) Brother

(*c*) Son-in-Law (*d*) Uncle

[Jan. 2021]

Solution :



Clearly the boy is cousin brother.

So (*b*) is correct.

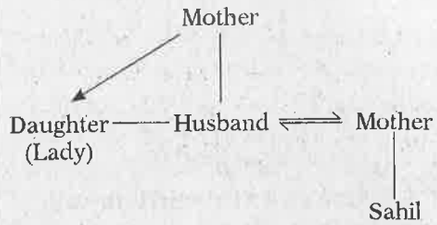
Q.24. Pointing to a lady, Sahil said, "She is the daughter of the women who is the mother of the husband of my mother". Who is the lady to Sahil?

(*a*) Aunt (*b*) Sister

(*c*) Daughter (*d*) Sister-in-Law

[Jan. 2021]

Solution :



Clearly Lady is the sister of Sahil's Father.

So the lady is Aunt of Sahil.

∴ (*a*) is correct.

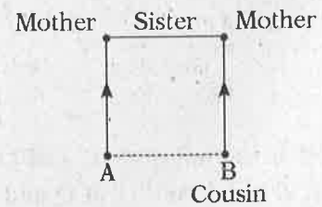
Q.25. Pointing towards "A", "B", said : "Your mother is the younger sister of my mother". "A" is related to "B" as

(*a*) Uncle (*b*) Cousin

(*c*) Nephew (*d*) Father

[July 2021]

Solution : (*b*) is correct

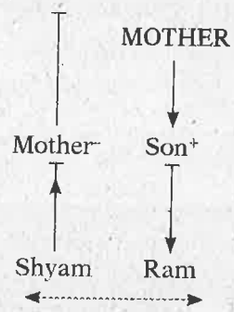


Q.26. Shyam's mother said to Shyam "my mother has a son whose son is Ram". Shyam is related to Ram as

(a) Uncle (b) Cousin
(c) Nephew (d) Grandfather

[July 2021]

Solution : (b) is correct

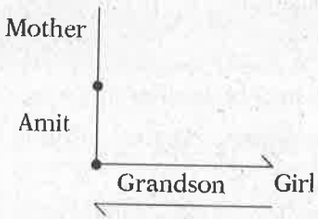


Q.27. Amit said "This girl is the wife of the grandson of my mother". How Amit related to the girl?

(a) Father-in-law
(b) Grandson
(c) Father
(d) Son

[July 2021]

Solution : (a) is correct

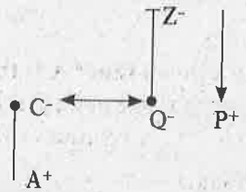


Q.28. A is the son of C; C and Q are sisters; Z is the mother of Q and P is the son of Z Which of the following statements is true?

(a) A and P are cousins
(b) C and P are sisters
(c) P is the maternal uncle of A
(d) A is the maternal uncle of P

[July 2021]

Solution : (c) is correct

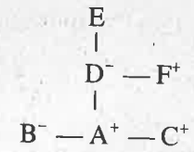


Q.29. D is daughter of E. A is son of D. C is a brother of A and B is sister of A. F is brother of D. How F is related to B?

(a) Father-in-Law
(b) Uncle
(c) Brother
(d) Mother-in-law

[Dec. 2021]

Solution : (b)



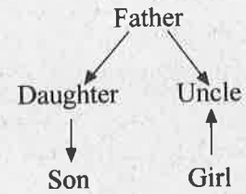
Clearly, F is Maternal Uncle of B.

Q.30. Introducing a boy a girl said, "He is the son of the daughter of the father of my uncle". Who is the boy to the girl?

(a) Brother (b) Nephew
(c) Uncle (d) Son-in-law

[Dec. 2021]

Solution : (a)



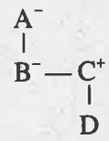
He = boy

Q.31. It is given that "A is the mother of B; B is the sister of C; C is the father of D". How is A related to D?

(a) Mother
(b) Grandmother
(c) Aunt
(d) Sister

[Dec. 2021]

Solution : (b)

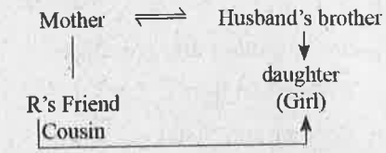


Q.32. R told to M as, "the girl, I met at the beach, was the youngest daughter of the brother-in-law of my friend's mother". How is the girl related to R's friend?

(a) Cousin (b) Daughter
(c) Niece (d) Aunt

[Dec. 2021]

Solution : (a)

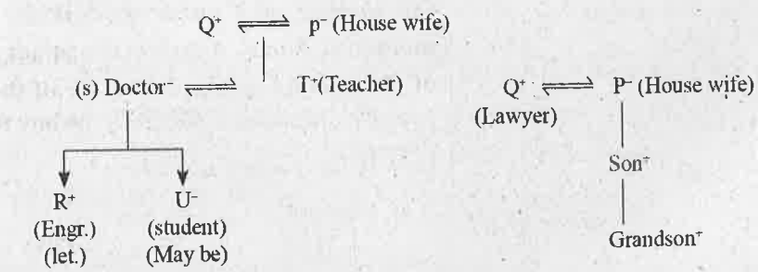


Q.33. P, Q, R, S, T, U are 6 members of a family in which there are two married couples. T, a teacher is married to a doctor who is mother of R and U, Q the lawyer is married to P. P has one son and one grandson. Of the two married ladies one is a housewife. There is also one student and one male engineer in the family. Which of the following is true about the granddaughter of the family?

(a) She is a lawyer
(b) She is an engineer
(c) She is a student
(d) She is a doctor

[Dec. 2021]

Solution : (c)



Q.34. A woman going with a boy is asked by another woman about the relationship between them. The woman replied " My maternal uncle and the uncle of his maternal uncle is the same". How is the lady related with that boy?

(*a*) Grand mother and grandson

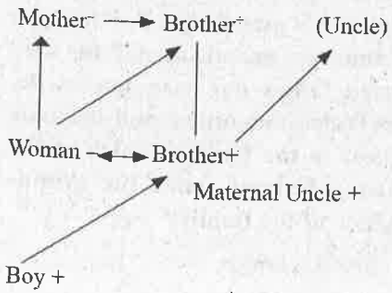
(*b*) Mother and Son

(*c*) Brother and Sister

(*d*) Aunt and Nephew

[June 2022]

Solution :



(*b*) is correct

Q.35. If X+ Y means X is the mother of Y; X- y means X is the brother y; X % Y means X is the father of Y and X × Y means X is the sister of y, Which of the following shows that A is the maternal uncle of B?

(*a*) B + D × C - A

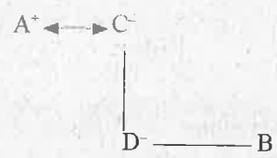
(*b*) B - D % A

(*c*) A-C + D × B

(*d*) A + C × D - B

[June 2022]

Solution :



Clearly A is Maternal Uncle of B.

(*c*) is correct

Q.36. B and C are siblings. M has two children, and he is son of E, who is father-in-law of H. H has only one son. C is not grand daughter of E. How's B related to E?

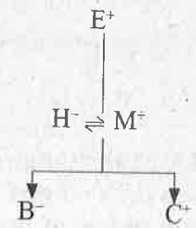
(*a*) Daughter

(*b*) Son

(*c*) Grand daughter

(*d*) Grand son ***[June 2022]***

Solution :



(*c*) is correct

Q.37. Ravi is a son of Aman's father's sister. Sahil is the son of Divya who is the mother of Gaurav and grand mother of Aman. Ashok is the father of Tanya and grand father of Ravi Divya is the wife of Ashok.

How is Ravi related to Divya?

(*a*) Nephew (*b*) Son

(*c*) Grandson (*d*) Father

[June 2022]

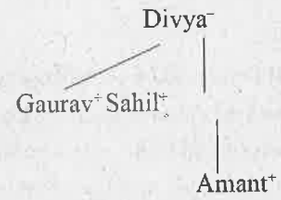
Solution :

(i) Father Sister

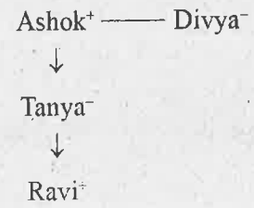
↓ ↓

Aman⁺ Ravi⁺

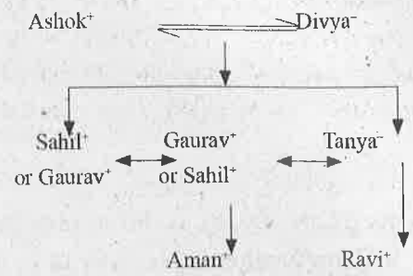
(ii)



(iii)



Combining points (i), (ii) and (iii) together



∴ (*c*) is correct

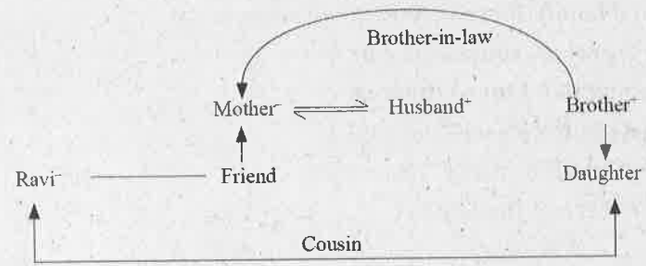
Q.38. Rani told Jaya, "The girl I met yesterday at the beach was the youngest daughter of the brother-in-law of my friend's mother." How is the girl related to Rani's friend?

(*a*) Cousin (*b*) Daughter

(*c*) Mother (*d*) Aunt

[June 2022]

Solution :



∴ (*a*) is correct

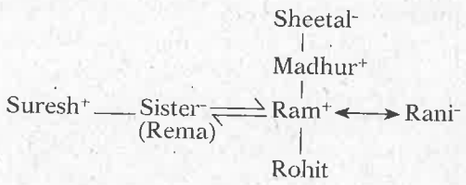
Q.39. Suresh's sister is the wife of Ram. Ram is Rani's brother. Ram's father is Madhur Sheetal is Ram's grandmother. Rema is sheetal's daughter-in-law. Rohit is Ram's brother's son. Who is Rohit to Suresh?

(*a*) Brother-in-law (*b*) Son

(*c*) Brother (*d*) Nephew

[Dec. 2022]

Solution :



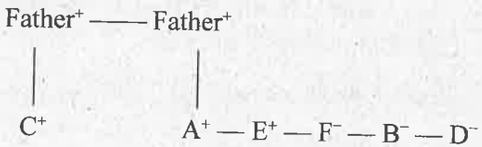
∴ (*d*) is correct

Q.40. There are six children playing football namely A, B, C, D, E and F. A & E are brothers. F is sister of E. C is the only son of A's uncle. B & D are daughters of the brother of C's father. How D is related to A?

(*a*) Uncle (*b*) Cousin (*c*) Niece (*d*) Sister

[Dec. 2022]

Solution :



Clearly D is the sister of A.

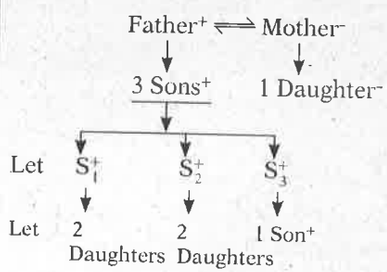
∴ (*d*) is correct

Q.41. In a point family, there are father, mother, a married sons and one unmarried daughter. Out of the sons two have 2 daughters each and one has a son only. How many female members are there in the family?

(*a*) 3 (*b*) 6
(*c*) 9 (*d*) 5

[Dec. 2022]

Solution:



∴ Total No. of Females = 1+ 1 + 2 + 2

= 6

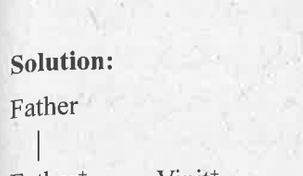
∴ (*b*) is correct

Q.42. When Rani was Vinit, she recollected that "He is the brother of my grandfather's son". How is Rani related to Vinit?

(*a*) Aunt (*b*) Daughter
(*c*) Sister (*d*) Niece

[Dec. 2022]

Solution:



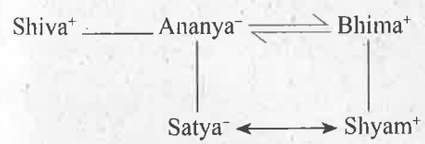
∴ (*d*) is correct.

Q.43. Annanya is mother of Satya and Shyam is the son of Bhima. Shiva is brother of Annanya. If Satya is sister of Shyam, how Bhima is related to Shiva?

(*a*) Son
(*b*) Cousin
(*c*) Brother-in-law
(*d*) Son-in-law

[Dec. 2022]

Solution:



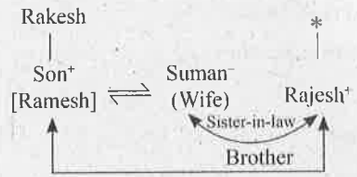
∴ (*c*) is correct.

Q.44. Suman is daughter-in-law of Rakesh and sister-in-law of Rajesh. Ramesh is the son of Rakesh and only brother of Rajesh. Find the relation of Suman with Ramesh.

(*a*) Sister-in-law (*b*) Cousin
(*c*) Aunt (*d*) Wife

[Dec. 2022]

Solution:



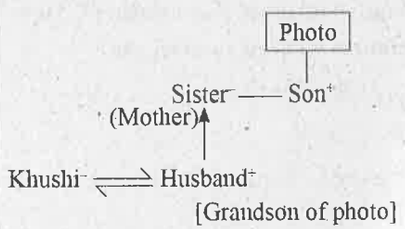
∴ (*d*) is correct.

Q.45. Pointing to a man in the photograph, Khushi says. "This man's son's sister is my mother-in-law." How is the Khushi's husband related to the man in the photograph?

(*a*) Grandson (*b*) Son
(*c*) Son-in-law (*d*) Cousin

[Dec. 2022]

Solution:



∴ (*a*) is correct

Q.46. P, Q, R, S, T and U are 6 members of a family in which there are two married couples. T, a teacher is married to a doctor, who is mother of R and U. Q, the lawyer is married to P. P has one son and one grandson. Of the two married ladies one is a housewife.

There is also one student and one male engineer in the family. Which of the following is true about the grandson of the family?

(*a*) He is a lawyer

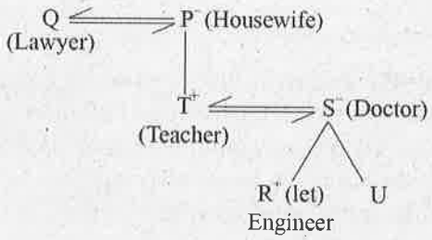
(*b*) He is an engineer

(*c*) He is a student

(*d*) He is a doctor

[June 2023]

Solution :



(*b*) is correct.

Q.47. Neelam, who is Deepak's daughter, says to Deepika "your mother-in-law. Rekha is younger daughter of Ramlal, who is my grandfather". How Neelam is related to Deepika?

(*a*) Sister

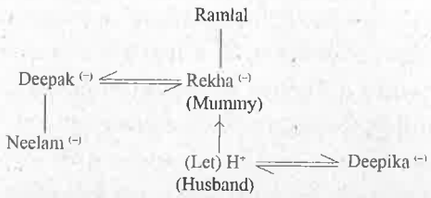
(*b*) Niece

(*c*) Sister-in-law

(*d*) Aunt

[June 2023]

Solution :



(*c*) is correct.

Q.48. Pointing on a photograph, a woman says "This man's son's sister is my mother-in-law". How is the women's husband related to the man in the photograph?

(*a*) Son

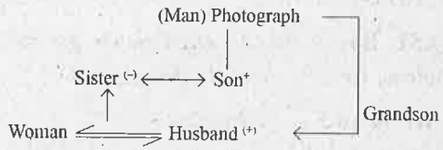
(*b*) Son-in-law

(*c*) Grandson

(*d*) Nephew

[June 2023]

Solution :



(*c*) is correct.

Q.49. If A is the brother of B, B is the daughter of C and D is the father of A, then how is C related to D?

(*a*) Husband

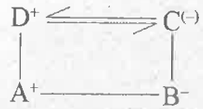
(*b*) Wife

(*c*) Granddaughter

(*d*) Grandfather

[June 2023]

Solution :



(*b*) is correct.

Q.50. X is the husband of Y. W is daughter of X. Z is husband of W. N is daughter of Z.

(*a*) Cousin

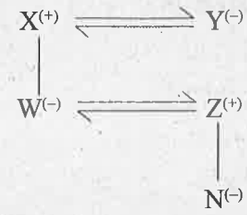
(*b*) Niece

(*c*) Daughter

(*d*) Grandmother

[June 2023]

Solution :



Y is Maternal Grandmother.

∴ (*d*) is correct.

Q.51. Based on the statements given below, find out who is the uncle of P?

(*i*) K and J are brothers

(*ii*) K's sister is M

(*iii*) P and N are siblings

(*iv*) N is the daughter of J

(*a*) K

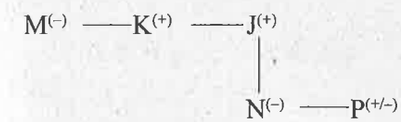
(*b*) J

(*c*) N

(*d*) M

[June 2023]

Solution :



Clearly K is uncle of "P".

∴ (*a*) is correct.

Q.52. There are six persons A, B, C, D, E and F are a family. A and B are a married couple and A is a male member. D is the only son of C, who is the brother of A, E is the sister of D. B is the daughter-in-law of F whose husband has died.

(*a*) A

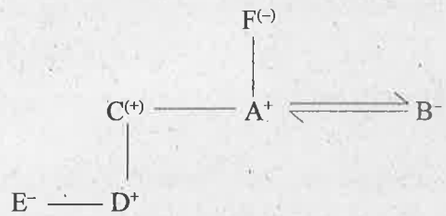
(*b*) E

(*c*) D

(*d*) F

[June 2023]

Solution :



∴ F is the mother of C.

∴ (*d*) is correct.

20 CHAPTER DESCRIPTION OF DATA

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. Cost of sugar in a month under the heads raw materials, labour, direct production and others were 12, 20, 35 and 23 units respectively, What is the difference between the central angles for the largest and smallest components of the cost of sugar?

(*a*) 72° (*b*) 48°

(*c*) 56° (*d*) 92°

[Aug. 2007, June 2012]

Solution : (*d*)

Total cost of sugar = (12 + 20 + 35 + 23) units = 90 units

Value of Largest component = 35 units

Value of Smallest component = 12 units

Difference between their central angles $= \frac{35-12}{90} \times 360^0 = 92^0$

Q.2. Arrange the dimensions of Bar diagram, Cube diagram, Pie diagram in sequence.

(*a*) 1,2,3 (*b*) 2,1,3

(*c*) 2,3,2 (*d*) 3,2,1

[Dec. 2009, June 2011]

Solution : (*c*)

Bar diagram : - has length and breadth, so two dimensional.

Cube diagram: - has length, breadth and height and hence it is three-dimensional.

Pie-diagram : Its area can be obtained. Hence two-dimensional

Therefore, if we arrange it in sequence we get

Pie-diagram; Bar, diagram and Cube-diagram *i.e.* 2,3,2.

Q.3. Nationality of a person is:

(*a*) Discrete variable

(*b*) An attribution

(*c*) Continuous variable

(*d*) None

[Dec. 2009]

Solution : (*b*)

Q.4. If we plot less than and more than type cumulative frequency distribution then the graph plotted is–

(*a*) Histogram
(*b*) Frequency curve
(*c*) Ogive
(*d*) None of these

[Dec. 2009]

Solution : (*c*)

Q.5. The primary rules that should be observed in classification

(*i*) As far as possible the class should be of equal width
(*ii*) The classes should be exhaustive
(*iii*) The classes should be unambiguously defined

Then which of the following is correct:

(*a*) Only (*i*) and (*ii*)
(*b*) Only (*ii*) and (*iii*)
(*c*) Only (*i*) and (*iii*)
(*d*) all (*i*), (*ii*) and (*iii*)

[June 2010]

Solution : (*b*)

Q.6. Using Ogive curve,we can determine:

(*a*) Median
(*b*) Quartile
(*c*) Both (*a*) and (*b*)
(*d*) None

[June 2010]

Solution : (*c*)

Q.7. With the help of Histogram one can find:

(*a*) Mean (*b*) Median
(*c*) Mode (*d*) First Quartile

[June 2010]

Solution : (*c*) Histogram is used to find Mode.

Q.8. Mode can be obtained from

(*a*) Frequency polygon
(*b*) Histogram
(*c*) Ogive
(*d*) All of the above

[Dec. 2010]

Solution : (*b*) Mode can be obtained from Histogram

Q.9. The most appropriate diagram to represent the data relating to the monthly expenditure on different items by a family is

(*a*) Histogram
(*b*) Pie-diagram
(*c*) Frequency polygon
(*d*) Line graph

[Dec. 2010]

Solution : (*b*) Pie diagram

Q.10. The data obtained by the internet are

(*a*) Primary data
(*b*) Secondary data
(*c*) Both (*a*) and (*b*)
(*d*) None of these

[Dec. 2010]

Solution : (*b*) Secondary data

Q.11. The Statistical measure computed from the sample observations alone have been termed as

(*a*) estimate (*b*) Parameter
(*c*) Statistic (*d*) Attribute

[Dec. 2010]

Solution : (*c*) Statistic

Q.12. When the two curves of Ogive intersect the point of intersection provides:

(*a*) First Quartile
(*b*) Second Quartile
(*c*) Third Quartile
(*d*) Mode

[June 2011]

Solution : (*b*)

Q.13. Frequency Density can be framed as:

(*a*) Class frequency to the cumulative frequency
(*b*) Class frequency to the total frequency
(*c*) Class frequency to the class length
(*d*) Class length to the class frequency

[June 2011]

Solution : (*c*) Frequency Density (F.D) = Class frequency/Class Length

Q.14. The Chronological classification of data are classified on the basis of:

(*a*) Attributes (*b*) Area
(*c*) Time (*d*) Class Interval

[June 2011]

Solution : (*c*)

Q.15. Arrange the following dimension wise : Pie-diagram, bar-digram and cubic diagram.

(*a*) 1,2,3 (*b*) 3,1,2
(*c*) 3,2,1 (*d*) 2,2,3

[June 2011]

Solution : (*d*)

Q.16. The frequency of class 20-30 in the following data is

Class	0-10	10-20	20-30	30-40	40-50
Cumulative Frequency	5	13	28	34	38

(*a*) 5 (*b*) 28 (*c*) 15 (*d*) 13

[Dec. 2011]

Solution : (*c*)

Frequency of class
interval 20 – 30
= c.f. of 20 – 30
– c.f. of 10 – 20
= 28 – 13 = 15

Here the frequency of class " 20 - 30 " = 15

Q.17. The Graphical representation by which median is calculated is called

(*a*) Ogive Curve
(*b*) Frequency Curve

(c) Line diagram

(d) Histogram

[Dec. 2011]

Solution : (a) The median is calculated by Ogive Curve

Q.18. Which of the following is not a two dimensional diagram?

(a) Square diagram

(b) Line diagram

(c) Rectangular diagram

(d) Pie-chart

[Dec. 2011]

Solution : (b) Line diagram is not two dimensional diagram because it has only length.

Q.19. From which graphical representation , we can calculate partition values ?

(a) Lorenz curve

(b) Ogive curve

(c) Histogram

(d) None of the above

[Dec, 2012]

Solution : (b) We can calculate Partition values with the help of Ogive Curve.

Q.20. The data given below refers to the marks gained by a group of students:

Marks	**Below** 10	**Below** 20	**Below** 30	**Below** 40	**Below** 50
No. of students	15	38	65	84	100

Then the No. of students getting marks more than 30 would be _____.

(a) 50 (b) 53 (c) 35 (d) 62

[June 2012]

Solution : (c) It is cumulative frequency data.

The No. of students getting marks more than 30 = 100 - 65 = 35

Q.21. Cost of sugar in a month under the heads raw materials, labour, direct production and others were 12, 20, 35 & 23 units respectively. The difference between their central angles for the largest & smallest components of the cost of Sugar is

(a) 92^0 (b) 72^0 (c) 48^0 (d) 56^0

[June 2012]

Solution : (a) Total = 90

Angle difference = $\frac{35-12}{90}\times 360^0 = 92^0$.

Q.22. What is a exclusive series ?

(a) In which both upper and lower limit are not included in class frequency.

(b) In which lower limit is not included in class frequency.

(c) In which upper limit is not included in class frequency

(d) None of the above

[Dec. 2012]

Solution : (c) In exclusive series,upper limit is not included in class frequency.

Q.23. A pie diagram used to represent the following data_____

Source	Customers	Excise	Income Tax	Wealth Tax
Revenue in Millions	120	180	240	180

The central angles corresponding to Income Tax and Wealth Tax are

(a) 90^0,120^0 (b) 120^0,90^0 (c) 60^0,120^0 (d) 90^0,60^0.

[June 2013]

Solution : (b) is correct

Total Revenue = 720

Central Angle for Income-tax $= \frac{240}{720}\times 360^0 = 120^0$

Central Angle for Wealth Tax = $\frac{180}{720}\times 360^0 = 90^0$

Q.24. The pair of averages whose value can be determined graphically?

(a) Mean & Median (b) Mode & Mean

(c) Mode & Median (d) None of the above

[Dec. 2013]

Solution : (c) is correct.

Q.25. The difference between upper limit and lower limit of a class is called:

(a) Class Interval (b) Class Boundaries

(c) Mid-value (d) Frequency

[Dec. 2013]

Solution : (a) Length of class- Interval

Q.26. If the class intervals are 10 - 14, 15 - 19, 20 - 24,..... Then the first class boundaries are:

(*a*) 9.5 - 14.5 (*b*) 10 - 15 (*c*) 9 - 15 (*d*) 10.5 - 15.5

[Dec. 2013]

Solution : (*a*) is correct

Since, D = 1 $\therefore LCB = l_1 - \frac{D}{2} = 10 - \frac{1}{2} = 9.5$; UCB = $l_2 + \frac{D}{2} = 14 + \frac{1}{2} = 14.5$

Q.27. The following data related to the marks of group of students

Marks	**No. of students**
More than 70%	7
More than 60%	18
More than 50%	40
More than 40%	60
More than 30%	75
More than 20%	100

How many students have got marks less than 50%?

(*a*) 60 (*b*) 82 (*c*) 40 (*d*) 53

[June 2014]

Solution : (*a*) is correct.

No .of students scoring marks less than 50% = 100 - No. of students scoring more than 50%

= 100 - 40 = 60

Q.28. To draw Histogram the frequency distribution should be

(*a*) Inclusive type

(*b*) Exclusive type

(*c*) Inclusive and Exclusive type

(*d*) None

[June 2014]

Solution : (*b*) is correct

Q.29. "The less than Ogive" is a:

(*a*) U-shaped curve

(*b*) J-shaped curve

(*c*) S- shaped curve

(*d*) Bell-shaped curve

[June 2014]

Solution : (*b*) is correct.

Q.30. There were 200 employees in an office in which 150 were married. Total male employees were 160 out of which 120 were married. What was the number of female unmarried employees?

(*a*) 30 (*b*) 10

(*c*) 40 (*d*) 50

[June 2014]

Solution : (*b*) is correct.

	Male	**Female**	**Total**
Married	120	30	150
Unmarried	40	10	50
Total	160	40	200

Unmarried Female = 10

Q.31. The most appropriate diagram to represent 5 year plan outlay of India in different economic sectors is

(*a*) Pie diagram

(*b*) Histogram

(*c*) Line diagram

(*d*) Frequency polygon

[Dec. 2014]

Solution : (*a*) is correct

Q.32. For construction of Histogram the class intervals of frequency distribution is

(*a*) Equal

(*b*) Unequal

(*c*) Either Equal or Unequal

(*d*) None

[Dec. 2014]

Solution : (*a*) is correct

Q.33. 100 persons are divided into number of male/female and employed/un-employed it refers to

(*a*) Cardinal Data

(*b*) Ordinal Data

(*c*) Spatial Data

(*d*) Temporal Data

[Dec. 2014]

Solution : (*b*) is correct

Q.34. The number of observations between 150 and 200 based on the following data is:

Value	More than 100	More than 150	More than 200	More than 250
No. of observations	70	63	28	05

(*a*) 46 (*b*) 35 (*c*) 28 (*d*) 23

[June 2015]

Solution : (*b*) is correct

No. of observation b/w 150 and 200 = 63-28=35.

Q.35.

Number of accidents	0	1	2	3	4	5	6	7
Frequency	12	9	11	13	8	9	6	3

In how many cases 4 or more accidents occur ?

(*a*) 32 (*b*) 41 (*c*) 26 (*d*) 18

[June 2015]

Solution : (*c*) is correct

No. of 4 or more accidents = 8+ 9 + 6+ 3 = 26

Q.36. The curve obtained by joining the points, whose X-coordinates are the upper limits of the class-intervals and Y-coordinates are the corresponding cumulative frequencies is called

(*a*) Ogive
(*b*) Histogram
(*c*) Frequency Polygon
(*d*) Frequency Curve

[June 2015]

Solution : (*a*)

Q.37. Histogram is used for the presentation of the following type of series:

(*a*) Time series
(*b*) Continuous frequency series
(*c*) Discrete series
(*d*) Individual series

[June 2015]

Solution : (*b*)

Q.38. The perpendicular line drawn from the intersection of two Ogives which touches at ___ point in X-axis.

(*a*) Median
(*b*) Mode
(*c*) Third quartile
(*d*) First quartile

[June 2015]

Solution : (*a*)

Q.39. Which is most common diagrammatic representation for grouped frequency distribution.

(*a*) Histogram
(*b*) Ogive
(*c*) Both (*a*) & (*b*)
(*d*) None of these

[June 2015]

Solution : (*a*) is correct.

Q.40. Classification is of _____ kinds:

(*a*) One (*b*) two
(*c*) three (*d*) Four

[June 2015]

Solution : (*d*) is correct.

Q.41. Quartiles can be found through which graph?

(*a*) Ogive
(*b*) Histogram
(*c*) Frequency polygon
(*d*) Frequency curve

[June 2015]

Solution : (*a*) is correct.

Q.42. The chart that user Logarithm of the variable is known as:

(*a*) Line chart
(*b*) Ratio chart
(*c*) Multiple line chart
(*d*) Component line chart

[June 2015]

Solution : (*b*) is correct.

Q.43. Find the number of observations between 250 and 300 from the following data:

Value	More than 200	More than 250	More than 300	More than 350
No. of obs.	56	38	15	0

(*a*) 56 (*b*) 23 (c) 15 (*d*) 8

[June 2015]

Solution : (*b*) is correct.

No. of observations = 38 - 15 = 23

Q.44. Data collected on religion from the census reports are

(*a*) Primary data
(*b*) Secondary data
(*c*) Sample data
(*d*) (a) or (b)

[June 2016]

Solution : (*b*) is correct.

Q.45. Different modes of presentations of data are _____

(*a*) Textual
(*b*) Tabular
(*c*) Both (a) & (b)
(*d*) None

[Dec. 2016]

Solution : (*c*) is correct

Q.46. For construction of Histogram the class intervals of frequency distribution is

(*a*) Equal
(*b*) Unequal
(*c*) Either Equal or Unequal
(*d*) None

[Dec. 2016]

Solution : (*a*) is correct.

Q.47. In _____ method(s) information can be gathered by the researcher himself by contacting the interviewee

(*a*) Personal Interview
(*b*) Telephone Interview
(*c*) Both (a) & (b)
(*d*) Indirect oral

[Dec. 2016]

Solution : (*c*) is correct.

Q.48. Profits made by XYZ Bank in different years refers to _____

(*a*) Attribute
(*b*) Discrete variable
(*c*) Continuous variable
(*d*) None

[Dec. 2016]

Solution : (*c*) is correct because Blue Chips company's profit always increases.

Q.49. The intersection of point of less than Ogive and more than Ogive gives

(*a*) Mean
(*b*) Mode
(*c*) Median
(*d*) None

[June 2017]

Solution : (*c*)

Q.50. Which of the following diagram is appropriate to represent the various heads in total cost ?

(*a*) Bar graph
(*b*) Pie chart
(*c*) Multiple line chart
(*d*) Scatter plot

[June 2017]

Solution : (*b*)

Q.51. Frequency density corresponding to a class interval is the ratio of ____

(*a*) Class frequency to the class length
(*b*) Class frequency to the total frequency
(*c*) Class length to the class frequency
(*d*) Class frequency to the cumulative frequency

[June 2017]

Solution : (*a*)

Q.52. Stub of a table is the :

(*a*) Right part of the table describing the columns
(*b*) Left part to the table describing the columns
(*c*) Right part of the table describing the row
(*d*) Left part of the table describing the rows

[Dec. 2017]

Solution : (*d*)

Q.53. Pie diagram is used for :

(*a*) Comparing different components and their relation to the total
(*b*) Representing quantitative data in circle
(*c*) Representing qualitative data in a circle
(*d*) Either (b) or (c)

[June 2018]

Solution : (*a*)

Q.54. Find the number of observation between 250-300 from the following data:

Value	More than 200	More than 250	More than 300	More than 500
No. of observation	56	38	15	0

(*a*) 38 (*b*) 23 (*c*) 15 (*d*) None of these

[June 2018]

Solution : (*b*)

No. of obs. = 38 - 15 = 23

Q.55. The graphical representation of median can be found by using :

(*a*) Frequency polygon
(*b*) Histogram
(*c*) Ogives
(*d*) Frequency curve

[June 2018]

Solution : (*c*)

Q.56. Frequency density is used in the construction of

(*a*) Histogram
(*b*) Ogive
(*c*) Frequency polygon
(*d*) None when the classes are of unequal width

[May 2018]

Solution : (*a*)

Q.57. Divided Bar Chart is considered for

(*a*) Comparing different components of a variable
(*b*) The relation of different components to the table
(*c*) (a) or (b)
(*d*) (a) and (b)

[May 2018]

Solution : (*d*)

Q.58. The following frequency distribution

X :	**12**	**17**	**24**	**36**	**45**
Y :	**2**	**5**	**3**	**8**	**9**

is classified as:

(*a*) Discrete distribution
(*b*) Continuous distribution
(*c*) Cumulative frequency distribution
(*d*) None of the above

[Nov. 2018]

Solution : (*a*)

Q.59. Histogram is useful to determine graphically the value of

(*a*) Arithmetic mean
(*b*) Mode
(*c*) Median
(*d*) None

[Nov. 2018]

Solution : (*b*)

Q.60. Data are said to be _____ if the investigator himself is responsible for the collection of the data.

(*a*) Primary data
(*b*) Secondary Data
(*c*) Mixed of primary and secondary data
(*d*) None

[Nov. 2018]

Solution : (*a*)

Q.61. A suitable graph for representing the portioning of total into sub parts in Statistics is

(*a*) A pictograph (*b*) A Pie Chart
(*c*) An Ogive (*d*) Histogram

[Nov. 2018]

Solution : (*b*)

Q.62. The number of times a particular items occurs in a class interval is called its

(*a*) Mean
(*b*) Cumulative frequency
(*c*) Frequency
(*d*) None of the above

[Nov. 2018]

Solution : (*c*)

Q.63. An Ogive is a graphical representation of

(*a*) Cumulative frequency distribution of
(*b*) Ungrouped data
(*c*) A frequency distribution
(*d*) None of the above

[Nov. 2018]

Solution : (*a*)

Q.64.

Class	0–10	10–20	20–30	30–40	40–50
Frequency	4	6	20	8	3

For the class 20-30, cumulative frequency is

(*a*) 26 (*b*) 10 (*c*) 41 (*d*) 30

[Nov. 2018]

Solution : (*d*) $\because$ cf = 4 + 6 + 20 = 30

Q.65. Which of the following graph is suitable for cumulative frequency distribution?

(*a*) 'O' give (*b*) Histogram (*c*) G.M (*d*) A.M

[June 2019]

Solution : (*a*)

Q.66. Histogram can be shown as

(*a*) Ellipse (*b*) Rectangle
(*c*) Hyperbola (*d*) Circle

[June 2019]

Solution : (*b*)

Q.67. _____ Series is continuous

(*a*) Open ended
(*b*) Exclusive
(*c*) Close ended
(*d*) Unequal call intervals

[June 2019]

Solution : (*b*)

Q.68. Ogive graph is used for finding

(*a*) Mean
(*b*) Mode
(*c*) Median
(*d*) None

[June 2019]

Solution : (*c*)

Q.69. Histogram is used for finding

(*a*) Mode (*b*) Mean
(*c*) First quartile (*d*) None

[June 2019]

Solution : (*a*)

Q.70. Histogram is used for presentation of the following type of series.

(*a*) Time Services
(*b*) Continuous Frequency Series
(*c*) Discrete Series
(*d*) Individual Series

[Dec. 2019]

Solution : (*b*)

Q.71. The graphical representation of cumulative frequency distribution is called-

(*a*) Histogram
(*b*) Pie Chart
(*c*) Frequency Polygon
(*d*) Ogive *[Dec. 2019]*

Solution : (*d*)

Q.72.

No. of Accidents	0	1	2	3	4	5	6	7
Frequency	36	27	33	29	24	27	18	9

In how many cases 4 or more accidents occur ?

(*a*) 96 (*b*) 133 (*c*) 78 (*d*) 54

[Dec. 2019]

Solution : (*c*)

No. of cases in which 4 or more accidents occur

= 24 + 27 + 18 + 9 = 78

Q.73. The difference between upper limit and lower limit of a class is called:

(*a*) Class interval
(*b*) Class boundaries
(*c*) Mid-value
(*d*) Frequency

[Dec. 2019]

Solution : (*a*)

Q.74. The average of salaries in a factory is ₹ 47,000. The statement that the average salary ₹ 47,000 is.....

(*a*) Descriptive statics
(*b*) Inferential
(*c*) Detailed
(*d*) Undetailed

[Dec. 2020]

Solution : (*a*)

Descriptive Statistics - Descriptive Statistics is the term given to the analysis of data that helps to describe position using a number of statistics, including the mode, Median and Mean. Inferential Statistics - Using sample data to make an inference or draw a conclusion of the population.

Q.75. Statistics cannot deal with ..... data.

(*a*) Quantitative
(*b*) Qualitative
(*c*) Textual
(*d*) Attribute

[Dec. 2020]

Solution : (*c*)

Q.76. Sweetness of a sweet dish is.....

(*a*) Attribute
(*b*) Discrete Variable
(*c*) Continuous Variable
(*d*) Variable

[Dec. 2020]

Solution : (*a*)

Q.77. Census reports are used as a source of..... data.

(*a*) Secondary (*b*) Primary
(*c*) Organize (*d*) Confidential

[Dec. 2020]

Solution : (*a*)

Q.78. Types of cumulative frequencies are.....

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

[Dec. 2020]

Solution : (*b*)

Q.79. You are an Auditor of a firm and the firm earns a profit ₹ 67,000 you stated to them that the annual profit is ₹ 67,000. This is .....type of statistics.

(*a*) Descriptive (*b*) Detailed
(*c*) Non-detailed (*d*) Inferential

[Dec. 2020]

Solution : (*a*)

Q.80. The.....are used usually when we wants to examine the relationship between two variables.

(*a*) Bar Graph (*b*) Pie Chart
(*c*) Line Chart (*d*) Scatter Plot

[Dec. 2020]

Solution : (*d*)

Q.81. A bar chart is drawn for

(*a*) Continuous data
(*b*) Nominal data
(*c*) Time series data
(*d*) Comparing different components

[Jan. 2021]

Solution : (*d*) A bar chart (may be vertical or horizontal) shows comparison of different components.

Q.82. A tabular presentation can be used for

(*a*) Continuous series data
(*b*) Nominal data
(*c*) Time series data for longer period
(*d*) Primary data

[Jan. 2021]

Solution : (*b*) Tabular presentation (Tabulation) is normally used for Nominal Data.

Q.83. A variable with qualitative characteristic is known as

(*a*) Quality Variable
(*b*) An attribute
(*c*) A discrete variable
(*d*) A continuous variable

[Jan. 2021]

Solution : (*b*) is correct

Q.84. The accuracy and consistency of data can be verified by

(*a*) Scrutiny
(*b*) Internal Checking
(*c*) External Checking
(*d*) Double Checking

[Jan. 2021]

Solution : (*a*) is correct

Q.85. From a histogram one cannot compute the approximate value of

(*a*) Mode
(*b*) Standard deviation
(*c*) Median
(*d*) Mean

[Jan. 2021]

Solution : (*b*) With the help of histogram approximate value of standard deviation cannot be obtained.

Q.86. The left part of a table providing the description of rows is called.

(*a*) Caption (*b*) Box - head
(*c*) Stub (*d*) Body

[Jan. 2021]

Solution : (*c*) is correct.

Q.87. Mode can be obtained from

(*a*) Frequency polygon
(*b*) Histogram
(*c*) O give
(*d*) All of the above

[Jan. 2021]

Solution : (*b*) is correct.

Q.88. Most of the Commonly used distributions provide a.

(*a*) Bell - Shaped
(*b*) U Shaped
(*c*) J - Shaped Curve
(*d*) Mixed Curve

[Jan. 2021]

Solution : (*a*) is correct.

Q.89. Which of the following is suitable for the graphical representation of a Cumulative frequency distribution ?

(*a*) Frequency polygon
(*b*) Histogram
(*c*) O give
(*d*) Pie chart

[Jan. 2021]

Solution : (*c*) is correct.

Q.90. Sweetness of sweet dish is.

(*a*) An Attribute
(*b*) A discrete variable
(*c*) A continuous variable
(*d*) A variable

[Jan. 2021]

Solution : (*a*) is correct.

because sweetness is the nature of sweets.

Q.91. There were 200 employees in an office in which 150 were married. Total male employees were 160 out of which 120 were married. What was the number of female unmarried employees?

(*a*) 30 (*b*) 40 (*c*) 50 (*d*) 10

[July 2021]

Solution : (*d*) is correct

Tabulation

	Male	Female	Total
Married	120	150 – 120 = 30	150
Unmarried	40	50 – 40 = 10	50
Total	160	40	200

Q.92. Data collected on religion from the census reports are

(*a*) Primary data
(*b*) Unclassified data
(*c*) Sample data
(*d*) Secondary data

[July 2021]

Solution : (*d*) is correct.

Census Report → Primary Data.

Data on Religion from Census = Secondary Data.

Q.93. Which of the following diagram is the most appropriate to represents various heads in total cost?

(*a*) Pie chart (*b*) Bar graph
(*c*) Multiple line chart (*d*) Scatter plot

[July 2021]

Solution : (*a*) is correct

Q.94. In a graphical representation of data, the largest numerical value is 4 the smallest numerical value is 25. If classes desired are 4 then which class interval is

(*a*) 45 (*b*) 5 (*c*) 20 (*d*) 7.5

[July 2021]

Solution : (*b*) is correct

Given

Smallest observation = 4 = S

Largest observation = 25 = L

No. of Class intervals desired = 4.

Formula

$$\text{No. of CIs} = \frac{\text{Range}}{i}$$

$$\therefore \quad \text{(length of the C.I.)} = i = \frac{L - S}{\text{No. of CIs}} = \frac{25 - 4}{4}$$

$$= 5.25 \cong 5.$$

Q.95. In graphical representation of data, ideographs are also called as

(*a*) Picto-graphs (*b*) Asymmetry graphs
(*c*) Symmetry graphs (*d*) Pictograms

[July 2021]

Solution : (*d*) is correct.

Q.96. ____ means separating items according to similar characteristics grouping them into various classes.

(*a*) Classification (*b*) Editing (*c*) Separation (*d*) Tabulation

[July 2021]

Solution : (*a*) is correct.

Q.97. Frequency density of a class interval is the ratio of _____

(*a*) Class frequency to the total frequency

(*b*) Class length to class frequency

(*c*) Class frequency to the cumulative frequency

(*d*) Frequency of that class interval to the corresponding class length

[July 2021]

Solution : (*d*)

$$\text{Frequency density} = \frac{\text{Frequency of class-interval}}{\text{Length of that class-interval}}$$

Q.98. A graph that uses vertical bars to represent data is called a

(*a*) Line graph (*b*) Scatter plot

(*c*) Vertical graphs (*d*) Bar

[July 2021]

Solution : (*d*) is correct.

Q.99. In a study about the male and female students of Commerce and Science departments of a college in 5 years, the following data's were obtained:

1995	2000
70% female students	75% female students
65% read Commerce	40% read Science
20% of male students read Science	50% of female students read Commerce
3000 total No. of students	3600 total No. of students

After combining 1995 and 2000 if x denotes the ratio of female commerce student to female Science student and y denotes the ratio of male commerce student to male Science student, then

(*a*) $x = y$ (*b*) $x > y$ (*c*) $x < y$ (*d*) $x \geq y$

[Dec. 2021]

Solution : (*c*)

	1995			2000		
	M.	F.	Total	M.	F.	Total
Commerce	10%	55%	65%	10%	50%	60%
Science	20%	15%	35%	15%	25%	40%
Total	30%	70%	100%	25%	75%	100%
		Total	=3000		Total	=3600

Total Female Commerce Students in 1995 & 2000 together

$$= 3000 \times \frac{55}{100} + 3600 \times \frac{50}{100} = 1650 + 1800 = 3450$$

$$\text{Female Science Students} = 3000 \times \frac{15}{100} + 3600 \times \frac{25}{100} = 450 + 900 = 1350$$

$$\therefore x = \frac{3450}{1350} = 2.555$$

Total Male Commerce Students

$$= 3000 \times \frac{10}{100} + 3600 \times \frac{10}{100}$$

$$= 300 + 360 = 660$$

Total Male Science Students

$$= 3000 \times \frac{20}{100} + 3600 \times \frac{15}{100}$$

$$= 600 + 540 = 1140$$

$$Y = \frac{660}{1140} = 0.5785$$

Clearly $x < y$

C Correct.

Q.100. A National Institute arrange its students data in accordance with different States. This arrangement of data is known as

(*a*) Temporal Data

(*b*) Geographical Data

(*c*) Ordinal Data

(*d*) Cardinal Data ***[Dec. 2021]***

Solution : (*b*)

Data arrange region wise are known as Geographical Data Example:– Arrangement of students States wise are Geographical Data

Q.101. A student marks in five subjects S_1, S_2, S_3, S_4 and S_5 are 86, 79, 90, 88 and 89. If we need to draw a Pie chart to represent these marks, then what will be the Central angle for S_3?

(*a*) 103.2° (*b*) 75°

(*c*) 105.6° (*d*) 94.8°

[Dec. 2021]

Solution : (*b*)

$$\text{Central Angle for } S_3 = \frac{S_3}{\text{Total}} \times 360°$$

$$= \frac{90}{86+79+90+88+89} \times 360°$$

$$= 75°$$

(*b*) is correct.

Q.102. The following data relate to the marks of a group of students:

Marks :	Below 10	Below 20	Below 30	Below 40	Below 50
No. of students:	15	38	65	84	100

How many students got marks more than 30?

(*a*) 65 (*b*) 50 (*c*) 35 (*d*) 43

[Dec. 2021]

Solution : (*c*)

No. of Students getting Marks more than 30

= 100–65 = 35

Q.103. The following data relate to the marks of 48 students in Statistics:

56	10	54	38	21	43	12	22
48	51	39	26	12	17	36	19
48	36	15	33	30	62	57	17
5	17	45	46	43	55	57	38
43	28	32	35	54	27	17	16
11	43	45	2	16	46	28	45

What are the frequency densities for the class intervals 30-39, 40-49, 50-59?

(*a*) 0.20, 0.50, 0.90 (*b*) 0.70, 0.90, 1.10

(*c*) 0.1875, 0.1667, 0.2083 (*d*) 0.90, 1.00, 0.80

[Dec. 2021]

Solution : (*d*)

Class Interval	Frequency	Frequency = f/i Density
30 – 39	9	$\frac{9}{10} = 0.9$
40 – 49	11	$\frac{11}{10} = 1.1.$
50 – 59	7	$\frac{7}{10} = 0.7$

Here Length of the class interval

$= i = l_2 - l_1 + D = 39 - 30 + 1 = 10$

Where D = Lower Limit of a class interval-upper limit of just pre-class interval.

Maximum matching value in option D

Q.104. Multiple axis line chart is considered when

(*a*) There is more than one time series

(*b*) The units of the variables are different.

(*c*) In any case

(*d*) If there are more than one time series and unit of variables are different.

[Dec. 2021]

Solution : (*d*)

Q.105. Sweetness of a sweet dish is

(*a*) An attribute

(*b*) A discrete variable

(*c*) A continuous variable

(*d*) A variable

[June 2022]

Solution : (*a*) is correct

Sweatness is the nature of Sweat.

So it is an Attribute

Q.106. Median of a distribution can be obtained from

(*a*) Frequency polygon

(*b*) Histogram

(*c*) Less than type ogives

(*d*) Pie chart

[June 2022]

Solution : (*c*) is correct

Ogive Curve/Cumulative Frequency Curve is suitable for any Partion Value (*i.e.* Median; Quartiles etc.)

Q.107. We get —by plotting cumulative frequency against the respective class boundary.

(*a*) Histogram (*b*) Polygon

(*c*) Pie chart (*d*) Ogives

[June 2022]

Solution : (*d*) is correct

Ogive curves are plotted by using cumulative frequencies and Class - Boundaries.

Q.108. Types of research data are

(*a*) Organised data and unorganized data

(*b*) Qualitative data and quantitative data

(*c*) Processed data and unprocessed data

(*d*) Discrete data and continuous data

[June 2022]

Solution : (*b*) is correct

Q.109. The collected information on which of the following characteristic do not form data?

(*a*) The number of files audited are 'less than 6'. 'between 5 and 10' and 'more than 9'

(*b*) The number of files audited are 'very less'. ' moderate' and 'very large'

(*c*) The number of files audits in a file

(*d*) The number of auditors who audited a file

[June 2022]

Solution : (*b*) is correct

Q.110. Histograms are drawn only when

(*a*) Frequencies in various class intervals are equal

(*b*) Class intervals are equal

(*c*) Class intervals are unequal

(*d*) For less than type cumulative frequencies

[June 2022]

Solution : (*b*) is correct

Histogram is drawn taking length of class-intervals equal.

Q.111. Which one of the following is not a mode of presentation of data?

(*a*) Textual presentation

(*b*) Tabular presentation

(*c*) External presentation

(*d*) Diagrammatic representation

[June 2022]

Solution : (*c*) is correct

Q.112. Which one of the following is a continuous variable?

(*a*) The quantum of days to get a cure from illness

(*b*) The quantum of oxygen cylinders used to treat a patient

(c) The quantum of drug injected in to a patient

(d) The quantum of tablets prescribe to a patient

[June 2022]

Solution : (c) is correct

Illness gradually up or down.

Q.113. Which one of the following is a source of primary data?

(a) Government records

(b) Research Articles

(c) Journals

(d) Questionnaire filled by enumerators

[Dec. 2022]

Solution: (d)

Q.114. Which is the left part of the table providing the description of the rows?

(a) Caption (b) Box head

(c) Stub (d) Body

[Dec. 2022]

Solution: (c)

Q.115. The suitable formula for computing the number of class intervals is :

(a) 3.322 logN

(b) 0.322 logN

(c) 1+3.322 logN

(d) 1–3.322 logN

[Dec. 2022]

Solution : (c) is correct

H.A. Sturges provides a formula for determining the approximation number of classes

$K = 1 + 3.322 \log N$

Where K = Number of classes

and logN = log of the total number of observations.

Q.116. Ogive for more than type and less than type distributions intersect at

(a) Mean (b) Median

(c) Mode (d) Origin

[Dec. 2022]

Solution: (b) is correct.

The perpendicular drawn from the point of intersection of less than and more than ogive curve cut X-axis at median.

Q.117. The shareholding pattern of ABC Ltd. is as follows:

Share-holders	Pro-moter	FII	DII	Govt.	Public
No. of shares in Millions	120	25	20	20	15

What is the difference between central angles (in degree) for shares held by Promoters and Public in pie chart?

(a) 216 (b) 189

(c) 180 (d) 99

[June 2022]

Solution :

Total = 120 + 25 + 20 + 20 + 15

= 200

Difference between the Central Angle of Promoters and Public in Pie Chart

$$= \left(\frac{120-15}{200}\right) \times 360^{o}$$

$= 189^{o}$

(b) is correct.

Q.118. What does an Ogive curve represent?

(a) The cumulative frequency and class boundary

(b) The frequency and class boundary

(c) The frequency and cumulative frequency

(d) The Frequency and Class Interval

[June 2022]

Solution :

(a) Ogive Curve (Cumulative frequency curve) represent cumulative frequency & Class boundary.

Q.119. The following is the data related to the daily income of 86 persons:

Income in ₹	500-999	1000-1499	1500-1999	2000-2499
No. of persons	15	28	36	7

What is the percentage of persons earning at least ₹ 1,500 per day?

(a) 50% (b) 45%

(c) 40% (d) 60%

[June 2022]

Solution : Total persons

= 15 + 28 + 36 + 7

= 86

No. of persons earning more than ₹ 1,500 per day

= 36 + 7 = 43

∴ Percentage of persons earning more than ₹ 1,500

$$= \frac{43}{86} \times 100 = 50\%$$

∴ (a) is correct.

Q.120. For tabulation, 'caption' is:

(a) The upper part of the table

(b) The lower part of the table

(c) The main part of the table

(d) The upper part of a table that describes the rows and sub-rows

[June 2022]

Solution : ∴ (a) is correct.

Q.121. The modes of presentation of data are:

(a) Textual, Diagrammatic and Internal presentation

(b) Tabular, Textual and Internal Presentation

(c) Textual, Tabular and Diagrammatic presentation

(d) Tabular, Diagrammatic and Internal Presentation

[June 2022]

Solution : (c) is correct.

21 CHAPTER

CENTRAL TENDENCY

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. The harmonic mean of 1, 1/2, 1/3.....1/n is

(*a*) 1/(n + 1) (*b*) 2/(n + 1)

(*c*) (n + 1)/2 (*d*) 1/(n - 1)

[June 2010]

Solution : (*b*) $H = \frac{n}{\sum\left(\frac{1}{x_1}\right)}$

$$= \frac{n}{1+2+3+.....+n}$$

$$= \frac{n}{\frac{n}{2}(n+1)} = \frac{2}{(n+1)}$$

Tricks : GBC

Q.2. The mean weight of 15 students is 110 kg. The mean weight of 5 of them is 100 kg. and that of another five students is 125 kg., then the mean weight of the remaining students is:

(*a*) 120 (*b*) 105

(*c*) 115 (*d*) None of these

[June 2010]

Solution : (*b*) Total weight of remaining 5 students

$= 15\times110 - 5\times100 - 5\times125 = 525$

$\therefore$ Mean weight of remaining 5 students $= \frac{525}{5} = 105$

Q.3. In a class of 11 students, 3 students were failed in a test. 8 students who passed secured 10,11, 20, 15, 12, 14, 26 and 24 marks respectively. What will be the median marks of the students:

(*a*) 12 (*b*) 15

(*c*) 13 (*d*) 13.5

[June 2010]

Solution : (*a*) Let x_1, x_2, x_3 be the marks of fail 3 students. Arranging marks in ascending order ; we get x_1, x_2, x_3, 10, 11, 12, 14, 15, 20, 24, 26 Median = Middle obs. = 12

Q.4. A lady travel at a speed of 20 km/h and returned at quicker speed. If her average speed of the whole journey is 24 km/h, find the speed of return journey (in km/h)

(*a*) 25 (*b*) 30

(*c*) 35 (*d*) 38

[Dec. 2010]

Solution : (*b*) **TRICKS** : Av. Speed = $\frac{2\times 20x}{x+20} = 24$

Where x = Retuning Speed

$24x + 480 = 40x$

$16x = 480$

$x = 30$ km/hr.

Q.5. Let the mean of the variable 'x' be 50, then the mean of u = 10+5x will be:

(*a*) 250 (*b*) 260

(*c*) 265 (*d*) 273

[Dec. 2010]

Solution : (*b*) $u = 10 + 5x$

Since mean changes w.r.t the change of Origin & Scale

$\therefore$ New Mean of $u = 10 + 5 \times 50 = 260$

Q.6. If the difference between mean and mode is 63, then the difference between Mean and Median will be_____

(*a*) 63

(*b*) 31.5

(*c*) 21

(*d*) None of the above

[June 2011]

Solution : (*c*) Given : Mode - Mean = 63

Since ; Empirical Relationship is

(Mode - Mean) = 3(Median - Mean)

$\therefore$ Median - Mean = $\frac{63}{3} = 21$

Q.7. If the Arithmetic mean between two numbers is 64 and the Geometric mean between them is 16. The Harmonic Mean between them is_____.

(*a*) 64 (*b*) 4

(*c*) 16 (*d*) 40

[June 2011]

Solution : (*b*) Given : A.M =A = 64

G.M. = G = 16; H.M = ?

We know, $(G.M.)^2 = A.M. \times H.M.$

$(16)^2 = 64 \times H.M.$

$\therefore$ HM = $\frac{256}{64} = 4$

Q.8. The average of 5 quantities is 6 and the average of 3 is 8. What is the average of the remaining two?

(*a*) 4 (*b*) 5

(*c*) 3 (*d*) 3.5

[June 2011]

Solution : (*c*)

Sum of Remaining two Nos. = $5 \times 6 - 3 \times 8 = 6$

$\therefore$ Av. of Remaining two = $\frac{6}{2} = 3$

Q.9. The median of following numbers, which are given in ascending order is 25. Find the Value of X if data is 11, 13, 15, 19, (x + 2), (x + 4), 30, 35, 39, 46

(*a*) 22 (*b*) 20

(*c*) 15 (*d*) 30

[Dec. 2011]

Solution : (*a*)

Numbers are in Ascending Order ; N = 10 (Even)

Median = Av. of Middle two obs.

$25 = \frac{1}{2}[(x+2)+(x+4)]$

$50 = 2x + 6$;

$2x = 50 - 6$

$x = 22$

Tricks :- GBC.

Q.10. The average age of a group of 10 students was 20 years. The average age are increased by two years when two new students joined the group. What is the average age of two new students who joined the group ?

