# CHEATSHEET
# ( फर्रा )

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Ratio

Proportion

Logarithm

Indices

# #1] Ratios

comparison of 2 or more quantities

same kind

same units

$\frac{a}{b}$ is written as $a : b$

$a$ = Numerator (First term) (Antecedent)

$b$ = Denominator (second term) (consequent)

If more then two quantities of same kind are given in Ratio then this is called continued Ratio

continued Ratio of $a, b$ & $c$ is denoted by $a : b : c$

If two quantities are in $a:b$
then First quantity $= ak$
& Second Quantity $= bk$

Q The internal angles of a triangle are in $1:2:3$. Find three angles.

Sol: Let three angles are $k, 2k$ & $3k$

So $(k) + (2k) + (3k) = 180^\circ$

$6k = 180^\circ$

$k = 30^\circ$

So angles are $30^\circ, 60^\circ$ & $90^\circ$

{ Terms of a Ratio can be multiplied or Divided by any non zero number }

eg $2 : 3$

multiply by 5

$10 : 15$

eg $\frac{2}{3} : 5$

multiply by 3

$\frac{6}{3} : 15$

$2 : 15$

eg $\frac{1}{2} : \frac{1}{3} : 5$

multiply by 6 (Lcm of 2 & 3)

$3 : 2 : 30$

# $a : b$ or $c : d$

which is greater?

use calculator to find $\frac{a}{b}$ & $\frac{c}{d}$

Q 4 : 5 & 7 : 9

which is greater ?

Sol: $\frac{4}{5} = 0.80$

$\frac{7}{9} = 0.77$

so $\frac{4}{5} > \frac{7}{9}$

# $a : b = 2 : 3$

$b : c = 5 : 4$

Find $a : b : c = ?$

Sol: $a : b$

$b : c$

$2 : 3$

$5 : 4$

$10 : 15 : 12$

# Inverse of a : b = b : a

# Ratio compounded of

$a : b$ & $c : d$ = $ac : bd$

i.e. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

# Duplicate Ratio of $a : b = a^2 : b^2$

# Triplicate Ratio of $a : b = a^3 : b^3$

# Sub Duplicate Ratio of $a : b = (a)^{\frac{1}{2}} : (b)^{\frac{1}{2}}$

# Sub Triplicate Ratio of $a : b = (a)^{\frac{1}{3}} : (b)^{\frac{1}{3}}$

# Commensurable Quantities

$\Downarrow$

If Ratio of two quantities is a rational number

g 5 & 2 are commensurable
as $\frac{5}{2}$ is a Rational no.

g $\sqrt{5}$ & 2 are incommensurable
as $\frac{\sqrt{5}}{2}$ is not a Rational no.

# #2] Proportion

Equality of two Ratios
make a proportion

Denoted by $a:b::c:d$

or $a:b = c:d$

or $\frac{a}{b} = \frac{c}{d}$

eg $\frac{3}{6} = \frac{8}{16}$

so $3:6 :: 8:16$

# If a, b, c & d are in Proportion

so $a : b :: c : d$

a, b, c & d are called terms

a & d => Extremes

b & c => means

# Continous Proportion

four 3 terms a, b & c

if $\frac{a}{b} = \frac{b}{c} \Rightarrow \boxed{b^2 = ac}$

then a, b & c are called in

Continous Proportion

where $a$ = First Proportional
$c$ = Third Proportional
$b$ = mean Proportional

# Continued Proportion

for more than 3 terms

$a, b, c, d, e, f ----$

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = ----$$

eg 2, 4, 8 & 16 are in continued proportion

as $\frac{2}{4} = \frac{4}{8} = \frac{8}{16} = ---$

# Properties of Proportion

1) Cross multiplication Rule

If $\frac{a}{b} = \frac{c}{d}$

then $ad = bc$

2) Invertendo Rule

If $\frac{a}{b} = \frac{c}{d}$

then $\frac{b}{a} = \frac{d}{c}$

3) Alternendo Rule

If $a : b = c : d$

then $a : c = b : d$

4) Componendo Rule

If $a : b = c : d$

i.e. $\frac{a}{b} = \frac{c}{d}$

then $\frac{a+b}{b} = \frac{c+d}{d}$

eg $\frac{x}{2} = \frac{y}{5}$

then $\frac{x+2}{2} = \frac{y+5}{5}$

5> Dividendo Rule

If $\frac{a}{b} = \frac{c}{d}$

then $\frac{a-b}{b} = \frac{c-d}{d}$

eg. $\frac{x}{5} = \frac{y}{6}$

then $\frac{x-5}{5} = \frac{y-6}{6}$

## 6> Componendo & Dividendo Rule

If $\frac{a}{b} = \frac{c}{d}$

then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

eg $\frac{x}{2} = \frac{y}{5}$

Then $\frac{x+2}{x-2} = \frac{y+5}{y-5}$

## 7> Addendo Rule

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = K$

then $\frac{a+c+e}{b+d+f} = K$

Eg $\frac{8}{4} = \frac{6}{3} = 2$

then $\frac{8+6}{4+3} = 2$

## 8> Subtrahendo Rule

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = K$

then $\frac{a-c-e}{b-d-f} = K$

# Allegation Rule

$P_1$ $\quad\quad\quad$ $P_2$

Avg. Price ($P_{12}$)

