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# Objective Probability — 5/6 marks

↓ chance | possibility

Experiment : when the answer is fixed
(conducting of a thing)

Random Experiment : when there are multiple answers

Events : tossing of a coin results

Simple / Elementary
- one ans only
- H | T

Composite
- multiple ans
- H/T or T/H

i) Mutually Exclusive / Incompatible Event
- अलग अलग answers [H आएगा तो T नहीं]

ii) Exhaustive event
- event होते उसके बाहर कोई Possibility नहीं [H, T]

iii) Equally Likely events
- भेद - भाव नहीं

For Finite elementary Events :- (Classic way)

$$\boxed{P(A) = \frac{n_A}{n} = \frac{\text{No. of equally likely events}}{\text{Total no. of events}}}$$

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## For Finite Composite Events :-

$$\boxed{P(A) = \frac{m_A}{m} = \frac{\text{No. of mutually favourable outcome}}{\text{Total no. of events}}}$$

### Important Points :-

i) Probability always lies b/w 0 to 1 $\boxed{0 \leq P(x) \leq 1}$

ii) happening of a event $= P(A)$
non-happening $= P(A')$ $\boxed{P(A) + P(A') = 1}$

iii) Odds in favour $= \dfrac{\text{No. of favourable events}}{\text{No. of unfavourable events}} = n_A : n_B$

↓

Odds in against = Inverse $= n_B : n_A$

$$P(A) = \frac{n_A}{n_A + n_B} \qquad P(B) = \frac{n_B}{n_A + n_B}$$

iv) $P(A) = 0$ (impossible event) $P(A) = 1$ (sure event)

### LIMITATIONS :-

i) Total no. of events should be finite
ii) When events are equally likely or equi-probable
iii) Limited real life examples

* Relative Frequency Distribution $= \dfrac{\text{favourable frequency}}{\text{Total frequency}}$

$$\boxed{P(A) = \frac{f_a}{n}}$$ $n = \infty$ (बहुत बड़ी value)

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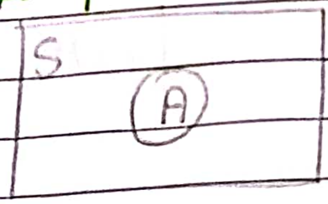
## ★ Set- Theoretic Approach to Probability :

Sample space

Total no. of events $= n(S)$




$$P(A) = \frac{n(A)}{n(S)}$$

**2 events are MUTUALLY EXCLUSIVE**

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{n(A) + n(B)}{n(S)} = P(A) + P(B)$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{0}{n(S)} = 0$$

**3 events are MUTUALLY EXCLUSIVE**

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$P(A \cap B) = P(B \cap C) = P(A \cap C) = P(A \cap B \cap C) = 0$$

**2 events are EXHAUSTIVE** $[n(A \cup B)] = n(S)$

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{n(S)}{n(S)} = 1$$

**3 events are EQUALLY LIKELY**

$$P(A) = P(B) = P(C)$$

## ★ Axiomatic approach

i) $A \subseteq S$ (subset)

ii) $P(S) = \frac{n(S)}{n(S)} = 1$

iii) $P(A_1 \cup A_2 \cup A_3 \cup \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$

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## Theorem 1 – Exclusive

या or $P(A \cup B)$ or $P(A+B) = P(A) + P(B)$

और and $P(A \cap B)$ or $P(A \cdot B) = 0$

No. of number divible by $x \Rightarrow x)\,20\,(q$ → no. divisible

Ex :- $3)\,20\,(6$ 6 no. are divisible by 3 less than 20
$\frac{18}{2}$

## Theorem 2 – Exclusive

$P(A_1 \cup A_2 \cup \ldots \cup A_K) = P(A_1) + P(A_2) + P(A_3) + \ldots$

## Theorem 3 – Mutually inclusive

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{n(A) + (n(B) - n(A \cap B)}{n(S)} = P(A) + P(B) - P(A \cap B)$$

## Theorem 4 – inclusive

$$P(A \cup B \cup C) = \frac{n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)}{n(S)}$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

## ☆ Compound Probability or Joint Probability

$P(A)$ = independent $P(B|A)$ — B dependent on A → conditional probability

| Independent event | Dependent event |
|---|---|
| Not dependency of 2 events | एक event दूसरे event पर depend करता है |
| $P(B\|A) = P(B)$ <br> $P(A\|B) = P(A)$ <br> $P(A \cap B) = P(A) \cdot P(B)$ | $P(A \cap B) = P(A) \times P(B/A)$ <br> $P(A\|B) = \frac{P(A \cap B)}{P(B)}$ |