(*a*) 22 years (*b*) 30 years

(*c*) 44 years (*d*) 32 years

[Dec. 2011]

Solution : (*d*)

Sum of age of two boys

$= (10+2)\times(20+2) - 10 \times 20$

$= 264 - 200 = 64$

Average Age of two boys = $\frac{64}{2} = 32$

Q.11. Geometric Mean of three observations 40, 50 and X is 10. The value of X is

(*a*) 2

(*b*) 4

(*c*) 1/2

(*d*) None of the above

[June 2010]

Solution : (*c*) **Tricks:** Product of obs. $=(GM)^3$

$(10)^3 = 40.50.X$

$1{,}000 = 40.50.X$

$X = \frac{10}{20} = \frac{1}{2}$

Q.12. The mean of first three terms is 14 and mean of next two terms is 18. The mean of all five terms is:

(*a*) 14.5 (*b*) 15

(*c*) 14 (*d*) 15.6

[June 2010]

Solution : (*d*) Formula

$\sum X = N.\bar{X}$

Sum of 1st 3 Numbers = $3 \times 14 = 42$

Sum of next 2 Numbers = $2 \times 18 = 36$

Sum of all these 5 Numbers = 42 + 36 = 78

Av. of these 5 Numbers = $\frac{78}{5} = 15.6$

Tricks : Combined Mean

$= \frac{3\times 14 + 2\times 18}{3+2} = 15.6$

Q.13. The mean salary of a group of 50 persons is ₹ 5,850. Later on it is discovered that the salary of one employee has been wrongly taken as ₹ 8,000 instead of ₹ 7,800. The corrected mean salary is

(*a*) ₹ 5,854

(*b*) ₹ 5,846

(*c*) ₹ 5,650

(*d*) None of the above

[Dec. 2012]

Solution : (*b*) Correct mean

$= \frac{\text{Correct}\sum x}{N}$

$\bar{x} = \frac{N.\bar{X} - \text{wrong obs.} + \text{correct obs.}}{N}$

$$= \bar{x} = \frac{50\times5,850 - 8000 + 7800}{50}$$

$= 5846$

TRICKS : Do with Calculator mentally

Q.14. If the mode of a data is 18 and mean is 24 then median is_____

(*a*) 18 (*b*) 24

(*c*) 22 (*d*) 21

[Dec. 2012]

Solution : (*c*) Mode = 18 ; Mean = 24

Mode = 3 Median - 2 Mean

$18 = 3$ Median $- 2 \times 24$

$18 = 3$ Median $- 48$

$18 + 48 = 3$ Median

$$\text{Median} = \frac{66}{3} = 22$$

Q.15. For data on frequency distribution of weights :

70, 73, 49, 57, 56, 44, 56, 71, 65, 62, 60, 50, 55, 49, 63 and 45

If we assume class length as 5, the number of class intervals would be

(*a*) 5 (*b*) 6

(*c*) 7 (*d*) 8

[Dec. 2012]

Solution : (*b*) L = Largest obs. = 73

S = Smallest obs. = 44

Range = L - S = 73 - 44 = 29

Length = i = 5

$$\text{No. of Class -Intervals} = \frac{\text{Range}}{\text{i}}$$

$$= \frac{29}{5} = 5.8 = 6$$

Q.16. The point of intersection of the "less then" and "more then" Ogives correspond to

(*a*) Mean (*b*) Mode

(*c*) Median (*d*) 10th percentile

[Dec. 2012]

Solution : (*c*) Median

Q.17. A man travels from Agra to Gwalior at an average speed of 30 km per hour and back at an average speed of 60 km per hour. What is his average speed?

(*a*) 38 km per hour

(*b*) 40 km per hour

(*c*) 45 km per hour

(*d*) 35 km per hour

[Dec. 2012]

Solution : (*b*); **Tricks :** Average speed = $\frac{2xy}{x+y}$

Given x = 30 km/h & y = 60 km/h

$$\text{Average speed} = \frac{2\times30\times60}{30+60}$$

$$= \frac{2\times30\times60}{90} = 40 \text{ km per hour}$$

Q.18. Which of the following measures of central tendency cannot be shown by graphical method?

(*a*) Mean (*b*) Median

(*c*) Mode (*d*) Quartiles

[June 2013]

Solution : (*a*) Mean Cannot be Shown with Graphical Method.

Q.19. GM of 8,4,2 is_____

(*a*) 4 (*b*) 2

(*c*) 8 (*d*) None

[June 2013]

Solution : (*a*) is correct

$G = (8.4.2)^{1/3} = (2^3.2^2.2)^{1/3}$

$= (2^6)^{1/3} = 2^2 = 4$

Tricks : Go by choices

Q.20. The average age of 15 students is 15 years. Out of these the average age of 5 students is 14 years and that of other 9 students is 16 years, then the age of 15th student is_____

(*a*) 11 years (*b*) 14 years

(*c*) 15 years (*d*) None of these

[June 2013]

Solution : (*a*) is correct

Age of 15th student = $15 \times 15 - 5 \times 14 - 9 \times 16 = 11$ years

Q.21. Which of the following statement is true?

(*a*) Median is based on all observations

(*b*) The Mode is the mid value

(*c*) The Median is the 2nd Quartile

(*d*) The Mode is the 5th decile

[June 2014]

Solution : (*c*) is correct

Median $M = Q_2$

Q.22. For two numbers A.M. = 10 and G.M. = 8; the H.M. ....?

(*a*) 9 (*b*) 8.9

(*c*) 6.4 (*d*) None

[Dec. 2014]

Solution : (*c*) is correct

$$\because AH = G^2 \Rightarrow H = \frac{G^2}{A}$$

$$= H = \frac{8^2}{10} = 6.4$$

Q.23. The 3rd decile for the values 15, 10, 20, 25, 18, 11, 9, 12 is

(*a*) 13 (*b*) 10.7 (*c*) 11 (*d*) 11.5

[Dec. 2014]

Solution : (*b*) is correct

Arranging in Ascending order

9, 10, 11, 12, 15, 18, 20, 25

$$N=8 \ \therefore D_3 = 3\left(\frac{N+1}{10}\right)^{th} obs. = 3\left(\frac{8+1}{10}\right) = 2.7^{th} obs.$$

$=2^{nd}$ obs.$+0.7(3^{rd}-2^{nd}$obs.$)$

$=10+0.7(11-10)$

$=10.7$

Q.24. The A.M. of square of first '2n' natural number is

(*a*) $\frac{1}{6}(2n+1)(4n-1)$

(*b*) $\frac{1}{6}(2n-1)(4n-1)$

(*c*) $\frac{1}{6}(2n-1)(4n+1)$

(*d*) $\frac{1}{6}(2n+1)(4n+1)$

[Dec. 2014]

Solution : (*d*) is correct

Formula : $S_n = \frac{n(n+1)(n+1)}{6}$

$S_{2n} = \frac{2n(2n+1)(2\times 2n+1)}{6}$

$= \frac{2n(2n+1)(4n+1)}{6}$

A.M. $= \frac{S_{2n}}{2n} = \frac{2n(2n+1)(4n+1)}{6\times 2n}$

$= \frac{1}{6}(2n+1)(4n+1)$

Tricks : GBC

Q.25. A Random variables x follows uniform distribution in the interval [-3,7]. Then the mean of distribution is

(*a*) 2 (*b*) 4

(*c*) 5 (*d*) 6

[Dec. 2014]

Solution : (*a*) is correct

Note:- Random variable has uniform distribution *i.e.*

Here, Arithmetic Mean is applied

$\therefore$ Mean $= \frac{-3+7}{2} = 2$

Q.26. If the Harmonic mean of two numbers is 4 and Arithmetic mean (A) and Geometric mean (G) satisfy the equation $2A+G^2 = 27$ then the two numbers are

(*a*) (1,3) (*b*) (9,5)

(*c*) (6,3) (*d*) (12,7)

[June 2015]

Solution : Tricks : (*a*) Go by choices

For option (*c*); H $= \frac{2ab}{a+b} = \frac{2\times 6\times 3}{6+3} = 4$

= H (true)

A $= \frac{6+3}{2} = 4.5$

G $= \sqrt{ab} = \sqrt{6\times 3} = \sqrt{18}$

It satisfies $2A + G^2 = 27$

$\therefore$ (*c*) is correct

Q.27. There were 50 students in a class. 10 failed whose average marks were 2.5. The total marks of class were 281. Find the average marks of students who passed?

(*a*) 6.4 (*b*) 25

(*c*) 256 (*d*) 86

[Dec. 2015]

Solution : (*a*) is correct.

Average Marks of students who passed

$= \frac{281-10\times 2.5}{50-10} = 6.4$

Q.28. If the mean of two numbers is 30 and Geometric Mean is 24 then what will be those two numbers ?

(*a*) 36 and 24 (*b*) 30 and 30

(*c*) 48 and 12 (*d*) None of these

[June 2016]

Solution : (*c*) is correct.

Tricks: Go by choices

(*c*) Arithmetic Mean $= \frac{48+12}{2} = 30$

Geometric Mean $= \sqrt{48\times 12} = 24$

It satisfies all conditions.

Q.29. The G.M. of observations 40, 50 and *x* is 10, then find the value of *x* ?

(*a*) 4 (*b*) 5

(*c*) 2 (*d*) $\frac{1}{2}$

[Dec. 2016]

Solution : (*d*) is correct.

Tricks : Product of observations = (GM) No. of obs.

$40\times 50\times x = 10^3$

So, $x = \frac{1}{2}$

Q.30. If the mean of data is 55.6 and the mode is 46, then the median is _____

(*a*) 50.4 (*b*) 40.7

(*c*) 52.4 (*d*) None

[Dec. 2016]

Solution : (*c*) is correct.

Since, Mode = 3Median - 2 Mean

46 = 3Median - 2×55.6

So, Median = (46 + 111.2)/3 = 52.4

Q.31. _____ is used for ordering the size of designed cloths.

(*a*) Mean (*b*) Median

(*c*) Mode (*d*) None

[Dec. 2016]

Solution : (*c*) is correct.

Q.32. The mean of 10 observations is 14.4. Out of these mean of 4 observations is 16.5, then find the mean of remaining observations?

(*a*) 13.6 (*b*) 13

(*c*) 13.8 (*d*) 12

[Dec. 2016]

Solution : (*b*) is correct.

No. of remaining observations = 10 - 4 = 6

Sum of these 6 observations = $14.4\times 10 - 4\times 16.5$

= 78

Their mean = 78/6 = 13

Q.33. The mean of 6, 4, 1, 5, 6, 10 and 3 is 5. If each number is added with 2, then the new mean is _____

(*a*) 7 (*b*) 5

(*c*) 6 (*d*) 10

[Dec. 2016]

Solution : (*a*) is correct.

Q.34. Which of the following is correct ?

(*a*) 3(Mean - Median) = Mean - Mode

(*b*) Mean - Median = 3(Mean - Mode)

(*c*) Mean - Median = 2(Mean - Mode)

(*d*) Mean - Mode = 2(Mean - Median)

[June 2017]

Solution : (*a*) is correct

Q.35. A person purchases 5 rupees worth of eggs from 10 different markets. You are to find average No. of eggs per rupee for all the markets taken together. What is the suitable form of average in this case?

(*a*) AM (*b*) GM

(*c*) HM (*d*) None

[June 2017]

Solution : (*c*) is correct.

Q.36. GM = 6, AM = 6.5 then HM =

(*a*) $\frac{6^2}{6.5}$ (*b*) $\frac{6}{6.5}$

(*c*) $\frac{6.5}{6}$ (*d*) None

[June 2017]

Solution : Formula ; $G^2 = A.H.$

$\Rightarrow \quad 6^2 = (6.5)H.$

or $H = \frac{6^2}{6.5}$

$\therefore$ (*a*) is correct.

Q.37. A company's past 10 years average earnings was ₹40 crores. For obtaining the same average earnings for 11 years including these 10 years how much earning (in ₹) must be made by the company in the 11th year ?

(*a*) 40 crores

(*b*) $\frac{40 \times 10}{11}$ crores

(*c*) More than 40 crores

(*d*) None

[June 2017]

Solution : Earning in 11^{th} year

$= 11 \times 40 - 10 \times 40 =$ ₹ 40 crores

(*a*) is correct.

Q.38. The rates of returns from three different shares are 100%, 200% and 400% respectively, the average rate of return will be _____

(*a*) 350% (*b*) 233.33%

(*c*) 200% (*d*) 300%

[June 2017]

Solution : (*c*) is correct.

$Gm = (100 \times 200 \times 400)^{1/3}$

$= 200\%$

Q.39. Mean of 7, 9, 12, x, 4, 11 & 5 is 9. Find the missing observation :

(*a*) 13 (*b*) 15

(*c*) 12 (*d*) None of these

[Dec. 2017]

Solution :

$x = 7 \times 9 - 7 - 9 - 12 - 4 - 11 - 5$

$= 15.$

Q.40. If all the frequencies are equal then which will doesn't exist :

(*a*) Mean (*b*) Median

(*c*) Mode (*d*) None of these

[Dec. 2017]

Solution : (*c*)

Q.41. _____is the reciprocal of the AM of reciprocal of observation :

(*a*) HM (*b*) GM

(*c*) Both (*d*) None of these

[Dec. 2017]

Solution : (*a*)

Q.42. Mean of n observation is $\bar{x}$, if first observation is increased by 1, 2nd observation is by 2 and so on, then new mean is _____

(*a*) $\bar{x} + n.n$ (*b*) $\bar{x} + \frac{n+1}{2}$ (*c*) $\bar{x} + \frac{n}{2}$ (*d*) $\bar{x} + n$

[June 2018]

Solution : (*b*)

$$\text{New Mean} = \frac{n\bar{x} + (1+2+3+.....+n)}{n}$$

$$= \bar{X} + \frac{n(n+1)}{2n} = \bar{X} + \frac{(n+1)}{2}$$

Q.43. GM. of 4, 8, 16, 32 and 64 is _____ :

(*a*) 8 (*b*) 16

(*c*) 32 (*d*) 64

[June 2018]

Solution : (*b*)

Tricks : 4, 8, 16, 32, 64 are in G.P.

$\therefore$ GM = Middle observation = 16.

Q.44. Which of the following results hold for a set of distinct positive observations?

(*a*) AM $\geq$ GM $\geq$ HM

(*b*) HM $\geq$ GM $\geq$ AM

(*c*) AM > GM > HM

(*d*) GM > AM > HM

[June 2018]

Solution : (*c*)

Q.45. For a moderately skewed distribution, which of the following relationship holds?

(*a*) Mean - Mode = 3 (Mean - Median)

(*b*) Median - Mode = 3 (Mean - Median)

(*c*) Mean - Median = 3 (Mean - Mode)

(*d*) Mean - Median = 3 (Median - Mode)

[June 2018]

Solution : (*a*)

Q.46. If the variables X and Z are so related that Z = aX + b for each $X = x_i$, where a and b are constant, then $\bar{Z} = a\bar{X} + b$

(*a*) True (*b*) False

(*c*) Both (*d*) None

[May 2018]

Solution : (*a*)

Q.47. Relation between mean, median and mode is

(*a*) Mean - mode = 2 (mean - median)

(*b*) mean - median = 3 (mean - mode)

(*c*) mean - median = 2 (mean - mode)

(*d*) mean - mode = 3 (mean - median)

[May 2018]

Solution : (*d*)

Q.48. If each item is reduced by 15 , A.M is

(*a*) reduced by 15 (*b*) Increased by 15

(*c*) reduced by 10 (*d*) None

[May 2018]

Solution : (*a*)

If each observation is reduced by 15 then new A.M. also reduced by 15, because A.M. changes with the shifting of origin.

Q.49. For 889, 999, 391, 384, 390, 480, 485, 760, 111, 240 Rank of median is

(*a*) 2.75 (*b*) 5.5

(*c*) 8.25 (*d*) None

[May 2018]

Solution : (*b*)

Here No. of observation (N) = 10

$$\text{Rank of Median } (m_e) = \left(\frac{N+1}{2}\right)^{th} \text{ observation}$$

$$= \left(\frac{10+1}{2}\right)^{th} \text{ term } = 5.5^{th} \text{ term}$$

Rank of median $(m_e) = 5.5$

Q.50. The average of a series of overlapping averages, each of which is based on a certain number of item within a series is known as

(*a*) Moving average

(*b*) Weighted average

(*c*) Simple average

(*d*) None

[May 2018]

Solution : (*a*)

Q.51. If the mean of the following distribution is 6 then the value of P is

X :	2	4	6	10	P + 5
f :	3	2	3	1	2

(*a*) 7 (*b*) 5

(*c*) 11 (*d*) 8

[Nov. 2018]

Solution : (*a*)

Tricks : Go by choices

Use Calculator

(*a*) If P = 7 $\Rightarrow$ P + 5 = 7 + 5 = 12

So; mean = press 2×3 = ; 4×2 = ; 6×3 = ; 10×1 = ; 12×2 = button, then GT button ÷ 11 = button 6 (True)

So, (*a*) is correct.

Q.52. If total frequencies of three series are 50,60 and 90 and their means are 12, 15 and 20 respectively, then the mean of their composite series is

(*a*) 15.5 (*b*) 16

(*c*) 14.5 (*d*) 16.5

[Nov. 2018]

Solution : (*d*)

Calculator Tricks:-

X_{123} = press $50\times12=$; $60\times15=$; $90\times20=$ button , then GT button, then

$\div(50+60+90=200)$ button

= 16.5

Q.53. If in a moderately skewed distribution the values of mode and mean are 32.1 and 35.4 respectively, then the value of the median is

(*a*) 33.3 (*b*) 34

(*c*) 34.3 (*d*) 33

[Nov. 2018]

Solution : (*c*)

$$M_o = 3M - 2\bar{X}$$

or $32.1 = 3M - 2\times35.4$

or $M = \frac{32.1+70.8}{3} = 34.3$

Q.54. The median of the data 5, 6, 7, 7, 8, 9, 10, 11, 11, 12, 15, 18, 18 and 19 is

(*a*) 10 (*b*) 10.5

(*c*) 11.5 (*d*) 11

[Nov. 2018]

Solution : (*b*) M_e = Average of middle two observations , if N = even No. of obs.

$$M_e = \frac{10+11}{2} = 10.5$$

Q.55. The means of 20 items of a data is 5 and if each item is multiplied by 3, then the new mean will be

(*a*) 20 (*b*) 5

(*c*) 15 (*d*) 10

[Nov. 2018]

Solution : (*c*)

New Mean = $5\times3=15$

Q.56. The Geometric mean of 3, 6, 24 and 48 is

(*a*) 6 (*b*) 8

(*c*) 12 (*d*) 24

[Nov. 2018]

Solution : (*c*)

$$GM = \sqrt[4]{3\times6\times24\times48} = 12$$

Q.57. The Algebraic sum of the deviation of a set of values from their arithmetic mean is

(*a*) >0 (*b*) =0

(*c*) <0 (*d*) None

[Nov. 2018]

Solution : (*b*)

Q.58. Which one of the following is not a central tendency ?

(*a*) Mean Deviation

(*b*) Arithmetic mean

(*c*) Median

(*d*) Mode

[Nov. 2018]

Solution : (*a*)

Q.59. The AM of 15 Observations is 9 and the AM of first 9 Observation is 11 and then AM of remaining Observation is

(*a*) 11 (*b*) 6

(*c*) 5 (*d*) 9

[June 2019]

Solution : (*b*)

Sum of remaining 6 observations =

Sum of 15 observations -

Sum of 9 observations

$= 9\times15 - 11\times9 = 135 - 99 = 36$

Average of 6 observations $= \frac{36}{6} = 6$

Q.60. In a moderately Skewed distribution the values of mean & median are 12 & 8 respectively. The value of mode is

(*a*) 0 (*b*) 12

(*c*) 15 (*d*) 30

[June 2019]

Solution : (*a*)

Mode = 3 Median - 2 Mean

$= 3\times8 - 2\times12 = 24 - 24 = 0$

Q.61. Which of the following is positional average ?

(*a*) Median (*b*) GM

(*c*) HM (*d*) AM

[June 2019]

Solution : (*a*)

Q.62. For a symmetric distribution

(*a*) Mean = Median = Mode

(*b*) Mode = 3 Median - 2 Mean

(*c*) Mode $= \frac{1}{3}$ Median $= \frac{1}{2}$ Mean

(*d*) None

[June 2019]

Solution : (*a*)

(*a*) For a symmetric distribution Mean = Median = Mode

Q.63. For the distribution

X	**1**	**2**	**3**	**4**	**5**	**6**
f	**6**	**9**	**10**	**14**	**12**	**8**

The value of median is

(*a*) 3.5 (*b*) 3 (*c*) 4 (*d*) 5

[June 2019]

Solution : (*c*)

x	1	2	3	4	5	6
f	6	9	10	14	12	8
C.F	6	15	25	39	51	59

N = 59

Median (Me) = The observation having cumulative frequency just equal to or just greater than $\left(\frac{N+1}{2}\right)^{th}$ term $= \left(\frac{59+1}{2}\right)^{th}$ term

= 30th term = 4

Q.64. $\sum_{i=1}^{m}\left(x-\bar{x}\right) = ?$

(*a*) 1 (*b*) 0 (*c*) -1 (*d*) None of these

[Dec. 2019]

Solution : (*b*)

The algebraic sum of deviations of all observations from mean is always equal to ZERO.

Q.65. The median of the following frequency distribution is equal to

X :	5	7	9	12	14	17	19	21
Y :	6	5	3	6	5	3	2	4

(*a*) 6 (*b*) 12 (*c*) 13 (*d*) 14

[Dec. 2019]

Solution : (*b*)

Ascending order

X	5	7	9	12	14	17	19	21
Y i.e. "f".	6	5	3	6	5	3	2	4
c.f.	6	11	14	20	25	28	30	34

N = 34

$M_e = \frac{N+1}{2} = \frac{34+1}{2} = 17.5^{th}$ obs.

$= 17^{th}$ obs. + 0.5 (18^{th} obs. - 17^{th} obs.)

= 12 + 0.5 (12 - 12)

= 12

Q.66. Find median from the following data:

Marks	0-10	10-30	30-60	60-80	80-90
No. of students	5	15	30	8	2

(*a*) 8 (*b*) 30 (*c*) 40 (*d*) 45

[Dec. 2019]

Solution : (*c*)

C.I	0-10	10-30	30-60	60-80	80-90
f	5	15	30	8	2
c.f.	5	20	50	58	60

$N = \sum f = 60.$

$$\frac{N}{2} = \frac{60}{2} = 30$$

$\therefore$ Median - Class = The class-interval having cumulative frequency just equal to or greater than $\frac{N}{2} = 30$ is 30 - 60

$\therefore\ f = 30\ ;\ c = 20\ ;\ i = 30.$

$$M_e = L + \frac{\frac{N}{2} - C}{f} \times i$$

$$= 30 + \frac{30 - 20}{30} \times 30$$

$$= 30 + 10 = 40$$

Q.67. Find the mode from the following data:

Class:	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	2	5	10	23	21	12	3

(*a*) 23 (*b*) 13.3 (*c*) 12.6 (*d*) 14.6

[Dec. 2019]

Solution : (*d*)

Clearly model - class = 12-15 because its frequency is highest.

$\therefore\ f_0 = 23\ ;\ f_{-1} = 10\ ;\ f_{+1} = 21$

$\therefore\ \Delta_1 = f_0 - f_{-1} = 23 - 10 = 13$

$\Delta_2 = f_0 - f_{+1} = 23 - 21 = 2$

Formula

$$M_0 = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i$$

$$= 12 + \frac{13}{13 + 2} \times 3$$

$$= 12 + \frac{13}{5} = 12 + 2.6$$

$$= 14.6$$

Q.68. Find the mode of the following distribution?

Class:	0-7	7-14	14-21	21-28	28-35	35-42	42-49
Frequency	19	25	36	72	51	43	28

(*a*) 24.3 (*b*) 25.4 (*c*) 72 (*d*) 21

[Dec. 2019]

Solution : (*b*)

Model - Class = 21 - 28

because its frequency is highest.

So; $f_0 = 72\ ;\ f_{-1} = 36\ ;\ f_{+1} = 51.$

$$\Delta_1 = f_0 - f_{-1} = 72 - 36 = 36$$

$$\Delta_2 = f_0 - f_{+1} = 72 - 51 = 21.$$

$$\therefore\ M_0 = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i$$

$$= 21 + \frac{36}{36 + 21} \times 7$$

$$= 25.4$$

Q.69. The arithmetic mean of two numbers is 30 and geometric mean is 24 find the two number

(*a*) 12 and 48 (*b*) 14 and 46 (*c*) 10 and 50 (*d*) 16 and 44

[Dec. 2019]

Solution : (*a*)

Trick s: GBC

Let (*a*) 12 ; 48 is correct.

$$\therefore\ AM = \frac{12 + 48}{2} = 30\ (\text{True})$$

$$GM = \sqrt{12 \times 48} = \sqrt{576} = 24\ (\text{Also True})$$

So; (*a*) is correct.

Q.70. Sum of the squares of deviations is minimum when deviations are taken from

(*a*) Mean (*b*) Median

(*c*) Mode (*d*) An arbitrary value

[Dec. 2019]

Solution : (*a*)

$\because \Sigma\left(X-\overline{X}\right)^2 \le \Sigma(X-M_e)^2$

Q.71. Given the weights for the numbers 1, 2, 3,.....n are respectively 1^2, 2^2, 3^2,....., n^2. Then weighted HM is.....

(*a*) $\frac{2n+1}{2}$ (*b*) $\frac{2n+1}{6}$ (*c*) $\frac{2n+1}{3}$ (*d*) $\frac{2n+1}{2}$

[Dec. 2020]

Solution : Tricks for 1st observation HM = 1

(No need of its Weight)

For n = 2 $\Rightarrow$ For 1st two observation

X	W	W/X
1	$1^2 = 1$	1
2	$2^2 = 4$	2
	$\Sigma W = 5$	$\frac{W}{X} = 3$

$$HW = \frac{\sum W}{\sum\left(W/X\right)} = \frac{5}{3}$$

Now GBC

(*a*) $HW = \frac{2n+1}{4} \Rightarrow$ if $n = 1 \Rightarrow HW = \frac{2\times1+1}{4} \neq 1.$

$\therefore$ (*a*) is False

(*b*) $n = 1 \Rightarrow HW = \frac{2\times1+1}{6} \neq 1.$

$\therefore$ (*b*) is also False.

(*c*) for $n = 1 \Rightarrow HW = \frac{2\times1+1}{3} = 1$ (True)

for $n = 2 \Rightarrow HW = \frac{2\times2+1}{3} = \frac{5}{3}$ (True also)

(*c*) is correct

Q.72. Which measure is suitable for open - end classification?

(*a*) Median (*b*) Mean (*c*) Mode (*d*) GM

[Dec. 2020]

Solution : (*a*) is Correct.

Q.73. 50th percentile is equal to

(*a*) Median (*b*) Mode (*c*) Mean (*d*) None

[Dec. 2020]

Solution : $P_{50} = Me = Q_2 = D_5$

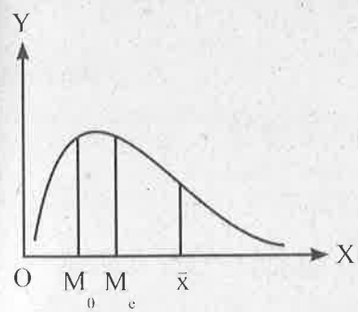
$\therefore$ (*a*) is correct.

Q.74. For a distribution Mean, Median and Mode are 23, 24 and 25.5 respectively, then it is most likely.....skewed distribution

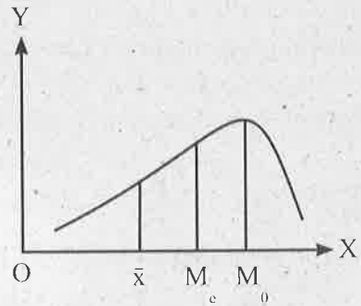
(*a*) Positively (*b*) Symmetrical (*c*) Asymptotically (*d*) Negatively

[Dec. 2020]

Solution : $\bar{x} = 23$; Me = 24, $M_o = 25.5$



(*i*) Positively Skewed



(*ii*) Negatively Skewed

Clearly This is Negatively Skewed.

(*d*) is correct.

Q.75. If any two numbers are in AP, then GM^2 =.....

(*a*) AM x HM (*b*) AM + HM (*c*) M x Z (*d*) AM x M

[Dec. 2020]

Solution : (*a*)

$G^2 = AH$

$\Rightarrow GM^2 = AM \times HM$

= (Arithmetic Mean) (Harmonic Mean)

Q.76. Two values yielded an arithmetic mean of 24 and a harmonic mean of 6. The geometric mean of these values is.....

(*a*) 8 (*b*) 12 (*c*) 14 (*d*) 16

[Dec. 2020]

Solution : AM = 24

HM = 6

$GM = \sqrt{AM \times HM} = \sqrt{24 \times 6}$

= 12

(*b*) is correct

Q.77. The HM of A and B is $\frac{1}{3}$ and HM of C and D is $\frac{1}{5}$. Then HM of A, B, C and D is .....

(*a*) $\frac{8}{15}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{15}{8}$ (*d*) $\frac{4}{15}$

[Dec. 2020]

Solution : A & B Means two observations $\Rightarrow N_1 = 2$

(*b*) $H_1 = \frac{1}{3}$

C &D Means two observations $\Rightarrow N_2 = 2$

and $H_2 = \frac{1}{5}$

HM of A,B,C &D means combined

HM of A & B and C & D

$$= \frac{N_1 + N_2}{N_1/H_1 + N_2/H_2} = \frac{2+2}{\frac{2}{\frac{1}{3}} + \frac{2}{\frac{1}{5}}} = \frac{4}{2\times3 + 2\times5} = \frac{4}{16} = \frac{1}{4}$$

$\therefore$ (*b*) is correct.

Q.78. Which one of these is least affected by extreme values?

(*a*) Mean (*b*) Median

(*c*) Mode (*d*) None

[Dec. 2020]

Solution : (*c*)

AM; GM; HM are more affected by extreme values. Median is not affected by extreme values. Mode is least affected by extreme values.

Q.79. A fire engine rushes to a place of fire accident with a speed of 110 kmph and after the completion of operation returned to the base at a speed of 35 kmph. The average speed per hour in per direction is obtained as ..... of those speeds.

(*a*) Speed Avg. of (*b*) HM of

(*c*) GM of (*d*) Half of HM of

[Dec. 2020]

Solution : (*b*)

Up & Down Distances are same. Speeds different.

Average speed = H.M. of those speeds.

$$= \frac{1+1}{\frac{1}{110} + \frac{1}{35}} = 53.1 \text{ kmph.}$$

Q.80. Ten matches data is given. Then which of the following cannot be found?

(*a*) Least Score (*b*) Highest Score

(*c*) Best Score (*d*) Median Score

[Dec. 2020]

Solution : Ten Matches data are given; means Lowest and highest scores are given. Their Median can be obtained. The data has equal frequency, So its Mode cannot be obtained. It means Best Score cannot be obtained.

So, (*c*) is correct.

Q.81. If the AM and HM of two numbers are 6 and 9 respectively, then GM is.....

(*a*) 7.35 (*b*) 8.5

(*c*) 6.75 (*d*) None

[Dec. 2020]

Solution : AM=6; HM=9

$AM \geq GM \geq HM$. (Always)

Here AM < HM

Hence data is not perfect.

So, its GM cannot be determined. (*d*) is correct.

Q.82. From the record on sizes of shoes sold in a shop, one can compute the following to determine the most preferred shoe size.

(*a*) Mean (*b*) Median

(*c*) Mode (*d*) Range

[Jan. 2021]

Solution : (*c*) Most preferred shoe size in sale, means most frequent. So, mode is suitable.

Q.83. Which of the following measure does not possess mathematical properties?

(*a*) Arithmetic mean

(*b*) Geometric mean

(c) Harmonic mean

(d) Median

[Jan. 2021]

Solution : (d) Median.

Q.84. If $y = 3 + (4.5)x$ and the mode for x-value is 20, then the mode for y-value is

(a) 3.225 (b) 12

(c) 24.5 (d) 93

[Jan. 2021]

Solution : (d) is correct.

$$Y = 3 + (4.5)\,x$$

$$\text{Mode (Y)} = 3 + 4.5 \times \text{Mode (x)}$$

$$= 3 + 4.5 \times 20$$

$$= 3 + 90 = 93$$

Q.85. If there are two groups with n_1 with n_2 observations and H_1 and H_2 are respective harmonic means, then the harmonic mean of combined observation is

(a) $\dfrac{n_1H_1 + n_2H_2}{n_1 + n_2}$

(b) $\dfrac{n_1H_1 + n_2H_2}{H_1 + H_2}$

(c) $\dfrac{n_1 + n_2}{n_1H_1 + n_2H_2}$

(d) $\dfrac{(n_1 + n_2)H_1 + H_2}{n_1H_2 + n_2H_1}$

[Jan. 2021]

Solution : (d) is correct

$$\text{Combined H.M} = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$$

$$= \frac{n_1 + n_2}{\left(\frac{n_1H_2 + n_2H_1}{H_1H_2}\right)} = \frac{(n_1 + n_2)(H_1H_2)}{n_1H_2 + n_2H_1}$$

Q.86. Expenditures of a Company (in Million Rupees) per item in various Years

Years	Item of Expenditures				
	Salary	Fuel and Transport	Bonus	Interest on Loans	Taxes
1998	288	98	3.00	23.4	83
1999	342	112	2.52	32.5	108
2000	324	101	3.84	41.6	74
2001	336	133	3.68	36.4	88
2002	420	142	3.96	49.4	98

What is the average amount of interest per year which the company had to pay during this period?

(a) 33.66 (b) 36.66 (c) 31.66 (d) 39.66

[July 2021]

Solution : (b) is correct

Average Amount of interest

$$= \frac{23{\cdot}4 + 32{\cdot}5 + 41{\cdot}6 + 36{\cdot}4 + 49{\cdot}4}{5}$$

$$= 36.66$$

Q.87. There are n numbers. When 50 is subtracted from each of these number the sum of the numbers so obtained is -10. When 46 is subtracted from each of the original n numbers, then the sum of numbers so obtained is 70. What is the mean of the original n numbers?

(a) 56.8 (b) 25.7 (c) 49.5 (d) 53.8

[July 2021]

Solution : (c) is correct.

From 1st condition

$$\bar{X} = 50 + \frac{\sum(X - 50)}{n} = 50 + \frac{10}{n}$$

$$= 50 - \frac{10}{n}$$

From 2nd condition

$$\bar{X} = 46 + \frac{\sum(X - 46)}{n}$$

$$= 46 + \frac{70}{n}$$

From 1st

$$50 - \frac{10}{n} = 46 + \frac{70}{n}$$

or;

$$4 = \frac{10}{n} + \frac{70}{n}$$

or

$$4 = 80 \quad \Rightarrow \quad n = 20$$

$\therefore$

$$\textbf{mean} = \bar{X} = 50 - \frac{10}{20} = 49.5$$

Q.88. The mean of 'n' observation is 'X'. If K is added to each observation, then the new mean is _____

(*a*) X (*b*) XK

(*c*) X-K (*d*) X + K

[July 2021]

Solution : (*d*) is correct

old mean = X

If K is added to each observation

Then new mean = X + K

Q.89. If y = 3 + 1.9 x, and mode of x is 15, then the mode of y is:

(*a*) 15.9 (*b*) 27.8

(*c*) 35.7 (*d*) 31.5

[July 2021]

Solution : (*d*) is correct.

Mode (Y) = 3 + 1.9(15)

= 31.5

Q.90. If there are 3 observations 15, 20, 25 then the sum of deviation of the observations from their AM is

(*a*) 0 (*b*) 5

(*c*) –5 (*d*) 10

[Dec. 2021]

Solution : (*a*)

Because sum of deviations of obs. from mean is always equal to zero.

$\therefore \Sigma\left(X-\overline{X}\right)=0$

[Note : No need to solve]

Q.91. If the AM and GM for 10 observations are both 15, then the value of HM is

(*a*) less than 15

(*b*) more than 15

(*c*) 15

(*d*) cannot be determined

[Dec. 2021]

Solution : (*c*)

If AM = GM = 15

$\Rightarrow$ AM = GM = HM = 15

Q.92. If average mark for a group of 30 girls is 80, a group of boys is 70 and combined average is 76, then how many are in the boy's group ?

(*a*) 21 (*b*) 20

(*c*) 22 (*d*) 19

[Dec. 2021]

Solution : (*b*)

Tricks : Go by choices (GBC)

$$\overline{x_{12}}=\frac{N_1\overline{X_1}+N_2\overline{X_2}}{N_1+N_2}$$

$$=\frac{30\times80+20\times70}{30+20}$$

Calculator Work

30×80 = button

20×70 = button GT button (Press)

÷ 50 = button

we get 76.

$\therefore$ (*b*) is correct.

Q.93. If two variables *a* and *b* are related by *c* = *ab* then G.M. of *c* is equal to

(*a*) G.M. of *a* + G.M. of *b*

(*b*) G.M. of *a* × G.M. of *b*

(*c*) G.M. of *a* – G.M. of *b*

(*d*) G.M. of *a* / G.M. of *b]*

[Dec. 2021]

Solution : (*b*)

GM(C) = GM(ab)

= GM(a). GM(b)

Q.94. For a moderately skewed distribution the median is twice the mean, then the mode is _____ times the median.

(*a*) 3 (*b*) 2

(*c*) 2/3 (*d*) 3/2

[Dec. 2021]

Solution : (*b*)

$\because$ Median = $2\overline{X}$

$M_o = 3M - 2\overline{X}$

$= 3 \times 2\overline{X} - 2\overline{X} = 4\overline{X}$

$= 2 \times 2\overline{X}$

= 2 Median

Q.95. The median value of the set of observations 48, 36, 72, 87, 19, 66, 56, 91 is

(*a*) 53 (*b*) 87

(*c*) 61 (*d*) 19

[Dec. 2021]

Solution : (*c*)

Arranging in ascending order; we get

19, 36, 48, 56, 66, 72, 87, 91

$$\text{Median}=\frac{56+66}{2}=61$$

Q.96. For a data having odd number of values, the difference between the first and the middle value is equal to the difference between the last and the middle value; similarly, the difference between the second and middle value is equal to that of second last and middle value so on. Therefore, the middle value is equal to

(*a*) Half of the range

(*b*) Half of standard deviation

(*c*) Mode

(*d*) Mean

[Dec. 2021]

Solution : (*d*)

Let us assume Nos. as

2, 3, 4, 5, 6 (odd No. of observations)

1st Condition

4–2 = 6–4

2 = 2 (True)

[Difference means Larger value – smaller value]

2nd condition

4–3 = 5–4 (Also True)

Clearly middle observation = 4

= Median or Mean (Here)

Median is not in option.

So Mean is Answer.

Q.97. One hundred participants expressed their opinion on recommending a new product to their friends using the attributes : most unlikely, unlikely, not sure, likely, most likely. The appropriate measure of central tendency that can be used here is

(*a*) Mean

(*b*) Mode

(c) Geometric mean

(d) Harmonic mean

[Dec. 2021]

Solution : (b)

Here, Quality is asked.

So mode is suitable

Q.98. Ogive curves cannot be used to determine

(a) Mean (b) Median

(c) Mode (d) Range

[Dec. 2021]

Solution : (d)

Ogive Curves means less than or more than cumulative frequency curve. Here, it is difficult to find Largest or smallest observations So, Range cannot be determined

Q.99. Along a road there are 5 buildings of apartments, marked as 1, 2, 3, 4, 5. Number of people residing in each building is available. A bus stop is to be setup near one of the buildings so that the total distance walked by the residents to the bus stop from their buildings must be kept minimum. One must consider involving _____ to find the position of the bus stop.

(a) Mean (b) Median

(c) Mode (d) Weighted mean

[Dec. 2021]

Solution : (b)

Q.100. Given that mean = 70.20 and mode = 70.50, the median is expected to be

(a) 70.15 (b) 70.20

(c) 7.30 (d) 70.35

[Dec. 2021]

Solution : (c)

$M_o = 3M - 2\bar{X}$

$70.50 = 3M - 2 \times 70.20$

or $3M = 70.50 + 140.40$

$= 210.9$

or $M = \frac{210.9}{3} = 70.30$

Q.101. _____Mean is calculated, when the values in series do not have equal importance.

(a) Arithmetic (b) Harmonic

(c) Geometric (d) Weighted

[June 2022]

Solution : (d)

When observations have not equal importance than weighted Mean is used.

Q.102. A seller of pearls kept the pearls in seven boxes labelled from one to seven. At the end of a day, he found that J labelled box contained J pearls, the average number of pearls per box is

(a) 4 (b) 6.5

(c) 7.5 (d) 8

[June 2022]

Solution : In J^{th} box has of J No. of pearls.

∴ No. of Pearls contained from 1 to 7 boxes are 1, 2, 3, 4, 5, 6, 7

∴ A. M. = Average

$$= \frac{1+2+3+4+5+6+7}{7} = 4$$

∴ (a) is correct

Q.103. The Mean of 100 students was 45. Later on, it was discovered that the marks of two students were misread as 85 and 54 instead of 58 and 45. Find out the correct mean.

(a) 68 (b) 36 (c) 44.64 (d) 52

[June 2022]

Solution :

Tricks

$$\text{Correct Mean} = \left(\frac{\text{N . Incorrect Mean} - \text{Incorrect obs.} + \text{correct obs.}}{\text{N}}\right)$$

$$= \frac{100 \times 45 - 85 - 54 + 58 + 45}{100}$$

$=44.64$

∴ (c) is correct.

Q.104. Calculate the value of 3rd quartile from the following data 40, 35, 51, 30, 21, 25, 16, 29, 27, 32.

(a) 36.25 (b) 30.25

(c) 25 (d) 35

[June 2022]

Solution : Arranging observations in ascending order, we get

16, 21, 25, 27, 29, 30, 32, 35, 40, 51

Here N = 10

$$Q_3 = 3\left(\frac{N+1}{4}\right)^{th} \text{ observation}$$

$$= 3\left(\frac{10+1}{4}\right) = 8.25^{th} \text{ observation}$$

$=8^{th}$ obs.$+0.25\ (9^{th} - 8^{th}$ obs.)

$= 35 + 0.25\ (40 - 35)$

$=36.25$

∴ (a) is correct

Q.105. If mean $(\bar{X})$ is = 10 and mode (Z) is = 7, then find out the value of median (M).

(a) 9 (b) 17

(c) 3 (d) 4.33

[Dec. 2022]

Solution: *Given :*

Mean = $\bar{X} = 10$; Mode = $Z = 7$

Formula :

$M_0 = 3M_e - 2\bar{X}$

or

$7 = 3M_e - 2 \times 10$

or $3M_e = 27 \quad \therefore M_e = 9$

∴ (a) is correct

Q.106. _____ is based on all the observations and _____ is based on the central fifty per cent of the observations.

(a) Mean deviation, Range

(b) Mean deviation, quartile deviation

(c) Range, standard deviation

(d) Quartile deviation, standard deviation

[Dec. 2022]

Solution : (b)

Q.107. The relationship between two variables x and y is given by 4x – 10y =20. If the median value of the variable x is 10 then what is median value of variable y?

(a) 1.0 (b) 2.0

(c) 3.0 (d) 4.0

[Dec. 2022]

Solution : *Given :*

$4x - 10y = 20$

and Me $(x) = 10$; $\therefore$ Me $(y) = ?$

$\therefore 4 \times 10 - 10$ Me $(y) = 20$

or; $40 - 20 = 10$ Me (y)

or; $20 = 10$ Me (y)

Me $(y) = 2$

$\therefore$ (b) is correct.

Q.108. The median of the observations 42, 72, 35, 92, 67, 85, 72, 81, 51, 56 is

(a) 69.5 (b) 72

(c) 64 (d) 61.5

[Dec. 2022]

Solution: Arranging in ascending order ;

We get

35, 42, 51, 56, 67, 72, 72, 81, 85, 92

N = 10 (Even No.)

Median = Me = Average of middle two observations

$$= \frac{67+72}{2} = \frac{139}{2} = 69.5$$

(a) is correct

Q.109. The mean of 50 observations is 36. If two observations 30 and 42 are to be excluded, than the mean of the remaining observations will be:

(a) 36 (b) 38

(c) 48 (d) 50

[Dec. 2022]

Solution: Mean of rest observations

$$= \frac{N\bar{x} - \text{Excluded observation}}{N - \text{No. of observations removed}}$$

$$= \frac{50 \times 36 - 30 - 42}{50 - 2} = 36$$

$\therefore$ (a) is correct

Q.110. If Arithmetic Mean and Geometric Mean between two numbers are 5 and 4 respectively, then these numbers are

(a) 2 & 3 (b) 2 & 8

(c) 4 & 6 (d) 1 & 16

[Dec. 2022]

Solution: *Given:* Arithmetic mean = 5

and Geometric mean = 4

Tricks: Go by choices

(b) 2; 8

$$\text{A.M} = \frac{2+8}{2} = 5 \text{ (True)}$$

and GM $= \sqrt{2 \times 8} = 4$ (Also is True)

$\therefore$ (b) is correct

Q.111. If Arithmetic between two numbers is 5 and Geometric mean is 4 then what is the value of Harmonic mean?

(a) 3.2 (b) 3.4

(c) 3.5 (d) 3.6

[Dec. 2022]

Solution: $\text{HM} = \frac{G^2}{A}$

$$= \frac{4^2}{5}$$

$= 3.2$

$\therefore$ Option (a) is correct.

Q.112. The average age of 15 students in a class is 9 years. Out of them, the average age of 5 students is 13 years and that of 8 students is 5 years. What is the average of remaining 2 students?

(a) 5 years (b) 9 years

(c) 10 years (d) 15 years

[Dec. 2022]

Solution : Total age = N. $\bar{x} = 15 \times 9 = 135$

Total age of 5 students $= 5 \times 13 = 65$

Total age of 8 students $= 8 \times 5 = 40$

$\therefore$ Age of rest 2 students = 135-65-40 = 30

$\therefore$ Av. age of rest 2 students = 30/2 = 15

$\therefore$ option (d) is correct.

Q.113. A Professor has given assignment to students in a Statistics class. A student Jagan computes the arithmetic mean and standard deviation for a set of 100 observations as 50 and 5 respectively. Later on, Sonali points out to Jagan that he has made a mistake in taking one observation as 100 instead of 50. What would be the correct mean if the wrong observation is corrected?

(a) 50.5 (b) 49.9

(c) 49.5 (d) 50.1

Solution:

Given

$$\left.\begin{matrix} N = 100; \bar{X} = 50 \\ \text{S.D.} = 5 \end{matrix}\right] \text{Incorrect}$$

Incorrect Observation = 100

Correct Observation = 50

$\therefore$ Correct Mean

$$= \frac{100 \times 50 - 100 + 50}{100}$$

$= 49.5$

$\therefore$ (c) is correct.

Q.114. Find the mean of the following data:

Class Interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	9	13	6	4	6	2	3

(a) 23.7 (b) 35.7 (c) 39.7 (d) 43.7

Solution :

Note :– No need to make this table. Do it directly on Calculator.

Class-Interval	Mid Value (X)	Frequency (f)	fx
10 - 20	15	9	
20 - 30	25	13	
30 - 40	35	6	
40 - 50	45	4	
50 - 60	55	6	
60 - 70	65	2	
70 - 80	75	3	

$\Sigma f = 43$

$$\overline{X} = \frac{\Sigma fx}{\Sigma f}$$

Calculator Tricks

Press $15 \times 9 =$ button

$25 \times 13 =$ "

$35 \times 6 =$ "

$45 \times 4 =$ "

$55 \times 6 =$ "

$65 \times 2 =$ "

$70 \times 3 =$ " Then

Press GT button Then $\div 43 =$ button; we get $35.697 = 35.7$

$\therefore$ (*b*) is correct.

Q.115. The Median of the following set of observations 24, 18, 36, 42, 30, 28, 21, 29, 25, 33 is:

(*a*) 26.5 (*b*) 27.5

(*c*) 28.5 (*d*) 29.5

Solution :

Arranging all observations in ascending order; we get

18, 21, 24, 25, 28, 29, 30, 33, 36, 42

Here N = 10 observations (Even)

$\therefore$ Median = A.M. of middle two observations

$$= \frac{28+29}{2} = 28.5$$

(*c*) is correct.

Q.116. Find the mode of the following data:

X	25-30	30-35	35-40	40-45	45-50	50-55
f(*x*)	20	53	42	42	41	43

(*a*) 31.75 (*b*) 30.75

(*c*) 33.75 (*d*) 35.75

Solution :

Here Modal-Class = 30 – 35

(Because highest frequency lie in this Class-Interval)

$i = 5$

And $f_o = 53$; $f_{-1} = 20$; $f_{+1} = 42$

$\Delta_1 = f_o - f_{-1} = 53 - 20 = 33$

$\Delta_2 = f_o - f_{+1} = 53 - 42 = 11$

$$\therefore M_O = L + \frac{\Delta_i}{\Delta_1 + \Delta_2} \times i$$

$$= 30 + \frac{33}{33+11} \times 5$$

$$= 33.75$$

$\therefore$ (*c*) is correct.

Q.117. For a moderately skewed distribution of marks in statistics for a group of 200 students, the mean marks and median marks were found to be 55.60 and 52.40, respectively. What are the modal marks?

(*a*) 54.43 (*b*) 48

(*c*) 53.56 (*d*) 46

Solution : Given

$\overline{X} = 55.60$; $M_e = 52.40$

$M_O = 3M_e - 2\overline{X}$

$= 3 \times 52.40 - 2 \times 55.60$

$= 46$

$\therefore$ (*d*) is correct.

Q.118. For a given data set: 5, 10, 3, 6, 4, 8, 9, 3, 15, 2, 9, 4, 19, 11, 4: what is the median?

(*a*) 8 (*b*) 6

(*c*) 4 (*d*) 9

Solution : Arranging in ascending order. We get

2, 3, 3, 4, 4, 4, 5, 6, 8, 9, 9, 10, 11, 15, 19.

N = 15

$$\text{Median} = \frac{N+1}{2} = \frac{15+1}{2} = 8\text{th obs.}$$

$= 6$

$\therefore$ (*b*) is correct.

Q.119. If the mean of two numbers is 30 and geometric mean is 24, then what will be the Harmonic mean of two numbers?

(*a*) 19.2 (*b*) 21.8

(*c*) 22.3 (*d*) 18.4

Solution : Given

A = 30; G = 24

$$\therefore H = \frac{G^2}{A} = \frac{24^2}{30} = 19.2$$

(*a*) is correct.

Q.120. The Geometric Mean of 3, 7, 11, 15, 24, 28, 30, 0 is:

(*a*) 6 (*b*) 0

(*c*) 9 (*d*) 12

Solution : If at least one observation of the data is Zero(0).

Then Geometric Mean is Not possible.

No option should be the Answer.

But as per formula

$= (3 \times 7 \times 11 \times 15 \times 24 \times 28 \times 30 \times 0)^{1/8}$

$= 0$

$\therefore$ (*b*) should be selected.

22 CHAPTER MEASURES OF DISPERSION

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. The variance of data : 3, 4, 5, 8 is

(*a*) 4.5 (*b*) 3.5 (*c*) 5.5 (*d*) 6.5

[Dec. 2010]

Solution : (*b*) X : 3, 4, 5, 8

$$\sum X = 20;\ \sum X^2 = 9 + 16 + 25 + 64 = 114$$

$$\text{variance} = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2$$

$$= \frac{114}{4} - 25 = 3.5$$

Q.2. Given the observations: 4, 9, 11, 14, 37. The Mean deviation about the Median is

(*a*) 11 (*b*) 8.5

(*c*) 7.6 (*d*) 7.45

[Dec. 2010]

Solution : (*c*) 4, 9, 11, 14, 37

Median = Middle obs. = 11

X	$\lvert d\rvert = \lvert x - 11\rvert$
4	7
9	2
11	0
14	3
37	26

$$\sum |d| = 38$$

Mean deviation about median

$$\text{M.D.} = \frac{\sum |d|}{n} = \frac{3}{2} = 7.6$$

Q.3. If all observations in a distribution are increased by 6, then the variance of the series will be _____.

(*a*) Increased (*b*) Decreased

(*c*) Unchanged (*d*) None of these

[Dec. 2010]

Solution : (*c*) RULE :- SD does not w.r.t. origin.

So; Variance will also remain unchanged.

Q.4. The standard deviation of the weights (in kg.) of the student of a class of 50 students was calculated to be 4.5 kg. Later on it was found that due to some fault in weighing machine, the weight of each student was under measured by 0.5 kg. The Correct standard deviation of the weight will be:

(*a*) Less than 4.5

(*b*) Greater than 4.5

(*c*) Equal to 4.5

(*d*) Cannot be determined

[Dec. 2011]

Solution : (*c*)

RULE :- S.D remains unaffected due to a change of origin but changes with respect to scale.

So, correct S.D. of 50 students = 4.5

Q.5. For Normal distribution the relation between Quartile Deviation (Q.D.) and Standard Deviation (S.D.) is

(*a*) Q.D > S.D

(*b*) Q.D < SD

(*c*) Q.D = S.D

(*d*) None of the above

[Dec. 2011]

Solution : (*b*) Since; $\text{Q.D} = \frac{2}{3}$

$\text{S.D} \Rightarrow Q.D < S.D$

Q.6. If standard deviation of first 'n' natural numbers is 2 then the value of 'n' is

(*a*) 10 (*b*) 7

(*c*) 6 (*d*) 5

[June 2010]

Solution : (*b*) S.D. of 1st .n. natural Numbers

$$= \sqrt{\frac{n^2-1}{12}}$$

$$\frac{2}{1} = \sqrt{\frac{n^2-1}{12}}, \text{ OR, } 4 = \frac{n^2-1}{12}$$

OR, $n^2 - 1 = 48$

OR, $n^2 = 49 \Rightarrow n = 7$

TRICKS : Go by choices.

Q.7. The standard deviation is independent of change of

(*a*) Scale

(*b*) Origin

(*c*) Both origin and scale

(*d*) None of these

[June 2010]

Solution : (*b*)

Q.8. If Standard deviation of X is σ, then Standard deviation of $\frac{ax+b}{c}$, where a, b and c are arbitrary constants, will be

(*a*) σ (*b*) $\frac{a\sigma+b}{c}$

(*c*) $\frac{a}{c}.\sigma$ (*d*) $\left|\frac{a}{c}\right|\sigma$

[Dec. 2012]

Solution : (*d*) S.D. of $X = \sigma$

$$\text{Let } y = \frac{ax+b}{c} = \frac{ax}{c} + \frac{b}{c}$$

$$\text{S.D. of } y = \left|\frac{a}{c}\right| \text{S.D. of } x = \left|\frac{a}{c}\right|\sigma$$

Q.9. Which of the following measures of dispersion is used for calculating the consistency between two series?

(*a*) Quartile deviation

(*b*) Standard Deviation

(*c*) Coefficient of variation

(*d*) None of them

[Dec. 2012]

Solution : (*c*)

Coefficient of variation (CV) is used for calculating the consistency between two series.

Q.10. $\sum x^2 = 3390, n = 30, \sigma = 7$; then $\bar{X} =$ _____

(*a*) 113 (*b*) 210 (*c*) 8 (*d*) None

[June 2013]

Solution : (*c*) is correct

$$\because \sigma^2 = \frac{\sum x^2}{N} - (\bar{X})^2$$

$$\text{or } 49 = \frac{3390}{30} - (\bar{X})^2$$

$$\text{or } (X)^2 = 113 - 49 = 64$$

$$\therefore \bar{X} = 8$$

Q.11. If the mean of frequency distribution is 100 and coefficient of variation is 45% then standard deviation is_____

(*a*) 45

(*b*) 0.45

(*c*) 0.045

(*d*) None

[June 2013]

Solution : (*a*) is correct

$$\text{C.V.} = \frac{\sigma}{\bar{X}} \times 100$$

$$\text{or, } 45 = \frac{\sigma}{100} \times 100, \quad \sigma = 45$$

Q.12. If Arithmetic Mean= $\frac{8+4}{2}$ then variance is

(*a*) 2 (*b*) 6

(*c*) 1 (*d*) 4

[Dec. 2013]

Solution : (*d*) is correct

$$\text{Arithmetic mean} = \frac{8+4}{2}$$

$\therefore$ Numbers are 8;4

$$\therefore \text{SD of 8 \& 4} = \frac{1}{2}|8-4| = 2$$

$$\text{Variance} = SD^2 = 4$$

Q.13. Coefficient of mean deviation about mean for the first 9 natural numbers is:

(*a*) $\frac{200}{9}$

(*b*) 80

(*c*) $\frac{400}{9}$

(*d*) 50

[Dec. 2013]

Solution : (*c*) is correct

X	$\lvert X-\overline{X}\rvert$
1	4
2	3
3	2
4	1
5	0
6	1
7	2
8	3
9	4

$\overline{X}=5$ $\sum\lvert X-\overline{X}\rvert=20$

$$\text{MD}=\frac{\sum\lvert X-\overline{X}\rvert}{N}=\frac{20}{9}$$

$$\text{Coefficient of MD}=\frac{\text{MD}}{\overline{X}}\times 100$$

$$=\frac{20\times 100}{9\times 5}=\frac{400}{9}$$

Q.14. Mean = 5, S.D = 2.6, Median = 5, Q.D = 1.5 then coefficient of Q.D is:

(*a*) 35 (*b*) 39

(*c*) 30 (*d*) 32

[Dec. 2013]

Solution : (*c*) is correct

$\because$ Mean = Median = 5

$\therefore$ Data is symmetrical

$$\therefore M=\frac{Q_3+Q_1}{2}=5$$

$$\therefore Q_3+Q_1=10$$

$$\text{Coefficient of QD}=\frac{Q_3-Q_1}{Q_3+Q_1}\times 100$$

$$=\frac{2\times QD}{Q_3+Q_1}\times 100=\frac{2\times 1.5}{10}\times 100$$

$= 30.$

Q.15. The difference between maximum and minimum value of the data is known as:

(*a*) Range (*b*) Size

(*c*) Width (*d*) Class

[Dec. 2013]

Solution : (*a*) Range = L-S.

Q.16. The formula for range of middle 50% items of a series is

(*a*) Q_3-Q_1

(*b*) Q_3-Q_2

(*c*) Q_2-Q_1

(*d*) $\frac{Q_3-Q_1}{2}$

[June 2014]

Solution : (*a*) is correct

Range of Middle 50% items

= Inter Quartile Range

$= Q_3-Q_1$

Q.17. What will be the probable value of mean deviation? When $Q_3=40$ and $Q_1=15$

(*a*) 17.50

(*b*) 18.75

(*c*) 15.00

(*d*) None of the above

[June 2014]

Solution : (*c*) is correct

6QD = 5 MD = 4 SD

or $\frac{6.(Q_3-Q_1)}{2}=5MD$

or 3(40 – 15) = 5 MD

or MD = 15.00

Q.18. 1st quartile is 142 Semi-inter Quartile 18. Then median is

(*a*) 151 (*b*) 160

(*c*) 178 (*d*) None

[Dec. 2014]

Solution : (*b*) is correct.

