BUSINESS MATHEMATICS, LOGICAL REASONING, STATICS

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NISH10

Chapter 1 – Ratio, Proportion, Indices, Logarithms

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S. No.	Ratio	Proportion	Indices	Logarithms
1.	Ratio exists only between quantities of same kind.	Cross Product Rule If $\frac{a}{b}=\frac{c}{d}$, then $ad=bc$.	$a^n = a \times a \times a \times a \times \ldots \times a$ (n times)	$2^3=8$ is expressed in terms of Logarithms as $\log_2 8=3$. It is read as log 8 to the base 2 is 3.
2.	Quantities to be compared must be in the same units.	Invertendo If $\frac{a}{b}=\frac{c}{d}$, then $\frac{b}{a}=\frac{d}{c}$.	$a^{-n}=\frac{1}{a^n}$	$\log_a 1=0$
3.	To compare ratios, use calculator.	Alternendo	$a^0 = 1$	$\log_a a=1$

		If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a}{c}=\frac{b}{d}$, or, $\frac{d}{b}=\frac{c}{a}$		
4.	If a quantity increases or decreases in the ratio *a* : *b*, then new quantity = *b* of the original quantity/*a*. The fraction by which the original quantity is multiplied to get a	Componendo If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{b}=\frac{c+d}{d}$.	$a^m \times a^n = a^{m+n}$	$\log_a(mn) = \log_a m + \log_a n$

	new quantity is called the factor multiplying ratio. (This is basically unitary method.)			
5.	**Inverse Ratio** – The inverse ratio of *a*/*b* is *b*/*a*.	Dividendo If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a-b}{b}=\frac{c-d}{d}$	$\frac{a^m}{a^n}=a^{m-n}$	$\log_a\left(\frac{m}{n}\right)=\log_a m-\log_a n$
6.	**Compound Ratio** – The multiplication of two or more ratios	Componendo and Dividendo	$\left(a^m\right)^n=a^{mn}=\left(a^n\right)^m$	$\log_a\left(m^n\right)=n\log_a m$

	is called compound ratio. The compound ratio of *a* : *b* and *c* : *d* is *ac* : *bd*.	If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$.		
7.	**Duplicate Ratio –** A ratio compounded of itself is called a Duplicate Ratio. The duplicate ratio of *a* : *b* is $a^2 : b^2$.	Addendo If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=...$, then each of these ratios is equal to $\frac{a+c+e+...}{b+d+f+...}$, i.e.,	$(ab)^n = a^n b^n$ $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$	$\log_a m=\frac{\log_b m}{\log_b a}$

		$\frac{a}{b}=\frac{a+c+e+...}{b+d+f+...}$; $\frac{c}{d}=\frac{a+c+e+...}{b+d+f+...}$; $\frac{e}{f}=\frac{a+c+e+...}{b+d+f+...}$.		
8.	**Sub-Duplicate Ratio** – The sub-duplicate ratio of a : b is $\sqrt{a}:\sqrt{b}$.	Subtrahendo If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=...$, then each of these ratios is equal to $\frac{a-c-e-...}{b-d-f-...}$, i.e.,	$a^{m/n}=\left(a^m\right)^{1/n}$, i.e., $a^{m/n}=\sqrt[n]{a^m}$ $=\left(\sqrt[n]{a}\right)^m$	$\frac{1}{\log_a m}=\log_m a$

		$\frac{a}{b}=\frac{a-c-e-...}{b-d-f-...}$; $\frac{c}{d}=\frac{a-c-e-...}{b-d-f-...}$; $\frac{e}{f}=\frac{a-c-e-...}{b-d-f-...}$		
9.	**Triplicate Ratio –** The triplicate ratio of $a : b$ is $a^3 : b^3$.			$a^{\log_a n}=n$
10.	**Sub-Triplicate Ratio –** The sub-			$\log_{a^q} n^p=\frac{p}{q}\log_a n$

	triplicate ratio of a : b is $\sqrt[3]{a} : \sqrt[3]{b}$.			

Question 1

If A : B = 3 : 5, B : C = 5 : 4, C : D = 2 : 3, and D is 50% more than E, find the ratio between A and E.

(a) 2 : 3 (b) 3 : 4 (c) 3 : 5 (d) 4 : 5

Solution

(b)

Let E be 1. Then, D will be 1.5. Therefore, D : E = 1.5 : 1, or, 3 : 2.

Thus, we have $\frac{A}{B}=\frac{3}{5};\ \frac{B}{C}=\frac{5}{4};\ \frac{C}{D}=\frac{2}{3};\ \frac{D}{E}=\frac{3}{2}$

Therefore, $\frac{A}{E} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D} \times \frac{D}{E} = \frac{3}{5} \times \frac{5}{4} \times \frac{2}{3} \times \frac{3}{2} = 0.75$

Question 2

Find the value of $\sqrt{6561}+\sqrt[4]{6561}+\sqrt[8]{6561}$

(a) 81 (b) 93 (c) 121 (d) 243

Solution

(b)

$$\sqrt{6561}+\sqrt[4]{6561}+\sqrt[8]{6561}=93$$

Question 3

Find the value of $\log\frac{x^n}{y^n}+\log\frac{y^n}{z^n}+\log\frac{z^n}{x^n}$.

(a) –1 (b) 0 (c) 1 (d) 2

Solution

(b)

$$\log\frac{x^n}{y^n}+\log\frac{y^n}{z^n}+\log\frac{z^n}{x^n}$$

$$\log x^n-\log y^n+\log y^n-\log z^n+\log z^n-\log x^n=0$$

Question 4

If $\frac{8^n \times 2^3 \times 16^{-1}}{2^n \times 4^2} = \frac{1}{4}$, then the value of n

(a) 1 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

Solution

(c)

$$\frac{8^n \times 2^3 \times 16^{-1}}{2^n \times 4^2} = \frac{1}{4}$$

$$\Rightarrow \frac{8^n \times 2^3}{2^n \times 4^2 \times 16} = \frac{1}{4}$$

$$\Rightarrow \frac{\left(2^3\right)^n \times 2^3}{2^n \times \left(2^2\right)^2 \times 2^4} = \frac{1}{4}$$

$$\Rightarrow \frac{2^{3n} \times 2^3}{2^n \times 2^4 \times 2^4} = \frac{1}{4}$$

$$\Rightarrow \frac{2^{3n+3}}{2^{n+4+4}} = \frac{1}{4}$$

$$\Rightarrow 2^{3n+3-n-4-4} = \frac{1}{4}$$

$$\Rightarrow 2^{2n-5} = \frac{1}{4}$$

We know that $\frac{1}{4}$ can be written as 2^{-2}.

Therefore, $2^{2n-5} = 2^{-2}$

Since the bases are same, powers can be equated.

Therefore, $2n - 5 = -2$

$$\Rightarrow 2n = 5 - 2$$

$$\Rightarrow 2n = 3$$

$$\Rightarrow n = \frac{3}{2}$$

Question 5

If $\log_{10} 5 + \log_{10}(5x+1) = \log_{10}(x+5) + 1$, then x is equal to:

(a) 1 (b) 3 (c) 5 (d) 10

Solution

(b)

$$\log_{10} 5 + \log_{10}(5x+1) = \log_{10}(x+5) + 1$$

$$\Rightarrow \log_{10} 5 + \log_{10}(5x+1) = \log_{10}(x+5) + \log_{10} 10$$

$\Rightarrow \log_{10}\{5\times(5x+1)\} = \log_{10}\{(x+5)\times 10\}$ (As $\log a + \log b = \log ab$)

Taking Anti-log on both sides, we'll get:

$5(5x+1) = 10(x+5)$

$\Rightarrow 5x+1 = 2(x+5)$

$\Rightarrow 5x+1 = 2x+10$

$\Rightarrow 5x-2x = 10-1$

$\Rightarrow 3x = 9$

$\Rightarrow x = \frac{9}{3} = 3$

Question 6

If $xy+yz+zx=-1$, then the value of $\left(\frac{x+y}{1+xy}+\frac{z+y}{1+zy}+\frac{x+z}{1+zx}\right)$ is:

(a) xyz (b) $-\frac{1}{yz}$ (c) $\frac{1}{xyz}$ (d) $\frac{1}{x+y+z}$

Solution

(c)

Given $xy+yz+zx=-1$

This means $1+xy=-yz-zx$ …Eq. (1)

$1+yz=-xy-zx \ldots$Eq. (2)

$1+zx=-xy-yz \ldots$Eq. (3)

$$\frac{x+y}{1+xy}+\frac{z+y}{1+zy}+\frac{x+z}{1+zx}$$

Substituting the values of $1+xy$, $1+zy$, and $1+zx$ above from Eqs. (1), (2), and (3), we get:

$$\frac{x+y}{-yz-zx}+\frac{z+y}{-xy-zx}+\frac{x+z}{-xy-yz}$$

$$\Rightarrow \frac{x+y}{-z(y+x)}+\frac{z+y}{-x(y+z)}+\frac{x+z}{-y(x+z)}$$

$$\Rightarrow \frac{-1}{z}+\frac{-1}{x}+\frac{-1}{y}$$

$$\Rightarrow -\left(\frac{1}{z}+\frac{1}{x}+\frac{1}{y}\right)$$

$$\Rightarrow -\left(\frac{xy+yz+zx}{xyz}\right)$$

$$\Rightarrow -\left(\frac{-1}{xyz}\right)$$

$$\Rightarrow \frac{1}{xyz}$$

Question 7

The salaries of *A*, *B* and *C* are in the ratio 2 : 3 : 5. If increments of 15%, 10% and 20% are allowed respectively to their salary, then what will be the new ratio of their salaries?

(a) 23 : 33 : 60 (b) 33 : 23 : 60 (c) 23 : 60 : 33 (d) 33 : 60 : 23

Solution

(a)

Since the ratio of the salaries of *A*, *B* and *C* is 2:3:5, let the salaries be 200, 300, and 500 respectively.

A's new salary = 200 + (15% of 200) = 230

B's new salary = 300 + (10% of 300) = 330

C’s new salary = 500 + (20% of 500) = 600

Therefore, clearly, the new ratio is 23:33:60.

Question 8

If $A : B = 5 : 3$, $B : C = 6 : 7$, and $C : D = 14 : 9$, then the value of $A : B : C : D$ is:

(a) 20 : 14 : 12 : 9 (b) 20 : 9 : 12 : 14 (c) 20 : 9 : 14 : 12 (d) 20 : 12 : 14 : 9

Solution

(d)

We have $\frac{A}{B} = \frac{5}{3}$ and $\frac{B}{C} = \frac{6}{7}$.

To make the Bs same, let's multiply $\frac{A}{B} = \frac{5}{3}$ with $\frac{2}{2}$

Now, $\frac{A}{B}=\frac{5}{3}\times\frac{2}{2}=\frac{10}{6}$ and $\frac{B}{C}=\frac{6}{7}$.

Also, we have $\frac{C}{D}=\frac{14}{9}$.

To make the Cs same, let's multiply $\frac{B}{C}=\frac{6}{7}$ with $\frac{2}{2}$.

Therefore, $\frac{B}{C}=\frac{6}{7}\times\frac{2}{2}=\frac{12}{14}$.

Now, we have $\frac{A}{B}=\frac{10}{6}$; $\frac{B}{C}=\frac{12}{14}$; $\frac{C}{D}=\frac{14}{9}$.

Again, to make the Bs same, let's multiply $\frac{A}{B}=\frac{10}{6}$ with $\frac{2}{2}$.

Therefore, $\frac{A}{B}=\frac{10}{6}\times\frac{2}{2}=\frac{20}{12}$

So, now we have $\frac{A}{B}=\frac{20}{12};\ \frac{B}{C}=\frac{12}{14};\ \frac{C}{D}=\frac{14}{9}$

Therefore, $A:B:C:D=20:12:14:9$

Question 9

X and Y have their present ages in the ratio 6 : 7. 14 years ago, the ratio of the ages of the two was 4 : 5. What will be the ratio of their ages 21 years from now?

(a) 7 : 11 (b) 9 : 10 (c) 8 : 11 (d) 11 : 13

Solution

(b)

Let the ages of X and Y be $6x$ and $7x$ respectively.

14 years, ago, their ages would have been $(6x-14)$, and $(7x-14)$.

It is given that the ratio of their ages 14 years ago was 4 : 5.

Therefore, $\frac{(6x-14)}{(7x-14)}=\frac{4}{5}$

$\Rightarrow 5(6x-14)=4(7x-14)$

$\Rightarrow 30x-70=28x-56$

$\Rightarrow 30x-28x=70-56$

$\Rightarrow 2x=14$

$\Rightarrow x=\frac{14}{2}=7$

Therefore, the present ages are 6 × 7 = 42, and 7 × 7 = 49 respectively.

Their ages after 21 years will be 42 + 21 = 63, and 49 + 21 = 70 respectively.

Therefore, the ratio of their ages after 21 years will be 63 : 70 = 0.9.

Now, try the options.

Option (a) → 7 : 11 = 0.6363

Option (b) → 9 : 10 = 0.9

Therefore, option (b) is the answer.

Question 10

If $x=\sqrt{3}+\frac{1}{\sqrt{3}}$, then $\left(x-\frac{\sqrt{126}}{\sqrt{42}}\right)\left(x-\frac{1}{x-\frac{2\sqrt{3}}{3}}\right)=?$

(a) 5/6 (b) 6/5 (c) 2/3 (d) –3/5

Solution

(a)

$$x=\sqrt{3}+\frac{1}{\sqrt{3}}=2.3094$$

$$x-\frac{\sqrt{126}}{\sqrt{42}}=2.3094-1.7321=0.5773$$

$$x-\frac{1}{x-\frac{2\sqrt{3}}{3}}=2.3094-\frac{1}{2.3094-1.1547}=1.4434$$

$$\left(x-\frac{\sqrt{126}}{\sqrt{42}}\right)\left(x-\frac{1}{x-\frac{2\sqrt{3}}{3}}\right)=0.5773\times1.4434=0.8333$$

Now, try the options.

Option (a) → 5/6 = 0.8333

Therefore, option (a) is the answer.

Question 11

Find the value of a from the following: $\left(\sqrt{9}\right)^{-5} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$

(a) 11 (b) 13 (c) 15 (d) 17

Solution

(d)

$$\left(\sqrt{9}\right)^{-5} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$$

$$3^{-5} \times \left(3^{\frac{1}{2}}\right)^{-7} = \left(3^{\frac{1}{2}}\right)^{-a}$$

$$3^{-5} \times 3^{-\frac{7}{2}} = 3^{-\frac{a}{2}}$$

$$3^{-5-\frac{7}{2}} = 3^{-\frac{a}{2}}$$

$$3^{\frac{-10-7}{2}} = 3^{-\frac{a}{2}}$$

$$3^{-\frac{17}{2}} = 3^{-\frac{a}{2}}$$

$$-\frac{17}{2} = -\frac{a}{2}$$

Question 12

If $\log_a(ab) = x$, then $\log_b(ab) = ?$

(a) $1/x$ (b) $\frac{x}{1+x}$ (c) $\frac{x}{x-1}$ (d) None

Solution

(c)

$\log_a(ab) = x$

$\log_a a + \log_a b = x \quad [\text{As } \log m + \log n = \log mn]$

$1 + \log_a b = x$

$\log_a b = x - 1 \ldots$Eq. (1)

We know that $\log_a b \times \log_b a = 1$

Putting the value of $\log_a b$ from eq. (1), we get:

$(x-1) \times \log_b a = 1$

$$\log_b a = \frac{1}{x-1}$$

$$\log_a (ab) = \frac{\log_b (ab)}{\log_b a}$$ [As per Base Change Formula]

$\log_b (ab) = \log_a (ab) \times \log_b a$

$$\log_b(ab) = x \times \left(\frac{1}{x-1}\right) \quad \left[\text{As } \log_b(ab) = x \text{ and } \log_b a = \frac{1}{x-1}\right]$$

$$\log_a(ab) = \frac{x}{x-1}$$

Question 13

A vessel contained a solution of acid and water in which water was 64%. Four litres of the solution were taken out of the vessel and the same quantity of water was added. If the resulting solution contains 30% acid, the quantity (in litres) of the solution, in the beginning in the vessel, was:

(a) 12 (b) 36 (c) 24 (d) 27

Solution

(c)

Let the initial total volume be V.

Water = $0.64V$; Acid = $0.36V$

Now, 4 litres were taken out.

Remaining Water = $0.64V - (0.64 \times 4) = 0.64V - 2.56$

Remaining Acid = $0.36V - (0.36 \times 4) = 0.36V - 1.44$

To the above, 4 litres of water was added. Therefore, the total volume of the vessel would be V – 4 litres + 4 litres = V.

Now, it is given that this resulting solution contains 30% of acid.

Therefore, $\frac{0.36V - 1.44}{V} = 0.30$

$\Rightarrow 0.36V - 1.44 = 0.30V$

$\Rightarrow 0.36V - 0.30V = 1.44$

$$\Rightarrow 0.06V = 1.44$$

$$\Rightarrow V = \frac{1.44}{0.06} = 24$$

Question 14

If $\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$, then the value of x is:

(a) 64 (b) 4 (c) 16 (d) 2

Solution

(c)

$$\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$$

$$\Rightarrow \log_{2^2} x + \log_{2^4} x + \log_{2^6} x + \log_{2^8} x = \frac{25}{6}$$

$$\Rightarrow \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x + \frac{1}{6}\log_2 x + \frac{1}{8}\log_2 x = \frac{25}{6}$$

$$\Rightarrow \log_2 x\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right) = \frac{25}{6}$$

$$\Rightarrow \log_2 x\left(\frac{12+6+4+3}{24}\right) = \frac{25}{6}$$

$$\Rightarrow \log_2 x\left(\frac{25}{24}\right) = \frac{25}{6}$$

$$\Rightarrow \log_2 x = \frac{25}{6} \times \frac{24}{25}$$

$$\Rightarrow \log_2 x = 4$$

$$\Rightarrow x = 2^4 = 16$$

Question 15

If $x^2 + y^2 = 7xy$, then $\log \frac{1}{3}(x+y) = ?$

(a) $(\log x + \log y)$ (b) $\frac{1}{2}(\log x + \log y)$ (c) $\frac{1}{3}(\log x + \log y)$ (d) $3(\log/ \log y)$

Solution

(b)

$x^2 + y^2 = 7xy$

$\Rightarrow x^2 + y^2 + 2xy - 2xy = 7xy$

$$\Rightarrow (x+y)^2 - 2xy = 7xy$$

$$\Rightarrow (x+y)^2 = 7xy + 2xy$$

$$\Rightarrow (x+y)^2 = 9xy$$

$$\Rightarrow x+y = \sqrt{9xy}$$

$$\Rightarrow x+y = 3\sqrt{xy}$$

$$\Rightarrow x+y = 3(xy)^{\frac{1}{2}}$$

We have to find the value of $\log\frac{1}{3}(x+y)$

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$$\Rightarrow \log \frac{1}{3} \times 3(xy)^{\frac{1}{2}}$$

$$\Rightarrow \log (xy)^{\frac{1}{2}}$$

$$\Rightarrow \frac{1}{2}(\log xy)$$

$$\Rightarrow \frac{1}{2}(\log x + \log y)$$

Question 16

Value of $\left[9^{n+\frac{1}{4}}.\frac{\sqrt{3.3^{n}}}{3.\sqrt{3^{-n}}}\right]^{\frac{1}{n}}$

(a) 9 (b) 27 (c) 81 (d) 3

Solution

(b)

We can see that none of the options contain *n*. This means that ultimately *n* has to get cancelled out. Therefore, we can assume the value of *n* to be anything, and we'll get the answer. For the sake of simplicity, let's assume the value of *n* to be 1.

Question 17

If $3^x = 5^y = 75^z$, then:

(a) $x + y - z = 0$ (b) $\frac{2}{x} + \frac{1}{y} = \frac{1}{z}$ (c) $\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$ (d) $\frac{2}{x} + \frac{1}{z} = \frac{1}{y}$

Solution

(c)

Question 18

A bag contains ₹187 in the form 1 rupee, 50 paise and 10 paise coins in the ratio 3:4:5. Find the number of each type of coins.

(a) 102, 136, 170 (b) 136, 102, 170 (c) 170, 102, 136 (d) None

(a)

Let the number of 1 rupee, 50 paise, and 10 paise coins be $3x$, $4x$ and $5x$ respectively.

Value of 1 rupee coins = $3x \times$ ₹1 = ₹$3x$

Value of 50 paise coins = $4x \times$ ₹0.50 = ₹$2x$

Value of 10 paise coins = $5x \times$ ₹0.10 = ₹$0.50x$

Total value = $3x + 2x + 0.5x = 5.5x$

We know that the total value is ₹187.

Therefore, $5.5x = 187$

$$\Rightarrow x = \frac{187}{5.5} = 34$$

Therefore, number of ₹1 coins = 3 × 34 = 102

Number of 50 paise coins = 4 × 34 = 136

Number of 10 paise coins = 5 × 34 = 170

Question 19

$\log_e x + \log(1+x) = 0$ is equivalent to:

(a) $x^2 + x + e = 0$ (b) $x^2 + x - e = 0$ (c) $x^2 + x + 1 = 0$ (d) $x^2 + x - 1 = 0$

(d)

In this question, the base of $\log(1+x)$ will be taken as *e*, as the base of the term log *x* is also *e*.

Therefore, the given equation can be written as:

$\log_e x + \log_e(1+x) = 0$

$\Rightarrow \log_e x + \log_e(1+x) = \log_e 1$

Also, we know that $\log a + \log b = \log ab$

Therefore, $\log_e x + \log_e (1+x) = \log_e 1$ can be written as:

$$\log_e \{x(1+x)\} = \log_e 1$$

$$\Rightarrow \log_e \{x + x^2\} = \log_e 1$$

$$\Rightarrow x + x^2 = 1$$

$$\Rightarrow x^2 + x - 1 = 0$$

Question 20

If $x = 3^{1/4} + 3^{-1/4}$, and $y = 3^{1/4} - 3^{-1/4}$, then the value of $3(x^2 + y^2)^2$ will be:

(a) 12 (b) 18 (c) 46 (d) 64

(d)

On calculator, press 3 → √ → √ → M+ → 1 → ÷ → MRC MRC = M+

Press 3 → √ → √ → + → MRC MRC = → × → =. This gives you the value of $x^2 = 4.3094$.

Press 3 → √ → √ → M+ → 1 → ÷ → MRC MRC = M+

Press 3 → √ → √ → – → MRC MRC = → × → =. This gives you the value of $y^2 = 0.3094$.

Press 4.3094 + 0.3094 = × = × 3 = 63.9999 ≈ 64

Question 21

Find the value of $(x+y)$, if $\left(x+\frac{y^3}{x^2}\right)^{-1}-\left(\frac{x^2}{y}+\frac{y^2}{x}\right)^{-1}+\left(\frac{x^3}{y^2}+y\right)^{-1}=\frac{1}{3}$.

(a) 1/3 (b) 3 (c) ½ (d) 2

(b)

$$\left(x+\frac{y^3}{x^2}\right)^{-1}-\left(\frac{x^2}{y}+\frac{y^2}{x}\right)^{-1}+\left(\frac{x^3}{y^2}+y\right)^{-1}=\frac{1}{3}$$

$$\left(\frac{x^3+y^3}{x^2}\right)^{-1}-\left(\frac{x^3+y^3}{xy}\right)^{-1}+\left(\frac{x^3+y^3}{y^2}\right)^{-1}=\frac{1}{3}$$

$$\left(\frac{x^2}{x^3+y^3}\right)-\left(\frac{xy}{x^3+y^3}\right)+\left(\frac{y^2}{x^3+y^3}\right)=\frac{1}{3}$$

$$\frac{x^2-xy+y^2}{x^3+y^3}=\frac{1}{3}$$

$$\frac{x^2-xy+y^2}{(x+y)\left(x^2-xy+y^2\right)}=\frac{1}{3}$$

$$\frac{1}{x+y}=\frac{1}{3}$$

$$3=x+y$$

$$x+y=3$$

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Question 22 – Ambiguous

If $pqr = a^x$, $qrs = a^y$, $rsp = a^z$, then find the value of $\left(pqrs\right)^{1/2}$.

(a) a^{x+y+z} (b) $a^{\sqrt{x+y+z}}$ (c) $a^{\sqrt[4]{x+y+z}}$ (d) $\left(a^{x+y+z}\right)^{1/4}$

(d)

$pqr = a^x$

$qrs = a^y$

$rsp = a^z$

Multiplying these equations, we have

$$\left(pqr\right)\times\left(qrs\right)\times\left(rsp\right)=a^{x}\times a^{y}\times a^{z}$$

$$p^{2}q^{2}r^{3}s^{2}=a^{x+y+z}$$

$$p^{2}q^{2}r.r^{2}s^{2}=a^{x+y+z}$$

$$r\left(p^{2}q^{2}r^{2}s^{2}\right)=a^{x+y+z}$$

$$r\left(pqrs\right)^{2}=a^{x+y+z}$$

Now, ICAI has simply ignored this additional *r* outside the bracket on the left-hand side. So, we'll also do the same. Therefore, we'll have:

$$\left(pqrs\right)^{2}=a^{x+y+z}$$

Taking fourth root on both sides of the equation, we have:

$$\left\{ \left(pqrs \right)^2 \right\}^{1/4} = \left(a^{x+y+z} \right)^{1/4}$$

$$\left(pqrs \right)^{2\times\frac{1}{4}} = \left(a^{x+y+z} \right)^{1/4}$$

$$\left(pqrs \right)^{1/2} = \left(a^{x+y+z} \right)^{1/4}$$

Question 23

The ratio of the earnings of two persons 3:2. If each saves $1/5^{th}$ of their earnings, the ratio of their savings is:

(a) 2 : 3 (b) 3 : 2 (c) 4 : 5 (d) 5 : 4

(b)

Let the earnings of two persons be ₹300 and ₹200 respectively.

$1/5^{th}$ of ₹300 = ₹60

$1/5^{th}$ of ₹200 = ₹40

Therefore, the ratio of their savings is 60:40 = 3:2.

Question 24

If $x = 5^{1/3} + 5^{-1/3}$, then $5x^3 - 15x$ is given by:

(a) 25 (b) 26 (c) 27 (d) 30

Solution

(b)

Question 25

The value of $\log_5\left(1+\frac{1}{5}\right)+\log_5\left(1+\frac{1}{6}\right)+-----+\log_5\left(1+\frac{1}{624}\right)$

(a) 2
(b) 3
(c) 5
(d) 0

Solution

(b)

$$\log_5\left(1+\frac{1}{5}\right)+\log_5\left(1+\frac{1}{6}\right)+-----+\log_5\left(1+\frac{1}{624}\right)$$

$$=\log\left(\frac{6}{5}\right)+\log\left(\frac{7}{6}\right)\log\left(\frac{8}{7}\right)+-----+\log\left(\frac{625}{624}\right)$$

$$=\log_5\left(\frac{5}{6}\times\frac{7}{6}\times\frac{8}{7}\times\ldots\times\frac{624}{625}\times\frac{625}{624}\right)$$

$$=\log_5\left(\frac{625}{5}\right)$$

$$=\log_5(125)=\log_5 5^3=3\log_5 5=3$$

CA NISHANT KUMAR

Question 26

$$\log_{2\sqrt{2}}\left(512\right):\log_{3\sqrt{2}}324=$$

(a) 128 : 81

(b) 2 : 3

(c) 3 : 2

(d) None

Solution

(c)

$$\log_{2\sqrt{2}}\left(512\right):\log_{3\sqrt{2}}324$$

$$=\frac{\log 512}{\log 2\sqrt{2}}:\frac{\log 324}{\log 3\sqrt{2}}$$

$$=\frac{\log(8)^{3}}{\log\sqrt{2\times2\times2}}:\frac{\log18^{2}}{\log\sqrt{3\times3\times2}}$$

$$=\frac{3\log8}{1/2\log8}:\frac{\log18^{2}}{1/2\log18}$$

$$=(3\times2):(2\times2)$$

$$=6:4$$

$$=3:2$$

Question 27

$\log_{0.01} 10{,}000$

(a) 2 (b) –2 (c) 4 (d) –4

Solution

(b)

Question 28 – MTP June, 2023

The value of $\dfrac{64\left(b^4a^3\right)^6}{\left[4\left(a^3b\right)^2\times\left(ab\right)^2\right]}$

(a) $16a^{10}b^{20}$ (b) $4a^{20}b^{10}$ (c) $8a^{10}b^{20}$ (d) $4a^{10}b^{20}$

Solution

(a)

$$\frac{64\left(b^4a^3\right)^6}{\left[4\left(a^3b\right)^2\times\left(ab\right)^2\right]}$$

$$=\frac{64\left(b^{4\times6}a^{3\times6}\right)}{\left[4\left(a^{3\times2}b^{2}\right)\times\left(a^{2}b^{2}\right)\right]}$$

$$=\frac{64\left(b^{24}a^{18}\right)}{\left[4\left(a^{6}b^{2}\right)\times\left(a^{2}b^{2}\right)\right]}$$

$$=\frac{64a^{18}b^{24}}{\left[4a^{6}b^{2}\times a^{2}b^{2}\right]}$$

$$=\frac{64a^{18}b^{24}}{4a^{8}b^{4}}$$

$$=16a^{18-8}b^{24-4}$$

$$=16a^{10}b^{20}$$

Question 29 – December, 2022; MTP June, 2023

Four persons A, B, C, D wish to share a sum in the ratio of 5:2:4:3. If D gets ₹1,000 less than C, then the share of B?

(a) ₹2,000 (b) ₹1,200 (c) ₹2,400 (d) ₹3,000

Solution

(a)

Now, we have four people, A, B, C, and D, who want to divide a sum of money in the ratio of 5:2:4:3. We also know that D's share is ₹1,000 less than C's share.

To find out the share of B, let's assign variables to represent the shares. Let A's share be $5x$, B's share be $2x$, Cs share be $4x$, and D's share be $3x$.

Now, we know that D's share is ₹1,000 less than C's share. So we can set up the following equation:

C’s share ($4x$) – D’s share ($3x$) = ₹1,000

Simplifying the equation, we get:

$4x - 3x$ = ₹1,000

That gives us:

x = ₹1,000

Now that we know the value of x, we can find the share of each person.

B's share is represented by $2x$, so substituting the value of x, we get:

B’s share = 2 × ₹1,000 = ₹2,000

Therefore, the share of B is ₹2,000.

Question 30 – MTP June, 2023

The mean proportional between $12x^2$ and $27y^2$ is:

(a) $18xy$ (b) $81xy$ (c) $8xy$ (d) $9xy$

Solution

(a)

Mean Proportional = $\sqrt{12x^2 \times 27y^2} = \sqrt{324x^2y^2} = 18xy$

Question 31 – MTP June, 2023

If thrice of A’s age 6 years ago be subtracted from twice his present age, the result would be equal to his present age. Find A’s present age.

(a) 7 (b) 8 (c) 9 (d) 6

Solution

(c)

Let’s assume A’s present age is x years.

According to the problem, if we subtract thrice of A’s age 6 years ago from twice his present age, the result would be equal to his present age.

Twice of A’s present age is $2 \times x = 2x$.

Thrice A's age 6 years ago is $3 \times (x - 6) = 3x - 18$.

Now, we can set up the equation:

$2x - (3x - 18) = x$

Let's simplify the equation:

$2x - 3x + 18 = x$

$-x + 18 = x$

To solve for x, let's bring x terms to one side and constants to the other side:

$18 = 2x$

$\Rightarrow x = 18 \div 2 = 9$

Question 32 – MTP June, 2023

If $\log_3 4.\log_4 5.\log_5 6.\log_6 7.\log_7 8.\log_8 9 = x$, then find the value of x.

(a) 4 (b) 2 (c) 3 (d) 1

Solution

(b)

$$\Rightarrow \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \frac{\log 7}{\log 6} \times \frac{\log 8}{\log 7} \times \frac{\log 9}{\log 8} = x$$

$$\Rightarrow \frac{\log 9}{\log 3} = x$$

$$\Rightarrow \log_3 9 = x$$

$$\Rightarrow 3^x = 9$$

$$\Rightarrow x = 2$$

Question 33 – MTP June, 2023

If $\frac{1}{2}\log_{10} 4 = y$, and if $\frac{1}{2}\log_{10} 9 = x$, then find the value of $\log_{10} 15$.

(a) $x - y + 1$ (b) $x + y - 1$ (c) $x + y + 1$ (d) $y - x + 1$

Solution

(a)

$$\frac{1}{2}\log_{10} 4 = y$$

$$\Rightarrow \frac{1}{2}\log_{10} 2^2 = y$$

$$\Rightarrow \frac{2}{2}\log_{10} 2 = y$$

$$\Rightarrow \log_{10} 2 = y \ldots \text{Eq. (1)}$$

$$\frac{1}{2}\log_{10} 9 = x$$

$$\Rightarrow \frac{1}{2}\log_{10} 3^2 = x$$

$$\Rightarrow \frac{2}{2}\log_{10} 3 = x$$

$$\Rightarrow \log_{10} 3 = x \ldots \text{Eq. (2)}$$

$$\log_{10} 15$$
$$= \log_{10}(3 \times 5)$$
$$= \log_{10} 3 + \log_{10} 5$$
$$= \log_{10} 3 + \log_{10}\left(\frac{10}{2}\right)$$
$$= \log_{10} 3 + \log_{10} 10 - \log_{10} 2$$
$$= x + 1 - y$$

Question 34 – December, 2022; MTP June, 2023

In a hostel, ration is stocked for 400 students upto 31 days. After 28 days 280 students were vacated the hostel. Find the number of days for which the remaining ration will be sufficient for the remaining students.

(a) 5 (b) 4 (c) 7 (d) 10

Solution

(d)

Here, Total men = 400, No. of days = 31

Total No. of unit of food for 400 men in 31 days

$= 400 \times 31 = 12400$ unit

Total No. of unit of food for 400 men in 28 days

= 400 × 28 = 11200 unit

Rest food = 12400 -11200 = 1200 unit

Remain men after 28 days = 400 - 280 = 120

No. of days for which the remaining food will be sufficient: $\frac{Total\ Remaining\ Food}{No.\ of\ Remaining\ Men} = \frac{1200}{120} = 10\ days$

Question 35 – MTP June, 2023

Two vessels containing water and milk in the ratio 2 : 3 and 4 : 5 are mixed in the ratio 1 : 2. The ratio of milk and water in the resulting mixture is:

(a) 58 : 77 (b) 77 : 58 (c) 68 : 77 (d) None

Solution

(b)

Let the mixture contain 10 litres of solution from first vessel, and 20 litres of solution from the second vessel.

The 10 litres of solution from the first vessel would contain 2/5 water and 3/5 milk. Therefore, portion of water out of 10 litres would be 2/5 × 10 = 4 litres, and the portion of milk out of 10 litres would be 3/5 × 10 = 6 litres.

The 20 litres of solution from the second vessel would contain 4/9 water and 5/9 milk. Therefore, portion of water from 20 litres would be 4/9 × 20 = 80/9 litres, and the portion of milk out of 20 litres would be 5/9 × 20 = 100/9 litres.

Total water = 4 litres (from first vessel) + 80/9 litres (from second vessel)

Total milk = 6 litres (from first vessel) + 100/9 litres (from second vessel)

Ratio of milk to water = $\dfrac{6+\dfrac{100}{9}}{4+\dfrac{80}{9}} = 1.327586$

Question 36 – MTP June, 2023

If $(x-9):(3x+6)$ is the duplicate ratio of 4 : 9, find the value of *x*.

(a) *x* = 9 (b) *x* = 16 (c) *x* = 36 (d) *x* = 25

Solution

(d)

$$\frac{x-9}{3x+6}=\frac{4^2}{9^2}$$

$$\Rightarrow \frac{x-9}{3x+6}=\frac{16}{81}$$

$$\Rightarrow 81(x-9)=16(3x+6)$$

$$\Rightarrow 81x-729=48x+96$$

$$\Rightarrow 81x-48x=729+96$$

$$\Rightarrow 33x=825$$

$$\Rightarrow x=\frac{825}{33}=25$$

Question 37 – MTP June, 2023

Value of $\left(a^{1/8}+a^{-1/8}\right)\left(a^{1/8}-a^{-1/8}\right)\left(a^{1/4}+a^{-1/4}\right)\left(a^{1/2}+a^{-1/2}\right)$ is:

(a) $a+\dfrac{1}{a}$ (b) $a-\dfrac{1}{a}$ (c) $a^2+\dfrac{1}{a^2}$ (d) $a^2-\dfrac{1}{a^2}$

Solution

(b)

$$\left(a^{1/8}+a^{-1/8}\right)\left(a^{1/8}-a^{-1/8}\right)\left(a^{1/4}+a^{-1/4}\right)\left(a^{1/2}+a^{-1/2}\right)$$

$$\Rightarrow\left\{\left(a^{1/8}+a^{-1/8}\right)\left(a^{1/8}-a^{-1/8}\right)\right\}\left(a^{1/4}+a^{-1/4}\right)\left(a^{1/2}+a^{-1/2}\right)$$

$$\Rightarrow \left\{ \left(a^{1/8}\right)^2 - \left(a^{-1/8}\right)^2 \right\} \left(a^{1/4} + a^{-1/4}\right)\left(a^{1/2} + a^{-1/2}\right)$$

$$\Rightarrow \left\{ a^{\frac{1}{8}\times 2} - a^{-\frac{1}{8}\times 2} \right\} \left(a^{1/4} + a^{-1/4}\right)\left(a^{1/2} + a^{-1/2}\right)$$

$$\Rightarrow \left(a^{1/4} - a^{-1/4}\right)\left(a^{1/4} + a^{-1/4}\right)\left(a^{1/2} + a^{-1/2}\right)$$

$$\Rightarrow \left\{ \left(a^{1/4} - a^{-1/4}\right)\left(a^{1/4} + a^{-1/4}\right) \right\} \left(a^{1/2} + a^{-1/2}\right)$$

$$\Rightarrow \left\{ \left(a^{1/4}\right)^2 - \left(a^{-1/4}\right)^2 \right\} \left(a^{1/2} + a^{-1/2}\right)$$

$$\Rightarrow \left\{ a^{\frac{1}{4}\times 2} - a^{-\frac{1}{4}\times 2} \right\} \left(a^{1/2} + a^{-1/2}\right)$$

$$\Rightarrow \left(a^{1/2} - a^{-1/2}\right)\left(a^{1/2} + a^{-1/2}\right)$$

$$\Rightarrow \left(a^{1/2}\right)^2 - \left(a^{-1/2}\right)^2$$

$$\Rightarrow a - a^{-1}$$

$$\Rightarrow a - \frac{1}{a}$$

Question 38 – MTP June, 2023

If $\left(25\right)^{150} = \left(25x\right)^{50}$, then the value of x will be:

(a) 5^3 (b) 5^4 (c) 5^2 (d) 5

Solution

(b)

$\left(25\right)^{150} = \left(25x\right)^{50}$

$\Rightarrow 25^{150} = 25^{50} \times x^{50}$

$\Rightarrow x^{50} = \dfrac{25^{150}}{25^{50}}$

$$\Rightarrow x^{50} = 25^{150-50}$$

$$\Rightarrow x^{50} = 25^{100}$$

$$\Rightarrow x^{50} = \left(5^2\right)^{100}$$

$$\Rightarrow x^{50} = 5^{200}$$

Now, try the options.