$Q_1 : Q_2 = |P_{12} - P_2| : |P_{12} - P_1|$

eg ₹ 10/kg ₹ 15/kg

Avg. Price = ₹ 12/kg

15−12 = 3

12−10 = 2

$Q_1 : Q_2 = 3 : 2$

# #3) Indices

If a number '$x$' is multiplied 'n' times

$$x \cdot x \cdot x \cdot x \cdots \text{n times} = x^n$$

then $x$ & $n$ ⇒ Power (Index) or Exponent

$x$ ⇒ Base

eg $2^5 = 2\times2\times2\times2\times2 = 32$

2 = Base
5 = Index

## Proporties

1) $a^0 = 1$ where $a \neq 0$

2) $a^{-1} = \frac{1}{a}$

Calculator
÷ =

3) $a^{-n} = \frac{1}{a^n}$

# Calculator Trick

$a^n = ?$

→ 'a' on the screen
→ Press 'x'
→ Press '=' (n-1) times

$2^{10}$
→ '2' on screen
→ $\boxed{\times}$
→ $\boxed{=}$ 9 times

# Calculator Trick

$a^{-n} = ?$

→ 'a' on the screen

→ Press $\boxed{\div}$

→ Press $\boxed{=}$ n times

eg $2^{-10} = ?$

→ 2 on screen

→ $\boxed{\div}$

→ $\boxed{=}$ '10' times

4) $a^m \cdot a^n = a^{m+n}$

(Same Base & multiplication then Power add)

eg $x^2 x^{\frac{1}{3}} = (x)^{2+\frac{1}{3}} = (x)^{7/3}$

5) $\dfrac{a^m}{a^n} = a^{m-n}$

(Same Base & Division then Power subtract)

eg $\dfrac{x^{10}}{x^3} = x^7$

6> $(a^m)^n = a^{mn}$

eg $(x^2)^5 = x^{10}$

Note $a^{m^n} \neq a^{mn}$

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7> $\sqrt[n]{x} = (x)^{1/n}$

($n^{th}$ Root of a number)

eg $\sqrt{x} = (x)^{1/2}$

eg $\sqrt[3]{x} = (x)^{1/3}$

eg $\sqrt[4]{x} = (x)^{1/4}$

Calculator Trick

→ √ 12 times

→ −1

→ ÷ by n

→ +1

→ $\boxed{x=}$ 12 times

eg $\sqrt[5]{32} = (32)^{\frac{1}{5}}$

$= (2^5)^{1/5}$

$= 2$

32 on screen of calculator

→ $\sqrt{\phantom{x}}$ 12 times

→ $-1$

→ $\div$ by 5

→ $+1$

→ $\boxed{x=}$ 12 times

calculator's answer
will not be exactly 2
(It will be very close to 2)

8) If $a^m = a^n$

then $m = n$

9) If $a^m = k$

then $a = (k)^{1/m}$

eg $x^3 = 10$

then $x = (10)^{1/3}$

eg $x^{\frac{1}{2}} = y^{1/5}$

then $x^5 = x^2$

10> $(abcd)^m = a^m b^m c^m d^m$

## Class 10th identities

$\Rightarrow (a+b)^2 = a^2 + b^2 + 2ab$

$a^2+b^2 = (a+b)^2 - 2ab$

$\Rightarrow (a-b)^2 = a^2 + b^2 - 2ab$

$a^2+b^2 = (a+b)^2 + 2ab$

$\Rightarrow (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$\Rightarrow (a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$\Rightarrow a^3+b^3 = (a+b)(a^2+b^2-ab)$