$\therefore Q_1=142$

Semi-inter Quartile Range

$$=\text{QD}=\frac{Q_3-Q_1}{2}=18$$

$\therefore Q_3-Q_1=36$

$Q_3-142=36$

$Q_3=142+36=178$

Median

$$=\text{M}=\frac{Q_3+Q_1}{2}=\frac{178+142}{2}=160$$

(**Remember :** Here it has assumed as symmetrical)

Q.19. Q.D is

(*a*) 2/3 .SD (*b*) 4/5 SD

(*c*) 5/6S.D (*d*) None

[Dec. 2014]

Solution : (*a*) is correct.

$\therefore$ 6QD = 5 MD = 4 SD

$$\text{QD}=\frac{4}{6}\text{SD}=\frac{2}{3}\text{SD}$$

Q.20. Co-efficient of QD is equal to_____

(*a*) $\frac{QD}{M}\times 100$ (*b*) $\frac{QD}{x}\times 100$

(*c*) $\frac{QD}{Z}\times 100$ (*d*) None

[June 2015]

Solution : (*a*) is correct.

$$\text{Co-efficient of QD}=\frac{Q_3-Q_1}{Q_3+Q_1}\times 100$$

$$=\frac{\frac{Q_3-Q_1}{2}}{\frac{Q_3+Q_1}{2}}\times 100=\frac{QD}{M}\times 100$$

where $\text{M}=\frac{Q_3+Q_1}{2}$ (Symmetrical)

Q.21. If every observation is increased by 5 then

(*a*) SD increases by 5

(*b*) MD increases by 5

(*c*) QD increases by 5

(*d*) None affected

[June 2015]

Solution : (*d*) is correct.

Rule : The value of QD; MD; SD; RANGE does not change with respect to the change of origin.

Q.22. The SD of X is known to be 10 then the SD of (5θ + 5 x) is

(a) 50 (b) 100
(c) 10 (d) 500

[June 2015]

Solution : SD of (50 + 5x)

$= 5 \times$ SD of x

$= 5 \times 10 = 50$

Q.23. Find the range of 6,5,4,3,1, 3,6,10,8.

(a) 6 (b) 3
(c) 9 (d) 10

[Dec. 2015]

Solution : (c) is correct.

Range = L – S = 10 – 1 = 9

Q.24. Find the mean deviation about mean of 4, 5, 6, 8, 3:

(a) 7.2 (b) 5.2
(c) 1.44 (d) 1.70

[Dec. 2015]

Solution : (c) is correct.

X	$\lvert X-\overline{X}\rvert$
3	2.2
4	1.2
5	0.2
6	0.8
8	2.8
$\sum X = 26$	$\sum \lvert X-\overline{X}\rvert = 7.2$

$$\overline{X} = \frac{26}{5} = 5.2$$

$$MD = \frac{\Sigma|X-\overline{X}|}{N} = \frac{7.2}{5} = 1.44$$

Q.25. If V(x) = 23 Find variance of (2x+10:)

(a) 104 (b) 110
(c) 92 (d) 85

[Dec. 2015]

Solution : (c) is correct.

$\text{Var.}(X) = \sigma^2 = 23 ; \therefore \sigma = \sqrt{23}$

$\text{SD of } (2X+10) = 2.\sigma = 2\sqrt{23}$

$\text{Var.}(2X+10) = \left(2\sqrt{23}\right)^2 = 92$

Q.26. The average of 2 numbers is 20 and their standard deviation 5. Find the two numbers?

(a) 15, 25 (b) 30, 40
(c) 10,15 (d) None of these

[Dec. 2015]

Solution : (a) is correct.

Tricks : Go by choices. For option (a)

$$\text{AV.} = \frac{15+25}{2} = 20$$

$$SD = \sigma = \frac{1}{2}|25-15| = 5$$

It satisfy both conditions which are given in question.

Q.27. If Variance = 125.6, $\overline{X}$ = 40, coefficient of variation =

(a) 28.02 (b) 314
(c) 40.02 (d) None of these

[Dec. 2015]

Solution : (a) is correct.

$\sigma = \sqrt{125.6} = 11.21$

$$\text{C.V} = \frac{\sigma}{\overline{X}} \times 100 = \frac{11.21}{40} \times 100 = 28.02$$

Q.28. If same amount is added to or subtracted from all the values of the individual series then the standard deviation and variance both shall be

(a) Changed (b) Unchanged
(c) Same (d) None of these

[June 2016]

Solution : (b)

Since, SD does not change with respect to the change of origin. So, variance will also remain unchanged.

Q.29. The SD of first n natural numbers is ____

(a) $\sqrt{\frac{n^2-1}{12}}$ (b) $\sqrt{\frac{n(n+1)}{12}}$
(c) $\sqrt{\frac{n(n-1)}{6}}$ (d) None of these

[June 2016]

Solution : (a)

Q.30. If mean and coefficient of variation of the marks of n students is 20 and 80 respectively. What will be variance of them

(a) 256 (b) 16
(c) 25 (d) None of these

[June 2016]

Solution : (a)

Given $\overline{X} = 20$; C.V = 80%

$$\therefore \text{C.V.} = \frac{\sigma}{\overline{X}} \times 100$$

$$\therefore 80 = \frac{\sigma}{20} \times 100$$

$$\therefore \sigma = \frac{80 \times 20}{100} = 16$$

Variance $= \sigma^2 = 16^2 = 256$

Q.31. If AM and CV of a random variable X are 10 & 40 respectively, then the variance of $\left(-15+\frac{3x}{2}\right)$:

(a) 64 (b) 81 (c) 49 (d) 36

[June 2017]

Solution : (d) is correct.

Given; $\frac{\sigma}{10} \times 100 = 40 \ \therefore \ \sigma = 4$

$$\therefore \text{SD of } \left(-15+\frac{3}{2}x\right) = \frac{3}{2} \times \text{SD}(x) = \frac{3}{2} \times 4 = 6$$

$$\therefore \text{Variance of } \left(-15+\frac{3}{2}x\right) = 6^2 = 36$$

Q.32. Mean deviation is least when deviations are taken from:

(*a*) Mean (*b*) Median

(*c*) Mode (*d*) None of these

[Dec. 2017]

Solution : (*b*)

Q.33. Mean of a series is equal to 100, coefficient of variation is 45% then the S.D. is _____.

(*a*) 45 (*b*) 0.45

(*c*) 4.5 (*d*) 40.5

[June 2018]

Solution : (*a*)

$$C.V. = \frac{\sigma}{\bar{X}} \times 100$$

$$\text{or } 45 = \frac{\sigma}{100} \times 100$$

$$\sigma = 45$$

Q.34. Coefficient of variation is a relative measure of _____ :

(*a*) Range

(*b*) Central Tendency

(*c*) Dispersion

(*d*) Q.D.

[June 2018]

Solution : (*c*)

Q.35. $\frac{(Q_3 - Q_1)}{(Q_3 + Q_1)}$ **is known as**

(*a*) Coefficient of Range

(*b*) Coefficient of Q.D.

(*c*) Coefficient of S.D.

(*d*) Coefficient of M.D.

[May 2018]

Solution : (*b*) Coefficient of Q.D.

$$= \frac{(Q_3 - Q_1)}{(Q_3 + Q_1)}$$

Q.36. If the S.D. of the 1st n natural Nos. is $\sqrt{30}$, **Then the value of n is**

(*a*) 19 (*b*) 20

(*c*) 21 (*d*) None

[May 2018]

Solution : (*a*) **Tricks** : GBC

For (a)

$$S.D = \sqrt{\frac{n^2 - 1}{12}} = \sqrt{\frac{19^2 - 1}{12}} = \sqrt{30}$$

(true)

Q.37. If two random variables x and y are related by Y = 2 – 3X, then the SD of Y is given by

(*a*) $-3 \times$ SD of X

(*b*) $3 \times$ SD of X

(*c*) $9 \times$ SD of X

(*d*) $2 \times$ SD of X

[May 2018]

Solution : (*b*)

$$S.D \text{ of } y\ (\sigma_y) = |b|\ S.D \text{ of } X\ (\sigma_x)$$

$$= |-3| \cdot \sigma_x$$

$$= 3\sigma_x$$

Q.38. If the variance of 5, 7, 9 and 11 is 4, then the coefficient of variation is

(*a*) 25 (*b*) 15

(*c*) 17 (*d*) 19

[Nov. 2018]

Solution : (*a*)

$$\sigma^2 = 4 \Rightarrow \sigma = 2$$

$$\bar{X} = (5+7+9+11) \div 4 = 8$$

$$\therefore CV = \frac{\sigma}{\bar{X}} \times 100 = \frac{2}{8} \times 100 = 25\%$$

Q.39. Standard Deviation for the marks obtained by a student in monthly test in Mathematics (out of 50) as 30, 35, 25, 20, 15 is

(*a*) 25 (*b*) 50

(*c*) $\sqrt{50}$ (*d*) $\sqrt{30}$

[Nov. 2018]

Solution : (*c*)

X	X^2
30	900
35	1225
25	625
20	400
15	225
Σx = 125	Σx^2 = 3375

Tricks : Use Calculator for direct calculation

$$6 = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

$$= \sqrt{\frac{3375}{5} - \left(\frac{125}{5}\right)^2}$$

$$= \sqrt{50}$$

Q.40. If the standard deviation for the marks obtained by a student in monthly test is 36, then the variance is

(*a*) 36

(*b*) 6

(*c*) 1296

(*d*) None of the above

[Nov. 2018]

Solution : (*c*)

$$\sigma = 36$$

$$\text{Variance} = \sigma^2 = 36^2 = 1296.$$

Q41. Which one of the following is not a Central Tendency?

(*a*) Mean Deviation

(*b*) Arithmetic mean

(*c*) Median

(*d*) Mode

[Nov. 2018]

Solution : (*a*)

Q.42. If the range of a set of values is 65 and maximum value in the set is 83, then the minimum value in the set is

(*a*) 74

(*b*) 9

(*c*) 18

(*d*) None of the above

[Nov. 2018]

Solution : (*c*)

Range = L – S

65 = 83 – S

$\therefore$ S = 83 – 65 = 18

Q.43. If $\sigma^2 = 100$ **and coefficient of variation = 20% then** $\bar{X}$ **=**

(*a*) 60 (*b*) 70

(*c*) 80 (*d*) 50

[June 2019]

Solution : (*a*) If $\sigma^2 = 100$, So

$\sigma = \sqrt{100} = 10$ C.V. $= \frac{S.D}{A.M} \times 100$

$20 = \frac{10}{\overline{X}} \times 100$

$20\overline{X} = 1000$; $\overline{X} = 50$

Q.44. If the points of inflexion of a Normal Curve are 40 and 60 respectively, then its mean deviation is

(*a*) 8 (*b*) 45

(*c*) 50 (*d*) 60

[June 2019]

Solution : (*a*)

If the point of inflexion of a Normal Distribution are 40 and 60.

Formula, $\mu - \sigma = 40$ .....(1)

$\mu + \sigma = 60$ .....(2)

Solving eqns. (1) and (2); we get

$\mu = 50$, $\sigma = 10$

Then M.D. = 0.8. $\sigma = 0.8 \times 10 = 8$

Q.45. Standard deviation is ___ times of $\sqrt{MD \times QD}$

(*a*) 2/3 (*b*) 4/5

(*c*) $\sqrt{\frac{15}{8}}$ (*d*) $\sqrt{\frac{8}{15}}$

[June 2019]

Solution : (*c*) We know that,

4 S.D = 5 M.D = 6 Q.D

Let, 4 S.D = 5 M.D = 6 Q.D

LCM of 4,5,6 = 60

S.D = 60/4 = 15

MD = 60/5 = 12

QD = 60/6 = 10

Let SD is x times of $\sqrt{MD \times QD}$

SD = x $\sqrt{MD \times QD}$

$15 = x\sqrt{12 \times 10}$

Squaring on both sides; we get

$225 = x^2 . 12 \times 10$

So, $x = \sqrt{\frac{225}{12 \times 10}} = \sqrt{\frac{15}{8}}$

Q.46. The Q.D of 6 numbers 15, 8, 36, 40, 38, 41 is equal to

(*a*) 12.5 (*b*) 25

(*c*) 13.5 (*d*) 37

[June 2019]

Solution : (*c*) Arranging obs. in Ascending order, we get

8, 15, 36, 38, 40, 41

Here, N = 6

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ term

$= \left(\frac{6+1}{4}\right)^{th}$ term $= 1.75^{th}$ term

$= 1^{st}$ term + 0.75 (2^{nd} term – 1^{st} term)

$= 8 + 0.75 \times (15 - 8)$

$= 8 + 0.75 \times 7$

$= 8 + 5.25$

$= 13.25$

$Q_3 = 3\left(\frac{N+1}{4}\right)^{th}$ obs.

$= \frac{3(6+1)^{th}}{4}$ term $= 5.25^{th}$ term

$= 5^{th}$ term + 0.25 (6^{th} term – 5^{th} term)

$= 40 + 0.25\,(41 - 40)$

$= 40 + 0.25 \times 1$

$= 40 + 0.25 = 40.25$

$QD = \frac{Q_3 - Q_1}{2}$

$= \frac{40.25 - 13.25}{2} = \frac{27}{2} = 13.5$

Q.47. S.D of first five consecutive natural numbers is

(*a*) $\sqrt{10}$ (*b*) $\sqrt{8}$

(*c*) $\sqrt{3}$ (*d*) $\sqrt{2}$

[June 2019]

Solution : (*d*)

S.D of 1st 'n' Natural No. $= \sqrt{\frac{n^2 - 1}{12}}$

n = 5

$S.D = \sqrt{\frac{5^2 - 1}{12}} = \sqrt{\frac{24}{12}} = \sqrt{2}$

Q.48. If the profit of a company remain same for the last ten months then the S.D. of profits of the company would be:

(*a*) Positive (*b*) Negative

(*c*) Zero (*d*) (a) or (c)

[June 2019]

Solution : (*c*)

If the profits of a company remain same for ten months.

then S.D = 0

(Since shifting of origin S.D is not changed)

Q.49. Coefficient of quartile deviation is 1/4 then Q_3/Q_1 is

(*a*) 5/3 (*b*) 4/3

(*c*) 3/4 (*d*) 3/5

[June 2019]

Solution : (*a*) Coeff. of Q.D = $\frac{1}{4}$

$\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{1}{4}$ [cross product]

or ; $4Q_3 - 4Q_1 = Q_3 + Q_1$

or ; $4Q_3 - Q_3 = Q_1 + 4Q_1$

or ; $3Q_3 = 5Q_1$

or; $\frac{Q_3}{Q_1} = \frac{5}{3}$

Q.50. The sum of mean and SD of a series is a + b, if we add 2 to each observation of the series then the sum of mean and SD is

(*a*) a + b + 2 (*b*) 6 – a + b

(*c*) 4 + a – b (*d*) a + b + 4

[June 2019]

Solution : (*a*) If a constant quantity is added to each observation, then value of mean changes but SD remains unchanged.

Since, Mean + S.D = (a + b)

⇒ Let mean = a and SD = b

If we add '2' in each observation New mean = a + 2 and new Sd = b, So new (mean + S.D) = (a + b + 2)

Q.51. What will be the probable value of mean deviation when Q_3 = 40 and Q_1 = 15?

(a) 17.50 (b) 18.75
(c) 15.00 (d) 16.00

[Dec. 2019]

Solution : (c)

$$\text{Q.D.} = \frac{Q_3 - Q_1}{2} = \frac{40-15}{2} = 12.5$$

Formula

$$6.\text{QD} = 5.\text{MD}$$

or; $6 \times 12.5 = 5.\text{MD}$

$$\therefore \text{MD} = \frac{6 \times 12.5}{5} = 15.$$

Q.52. Find the mean deviation about mean of 4, 5, 6, 8, 3

(a) 5.20 (b) 7.20
(c) 1.44 (d) 2.33

[Dec. 2019]

Solution : (c)

X	$\lvert X - \bar{X} \rvert$
4	1.2
5	0.2
6	0.8
8	2.8
3	2.2

$\Sigma X = 26 \qquad \Sigma \lvert X - \bar{X} \rvert = 7.2$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{26}{5} = 5.2$$

$$\text{MD.} = \frac{\Sigma \lvert X - \bar{X} \rvert}{N} = \frac{7.2}{5} = 1.44$$

Q.53. The mean and coefficient of variance are 20 and 80. Find the value of variance

(a) 16 (b) 256
(c) 36 (d) none

[Dec. 2019]

Solution : (b)

$$\text{C.V} = \frac{\sigma}{\bar{X}} \times 100$$

or ; $80 = \frac{\sigma}{20} \times 100 \Rightarrow \sigma = 16$

Variance $= \sigma^2 = 16^2 = 256.$

Q.54. Find SD of 1, 2, 3, 4, 5, 6, 7, 8, 9

(a) $\sqrt{\frac{20}{3}}$ (b) $\sqrt{\frac{81}{3}}$

(c) $\sqrt{\frac{20}{5}}$ (d) None of these

[Dec. 2019]

Solution : (a)

Formula; S.D of 1st n natural Nos. = $\sqrt{\frac{n^2-1}{12}}$

$$= \sqrt{\frac{9^2-1}{12}} = \sqrt{\frac{80}{12}} = \sqrt{\frac{20}{3}}$$

Q.55. The standard deviation for the set of numbers 1,4,5,7,8, is 2.45 nearly. If 10 is added to each number then new standard deviation is

(a) 24.45 (b) 12.45
(c) 2.45 (d) 0.245

[Dec. 2019]

Solution : (c)

Note:- If a constant quantity is added to each observation then the value of S.D. does not change.

Q.56. If every observation is increased by 5 then:

(a) SD increase by 5
(b) MD increased by 5
(c) QD increases by 5
(d) Dispersion Not affected

[Dec. 2019]

Solution : (d)

Note:- If every observation is increased by a constant quantity then the value of dispersion does not change.

Q.57. For given distribution the arithmetic mean is 15 and the standard deviation is 9 then the coefficient of variation?

(a) $\frac{15}{9} \times 100$ (b) $\frac{15}{9}$

(c) $\frac{9}{15}$ (d) $\frac{9}{15} \times 100$

[Dec. 2019]

Solution : (d)

Co-efficient of variation

$$\therefore \text{C.V} = \frac{\sigma}{\bar{X}} \times 100 = \frac{9}{15} \times 100$$

$= 60\ \%$

Q.58. The mean of a distribution is 14 and the standard deviation is 5. What is the value of the coefficient of variation?

(a) 60.4% (b) 70%
(c) 35.7% (d) 27.8%

[Dec. 2019]

Solution : (c)

$$\text{C.V} = \frac{\sigma}{\bar{X}} \times 100 = \frac{5}{14} \times 100$$

$= 35.7\%$

Q.59. Which of the following measure of dispersion is based on absolute deviations?

(a) Range
(b) SD
(c) Mean Deviation
(d) Quartile Deviation

Solution : (c)

Q.60. The best statistical measure used for comparing two series is

(a) Mean absolute deviation
(b) Range
(c) Coefficient of variation
(d) Standard deviation

[Jan. 2021]

Solution : (c) Coefficient of variation (C.V.) (a relative dispersion) is assumed as the best measure for comparing two series.

Q.61. Which of the following measure of dispersion is based on absolute deviations?

(a) Range
(b) SD
(c) Mean Deviation
(d) Quartile Deviation

[Dec. 2019]

Solution : (c)

Q.62. The relationship between P-series and Q series is given by 2P- 3Q = 10. If the rage of P- Series is 18. What would be the range of Q?

(*a*) 10 (*b*) 15

(*c*) 9 (*d*) 12

[Jan. 2021]

Solution : (*d*) is correct.

Given eqn. is

$$2P - 3Q = 10$$

Tricks 2 Range (P) = 3 Range (Q)

[∴ Range does not change with respect to the change of origin but changes with respect to scale.]

So, $2 \times 18 = 3$ Range (Q)

$$\therefore \quad \text{Range(Q)} = \frac{2\times 18}{3} = 12$$

Q.63. It is given that the mean $(\bar{X})$ is 10 and standard deviation (s.d.) is 3.2. If the observations are increased by 4, then the new mean and standard deviations are:

(*a*) $(\bar{X}) = 10$, s.d. = 7.2

(*b*) $(\bar{X}) = 10$, s.d. = 3.2

(*c*) $(\bar{X}) = 14$, s.d. = 3.2

(*d*) $(\bar{X}) = 14$, s.d. = 7.2

[Jan. 2021]

Solution : (*c*)

Mean	S.D.
10	3.2
(Each observation + 4)	3.2
	No change

$= 10 + 4 = 14$

(*c*) is correct

Q.64. Which of the following is a relative measure of dispersion?

(*a*) Range

(*b*) Mean deviation

(*c*) Standard deviation

(*d*) Coefficient of quartile deviation

[Jan. 2021]

Solution : (*d*) is correct

Q.65. Find the coefficient of mean deviation about mean for the data: 5, 7, 8, 10, 11, 13, 19

(*a*) 17.28 (*b*) 28.57 (*c*) 32.11 (*d*) 18.56

[Jan. 2021]

Solution : (*c*) is correct

$$\text{Mean} = \bar{X} = \frac{\text{sum of observations}}{\text{No. of observations}}$$

$$= \frac{5+7+8+10+11+13+19}{7}$$

$$= \frac{73}{7} = 10.42 \text{ [Find by calculator]}$$

$$\text{Mean Deviation} = \text{MD} = \frac{\sum |X - \bar{X}|}{N}$$

$$= \frac{[|5-10.42|+|7-10.42|+|8-10.42|+|10-10.42|+|11-10.42|+|13-10.42|+|19-10.42|]}{7}$$

$= 3.3457$

On Calculator [Minimum Time]

Do as 5-10.42 = [+/−] button Then M + (Press)

[because MD is always positive]

Type another observation 7 = [+/−]

Then M + button (Press)

[Note : No need of typing 10.42 again and again].

Then type 8 = [+/−] M + (Press)

10 = [+/−] M + (Press)

11 = M + (Press)

[Here difference is positive; so never press = [+/−] button]

13 = M + (Press)

19 = M + (Press)

Press MRC button two times at the end. It will give you numerator value.

Divide it by 7; we will get.

$$\text{Coefficient of MD} = \frac{\sum |X - \bar{X}|}{\bar{X}} \times 100$$

$$= \frac{3.3457}{10.42} \times 100$$

$$= 32.108 = 32.11\%$$

[95% Calculations can be made without noting/writing any thing else.]

Q.66. The mean deviation of the numbers 3, 10, 6, 11, 14, 17, 9, 8, 12 about the mean is (correct to one decimal place)

(*a*) 8.7 (*b*) 4.2 (*c*) 3.1 (*d*) 9.8

[July 2021]

Solution : (*c*) is correct.

[Do everything on calculation]

[Note: – Never write in this way in exam. Only type difference on calculator and divide result by 14]

= 3.43 Approx.

Q.72. If the relationship between x and y is given by 2x + 3y = 10 and the range of y is 10, then what is the range of x?

(*a*) 10 (*b*) 18 (*c*) 8 (*d*) 15

[July 2021]

Solution : (*d*) is correct.

Tricks:

2. Range (X) = 3. Range (Y)

[∵ value of Range is always positive.]

or; 2. Range (X) = 3 × 10

$$\text{or Range (X)} = \frac{3\times10}{2} = 15$$

Q.73. The marks secured by 5 students in a subject are 82, 73, 69, 84, 66. What is the coefficient of Range

(*a*) 0.12 (*b*) 12 (*c*) 120 (*d*) 0.012

[Dec. 2021]

Solution : (*b*)

$$\text{Coefficient of Range} = \frac{84-66}{84+66}\times100$$

$$= 12$$

Q.74. Which one of the following is not a measure of central tendency?

(*a*) Median (*b*) Range

(*c*) Arithmetic Mean (*d*) Harmonic Mean

[June 2022]

Solution : (*b*) Range is not a measure of central-Tendency.

Q.75. What is mean deviation about mean of the following numbers? 11, 8, 10, 10, 12, 9

(*a*) 2 (*b*) 1 (*c*) 1.5 (*d*) 1.8

[June 2022]

Solution : Mean = $\bar{X} = \frac{11+8+10+10+12+9}{6} = 10$

$$MD_{\bar{x}} = \delta_{\bar{x}} = \frac{\Sigma|X-\bar{X}|}{N}$$

$$= \frac{|11-10|+|8-10|+|10-10|+|10-10|+|12-10|+|9-10|}{6}$$

$$= \frac{1+2+0+0+2+1}{6} = \frac{6}{6} = 1$$

∴ (*b*) is correct

Q.76. Following are the ages of 8 employees of a small old age home expressed in 96, 50, 67, 75, 71, 69, 64, 66. Find the range and its coefficient.

(*a*) 46, 31.51 respectively (*b*) 51, 37.67 respectively

(*c*) 43, 29.49 respectively (*d*) 49, 36.42 respectively

[June 2022]

Solution : Smallest observation = S = 50

Largest observation = L = 96

∴ Range = L–S = 96 – 50 = 46

$$\text{Coefficient of Range} = \frac{L-S}{L+S}\times100$$

$$= \frac{96-50}{96+50}\times100 = 31.51\%$$

∴ (*a*) is correct

Q.77. Find the standard deviation and coefficient of variation of 1, 6, 5, 9, 8.

(*a*) 2.78 and 40.83 respectively (*b*) 2.45 and 47.93 respectively

(*c*) 2.78 and 47.93 respectively (*d*) 2.87 and 49.37 respectively

[June 2022]

Solution : Observations are x : 1, 5, 6, 8, 9

∴ $\Sigma x = 1 + 5 + 6 + 8 + 9 = 29$

$\Sigma x^2 = 1^2 + 5^2 + 6^2 + 8^2 + 9^2 = 207$

$$\therefore \sigma = \sqrt{\left(\frac{\Sigma x^2}{N}\right)-\left(\frac{\Sigma x}{N}\right)^2} = \sqrt{\frac{207}{5}-\left(\frac{29}{5}\right)^2}$$

$$= \sqrt{41.4-(5.8)^2} = 2.78$$

$$\bar{X} = \frac{\Sigma x}{N} = \frac{29}{5} = 5.8$$

$\therefore$ Coefficient of variation = C.V. = $\frac{\sigma}{\bar{X}} \times 100$

$$= \frac{2.78}{5.8} \times 100 = 47.93$$

$\therefore$ (*c*) is correct

Q.78. The arithmetic mean and coefficient of variation of data set X are respectively, 10 and 30. The variance of 30 - 2X is

(*a*) 28 (*b*) 32.

(*c*) 34 (*d*) 36

[June 2022]

Solution : Given $\bar{X} = 10$; CV =30

$\because$ CV = $\frac{\sigma}{\bar{X}} \times 100 = 30$

or $\frac{\sigma}{10} \times 100 = 30$

$\therefore \sigma_x = 3$

Let $y = 30 - 2x$

$\sigma_y = |-2| . \sigma_x = 2 \times 3 = 6$

Variance (y) = σ_y^2 = var. (30–2x)

$= 6^2 = 36$

$\therefore$ (*d*) is correct.

Q.79. Which measure of dispersion is base on the absolute deviation only?

(*a*) Range

(*b*) Standard deviation

(*c*) Mean deviation

(*d*) Quartile deviation

[June 2022]

Solution : (*c*) is correct

$\because$ MD = $\frac{\Sigma|x - \bar{x}|}{N}$

Hence only MD is based on absolute deviation.

Q.80. The coefficient of deviation based on 25th and 75th percentiles of 6, 9, 3, 8, 4, 5, 8 and 4 is

(*a*) 50 (*b*) 100/3

(*c*) 30 (*d*) 25

[June 2022]

Solution : Arranging observations in ascending order than we get

3, 4, 4, 5, 6, 8, 8, 9

Here N = 8

$P_{25} = Q_1 = 25 \left(\frac{N+1}{100}\right)^{th}$ observation

$= 25 \left(\frac{8+1}{100}\right) = 2.25^{th}$ obs.

$= 2^{nd}$ obs.+ 0.25 (3^{rd} obs – 2^{nd} obs.)

$= 4 + 0.25 (4–4) = 4$

$P_{75} = Q_3 = 75 \left(\frac{N+1}{100}\right)^{th}$ observation.

$= 75 \left(\frac{8+1}{100}\right) = 6.75^{th}$ obs.

$= 6^{th}$ obs. + 0.75 (7^{th} obs. – 6^{th} obs.)

$= 8 + 0.75 (8 - 8)$

$= 8$

$\therefore$ Coefficient of Deviation

$$= \frac{P_{75} - P_{25}}{P_{75} + P_{25}} \times 100 = \frac{8-4}{8+4} \times 100$$

$$= \frac{100}{3}\ \%$$

$\therefore$ (*b*) is correct

Q.81. If the coefficient of variation and standard deviation are 30 and 12 respectively, then the arithmetic mean of the distribution is:

(*a*) 40 (*b*) 36

(*c*) 25 (*d*) 19

[Dec. 2022]

Solution : *Given:*

CV = 30; $\sigma = 12$

$\because$ C.V. = $\frac{\sigma}{\bar{x}} \times 100$

$\therefore \quad \bar{x} = \frac{\sigma}{CV} \times 100 = \frac{12}{30} \times 100$

$= 40$

(*a*) is correct

Q.82. Which one of the following is not a method of measures of dispersion?

(*a*) Standard deviation

(*b*) Mean deviation

(*c*) Range

(*d*) Concurrent deviation method

[Dec. 2022]

Solution : (*d*)

Q.83. Mean deviation is minimum when deviation are taken from:

(*a*) Mean (*b*) Median

(*c*) Mode (*d*) Range

[Dec. 2022]

Solution: (*b*) is correct

$\sum|X - Me| < \sum|X - \bar{X}|$

Dividing by N on both sides;

we get

$$\frac{\sum|X - Me|}{N} < \frac{\sum|X - \bar{X}|}{N}$$

$\therefore$ MD from median (minimum) < MD from (Maximum)

Q.84. If the first quartile is 56.50 and the third quartile is 77.50, then the coefficient of quartile deviation is:

(*a*) 638.09 (*b*) 15.67

(*c*) 63.80 (*d*) 156.71

[Dec. 2022]

Solution : *Given*

$Q_1 = 56.50$

$Q_3 = 77.50$

Co-efficient of QD

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

$$= \frac{77.50 - 56.50}{77.50 + 56.50} \times 100$$

$$= \frac{21}{134} \times 100 = 15.67$$

$\therefore$ (*b*) is correct

Q.85. If the sum of square of the values equals to 3390, Number of observations are 30 and Standard deviation is 7, what is the mean value of the above observations?

(*a*) 14 (*b*) 11

(*c*) 8 (*d*) 5

[Dec. 2022]

Solution : *Given,*

$= \Sigma x^2 = 3390$; N = 30; $SD = \sigma = 7$

Formula

$$\sigma^2 = \frac{\Sigma x^2}{N} - (\bar{x})^2$$

or; $7^2 = \frac{3390}{30} - (\bar{x})^2$

or; $49 = 113 - (\bar{x})^2$

or; $(\bar{x})^2 = 113 - 49 = 64$

or; $\bar{x} = \sqrt{64} = 8$

(*c*) is correct

Q.86. If the variance of a random variable '*x*' is 17 then what is variance of y = 2x + 5?

(*a*) 34 (*b*) 39

(*c*) 68 (*d*) 78

[Dec. 2022]

Solution : $\sigma_x = Sd = \sqrt{17}$

$\sigma_y = 2.\ \sigma_x = 2\sqrt{17}$

$\therefore$ Ver. (4) $= \sigma_{y2} = 4 \times 17$

$= 68$

So, (*c*) is correct.

Q.87. If the variance of given data is 12, and their mean value is 40, what is Coefficient of Variation (CV)?

(*a*) 5.66% (*b*) 6.66%

(*c*) 7.50% (*d*) 8.65%

[Dec. 2022]

Solution: $\sigma^2 = 12$

$\sigma = \sqrt{12}$

$\bar{x} = 40$

$\therefore$ CV $= \frac{\sigma}{\bar{x}} \times 100$

$= \frac{\sqrt{12}}{40_2} \times \cancel{100}^{5}$

$= 8.66\%$

So, (*d*) is correct.

Q.88. In a given set if all data are of same value then variance would be:

(*a*) 0 (*b*) 1

(*c*) –1 (*d*) 0.5

[Dec. 2022]

Solution: So, S.D. = 0 [$\because$ All observations are same]

$\therefore$ Variance $= 0^2 = 0$

$\therefore$ Option (*a*) is correct

Q.89. If *x* and *y* are related as 4*x* + 3*y* + 11 = 0 and mean deviation of *y* is 7.20, what is the mean deviation of x?

(*a*) 2.70 (*b*) 7.20

(*c*) 4.50 (*d*) 5.40

Solution:

Tricks

4 MD of X = + 3 MD of Y

(Always Positive)

or; 4 MD(x) = 3 × 7.20

or $MD(x) = \frac{3 \times 7.20}{4}$

$= 5.40$

(*d*) is correct.

Q.90. The mean deviation about the mean for the data 12, 16, 24, 30, 35, 39, 40 is:

(*a*) 9.14 (*b*) 9.41

(*c*) 8.91 (*d*) 9.81

Solution:

X	$\lvert X-\bar{X}\rvert$
12	16
16	12
24	4
30	2
35	7
39	11
40	12
$\Sigma X = 196$	$\Sigma\lvert X - \bar{X}\rvert = 64$

$\bar{X} = \frac{\Sigma X}{N} = \frac{196}{7} = 28$

$\therefore$ Mean Deviation $= \frac{\Sigma\lvert X - \bar{X}\rvert}{N}$

$= \frac{64}{7} = 9.14$

(*a*) is correct.

Q.91. If the Standard Deviation of data 2, 4, 5, 6, 8, 17 is 4.47, then Standard Deviation of the data 4, 8, 10, 12, 16, 34 is:

(*a*) 4.47 (*b*) 8.94

(*c*) 13.41 (*d*) 2.24

Solution:

SD of given data is 4.47.

If all observations are multiplied by 2; we get the resulting data.

$\therefore$ SD of New Data

$= 2 \times 4.47 = 8.94$

$\therefore$ (*b*) is correct.

Q.92. The mean and variance of a group of 100 observations are 8 and 9, respectively. Out of 100 observations, the mean and standard deviation of 60 observations are 10 and 2, respectively. Find the standard deviation of remaining 40 observations?

(*a*) 4.5

(*b*) 3.5

(*c*) 2.5

(*d*) 1.5

Solution:

$N = 100;\ \bar{X}_{12} = 8;$

$\text{Var.} = S_{12}^2 = 9$

$\therefore S_{12} = 3$

$N_1 = 60;\ \bar{X}_1 = 10;\ S_1 = 2$

$N_2 = 100 - N_1 = 100 - 60 = 40$

$\therefore \bar{X}_2 = \frac{N\bar{X}_{12} - N_1\bar{X}_1}{N_2}$

$= \frac{100 \times 8 - 60 \times 10}{40} = 5$

$d_1 = \bar{X}_1 - \bar{X}_2 = 10 - 8 = 2$

$d_2 = \overline{X}_2 - \overline{X}_{12} = 5 - 8 = -3$

$\therefore S_{12}^2 = \frac{N_1(S_1^2 + d_1^2) + N_2(S_2^2 + d_2^2)}{N}$

or; $3^2 = \frac{60(2^2 + 2^2) + 40(S_2^2 + (-3)^2)}{100}$

or; $900 = 480 + 40\,(S_2^2 + 9)$

or; $420 = 40\,(S_2^2 + 9)$

or; $S_2^2 + 9 = \frac{420}{40} = 10.5$

or; $S_2^2 = 1.5$

$\therefore S_2 = \sqrt{1.5} = 1.22$

No option.

Most approximate value of

$S_2 = 1.22$ can be taken 1.5.

$\therefore$ (*d*) should be correct.

Q.93. For a given set of normally distributed data, the following statistical parameters are known: Mean = 6; Standard deviation = 2.6; Median = 5 and Quartile deviation = 1.5, then the coefficient of quartile deviation equals to:

(*a*) 30 (*b*) 32

(*c*) 25 (*d*) 39

Solution:

Co-efficient of QD

$= \frac{\frac{Q_3 - Q_1}{2}}{M_e} \times 100$

$= \frac{QD}{M_e} \times 100 = \frac{1.5}{5} \times 100$

$= 30\%$

$\therefore$ (*a*) is correct.

Q.94. If the first quartile is 42.75 and the third quartile is 74.25, then the coefficient of quartile deviation is:

(*a*) 29.62 (*b*) 15.75

(*c*) 17.57 (*d*) 29.62

Solution:

Given

$Q_1 = 42.75;\ Q_3 = 74.25$

$\therefore$ Co-eff. of Q.D

$= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

$= \frac{74.25 - 42.75}{74.25 + 42.75} \times 100$

$= \frac{31.75}{117} \times 100 = 26.92\%$

$\therefore$ (*d*) is correct.

23 CHAPTER PROBABILITY

Experiment:- The performance which produces the certain results is called Experiment.

Random Experiment:- If the results of the experiment depend on chance only, then the experiment is called Random Experiment. Example:- Tossing a coin; Throwing a die; drawing a card from well - shuffled pack of 52 cards etc.

Sample Space:- The set of all possible distinct outcomes of a random experiment is called SAMPLE SPACE (or Event Space). It is denoted by capital letter "S".

Ex-1 : A coin is tossed at random then S = {H, T} $\Rightarrow$ n(S) = 2

Either Head (H) or Tail (T) can occur on upper face of the coin.

Ex-2 : A die is thrown at random then S = {1, 2, 3, 4, 5, 6} $\Rightarrow$ n(S) = 6

$\because$ A die has 6 faces with face number 1, 2, 3, 4, 5, 6. One of them can occur at a time.

Ex-3 : Two coins are tossed together then S = {HH, HT; TH; TT} i.e. If both coins are tossed together then heads on both coins or Tails on both coins or Head on one coin and Tail on another one coin can occur. Another way to find Sample Space.

Total Sample Space "S" = cross - product of individual Sample - Space.

$\therefore$ S = {H, T} $\times$ {H, T}

= {H H, HT, TH, TT}

$\therefore$ $n(S) = n(S_1).n(S_2) = 2 \times 2 = 4$

Tricks : For Coins $n(S) = 2^{\text{No. of coins tossed together}}$

Ex-1 For 2 coins

$n(S) = 2^2 = 4$

Ex-3. For 3 coins tossed together

$n(S) = 2^3 = 8$.

For Dice

Ex-: If two dice are thrown together then, S = {1, 2, 3, 4, 5, 6} × {1, 2, 3, 4, 5, 6}
= {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6);
(2, 1), (2, 2), ...; (2, 3);
(3, 1), (3, 2);.....;(3, 6);
(4, 1), (4, 2);.....;(4, 6);
(5, 1), (5, 2);.....;(5, 6);
(6, 1), (6, 2);.....;(6, 6)}

$\therefore\ n(s) = n(s_1).\ n(s_2) = 6 \times 6 = 36.$

Tricks: For Dice

$n(S) = 6^{\text{No. of dice Thrown together}}$

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. In a pack of playing cards with two jokers probability of getting king of spade is

(*a*) 4/13 (*b*) 4/52
(*c*) 1/52 (*d*) 1/54

[June 2010]

Solution : (*d*) Total No. of playing cards = 52 + 2 = 54

Sample Space $n(S) = {}^{54}c_1 = 54$

Total no. of spade king = 1

E = Event of getting 1 spade king = 1

$\therefore P(E) = \frac{1}{54}$

Q.2. Consider two events A and B not mutually exclusive, such that P(A) = 1/4, P(B) = 2/5, $P(A \cup B) = 1/2$, then $P(A\bar{B})$ is

(*a*) 3/7 (*b*) 2/10
(*c*) 1/10 (*d*) None of them

[June 2010]

Solution : (*d*) Since events A & B are not mutually exclusive. So, they are independent events.

$P(AB) = P(A \cap B) = P(A).P(B)$

$\therefore P(A\bar{B}) = P(A) - P(A \cap B) = P(A) - P(A).P(B)$

$= P(A).[1 - (B)]$

$= \frac{1}{4}\cdot\left(1-\frac{2}{5}\right) = \frac{1}{4}\cdot\frac{3}{5} = \frac{3}{20}$

Q.3. If x be the sum of two numbers obtained when two dice are thrown simultaneously then $P(x \geq 7)$ is

(*a*) 5/12 (*b*) 7/12
(*c*) 11/15 (*d*) 3/8

[June 2010]

Solution : (*b*) Sample Space n (S) = 36

E = Event of getting X > = 7

= {(1,6);(2,5);(3,4);(4;3);(5,2);(6,1);
(2,6);(3,5);(4,4);(5,3);(6,2);.....;
(6,6)}; ∴ n (E) = 21

∴ Required Probability P(E) = $\frac{21}{36} = \frac{7}{12}$

Tricks :- See Nov. 2019 Qts.

Q.4. If P(A/B) = P(A), then A and B are

(*a*) Mutually exclusive events
(*b*) Dependent events
(*c*) Independent events
(*d*) Composite events ***[Dec. 2010]***

Solution : (*c*) P(A/B) = P(A)

Hence, A and B are independent events.

Q.5. A bag contains 3 white and 5 black balls and second bag contains 4 white and 2 black balls. If one ball is taken from each bag, the probability that both the balls are white is_____

(*a*) 1/3 (*b*) 1/4
(*c*) 1/2 (*d*) None of these

[Dec. 2010]

Solution : (*b*) Let A = Event of getting 1 white ball from 1st bag.

B = Event of getting 1 white ball from 2nd bag.

A & B are Independent Events.

$P(A \cap B) = P(A).P(B)$

$= \frac{{}^3C_1}{{}^8C_1}\cdot\frac{{}^4C_1}{{}^6C_1} = \frac{3}{8}\times\frac{4}{6} = \frac{1}{4}$

Q.6. The odds in favour of A solving a problem is 5:7 and Odds against B solving the same problem is 9:6. What is the probability that if both of them try, the problem will be solved?

(*a*) 117/180 (*b*) 181/200
(*c*) 147/180 (*d*) 119/180

[Dec. 2010]

Solution : (*a*) $P(A) = \frac{5}{12}$; $P(B) = \frac{6}{15}$

Probability that problem will be Solved
= 1 – Prob. that problem not solved
$= 1 - P(A').P(B')$
$= 1 - [1 - P(A)][1 - P(B)]$

$= 1 - \left(1-\frac{5}{12}\right)\left(1-\frac{6}{15}\right)$

$= 1 - \frac{7}{12}\times\frac{9}{15} = \frac{180-63}{180} = \frac{117}{180}$

Q.7. Consider Urn 1 : 2 white balls, 3 black balls; Urn II : 4 white balls, 6 black balls. One ball is randomly transferred from first to second Urn, then one ball is drawn from II Urn. The probability that drawn ball is white is

(*a*) 22/65 (*b*) 22/46
(*c*) 22/55 (*d*) 21/45

[Dec. 2010]

Solution : (*c*)

Case I : 1 Black ball is transferred from Urn I to Urn II and then a white ball is picked from Urn II

Prob. $= \frac{3}{5}\times\frac{4}{11} = \frac{12}{55}$

Case II : 1 White ball is transferred from Urn I to II and then a white ball is picked from Urn II

Prob. $= \frac{2}{5}\times\frac{5}{11} = \frac{10}{55}$

From Case I + Case II;

P(White ball) $= \frac{12}{55} + \frac{10}{55} = \frac{22}{55}$

Q.8. If P $(A \cup B)$= P(A), Find P $(A \cap B)$.

(*a*) P(A).P(B) (*b*) P(A) + P(B)

(*c*) 0 (*d*) P(B)

[June 2011]

Solution : (*d*) Given : $P(A \cup B) = P(A)$

we know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore$ we get $P(A \cap B)$ = P(B)

Q.9. A bag contains 5 Red balls, 4 Blue Balls and 'm' Green Balls. If the random probability of picking two green balls is 1/7. What is the No. of green Balls (m)

(*a*) 5 (*b*) 7

(*c*) 6 (*d*) None of above

[June 2011]

Solution : (*c*) Total balls = 5 + 4 + m = m + 9

$\therefore$ Probability of picking two green balls = $\frac{1}{7}$

TRICKS : Go by choices

For (c) Total = 6 + 9 = 15

$$P = \frac{{}^{6}c_2}{{}^{15}c_2}$$

$$= \frac{15}{105} = \frac{1}{7} \text{ (True)}$$

Q.10. The probability of Girl getting scholarship is 0.6 and the same probability for Boy is 0.8 Find the probability that at least one of the categories getting scholarship.

(*a*) 0.32 (*b*) 0.44

(*c*) 0.92 (*d*) None of these

[June 2011]

Solution : (*c*) Given

Probability of Girl getting scholarship P(A) = 0.6

Probability of Boy getting scholarship P(B) = 0.8

Required of at least one category getting scholarship:

= 1 – P(None getting scholarship)

$= 1 - P(A').P(B') = 1 - (1 - 0.6).(1 - 0.8)$

= 1 -(0.4). (0.2) = 1- 0.08 = 0.92

Q.11. A coin is tossed 5 times, what is the probability that exactly 3 heads will occur.

(*a*) $\frac{5}{16}$ (*b*) $\frac{1}{32}$ (*c*) $\frac{5}{36}$ (*d*) $\frac{3}{32}$

Solution : (*a*) Total No. of Tails (n) = 5

r = 3

Probability of getting Head (p) = 1/2

Probability of getting Tail (q) = 1/2

By Binomial Distribution,

$P(x = r) = {}^{n}C_r p^r q^{n-r}$

$$P(r = 3) = {}^{5}c_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} = 10.\frac{1}{2^5} = \frac{10}{32} = \frac{5}{16}$$

Q.12. Arun & Tarun appear for an interview for two vacancies. The probability of Arun's selection is 1/3 and that of Tarun's selection is 1/5. Find the probability that only one of them will be selected.

(*a*) 2/5 (*b*) 4/5

(*c*) 6/5 (*d*) 8/5

[June 2012]

Solution : (*a*)

Let A & T are events of selection of Arun and Tarun respectively.

$$P(A) = \frac{1}{3}$$

$$\therefore P(\bar{A}) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(T) = \frac{1}{5}$$

$$\therefore P(\bar{T}) = 1 - \frac{1}{5} = \frac{4}{5}$$

$\therefore$ Probability that only one will be selected

$= P(A).\ P(\bar{T}) + P(\bar{A}).P(T)$

$$= \frac{1}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{5} = \frac{4}{15} + \frac{2}{15} = \frac{6}{15} = \frac{2}{5}$$

Q.13. A company employed 7 CA's, 6 MBA's and 3 Engineer's. In how many ways the company can form a committee if the committee has two members of each type.

(*a*) 900 (*b*) 1,000

(*c*) 787 (*d*) 945

[June 2012]

Solution : (*d*) The number of ways to make a committee containing two members of each type

$= {}^{7}C_2 \times {}^{6}C_2 \times {}^{3}C_2$

$= 21 \times 15 \times 3$

= 945

Q.14. Two dice are thrown together. Find the probability of getting a multiple of 2 on one dice and multiple of 3 on the other.

(*a*) 2/3

(*b*) 1/6

(*c*) 1/3

(*d*) None of the above

[Dec. 2012]

Solution : (*b*)

Sample space n(S) = 36

Let 'E' = 'Events of getting a multiple of 2 on the Ist die and multiple of 3 on the IInd die

= {(2,3);(2,6);(4,3);(4,6);(6,3);(6,6)}

n(E) = 6

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Q.15. The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5. What is the probability that the problem will be solved if they both try?

(*a*) 15/21 (*b*) 16/21

(*c*) 17/21 (*d*) 13/21

[Dec. 2012]

Solution : (*b*) The odd against A solving a certain problem = 4 : 3

P(A) = Prob (to solve the problem) =

$$\frac{3}{4+3} = \frac{3}{7}$$

$P(\bar{A})$ = prob (not to solve the problem) = $\frac{4}{4+3}=\frac{4}{7}$

The odds in favour of B solving the same problem = 7 : 5

P(B) = Prob (to solve the problem) = $\frac{7}{7+5}=\frac{7}{12}$

$P(\bar{B})$ = Prob (not to solve the problem) = $\frac{5}{7+5}=\frac{5}{12}$

Probability (the problem is solved)

$1-P(\bar{A}\cap\bar{B})=1-P(\bar{A}).P(\bar{B})=1-\frac{4}{7}.\frac{5}{12}$

Probability (problem is solved) = $1-\frac{5}{21}=\frac{16}{21}$

Q.16. A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball from the bag is twice that of a red ball, find the number of blue balls in the bag

(*a*) 10 (*b*) 12

(*c*) 14 (*d*) 16

[Dec. 2012]

Solution : (*b*) Let No. of Blue ball, = X

Total Ball in the Bag = (6 + X)

Prob of a Red ball P(R)

$=\frac{^6C_1}{^{6+x}C_1}=\frac{6}{6+x}$

and prob of a Blue Ball P(B)

$=\frac{^xC_1}{^{6+x}C_1}=\frac{x}{6+x}$

Given, P(B) = 2 P(R)

$\frac{X}{(6+X)}=\frac{2\times6}{(6+X)}\Rightarrow X=12$

Tricks : GBC

Q.17. The odds that a book will be received favourably by 3 independent reviewers are 5 to 2, 3 to 4, 4 to 3 respectively, then the probability that out of 3 critics the majority will be favorable is_____

(*a*) $\frac{209}{343}$ (*b*) $\frac{209}{434}$

(*c*) $\frac{209}{443}$ (*d*) $\frac{209}{350}$

[June 2013]

Solution : (*a*) is correct

Let A; B and C are three independent reviewers (Event)

Given: odds in favour of Events

$i.e.\frac{P(A)}{P(A^1)}=\frac{5}{2};\frac{P(B)}{P(B^1)}=\frac{3}{4};\frac{P(C)}{P(C^1)}=\frac{4}{3}$

$\therefore P(A)=\frac{5}{7};P(A^1)=\frac{2}{7}$

$P(B)=\frac{3}{7};P(B^1)=\frac{4}{7}$

$P(C)=\frac{4}{7};P(C^1)=\frac{3}{7}$

Prob.(Majority will be favourable) =

$P(A).P(B).P(C^1)+P(A)P(B^1)P(C)$

$+P(A^1).P(B).P(C)+P(A).P(B).P(C)$

$=\frac{5}{7}.\frac{3}{7}.\frac{3}{7}+\frac{5}{7}.\frac{4}{7}.\frac{4}{7}+\frac{2}{7}.\frac{3}{7}.\frac{4}{7}+\frac{5}{7}.\frac{3}{7}.\frac{4}{7}$

$=\frac{45+80+24+60}{343}=\frac{209}{343}$

Q.18. Find the probability of drawing spade on each of 2 consecutive draws from a well shuffled pack of cards when the draws are without replacement.

(*a*) $\frac{2}{51}$ (*b*) $\frac{3}{51}$

(*c*) $\frac{4}{51}$ (*d*) $\frac{4}{51}$

[June 2013]

Solution : (*b*) is correct

P(spades on consecutive draws of 2 cards)

$=\frac{13_{c_1}}{52_{c_1}}\times\frac{12_{c_1}}{51_{c_1}}=\frac{13}{52}\times\frac{12}{51}$

$=\frac{3}{51}=\frac{1}{17}$

Q.19. A bag contains 2 red 3 green and 2 blue balls. If 2 balls are drawn at random from the bag find the probability that none of them will be blue.

(*a*) $\frac{11}{21}$ (*b*) $\frac{5}{7}$

(*c*) $\frac{10}{21}$ (*d*) $\frac{2}{7}$

[June 2013]

Solution : (*c*) is correct

Sample Space = n (S) = $^7c_2=21$

p(None are blue) = $\frac{5_{c_2}}{7_{c_2}}=\frac{10}{21}$

Q.20. If P(A) = 0.45, P(B) = 0.35, P(A and B) = 0.25 then P(A/B)

(*a*) 1.4 (*b*) 1.8

(*c*) 0.714 (*d*) 0.556

[Dec. 2013]

Solution :.(*c*) is correct

$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{0.25}{0.35}$

= 0.714

Q.21. Two coins are tossed simultaneously then the probability of getting exactly one head is

(*a*) $\frac{3}{4}$ (*b*) $\frac{2}{3}$

(*c*) $\frac{1}{4}$ (*d*) $\frac{1}{2}$

[Dec. 2013]

Solution : (*d*) is correct

P(Exactly 1 head) = P(H).P(T) + P(T).P(H)

$=\frac{1}{2}.\frac{1}{2}+\frac{1}{2}.\frac{1}{2}=\frac{2}{4}=\frac{1}{2}$

Q.22. The probability that a cricket team winning a match at Kanpur is 2/5 and loosing a match at Delhi is 1/7. What is the probability of the team winning at least one match?

(*a*) $\frac{3}{35}$ (*b*) $\frac{32}{35}$

(*c*) $\frac{18}{35}$ (*d*) $\frac{17}{35}$

[Dec. 2013]

Solution : (*b*) is correct

P(At least 1 match win)

= 1- P (No match win)

$=1-\left(1-\frac{2}{5}\right)\frac{1}{7}$=1-P (Loosing at Kanpur).P (Loosing at Delhi)

$=1-\frac{3}{5}.\frac{1}{7}=1-\frac{3}{35}=\frac{32}{35}$

Q.23. For any two events

A_1, A_2; let $P(A_1)=\frac{2}{3}, P(A_2)=\frac{3}{8}$, $P(A_1 \cap A_2)=\frac{1}{4}$ then A_1, A_2 are

(*a*) Mutually Exclusive but not independent events

(*b*) Mutually Exclusive and independent events

(*c*) Independent but not Mutually Exclusive

(*d*) None *[June 2014]*

Solution : (*c*) is correct

$$(A_1 \cap A_2)=\frac{1}{4} \neq 0$$

So, A_1 and A_2 are not Mutually Exclusive Events.

$$P(A_1 \cap A_2)=P(A_1).P(A_2)$$

$$=\frac{2}{3}\times\frac{3}{8}=\frac{1}{4}(given)$$

$\therefore A_1$ & A_2 are clearly Independent Events

Q.24. If a pair of dice is thrown what is the probability of occurring neither 7 nor 11?

(*a*) $\frac{1}{6}$ (*b*) $\frac{1}{8}$

(*c*) $\frac{2}{9}$ (*d*) $\frac{7}{9}$

[June 2014]

Solution : (*d*) is correct

$n(s) = 6^2 = 36$

Let A Event of getting Their sum = 7

$= \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$n(A) = 6$

Let B = Event of getting their sum = 11

$= \{(5,6);(6,5)\}$

$n(B) = 2$

$$P(A \cup B)=\frac{n(A)+n(B)}{n(s)}$$

[$\because$ A & B are mutually Exclusive Events]

$$=\frac{6+2}{36}=\frac{8}{36}=\frac{2}{9}$$

P(Neither 7 nor 11) = $P(A \cup B)^1$

$= 1-p(A \cup B)$

$= 1-\frac{2}{9}=\frac{7}{9}$

Q.25. An urn contains 2 red and 1 green balls, another urn contains 2 red and 2 green balls. An urn was selected at random and then a ball was drawn from it. If it was found to be red then the probability that it has been drawn from first urn is

(*a*) $\frac{4}{7}$ (*b*) $\frac{3}{7}$

(*c*) $\frac{2}{3}$ (*d*) $\frac{7}{12}$

[June 2014]

Solution : (*a*) is correct

Baye's Theorem Question

Let A & B are selection of 1st & 2nd urns respectively.

$\therefore P(A) = P(B) = \frac{1}{2}$

E = Event of getting red ball.

$P(E/A) = \frac{2}{3}$; $P(E/B) = \frac{2}{4}=\frac{1}{2}$

$$\therefore P(A/E)=\frac{P(A).P(E/A)}{P(A).P(E/A)+P(B).P(E/B)}$$

$$=\frac{\frac{1}{2}\cdot\frac{2}{3}}{\frac{1}{2}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{1}{2}}=\frac{\frac{2}{3}}{\frac{2}{3}+\frac{1}{2}}=\frac{2}{3}\times\frac{6}{7}=\frac{4}{7}$$

Q.26. If 6 coins are tossed simultaneously then the probability of obtaining exactly 2 heads is

(*a*) $\frac{1}{64}$ (*b*) $\frac{63}{64}$

(*c*) $\frac{15}{64}$ (*d*) None

[Dec. 2014]

Solution : (*c*) is correct

$n = 6$; $p=\frac{1}{2}$ (prob. of getting head)

$q = 1-p = 1-\frac{1}{2}=\frac{1}{2}$

$p(x = 2)$

$$={}^6c_2.p^2.q^4 = 15.\left(\frac{1}{2}\right)^2.\left(\frac{1}{2}\right)^4=\frac{15}{64}$$

Q.27. A die is thrown twice then the probability that the sum of the number is divisible by 4 is

(*a*) $\frac{1}{9}$ (*b*) $\frac{1}{3}$

(*c*) $\frac{11}{36}$ (*d*) $\frac{1}{4}$

[Dec. 2014]

Solution : (*d*) is correct

$n(s) = 6^2 = 36$

Let E = Event that the sum of numbers is divisible by 4.

$= \{(1,3),(2,2),(3,1),(2,6),(3,5),(4,4),$
$(5,3),(6,2),(6,6)\}$

$n(E) = 9$

$$\therefore P(E)=\frac{n(E)}{n(S)}=\frac{9}{36}=\frac{1}{4}$$

Q.28. There are 6 positive and 8 negative numbers. Four number are selected at random without replacement and multiplied. Find the probability that the product is positive.

(*a*) $\frac{420}{1001}$ (*b*) $\frac{409}{1001}$

(*c*) $\frac{70}{1001}$ (*d*) $\frac{505}{1001}$

[June 2015]

Solution : (*d*) is correct

Let 6 positive Nos. are 1, 2, 3, 4, 5, 6, and 8 negative Nos. are $-1,-2,-3,$ .....; -8

$\therefore$ Sample space

$$= n(s) = {}^{14}C_4 = \frac{14!}{(4!)(10!)}$$

$$=\frac{14.13.12.11.10!}{4.3.2.1.10!}$$

$= 1001$

Let Event = E = such that product of them is positive

$\therefore n(E) = {}^6C_4+{}^6C_2.{}^8C_2+{}^8C_4$ = (All +ve) (Two +ve & two -ve) + (All 4 are -ve)

$= 15 + 15 \times 28 + 70$

$= 505$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{505}{1001}$

Q.29. $P(A^1) = 3/8; P(A^2) = 2/3; P(A^1 \cap A^2) = 1/4$ then A_1 and A_2 will be

(*a*) Mutually exclusive & independent

(*b*) Exclusive but not independent

(*c*) Independent but not exclusive

(*d*) None *[June 2015]*

Solution : (*c*) is correct

$\because P(A_1 \cap A_2) = \frac{1}{4}$ (given) $\neq 0$

$\therefore A_1$ & A_2 are not Mutually Exclusive Events

$P(A_2 \cap A_2) = P(A_1).P(A_2) = \frac{3}{8} \times \frac{2}{3} = \frac{1}{4}$

Clearly A_1 and A_2 are Independent Events

Q.30. The sum of two numbers obtained in a single throw of two dice is 'S'. Then the probability of 's' will be maximum when 'S' =

(*a*) 5 (*b*) 7

(*c*) 6 (*d*) 8 *[June 2015]*

Solution : (*b*) is correct

S=Sum of face values of two due.