Option (b) $\rightarrow 5^4$

LHS: $\left(5^4\right)^{50} = 5^{4\times 50} = 5^{200} = \text{RHS}$

Therefore, option (b) is the answer.

Question 39 – MTP June, 2023

$$7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)$$ is equal to:

(a) 0 (b) 1 (c) $\log 2$ (d) $\log 3$

Solution

(c)

$$7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)$$

$$\Rightarrow \log\left(\frac{16}{15}\right)^7 + \log\left(\frac{25}{24}\right)^5 + \log\left(\frac{81}{80}\right)^3$$

$$\Rightarrow \log\left(\frac{16^7}{15^7}\right) + \log\left(\frac{25^5}{24^5}\right) + \log\left(\frac{81^3}{80^3}\right)$$

$$\Rightarrow \log\left(\frac{16^7}{15^7} \times \frac{25^5}{24^5} \times \frac{81^3}{80^3}\right)$$

$$\Rightarrow \log 2$$

Question 40 – MTP June, 2023

$\log_4(x^2+x)-\log_4(x+1)=2$. Find x.

(a) 16 (b) 0 (c) –1 (d) None

Solution

(a)

$$\log_4(x^2+x)-\log_4(x+1)=2$$

$$\Rightarrow \log_4\left(\frac{x^2+x}{x+1}\right)=2$$

$$\Rightarrow \log_4\left(\frac{x(x+1)}{x+1}\right)=2$$

$$\Rightarrow \log_4 x = 2$$

$$\Rightarrow x = 4^2 = 16$$

Chapter 2 – Equations

CA NISHANT KUMAR

Concepts at a Glance

Simple Equations

- An equation with only one variable with a linear power is known as a Simple Equation.
- Try the options to solve the questions.

Simultaneous Linear Equations in Two Variables

- Two equations with two variables with linear power are known as simultaneous linear equations.
- Try the options to solve the questions.

Quadratic Equations

- A quadratic equation is an equation in which the highest power of the variables is 2.
- A quadratic equation is of the form $ax^2 + bx + c = 0$.
- x is a variable while a, b and c are constants.
- A quadratic equation has two solutions/roots.

Methods of Solving Quadratic Equations

There are three methods of solving any quadratic equation:

1. Factorization Method
2. Quadratic Formula

$$\text{Quadratic Formula} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If we call the roots α, and β, then,

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Sum of Roots $\left(\alpha + \beta\right) = -\frac{b}{a}$

Product of Roots $\alpha\beta = \frac{c}{a}$

3. Fastest Method

Solve the equation $x^2 - 5x + 6 = 0$ using fastest method.

Here, $a = 1$; $b = -5$; $c = 6$

Sum of Roots $= -\frac{b}{a} = -\frac{-5}{1} = 5$

Product of Roots $= \frac{c}{a} = \frac{6}{1} = 6$

Now, take the sum of the roots, divide it by half, and add x to it. You'll get $\left(\frac{5}{2} + x\right)$.

Similarly, take the sum of the roots, divide it by half, and subtract x from it. You'll get $\left(\frac{5}{2} - x\right)$. Multiply these two and equate with the product, i.e. 6.

$$\left(\frac{5}{2} + x\right)\left(\frac{5}{2} - x\right) = 6$$

$$\Rightarrow \left(\frac{5}{2}\right)^2 - x^2 = 6$$

$$\Rightarrow \frac{25}{4} - x^2 = 6$$

$$\Rightarrow x^2 = \frac{25}{4} - 6$$

$$\Rightarrow x^2 = 6.25 - 6$$

$$\Rightarrow x^2 = 0.25$$

$$\Rightarrow x = \sqrt{0.25}$$

$$\Rightarrow x = 0.5$$

Now, put the value of $x = 0.5$ in the factors $\left(\frac{5}{2}+x\right)$, and $\left(\frac{5}{2}-x\right)$. You'll get the roots.

Therefore, $\alpha=\frac{5}{2}+0.5=3;\ \beta=\frac{5}{2}-0.5=2.$

This method applies to complicated roots as well.

Important Rule

If α and β are the roots of the equation, the equation is given by:

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

In other words,

$$x^2 - (Sum\,of\ Roots)x + Product\,of\ Roots = 0.$$

Nature of Roots

We know that the quadratic formula gives us the value of *x* as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

In this formula, the term $b^2 - 4ac$ plays a very important role. The nature of the roots is dependent on $b^2 - 4ac$.

1. If $b^2 - 4ac = 0$, the roots are real and equal.
2. If $b^2 - 4ac > 0$, the roots are real and unequal.
    a. If $b^2 - 4ac$ is a perfect square, the roots are real, rational, and unequal.

b. If $b^2 - 4ac$ is not a perfect square, the roots are real, irrational, and unequal.

3. If $b^2 - 4ac < 0$, the roots are imaginary and unequal.

Since $b^2 - 4ac$ discriminates the roots, it is known as the discriminant.

Points to be noted –

1. A real number is a number which can be expressed on a number line. Therefore, every number is a real number, including negative numbers.
2. An imaginary number is a number multiplied by a unit "*i*", which is identified by its property $i^2 = -1$.
3. An integer is a number without any fractional part. It includes positive as well as negative numbers.

4. A rational number is a number which can be expressed as a fraction of two integers. The decimal expansion of a rational number either terminates after a finite number of digits, or begins to repeat the same finite sequence of digits over and over. Examples:

   a. 2 is a rational number as it can be expressed in the form of $\frac{2}{1}$.

   b. $\frac{5}{2}$ is a rational number as its decimal expansion 2.5 terminates after a finite number of digits.

   c. $\frac{2}{9}$ is a rational number as its decimal expansion comes to 0.222…, i.e. it begins to repeat itself over and over.

d. $-\frac{5}{2}, -\frac{2}{9}$ are also rational numbers.

5. An irrational number is a number whose decimal expansion either does not terminate after a finite number of digits or does not repeat itself over and over.
   Examples:
   a. π is an irrational number as its decimal expansion is 3.14159265359…, i.e. it neither terminates after a finite number of digits nor does it repeat itself over and over.
   b. $\sqrt{2}$ is an irrational number as its decimal expansion is 1.41421356237…, i.e. it neither terminates after a finite number of digits nor does it repeat itself over and over.
6. Irrational roots occur in conjugate pairs, i.e. if $\left(m+\sqrt{n}\right)$ is a root, then $\left(m-\sqrt{n}\right)$ is the other root of the same equation.

7. If one root is reciprocal to the other root, then their product is 1 and so $\frac{c}{a}=1$, i.e. $c=a$.
8. If one root is equal to the other root but opposite in sign, then their sum = 0, i.e. $-\frac{b}{a}=0 \Rightarrow b=0$.

Cubic Equations

- An equation with the highest power of the variables as 3 is known as a cubic equation.
- Try the options to solve such an equation.

Questions Based on Simple Equations

Question 1

The denominator of a fraction exceeds the numerator by 5 and if 3 be added to both the fraction becomes $\frac{3}{4}$. Find the fraction.

(a) $\frac{12}{17}$ (b) $\frac{13}{17}$ (c) $\frac{14}{18}$ (d) $\frac{15}{19}$

Solution

(a)

Try the options.

Option (a) → $\frac{12}{17}$

17 – 12 = 5

Therefore, denominator exceeds the numerator by 5.

If 3 be added to both, the fraction becomes:

$$\frac{12+3}{17+3}=\frac{15}{20}=0.75$$

Also, $\frac{3}{4}=0.75$

Therefore, it can be concluded that if 3 be added to both the numerator and the denominator, the fraction becomes ¾.

Question 2

If thrice of A's age 6 years ago, be subtracted from twice his present age, the result would be equal to his present age. Find A's present age.

(a) 8 (b) 9 (c) 10 (d) 11

Solution

(b)

Let A's present age be x years.

A's age 6 years ago would be $x-6$.

Twice of A's present age would be $2x$

As per the question, $2x-3(x-6)=x$

Now, try the options.

Option (a) → 8

LHS = $\left(2\times 8\right)-3\left(8-6\right)=10\neq 8$

Option (b) → 9

LHS = $\left(2\times 9\right)-3\left(9-6\right)=9$

Therefore, option (b) is the answer.

Question 3

A number consists of two digits. The digit in the ten's place is twice the digit in the unit's place. If 18 be subtracted from the number, the digits are reversed. Find the number.

(a) 63 (b) 84 (c) 42 (d) 21

Solution

(c)

By trying the options, we'll find that option (c) is the answer.

Clearly, the digit in the ten's place is twice the digit in the unit's place.

Also, $42 - 18 = 24 \Rightarrow$ the digits are reversed.

Question 4

For a certain commodity, the demand equation giving demand *'d'* in kg, for a price *'p'* in rupees per kg. is $d = 100(10 - p)$. The supply equation giving the supply s in kg. for a price p in rupees per kg. is $s = 75(p - 3)$. The market price is such at which demand equals supply. Find the market price and quantity that will be bought and sold.

(a) 10, 400, 400 (b) 9, 500, 500 (c) 8, 340, 440 (d) 7, 300, 300

Solution

(d)

Demand = Supply

$100(10 - p) = 75(p - 3)$

$$1000-100p=75p-225$$

$$100p+75p=1000+225$$

$$175p=1225$$

$$p=\frac{1225}{175}=7$$

Now, $d=100(10-p)$

$$d=100(10-7)=300$$

$$s=75(p-3)$$

$$s=75(7-3)=300$$

Question 5

The sum of two numbers is 52 and their difference is 2. The numbers are:

(a) 17 and 15 (b) 12 and 10 (c) 27 and 25 (d) None

Solution

(c)

By trying the options, we'll find that option (c) is the answer.

Option (c) → 27 and 25

Clearly, 27 + 25 = 52; and 27 – 25 = 2

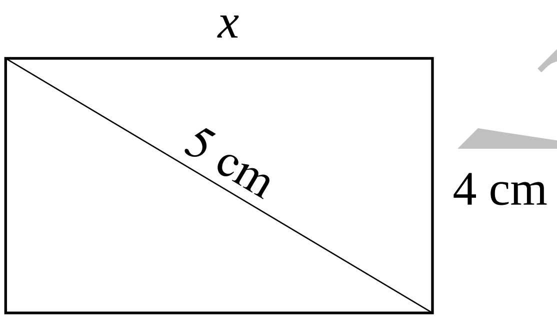
Question 6

The diagonal of a rectangle is 5 cm and one of at sides is 4 cm. Its area is:

(a) 20 sq. cm. (b) 12 sq. cm. (c) 10 sq. cm. (d) None

Solution

(b)



As per Pythagoras' Theorem,

$$x^2 + 4^2 = 5^2$$

$$\Rightarrow x = \sqrt{5^2 - 4^2} = 3$$

Therefore, the area of the rectangle is 3 × 4 = 12 sq. cm.

Question 7

Divide 56 into two parts such that three times the first part exceeds one third of the second by 48. The parts are:

(a) (20, 36) (b) (25, 31) (c) (24, 32) (d) None

Solution

(a)

Question 8

The sum of the digits of a two-digit number is 10. If 18 be subtracted from it, the digits in the resulting number will be equal. The number is:

(a) 37 (b) 73 (c) 75 (d) None

Solution

(b)

Question 9

The fourth part of a number exceeds the sixth part by 4. The number is:

(a) 84 (b) 44 (c) 48 (d) None

Solution

(c)

Question 10

Ten years ago, the age of a father was four times of his son. Ten years hence, the age of the father will be twice that of his son. The present ages of the father and the son are:

(a) (50, 20) (b) (60, 20) (c) (55, 25) (d) None

Solution

(a)

Question 11

The product of two numbers is 3200 and the quotient when the larger number is divided by the smaller is 2. The numbers are:

(a) (16, 200) (b) (160, 20) (c) (60, 30) (d) (80, 40)

Solution

(d)

Question 12

The denominator of a fraction exceeds the numerator by 2. If 5 be added to the numerator, the fraction increases by unity. The fraction is:

(a) $\frac{5}{7}$ (b) $\frac{1}{3}$ (c) $\frac{7}{9}$ (d) $\frac{3}{5}$

Solution

(d)

Question 13

Three persons Mr. Roy, Mr. Paul and Mr. Singh together have ₹51. Mr. Paul has ₹4 less than Mr. Roy and Mr. Singh has got ₹5 less than Mr. Roy. They have the money as:

(a) (₹20, ₹16, ₹15) (b) (₹15, ₹20, ₹16) (c) (₹25, ₹11, ₹15) (d) None

Solution

(a)

Question 14

A number consists of two digits. The digits in the ten's place is 3 times the digit in the unit's place. If 54 is subtracted from the number, the digits are reversed. The number is:

(a) 39 (b) 92 (c) 93 (d) 94

Solution

(c)

Question 15

One student is asked to divide a half of a number by 6 and other half by 4 and then to add the two quantities. Instead of doing so, the student divides the given number by 5. If the answer is 4 short of the correct answer, then the number was:

(a) 320 (b) 400 (c) 480 (d) None

Solution

(c)

Question 16

If a number of which the half is greater than $1/5^{th}$ of the number by 15, then the number is:

(a) 50 (b) 40 (c) 80 (d) None

Solution

(a)

Questions Based on Simultaneous Linear Equations in Two Variables

CA NISHANT KUMAR

Question 16

The point of intersection between the lines $3x+4y=7$ and $4x-y=3$ lie in the:

(a) 1st Quadrant (b) 2nd Quadrant (c) 3rd Quadrant (d) 4th Quadrant

Solution

(a)

We have:

$3x+4y=7$ …Eq. (1)

$4x-y=3$…Eq. (2)

Multiplying Eq. (2) with 4, we'll get:

$\left(4x-y=3\right)\times 4$

$\Rightarrow 16x-4y=12\ldots$Eq. (3)

Adding Equations (1) and (3), we'll get:

$3x+16x+4y-4y=7+12$

$\Rightarrow 19x=19$

$\Rightarrow x=1$

Putting this value of x in Eq. (1), we'll get:

$3\left(1\right)+4y=7$

$\Rightarrow 3+4y=7$

$$\Rightarrow 4y = 7 - 3$$

$$\Rightarrow 4y = 4$$

$$\Rightarrow y = 1$$

Since both *x* and *y* are positive, they lie in the 1st quadrant.

Question 17

If the numerator of a fraction is increased by 2 and the denominator by 1, it becomes 1. Again, if the numerator is decreased by 4 and the denominator by 2, it becomes 1/2. Find the fraction.

(a) 2/3 (b) 4/5 (c) 7/8 (d) None

Solution

(c)

Question 18

The age of a man is three times the sum of the ages of his two sons and 5 years hence his age will be double the sum of their ages. Find the present age of the man?

(a) 23 (b) 45 (c) 78 (d) None

Solution

(b)

Question 19

A number consist of three digits of which the middle one is zero and the sum of the other digits is 9. The number formed by interchanging the first and third digits is more than the original number by 297. Find the number.

(a) 306 (b) 207 (c) 702 (d) None

Solution

(a)

Question 20

Monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenses are in the ratio 7 : 9. If each saves ₹50 per month find their monthly incomes.

(a) (500, 400) (b) (400, 500) (c) (300, 600) (d) (350, 550)

Solution

(b)

Question 21

Find the fraction which is equal to 1/2 when both its numerator and denominator are increased by 2. It is equal to 3/4 when both are increased by 12.

(a) 3/8 (b) 5/8 (c) 2/8 (d) 2/3

Solution

(a)

Question 22

The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age.

(a) 60 years (b) 52 years (c) 51 years (d) 50 years

Solution

(d)

Question 23

A number between 10 and 100 is five times the sum of its digits. If 9 be added to it the digits are reversed find the number.

(a) 54 (b) 53 (c) 45 (d) 55

Solution

(c)

Question 24

The wages of 8 men and 6 boys amount to ₹33. If 4 men earn ₹4.50 more than 5 boys determine the wages of each man and boy.

(a) (₹1.50, ₹3) (b) (₹3, ₹1.50) (c) (₹2.50, ₹2) (d) (₹2, ₹2.50)

Solution

(b)

Question 25

A number consisting of two digits is four times the sum of its digits and if 27 be added to it the digits are reversed. The number is:

(a) 63 (b) 35 (c) 36 (d) 60

Solution

(c)

Question 26

Of two numbers, 1/5th of the greater is equal to 1/3rd of the smaller and their sum is 16. The numbers are:

(a) (6, 10) (b) (9, 7) (c) (12, 4) (d) (11, 5)

Solution

(a)

Question 27

y is older than x by 7 years. 15 years back, x's age was $3/4^{th}$ of y's age. Their present ages are:

(a) ($x = 36, y = 43$) (b) ($x = 50, y = 43$) (c) ($x = 43, y = 50$) (d) ($x = 40, y = 47$)

Solution

(a)

Question 28

The sum of the digits in a three digit number is 12. If the digits are reversed, the number is increased by 495 but reversing only of the tens and units digits increases the number by 36. The number is:

(a) 327 (b) 372 (c) 237 (d) 273

Solution

(c)

Question 29

Two numbers are such that twice the greater number exceeds twice the smaller one by 18 and 1/3rd of the smaller and 1/5th of the greater number are together 21. The numbers are:

(a) (36, 45) (b) (45, 36) (c) (50, 41) (d) (55, 46)

Solution

(b)

Question 30

The demand and supply equations for a certain commodity are $4q + 7p = 17$ and $p = \frac{q}{3} + \frac{7}{4}$ respectively where p is the market price and q is the quantity. The equilibrium price and quantity are:

(a) $2, \frac{3}{4}$ (b) $3, \frac{1}{2}$ (c) $5, \frac{3}{5}$ (d) None

Solution

(a)

Question 31

The cab bill is partly fixed and partly varies on the distance covered. For 456 km, the bill is ₹8252, for 484 km the bill is ₹8728. What will the bill be for 500 km?

(a) ₹8876 (b) ₹9156 (c) ₹9472 (d) ₹9000

Solution

(d)

Variable Cost per unit = $\frac{8728-8252}{484-456}=17$

Therefore, Fixed Cost = 8252 – (17 × 456) = 8252 – 7752 = ₹500

The bill for 500 km will be ₹500 + (500 × ₹17 p.u.) = ₹500 + ₹8,500 = ₹9,000

Alternatively,

Let the fixed cost be x and variable cost per unit be y.

For 456 km, we have $x + 456y = 8252$…Eq. (1)

For 484 km, we have $x + 484y = 8728$…Eq. (2)

Subtracting Eq. (1) from Eq. (2), we have:

$x - x + 484y - 456y = 8728 - 8252$

$$\Rightarrow y\left(484 - 456\right) = 476$$

$$\Rightarrow y \times 28 = 476$$

$$\Rightarrow y = \frac{476}{28} = 17$$

Putting this value in Eq. (1), we have:

$$x+\left(456\times17\right)=8252$$

$$\Rightarrow x=8252-\left(456\times17\right)=8252-7752=500$$

Therefore, for 500 km, the bill will be $x+500y=500+\left(500\times17\right)=500+8500=9000$

Question 32

The value of k for the system of equations $kx+2y=5$ and $3x+y=1$ has no solution is:

(a) 5 (b) 2/3 (c) 6 (d) 3/2

Solution

(c)

$kx+2y=5$…Eq. (1)

$3x+y=1$…Eq. (2)

Multiplying Eq. (2) with 2, we'll get:

$6x+2y=2$…Eq. (3)

Subtracting Eq. (3) from Eq. (1), we'll get:

$$kx - 6x + 2y - 2y = 5 - 2$$

$$\Rightarrow kx - 6x = 3$$

$$\Rightarrow x(k - 6) = 3$$

$$\Rightarrow x = \frac{3}{k-6}$$

Now, clearly, if k takes the value 6, then denominator becomes zero, and x becomes not defined, and hence the system of equations won't have any solution.

Questions Based on Quadratic Equations

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Question 33

If α, β be the roots of the equation $2x^2 - 4x - 3 = 0$, then the value of $\alpha^2 + \beta^2$ is:

(a) 5 (b) 7 (c) 3 (d) –4

Solution

(b)

Method 1 – Solve the quadratic, find the roots and calculate $\alpha^2 + \beta^2$.

$2x^2 - 4x - 3 = 0$

$a = 2;\ b = -4;\ c = -3$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-4}{2} = 2$$

$$\alpha\beta = \frac{c}{a} = \frac{-3}{2} = -1.5$$

$$\left(\frac{2}{2} + x\right)\left(\frac{2}{2} - x\right) = -1.5$$

$$1^2 - x^2 = -1.5$$

$$x^2 = 1 + 1.5 = 2.5$$

$$x = \sqrt{2.5}$$

$$\alpha = 1 + \sqrt{2.5}$$

$$\beta = 1 - \sqrt{2.5}$$

$$\alpha^2 + \beta^2 = \left(1 + \sqrt{2.5}\right)^2 + \left(1 - \sqrt{2.5}\right)^2 = 7$$

Method 2 –

Given equation:

$$2x^2 - 4x - 3 = 0$$

$$a = 2;\ b = -4;\ c = -3$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-4}{2} = 2$$

$$\alpha\beta = \frac{c}{a} = \frac{-3}{2} = -1.5$$

We know that $\left(\alpha+\beta\right)^2=\alpha^2+\beta^2+2\alpha\beta$

$\Rightarrow \alpha^2+\beta^2=\left(\alpha+\beta\right)^2-2\alpha\beta=\left(2\right)^2-\left(2\times -1.5\right)=7$

Question 34

Examine the nature of roots of the following equation: $3x^2 - 8x + 4 = 0$.

(a) Real and Equal
(b) Real and Unequal
(c) Imaginary and Unequal
(d) Real, Rational, Unequal

Solution

(d)

$$3x^2 - 8x + 4 = 0$$

$$a = 3;\ b = -8;\ c = 4$$

$$b^2 - 4ac = (-8)^2 - (4)(3)(4) = 16$$

Since D > 0, the roots are real and unequal. Also, since D is a perfect square, the roots are rational.

Question 35

Examine the nature of roots of the following equation: $5x^2 - 4x + 2 = 0$.

(a) Real and Equal

(b) Real and Unequal

(c) Imaginary and Unequal

(d) Real, Rational, Unequal

Solution

(c)

$$5x^2 - 4x + 2 = 0$$

$$a = 5;\ b = -4;\ c = 2$$

$$b^2 - 4ac = (-4)^2 - (4)(5)(2) = -24$$

Since D < 0, the roots are imaginary and unequal.

Question 36

If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal, then the value of *m* is:

(a) –3 (b) –1 (c) 1 (d) –2

Solution

(d)

$$2x^{2+} + 8x - m^3 = 0$$

$$a = 2;\ b = 8;\ c = -m^3$$

$$b^2 - 4ac = 0$$

$$(8)^2 - (4)(2)(-m^3) = 0$$

$$64-\left(-8m^3\right)=0$$

$$64+8m^3=0$$

$$8m^3=-64$$

$$m^3=\frac{-64}{8}=-8$$

$$\Rightarrow m=-2$$

Question 37

The equation $x^2 - (p+4)x + 2p + 5 = 0$ has equal roots. The value of p will be:

(a) ± 1 (b) 2 (c) ± 2 (d) –2

Solution

(c)

$$x^2 - (p+4)x + 2p + 5 = 0$$

$$a = 1;\ b = -(p+4);\ c = 2p+5$$

$$b^2 - 4ac = 0$$

$$\left\{-\left(p+4\right)\right\}^2-\left(4\right)\left(1\right)\left(2p+5\right)=0$$

$$\left\{p^2+4^2+\left(2\times p\times 4\right)\right\}-\left\{4\left(2p+5\right)\right\}=0$$

$$\left\{p^2+16+8p\right\}-\left\{8p+20\right\}=0$$

$$p^2+16+8p-8p-20=0$$

$$p^2-4=0$$

$$p^2=4$$

$$p=\sqrt{4}=\pm 2$$

Question 38

The roots of the equation $x^2 + (2p-1)x + p^2 = 0$ are real if:

(a) $p \geq 1$ (b) $p \leq 4$ (c) $p \geq 1/4$ (d) $p \leq 1/4$

Solution

(d)

$x^2 + (2p-1)x + p^2 = 0$

$a = 1;\ b = 2p-1;\ c = p^2$

For the roots to be real, $b^2 - 4ac \geq 0$

$$(2p-1)^2-(4)(1)\left(p^2\right)\geq 0$$

$$\left\{(2p)^2+(1)^2-(2)(2p)(1)\right\}-4p^2\geq 0$$

$$4p^2+1-4p-4p^2\geq 0$$

$$1-4p\geq 0$$

$$1\geq 4p$$

$$\frac{1}{4}\geq p$$

$$\Rightarrow p\leq\frac{1}{4}$$

Question 39

If $L+M+N=0$, and L, M, and N are rationals, the roots of the equation $\left(M+N-L\right)x^2+\left(N+L-M\right)x+\left(L+M-N\right)=0$ are:

(a) Real and Irrational
(b) Real and Rational
(c) Imaginary and Equal
(d) Real and Equal

Solution

(b)

We have

$$\left(M+N-L\right)x^2+\left(N+L-M\right)x+\left(L+M-N\right)=0$$

We know that

$$L + M + N = 0$$

Therefore,

$$M + N = -L;\ N + L = -M;\ L + M = -N;\ M = -N - L$$

Therefore, we have

$$(-L - L)x^2 + (-M - M)x + (-N - N) = 0$$

$$\Rightarrow -2Lx^2 - 2Mx - 2N = 0$$

$$\Rightarrow -2\left(Lx^2 + Mx + N\right) = 0$$

$$\Rightarrow Lx^2 + Mx + N = 0$$

Here, $a = L;\ b = M;\ c = N$

$$b^2 - 4ac = M^2 - (4)(L)(N)$$

$$= (-N - L)^2 - 4LN$$

$$= \{-(N + L)\}^2 - 4LN$$

$$= (N + L)^2 - 4LN$$

$$= N^2 + L^2 + 2LN - 4LN$$

$$= N^2 + L^2 - 2LN$$

$$= (N - L)^2$$

Therefore, D is a perfect square. Hence, the roots are rational. Also, the roots are real. This is because even if N – L comes to be a negative figure, squaring it would make it positive, and thereafter, its square root will be determined in the quadratic formula. Therefore, the roots are Real and Rational.

Question 40

If one root of the equation is $2-\sqrt{3}$, form the equation given that the roots are irrational.

(a) $x^2-4x+2=0$

(b) $x^2-3x+9=0$

(c) $x^2-5x+2=0$

(d) $x^2-4x+1=0$

Solution

(d)

Irrational roots always occur in conjugate pairs. This means that if one root is $2-\sqrt{3}$, the other root will be $2+\sqrt{3}$.

When the roots are known, the equation is given by:

$$x^2 - \left(Sum\, of\ Roots\right)x + Product\, of\ Roots = 0$$

$$x^2 - \left\{\left(2-\sqrt{3}\right)+\left(2+\sqrt{3}\right)\right\}x + \left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right) = 0$$

$$x^2 - \left\{2-\sqrt{3}+2+\sqrt{3}\right\}x + \left\{\left(2\right)^2 - \left(\sqrt{3}\right)^2\right\} = 0$$

$$x^2 - \left\{4\right\}x + \left\{4-3\right\} = 0$$

$$x^2 - 4x + 1 = 0$$

Question 41

If the roots of the equation $p(q-r)x^2+q(r-p)x+r(p-q)=0$ are equal, find the value of $\frac{1}{p}+\frac{1}{r}$.

(a) $\frac{2}{q}$ (b) $\frac{1}{q}$ (c) $\frac{1}{2}$ (d) None

Solution

(a)

Here, $a=p(q-r);\ b=q(r-p);\ c=r(p-q)$

Since the roots of this equation are equal, $b^2 - 4ac = 0$.

$$\{q(r-p)\}^2 - (4)\{p(q-r)\}\{r(p-q)\} = 0$$

$$q^2(r-p)^2 - [4pr(q-r)(p-q)] = 0$$

$$q^2(r^2 + p^2 - 2rp) - [4pr(qp - q^2 - pr + qr)] = 0$$

$$q^2r^2 + q^2p^2 - 2rpq^2 - [4p^2qr - 4pq^2r - 4p^2r^2 + 4pqr^2] = 0$$

$$q^2r^2 + q^2p^2 - 2rpq^2 - 4p^2qr + 4pq^2r + 4p^2r^2 - 4pqr^2 = 0$$

$$q^2r^2 + q^2p^2 + 4pq^2r - 2rpq^2 - 4p^2qr + 4p^2r^2 - 4pqr^2 = 0$$

$$q^2r^2 + q^2p^2 + 2pq^2r - 4p^2qr + 4p^2r^2 - 4pqr^2 = 0$$

We know that $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

If we look closely at the LHS of the following equation, $q^2r^2+q^2p^2+2pq^2r-4p^2qr+4p^2r^2-4pqr^2=0$, we'll find that it is the expansion of $(qr+qp-2pr)^2$.

Therefore,

$$(qr+qp-2pr)^2=0$$

$$\Rightarrow qr+qp-2pr=0$$

$$\Rightarrow qr+qp=2pr$$

Dividing the entire equation by *pqr*, we get:

$$\frac{qr}{pqr}+\frac{qp}{pqr}=\frac{2pr}{pqr}$$

$$\Rightarrow \frac{1}{p}+\frac{1}{r}=\frac{2}{q}$$

Question 42

If α and β be the roots of $x^2 + 7x + 12 = 0$, find the equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$.

(a) $x^2+50x+49=0$

(b) $x^2-24x+144=0$

(c) $x^2-50x+49=0$

(d) $x^2-19x+49=0$

Solution

(c)

$x^2+7x+12=0$

Here, $a = 1$; $b = 7$; $c = 12$

$$\alpha + \beta = -\frac{b}{a} = -\frac{7}{1} = -7$$

$$\alpha\beta = \frac{c}{a} = \frac{12}{1} = 12$$

As per the fastest method,

$$\left(\frac{-7}{2} + x\right)\left(\frac{-7}{2} - x\right) = 12$$

$$\left(\frac{-7}{2}\right)^2 - x^2 = 12$$

$$x^2 = \frac{49}{4} - 12 = \frac{49 - 48}{4} = \frac{1}{4}$$

$$x = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Therefore, $\alpha = \frac{-7}{2} + \frac{1}{2} = -\frac{6}{2} = -3$

$$\beta = \frac{-7}{2} - \frac{1}{2} = -\frac{8}{2} = -4$$

The roots of the new equation will be:

$(\alpha + \beta)^2 = (-3 - 4)^2 = 49$, and

$$(\alpha - \beta)^2 = \{-3 - (-4)\}^2 = 1$$

When roots of the equation are known, the equation is given by:

$$x^2 - (Sum\ of\ Roots)x + Product\ of\ Roots = 0$$

Therefore, the equation will be $x^2 - (49+1)x + (49 \times 1) = 0$

$$\Rightarrow x^2 - 50x + 49 = 0$$

Question 43

If α, β are the two roots of the equation $x^2 + px + q = 0$, form the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

(a) $qx^2 - (p^2 - 2q)x + q = 0$

(b) $px^2 - (p^2 - 2q)x + q = 0$

(c) $qx^2 - (p^2 - 2q)x + p = 0$

(d) $qx^2 + (p^2 - 2q)x + p = 0$

Solution

(a)

$x^2 + px + q = 0$

$$\Rightarrow \alpha + \beta = -\frac{b}{a} = -\frac{p}{1} = -p$$, and

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

We need an equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

Quadratic equation is given by: $x^2 - (Sum\, of\ Roots)x + Product\, of\ Roots = 0$

Therefore,

$$x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \left(\frac{\alpha}{\beta} \times \frac{\beta}{\alpha}\right) = 0$$

$$\Rightarrow x^2 - \left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)x + 1 = 0$$

$$\Rightarrow x^2 - \left(\frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}\right)x + 1 = 0$$

$$\Rightarrow x^2 - \left\{\frac{\left(\alpha^2 + \beta^2 + 2\alpha\beta\right) - 2\alpha\beta}{\alpha\beta}\right\}x + 1 = 0$$

$$\Rightarrow x^2 - \left\{\frac{\left(\alpha + \beta\right)^2 - 2\alpha\beta}{\alpha\beta}\right\}x + 1 = 0$$

$$\Rightarrow x^2 - \left\{ \frac{(-p)^2 - (2q)}{q} \right\} x + 1 = 0$$

$$\Rightarrow x^2 - \left\{ \frac{p^2 - 2q}{q} \right\} x + 1 = 0$$

Multiplying the entire equation with q, we get:

$$qx^2 - q\left\{ \frac{p^2 - 2q}{q} \right\} x + q = 0$$

$$\Rightarrow qx^2 - \left(p^2 - 2q\right)x + q = 0$$

Question 44

If one root of $5x^2 + 13x + p = 0$ be reciprocal of the other, then the value of p is:

(a) –5 (b) 5 (c) 1/5 (d) –1/5

Solution

(b)

$5x^2 + 13x + p = 0$

$a = 5;\ b = 13;\ c = p$

If one root is the reciprocal of the other, then the product of roots = 1

Therefore, $\frac{c}{a}=1 \Rightarrow c=a \Rightarrow p=5$

Question 45

If α and β are the roots of $x^2 = x + 1$, then the value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ is:

(a) $2\sqrt{5}$
(b) $\sqrt{5}$
(c) $3\sqrt{5}$
(d) $-2\sqrt{5}$

Solution

(d)

$x^2 = x + 1$

$x^2 - x - 1 = 0$

$$a = 1;\ b = -1;\ c = -1$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-1}{1} = 1$$

$$\alpha\beta = \frac{c}{a} = \frac{-1}{1} = -1$$

$$\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha} = \frac{\alpha^3 - \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha - \beta)\left(\alpha^2 + \alpha\beta + \beta^2\right)}{\alpha\beta}$$

$$= \frac{(\alpha - \beta)\left(\alpha^2 + \alpha\beta + \beta^2 + \alpha\beta - \alpha\beta\right)}{\alpha\beta}$$

$$= \frac{(\alpha - \beta)\left(\alpha^2 + 2\alpha\beta + \beta^2 - \alpha\beta\right)}{\alpha\beta}$$

$$= \frac{(\alpha - \beta)\left\{(1)^2 - (-1)\right\}}{-1}$$

$$= -\frac{(\alpha - \beta)\{1 + 1\}}{-1}$$

$$=-2\left(\alpha-\beta\right)$$

$$=-2\sqrt{\left(\alpha-\beta\right)^2}$$

$$=-2\sqrt{\alpha^2+\beta^2-2\alpha\beta}$$

$$=-2\sqrt{\alpha^2+\beta^2-2\alpha\beta+2\alpha\beta-2\alpha\beta}$$

$$=-2\sqrt{\left(\alpha+\beta\right)^2-4\alpha\beta}$$

$$= -2\sqrt{(1)^2 - (4)(-1)}$$

$$= -2\sqrt{5}$$

Alternatively, as per the fastest method:

$$\left(\frac{1}{2} + x\right)\left(\frac{1}{2} - x\right) = -1$$

$$\left(\frac{1}{2}\right)^2 - x^2 = -1$$

$$\frac{1}{4}+1=x^2$$

$$x^2=\frac{5}{4}$$

$$x=\sqrt{\frac{5}{4}}=\frac{\sqrt{5}}{2}$$

$$\alpha=\frac{1}{2}+\frac{\sqrt{5}}{2}=\frac{1+\sqrt{5}}{2}$$

$$\beta=\frac{1}{2}-\frac{\sqrt{5}}{2}=\frac{1-\sqrt{5}}{2}$$

$$\frac{\alpha^2}{\beta}-\frac{\beta^2}{\alpha}=\left[\left\{\left(\frac{1+\sqrt{5}}{2}\right)^2\right\}\div\left(\frac{1-\sqrt{5}}{2}\right)\right]-\left[\left\{\left(\frac{1-\sqrt{5}}{2}\right)^2\right\}\div\left(\frac{1+\sqrt{5}}{2}\right)\right]$$

$$\frac{\alpha^2}{\beta}-\frac{\beta^2}{\alpha}=-4.2361-0.2361=-4.4722=-2\sqrt{5}$$

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Question 46

If α, β be the roots of $2x^2 - 4x - 1 = 0$, find the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$.

(a) –22 (b) 23 (c) –23 (d) None

Solution

(a)

Question 47

The value of $4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\ldots\infty}}}$ is:

(a) $1\pm\sqrt{2}$ (b) $2+\sqrt{5}$ (c) $2\pm\sqrt{5}$ (d) None

Solution

(b)

$$\text{Let } 4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\ldots\infty}}}=x$$

$$x=4+\frac{1}{x}$$

$$x=\frac{4x+1}{x}$$

$$x^2=4x+1$$

$$x^2-4x-1=0$$

$$a=1;\ b=-4;\ c=-1$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-4}{1} = 4$$

$$\alpha\beta = \frac{c}{a} = \frac{-1}{1} = -1$$

$$\left(\frac{4}{2} + x\right)\left(\frac{4}{2} - x\right) = -1$$

$$(2)^2 - x^2 = -1$$

$$x^2 = 4 + 1 = 5$$

$$x = \sqrt{5}$$

$$\alpha = \frac{4}{2} + \sqrt{5} = 2 + \sqrt{5} = 4.23$$

$$\beta = \frac{4}{2} - \sqrt{5} = 2 - \sqrt{5} = -0.24$$

Clearly, the answer cannot be negative. Therefore, option (b) is the answer.