$\Rightarrow a^3-b^3 = (a-b)(a^2+b^2+ab)$

# If $x = a^{\frac{1}{3}} + a^{-\frac{1}{3}}$

then $x^3 - 3x = a + \frac{1}{a}$

& If $x = a^{\frac{1}{3}} - a^{-\frac{1}{3}}$

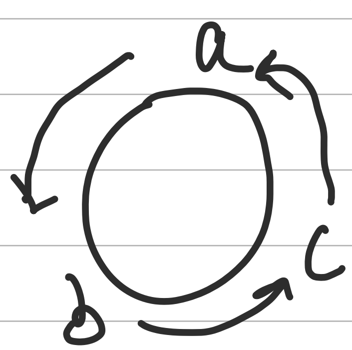
then $x^3 + 3x = a - \frac{1}{a}$

Eg $x = 5^{\frac{1}{3}} + 5^{-\frac{1}{3}}$

then $x^3 - 3x = ?$

Sol: $x^3 - 3x = 5 + \frac{1}{5} = \frac{26}{5}$

# Cyclic order Tricks




$\rightarrow (a-b) + (b-c) + (c-a) = 0$

$\rightarrow a(b-c) + b(c-a) + c(a-b) = 0$

$\rightarrow (a^2-b^2) + (b^2-c^2) + (c^2-a^2) = 0$

$\rightarrow (a^3-b^3) + (b^3-c^3) + (c^3-a^3) = 0$

$\rightarrow (b-c)(b+c-a)+(c-a)(c+a-b)$
$+(a-b)(a+b-c)=0$

$\rightarrow \frac{1}{(a-b)(b-c)}+\frac{1}{(b-c)(c-a)}+\frac{1}{(c-a)(a-b)}=0$

Q $x^{a-b}\ x^{b-c}\ x^{c-a}$

$= x^{a-b+b-c+c-a}$

$= x^{0} = 1$

Q $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \cdot \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$

$= (x)^{(a-b)(a^2+ab+b^2)}\ (x)^{(b-c)(b^2+bc+c^2)}\ (x)^{(c-a)(c^2+ac+a^2)}$

$= (x)^{a^3-b^3}\ (x)^{b^3-c^3}\ (x)^{c^3-a^3}$

$= x^{0} = 1$

# #4] Logarithm

If $\log_a(x) = y$ then $x = (a)^y$

$a$ → Base, $x$ → value, $y$ → Power

where
$x > 0$
$a > 0$
$a \neq 1$

a को Power क्या रखे जिससे value $x$ मिले

Eg $\log_2(8) = ?$

2 को Power मे क्या रखें जिससे answer 8 मिले ?

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$

so $\log_2(8) = 3$

or

$\log_2(8) = y$

$\underbrace{8} = (2)^y$

$(2)^3 = (2)^y$

$\boxed{y = 3}$

याद रखो

$\log_a(x) = y$

$x = a^y$

# Common Log

$y = \log_{10}(x)$

⇓

Calculator Trick

→ √ 19 times

→ −1

→ ×227695

There are many other tricks to find Log.

# Natural log

$y = \log_e(x)$

⇓

used in calculus

$\log(200) = 2.3010$

$= \underbrace{1}_{\text{character}} + \underbrace{0.3010}_{\text{mantissa}}$

mantissa → always positive (Lies b/w 0 & 1)

$\log(20) = 1.3010$

$= \underbrace{1}_{\text{character}} + \underbrace{0.3010}_{\text{mantissa}}$

$\log(2) = 0.3010$

$= \underbrace{0}_{\text{character}} + \underbrace{0.3010}_{\text{mantissa}}$

* $\log(0.2) = -0.699$

special case → when log is negative

$= \underbrace{0}_{\text{subtract 1}} - \underbrace{0.699}_{\text{Add 1}}$

$= \underbrace{-1+0}\ - 0.699 + 1$

$= \underbrace{-1}_{\text{character}} + \underbrace{0.3010}_{\text{mantissa}}$

negative character

$= \bar{1}.3010$

$$\log(0.02) = -1.699$$

$$= \underbrace{-1}_{\text{subtract } 1} - \underbrace{0.699}_{\Downarrow \text{ add } 1}$$

$$= \underbrace{-1-1} - \underbrace{0.699+1}$$

$$= \underbrace{-2}_{\text{character}} + \underbrace{0.3010}_{\text{mantissa}}$$

$$= \bar{2}.3010$$

# Properties of Log

1> $\log_a(1) = 0$

2> $\log_a(a) = 1$

3> $\log(xy) = \log x + \log(y)$

4> $\log\left(\frac{x}{y}\right) = \log x - \log y$

5> $\log(x)^n = n\log x$

note:

$$\boxed{[\log(x)]^n \neq n\log x}$$

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6> $\log_a(x) = \dfrac{\log_b(x)}{\log_b a}$

7> $\log_x(y) \times \log_y(x) = 1$

8> $\log_{a^b}(x) = \frac{1}{b}\log_a(x) = \log_a(x)^{1/b}$

9> $\log_{a^b}(x)^y = \dfrac{y}{b}\log_a(x)$

10> $a^{\log_a(x)} = x$