Whenever que. doesn't mention anything → Treat as Independent events

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## Independent

$P(A \text{ and } B') = P(A \cap B') = P(A) \cdot P(B') = P(A) \cdot (1 - P(B))$

$P(A' \cap B) = P(A') \cdot P(B) = (1 - P(A)) \cdot P(B)$

$P(A' \cap B') = P(A \cap B)' = P(A') \cdot P(B')$

## Probability of Three Events :

### Dependent

$P(A \cap B \cap C) = P(A) \cdot P(B/A) \cdot P(C/A \cap B)$

### Independent

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

☆ $P(A \cup B \cup C) = P(\text{Total}) - P(A' \cap B' \cap C')$

Stochastic variable

## RANDOM VARIABLE

All possible scenarios

Ex:- Tossing of 3 coins

No. of heads : 0 , 1 , 2 , 3

$\frac{1}{8}$ $\frac{3}{8}$ $\frac{3}{8}$ $\frac{1}{8}$ [Probability distribution $P(X)$]

(Random variable) X    Real number

### 2 types

I) Discrete – fixed value

II) Continuous — interval

### Important points

I) $P_i \geq 0$

II) $\Sigma P_i = 1$

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iii)

| $X$ | $X_1$ | $X_2$ | $X_3$ | ... | $X_n$ |
|---|---|---|---|---|---|
| $P(X)$ | $P(X_1)$ or $P_1$ | $P(X_2)$ or $P_2$ | $P(X_3)$ or $P_3$ | ... | $P(X_n)$ or $P(X_n)$ |

$$\Sigma P(X) = P(X_1) + P(X_2) + P(X_3) + \dots P(X_n) = 1$$

## Probability Density Function (PDF)

i) $f(x) \geq 0$ for $x \in [\alpha, \beta]$

ii) $\int_{\alpha}^{\beta} f(x)\,dx = 1$ $\quad f(x) \geq 0$

integration

* **Expected Value**

- Mean ($\mu$) $= \dfrac{\Sigma P_i x_i}{\Sigma P_i} = \Sigma P_i x_i$ $[\Sigma P_i = 1]$
(Expected value of X)

- Expected value of $E(x_i^2) = \Sigma P_i x_i^2$
- If $g(x) = ax + b$ $\quad E g(x) = \Sigma P_i (g(x_i))$

- If Expected value of $\sigma^2 = \dfrac{\Sigma f_i x_i^2}{\Sigma f_i} - \left(\dfrac{\Sigma f_i x_i}{\Sigma f_i}\right)^2$

$$= \Sigma P_i x_i^2 - (\Sigma P_i x_i)^2$$
$$= E(x^2) - (E(x))^2$$

- $E(\sigma) = \sqrt{\Sigma P_i x_i^2 - (\Sigma P_i x_i)^2}$
$$= \sqrt{E(x^2) - (E(x))^2}$$

**PROPERTIES :-**

* If $y = a + bx$
$\mu_y = a + b\mu_x$

* If $\sigma_y = |b|\,\sigma_x$
$\sigma_y = |b|\,\sigma_x$ and $\sigma_y^2 = |b^2|\,\sigma_x^2$

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# Probability Mass Function

$$\mu = E(x) = \Sigma P_i x_i = \Sigma f(x) \cdot x$$

$$E(x^2) = \Sigma P_i x_i^2 = \Sigma f(x) \cdot x^2$$

DISCRETE

$$\sigma^2 = E(x^2) - (E(x))^2$$

$$\sigma^2 = E(x_i - E(x_i))^2$$

$$\sigma^2 = \Sigma P_i (x_i - \mu)^2$$

$$\mu = E(x) = \int_{-\infty}^{\infty} (f(x) \cdot x_i) dx$$

CONTINUOUS

$$E(x^2) = \int_{-\infty}^{\infty} (f(x) \cdot x^2) dx$$

$$\sigma^2 = E(x^2) - (E(x))^2$$

## Important points

i) Expected value, when all value is constant $K$ is

$$E(x) = \mu = K \text{ and } \sigma = \sigma^2 = 0$$

ii) Sum of two variables, then

$$E(x+y) = \Sigma P(x+y) = \Sigma Px + \Sigma Py = E(x) + E(y)$$

iii) $E(Kx) = \Sigma P_i (Kx) = K \Sigma Px = K (E(x))$

iv) $E(xy) = E(x) \cdot E(y)$

v) $E(x-y) = E(x) - E(y)$