Question 48

The value of $\sqrt{6+\sqrt{6+\sqrt{6+...\infty}}}$ is:

(a) –3 (b) 2 (c) 3 (d) 4

Solution

(c)

Let $x=\sqrt{6+\sqrt{6+\sqrt{6+...\infty}}}$

Question 49

If the sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, then $\frac{b^2}{ac}+\frac{bc}{a^2}$ is:

(a) 2 (b) –2 (c) 1 (d) –1

Solution

(a)

$ax^2+bx+c=0$

$\alpha+\beta=-\frac{b}{a}$

$$\alpha\beta = \frac{c}{a}$$

Given: $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$-\frac{b}{a} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$$

$$-\frac{b}{a} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$-\frac{b}{a}=\frac{\left(-\frac{b}{a}\right)^2-\left(2\times\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$

$$-\frac{b}{a}=\left(\frac{b^2}{a^2}-\frac{2c}{a}\right)\div\frac{c^2}{a^2}$$

$$-\frac{b}{a}=\left(\frac{b^2-2ac}{a^2}\right)\times\frac{a^2}{c^2}$$

$$-\frac{b}{a}=\frac{b^2-2ac}{c^2}$$

$$-bc^2 = ab^2 - 2a^2c$$

$$ab^2 + bc^2 = 2a^2c$$

Dividing both sides by a^2c

$$\frac{ab^2}{a^2c} + \frac{bc^2}{a^2c} = \frac{2a^2c}{a^2c}$$

$$\frac{b^2}{ac} + \frac{bc}{a^2} = 2$$

Question 50

If p and q are the roots of the $x^2+2x+1=0$, then the values of p^3+q^3 becomes:

(a) 2 (b) –2 (c) 4 (d) –4

Solution

(b)

$$x^2+2x+1=0$$

$$a=1;\ b=2;\ c=1$$

$$p+q=-\frac{b}{a}=-\frac{2}{1}=-2$$

$$pq = \frac{c}{a} = \frac{1}{1} = 1$$

Therefore, obviously $p = -1$ and $q = -1$

$$p^3 + q^3 = (-1)^3 + (-1)^3 = -2$$

Alternatively,

We know that $(p+q)^3 = p^3 + q^3 + 3pq(p+q)$

$$\Rightarrow p^3 + q^3 = (p+q)^3 - 3pq(p+q)$$

$$p^3 + q^3 = (-2)^3 - (3)(1)(-2) = -2$$

Question 51

If $p \neq q$ and $p^2 = 5p - 3$ and $q^2 = 5q - 3$, the equation having the roots as $\frac{p}{q}$ and $\frac{q}{p}$ is:

(a) $x^2 - 19x + 3 = 0$
(b) $3x^2 - 19x - 3 = 0$
(c) $3x^2 - 19x + 3 = 0$
(d) $3x^2 + 19x + 3 = 0$

Solution

(c)

We have $p^2 = 5p - 3$

$p^2 - 5p + 3 = 0$

$$a = 1;\ b = -5;\ c = 3$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-5}{a} = 5$$

$$\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$$

$$\left(\frac{5}{2} + x\right)\left(\frac{5}{2} - x\right) = 3$$

$$\left(\frac{5}{2}\right)^2 - x^2 = 3$$

$$x^2 = \frac{25}{4} - 3$$

$$x^2 = \frac{25-12}{4} = \frac{13}{4}$$

$$x = \sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{2}$$

$$\alpha = \frac{5}{2} + \frac{\sqrt{13}}{2} = \frac{5+\sqrt{13}}{2}$$

$$\beta = \frac{5}{2} - \frac{\sqrt{13}}{2} = \frac{5-\sqrt{13}}{2}$$

Therefore, $p = \frac{5+\sqrt{13}}{2}, \frac{5-\sqrt{13}}{2}$

Also, we have $q^2 = 5q - 3$

Since this is exactly the same as $p^2 = 5p - 3$, it's obvious that q will also have the same two values.

Since it is given in the question that $p \neq q$, therefore, we'll have to take different values.

So, let $p = \frac{5+\sqrt{13}}{2}$, and $q = \frac{5-\sqrt{13}}{2}$

Now, we need to find the equation whose roots are $\frac{p}{q}$ and $\frac{q}{p}$.

$$\frac{p}{q}=\frac{\frac{5+\sqrt{13}}{2}}{\frac{5-\sqrt{13}}{2}}=\frac{5+\sqrt{13}}{5-\sqrt{13}}=6.1713$$

$$\frac{q}{p}=\frac{\frac{5-\sqrt{13}}{2}}{\frac{5+\sqrt{13}}{2}}=\frac{5-\sqrt{13}}{5+\sqrt{13}}=0.1620$$

If the roots are given, the equation is given by:

$$x^2-(Sum\,of\;Roots)x+Product\,of\;Roots=0$$

Therefore, the equation is:

$$x^2 - (6.1713 + 0.1620)x + (6.1713 \times 0.1620) = 0$$

$$x^2 - 6.333x + 1 = 0$$

Now, try the options.

Option (a) cannot be the answer.

Option (b) cannot be the answer as the last term has a negative sign.

Option (c) $\rightarrow 3x^2 - 19x + 3 = 0$

Dividing the entire equation by 3, we'll get:

$$\frac{3x^2}{3} - \frac{19}{3}x + \frac{3}{3} = 0$$

$$\Rightarrow x^2 - 6.333x + 1 = 0$$

Question 52

If the root of the equation $x^2 - 8x + m = 0$ exceeds the other by 4, then the value of m is:

(a) 10 (b) 11

(c) 9 (d) 12

Solution

(d)

$$x^2 - 8x + m = 0$$

$$a = 1;\ b = -8;\ c = m$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-8}{1} = 8$$

$$\alpha\beta = \frac{c}{a} = \frac{m}{1} = m$$

$$\alpha - \beta = 4$$

Now, we have two equations:

$$\alpha + \beta = 8 \ldots \text{Eq. (1)}$$

$$\alpha - \beta = 4 \ldots \text{Eq. (2)}$$

Adding these two, we'll get:

$$2\alpha = 12 \Rightarrow \alpha = \frac{12}{2} = 6$$

Putting this value in Eq. (1), we'll get:

$$6+\beta=8 \Rightarrow \beta=8-6=2$$

Now, $\alpha\beta=m \Rightarrow 6\times 2=m \Rightarrow m=12$

Question 53

If arithmetic mean between roots of a quadratic equation is 8 and the geometric mean between them is 5, the equation is ________.

(a) $x^2 - 16x - 25 = 0$ (b) $x^2 - 16x + 25 = 0$ (c) $x^2 + 16x + 25 = 0$ (d) None

Solution

(b)

$$\frac{\alpha+\beta}{2} = 8 \Rightarrow \alpha+\beta = 8\times 2 \Rightarrow \alpha+\beta = 16$$

$$\sqrt{\alpha\beta} = 5 \Rightarrow \alpha\beta = 5^2 \Rightarrow \alpha\beta = 25$$

Now, try the options.

Option (a) → $x^2 - 16x - 25 = 0$

Here, $\alpha + \beta = -\frac{b}{a} = -\frac{-16}{1} = 16$

$$\alpha\beta = \frac{c}{a} = \frac{-25}{1} = -25$$

Therefore, option (a) cannot be the answer.

Option (b) → $x^2 - 16x + 25 = 0$

Here, $\alpha + \beta = -\frac{b}{a} = -\frac{-16}{1} = 16$

$$\alpha\beta = \frac{c}{a} = \frac{25}{1} = 25$$

Therefore, option (b) is the answer.

Question 54

The harmonic mean of the roots of the equation $\left(5+\sqrt{2}\right)x^2-\left(4+\sqrt{5}\right)x+8+2\sqrt{5}=0$ is:

(a) 2 (b) 4 (c) 6 (d) 8

Solution

(b)

$$\left(5+\sqrt{2}\right)x^2-\left(4+\sqrt{5}\right)x+8+2\sqrt{5}=0$$

Here,

$$a=5+\sqrt{2};\ b=-\left(4+\sqrt{5}\right);\ c=8+2\sqrt{5}$$

Therefore, $\alpha+\beta=-\frac{b}{a}=-\frac{-\left(4+\sqrt{5}\right)}{5+\sqrt{2}}=0.9722$

$$\alpha\beta=\frac{c}{a}=\frac{8+2\sqrt{5}}{5+\sqrt{2}}=1.9444$$

$$HM=\frac{2\alpha\beta}{\alpha+\beta}=\frac{2\times1.9444}{0.9722}=4$$

Question 55

Difference between a number and its positive square root is 12; find the numbers.

(a) 4, 16 (b) 16, 4 (c) 22, 35 (d) Both (a) and (b)

Solution

(d)

Question 56

A piece of iron rod costs ₹60. If the rod was 2 metre shorter and each metre costs ₹1.00 more, the cost would remain unchanged. What is the length of the rod?

(a) 10 m (b) 14 m (c) 12 m (d) None

Solution

(c)

Question 57

Divide 25 into two parts so that sum of their reciprocals is 1/6.

(a) 8 and 17 (b) 10 and 15 (c) 20 and 5 (d) None

Solution

(b)

Question 58

The sum of two numbers is 8 and the sum of their squares is 34. Taking one number as x form an equation in x and hence find the numbers. The numbers are:

(a) (7, 10) (b) (4, 4) (c) (3, 5) (d) (2, 6)

Solution

(c)

Question 59

The difference of two positive integers is 3 and the sum of their squares is 89. Taking the smaller integer as x form a quadratic equation and solve it to find the integers. The integers are:

(a) (7, 4) (b) (5, 8) (c) (3, 6) (d) (2, 5)

Solution

(b)

Question 60

Five times of a positive whole number is 3 less than twice the square of the number. The number is

(a) 3 (b) 4 (c) –3 (d) 2

Solution

(a)

Question 61

The area of a rectangular field is 2000 sq.m. and its perimeter is 180 m. Form a quadratic equation by taking the length of the field as *x* and solve it to find the length and breadth of the field. The length and breadth are:

(a) (205 m, 80 m) (b) (50 m, 40 m) (c) (60 m, 50 m) (d) None

Solution

(b)

Question 62

Two squares have sides *p* cm and (*p* + 5) cms. The sum of their squares is 625 sq. cm. The sides of the squares are:

(a) (10 cm, 30 cm) (b) (12 cm, 25 cm) (c) (15 cm, 20 cm) (d) None

Solution

(c)

Question 63

Divide 50 into two parts such that the sum of their reciprocals is 1/12. The numbers are:

(a) (24, 26) (b) (28, 22) (c) (27, 23) (d) (20, 30)

Solution

(d)

Question 64

There are two consecutive numbers such that the difference of their reciprocals is 1/240. The numbers are:

(a) (15, 16) (b) (17, 18) (c) (13, 14) (d) (12, 13)

Solution

(a)

Question 65

The hypotenuse of a right–angled triangle is 20 cm. The difference between its other two sides be 4 cm. The sides are:

(a) (11 cm, 15 cm) (b) (12 cm, 16 cm) (c) (20 cm, 24 cm) (d) None

Solution

(b)

Question 66

The sum of two numbers is 45 and the mean proportional between them is 18. The numbers are:

(a) (15, 30) (b) (32, 13) (c) (36, 9) (d) (25, 20)

Solution

(c)

Question 67

The sides of an equilateral triangle are shortened by 12 units 13 units and 14 units respectively and a right-angle triangle is formed. The side of the equilateral triangle is:

(a) 17 units (b) 16 units (c) 15 units (d) 18 units

Solution

(a)

Question 68

A distributor of apple Juice has 5000 bottles in the store that it wishes to distribute in a month. From experience it is known that demand D (in number of bottles) is given by $D=-2000p^2+2000p+17000$. The price per bottle that will result zero inventory is:

(a) ₹3 (b) ₹5 (c) ₹2 (d) None

Solution

(a)

Question 69

The sum of two irrational numbers multiplied by the larger one is 70 and their difference is multiplied by the smaller one is 12; the two numbers are:

(a) $3\sqrt{2}, 2\sqrt{3}$ (b) $5\sqrt{2}, 3\sqrt{5}$ (c) $2\sqrt{2}, 5\sqrt{2}$ (d) None

Solution

(c)

Questions Based on Cubic Equations

Question 70

$x,\ x-4,\ x+5$ are the factors of the left-hand side of the equation:

(a) $x^3+2x^2-x-2=0$

(b) $x^3+x^2-20x=0$

(c) $x^3-3x^2-4x+12=0$

(d) $x^3-6x^2+11x-6=0$

Solution

(b)

Question 71

The equation $3x^3 + 5x^2 = 3x + 5$ has got 3 roots and hence the factors of the left-hand side of the equation $3x^3 + 5x^2 - 3x - 5 = 0$ are:

(a) $x-1,\ x-2,\ x-5/3$

(b) $x-1,\ x+1,\ 3x+5$

(c) $x+1,\ x-1,\ 3x-5$

(d) $x-1,\ x+1,\ x-2$

Solution

(b)

Question 72

The roots of $x^3 + x^2 - x - 1 = 0$ are:

(a) (–1, –1, 1) (b) (1, 1, –1) (c) (–1, –1, –1) (d) (1, 1, 1)

Solution

(a)

Question 73

If $4x^3 + 8x^2 - x - 2 = 0$, then the value of $(2x+3)$ is given by:

(a) 4, –1, 2 (b) –4, 2, 1 (c) 2, –4, –1 (d) None

Solution

(a)

Question 74

The value of k is _________, if 2 is the root of the following cubic equation $x^3-(k+1)x+k=0$.

(a) 2 (b) 6 (c) 1 (d) 4

Solution

(b)

$x^3-(k+1)x+k=0$

$\Rightarrow 2^3-(k+1)2+k=0$

$\Rightarrow 8-2k-2+k=0$

$$\Rightarrow 6-k=0$$

$$\Rightarrow k=6$$

Chapter 3 – Linear Inequalities

CA NISHANT KUMAR

Question 1

On the average, an experienced person does 7 units of work while a fresh one work 5 units of work daily but the employer has to maintain an output of at least 35 units of work per day. The situation can be expressed as:

(a) $7x + 5y < 35$ (b) $7x + 5y \leq 35$ (c) $7x + 5y > 35$ (d) $7x + 5y \geq 35$

Solution

(d)

Question 2

The solution space of the inequalities $2x + y \leq 10$ and $x - y \leq 5$:

1. Includes origin
2. Includes the point (4, 3)

Which one is correct:

(a) Only 1 (b) Only 2 (c) Both 1 and 2 (d) None

Solution

(a)

Origin means (0, 0).

Try this point in the first equation.

$$2x + y \leq 10$$

$$2(0) + (0) = 0 + 0 = 0 \leq 10$$

Also,

$$x - y \leq 5$$

$$0 - 0 = 0 \leq 5$$

Since both the inequations are being satisfied with origin, the solution space definitely contains the origin.

Now, try the point (4, 3).

Put it in the first equation

$$2x + y \leq 10$$

$$2(4) + (3) = 8 + 3 = 11$$

This is neither less than nor equal to 10. Therefore, this point does not satisfy the first inequation. Hence, there is no point in checking the second equation.

Clearly, option (a) is the answer.

Question 3

The solution of the inequality $\frac{(5-2x)}{3} \leq \frac{x}{6} - 5$ is:

(a) $x \geq 8$ (b) $x \leq 8$ (c) $x = 8$ (d) None

Solution

(a)

Try the options.

Option (a) → $x \geq 8$

Take $x = 8$ first and see if the inequation satisfies.

LHS = $\frac{5-(2\times8)}{3}=\frac{5-16}{3}=-3.67$

RHS = $\frac{8}{6}-5=-3.67$

Since LHS = RHS, the inequation satisfies.

Now, let’s check if the inequation satisfies for a value greater than 8. Let’s try with 9.

LHS = $\frac{5-(2\times9)}{3}=\frac{5-18}{3}=-4.33$

RHS = $\frac{9}{6}-5=-3.5$

Clearly LHS < RHS, and therefore, the inequation is satisfied.

Therefore, option (a) is the answer.

Question 4

On the average, an experienced person does 5 units of work while a fresh one work 3 units of work daily but the employer has to maintain an output of at least 30 units of work per day. The situation can be expressed as:

(a) $5x + 3y \leq 30$ (b) $5x + 3y \geq 30$ (c) $5x + 3y = 30$ (d) None

Solution

(b)

Question 5

The solution set of the inequation $x+2>0$ and $2x-6>0$ is:

(a) $(-2,\infty)$ (b) $(3,\infty)$ (c) $(-\infty,-2)$ (d) $(-\infty,-3)$

(b)

$x+2>0 \Rightarrow x>-2$

Therefore, any value greater than –2 till infinity will satisfy this equation.

$2x-6>0$

$2x>6$

$$x > \frac{6}{2} \Rightarrow x > 3$$

Therefore, any value greater than 3 till infinity will satisfy this equation.

Therefore, the common solution set is $(3, \infty)$.

Question 6

A company produces two products A and B, each of which requires processing in two machines. The first machine can be used at most for 60 hours, the second machine can be used at most for 40 hours. The product A requires 2 hours on machine one and one hour on machine two. The product B requires one hour on machine one and two hours on machine two. Express above situation using linear inequalities.

(a) $2x+y\leq 60$ and $x+2y\geq 40$
(b) $2x+y\geq 60$ and $x+2y\geq 40$
(c) $2x+y\leq 60$ and $x+2y\leq 40$
(d) $2x+y\geq 60$ and $x+2y\leq 40$

(c)

	Machine 1	Machine 2
Product A (x)	2	1
Product B (y)	1	2
Maximum Capacity	60	40

Clearly, $2x + y \leq 60$ and $x + 2y \leq 40$

Therefore, option (c) is the answer.

Question 7

Mr. A plans to invest up to ₹30,000 in two stocks *X* and *Y*. Stock *X* (x) is priced at ₹175 and Stock *Y* (y) at ₹95 per share. This can be shown by:

(a) $175x + 95y < 30,000$ (b) $175x + 95y > 30,000$ (c) $175x + 95y = 30,000$ (d) None

Solution

(a)

Question 8

The solution of the inequality $8x + 6 < 12x + 14$ is:

(a) $(-2, 2)$ (b) $(0, -2)$ (c) $(2, \infty)$ (d) $(-2, \infty)$

Solution

(d)

Question 9

The rules and representations demand that employer should employ not more than 8 experienced leads to 1 fresh one and this fact can be expressed as:

(a) $y \geq x/8$ (b) $8y \leq x$ (c) $8y = x$ (d) $y = 8x$

Solution

(a)

Question 10

A manufacturer produces two items A and B. He has ₹10,000 to invest and a space to store 100 items. A table costs him ₹400 and a chair ₹100. Express this in the form of linear inequalities.

(a) $x + y \leq 100$, $4x + y \leq 100$, $x \geq 0$, $y \geq 0$
(b) $x + y \leq 1000$, $2x + 5y < 1000$, $x \geq 0$, $y \geq 0$
(c) $x + y > 100$, $4x + y \geq 100$, $x \geq 0$, $y \geq 0$
(d) None

Solution

(a)

Question 11

The common region in the graph of the inequalities $x + y \leq 4,\ x - y \leq 4,\ x \geq 2$ is

(a) Equilateral triangle
(b) Isosceles triangle
(c) Quadrilateral
(d) Square

Solution

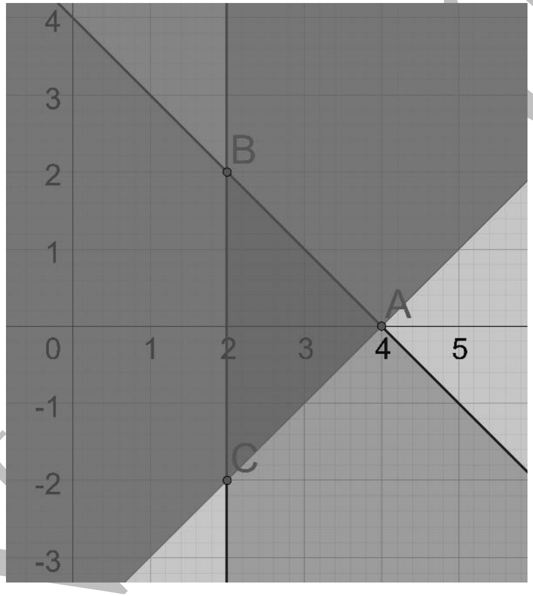
(b)

Inequalities graph for

$x + y \leq 4,$ $x - y \leq 4$ $x \geq 2$

x	0	4
y	4	0

x	0	4
y	–4	0



Common Area in the graph is Δ ABC

Clearly, it is an isosceles triangle.

Question 12

Solve for x of the Inequalities

$$2 \le \frac{3x-2}{5} \le 4 \text{ where } x \to N$$

(a) {5, 6, 7}
(b) {3, 4, 5, 6}
(c) {4, 5, 6}
(d) None

Solution

(d)

Given:

$$2 \le \frac{3x-2}{5} \le 4$$

Multiplying the entire equation with 5, we get:

$$(2\times5) \le \left\{\frac{(3x-2)}{5}\times5\right\} \le (4\times5)$$

$$10 \le 3x-2 \le 20$$

Adding 2 to the entire equation, we get:

$$10+2 \le 3x-2+2 \le 20+2$$

$$12 \le 3x \le 22$$

Dividing the entire equation by 3, we get:

$$\frac{12}{3} \leq \frac{3x}{3} \leq \frac{22}{3}$$

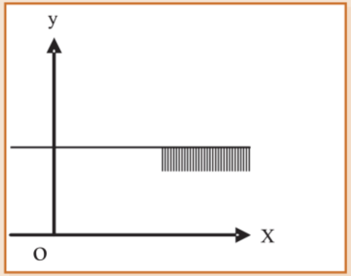
$$4 \leq \times \leq 7.33$$

So, solution set is $x = \{4, 5, 6, 7\}$

Question 13

(vii) The graph to express the inequality $y \leq \left(\frac{1}{2}\right) x$ is indicated by

(a)



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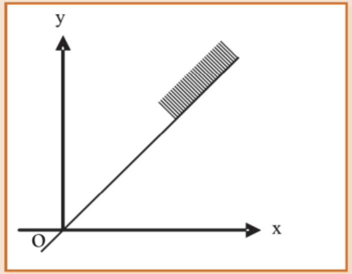

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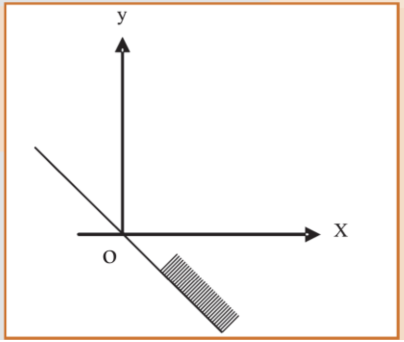
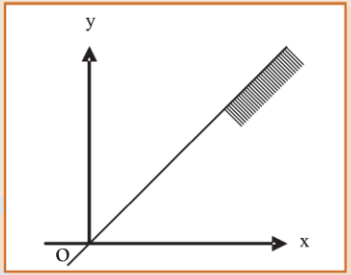
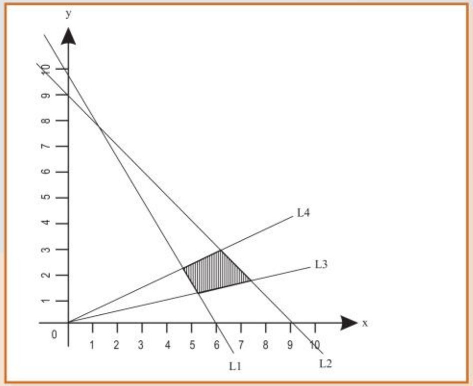


(d)



Question 14

(viii)



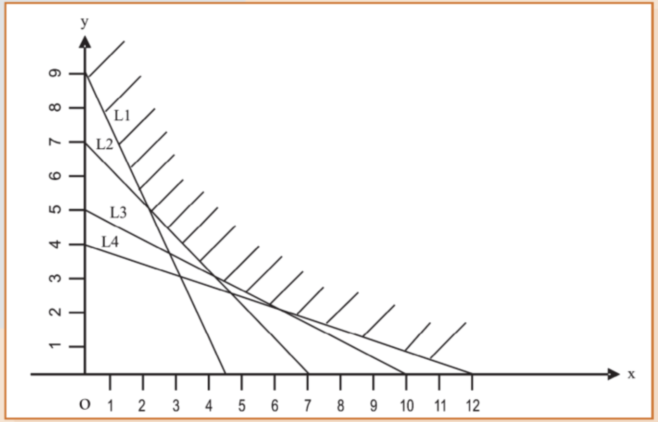
L1 : 5x + 3y = 30   L2 : x+y = 9   L3 : y = x/3   L4 : y = x/2

The common region (shaded part) shown in the diagram refers to

(a)	(b)	(c)	(d)	(e)
$5x + 3y \le 30$	$5x + 3y \ge 30$	$5x + 3y \ge 30$	$5x + 3y > 30$	None of these
$x + y \le 9$	$x + y \le 9$	$x + y \ge 9$	$x + y < 9$	
$y \le 1/5\ x$	$y \ge x/3$	$y \le x/3$	$y \ge 9$	
$y \le x/2$	$y \le x/2$	$y \ge x/2$	$y \le x/2$	
	$x \ge 0, y \ge 0$	$x \ge 0, y \ge 0$	$x \ge 0, y \ge 0$	

Question 15

3. Graphs of the inequations are drawn below :



L1 : 2x +y = 9 L2 : x + y = 7 L3 : x+2y= 10 L4 : x + 3y = 12

The common region (shaded part) indicated on the diagram is expressed by the set of inequalities

(a) $2x + y \leq 9$
$x + y \geq 7$
$x + 2y \geq 10$
$x + 3y \geq 12$

(b) $2x + y \geq 9$
$x + y \leq 7$
$x + 2y \geq 10$
$x + 3y \geq 12$

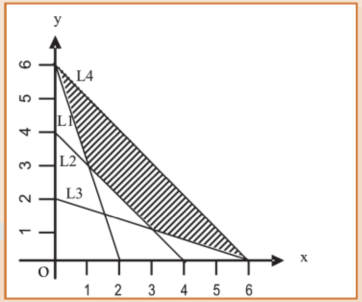
(c) $2x + y \geq 9$
$x + y \geq 7$
$x + 2y \geq 10$
$x + 3y \geq 12$
$x \geq 0, y \geq 0$

(d) none of these

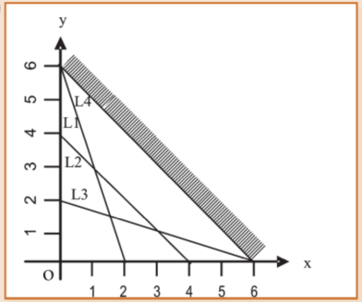
Question 16

4. The common region satisfied by the inequalities L1: $3x + y \geq 6$, L2: $x + y \geq 4$, L3: $x + 3y \geq 6$, and L4: $x + y \leq 6$ is indicated by

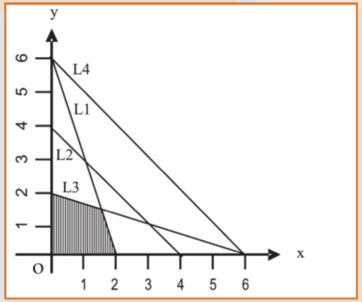
(a)



(b)



(c)



(d) none of these

Question 17

If $3x+2<2x+5$ and $4x-5\geq 2x-3$, then x can take from the following values:

(a) 3 (b) –1 (c) 2 (d) –3

Solution

(c)

$3x+2<2x+5$

$\Rightarrow 3x-2x<5-2$

$\Rightarrow x<3\ldots\text{Eq. (1)}$

$4x-5\geq 2x-3$

$\Rightarrow 4x - 2x \geq -3 + 5$

$\Rightarrow 2x \geq 2$

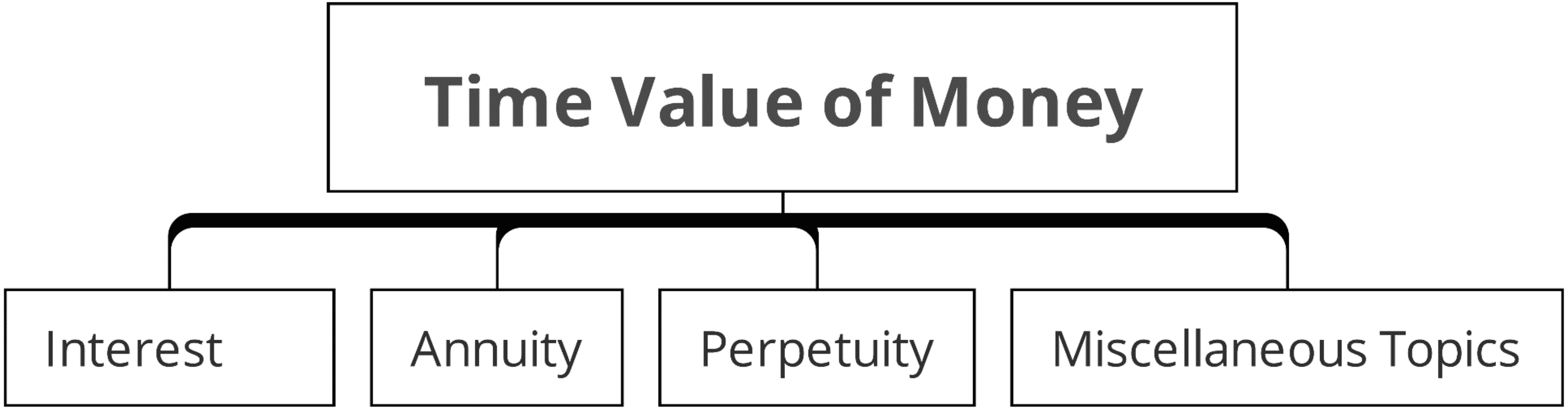
$\Rightarrow x \geq 1$...Eq. (2)

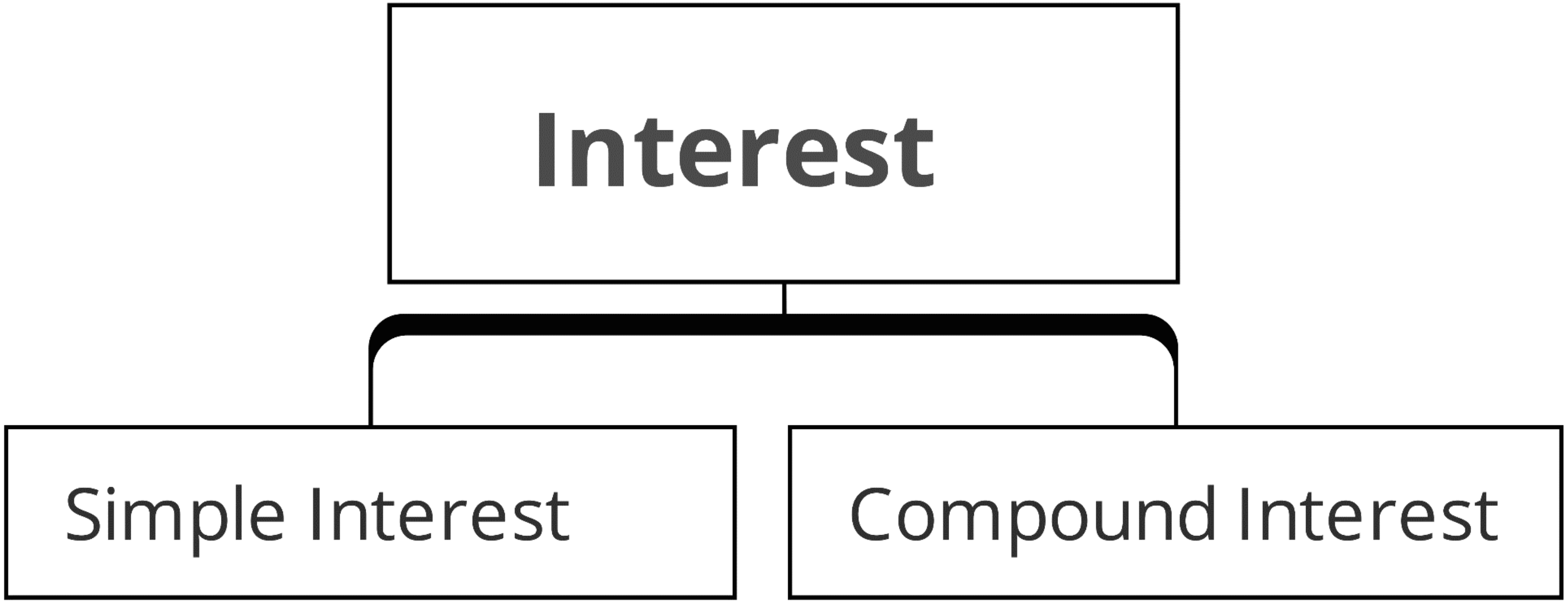
From Equations (1), and (2), x can take values between 1 and 3 (including 1, but excluding 3).

Therefore, option (c) is the answer.

Chapter 4 – Time Value of Money

CA NISHANT KUMAR


Time Value of Money
Interest
Annuity
Perpetuity
Miscellaneous Topics


Interest
Simple Interest
Compound Interest

Simple Interest

- **Meaning**
  - Interest accrues only on the original Principal
- **Formulas**
  - $I = Pit$
  - $A = P + I$
  - Derivatives of these Formulas
    - $A = P(1 + it)$
    - $i = \frac{A - P}{Pt}$
    - $t = \frac{A - P}{Pi}$
- **Important Points**
  - P = Principal
  - i = Rate of Interest (in Decimal)
  - t = Time (in years)
  - To convert months into years, divide the given months by 12

CA NISHANT KUMAR

Questions on Simple Interest

Question 1 – ICAI SM

Sania deposited ₹50,000 in a bank for two years with the interest rate of 5.5% p.a. What will be the final value of investment?

(a) ₹55,800 (b) ₹5,500 (c) ₹55,500 (d) ₹5,800

Solution

(c)

Question 2 – ICAI SM

The sum required to earn a monthly interest of ₹1,200 at 18% p.a. SI is:

(a) ₹50,000 (b) ₹60,000 (c) ₹80,000 (d) None

Solution

(c)

Question 3

₹8,000 becomes ₹10,000 in 1 year 8 months at simple interest. The amount that will become ₹6,875 in 2 years 7 months at the same rate of interest is:

(a) ₹4,850 (b) ₹5,000 (c) ₹4,955 (d) ₹5,275

Solution

(c)

First, let's calculate rate of interest.

$$i = \frac{A-P}{Pt} = \frac{1000-8000}{8000\times\left(1+\frac{8}{12}\right)} = 0.15$$

Now, let's calculate P.

$$P = \frac{A}{1+it} = \frac{6,875}{1+\left\{0.15\times\left(2+\frac{7}{12}\right)\right\}} = 4,955$$

Question 4 – ICAI SM

P = ₹8,500, A = ₹10,200, R = 12½% SI, t will be:

(a) 1 year 7 months (b) 2 years (c) 1½ year (d) None

Solution

(a)

Question 5 – MTP December, 2021

A sum of money gets doubled in 5 years at x% simple interest. If the interest was y%, the sum of money would have become ten-fold in thirty years. What is $y - x$ (in %)?

(a) 10 (b) 5 (c) 8 (d) None

Solution

(a)

Let's find out x first.

We have $P = 100$; $A = 200$; $t = 5$; $i = x/100$

$I = A - P = 200 - 100 = 10$

We know that $I = Pit$

Therefore, $100 = 100 \times \frac{x}{100} \times 5$

$\Rightarrow \frac{100 \times 100}{100 \times 5} = x$

$\Rightarrow x = 20\%$

Now, let's find out y.

$P = 100$; $A = 10 \times 100 = 1{,}000$; $t = 30$; $i = y/100$

$I = A - P = 1{,}000 - 100 = 900$

We know that $I = Pit$

Therefore, $900 = 100 \times \frac{y}{100} \times 30$

$\Rightarrow \frac{900 \times 100}{100 \times 30} = y$

$\Rightarrow y = 30\%$

Therefore, $y - x = 30\% - 20\% = 10\%$

Question 6 – MTP November, 2019

A person deposited a sum of ₹10,000 in a bank. After 2 years, he withdrew ₹4,000 and at the end of 5 years, he received an amount of ₹7,900; then the rate of simple interest is:

(a) 6% (b) 5% (c) 10% (d) None

(b)

For the first two years, $P = 10,000,\ t = 2,\ i = ?$

Therefore, $I = Pit = 10,000 \times i \times 2 = 20,000i$

Amount = $20,000i + 10,000$

For the next three years, $P = 10,000 - 4,000 = 6,000,\ t = 3,\ i = ?$

Therefore, $I = Pit = 6,000 \times i \times 3 = 18,000i$

Total Interest = $20,000i + 18,000i = 38,000i$

Amount = Principal + Interest

₹7,900 = ₹6,000 + $38,000i$

$38,000i = 7,900 - 6,000 = 1,900$

$$i = \frac{1,900}{38,000} = 0.05$$

Therefore, rate of interest is 5%.

Alternatively, try the options:

Option (a) → 6%

Interest for first two years = $10,000 \times 0.06 \times 2 = 1,200$

Interest for next three years = $6,000 \times 0.06 \times 3 = 1,080$

Total Interest = $1,200 + 1,080 = 2,280$

Total Amount = ₹6,000 + ₹2,280 = ₹8,280

Therefore, option (a) cannot be the answer.

Option (b) → 5%

Interest for first two years = $10,000 \times 0.05 \times 2 = 1,000$

Interest for next three years = $6,000 \times 0.05 \times 3 = 900$

Total Interest = $1,000 + 900 = 1,900$

Total Amount = ₹6,000 + ₹1,900 = ₹7,900

Therefore, option (b) is the answer.

Question 7 – May, 2018; MTP June, 2021

A person borrows ₹5,000 for 2 years at 4% p.a. simple interest. He immediately lends to another person at 6¼% p.a. for 2 years. Find his gain in the transaction per year.

(a) ₹112.50 (b) ₹125 (c) ₹225 (d) ₹167.50

Solution

(a)

Question 8 – June, 2011; MTP June, 2021

If a simple interest on a sum of money at 6% p.a. for 7 years is equal to twice of simple interest on another sum for 9 years at 5% p.a., the ratio will be:

(a) 2 : 15 (b) 7 : 15 (c) 15 : 7 (d) 1 : 7

Solution

(c)

$$P_1 \times 0.06 \times 7 = 2 \times P_2 \times 0.05 \times 9$$

$$\frac{P_1}{P_2} = \frac{2 \times 0.05 \times 9}{0.06 \times 7} = 2.1428$$

Question 9 – MTP June, 2021

A sum of money amounts to ₹20,800 in 5 years and ₹22,720 in 7 years. Find the principal and rate of interest.

(a) ₹5,000; 6% (b) ₹16,000; 6% (c) ₹80,000; 8% (d) ₹10,000; 10%

Solution

(b)

Since the question is silent about Simple Interest or Compound Interest, we'll try both. First, let's try Simple Interest.

$A = P + I$

Try the options:

Option (b) → 16,000; 6%

Amount after 5 years:

$A = 16{,}000 + (16{,}000 \times 0.06 \times 5) = ₹20{,}800$

Amount after 7 years:

$A = 16{,}000 + (16{,}000 \times 0.06 \times 7) = ₹22{,}720$

Therefore, option (b) is the answer.

Question 10 – MTP June, 2021

Two equal sums were lent out at 7% and 5% simple interest respectively. The interest earned on the two loans adds upto ₹960 for four years. Find the sum lent out.

(a) ₹4,000 (b) ₹3,000 (c) ₹5,000 (d) ₹6,000

Solution

(a)

Let the sum lent out be x each.

Interest from 7% = $x \times 0.07 \times 4 = 0.28x$

Interest from 5% = $x \times 0.05 \times 4 = 0.2x$

As per the question, $0.28x + 0.2x = 960 \Rightarrow 0.48x = 960 \Rightarrow x = 960 \div 0.48 =$ ₹2,000

Therefore, total sum lent out = ₹2,000 + ₹2,000 = ₹4,000

Question 11 – MTP November, 2019

A trust fund has invested ₹30,000 in two different types of bonds which pays 5% and 7% interest respectively. Determine how much amount is invested in each type of bond if trust obtains an annual total interest of ₹1,600.

(a) ₹5,000 (b) ₹6,000 (c) ₹7,000 (d) ₹8,000

(a)

Let the amount invested in the 5% bond be *x*. Then, the amount invested in the 7% bond will be (₹30,000 – *x*).

Total Interest = $\left(\frac{5}{100} \times x\right) + \left\{\frac{7}{100} \times (30,000 - x)\right\} = 1,600$

$$\Rightarrow 0.05x + \{0.07(30,000 - x)\} = 1,600$$

$$\Rightarrow 0.05x + 2,100 - 0.07x = 1,600$$

$$\Rightarrow -0.02x = -500$$

$$\Rightarrow x = \frac{500}{0.02} = 25,000$$

Therefore, in the 5% bond, the amount invested is ₹25,000, and so, the amount invested in the 7% bond is ₹30,000 – ₹25,000 = ₹5,000.