S={(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)}

n(s) = 6 it is maximum if sum of their face is 7.

Q.31. When an unbiased dice is rolled, find the odds in favour of getting of multiple of 3.

(*a*) 1/6 (*b*) 1/4

(*c*) 1/2 (*d*) 1/3 *[Dec. 2015]*

Solution : (*c*) is correct

S = {1, 2, 3, 4, 5, 6}

Let E = {3,6}; E' = {1,2,3,4,5,6}-{3,6}

= {1,2,4,5} $\Rightarrow$ $n(E) = 2$; $n(E') = 4$

Odds in favour of Event

$E = \frac{n(E)}{n(E')} = \frac{2}{4} = \frac{1}{2}$

Q.32. Three coins are rolled, what is the probability of getting exactly two heads:

(*a*) 1/8 (*b*) 3/8

(*c*) 7/8 (*d*) 5/8 *[Dec. 2015]*

Solution : (*b*) is correct.

Given $n = 3$

P = Prob. (head) in 1 trial $= \frac{1}{2}$.

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$P(X = 2) = {}^3C_2.p^2.q^1 = 3.\left(\frac{1}{2}\right)^2.\frac{1}{2} = \frac{3}{8}$

Q.33. If a random sample of 500 Oranges produces 25 rotten oranges. Then the estimate of the proportion of rotten oranges in the sample is:

(*a*) 0.01 (*b*) 0.05

(*c*) 0.028 (*d*) 0.0593 *[Dec. 2015]*

Solution : (*b*) is correct.

$P = \frac{25}{500} = 0.05$

Q.34. Two letter are drawn at random from word "HOME" find the probability that there is no vowel.

(*a*) 5/6 (*b*) 1/6

(*c*) 1/3 (*d*) None *[Dec. 2015]*

Solution : (*b*) is correct.

$n(S) = {}^4C_2 = 6$.

E = Event of no vowel = {H; M}

$\therefore n(E) = {}^2C_2 = 1$.

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$

Q.35. A bag contains 15 one rupee coins, 25 two rupee coins and 10 five rupee coins. If a coin is selected at random from the bag, then the probability of not selecting a one rupee coin is:

(*a*) 0.30 (*b*) 0.70

(*c*) 0.25 (*d*) 0.20 *[Dec. 2015]*

Solution : (*b*) is correct.

$P = \frac{25+10}{50} = 0.70$

Q.36. If $P(A) = \frac{2}{3}, P(B) = \frac{3}{5}, P(A \cup B) = \frac{5}{6}$, then $P(A/B')$ is

(*a*) $\frac{7}{12}$ (*b*) $\frac{5}{12}$

(*c*) $\frac{1}{4}$ (*d*) $\frac{1}{2}$ *[June 2016]*

Solution : (*b*)

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{2}{3} + \frac{3}{5} - \frac{5}{6}$

$= \frac{20+18-25}{30} = \frac{13}{30}$

$P\left(\frac{A}{B'}\right) = \frac{P(A \cap B')}{P(B')}$

$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$

$= \frac{\frac{2}{3} - \frac{13}{30}}{1 - \frac{3}{5}} = \frac{\frac{20-13}{30}}{\frac{5-3}{5}} = \frac{7}{30} \times \frac{5}{2} = \frac{7}{12}$

Q.37. Two dice are tossed what is the probability that the total is divisible by 3 or 4

(*a*) $\frac{20}{36}$ (*b*) $\frac{21}{36}$

(*c*) $\frac{14}{36}$ (*d*) None *[June 2016]*

Solution : (*a*)

$n(S) = 36$

A = Event of getting Nos. such that the sum of their face Nos. is divisible by 3 = (sum 3 + sum 6 + sum 9 + sum 12)

$= \{(1,2),(2,1);(1,5),(2,4),(3,3),(4,2),(5,1),(3,6),(4,5),(5,4),(6,3),(6,6)\}$

$n(A) = 12$

B = Event of getting Nos. such that the sum of their face Nos. is divisible by 4 = (sum 4 + sum 8 + sum 12)

$= \{(1,3),(2,2);(3,1);(2,6),(3,5),(4,4);(5,3);(6,2);(6,6)\}$

$n(B) = 9$

$A \cap B$ = Divisible by LCM of 3 & 4 = 12

= sum 12 = $\{(6,6)\}$

$n(A \cap B) = 1$

$$P(A \cup B) = \frac{n(A) + n(B) - n(A \cap B)}{n(S)}$$

$$= \frac{12+9-1}{36} = \frac{20}{36} = \frac{5}{9}$$

(*a*) is correct.

Q.38. If 2 dice are rolled simultaneously then the probability that their sum is neither 3 nor 6 is

(*a*) 0.5 (*b*) 0.75

(*c*) 0.25 (*d*) 0.80

[June 2016]

Solution : (*d*)

A = event that sum is 3

$= \{(1,2);(2,1)\}$

$n(A) = 2$

B = event that sum is 6

$= \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$

$n(B) = 5$

$A \cap B = \phi$

$\therefore n(A \cap B) = 0$

$$P(A \cup B) = \frac{n(A) + n(B) - n(A \cap B)}{n(S)}$$

$$= \frac{2+5-0}{36} = \frac{7}{36}$$

$$P(A \cup B)' = 1 - P(A \cup B) = 1 - \frac{7}{36}$$

$$= \frac{29}{36} = 0.80$$

Q.39. In a game, cards are thoroughly shuffled and distributed equally among four players. What is the probability that a specific player gets all the four kings?

(*a*) $\frac{52_{c_4} \times 48_{c_{13}}}{52_{c_{11}}}$ (*b*) $\frac{4_{c_4} \times 48_{c_9}}{52_{c_{13}}}$

(*c*) $\frac{13_{c_9} \times 39_{c_9}}{52_{c_{13}}}$ (*d*) $\frac{4_{c_4} \times 39_{c_9}}{52_{c_{13}}}$

[June 2016]

Solution : (*b*)

Each candidate will get 13 cards.

$\therefore n(S) = {}^{52}C_{13}$

let E = Event of getting all 4 kings and rest 9 cards by a player out of 48.

$\therefore n(E) = {}^{4}C_4 \cdot {}^{48}C_9$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{{}^{4}C_4 \cdot {}^{48}C_9}{{}^{52}C_{13}}$$

(*b*) is correct

Q.40. A bag contains 4 red and 5 black balls. Another bag contains 5 red, 3 black balls. If one ball is drawn at random from each bag. Then the probability that one red and one black ball drawn is _____.

(*a*) $\frac{12}{72}$ (*b*) $\frac{25}{72}$

(*c*) $\frac{37}{72}$ (*d*) $\frac{13}{72}$

[June 2016]

Solution : Prob. (1 red and 1 black balls)

$= P(R_1) \cdot P(B_2) + P(B_1) \cdot P(R_2)$

Where R_1 = event of getting red ball from 1st bag.

R_2 = " " " " 2nd "

B_1 = " " Black ball from 1st bag.

B_2 = " " " 2nd "

$$\therefore \text{Prob.} = \frac{{}^{4}C_1}{{}^{9}C_1} \times \frac{{}^{3}C_1}{{}^{8}C_1} + \frac{{}^{5}C_1}{{}^{9}C_1} \times \frac{{}^{5}C_1}{{}^{8}C_1}$$

$$= \frac{4}{9} \times \frac{3}{8} + \frac{5}{9} \times \frac{5}{8} = \frac{12+25}{72} = \frac{37}{72}$$

Q.41. In a discrete random variable follows uniform distribution and assumes only the value 8, 9, 11, 15, 18, 20. Then $P(X \le 15)$ is _____

(*a*) 1/2 (*b*) 1/3

(*c*) 2/3 (*d*) 2/7

[June 2016]

Solution : (*c*)

E = Event of Nos. $\le 15 = \{8, 9, 11, 15\}$

$$P(X \le 15) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

Q.42. A bag contains 6 green and 5 red balls. One ball is drawn at random. The probability of getting a red ball is?

(*a*) $\frac{5}{11}$ (*b*) $\frac{6}{11}$

(*c*) $\frac{5}{6}$ (*d*) None

[Dec. 2016]

Solution : (*a*) is correct.

$n(S) = {}^{11}C_1 = 11$

Let E = Event of getting a red ball.

$n(E) = {}^{5}C_1 = 5$

$$P(E) = \frac{5}{11}$$

Q.43. If two events A, B $P(A) = \frac{1}{2}$; $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{2}{3}$, then find $P(A \cap B)$?

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{6}$

(*c*) $\frac{2}{3}$ (*d*) $\frac{1}{2}$

[Dec. 2016]

Solution : (*b*) is correct.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{2}{3} = \frac{1}{6}$$

Q.44. If $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{8}$, $P(A \cap B) = \frac{1}{4}$, then the events A & B are_____

(*a*) Independent and mutually exclusive

(*b*) Independent but not mutually exclusive

(*c*) Mutually exclusive but not independent

(*d*) Neither Independent nor exclusive *[Dec. 2016]*

Solution : (*b*) is correct.

Since, $P(A \cap B) \neq 0$, So A & B are not mutually exclusive events.

Now, $P(A \cap B) = P(A).P(B) = \frac{2}{3}.\frac{3}{8} = \frac{1}{4}$

Hence, Events A & B are Independent events.

So, (*b*) is correct.

Q.45. The probability of getting atleast one 6 from 3 throws of a perfect die is

(*a*) $\frac{5}{6}$ (*b*) $\left(\frac{5}{6}\right)^3$

(*c*) $1-\left(\frac{1}{6}\right)^3$ (*d*) $1-\left(\frac{5}{6}\right)^3$

[June 2017]

Solution : (*d*) is correct

$n = 3$ (Trials)

P = Prob. of getting 6 in 1 trial $= \frac{1}{6}$

$q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0) = 1 - {}^3C_0 . P^0 . q^3$

$= 1 - 1.1.\left(\frac{5}{6}\right)^3 = 1 - \left(\frac{5}{6}\right)^3$

Q.46. For any two events A and B

(*a*) P(A-B) = P(A)-P(B)

(*b*) P(A-B) = P(A) - P(A ∩ B)

(*c*) P(A-B) = P(B) - P(A ∩ B)

(*d*) P(B-A) = P(B) + P(A ∩ B)

[June 2017]

Solution : (*b*)

Q.47. If $P(A) = \frac{2}{3}, P(B) = \frac{1}{4}, P(A \cap B) = \frac{1}{12}$ then $P\left(\frac{B}{A}\right) =$ ____

(*a*) $\frac{1}{8}$ (*b*) $\frac{7}{8}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{2}{5}$

[June 2017]

Solution : (*a*) is correct

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{1/12}{2/3} = \frac{1}{12} \times \frac{3}{2} = \frac{1}{8}$$

Q.48. For the events A & B if $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$ then $P\left(\frac{A}{B}\right) =$

(*a*) 1/2 (*b*) 1/6 (*c*) 2/3 (*d*) 3/4

[Dec. 2017]

Solution : (*d*)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$$

Q.49. If A & B are two mutually exclusive events such that

$P(A \cup B) = \frac{2}{3}$, $P(A) = \frac{2}{5}$, then P (B):

(*a*) 4/15 (*b*) 4/9

(*c*) 5/9 (*d*) 7/15

[Dec. 2017]

Solution : (*a*)

$P(A \cup B) = P(A) + P(B)$

[$\because$ A & B are two mutually exclusive events]

$$\frac{2}{3} = \frac{2}{5} + P(B)$$

or ; $P(B) = \frac{2}{3} - \frac{2}{5} = \frac{10-6}{15} = \frac{4}{15}$

Q.50. If a brother and a sister are applied for 2 vacancies in the same post. The probability that brother will select is 1/7 and that of sister is 1/5, then the probability that (i) Both will select (ii) Only one will select, (iii) None of them will select:

(*a*) $\frac{1}{35}, \frac{10}{35}, \frac{24}{35}$ (*b*) $\frac{24}{35}, \frac{7}{35}, \frac{14}{35}$

(*c*) $\frac{3}{35}, \frac{24}{35}, \frac{11}{35}$ (*d*) $\frac{24}{35}, \frac{6}{35}, \frac{20}{35}$

[Dec. 2017]

Solution : (*a*)

Let A and B are events of selection of brother and sister respectively.

$\because$ Both events are independent.

(*i*) P(Both selected)

$= P(A \cap B) = P(A) \cdot P(B)$

$= \frac{1}{7} \cdot \frac{1}{5} = \frac{1}{35}$

(*ii*) P(Only one will be selected]

$= \{P(A) - P(A \cap B)\} + \{P(B) - P(A \cap B)\}$

$= \frac{1}{7} - \frac{1}{35} + \frac{1}{5} - \frac{1}{35}$

$= \frac{1}{5} + \frac{1}{7} - \frac{2}{35}$

$= \frac{7+5-2}{35} = \frac{10}{35}$

(*iii*) P(None of them will be selected)

$$= P(A^1) \cdot P(B^1) = \left(1-\frac{1}{7}\right) \cdot \left(1-\frac{1}{5}\right)$$

$$= \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$$

Q.51. If 4 letters are put randomly among the 4 envelopes then the probability that all are not put in correct envelopes:

(*a*) 1/24 (*b*) 1

(*c*) 23/24 (*d*) 9/24

[June 2018]

Solution : (*c*)

n(s) = 4! = 24

Let E = Events of putting letter in right envelop.

$\therefore$ n(E) = 1.1.1.1 = 1

$$p(E) = \frac{n(E)}{n(S)} = \frac{1}{24}$$

$$p(E^1) = 1 - 1/24 = 23/24$$

Q.52. Two broad divisions of probability are:

(*a*) Subjective probability and objective probability

(*b*) Deductive probability and mathematical probability

(*c*) Statistical probability and mathematical probability

(*d*) None of these

[May 2018]

Solution : (*a*)

Two broad divisions of Probability are

(i) Subjective Probability

(ii) Objective Probability

Q.53. The term "chance" and probability are synonyms:

(*a*) True (*b*) False

(*c*) Both (*d*) None

[May 2018]

Solution : (*a*)

Q.54. The theorem of Compound Probability states that for any two events A and B

(*a*) $P(A \cap B) = P(A) \times P(B/A)$

(*b*) $P(A \cup B) = P(A) \times P(B/A)$

(*c*) $P(A \cap B) = P(A) \times P(B)$

(*d*) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ***[CA (F) May 2018]***

Solution :

(*a*) The theorem of Compound Probability states that for two events A and B

$$P(A \cap B) = P(A) \times P(B/A)$$

Q.55. Variance of random variable x is given by

(*a*) $E(X-\mu)^2$

(*b*) $E[X-E(X)]^2$

(*c*) $E(X^2-\mu)$

(*d*) (*a*) or (*b*)

[May 2018]

Solution :

(*d*) Variance of a random variable x is given by

$$V(x) = E(X-\mu)^2$$

or

$$V(x) = E[X-E(X)]^2$$

Note : $\because \mu$ = E(X)

Q.56. What is the probability of having at least one 'six' appear in 3 throws of a perfect die?

(*a*) 5/6 (*b*) $(5/6)^3$

(*c*) $1-(1/6)^3$ (*d*) $1-(5/6)^3$

[May 2018]

Solution :

(*d*) For a die Probability of getting Six

$$P(A) = \frac{1}{6} = p$$

$$P(\overline{A}) = 1 - \frac{1}{6} = \frac{5}{6} = q$$

Here n = 3

P(getting at least '1' Six) = $P(X \geq 1)$

$$= 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - {}^3C_0 \cdot \left(\frac{1}{6}\right)^0 \cdot \left(\frac{1}{6}\right)^{3-0}$$

$$= 1 - 1 \times 1 \times \left(\frac{5}{6}\right)^3$$

$$= 1 - \left(\frac{5}{6}\right)^3$$

Q.57. Sum of all probabilities of mutually exclusive and exhaustive events is equal to

(*a*) 0 (*b*) 1/2

(*c*) 1/4 (*d*) 1

[Nov. 2018]

Solution : (*d*)

If events are mutually exclusive and exhaustive events then

Sum of all probabilities = 1.

Q.58. If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, and $P(A \cap B) = \frac{1}{4}$ then $P(A \cup B)$ is equal to

(*a*) $\frac{11}{12}$ (*b*) $\frac{7}{12}$ (*c*) $\frac{10}{12}$ (*d*) $\frac{1}{6}$

[Nov. 2018]

Solution : (*b*)

$$P(A \cap B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{6+4-3}{12} = \frac{7}{12}$$

Q.59. Two different dice are thrown simultaneously, then the probability, that the sum of two numbers appearing on the top of dice is 9 is

(*a*) $\frac{1}{9}$

(*b*) $\frac{8}{9}$

(*c*) $\frac{7}{9}$

(*d*) None of the above

[Nov. 2018]

Solution : (*a*)

$$n(S) = 6^2 = 36$$

E = {(3, 6), (4, 5), (5, 4), (6, 3)}

n(E) = 4

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Q.60. If $(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$ then $P(\overline{A}) + P(\overline{B})$ is equal to:

(*a*) 0.3 (*b*) 0.5

(*c*) 0.9 (*d*) 0.7

[Nov. 2018]

Solution : (*c*)

$$P(A \cap B) = P(A) + P(B) - P(A \cap B)$$

or ; 0.8 = P(A) + (B) – 0.3

or ; P(A) + P(B) = 0.8 + 0.3 = 1.1

$$\therefore P(\overline{A}) + P(\overline{B}) = 1 - P(A) + 1 - P(B)$$

$$= 2 - [P(A) + P(B)]$$

$$= 2 - 1.1 = 0.9$$

Q.61. The probability that a leap year has 53 Wednesday is

(*a*) $\frac{2}{7}$ (*b*) $\frac{3}{5}$

(*c*) $\frac{1}{7}$ (*d*) $\frac{2}{3}$

[Nov. 2018]

Solution : (*a*)

1 Leap year = 366 days = 52 weeks & 2 days

∴ S = {(Sun ; Mon) ; (Mon ; Tues) ; (Tues ; Wed) ; (Wed ; Thurs) ; (Thurs ; Fri) ; (Fri ; Sat) ; (Sat ; Sun)

∴ n(S) = 7

E = Event of getting wednesday

= {Tues ; Wed) ; (Wed ; Thurs)}

n(E) = 2

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$$

Q.62. A coin is tossed six times, then the probability of obtaining heads and tails alternatively is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{32}$

(*c*) $\frac{1}{64}$ (*d*) $\frac{1}{16}$

[Nov. 2018]

Solution : (*b*)

P(Head & Tail alternatively)

= P(H) . P(T) . P(H) . P(T) . P(H) . P(T).

+P(T) . P(H) . P(T) . P(H) . P(T) . P(H)

$$= \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6$$

$$= \frac{1}{64} + \frac{1}{64} = \frac{2}{64} = \frac{1}{32}$$

Q.63. Ram is known to hit a target in 2 out of 3 shots where as Shyam is known to hit the same target in 5 out of 11 shots. What is the probability that the target would be hit if they both try?

(*a*) $\frac{9}{11}$ (*b*) $\frac{6}{11}$

(*c*) $\frac{10}{33}$ (*d*) $\frac{3}{11}$

[Nov. 2018]

Solution : (*a*)

$$P(\text{Ram}) = P(R) = \frac{2}{3}$$

$$P(R^1) = 1 - \frac{2}{3} = \frac{1}{3}$$

and $P(\text{Shyam}) = P(S) = \frac{5}{11}$

$$\Rightarrow P(S^1) = 1 - \frac{5}{11} = \frac{6}{11}$$

∴ P (Target hit) = 1 – P (Target not hit)

$$= 1 - P(R^1) \cdot P(S^1) = 1 - \frac{1}{3} \cdot \frac{6}{11}$$

$$= 1 - \frac{2}{11} = \frac{9}{11}$$

Q.64. The probability that a student is not a swimmer is $\frac{1}{5}$, then the probability that out of five students four are swimmer is

(*a*) $\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$ (*b*) ${}^5C_1\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)$

(*c*) ${}^5C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$ (*d*) None

[Nov. 2018]

Solution : (*c*)

Given that

let q = Prob. that a student is not a swimmer $= \frac{1}{5}$

∴ P = Prob. (Swimmer) = 1 – *q* =

$$1 - \frac{1}{5} = \frac{4}{5}$$

n = 5

$$\therefore p(X = 4) = {}^5C_4 \cdot p^4 \cdot q^1$$

$$= {}^5C_4 \cdot \left(\frac{4}{5}\right)^4 \cdot \left(\frac{1}{5}\right)^1$$

Q.65. If Y ≥ x then mathematical expectation is

(*a*) E (X) > E (Y)

(*b*) E (X) ≤ E (Y)

(*c*) E (X) = E (Y)

(*d*) E (X) . E (Y) = 1

[June 2019]

Solution :

(*b*) If $y \geq x$

then $E(y) \geq E(x)$

$E(x) \leq E(y)$

Q.66. Two event A and B are such that they do not occurs simultaneously then they are called _____ events

(*a*) Mutually exhaustive

(*b*) Mutually exclusive

(*c*) Mutually independent

(*d*) Equally likely

[June 2019]

Solution : (*b*)

Q.67. According to Baye's theorem.

$$P(E_K / A) = \frac{P(E_K)P(A/E_K)}{\sum_{i\,1}^{n} P(E_1)P(A/E_1)}$$

(*a*) E_1, E_2 ..... are mutually exclusive

(*b*) $P(E/A_1), P(E/A_2)$, ... are equal to 1

(*c*) $P(A_1 / E), P(A_2 / E)$, ... are equal to 1

(*d*) A & E_i's are disjoint sets.

[June 2019]

Solution :

(*b*) According to Baye's Theorem

$$P(E_K / A) = \frac{P(E_K)P(A/E_K)}{\sum_{i=1}^{i=n} P(E_i)\cdot P(A/E_i)}$$

Where, E_1, E_2, E_3 ..... are Mutually Exclusive.

Q.68. If a coin is tossed 5 times then the probability of getting Tail and Head Occurs alternatively is

(*a*) $\frac{1}{8}$ (*b*) $\frac{1}{16}$

(*c*) $\frac{1}{32}$ (*d*) $\frac{1}{64}$

[June 2019]

Solution : (*b*)

P(getting tail and Head occurs Alternatives)

= P(HTHTH) or P(THTHT)

$$= \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$$

$$= \frac{1}{32}+\frac{1}{32} = \left(\frac{1+1}{32}\right)$$

$$= \frac{2}{32} = \frac{1}{16}$$

Q.69. When 2 - dice are thrown Simultaneously then the probability of getting at least one 5 is

(*a*) $\frac{11}{36}$ (*b*) $\frac{5}{36}$

(*c*) $\frac{8}{15}$ (*d*) $\frac{1}{7}$

[June 2019]

Solution :

(*a*) If two dice are thrown then sample space n(S) = 36

Events 'A' = getting at least one '5'

'A' = {(5,1) ; (5,2) ; (5,3) ; (5. 4) ; (5, 5) ; (5, 6)

(1,5) ; (2,5) ; (3,5) ; (4. 5) ; (6, 5)}

n (A) = 11

$$p(A) = \frac{n(A)}{n(S)} = \frac{11}{36}$$

Q.70. If two letters are taken at random from the word HOME, what is the Probability that none of the letters would be vowels:

(*a*) 1/6 (*b*) 1/2

(*c*) 1/3 (*d*) 1/4

[Dec. 2019]

Solution : (*a*)

$n(S) = {}^4C_2 = 6$

Let E = Event of getting 2 consonants *i.e.* not vowel.

$\therefore n(E) = {}^2C_2 = 1$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

Q.71. A bag contains 15 one rupee coins, 25 two rupee coins and 10 five rupee coins. If a coin is selected at random from the bag, then the probability of not selecting a one rupee coin is

(*a*) 0.30 (*b*) 0.70

(*c*) 0.25 (*d*) 0.20

[Dec. 2019]

Solution : (*b*)

Total coins = 15 + 25 + 10 = 50

$n(S) = {}^{50}C_1 = 50.$

Let E = Event of selecting no one rupee coin.

$\therefore n(E) = {}^{25+10}C_1 = {}^{35}C_1 = 35$

$$\therefore P(E) = \frac{n(E)}{n(s)} = \frac{35}{50} = 0.70.$$

Q.72. The chance of getting 7 or 11 in a throw of 2 dice is

(*a*) 7/9 (*b*) 5/9

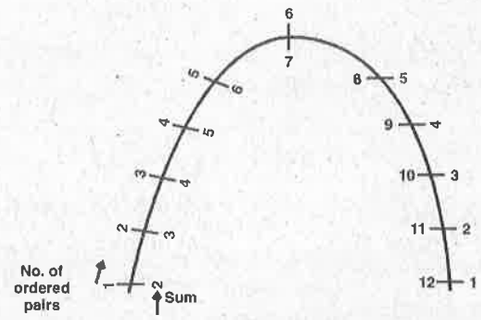
(*c*) 2/9 (*d*) None of these

[Dec. 2019]

Solution : (*c*)

Tricks:

Remember this diagram.



$n(s) = 6^2 = 36$

Let E = Sum 7 or 11

$n(E) = 6 + 2 = 8$

$P(E) = \frac{n(E)}{n(s)} = \frac{8}{36} = \frac{2}{9}$

Q.73. When 2 fair dice are thrown what is the probability of getting the sum which is a multiple of 3?

(*a*) 4/36 (*b*) 8/36 (*c*) 2/36 (*d*) 12/36

[Dec. 2020]

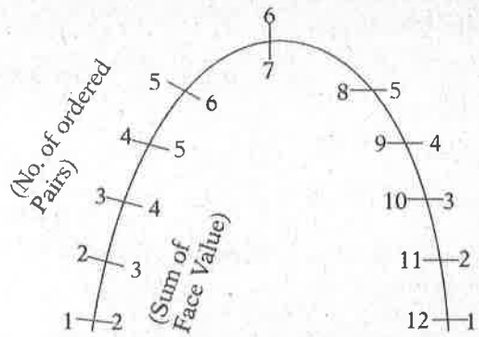
Solution :

$n(s)$ = sample space = $6^2 = 36$

E = The sum which is multiple of 3

$n(E)$ = sum (3) + sum (6) + sum (9) + sum (12)

= 2 + 5 + 4 + 1 = 12



$P(E) = \frac{n(E)}{n(S)}$

$= \frac{12}{36}$

(*d*) is correct.

Q.74. When two coins are tossed simultaneously the probability of getting atleast one tail?

(*a*) 1 (*b*) 0.75

(*c*) 0.5 (*d*) 0.25

[Dec. 2020]

Solution :

Sample Space = S = {HH; HT, TH; TT}

$n(S) = 4$

E = Event of at least 1 tail

= {HT, TH,TT}

$n(E) = 3$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{4} = 0.75$

(*b*) is correct

Q.75. When 3 dice are rolled simultaneously the probability of a number on the third die is greater than the sum of the numbers on two dice is

(*a*) 12/216 (*b*) 36/216

(*c*) 48/216 (*d*) 60/216

[Dec. 2020]

Solution :

Sample Space = $n(S) = 6^3 = 216$

E = Event that Number on third die is greater that sum of the number on two dice.

$\therefore n(E) = 3[n(1 > \text{sum}) + n(2 > \text{sum}) + n(3 > \text{sum}) + n(4 > \text{sum}) + n(5 > \text{sum}) + n(6 > \text{sum})]$

$= 3[0 + 0 + 1 + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4)] = 3 \times [1 + 3 + 6 + 10]$

$= 20 \times 3 = 60$

$\therefore P(E) = \frac{n(E)}{n(s)} = \frac{60}{216}$

(*d*) is correct

Q.76. If A speaks 75% of truth and B speaks 80% of truth. In what percentage both of them likely contradict with each other in narrating the same questions?

(*a*) 0.60 (*b*) 0.45

(*c*) 0.65 (*d*) 0.35

[Dec. 2020]

Solution :

Let A = Event of Speaking truth by A

B = Event of Speaking truth by B

$\therefore P(A) = 75\% = 0.75; P(A') = 1 - P(A) = 1 - 0.75 = 0.25$

$P(B) = 80\% = 0.80;$

$P(B') = 1 - P(B) = 1 - 0.80 = 0.20$

$P(\text{Contradict}) = P(A)P(B') + P(A')P(B)$

$= 0.75 \times 0.20 + 0.25 \times 0.80$

$= 0.35 = 35\%$

(*d*) is correct

Q.77. An event that can be sub-divided into further events is called as.

(*a*) A composite event

(*b*) A complex event

(*c*) A mixed event

(*d*) A simple event

[Jan. 2021]

Solution : (*a*) is correct

An event that can be sub-divided into further events is called composite or compound Events.

Example : In the experiment of throwing a die: The event of getting an even number is a composite number. This event can be further sub-divided or broken down into 3 simpler event

1. The event of getting 2.
2. The event of getting 4 and
3. The event of getting 6.

Q.78. Three identical and balanced dice are rolled. The probability that the same number will appear on each of them is.

(*a*) $\frac{1}{6}$ (*b*) $\frac{1}{18}$

(*c*) $\frac{1}{36}$ (*d*) $\frac{1}{24}$

[Jan. 2021]

Solution : (*c*) is correct

Sample space = n(s) = $6^3 = 216$.

Let A = Event of getting same number on all 3 dice

= {(1, 1, 1); (2, 2, 2); (3, 3, 3); ...; (6, 6, 6)}

$\therefore \quad n(A) = 6.$

$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{6}{216} = \frac{1}{36}$

Q.79. A basket contains 15 white balls, 25 red balls and 10 blue balls. If a ball is selected at random, the probability of selecting not a white ball.

(*a*) 0.20 (*b*) 0.25

(*c*) 0.60 (*d*) 0.70

[Jan. 2021]

Solution : (*d*) is correct

Total balls = 15 + 25 + 10 = 50

Sample space of getting 1 ball

$= n(S) = 50c_1 = 50$

Let A = Event of getting not a white ball means event of getting a ball out of 10 blue and 25 red balls

$\therefore \quad n(A) = 35c_1 = 35$

$\therefore \quad P(A) = \frac{n(A)}{n(S)}$

$= \frac{35}{50} = 0.70$

Q.80. Two dice are thrown simultaneously. The probability of a total score of 5 from the outcomes of dice is.

(*a*) $\frac{1}{18}$ (*b*) $\frac{1}{12}$

(*c*) $\frac{1}{9}$ (*d*) $\frac{2}{5}$

[Jan. 2021]

Solution : (*c*) is correct.

Sample space = $n(S) = 6^2 = 36$

Let A = Event of getting numbers on both dice such that their sum is equal to 5.

= {(1,4), (2, 3), (3, 2), (4, 1)}

n(A) = 4

$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

Q.81. Two dice are thrown simultaneously. The probability of a total score of 5 from the outcomes of dice is.

(*a*) $\frac{1}{18}$ (*b*) $\frac{1}{12}$

(*c*) $\frac{1}{9}$ (*d*) $\frac{2}{5}$

[July 2021]

Solution : (*c*) is correct.

Sample space = $n(S) = 6^2 = 36$

Let A = Event of getting numbers on both dice such that their sum is equal to 5.

= {(1, 4), (2, 3), (3, 2), (4, 1)}

n(A) = 4

$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

Q.82. A biased coin is such that the probability of getting a head is thrice the probability of getting a tail. If the coin is tossed 4 times, what is the probability of getting a head all the times?

(*a*) 2/5 (*b*) 81/128

(*c*) 81/256 (*d*) 81/64

[July 2021]

Solution : (*c*) is correct. Given n = 4

Let probability of getting head = p

and probability of getting tail = q.

Given p = 3q.

$\because \quad p + q = 1$

or $\quad 3q + q = 1$

or $\quad 4q = 1$

or $\quad q = \frac{1}{4}$

$\therefore \quad p = 3q = 3 \times \frac{1}{4} = \frac{3}{4}$

$p(X = 4) = {}^4c_4.p^4.q^0$

$= 1.\left(\frac{3}{4}\right)^4.1 = \frac{81}{256}$

Q.83. If there are 16 phones, 10 of them are Android and 6 of them are of Apple, then the probability of 4 randomly selected phones to include 2 Android and 2 Apple phone is

(*a*) 0.47 (*b*) 0.51

(*c*) 0.37 (*d*) 0.27

[July 2021]

Solution : (*c*) is correct.

n(*s*) = Sample Space

$= {}^{16}C_4 = \frac{\lfloor 16}{\lfloor 4 \lfloor 12} = 1820$

Let E = Event of getting 2 Android and 2 Apple phones

$n(E) = {}^{10}C_2 \times {}^6C_2 = 45 \times 15 = 675$

$\therefore p(E) = n(E)\ \frac{n(E)}{n(S)} = \frac{675}{1820} = 0.37$

Q.84. If there are 48 marbles marked with numbers 1 to 48, then the probability of selecting a marble having the number divisible by 4 is

(*a*) 1/2 (*b*) 2/3

(*c*) 1/3 (*d*) 1/4

[July 2021]

Solution : (*d*) is correct

$n(s)$ = Sample Space = ${}^{48}C_1 = 48$

Let E = Event of getting a number divisible by 4

$= \{4, 8, 12, \ldots 48\}$

$n(E) = 12$

$$p(E) = \frac{n(E)}{n(S)} = \frac{12}{48} = \frac{1}{4}$$

Q.85. If in a class, 60% of the student study Mathematics and Science and 90% of the student study Science, then the probability of a student studying Mathematics given that he/she is already studying Science is

(*a*) 1/4 (*b*) 2/3

(*c*) 1 (*d*) 1/2

[July 2021]

Solution : (*b*) is correct.

Let A = Event of studying Mathematics

B = Event of Studying Science.

Given

$p(A \cap B) = 60\% = 0.60$

and $p(B) = 90\% = 0.90$

$$p\left(\frac{A}{B}\right) = \frac{p(A \cap B)}{p(B)} = \frac{0.60}{0.90}$$

$$= \frac{6}{9} = \frac{2}{3}$$

Q.86. A bag contains 7 Blue and 5 Green bails One ball is drawn at random. The probability of getting a Blue ball is

(*a*) 5/12 (*b*) 12/35

(*c*) 7/12 (*d*) 0

[July 2021]

Solution : (*c*) is correct.

Let Sample Space = $n(S) = {}^{12}C_1 = 12$

Let E = Event of getting a blue ball

$\therefore \quad n(E) = {}^{7}C_1 = 7$

$$\therefore \quad p(E) = \frac{n(E)}{n(S)} = \frac{7}{12}$$

Q.87. The probability that a football team loosing a match at Kolkata is 3/5 and winning a match at Bengaluru is 6/7, the probability of the team winning at least one match is

(*a*) 3/35 (*b*) 18/35

(*c*) 32/35 (*d*) 17/35

[July 2021]

Solution : (*c*) is correct.

Let p(A′) = probability of loosing match in Kolkata = $\frac{3}{5}$

$$\therefore \quad p(A) = 1 - p(A') = 1 - \frac{3}{5} = \frac{2}{5}$$

Let p(B) = proof. of winning match in Bengaluru.

$$= \frac{6}{7}$$

$$\therefore \quad p(B') = 1 - p(B) = 1 - \frac{6}{7} = \frac{1}{7}$$

p(At least one match winning)

$= 1 - p$(Loosing both match)

$= 1 - p(A').\ p(B')$

$$= 1 - \frac{3}{5} \cdot \frac{1}{7} = \frac{35-3}{35} = \frac{32}{35}$$

Q.88. For any two dependent events A and B, P(A) = 5/9 and P(B) = 6/11 and P(A∩B) = 10/33. What are the values of P(A/B) and P(B/A)?

(*a*) 5/9, 6/11 (*b*) 5/6, 6/11

(*c*) 1/9, 2/9 (*d*) 2/9, 4/9

[Dec. 2021]

Solution : (*a*)

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{10}{33}}{\frac{6}{11}}$$

$$= \frac{\cancel{10}^{5}}{\cancel{33}_{3}} \times \frac{\cancel{11}}{\cancel{6}_{3}} = \frac{5}{9}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{10}{33}}{\frac{5}{9}}$$

$$= \frac{\cancel{10}^{2}}{\cancel{33}_{11}} \times \frac{\cancel{9}^{3}}{5} = \frac{6}{11}$$

Q.89. Which of the following pair of events E and F are mutually exclusive?

(*a*) E = {Ram's age is 13} and F = {Ram is studying in a college}

(*b*) E = {Sita studies in a school} and F = {Sita is a play back singer}

(*c*) E = {Raju is an elder brother in a family} and F = {Raju's father has more than one son}

(*d*) E = {Banu studied B.A. English literature} and F = {Banu can read English novels}

[Dec. 2021]

Solution : (*a*)

Note : 13 years old student cannot get admission in college.

Q.90. Four unbiased coins are tossed simultaneously. The expected number of heads is:

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

[Dec. 2021]

Solution : (*b*)

Given

n = 4; P = Probability of getting head

$= \frac{1}{2}$

∴ Expected No. of heads

= Mean of getting head

$= np = 4 \times \frac{1}{2} = 2$

Q.91. Assume that the probability for rain on a day is 0.4. An umbrella salesman can earn Rs. 400 per day in case of rain on that day and will lose Rs. 100 per day if there is no rain. The expected earnings (in Rs.) per day of the salesman is

(*a*) 400 (*b*) 200

(*c*) 100 (*d*) 0

[Dec. 2021]

Solution : (*c*)

Rain (x)	P	Px
400	0.4	160
−100	0.6	− 60
		$\sum Px = 100$

∴ Expected earning = $\sum Px = 100$

Q.92. The probability distribution of a random variable *x* is given below:

x:	1	2	4	5	6
P:	0.15	0.25	0.2	0.3	0.1

What is the standard deviation of x?

(*a*) 1.49 (*b*) 1.56

(*c*) 1.69 (*d*) 1.72

[Dec. 2021]

Solution : (*c*)

Variance $= \sigma^2 = \Sigma Px^2 - (\Sigma Px)^2$

Calculator Tricks

*$\Sigma Px^2 = 1^2 \times 0.15$ = button (Press)

4×0.25 = button (Press)

16×0.2 = button (Press)

25×0.3 = button (Press)

36×0.1 = button (Press) then GT button

= 15.45 (M+) button

*For ΣPx = 1×0.15 = button (Press)

2×0.25 = button (Press)

4×0.2 = button (Press)

5×0.3 = button (Press)

6×0.1 = button (Press) n GT button

$= 3 \times 55 \times n$ = button

(M–) button.

Finally press MRC button 2 times

Then Press √ button

we get SD = σ = 1.687

= 1.69

Q.93. In a group of 20 males and 15 females 12 males and 8 females are service holders. What is the probability that a person selected at random from the group is a service holder given that the selected person is a male?

(*a*) 0.40 (*b*) 0.60

(*c*) 0.45 (*d*) 0.55

[Dec. 2021]

Solution : (*b*)

	Males	Females
Total	20	15
Service holder	12	8
Non-Service holder	8	7

Probability of male service holder

$$= \frac{12}{20} = 0.60$$

Q.94. There are 3 boxes with the following composition:

Box I : 7 Red + 5 White + 4 Blue balls

Box II : 5 Red + 6 White + 3 Blue balls

Box III : 4 Red + 3 White + 2 Blue balls

One of the boxes is selected at random and a ball is drawn from it.

What is the probability the drawn ball is red?

(*a*) 1249/3024 (*b*) 1247/3004

(*c*) 1147/3024 (*d*) 1/2

[Dec. 2021]

Solution : (*a*)

Probability of selection of each box $= \frac{1}{3}$

Prob. of getting red ball

$$= P(\text{Box I}) - P\left(\frac{\text{Red}}{\text{Box I}}\right) + P(\text{Box} - \text{II}).P\left(\frac{\text{Red}}{\text{Box II}}\right)$$

$$+ P(\text{Box} - \text{III}).P\left(\frac{\text{Red}}{\text{Box III}}\right)$$

$$= \frac{1}{3}.\frac{7}{16} + \frac{1}{3}.\frac{5}{14} + \frac{1}{3}.\frac{4}{9}$$

$$= \frac{1}{3}\left[\frac{7}{16} + \frac{5}{14} + \frac{4}{9}\right]$$

= 0.41302910052

= 0.4130

$$\text{GBC} \quad A = \frac{1249}{3024} = 0.41302910052$$

$\therefore$ (*a*) is correct.

Q.95. For a probability distribution, probability is given by, $P(X) = \frac{X_i}{k}$; $X_i = 1, 2, \ldots\ldots 9$. The value of *k* is:

(*a*) 55 (*b*) 9

(*c*) 45 (*d*) 81

[Dec. 2021]

Solution : (*b*)

$$P(x) = \frac{x_i}{K} = \frac{n(E)}{n(S)}$$

Here $n(S) = K = {}^9C_1 = 9$

Q.96. If P (A) = 0.3, P (B) = 0.8 and P (B/A) = 0.5. Find P (A∪B).

(*a*) 0.7 (*b*) 0.95

(*c*) 0.60 (*d*) 0.59

[June 2022]

Solution : $\because P(B/A) = \frac{P(A \cap B)}{P(A)} = 0.5$

or $P(A \cap B) = 0.5\ P(A)$

$= 0.5 \times 0.3 = 0.15$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.8 - 0.15 = 0.95$

$\therefore$ (*b*) is correct.

Q.97. What is the chance that a leap year selected at random will contain 53 Fridays?

(*a*) 3/7 (*b*) 1/7

(*c*) 2/7 (*d*) 4/7

[June 2022]

Solution : (*c*) is correct

No. of days in a leap year = 366

= 52 weeks & 2 days

Means 52 Fridays will be sure but in rest two days Friday lies or not.

So sample Space for those 2 days

S = {(Sun, Mon); (Mon; Tues);

(Tues; wed); (Wed; Thurs); (Thurs; Fri); (Fri; Sat); (Sat; Sun)}

$\therefore n(s) = 7$

E = Event of getting Friday

= {(Thurs; Fri); (Fri; Sat.)}

$\therefore n(E) = 2$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$$

Q.98. Two balanced dice are rolled. The probability of getting 1 in at lest one dice is x/36 where x is

(*a*) 12 (*b*) 1

(*c*) 11 (*d*) 2

[June 2022]

Solution : (*c*) is correct

Sample Space = $n(s) = 6^2 = 36$

Event E = {(1,1), (1,2), (1,3),....; (1,6); (6,1), (5,1), (4,1), (3,1), (2,1)}

$\therefore n(E) = 11$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{11}{36} = \frac{x}{36}$$

$\therefore x = 11$

Q.99. Thirty balls are serially numbered and placed in a bag. Find chance that the first ball drawn is a multiple of 3 or 5.

(*a*) 8/15 (*b*) 2/15

(*c*) 1/2 (*d*) 7/15

[June 2022]

Solution : $n(s) = {}^{30}c_1 = 30$

Let A = Event of getting No. multiple of 3

= {3, 6, 9, 12, 15, 18, 21, 24, 27, 30}

n(A) =10

B = Event of getting No. multiple of 5

= {5, 10, 15, 20, 25, 30}

n(B) = 6

$A \cap B$ = Multiple of 3 & 5 both

= LCM of 3 & 5 = 15

$A \cap B = \{15; 30\}$

$n(A \cap B) = 2$

$$P(A \cup B) = \frac{n(A \cup B) - n(A) + n(B) - n(A \cap B)}{n(s)}$$

$$= \frac{10+6-2}{30} = \frac{14}{30} = \frac{7}{15}$$

(*d*) is correct.

Q.100. The odds in favour of an event A is 2 : 3 and odds against an event B is 6 : 4 the probability that only one of A and B occurs is y/25 where y is

(*a*) 12 (*b*) 15

(*c*) 18 (*d*) 9

[June 2022]

Solution : Given:

$$\frac{P(A)}{P(A^1)} = \frac{2}{3} \Rightarrow P(A) = \frac{2}{2+3} = \frac{2}{5}$$

$$P(B) = \frac{3}{2+3} = \frac{3}{5}$$

And $\frac{P(B^1)}{P(B)}$

$$= \frac{6}{4} = \frac{3}{2} \Rightarrow P(B) = \frac{2}{3+2} = \frac{2}{5}$$

$$P(B^1) = \frac{3}{3+2} = \frac{3}{5}$$

Events should be independent

So, P (Only one of A & B occurs)

$= P(A).P(B^1) + P(A^1).P(B)$

$$= \frac{2}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5}$$

$$= \frac{6+6}{25} = \frac{12}{25} = \frac{y}{25} \text{ (given)}$$

$\therefore y = 12$

(*a*) is correct

Q.101. The odds in favour of event A, in a trial, is 3:1. In a three independent trials, the probability of no occurrence of the event A is

(*a*) 1/64 (*b*) 1/32

(*c*) 1/27 (*d*) 1/8

[June 2022]

Solution : Given:

n = No. of trials = 3

Odds in Favour of event A

$$\frac{P(A)}{P(A')} = \frac{3}{1}$$

$$= P(A) = \frac{3}{3+1} = \frac{3}{4}$$

Here $P = P(A) = \frac{3}{4}$

Then $q = P(A^1) = 1 - P(A) = 1 - \frac{3}{4} = \frac{1}{4}$

$\therefore P(x = 0) = 3c_0.p^0q^3$

$$= 1.1.\left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

$\therefore$ (*a*) is correct

Q.102. A machine is made of two parts A and B The manufacturing process of each part is such that probability of defective in part A is 0.08 and that B is 0.05. What is the probability that the assembled part will not have any defect?

(*a*) 0.934 (*b*) 0.864

(*c*) 0.85 (*d*) 0.874

[Dec. 2022]

Solution : *Given:*

$P(A') = 0.08 \Rightarrow P(A) = 1 - P(A')$

$= 1 - 0.08 = 0.92$

and $P(B') = 0.05 \Rightarrow P(B) = 1 - P(B')$

$= 1 - 0.05 = 0.95$

P(No defective) = P(A).P(B)

$= 0.92 \times 0.95 = 0.874$

$\therefore$ (*d*) is correct

Q.103. If $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$ then $P\left(\frac{B}{A}\right)$ is :

(*a*) $\frac{1}{6}$ (*b*) $\frac{4}{9}$

(*c*) $\frac{1}{2}$ (*d*) $\frac{1}{8}$

[Dec. 2022]

Solution : $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{1}{3} + \frac{3}{4} - \frac{11}{12}$$

$$= \frac{4+9-11}{12} = \frac{2}{12} = \frac{1}{6}$$

$$\therefore P(B/A) = P\frac{(A \cap B)}{P(A)} = \frac{1/6}{1/3} = \frac{1}{6} \times \frac{3}{1} = \frac{1}{2}$$

$\therefore$ (*c*) is correct

Q.104. The probability that a leap year has 53 Monday is:

(a) $\frac{1}{7}$ (b) $\frac{2}{3}$

(c) $\frac{2}{7}$ (d) $\frac{3}{5}$

[Dec. 2022]

Solution : A Leap year = 366 days

= 52 weeks & 2 days

Means 52 Mondays (Sure) but doubts on rest 2 days

So Sample Space = S = {(Sun, Mon); (Mon, Tues); (Tues, Wed); (Wed, Thurs); (Thurs, Fri); (Fri, Sat); (Sat, Sun)}

$\therefore \quad n(s) = 7$

Let E = Event of getting Monday

= {(Sun, Mon); (Mon, Tues)}

$\therefore \quad n(E) = 2$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$$

$\therefore$ (c) is correct

Q.105. Suppose A and B are two independent events with probabilities P(A) # 0 and P(B) # 0. Let A' and B' be their complements. Which one of the following statements is FALSE?

(a) $P(A \cap B) = P(A)P(B)$

(b) $P(A / B) = P(A)$

(c) $P(A \cup B) = P(A) + P(B)$

(d) $P(A' \cap B') = P(A')P(B')$

[Dec. 2022]

Solution : $\therefore$ A and B are two Independent variables.

$\therefore \quad P(A \cap B) = P(A).P(B) \neq 0$

Because P(A); P(B) $\neq 0$

So $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\neq P(A) + P(B)$

$\therefore$ (c) is correct

Q.106. The Theorem of compound Probability states that for any two events A and B

(a) $P(A \cap B) = P(A) \times P(B / A)$

(b) $P(A \cup B) = P(A) \times P(B / A)$

(c) $P(A \cap B) = P(A) \times P(B)$

(d) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

[Dec. 2022]

Solution : (a) $P(A \cap B) = P(A).P(B/A)$

Q.107. If a number is selected at random from the first 50 natural numbers, what will be the probability that the selected is a multiple of 3 and 4?

(a) 5/50 (b) 2/25

(c) 3/50 (d) 4/25

[Dec. 2022]

Solution : Sample space = $n(S) = {}^{50}C_1 = 50$

Let E = Event of getting a No. divisible by 3 & 4

i.e. LCM of 3 & 4 = 12

E = {12, 24, 36, 48}

$n(E) = {}^{4}C_1 = 4$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{50} = \frac{2}{25}$$

$\therefore$ (b) is correct

Q.108. If three coins are tossed simultaneously, what is the probability of getting two heads together.

(a) 1/4 (b) 1/8

(c) 5/8 (d) 3/8

[Dec. 2022]

Solution : Sample space = $n(S) = 2^3 = 8$

3 coins means 3 heads are considered.

Let E = Event of getting 2 heads

$\therefore \quad n(E) = {}^{3}C_2 = 3$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

(d) is correct

Q.109. Company 'A' produces 10% defective products, company 'B' produces 20% defective products and company 'C' produces 5% defective products. If choosing a company is an equally likely event, what is probability that product chosen is free from defect?

(a) 0.88 (b) 0.80

(c) 0.79 (d) 0.78

[June 2023]

Solution : Selection of Company A = $\frac{1}{3}$

" " " B = $\frac{1}{3}$

& " " " C = $\frac{1}{3}$

Let E = Event of Selecting Non-defective Product

$$\therefore p(E/A) = 1 - \frac{10}{100} = 0.9$$

$$p(E/B) = 1 - \frac{20}{100} = 0.8$$

$$p(E/C) = 1 - \frac{5}{100} = 0.95.$$

$\therefore$ Probability of getting Non-defective Product =

$p(A) \times p(E/A) + p(B) \times p(E/B) + p(C) \times p(E/C)$

$$= \frac{1}{3} \times 0.9 + \frac{1}{3} \times 0.8 + \frac{1}{3} \times (0.95)$$

$$= \frac{1}{3}(0.9 + 0.8 + 0.95) = 0.88333...$$

$= 0.88$

$\therefore$ (a) is correct.

Q.110. For any two events 'A' and 'B' it is known that P(A) = 2/3, P(B) = 3/8 and P(A $\cap$ B) = 1/4, then the events A and B are:

(a) Mutually exclusive and Independent

(b) Mutually not exclusive and Independent

(c) Mutually exclusive but not independent

(d) Neither independent nor mutually exclusive

[June 2023]

Solution : $P(A \cap B) = P(A) \times P(B)$

$$= \frac{1}{4} = \frac{2}{3} \times \frac{3}{8}$$

$$\Rightarrow \frac{1}{4} = \frac{1}{4} \text{ (True)}$$

$\therefore$ (b) is correct.

Q.111. The probability that a four digit number comprising the digits 2, 5, 6 and 7, without repetition of digits, would be divisible by 4 is:

(*a*) 1/2 (*b*) 3/4

(*c*) 1/4 (*d*) 1/3

[June 2023]

Solution : Total No. of Numbers n(s) = $\lfloor 4 = 24$

Note: If last two digits of a Number is divisible by 4 then that whole No. is divisible by 4.

Number ending with 52; 56; 72; 76; are divisible by 4.

So n(E) = Event

$$= \underset{\text{Fix}}{\lfloor 2(52)} + \underset{\text{Fix}}{\lfloor 2(56)} + \underset{\text{Fix}}{\lfloor 2(72)} + \underset{\text{Fix}}{\lfloor 2(76)}$$

$= 2 \times 1 + 2 \times 1 + 2 \times 1 + 2 \times 1$

$= 8$

$$\therefore p(E) = \frac{n(E)}{n(S)} = \frac{8}{24} = \frac{1}{3}$$

$\therefore$ (*d*) is correct.

Q.112. Four persons are chosen at random from a group of 3 men, 2 women and 4 children. The probability that exactly 2 of them are children, is:

(*a*) $\frac{10}{21}$ (*b*) $\frac{1}{12}$

(*c*) $\frac{1}{5}$ (*d*) $\frac{1}{9}$

[June 2023]

Solution : Total persons = 3 + 2 + 4 = 9

Sample Space = $n(S) = {}^9C_4 = 126$

Let E = Event of getting exactly 2 children.

$n(E) = {}^4C_2 \times {}^5C_2 = 6 \times 10 = 60$

$$\therefore p(E) = \frac{n(E)}{n(S)} = \frac{60}{126} = \frac{10}{21}$$

(*a*) is correct.

Q.113. If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(A/B) = \frac{1}{6}$, the probability $P\left(\frac{B}{A}\right)$ is:

(*a*) $\frac{1}{8}$ (*b*) $\frac{1}{4}$

(*c*) $\frac{3}{8}$ (*d*) $\frac{1}{2}$

[June 2023]

Solution : $p(A \cap B) = p(B) \times p(A/B)$

$$= \frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$$

$$\therefore p(B/A) = \frac{p(A \cap B)}{p(A)} = \frac{\frac{1}{24}}{\frac{1}{3}}$$

$$\frac{1}{24} \times \frac{3}{1} = \frac{1}{8}$$

$\therefore$ (*a*) is correct.

24 CHAPTER PROBABILITY (THEORETICAL) DISTRIBUTION

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. The Variance of standard normal distribution is

(*a*) 1 (*b*) μ

(*c*) σ^2 (*d*) 0

[June 2010]

Solution : (*c*)

Q.2. In Binomial distribution, n = 9 and P = 1/3 what is the value of variance:

(*a*) 8 (*b*) 4

(*c*) 2 (*d*) 16

[June 2010]

Solution : (*c*) q = 1 – p = 1 – 1 / 3 = 2 / 3

So, Variance = $npq = 9.\frac{1}{3}.\frac{2}{3} = 2$

Q.3. If standard deviation of a poisson distribution is 2, then its

(*a*) Mode is 2

(*b*) Mode is 4

(*c*) Modes are 3 and 4

(*d*) Modes are 4 and 5

[Dec. 2010]

Solution : (*c*) $\because$ S.D = 2 $\Rightarrow$ Variance = $\sigma^2 = 4$

$\because$ In poisson distribution, Mean = variance = 4 (Integer)

So; it is bi-modal

$\therefore$ Modes are m and (m-1)

i.e., Mo = 4 and 3,

Q.4. The area under the Normal curve is

(*a*) 1 (*b*) 0

(*c*) 0.5 (*d*) –1

[Dec. 2010]

Solution : (*a*)

Q.5. For a normal distribution $N(\mu, \sigma^2)$, $P(\mu - 3\sigma < x < \mu + 3\sigma)$ is equal to

(*a*) 0.9973 (*b*) 0.9546

(*c*) 0.9899 (*d*) 0.9788

[Dec. 2010]

Solution : (*a*)

$\because$ Area in $P(\mu - 36 < x < \mu + 36)$ = 0.9973

Q.6. If for a Binomial distribution B (n,p) the mean = 6 and Variance = 2 then 'p' is

(*a*) 2/3 (*b*) 1/3

(*c*) 3/5 (*d*) 1/4

[Dec. 2010]

Solution : (*a*) Mean = 6 = np

& Variance = 2 = npq $\Rightarrow$ 6q = 2

$\Rightarrow$ q = 1 / 3

So, $p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow p = \frac{2}{3}$

Q.7. If the inflexion points of a Normal Distribution are 6 and 14. Find its Standard Deviation?

(*a*) 4 (*b*) 6

(*c*) 10 (*d*) 12

[June 2011]

Solution : (*a*)

Upper point of inflexion

$= \mu + \sigma = 14$ ..... (i)

& Lower point of inflexion

$= \mu - \sigma = 6$ .....(ii)

Eqn . (i) – (ii), we get

$2\sigma = 14 - 6 = 8$; So, $\sigma = 4$

Hence, S.D $(\sigma) = 4$

Q.8. In a Binomial Distribution, if mean is k-times the variance, then the value of 'k' will be_____.