Since we don't have ₹25,000 in the options, we'll mark ₹5,000.

Therefore, option (a) is the answer.

Question 12 – December, 2022

A farmer borrowed ₹3,600 at the rate of 15% simple interest per annum. At the end of 4 years, he cleared this account by paying ₹4,000 and a cow. The cost of the cow is:

(a) ₹1,000 (b) ₹1,200 (c) ₹1,550 (d) ₹1,760

Solution

(d)

$$A = P(1 + it)$$

$$A = 3,600\{1 + (0.15 \times 4)\} = 5,760$$

Cost of Cow = ₹5,760 – ₹4,000 = ₹1,760

Question 13 – July, 2021

A certain sum amounts to ₹15,748 in 3 years at simple interest at r% p.a. The same sum amounts to ₹16,510 at $(r+2)$% p.a. simple interest in the same time. What is the value of r?

(a) 10% (b) 8% (c) 12% (d) 6%

Solution

(d)

We know that $A = P(1+it)$

Therefore, $15,748 = P(1+3i)$…Eq. (1)

Also,

$16,510 = P\left\{1+3\left(i+0.02\right)\right\}$ [Note: We added 0.02 because we need to take the interest in decimal]

$\Rightarrow 16,510 = P\left\{1+3\left(i+0.02\right)\right\}$

$\Rightarrow 16,510 = P\left(1.06+3i\right)$...Eq. (2)

Dividing Eq. (1) by Eq. (2), we have:

$$\frac{15,748}{16,510} = \frac{P\left(1+3i\right)}{P\left(1.06+3i\right)}$$

$$\Rightarrow \frac{15,748}{16,510} = \frac{1+3i}{1.06+3i}$$

$$\Rightarrow 0.9538 = \frac{1+3i}{1.06+3i}$$

Now, try the options.

Option (a) $\rightarrow$ 10%

$$RHS = \frac{1+3(0.10)}{1.06+3(0.10)} = \frac{1.3}{1.36} = 0.9559 \neq 0.9538$$

Option (b) $\rightarrow$ 8%

$$\frac{1+3(0.08)}{1.06+3(0.08)} = \frac{1.24}{1.3} = 0.9538 = LHS$$

Therefore, option (b) is the answer.

Question 14 – December, 2021

An amount is lent at R% simple interest for R years and the simple interest amount was one-fourth of the principal amount. Then R is _______.

(a) 5 (b) 6 (c) 5½ (d) 6½

Solution

(a)

We know that $I = Pit$

Given: $I = \frac{P}{4}$; $i = \frac{R}{100}$; $t = R$

$$I = Pit$$

$$\Rightarrow \frac{P}{4} = P \times \frac{R}{100} \times R$$

$$\Rightarrow \frac{1}{4} = \frac{R^2}{100}$$

$$\Rightarrow 100 = 4R^2$$

$$\Rightarrow R^2 = \frac{100}{4} = 25$$

$$\Rightarrow R = \sqrt{25} = 5$$

Compound Interest

Meaning

Interest is calculated not only on the original Principal, but also on the interest accrued on it.

Formulas

$$A = P\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}$$

$$CI = P\left[\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY} - 1\right]$$

Important Points

NOCPPY = No. of Conversion Periods Per Year

Compounded Annually	NOCPPY = 1
Compounded Semi-Annually	NOCPPY = 2
Compounded Quarterly	NOCPPY = 4
Compounded Monthly	NOCPPY = 12

$n = t \times NOCPPY$

Questions on Compound Interest

CA NISHANT KUMAR

Question 15 – July, 2021

A sum of ₹x amounts to ₹27,900 in 3 years and to ₹41,850 in 6 years at a certain rate percent per annum, when the interest is compounded yearly. The value of x is:

(a) 16,080 (b) 18,600 (c) 18,060 (d) 16,800

Solution

(b)

From the given information, it is clear that interest from the 3rd year to the 6th year is ₹41,850 – ₹27,900 = ₹13,950

Therefore, for this duration, we have P = ₹27,900; t = 3 years; A = ₹41,850; i = ?

We know that $A = P\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}$

Therefore, $41{,}850 = 27{,}900\left(1 + \frac{i}{1}\right)^{3 \times 1}$

$$\Rightarrow \frac{41{,}850}{27{,}900} = (1+i)^3$$

$$\Rightarrow 1.5 = (1+i)^3 \ldots \text{Eq. } (1)$$

Now, for the first 3 years, we have: $P = x$; $t = 3$ years; $A = 27{,}900$; $(1+i)^3 = 1.5$

We know that $A = P\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}$

Therefore, $P = \frac{A}{\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}}$

$$\Rightarrow P = \frac{27,900}{\left(1 + \frac{i}{1}\right)^{3 \times 1}} = \frac{27,900}{(1+i)^3} = \frac{27,900}{1.5} = 18,600$$

[We know from Eq. (1) that $(1+i)^3 = 1.5$]

Question 16 – ICAI SM

Mr. X borrowed ₹5,120 at 12½% p.a. C.I. At the end of 3 years, the money was repaid along with the interest accrued. The amount of interest paid by him is:

(a) ₹2,100 (b) ₹2,170 (c) ₹2,000 (d) None

Solution

(b)

Question 17 – ICAI SM

If A = ₹1,000; n = 2 years; R = 6% p.a. compound interest payable half-yearly, then Principal (P) is:

(a) ₹888.50 (b) ₹885 (c) ₹800 (d) None

Solution

(a)

Even though in this question, we are directly given the value of n, but this is wrong as n cannot have a unit. n means total number of conversions. In this question, they were supposed to write t = 2 years, and not n. Also, since we are given the compounding as well in the question as "half-yearly", this further goes to show that this n should have been t. We will solve the question taking it to be t only.

So, we have $A = 1{,}000;\ t = 2;\ i = 0.06;\ NOCPPY = 2;\ P = ?$

$$A = P\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}$$

$$P = \frac{A}{\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}} = \frac{1{,}000}{\left(1 + \frac{0.06}{2}\right)^{2 \times 2}} = 888.48$$

Question 18 – ICAI SM

The compound interest on half-yearly rests on ₹10,000 the rate for the first and second years being 6% and for the third year 9% p.a. is:

(a) ₹2,200 (b) ₹2,287 (c) ₹2,285 (d) ₹2,290.84

(d)

For the first two years, we have P = ₹10,000; $i = 0.06$; $t = 2$; $NOCPPY = 2$.

We know that $A = P\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}$

Amount at the end of two years = $10,000\left(1+\frac{0.06}{2}\right)^{2\times 2} = 11,255.0881$

For the third year, we have *P* = 11,255.0881; *i* = 0.09; NOCPPY = 2; t = 1 year

Amount at the end of the third year = $11,255.0881\left(1+\frac{0.09}{2}\right)^{1\times 2} = 12,290.84$

We know that Compound Interest = Amount – Principal

Therefore, Compound Interest = ₹12,290.84 – ₹10,000 = ₹2,290.84

Question 19 – ICAI SM

On what sum will the compound interest at 5% p.a. for two years compounded annually be ₹1,640?

(a) ₹2,200 (b) ₹1,487.53 (c) ₹16,000 (d) None

Solution

(c)

$$CI = P\left[\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY} - 1\right]$$

$$P = \frac{CI}{\left[\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY} - 1\right]} = \frac{1,640}{\left[\left(1+\frac{0.05}{1}\right)^{2\times 1} - 1\right]} = 16,000$$

Question 20 – ICAI SM

What annual rate of interest compounded annually doubles an investment in 7 years? Given that $2^{\frac{1}{7}} = 1.104090$.

(a) 11.41% (b) 10% (c) 10.41% (d) None

Solution

(c)

Question 21 – ICAI SM

The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is:

(a) 7 years (b) 10 years (c) 17 years (approx.) (d) None

Solution

(c)

Question 22 – ICAI SM

The annual birth and death rates per 1,000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is:

(a) 35 years (b) 30 years (c) 25 years (d) None

Solution

(a)

Question 23 – ICAI SM

A = ₹5,200, R = 5% p.a., T = 6 years, P will be

(a) ₹2,000 (b) ₹3,880 (c) ₹3,000 (d) None

(b)

It is not mentioned in the question whether we have to use Simple Interest or Compound Interest. So, we'll try both.

First, let's try Simple Interest.

$$I = Pit$$

$$A = P + I$$

$$A = P + Pit$$

$$A = P(1 + it)$$

$$P = \frac{A}{1 + it}$$

$$P = \frac{A}{1 + it} = \frac{5,200}{1 + (0.05 \times 6)} = ₹4,000$$

Clearly, ₹4,000 is not present in any of the options. Now, don't just straightaway mark the option (d). Try with Compound Interest first.

$$A = P\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}$$

$$P = \frac{A}{\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}}$$

$$P = \frac{A}{\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}} = \frac{5,200}{\left(1 + \frac{0.05}{1}\right)^{6 \times 1}} = ₹3,880$$

Therefore, option (b) is the answer.

Question 24 – MTP December, 2020

A man borrows ₹4,000 from a bank at 10% compound interest. At the end of every year ₹1,500 as part of repayment of loan and interest. How much is still owed to the bank after three such instalments [Given: $(1.1)^3 = 1.331$]

(a) ₹359 (b) ₹820 (c) ₹724 (d) ₹720

(a)

Amount owed at the end of first year before payment of instalment =

$$4,000\left(1+\frac{0.10}{1}\right)^{1\times 1} = 4,400$$

From this, instalment of ₹1,500 is paid.

Therefore, amount owed at the end of first year after payment of instalment = ₹4,400 – ₹1,500 = ₹2,900

Now, amount owed at the end of the second year before payment of instalment =

$$2,900\left(1+\frac{0.10}{1}\right)^{1\times1} = 3,190$$

From this, instalment of ₹1,500 is paid.

Therefore, amount owed at the end of the second year after payment of instalment = ₹3,190 – ₹1,500 = ₹1,690

Now, amount owed at the end of the third year before payment of instalment =

$$1,690\left(1+\frac{0.10}{1}\right)^{1\times1} = 1,859$$

From this, instalment of ₹1,500 is paid.

Therefore, amount owed at the end of the third year after payment of instalment = ₹1,859 – ₹1,500 = ₹359

Therefore, amount owed after payment of the third instalment = ₹359.

Question 25 – December, 2022

A trust fund has invested ₹27,000 money in two schemes ‘A’ and ‘B’ offering compound interest at the rate of 8% and 9% per annum respectively. It the total amount of interest accrued through these two schemes together in two years was ₹4,818.30, what was the amount invested in scheme ‘A’?

(a) ₹12,000 (b) ₹12,500 (c) ₹13,000 (d) ₹12,500

Solution

(a)

Let the amount invested in Scheme A be x; then the amount invested in Scheme B = ₹27,000 – x

We know that $CI = P\left[\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY} - 1\right]$

Interest after 2 years from Scheme A:

$$CI = x\left[\left(1+\frac{0.08}{1}\right)^{2\times 1} - 1\right] = 0.1664x$$

Interest after 2 years from Scheme B:

$$CI = (27,000 - x)\left[\left(1+\frac{0.09}{1}\right)^{2\times 1} - 1\right] = 0.1881(27,000 - x)$$

$$CI = 5,078.7 - 0.1881x$$

Total interest = ₹4,818.30

$$0.1664x + 5,078.7 - 0.1881x = 4,818.30$$

$$\Rightarrow -0.0217x = 4,818.30 - 5,078.7$$

$$\Rightarrow -0.0217x = -260.4$$

$$\Rightarrow x = \frac{260.4}{0.0217} = 12,000$$

Question 26 – December, 2022

A sum of money invested of compound interest double itself in four years. In how many years it become 32 times of itself at the same rate of compound interest.

(a) 12 years (b) 16 years (c) 20 years (d) 18 years

Solution

(c)

Let $P = 100$; $A = 200$; $t = 4$; $i = ?$

$$A = P\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}$$

$$\Rightarrow 200 = 100\left(1+\frac{i}{1}\right)^{4\times 1}$$

$$\Rightarrow \frac{200}{100} = (1+i)^{4}$$

$$\Rightarrow (1+i)^{4} = 2$$

$$\Rightarrow 1+i = 2^{\frac{1}{4}} \ldots \text{Eq. (1)}$$

Now, we have P = 100; A = 32 × 100 = 3,200; t = ?

$$A = P\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}$$

$$\Rightarrow 3,200 = 100\left(1+\frac{i}{1}\right)^{t\times 1}$$

$$\Rightarrow \frac{3,200}{100} = \left(1+i\right)^{t}$$

$$\Rightarrow 32 = \left(1+i\right)^{t}$$

$$\Rightarrow \left(1+i\right)^{t} = 32$$

$$\Rightarrow 1+i = 32^{\frac{1}{t}} \ldots \text{Eq. (2)}$$

From Eqs. (1) and (2), we have:

$$2^{\frac{1}{4}} = 32^{\frac{1}{t}}$$

$$\Rightarrow 2^{\frac{1}{4}} = \left(2^5\right)^{\frac{1}{t}}$$

$$\Rightarrow 2^{\frac{1}{4}} = 2^{\frac{5}{t}}$$

$$\Rightarrow \frac{1}{4} = \frac{5}{t}$$

$$\Rightarrow t = 5 \times 4 = 20$$

Compound Interest

Applications

- **Depreciation**
  - Important Points
    - Principal = Cost of Machine
    - Amount = Scrap Value
    - Rate of Depreciation has to be written in negative
- **Effective Rate of Interest**
  - Meaning
    - Ignoring the compounding of Annually/ Semi-Annually/ Quarterly/ Monthly, etc., what is the actual rate of interest which will fetch you the effective interest in a year
  - Formula

$$E = \left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY} - 1$$

- **Difference between CI and SI**
  - Formula

$$CI - SI = P\left[\left\{(1+i)^t - 1\right\} - it\right]$$

Questions Based on Depreciation

Question 27 – MTP December, 2020

A Machine was purchased for ₹10,000. Its rate of depreciation is 10% in the first year and 5% per annum afterwards. Find the depreciated value of Machine after 7 years of purchase. $\left\{Given:(0.95)^6 = 0.7351\right\}$

(a) ₹6,606 (b) ₹6,616 (c) ₹6,660 (d) ₹6,661

(b)

Depreciated Value $= 10,000(1-0.10)^1(1-0.05)^6$

$= 10,000(0.90)(0.95)^6$

$= 10,000 \times 0.90 \times 0.7351$

$$= 6,615.90$$

$$\approx 6,616$$

Question 28 – December, 2022

A machine worth ₹4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹2,00,750?

(a) 5 years 5 months (b) 5 years 6 months (c) 5 years 7 months (d) 5 years 8 months

Solution

(b)

P = ₹4,90,740; A = ₹2,00,750; $i = -0.15$; t = ?

$$2,00,750 = 4,90,740\left(1+\frac{-0.15}{1}\right)^{t\times 1}$$

$$\Rightarrow \frac{2,00,750}{4,90,740} = 0.85^t$$

$$\Rightarrow 0.4091 = 0.85^t$$

Now try the options.

Question 29 – ICAI SM

A machine worth ₹4,90,740 is depreciated at 15% of its opening value each year. When its value would reduce by 90%?

(a) 11 years 6 months

(b) 11 years 7 months

(c) 11 years 8 months

(d) 14 years 2 months

Solution

(d)

P = ₹4,90,740; A = 10% × ₹4,90,740 = ₹49,074; $i = -0.15$; $t = ?$

$$49,074 = 4,90,740\left(1 + \frac{-0.15}{1}\right)^{t\times 1}$$

$$\frac{49,074}{4,90,740}=(0.85)^t$$

$$(0.85)^t=0.10$$

Now, try the options.

Options (a), (b), (c) are nearly the same. Try with option (d)

Option (d) → 14 years 2 months $=\left(14+\frac{2}{12}\right)$ years

$$(0.85)^{\left(14+\frac{2}{12}\right)}=0.09996\approx 0.10$$

Therefore, option (d) is the answer.

Questions Based on Difference Between Compound Interest and Simple Interest

CA NISHANT KUMAR

Question 30 – June, 2006; MTP June, 2021

The difference between the simple and compound interest on a certain sum for 3 years at 5% p.a. is ₹228.75. The compound interest on the sum for 2 years at 5% p.a. is:

(a) ₹3,175 (b) ₹3,075 (c) ₹3,275 (d) ₹2,975

Solution

(b)

Question 31 – MTP December, 2020

The difference between simple interest and compound interest on a sum of ₹6,00,000 for two years is ₹6,000. What is the annual rate of interest?

(a) 8% (b) 10% (c) 6% (d) 12%

Solution

(b)

$$CI - SI = P\left[\left\{(1+i)^t - 1\right\} - it\right]$$

$$6,000 = 6,00,000\left[\left\{(1+i)^2 - 1\right\} - (i \times 2)\right]$$

$$\frac{6,000}{6,00,000} = \left\{(1+i)^2 - 1\right\} - (i \times 2)$$

$$0.01 = \left\{(1+i)^2 - 1\right\} - (i \times 2)$$

$$\left\{(1+i)^2 - 1\right\} - (i \times 2) = 0.01$$

Try the options.

Option (a) → 8%

$$\left\{(1+0.08)^2 - 1\right\} - (0.08 \times 2) = 0.0064 \neq 0.01$$

Option (b) → 10%

$$\left\{(1+0.10)^2-1\right\}-(0.10\times 2)=0.01$$

Therefore, option (b) is the answer.

Question 32 – July, 2021

What is the difference (in ₹) between the simple interest and the compound interest on a sum of ₹8,000 for $2\frac{2}{5}$ years at the rate of 10% p.a., when the interest is compounded yearly?

(a) 136.12 (b) 129.50 (c) 151.75 (d) 147.20

Solution

(a)

We have P = ₹8,000; $t = \left(2+\frac{2}{5}\right) = 2.4$ years; $i = 0.10$

$$CI - SI = P\left[\left\{(1+i)^{t} - 1\right\} - it\right]$$

$$CI - SI = 8,000\left[\left\{(1+0.10)^{2.4} - 1\right\} - (0.10 \times 2.4)\right] = 136.12$$

Questions Based on Effective Rate of Interest

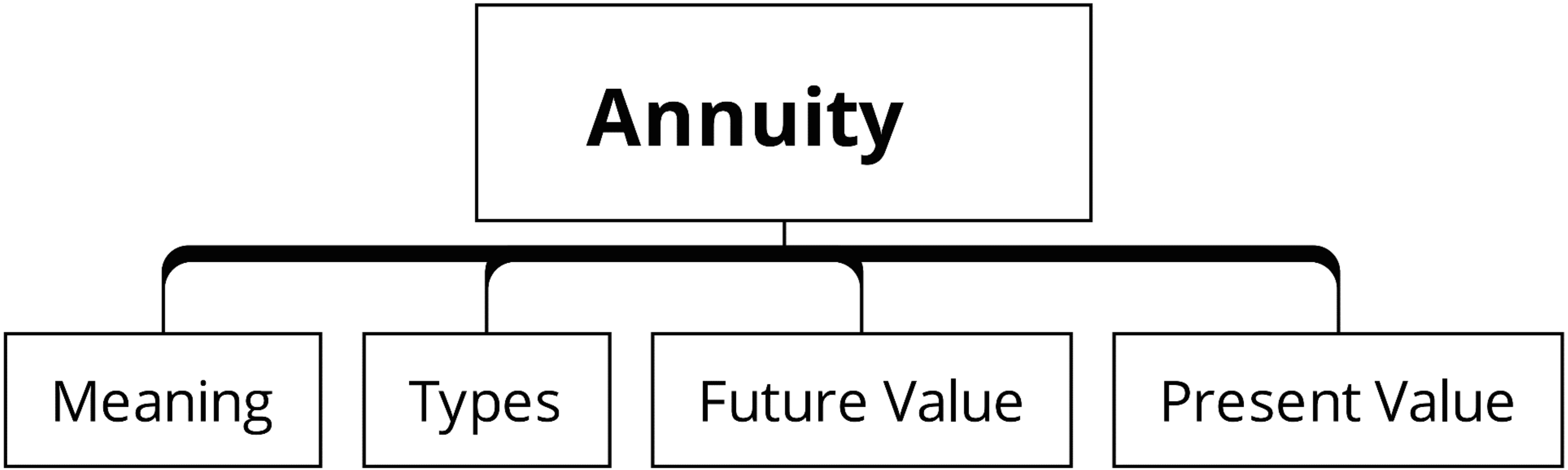
Question 33 – ICAI SM

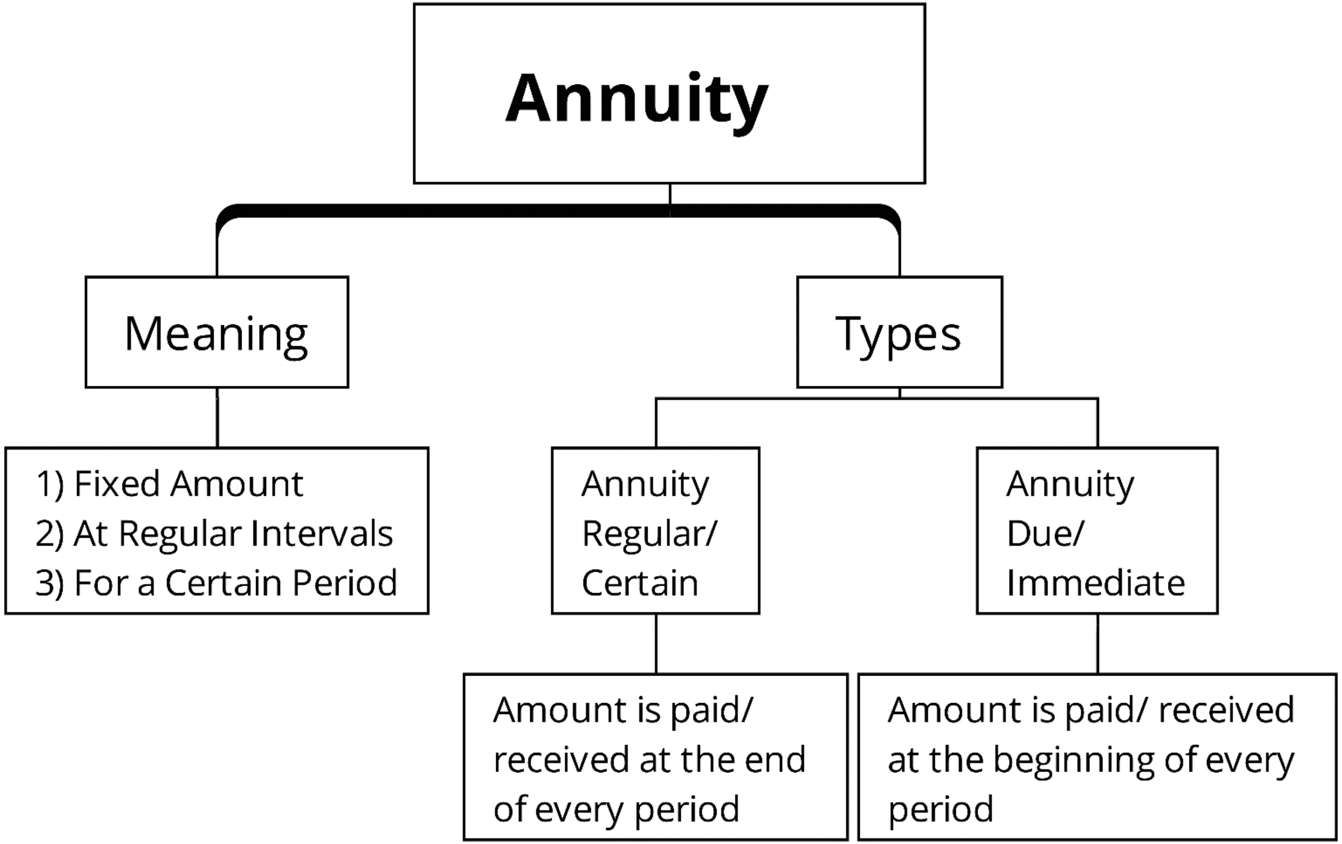
Which is a better investment 3% per year compounded monthly or 3.2% per year simple interest? Given that $(1 + 0.0025)^{12} = 1.0304$.

(a) Compound Interest (b) Simple Interest (c) Don’t Know (d) None

Solution

(b)


Annuity
Meaning
Types
Future Value
Present Value



CA NISHANT KUMAR

Annuity

Future Value

Annuity Regular

Formula

$$FV = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}}\right]$$

Applications

Sinking Fund

The amount required from the Sinking Fund is the Future Value of Annuity

Annuity Due

Formula

$$FV = \left[A\left\{\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}}\right\}\right]\times\left(1+\frac{i}{NOCPPY}\right)$$

Questions Based on Future Value of Annuity

Question 34 – June, 2022

Ankit invests ₹3,000 at the end of each quarter receiving interest @ 7% per annum for 5 years. What amount will be received at the end of the period?

(a) ₹71,200.20 (b) ₹71,104.83 (c) ₹73,204.83 (d) None

Solution

(b)

Given $A = 3{,}000;\ i = 0.07;\ NOCPPY = 4;\ t = 5$

$$FVAR = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}}\right]$$

$$FVAR = 3,000\left[\frac{\left(1+\frac{0.07}{4}\right)^{5\times 4}-1}{\frac{0.07}{4}}\right] = 71,104.83$$

Question 35 – ICAI SM; MTP May, 2019

₹200 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10^{th} payment? Given that $(1.005)^{10} = 1.0511$.

(a) ₹2,047 (b) ₹2,046 (c) ₹2,044 (d) ₹2,045

Solution

(c)

Question 36 – MTP November, 2019

At six months' intervals A deposited ₹1,000 in a savings account which credits interest at 10% p.a., compounded semi-annually. The first deposit was made when A's son was 6 months old and last deposit was made when his son was 8 years old. The money remained in the account and was presented to the son on his 10^{th} birthday. How much did he receive? ($(1.06)^{16} = 2.1829$)

(a) ₹25,740 (b) ₹23,740 (c) ₹25,860 (d) ₹25,760

(b)

This is the question on Future Value of Annuity Regular. If A's son is born today, he deposited ₹1,000 after 6 months. He continued to deposit ₹1,000 at the end of every 6 months for 8 years. Therefore,

$$\text{Annuity}(A) = 1,000;\ t = 8\, years;\ i = 0.10$$

$$\text{No. of Conversion Periods Per Year } (NOCPPY) = 2$$

$$FV = A\left[\frac{\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY} - 1}{\left(\frac{i}{NOCPPY}\right)}\right]$$

$$FV = 1,000\left[\frac{\left(1 + \frac{0.10}{2}\right)^{8 \times 2} - 1}{\left(\frac{0.10}{2}\right)}\right] = 23,657$$

This 23,657 is the amount after 8 years. Now, the option (b) is the closest to it. Therefore, option (b) is the answer.

However, there are plenty of problems with this question.

First of all, if ICAI wants the answer to be exactly ₹23,740, then, the interest rate should be 10.085723883609%.

Secondly, where it is given $(1.06)^{16}$, it is actually $(1.05)^{16}$.

Thirdly, when the amount remained in that savings account for two years (from age 8 to age 10), interest should have been compounded to it after every six months. However, ICAI has ignored this. The money remained in the savings account for two years and no interest accrued on it.

All said and done, if this question comes in the exam, mark the option (b), i.e. ₹23,740.

Question 37 – December, 2022

How much amount is required to be invested every year so as to accumulate ₹5,00,000 at the end of 12 years if interest is compounded annually at 10% {Where A(12, 0.1) = 21.384284}

(a) ₹23,381.65 (b) ₹24,385.85 (c) ₹26,381.65 (d) ₹28,362.75

Solution

(a)

The value of A(12, 0.1) given in the question is incorrect. The correct value should be 21.384283767.

We have $FVAR = 5,00,000;\ t = 12;\ i = 0.10;\ NOCPPY = 1;\ A = ?$

$$FVAR = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}}\right]$$

$$A = \frac{FVAR}{\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}}\right]}$$

$$A = \frac{5,00,000}{\left[\frac{\left(1+\frac{0.10}{1}\right)^{12\times1}-1}{\frac{0.10}{1}}\right]} = \frac{5,00,000}{21.384283767} = 23,381.66$$

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Question 38 – ICAI SM

A machine costs ₹5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of ₹25,000. What amount should be set aside every year if the sinking fund investments accumulate at 3.5% compound interest p.a.?

(a) ₹16,000 (b) ₹16,500 (c) ₹16,050 (d) ₹16,005

Solution

(c)

Question 39 – December, 2022

Sinking fund factor is the reciprocal of:

(a) Present value interest factor of a single cash flow
(b) Present value interest factor of an annuity
(c) Future value interest factor of an annuity
(d) Future value interest factor of a single cash flow.

Solution

(c)

Following is the formula for calculating the future value of annuity regular:

$$FVAR = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}}\right]$$

Here, *A* stands for annuity, i.e., the amount invested at regular intervals.

$\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}}\right]$ is called the future value interest factor of the annuity, because, when it is multiplied by the annuity, we get the future value.

Now, if we are required to calculate the amount to be invested every period to meet a specific goal at the end of certain periods, we'll re-write the formula like this:

$$A=\frac{FVAR}{\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}}\right]}$$

We can also write it like this: $A = FVAR \times \dfrac{1}{\left[\dfrac{\left(1+\dfrac{i}{NOCPPY}\right)^{t \times NOCPPY} - 1}{\dfrac{i}{NOCPPY}}\right]}$

We can see that we are multiplying *FVAR* with $\frac{1}{\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}}\right]}$ to find out the amount required to be invested every period. This $\frac{1}{\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}}\right]}$ is nothing but the sinking fund factor, as it helps you determine what amount is required to be invested in a sinking fund to achieve the desired value in future. As we can see, this is clearly the reciprocal of the future value interest factor of the annuity.

Question 40 – December, 2022

Raju invests ₹20,000 every year in a deposit scheme starting from today for next 12 years. Assuming that interest rate on this deposit is 7% per annum compounded annually. What will be the future value of this annuity? Given that $(1 + 0.07)^{12} = 2.25219150$

(a) ₹5,40,576 (b) ₹3,82,813 (c) ₹6,43,483 (d) ₹3,57,769

Solution

(b)

We have $A = 20,000;\ t = 12;\ i = 0.07;\ NOCPPY = 1;\ FVAD = ?$

$$FVAD = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}}\right]\times\left(1+\frac{i}{NOCPPY}\right)$$

$$FVAD = 20,000\left[\frac{\left(1+\frac{0.07}{1}\right)^{12\times 1}-1}{\frac{0.07}{1}}\right]\times\left(1+\frac{0.07}{1}\right)$$

$$FVAD = 20,000\left[\frac{2.25219150-1}{0.07}\right]\times 1.07 = 3,82,812.83$$

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Question 41 – ICAI SM

Raja aged 40 wishes his wife Rani to have ₹40 lakhs at his death. If his expectation of life is another 30 years and he starts making equal annual investments commencing now at 3% compound interest p.a. how much should he invest annually?

(a) ₹84,448 (b) ₹84,450 (c) ₹84,449 (d) ₹84,080

Solution

(d)

Present Value

Annuity Regular

Formula

$$PV = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}\times\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]$$

Annuity Due

Formula

PV of Annuity Due = Initial Receipt/ Payment + PV of Annuity Regular for (n - 1) periods

Questions Based on Present Value

CA NISHANT KUMAR

Question 42 – ICAI SM

The present value of ₹10,000 due in 2 years at 5% p.a. compound interest when the interest is paid on half-yearly basis is ₹_________.

(a) ₹9,070 (b) ₹9,069 (c) ₹9,061 (d) None

Solution

(c)

Questions Based on Present Value of Annuity Regular

Question 43 – MTP June, 2023

Find the present value of an ordinary annuity of 8 quarterly payments of ₹500 each, the rate of interest being 8% p.a. compound quarterly.

(a) 4275.00 (b) 4725.00 (c) 3662.50 (d) 3266.50

Solution

(c)

We have $A = 500;\ t = 2;\ i = 0.08;\ NOCPPY = 4;\ PVAR = ?$

$$PVAR = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}\times\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]$$

$$=500\left[\frac{\left(1+\frac{0.08}{4}\right)^{2\times 4}-1}{\frac{0.08}{4}\times\left(1+\frac{0.08}{4}\right)^{2\times 4}}\right]=3,662.74$$

Applications of Present Value of Annuity Regular

Loan

PV of all installments is equal to the Loan Amount.

Capital Expenditure

When a project is undertaken, the present value of all the inflows is compared with the present value of outflows. If the PV of inflows is greater than the PV of outflows, the project should be undertaken.

Net Present Value

When a project is undertaken, the present value of all the inflows is compared with the present value of the outflows. If the PV of Inflows is greater than PV of Outflows, the Net Present Value (NPV) is positive, and the project should be undertaken, otherwise, the project should not be undertaken.

Leasing

Leasing means Renting. When you're willing to take a machine on rent, you should compare it with its cost. If the cost is more than the Present Value of all the lease rentals that you're going to pay, then obviously, go for leasing, otherwise go for buying.

When you're willing to give a machine on rent, you should compare it with its selling price. If the selling price is more than the present value of all the lease rentals that you're going to receive, then obviously, go for selling, otherwise go for leasing.

Bond Valuation

Bond is a loan document which provides you interest at the coupon rate. It is redeemed at the end of the period at Par (as per your syllabus). The price of the bond should be either equal to or less than the Present Value of all the inflows.

Questions Based on Applications of Present Value of Annuity Regular

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Question 44 – June, 2022

₹2,500 is paid every year for 10 years to pay off a loan. What is the loan amount if the interest rate is 14% per annum compounded annually?

(a) ₹15,841.90 (b) ₹13,040.27 (c) ₹14,674.21 (d) ₹14,010.90

Solution

(b)

$$PVAR = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}\times\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]$$

$$PVAR = 2,500\left[\frac{\left(1+\frac{0.14}{1}\right)^{10\times1}-1}{\frac{0.14}{1}\times\left(1+\frac{0.14}{1}\right)^{10\times1}}\right]=13,040.29$$

Question 45 – ICAI SM

Appu retires at 60 years receiving a pension of ₹14,400 a year paid in half-yearly installments for rest of his life after reckoning his life expectation to be 13 years and that interest at 4% p.a. is payable half-yearly. What single sum is equivalent to his pension?

(a) ₹1,45,000 (b) ₹1,44,900 (c) ₹1,44,800 (d) ₹1,44,700

Solution

(b)

Question 46 – MTP December, 2021

A took a loan from B. The loan is to be repaid in annual installments of ₹2,000 each. The first instalment is to be paid three years from today and the last one is to be paid 8 years from today? What is the value of loan today, using a discount rate of eight percent?

(a) ₹9,246 (b) ₹7,927 (c) ₹8,567 (d) None

Solution

(b)

The first instalment is to be paid at the end of 3rd year, and the last instalment is to be paid at the end of 8th year. Therefore, total number of instalments = 6.

If we calculate the present value of this annuity regular, we'll get the value at the end of 2nd year.

$$PVAR = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\frac{i}{NOCPPY}\times\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]$$

$$\Rightarrow PVAR = 2,000\left[\frac{\left(1+\frac{0.08}{1}\right)^{6\times 1}-1}{\left(\frac{0.08}{1}\right)\times\left(1+\frac{0.08}{1}\right)^{6\times 1}}\right]=9,246$$

Now, this amount is standing at the end of 2nd year.

Let's calculate the Present Value of this amount now:

$$P = \frac{A}{\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}}$$

$$\Rightarrow P = \frac{9,246}{\left(1 + \frac{0.08}{1}\right)^{2 \times 1}} = 7,927$$

Question 47 – July, 2021

A loan of ₹1,02,000 is to be paid back in two equal annual instalments. If the rate of interest is 4% p.a., compounded annually, then the total interest charged (in ₹) under this instalment plan is:

(a) 6,160 (b) 8,120 (c) 5,980 (d) 7,560

Solution

(a)

We have $PV = 1{,}02{,}000$; $t = 2$ years; $NOCPPY = 1$; $i = 0.04$; $A = ?$

We know that $PV = A\left[\dfrac{\left(1+\dfrac{i}{NOCCPY}\right)^{t\times NOCPPY}-1}{\dfrac{i}{NOCPPY}\times\left(1+\dfrac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]$

Therefore,

$$A=\frac{PV}{\left[\dfrac{\left(1+\dfrac{i}{NOCCPY}\right)^{t\times NOCPPY}-1}{\dfrac{i}{NOCPPY}\times\left(1+\dfrac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]}=\frac{1,02,000}{\left[\dfrac{\left(1+\dfrac{0.04}{1}\right)^{2\times 1}-1}{\dfrac{0.04}{1}\times\left(1+\dfrac{0.04}{1}\right)^{2\times 1}}\right]}=54,080$$

Therefore, total amount paid = 54,080 + 54,080 = 1,08,160

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Interest = 1,08,160 – 1,02,000 = 6,160

Question 48 – ICAI SM; MTP June, 2023

Mr. Paul borrows ₹20,000 on condition to repay it with C.I. at 5% p.a. in annual installments of ₹2,000 each. The number of years for the debt to be paid off is:

(a) 10 years (b) 12 years (c) 11 years (d) 14.2 years

Solution

(d)

Question 49 – ICAI SM

A man purchased a house valued at ₹3,00,000. He paid ₹2,00,000 at the time of purchase and agreed to pay the balance with interest at 12% per annum compounded half yearly in 20 equal half-yearly instalments. If the first instalment is paid after six months from the date of purchase then the amount of each instalment is:

(a) ₹8,718.45 (b) ₹8,769.21 (c) ₹7,893.13 (d) None

(a)

The value of the house at the time of purchase is ₹3,00,000. The man has paid ₹2,00,000 upfront, and ₹1,00,000 is pending. This is the present value of all the instalments that he is going to pay. We need to find out the amount of each instalment. Therefore, we have

PV = ₹1,00,000; i = 0.12; $NOCPPY$ = 2; t = 10 years (since there are 20 half yearly instalments); A = ?

$$PV = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\left(\frac{i}{NOCPPY}\right)\times\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]$$

$$A=\frac{PV}{\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\left(\frac{i}{NOCPPY}\right)\times\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]}$$

$$A = \frac{1,00,000}{\left[\dfrac{\left(1+\dfrac{0.12}{2}\right)^{10\times 2} - 1}{\left(\dfrac{0.12}{2}\right)\times\left(1+\dfrac{0.12}{2}\right)^{10\times 2}}\right]}$$

$$A = \frac{1,00,000}{\dfrac{(1.06)^{20} - 1}{0.06\times(1.06)^{20}}} = 8718.45$$

Question 50 – ICAI SM; MTP May, 2020

A person bought a house paying ₹20,000 cash down and ₹4,000 at the end of each year for 25 yrs. at 5% p.a. C.I. The cash down price is:

[Given $(1.05)^{25} = 3.386355$]

(a) ₹75,000 (b) ₹76,000 (c) ₹76,375.80 (d) None

(c)

Cash Down Price = Down Payment + Present Value of Annual Instalments

$$= ₹20,000 + A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1}{\left(\frac{i}{NOCPPY}\right)\times\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}\right]$$

$$= ₹20,000 + 4,000\left[\frac{\left(1+\frac{0.05}{1}\right)^{25\times 1}-1}{\left(\frac{0.05}{1}\right)\times\left(1+\frac{0.05}{1}\right)^{25\times 1}}\right]$$

$$= ₹20,000 + 4,000\left[\frac{(1.05)^{25}-1}{0.05\times(1.05)^{25}}\right]$$

$$= ₹20{,}000 + 4{,}000\left[\frac{3.386355-1}{0.05\times 3.386355}\right]$$

$$= ₹20{,}000 + ₹56{,}375.778$$

$$= ₹76{,}375.778 \approx ₹76{,}375.80$$

Therefore, option (c) is the answer.