(*a*) p (*b*) q

(*c*) 1 – p (*d*) $\frac{1}{1-P}$

[June 2011]

Solution : (*d*)

We know that in Binomial Distribution:

Mean = np & Variance = npq

From question

Mean = k(Variance) $\Rightarrow$ np = K.(npq) $\Rightarrow$ kq = 1

$\therefore$ K = 1/q $\therefore K = \frac{1}{1-p}$

Q.9. The mean of Binomial distribution is 20 and Standard deviation is 4 then;

(*a*) n = 100, p = 1/5, q = 4/5

(*b*) n = 50, p = 2/5, q = 2/5

(*c*) n = 100, p = 2/5, q = 4/5

(*d*) n = 100, p = 1/5, q = 3/5

[Dec. 2011]

Solution : (*a*)

Given, Mean = 20; S.D = 4

$\Rightarrow$ np = 20; Variance = npq = $(4)^2$

$\Rightarrow$ npq = 16

$\Rightarrow$ 20q = 16; $q = \frac{4}{5}$

Hence $p = 1 - q = 1 - \frac{4}{5}$; $p = \frac{1}{5}$

So, $np = n \times \frac{1}{5} = 20 \Rightarrow n = 20 \times 5 = 100$

Q.10. A Company has two cars which it hires out during the day. The number of Cars demanded with mean 1.5. Then percentage of days on which only one car was in demand is equal to

(*a*) 23.26 (*b*) 33.47

(*c*) 44.62 (*d*) 46.40

[Dec. 2011]

[Given Exp. (–1.5) = 0.2231]

Solution : (*b*)

Given; mean = m = 1.5

Formula , $P(x) = \frac{e^{-m}.m^x}{x!}$

So, $P(X = 1) = \frac{e^{-1.5}.(1.5)^1}{1.!} = \frac{0.2231 \times 1.5}{1}$

$= 0.33465 = 0.3347 = 33.47\ \%$

Q.11. The binomial distribution with mean 3 & variance 2 is:

(*a*) $\left(\frac{2}{3}+\frac{1}{4}\right)^{2-9}$ (*b*) $\left(\frac{2}{6}+\frac{1}{6}\right)^{2-9}$

(*c*) $\left(\frac{2}{3}+\frac{1}{3}\right)^{9}$ (*d*) $\left(\frac{2}{5}+\frac{1}{5}\right)^{2-9}$

[Dec. 2011]

Solution : (*c*) Given mean = np = 3

Variance = npq = 2

3q = 2; So, q = 2 / 3; Hence $p = 1 - \frac{2}{3} = \frac{1}{3}$

Since, $np = n \times \frac{1}{3} = 3$; So, n = 9

The binomial distribution is $(q+p)^n = \left[\frac{2}{3}+\frac{1}{3}\right]^9$

Q.12. For binomial distribution

(*a*) Variance < Mean (*b*) Variance = Mean

(*c*) Variance > Mean (*d*) None of the above

[June 2012]

Solution : (*a*) For Binomial distribution

npq < np $\Rightarrow$ Variance < Mean

Q.13. If X is a Poisson variate and E(x) = 1, then P (x > 1) is

(*a*) $1 - \frac{e^{-1}}{2}$ (*b*) $1 - e^{-1}$ (*c*) $1 - 2e^{-1}$ (*d*) $1 - \frac{5}{2}e$

[June 2012]

Solution : (*c*) Since, $P(x) = \frac{e^{-m}m^x}{x!}$; $E(X) = m = 1 = \text{mean}$

$\therefore P(x > 1) = 1 - P(x < 1)$

$= 1 - [P(x = 0) + P(X = 1)]$

$= 1 - \left[\frac{e^{-1}.1^0}{0!} + \frac{e^{-1}.1^1}{1!}\right]$

$= 1 - [e^{-1} + e^{-1}] = 1 - 2e^{-1}$

Q.14. The mean and the variance of a random variable X having the probability density function P(X = x) = exp. $\{-(x-4)^2\}/\sqrt{\pi}, -\infty < x < \infty$ is

(*a*) $4, \frac{1}{2}$ (*b*) $4, \frac{1}{\sqrt{2}}$ (*c*) 2,2 (*d*) $2, \frac{1}{2}$

[June 2012]

Solution : (*a*) Normal distribution - density function is:

$$P(X = x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu)^2}{\sigma})^2}, -\infty < X < \infty$$

Given equation:

$$P(X = x) = \frac{1}{\sqrt{\pi}} e^{-(x-4)^2} = \frac{1}{\frac{1}{\sqrt{2}}\sqrt{2\pi}} . e^{-\frac{1}{2}\left(\frac{x-4}{1/\sqrt{2}}\right)^2}$$

Comparing given function with the standard form, we get Mean (u) = 4

$$S.D\ (\sigma) = \frac{1}{\sqrt{2}}$$

$$\therefore Variance(\sigma^2) = \frac{1}{2}$$

Q.15. In a Normal Distribution

(*a*) The first and second quartile are equidistant from median

(*b*) The second and third quartiles are equidistant from the median

(*c*) The first and third quartiles are equidistant from the median

(*d*) None of the above

[Dec. 2012]

Solution : (*c*) In a Normal Distribution: "The first and third quartiles are equidistant from the median".

Q.16. If a parameters of a binomial distribution are n and p then, this distribution tends to a poisson distribution when

(*a*) $n \to \infty, p \to 0$

(*b*) $p \to 0, np = \lambda$

(*c*) $p \to \infty, np = \lambda$

(*d*) $n \to \infty, p \to 0, np = \lambda$ ***[Dec. 2012]***

where 'λ' is a finite constant

Solution : (*d*)

Q.17. If a random variable x follows Poisson distribution such that E(x)=30, then the variance of the distribution is

(*a*) 7 (*b*) 5

(*c*) 30 (*d*) 20

[Dec. 2012]

Solution : (*c*) In Poisson distribution,

Mean = Variance

E(X) = 30 = Mean ; So, Variance = 30

Q.18. In a normal distribution quartile deviation is 6 the standard deviation will be

(*a*) 4 (*b*) 9

(*c*) 7.5 (*d*) 6

[Dec. 2012]

Solution : (*b*) In Normal distribution

4 S.D. = 6. Q.D.

$$S.D. = \frac{6}{4} Q.D. = \frac{6}{4} \times 6 = 9$$

Q.19. Which of the following is false in case of Normal distribution.

(*a*) it is multi model

(*b*) mean = median = mode

(*c*) it is symmetric

(*d*) Total area is 1

[June 2013]

Solution : (*a*) No; it is uni-modal not multi-modal.

Q.20. If x~B (5,p) and p(x = 2) = 0.4362 and p (x = 3) = 0.2181 then p =

(*a*) 3/4 (*b*) 1/3 (*c*) 2/3 (*d*) 1/4

[June 2013]

Solution : (*b*) is correct

Given; n = 5

$P(X = 2) = {}^5c_2.p^2q^3 = 10.p^2q^3 = 0.4362$ _____(1)

$P(X = 3) = {}^5c_3.p^3.q^2 = 0.2181$

$10p^3q^2 = 0.2181$ _____(2)

Eqn.(1) ÷(2); We get.

$\frac{10p^2q^3}{10p^3q^2} = 2$; or $\frac{q}{p} = 2$; or $2p = q$

or $2p = 1 - p$; or $2p + p = 1$; or $p = \frac{1}{3}$

Q.21. In a poisson distribution

(*a*) Mean & SD are equal

(*b*) Mean, variance are equal

(*c*) SD & variance are equal

(*d*) both (*a*) and (*b*)

[June 2013]

Solution : (*b*) Mean = Variance.

Q.22. In Binomial Distribution, $\mu=4$ and $\sigma^2=3$ then mode =

(*a*) 4 (*b*) 4.25

(*c*) 4.5 (*d*) 4.1

[June 2013]

Solution : (*a*) is correct.

Since, Mean = np = 4 & npq = 3 (Given)

So, 3 q = 4; So, q = 3 / 4 & p = 1 – q = 1/4

n.(1/4) = 4; Hence n = 16

Now, (n +1) p = (16 +1) (1/4) = 4.25

Rule : Mo = The integral part of (n + 1) p = 4.25 = 4; So, Mo = 4

Q.23. In Normal distribution mean median and mode are

(*a*) Equal

(*b*) Not equal

(*c*) Zero

(*d*) None of the above

[Dec. 2013]

Solution : (*a*) is correct.

Q.24. If the points of inflexion of a normal curve are 6 and 14 then standard deviation is

(*a*) 4 (*b*) 8

(*c*) 16 (*d*) 32

[Dec. 2013]

Solution : (*a*) is correct.

Given that, Lower point of inflexion $= \mu-\sigma = 6$ .....(i)

Upper point of inflexion $= \mu+\sigma = 14$ .....(ii)

Eqn. (ii) – (i); we get

$2\sigma = 8$

$\sigma = 4$

Q.25. There are 75 students in a class and their average marks is 50 and S.D of marks is 5. Number of students who have secured more than 60 marks (Given that area under the normal curve for Z = 2 is 0.4772) is_____

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

[Dec. 2013]

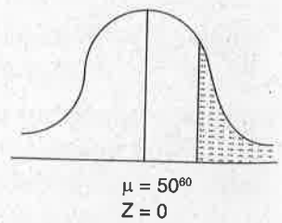
Solution : (*b*) is correct

Given N = 75;

Mean = $\mu = 50$; SD = $\sigma = 5$

Z at $x = 60 = \dfrac{X-\mu}{\sigma} = \dfrac{60-50}{5} = 2$

$P(X > 60) = P(z > 2)$



$= 0.5 - p(0 \le z \le 2)$

$= 0.5 - 0.4772 = 0.0228$

Total No. of students scoring more than 60 marks = N.P(x > 60)

$= 75 \times 0.0228 = 1.71 = 2$

Q.26. If a variate X has, Mean > variance, then its distribution will be

(*a*) Binomial

(*b*) Poisson

(*c*) Normal

(*d*) t-distribution

[June 2014]

Solution : (*a*) is correct

Note :- np > npq :- Mean > Variance

Q.27. Mean & variance of a Binomial variate are 4 and $\frac{4}{3}$ respectively then P(x ≥ 1) will be

(*a*) $\dfrac{728}{729}$ (*b*) $\dfrac{1}{729}$

(*c*) $\dfrac{723}{729}$ (*d*) None

[June 2014]

Solution : (*a*) is correct

Mean = np = 4; Variance = npq = $\dfrac{4}{3}$

or $4q = \dfrac{4}{3}$; or, $q = \dfrac{1}{3}$

$p = 1 - q = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

Since, np = 4; So, $n.\dfrac{2}{3} = 4$ or n = 6

$\therefore P(X \ge 1) = 1 - p(X < 1)$

$= 1 - p(x = 0)$

$= 1 - {}^6C_0 . p^0 . q^6$

$= 1 - 1.1.(1/3)^6 = 1 - \dfrac{1}{729} = \dfrac{728}{729}$ N

Q.28. 5,000 students were appeared in an examination. The mean of marks was 39.5 with standard deviation 12.5 marks. Assuming the distribution to be normal, find the number of students recorded more than 60% marks. [Given when Z = 1.64 area of normal curve = 0.4494]

(*a*) 1000 (*b*) 505

(*c*) 253 (*d*) 2227

[June 2014]

Solution : (*c*) is correct

Given $\mu = 39.5$; $\sigma = 12.5$

Z at $x = 60 = \dfrac{X-\mu}{\sigma} = \dfrac{60-39.5}{12.5} = 1.64$

$P(X > 60\%) = 0.5 - p(39.5 \le X \le 60)$

$= 0.5 - p(0 \le z \le 1.64)$

$= 0.5 - 0.4494 = 0.0506$

No. of students scoring more than 5,000 students = N.P (x > 60%)

$= 5000 \times 0.0506 = 253$

Q.29. Let the distribution function of a random variable x be F(x) = P(x ≤ x), then F(5)-F(2)

(*a*) $P(2 \le x < 5)$

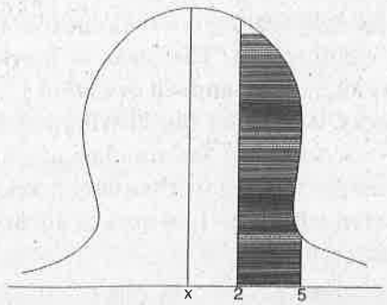
(*b*) $P(2 \le x \le 5)$

(c) $P(2 \le x \le 5)$

(d) $P(2 < x < 5)$

[Dec. 2014]

Solution : (b) is correct



$F(5) - F(2)$

$= p(0 \le \times \le 5) - p(0 \le \times \le 2)$

$= p(2 < \times \le 5).$

Q.30. For a Binomial distribution mean is 4 and variance is 3 then, 3rd central moment is

(a) 5/2 (b) 7/4

(c) 3/2 (d) 1/3

[Dec. 2014]

Solution : (c) is correct

np = 4; npq = 3

$\therefore 4q = 3 \Rightarrow q = \frac{3}{4}; p = 1 - q$

$= 1 - \frac{3}{4} = \frac{1}{4}$

Formula ; 3rd Central Moment

$\mu_3 = npq(q - p) = 3\left(\frac{3}{4} - \frac{1}{4}\right) = 3 \times \frac{2}{4}$

$= 3/2$

Q.31. In a Normal distribution mean = 2 and variance = 4 then, 4th central moment is

(a) 16 (b) 32

(c) 48 (d) 64

[Dec. 2014]

Solution : Mean = 2; var. = $\sigma^2 = 4$

Formula; 4th Central Moment

$= \mu_4 = 3.\sigma^4 = 3.(\sigma^2)^2 = 3 \times 4^2 = 48$

Q.32. X and Y are two independent Normal variables, then the distribution of x + y is_____

(a) Normal distribution

(b) t-distribution

(c) Chi-Square distribution

(d) F-distribution

[Dec. 2014]

Solution : (a) is correct.

X and Y are two independent variables which follows Normal-distribution

$\therefore$ x + y also follows Normal distribution.

Q.33. In the Binomial distribution the parameters are n and p, then X assumes values

(a) Between 0 and n

(b) Between 0 and n both inclusive

(c) Between 0 and 1

(d) Between 0 and ∞

[June 2015]

Solution : (b) is correct.

$P(x = r) = {}^nC_r.p^r.q^{n-r}$

Where r = 0 , 1, 2 ,.....; n

Q.34. In _____distribution, Mean = Variance

(a) Binomial (b) Poisson

(c) Normal (d) None

[June 2015]

Solution : (b) is correct.

Q.35. Under normal curve $\mu \pm 3\sigma$ covers ___ of the area of items.

(a) 100% (b) 99%

(c) 99.73% (d) 99.37%

[June 2015]

Solution : (c) is correct.

Q.36. Wages paid to workers follows

(a) Binominal distribution

(b) Poisson distribution

(c) Normal

(d) Chi-Square

[Dec. 2015]

Solution : (c) is correct.

Q.37. For a Binominal distribution, the parameters are 15 and 1/3 Find mode:

(a) 5 and 6 (b) 5.5

(c) 5 (d) 6

[Dec. 2015]

Solution : (c) is correct.

Given $n = 15; p = \frac{1}{3}$

$\therefore$ Mode = Integral part of $(n+1)p$

$= (15+1)\frac{1}{3} = 5.33$

$\therefore$ Mo = 5

Q.38. Standard Deviation of Binominal distribution is -

(a) npq (b) $(npq)^2$

(c) $\sqrt{npq}$ (d) $n^2p^2q^2$

[Dec. 2015]

Solution : (c) is correct.

Q.39. The Normal curve is

(a) Positively skewed

(b) Negatively skewed

(c) Symmetrical

(d) All these

[June 2016]

Solution : (c) is correct.

Q.40. For a poisson variate X, P(X = 1) = P(X = 2). What is the mean of X?

(a) 1 (b) 3/2

(c) 2 (d) 5/2

[June 2016]

Solution : (c)

$\therefore P(X = 1) = P(X = 2)$

$\frac{m^1 \cdot e^{-m}}{1!} = \frac{m^2 \cdot e^{-m}}{2!} \Rightarrow 1 = \frac{m}{2} \quad \therefore m = 2$

So, m = mean of x = 2

Q.41. In_____distribution mean = variance.

(a) Binomial (b) Normal

(c) Poisson (d) t

[Dec. 2016]

Solution : (c) is correct.

Q.42. _____ is/are Bi-parametric distribution(s)

(a) Binomial (b) Poisson
(c) Normal (d) Both (a) & (c)

[Dec. 2016]

Solution : (d) correct.

Q.43. In Poisson distribution $\mu_4 = 2$, then find μ_2.

(a) 2 (b) 4
(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

[Dec. 2016]

Solution : (c) correct

$\because$ Fourth moment of poisson - Distribution

$\mu_4 = 3\mu_2$

$2 = 3\sigma_2$

$\Rightarrow \sigma_2 = \frac{2}{3}$

Q.44. The second & third moments of observations (–6, –4, –2, 0, 2, 4, 6) are _____

(a) (12, 0) (b) (0, 12) (c) (16, 0) (d) (0, 16)

[Dec. 2016]

Solution : (c) is correct; $\mu_2 = \frac{\sum (x - \overline{X})^2}{N} = \sigma^2 = 16$; $\mu_3 = 0$ (Always)

Q.45. If X & Y are two independent Normal variates with means μ_1 and μ_2 and standard deviations σ_1 & σ_2 respectively, then X + Y follows _____

(a) Mean $= \mu_1 + \mu_2$, S.D $= 0$

(b) Mean $= \mu_1 + \mu_2$, S.D $= \sigma_1^2 + \sigma_2^2$

(c) Mean $= 0$, S.D $= \sigma_1^2 + \sigma_2^2$

(d) Mean $= \mu_1 + \mu_2$, S.D $= \sqrt{\sigma_1^2 + \sigma_2^2}$

[Dec. 2016]

Solution : (d) is correct.

Q.46. In _____ distribution, mean = variance.

(a) Binomial (b) Poisson
(c) Normal (d) None

[June 2017]

Solution : (b)

Q.47. In Binomial distribution, if variance = mean² then n & p are:

(a) $1, \frac{1}{2}$ (b) 1, 1
(c) $2, \frac{1}{2}$ (d) $3, \frac{1}{2}$

[June 2017]

Solution : (a) is correct

Tricks : GBC.

Mean $= np = 1.\frac{1}{2} = \frac{1}{2}$

Var. $= npq = 1 . \frac{1}{2} . \left(\frac{1}{2}\right) = (\text{mean})^2$

Q.48. If X ~ N (50, 16) then which of the following is not possible.

(a) P(X > 60) = 0.30 (b) P(X < 50) = 0.50
(c) P(X < 60) = 0.40 (d) P(X > 50) = 0.50

[June 2017]

Solution : (c)

for (c)

$P(x < 60) = P(x < 50) + P(50 \le x \le 60)$

$= 0.0.5 + P(50 \le x \le 60)$

. definitely greater than 50%.

$\therefore$ (c) not possible.

Q.49. The distribution of demand is as follows:

Demand	5	6	7	8	9	10
Probability	0.05	0.1	0.3	0.4	0.1	0.05

The mean is given by

(a) 7.55 (b) 7.85 (c) 1.25 (d) 8.35

[Dec. 2017]

Solution : (a) Mean = $\sum Px$

= 5 (0.05) + 6 (0.1) + 7 (0.3) + 8 (0.4)
+ 9 (0.1) + 10 (0.05)

= 7.55

Q.50. In _____ distribution, mean = variance:

(a) Binomial (b) Poisson
(c) Normal (d) None of these

[Dec. 2017]

Solution : (b)

Q.51. An example of a bi-parametric discrete probability distribution is:

(a) Binomial distribution
(b) Poisson distribution

(c) Normal distribution

(d) Both (a) & (b)

[Dec. 2017]

Solution : (a)

Q.52. In Normal distribution 95% observation lies between _____ & _____:

(a) $(\mu-2\sigma, \mu+2\sigma)$

(b) $(\mu-3\sigma, \mu+3\sigma)$

(c) $(\mu-1.96\sigma, \mu+1.96\sigma)$

(d) $(\mu-2.58\sigma, \mu+2.58\sigma)$

[Dec. 2017]

Solution : (c)

Q.53. If x is a poisson variate with mean m then $z=\frac{x-m}{\sqrt{m}}$ follows _____ distribution:

(a) Normal

(b) Binomial

(c) Bernoulli

(d) None of the above

[June 2018]

Solution : (a)

Q.54. The mean of a Binomial distribution is _____ :

(a) np (1 – p)

(b) np

(c) $\sqrt[n]{p(1-p)}$

(d) None of the above

[June 2018]

Solution : (b)

Q.55. Mean of poisson distribution is 6 then variance is _____ :

(a) 6 (b) $\sqrt{6}$

(c) 4 (d) 3

[June 2018]

Solution : (a)

Q.56. For a Poisson variate X, P(X = 2) = 3P (X = 4), then the standard deviation of X is

(a) $\sqrt{2}$ (b) 3

(c) 4 (d) 5

[Nov. 2018]

Solution : (a)

$\because$ P(X = 2) = 3 P(X = 4)

$$\text{or}; \frac{m^2 . e^{-m}}{2!} = 3 . \frac{m^4 . e^{-m}}{4!}$$

$$\Rightarrow \frac{1}{2} = \frac{3.m^2}{24}$$

$$\text{or } m^2 = 4 \Rightarrow m = 2$$

$\therefore$ Var. m = $\sigma^2 = 2$

$\sigma = \sqrt{2}$

Q.57. The mean of the Binomial distribution $B\left(4, \frac{1}{3}\right)$ is equal to

(a) $\frac{3}{5}$ (b) $\frac{4}{3}$

(c) $\frac{8}{3}$ (d) $\frac{3}{4}$

[Nov. 2018]

Solution : (b)

Given; n = 4; P = $\frac{1}{3}$;

$$\text{Mean} = np = 4.\frac{1}{3} = \frac{4}{3}$$

Q.58. If for a Normal distribution Q_1= 54.52 and Q_3 = 78.86, then the median of the distribution is

(a) 12.17 (b) 66.69 (c) 39.43 (d) None of these

[Nov. 2018]

Solution : (b)

Given; $Q_1 = 54 \cdot 52$; $Q_3 = 78 \cdot 86$.

It is normally distributed *i.e.* symmetrical data.

$$\therefore \text{Mean} = \text{Mode} = m_e = \frac{Q_1 + Q_3}{2} = \frac{54 \cdot 52 + 78 \cdot 86}{2} = 66.69.$$

Q.59. What is the mean of X having the following density function?

$$f(x) = \frac{1}{4\sqrt{2\pi}} e^{\frac{(x-10)^2}{32}} \text{ for } -\infty < x < \infty$$

(a) 4 (b) 10

(c) 40 (d) None of the above

[Nov. 2018]

Solution : (b)

Comparing it $f(x) = \frac{1}{4\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-10}{4}\right)^2}$ with the Standard formula

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Where $-\infty < x < \infty$.

We get ; Mean = $\mu = 10$ and $\sigma = 4$

Q.60. 4 coins were tossed 1,600 times. What is the probability that all 4 coins do not turn head upward at a time?

(a) $1600\ e^{-100}$ (b) $1000\ e^{-100}$ (c) $100\ e^{-1600}$ (d) e^{-1600}

[June 2019]

Solution : (d) Probability of getting a head in a throw of a coin = $\frac{1}{2}$

Probability of getting 4 heads upward in a throw of four coins = $\frac{1}{2^4} = \frac{1}{16}$ = p(let)

Given that

Here, n = 1600

$\therefore$ Mean = m = np

$= 1600 \times \frac{1}{16} = 100$

$$P(\text{No Head}) = P(X = 0) = \frac{e^{-100} \cdot (100)^0}{0!} = \frac{e^{-100} \cdot 1}{1}$$

$= e^{-100}$

Q.61. If mean and variance are 5 and 3 respectively then relation between p and q is:

(*a*) p > q (*b*) p < q

(*c*) p = q (*d*) p is symmetric

[June 2019]

Solution :

(*b*) Mean = 5, $\Rightarrow$ np = 5; Variance = 3

$\Rightarrow$ npq = 3

$\Rightarrow$ 5q = 3

$\Rightarrow$ q = 3/5

p = 1–q = 1–3/5 = 2/5

Hence, p < q

Q.62. In a Poisson distribution if P(x = 4) = P(x = 5) then the parameter of Poisson distribution is:

(*a*) $\frac{4}{5}$ (*b*) $\frac{5}{4}$

(*c*) 4 (*d*) 5

[June 2019]

Solution : (*d*)

Given that in Poisson distribution

P(x = 4) = P (x = 5)

$$\Rightarrow \frac{e^{-m} \cdot m^4}{4!} = \frac{e^{-m} \cdot m^5}{5!}$$

$$\Rightarrow \frac{1}{4!} = \frac{m}{5!}$$

$$\Rightarrow \frac{1}{24} = \frac{m}{120}$$

Hence m = 5

Q.63. Area between – 1.96 to + 1.96 in a normal distribution is:

(*a*) 95.45% (*b*) 95% (*c*) 96% (*d*) 99%

[June 2019]

Solution : (*b*)

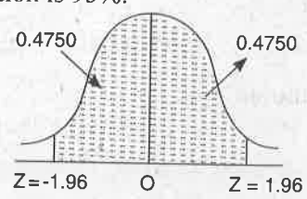
Area between – 1.96 to + 1.96 in a Normal distribution is 95%.

$\because$ P(z = 1.96) = 0.4750

$\therefore$ P(–1.96 $\le$ z $\le$ 1.96) = 0.4750 × 2

= 0.9500

= 95%



Q.64. In normal distribution what is the ratio of QD:MD:SD

(*a*) 12:10:15 (*b*) 15:10:12

(*c*) 10:15:12 (*d*) 10:12:15

[Dec. 2019]

Solution : (*d*)

$\because$ 6 QD = 5 MD = 4 SD.

Tricks: GBC

Let (*a*) is correct.

So; QD = 12; MD = 10 & SD = 15

but $6 \times 12 \neq 5 \times 10 \neq 4 \times 15$

So; (*a*) is not correct.

for (d)

$\because$ QD : MD : SD = 10 : 12 : 15

$\therefore$ QD = 10; MD = 12 & SD = 15

can be assumed.

So, $6 \times 10 = 5 \times 12 = 4 \times 15$ (True)

i.e. QD : MD : SD = 10 : 12 : 15

satisfies condition 6 QD = 5 MD = 4 SD

$\therefore$ (*d*) is correct.

Q.65. For a normal distribution $\sqrt{\frac{2}{\pi}} e^{-2(x-3)^2}$ mean and standard deviation will be -

(*a*) 3, ½ (*b*) $3, \frac{1}{\sqrt{2}}$

(*c*) $3, \sqrt{2}$ (*d*) None of these

[Dec. 2019]

Solution : (*a*)

$$f(x) = \sqrt{\frac{2}{\pi}} \cdot e^{-2(x-3)^2}$$

$$= \sqrt{\frac{4}{2\pi}} \cdot e^{-\frac{4}{2}(x-3)^2}$$

$$= \frac{2}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2} \cdot \frac{(x-3)^2}{1/4}}$$

$$= \frac{1}{\frac{1}{2}\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-3}{1/2}\right)^2}$$

Comparing it with standard form

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

We get; $\sigma = \frac{1}{2}; \mu = 3$

$\therefore$ Mean = 3; $\sigma = \frac{1}{2}$

Q.66. Area covered normal curve by $(\mu \pm 3\sigma)$

(*a*) 68.28% (*b*) 95.96%

(*c*) 99.73% (*d*) 99.23%

[Dec. 2019]

Solution : (*c*)

Area between $\mu - 3\sigma$ and $\mu + 3\sigma$ in normal curve

= 99.73%

i.e. All except 100 – 99.73

= 0.27%

Q.67. If x is binomial variate with parameter 15 and 1/3 what is the value of mode of the distribution

(*a*) 5 & 6 (*b*) 5.5

(*c*) 5 (*d*) 6

[Dec. 2019]

Solution : (*c*)

Given that

$n = 15;\ P = \frac{1}{3}$

$\therefore\ (n+1)P = (15+1)\cdot\frac{1}{3} = 5.33.....$

Formula

M_0 = Integral part of $(n + 1)\ P = 5$

If $(n + 1)\ P$ is in fraction.

Q.68. In Poisson distribution which of the following is same.

(*a*) Mean and variance

(*b*) Mean and SD

(*c*) Both

(*d*) None of these

[Dec. 2019]

Solution : (*a*)

Q.69. If for a Binomial distribution B(n,p); n = 4 and also P(x = 2) = 3 P(x = 3) then the value of P is equal to

(*a*) $\frac{9}{11}$ (*b*) 1

(*c*) $\frac{1}{3}$ (*d*) $\frac{1}{9}$

[Dec. 2019]

Solution : (*c*)

$\because\ P(X = 2) = 3.\ P(X = 3)$

or ; $^4C_2 \cdot p^2 \cdot q^2 = 3.\ ^4C_3 \cdot p^3 \cdot q^1$

or ; $6 . q = 3 . 4 . p$

or ; $6\ q = 12\ p$

or ; $q = 2p$

$\because\ p + q = 1$

So, $p + 2p = 1 \Rightarrow 3\ p = 1$

$\therefore\ p = \frac{1}{3}$

Q.70. Let x be a Poisson random variable with parameter λ. Then p(x) is equal to

(*a*) $\frac{e^{\lambda} - e^{-\lambda}}{2}$ (*b*) $\frac{e^{\lambda} + e^{-\lambda}}{2}$

(*c*) $\frac{e^{2\lambda} - 1}{2}$ (*d*) $\frac{\lambda^x . e^{-\lambda}}{x!}$

[Dec. 2019]

Solution : (*d*)

Q.71. Which of the following is uniparametric distribution?

(*a*) Normal

(*b*) Poisson

(*c*) Binomial

(*d*) Hyper geometric

[Dec. 2020]

Solution : (*b*)

Q.72. If the probability of success in a binomial distribution is less than one-half, then the binomial distribution.....

(*a*) is skewed to left

(*b*) is skewed to right

(*c*) has two modes

(*d*) has median at a point > mean + ½

[Dec. 2020]

Solution : (*b*)

For Binomial Distribution

(*i*) if P = Probability of Success $< \frac{1}{2}$

The Binomial Distribution is positively skewed i.e. skewed to right.

(*ii*) if P = Probability of success $> \frac{1}{2}$

Then Binomial Distribution is negatively skewed *i.e.* skewed to left.

(*b*) is correct.

Q.73. If we change the parameter(s) of a ..... distribution the Shape of probability curve does not change.

(*a*) Binomial (*b*) Normal

(*c*) Poisson (*d*) Non-Gaussian

[Dec. 2020]

Solution : (*b*)

Q.74. Which one of the following has Poisson distribution?

(*a*) The number of days to get a complete cure.

(*b*) The number of defects per meter on long roll of coated polythene sheet.

(*c*) The errors obtained in repeated measuring of the length of a rod.

(*d*) The number of claims rejected by an insurance agency.

[Dec. 2020]

Solution : (*b*)

Q.75. For a Poisson distributed variable X, we have P(X = 7) = 8. P (X = 9), the mean of the distribution is

(*a*) 4 (*b*) 3

(*c*) 7 (*d*) 9

[Dec. 2020]

Solution : Let Mean of Poisson distribution = *m*

$P(x = 7) = 8.P(x = 9)$

$\frac{m^7 e^{-m}}{\lfloor 7} = 8.\frac{m^9 e^{-m}}{\lfloor 9}$

or $\frac{1}{\lfloor 7} = \frac{8.m^2}{9.8.\lfloor 7}$

or $9 = m^2 \Rightarrow m = \sqrt{9} = 3$

Mean = m = 3

(*b*) is correct.

Q.76. The quartile deviation of a normal distribution with mean 10 and standard deviation 4 is.....

(*a*) 54.24 (*b*) 23.20

(*c*) 0.275 (*d*) 2.70

[Dec. 2020]

Solution : QD = Quartile Deviation

$= 0.675.s$

$= 0.675 \times 4 = 2.70$

(*d*) is correct.

Q.77. If the parameter of poisson distribution is m and mean + S.D.= $\frac{6}{25}$ then find m.

(*a*) $\frac{3}{25}$ (*b*) $\frac{1}{25}$

(*c*) $\frac{4}{25}$ (*d*) $\frac{3}{5}$

[Dec. 2020]

Solution : (*a*) is correct

Given

$$P(X=1) = P(X=2)$$

$$\Rightarrow \quad \frac{m^1 \cdot e^{-m}}{\lfloor 1} = \frac{m^2 \cdot e^{-m}}{\lfloor 2}$$

or $$\frac{1}{1} = \frac{m}{2}$$

or $$m = 2$$

Now $$P(X=4) = \frac{m^4 \cdot e^{-m}}{\lfloor 4}$$

$$= \frac{2^4 \cdot e^{-2}}{24} = \frac{16 \cdot e^{-2}}{24}$$

$$= \frac{2 \cdot e^{-2}}{3}$$

Q.82. Which one of the following is an uniparametric distribution?

(*a*) Poisson (*b*) Normal

(*c*) Binomial (*d*) Hyper geometric

[Jan. 2021]

Solution : (*a*) is correct

Parameter of Poisson distribution

= *m* (Mean) only

Q.83. For a normal distribution, the value of third moment about mean is.

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

[Jan. 2021]

Solution : (*a*) is correct

For normal distribution

1st moment = $\mu_1 = \frac{\sum (X - \overline{X})}{N} = 0$ [because $\Sigma(X - \overline{X}) = 0$]

2nd Central Moment = $\mu_2 = \frac{\sum (X - \overline{X})^2}{N}$ = Variance = σ^2

3rd Central Moment = $\mu_3 = \frac{\sum (X - \overline{X})^3}{N} = 0$

because $\Sigma(X - \overline{X})^3 = 0$ [Always]

Q.84. The value of K for the probability density function of a variate X is equal to

X	0	1	2	3	4	5	6
P (X)	5K	3K	4K	6K	7K	9K	11K

(*a*) 39 (*b*) 1/40 (*c*) 1/49 (*d*) 1/45

[July 2021]

Solution : (*d*) is correct.

∵ For a Probability Density Function

$$\Sigma p(X) = 1$$

$$\therefore \quad 5K + 3K + 4K + 6K + 7K + 9K + 11K = 1$$

$$\Rightarrow \quad 45K = 1$$

$$\therefore \quad K = \frac{1}{45}$$

Q.85. In normal distribution, Mean. Median and Mode are

(*a*) Zero (*b*) Not Equal (*c*) Equal (*d*) Null

[July 2021]

Solution : (*c*) is correct.

Q.86. If X is a Poisson variate such that P(x = 1) = 0.7, P(x = 2) = 0.3, then P(x = 0) =

(*a*) $e^{6/7}$ (*b*) $e^{-6/7}$ (*c*) $e^{-2/3}$ (*d*) $e^{-1/3}$

[July 2021]

Solution : (*b*) is correct.

Given

$$p(X = 1) = 0.7$$

or $$\frac{m^1 e^{-m}}{\lfloor 1} = me^{-m} = 0.7 \qquad (1)$$

and ∵ $$p(X = 2) = \frac{m^2 e^{-m}}{\lfloor 2} m = \frac{m^2 e^{-m}}{2} = 0.3 \qquad (2)$$

Eqn. (2), Eqn. (1); we get

$$\frac{m^2e^{-m}}{2.me^{-m}} = \frac{0.3}{0.7}$$

or $$\frac{m}{2} = \frac{3}{7} \Rightarrow m = \frac{6}{7}$$

$\therefore$ $$p(X = 0) = \frac{m^0e^{-m}}{\lfloor 0} = 1\frac{e^{-6/7}}{1}$$

$$= e^{-6/7}$$

Q.87. If X is a binomial variate with p = 1/3, for the experiment of 90 trials, then the standard deviation is equal to

(*a*) $-\sqrt{5}$ (*b*) $\sqrt{5}$

(*c*) $\sqrt[2]{5}$ (*d*) $\sqrt{15}$

[July 2021]

Solution : (*c*) is correct.

Given $p = \frac{1}{3} \Rightarrow q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

and n = 90

Variance $= \sigma^2 = npq = 90.\frac{1}{3}.\frac{2}{3} = 20$

SD $= \sigma = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

Q.88. For a certain type of mobiles, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. A person owns one of these mobiles and wants to know the probability that the length of time will be between 50 and 70 hours is (Given $\phi(1.33) = 0.9082$, $\phi(0) = 0.5$)

(*a*) –0.4082 (*b*) 0.5 (*c*) 0.4082 (*d*) –0.5

[July 2021]

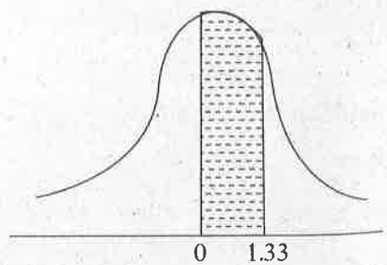
Solution : (*c*) is correct.

Given

Mean $= \mu = 50$ hours

and SD $= \sigma = 15$ hours.

$$p(50 \le X \le 70) = p\left(\frac{50-50}{15} \le Z \le \frac{70-50}{15}\right)$$



where $$z = \frac{x-\mu}{\sigma}$$

$$= p(0 \le z \le 1.33)$$

$$= \phi(1.33) - \phi(0)$$

$$= 0.9082 - 0.5 = 0.4082$$

Q.89. The average number of advertisements per page appearing in a newspaper is 3. What is the probability that in a particular page zero number of advertisements are there?

(*a*) e^{-3} (*b*) e^0

(*c*) e^{+3} (*d*) e^{-1}

[Dec. 2021]

Solution : (*a*)

Given: Mean = m = 3

$$P(X = 0) = \frac{m^0e^{-m}}{\lfloor 0} = \frac{1.e^{-m}}{1}$$

$$= e^{-m} = e^{-3}$$

Q.90. If, for a Poisson distributed random variable X, the probability for X taking value 2 is 3 times the probability for X taking value 4, then the variance of X is

(*a*) 4 (*b*) 3

(*c*) 2 (*d*) 5

[Dec. 2021]

Solution : (*c*)

Given

$P(x = 2) = 3P(x = 4)$

or $$\frac{m^2 \cdot e^{-m}}{\lfloor 2} = 3 - \frac{m^4 e^{-m}}{\lfloor 4}$$

or $$\frac{1}{2} = \frac{3}{\frac{24}{12}}m^2$$

$$4$$

$\Rightarrow m^2 = 4$ $\therefore$ m = 2

Mean = Variance = 2

Q.91. Let X be normal distribution with mean 2.5 and variance 1. If $P[a < X < 2.5] = 0.4772$ and that the cumulative normal probability value at 2 is 0.9772, then a =?

(*a*) 1.5 (*b*) 3

(*c*) –3.5 (*d*) –4.5

[Dec. 2021]

Solution : No option

Given $P(Q) = P(Z \le 2) = 0.9772$

$\therefore$ $P(Z = 2) = 0.9772 - 0.5 = 0.4772$

Mean $= \mu = 2.5$

Variance $= \sigma^2 = 1$

$\Rightarrow \sigma = 1$

$$Z \text{ at } X = a = \frac{x-\mu}{\sigma} = \frac{a-2.5}{1}$$

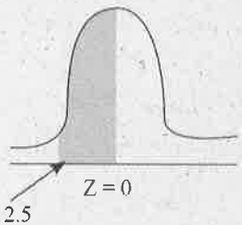
$$Z \text{ at } X = 2.5 = \frac{2.5-2.5}{1} = 0$$

$P(a < X < 2.5) = P(a - 2.5 < Z < 0)$

$= 0.4772$

$\Rightarrow P(a - 2.5 < Z < 0)$

$= P(-2 < Z < 0)$



$\therefore$ $a - 2.5 = -2$

or $a = 2.5 - 2$

$= 0.5$

No. option.

Its nearest value is 1.5.

So **(A)** may be Answer.

Q.92. The manufacturer of a certain electronic component is certain that 2% of his product is defective. He sells the components in boxes of 120 and guarantees that not more than 2% in any box will be defective.

Find the probability that a box, selected at random would fail to meet the guarantee? (Given that $e^{-2.4} = 0.0907$)

(*a*) 0.49 (*b*) 0.39

(*c*) 0.37 (*d*) 0.43

[Dec. 2021]

Solution : (*d*)

Let m = mean = n.p = $120 \times \frac{2}{100} = 2.4$

P (Fail the guarantee) = P (X > 2.4)

$= 1 - P(X \le 2.4)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[\frac{m^0 . e^{-m}}{\lfloor 0} + \frac{m^1 . e^{-m}}{\lfloor 1} + \frac{m^2 . e^{-m}}{\lfloor 2}\right]$

$= 1 - e^{-m}\left[1 + m + \frac{m^2}{2}\right]$

$= 1 - e^{-2.4}\left[1 + 2.4 + \frac{(2.4)^2}{2}\right]$

$= 1 - 0.907\ (6.28)$

$= 0.430404$

$= 0.43$

Q.93. A renowned hospital usually admits 200 patients everyday. One per cent patients, on an average, require special room facilities. On one particular morning, it was found that only one special room is available. What is the probability that more than 3 patients would require special room facilities?

(*a*) 0.1428 (*b*) 0.1732 (*c*) 0.2235 (*d*) 0.3450

[Dec. 2021]

Solution : (*a*)

Given

n = 200

p = 1% = 0.01.

Let mean = m = np = 200 × 0.01

= 2.

So, P(*x* > 3) = ?

$p(x>3) = 1 - p(x \le 3)$

$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$

$= 1 - \left[\frac{m^0\, e^{-m}}{\lfloor 0} + \frac{m^1\, e^{-m}}{\lfloor 1} + \frac{m^2\, e^{-m}}{\lfloor 2} + \frac{m^3\, e^{-m}}{\lfloor 3}\right]$

$= 1 - e^{-m}\left[\frac{1}{1} + \frac{2}{1} + \frac{2^2}{2} + \frac{2^3}{6}\right]$

$= 1 - (2.7183)^{-2} \cdot [1 + 2 + 2 + 4/3]$

$1 - (2.7183)^{-2}\left(\frac{19}{3}\right)$

Calculator

Press button

2.7183 ÷ = 2 times × 19 ÷ 3 +/– button + 1 = button

We get 0.1428

(*a*) is correct

Q.94. The binomial distribution, having mean and standard deviation as 3 and 1.5, has number of trials equal to

(*a*) 3 (*b*) 6

(*c*) 8 (*d*) 12

[June 2022]

Solution : Mean = np =3 (given)

and $\sqrt{npq} = 1.5$

$\therefore$ npq = $(1.5)^2 = 2.25$

or 3q = 2.25

$\therefore\ q = \frac{2.25}{3} = 0.75$

$\therefore$ p = 1 – q = 1– 0.75 = 0.25

$\therefore$ np =3

or $n = \frac{3}{p}$ $\therefore\ n = \frac{3}{0.25} = 12$

$\therefore$ (*d*) is correct

Q.95. The mean of binomial distribution is

(*a*) Always less than its variance

(*b*) Always more than its variance

(*c*) Always equal to its variance

(*d*) Always equal to its standard deviation

[June 2022]

Solution : Mean of Binomial Distribution = np

Its Variance = npq

Clearly np>npq because 0<q<1

$\therefore$ (*b*) is correct

Q.96. The variance of a normal distribution is given to be 16. The mean deviation about mode is

(*a*) 3.2 (*b*) 8

(*c*) 12.8 (*d*) 12

[June 2022]

Solution : Var = $\sigma^2 = 16$

$\sigma = 4$

$\therefore$ MD = 0.8.σ

$= 0.8 \times 4 = 3.2$

$\therefore$ MD about Mean = Median = Mode = 3.2

($\bar{x} = M_e = M_0$) because it is Normally distributed.

$\therefore$ (*a*) is correct.

Q.97. The standard deviation of a Poisson variate X is 1.732. The P [–2.9 < X < 3.54] is

(*a*) $13e^{-3}$ (*b*) $9e^{-3}$

(*c*) $4e^{-2}$ (*d*) e^{-6}

[June 2022]

Solution : Given

SD = $\sqrt{np} = \sqrt{m} = 1.732 = \sqrt{3}$

$\therefore$ m = 3

$$P(x = r) = \frac{m^r . e^{-m}}{\lfloor r}$$

Where r = 0,1,2,3,——(Whole No.)

P = (–2.9 < X < 3.54)

= P (Whole Nos. b/w – 2.9 & 3.54)

= P (X = 0) + P(X = 1) + P(X = 2) + P (X =3)

$$= \frac{m^0 . e^{-m}}{\lfloor 0} + \frac{m^1 . e^{-m}}{\lfloor 1} + \frac{m^2 . e^{-m}}{\lfloor 2} + \frac{m^3 . e^{-m}}{\lfloor 3}$$

$$= e^{-m}\left(m^0 + \frac{m}{1} + \frac{m^2}{2} + \frac{m^3}{6}\right)$$

$$= e^{-3}\left(1 + \frac{3}{1} + \frac{3^2}{2} + \frac{3^3}{6}\right)$$

$= e^{-3} \times 13 = 13e^{-3}$

∴ (*a*) is correct.

Q.98. For a normal distribution, the first and third quartiles are given to be 37 and 49, the mode of the distribution is

(*a*) 37 (*b*) 49

(*c*) 43 (*d*) 45

[June 2022]

Solution : For Normal Distribution

$Q_1 = 37; Q_3 = 49$

$$\therefore \text{Mean} = \mu = \frac{Q_1 + Q_3}{2} = \frac{37 + 49}{2} = 43$$

∴ It is Normally Distributed

∴ Mean = Median = Mode = 43

∴ (*c*) is correct.

Q.99. Skewness of normal distribution is:

(*a*) Negative (*b*) Positive

(*c*) Zero (*d*) Undefined

[Dec. 2022]

Solution: (*c*) is correct.

Q.100. If a Poisson distribution is such that P(X = 2) = P(X = 3) then the variance of the distribution is

(*a*) $\sqrt{3}$ (*b*) 3

(*c*) 6 (*d*) 9

[Dec. 2022]

Solution: Let mean = *m* = variance

∵ P(X = 2) = P(X = 3)

[∵ It follows poisson Distribution]

$$\therefore \frac{m^2 . e^{-m}}{\lfloor 2} = \frac{m^3 . e^{-m}}{\lfloor 3}$$

$$\text{or; } \frac{1}{\lfloor 2} = \frac{m}{3 \times \lfloor 2}$$

or; *m* = 3

∴ Variance = *m* = 3

∴ (*b*) is correct

Q.101. The standard Deviation of Binomial distribution is:

(*a*) *npq* (*b*) $\sqrt{npq}$

(*c*) *np* (*d*) $\sqrt{np}$

[Dec. 2022]

Solution: Variance = $\sigma^2 = npq$

$\therefore \sigma = SD = \sqrt{npq}$

(*b*) is correct

Q.102. The speeds of a number of bikes follow a normal distribution model with a mean of 83 km/hr and a standard deviation of 9.4 km/hr. Find the probability that a bike picked at random is travelling at more than 95 km/hr.?

(*a*) 0.1587 (*b*) 0.38

(*c*) 0.49 (*d*) 0.278

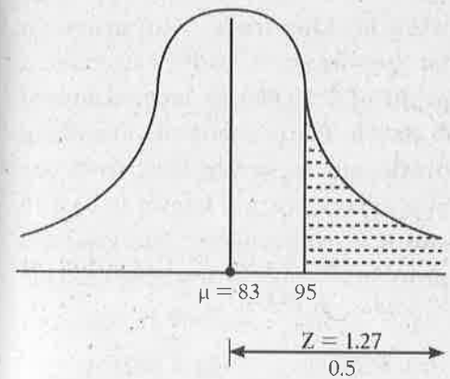
[Dec. 2022]

Solution: *Given* $\mu = 83; \sigma = 9.4$

$$Z \text{ at } x = 95 = \frac{x - \mu}{\sigma} = \frac{95 - 83}{9.4}$$

= 1.2765 ..... = 1.27

P(Z = 1.27)



= 0.3980 = 0.40

= 0.40%

∴ P(X > 95)

= P(Z > 1.27)

= 0.5 – P(0 ≤ Z ≤ 1.27)

= 0.5 – 0.3980

= 0.102

[No option but it is most nearest to 0.1587]. So option (*a*) is correct.

Q.103. Between 9 AM and 10 AM, the average number of phone calls per minute coming into the switchboard of a company is 4. Find the probability that during one particular minute, there will be either 2 phone calls or no phone calls (given $e^{-4} = 0.018316$).

(*a*) 0.156 (*b*) 0.165

(*c*) 0.149 (*d*) 0.194

Solution: Given mean = m = 4

Here Poisson Distribution is suitable.

∴ p (2 phone calls or no phone calls)

= p (x = 2 or x = 0)

= p (x = 2) + p (x = 0)

$$= \frac{m^2 \times e^{-m}}{\lfloor 2} + \frac{m^0 \times e^{-m}}{\lfloor 0}$$

$$= e^{-m}\left[\frac{m^2}{2} + \frac{1}{1}\right]$$

$$= e^{-4}\left[\frac{4^2}{2} + 1\right]$$

= 0.018316(8+1) = 0.164843

= 0.165

∴ (*b*) is correct.

Q.104. The probability distribution of *x* is given below :

Value of x	1	0	Total
Probability	p	1 – p	1

Mean is equally to:

(*a*) p (*b*) 1 – p

(*c*) 0 (*d*) 1

Solution:

Mean = Expected Value

$= \sum px = 1 \times p + 0(1-p)$

$= p + 0 = p$

$\therefore$ (*a*) is correct.

Q.105. If a Poisson distribution is such that $P(X = 2) = \frac{1}{3}P(X = 3)$, then the standard deviation of the distribution is:

(*a*) $\sqrt{3}$ (*b*) 3

(*c*) 2 (*d*) 1

Solution:

$P(x = 2) = \frac{1}{3}P(x = 3)$

or; $\frac{m^2 \times e^{-m}}{\lfloor 2} = \frac{1}{3} \times \frac{m^3 \times e^{-m}}{\lfloor 3}$

where m = mean.

$\Rightarrow \frac{1}{2} = \frac{1}{3} \times \frac{m}{3}$

$\therefore m = 9$

Mean = Variance = $SD^2 = 9$

$\therefore SD = \sqrt{9} = 3$

$\therefore$ (*b*) is correct.

Q.106. If a random variable *X* has the following probability distribution, then the expected value of *X* is:

X	–1	–2	0	1	2
f(*x*)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{3}$

(*a*) $\frac{3}{2}$ (*b*) $\frac{1}{2}$

(*c*) $\frac{1}{6}$ (*d*) $\frac{1}{3}$

Solution:

Expected value = $\sum px$

$= \sum\{x \times f(x)\}$

$= (-1)\times\frac{1}{3} + (-2)\times\frac{1}{6} + 0\times\frac{1}{5} + 1\times\frac{1}{6} + 2\times\frac{1}{3}$

$= \frac{1}{3}(2-1) + \frac{1}{6}(-2+1) + 0$

$= \frac{1}{3} - \frac{1}{6} = \frac{2-1}{6} = \frac{1}{6}$

$\therefore$ (*c*) is correct.

Q.107. On a commodity exchange when booking trades with provision for stop-losses, a trader can make a profit of ₹ 50,000 or incur a loss of ₹ 20,000. The probabilities of making profit and incurring loss, from the past experience, are known to be 0.75 and 0.25 respectively. The expected profit to be made by trader should be:

(*a*) ₹ 32,500 (*b*) ₹ 35,000

(*c*) ₹ 30,000 (*d*) ₹ 40,000

Solution: Expected Profit

$= \sum Px = 50{,}000 \times 0.75 + (-20{,}000) \times 0.25$

= ₹ 32,500

$\therefore$ (*a*) is correct.

Q.108. The incidence of skin diseases in a chemical plant occurs in such a way that the workers have 20% chance of suffering from it. What is the probability that out of 6 workers 4 or more will have skin diseases?

(*a*) 0.1696 (*b*) 0.01696

(*c*) 0.1643 (*d*) 0.01643

Solution: Given

p = 20% = 0.2

q = 1 – p = 1 – 0.2 = 0.8

n = 6

$\therefore p(x \geq 4) = p(x = 4) + p(x = 5) + p(x = 6)$

$= {}^6C_4 \times p^4 \times q^2 + {}^6C_5 \times p^5 \times q + {}^6C_6 \times p^6 \times q^0$

$= 15 \times (0.2)^4 \times (0.8)^2 + 6 \times (0.2)^5 \times (0.8) + 1 \times (0.2)^6 \times 1$

= 0.01696

$\therefore$ (*b*) is correct.

25 CHAPTER

SAMPLING THEORY OF ESTIMATION

Some Important Terms.

CENSUS:- A large volume of statistical information is collected by way of periodic enumerations is called Census. Example Census of the voters of the population of eligible voters.

POPULATION OR UNIVERSE :- The aggregate of statistical information on a particular character of all the members covered by an investigation/enquiry is called **Population or Universe.** For example, marks obtained by students in CA CPT exam. constitute population.

SAMPLE :- The part of the population which is actually selected in the course of an investigation/enquiry to ascertain the characteristics of the population is called **Sample.**

SAMPLING :- The methods or techniques by which samples are drawn is called Sampling.

Sampling is used in various areas such as -

(*a*) In Marketing Research for assessing customers' behavior.

(*b*) In Industry for Statistical Quality Control.

(*c*) In Business for inspecting the incoming lots of materials from suppliers.

(*d*) In Auditing for test checking the accounts/transactions.

Remember some useful notations

	Population (Parameter)	Sample (Statistic)
Size	**N**	**n**
Mean	μ	$\overline{X}$
Proportion	**P**	**p**
Standard Deviation	σ	**s**

PARAMETER ;- A statistical measure based on each and every item of the universe/population is called **Parameter**. They are N, μ, P, σ. It shows the characteristic of the universe/ population. Since the parameter remains a constant, it has **neither a sampling fluctuation nor sampling distribution nor a standard error.** They are used as estimates of parameters as to calculate the standard error of statistic.

STATISTIC:- A statistical measure based on items/observation of a sample

is called **Statistic.** They are n, $\overline{X}$, p , s. It shows the characteristic of the universe/ population. It varies from sample to sample, it has **sampling fluctuation, sampling distribution and standard error.** Sampling distribution of a statistic is the probability distribution of that statistic and **standard error** is the standard deviation of the sampling of a statistic distribution. Usually parameters are unknown and statistics are used as estimates of parameters.

OBJECTIVES OF SAMPLING :-

1. To obtain the maximum information about the population with the minimum effort; (e.g. to estimate the unknown characteristics of the population)
2. To state the limits of accuracy of estimates based on sample.
3. To draw inferences on the behavior of the population.

When units are so chosen from the universe that the selected unit will not figure again, the sampling is said done **without replacement,** otherwise known as **with replacement.**

Without Replacement :- No. of samples with size **n** observations out of Population size N

$$= {}^{N}c_{n} = \frac{N!}{n!.(N-n)!}$$

With Replacement :- No. of samples with size **n** observations out of Population size N

$$= N^{n}$$

Example:- How many No. of samples of 2 observations out of 5 can be made. (*i*) with replacement,

(*ii*) Without Replacement

Soln.:- Given that N = 5, n = 2

(*i*) with replacement :- No. samples = $N^{n} = 5^{2} = 25$

(*ii*) Without Replacement :- No. samples $= {}^{N}c_{n} = \frac{N!}{n!.(N-n)!}$

$${}^{5}c_{2} = \frac{5!}{2!.(5-2)!} = \frac{5.4.3!}{2.1.3!} = 10$$

METHODS OF SAMPLING

The different methods of sampling are discussed below:

1. Deliberate, Purposive or Judgment Sampling

Meaning - Under this method selection of sample items is often based on certain predetermined criteria fixed by the individual judgment of the sampler.

Advantages

1. A purposive sample may not vary widely from the average.
2. It is economical and useful if the sample size is small.

Disadvantages

1. This is much scope for personal bias.
2. Degree of accuracy of the estimates is not known.
3. As the sample size increases, the estimates become unreliable due to accumulation of bias.

2. Block or Cluster Sampling

Meaning - Under this method, certain blocks or clusters of higher concentration are selected for complete enquiry e.g. all transactions of a particular period in a year. These clusters are used often in multistage sampling wherein sampling is done in stages.

Suitability - It is suitable where there is an unequal concentration of individual units in the universe.

3. Area Sampling

Meaning - Under this method, the total geographical area (if big) is divided into a number of smaller non-overlapping areas and then some of the smaller areas are selected and all units of the selected areas constitute the sample.

Advantage - It generally makes field interviewing efficient.

Suitability - It is suited in inquiries to be conducted over a large area, when the list of population concerned is not available.

4. Quota Sampling

Meaning - Under this method, each person engaged in the primary selection of data is assigned a fixed quota of investigations e.g. 50 salaried persons in the age group of 25–30 years. Within the quota, the selection of sample items depends entirely on personal judgment.

Advantage - The benefits of stratification are available. Disadvantage-There is scope for personal bias.

Suitability - It is suitable in marketing research studies where it is not possible to stick to it without delay and expenditure.

5. Random (or Probability) Sampling

Meaning - Under this method, selection of sample items is based on chance in such a manner that each unit of the population has an equal chance of being included in the sample. The methods of obtaining a random sample include Lottery System, Random Tables, Nth number etc.

Advantages

1. There is no scope of personal bias.
2. Each item has an equal chance of being selected.
3. It provides more accurate and reliable data.
4. It becomes possible to have an idea about the errors of estimation.

Disadvantages - It is not suitable if the field of enquiry is small.

Suitability - It is suitable when the population is more or less homogeneous with respect to characteristics under study.

Usefulness - The theories of sampling distribution and test of significance are based on random sampling only.

6. Systematic Sampling

Meaning - Under this method, selection of sampling items is done at uniform intervals of time, space or order of occurrence.

Advantages - Actual selection of the sample is easier and quicker. A systematic sample is practically equivalent to a random sample if the characteristic under study is independent of the order of arrangement of the units.

Disadvantage - The sample may be biased if there are periodic features associated with the sampling interval.

7. Stratified Sampling

Meaning - Under this method, the population is sub-divided into several groups (*called strata*) on the basis of purposive sampling and then samples of desired size are selected from each of them on the basis of random sampling. All the samples combined together give the stratified sample. Thus, it is a mixture of both purposive and random sampling.

Purposes - The main purposes of stratification are:

(*a*) to increase the overall estimates,

(*b*) to ensure that all sections of the population are adequately represented.

(*c*) to avoid a large size of the population and

(*d*) to avoid the heterogeneity of the population.

Advantages

1. It eliminates the difference between strata and thereby reduces the sampling error.
2. It brings about a gain in the precision of the sample estimate the strata variability is the least.
3. Independent estimates for different strata can be prepared.
4. There is not much scope for personal bias.

Disadvantage - The results may be misleading if the basis of stratification is not properly decided.

8. Multi Stage Sampling

Meaning - Under this method sampling is done in several stages starting from the larger units, intermediate units and finally reaching the ultimate units of selection.

Advantage - Usually, considerable saving in cost is achieved.

9. Sequential Sampling

Meaning - Under this method, a relatively small sample is tested for drawing a decision and if the first sample does not give evidence for a definite decision, more units are chosen at random and added to sample until a decision is possible using enlarged sample.

Usefulness - It is used to draw inference on the behavior of the population and in estimating the unknown characteristics of the population.

Advantages of Using Sampling Methods

1. These facilitate quick results.
2. These facilitate more skilled analysis.
3. These facilitate following up of non-responsive units
4. These facilitate the error estimation.
5. These involve lower costs.
6. These provide higher quality data.
7. These are more scientific as compared to census.

STATISTICAL LAWS

The possibility of reaching valid conclusions about the population on the basis of sample is based on the following two important laws:

1. Law of Statistical Regularity - It states that a sample of reasonably large size when selected at random is almost sure to represent the characteristics of the population. The selection is said to be at random, 'when every item in the universe has an equal chance of being selected. The larger the size of the sample more reliable is the result, because the sampling error is inversely proportional to the square root of the number of items in the sample.

2. Law of Inertia of large Numbers - It states that samples of large size shows a high degree of stability i.e. the results obtained from the samples are expected to be very close to the population characteristics. The greater the size of the sample, the greater will be compensation or tendency to neutralize extreme values and consequently more stable would be the result. For example, birth rate, death rate, etc., may vary from place to place, but for the country as a whole they will be found somewhat stable over a number of years.