Question 51 – MTP December, 2021

Arun purchased a vacuum cleaner by giving ₹1700 as cash down payment, which will be followed by five EMIs of ₹480 each. The vacuum cleaner can also be bought by paying ₹3900 cash. What is the approx. rate of interest p.a. (at simple interest) under this instalment plan?

(a) 18% (b) 19% (c) 22% (d) 20%

Solution

(c)

Cash Down Price = ₹3,900

Down Payment = ₹1,700

Loan Amount = ₹3,900 – ₹1,700 = ₹2,200

Total amount paid in instalments = ₹480 × 5 = ₹2,400

Therefore, interest paid = ₹2,400 – ₹2,200 = ₹200

Now, P = ₹2,200; t = 5/12 years; A = ₹2,400; i = ?

$$i = \frac{A-P}{Pt} = \frac{2400-2200}{2200\times\frac{5}{12}} = 0.21818 = 21.82\% \approx 22\%$$

Question 52 – MTP June, 2021

A company is considering proposal of purchasing a machine either by making full payment of ₹4,000 or by leasing it for four years at an annual rate of ₹1,250. Which course of action is preferable if the company can borrow money at 14% compounded annually?

(a) Leasing (b) Purchasing (c) Don't Know (d) None

Solution

(a)

Question 53 – June, 2019 (Similar)

ABC Ltd. wants to lease out an asset costing ₹3,60,000 for a five-year period. It has fixed a rental of ₹1,05,000 per annum payable annually starting from the end of first year. Suppose rate of interest is 14% per annum compounded annually on which money can be invested by the company. Is this agreement favourable to the company?

(a) No (b) Yes (c) Don't Know (d) None

Solution

(b)

Question 54 – MTP June, 2023; ICAI SM

A machine with useful life of seven years costs ₹10,000 while another machine with useful life of five years costs ₹8,000. The first machine saves labour expenses of ₹1,900 annually and the second one saves labour expenses of ₹2,200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded per annum.

(a) First Machine (b) Second Machine (c) Don't Know (d) None

Solution

(b)

Question 55 – July, 2021

If the cost of capital be 12% per annum, then the Net Present Value (in nearest ₹) from the given cash flow is given as:

Year	0	1	2	3
Operating Profit (in thousand ₹)	(100)	60	40	50

(a) ₹34,048 (b) ₹34,185 (c) ₹51,048 (d) ₹21,048

Solution

(d)

$$\text{Present Value of Inflows} = \frac{60,000}{\left(1+\frac{0.12}{1}\right)^{1\times1}} + \frac{40,000}{\left(1+\frac{0.12}{1}\right)^{2\times1}} + \frac{50,000}{\left(1+\frac{0.12}{1}\right)^{3\times1}} = 1,21,048$$

Net Present Value = PV of Inflows – PV of Outflows

Net Present Value = ₹1,21,048 – ₹1,00,000 = ₹21,048

Question 56 – ICAI SM

An investor intends purchasing a three-year ₹1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

(a) ₹907.125 (b) ₹800.125 (c) ₹729.12 (d) None

Solution

(a)

Question 57 – MTP June, 2023

Find the purchase price of a ₹1,000 bond redeemable at the paying annual dividends at 4% if the yield rate is to be 5% effective.

(a) ₹884.16 (b) ₹984.17 (c) ₹1,084.16 (d) None

Solution

(b)

We'll assume that the bond is redeemable at par. Also, since time is not given, we'll have to assume that it is a 1-year bond.

The cash flow at the end of 1 year would be the yield from the bond + the face value of the bond.

Yield from the bond $= 0.04 \times 1,000 = 40$

Face Value of the Bond = ₹1,000

Therefore, cash flow at the end of 1 year = ₹1,000 + ₹40 = ₹1,040

$$\text{Present Value} = \frac{A}{\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}}$$

$$= \frac{1,040}{\left(1 + \frac{0.05}{1}\right)^{1 \times 1}} = 990.47$$

Question 58 – ICAI SM

Suppose your mom decides to gift you ₹10,000 every year starting from today for the next five years. You deposit this amount in a bank as and when you receive and get 10% per annum interest rate compounded annually. What is the present value of this annuity?

(a) ₹91,000 (b) ₹79,489 (c) ₹41,698.70 (d) None

Solution

(c)

Perpetuity

Normal Perpetuity

$$\frac{A}{i / NOCPPY}$$

Growing Perpetuity

$$\frac{A}{i - g}$$

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Questions Based on Perpetuity

Question 59 – ICAI SM

Ramesh wants to retire and receive ₹3,000 a month. He wants to pass this monthly payment to future generations after his death. He can earn an interest of 8% compounded annually. How much will he need to set aside to achieve his perpetuity goal?

(a) ₹4,30,000 (b) ₹4,50,000 (c) ₹4,20,000 (d) None

Solution

(b)

Question 60 – July, 2021

If a person bought a house by paying ₹45,00,000 down payment and ₹80,000 at the end of each year till the perpetuity. Assuming the rate of interest as 16% the present value of house (in ₹) is given as:

(a) 47,00,000 (b) 45,00,000 (c) 57,80,000 (d) 50,00,000

Solution

(d)

Value of House = Down Payment + Present Value of Perpetuity

$$\text{Value of House} = 45{,}00{,}000 + \frac{80{,}000}{0.16}$$

Value of House = 45,00,000 + 5,00,000 = 50,00,000

Question 61 – December, 2020

A stock pays annually an amount of ₹10 from 6th year onwards. What is the present value of the perpetuity, if the rate of return is 20%?

(a) 20.1 (b) 19.1 (c) 21.1 (d) 22.1

Solution

(a)

Since the stock starts paying annually from 6th year onwards, if we use the present value of perpetuity formula to find out the present value, it'll give us the value at the 5th year. Think about it logically. In all the questions on perpetuity that we've done so far, the amount was supposed to be received from the end of the first year, and then, when we calculated the present value, it gave us the value at the beginning of the first year. In similar lines, if the stock will start paying the interest from the end of the 6th year, and we use the same formula to calculate the present value, it'll give the present value of only one year before, i.e., at the end of the fifth year.

Let's first calculate that:

$$PV = \frac{A}{i / NOCPPY} = \frac{10}{0.20 / 1} = 50$$

Now, this ₹50 is the amount standing at the end of the 5th year. Since we are required to find out the present value, we need to discount it to the present. Again, think about it logically. This is the amount that is standing at the end of the 5th year. We need to find out the sum that we could invest right now so as to get this 50 at the end of the 5th year. Therefore, this 50 is the amount, and we need to find out the principal.

$$P = \frac{A}{\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}}$$

$$P = \frac{50}{\left(1 + \frac{0.20}{1}\right)^{5 \times 1}} = 20.09$$

Question 62 – June, 2022

Assuming that the discount rate is 7% per annum, how much would you pay to receive ₹50, growing at 5%, annually, forever?

(a) ₹4,300 (b) ₹2,500 (c) ₹4,200 (d) None

Solution

(b)

Miscellaneous Topics

Nominal Rate of Interest

Real Rate + Inflation Rate = Nominal Rate

Compound Annual Growth Rate

Consider the first figure as Principal and the last figure as Amount. Then see how many years have lapsed in between. Find the rate of interest using the formula for Amount that you learned in Compound Interest.

Questions Based on Nominal Rate of Return

Question 63 – ICAI SM

Real Rate of Return = 5%; Inflation Rate = 2%. What is the Nominal Rate of Return?

(a) 7% (b) 9% (c) 11% (d) None

Solution

(a)

Question 64 – July, 2021

The nominal rate of growth is 17% and inflation is 9% for the five years. Let *P* be the Gross Domestic Product (GDP) amount at the present year, then the projected real GDP after 6 years is:

(a) 1.587*P* (b) 1.921*P* (c) 1.403*P* (d) 2.51*P*

Solution

(a)

Nominal Rate = Real Rate + Inflation Rate

17% = Real Rate + 9%

Real Rate = 17% – 9% = 8%

Present GDP = P

GDP after 6 years = $P(1.08)^6 = 1.5869P \approx 1.587P$

Questions Based on Compound Annual Growth Rate (CAGR)

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Question 65 – June, 2022

The CAGR of initial value of an investment of ₹15,000 and final value of ₹25,000 in 3 years is:

(a) 19% (b) 18.56% (c) 17.56% (d) 17%

Solution

(b)

$$A = P\left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY}$$

$$\Rightarrow 25,000 = 15,000\left(1+\frac{i}{1}\right)^{3\times 1}$$

$$\Rightarrow \frac{25,000}{15,000} = \left(1+i\right)^{3}$$

$$\Rightarrow i = \left(\frac{25,000}{15,000}\right)^{\frac{1}{3}} - 1 = 0.1856$$

Question 66 – December, 2022

10 years ago, the earning per share (EPS) of ABC Ltd. was ₹5 share its EPS for this year is ₹22. Compute at what rate, EPS of the company grows annually?

(a) 15.97% (b) 16.77% (c) 18.64% (d) 14.79%

Solution

(a)

EPS stands for Earnings Per Share. We simply need to find at which rate of interest compounded annually, the amount of ₹5 becomes ₹22 in 10 years.

Therefore, we have $P = 5;\ A = 22;\ t = 10;\ NOCPPY = 1;\ i = ?$

$$A = P\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}$$

$$\Rightarrow 22 = 5\left(1+\frac{i}{1}\right)^{10\times 1}$$

$$\Rightarrow \frac{22}{5} = \left(1+i\right)^{10}$$

$$\Rightarrow 4.40 = \left(1+i\right)^{10}$$

Now, try the options.

Option (a) → 15.97%

RHS = $(1+0.1597)^{10} = 4.40 = LHS$

Therefore, option (a) is the answer.

Question 67 – July, 2021

Let the operating profit of a manufacturer for five years is given as:

Years	1	2	3	4	5	6
Operating Profit (in lakh ₹)	90	100	106.4	107.14	120.24	157.34

The Compound Annual Growth Rate (CAGR) of Operating Profit for year 6 with respect to year 2 is:

(a) 9% (b) 12% (c) 11% (d) 13%

Solution

(b)

We need to find out the CAGR with respect to Year 2 as base. Therefore, let the profit of year 2 be *P*. Then the profit of year 6 will be *A*.

We have *P* = 100; *A* = 157.34; *t* = 4 years; *NOCPPY* = 1

$$A = P\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}$$

$$157.34 = 100\left(1+\frac{i}{1}\right)^{4\times 1}$$

Now, let's try the options.

Option (a) → 9%

$$\text{RHS} = 100\left(1+\frac{0.09}{1}\right)^{4\times 1} = 141.16 \neq 157.34$$

Therefore, option (a) cannot be the answer.

Option (b) $\rightarrow$ 12%

$$\text{RHS} = 100\left(1+\frac{0.12}{1}\right)^{4\times 1} = 157.35 = LHS$$

Therefore, option (b) is the answer.

Chapter 5 – Permutations and Combinations

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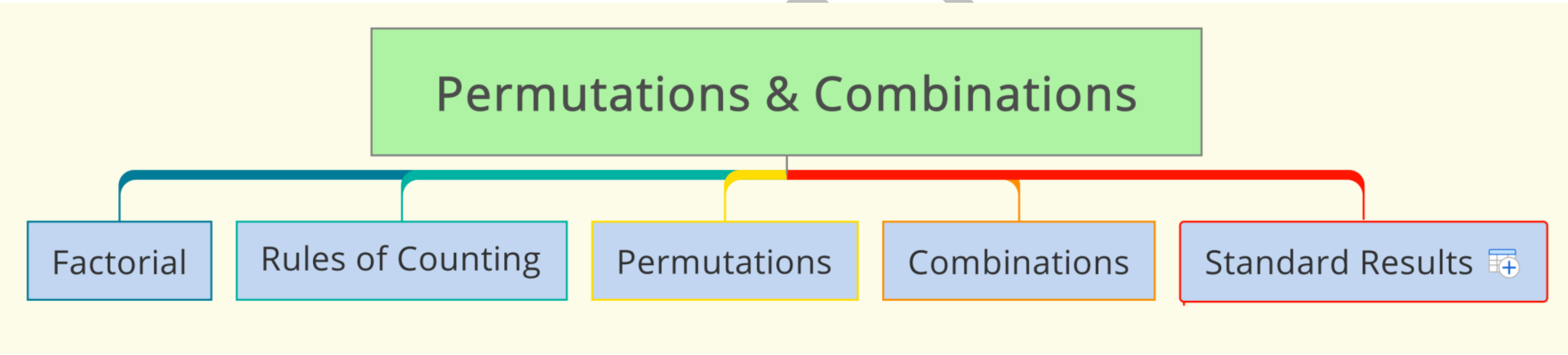
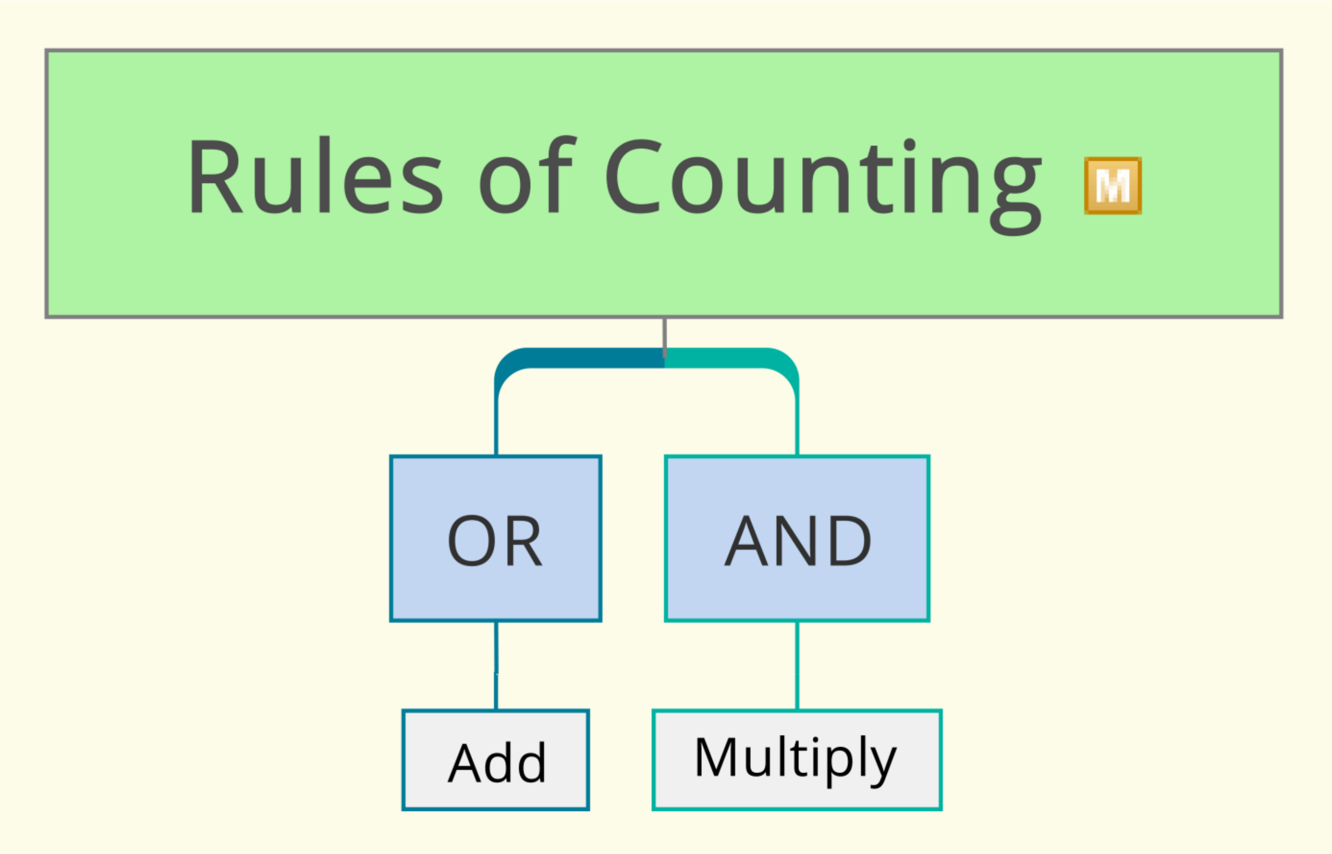
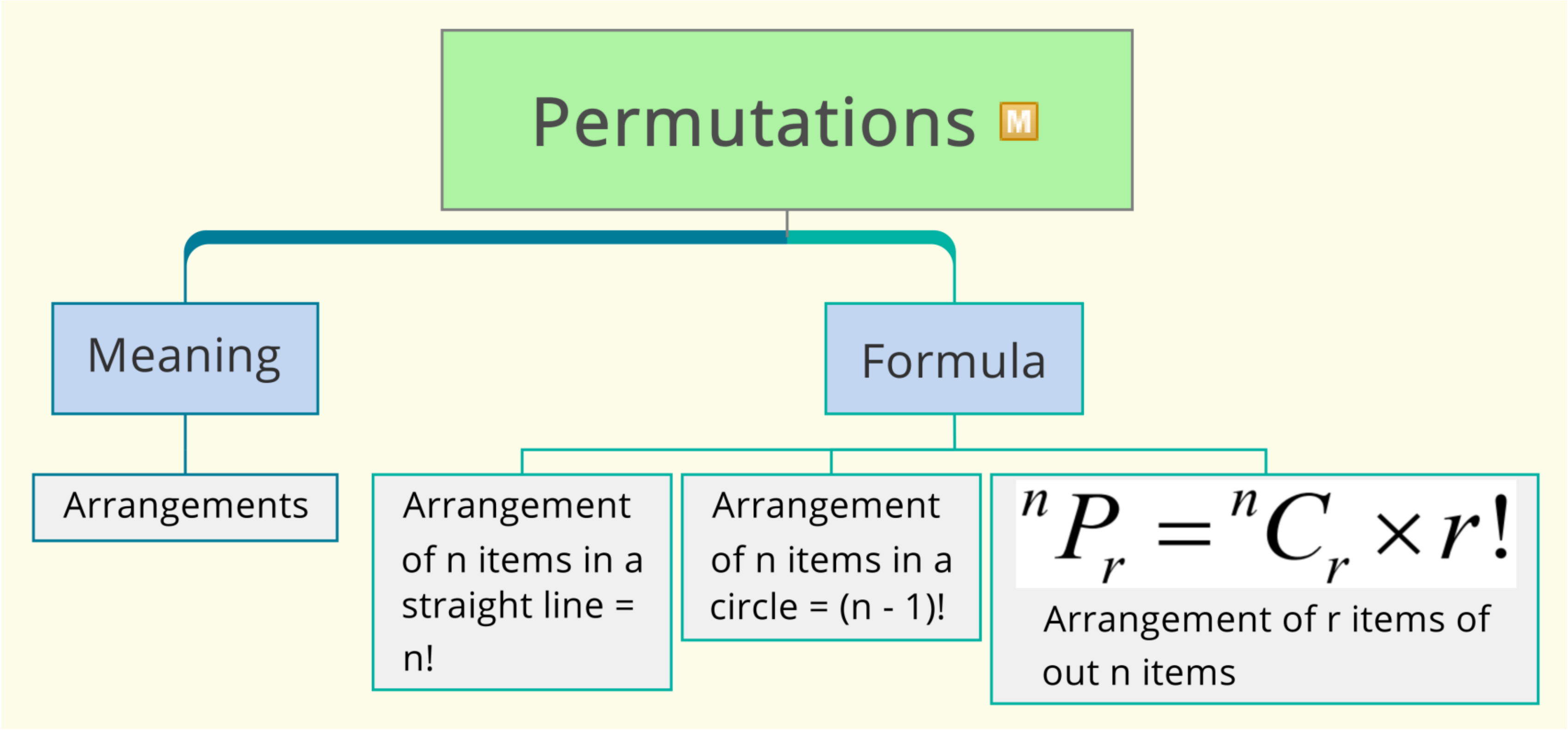

Permutations & Combinations
Factorial
Rules of Counting
Permutations
Combinations
Standard Results

Factorial

$$n! = 1 \times 2 \times 3 \times \ldots \times n$$


Rules of Counting
OR
AND
Add
Multiply



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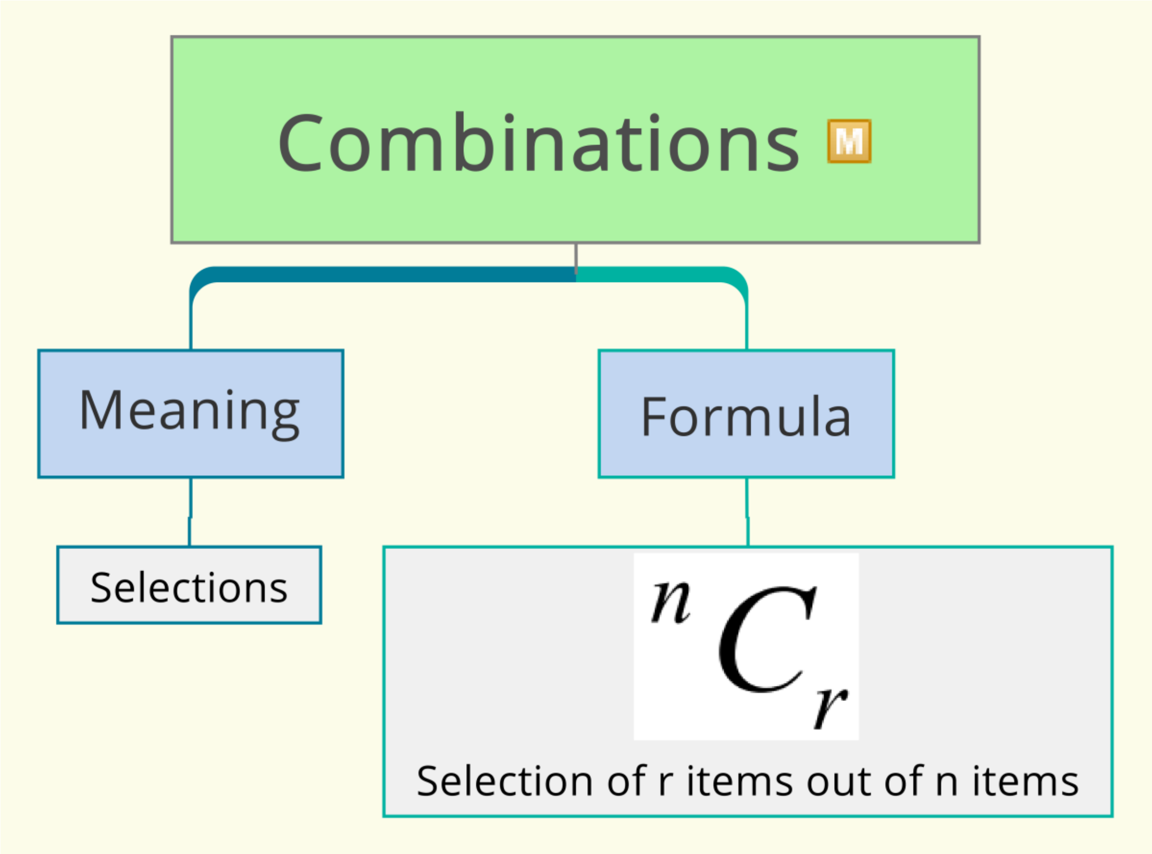

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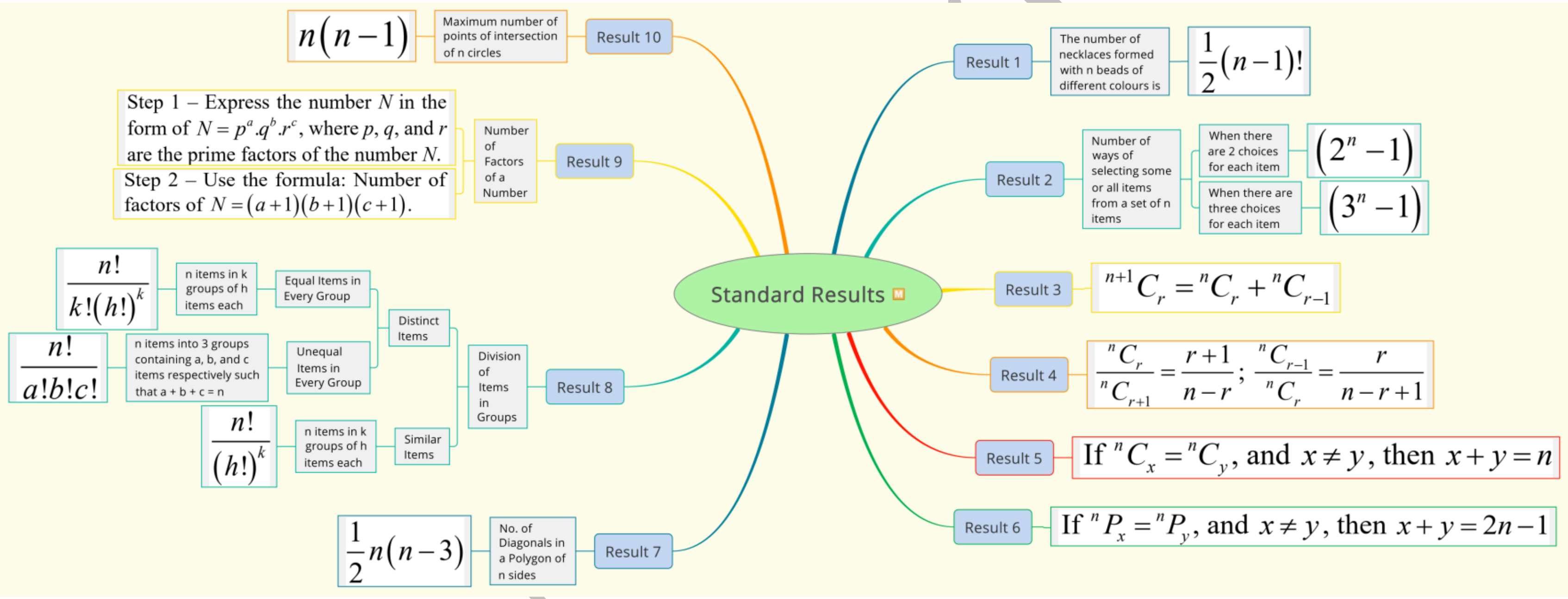

Question 1

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Find x if $1/9! + 1/10! = x/11!$

(a) 119 (b) 120 (c) 121 (d) None

Solution

(c)

Question 2

$(n+1)! - n! = ?$

(a) $n.n!$ (b) $n(n-1)!$ (c) $(n-1)!.n$ (d) None

Solution

(a)

Question 3

The number of ways the letters of the word 'COMPUTER' can be rearranged is:

(a) 40,320 (b) 40,319 (c) 40,318 (d) None

Solution

(b)

Question 4

If ${}^{18}C_r = {}^{18}C_{r+2}$, the value of ${}^{r}C_5$ is:

(a) 55 (b) 50 (c) 56 (d) None

Solution

(c)

Question 5

In ${}^{n}P_{r}$, the restriction is:

(a) $n > r$ (b) $n \geq r$ (c) $n \leq r$ (d) None

Solution

(b)

Question 6

In ${}^{n}P_{r} = n(n-1)(n-2)...(n-r+1)$, the number of factors is:

(a) n (b) $r - 1$ (c) $n - r$ (d) r

Solution

(d)

Question 7

If six times the number permutations of *n* things taken 3 at a time is equal to seven times the number of permutations of (*n* – 1) things taken 3 at a time, find *n*.

(a) 20 (b) 21 (c) 22 (d) None

Solution

(b)

Question 8

In a group of boys the number of arrangement of 4 boys is 12 times the number of arrangements of 2 boys. The number of boys in the group is:

(a) 10 (b) 8 (c) 6 (d) None

Solution

(c)

Question 9

A committee is to be formed of 3 persons out of 12. Find the number of ways of forming such a committee.

(a) 215 (b) 220 (c) 225 (d) None

Solution

(b)

Question 10

How many three letter words can be formed using the letters of the words HEXAGON?

(a) 120 (b) 210 (c) 130 (d) None

Solution

(b)

Question 11

First, second and third prizes are to be awarded at an engineering fair in which 13 exhibits have been entered. In how many different ways can the prizes be awarded?

(a) 1,716 (b) 1,720 (c) 1,270 (d) None

Solution

(a)

Question 12

In how many different ways can 3 students be associated with 4 chartered accountants, assuming that each chartered accountant can take at most one student?

(a) 24 (b) 25 (c) 30 (d) None

Solution

(a)

Question 13

When Dr. Ram arrives in his dispensary, he finds 12 patients waiting to see him. If he can see only one patient at a time, find the number of ways, he can schedule his patients (a) if they all want their turn, and (b) if 3 leave in disgust before Dr. Ram gets around to seeing them.

(a) ${}^{12}P_{12}$, ${}^{12}P_9$ (b) 12!, ${}^{12}P_9$ (c) Both (d) None

Solution

(c)

Question 14

Mr. X and Mr. Y enter into a railway compartment having six vacant seats. The number of ways in which they can occupy the seats is:

(a) 25 (b) 31 (c) 32 (d) 30

Solution

(d)

Question 15

The number of arrangements of the letters in the word 'FAILURE', so that vowels are always coming together is:

(a) 576 (b) 575 (c) 570 (d) None

Solution

(a)

Question 16

10 examination papers are arranged in such a way that the best and worst papers never come together. The number of arrangements is:

(a) $9\underline{\lvert 8}$ (b) $\underline{\lvert 10}$ (c) $8\underline{\lvert 9}$ (d) None

Solution

(c)

Question 17

n articles are arranged in such a way that 2 particular articles never come together. The number of such arrangements is:

(a) $(n-2)\lfloor\underline{n-1}$ (b) $(n-1)\lfloor\underline{n-2}$ (c) $\lfloor\underline{n}$ (d) None

Solution

(a)

Question 18

The number of 4-digit numbers greater than 5,000 can be formed out of the digits 3, 4, 5, 6 and 7 (No digit is repeated). The number of such is:

(a) 72 (b) 27 (c) 70 (d) None

Solution

(a)

Question 19

The number of ways the letters of the word ‘TRIANGLE’ to be arranged so that the word ‘angle’ will be always present is:

(a) 20 (b) 60 (c) 24 (d) 32

Solution

(c)

Question 20

If the letters word 'DAUGHTER' are to be arranged so that vowels occupy the odd places, then number of different words are:

(a) 2,880 (b) 676 (c) 625 (d) 576

Solution

(a)

Question 21

How many arrangements can be made out of the letters of the word 'DRAUGHT', the vowels never beings separated?

(a) 1,440 (b) 676 (c) 625 (d) 576

Solution

(a)

Question 22

There are 6 books on Economics, 3 on Mathematics and 2 on Accountancy. In how many ways can these be placed on a shelf if the books on the same subject are to be together?

(a) 1,440 (b) 51,840 (c) 52,740 (d) None

Solution

(b)

Question 23

A family of 4 brothers and three sisters is to be arranged for a photograph in one row. In how many ways can they be seated if (i) all the sisters sit together, (ii) no two sisters sit together?

(a) 720; 1,440 (b) 1,440; 720 (c) 840; 720 (d) None

Solution

(a)

Question 24

Six boys and five girls are to be seated for a photograph in a row such that no two girls sit together and no two boys sit together. Find the number of ways in which this can be done.

(a) 72,000 (b) 14,440 (c) 86,400 (d) None

Solution

(c)

Question 25

The number of arrangements of 10 different things taken 4 at a time in which one particular thing always occurs is:

(a) 2015 (b) 2016 (c) 2014 (d) None

Solution

(b)

Question 26

The number of permutations of 10 different things taken 4 at a time in which one particular thing never occurs is:

(a) 3,020 (b) 3,025 (c) 3,024 (d) None

Solution

(c)

Question 27

The number of numbers lying between 10 and 1000 can be formed with the digits 2, 3, 4, 0, 8, 9 is:

(a) 124 (b) 120 (c) 125 (d) None

Solution

(c)

Question 28

The total number of 9 digit numbers of different digits is:

(a) $10\underline{|9}$ (b) $8\underline{|9}$ (c) $9\underline{|9}$ (d) None

Solution

(c)

Question 29

There are 5 speakers A, B, C, D and E. The number of ways in which A will speak always before B is:

(a) 24 (b) $\lfloor\underline{4} \times \lfloor\underline{2}$ (c) $\lfloor\underline{5}$ (d) None

Solution

(a)

Question 30

There are 10 trains plying between Calcutta and Delhi. The number of ways in which a person can go from Calcutta to Delhi and return by a different train is:

(a) 99 (b) 90 (c) 80 (d) None

Solution

(b)

Question 31

Number ways of painting of a face of a cube by 6 colours is:

(a) 36 (b) 6 (c) 24 (d) 20

Solution

(b)

Question 32

How many six-digit telephone numbers can be formed by using 10 distinct digits?

(a) 10^8 (b) 6^{10} (c) ${}^{10}C_9$ (d) ${}^{10}P_6$

Solution

(d)

Question 33

The number of ways in which 8 sweats of different sizes can be distributed among 8 persons of different ages so that the largest sweat always goes to be younger assuming that each one of then gets a sweat is:

(a) $\underline{|8}$ (b) 5040 (c) 5039 (d) None

Solution

(b)

Question 34

The number of arrangements in which the letters of the word 'MONDAY' be arranged so that the words thus formed begin with M and do not end with N is:

(a) 720 (b) 120 (c) 96 (d) None

Solution

(c)

Question 35

How many numbers of seven-digit numbers which can be formed from the digits 3, 4, 5, 6, 7, 8, 9, no digit being repeated are not divisible by 5?

(a) 4320 (b) 4690 (c) 3900 (d) 3890

Solution

(a)

Question 36

Eight guests have to be seated 4 on each side of a long rectangular table. 2 particular guests desire to sit on one side of the table and 3 on the other side. The number of ways in which the sitting arrangements can be made is:

(a) 1732 (b) 1728 (c) 1730 (d) 1278

Solution

(b)

Question 37

The number of even numbers greater than 300 can be formed with the digits 1, 2, 3, 4, 5 without repetition is:

(a) 110 (b) 112 (c) 111 (d) None

Solution

(c)

Question 38

The letters of the words 'CALCUTTA' and 'AMERICA' are arranged in all possible ways. The ratio of the number of these arrangements is:

(a) 1 : 2 (b) 2 : 1 (c) 2 : 2 (d) None

Solution

(b)

Question 39

The number of words that can be made by rearranging the letters of the word APURNA so that vowels and consonants appear alternate is:

(a) 18 (b) 35 (c) 36 (d) None

Solution

(c)

Question 40

Every two persons shakes hands with each other in a party and the total number of handshakes is 66. The number of guests in the party is:

(a) 11 (b) 12 (c) 13 (d) 14

Solution

(b)

Question 41

There are 12 points in a plane of which 5 are collinear. The number of triangles is:

(a) 200 (b) 211 (c) 210 (d) None

Solution

(c)

Question 42

The total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two '–' signs occur together is:

(a) $\underline{|7} / \underline{|3}$ (b) $\underline{|6} \times \underline{|7} / \underline{|3}$ (c) 35 (d) None

Solution

(c)

Question 43

A person has 12 friends of whom 8 are relatives. In how many ways can he invite 7 guests such that 5 of them are relatives?

(a) 420 (b) 446 (c) 336 (d) None

Solution

(c)

Question 44

The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is:

(a) 6 (b) 18 (c) 12 (d) 9

Solution

(b)

Question 45

The Supreme Court has given a 6 to 3 decision upholding a lower court; the number of ways it can give a majority decision reversing the lower court is:

(a) 256 (b) 276 (c) 245 (d) 226

Solution

(a)

Question 46

Five bulbs of which three are defective are to be tried in two bulb points in a dark room. Number of trials the room shall be lighted is:

(a) 6 (b) 8 (c) 5 (d) 7

Solution

(d)

Question 47

The number of different words that can be formed with 12 consonants and 5 vowels by taking 4 consonants and 3 vowels in each word is:

(a) ${}^{12}C_4 \times {}^{5}C_3$ (b) ${}^{17}C_7$ (c) $4950 \times 7!$ (d) None

Solution

(c)

Question 48

Out of 7 gents and 4 ladies a committee of 5 is to be formed. The number of committees such that each committee includes at least one lady is:

(a) 400 (b) 440 (c) 441 (d) None

Solution

(c)

Question 49

There are 7 men and 3 ladies. Find the number of ways in which a committee of 6 can be formed of them if the committee is to include atleast two ladies?

(a) 420 (b) 140 (c) 168 (d) None

Solution

(b)

Question 50

A committee of 7 members is to be chosen from 6 Chartered Accountants, 4 Economists and 5 Cost Accountants. In how many ways can this be done if in the committee, there must be at least one member from each group and at least 3 Chartered Accountants?

(a) 3,570 (b) 3,750 (c) 7,350 (d) None

Solution

(a)

Question 51

A committee of 3 ladies and 4 gents is to be formed out of 8 ladies and 7 gents. Mrs. X refuses to serve in a committee in which Mr. Y is a member. The number of such committees is:

(a) 1530 (b) 1500 (c) 1520 (d) 1540

Solution

(d)

Question 52

A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics Part II unless Mathematics Part I is also borrowed. In how many ways can he choose the three books to be borrowed?

(a) 41 (b) 51 (c) 61 (d) 71

Solution

(a)

Question 53

The ways of selecting 4 letters from the word ‘EXAMINATION’ is

(a) 136 (b) 130 (c) 125 (d) None

Solution

(a)

Question 54

The number of 4-digit numbers formed with the digits 1, 1, 2, 2, 3, 4 is:

(a) 100 (b) 101 (c) 201 (d) None

Solution

(d)

Question 55

In how many ways can 8 persons be seated at a round table? In how many cases will 2 particular persons sit together?

(a) 5,040; 1,440 (b) 5,040; 720 (c) 5,040; 120 (d) None

Solution

(a)

Question 56

The number of ways in which 7 boys sit in a round table so that two particular boys may sit together is:

(a) 240 (b) 200 (c) 120 (d) None

Solution

(a)

Question 57

3 ladies and 3 gents can be seated at a round table so that any two and only two of the ladies sit together. The number of ways is:

(a) 70 (b) 27 (c) 72 (d) None

Solution

(c)

Question 58

5 persons are sitting in a round table in such way that Tallest Person is always on the right–side of the shortest person; the number of such arrangements is:

(a) 6 (b) 8 (c) 24 (d) None

Solution

(a)

Question 59

If 15 persons are to be seated around 2 round tables, one occupying 8 persons and another 7 persons. Find the number of ways in which they can be seated.