STANDARD ERROR

Meaning :- Standard Error of a given statistic is the standard deviation of sampling-distribution of that statistic.

Reason to arises standard Error : - The standard error arises due to use of sampling (which is based on some items of the population) as against the complete enumeration censes enquiry (which is based on all items of the population).

Factor affecting - Standard Error depends on -

1. the sample size
2. the nature of the statistic *e.g.* mean, variance, etc.
3. the mathematical form of the sampling distribution
4. the values of some of the parameters used in the sampling distribution.

USEFULNESS OF STANDARD ERROR

1. It is used to find confidence limits within which parameters are expected to lie.

For example :- mean ± 1 S.E. will give 68.27% values,

mean ± 2 S.E. will give 95.45% of values

mean ± 3 S.E. will give 99.73% of values,

$\overline{X} \pm Z.\,S.E.(\overline{X})$, S ± Z S.E.(s) will give the confidence limit.

2. It is used in testing a given statistical hypothesis at different levels of Significance.

For example.

Confidence Level	Level of significance	Difference between observed and expected values	Whether or not Considered significant
95 %	5%	If the difference is more than 1.96 S.E.	Significant

Confidence Level	Level of significance	Difference between observed and expected values	Whether or not Considered significant
95%	5%	If the difference is less than 1.96 S.E.	Not significant
99%	1%	If the difference is more than 2.58 P.E.	Significant
99%	1%	If the difference is less than 2.58 P.E.	Not significant

Note: In practice, usually the hypotheses are tested at 5% level of significance. Unless otherwise stated in the examination.

COMPUTATION OF STANDARD ERROR OF THE MEAN

Population Size	When "σ" is known	When "σ" is unknown i.e."s" known
(I) * N is Large ** N is unknown *** $\frac{n}{N} < 0.05$ **** **SRSWR** (Simple Random Sampling with Replacement)	$S.E._{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ Where, σ = Population S.D. n = Sample size	$S.E._{\bar{x}} = \frac{s}{\sqrt{n-1}}$ Where, S = Sample S.D. n = Sample size
(II) * $\frac{n}{N} \geq 0.05$ ** **SRSWOR** (Simple Random Sampling Without Replacement)	$SE(\bar{X}) = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$	$SE(\bar{X}) = \frac{s}{\sqrt{n-1}}\sqrt{\frac{N-n}{N-1}}$

COMPUTATION OF STANDARD ERROR OF THE PROPORTION

Standard Error of the proportion (P) = SE(p) as follows :

Population Size	When Population proportion is known	When Population proportion is not known
(I) * N is Large ** N is unknown *** $\frac{n}{N} < 0.05$ **** **SRSWOR** (Simple Random Sampling without Replacement)	$SE(p) = \sqrt{\frac{PQ}{n}}$ *where, P* = Population proportion $Q = 1-P$, n = Sample size	$SE(p) = \sqrt{\frac{pq}{n}}$ *where, p* = Sample proportion $q = 1-p$, n = Sample size
(II) * $\frac{n}{N} \geq 0.05$ ** **SRSWOR** (Simple Random Sampling Without Replacement)	$SE(p) = \sqrt{\frac{PQ}{n}}\sqrt{\frac{N-n}{N-1}}$	$SE(p) = \sqrt{\frac{pq}{n}}\sqrt{\frac{N-n}{N-1}}$

MEANING AND TWO WAYS OF ESTIMATION

Meaning - In the context of statistics, estimation is a statistical technique of estimating unknown population parameters from the corresponding sample statistic.

Two ways - A population parameter can be estimated in two ways:

1. Point Estimation, and
2. Interval Estimation

POINT ESTIMATION

Meaning - It provides a single value of a statistic that is used to estimate an unknown population parameter.

Estimator - The statistic which is used to obtain a point estimate is called estimator.

Estimate - The value of statistic is the estimate.

Example - If sample mean ($\bar{X}$) is used for estimating the population mean (μ), then $\bar{X}$ is called as estimator and the value of is called an estimate.

Criteria for a Good Estimator - According to R. A. Fisher, the criteria for a good estimator are:

(*a*) Unbiasedness, (*b*) Consistency, (*c*) Efficiency, (*d*) Sufficiency

(*a*) Unbiasedness - A statistic is said to be an unbiased estimator of parameter if its expected value is equal to the value of the parameter. The expected value of the statistic expressed as 'E' is the arithmetic mean of the sampling distribution of the statistic.

Thus,

1. The sample mean ($\overline{X}$) is an unbiased estimator of the population mean (μ) because mean of the sampling distribution of several means is equal to population mean (μ).

2. The sample variance (S^2) is a biased estimator of the population variance (σ^2) because the expected value of sample variance [$E(S^2)$] is not equal to population variance (σ^2).

3. An unbiased estimator of the population variance (σ^2) is given by –

$$\hat{S}^2 = S^2\left(\frac{n}{n-1}\right) = \Sigma (x_i - \overline{x})^2 \left(\frac{n}{n-1}\right)$$

This is because $E\ (\hat{S}^2) = \sigma^2$.

(*b*) Consistency - A statistic is said to be a consistent estimator of a parameter if it comes closer to the value of parameter as the sample size (n) tends to infinity. *For example,* in random sampling from a Normal population, both the sample mean and the sample median are consistent estimators of population mean *(μ).*

(*c*) Efficiency - A consistent statistic is said to be 'Most efficient' estimator of a parameter if its sampling variance is less than that of any other consistent estimator. *For example,* sample mean *(X)* is more efficient then median (Med.) in estimating the population mean (μ) since the variance of mean is smaller than variance of median.

A statistic which has the minimum variance among all estimators of population parameter is called the **Minimum Variance (MV) estimator.**

A statistic which is unbiased and has also minimum variance (i.e. most efficient) is called the **Minimum Variance Unbiased Estimator (MVUE).**

Sufficiency - A statistic is said to be a 'sufficient estimator' of a parameter if it contains all information in the sample about the population parameter. *For example* in random sampling from a normal population, the sample means *()* is a sufficient estimator of μ.

Two Methods of Point Estimation - There are two methods of Point estimation as follows

I. Method of Maximum Likelihood
II. Method of Moments.

I. Method of Maximum Likelihood - It is a process of choosing as an estimator of population parameter, (Ø) that statistic which when substituted for population parameter (Ø), maximizes the likelihood function *L.*

$L = f(x_1, Ø)f(x_2, Ø),\ldots\ldots f(x, Ø)$

The statistic which maximizes the likelihood function *L* is called a **Maximum Likelihood Estimator (MLE).**

Properties of MLE

1. It is consistent, most efficient and also sufficient provided a sufficient estimator exists.

2. It tends to be distributed normally for large samples.

3. It is not necessarily unbiased. A biased MLE can be converted into an unbiased estimator by a slight modification.

4. It is invariant under functional transformations.

INTERVAL ESTIMATION

Meaning - Interval Estimation provides an interval of finite width centered at the point estimate of the parameter, within which unknown parameter is expected to lie with a specified probability. Such an interval is called a confidence interval for population parameter.

Confidence Limits - The lower and upper limits of the confidence interval are called confidence limits.

Confidence Level (say 95%, 99%) and Confidence coefficient (i.e. value of Z from the Normal Distribution Table or value of t from t-distribution Table) corresponding to that confidence level.

The following table summaries when to use which value (Z or *t*):

Sample Size	**When Population S. D. is known**	**When Population S. D. Is not known**
$n > 30$	Value of Z (Z-Test)	Value of Z (Z-Test)
$n \leq 30$	Value of Z (Z-Test) ***Note:*** The population must be normal	Value of *t* (t-Test)

Confidence Interval as follows

Sample Size	**When Population S. D. is known**	**When Population S. D. is not known**
$n > 30$	$\overline{X} \pm Z_\alpha \cdot SE(\overline{X})$	$\overline{X} \pm Z_\alpha \cdot SE(\overline{X})$
$n \leq 30$	$\overline{X} \pm Z_\alpha \cdot SE(\overline{X})$	$\overline{X} \pm t_{\alpha,\nu} \cdot SE(\overline{X})$

Degree of freedom (ν) = n-1

Confidence Coefficients Z from the table of areas under the standard normal probability distribution at various confidence levels are given below

Confidence Level	90%	95%	98%	99%	Almost sure level
Significance Level	10%	5%	2%	1%	
Confidence Coefficient Z	1.64	1.96	2.33	2.58	3.00

Illustrative Examples:-

Example 1. The quality control manager of a tyre company has sample of 100 tyres and has found the mean life time to be 30,214 km. The population s.d. is 860. Construct a 95% confidence interval for the mean life time for this particular brand of tyres.

Solution. Here $n = 100$, $\bar{x} = 30214$, a = 860.

Also $(1 - \alpha)\% = 95\%$ so that $z_{\alpha=1.96}$ [From table]

$$\therefore S.E.(\bar{x}) = \frac{a}{\sqrt{n}} = \frac{860}{\sqrt{100}} = \frac{860}{10} = 86$$

$\therefore$ **Confidence interval :- Lower Confidence Level** $= (\bar{X}) - Z_\alpha . SE(\bar{X})$

$= 30214 - 86 \times 1.96$

$= 30214 - 168.56 =$ **30045.44**

Upper Confidence Level

$= \bar{X} + Z_\alpha . SE(\bar{X}) = 30214 + 86 \times 1.96$

$= 30214 + 168.56 =$ **30382.56**

Example 2. In a random selection of 64 of 600 road crossing in a town, the mean number of automobile accidents per year was found to 4.2 and the sample s.d was 0.8. Construct a 95% confidence interval for the mean number of automobile accidents per crossing per year.

Solution. Here n = 64, N = 600, s.d of sample **s** = 0.8,

$(1-\alpha)\% = 95\%$, so the value of $z_a = 1.96$

Since we have a finite population of size 600 and $\frac{n}{N} = 64/600 = 1.106$, which is more than 0.05, so we use the following result to calculate the standard error of $\bar{X}$

$$S.E.(\bar{x}) = \frac{s}{\sqrt{n-1}}\sqrt{\frac{N-n}{N-1}} = \frac{0.8}{\sqrt{63}}\sqrt{\frac{600-64}{600-1}}$$

$$\frac{0.8}{7.39}\sqrt{\frac{536}{599}} = \frac{0.8}{7.93} \times 0.894 = 0.091$$

Lower Confidence Level

$= \bar{X} - Z_\alpha \cdot SE(\bar{X}) = 4.2 - 0.091 \times 1.96 = 4.2 - 0.177 =$ **4.023**

Upper Confidence Level

$= \bar{X} + Z_\alpha \cdot SE(\bar{X}) = 4.2 + 0.091 \times 1.96 = 4.2 + 0.177 =$ **4.377**

Example 3. A random sample of size 100 has mean 15, the population variance being 25. Find the interval estimate of the population mean with a confidence level of (i) 99% and (ii) 95%.

[C.A. (Intermediate); Nov. 1988]

Solution. Here $n = 100$, $\sigma^2 = 25$, $\bar{X} = 15$.

$$S.E.(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{100}} = \frac{5}{10} = 0.5$$

***(i)* 99% confidence level** $z_a = 2.58$

Confidence Intervals

$= \bar{X} \pm S.E.(\bar{X}) \times z_\alpha$

$= 15 \pm 0.5 \times 2.58 = 15 \pm 1.29$ *i.e.*, 13.71 to 16.29.

***(ii)* 95% Confidence level** $z_\alpha = 1.96$.

Confidence Intervals

$= \bar{X} \pm S.E.(\bar{X}) \times z_\alpha = 15 \pm (0.5) \times 1.96$

$= 15 \pm 0.98$ *i.e.* 14.02 to 15.98

Example 4. A sample of 100 gave a mean of 7.4 kg and a standard deviation of 1.2 kg. Find 95% confidence limits for population mean.

[C.A. (Found.); Nov. 2001]

Solution:- Standard error of mean S.E.

$$(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{1.2}{\sqrt{100}} = .02.$$

95% confidence limits for population mean are given by :

$$\bar{x} \pm 1.96\,\frac{a}{x},\ i.e.\ 7,4 \pm 1.96 \times 0.12$$

The confidence limits are: 7.1648 and 7.6352.

Example 5. (*a*) A random sample of size 65 was taken to estimate the mean annual income, of 1000'families and the mean and standard deviation were found to the Rs. 6300 and Rs. 9.5 respectively. Find a 95% confidence interval for the population.

[C.A. (Found); May 1996]

Solution. (*a*) We have n = sample size = 65 (large), N = 1000, $\bar{X} = 6300$, s = 9.5.

Since the sample is large, so we can assume that the sampling distribution is Normal.

Since $\frac{n}{N} = \frac{65}{1000} = 0.065 > .05$ therefore, the finite Population correction factor is used.

Thus we shall use the following result to calculate the standard Error of i.e.,

$$S.E.(\bar{X}) = \frac{s}{\sqrt{n-1}}\sqrt{\frac{N-n}{N-1}} = \frac{95}{\sqrt{64}} \times \sqrt{\frac{1000-65}{1000-1}}$$

$$= \frac{9.5}{8}\sqrt{\frac{935}{999}} = \frac{9.5\sqrt{.936}}{8} = \frac{9.5 \times 0.97}{8} = 1.15$$

Confidence interval

$= \left[\bar{x} - S.E.(\bar{x})z_a, \bar{x} + S.E.(\bar{x})z_a\right]$

$= [6300 - 1.15 \times 1.96;\ 6300 + 1.15 \times 1.96]$

i.e., $[6300 - 2.25,\ 6300 + 2.25]$ or $[6297.75,\ 6302.25]$.

Example 6. If sample mean is 20, population standard deviation is 3 and sample size is 64, find the interval estimate of the mean.

[C.A. (Found); Nov. 1999]

Solution:- Interval estimate of the sample mean is:

$$\bar{x} \pm \frac{a}{\sqrt{n}} \times 1.96 = 20 \pm \frac{3}{8} \times 1.96$$

Hence the interval estimate of mean is: [19.265, 20.735]

CONFIDENCE INTERVAL ESTIMATE OF THE PROPORTION

Population proportion. The population proportion P is the ratio of the number of elements possessing a characteristic to the total number of elements in the population, i.e.,

$$P = \frac{\text{Number of elements possessing the characteristic}}{\text{Total number of elements in the Population}}$$

If we multiply the population by 100, then we get the percentage and we may make use of percentage for the proportion and vice - versa.

Sample Proportion. The sample proportion p is the ratio of the number of elements possessing a characteristic to the total number of elements 'n' in the sample.

$$\textbf{Sample Proportion: } p = \frac{\text{Number of elements possessing the characteristic}}{n = \text{Total number of elements in the Population}}$$

It is important to note that the mean of sampling distribution of p equals the population proportion, i.e., E (P) = P.

Example 7. A random sample of 800 units from a large consignment showed that 200 were damaged. Find 95% confidence limits for the population proportion of damaged units in the consignment.

Solution. Here n = 800, Sample proportion

$$p = \frac{200}{800} = 0.25, q = 1 - 0.25 = 0.75.$$

$$S.E.(p) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.25 \times 0.75}{800}} = 0.031$$

Also $(1 - a)\% = 95\%$ so that $z_a = 1.69$

Confidence limits for the population proportion are: $p \pm z_a \times S.E.(p)$.

Lower limit =

$$p - z_a \times S.E.(p) = 0.25 - 1.96 \times 0.031$$

$$= 0.25 - 0.06 = 0.19.$$

Upper limit $= p + z_a \times S.E.(p) = 0.25 + 1.96 \times 0.031 = 0.25 + 0.06 = 0.31.$

Hence, 95 % confidence limits for the population proportion of damaged units in the consignment are 0.19 and 0.31.

Example 8. Out of 300 households in a town 123 have T. V. sets. Find 95% confidence limits to the true value of the proportion of the households with T. V. sets in the whole town.

Solution: Here n = 300, Sample proportion $p = \frac{123}{300} = 0.41; q = 1 - 0.41 = 0.59$

$$S.E.p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.41 \times 0.59}{300}} = 0.0283.$$

Also $(1 - a)\% = 95\%$ therefore, $z_a = 1.96$.

Confidence limits $= p \pm z_a \times S.E.(p)$

$$= 0.41 \pm 1.96 \times 0.0283 = 0.41 \pm 0.055$$

Lower limit $= 0.41 - 0.055 = 0.355$

Upper limit $= 0.41 + 0.055 = 0.465.$

Hence, 95 % confidence limits to the true value of the proportion of the households with T.V. sets in the whole town are 0.355 and 0.465.

Example 9. A factory is producting 50,000 pairs of shoes daily. From a sample of 500 pairs, 2% were found to be of substandard quality. Estimate the number of pairs that can be reasonably expected to be spoiled in the daily production and assign limits at 95% level of confidence. **[C.A. (Intermediate); May 1979, May 1987]**

Solution. Here n = 500, Sample percentage p = 2%

$$S.E.(p) = \sqrt{\frac{p(100 - p)}{n}} = \sqrt{\frac{2 \times 98}{500}} = 0.63$$

Also $(1 - a)\% = 95\% \Rightarrow z_a = 1.96.$

$\therefore$ 95% confidence limits for the percentage of substandard quality are:

$$p \pm S.E.(p) z_a = 2 \pm (1.96 \times 0.63) 2 \pm 1.23$$

$= 2 - 1.23$ and $2 + 1.23 = 0.77$ and 3.23.

Number of pairs of shoes that can be expected to be spoiled in daily production lies between

$$\frac{0.77}{100} \times 50{,}000 = 385$$

and $\frac{3.23}{100} \times 50{,}000 = 1615$

Example 10. Out of 20,000 customer's ledger accounts, a sample of 600 accounts was taken to test the accuracy of posting and balancing where 45 mistakes were found. Assign limits within which the number of defective cases can be expected at 5% level.

[C.A. (Intermediate); May 1976]

Solution. Here n = 600, Sample proportion $p = \frac{45}{600} = 0.075$

$$q = 1 - 0.075 = .952.$$

$$S.E.(p) = \sqrt{\frac{pq}{n}} \sqrt{\frac{0.075 \times 0.925}{600}} = 0.011$$

Also $(1 - a)\% = 95\% \Rightarrow z_a = 1.96.$

95% confidence limits are $p \pm z_a \times S.E(p)$

$= 0.075 \pm 1.96 \times 0.011 = 0.075 \pm 0.22 =$ 0.053 to 0.97

Hence at 5% level of significance the expected number of mistakes would lie between 0.053 and 0.097. The number of defective cases will lie between $0.53 \times 20{,}000 = 1060$ and $.097 \times 20{,}000 = 1940$.

Example 11. A random sample of 500 pineapples was taken from large consignment and 65 of them were found to be bad. Show that the standard deviation of the population of bad one in a sample of this size is 0.015 and deduce that the percentage of bad pineapples in the consignment lie between 8.5 and 17.5.

Solution. Here n = 500, Sample proportion

$$p = \frac{65}{500} = 0.13 \quad q = 1 - 0.13 = 0.87.$$

Standard deviation or standard error of population of bad pineapples

$$S.E.(p) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.13 \times 0.87}{500}} = 0.015$$

Here the level is not specific so we take Z = 3.

$\therefore$ 99.73% confidence limits for the proportion of bad pineapples

$p = \pm S.E.(p) z_a = 0.13 \pm (3*0.015) = 0.13 \pm 0.045$ *i.e.* 0.085 to 0.175

Hence 99.73% confidence limits for percentage of bad pine apples 0.085 × 100 and 0.175 × 100 i.e., between 8.5% and 17.5%.

t - DISTRIBUTION

Example 12. A sample of size 9 from a normal population gave $\bar{x} = 15.8$ and $s_x^2 = 10.3$. Find a 99% interval for population mean.

[C.A. (Intermediate), November 1985]

Solution. We are given $\bar{x} = 15.5$. $s_x^2 = 10.3$ and n = 9.

$\therefore$ Degree of freedom = n – 1 = 9 – 1 = 8.

Also $t_{0.01}$ for 8 *d.f* = 3.36 **[From t table]**

99% confidence limits for the population mean $\bar{x}$ are $\bar{x} \pm t_{0.01} \frac{s}{\sqrt{n-1}}$

$$= 15.8 \pm 3.36 \times \sqrt{\frac{10.3}{8}}$$

$$= 15.8 \pm 3.36 \times 1.135 = 15.8 \pm 3.8136$$

$$= 11.9864, 196136.$$

Hence 99% confidence interval = [11.9864, 19.6136]

DETERMINATION OF SAMPLE SIZE

The determination of sample size for estimating a mean or proportion is a crucial question. By selecting a sample size lower than the correct size may affect reliability and a higher one will mean more cost and time. The determination of the size of a sample is the most important factor for the purposes of estimation of the value of the population parameters. We have following formula for it.

Sample size for Estimation a Mean

In order to determine the sample size for estimating a population mean, the following factors must be known:

***(i)* The desired confidence level.**

***(ii)* The permissible sampling error E = $\bar{x} - u$.**

***(iii)* The standard deviation 0'.**

After having known the above mentioned three factors, **the size of sample mean n is given by** $n = \left(\frac{\sigma z_\alpha}{E}\right)^2$

Sample size for Estimating a Proportion

In this case we must know the following three factors:

(*i*) the desired confidence level.

(*ii*) the permissible sampling error E = difference between the estimate from the sample p and the parameter P to be estimated = P - p.

(*iii*) the estimated true proportion of success.

The sample size n is given by

$$n = \frac{z^2 pq}{E^2} \text{ where } q = 1 - p$$

Example 13. It is known that the population standard deviation in waiting time for L.P.G. gas cylinder in Delhi is 15 days. How large a sample should be chosen to be 95% confident, the waiting time is within 7 days of true average.

Solution. The required sample size is

$$n = \left(\frac{\sigma z}{E}\right)^2 = \left(\frac{15 \times 1.96}{7}\right)^2 = 17.64.$$

Hence, the size of the sample is 18.

Example 14. A manufacturing concern wants to estimate average amount of purchase of its product in a month by the customers whose standard error is Rs. 10. Find the sample size if the maximum error is not to, exceed Rs.3 with a probability of 0.99.

Solution. It is given that'

$$p[1\bar{x} - u1,3] = 0.99$$

But $p\left[1\bar{x} - u1,258\frac{\sigma}{\sqrt{n}}\right] - 0.09....(II)$

From (I) and (II), we have

$$2.58\frac{\sigma}{\sqrt{n}} = 3 \Rightarrow \sqrt{n} = \frac{2.58 \times 10}{3}$$

or $n = \left(\frac{2.58 \times 10}{3}\right)^2 = (8.6)2$

$$= 73.96 \cong 74.$$

Hence, the sample size should be 74.

Example 15. Mr. X wants to determine on the basis of sample study the mean time required to complete a certain job so that he may be 95% confident that the mean may remain within ± days of the true mean. As per the available records the population variance is 64 days. How large should the sample be for his study?

[C.A. (Intermediate); May 1987]

Solution. Here $\sigma = \sqrt{64} = 8$. Also Z is N (0, 1).

It is given that

$$p[\bar{x} - u1 < 2] = 0.95 \ldots\ldots\ldots\ldots\ldots\ldots\text{(I)}$$

But $p\left[(\bar{x} - u) \le 1.96\frac{\sigma}{\sqrt{n}}\right] = 0.95 \ldots (II)$

From (I) and (II), we get

$$\frac{1.96 \times \sigma}{\sqrt{n}} = 2 \Rightarrow n = \left(\frac{1.96 \times 8}{2}\right)^2$$

$$= (7.84)2 = 61.42 \approx 62$$

Hence, the sample size should be 62.

Example 16. The business manager of a large company wants to check the inventory records against the physical inventories by a sample survey. He wants to almost assure that the maximum sampling error should not be more than 5% or below the true proportion of accurate records. The proportion of the accurate records is estimated at 35% from past experience. Determine the sample size.

Solution:- Here $p = 0.35, E = 5\% = 0.05$. Confidence coefficient $1 - \sigma \cong 1$

We know that

$p[|z| \geq 3] = 0.9973 \cong 1$

[From standard normal Table]

$\Rightarrow z$ lies between –3 and +3 almost surely.

Here Z = 3.

Now, sample size $n = \frac{PQz_{\alpha}^2}{E^2}$

$= \frac{0.35(1-0.35)\times 3^2}{(0.05)^2} = \frac{0.35\times 0.65\times 9}{0.0025}$

$= \frac{2.0475}{0.0025} = 819$

Hence, the sample size is 819.

Q.17. In measuring reaction time, a psychologist estimated that the standard deviation is 1.08 seconds. What should be the size of the sample in order to be 99% confident that the error of her estimates of mean would not exceed 0.18 seconds?

(*a*) 240 (*b*) 210

(*c*) 300 (*d*) None

Ans.: (*a*)

Q.18. The incidence of a particular disease in an area in such that 20 % people of that area suffers from it. What size of sample should be taken so as to ensure that the error of estimation of the proportion should not be more than 5 % with 95% confidence ?

(*a*) 206 (*b*) 246

(*c*) 305 (*d*) None

Ans.: (*b*)

PREVIOUS YEAR QUESTIONS

Q.1. Sampling fluctuations may be described as:

(*a*) The variation in the values of a sample

(*b*) The differences in the values of a parameter

(*c*) The variation in the values of a statistic

(*d*) The variation in the values of observations

[Nov. 2006]

Solution : (*c*) If we compute the value of a statistic say, mean, it is quite natural that the value of the sample mean may vary from sample to sample as the sampling units of one sample may be different from that of the another sample.

Therefore, sampling fluctuations may be described as the variation in the values of a statistic.

Q.2. A random sample of size 17 has 52 as mean. The variance is 160. The 99% confidence for the mean are:

(*a*) [42.77,61.23]

(*b*) [44,58]

(*c*) [49,51]

(*d*) [37, 18]

[Nov. 2006]

Solution : (*a*) Given : $S = \sqrt{160} = 12.65$;

Mean $\mu = 52$ and sample size *i.e.* $n = 17$

Standard Error = $SE(\bar{X}) = \frac{s}{\sqrt{n^2 - 1}}$

$\frac{12.65}{\sqrt{17-1}} = 3.1625$

'S' known and n < 30 , So t - Test will used. Degree of freedom = d.f. = n - 1 = 17-1= 16

$t_{0.005,16\,df} = 2.92$ (From Table)

$LCL = \bar{X} - t_{0.005,16} . SE(\bar{X})$

= 52 - (2.92).(3.1625) = 42.7655 = 42.77

$UCL = \bar{X} + t_{0.005,16} . SE(\bar{X})$

= 52 + (2.92).(3.1625) = 61.2345

= 61.23

Therefore, the confidence limits are given by [42.77,61.23]

Q.3. Which sampling provides separate estimates for population means for different purposes and also an over all estimate ?

(*a*) Multistage sampling

(*b*) Simple random sampling

(*c*) Systematic sampling

(*d*) Stratified sampling

[Nov. 2006]

Solution : (*d*) Stratified sampling provides separate for population means for differents segments and also an overall estimate.

Q.4. The criteria for an ideal estimator are:

(*a*) Unbiasedness, expectation, estimation and sampling

(*b*) Estimation, efficiency, expectation and sufficiency

(*c*) Consistency, efficiency, sufficiency and unbiasedness

(*d*) Estimation, consistency, efficiency and sufficiency.

[Feb. 2007]

Solution : (*c*) The criteria for an ideal estimator are consistency, efficiency, sufficiency and unbiasedness.

Q.5. The permissible sampling error is required to determine sample size for:

(*a*) Estimating a Proportion

(*b*) Estimating a Mean

(*c*) Both (*a*) & (*b*)

(*d*) None of these

[May 2007]

Solution : (*c*) The permissible sampling error is required to determine sample size for estimating a proportion and mean.

Q.6. If the population S.D. is known to be 5 for a population containing 80 units, then the standard error of sample mean for a sample of size 25 without replacement is:

(*a*) 0.83 (*b*) 0.80

(*c*) 0.93 (*d*) 0.74

[May 2007]

Solution : (*a*) $\sigma = 5, N = 80, n = 25$

In case of WITHOUT REPLACEMENT

$\therefore S.E.(\bar{X}) = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$

$= \frac{5}{\sqrt{25}} \times \sqrt{\frac{80-25}{80-1}} = \frac{5}{5} \times \sqrt{0.696} = 0.834$

=0.83

Q.7. A researcher wishes to estimate the mean of a population by using sufficiently large sample. The probability is 0.95 that the sample mean will not differ from the true mean by more than 25% of the standard deviation. How large sample should be taken?

(*a*) 72 (*b*) 62

(*c*) 42 (*d*) 32

[May 2007]

Solution : (*b*) Given; Confidence Limit = 95%

$Z_{0.05} = 1.96$

Error = E = 0.25 of $\alpha = 0.25\,\alpha$

$$n = \left(\frac{Z_{0.05}\sigma}{E}\right)^2 = \left(\frac{1.96\sigma}{0.25\sigma}\right)^2$$

$$= 61.4656$$

$\Rightarrow n \approx 62$

Q.8. In determining the sample size for estimating a population mean, the number of factors must be known is:

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

[Aug. 2007]

Solution : (*b*) 3 factors must be known for determining the sample size for estimating a population mean *i.e.*

(*i*) the desired confidence level

(*ii*) permissible sampling error

$E = \bar{X} - \mu$

(*iii*) the standard deviation σ

Q.9. When we have an idea that the error might be involved, we use:

(*a*) Point Estimate

(*b*) Interval Estimate

(*c*) Both (*a*) & (*b*)

(*d*) None of these

[Aug. 2007]

Solution : (*b*) Interval estimation is when we have an idea that the error might be involved.

Q.10. A sample of size 3 is taken from a population of 10 members with replacement. If the sample observations are 1, 3 and 5 what is the estimate of the standard error of sample mean?

(*a*) 1.02 (*b*) 1.92

(*c*) 2.37 (*d*) 3.01

[Nov. 2007]

Solution : (*a*) Here, $\sum X = 1+3+5 = 9$

$$\sum X^2 = 1+9+25 = 35$$

$$S.D. = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$$

$$= \sqrt{\frac{35}{3} - \left(\frac{9}{3}\right)^2} = \sqrt{2.666} = 1.633$$

$\therefore S.E(\bar{X})$ without replacement

$$= \frac{s}{\sqrt{n-1}} \times \sqrt{\frac{N-n}{N-1}}$$

$$= \frac{1.633}{\sqrt{3-1}} \times \sqrt{\frac{10-3}{10-1}}$$

$$= \frac{1.633}{\sqrt{2}} \times 0.882 = 1.02$$

Q.11. For a given sample of 200 items drawn from a large population, the mean is 65 and the standard deviation is 8. Find the 95% confidence limits for the population mean:

(*a*) [67.11,63.89]

(*b*) [66.11,63.89]

(*c*) [68.11, 65.89]

(*d*) None of these

[Nov. 2007]

Solution : (*b*) Given , $\bar{X} = 65$, S = 8

95% = confidence limits for population mean

So, Significance Level = 5 %

$$\text{Confidence Level} = \bar{X} \pm \frac{S}{\sqrt{n-1}} . Z_\alpha$$

(fpc is ignored since population is large)

$$= 65 \pm \frac{6}{\sqrt{200-1}} \times 1.96 = 65 \pm 1.11$$

= (66.11, 63.89)

Q.12. An unbiased estimator:

(*a*) Has the smallest variance among all estimator

(*b*) Is always the best estimator

(*c*) Has an expected value equal to the true parameter value

(*d*) Always generates the true value of the parameter

[Nov. 2007]

Solution : (*c*) An unbiased estimator has an expected value equal to the true parameter value.

OR

A statistic 't' is said to be an unbiased estimate of the corresponding population parameter 0, if E(t) = 0

$\Rightarrow$ the mean value of the sampling distribution of the statistic 't' is equal to the parameter of the population.

Q.13. A simple random sample of size 10 is drawn without replacement from a universe containing 85 units. If the mean 90 and S.D. 4 is obtained from the sample, what is the estimate of the standard error of sample mean?

(*a*) 0.1176 (*b*) 0.58

(*c*) 1.19 (*d*) 1.26

[Feb. 2008]

Solution : (*d*) Here, n = 10, N = 85,

$\bar{X} = 90$. s = 4

Since, Samples are drawn

WITHOUT REPLACEMENT

OR, $\frac{n}{N} = \frac{10}{85} = 0.1176 > 0.05$, so fpc is required.

$$S.E.(\bar{X}) = \frac{3}{\sqrt{n-1}} \times \sqrt{\frac{N-n}{N-1}}$$

$$= \frac{4}{\sqrt{10-1}} \times \sqrt{\frac{85-10}{85-1}}$$

$$= \frac{4}{\sqrt{9}} \times \sqrt{\frac{75}{84}} = \frac{4}{3} \times \sqrt{0.893}$$

$$= \frac{4}{3} \times 0.945$$

$$= 1.26$$

Q.14. A simple random sample of size 66 was drawn in the process of estimating the mean annual income of

950 families of a certain township. The mean and standard deviation of the samples were found to be Rs. 4730 and Rs. 7.65 respectively. Find a 95% confidence interval for the population mean:

(*a*) [4782.15,4731.85]

(*b*) [4728.15,4731.85]

(*c*) [4793.85,4801.85]

(*d*) None of these

[Feb. 2008]

Solution : (*b*) Given ; 95% = confidence limits for population mean

$$\text{S.E.}(\bar{X}) = \frac{S}{\sqrt{n-1}} (\because \text{Sample mean is given})$$

$$= \frac{7.65}{\sqrt{66-1}} = \frac{7.65}{\sqrt{65}} = 0.949$$

Now confidence limits are :

$$[\bar{X} \pm Z_\alpha \times S.E.(\bar{X})]$$

$$= [4730 \pm 0.948 \times 1.96]$$

$$= [4730 \pm 1.85] = [4728.15, 4728.85]$$

Q.15. The sample mean is:

(*a*) An MVUE for population mean

(*b*) A sufficient estimator for population mean

(*c*) A consistent and efficient estimator for population mean

(*d*) All of these

[Feb. 2008]

Solution : (*d*) The Sample Mean fulfils all the conditions of a good estimator.

1. MVUE (Min. var. unbiased Estimator)
2. Sufficiency
3. Consistency
4. Efficiency

Q.16. Which sampling adds flexibility to the sampling process?

(*a*) Systematic sampling

(*b*) Multistage sampling

(*c*) Stratified sampling

(*d*) Simple random sampling

[Feb. 2008]

Solution : (*b*) Multistage Sampling adds flexibility into the sampling process which is lacking in other schemes.

Note : Students are advised to take note of the final result only and not deliberate on the calculations.

Q.17. For any unknown parameter, how many interval estimates exist?

(*a*) 1 (*b*) 2

(*c*) Many (*d*) 3

[June 2008]

Solution : (*c*) For any unknown parameter many interval estimates exist i.e. a range of values.

Q.18. If it is known that the 95% L.C.L. and U. C. L. to population mean are 48.04 and 51.96 respectively, what is the value of the population variance when the sample size is 100?

(*a*) 9 (*b*) 11

(*c*) 12 (*d*) 100

[June 2008]

Solution : (*d*) As we know, 95 % confidence limits are.

$$\Rightarrow \bar{x} \pm \text{S.E.}(\bar{x}).z_\alpha$$

$$\Rightarrow \bar{x} - S.E.(\bar{x}) \times 1.96 = 48.04 \ldots\ldots \quad (1)$$

$$\Rightarrow \bar{x} + S.E.(\bar{x}) \times 1.96 = 51.96 \ldots\ldots \quad (2)$$

Subtracting (1) from (2)

$$2.\ \text{SE}(\bar{x}) \times 1.96 = 3.92$$

$$\therefore S.E.(\bar{x}) = 1$$

But $\text{S.E.} = (\bar{x}) = \frac{\sigma}{\sqrt{n}}$ (from above)

$$\Rightarrow \frac{\sigma}{\sqrt{n}} = 1$$

$$\Rightarrow \frac{\sigma}{\sqrt{100}} = 1$$

$$\sigma^2 = 100$$

Q.19. The hypothesis which is tested for possible rejection under assumption, that is known is:

(*a*) Null Hypothesis

(*b*) Alternate Hypothesis

(*c*) Simple Hypothesis

(*d*) Composite Hypothesis

[Dec. 2008]

Solution : (*a*) In tests of hypothesis, we always begin with an assumption or hypothesis (*i.e.* an assumed value of a population parameter). This is called. Null Hypothesis. R.A. Fisher defined :—

Null Hypothesis as "the Hypothesis which is tested for possible rejection under the assumption that it is true."

Q.20. A Population comprises of 20 members. The number of all possible samples of size 2 that can be drawn from it without replacement.

(*a*) 210 (*b*) 380

(*c*) 190 (*d*) 400

[Dec. 2008]

Solution : (*c*) Sampling distribution without replacement:

As N = 20 and n = 2, the total number of possible samples WOR

$$= {}^{N}C_2 = {}^{20}C_2 = 190$$

Q.21. For an unbiased estimator which of these is not an absolute condition.

(*a*) Efficiency

(*b*) Sufficiency

(*c*) Unbiasedness

(*d*) Minimum variance

[Dec. 2008]

Solution : (*d*) For an unbiased estimator, minimum variance is not an absolute condition.

Q.22. In sampling, standard deviations are known as:

(*a*) Expectation

(*b*) Sampling Errors

(*c*) Standard Error

(*d*) All of the above

[June 2009]

Solution : (*c*) The Sampling Distribution possesses different characteristics. The mean of statistic, as obtained from its sampling distribution, is known as "expectation" and the standard deviation of the statistic T is known as the "Standard Error (SE)"

Q.23. When every member in population has an equal chance of being

selection, then that sampling is called_____.

(*a*) Restrictive

(*b*) Purposive

(*c*) Subjective

(*d*) Non-restrictive

[June 2009]

Solution : (*d*) When the units are selected independent of each other in such a way that each unit belonging to the population has an equal chance of being a part of the sample, the sampling is known as Simple Random Sampling.

Simple Random Sampling is also known as Non-Restrictive sampling as the sample is selected randomly and there is no restriction.

Q.24. Except sampling error, other errors in sampling are:

(*a*) Non-sampling errors

(*b*) Standard errors

(*c*) Sampling fluctuations

(*d*) All of these

[June 2009]

Solution : (*a*) Errors or biases in a survey may be defined as the deviation between the value of population parameter as obtained from a sample and its observed value. Errors are of two types:

(*i*) Sampling Errors-

Since only a part of the population is investigated, every sampling design is subjected to this of error. Factors contributing to sampling error are:

(*a*) Errors arising out due to defective planning design.

(*b*) Errors arising out due to substitution.

(*c*) Errors owing to faulty demarcation of units, etc.

(*ii*) Non-Sampling Errors-

This type of errors happen both in sampling and complete enumeration. Some factors responsible for this particular kind of biases are lapse of memory, preference for certain digits, ignorance, etc.

Q.25. Distribution formed of all possible value of statistics is called____.

(*a*) Sampling Distribution

(*b*) Classification

(*c*) Tabulation

(*d*) None

[Dec. 2009]

Solution : (*a*) Distribution formed of all possible value of statistics is called Sampling Distribution [Self-Explanatory].

Q.26. In sampling, standard error is:

(*a*) Standard deviation

(*b*) Quartile deviation

(*c*) Mean deviation

(*d*) Coefficient of variation

[Dec. 2009]

Solution : (*a*) The standard deviation of the statistic, as obtained from its sampling distribution is known as "Standard Error".

Therefore, in sampling, standard error is Standard Deviation.[Self-Explanatory].

Q.27. If every 9th unit is selected from universal set then this type of sampling is known as:

(*a*) Quota Sampling

(*b*) Systematic Sampling

(*c*) Stratified Sampling

(*d*) None of these

[Dec. 2009]

Solution : (*b*) Systematic sampling refers to a sampling scheme where the units constituting the sample are selected at regular interval after selecting the very first unit at random *i.e.* with equal probability. Therefore, if 9th unit is selected from universal set, then it is known as systematic sampling.

Q.28. The Standard deviation of the sampling distribution of a statistical distribution is a:

(*a*) Critical value

(*b*) Biased estimate

(*c*) Unbiased estimate

(*d*) Standard error

[Dec. 2010]

Solution : (*d*) The standard deviation of the statistic, as obtained from its sampling distribution is known as "STANDARD ERROR".

Therefore in sampling, standard error is Standard Deviation. [Self-Explanatory]

Q.29. The sampling is said to be large sampling if the size of the sample is:

(*a*) Greater than or equal to 30

(*b*) Less than 30

(*c*) Less than or equal to 35

(*d*) Less than 25

[Dec. 2010]

Solution : (*a*) Sampling is sad to be large sampling if the size of sample is greater than or equal to 30. [Self-Explanatory]

Q.30. The errors other than sampling error are termed as:

(*a*) Response error

(*b*) Non-response error

(*c*) Non-sampling error

(*d*) Type II error

[Dec. 2010]

Solution : (*c*) Errors are of two types-

(1) Sampling errors

(2) Non-sampling errors

Q.31. The method of sampling in which each unit of the population has an equal chance of being selected in the sample is:

(*a*) Random sampling

(*b*) Stratified sampling

(*c*) Systematic sampling

(*d*) None of the above

[Dec. 2010]

Solution : (*a*) Simple Random Sampling (SRS) :

When units are selected independent of each other in such a way that each unit belonging to the population has an equal chance of being a part of the samples, the sampling is known as Simple Random Sampling or just Random Sampling.

Q.32. The hypothesis which is tested for possible rejection under the assumption that it is known as:

(*a*) Biased hypothesis

(*b*) Alternative hypothesis

(c) Null hypothesis

(d) None of these

[Dec. 2010]

Solution : (c) Null hypothesis. (As per definition)

Q.33. A selection procedure of a sample, having no involvement of probability is known as:

(a) Purposive Sampling

(b) Judgment Sampling

(c) Subjective Sampling

(d) All of the above

[Dec. 2010]

Solution : (d) All of the above (as per definition)

Q.34. Standard Deviation of a Sampling Distribution is called_____.

(a) Standard Error

(b) Statistic

(c) Parameter

(d) None of the above

[June 2011]

Solution : (a) When the average amount of the variability of the observations of a population is computed, it is called the Standard Deviation. But when the average amount of the variability of the observations of a sampling distribution of a statistic is computed, it is known as Standard Error.

Thus, the Standard Deviation of the Sampling Distribution is known as Standard Error.

Q.35. If the standard error is 1/2 and the parameter of Normal population given to be S.D = 4 then, find the sample size ?

(a) 1024 (b) 16

(c) 64 (d) 36

[June 2011]

Solution : (c) Since, Standard Error

$$S.E = (\overline{X}) = \frac{\sigma}{\sqrt{n}}$$

n = sample size = ?

Given σ = S.D = 4 and S.E = 1/2

$$\therefore \frac{1}{2} = \frac{4}{\sqrt{n}} \text{ ; or, } n = 64$$

Q.36. The total number of samples of size n drawn from a population of size N units by simple random sampling without replacement is:

(a) N^n (b) ${}^NC_n$

(c) ${}^nC_N$ (d) n^N

[Dec. 2011]

Solution : (b) The total number of possible sample of size n out of N without Replacement = ${}^NC_n$

Q.37. Type II errors are made while we accept a null hypothesis which is _____.

(a) True (b) False

(c) Doubtful (d) Not Defined

[Dec. 2011]

Solution : (b) Type II errors are made by accepting the Null Hypothesis (H_0) when it is FALSE, that is when a Null Hypothesis is false but it is accepted due to insignificant difference between observed & expected values.

Q.38. In order to test the quality of chalks the best suitable method will be_____.

(a) Complete enumeration

(b) Simple random sampling

(c) Systematic sampling

(d) Stratified sampling

[June 2012]

Solution : (b) The correct answer is Simple Random Sampling because the population is Homogeneous & not very large.

Q.39. In a factory there are 48 employees with employee code from 1 to 48, the employer desires to take the sample of every sixth employee under the systematic sampling technique, the sample size will be:

(a) 6 (b) 8

(c) 10 (d) 7

[June 2012]

Solution : (b) Under Systematic Sampling Technique, we know:

N = nk ($\because$ 'k' < n)

Where N = Population size = 48

n = Sample size = ?

K = Sample interval = 6

So, 48 = n × 6

n = 8

Q.40. The statistic T is said to be consistent estimator of the population parameter 'Q' if:

(a) E(T) = Q

(b) $V(T) \to 0 as n \to \infty$

(c) Both (a) and (b)

(d) None of the above

[June 2012]

Solution : (c) Consistency and efficiency:

"A statistic 't' is known to be consistent estimator of the parameter '0' if the difference between T and 0 can be made smaller and smaller by taking the sample size 'n' large and large,"

Mathematically, T is consistent for 0 if

E(T) = 0

and $V(T) \to 0$ as $n \to \infty$

Q.41. Sampling is a tool which helps to know the characteristics of the ____ by examining only a small part of it:

(a) Population (b) Data

(c) Sample (d) All the above

[Dec. 2012]

Solution : (c) Sampling is a tool which helps to know the characteristic of the sample by examining only a small part of it.

Q.42. The criteria for an ideal estimator are:

(a) Unbiased and minimum variance

(b) Consistency and efficiency

(c) Sufficiency

(d) All of the above

[Dec. 2012]

Solution : (d) The criteria for an ideal estimator are unbiased and minimum variance consistency and efficiency and sufficiency.

Q.43. If N, n denote sizes of the population and its factor of finite population, correction is given by:

(*a*) $\sqrt{(N-n)/(N-1)}$

(*b*) $\sqrt{(N-1)/(N-n)}$

(*c*) $\sqrt{(1-N)/(N-n)}$

(*d*) $\sqrt{(N+n)/(N-1)}$

[Dec. 2012]

Solution : (*a*) Factor of finite population correction is given by $\sqrt{\frac{N-n}{N-1}}$

MODEL EXAM QUESTIONS (FOR PRACTICE)

Type - I

Select the correct alternative out of the given ones:

Q.1. Sampling errors are:

(*a*) caused by inaccurate measurement.

(*b*) the result of the chance selection of the sampling units.

(*c*) of no great concern.

(*d*) larger for a census than for a sample.

Q.2. Non-sampling errors are

(*a*) caused by inaccurate measurement.

(*b*) the result of the chance selection of the sampling units.

(*c*) of no great concern.

(*d*) always larger for a census than fen a sample.

Q.3. If μ_x w is the population mean, and σ_x^2 is the population variance, then the mean and variance of a sample are equal to

(*a*) μ_x and σ_x^2

(*b*) μ_x/n and σ_x^2/n

(*c*) μ_x/n and σ_x^2/n^2

(*d*) μ_x and σ_x^2/n.

Q.4. A sample consists of

(*a*) all units of the population.

(*b*) 50 per cent units of the population.

(*c*) 5 per cent units of the population.

(*d*) any fraction of the population.

Q.5. If we sample without replacement,

(*a*) it is important to consider the size of the sample relative to the size of the population.

(*b*) a larger sample relative to the size of the population is preferred because it will reduce the sampling error.

(*c*) the sample size is not important.

(*d*) use a smaller sample.

Q.6. If we sample without replacement and the sample IS large relative to the population,

(*a*) the sample variance will be small.

(*b*) the sample mean will be large.

(*c*) the sample variance will large

(*d*) the sample variance must be adjusted

Q.7. Probability of selection varies at each subsequent draw in:

(*a*) sampling without replacement.

(*b*) sampling with replacement.

(*c*) both (*a*) and (*b*).

(*d*) neither (*a*) nor (*b*).

Q.8. The number of possible samples of size 11 from a population of N units with replacement is:

(*a*) n^2 (*b*) N^2

(*c*) ∞ (*d*) none of these.

Q.9. A function of variates for estimating a parameter is called:

(*a*) an estimate

(*b*) an estimator

(*c*) a frame

(*d*) a statistic.

Q.10. Let the standard error of an estimator T under srswor is more than the standard error of T under stratified randomly sampling. Then T under stratified sampling as compared to T under srswor is:

(*a*) more reliable.

(*b*) equally reliable.

(*c*) less reliable.

(*d*) not comparable.

Q.11. Which of the following basis distinguishes cluster sampling from stratified sampling?

(*a*) A sample is always drawn from each stratum whereas no sample of elementary units is drawn from clusters.

(*b*) Clusters are preferably heterogeneous whareas strata are taken as homogeneous as possible.

(*c*) Small size clusters are better whereas there is no such restriction for stratum size.

(*d*) all of these.

Q.12. Non-response in surveys mean:

(*a*) non-return of questionnaire by the respondents.

(*b*) non-availability of respondents.

(*c*) refusal to give information by the respondents.

(*d*) all of these.

Q.13. Choose the pair of symbols that best completes this sentence: _____ is a parameter, whereas _____ is a static.

(*a*) N, μ (*b*) N, n

(*c*) σ, s (*d*) all of these.

Q.14. Regarding the number of strata, which statement is true?

(*a*) More the number of strata, poorer it is.

(*b*) Lesser the number of strata, better it is.

(*c*) More the number of strata, better it is.

(*d*) Not more than ten items should be there in a stratum.

Q.15. In random sampling, we can describe mathematically how objective our estimates are. Why is this?

(*a*) We always know the chance that a population element will be included in the sample.

(*b*) Every sample always has an equal chance of being selected.

(*c*) All the samples are of exactly the same size and can be counted.

(*d*) (*a*) and (*b*) but not (*c*).

Q.16. The magnitude of the standard error of an estimate is an index of its:

(*a*) accuracy (*b*) precision

(*c*) efficiency (*d*) all of these.

Q.17. An estimate based on a fixed set of values of a sample always possess:

(*a*) a single value

(*b*) any value

(*c*) a value equal to one

(*d*) all of these.

Q.18. If the sample values are 1, 3, 5, 7, 9, the standard error of sample mean is :

(*a*) S.E. = $\sqrt{2}$

(*b*) S.E. = $1/\sqrt{2}$

(*c*) S.E. = 2.0

(*d*) S.E. = 1/2.

Q.19. Which of the following statements does not hold good in case of stratified sampling?

(*a*) Stratified sampling is always good.

(*b*) Stratified sampling is convenient.

(*c*) Reduces error for fixed cost.

(*d*) Enables to gather information about different stratum separately.

Q.20. What precaution(s) make(s) cluster sampling more efficient?

(*a*) Choosing clusters having largest within variation.

(*b*) By taking clusters of small size.

(*c*) Choosing clusters having least variation clusters.

(*d*) All of these.

Q.21. Suppose you are performing stratified sampling on a particular population and have divided the population into strata of different sizes. How can you now make your sample selection?

(*a*) Select at random an equal number of elements from each stratum.

(*b*) Draw an equal number of elements from each stratum and give weights to the results.

(*c*) Draw a number of elements from each stratum proportional to its weight in the population.

(*d*) (*b*) and (*c*) only.

Q.22. A population is divided into clusters and it has been found that all items within a cluster are alike. Which of the following sampling procedures would you adopt?

(*a*) Cluster sampling.

(*b*) Simple random sampling.

(*c*) Systematic sampling

(*d*) Stratified sampling.

Q.23. In which of the following situations would $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ be the correct formula to use for computing $\sigma_{\bar{x}}$?

(*a*) Sampling is from a finite population with replacement.

(*b*) Sampling is from an infinite population.

(*c*) Sampling is from a finite population without replacement.

(*d*) (*a*) and (*b*) only.

Q.24. If population variance of an infinite population is 2 Cr and a sample of n items is selected from this population, the standard error of sample mean is equal to:

(*a*) σ/n (*b*) σ^2/n

(*c*) $\sigma/\sqrt{n}$ (*d*) σ.

25. A population is perfectly homogeneous in respect of a characteristic. What size of sample would you prefer?

(*a*) A large sample

(*b*) A small sample

(*c*) A single item

(*d*) No item.

Q.26. A systematic sample does not yield good results if:

(*a*) variation in units is periodic.

(*b*) units at regular intervals are corrected.

(*c*) both (*a*) and (*b*).

(*d*) none of (*a*) and (*b*).

Q.27. Double sampling is also known as:

(*a*) two stage sampling.

(*b*) two phase sampling.

(*c*) two directional sampling.

(*d*) all of these.

Q.28. If we sample with replacement,

(*a*) it is important to consider the size of the sample relative to the size of the population.

(*b*) a larger sample is preferred.

(*c*) the sample variance will not be biased.

(*d*) a smaller sample is preferred.

Q.29. If we sample with replacement and the sample is large relative to the population.

(*a*) the sample mean will be biased.

(*b*) the sample variance will be biased.

(*c*) the sample mean will be large.

(*d*) no adjustments need to be made.

Q.30. The number of possible samples of size n out of N population units without replacement is:

(*a*) ${}^{N}C_{n}$ (*b*) $(N)^n$

(*c*) n^2 (*d*) n !.

Q.31. Probability of anyone sample of size n being drawn out of N units is:

(*a*) 1/N (*b*) n/N

(*c*) 1/n ! (*d*) $1/\binom{N}{n}$

32. Two stage sampling design is more efficient than single stage sampling if the correlation between units in the first stage is:

(*a*) negative (*b*) positive

(*c*) zero (*d*) none of these.

Q.33. The discrepancy between estimates and population parameters is known as:

(*a*) human error.

(*b*) enumeration error.

(*c*) sampling error.

(*d*) formula error.

Q.34. If larger units have greater probability of their inclusion in the sample, it is known as:

(*a*) selection with replacement.

(*b*) selection with probability proportional to size.

(*c*) probability selection.

(*d*) selection with constant probability.

Q.35. If there is a certain number of very high values in a sample, then it is preferable to calculate:

(*a*) standard deviation.

(*b*) standard error.

(*c*) variance.

(*d*) all of these.

Q.36. A population consisting of the results of the conceptually repeated trials is known as:

(*a*) hypothetical population.

(*b*) infinite population.

(*c*) finite population.

(*d*) real population.

Q.37. The dispersion among sample means is less than the dispersion among the sampled items themselves because:

(*a*) each sample is smaller than the population from which it is drawn.

(*b*) very large values are averaged down, and very small values are averaged up.

(*c*) the sampled items are all drawn from the same population.

(*d*) none of these.

Q.38. A selection procedure of a sample having no involvement of probability is known as:

(*a*) judgement sampling.

(*b*) purposive sampling.

(*c*) subjective sampling.

(*d*) all of these.

Q.39. Sampling is inevitable in the situation (s):

(*a*) when the population is infinite.

(*b*) blood test of a person.

(*c*) testing of life of dry battery cells.

(*d*) all of these.

Q.40. An unordered sample of size n can occur m:

(*a*) n ways (*b*) n ! ways

(*c*) n^2 ways (*d*) one way.

Q.41. Suppose that a population with N = 144 has $\mu = 24$. What is the mean of the sampling distribution of the mean for samples of size 25?

(*a*) 24

(*b*) 2

(*c*) 4.8

(*d*) Cannot be determined from the information given.

Q.42. In simple random sampling with replacement, the same sampling unit may be included in the sample:

(*a*) only twice

(*b*) only once

(*c*) more than once

(*d*) none of these.

Q.43. If each and every unit of a population has equal chance of being included in the sample, it is known as:

(*a*) purposive sampling

(*b*) restricted sampling.

(*c*) subjective sampling.

(*d*) unrestricted sampling.

Q.44. The errors emerging out of faulty planning of surveys are categorised as:

(*a*) non-sampling errors.

(*b*) non-response errors.

(*c*) absolute error.

(*d*) sampling errors.

Q.45. When an investigator wants a sample containing m units which possess a rare attribute, the appropriate sampling procedure is:

(*a*) stratified sampling.

(*b*) srswor.

(*c*) inverse sampling.

(*d*) all of these.

Q.46. Probability of drawing a unit at each selection remains same in :

(*a*) srswor.

(*b*) srswr.

(*c*) both (*a*) and (*b*)

(*d*) none of (*a*) and (*b*).

Q.47. The central limit theorem assures us that the sampling distribution of the mean:

(*a*) is always normal.

(*b*) is always normal for large sample Sizes.

(*c*) approaches normality as sample Size increases.

(*d*) appears normal only when N is greater than 1,000.

Q.48. Simple random, sample can be drawn with the help of:

(*a*) random number tables

(*b*) chit method

(*c*) roulette wheel

(*d*) all of these.

Q.49. Suppose that, for a certain population, $\sigma_{\bar{x}}$ is calculated as 20 when samples of size 25 are taken and as 10 when samples of size 100 are taken. A quadrupling of sample size, then, only halved $\sigma_{\bar{x}}$. We can conclude that increasing sample size is:

(*a*) always cost-effective.

(*b*) sometimes cost-effective.

(*c*) never cost-effective.

(*d*) none of these.

Q.50. Increase in reliability and accuracy of results from a sampling study with the increase in sample size is known as the principle of:

(*a*) statistical regularity.

(*b*) optimisation.

(*c*) law of increasing returns.

(*d*) inertia of large numbers.

Q.51. Suppose that, for a certain population, $\sigma_{\bar{x}}$ is calculated as 20 when samples of size 25 are taken and as 10 when samples of size 100 are taken. A quadrupling of sample size, then, only halved $\sigma_{\bar{x}}$. What must be the value of σ for this infinite population?

(*a*) 1,000 (*b*) 500

(*c*) 377.5 (*d*) 100

Q.52. A population consisting of all the items which are physically present is called:

(*a*) hypothetical population.

(*b*) real population.

(*c*) infinite population.

(*d*) none of these.

Q.53. A population consisting of all real numbers is an example of:

(*a*) an infinite population.

(*b*) a finite population.

(*c*) an imaginary population.

(*d*) none of these.

Q.54. The number of all possible samples of size two from a population of 4 units is :

(*a*) 2 (*b*) 4

(*c*) 8 (*d*) 12.

Q.55. Probability of including a specified unit in a sample of size n selected out of N units is:

(*a*) l/n (*b*) 1/N

(*c*) n/N (*d*) N/n.

Q.56. What sampling design is most appropriate for cluster sampling?

(*a*) Simple random sampling without replacement.

(*b*) Stratified random sampling.

(*c*) Simple random sampling with replacement.

(*d*) Quota sampling.

Q.57. To meet requirement of the principle of validity of sampling methods, one must adopt:

(*a*) purpose sampling.

(*b*) restricted sampling.

(*c*) probability sampling.

(*d*) none of these.

Q.58. An estimator can possess :

(*a*) a fixed value

(*b*) any value

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.59. The finite population multiplier does not have to be used when the sampling fraction is:

(*a*) greater than, 0.05.

(*b*) greater than 0.50.

(*c*) less than 0.50.

(*d*) none of these.

Q.60. If the items are destroyed under investigation, we have to go for :

(*a*) complete enumeration

(*b*) sampling studies

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.61. What distinction exists between cluster sampling and two stage sampling ?