(a) $\frac{15!}{18!}$

(b) ${}^{15}C_7 \frac{7!}{8!}$

(c) 7!.8!

(d) $2.{}^{15}C_7\ 6!\ 7!$

Solution

(d)

Question 60

The sum of all 4 digit number containing the digits 1, 3, 5, 7, without repetitions is:

(a) 1,49,550 (b) 1,06,656 (c) 1,07,750 (d) 1,06,556

Solution

(b)

Question 61

The results of 8 matches (Win, Loss or Draw) are to be predicted. The number of different forecasts containing exactly 6 correct results is:

(a) 316 (b) 214 (c) 112 (d) None

Solution

(c)

Question 62

Eight chairs are numbered from 1 to 8. Two women and three men are to be seated by allowing one chair for each. First, the women choose the chairs from the chairs numbered 1 to 4 and then men select the chairs from the remaining. The number of possible arrangements is:

(a) 120 (b) 288 (c) 32 (d) 1440

Solution

(d)

Question 63

n locks and n corresponding keys are available but the actual combination is not known. The maximum number of trials that are needed to assigns the keys to the corresponding locks is:

(a) ${}^{(n-1)}C_2$ (b) ${}^{(n+1)}C_2$ (c) $\sum_{k=2}^{n}(k-1)$ (d) $\sum_{k=2}^{n}k$

Solution

(c)

Some Standard Results

1. The number of necklaces formed with n beads of different colours is $\frac{1}{2}(n-1)!$.
2. Number of ways of selecting some or all items from a set of n items –
   a. When there are 2 choices for each item: $\left(2^n-1\right)$.
   b. When there are 3 choices for each item: $\left(3^n-1\right)$.
3. $^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$
4. $\frac{^nC_r}{^nC_{r+1}} = \frac{r+1}{n-r}$; $\frac{^nC_{r-1}}{^nC_r} = \frac{r}{n-r+1}$
5. If $^nC_x = {}^nC_y$, and $x \neq y$, then $x+y=n$.
6. If $^nP_x = {}^nP_y$, and $x \neq y$, then $x+y=2n-1$.

7. The number of diagonals in a polygon of n sides is $\frac{1}{2}n(n-3)$.

8. Division of Items in Groups –

a. Division of Distinct Items in Groups –

i. Equal items in every group – The number of ways to divide n students into k groups of h students each is given by $\frac{n!}{k!(h!)^k}$.

ii. Unequal items in every group – The number of ways to divide n items into 3 groups → one containing a items, the second containing b items, and the third containing c items, such that $a+b+c=n$, is given by $\frac{n!}{a!b!c!}$.

b. Division of Identical Items in Groups – The number of ways to divide n identical objects into k groups of h items each is given by $\frac{n!}{(h!)^k}$.

9. Number of Factors of a number – Factors of a number N refers to all the numbers which divide N completely.

   Step 1 – Express the number N in the form of $N = p^a.q^b.r^c$, where p, q, and r are the prime factors of the number N.

   Step 2 – Use the formula: Number of factors of $N = (a+1)(b+1)(c+1)$.

10. The maximum number of points of intersection of n circles will be $n(n-1)$.

Question 64

The number of ways in which 8 different beads be strung on a necklace is:

(a) 2500 (b) 2520 (c) 2250 (d) None

Solution

(b)

Question 65

An examination paper with 10 questions consists of 6 questions in Algebra and 4 questions in Geometry. At least one question from each section is to be attempted. In how many ways can this be done?

(a) $\left(2^6-1\right)\left(2^4-1\right)$ (b) $\left(2^6-1\right)$ (c) $\left(2^4-1\right)$ (d) $\left(2^{10}-1\right)$

Solution

(a)

There are two choices for each question in Algebra – either to attempt, or, to not attempt. Therefore, $\left(2^6-1\right)$. Further, there are two choices for each question in Geometry – either to attempt, or to not attempt. Therefore, $\left(2^4 - 1\right)$. So, the total number of ways = $\left(2^6 - 1\right)\left(2^4 - 1\right)$.

Question 66

There are 12 questions to be answered in Yes or No. How many ways can these be answered?

(a) 1024 (b) 2048 (c) 4096 (d) None

Solution

(c)

Every question can be answered in 2 ways, i.e., Yes, or No.

Therefore, all the 12 questions can be answered in $2^{12} = 4096$ ways.

Question 67

In an examination, a candidate has to pass in each of the 4 papers. In how many different ways can be failed?

(a) 14 (b) 16 (c) 15 (d) None

Solution

(c)

The candidate would be failed if he fails in one or more papers.

No. of ways of selecting one or more items from *n* items is given by $2^n - 1$.

Therefore, no. of ways he can be failed $= 2^4 - 1 = 16 - 1 = 15$

Question 68

In an election the number of candidates is one more than the number of members to be elected. If a voter can vote in 254 different ways; find the number of candidates.

(a) 8 (b) 10 (c) 7 (d) None

Solution

(a)

In an election the number of candidates is one more than the number of members to be elected. This means that if, suppose the total number of candidates is 11, then only 10 are to be selected. In other words, if, suppose the total number of candidates is n, then the number of candidates to be voted for are $n - 1$.

Given that a voter can vote in 254 different ways, it is clear that the voter can vote for one or more members.

Now, number of ways of selecting one or more items from a set of n items = $2^n - 1$. However, this would also consider the one extra candidate which cannot be voted for. Therefore, we need to subtract that one extra candidate as well.

$\Rightarrow 2^n - 1 - 1$

$= 2^n - 2$

Given that a voter can vote in 254 different ways.

$\Rightarrow 2^n - 2 = 254$

$\Rightarrow 2^n = 254 + 2$

$$\Rightarrow 2^n = 256$$

$$\Rightarrow 2^n = 2^8$$

$$\Rightarrow n = 8$$

Question 69

$^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 + {}^{n}C_4 + \ldots + {}^{n}C_n$ equals

(a) $2^n - 1$ (b) 2^n (c) $2^n + 1$ (d) None

Solution

(a)

Question 70

A question paper contains 6 questions, each having an alternative. In how many ways can an examinee answer one or more questions?

(a) 720 (b) 728 (c) 729 (d) None

Solution

(b)

There are three choices for each question – either to attempt alternative 1, or, to attempt alternative 2, or, to not attempt the question. Therefore, $3^n - 1 = 3^6 - 1 = 728$ ways.

Question 71

Find x if ${}^{12}C_5 + 2\,{}^{12}C_4 + {}^{12}C_3 = {}^{14}C_x$

(a) 5 (b) 6 (c) 7 (d) 8

Solution

(a)

$${}^{12}C_5 + 2\,{}^{12}C_4 + {}^{12}C_3$$

$${}^{12}C_5 + {}^{12}C_4 + {}^{12}C_4 + {}^{12}C_3$$

$$\left({}^{12}C_5 + {}^{12}C_4\right) + \left({}^{12}C_4 + {}^{12}C_3\right)$$

$$\left({}^{12+1}C_5\right)+\left({}^{12+1}C_4\right)$$

$${}^{13}C_5 + {}^{13}C_4$$

$${}^{13+1}C_5$$

$${}^{14}C_5$$

Question 72

If ${}^{n}C_{r-1}=56,\ {}^{n}C_{r}=28$, and ${}^{n}C_{r+1}=8$, find the value of r.

(a) 8 (b) 6 (c) 5 (d) None

Solution

(b)

$$\frac{{}^{n}C_{r}}{{}^{n}C_{r+1}}=\frac{r+1}{n-r}\Rightarrow\frac{r+1}{n-r}=\frac{28}{8}\Rightarrow\frac{r+1}{n-r}=\frac{7}{2}\Rightarrow 7n-9r=2\ldots\text{ Eq. (1)}$$

Now, $\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}}=\frac{r}{n-r+1}\Rightarrow\frac{56}{28}=\frac{r}{n-r+1}\Rightarrow\frac{r}{n-r+1}=2\Rightarrow 2n-3r=-2\ldots\text{Eq. (2)}$

Solving both the equations, $r = 6$.

Question 73

If ${}^{n}C_{10} = {}^{n}C_{14}$, then what is the value of ${}^{25}C_{n}$?

(a) 24 (b) 25 (c) 1 (d) None

Solution

(b)

We know that if ${}^{n}C_{x} = {}^{n}C_{y}$, and $x \neq y$, then $x + y = n$. Therefore, n = 10 + 14 = 24.

$\Rightarrow {}^{25}C_{24} = 25$

Question 74

If ${}^nP_r = {}^nP_{r+1}$, and ${}^nC_r = {}^nC_{r-1}$, then find the value of n.

(a) 3 (b) 4 (c) 5 (d) 6

Solution

(a)

We know that if ${}^nP_x = {}^nP_y$, and $x \neq y$, then $x + y = 2n - 1$. Therefore, $r + r + 1 = 2n - 1$.

$$\Rightarrow 2r + 1 = 2n - 1 \Rightarrow 2r = 2n - 2 \Rightarrow r = \frac{2n-2}{2} \Rightarrow r = n - 1.$$

Also, we know that if ${}^nC_x = {}^nC_y$, and $x \neq y$, then $x + y = n$. Therefore, $r + r - 1 = n$.

$$\Rightarrow 2r-1=n\Rightarrow 2r=n+1\Rightarrow r=\frac{n+1}{2}.$$

Therefore, $n-1=\frac{n+1}{2}\Rightarrow 2n-2=n+1\Rightarrow 2n-n=1+2\Rightarrow n=3.$

Question 75

The number of diagonals in a decagon is:

(a) 30 (b) 35 (c) 45 (d) None

Solution

(b)

The number of diagonals in a polygon of n sides is $\frac{1}{2}n(n-3)$.

$$\frac{1}{2}n(n-3) = \frac{1}{2}\times 10\times (10-3) = 35$$

Question 76

If a polygon has 44 diagonals, then the number of sides are:

(a) 6 (b) 7 (c) 8 (d) 11

Solution

(d)

The number of diagonals in a polygon of n sides is $\frac{1}{2}n(n-3)$.

$$\frac{1}{2}n(n-3)=44$$

$$n(n-3) = 44 \times 2 = 88$$

$$n(n-3) = 88$$

Try the options.

Option (d) $\rightarrow$ 11

Question 77

In how many number of ways can 12 students be equally divided into three groups?

(a) 5775 (b) 7575 (c) 7755 (d) None

Solution

(a)

The number of ways to divide *n* students into *k* groups of *h* students each is given by $\frac{n!}{k!(h!)^k}$.

We have, $n = 12$; $k = 3$; $h = 4$.

Therefore, $\frac{n!}{k!(h!)^k} = \frac{12!}{3!(4!)^3} = 5,775.$

Question 78

The number of ways in which 9 things can be divided into twice groups containing 2, 3, and 4 things respectively is:

(a) 1250 (b) 1260 (c) 1200 (d) None

Solution

(b)

The number of ways to divide *n* items into 3 groups → one containing *a* items, the second containing *b* items, and the third containing *c* items, such that $a+b+c=n$, is given by $\frac{n!}{a!b!c!}$.

Here, $n = 9$; $a = 2$; $b = 3$; $c = 4$

$$\frac{n!}{a!b!c!} = \frac{9!}{2! \times 3! \times 4!} = 1,260$$

Question 79

In how many number of ways can 15 mangoes be equally divided among 3 students?

(a) $\underline{|15} / \underline{|(5)^4}$ (b) $\underline{|15} / \underline{|(5)^3}$ (c) $\underline{|15} / \underline{|(5)^2}$ (d) None

Solution

(b)

The number of ways to divide *n* identical objects into *k* groups of *h* items each is given by $\frac{n!}{(h!)^k}$.

We have, $n = 15$; $k = 3$; $h = 5$.

Therefore, $\frac{n!}{(h!)^k} = \frac{15!}{(5!)^3} = 7,56,756$.

Question 80

The number of different factors of the number 75,600 has is:

(a) 120 (b) 121 (c) 119 (d) None

Solution

(c)

$75,600 = 2^4.3^3.5^2.7$

Therefore, total number of factors of 75,600 $= (4+1)(3+1)(2+1)(1+1) = 120$

However, the question has asked us the ***different*** factors of the number 75,600. Since one of the factors is the number itself, the different factors would be determined by subtracting 1 from the total number of factors.

Therefore, no. of different factors of the number 75,600 = 120 – 1 = 119.

Question 81

The maximum number of points of intersection of 10 circles will be ________.

(a) 90 (b) 100 (c) 110 (d) 120

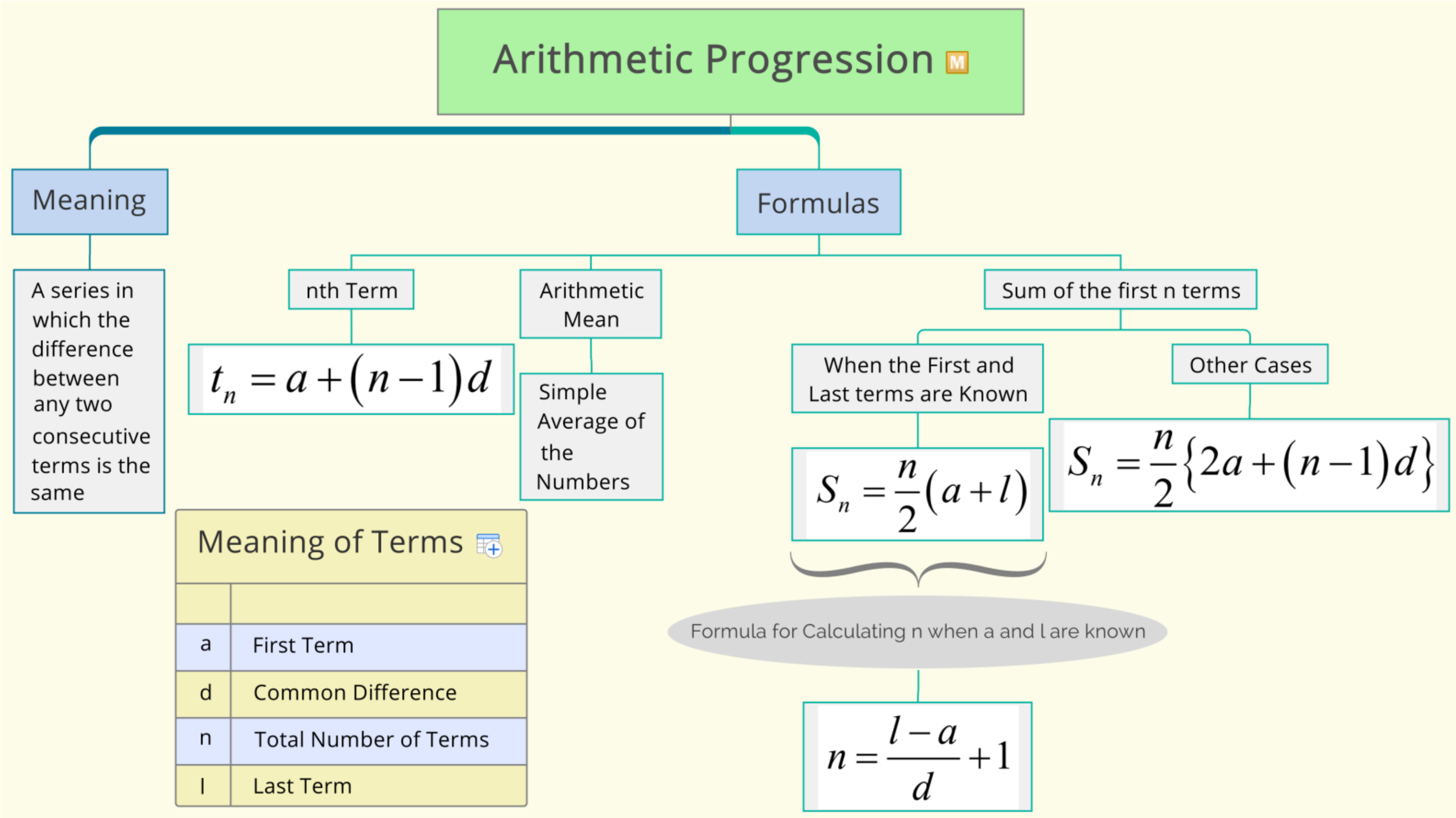
Solution

The maximum number of points of intersection of n circles will be $n(n-1)$.

Therefore, $10(10-1) = 10 \times 9 = 90$.

Chapter 6 – Sequence and Series

CA NISHANT KUMAR



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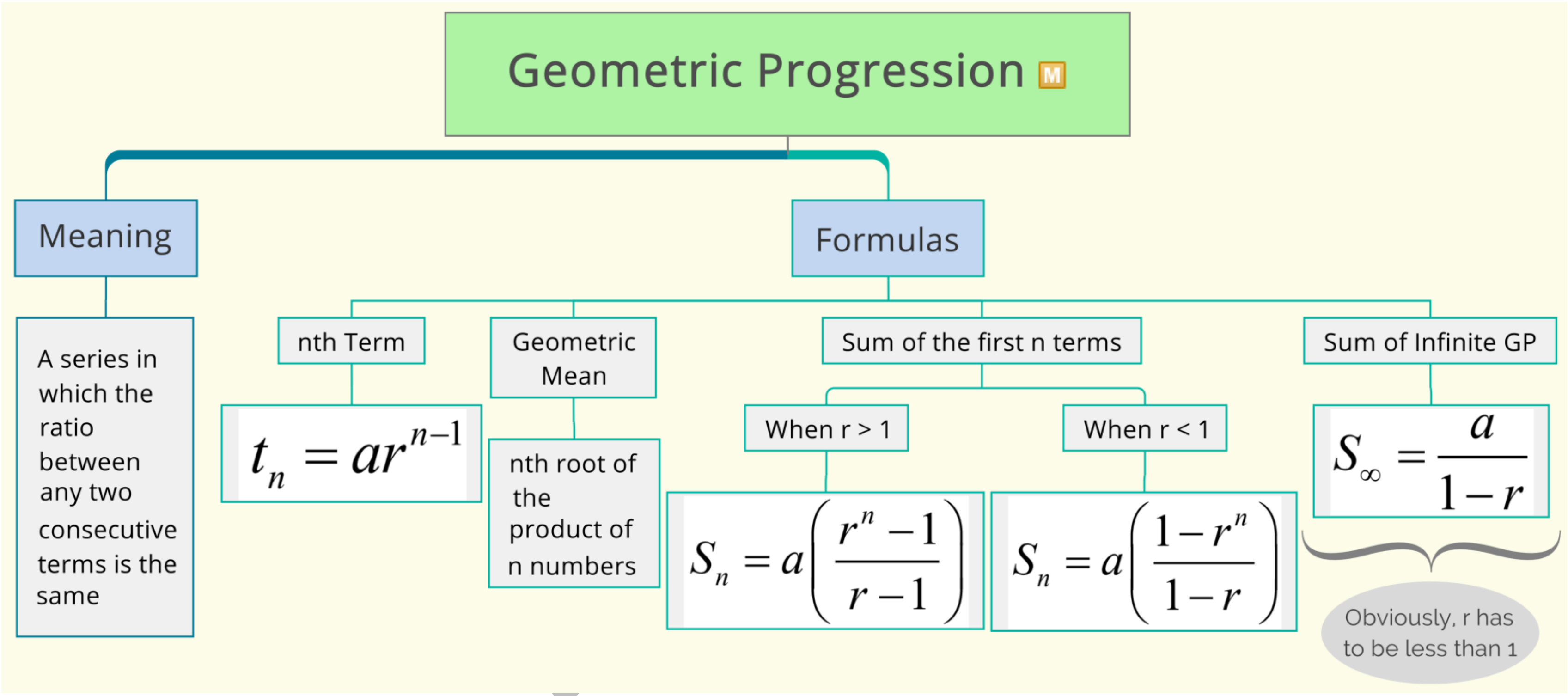

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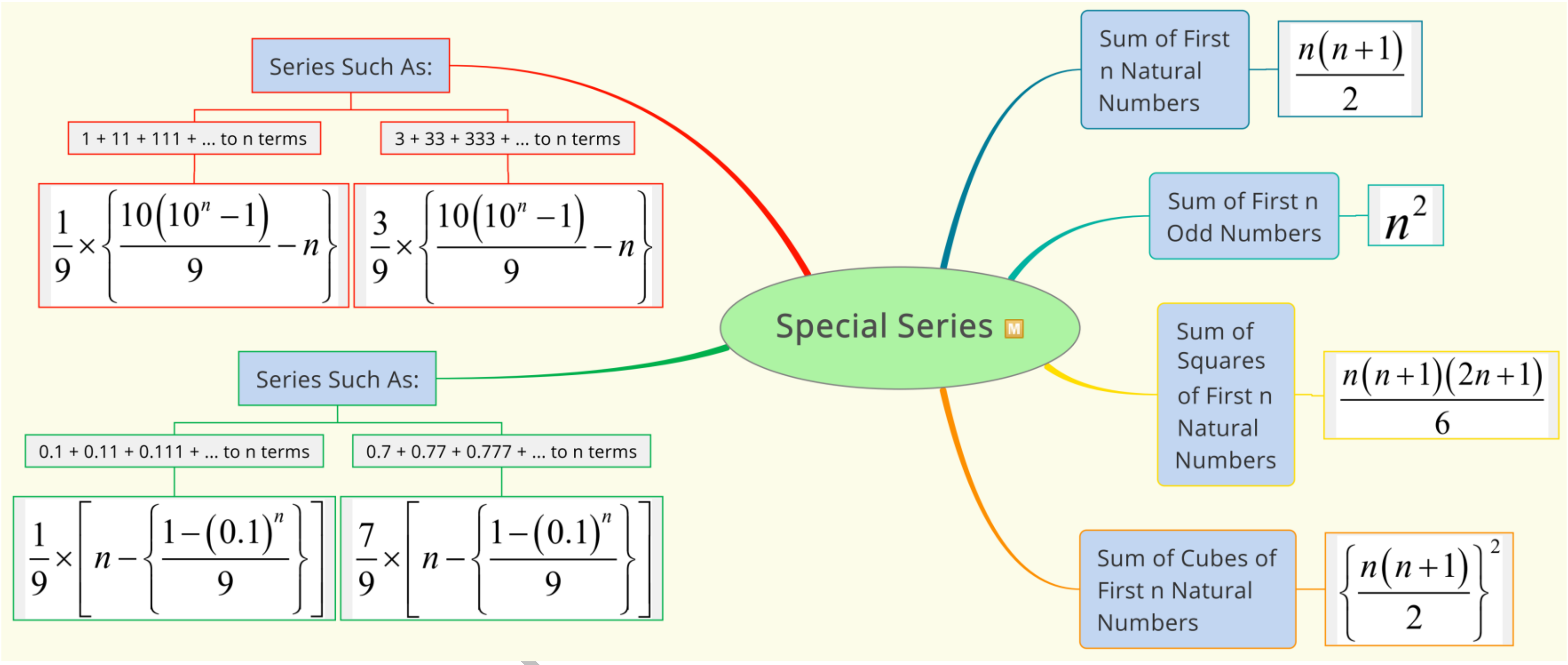
Special Series
Series Such As:
1 + 11 + 111 + ... to n terms
$\frac{1}{9}\times\left\{\frac{10\left(10^{n}-1\right)}{9}-n\right\}$
3 + 33 + 333 + ... to n terms
$\frac{3}{9}\times\left\{\frac{10\left(10^{n}-1\right)}{9}-n\right\}$
Series Such As:
0.1 + 0.11 + 0.111 + ... to n terms
$\frac{1}{9}\times\left[n-\left\{\frac{1-(0.1)^{n}}{9}\right\}\right]$
0.7 + 0.77 + 0.777 + ... to n terms
$\frac{7}{9}\times\left[n-\left\{\frac{1-(0.1)^{n}}{9}\right\}\right]$
Sum of First n Natural Numbers
$\frac{n(n+1)}{2}$
Sum of First n Odd Numbers
n^{2}
Sum of Squares of First n Natural Numbers
$\frac{n(n+1)(2n+1)}{6}$
Sum of Cubes of First n Natural Numbers
$\left\{\frac{n(n+1)}{2}\right\}^{2}$

Questions Based on Arithmetic Progression

Question 1

The value of x such that $8x + 4$, $6x - 2$, $2x + 7$ will form an AP is:

(a) 15 (b) 2 (c) 15/2 (d) None

Solution

(c)

Question 2

The last term of the A.P. 0.6, 1.2, 1.8, … to 13 terms is:

(a) 8.7 (b) 7.8 (c) 7.7 (d) None

Solution

(b)

Question 3

Which term of the progression –1, –3, –5, … is –39?

(a) 21^{st} (b) 20^{th} (c) 19^{th} (d) None

Solution

(b)

Question 4

The number of numbers between 74 and 25,556 divisible by 5 is:

(a) 5090 (b) 5097 (c) 5095 (d) None

Solution

(b)

Question 5

The n^{th} element of the sequence –1, 2, –4, 8, … is:

(a) $(-1)^n 2^{n-1}$ (b) 2^{n-1} (c) 2^n (d) None

Solution

(a)

Question 6

The n^{th} term of the series 3 + 7 + 13 + 21 + 31 + … is:

(a) $4n-1$ (b) n^2+2n (c) n^2+n+1 (d) n^3+2

Solution

(c)

Try the options.

Question 7

The two arithmetic means between –6 and 14 is:

(a) 2/3, 1/3 (b) 2/3, $7\frac{1}{3}$ (c) $-2/3, -7\frac{1}{3}$ (d) None

Solution

(b)

Question 8

The 4 arithmetic means between –2 and 23 are

(a) 3, 13, 8, 18 (b) 18, 3, 8, 13 (c) 3, 8, 13, 18 (d) None

Solution

(c)

Question 9

The sum of the series 9, 5, 1, … to 100 terms is:

(a) –18,900 (b) 18,900 (c) 19,900 (d) None

Solution

(a)

Question 10

The sum of all natural numbers between 500 and 1000 which are divisible by 13, is:

(a) 28,405 (b) 24,805 (c) 28,540 (d) None

Solution

(a)

Question 11

The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 and 5 is:

(a) 2,200 (b) 2,000 (c) 2,220 (d) None

Solution

(a)

Question 12

The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 or 5 is:

(a) 10,200 (b) 15,200 (c) 16,200 (d) None

Solution

(c)

Question 13

A person is employed in a company at ₹3,000 per month and he would get an increase of ₹100 per year. Find the total amount which he receives in 25 years and the monthly salary in the last year.

(a) ₹14,60,000 (b) ₹13,60,000 (c) ₹12,60,000 (d) None

Solution

(c)

Question 14

A sum of ₹6240 is paid off in 30 instalments such that each instalment is ₹10 more than the preceding installment. The value of the 1st instalment is:

(a) ₹36 (b) ₹30 (c) ₹60 (d) None

Solution

(d)

Question 15

A person saved ₹16,500 in ten years. In each year after the first year, he saved ₹100 more than he did in the preceding year. The amount of money he saved in the 1st year was:

(a) ₹1,000 (b) ₹1,500 (c) ₹1,200 (d) None

Solution

(c)

Question 16

The sum of a certain number of terms of an AP series –8, –6, –4, … is 52. The number of terms is:

(a) 12 (b) 13 (c) 11 (d) None

Solution

(b)

Question 17

The first and the last term of an AP are –4 and 146. The sum of the terms is 7171. The number of terms is:

(a) 101 (b) 100 (c) 99 (d) None

Solution

(a)

Question 18

The number of terms of the series $10 + 9\frac{2}{3} + 9\frac{1}{3} + 9 + ...$ will amount to 155 is:

(a) 30 (b) 31 (c) 32 (d) Both (a) and (b)

Solution

(d)

Question 19

If 8th term of an AP is 15, the sum of its first 15 terms is:

(a) 15 (b) 0 (c) 225 (d) 225/2

Solution

(c)

$$t_8 = a + 7d = 15$$

$$S_{15} = \frac{15}{2}\left\{2a + 14d\right\}$$

$$S_{15} = \frac{15}{2}\left\{2\left(a + 7d\right)\right\}$$

$$S_{15} = \frac{15}{2}\left\{2 \times 15\right\} = 225$$

Question 20

A person pays ₹975 by monthly instalment each less than the former by ₹5. The first instalment is ₹100. The time by which the entire amount will be paid is:

(a) 10 months (b) 15 months (c) 14 months (d) None

Solution

(b)

Question 21

The n^{th} term of the series whose sum to n terms is $5n^2 + 2n$ is:

(a) $3n - 10$ (b) $10n - 2$ (c) $10n - 3$ (d) None

Solution

(c)

Question 22

The p^{th} term of an AP is $(3p - 1)/6$. The sum of the first n terms of the AP is:

(a) $n(3n+1)$ (b) $n(3n+1)/12$ (c) $n/12(3n-1)$ (d) None

Solution

(b)

Question 23

If 5th and 12th terms of an AP are 14 and 35 respectively, find the AP.

(a) 2, 5, 8, 11 (b) 2, 5, 8, 9 (c) 2, 5, 9, 13 (d) None

Solution

(a)

Question 24

$\sum_{i=4}^{7} \sqrt{2i-1}$ can be written as:

(a) $\sqrt{7}+\sqrt{9}+\sqrt{11}+\sqrt{13}$

(b) $2\sqrt{7}+2\sqrt{9}+2\sqrt{11}+2\sqrt{13}$

(c) $2\sqrt{7}+2\sqrt{9}+2\sqrt{11}+2\sqrt{13}$

(d) None

Solution

(a)

Question 25

The sum to ∞ of the series –5, 25, –125, 625, … can be written as:

(a) $\sum_{k=1}^{\infty}(-5)^k$ (b) $\sum_{k=1}^{\infty}5^k$ (c) $\sum_{k=1}^{\infty}-5^k$ (d) None

Solution

(a)

Question 26

The m^{th} term of an AP is n and the n^{th} term is m. The r^{th} term of it is:

(a) $m+n+r$ (b) $n+m-2r$ (c) $m+n+r/2$ (d) $m+n-r$

Solution

(d)

Question 27

The first term of an A.P is 14 and the sums of the first five terms and the first ten terms are equal in magnitude but opposite in sign. The 3rd term of the AP is:

(a) $6\frac{4}{11}$ (b) 6 (c) 4/11 (d) None

Solution

(a)

Question 28

If unity is added to the sum of any number of terms of the A.P. 3, 5, 7, 9, … the resulting sum is:

(a) a perfect cube (b) a perfect square (c) a number (d) None

Solution

(b)

Question 29

The sum of the progression $(a+b),\ a,\ (a-b)\ldots n$ terms is:

(a) $\frac{n}{2}\left[2a+(n-1)b\right]$ (b) $\frac{n}{2}\left[2a+(3-n)b\right]$ (c) $\frac{n}{2}\left[2a+(3-n)\right]$ (d) $\frac{n}{2}\left[2a+(n-1)\right]$

Solution

(b)

Here, the first term is $(a+b)$, and the common difference is $a-(a+b)=a-a-b=-b$

Therefore, $a=(a+b);\ d=-b$

Sum to n terms is given by: $S_n = \frac{n}{2}\left\{2a + \left(n-1\right)d\right\}$

$$\Rightarrow S_n = \frac{n}{2}\left\{2\left(a+b\right) + \left(n-1\right)\left(-b\right)\right\}$$

$$\Rightarrow S_n = \frac{n}{2}\left\{2a + 2b + \left(-bn + b\right)\right\}$$

$$\Rightarrow S_n = \frac{n}{2}\left\{2a + 2b - bn + b\right\}$$

$$\Rightarrow S_n = \frac{n}{2}\left\{2a + 3b - bn\right\}$$

$$\Rightarrow S_n = \frac{n}{2}\left\{2a + b\left(3 - n\right)\right\}$$

Question 30

Find the sum of first twenty-five terms of A.P. series whose n^{th} term is $\left(\frac{n}{5}+2\right)$.

(a) 105 (b) 115 (c) 125 (d) 135

Solution

(b)

$$t_n = \frac{n}{5} + 2$$

$$t_1 = \frac{1}{5} + 2 = \frac{1+10}{5} = \frac{11}{5}$$

$$t_2 = \frac{2}{5} + 2 = \frac{2+10}{5} = \frac{12}{5}$$

$$t_3 = \frac{3}{5} + 2 = \frac{3+10}{5} = \frac{13}{5}$$

Therefore, $a = \frac{11}{5};\ d = \frac{12}{5} - \frac{11}{5} = \frac{1}{5};\ n = 25$

We know that $S_n = \frac{n}{2}\left\{2a + (n-1)d\right\}$

$$S_{25} = \frac{25}{2}\left\{\left(2\times\frac{11}{5}\right) + (25-1)\left(\frac{1}{5}\right)\right\} = 115$$

Question 31

The sum of the first 3 terms in an AP is 18 and that of the last 3 is 28. If the AP has 13 terms, what is the sum of the middle three terms?

(a) 23 (b) 18 (c) 19 (d) None

Solution

(a)

Let the first term be *a* and the common difference be *d*.

$t_1 + t_2 + t_3 = 18$

$\Rightarrow (a) + (a + d) + (a + 2d) = 18$

$$\Rightarrow a + a + d + a + 2d = 18$$

$$\Rightarrow 3a + 3d = 18$$

$$\Rightarrow 3(a + d) = 18$$

$$\Rightarrow a + d = \frac{18}{3} = 6$$

$$\Rightarrow a + d = 6 \ldots \text{Eq. (1)}$$

$$t_{11} + t_{12} + t_{13} = 28$$

$$\Rightarrow (a + 10d) + (a + 11d) + (a + 12d) = 28$$

$$\Rightarrow a + 10d + a + 11d + a + 12d = 28$$

$$\Rightarrow 3a + 33d = 28$$

$$\Rightarrow 3\left(a+11d\right)=28$$

$$\Rightarrow a+11d=\frac{28}{3} \ldots \text{Eq. (2)}$$

Subtracting Eq. (2) from Eq. (1), we get:

$$d-11d=6-\frac{28}{3}$$

$$\Rightarrow -10d=\frac{18-28}{3}$$

$$\Rightarrow -10d=-\frac{10}{3}$$

$$\Rightarrow d = \frac{1}{3}$$

Putting this value in Eq. (1), we get:

$$a + \frac{1}{3} = 6$$

$$\Rightarrow a = 6 - \frac{1}{3} = \frac{18-1}{3} = \frac{17}{3}$$

Therefore, $a = \frac{17}{3}$; $d = \frac{1}{3}$

Middle three terms of the series are t_6, t_7, and t_8

$$t_6 + t_7 + t_8$$

$$= (a + 5d) + (a + 6d) + (a + 7d)$$

$$= a + 5d + a + 6d + a + 7d$$

$$= 3a + 18d$$

$$= \left(3 \times \frac{17}{3}\right) + \left(18 \times \frac{1}{3}\right)$$

$$= 17 + 6 = 23$$

Question 32

The first term of an A.P. is 100 and the sum of whose first 6 terms is 5 times the sum of the next 6 terms, then the c.d. is:

(a) –10 (b) 10 (c) 5 (d) None

(a)

Try the options.

Option (a) → –10

If the common difference is –10, the series is:

100, 90, 80, 70, 60, 50, 40, 30, 20, 10, 0, –10

Sum of the first 6 terms = 100 + 90 + 80 + 70 + 60 + 50 = 450

Sum of the next 6 terms = 40 + 30 + 20 + 10 + 0 + (–10) = 90

Since 450 = 5 × 90, therefore, clearly sum of the first 6 terms, i.e., 450, is 5 times the sum of the next 6 terms, i.e. 90.

Therefore, option (a) is the answer.

Question 33

If $\frac{1+3+5+...+n \text{ terms}}{2+4+6+...+50 \text{ terms}} = \frac{2}{51}$, the value of n is:

(a) 9 (b) 10 (c) 12 (d) 13

Solution

(b)

Try the options.

Option (b) → 10

This becomes the sum of first 10 odd numbers

Numerator → 1 + 3 + 5 + … 10 terms

$$S_{10} = \frac{10}{2}\left\{\left(2\times1\right)+\left(10-1\right)2\right\}=100$$

Denominator → $S_{50} = \frac{50}{2}\left\{\left(2\times2\right)+\left(50-1\right)2\right\}=2550$

On calculator $\frac{100}{2550}=\frac{2}{51}$

Question 34

The sum of n terms of an A.P. is $3n^2 + n$; then its p^{th} term is:

(a) $6p+2$ (b) $6p-2$ (c) $6p-1$ (d) None

(b)

Given $S_n = 3n^2 + n$

Therefore, $S_1 = 3(1)^2 + 1 = 3 + 1 = 4$. This means that the sum of the first term is 4.

Now, obviously, the sum of the first term is the first term itself.

This means that the first term is 4. Therefore, $a = 4$

Also, $S_2 = 3(2)^2 + 2 = (3 \times 4) + 2 = 14$.

This means that the sum of the first two terms is 14.

Clearly, second term is 14 – 4 = 10.

Therefore, first term is 4, and the second term is 10. Clearly, common difference is 10 – 4 = 6.

Therefore, we have $a = 4$, and $d = 6$.

Therefore, the p^{th} term is given by:

$$t_p = a + (p-1)d$$

$$= 4 + (p-1)6$$

$=4+6p-6$

$=6p-2$

Questions Based on Geometric Progression

Question 35

t_{12} of the series –128, 64, –32, … is:

(a) –1/16 (b) 16 (c) 1/16 (d) None

Solution

(c)

Question 36

The last term of the series 1, –3, 9, –27 up to 7 terms is:

(a) 297 (b) 729 (c) 927 (d) None

Solution

(b)

Question 37

The last term of the series x^2, x, 1, …. to 31 terms is:

(a) x^{28} (b) $1/x$ (c) $1/x^{28}$ (d) None

Solution

(c)

Question 38

Which term of the progression 1, 2, 4, 8, … is 256?

(a) 9^{th} (b) 10^{th} (c) 11^{th} (d) None

Solution

(a)

Question 39

Insert 3 geometric means between 1/9 and 9.

(a) 1/3, 1, 3 (b) 1/9, 1, 9 (c) ¼, 1, 4 (d) None

Solution

(a)

Question 40

The sum of the series –2, 6, –18, …. to 7 terms is:

(a) –1094 (b) 1094 (c) –1049 (d) None

Solution

(a)

Question 41

The sum of the series 243, 81, 27, …. to 8 terms is:

(a) 36 (b) $\left(36\frac{13}{30}\right)$ (c) $36\frac{1}{9}$ (d) None

Solution

(d)

Question 42

The sum of the series $\frac{1}{\sqrt{3}}+1+\frac{3}{\sqrt{3}}+...$ to 18 terms is:

(a) $9841\frac{\left(1+\sqrt{3}\right)}{\sqrt{3}}$ (b) 9841 (c) $\frac{9841}{\sqrt{3}}$ (d) None

Solution

(a)

Question 43

If you save 1 paise today, 2 paise the next day 4 paise the succeeding day and so on, then your total savings in two weeks will be:

(a) ₹163 (b) ₹183 (c) ₹163.83 (d) None

Solution

(c)

Question 44

The sum of the series 1 + 2 + 4 + 8 + … to *n* terms is:

(a) $2^n - 1$ (b) $2n - 1$ (c) $1/2^n - 1$ (d) None

Solution

(a)

Question 45

The number of terms to be taken so that 1 + 2 + 4 + 8 + will be 8191 is:

(a) 10 (b) 13 (c) 12 (d) None

Solution

(b)

Question 46

The sum of the infinite GP 14, – 2, + 2/7, – 2/49, + … is:

(a) $4\frac{1}{12}$ (b) $12\frac{1}{4}$ (c) 12 (d) None

Solution

(b)

Question 47

The sum of the infinite G. P. 1 – 1/3 + 1/9 – 1/27 + … is:

(a) 0.33 (b) 0.57 (c) 0.75 (d) None

Solution

(c)

Question 48

The n^{th} term of the series 16, 8, 4, … is $1/2^{17}$. The value of n is:

(a) 20 (b) 21 (c) 22 (d) None

Solution

(c)

Question 49

The sum of $1 + 1/3 + 1/3^2 + 1/3^3 + \ldots + 1/3^{n-1}$ is:

(a) 2/3 (b) 3/2 (c) 4/5 (d) None

Solution

(d)

If we take the first 4 terms, we can see that in the last term, the power of 3 is 3, which is one less than the total number of terms (4). Therefore, when $n = 4$, the power of the last 3 is (4 – 1). Similarly, when the total number of terms is n, the power of the last 3 is (n – 1).

Therefore, we have $a = 1;\ r = 1/3;\ n = n$

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

$$S_n = 1\left(\frac{1-(1/3)^n}{1-(1/3)}\right)$$

$$S_n = 1\left(\frac{1-(1/3)^n}{2/3}\right)$$

$$S_n = \frac{3}{2}\times\left\{1-\left(\frac{1}{3}\right)^n\right\}$$

Question 50

The sum of $1.03 + (1.03)^2 + (1.03)^3 + \ldots$ to n terms is:

(a) $103\left\{(1.03)^n - 1\right\}$ (b) $103/3\left\{(1.03)^n - 1\right\}$ (c) $(1.03)^n - 1$ (d) None

Solution

(b)

Here $a = 1.03;\ r = 1.03$

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$

$$S_n = 1.03\left(\frac{(1.03)^n - 1}{1.03 - 1}\right)$$

$$S_n = 1.03\left(\frac{(1.03)^n - 1}{0.03}\right)$$

$$S_n = \frac{1.03}{0.03}\left\{(1.03)^n - 1\right\}$$

$$S_n = \frac{103}{3}\left\{(1.03)^n - 1\right\}$$

Question 51

The sum of the infinite series 1 + 2/3 + 4/9 + … is:

(a) 1/3 (b) 3 (c) 2/3 (d) None

Solution

(b)

Question 52

Find the G.P where 4^{th} term is 8 and 8^{th} term is 128/625:

(a) 125, 50, 20, … (b) –125, 50, –20 (c) 120, 60, 30, … (d) Both (a) and (b)

Solution

(d)

Question 53

If x, y, and z are in GP, then:

(a) $y^2 = xz$ (b) $y\left(z^2 + x^2\right) = x\left(z^2 + y^2\right)$ (c) $2y = x + z$ (d) None

Solution

(a)

Question 54

In a G.P., the product of the first three terms 27/8. The middle term is:

(a) 3/2 (b) 2/3 (c) 2/5 (d) None

Solution

(a)

Question 55

The sum of the first 20 terms of a G.P. is 244 times the sum of its first 10 terms. The common ratio is:

(a) $\pm\sqrt{3}$ (b) ± 3 (c) $\sqrt{3}$ (d) None

Solution

(a)

Question 56

The sum of the first two terms of a G.P. is 5/3 and the sum to infinity of the series is 3. The common ratio is:

(a) 1/3 (b) 2/3 (c) –2/3 (d) Both (b) and (c)

Solution

(d)

Question 57

If $y = 1 + x + x^2 + ... + \infty$, then x =

(a) $\frac{y-1}{y}$ (b) $\frac{y+1}{y}$ (c) $\frac{y}{y+1}$ (d) $\frac{y}{y-1}$

Solution

(a)

$y = S_\infty$

$\Rightarrow y = \frac{a}{1-r}$

$$\Rightarrow y = \frac{1}{1-x}$$

$$\Rightarrow y(1-x) = 1$$

$$\Rightarrow y - xy = 1$$

$$\Rightarrow xy = y - 1$$

$$\Rightarrow x = \frac{y-1}{y}$$

Question 58

Sum upto infinity of series: $\frac{1}{2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}+\frac{1}{2^5}+...$

(a) 19/24 (b) 24/19 (c) 5/24 (d) None

Solution

(a)

This is a combination of two separate series:

$$\left(\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}+...\infty\right)+\left(\frac{1}{3^2}+\frac{1}{3^4}+...\infty\right)$$

$$=\frac{1/2}{1-\left(1/4\right)}+\frac{1/3^2}{1-\left(1/3^2\right)}$$

$$=\frac{1/2}{3/4}+\frac{1/9}{8/9}$$

$$=\left(\frac{1}{2}\times\frac{4}{3}\right)+\left(\frac{1}{9}\times\frac{9}{8}\right)$$

$$=\frac{2}{3}+\frac{1}{8}=\frac{16+3}{24}=\frac{19}{24}$$

Question 59

If 2 + 6 + 10 + 14 + 18 + … + x = 882 then the value of x

(a) 78 (b) 80 (c) 82 (d) 86

Solution

(c)

We have $a=2;\ d=4;\ S_n=882$

$$S_n=\frac{n}{2}\{2a+(n-1)d\}$$

$$\Rightarrow 882=\frac{n}{2}\{(2\times 2)+(n-1)(4)\}$$

$$\Rightarrow 882\times 2 = n\{4+4n-4\}$$

$$\Rightarrow 882\times 2 = n\{4n\}$$

$$\Rightarrow 882\times 2 = 4n^2$$

$$\Rightarrow n^2 = \frac{882\times 2}{4}$$

$$\Rightarrow n = \sqrt{\frac{882\times 2}{4}} = 21$$

$$x = t_{21}$$

$$\Rightarrow t_{21} = a + 20d$$

$$\Rightarrow t_{21} = 2 + \left(20 \times 4\right) = 82$$

$$\Rightarrow x = 82$$

Question 60

The sum of *n* terms of a G.P. whose first term is 1 and the common ratio is 1/2, is equal to $1\frac{127}{128}$. The value of *n* is:

(a) 7 (b) 8 (c) 6 (d) None

Solution

(b)

We have $a = 1$; $r = \frac{1}{2}$; $S_n = 1\frac{127}{128} = \frac{255}{128}$

$$S_n = a\left[\frac{1-r^n}{1-r}\right]$$

$$\Rightarrow \frac{255}{128} = 1\left[\frac{1-(1/2)^n}{1-1/2}\right]$$

$$\Rightarrow \frac{255}{128} = \frac{1}{1/2}\left[1-\left(\frac{1}{2}\right)^n\right]$$

$$\Rightarrow \frac{1/2\times 255}{128} = 1-\left(\frac{1}{2}\right)^n$$

$$\Rightarrow 0.99609375 = 1 - \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \left(\frac{1}{2}\right)^n = 1 - 0.99609375 = 0.00390625$$

Now, try the options.

$$\left(\frac{1}{2}\right)^8 = 0.00390625$$

$$\Rightarrow n = 8$$

Question 61

In a G.P., if the fourth term is ‘3’ then the product of first seven terms is:

(a) 3^5 (b) 3^7 (c) 3^6 (d) 3^8

Solution

(b)

$$t_4 = 3$$

$$t_4 = ar^3$$

$$\Rightarrow ar^3 = 3$$

$$t_1 \times t_2 \times t_3 \times t_4 \times t_5 \times t_6 \times t_7$$

$$= a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6$$

$$= a^{1+1+1+1+1+1+1} r^{1+2+3+4+5+6}$$

$$= a^7 r^{21}$$

$$= \left(ar^3\right)^7$$

$$= 3^7$$

Question 62

If t_4 of a GP is x, $t_{10} = y$, and $t_{16} = z$, then,

(a) $x^2 = yz$ (b) $z^2 = xy$ (c) $y^2 = zx$ (d) None

Solution

(c)

$ar^3 = x$; $ar^9 = y$; $ar^{15} = z$

Try the options.

Option (a) → $x^2 = yz$

LHS $\rightarrow \left(ar^3\right)^2 = a^2r^6$

RHS $\rightarrow ar^9 \times ar^{15} = a^2r^{9+15} = a^2r^{24}$

Option (b) $\rightarrow z^2 = xy$

LHS $\rightarrow \left(ar^{15}\right)^2 = a^2r^{30}$

RHS $\rightarrow ar^3 \times ar^9 = a^2r^{3+9} = a^2r^{12}$

Option (c) $\rightarrow y^2 = zx$

LHS $\rightarrow \left(ar^9\right)^2 = a^2r^{18}$

RHS $\rightarrow ar^{15} \times ar^3 = a^2r^{15+3} = a^2r^{18}$

Therefore, option (c) is the answer.

Alternatively,

We can see that t_{10} is the middle term between t_4 and t_{16}. Therefore, t_{10} is the geometric mean. Therefore, $\left(t_{10}\right)^2 = t_4 \times t_{16} \Rightarrow y^2 = xz$

Question 63

If p, q and r, are in A.P. and x, y, z are in G.P., then $x^{q-r}.y^{r-p}.z^{p-q}$ is equal to:

(a) 0 (b) –1 (c) 1 (d) None

Solution

(c)

Since p, q, and r, are in AP, we have $q-p=r-q=d$

$\therefore q-p=d \Rightarrow p-q=-d$

And $r-q=d \Rightarrow q-r=-d$

Also, $r-p=(r-q)+(q-p)=d+d=2d$

Also, since x, y, and z are in GP, we have $y^2 = xz$

Now, we have:

$$x^{q-r}.y^{r-p}.z^{p-q}$$

$$x^{-d}.y^{2d}.z^{-d} \text{ (Since } q-r=-d;\ r-p=2d;\ p-q=-d)$$

$$(xz)^{-d}.y^{2d}$$

$$(y^2)^{-d}.y^{2d} \text{ (Since } y^2 = xz)$$

$$y^{-2d}.y^{2d} = 1$$

Question 64

Given x, y, and z are in GP and $x^p = y^q = z^\sigma$, then $1/p$, $1/q$, $1/\sigma$ are in:

(a) AP (b) GP (c) Both (d) None

Solution

(a)

Let $x^p = y^q = z^\sigma = k$

$\Rightarrow x^p = k \Rightarrow x = k^{\frac{1}{p}}$

$\Rightarrow y^q = k \Rightarrow y = k^{\frac{1}{q}}$

$$\Rightarrow z^{\sigma} = k \Rightarrow z = k^{\frac{1}{\sigma}}$$

Since x, y, and z are in GP, $y^2 = xz$

$$\Rightarrow \left(k^{\frac{1}{q}}\right)^2 = k^{\frac{1}{p}} \times k^{\frac{1}{\sigma}}$$

$$\Rightarrow k^{\frac{2}{q}} = k^{\frac{1}{p}+\frac{1}{\sigma}}$$

$$\Rightarrow \frac{2}{q} = \frac{1}{p} + \frac{1}{\sigma}$$

$$\Rightarrow \frac{1}{q} + \frac{1}{q} = \frac{1}{p} + \frac{1}{\sigma}$$

$$\Rightarrow \frac{1}{p} - \frac{1}{q} = \frac{1}{q} - \frac{1}{\sigma}$$

Therefore, they are in AP.

Question 65

If A be the A.M. of two positive unequal quantities x and y and G be their G.M., then:

(a) $A < G$ (b) $A > G$ (c) $A \geq G$ (d) $A \leq G$

Solution

(b)

Question 66

If x, y, z, are in A.P. and x, y, $(z + 1)$ are in G.P., then:

(a) $(x-z)^2 = 4x$ (b) $z^2 = x - y$ (c) $z = x - y$ (d) None

Solution

(a)

Since x, y, and z are in AP, $y = \frac{x+z}{2}$ …Eq. (1)

Also, since x, y, $(z + 1)$ are in G.P., $y^2 = x(z+1)$…Eq. (2)

Putting the value of y from Eq. (1) in Eq. (2), we have:

$$\left(\frac{x+z}{2}\right)^2 = xz + x$$

$$\frac{x^2+z^2+2xz}{4} = xz + x$$

$$x^2+z^2+2xz = 4xz+4x$$

$$x^2+z^2+2xz-4xz = 4x$$

$$x^2+z^2-2xz = 4x$$

$$(x-z)^2 = 4x$$

Question 67

The numbers x, 8, y are in G.P. and the numbers x, y, –8 are in A.P. The value of x and y are:

(a) (–8, –8) (b) (16, 4) (c) (8, 8) (d) Both (a) and (b)

Solution

(d)

Try the options.

Question 68

The series $1+10^{-1}+10^{-2}+10^{-3}...$ to ∞ is:

(a) 9/10 (b) 1/10 (c) 10/9 (d) None

Solution

(c)

Given series $1+10^{-1}+10^{-2}+10^{-3}...$

$$\Rightarrow 1+\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+...\infty$$

Here, $a = 1;\ r = \frac{1}{10}$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{10}} = \frac{1}{\frac{9}{10}} = \frac{10}{9}$$

Question 69

The sum of the first two terms of an infinite geometric series is 15 and each term is equal to the sum of all the terms following it; then the sum of the series is:

(a) 20 (b) 15 (c) 25 (d) None

(a)

Let the first term of the GP be a, and the second term of the GP be ar.

Given:

$a + ar = 15$

$\Rightarrow a(1 + r) = 15$

$\Rightarrow a = \frac{15}{1+r}$ ...Eq. (1)

Also, we are given that every term is equal to the sum of all the terms following it. This means that $t_2 = S_\infty - S_2$.

Now, we know that $S_\infty = \frac{a}{1-r}$, and S_2 is given as 15.

Therefore, $t_2 = \frac{a}{1-r} - 15$

Also, we know that $t_2 = ar$

Therefore, $ar = \frac{a}{1-r} - 15$ ...Eq. (2)

Putting the value of *a* from Eq. (1) to Eq. (2), we get:

$$\frac{15}{1+r}\times r=\frac{\frac{15}{1+r}}{1-r}-15$$

$$\frac{15r}{1+r}=\left(\frac{15}{1+r}\div 1-r\right)-15$$

$$\frac{15r}{1+r}=\left(\frac{15}{1+r}\times\frac{1}{1-r}\right)-15$$

$$\frac{15r}{1+r}=\frac{15}{(1+r)(1-r)}-15$$

$$\frac{15r}{1+r}=\frac{15-15(1+r)(1-r)}{(1+r)(1-r)}$$

$$15r=\frac{15\{1-(1+r)(1-r)\}}{1-r}$$

$$r=\frac{\{1-(1-r^2)\}}{1-r}$$

$$r(1-r)=1-1+r^2$$

$$r-r^2=r^2$$

$$r^2+r^2-r=0$$

$$2r^2 - r = 0$$

$$r(2r-1) = 0$$

Therefore, either $r = 0$, or $r = \frac{1}{2}$

Since r cannot be 0, it'll be ½.

Putting the value of r in Eq. (1), we get:

$$a = \frac{15}{1+\frac{1}{2}} = 10$$

Therefore, we have a = 10, and r = ½.

$$S_\infty = \frac{a}{1-r} = \frac{10}{1-\frac{1}{2}} = 20$$

Therefore, option (a) is the answer.

Question 70

If the p^{th} term of a GP is *x* and the q^{th} term is *y*, then find the n^{th} term.

(a) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]$ (b) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]^{(p-q)}$ (c) 1 (d) $\left[\frac{x^{(n-q)}}{y^{(n-p)}}\right]^{\frac{1}{p-q}}$

Solution

(d)

$t_p = ar^{p-1} = x$ …Eq. (1)

$t_q = ar^{q-1} = y$ …Eq. (2)

Dividing Eq. (1) by Eq. (2)

$$\frac{ar^{p-1}}{ar^{q-1}} = \frac{x}{y}$$

$$r^{p-1-(q-1)} = \frac{x}{y}$$

$$r^{p-1-q+1} = \frac{x}{y}$$

$$r^{p-q} = \frac{x}{y}$$

$$r = \left(\frac{x}{y}\right)^{\frac{1}{p-q}}$$

$$t_n = ar^{n-1}$$

Adding *p* and subtracting *p* in the power of *r*, we get:

$$t_n = ar^{n-1+p-p}$$

$$t_n = ar^{(n-p)+(p-1)}$$

$$t_n = ar^{(p-1)}r^{(n-p)}$$

We know that $ar^{p-1} = x$ and $r = \left(\frac{x}{y}\right)^{\frac{1}{p-q}}$. Putting these values above, we get:

$$t_n = x\left[\left(\frac{x}{y}\right)^{\frac{1}{p-q}}\right]^{n-p}$$

$$t_n = x\left(\frac{x}{y}\right)^{\frac{n-p}{p-q}}$$

$$t_n = x\left(\frac{x^{\frac{n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}\right)$$

$$t_n = \frac{x.x^{\frac{n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \frac{x^{1+\frac{n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \frac{x^{\frac{p-q+n-p}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \frac{x^{\frac{n-q}{p-q}}}{y^{\frac{n-p}{p-q}}}$$

$$t_n = \left(\frac{x^{n-q}}{y^{n-p}}\right)^{\frac{1}{p-q}}$$

Question 71

The sum of three numbers in a geometric progression is 28. When 7, 2, and 1 are subtracted from the first, second, and the third numbers respectively, the resulting numbers are in Arithmetic Progression. What is the sum of squares of the original three numbers?

(a) 510 (b) 456 (c) 400 (d) 336

Solution

(d)

Let the numbers in GP be $\frac{a}{r}$, a, and ar respectively.

Given that the sum is 28.

Therefore, $\frac{a}{r}+a+ar=28$

$\Rightarrow a\left(\frac{1}{r}+1+r\right)=28\ldots$Eq. (1)

Also, given that if we subtract 7, 2, and 1 from the first, second and third terms respectively, we get an AP.

On subtracting 7, 2, and 1 from first, second and third terms, we get:

$\left(\frac{a}{r}-7\right)$, $(a-2)$, and $(ar-1)$

Since these numbers are in AP, we have $\left(a-2\right)-\left(\frac{a}{r}-7\right)=\left(ar-1\right)-\left(a-2\right)$

$$\Rightarrow a-2-\frac{a}{r}+7=ar-1-a+2$$

$$\Rightarrow a-\frac{a}{r}+5=ar-a+1$$

$$\Rightarrow a-\frac{a}{r}-ar+a=1-5$$

$$\Rightarrow 2a-\frac{a}{r}-ar=-4$$

$$\Rightarrow a\left(2-\frac{1}{r}-r\right)=-4 \ldots \text{Eq. (2)}$$

Dividing Eq. (1) by Eq. (2), we get:

$$\frac{a\left(\frac{1}{r}+1+r\right)}{a\left(2-\frac{1}{r}-r\right)}=\frac{28}{-4}$$

$$\Rightarrow \frac{\frac{1+1r+r^2}{r}}{\frac{2r-1-r^2}{r}}=-7$$

$$\Rightarrow \frac{1+r+r^2}{2r-1-r^2} = -7$$

$$\Rightarrow 1+r+r^2 = -7\left(2r-1-r^2\right)$$

$$\Rightarrow 1+r+r^2 = -14r+7+7r^2$$

$$\Rightarrow 7r^2+7-14r-1-r-r^2 = 0$$

$$\Rightarrow 6r^2-15r+6 = 0$$

Here, $a=6;\ b=-15;\ c=6$

$$\alpha+\beta = -\frac{b}{a} = -\frac{-15}{6} = \frac{15}{6}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{6} = 1$$

As per fastest method, $\left(\frac{15}{6\times 2} + x\right)\left(\frac{15}{6\times 2} - x\right) = 1$

$$\Rightarrow \left(\frac{15}{12}\right)^2 - x^2 = 1$$

$$x^2 = \left(\frac{15}{12}\right)^2 - 1 = 1.5625 - 1 = 0.5625$$

$$x = \sqrt{0.5625} = 0.75$$

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$$\alpha = \frac{15}{12} + 0.75 = 2$$

$$\beta = \frac{15}{12} - 0.75 = 0.5$$

Therefore, common ratio could either be 2, or 0.5

Taking the common ratio to be 2, let's find out the value of *a*.

Putting the value of $r = 2$ in Eq. (1), we'll get:

$$a\left(\frac{1}{2} + 1 + 2\right) = 28$$

$$\Rightarrow a(3.5) = 28$$

$$\Rightarrow a = \frac{28}{3.5} = 8$$

Therefore, the GP will be $\frac{8}{2}, 8, 8 \times 2 = 4, 8, 16$

We can see that the sum of these numbers = 4 + 8 + 16 = 28

Subtracting 7, 2, and 1 from first, second, and third terms, we'll get 4 – 7 = –3, 8 – 2 = 6, 16 – 1 = 15.

These terms are clearly in AP as 15 – 6 = 6 – (–3) = 9

The sum of squares of the numbers 4, 8, and 16 = $4^2 + 8^2 + 16^2 = 336$

Now, taking 0.5 as the common ratio, let's find out the value of *a*.

Putting the value of r = 0.5 in Eq. (1), we'll get:

$$a\left(\frac{1}{r}+1+r\right)=28$$

$$\Rightarrow a\left(\frac{1}{0.5}+1+0.5\right)=28$$

$$\Rightarrow a(3.5)=28$$

$$\Rightarrow a=\frac{28}{3.5}=8$$

Therefore, the GP will be $\frac{8}{0.5}$, 8, $8\times 0.5=16$, 8, 4

We can see that the sum of these numbers = 16 + 8 + 4 = 28

Subtracting 7, 2, and 1 from first, second, and third terms, we'll get 16 – 7 = 9, 8 – 2 = 6, 4 – 1 = 3.

These terms are clearly in AP as 6 – 9 = 3 – 6 = –3

The sum of squares of the numbers 16, 8, and 4 = $16^2 + 8^2 + 4^2 = 336$

Special Series

Following are some of the Standard Results:

1. Sum of first n natural or counting numbers $\left(1+2+3+4+...+n\right)=\dfrac{n\left(n+1\right)}{2}$
2. Sum of first n odd numbers $\left\{1+3+5+...+\left(2n-1\right)\right\}=n^2$
3. Sum of the Squares of first n natural numbers

$$\left(1^2+2^2+3^2+4^2+...+n^2\right)=\frac{n\left(n+1\right)\left(2n+1\right)}{6}$$

4. Sum of the Cubes of first n natural numbers $\left(1^3+2^3+3^3+4^3+...+n^3\right)=\left\{\dfrac{n\left(n+1\right)}{2}\right\}^2$

5. Sum of the series such as: 1 + 11 + 111 + … to *n* terms, or 2 + 22 + 222 + … to *n* terms, or 3 + 33 + 333 + … to *n* terms, and so on: $\frac{Number}{9} \times \left\{ \frac{10\left(10^n - 1\right)}{9} - n \right\}$. For example:

a. 1 + 11 + 111 + … to *n* terms = $\frac{1}{9} \times \left\{ \frac{10\left(10^n - 1\right)}{9} - n \right\}$

b. 2 + 22 + 222 + … to *n* terms = $\frac{2}{9} \times \left\{ \frac{10\left(10^n - 1\right)}{9} - n \right\}$

c. 3 + 33 + 333 + … to n terms = $\frac{3}{9}\times\left\{\frac{10\left(10^n-1\right)}{9}-n\right\}$

6. Sum of the series 0.1 + 0.11 + 0.111 + … to n terms = $\frac{1}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$.

Example: Calculate the sum of 0.7 + 0.77 + 0.777 + … to n terms.

Solution:

0.7 + 0.77 + 0.777 + … to n terms = 7 × (0.1 + 0.11 + 0.111 + … to n terms)

Therefore, 0.7 + 0.77 + 0.777 + … to n terms = $\frac{7}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$

Similarly, sum of series 0.2 + 0.22 + 0.222 + … to n terms = $\frac{2}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$

Sum of series 0.4 + 0.44 + 0.444 + … to n terms = $\frac{4}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$.

Question 72

The ratio of the sum of first *n* natural numbers to that of the sum of cubes of first *n* natural numbers is:

(a) 3 : 16 (b) $n(n+1)/2$ (c) $2/n(n+1)$ (d) None

Solution

(c)

Sum of first *n* natural numbers = $\frac{n(n+1)}{2}$

Sum of cubes of first n natural numbers = $\left\{\frac{n(n+1)}{2}\right\}^2$

$$\text{Ratio} = \frac{n(n+1)}{2} \div \left\{\frac{n(n+1)}{2}\right\}^2$$

$$= \frac{n(n+1)}{2} \div \left\{\frac{n(n+1)}{2} \times \frac{n(n+1)}{2}\right\}$$

$$= \frac{n(n+1)}{2} \div \left\{\frac{n^2(n+1)^2}{4}\right\}$$

$$= \frac{n(n+1)}{2} \times \frac{4}{n^2 (n+1)^2}$$

$$= \frac{2}{n(n+1)}$$

Question 73

Find the sum to *n* terms of 6 + 27 + 128 + 629 + …

(a) $\left\{5\left(5^n - 1\right)\right\} + \left\{n\left(n+1\right)\right\}$

(b) $\left\{\frac{5}{4}\left(5^n - 1\right)\right\} + \left\{\frac{n\left(n+1\right)}{2}\right\}$

(c) $\left\{5\left(5^n - 1\right)\right\} + \left\{\frac{n\left(n+1\right)}{2}\right\}$

(d) None

Solution

(b)

6 + 27 + 128 + 629 + …

$$\Rightarrow (5+1)+(25+2)+(125+3)+(625+4)+\ldots$$

$$\Rightarrow (5+25+125+625+\ldots)+(1+2+3+4+\ldots)$$

$$\Rightarrow \left(5+5^2+5^3+5^4+\ldots+5^n\right)+(1+2+3+4+\ldots+n)$$

The first bracket is a Geometric Progression with a = 5, and r = 5

$$\Rightarrow \left\{5\left(\frac{5^n-1}{5-1}\right)\right\}+\left\{\frac{n(n+1)}{2}\right\}$$

$$\Rightarrow \left\{5\left(\frac{5^n-1}{4}\right)\right\}+\left\{\frac{n(n+1)}{2}\right\}$$

$$\Rightarrow \left\{ \frac{5}{4}\left(5^n - 1\right) \right\} + \left\{ \frac{n\left(n+1\right)}{2} \right\}$$

Question 74

Find the sum to *n* terms of the series 3 + 33 + 333 + 3333 + …

(a) $\frac{1}{27}\times\left(10^{n+1}-9n-10\right)$

(b) $\frac{1}{27}\times\left(10^{n+1}-9n+10\right)$

(c) $\frac{1}{27}\times\left(10^{n+1}+9n+10\right)$

(d) None

Solution

(a)

The sum of such type of series is given by $\frac{Number}{9}\times\left\{\frac{10\left(10^{n}-1\right)}{9}-n\right\}$

Therefore, sum of 3 + 33 + 333 + 3333 + … is given by: $\frac{3}{9}\times\left\{\frac{10\left(10^{n}-1\right)}{9}-n\right\}$

$$\Rightarrow \frac{3}{9}\times\left\{\frac{10\left(10^{n}-1\right)-9n}{9}\right\}$$

$$\Rightarrow \frac{3}{81}\times\left\{10\left(10^{n}-1\right)-9n\right\}$$

$$\Rightarrow \frac{1}{27}\times\left\{10\times10^{n}-10-9n\right\}$$

$$\Rightarrow \frac{1}{27}\times\left(10^{n+1}-10-9n\right)$$

$$\Rightarrow \frac{1}{27}\times\left(10^{n+1}-9n-10\right)$$

Question 75

Find the sum to n terms of the series 0.7 + 0.77 + 0.777 + 0.7777 + …

(a) $\frac{7}{81}\times\left\{9n-1-10^{-n}\right\}$

(b) $\frac{7}{81}\times\left\{9n-1+10^{n}\right\}$

(c) $\frac{7}{81}\times\left\{9n-1+10^{-n}\right\}$

(d) None

Solution

(c)

The sum to such series is given by $\frac{7}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$

$$\Rightarrow \frac{7}{9}\times\left[\frac{9n-\left\{1-(0.1)^n\right\}}{9}\right]$$

$$\Rightarrow \frac{7}{81}\times\left\{9n-1+(0.1)^n\right\}$$

$$\Rightarrow \frac{7}{81}\times\left\{9n-1+\left(\frac{1}{10}\right)^n\right\}$$

$$\Rightarrow \frac{7}{81} \times \left\{ 9n - 1 + \frac{1}{10^n} \right\}$$

$$\Rightarrow \frac{7}{81} \times \left\{ 9n - 1 + 10^{-n} \right\}$$

Question 76

Evaluate $0.21\dot{7}\dot{5}$ using the sum of an infinite geometric series.

(a) $\dfrac{357}{1650}$ (b) $\dfrac{358}{1650}$ (c) $\dfrac{359}{1650}$ (d) None

Solution

(c)

Try the options.

Question 77

A person borrows ₹8,000 at 2.76% Simple Interest per annum. The principal and the interest are to be paid in the 10 monthly instalments. If each instalment is double the preceding one, find the value of the first and the last instalment.

(a) 8; 4,095 (b) 2; 4,096 (c) 8; 4,096 (d) None

Solution

(c)

Total amount to be paid = $8,000+\left(8,000\times 0.0276\times\frac{10}{12}\right)=8,184$

Since each instalment is to be double the preceding one, it is clearly a GP with r = 2.

Therefore, we have $n = 10;\ r = 2;\ S_{10} = 8{,}184$

Since $r > 1,\ S_n = a\left(\dfrac{r^n - 1}{r - 1}\right)$

$$a = \frac{S_n}{\left(\dfrac{r^n - 1}{r - 1}\right)} = \frac{8{,}184}{\left(\dfrac{2^{10} - 1}{2 - 1}\right)} = 8$$

Therefore, the first instalment is 8.

Now, let's calculate the last instalment.

$$t_{10} = ar^9 = 8 \times 2^9 = 4{,}096$$

Chapter 7 – Sets, Relations, Functions

CA NISHANT KUMAR

Questions Based on Sets

Question 1

$\left\{1-(-1)^{x}\right\}$ for all integral *x* is the set:

(a) {0} (b) {2} (c) {0, 2} (d) None

Solution

(c)

For $x=1,\ 1-(-1)^{x}=1-(-1)^{1}=1-(-1)=1+1=2$

For $x=2,\ 1-(-1)^{x}=1-(-1)^{2}=1-(1)=1-1=0$

For $x=3,\ 1-(-1)^{x}=1-(-1)^{3}=1-(-1)=1+1=2$

For $x=4,\ 1-\left(-1\right)^{x}=1-\left(-1\right)^{4}=1-\left(1\right)=1-1=0$, and so on…

Therefore, it can be clearly seen that the set is {0, 2}

Question 2

The set {0, 2, 4, 6, 8, 10} can be written as:

(a) $\{2x \mid 0 < x < 5\}$ (b) $\{x : 0 < x < 5\}$ (c) $\{2x : 0 \leq x \leq 5\}$ (d) None

Solution

(c)

Question 3

The null set is represented by:

(a) $\{\phi\}$ (b) $\{0\}$ (c) ϕ (d) None

Solution

(c)

Question 4

If $A = \{1, 2, 3, 5, 7\}$, and $B = \{x^2 : x \in A\}$, then:

(a) $n(B) = n(A)$ (b) $n(B) > n(A)$ (c) $n(A) = n(B)$ (d) $n(A) < n(B)$

Solution

(c)

Question 5

The sets $V=\{x \mid x+2=0\}$, $R=\{x \mid x^2+2x=0\}$, and $S=\{x : x^2+x-2=0\}$ are equal to one another if *x* is equal to:

(a) –2 (b) 2 (c) ½ (d) None

Solution

(a)

Try the options.

Option (a) → –2

Set V = –2 + 2 = 0 ⇒ V = {0}

Set $R = (-2)^2 + 2(-2) = 4 - 4 = 0 \Rightarrow R = \{0\}$

Set $S = (-2)^2 + (-2) - 2 = 4 - 2 - 2 = 4 - 4 = 0 \Rightarrow S = \{0\}$

Therefore, option (a) is the answer.

Question 6

If R is the set of positive rational number and E is the set of real numbers then:

(a) $R \subseteq E$ (b) $R \subset E$ (c) $E \subset R$ (d) None

Solution

(b)

Question 7

If I is the set of isosceles triangles and E is the set of equilateral triangles, then:

(a) $I \subset E$ (b) $E \subset I$ (c) $E = I$ (d) None

Solution

(b)

Question 8

If R is the set of isosceles right-angled triangles and I is set of isosceles triangles, then:

(a) $R = I$ (b) $R \supset I$ (c) $R \subset I$ (d) None

Solution

(c)

Question 9

Two finite sets respectively have *x* and *y* number of elements. The total number of subsets of the first is 56 more than the total number of subsets of the second. The value of *x* and *y* respectively?

(a) 6 and 3 (b) 4 and 2 (c) 2 and 4 (d) 3 and 6

Solution:

(a)

Question 10

The numbers of proper subsets of the set {3, 4, 5, 6, 7} is:

(a) 32
(b) 31
(c) 30
(d) 25

Solution:

(b)

Question 11

Let $A = \{a, b\}$. Set of subsets of A is called power set of A denoted by $P(A)$. Now, $n(P(A))$ is:

(a) 2 (b) 4 (c) 3 (d) None

Solution

(b)

Question 12

If *E* is a set of positive even numbers and *O* is a set of positive odd numbers, then $E \cup O$ is a:

(a) set of whole numbers (b) *N* (c) set of rational numbers (d) None

Solution

(b)

Question 13

$A \cup A$ is equal to:

(a) A (b) E (c) ϕ (d) None

Solution

(a)

Question 14

$A \cup E$ is equal to (E is a superset of A):

(a) A (b) E (c) ϕ (d) None

Solution

(b)

Question 15

$E \cup E$ is equal to (E is a superset of A):

(a) E (b) ϕ (c) $2E$ (d) None

Solution

(a)

Question 16

$A \cap A$ is equal to:

(a) ϕ (b) A (c) E (d) None

Solution

(b)

Question 17

$A \cap E$ is equal to (E is a superset of A):

(a) A (b) E (c) ϕ (d) None

Solution

(a)

Question 18

$A \cap \phi$ is equal to (E is a superset of A):

(a) A (b) E (c) ϕ (d) None

Solution

(c)

Question 19

If $A\Delta B = (A - B) \cup (B - A)$, and A = {1, 2, 3, 4}, B = {3, 5, 7} then $A\Delta B$ is:

(a) {1, 2, 4, 5, 7} (b) {3} (c) {1, 2, 3, 4, 5, 7} (d) None

Solution

(a)

Question 20

$A \cap E'$ is equal to (E is a superset of A)

(a) E (b) ϕ (c) A (d) None

Solution

(b)

Question 21

$A \cup A'$ is equal to (E is a superset of A)

(a) E (b) ϕ (c) A (d) None

Solution

(a)

Question 22

If the universal set E = {x | x is a positive integer < 25}, A = {2, 6, 8, 14, 22}, B = {4, 8, 10, 14}, then

(a) $(A \cap B)' = A' \cup B'$ (b) $(A \cap B)' = A' \cap B'$ (c) $(A' \cap B)' = \phi$ (d) None

Solution

(a)

Question 23

A survey shows that 74% of the Indians like grapes, whereas 68% like bananas. What percentage of the Indians like both grapes and bananas?

(a) 36% (b) 42% (c) 55% (d) None

Solution

(b)

Question 24

In a class of 60 students, 40 students like Maths, 36 like Science, and 24 like both the subjects. Find the number of students who like either Maths or Science.

(a) 36 (b) 42 (c) 52 (d) None

Solution

(c)

Question 25

In a class of 60 students, 40 students like Maths, 36 like Science, and 24 like both the subjects. Find the number of students who like neither Maths nor Science.

(a) 8 (b) 60 (c) 52 (d) None

Solution

(a)

Question 26

At a certain conference of 100 people there are 29 Indian women and 23 Indian men. Out of these Indian people 4 are doctors and 24 are either men or doctors. There are no foreign doctors. The number of women doctors attending the conference is:

(a) 2 (b) 4 (c) 1 (d) None

Solution

(c)

We have $n(U) = 100$, $n(W) = 29$, $n(M) = 23$, $n(D) = 4$, $n(M \cup D) = 24$

We have to find out the number of female doctors, i.e., $n(W \cap D)$. We have $n(W)$, and $n(D)$, but we don't have $n(W \cup D)$. Therefore, we cannot apply the formula $n(W \cap D) = n(W) + n(D) - n(W \cup D)$.

However, if we find out the number of Male Doctors, we can then subtract them from the total doctors to find out the number of female doctors.

$$n(M \cap D) = n(M) + n(D) - n(M \cup D)$$

$$\Rightarrow n(M \cap D) = 23 + 4 - 24 = 3$$

Therefore, number of female doctors = 4 – 3 = 1.

Question 27

In a class of 60 students, 40 students like Maths, 36 like Science, and 24 like both the subjects. Find the number of students who Maths only.

(a) 16 (b) 42 (c) 52 (d) None

Solution

(a)

Question 28

In a survey of 300 companies, the number of companies using different media – Newspapers (*N*), Radio (*R*) and Television (*T*) are as follows: $n(N)=200$, $n(R)=100$, $n(T)=40$, $n(N\cap R)=50$, $n(R\cap T)=20$, $n(N\cap T)=25$, $n(N\cap R\cap T)=5$. Find the number of companies using none of these media.