(*a*) In cluster sampling, one studies each unit of the selected cluster, whereas in two stage sampling one selects a sample of elementary units from each cluster.

(*b*) In two stage, sampling one draws a sample in two stages, whereas in cluster sampling only a sample of clusters is selected.

(*c*) Both (*a*) and (*b*).

(*d*) none of these.

Q.62. A population was divided into clusters and it was found that within cluster variation was less than the variation between clusters. If a sample of units was selected from each cluster, the sampling procedure used was:

(*a*) multistage sampling.

(*b*) stratified sampling.

(*c*) cluster sampling.

(*d*) none of these.

Q.63. Systematic sampling means:

(*a*) selection of n contiguous units.

(*b*) selection of n units situated at equal distances.

(*c*) selection of n largest units.

(*d*) none of these.

Q.64. The standard error of the mean for a sample size of two or more is:

(*a*) always greater than the standard deviation of the population.

(*b*) generally, greater than the standard deviation of the population.

(*c*) usually, less than the standard deviation of the population.

(*d*) none of these.

Q.65. In which of the following situation(s) cluster sampling is appropriate?

(*a*) When the units are situated for apart.

(*b*) When sampling frame is not available.

(*c*) When all the elementary units are not easily identifiable.

(*d*) All of these.

Q.66. The selected items of a sample resulted into same values pertaining to a character. The variance of the sample is:

(*a*) 2 (*b*) 0

(*c*) 1 (*d*) none of these.

Q.67. A sample of 16 items from an infinite population having S.D. = 4, yielded total scores as 160. The standard error of sampling distribution of mean is:

(*a*) 1 (*b*) 20

(*c*) 30 (*d*) none of these.

Q.68. Double sampling has its utility in:

(*a*) stratified sampling.

(*b*) ratio method of estimation.

(*c*) regression method of estimation.

(*d*) all of these.

Q.69. A border patrol checkpoint which stops every passenger van is utilising:

(*a*) simple random sampling.

(*b*) systematic sampling.

(*c*) stratified sampling.

(*d*) complete enumeration.

Q.70. Selected units of a systematic sample are:

(*a*) not easily locateable.

(*b*) easily locateable.

(*c*) not representing the whole population.

(*d*) all of these.

Q.71. Greatest drawback of systematic sampling is that:

(*a*) one requires a large sample.

(*b*) data are not easily accessible.

(*c*) no single reliable formula for standard error of mean is available.

(*d*) none of these.

Q.72. In a normally distributed population, the sampling distribution of the mean:

(*a*) is normally distributed.

(*b*) has a mean equal to the population mean.

(*c*) has standard deviation equal to the population standard deviation divided by the square root of the simple size.

(*d*) all of these.

Q.73. Under proportional allocation, the size of the sample from each stratum depends on:

(*a*) size of the stratum.

(*b*) total sample size.

(*c*) population size.

(*d*) all of these.

Q.74. Which of the following statements is true?

(*a*) All sampling procedures involve sampling with constant probability.

(*b*) There exists sampling procedure in which the units are selected with varying probability.

(*c*) Every selection procedure of a sample involves probability.

(*d*) none of these.

Q.75. The central limit theorem:

(*a*) requires some knowledge of the frequency distribution.

(*b*) permits us to use sample statistics to make inferences about population parameters.

(*c*) relates the shape of a sampling distribution of the mean to the mean of the sample.

(*d*) requires a sample to contain fewer than 30 observations.

Q.76. Under equal allocation in stratified sampling, the sample from each stratum IS:

(*a*) proportional to stratum size.

(*b*) of same size from each stratum.

(*c*) in proportion to the per unit cost of survey of the stratum.

(*d*) none of these.

Q.77. The errors in a survey other than sampling errors are called

(*a*) planning error

(*b*) formula errors

(*c*) non-sampling error

(*d*) none of these.

Q.78. The most important factor in determining the size of a sample is:

(*a*) purpose of the survey.

(*b*) the availability of resources.

(*c*) heterogeneity of population.

(*d*) none of these.

Q.79. Stratified sampling belongs to the category of:

(*a*) subjective sampling.

(*b*) judgement sampling.

(*c*) controlled sampling.

(*d*) non-random sampling.

Q.80. There are more chances of non-sampling errors than sampling errors in case of:

(*a*) studies of large samples.

(*b*) complete enumeration.

(*c*) inefficient investigators.

(*d*) all of these.

Q.81. Stratified sampling comes under the category of:

(*a*) unrestricted sampling.

(*b*) subjective sampling.

(*c*) purposive sampling.

(*d*) restricted sampling.

Q.82. Which one problem out of the four is not related to stratified sampling?

(*a*) Fixing the number of strata.

(*b*) Fixing the criterion for stratification.

(*c*) Fixing the sample size.

(*d*) Fixing the points of demarcation between strata.

Q.83. In what situation, two stage sampling is better than single stage sampling?

(*a*) When the elements in the same stage are positively correlated.

(*b*) When the elements in the same stage are negatively correlated.

(*c*) When the elements in the same stage.

(*d*) none of these.

Q.84. Sampling error can be reduced by:

(*a*) choosing a proper probability sampling.

(*b*) selecting a sample of adequate size.

(*c*) using a suitable formula for estimation.

(*d*) all of these.

Q.85. Which of the following statement is correct?

(*a*) Simple random sample is inferior than systematic sample.

(*b*) Systematic sample is superior than stratified random sample.

(*c*) Stratified random sample is better than systematic sample.

(*d*) None of these.

Q.86. In what situation(s), a systematic sample is more preferred than others?

(*a*) When the items are in row.

(*b*) When the data are on cards.

(*c*) When the items situated at equal distances are uncorrelated.

(*d*) all of these.

Q.87. Which of the following advantage of systematic sampling you approve?

(*a*) Economical

(*b*) Easy selection of sample

(*c*) Spread of sample over the whole population

(*d*) all of these.

Q.88. Sampling is the process of obtaining a

(*a*) population (*b*) sample

(*c*) frequency (*d*) none of these.

Q.89. Sampling can be described as a statistical procedure

(*a*) to infer about the unknown universe from a knowledge of any sample.

(*b*) to infer about the known universe from a knowledge of a sample drawn from it.

(*c*) to infer about the unknown.

(*d*) Both (*a*) and (*b*).

Q.90. A parameter is a characteristic of

(*a*) Population

(*b*) Sample

(*c*) Both (*a*) and (*b*)

(*d*) none of these.

Q.91. Statistical decision about an unknown universe is taken on the basis of

(*a*) sample observations.

(*b*) a sampling frame.

(*c*) sample survey.

(*d*) compute enumeration.

Q.92. Statistical data may be collected by complete enumeration called

(*a*) census inquiry.

(*b*) sample inquiry.

(*c*) Both

(*d*) none

Q.93. By using sampling methods we have:

(*a*) the error estimation and less quality data.

(*b*) higher quality data and higher costs.

(*c*) the error estimation and higher quality data.

(*d*) less quality data and lower costs.

Q.94. The main object of sampling is to state the limits of accuracy of estimates base on samples.

(*a*) Yes.

(*b*) No.

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.95. A sample survey is prone to:

(*a*) Non-sampling errors.

(*b*) Sampling errors.

(*c*) Either (*a*) or (*b*).

(*d*) Both (*a*) and (*b*).

Q.96. A sample is a selected part of the

(*a*) estimation (*b*) population

(*c*) both (*d*) none of these.

Q.97. Two basic Statistical laws concerning a population are

(*a*) the law of statistical irregularity and the law of inertia of large numbers.

(*b*) the law of statistical regularity and the law of inertia of large numbers.

(*c*) the law of statistical regularity and the law of inertia of small numbers.

(*d*) the law of statistical irregularity and the law of inertia of small numbers.

Q.98. Sampling Fluctuations may be described as:

(*a*) the variation in the values of a sample.

(*b*) the variation in the values of a sample.

(*c*) the differences in the values of a parameter.

(*d*) the variation in the values of observations.

Q.99. The Law of Statistical Regularity says that:

(*a*) Sample drawn from the population under discussion possesses the characteristics of the population

(*b*) A large sample drawn at random from the population would possess the characteristics of the population.

(*c*) A large sample drawn at random from the population would possess the characteristics of the population on an average.

(*d*) An optimum level of efficiency can be attained at a minimum cost.

Q.100. The population of roses in Salt Lake City is an example of:

(*a*) A hypothetical population

(*b*) An infinite population

(*c*) A finite population

(*d*) An imaginary population.

Q.101. Which sampling is subjected to the discretion of the sampler?

(*a*) Simple random sampling

(*b*) Systematic sampling

(*c*) Purposive sampling

(*d*) Quota sampling.

Q.102. Which sampling provides separate estimates for population means for different segments and also overall estimate?

(*a*) Multistage sampling

(*b*) Stratified sampling

(*c*) Systematic sampling

(*d*) Simple random sampling

Q.103. Which sampling adds flexibility to the sampling process?

(*a*) Multistage sampling

(*b*) Simple random sampling

(*c*) Stratified sampling

(*d*) Systematic sampling.

Q.104. The difference of the actual value and the expected value using a model is:

(*a*) Error in statistics

(*b*) Absolute error.

(*c*) Percentage error.

(*d*) Relative error.

Q.105. Which sampling is affected most if the sampling frame contains an undetected periodicity?

(*a*) Simple random sampling.

(*b*) Multistage sampling.

(*c*) Stratified sampling.

(*d*) Systematic sampling.

Q.106. Sample mean is an example of:

(*a*) parameter (*b*) statistic

(*c*) both (*d*) none.

Q.107. Large sample is that sample whose size is:

(*a*) greater than 30.

(*b*) greater than or equal to 30.

(*c*) less than 20.

(*d*) less than or equal to 30.

Q.108. Population mean is an example of:

(*a*) parameter

(*b*) statistic

(*c*) both (*a*) and (*b*)

(*d*) none.

Q.109. The finite population multiplier is ignored when the sampling fraction is:

(*a*) greater than 0.05.

(*b*) less than 0.6.

(*c*) less than 0.05.

(*d*) greater than 0.6.

Q.110. The ways of selecting a sample are:

(*a*) Random sampling

(*b*) Multi-stage sampling

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.111. Random sampling implies

(*a*) Probability sampling

(*b*) Haphazard sampling

(*c*) Systematic sampling

(*d*) Sampling with the same probabililty for each unit.

Q.112. Simple random sampling is:

(*a*) a probabilistic sampling

(*b*) a mixed sampling.

(*c*) a non-probabilistic sampling.

(*d*) Both (*b*) and (*c*).

Q.113. If random sampling with replacement is applied, then the mean of sample means will be ..... the population mean

(*a*) greater than

(*b*) less than

(*c*) exactly equal to

(*d*) none of these.

Q.114. Simple random sampling is very effective if

(*a*) the population is not very large.

(*b*) the population is not much heterogeneous.

(*c*) the population is partitioned into several sections.

(*d*) Both (*a*) and (*b*).

Q.115. The number of types of random sampling is:

(*a*) 2 (*b*) 3

(*c*) 1 (*d*) 4.

Q.116. Random sampling is called lottery sampling.

(*a*) True

(*b*) False

(*c*) Both (*a*) and (*b*)

(*d*) none of these.

Q.117. Stratified random sampling is appropriate when the universe is not homo-geneous.

(*a*) True

(*b*) False

(*c*) Both (*a*) and (*b*)

(*d*) none of these.

Q.118. Random numbers are also called Random sampling number.

(*a*) True

(*b*) False

(*c*) Both (*a*) and (*b*)

(*d*) none of these.

Q.119. Cluster sampling is ideal in case the data are widely scattered.

(*a*) True

(*b*) False

(*c*) Both (*a*) and (*b*)

(*d*) none of these.

Q.120. In stratified sampling, the sampling is subdivided into several parts, called

(*a*) strata. (*b*) strati.

(*c*) start. (*d*) none of these.

Q.121. The Exploratory sampling is known as:

(*a*) Estimation sampling

(*b*) Acceptance sampling

(*c*) Discovery sampling

(*d*) none of these.

Q.122. Deliberate sampling is free from bias.

(*a*) True

(*b*) False

(*c*) Both (*a*) and (*b*)

(*d*) none of these.

Q.123. Single, double, multiple and sequential are several types of:

(*a*) Discovery sampling method

(*b*) Acceptance sampling method

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.124. Purposive selection is resorted to in case of judgment sampling.

(*a*) True

(*b*) False

(*c*) Both (*a*) and (*b*)

(*d*) none of these.

Q.125. The ratio of the number of elements possessing a characteristic to the total number of elements in a sample is known as:

(*a*) characteristic proportion

(*b*) sample proportion

(*c*) Both (*a*) and (*b*)

(*d*) none of these.

Q.126. Finite population multiplier is:

(*a*) square of $(N - 1)/(N - n)$

(*b*) square root of $(N - n)/(N - 1)$

(*c*) square root of $(N - 1)/(N - n)$

(*d*) square of $(N - n)/(N - 1)$.

Q.127. Standard error can be described as:

(*a*) the error committed in sampling.

(*b*) the error committed in sample survey.

(*c*) the error committed in estimating a parameter.

(*d*) Standard deviation of a statistic.

Q.128. A measure of precision obtained by sampling is given by:

(*a*) Standard error

(*b*) Expectation

(*c*) Sampling distribution

(*d*) Sampling fluctuation.

Q.129. The standard deviation in the sampling deviation is called:

(*a*) standard error.

(*b*) absolute error.

(*c*) relative error.

(*d*) none of these.

Q.130. As the sample size increases, standard error

(*a*) increases

(*b*) decreases

(*c*) remains constant

(*d*) decreases proportionately.

Q.131. The magnitude of standard error increase both by absolute and relative size of the sample.

(*a*) True

(*b*) False

(*c*) Both (*a*) and (*b*)

(*d*) none of these.

Q.132. A population comprises 5 members. The number of all possible samples of size 2 that can be drawn from it with replacement is:

(*a*) 100 (*b*) 25

(*c*) 125 (*d*) 25.

Q.133. Standard error of mean may be defined as the standard deviation in the sampling distribution of:

(*a*) mean (*b*) median

(*c*) mode (*d*) none of these.

Q.134. If from a population with 25 members, a random sample without replacement of 2 members is taken, the number of all such samples is:

(*a*) 300 (*b*) 725

(*c*) 150 (*d*) 540.

Q.135. The sample proportion is taken as an estimate of the population proportion of defectives.

(*a*) True

(*b*) False

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.136. Standard deviation of a sampling distribution is itself the standard error.

(*a*) True

(*b*) False

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.137. The standard error of the mean for finite population is very close to the standard error of the mean for infinite population when the sampling fraction is:

(*a*) small (*b*) moderate

(*c*) large (*d*) none of these.

Q.138. Sampling error increases with an increase in the size of the sample.

(*a*) True

(*b*) False

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.139. Testing the assumption that an assumed population is located at a known level of significance is known as

(*a*) confidence testing

(*b*) point estimation

(*c*) interval estimation

(*d*) hypothesis testing.

Q.140. The Standard deviation of the ..... distribution is called standard error.

(*a*) Normal (*b*) Poisson

(*c*) Binomial (*d*) Sampling.

Q.141. Under ..... method selection is often based on certain predetermined criteria.

(*a*) Area sampling

(*b*) Block or Cluster sampling

(*c*) Quota sampling

(*d*) Deliberate, purposive or judgment sampling.

Q.142. Which would you prefer ..... if higher degree of confidence is desired?

(*a*) small sample (*b*) larger sample

(*c*) both (*d*) none of these.

Q.143. Which would you prefer for ..... when "The universe is large"?

(*a*) Full enumeration

(*b*) Sampling

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.144. A ..... is a complete or whole set or possible measurements data corresponding to the entire collection of units.

(*a*) Sample

(*b*) Population

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.145. The finite population correction factors should be used when the population is:

(*a*) infinite

(*b*) finite and large

(*c*) finite and small

(*d*) none of these.

Q.146. A statistic is a ..... variable

(*a*) compound (*b*) simple

(*c*) random (*d*) none of these

Q.147. Which would you prefer for .... "The Statistical inquiry is in depth".

(*a*) Full enumeration

(*b*) Sampling

(*c*) both (*a*) and (*b*)

(*d*) none.

Q.148. The primary object of sampling is to obtain ..... information about population with ..... effort.

(*a*) maximum, minimum

(*b*) minimum, maximum

(*c*) some, less

(*d*) none.

Q.149. The measure of divergence is ..... as the size of the sample approaches that of the population.

(*a*) more (*b*) less

(*c*) same (*d*) none of these.

Q.150. For ..... samples, the sample proportion is an unbiased estimate of the population proportion.

(*a*) large (*b*) small

(*c*) moderate (*d*) none of these.

Q.151. Value of a .....is different for different samples.

(*a*) statistic

(*b*) skill

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.152. Sampling error is..... proportional to the square root M the number of items in the sample.

(*a*) inversely (*b*) directly

(*c*) equally (*d*) none.

Q.153. Which would you prefer for ....."Where testing destroys the quality of the product".

(*a*) Full enumeration

(*b*) Sampling

(*c*) both (*a*) and (*b*)

(*d*) none of these.

Q.154. The distribution of sample ..... is normally or approximately normally distributed about the population

(*a*) median (*b*) mode

(*c*) mean (*d*) none of these.

Q.155. The standard error of the .....is the standard deviation of sample means

(*a*) mode (*b*) median

(*c*) moderate (*d*) none of these.

Q.156. In test for means of Paired data, if the computed value is ..... then the table value the difference is considered significant.

(*a*) lesser (*b*) greater

(*c*) moderate (*d*) none of these.

Q.157. ..... sampling is the most appropriate in cases when the population is more or less homogeneous with respect to the characteristic under study.

(*a*) Stratified (*b*) Multi - stage

(*c*) Random (*d*) noneof these.

Q.158. The mean of the sampling distribution of sample proportion is ..... the population proportion.

(*a*) greater than (*b*) less than

(*c*) equal to (*d*) none of these.

Q.159. sampling is similar to cluster sampling.

(*a*) Judgment (*b*) Quota

(*c*) Area (*c*) none of these.

Q.160. Which would you prefer ..... Previous experiences reveals a low rate of error.

(*a*) Larger Sample

(*b*) Small sample

(*c*) both (A) and (B)

(*d*) none of these.

ANSWERS

1.	(*b*)	**2.**	(*a*)	**3.**	(*d*)	**4.**	(*d*)	**5.**	(*a*)	**6.**	(*d*)	**7.**	(*a*)
8.	(*d*)	**9.**	(*b*)	**10.**	(*a*)	**11.**	(*d*)	**12.**	(*b*)	**13.**	(*c*)	**14.**	(*d*)
15.	(*d*)	**16.**	(*b*)	**17.**	(*a*)	**18.**	(*a*)	**19.**	(*b*)	**20.**	(*d*)	**21.**	(*d*)
22.	(*d*)	**23.**	(*d*)	**24.**	(*c*)	**25.**	(*c*)	**26.**	(*c*)	**27.**	(*b*)	**28.**	(*c*)
29.	(*d*)	**30.**	(*a*)	**31.**	(*d*)	**32.**	(*b*)	**33.**	(*c*)	**34.**	(*b*)	**35.**	(*b*)
36.	(*a*)	**37.**	(*d*)	**38.**	(*d*)	**39.**	(*d*)	**40.**	(*b*)	**41.**	(*a*)	**42.**	(*c*)
43.	(*d*)	**44.**	(*a*)	**45.**	(*c*)	**46.**	(*b*)	**47.**	(*c*)	**48.**	(*d*)	**49.**	(*b*)
50.	(*d*)	**51.**	(*d*)	**52.**	(*b*)	**53.**	(*a*)	**54.**	(*b*)	**55.**	(*b*)	**56.**	(*a*)
57.	(*a*)	**58.**	(*c*)	**59.**	(*b*)	**60.**	(*d*)	**61.**	(*b*)	**62.**	(*c*)	**63.**	(*b*)

64.	(*b*)	**65.**	(*d*)	**66.**	(*b*)	**67.**	(*a*)	**68.**	(*d*)	**69.**	(*d*)	**70.**	(*b*)
71.	(*c*)	**72.**	(*d*)	**73.**	(*d*)	**74.**	(*b*)	**75.**	(*b*)	**76.**	(*b*)	**77.**	(*c*)
78.	(*c*)	**79.**	(*c*)	**80.**	(*d*)	**81.**	(*d*)	**82.**	(*c*)	**83.**	(*b*)	**84.**	(*d*)
85.	(*d*)	**86.**	(*d*)	**87.**	(*d*)	**88.**	(*b*)	**89.**	(*c*)	**90.**	(*a*)	**91.**	(*a*)
92.	(*a*)	**93.**	(*a*)	**94.**	(*a*)	**95.**	(*d*)	**96.**	(*b*)	**97.**	(*b*)	**98.**	(*a*)
99.	(*c*)	**100.**	(*b*)	**101.**	(*c*)	**102.**	(*b*)	**103.**	(*d*)	**104.**	(*a*)	**105.**	(*d*)
106.	(*b*)	**107.**	(*b*)	**108.**	(*a*)	**109.**	(*c*)	**110.**	(*c*)	**111.**	(*d*)	**112.**	(*a*)
113.	(*c*)	**114.**	(*d*)	**115.**	(*a*)	**116.**	(*a*)	**117.**	(*b*)	**118.**	(*a*)	**119.**	(*b*)
120.	(*a*)	**121.**	(*c*)	**122.**	(*b*)	**123.**	(*b*)	**124.**	(*a*)	**125.**	(*b*)	**126.**	(*b*)
127.	(*d*)	**128.**	(*a*)	**129.**	(*a*)	**130.**	(*b*)	**131.**	(*a*)	**132.**	(*c*)	**133.**	(*a*)
134.	(*a*)	**135.**	(*a*)	**136.**	(*a*)	**137.**	(*a*)	**138.**	(*b*)	**139.**	(*d*)	**140.**	(*d*)
141.	(*d*)	**142.**	(*c*)	**143.**	(*b*)	**144.**	(*b*)	**145.**	(*c*)	**146.**	(*c*)	**147.**	(*b*)
148.	(*a*)	**149.**	(*b*)	**150.**	(*a*)	**151.**	(*a*)	**152.**	(*a*)	**153.**	(*b*)	**154.**	(*c*)
155.	(*c*)	**156.**	(*b*)	**157.**	(*c*)	**158.**	(*c*)	**159.**	(*c*)	**160.**	(*b*)		

26 CHAPTER

CORRELATION

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. If 'ρ' is the simple correlation coefficient, the quantity ρ^2 is known as :

(*a*) Coefficient of determination

(*b*) Coefficient of Non-determination

(*c*) Coefficient of alienation

(*d*) None of the above.

[June 2010]

Solution : (*a*)

Q.2. If the correlation coefficient between X and Y is r, & $U = \frac{X-5}{10}$ and $V = \frac{Y-7}{2}$ then r_{uv} is

(*a*) r (*b*) -r

(*c*) (r-5)/2 (*d*) (r-7)/10

[June 2010]

Solution : (*a*) $U = \frac{X-5}{10} = \frac{X}{10} - \frac{5}{10}$ & $V = \frac{Y}{2} - \frac{7}{2}$

TRICKS : Since X & Y have same sign.

So, $r_{xy} = r_{uv} = r$

Q.3. If the sum of the product of deviations of x and y series from their mean is zero, then the coefficient of correlation will be

(*a*) 1 (*b*) -1

(*c*) 0 (*d*) None of these

[Dec. 2010]

Solution : (*c*) Given

$$\sum\left(X-\overline{X}\right)\left(Y-\overline{Y}\right)=0$$

Formula, $r = \frac{\sum(X-\overline{X})(Y-\overline{Y})}{N\times\sigma_x\times\sigma_y}$

$$= \frac{0}{N\times\sigma_x\times\sigma_y} = 0$$

Q.4. The ranks of five participants given by two judges are

	Participants				
	A	**B**	**C**	**D**	**E**
Judge 1	**1**	**2**	**3**	**4**	**5**
Judge 2	**5**	**4**	**3**	**2**	**1**

Rank correlation coefficient between ranks will be

(*a*) 1 (*b*) 0

(*c*) -1 (*d*) 1/2

[Dec. 2010]

Solution : (*c*) Ranks given by two Judges are in reverse order in same ratio.

∴ R = - 1 [It is always -1; No need to proof.]

Q.5. The covariance between two variables X and Y is 8.4 and their variances are 25 and 36 respectively. Calculate Karl Pearson's coefficient of correlation between them.

(*a*) 0.82 (*b*) 0.28

(*c*) 0.01 (*d*) 0.09

[June 2011]

Solution : (*b*) Given : Cov. (x,y) = 8.4

$\sigma_x = \sqrt{25} = 5,\ \sigma_y = \sqrt{36} = 6$

$r = \frac{Cov.(x,y)}{\sigma_x, \sigma_y} = \frac{8.4}{5\times 6} = 0.28$

Q.6. In Spearman's Correlation Coefficient, the sum of the differences of ranks between two variables shall be_____.

(*a*) 0 (*b*) 1

(*c*) -1 (*d*) None of them

[Dec. 2012]

Solution : (*a*) In Spearman's correlation coefficient the sum of the differences of ranks between two variable shall be **Zero.**

$$\sum D = \sum (R_1 - R_2) = 0$$

Q.7. The coefficient of correlation between X and Y series is - 0.38 The linear relation between U & V are 3X + 5U = 3 and -8Y - 7V = 44, what is the coefficient of correlation between U & V?

(*a*) 0.38 (*b*) -0.38

(*c*) 0.40 (*d*) None of them

[Dec. 2012]

Solution : (*b*) Given $r_{xy} = -0.38$

TRICKS : 3X + 5U = 3 and

-8Y - 7V = 44

8Y + 7V = 44

r = - 0.38

Q.8. Two variables X and Y are related as 4x + 3y = 7 then correlation between x and y is_____

(*a*) Perfect positive

(*b*) Perfect negative

(*c*) Zero

(*d*) None of these

[June 2013]

Solution : (*b*)

Since, $4x + 3y = 7$

∴ 3y = -4x + 7; ∴ r = -1

∴ X and are y perfectly negative because x and y have opposite sign.

Q.9. If r is the karl pearson's coefficient of correlation in a bivariate distribution the two regression lines are at right angles when_____

(*a*) $r = \pm 1$ (*b*) r = 0

(*c*) $r = \pm\infty$ (*d*) None

[June 2013]

Solution : (*b*) If r = 0 ; Two Regression Lines are perpendicular to each other.

Q.10. If r = 0.28, Cov. (x,y) = 7.6, V(x) = 9 then σ_y =

(*a*) 8.75 (*b*) 9.04

(*c*) 6.25 (*d*) None

[June 2013]

Solution : (*b*) is correct

$r = \frac{Cov.(x;y)}{\sigma_x.\sigma_y}$

or; $0.28 = \frac{7.6}{3.\sigma_y}$ or; $\sigma_y = 9.04$

Q.11. Price and Demand is example for:

(*a*) No correlation

(*b*) Positive correlation

(*c*) Negative correlation

(*d*) None of these

[Dec. 2013]

Solution : (*c*)

Q.12. Determine the coefficient of correlation between x and y series

	X-Series	Y-Series
Number of items	15	15
Arithmetic mean	25	18
Sum of Square of deviation of mean	136	138

Sum of product deviation of X and Y series from mean = 122

(*a*) -0.89 (*b*) 0.89

(*c*) 0.69 (*d*) -0.69

[Dec. 2013]

Solution : (*b*) is correct

Given N = 15; $\overline{X} = 25; \overline{y} = 18$

$\sum (x - \overline{x})^2 = \sum x^2 = 136;$

$\sum (y - \overline{y})^2 = \sum y^2 = 138$

and $\sum (x - \overline{x})(y - \overline{y}) = \sum xy = 122$

$\therefore r = \frac{\sum xy}{\sqrt{\sum x^2}\sqrt{\sum y^2}} = \frac{122}{\sqrt{136}\sqrt{138}}$

= 0.89

Tricks : Use calculator (Without writing)

Q.13. When each individual gets the exactly opposite rank by the two judges then the rank correlation will be_____

(*a*) -1 (*b*) 0

(*c*) +1 (*d*) +1/2

[June 2014]

Solution : (*a*) is correct

Note :- Opposite Ranks Means r = -1

Q.14. Correlation coefficient between x and y is 1, then correlation coefficient between x - 2 and (-y/2)+1 is.

(*a*) 1 (*b*) -1

(*c*) -1/2 (*d*) 1/2

[Dec. 2014]

Solution : (*b*) is correct.

∵ r = + 1 (given)

For variables (x-2) and $\left(-\frac{y}{2}+1\right)$

r = -1 (because sign of x & y are opposite)

Q.15. When r = 1, all the points in a scatter diagram would lie:

(*a*) On a straight line directed from lower left to upper right

(*b*) On a straight line

(*c*) On a straight line directed from upper left to lower right

(*d*) Both (*a*) and (*b*)

[June 2015]

Solution : (*a*) is correct.

Q.16. In case 'Insurance Companies' Profits and the No. of claims they have to pay:

(*a*) Positive correlation

(*b*) Negative correlation

(*c*) No correlation

(*d*) None of these

[Dec. 2015]

Solution : (*b*) is correct.

Q.17. If r = 0.6 then the coefficient of non-determination is _____

(*a*) 0.4 (*b*) - 0.6

(*c*) 0.36 (*d*) 0.64

[June 2016]

Solution : (*d*)

co-efficient of Non-determination

$= 1 - r^2 = 1 - (0.6)^2 = 0.64$

$= 64\%$

Q.18. If the coefficient of correlation between x and y variables is -0.90 then what will be the coefficient of determination

(*a*) 0.10 (*b*) 0.81

(*c*) 0.94 (*d*) None

[June 2016]

Solution : (*b*)

co-eff. of determination $= r^2$

$= (-0.90)^2 = 0.81$

Q.19. If the sum of the squares of Rank differences in the marks of 10 students in two students is 44, then the coefficient of rank correlation is _____

(*a*) 0.78 (*b*) 0.73

(*c*) 0.87 (*d*) None

[Dec. 2016]

Solution : (*b*) is correct.

Given, N = 10 & $\sum D^2 = 44$

$$R = 1 - \frac{6\sum D^2}{N^3 - N} = 1 - \frac{6 \times 44}{10^3 - 10} = 0.73$$

Q.20. Correlation between temperature and power consumption is

(*a*) Positive (*b*) Negative

(*c*) Zero (*d*) None

[June 2017]

Solution : (*a*)

Q.21. Coefficient of correlation between X & Y is 0.6. If both X and Y are multiplied by -1. Then resultant co-efficient of correlation is

(*a*) 0.6 (*b*) Negative

(*c*) 1/0.6 (*d*) None

[June 2017]

Solution : (*a*) [$\because$ *r* does not change with respect to the change of origin and scale]

Q.22. If r = 0.6 then the coefficient of non-determination is :

(*a*) 0.4 (*b*) -0.6

(*c*) 0.36 (*d*) 0.64

[Dec. 2017]

Solution : (*d*)

Co-efficient of Non-determination

$= 1 - r^2 = 1 - (0.6)^2$

$= 0.64$

Q.23. If there is a constant increase in the series then the obtained graph is :

(*a*) Convex

(*b*) Concave

(*c*) Parabola

(*d*) Straight line from left to right

[Dec. 2017]

Solution : (*d*)

Q.24. If r = 0.58, correlation coefficient of u = -5x + 3 and v = y + 2 is _____:

(*a*) 0.58 (*b*) -0.58

(*c*) 0.62 (*d*) None

[June 2018]

Solution : (*b*)

The value of "r" does not change with respect to the change of origin and scale but sign may change.

So $r_{uv} = -0.58$

[Note : Here; sign of x & y in both eqns. are opposite, so sign of "r" changes].

Q.25. If the sum of squares of deviations of ranks of 8 students is 50 then the rank correlation coefficient is _____ :

(*a*) 0.40 (*b*) 0.45

(*c*) 0.5 (*d*) 0.8

[June 2018]

Solution : (*a*)

Given, $N = 8$; $\sum D^2 = 50$

$$R = 1 - \frac{6\sum D^2}{N^3 - N} = 1 - \frac{6 \times 50}{8^3 - 8} = 0.40$$

Q.26. If the plotted points in a scatter diagram are evenly distributed, then the correlation is

(*a*) Zero (*b*) Negative

(*c*) Positive (*d*) (a) or (b)

[May 2018]

Solution : (*a*)

Q.27. The covariance between two variables is

(*a*) Strictly positive

(*b*) Strictly negative

(*c*) Always Zero

(*d*) Either positive or negative or zero

[May 2018]

Solution : (*d*)

Q.28. The coefficient of determination is defined by the formula

(*a*) $r^2 = \dfrac{1 - \text{unexplained variance}}{\text{total variance}}$

(*b*) $r^2 = \dfrac{\text{explained variance}}{\text{total variance}}$

(*c*) both (*a*) and (*b*)

(*d*) None

[May 2018]

Solution : (*c*)

Q.29. In the method of Concurrent Deviations, only the directions of change (Positive direction/Negative direction) in the variables are taken into account for calculation of

(*a*) Coefficient of S.D

(*b*) Coefficient of regression

(*c*) Coefficient of correlation

(*d*) None

[May 2018]

Solution : (*c*)

Q.30. Correlation coefficient is _____ _____of the units of measurement.

(*a*) dependent (*b*) independent

(*c*) both (*d*) None

[May 2018]

Solution : (*b*)

Q.31. In case speed of an automobile and the distance required to stop the car after applying brakes correlation is.....

(*a*) Positive (*b*) Negative

(*c*) Zero (*d*) None

[May 2018]

Solution : (*a*)

Q.32. A relationship $r^2 = 1 - \frac{500}{300}$ is not possible

(*a*) True (*b*) False

(*c*) Both (*d*) None

[May 2018]

Solution : (*a*) Given :

$r^2 = 1 - \frac{500}{300} = \frac{-200}{300}$ is not possible

[$\because$ r^2 is always positive.]

Q.33. Rank correlation coefficient lies between

(*a*) 0 to 1

(*b*) -1 to +1 inclusive of these values

(*c*) -1 to 0

(*d*) Both

[May 2018]

Solution : (*b*)

Q.34. If the correlation coefficient between the variables X and Y is 0.5, then the correlation coefficient between the variables $2x - 4$ and $3 - 2y$ is

(*a*) 0.5 (*b*) 1

(*c*) –0.5 (*d*) 0

[Nov. 2018]

Solution : (*c*)

Tricks : See Quicker BMLRS

Chapter : Correlation.

r = -0.5 (Sign. of X & Y in both are opposite).

Q.35. A.M. of regression coefficient is

(*a*) Equal to r

(*b*) Greater than or equal to r

(*c*) Half of r

(*d*) None

[June 2019]

Solution : (*b*)

Q.36. Given that

X	-3	-3/2	0	3/2	3
Y	9	9/4	0	9/4	9

Then Karl Pearson's coefficient of correlation is

(*a*) Positive (*b*) Zero

(*c*) Negative (*d*) None

[June 2019]

Solution : (*b*)

If graph of this data is drawn then it will make a curvilinear relation. In this case r = 0

Hence Karl Pearson's Coefficient of Correlation = r = "Zero". Because it is equally distributed.

Q.37. Find the probable error if $r = \frac{2}{\sqrt{10}}$ and N = 36

(*a*) 0.6745 (*b*) 0.067

(*c*) 0.5287 (*d*) None

[June 2019]

Solution : (*b*)

$r = \frac{2}{\sqrt{10}}$, N= 36, P.E = ?

Probable Error $P.E = 0.6745\frac{1-r^2}{\sqrt{N}}$

$= 0.6745\left[\frac{1-\left(\frac{2}{\sqrt{10}}\right)^2}{\sqrt{36}}\right] = 0.6745\frac{\left(1-\frac{4}{10}\right)}{6}$

$= 0.06745 = 0.067$

Q.38. Given the following series :

X	10	13	12	15	8	15
Y	12	16	18	16	7	18

The rank correlation coefficient R =

(*a*) $1 - \frac{6\sum d^2 \sum_{i=1}^{2} \frac{m_i(m_i^2-1)}{12}}{N(N^2-1)}$

(*b*) $1 - \frac{6\left[\sum d^2 + \sum_{i=1}^{2} \frac{m_i(m_i^2-1)}{12}\right]}{N(N^2-1)}$

(*c*) $1 - 6\sum d^2 + \sum_{i=1}^{2} \frac{\frac{m_i(m_i^2-1)}{12}}{N(N^2-1)}$

(*d*) $1 - 6\sum d^2 + \sum_{i=1}^{3} \frac{\frac{m_i(m_i^2-1)}{12}}{N(N^2-1)}$

[June 2019]

Solution : (*b*)

Here two observations 15 & 16 has been repeated two times. So, we use the formula of the Rank Correlation Coefficient.

$$R = 1 - \frac{6\left[\sum d^2 + \sum_{i=1}^{2} \frac{m_i(m_i^2-1)}{12}\right]}{N(N^2-1)}$$

Q.39. Determine Spearman's rank correlation coefficient from the given data $\sum D^2 = 30$, N = 10

(*a*) R = 0.82 (*b*) R = 0.32

(*c*) R = 0.40 (*d*) None of these

[June 2019]

Solution : (*a*)

Here, $\sum D^2 = 30$, $N = 10$

Spearman's rank correlation

$$R = 1 - \frac{6\sum D^2}{N(N^2-1)}$$

$$= 1 - \frac{6\times 30}{10(10^2-1)} = 1 - \frac{180}{990} = 1 - \frac{2}{11} = \frac{9}{11}$$

$= 0.82$

Q.40. Find correlation coefficient

X	5	4	3	2	1
Y	1	2	3	4	5

(*a*) 1 (*b*) -1

(*c*) 0 (*d*) None of these

[Dec. 2019]

Solution : (*b*)

Tricks: Each observation of X decreased by 1 but that of Y increase by 1.

So; r = -1. (Inverse relation)

Q.41. If scatter diagram from a line move from lower left to upper right corner then the correlation is.

(*a*) Perfect positive

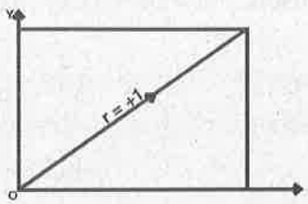
(*b*) Perfect negative

(*c*) Simple positive

(*d*) No correlation

[Dec. 2019]

Solution : (*a*)



Q.42. If correlation coefficient between x and y is 0.5, then find the correlation coefficient between 2x-3 and 3-5y is

(*a*) 0.5 (*b*) -0.5

(*c*) 2.5 (*d*) -2.5

[Dec. 2019]

Solution : (*b*)

The value of "r" does not change if a constant quantity is added to or subtracted from all observation.

The value of "r" also does not change if all observations are multiplied or divided by a constant quantity. X has been multiplied by +2 but Y by -"5" *i.e.* X & Y have opposite sign. So; only sign of "r" will change not its value.

$\therefore$ r = - 0.5

Q.43. If the equation of the two regression lines are 2x - 3y = 0 and 4y -5x = 8 then the correlation coefficient between x and y is equal to

(*a*) $\sqrt{\frac{15}{8}}$ (*b*) $\sqrt{\frac{8}{15}}$

(*c*) $\sqrt{\frac{6}{15}}$ (*d*) $\sqrt{\frac{1}{15}}$

[Dec. 2019]

Solution : (*b*)

$2X - 3Y = 0 \Rightarrow 2X = 3Y$

$-5X + 4Y = 8 \Rightarrow 5X = 4Y + 8$

$r = +\sqrt{\frac{2\times4}{3\times5}}$ $\because$ (Put + Sign because X & Y have same sign in both equation.)

[Do cross - product. Write smaller in numerator and larger in denominator]

Q.44. Which of the following is spurious correlation?

(*a*) Correlation between two variables having no causal relationship

(*b*) Negative Correlation

(*c*) Bad relation between two variables

(*d*) Very low correlation between two variables ***[Dec. 2020]***

Solution : (*a*)

Q.45. Scatter diagram does not help us to

(*a*) Find the type of correlation

(*b*) Identify whether variables correlated or not

(*c*) Determine the linear (or) non-linear correlation

(*d*) Find the numerical value of correlation coefficient ***[Dec. 2020]***

Solution : (*d*)

Q.46. The Covariance between two variables is

(*a*) Strictly Positive

(*b*) Strictly Negative

(*c*) Always Zero

(*d*) Either positive (or) Negative (or) Zero ***[Dec. 2020]***

Solution : (*d*)

Q.47. For the set of observations {(1, 2), (2, 5), (3, 7), (4, 8), (5, 10)} the value of karl-person's coefficient of correlation is approximately given by

(*a*) 0.755 (*b*) 0.655 (*c*) 0.525 (*d*) 0.985

[Jan. 2021]

Solution : (*d*) is correct

[Do these on calculator. No need of this Table]

X	Y	XY	X^2	Y^2
1	2	2	1	4
2	5	10	4	25
3	7	21	9	49
4	8	32	16	64
5	10	50	25	100
$\Sigma x = 15$	$\Sigma Y = 32$	$\Sigma xy = 115$	$\Sigma x^2 = 55$	$\Sigma Y^2 = 242$

Coefficient of correlation

$$= r = \frac{N\sum XY - \sum X \cdot \sum Y}{\sqrt{N\sum X^2 - \left(\sum X\right)^2} \cdot \sqrt{N\sum Y^2 - \left(\sum Y\right)^2}}$$

$$= \frac{5\times115 - 15\times32}{\sqrt{5\times55 - (15)^2} \cdot \sqrt{5\times242 - (32)^2}}$$

$$= \frac{575 - 480}{\sqrt{275 - 225} \cdot \sqrt{1210 - 1024}}$$

$$= \frac{+95}{\sqrt{50} \cdot \sqrt{186}} = +0.985$$

Q.48. The coefficient of correlation between *x* and *y* is 0.5 the covariance, is 16 and the standard deviation of *y* is if S.D. of *x* is 4.

(*a*) 4 (*b*) 8

(*c*) 16 (*d*) 64

[Jan. 2021]

Solution : (*b*) is correct

Formula $r = \frac{\text{cov.}(x; y)}{\sigma_x \cdot \sigma_y}$

or $0.5 = \frac{16^4}{4.\sigma_y}$

or $\sigma_y = \frac{4}{0.5} = 8$

Q.49. If the sum of the product of the deviation of and Y from their means is zero, the correlation coefficient between X and is:

(*a*) Zero (*b*) Positive

(*c*) Negative (*d*) 10

[July 2021]

Solution : (*a*) is correct.

Given that

$$\Sigma(X - \bar{X})(Y - \bar{Y}) = 0$$

Coefficient of correlation

$$= r = \frac{\sum(X - \bar{X})(Y - \bar{Y})}{N.\sigma_x.\sigma_y} \text{ (Formula)}$$

$$= \frac{0}{N.\sigma_x.\sigma_y} = 0$$

Q.50. If the data points of (X, Y) series on a scatter diagram lie along a straight line that goes downwards as X-values move from left to right, then the data exhibit _____ correlation.

(*a*) Direct

(*b*) Imperfect indirect

(*c*) Indirect

(*d*) Imperfect direct

[Dec. 2021]

Solution : (*c*)

Q.51. If the plotted points in a scatter diagram lie from lower left to upper right, then the correlation is

(*a*) Negative

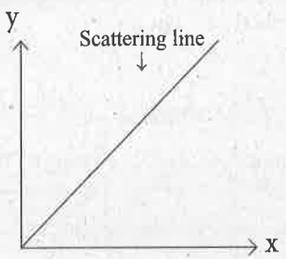
(*b*) Perfect Negative

(*c*) Zero

(*d*) Positive

[June 2022]

Solution :



This line shows positively corrected.

∴ (*d*) is correct.

Q.52. For finding correlation between two qualitative characteristics, we use

(*a*) Coefficient of rank correlation

(*b*) Scatter diagram

(*c*) Coefficient of concurrent deviation

(*d*) Product moment correlation coefficient

[June 2022]

Solution :

(*a*) is correct

For Qualitative Nature; Rank correlation coefficient is used.

Q.53. Karl Pearson's coefficient is defined from

(*a*) Grouped data

(*b*) Ungrouped data

(*c*) Any data

(*d*) Scattered data

[June 2022]

Solution : (*b*)

Karl Pearson's correlation Coefficient is defined for ungrouped data.

Q.54. For n pairs of observations, the coefficient of concurrent deviation is calculated as $\frac{1}{\sqrt{5}}$ If there are six concurrent deviations, then n =

(*a*) 11 (*b*) 10

(*c*) 9 (*d*) 8

[June 2022]

Solution : Given : $r_c = \frac{1}{\sqrt{5}}$ & c = 6.

Formula

$$r_c = \pm\sqrt{\pm\frac{2c - n}{n}}$$

or $\frac{1}{\sqrt{5}} = +\sqrt{\frac{+2\times6-n}{n}}$

Squaring on both sides; we get

$$\frac{1}{5} = \frac{12-n}{n}$$

or n = 60 – 5n

or n+5n = 60

or 6n = 60 ∴ n =10

∴ N = No. of observation = n + 1

= 10 + 1 = 11

∴ (*b*) is correct.

Q.55. The coefficient of rank correlation between the ranking of following 6 students in two subjects.

Mathematics and Statistics is :

Mathematics	3	5	8	4	7	10
Statistics	6	4	9	8	1	2

(*a*) 0.25 (*b*) 0.35 (*c*) 0.38 (*d*) 0.20

[Dec. 2022]

Solution:

Mathematics (X)	Statistics (Y)	R_1	R_2	$D^2 = (R_1 - R_2)^2$
3	6	1	4	9
5	4	3	3	0
8	9	5	6	1
4	8	2	5	9
7	1	4	1	9
10	2	6	2	16
				$\sum D^2 = 44$

$$\therefore R = 1 - \frac{6\sum D^2}{N^3 - N}$$

$$= 1 - \frac{6 \times 44}{6^3 - 6}$$

$$= 1 - 1.257$$

$$= -0.257$$

No option

Q.56. Pearson's Correlation co-efficient between x and y is:

(a) $\frac{\text{cov}(x^1, y)}{S_x S_y}$ (b) $\frac{\text{cov}^2(x_2, y)}{S_x S_y}$

(c) $\frac{(S_x S_y)^3}{\text{cov}(x, y)}$ (d) $\frac{S_x S_y}{\text{cov}(x, y)}$

[Dec. 2022]

Solution: (*a*) is correct.

Karl Pearson's correlation co-efficient

$$= r = \frac{Cov.(x;y)}{\sigma_x . \sigma_y} = \frac{Cov.(x;y)}{Sx.Sy}$$

Where $\sigma_x = s_x$ = SD of x

$\sigma_y = s_y$ = SD of y

Q.57. Given that r = 0.4 and n = 81, determine the limits for the population correlation coefficient.

(a) (0.333, 0.466)

(b) (0.367, 0.433)

(c) (0.337, 0.463)

(d) (0.373, 0.427)

[June 2023]

Solution: $PE = 0.6745 \frac{1 - r^2}{\sqrt{N}}$

$$= 0.06745 \left(\frac{1 - (0.4)^2}{\sqrt{81}} \right)$$

$$= 0.06295$$

$$= 0.063$$

Lower Limit = r – PE

$$= 0.4 - 0.063$$

$$= 0.337$$

Upper Limit = r + PE

$$= 0.4 + 0.063$$

$$= 0.463$$

$\therefore$ (*c*) is correct.

Q.58. Spearman's rank correlation co-efficient r_R is given by:

(a) $1 - \frac{6\sum d_i^2}{n(n^2+1)}$

(b) $1 + \frac{6\sum d_i^2}{n(n^2-1)}$

(c) $1 + \frac{6\sum d_i^2}{n(n^2+1)}$

(d) $1 - \frac{6\sum d_i^2}{n(n^2-1)}$

[June 2023]

Solution: Formula.

$$R = 1 - \frac{6\sum D^2}{N^3 - N}$$

$$= 1 - \frac{6\sum D^2}{N(N^2 - 1)}$$

$\therefore$ (*d*) is correct.

27 CHAPTER REGRESSION ANALYSIS

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. _____ of the regression Coefficient is greater than the correlation coefficient

(a) Combined mean

(b) Harmonic mean

(c) Geometric mean

(d) Arithmetic mean.

[June 2010]

Solution : (*d*) $r = \pm\sqrt{b_{xy} . b_{yx}}$ = GM of Regression Coefficients.

$\because$ AM > GM> HM

$\therefore$ AM of regression coefficients is greater than correlation coefficient.

Q.2. Regression coefficient are_____

(a) dependent of change of origin and of scale.

(b) independent of both change of origin and of scale.

(c) dependent of change of origin but not of scale.

(d) independent of change of origin but not of scale

[Dec. 2010]

Solution : (*d*) Regression coefficient are independent of change of origin but changes with respect to scale. [Properties of regression co-efficients]

Q.3. Given :

$\overline{X} = 16$, $\sigma_X = 4.8$,

$\overline{Y} = 20$, $\sigma_Y = 9.6$

The coefficient of correlation between x and y is 0.6. What will be the regression coefficient of 'x' on 'y' ?

(a) 0.03 (b) 0.3

(c) 0.2 (d) 0. 05

[Dec. 2010]

Solution : (*b*) $b_{xy} = r \times \frac{\sigma_x}{\sigma_y}$

$$b_{xy} = 0.6 \times \frac{4.8}{9.6} = 0.3$$

Q.4. If the two line of regression are x + 2y - 5 = 0 and 2x + 3y - 8 =0. The regression line of y on x is

(a) x + 2y - 5 = 0

(b) 2x + 3y - 8 = 0

(*c*) Any of the two line

(*d*) None of the two line

[Dec. 2010]

Solution : (*a*) Let x + 2y - 5 = 0 be Regression equation of Y on X .

$$b_{yx} = \frac{-\text{coeff. of x}}{\text{coff. of y}} = \frac{-1}{2} = -0.5$$

Let 2x + 3y - 8 = 0 be Regression equation of X on Y.

$$b_{xy} = -\frac{3}{2} = -1.5$$

$b_{xy}.b_{xy}$

$=(-0.5).(-1.5)$

$= 0.75$

$\therefore b_{yx}.b_{xy} < 1$

$\therefore$ 1st line is Y on X.

Q.5. For a bivariate data two lines of regression are 40x - 18y = 214 and 8x - 10y + 66 = 0, then find the values of x and y

(*a*) 17 and 13 (*b*) 13 and 17

(*c*) 13 and –17 (*d*) –13 and 17

[June 2011]

Solution : (*b*) **TRICKS :** Go by choices, X = 13 and Y = 17; satisfy both Regression Lines.

$\therefore \overline{X} = 13$ and $\overline{Y} = 17$

Q.6. Out of the following which one affects the regression co-efficient.

(*a*) Change of origin only

(*b*) Change of scale only

(*c*) Change of scale & origin both

(*d*) Neither change of origin nor change of scale ***[Dec. 2011]***

Solution : (*b*) The regression coefficients does not change due to a shift of origin but changes due to a shift of scale.

Q.7. For a bivariate data, the line of regression of Y on X, and of X on Y are respectively 2.5Y - X = 35 and 10 X - Y = 70, then correlation coefficient r is equal to:

(*a*) 0.2 (*b*) - 0.2

(*c*) 0.5 (*d*) - 0.5

[Dec. 2011]

Solution : (*a*) The equation of regression line Y on X

is 2.5 Y - X = 35

$$b_{yx} = -\left(\frac{-1}{2.5}\right) = \frac{1}{2.5} = 0.4$$

The equation of Regression line X on Y is 10x – y = 70

$$b_{xy} = -\left(\frac{-1}{10}\right) = \frac{1}{10} = 0.1$$

Both Regression Coefficients are positive.

$$r = +\sqrt{b_{xy} \times b_{yx}} = \sqrt{(0.4).(0.1)} = 0.2$$

Q.8. If one of regression coefficient is _____ unity, the other must be_____unity.

(*a*) more than, more than

(*b*) Less than, Less than

(*c*) more than, less than

(*d*) Positive, Negative

[Dec. 2011]

Solution : (*c*) If one regression coefficient is more than unity, the other must be less than unity.

Q.9. If Y is dependent variable and X is Independent variable and the S.D of X and Y are 5 and 8 respectively and Co-efficient of co-relation between X and Y is 0.8. Find the Regression co-efficient of Y on X.

(*a*) 0.78 (*b*) 1.28

(*c*) 6.8 (*d*) 0.32

[Dec. 2011]

Solution : (*b*) Given , $\sigma_x = 5$; $\sigma_y = 8$; r = 0.8

Regression Co- eff. of Y on X

$$b_{yx} = r.\frac{\sigma_y}{\sigma_x} = \frac{0.8 \times 8}{5} = \frac{6.4}{5} = 1.28$$

Q.10. If the regression lines are 8x – 10y + 66 = 0 and 40x – 18y = 214, the correlation coefficient between 'x' and 'y' is :

(*a*) 1 (*b*) 0.6

(*c*) –0.6 (*d*) –1

[June 2012]

Solution : (*b*) 8x - 10y + 66 = 0 be the Regression eqn. of Y on X

$$b_{yx} = -\frac{8}{-10} = 0.8 \text{ and}$$

40x – 18y = 214; be Regression eqn. of X on Y.

$$b_{xy} = -\frac{-18}{40} = 0.45$$

$r = \pm\sqrt{b_{yx} \times b_{xy}}$ [Both Regr. Coef. are + ve.]

$= +\sqrt{0.8 \times 0.45} = +0.6$

[r is also +ve.]

Q.11. The coefficients of correlation between two variables X and Y is the simple _____ of the two regression.

(*a*) Arithmetic Mean

(*b*) Geometric Mean

(*c*) Harmonic Mean

(*d*) None of the above

[June 2012]

Solution : (*b*) The coefficient of correlation between two variables X and Y is the simple geometric mean of the two regression coefficient.

Q.12. If 2 variables are uncorrelated, their regression lines are:

(*a*) Parallel

(*b*) Perpendicular

(*c*) Coincident

(*d*) Inclined at 45 degrees

[June 2012]

Solution : (*b*) If two variables are uncorrelated, (it means r = 0). Hence regression lines are perpendicular

Q.13. If x, y denote the arithmetic means, σ_x ; σ_y denote the standard the deviations, b_{xy} ; b_{yx} denote the regression coefficients of the variables 'x' and 'y' respectively, then the point of intersection of regression lines X on Y & Y on X is _____.

(*a*) $(\overline{X}; \overline{Y})$ (*b*) (σ_x, σ_y)

(*c*) (σ_x, σ_y) (*d*) (σ_x^2, σ_y^2)

[June 2012]

Solution : (*a*) $\because$ Two lines of regression pass through the point of intersection of Regression lines $(\overline{X}, \overline{Y})$.

Q.14. For certain x and y series which are correlated, the two line of regression are

$5x - 6y + 9 = 0$

$15x - 8y - 130 = 0$

The correlation coefficient is

(*a*) 4/5 (*b*) 3/4

(*c*) 2/3 (*d*) 1/2

[Dec. 2012]

Solution : (*c*) Let $5x - 6y + 9 = 0$; be a Regression eqn. of Y on X .

$$b_{yx} = -\frac{5}{-6} = \frac{5}{6}$$

And $15x - 8y - 130 = 0$, be a Regression eqn. of X on Y

$$b_{xy} = \frac{8}{15}$$

$$r = \pm\sqrt{b_{yx} \times b_{xy}} = +\sqrt{\frac{5}{6} \times \frac{8}{15}} = +\frac{2}{3}$$

(r is positive because regression coefficients are positive).

Q.15. The coefficient of correlation between X and Y series is – 0.38. The linear relation between X & V are 3X + 5U = 3 and -8Y – 7V = 44, what is the coefficient of correlation between U & V ?

(*a*) 0.38 (*b*) –0.38

(*c*) 0.40 (*d*) None of these

[Dec. 2012]

Solution : (*b*) Given $r_{xy} = -0.38$

TRICKS : $3X + 5U = 3$ and $-8Y - 7V = 44$

$8Y + 7V = 44$

$r = -0.38$

Note :- See QUICKER BMLRS Examples .

Q.16. If Y = 18X + 5 is the regression line of X on Y ; The value of b_{xy} is

(*a*) 5/18 (*b*) 18

(*c*) 5 (*d*) 1/18

[Dec. 2012]

Solution : (*d*) If $Y = 18X + 5$

$18X - Y - 5 = 0$

$$b_{xy} = -\left(\frac{-1}{18}\right) = \frac{1}{18}$$

Q.17. 8x - 3y + 7 = 0, 14x - 7y + 6 = 0 are two regression equation then the correlation coefficient , r =

(*a*) 0.86 (*b*) –0.86

(*c*) 0.45 (*d*) –0.45

[June 2013]

Solution : (*a*) is correct

Let $8x - 3y + 7 = 0$; be Regression Eqn. of y on x

$$\therefore b_{yx} = -\frac{8}{-3} = \frac{8}{3}$$

and $14x - 7y + 6 = 0$; the Regression Eqn. of x on y

$$\therefore b_{xy} = -\frac{-7}{14} = \frac{7}{14} = \frac{1}{2}$$

$$b_{xy} \cdot b_{xy} = \frac{8}{3} \times \frac{1}{2} = \frac{4}{3} > 1$$

:-Our Assumption is wrong

So; $b_{xy} = \frac{2}{1} = 2$ & $b_{yx} = \frac{3}{8}$

$$\therefore r^2 = b_{xy} \cdot b_{yx} = 2 \cdot \frac{3}{8} = \frac{3}{4} = 0.75$$

:- $r = +0.86$

Q.18. If r = + 1 or – 1 then the two regression lines_____

(*a*) Have 30% angle between them

(*b*) Have 45% angle between them

(*c*) Coincide

(*d*) Perpendicular to each other

[Dec. 2013]

Solution : (*c*) For r = + 1 or – 1

Regression Lines Coincide.

Q.19. If mean of X and Y variables is 20 and 40 respectively and the regression coefficient Y on X is 1.608 then the regression line of Y on X is:

(*a*) $Y = 1.608X + 7.84$

(*b*) $Y = 1.56X + 4.84$

(*c*) $Y = 1.608X + 4.84$

(*d*) $Y = 1.56X + 7.84$ ***[Dec. 2013]***

Solution : (*a*) is correct

check which option is correct

for $x = 20$; $y = 40$

For (a); $\bar{y} = 1.608 \times 20 + 7.84 = 40$ which is correct

Q.20. The equations two lines of regression for x & y are 5x = 22 + y and 64x = 24 + 45y, then the value of regression coefficient of y on x will be_____

(*a*) 5 (*b*) $\frac{1}{5}$

(*c*) $\frac{64}{45}$ (*d*) $\frac{45}{64}$

[June 2014]

Solution : (*c*) is correct

Let $5x = 22 + y$ be a regression Eqn. of X on Y

$$\therefore b_{xy} = \frac{1}{5}$$

and $64x = 24 + 45y$ be a Regression Eqn. of y on x

$\therefore 45y = -24 + 64x$

$$b_{yx} = \frac{64}{45}$$

$$\therefore r^2 = b_{xy} \cdot b_{yx} = \frac{1}{5} \cdot \frac{64}{45} = \frac{64}{225} < 1$$

$\therefore$ Our assumption is correct

$$\therefore b_{yx} = \frac{64}{45}$$

Q.21. Two regression lines for a bivariate data are 2x – 5y + 6 = 0 and 5x – 4y + 3 = 0. Then the coefficient correlation shall be___.

(*a*) $\frac{-2\sqrt{2}}{5}$ (*b*) $\frac{2}{5}$

(*c*) $\frac{+2\sqrt{2}}{5}$ (*d*) $\frac{\sqrt{2}}{5}$

[June 2014]

Solution : (*c*) is correct

Let $2x - 5y + 6 = 0$ be a Regression Eqn. of x on y

$$b_{xy} = -\frac{(-5)}{2} = \frac{5}{2}$$

and $5x - 4y + 3 = 0$ be a Regression Eqn. of y on x

$$b_{yx} = -\frac{5}{-4} = \frac{5}{4}$$

$$r^2 = b_{xy} \cdot b_{yx} = \frac{5}{2} \cdot \frac{5}{4} = \frac{25}{8} > 1$$

$\therefore$ Our assumption is incorrect

$\therefore$ Correct $b_{xy} = \frac{4}{5}$ and $b_{yx} = \frac{2}{5}$

$\therefore\ r^2 = b_{xy}.b_{yx} = \frac{4}{5}\cdot\frac{2}{5}$

$\therefore\ r = +\frac{2\sqrt{2}}{5}$

Q.22. If the mean of two variables x & y are 3 and 1 respectively. Then the equation of two regression lines are_____

(*a*) 5x+7y-22=0 & 6x+2y-20=0

(*b*) 5x+7y-22=0 & 6x+2y+20=0

(*c*) 5x+7y+22=0 & 6x+2y-20=0

(*d*) 5x+7y+22=0 & 6x+2y+20=0

[June 2014]

Solution : (*a*) is correct

Tricks : Go by choices

For (a) x = 3; y = 1 Satisfy eqns. of (a)

As LHS = 5 × 3+7-22 = 0 (RHS)

and LHS = 6 × 3+2 × 1-20=0 (RHS)

∴ (*a*) is correct.

Q.23. Two regression equations are x + y = 6 and x + 2y = 10 then correlation coefficient between X any Y is

(*a*) –1/2 (*b*) 1/2

(*c*) $-\frac{1}{\sqrt{2}}$ (*d*) $\frac{1}{\sqrt{2}}$

[Dec. 2014]

Solution : (*c*) is correct

Tricks : See Quicker QA book

$1x+1y=6 \Rightarrow y=-x+6$

$1x+2y=10 \Rightarrow 2y=-x+10$

$r=-\sqrt{\frac{1\times1}{1\times2}}=-\frac{1}{\sqrt{2}}$ (x & y have opposite signs.)

Q.24. Correlation coefficient between x and y is zero the two regression lines are

(*a*) Perpendicular to each other

(*b*) Coincide to each other

(*c*) Parallel to each other

(*d*) None of these ***[Dec. 2014]***

Solution : (*a*) is correct.

Q.25. The two regression lines are 16x – 20y + 132 = 0 and 80x – 30y – 428 = 0, the value of correlation coefficient is

(*a*) 0.6 (*b*) –0.6

(*c*) 0.54 (*d*) 0.45

[June 2015]

Solution : (*c*) is correct

Tricks $r=+\sqrt{\frac{16\times30}{20\times80}}=0.547$

Q.26. Which of the following is true:

(*a*) $b_{xy}=r.\frac{\sigma_y}{\sigma_x}$

(*b*) $b_{xy}=r.\frac{\sigma_x}{\sigma_y}$

(*c*) $b_{xy}=\pi.\frac{\sum xy}{\sigma_x}$

(*d*) $b_{xy}=\pi.\frac{\sum xy}{\sigma_y}$

[Dec. 2015]

Solution : (*b*) is correct.

Q.27. The regression are as follows

Regression equation of X on Y : 6X - Y = 28

Regression equation of Y on X : 64X - 45Y = 24

What will be the mean X and Y ?

(*a*) $\overline{X}=8, \overline{Y}=6$ (*b*) $\overline{X}=6, \overline{Y}=6$

(*c*) $\overline{X}=6, \overline{Y}=8$ (*d*) $\overline{X}=8, \overline{Y}=8$

[June 2016]

Solution : (*c*) is correct.

Tricks: Go by Choices

X = 6 and Y = 8 satisfy both Regression Eqns.

$\therefore \overline{X}=6;\ \overline{Y}=8$ is correct

Q.28. The two lines of regression become identical when

(*a*) r = 1 (*b*) r = –1

(*c*) r = 0 (*d*) (*a*) or (*b*)

[June 2016]

Solution : (*d*) is correct.

Q.29. Regression coefficients are affected by _____

(*a*) Change of origin

(*b*) Change of Scale

(*c*) Both origin & scale

(*d*) Neither origin nor scale

[Dec. 2016]

Solution : (*b*) is correct.

Q.30. Regression lines are passes through the_____ points

(*a*) Mean

(*b*) Standard deviation

(*c*) Both (*a*) & (*b*)

(*d*) None

[Dec. 2016]

Solution : (*a*) is correct.

Q.31. If the regression line of *x* on *y* is $3x+2y=100$, then find the value of b_{xy} ?

(*a*) $\frac{-2}{3}$ (*b*) $\frac{10}{3}$

(*c*) $\frac{3}{2}$ (*d*) $\frac{2}{3}$

[Dec. 2016]

Solution : (*a*) is correct.

$b_{xy}=-\frac{2}{3}$

Q.32. If the two regression lines are x + y = 1 and x - y = 1 then $\bar{x}$ and $\bar{y}$ are

(*a*) 1, 0 (*b*) 0, 1

(*c*) 1, 1 (*d*) None

[June 2017]

Solution : (*d*) is correct.

Sign of both Regression lines x + y = 1 and x – y = 1 are different. It means ; x & y are not correlated ∴ $(\bar{x}\ ;\ \bar{y})$ cannot be determined.

Q.33. The correlation coefficient is the _____of the two regression coefficients b_{yx} and b_{xy} :

(*a*) AM (*b*) GM

(*c*) HM (*d*) None of these

[Dec. 2017]

Solution : (*b*)

Q.34. Regression coefficient are independent of _____ :

(*a*) Change of origin

(*b*) Change of scale

(c) Both (a) & (b)

(d) None of these

[Dec. 2017]

Solution : (a)

Q.35. 5y = 9x – 22 & 20x = 9y + 350 are two regression lines. Find the correlation coefficient between x & y :

(a) 0.9 (b) 0.1

(c) –0.9 (d) –0.1

[Dec. 2017]

Solution : (a)

$9x - 5y = 22$

$20x - 9y = 350$

$$\therefore r = +\sqrt{\frac{81}{100}} = \frac{9}{10} = 0.9$$

Q.36. Regression lines are parallel then r = :

(a) ±1 (b) –1/2

(c) 0 (d) None

[June 2018]

Solution : (a)

Q.37. The two lines of regression intersect at the point:

(a) Mean

(b) Median

(c) Mode

(d) None of the these

[Nov. 2018]

Solution : (a)

Q.38. If the two lines of regression are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$, then the regression line of y on x is

(a) x + 2y – 5 = 0

(b) x + 2y = 0

(c) 2x + 3y – 8 = 0

(d) 2x + 3y = 0

[Nov. 2018]

Solution : (a) Let $x + 2y - 5 = 0$ is the Regression Eqn. of y on x then $2x + 3y - 8 = 0$ should be the Regression

Eqn. of x on y.

$$\therefore b_{yx} = -\frac{1}{2}$$

and $$b_{xy} = -\frac{3}{2}$$

Here

$$b_{xy} \cdot b_{yx} = \left(-\frac{1}{2}\right)\cdot\left(-\frac{3}{2}\right) = \frac{3}{4} < 1$$

$\therefore$ Our assumption is correct.

So; $x + 2y - 5 = 0$ is the Regression Eqn. of y on x.

Q.39. If the two regression lines are 3X = Y and 8Y = 6X, then the value of correlation coefficient is

(a) – 0.5 (b) 0.5

(c) 0.75 (d) –0.80

[Nov. 2018]

Solution : (b)

Regr. Eqns. are

$3x - y = 0$

& $6x - 8y = 0$

$$r = +\sqrt{\frac{1\times 6}{3\times 8}} = +\sqrt{\frac{1}{4}} = +\frac{1}{2} = 0.5$$

"+" Sign because X & Y have same sign.

Q.40. The regression coefficient is independent of the change of

(a) Origin

(b) Scale

(c) Scale and origin both

(d) None of these

[Nov. 2018]

Solution : (a)

Q.41. A.M of regression coefficient is

(a) Equal to r

(b) Greater than or equal to r

(c) Half of r

(d) None

[June 2019]

Solution : (b)

Q.42. If the regression line of Y on X is given by Y = X + 2 and Karl Pearson's coefficient of correlation is 0.5 then

$\frac{\sigma_y^2}{\sigma_x^2}$ = ___.

(a) 3 (b) 2

(c) 4 (d) None

[June 2019]

Solution : (c) The regression line of y on x is given by y = x + 2 [It is in the form Y = a + bX]

b_{yx} = Coefficient of X = 1

Coeff. of correlation (r) = 0.5

then Regression coefficient Y on X

$$= b_{yx} = r\frac{\sigma_y}{\sigma_x}$$

$$1 = 0.5\frac{\sigma_y}{\sigma_x}$$

$$\frac{\sigma_y}{\sigma_x} = \frac{1}{0.5} = \frac{10}{5} = 2$$

$$\left(\frac{\sigma_y}{\sigma_x}\right)^2 = (2)^2$$

$$\frac{\sigma_y^2}{\sigma_x^2} = 4.$$

Q.43. Consider to regression line 3x + 2y = 26, 6x + y = 31. Find the correlation coefficient between x and y

(a) 0.5 (b) –0.5

(c) 0.25 (d) –0.25

[Dec. 2019]

Solution : (b)

$3X + 2Y = 26 \Rightarrow 3X = -2Y + 26$

$6X + 1.Y = 31 \Rightarrow 6X = -Y + 31.$

Doing Cross-Multiplication

$3\times 1 = 3$ is smaller than $6\times 2 = 12$

So; $$r = -\sqrt{\frac{3\times 1}{6\times 2}} = -\sqrt{\frac{1}{4}} = -\frac{1}{2}$$

= –0.5

[Put sing "–" because X and Y have opposite sign in both Regression Eqn.]

Q.44. The interesting point of the two regression lines: y on x and x on y is

(a) (0, 0) (b) $(\bar{x}, \bar{y})$

(c) (b_{yx}, b_{xy}) (d) (1, 1)

[Jan. 2021]

Solution : (b) is correct

Q.45. Given that the variance of x is equal to the square of standard deviation x and the regression line of y on x is $y = 40 + 0.5\,(x - 30)$.

Then regression line of x on y is

(*a*) $y = 40 + 4\,(x - 30)$

(*b*) $y = 40 + (x - 30)$

(*c*) $y = 40 + 2\,(x - 30)$

(*d*) $x = 30 + 2\,(y - 40)$

[Jan. 2021]

Solution : (*d*) is correct

Given

Regression Eqn. of

y on x is

$$Y = 40 + 0{\cdot}5\,(x - 30)$$

or $$y - 40 = 0{\cdot}5\,(x - 30)$$

Comparing it with standard Form.

$$y - \bar{y} = b_{yx}\,(x - \bar{x})$$

We get $\bar{X} = 30$; $\bar{y} = 40$ and $b_{yx} = 0.5$

We know that

$$r^2 = b_{xy}.b_{yx} = 1 \qquad [\because \text{For one regression linc, } r = \pm 1]$$

or $$b_{xy}.(0.5) = 1$$

or $$b_{xy} = \frac{1}{0.5} = 2$$

Regression Eqn. of X on Y is

$$X - \bar{X} = b_{xy}\,(Y -)$$

$$x - 30 = 2(y - 40)$$

or $$x = 30 + 2\,(y - 40)$$

$\therefore$ (*d*) correct

Q.46. The regression coefficients remain unchanged due to

(*a*) A shift of scale

(*b*) A shift of origin

(*c*) Replacing x - values by $\frac{1}{x}$

(*d*) Replacing y values by $\frac{1}{y}$

[Jan. 2021]

Solution : (*b*) is correct

Since, Regression co-efficient does not change with respect to the change of origin but changes with respect to scale.

Q.47. If the slope of the regression line is calculated to be 5.5 and the intercept 15 then the value of Y when X is 6 is

(*a*) 88 (*b*) 48

(*c*) 18 (*d*) 78

[July 2021]

Solution : (*b*) is correct.

Regression Eqn. of Y on X is

$$Y = a + bX$$

where a = Y intercept = 15 (given)

b = slope = 5.5 (given)

Regression Eqn. of Y on X is

$$Y = 15 + 5.5X$$

when X = 6 Then

$$Y = 15 + 5.5(6) = 48.$$

Q.48. If Y = 9X and X = 0.01Y, then r is equal to:

(*a*) –0.1 (*b*) 0.1

(*c*) 0.3 (*d*) –0.3

[July 2021]

Solution : (*c*) is correct.

$$\therefore Y = 9X \Rightarrow b_{yx} = 9$$

and $$X = 0.01Y \Rightarrow b_{xy} = 0.01$$

$$r = \sqrt{b_{xy}b_{yx}} = +\sqrt{0.01 \times 9}$$

$$= +0.3$$

Because b_{xy} & b_{yx} are positive.

Q.49. The straight-line graph of the linear equation Y = a + b X, slope is horizontal if:

(*a*) b = 1 (*b*) b ≠ 0

(*c*) b = 0 (*d*) a = b ≠ 0

[July 2021]

Solution : (*c*) is correct.

$\because$ $Y = a + bX$

Slope = b.

If b = 0 then Y = a.

So, its graph is parallel to X – axis.

Q.50. If $b_{yx} = -1.6$ and $b_{xy} = -0.4$, then r_{xy} will be

(*a*) 0.4 (*b*) -0.8

(*c*) 0.64 (*d*) 0.8

[July 2021]

Solution : (*b*) is correct

$$r_{xy} = \sqrt{b_{xy} \cdot b_{yx}} = -\sqrt{(-1.6)\,(-0.4)}$$

$$= -\sqrt{0.64} = -0.8$$

Q.51. For any two variables x and y the regression equations are given as $2x + 5y - 9 = 0$ and $3x - y - 5 = 0$. What are the A.M. of x and y?

(*a*) 2, 1 (*b*) 1, 2

(*c*) 4, 2 (*d*) 2, 4

[Dec. 2021]

Solution : (*a*)

Trick Go by choices (GBC)

(A) If $\overline{X} = 2$, $\overline{Y} = 1$

$2 \times 2 + 5 \times 1 - 9 = 0$ (True)

2nd eqn. $3 \times 2 - 1 - 5 = 0$ (Also True)

(*a*) is correct.

Q.52. The intersecting point of two regression lines falls at X-axis. If the mean of X-values is 16, the standard devaluation of X and Y are respectively, 3 and 4, then the mean of Y-values is

(*a*) 16/3 (*b*) 4 (*c*) 0 (*d*) 1

[Dec. 2021]

Solution : (*c*)

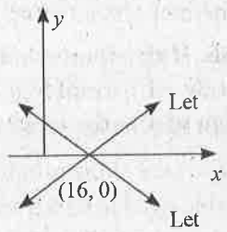
Intersecting point lies on x-axis.

So y Co-ordinate = 0

Hence, Mean Point is (16 ; 0)

$\Rightarrow \overline{X} = 16$; $\overline{Y} = 0$

(Clearly)



Q.53. The regression coefficients remain unchanged due to

(*a*) Shift of origin

(*b*) Shift of scale

(*c*) Always

(*d*) Never

[Dec. 2021]

Solution : (*a*)

Regression Coefficient does not change with respect to the change of origin but changes with respect to scale

Q.54. For positive and perfectly correlated random variables, one of the regression coefficient is 1.3 and the standard deviation of X is 2, the variance of Y is

(*a*) 2.66 (*b*) 6.76

(*c*) 6.56 (*d*) 3.16

[June 2021]

Solution : Given : $b_{yx} = r.\frac{\sigma_y}{\sigma_x} = 1.3$

$\because$ It is perfectly positively correlated

$\therefore r = 1$

$\therefore$ by $x = 1.\frac{\sigma_y}{2} = 1.3$

$\therefore \sigma_y = 2.6$

$\therefore$ (*a*) is correct

Q.55. The equations of the two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find the correlation coefficient between *x* and *y*.

(*a*) – 0.75 (*b*) 0.25

(*c*) – 0.92 (*d*) 1.25

[Dec. 2022]

Solution: Two regression eqns. are

$4x + 3y + 7 = 0 \Rightarrow 4x = -3y - 7$

(–vely correlated)

$3x + 4y + 8 = 0$

Tricks :

$$r = -\sqrt{\frac{3\times 3}{4\times 4}} = -\frac{3}{4} = -0.75$$

(*a*) is correct.

[Note : See cross product, smaller in numerator and larger in denominator]

Q.56. If the regression equations are $2x + 3y + 1 = 0$ and $5x + 6y + 1 = 0$, then Mean of *x* and *y* respectively are

(*a*) –1, –1 (*b*) –1, 1

(*c*) 1, –1 (*d*) 2, 3

[Dec. 2022]

Solution: Go by choices

(*c*) For $2x + 3y + 1 = 0$

$2 \times 1 + 3(-1) + 1 = 3 - 3 = 0$ (True)

and $5x + 6y + 1 = 0$

$\Rightarrow 5 \times 1 + 6(-1) + 1 = 6 - 6 = 0$ (True)

(1, – 1) satisfies both eqns.

$\therefore$ (*c*) is correct

Q.57. If $b, yx = 0.5, b, xy = 0.46$ then the value of correlation coefficient *r* is:

(*a*) 0.23 (*b*) 0.25

(*c*) 0.39 (*d*) 0.48

[Dec. 2022]

Solution: $r = \pm\sqrt{b_{xy}.b_{yx}}$

$r = +\sqrt{(0.5).(0.46)}$

$= 0.479 = 0.48$

(*d*) is correct.

Q.58. For variables *X* and *Y*, we collect the four observations with $\Sigma X = 10$; $\Sigma Y = 14$; $\Sigma X^2 = 65$; $\Sigma Y^2 = 5$ and $\Sigma XY = 3$. What is the regression line of *Y* on *X*?

(*a*) $Y = -0.8X - 5.5$

(*b*) $Y = 0.8X - 5.5$

(*c*) $Y = -0.8X + 5.5$

(*d*) $Y = 0.8X + 5.5$

Solution:

$$b_{YX} = \frac{N\Sigma XY - \Sigma X\Sigma Y}{N\Sigma X^2 - (\Sigma X)^2}$$

$$= \frac{4\times 3 - 10\times 14}{4\times 65 - (10)^2}$$

$= -0.8$

$$\overline{X} = \frac{\Sigma X}{N} = \frac{10}{4} = 2.5$$

$$\overline{Y} = \frac{\Sigma Y}{N} = \frac{14}{4} = 3.5$$

Regression line of Y on X is

$Y - \overline{Y} = b_{YX}(X - \overline{X})$

$\therefore Y - 3.5 = -0.8(X - 2.5)$

$Y = -0.8x + 5.5$

$\therefore$ (*c*) is correct.

Q.59. The regression lines will be perpendicular to each other when the value of *r* is:

(*a*) 1 (*b*) –1

(*c*) 1/2 (*d*) 0

Solution: (*d*) is correct.

Q.60. If the regression equations are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$, then the mean of x and the mean of y are _____, respectively:

(*a*) –3 and 4

(*b*) 2 and 4

(*c*) 1 and 2

(*d*) 2 and 1

Solution:

Tricks

GBC (Go by Choices)

(*c*) For $\overline{X} = 1; \overline{Y} = 2$

$x + 2y - 5 = 0$

$1 + 2 \times 2 - 5 = 0$ (True)

and $2 \times 1 + 3 \times 2 - 8 = 0$ (Also True)

$\therefore$ (*c*) is correct.

28 CHAPTER INDEX NUMBERS

PREVIOUS YEAR EXAM QUESTIONS (MEMORY BASED)

Q.1. In the data group Bowley's and Laspeyre's index number is as follows. Bowley's index number = 150, Laspeyre's index number = 180 then Paasche's index number is

(*a*) 120 (*b*) 30

(*c*) 165 (*d*) None of these

[June 2010]

Solution : (*a*) $\because$ Dorbish and Bowley's Index No. =

$$\frac{\text{Laspeyere's} + \text{Paasche's I.No.}}{2}$$

$$\therefore 150 = \frac{180 + \text{Paasche's}}{2}$$

$\Rightarrow 180 + \text{Paasche's} = 300$

$\therefore$ Paasche's Index No. = 120

Q.2. Consumer price index is commonly known as

(*a*) Chain Based index

(*b*) Ideal index

(*c*) Wholesale price index

(*d*) Cost of living index.

[June 2010]

Solution : (*d*)

Q.3. Find the Paasche's index number for prices from the following data taking 1970 as the base year.

Commodity	1970		1975	
	Price	quantity	Price	quantity
A	1	6	3	5
B	3	5	8	5
C	4	8	10	6

(*a*) 261.36 (*b*) 265.48 (*c*) 274.32 (*d*) 282

[June 2010]

Solution : (*a*)

P_0	Q_0	P_1	Q_1	P_1Q_1	P_0Q_1
1	6	3	5	15	5
3	5	8	5	40	15
4	8	10	6	60	24
				$\sum P_1Q_1 = 115$	$\sum P_0Q_1 = 44$

$$\text{Paasche's Index No.} = \frac{\sum P_1Q_1}{\sum P_0Q_1} \times 100 = \frac{115}{44} \times 100 = 261.36$$

Q.4. The life expectancy, E of male is a linear function of time (year). It is given that in 1980 the life expectancy was 70 years and in 2000 it was 75 years. Make a prediction of life expectancy in 2012.

(*a*) 78 (*b*) 80

(*c*) 82 (*d*) 84

[June 2010]

Solution : (*a*) Given E = at + c

Where t = Deviation of time from base year 1980

(E being a liner function of time t)

In 1980, t = 1980 - 1980 = 0

The eqn. is

E = a.0 + c = 70, So, c = 70

In year 2000, t =2000 - 1980 = 20

E = a.20 + c = 75

So, 20 a + 70 = 75

a = 5/20 = 0.25

The Eqn. is E = 0.25t + 70

For year 2012, t = 2012 - 1980 = 32

E = (0.25).(32) + 70 = 78

Q.5. If Laspeyre's index number is 90 and Paasche's index number is 160 then Fisher's index number will_____.

(*a*) 144 (*b*) 120

(*c*) 125 (*d*) None of these

[Dec. 2010]

Solution : (*b*) Fisher's index No.= $\sqrt{\text{Laspeyre's I.No.} \times \text{Paasche's I.No.}}$

Fisher's index no. = $\sqrt{90 \times 160} = 120$

Q.6. Wholesale Price Index (WPI) is given by:

(*a*) Marshall-Edgeworth Index

(*b*) Laspeyre's Index

(*c*) Paasche's Index

(*d*) None

[June 2011]

Solution : (*b*)

Q.7. Fisher's Ideal index is obtained by:

(*a*) Arithmetic Mean of Laspeyre's & Paasche's index

(*b*) Geometric Mean of Laspeyre's & Paasche's index

(*c*) Sum of Laspeyre's & Paasche's index

(*d*) None of them ***[June 2011]***

Solution : (*b*)

Fisher ideal index = $\sqrt{\text{Laspeyre's Price Index} \times \text{Paasche's Price Index}}$

Q.8. The index number of prices at a place in the year 2008 is 225 with 2004 as the base year then there is:

(*a*) average 125% increase in prices

(*b*) average 225% increase in prices.

(*c*) average 100% increase in prices

(*d*) None of the above. ***[June 2011]***

Solution : (*a*) Let the price of base year 2004 = 100

∴ the Price of current year 2008 = 225

% Increase in Price = 225 - 100 = 125 %

Q.9. Fishers Ideal Index Number not satisfies_____

(*a*) Unit Test

(*b*) Time Reversal Test

(*c*) Circular Test

(*d*) Factor Reversal Test

[Dec. 2011]

Solution : (*c*) Fishers Ideal index Number does not satisfies Circular Test.

Q.10. If the price of all commodities in a place has increased 20% in Comparison to the base period prices, then the index number of prices for the place is now_____

(*a*) 100 (*b*) 120

(*c*) 20 (*d*) 150

[Dec. 2011]

Solution : (*b*)

Index No. of current year = 100 + 20 = 120

Q.11. If $\sum P_0Q_0 = 116, \sum P_0Q_1 = 140 \sum P_1Q_0 = 97, \sum P_1Q_1 = 117$ then Fisher's ideal index number is.....

(*a*) 184 (*b*) 83.59 (*c*) 119.66 (*d*) 120

[June 2012]

Solution : (*b*)

$$\therefore \text{Fisher's index formula} = \sqrt{\frac{\sum P_1Q_0 \cdot \sum P_1Q_1}{\sum P_0Q_0 \sum P_0Q_1}} \times 100 = \sqrt{\frac{97 \times 117}{116 \times 140}} \times 100 = 83.59$$

Q.12. Find the Paasche's Index number for price from the following data taking 1970 as the base year.

[June 2012]

Solution : (*a*) Given :

Commodity	1970 Price (P_0)	1970 Qty (Q_0)	1975 Price (P_1)	1975 Qty (Q_1)	P_1Q_1	P_0Q_1
A	1	6	3	5	15	5
B	3	5	8	5	40	15
C	4	8	10	6	60	24
					$\sum P_1Q_1 = 115$	$\sum P_0Q_1 = 44$

$$\therefore \text{Paasche's index} = \frac{\sum P_1Q_1}{\sum P_0Q_1} \times 100 = \frac{115}{44} \times 100 = 261.36$$

Q.13. If Fisher's index = 150 and Paasche's Index = 144, then Laspeyre's index is_____

(*a*) 147 (*b*) 156.25

(*c*) 104.17 (*d*) 138

[Dec. 2012]

Solution : (*b*) Given Fisher's index = 150 and Paasche's index = 144

$\therefore$ Fisher's index = $\sqrt{\text{Laspeyre's} \times \text{Paasche's}}$

$150 = \sqrt{\text{Laspeyre's} \times 144}$

Squaring on both sides; we get

$150 \times 150 = \text{Laspeyre's} \times 144$

$$\text{Laspeyre's index} = \frac{150 \times 150}{144} = 156.25$$

Tricks : GBC

Q.14. Net monthly of an employees was ₹ 3,000. The consumer price index number in 1985 is 250 with rightly compensated then the additional dearness allowance to be paid to the employee is:

(*a*) ₹ 4,000 (*b*) ₹ 4,800

(*c*) ₹ 5,500 (*d*) ₹ 4,500

[Dec. 2012]

Solution : (*d*) Given,

Net monthly salary = ₹3,000 in 1980

Consumer price index in 1985 with 1980 as the base year = 250

$\therefore$ In 1985 then his monthly salary will be $= \frac{250 \times 3,000}{100} = ₹\ 7,500$

$\therefore$ The Dearness allowance to be paid to the employee

= ₹ (7,500 - 3,000) = ₹ 4,500

Q.15. Time Reversal Test is satisfied by_____

(*a*) Fisher's ideal index

(*b*) Dorbish Bowley's index

(*c*) Laspeyre's index

(*d*) None of these

[June 2013]

Solution : (*a*) is correct

Q.16. Bowley's Index Number = 150, Laspeyre's index = 180 then Paasche's index number is__

(*a*) 120 (*b*) 130

(*c*) 105 (*d*) None

[June 2013]

Solution : (*a*) is correct

Tricks : Go by choices

$$\text{Bowley's Index No.} = \frac{L+P}{2}$$

$$= \frac{120+180}{2} = 150$$

$\therefore$ (*a*) is correct.

Q.17. In 2005 price index is 286 with base 1995 then how much price increased in 2005 with base 1995?

(*a*) 286% (*b*) 386%

(*c*) 86% (*d*) 186%

[June 2013]

Solution : (*d*) is correct

% Increase in price = 286 - 100 = 186%

Q.18. What is the formula for calculating the deflated index:

(*a*) $\frac{\text{Current Value}}{\text{Price Index of current year}} \times 100$

(*b*) $\frac{\text{Current Value}}{\text{Price Index of Last year}} \times 100$

(*c*) $\frac{\text{Current Value}}{\text{Price Index of current year}} \times 100$

(*d*) $\frac{\text{Current Value}}{\text{Price Index of Last year}} \times 100$

[Dec. 2013]

Solution : (*c*) is correct

Q.19. The index number for the year 2012 taking 2011 as base using simple average of price relatives method from data given below is:

Commodity	A	B	C	D	E
Price in 2011 (P_0)	115	108	95	80	90 $\Sigma P_0 = 488$
Price in 2012 (P_1)	125	117	108	95	95 $\Sigma P_1 = 540$

(*a*) 112 (*b*) 117

(*c*) 120 (*d*) 111

[Dec. 2013]

Solution : (*d*) is correct

$$P_{01} = \frac{\sum p_1}{\sum p_0} \times 100 = \frac{540}{488} \times 100$$

= 110.65 = 111

Q.20. An index time series is a list of ___numbers for two or more periods of time

(*a*) Index (*b*) absolute

(*c*) Relative (*d*) None

[Dec. 2013]

Solution : (*a*) is correct.

Q.21. Circular test is satisfied by which index number?

(*a*) Laspeyre's

(*b*) Paasche's

(*c*) Fisher's

(*d*) None of the above

[June 2014]

Solution : (*d*) is correct

Note:- Circular test is satisfied by Simple Geometric mean of price Relatives and the aggregative with Fixed weights.

Q.22. Fisher's index number is_____ of Laspayre's and Paasche's index numbers

(*a*) A.M. (*b*) G.M.

(*c*) H.M. (*d*) None

[June 2014]

Solution : (*b*) is correct.

Q.23. Which of the following statement is true?

(*a*) Paasche's index number is based on base year quantity

(*b*) Fisher's index satisfies the circular test

(*c*) Arithmetic mean is the most appropriate average for constructing the index number

(*d*) Splicing means constructing one continuous series from two different indices on the basis of common base

[June 2014]

Solution : (*d*) is correct

Q.24. Monthly salary of an employee was ₹10,000 in the year 2000 and it was increased to ₹20,000 in the year 2013 while the consumer price index number is 240 in year 2013 with the base year 2000, what should be his salary in comparison of consumer price index in the year 2013?

(*a*) 2,000 (*b*) 16,000

(*c*) 24,000 (*d*) None

[June 2014]

Solution : (*c*) is correct

$p_0 = 10{,}000$

Let Salary in yr. 2013 = p_1

$\because$ C.L.I $= \frac{p_1}{p_0} \times 100;$

or $240 = \frac{p_1}{10{,}000} \times 100;$

$\therefore$ $p_1 = ₹ 24{,}000.$

Q.25. $\sum p_1q_0 = 1180, \sum p_0q_0 = 1170, \sum p_1q_1 = 1064, \sum p_0q_1 = 1100,$ **then Fisher ideal index number is**

(*a*) 96.73 (*b*) 98.795 (*c*) 98.77 (*d*) 100.86

[Dec. 2014]

Solution : (*c*) is correct

$$P_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100$$

$$= \sqrt{\frac{1180}{1170} \times \frac{1064}{1100}} \times 100 = 98.769 = 98.77$$

Q.26. When the prices are decreased by 30% then the index number is now

(*a*) 50 (*b*) 60

(*c*) 70 (*d*) 30

[Dec. 2014]

Solution : (*c*) is correct

New I. No = . 100 – 30 = 70.

Q.27. _____play a very important role in the construction of index number.

(*a*) Weights (*b*) Classes

(*c*) Estimate (*d*) None

[June 2015]

Solution : (*a*) is correct

Q.28. Factor reversal test is

(*a*) $\frac{\sum p_1q_1}{\sum p_0q_0}$

(*b*) $\frac{\sum p_1q_1}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}$

(*c*) $\frac{\sum p_1q_1}{\sum p_0q_1}$

(*d*) $\frac{\sum q_1p_0}{\sum q_0p_0} \times \frac{\sum q_1p_1}{\sum q_0p_1}$

[June 2015]

Solution : (*a*) is correct

Q.29. If with a rise of 10% in prices the wages are increased by 20% the real wage increases by

(*a*) 10%

(*b*) More than 10%

(*c*) 20%

(*d*) Less than 10%

[June 2015]

Solution : (*d*) is correct

$$\text{Real wage} = \frac{100+20}{100+10} \times 100 = 109.09$$

:- Real wage increases by 9.09% *i.e.* less than 10%

Q.30. Consumer Price index number for the year 1957 was 313 with 1940 as the base year. The Average Monthly wages in 1957 of the workers in to factory be ₹ 160/- their real wages is:

(*a*) ₹ 48.40 (*b*) ₹ 51.12

(*c*) ₹ 40.30 (*d*) None of these

[Dec. 2015]

Solution : (*b*) is correct.

$$\text{Real Wage} = \frac{160}{313} \times 100 = 51.12$$

Q.31. Purchasing power of money is

(*a*) Reciprocal of price index number

(*b*) Equal to price index number

(*c*) Unequal to price index number

(*d*) None of these

[June 2016]

Solution : (*a*)

Q.32. In the year 2010 the monthly salary was ₹24,000. The consumer price index number was 140 in the year 2010 which rises to 224 in the year 2016. If he has to be rightly compensated what additional monthly salary to be paid to him

(a) ₹14,400 (b) ₹38,400
(c) ₹7,200 (d) None of these

[June 2016]

Solution : (a)

Required salary in 2016 $= \frac{224}{140} \times 24000$
$= ₹ 38,400.$

$\therefore$ Additional monthly salary = 38400 - 24000 = ₹14,400/-

Q.33. The suitable index numbers for the comparison of every year is _____

(a) Fixed base index number
(b) Fisher's ideal index number
(c) Chain base index number
(d) Both (a) or (c)

[June 2016]

Solution : (c)

Q.34. Index numbers are used in

(a) Economics
(b) Statistics
(c) Both (a) & (b)
(d) None

[Dec. 2016]

Solution : (c) is correct

Q.35. From the following data

	Commodity	A	B	C	D
1992 Base	Price	3	5	4	1
year	Quantity	18	6	20	14
1993 Current	Price	4	5	6	3
year	Quantity	15	9	26	15

The Paasche's price index number is :

(a) 146.41 (b) 148.25 (c) 144.25 (d) None

[Dec. 2016]

Solution : (a) is correct.

Q.36. The time reversal test is satisfied by _____ Index number.

(a) Laspeyre's (b) Paasche's
(c) Fisher's (d) None

[Dec. 2016]

Solution : (c) is correct.

Q.37. Fisher's index number does not satisfy

(a) Unit test
(b) Circular test
(c) Time reversal test
(d) Factor reversal test

[June 2017]

Solution : (b)

Q.38. If Laspeyre's index is L and Paasche's index is P then Fisher's index F is

(a) $F = L \times P$
(b) $F^2 = L \times P$
(c) $F^2 = \sqrt{L+P}$
(d) $F = \frac{1}{L \times P}$

[June 2017]

Solution : (b)

$F = \sqrt{L.P} \Rightarrow F^2 = L.P$

Q.39. The monthly income of a person in the year 2014 was ₹8,000 and CPI was 160. The CPI is 200 in the year 2017. What will be the additional dearness allowance for the year 2017?

(a) 2400 (b) 2750
(c) 2500 (d) None

[June 2017]

Solution : (d)

Monthly Income in 2017 =

$\frac{200}{100} \times 8000 = ₹ 16,000$

$\therefore$ DA = 16000 - 8000 = ₹ 8000

Q.40. For knowing consumers price index number we want to collect data from :

(a) Retail shop prices
(b) Wholesale shop prices
(c) Fair Prices
(d) Government depots

[Dec. 2017]

Solution : (a)

Q.41. The circular test is an extension of :

(a) The time reversal test
(b) The factor reversal test
(c) The unit test
(d) None of these

[Dec. 2017]

Solution : (a)

Q.42. Fisher's ideal index number is :

(a) The arithmetic mean of Laspeyre's and Paasche's index
(b) The median of Laspeyre's and Paasche's index
(c) The mode of Laspeyre's and Paasche's index
(d) None of these

[Dec. 2017]

Solution : (d)

Q.43. Price relative is equal to :

(a) $\frac{\text{Price in the given year}}{\text{Price in the base year}} \times 100$

(b) $\frac{\text{Price in the base year}}{\text{Price in the given year}} \times 100$

(c) Price in the given year × 100
(d) Price in the base year × 100

[Dec. 2017]

Solution : (a)

Q.44. GM of Laspeyre's and Paasche's Price Index number is ____price index number :

(a) Kelly's (b) Fisher's
(c) Bowley's (d) None

[June 2018]

Solution : (b)

Q.45. Paasche's index number is expressed in terms of :

(a) $\frac{\sum p_n q_n}{\sum p_o q_n}$

(b) $\frac{\sum p_o q_o}{\sum p_n q_n}$

(c) $\frac{\sum p_n q_n}{\sum p_o q_n} \times 100$

(d) $\frac{\sum p_n q_o}{\sum p_o q_o} \times 100$

[June 2018]

Solution : (c)

Q.46. To overcome the disadvantage of a simple average of relative method, we can use _____ :

(a) Weighted average of relative method

(b) Chain base index number

(c) Simple aggregative method

(d) Fixed base index number

[June 2018]

Solution : (a)

Q.47. Time reversal & factor reversal are :

(a) Quantity Index

(b) Ideal Index

(c) Price Index

(d) Test of consistency

[May 2018]

Solution : (d)

Q.48. A series of numerical figures which show the relative position is called

(a) Index number

(b) Relative number

(c) Absolute number

(d) None

[May 2018]

Solution : (a)

Q.49. The number of test of Adequacy is

(a) 2 (b) 5

(c) 3 (d) 4

[May 2018]

Solution : (d)

Q.50. P_{01} is the index for time

(a) 1 on 0 (b) 0 on 1

(c) 1 on 1 (d) 0 on 0

[May 2018]

Solution : (a) P_{01} is the index number of 1 on 0.

Q.51. The circular test is an extension of

(a) The time is reversal test

(b) The factor reversal test

(c) The unit test

(d) None of these

[May 2018]

Solution : (a)

Q.52. If $\sum P_0 Q_0 = 1360$, $\sum P_n Q_0 = 1900$, $\sum P_0 Q_n = 1344$, $\sum P_n Q_n = 1880$ then the Laspeyre's Index Number is

(a) 0.71 (b) 1.39 (c) 1.75 (d) None of these

[May 2018]

Solution : (b)

Laspeyre's Index No. $= \frac{\sum P_n Q_0}{\sum P_0 Q_0} = \frac{1900}{1360} = 1.3970$ [In Decimal form]

Q.53. Price Relative is expressed in term of

(a) $P = \frac{P_n}{P_0}$ (b) $P = \frac{P_0}{P_n}$

(c) $P = \frac{P_n}{P_0} \times 100$ (d) $P = \frac{P_0}{P_n} \times 100$

[May 2018]

Solution : (c) Price Relative $(P) = \frac{P_n}{P_0} \times 100$

Q.54. Circular test is satisfied by

(a) Laspeyre's Index Number

(b) Paasche's Index Number

(c) The Simple Geometric Mean of Price Relatives and the Weighted Aggregative with fixed weights.

(d) None of these

[May 2018]

Solution : (c)

Q.55. If the 1970 index with base 1965 is 200 and 1965 index with base 1960 is 150, the index 1970 on base 1960 will be :

(a) 700 (b) 300 (c) 500 (d) 600

[May 2018]

Solution : (b)

Year	Link Relative	Chain Base Index Number (CBI)
1960	100	100
1965	150	$\frac{150 \times 100}{100} = 150$
1970	200	$\frac{200 \times 150}{100} = 300$

CBI = FBI = Index Number of 1970 on Base 1960 = 300

Note : - Chain Base Index Number =

$$\frac{\text{Link relative of current year} \times \text{Chain Index of Previous Year}}{100}$$

Q.56. Which of the following statement is true?

(a) Paasche's Index Number is based on the base year quantity

(b) Fisher's Index Number is the Arithmetic Mean of Laspeyre's Index Number and Paasche's Index Numbers

(c) Arithmetic Mean is the most appropriate average for constructing the index number

(d) Fisher's Index Number is an Ideal Index Number

[Nov. 2018]

Solution : (d)

Q.57. If Laspeyre's Index Number is 250 and Paasche's Index Number is 160. Then Fisher's index number is:

(a) 40000 (b) $\frac{25}{16}$

(c) 200 (d) $\frac{16}{25}$

[Nov. 2018]

Solution : (c)

Fisher's I. No. $= \sqrt{L.P} = \sqrt{250 \times 160}$

$= 200$

Q.58. The simple average method is used to calculate:

(a) Trend Variation

(b) Cyclical Variation

(c) Seasonal Variation

(d) Irregular Variation

[Nov. 2018]

Solution : (c)

Q.59. If $\sum P_0 Q_0 = 240$, $\sum P_1 Q_1 = 480$, $\sum P_1 Q_0 = 600$, and $\sum P_0 Q_1 = 192$, then Laspeyre's Index Number is:

(a) 250 (b) 300 (c) 350 (d) 200

[Nov. 2018]

Solution : (a)

$$\text{Laspeyre's I. No.} = \frac{\sum P_1 Q_1}{\sum P_0 Q_0} \times 100$$

$$= \frac{480}{192} \times 100 = 250$$

Q.60. Which one is called an ideal index number

(a) Laspeyre's index number

(b) Paasche's index number

(c) Fisher's index number

(d) Marshall Edgeworth index number

[June 2019]

Solution : (c)

Fisher's Index No. is called an Ideal Index Number.

Q.61. Which is not satisfied by Fisher's Ideal index number ?

(a) Factor Reversal Test

(b) Time Reversal Test

(c) Circular Test

(d) None of the above

[June 2019]

Solution : (c)

Q.62. The prices and quantities of 3 commodities in base and current years are as follows :

p_0	p_1	q_0	q_1
12	14	10	20
10	8	20	30
8	10	30	10

The Laspeyre's Price Index Number is

(a) 118.13 (b) 107.14 (c) 120.10 (d) None

[June 2019]

Solution :

(b) Here

p_0	p_1	q_0	q_1	p_0q_0	p_1q_0
12	14	10	20	120	140
10	8	20	30	200	160
8	10	30	10	240	300

$\sum p_0 q_0 = 560$ $\sum p_1 q_0 = 600$

$$\text{Lespeyre's Price Index No. } (P_{01}) = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

$$= \frac{600}{560} \times 100$$

$$= 107.14$$

Q.63. The cost of living index numbers in years 2015 and 2018 were 97.5 and 115 respectively. The Salary of a worker in 2015 was ₹ 19500. How much additional salary was required for him in 2018 to maintain the same standard of living as in 2015?

(a) 3000 (b) 4000

(c) 3500 (d) 4500

[June 2019]

Solution : (c)

Here

Yrs.	CLI	Salary
2015	97.5	19,500
2018	115	?

Salary Required in year 2018 =

$$\frac{\text{CLI of 2018}}{\text{CLI of 2015}} \times \text{Salary in 2015}$$

$$= \frac{115}{97.5} \times 19500 = ₹\ 23000$$

Addition Salary Required = 23,000 - 19,500 = ₹ 3,500

Q.64. For year 2015, price index was 267% with base year 2005. The percentage increase in price index over base year 2005 is:

(a) 267% (b) 67%
(c) 167% (d) None of these

[Dec. 2019]

Solution : (c)

Percentage increase

= 267 - 100 = 167%

Q.65. The value of the base time period serves as a standard point of comparison.

(a) True (b) False
(c) Both (d) None of these

[Dec. 2019]

Solution : (a)

Q.66. Fisher's ideal formula does not satisfy _____test?

(a) Unit test
(b) circular test
(c) Time reversal test
(d) None of these

[Dec. 2019, Dec. 2020]

Solution : (b) Circular test

Q.67. Index Number are expressed as .....

(a) Squares (b) Ratios
(c) Percentages (d) Combinations

[Dec. 2020]

Solution : (c)

Q.68. If Laspeyre's index number is 110 and Fisher's ideal Index number is 109. Then Paasche's Index number is

(a) 108 (b) 110
(c) 109 (d) 118

[Dec. 2020]

Solution : (a)

$$\text{Fisher's I. No} = \sqrt{\text{Laspeyre's I. No.} \times \text{Paasche's I. No}}$$

$$\Rightarrow \quad 109 = \sqrt{110 \times \text{Paasche's I. No.}}$$

$$\Rightarrow \quad 109^2 = 110 \times \text{Paasche's I. No.}$$

$$\therefore \quad \text{Passche's I. No} = \frac{109 \times 109}{110} = 108$$

Q.69. The cost of living index is always

(a) Price index number
(b) Quantity index number
(c) Weighted index number
(d) Value index number

[Jan. 2021]

Solution : (c) Cost of living Index Number is nothing but weighted Index No.

Q.70. Fisher's index number does not satisfy

(a) Unit test
(b) Circular test
(c) Time reversal test
(d) Factor reversal test

[Jan. 2021]

Solution : (b) Fisher's Index Number satisfies all the three tests except circular test.

Q.71. When the prices for quantities consumed of all commodities are changing in the same ratio, then the index numbers due to Laspeyre's and Paasche's will be

(a) Equal
(b) Unequal
(c) Reciprocal of Marshall Edge worth Index Number
(d) Reciprocal of Fisher Index Number

[Jan. 2021]

Solution : (a) is correct

Let $\frac{Q_1}{Q_0} = x$

or; $Q_1 = xQ_0$

Now Laspeyre's I. No.

$$= P_{01} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

And Paasche's I. No.

$$= P_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$$

$$= \frac{\sum P_1 x Q_0}{\sum P_0 x Q_0} \times 100$$

$$= \frac{x\sum P_1 Q_0}{x\sum P_0 Q_0} \times 100$$

$$= \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$\therefore$ Paasche's I. No. = Laspeyre's I. No.

Q.72. The consumer price Index goes up from 120 to 180 when salary goes up from 240 to 540, what is the increase in real terms?

(a) 80 (b) 150
(c) 120 (d) 240

[July 2021]

Solution : (c) is correct

$$\text{Actual Salary} = \frac{180}{120} \times 240 = 360$$

Salary increase in real terms

= 360 – 240 = 120

Q.73. The weighted aggregative price index numbers for 2001 with 2000 as the base year using Paasche's Index Number is

Commodity	Price (in ₹)		Quantities	
	2000	2001	2000	2001
A	10	12	20	22
B	8	8	16	18
C	5	6	10	11
D	4	4	7	8

(a) 112.32 (b) 112.38 (c) 112.26 (d) 112.20

[July 2021]

Solution : (*d*) is correct

Commodity	Price (in ₹)		Quantities			
	2000	2001	2000	2001		
	p_0	p_1	q_0	q_1	$p_1 q_1$	$p_0 q_1$
A	10	12	20	22	264	220
B	8	8	16	18	144	144
C	5	6	10	11	66	55
D	4	4	7	8	32	32
					$\Sigma p_1 q_1 = 506$	$\Sigma p_0 q_1 = 451$

Paasche's Index No.

$$p_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

$$= \frac{506}{451} \times 100 = 112.195 = 112.20$$

[Note : Never make Table like above in exam. Roughly Do on Calculator as:

For $\Sigma p_1 q_1$ = Type $12 \times 22 =$ button

$8 \times 18 =$ button

$6 \times 11 =$ button

$4 \times 8 =$ button

Then press GT button.

Similarly do for $\Sigma p_0 q_1$ and Then apply paasche's Formula].

Q.74. The weighted aggregative price index numbers for 2001 with 2000 as the base year using Marshal - Edgeworth Index Number is

Commodity	Price (in ₹)		Quantities	
	2000	2001	2000	2001
A	10	12	20	22
B	8	8	16	18
C	5	6	10	11
D	4	4	7	8

(*a*) 112.26 (*b*) 112.20 (*c*) 112.32 (*d*) 112.38

[July 2021]

Solution : (*a*) is correct.

Commodity	Price (in ₹)		Quantities			
	2000	2001	2000	2001		
	(p_0)	(p_1)	(q_0)	(q_1)	$p_1(q_0 + q_1)$	$p_0(q_0 + q_1)$
A	10	12	20	22	504	420
B	8	8	16	18	272	272
C	5	6	10	11	126	105
D	4	4	7	8	60	60
					$\Sigma p_1(q_0 + q_1) = 962$	$\Sigma p_0(q_0 + q_1) = 557$

Marshal Edgeworth Formula

$$p_{01} = \frac{\sum p_1 (q_0 + q_1)}{\sum p_0 (q_0 + q_1)} \times 100 = \frac{962}{857} \times 100$$

$$= 112.25 \approx 112.26$$

[Note : – Never make table like above. Time will loss. On calculator find $\Sigma p_1(q_0 + q_1)$ on calculator as

$(20 + 22) \times 12 =$

$(16 + 18) \times 8 =$

$(10 + 11) \times 6 =$

$(7 + 8) \times 4 =$ GT button (Press).

It will give $\Sigma p_1(q_0 + q_1) = 962$.

Do again as above for $\Sigma p_0(q_0 + q_1)$ by calculator. Then use Formula].

Q.75. The weighted aggregative price index number for 2001 with 2000 as the base year using Fisher's Index Number is

Commodity	Price (in ₹)		Quantities	
	2000	2001	2000	2001
A	10	12	20	22
B	8	8	16	18
C	5	6	10	11
D	4	4	7	8

(*a*) 112.32 (*b*) 112.20 (*c*) 112.38 (*d*) 112.26

[July 2021]

Solution : (*d*) is correct.

$$\text{Fisher's Index No.} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100$$

Calculator work.

Find $\sum p_1q_0$ as = 12 × 20 = ; 8 × 16 = ;

6 × 10 = ; 4 × 7 = button Then press GT button. We get $\sum p_1q_0 = 456$, Similarly find rest.

$\sum p_0q_0 = 406$; $\sum p_1q_1 = 506$; $\sum p_0q_1 = 451$

$$\therefore \quad p_{01} = \sqrt{\frac{456}{406} \times \frac{506}{451}} \times 100$$

$$= 112.26$$

Q.76. If P_{10} and P_{01} are index for 1 on 0 and 0 on 1 respectively then formula $P_{01} \times P_{10} = 1$ is used for

(*a*) Unit Test

(*b*) Time Reversal Test

(*c*) Factor Reversal Test

(*d*) Circular Test

[Dec. 2021]

Solution : (*b*)

Q.77. The weighted averaged of price relatives of commodities, when the weights are equal to the value of commodities in the current year, yields _____ index number.

(*a*) Fisher's ideal

(*b*) Laspeyres's

(*c*) Paasche's

(*d*) Marshall-Edgeworth

[Dec. 2021]

Solution : (*c*)

Q.78. From the following data base year:

Commodity	Base year		Current year	
	Price	Quantity	Price	Quantity
A	4	3	6	2
B	5	4	6	4
C	7	2	9	2
D	2	3	1	5

Fisher's Ideal Index is

(*a*) 117.30 (*b*) 115.43 (*c*) 118.35 (*d*) 116.48

[Dec. 2021]

Solution : (*a*)

Commodities	Base year		Current year					
	P_0	q_0	P_1	q_1	P_0q_0	P_1q_1	P_1q_0	P_0q_1
A	4	3	6	2	12	12	18	8
B	5	4	6	4	20	24	24	20
C	7	2	9	2	14	18	18	14
D	2	3	1	5	6	5	3	10
					ΣP_0q_0=52	ΣP_1q_1=59	ΣP_1q_0=63	ΣP_0q_1=52

Fisher's Index No.

$$P_{01} = \left\{\sqrt{\frac{\sum P_1q_0}{\sum P_0q_0} \times \frac{\sum P_1q_1}{\sum P_0q_1}}\right\} \times 100$$

$$= \left(\sqrt{\frac{63}{52} \times \frac{59}{52}}\right) \times 100$$

$$= 117.244 \cong 117.30$$

Q.79. Index numbers are not helpful in

(*a*) Framing economic policies

(*b*) Revealing trend

(*c*) Forecasting

(*d*) Identifying errors

[Dec. 2021]

Solution : (*d*)

Q.80. The three index numbers, namely, Laspeyre, Paasche and Fisher do not satisfy _____ test.

(*a*) Time reversal

(*b*) Factor reversal

(*c*) Unit

(*d*) Circular

[Dec. 2021]

Solution : (*d*)

Q.81. The test of shifting the base is called

(*a*) Unit (*b*) Circular

(*c*) Time reversal (*d*) Factor reversal

[June 2022]

Solution : (*b*) is correct

Q.82. Let P_0 and P_1 be prices of a commodity in the base and current years respectively. The price relative with respect to base year is

(*a*) P_1/P_0 (*b*) P_0/P_1

(*c*) $\frac{p_1 - p_0}{p_0}$ (*d*) $\frac{p_1 - p_0}{p_1}$

[June 2022]

Solution : (*a*) is correct

$$\text{Price relative} = \text{I. No.} = \frac{P_1}{P_0} \times 100 \text{ (In \%)}$$

$$= \frac{P_1}{P_0} \text{ (In decimal form)}$$

Q.83. Laspeyre's index number is a weighted aggregate method by taking_____ as weights.

(*a*) Quantity consumed in the base year

(*b*) Quantity consumed in the current year

(*c*) value of items consumed in the base year

(*d*) Value of items consumed in the current year

[June 2022]

Solution : (*a*) is correct

$$P_{01} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100$$

Here q_0 = base year quantity.

Q.84. Which one of the following method is based on geometric mean for calculating index number?

(*a*) Fishers' method

(*b*) Kelley's method

(*c*) Paasche's method

(*d*) Laspeyre's method

[June 2022]

Solution :

∴ (*a*) is correct

Fisher's Index No. is the GM of

Laspeyre's Index No. and Paasche's Index No.

Q.85. Which one of the following test is not applied for selecting an appropriate index number?

(*a*) Time reversal

(*b*) Price Relative

(*c*) Factor Reversal

(*d*) Circular

[June 2022]

Solution : (*b*) is correct

Test of Adequacy are:-

(*i*) Unit Test

(*ii*) Time Reversal Test

(*iii*) Factor Reversal Test

(*iv*) Circular Test

Q.86. From the following data extract the Index number by Laspeyre's method

$\Sigma P_1Q_1 = 99$, $\Sigma P_0Q_1 = 76$, $\Sigma P_0Q_0 = 73$, $\Sigma P_1Q_0 = 96$

(*a*) 130.36 (*b*) 131.51

(*c*) 130.88 (*d*) 76.04

[Dec. 2022]

Solution : *Given :*

$\sum P_oQ_o = 73$; $\sum P_1Q_0 = 96$

$$\text{Laspeyre's Index No.} = \frac{\sum P_1Q_0}{\sum P_0Q_0} \times 100$$

$$= \frac{96}{73} \times 100 = 131.5068$$

$$= 131.51$$

∴ (*b*) is correct

Q.87. Which of the following index measures the change from month to month in the cart of a representative "basket" of goods and services of the type bought by a typical household?

(*a*) Retail Price Index

(*b*) Laspeyre's Index

(*c*) Fisher's Index

(*d*) Paasche's Index

[Dec. 2022]

Solution : (*a*)

Q.88. Fisher's index number is called an ideal index number because it is satisfying

(*a*) Fector reversal test

(*b*) Time reversal Test

(*c*) Both factor and time reversal test

(*d*) Circular test

[Dec. 2022]

Solution : (*c*)

Q.89. If Laspeyre's Index is 119 and Paasche's Index is 112 then Fisher's Index number will be:

(*a*) 113.99 (*b*) 115.45

(*c*) 115.89 (*d*) 151.98

[Dec. 2022]

Solution : L = Laspeyre's I. No. = 119

P = Paasche's I. No. = 112

∴ Fisher's I. No. = $\sqrt{L.P}$

$= \sqrt{119 \times 112} = 115.446$

$= 115.45$

(*b*) is correct.

Q.90. In price index, when a new commodity is required to be added which of the following index is used?

(*a*) Shifted price index

(*b*) Splicing price index

(*c*) Deflating price index

(*d*) Value price index

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Solution : (*b*) is correct.

Q.91. Which of the following index is computed by taking the average of base year and current year?

(*a*) Marshall-Edgeworth index

(*b*) Paasche's Index

(*c*) Laspeyre's Index

(*d*) Fisher's Index

Solution : (*a*) is correct.

Q.92. Consider the data:

Year	Base year		Current year	
Commodities	Price	Quantity	Price	Quantity
A	10	5	20	2
B	15	4	25	8
C	40	2	60	6
D	25	3	40	4

Laspeyre's index is:

(*a*) 166.04

(*b*) 166.40

(*c*) 164.04

(*d*) 164.40

Solution :

Commodities	P_o	Q_0	P_1	Q_1	P_0Q_o	P_1Q_0
A	10	5	20	2	50	100
B	15	4	25	8	60	100
C	40	2	60	6	80	120

Commodities	P_0	Q_0	P_1	Q_1	P_0Q_0	P_1Q_0
D	25	3	40	4	75	120
					ΣP_0Q_0 = 265	ΣP_1Q_0 = 440

Laspeyre's Index No.

$$\frac{\Sigma P_1Q_0}{\Sigma P_1Q_0} \times 100 = \frac{440}{265} \times 100$$

$= 166.0377 = 166.04$

(*a*) is correct.

Q.93. The index number of prices for a country at a given date is 250. In comparison to the base period price, the price of all commodities in the country has increased by _____ _____times.

(*a*) 1.25

(*b*) 1.5

(*c*) 2

(*d*) 2.5

Solution :

Index No. $= \frac{250}{100} \times 100 = 250$

= (2.5) times of 100 base price

(*d*) is correct.

Q.94. Weighted geometric mean of relative formula satisfies _____ _____ test while Factor Reversal test is satisfied by _____.

(*a*) Time Reversal, Fisher's Ideal Index

(*b*) Time Reversal, Laspeyre's Index

(*c*) Factor Reversal, Paasche's Index

(*d*) Factor Reversal, Fisher's Ideal Index

Solution : (*a*) is correct.