(a) 20 (b) 250 (c) 30 (d) 50

Solution

(d)

Question 29

Out of 2000 employees in an office, 48% preferred Coffee (*C*), 54% liked Tea (*T*), 64% used to smoke (*S*). Out of the total 28% used *C* and *T*, 32% used *T* and *S* and 30% preferred *C* and *S*, only 6% did none of these. The number having all the three is:

(a) 360 (b) 300 (c) 380 (d) None

Solution

(a)

$$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(A\cap C)+n(A\cap B\cap C)$$

We have

$$n(C) = 48\% \times 2,000 = 960,$$

$$n(T) = 54\% \times 2,000 = 1,080,$$

$$n(S) = 64\% \times 2,000 = 1,280,$$

$$n(C \cap T) = 28\% \times 2,000 = 560,$$

$$n(T \cap S) = 32\% \times 2,000 = 640,$$

$$n(C \cap S) = 30\% \times 2,000 = 600,$$

$$n(C \cup T \cup S)' = 6\% \times 2,000 = 120,$$

$$n(C \cup T \cup S) = 2,000 - 120 = 1,880,$$

We know that:

$$n(C \cup T \cup S) = n(C) + n(T) + n(S) - n(C \cap T) - n(C \cap S) - n(T \cap S) + n(C \cap T \cap S)$$

or,

$$n(C \cap T \cap S) = n(C \cup T \cup S) - n(C) - n(T) - n(S) + n(C \cap T) + n(C \cap S) + n(T \cap S)$$

$$n(C \cap T \cap S) = 1{,}880 - 960 - 1{,}080 - 1{,}280 + 560 + 600 + 640 = 360$$

Question 30

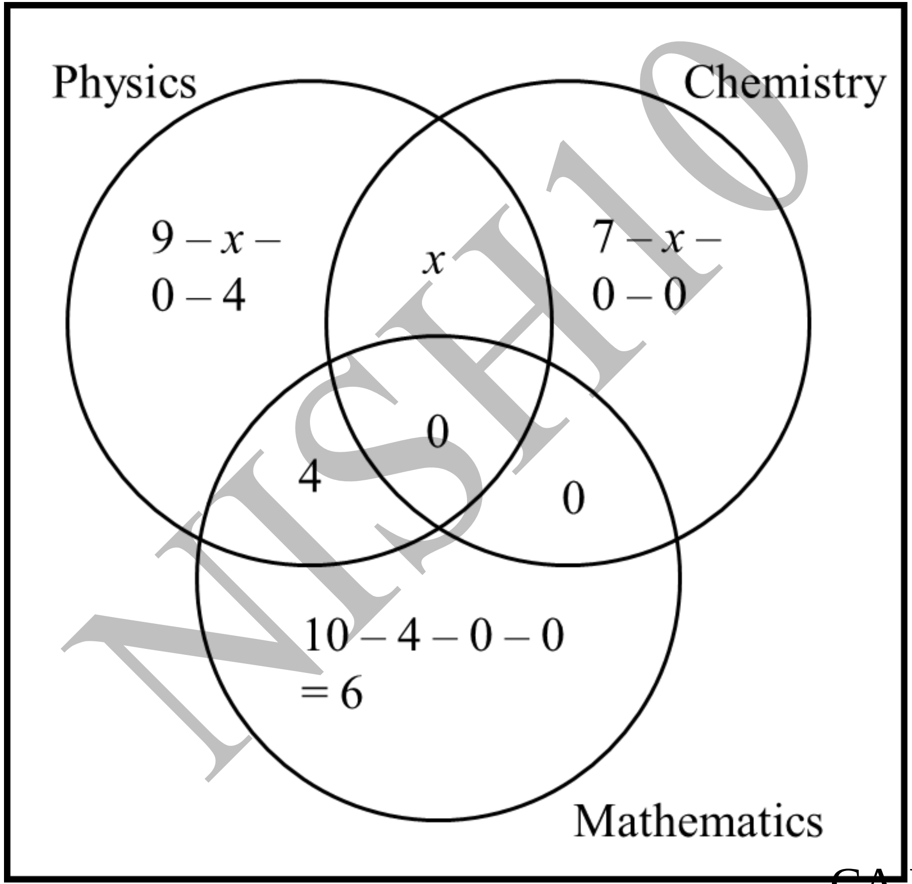
Out of a group of 20 teachers in a school, 10 teach Mathematics, 9 teach Physics and 7 teach Chemistry. 4 teach Mathematics and Physics but none teach both Mathematics and Chemistry. How many teach Chemistry and Physics? How many teach only Physics?

(a) 3; 2 (b) 2; 3 (c) 4; 5 (d) None

Solution

(b)

Let the number of teachers teaching both Physics and Chemistry be x.


Physics
Chemistry
9 – x –
0 – 4
x
7 – x –
0 – 0
0
4
0
10 – 4 – 0 – 0
= 6
Mathematics

In the absence of information, it is safe to assume that all the teachers teach at least one of the subjects. Therefore,

$9 - x - 0 - 4 + x + 7 - x - 0 - 0 + 4 + 0 + 0 + 6 = 20$

$\Rightarrow 9 - 4 + 7 + 4 + 6 - x + x - x = 20$

$\Rightarrow 22 - x = 20$

$\Rightarrow x = 22 - 20 = 2$

Therefore, number of teachers teaching both Physics and Chemistry = 2.

Number of teachers teaching only Physics = 9 – 2 – 4 = 3

Question 31

Let Z be the universal set for two sets – A and B. If $n(A) = 300$, $n(B) = 400$ and $n(A \cap B) = 200$, then $n(A' \cap B')$ is equal to 400 provided $n(Z)$ is equal to:

(a) 900 (b) 800 (c) 700 (d) 600

Solution

(a)

Given: $n(A) = 300$; $n(B) = 400$; $n(A \cap B) = 200$; $n(A' \cap B') = 400$; $n(Z) = ?$

$n(A' \cap B') = n(A \cup B)' = n(Z) - n(A \cup B)$

$$\Rightarrow n(A' \cap B') = n(Z) - n(A \cup B)$$

$$\Rightarrow n(Z) = n(A' \cap B') + n(A \cup B)$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\Rightarrow n(A \cup B) = 300 + 400 - 200 = 500$$

$$n(Z) = n(A' \cap B') + n(A \cup B)$$

$$\Rightarrow n(Z) = 400 + 500 = 900$$

Question 32

The number of integers from 1 to 100 which are neither divisible by 3 nor by 5 nor by 7 is:

(a) 67 (b) 55 (c) 45 (d) 33

Solution

(c)

Questions Based on Relations

Question 33

Let $P = \{1, 3, 6\}$ and $Q = \{3, 5\}$, find $P \times Q$.

(a) $\{(1, 3), (1, 5), (3, 3), (5, 5), (6, 3), (6, 5)\}$
(b) $\{(1, 3), (1, 5), (3, 3), (3, 5), (6, 3), (5, 6)\}$
(c) $\{(1, 3), (1, 5), (3, 3), (3, 5), (6, 3), (6, 5)\}$
(d) None

Solution

(c)

Question 34

Given $A = \{2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$ then $A \times (B \cap C)$ is:

(a) $\{(2, 5), (3, 5)\}$ (b) $\{(5, 2), (5, 3)\}$ (c) $\{(2, 3), (5, 5)\}$ (d) None

Solution

(a)

Question 35

If $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$, find A and B.

(a) $A = \{3, 5\}; B = \{2, 4\}$ (b) $A = \{2, 4\}; B = \{3, 5\}$ (c) $A = \{1\}; B = \{2\}$ (d) None

Solution

(a)

Question 36

If the set P has 3 elements, Q four and R two then the set $P \times Q \times R$ contains:

(a) 9 elements (b) 20 elements (c) 24 elements (d) None

Solution

(c)

Question 37

If $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7,6\}$, then cardinal number of the set $A \times B$ is:

(a) 7 (b) 1 (c) 16 (d) None

Solution

(d)

Question 38

For the relation $R = \{(1, 2), (1, 4), (3, 2), (3, 4)\}$, find the Domain and Range.

(a) $Dom(R) = \{1, 3\}; Range(R) = \{2, 4\}$

(b) $Dom(R) = \{1, 4\}; Range(R) = \{2, 4\}$

(c) $Dom(R) = \{1, 3\}; Range(R) = \{2, 3\}$

(d) None

Solution

(a)

Question 39

Consider the relation $R = \{(1, 1), (2, 2), (3, 3)\}$ on set $A = \{1, 2, 3\}$. This relation is:

(a) Identity Relation (b) Reflexive Relation (c) Transitive Relation (d) None

Solution

(a)

Question 40

Let $A = \{1, 2, 3\}$, then $R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$

(a) Only Reflexive
(b) Reflexive & Symmetric
(c) Reflexive & Transitive
(d) Equivalence

Solution

(c)

1. This is a Reflexive relation because every element in set A is related to itself. Also, there exists an additional ordered pair (1, 2) and both the elements belong to set A only.
2. This relation is not a symmetric relation because we do not have (2, 1) corresponding to (1, 2).

3. This relation is a transitive relation because we are unable to prove that it is not a transitive relation.
    a. For the first ordered pair, (1, 1), let’s find another ordered pair which starts with 1. Another ordered pair which starts with 1 is (1, 2). Now, we have two ordered pairs (1, 1), and (1, 2). In order for the relation to be Transitive, it must contain the pair (1, 2). Clearly it contains this ordered pair, and hence this is a Transitive relation.
    b. For the second ordered pair, (2, 2), we do not have any other ordered pair which starts with 2. So, there’s no way of proving that this is not a Transitive relation.
    c. For the third ordered pair, (3, 3), we do not have any other ordered pair which starts with 3. So, there’s no way of proving that this is not a Transitive relation.
    d. For the fourth ordered pair, (1, 2), let’s find another ordered pair which starts with 2. Another ordered pair which starts with 2 is (2, 2). Now, we h ave two

ordered pairs (1, 2), and (2, 2). In order for the relation to be Transitive, it must contain the ordered pair (1, 2). Clearly it contains this ordered pair, and hence this is a Transitive relation.

Question 41

Let $A = \{1, 2, 3\}$, then $R_2 = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$

(a) Only Symmetric
(b) Reflexive & Symmetric
(c) Reflexive & Transitive
(d) Symmetric & Transitive

Solution

(d)

1. This is not a Reflexive relation because (3, 3) is not present in the relation.
2. This relation is a symmetric relation because:
    a. For the first ordered pair (1, 1), we need (1, 1) to make the relation symmetric. We already have it.

b. For the second ordered pair (2, 2), we need (2, 2) to make the relation symmetric. We already have it.
c. For the third ordered pair (1, 2), we need (2, 1) to make the relation symmetric. We have that as well.
d. For the fourth ordered pair (2, 1), we need (1, 2) to make the relation symmetric. We have that as well.

3. This relation is a transitive relation because we are unable to prove that it is not a transitive relation.
   a. For the first ordered pair, (1, 1), let's find another ordered pair which starts with 1. Another ordered pair which starts with 1 is (1, 2). Now, we have two ordered pairs (1, 1), and (1, 2). In order for the relation to be Transitive, it must contain the pair (1, 2). Clearly it contains this ordered pair, and hence this is a Transitive relation.

b. For the second ordered pair, (2, 2), let's find another ordered pair which starts with 2. Another ordered pair which starts with 2 is (2, 1). Now, we have two ordered pairs (2, 2), and (2, 1). In order for the relation to be Transitive, it must contain the pair (2, 1). Clearly it contains this ordered pair, and hence this is a Transitive relation.
c. For the third ordered pair, (1, 2), let's find another ordered pair which starts with 2. We have two ordered pairs which start with 2 → (2, 2), and (2, 1).
   i. First, let's look at (1, 2), and (2, 2). For the relation to be transitive, we need (1, 2). Since we have it, it is a transitive relation.
   ii. Next, let's look at (1, 2), and (2, 1). For the relation to be transitive, we need (1, 1). Since we have it, it is a transitive relation.
d. For the fourth ordered pair, (2, 1), let's find another ordered pair which starts with 1. We have two ordered pairs which start with 2 → (1, 1), and (1, 2).

i. First, let's look at (2, 1), and (1, 1). For the relation to be transitive, we need (2, 1). Since we have it, it is a transitive relation.
ii. Next, let's look at (2, 1), and (1, 2). For the relation to be transitive, we need (2, 2). Since we have it, it is a transitive relation.

Question 42

Let $A = \{1, 2, 3\}$, then $R_3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$

(a) Only Symmetric
(b) Reflexive & Symmetric
(c) Reflexive & Transitive
(d) Symmetric & Transitive

Solution

(b)

1. This is a Reflexive relation because every element in set A is related to itself. Also, there exists some additional pairs and both the elements of these additional pairs belong to set A.
2. This relation is a symmetric relation because:

a. For the first ordered pair (1, 1), we need (1, 1) to make the relation symmetric. We already have it.
b. For the second ordered pair (2, 2), we need (2, 2) to make the relation symmetric. We already have it.
c. For the third ordered pair (3, 3), we need (3, 3) to make the relation symmetric. We already have it.
d. For the fourth ordered pair (1, 2), we need (2, 1) to make the relation symmetric. We have that as well.
e. For the fifth ordered pair (2, 1), we need (1, 2) to make the relation symmetric. We have that as well.
f. For the sixth ordered pair (2, 3), we need (3, 2) to make the relation symmetric. We have that as well.

g. For the seventh ordered pair (3, 2), we need (2, 3) to make the relation symmetric. We have that as well.

3. This relation is a NOT transitive relation because:
   a. For the first ordered pair, (1, 1), let's find another ordered pair which starts with 1. Another ordered pair which starts with 1 is (1, 2). Now, we have two ordered pairs (1, 1), and (1, 2). In order for the relation to be Transitive, it must contain the pair (1, 2). Clearly it contains this ordered pair, and hence this is a Transitive relation.
   b. For the second ordered pair, (2, 2), let's find another ordered pair which starts with 2. We have two ordered pairs which start with 2 → (2, 1), and (2, 3).
      i. First, let's look at (2, 2), and (2, 1). For the relation to be transitive, we need (2, 1). Since we have it, it is a transitive relation.

ii. Next, let's look at (2, 2), and (2, 3). For the relation to be transitive, we need (2, 3). Since we have it, it is a transitive relation.

c. For the third ordered pair, (3, 3), let's find another ordered pair which starts with 3. Another ordered pair which starts with 3 is (3, 2). Now, we have two ordered pairs (3, 3), and (3, 2). In order for the relation to be Transitive, it must contain the pair (3, 2). Clearly it contains this ordered pair, and hence this is a Transitive relation.

d. For the fourth ordered pair, (1, 2), let's find another ordered pair which starts with 2. We have three ordered pairs which start with 2 → (2, 2), (2, 1) and (2, 3).

i. First, let's look at (1, 2), and (2, 2). For the relation to be transitive, we need (1, 2). Since we have it, it is a transitive relation.

ii. Next, let's look at (1, 2), and (2, 1). For the relation to be transitive, we need (1, 1). Since we have it, it is a transitive relation.
iii. Next, let's look at (1, 2), and (2, 3). For the relation to be transitive, we need (1, 3). Since we don't have it, it is NOT a transitive relation.

Question 43

“is perpendicular to” over the set of straight lines in a given plane is:

(a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence

Solution

(b)

1. Since no line is perpendicular to itself, this relation cannot be Reflexive.
2. If line A is perpendicular to line B, then obviously line B will also be perpendicular to line A. Therefore, this relation is Symmetric.
3. If line A is perpendicular to line B, and line B is perpendicular to line C, then lines A and C will be parallel to each other and not perpendicular. Therefore, this relation is not transitive.

Question 44

"is the reciprocal of" over the set of non-zero real numbers is:

(a) Symmetric (b) Reflexive (c) Transitive (d) None

Solution

(a)

1. Since no number is reciprocal of itself, this relation cannot be Reflexive.
2. If, say, 2 is the reciprocal of ½, then, obviously, ½ is the reciprocal of 2. Therefore, this relation is Symmetric.
3. Clearly, this relation is not transitive.

Question 45

"Is smaller than" over the set of eggs in a box is:

(a) Transitive (b) Symmetric (c) Reflexive (d) Equivalence

Solution

(a)

1. Since no egg can be smaller than itself, this relation cannot be reflexive.
2. If, say, egg A is smaller than egg B, then obviously, egg B cannot be smaller than egg A. Therefore, this relation is not symmetric.
3. If, say, egg A is smaller than egg B, and egg B is smaller than egg C, then, obviously, egg A would be smaller than egg C. Therefore, this relation is transitive.

Question 46

"Is parallel to" over the set of straight lines is:

(a) Transitive (b) Symmetric (c) Reflexive (d) Equivalence

Solution

(d)

Question 47

“Is equal to” over the set of all rational numbers is

(a) Transitive (b) Symmetric (c) Reflexive (d) Equivalence

Solution

(d)

Question 48

“has the same father as” over the set of children:

(a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence

Solution

(d)

Question 49

$\{(x, y) : y = x\}$ is:

(a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence

Solution

(d)

Question 50

$\{(x, y) : x + y = 2x$ where x and y are positive integers$\}$, is:

(a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence

Solution

(d)

Question 51

"Is the square of" over *n* set of real numbers is:

(a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence

Solution

(d)

Question 52

Let A = {1, 2, 3} and R = {(1, 2), (2, 2), (3, 1), (3, 2)}. Find the Domain and Range of R^{-1}.

(a) $Dom\left(R^{-1}\right)=\{2, 1\}$; $Range\left(R^{-1}\right)=\{1, 2, 3\}$

(b) $Dom\left(R^{-1}\right)=\{2, 3\}$; $Range\left(R^{-1}\right)=\{1, 2, 3\}$

(c) $Dom\left(R^{-1}\right)=\{1, 3\}$; $Range\left(R^{-1}\right)=\{1, 2, 3\}$

(d) None

Solution

(a)

$R^{-1}=\{(2, 1), (2, 2), (1, 3), (2, 3)\}$

$$Dom\left(R^{-1}\right) = \{2, 1\}$$

$$Range\left(R^{-1}\right) = \{1, 2, 3\}$$

Questions Based on Functions

CA NISHANT KUMAR

Question 53

If $f(x)=x^2-1$, and $g(x)=\frac{x+1}{2}$, then $\frac{f(3)}{f(3)+g(3)}$ is:

(a) 5/4 (b) 4/5 (c) 3/5 (d) 5/3

Solution

(b)

$$\frac{f(3)}{f(3)+g(3)}=\frac{3^2-1}{(3^2-1)+\frac{3+1}{2}}=\frac{9-1}{(9-1)+\frac{4}{2}}=\frac{8}{8+2}=\frac{8}{10}=\frac{4}{5}$$

Question 54

If $f(x)=\left(\frac{x^2-4}{x-2}\right)$, then $f(2)$ is:

(a) 0 (b) 2 (c) 4 (d) 1

Solution

(c)

Question 55

If $f(x) = x + 3,\ g(x) = x^2$, then $f(x).g(x)$ is:

(a) $(x+3)^2$ (b) $x^2 + 3$ (c) $x^3 + 3x^2$ (d) None

Solution

(c)

Question 56

Let $f: R \rightarrow R$ be defined by:

$$f(x)=\begin{cases} 2x \text{ for } x>3 \\ x^2 \text{ for } 1<x\leq 3 \\ 3x \text{ for } x\leq 1 \end{cases}$$

The value of $f(-1)+f(2)+f(4)$ is:

(a) 9 (b) 14 (c) 5 (d) 6

Solution:

(a)

Question 57

Let N be the set of all natural numbers; then is the rule $f: N \to N : f(x) = 2x \forall x \in N$ a function?

(a) Yes (b) No (c) Can't Say (d) None

Solution

(a)

Question 58

Let $X = \{1, 2, 3, 4\}$ and $Y = \{1, 2, 3\}$. Is the relation $\{(1, 2), (1, 3), (2, 3)\}$ a function from X to Y?

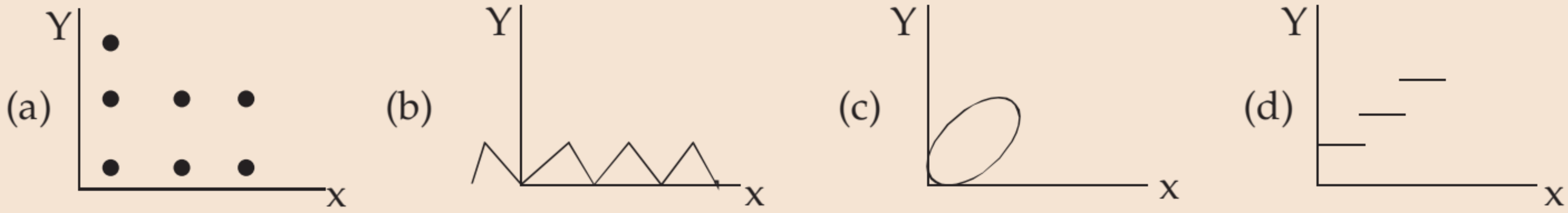
(a) Yes (b) No (c) Can't Say (d) None

Solution

(b)

Question 59

Which of the diagram is graph of a function



Solution

(b)

Question 60

Let A = {1, 2, 3, 4} and B = {1, 4, 9, 16, 25}. Consider the rule $f(x) = x^2$. Find the domain and range of the function.

(a) Domain = {1, 2, 3, 4}; Range = {1, 4, 9, 16}
(b) Domain = {1, 2, 3, 4}; Range = {1, 4, 9, 16, 25}
(c) Domain = {1, 2, 3, 4}; Range = {1, 4, 9}
(d) None

Solution

(a)

Question 61

The domain and range of $\left\{(x,\ y) : y = x^2\right\}$ where $x, y \in R$ is:

(a) (Reals, Natural Numbers)
(b) (Reals, Non-Negative Reals)
(c) (Reals, Reals)
(d) None

Solution

(b)

Question 62

The range of {(3, 0), (2, 0), (1, 0), (0, 0)} is:

(a) {0, 0} (b) {0} (c) {0, 0, 0, 0} (d) None

Solution

(b)

Question 63

The range of the function $f\left(x\right)=\log_{10}\left(1+x\right)$ for the domain of real values of x when $0\leq x\leq 9$ is:

(a) $\left[0, 1\right]$ (b) $\left[0, 1, 2\right]$ (c) $\left\{0, -1\right\}$ (d) None

Solution

(a)

Question 64

For the function $h(x)=10^{1+x}$, the domain of real values of x where $0\le x\le 9$, the range is:

(a) $10\le h(x)\le 10^{10}$ (b) $0\le h(x)\le 10^{10}$ (c) $0<h(x)<10$ (d) None

Solution

(a)

Question 65

Let A = {1, 2, 3} and B = {2, 4, 6}. Consider $f : A \to B : f(x) = 2x$. Is this a one-one function?

(a) Yes (b) No (c) Can't Say (d) None

Solution

(a)

Question 66

$\{(x,\ y) \mid x + y = 5\}$ where $x, y \in R$ is:

(a) Not a function (b) Composite function (c) One-one mapping (d) None

Solution

(c)

Question 67

The function $f\left(x\right)=2^{x}$ is:

(a) One-one mapping (b) One-many (c) Many-one (d) None

Solution

(a)

Question 68

Let N be the set of all natural numbers and E be the set of all even natural numbers. Then, the function $f : N \to E : f(x) = 2x \forall x \in N$ is:

(a) Onto (b) Into (c) Can't Say (d) None

Solution

(a)

Question 69

Let A = {2, 3, 5, 7}, B = {0, 1, 3, 5, 7}. Then, the function $f : A \rightarrow B : f(x) = x - 2$ is:

(a) Onto (b) Into (c) Can't Say (d) None

Solution

(b)

Question 70

Let $A = \{1, 2, 3\}$, $B = \{5, 7, 9\}$. Then, the function $f: A \to B: f(x) = 5$ for all $x \in A$ is:

(a) One-one (b) Onto (c) Constant function (d) None

Solution

(c)

Question 71

If $f\left(x\right)=1/1-x$ and $g\left(x\right)=\left(x-1\right)/x$, then $fog\left(x\right)$ is:

(a) x (b) $1/x$ (c) $-x$ (d) None

Solution

(a)

Question 72

If $f(x) = 1/1 - x$ and $g(x) = (x-1)/x$, then $gof(x)$ is:

(a) $x - 1$ (b) x (c) $1/x$ (d) None

Solution

(b)

Question 73

If $f(x) = x + 3$, and $g(x) = x^2$, then $fog(x)$

(a) $x^2 + 3$ (b) $x^2 + x + 3$ (c) $(x+3)^2$ (d) None

Solution

(a)

Question 74

If $f(x) = x + 3,\ g(x) = x^2$, then $gof(x)$ is:

(a) $(x+3)^2$ (b) $x^2 + 3$ (c) $x^2(x+3)$ (d) None

Solution

(a)

Question 75

Find gof for the functions $f(x)=\sqrt{x},\ g(x)=2x^2+1$

(a) $2x^2+1$ (b) $2x+1$ (c) $\left(2x^2+1\right)\left(\sqrt{x}\right)$ (d) $\sqrt{x}$

Solution

(b)

$$gof(x)=2\times\{f(x)\}^2+1$$

$$\Rightarrow gof(x)=2\times\left(\sqrt{x}\right)^2+1$$

$$\Rightarrow gof\left(x\right)=2\times x+1$$

$$\Rightarrow gof\left(x\right)=2x+1$$

Question 76

Let R be the set of real numbers such that the function $f:R\to R$ and $g:R\to R$ are defined by $f(x)=x^2+3x+1$ and $g(x)=2x-3$. Find (fog).

(a) $4x^2+6x+1$ (b) x^2+6x+1 (c) $4x^2-6x+1$ (d) x^2-6x+1

Solution

(c)

Question 77

If $A = \{1, 2, 3, 4\}$; $B = \{2, 4, 6, 8\}$; $f(1) = 2$; $f(2) = 4$; $f(3) = 6$; $f(4) = 8$; and $f : A \to B$, then find f^{-1}.

(a) $f^{-1} = \{(2, 1), (4, 2), (6, 3), (8, 4)\}$

(b) $f^{-1} = \{(2, 1), (4, 2), (6, 3), (3, 4)\}$

(c) $f^{-1} = \{(3, 1), (4, 2), (6, 3), (3, 4)\}$

(d) None

Solution

(a)

Question 78

Find the inverse of $f\left(x\right)=2x$ is:

(a) $1/2x$ (b) $\frac{x}{2}$ (c) $1/x$ (d) None

Solution

(b)

Question 79

The inverse h^{-1} when $h(x) = \log_{10} x$ is:

(a) $\log_{10} x$ (b) 10^x (c) $\log_{10}(1/x)$ (d) None

Solution

(b)

Question 80

If $f\left(x\right)=1/1-x$, then $f^{-1}\left(x\right)$ is:

(a) $1-x$ (b) $\left(x-1\right)/x$ (c) $x/\left(x-1\right)$ (d) None

Solution

(b)

Question 81

The inverse function f^{-1} of $f(y) = 3y$ is:

(a) $1/3y$

(b) $y/3$

(c) $-3y$

(d) $1/y$

Solution:

(b)

Question 82

A function $f\left(x\right)$ is an even function, if:

(a) $-f\left(x\right)=f\left(x\right)$ (b) $f\left(-x\right)=f\left(x\right)$ (c) $f\left(-x\right)=-f\left(x\right)$ (d) None

Solution

(b)

Number Series, Coding-Decoding, Odd Man Out

CA NISHANT KUMAR

Question 1 – January, 2021

In a certain code RIPPLE is written as 613382 and LIFE is written as 8192. How will RIFFLE be written in that code?

(a) 618892 (b) 689912 (c) 619982 (d) 629981

Solution

(c)

Question 2 – January, 2021

In a certain code language, BEAT is written as YVZG, then what will be code for MILD?
(a) ONRW (b) NOWR (c) ONWR (d) NROW

Solution

(d)

Backward	26	25	24	23	22	21	20	19	18	17	16	15	14
Forward	1	2	3	4	5	6	7	8	9	10	11	12	13
	A	B	C	D	E	F	G	H	I	J	K	L	M
	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Forward	14	15	16	17	18	19	20	21	22	23	24	25	26
Backward	13	12	11	10	9	8	7	6	5	4	3	2	1

B is the second letter moving forwards, and Y is the second letter moving backwards. Similarly, E is the 5^{th} letter moving forwards, V is the 5^{th} letter moving backwards. A is the first letter moving forwards, and Z is the first letter moving backwards. T is the 20^{th} letter moving forwards, and G is the 20^{th} letter moving backwards.

Now, we have to code MILD.

M is the 13^{th} letter moving forwards, N is the 13^{th} letter moving backwards. I is the 9^{th} letter moving forwards, R is the 9^{th} letter moving backwards. L is the 12^{th} letter moving forwards, O is the 12^{th} letter moving backwards. D is the 4^{th} letter moving forwards, W is the 4^{th} letter moving backwards.

Therefore, the code for MILD is NROW.

Question 3 – January, 2021

Find out the odd man out in the sequence 8, 25, 64, 125, 216.

(a) 25 (b) 64 (c) 125 (d) 216

Solution

(a)

The sequence is $2^3 = 8$; $3^3 = 27$; $4^3 = 64$; $5^3 = 125$; $6^3 = 216$

In place of 27, we have 25, therefore, 25 is the odd one out.

Question 4 – January, 2021

Find the missing term: P3C, R5F, T8I, V12L, ?

(a) Y17O (b) X17M (c) X17O (d) X16O

Solution

(c)

First letter → P + 2 = R; R + 2 = T; T + 2 = V; V + 2 = X

Number → 3 + 2 = 5; 5 + 3 = 8; 8 + 4 = 12; 12 + 5 = 17

Third letter → C + 3 = F; F + 3 = I; I + 3 = L; L + 3 = O

Therefore, X17O

Question 5 – December, 2020

Find the next term 1, 5, 21, 57, ?

(a) 105 (b) 138 (c) 121 (d) 101

Solution

(c)

1 → 5 → 21 → 57 → 121

+4 +16 +36 +64

⇓ ⇓ ⇓ ⇓

2^2 4^2 6^2 8^2

Question 6 – December, 2020

Find the wrong term in G4T, J10R, M20P, P43N, S90L

(a) M20P (b) P43N (c) J10R (d) G4T

Solution

(c)

Look at the numbers 4, 10, 20, 43, 90

The pattern of this series is as follows:

4

$(4 \times 2) + 1 = 8 + 1 = 9$

$(9 \times 2) + 2 = 18 + 2 = 20$

$$\left(20\times 2\right)+3=40+3=43$$

$$\left(43\times 2\right)+4=86+4=90$$

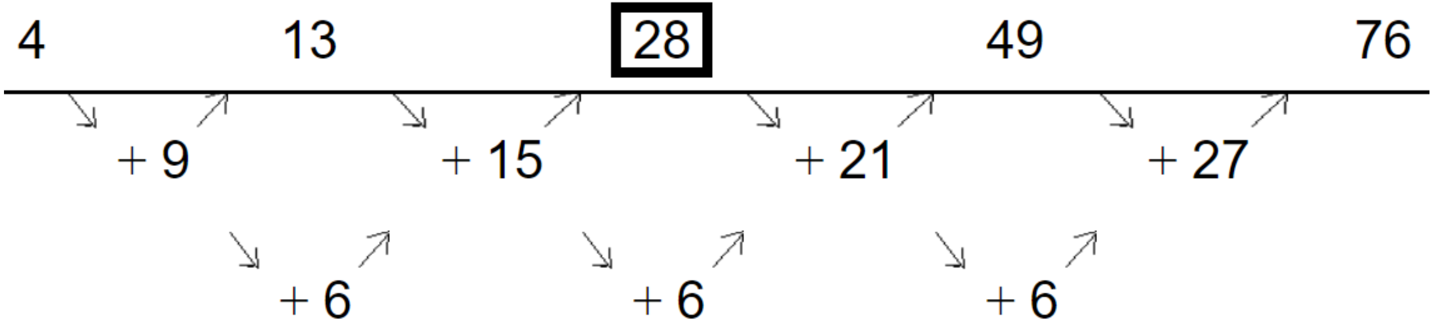
Question 7 – December, 2021

The missing term of the series 4, 13, _________, 49, 76 is:

(a) 26 (b) 28 (c) 30 (d) 32

Solution

(b)



Question 8 – MTP June, 2023

If GOODNESS is coded as HNPCODTR, then how GREATNESS can be written in that code?

(a) HQZSMFRT (b) HQFZUFRTM (c) HQFZUODTR (d) HQFZUMFRT

Solution

(d)

G	O	O	D	N	E	S	S
↓+1	↓–1	↓+1	↓–1	↓+1	↓–1	↓+1	↓–1
H	N	P	C	O	D	T	R

G	R	E	A	T	N	E	S	S
↓+1	↓–1	↓+1	↓–1	↓+1	↓–1	↓+1	↓–1	↓+1
H	Q	F	Z	U	M	F	R	T

Question 9 – MTP June, 2023

In certain code language, if TOUR, is written as 1234, CLEAR is written 5678 and SPARE is written as 90847, Find the code for TEARS?

(a) 17847 (b) 14847 (c) 15247 (d) 17849

Solution

(d)

T	O	U	R		C	L	E	A	R		S	P	A	R	E
↓	↓	↓	↓		↓	↓	↓	↓			↓	↓	↓	↓	↓
1	2	3	4		5	6	7	8			9	0	8	4	7

From the above, we can see that the code of R is the word CLEAR should also have been 4, as R is coded 4 is TOUR as well as in SPARE.

Therefore, the code for TEARS would be:

T	E	A	R	S
↓	↓	↓	↓	↓
1	7	8	4	9

Question 10 – MTP June, 2023

Find the missing number in the following series: 2, 5, 10, 17, 26, ?

(a) 49 (b) 47 (c) 37 (d) 36

Solution

(c)

Clearly, we can see that every number is increasing by +3 , +5, +7, +9, and so on…

Therefore,

2 + 3 = 5

5 + 5 = 10

10 + 7 = 17

17 + 9 = 26

26 + 11 = 37

Question 11 – MTP June, 2023

Find the odd man out: 34, 105, 424, 2125, 12755

(a) 12755 (b) 2125 (c) 424 (d) 34

Solution

(a)

The pattern is:

(34 × 3) + 3 = 102 + 3 = 105

(105 × 4) + 4 = 420 + 4 = 424

(424 × 5) + 5 = 2,120 + 5 = 2,125

(2,125 × 6) + 6 = 12,750 + 6 = 12,756

Therefore, the last term should be 12,756, and not 12,755. Hence, 12,755 is the odd one out.

Question 12 – MTP June, 2023

Find next term of the series 10, 69, 236, 595, ?

(a) 1254 (b) 1020 (c) 1320 (d) 1200

Solution

(a)

10	69	236	595	1254
↓	↓	↓	↓	↓
$3^2 + 1$	$8^2 + 5$	$15^2 + 11$	$24^2 + 19$	$35^2 + 29$

+5 +4 +7 +6 +9 +8 +11 +10

Question 13 – MTP June, 2023

In certain code language, BOARD is coded as CQDVI, what is the code for the word CONSULTING?

(a) DQQWZRARNQ (b) DQQWZARQWQ (c) DQQWZRAQWQ (d) None

Solution

(c)

B	O	A	R	D
↓+1	↓+2	↓+3	↓+4	↓+5
C	Q	D	V	I

C	O	N	S	U	L	T	I	N	G
↓+1	↓+2	↓+3	↓+4	↓+5	↓+6	↓+7	↓+8	↓+9	↓+10
D	Q	Q	W	Z	R	A	Q	W	Q

Question 14 – MTP June, 2023

In a certain code language if CAMP is written as 9, then in the same code how will the word TEAM be written?

(a) 14 (b) 19 (c) 27 (d) 33

Solution

(c)

C = 3; A = 1; M = 13; P = 16

Adding these, we get 3 + 1 + 13 + 16 = 33

Multiplying these digits, we get 3 × 3 = 9.

Therefore, the code of TEAM will be:

T = 20; E = 5; A = 1; M = 13

Adding these, we get 20 + 5 + 1 + 13 = 39

Multiplying these digits, we get $3 \times 9 = 27$.

Question 15 – MTP June, 2023

Which number will come next in the following series: 675, 623, 573, 525?

(a) 491 (b) 479 (c) 423 (d) 456

Solution

(b)

To identify the next number in the given series 675, 623, 573, 525, we need to analyze the pattern or rule behind the sequence. Let's examine the differences between consecutive numbers in the series:

675 – 623 = 52

623 – 573 = 50

573 – 525 = 48

From the differences, we observe a decreasing pattern where each difference is decreasing by 2. Therefore, we can assume that the next difference should be 48 – 2 = 46.

To find the next number, we subtract 46 from the last number in the series:

525 – 46 = 479

Hence, the next number in the series should be 479.

Question 16 – MTP June, 2023

105, 115.5, 150, 162.5, 203, ?

(a) 217 (b) 217.5 (c) 210.5 (d) None

Solution

(b)

105 115.5 150 162.5 203 217.5

+10.5 +34.5 +12. 5 +40.5 +14.5

+2 +6 +2

Question 17 – MTP May, 2019

GO = 32, SHE = 49, then SOME will be equal to:

(a) 56 (b) 58 (c) 62 (d) 64

Solution

(a)

Backward	26	25	24	23	22	21	20	19	18	17	16	15	14
Forward	1	2	3	4	5	6	7	8	9	10	11	12	13
	A	B	C	D	E	F	G	H	I	J	K	L	M
	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Forward	14	15	16	17	18	19	20	21	22	23	24	25	26
Backward	13	12	11	10	9	8	7	6	5	4	3	2	1

Step 1 – Decoding GO and SHE

G is the 20th alphabet from back; O is the 12th alphabet from back. Add 20 and 12, you get 32.

S is the 8th alphabet from back; H is the 19th alphabet from back; E is the 22nd alphabet from back. Add 8, 19, and 22, you get 49.

Step 2 – Coding SOME

S is the 8th alphabet from back; O is the 12th alphabet from back; M is the 14th alphabet from back; E is 22nd alphabet from back. Add 8, 12, 14, and 22, you get 56.

Question 18 – MTP November, 2019

a_c_ba_ca_cb

(a) *abcc* (b) *acba* (c) *bcaa* (d) *bcba*

Solution

(d)

The sequence is made of repetition of *abc* in different orders.

Therefore, identify the missing alphabet in *abc*.

The first three positions are *a_c*. Clearly, the letter *b* is missing.

The next three positions are _*ba*. Clearly, the letter *c* is missing.

The next three positions are _*ca*. Clearly, the letter *b* is missing.

The next three positions are _*cb*. Clearly, the letter *a* is missing.

Therefore, the missing letters are *bcba*.

Question 19 – MTP December, 2020

Find the alphabet missing series *ac_cab_baca_aaa_aba*

(a) *aabc* (b) *aacb* (c) *babb* (d) *bcbb*

(a)

The series is *aca / cab / aba / cab / aaa / cab / a*

Question 20 – ICAI SM

aab__aaa__bba__

(a) *baa* (b) *abb* (c) *bab* (d) *bba*

Solution

(a)

The series is *aabb*, *aaaa*, *bbaa*

Therefore, the answer is *baa*.

Direction Sense Test

CA NISHANT KUMAR

Question 1

One day, Ram ran away from home. He first ran 10 km to South, then turned right at 45°, and ran for 10 km, then turned right at 45° and ran for 10 km. In which direction is he facing now?

(a) North (b) East (c) West (d) South

Solution

(c)

Question 2

I started walking down a road in the morning facing the Sun. After walking for sometime I turned to my left. Then I turned to my right. In which direction am I from the starting point?

(a) North-East (b) North-West (c) South-West (d) South-East

Solution

(a)

Question 3 – January, 2021

Ms. N walks 10 km towards North from there she walks 6 km towards South. Then she walks 3 km towards East. How far and in which direction is she with reference to her starting point?

(a) 4 km West (b) 6 km West (c) 3 km West (d) 5 km North-East

(d)

Question 4 – December, 2020

A man can walk by having long, medium and short steps. He can cover 60 meters by 100 long steps, 100 meters by 200 medium steps and 80 meters by 200 short steps. He walks taking 5000 long steps, then he turns left and walks by taking 6000 medium steps. He then turns right and walk by taking 2500 short steps. How far (in meters) is he away from his starting point?

(a) 5000 m (b) 4000 m (c) 6000 m (d) 7000 m

(a)

Question 5 – ICAI SM

Sun rises behind the tower and sets behind the railway station. In which direction is the tower from railway station?

(a) North (b) South (c) East (d) West

(c)

Question 6 – ICAI SM

A car travelling from south covers a distance of 8 kms, then turns right and runs another 9 kms and again turns to the right and was stopped. Which direction does it face now?

(a) South (b) North (c) West (d) East

Solution

(a)

Question 7 – MTP November, 2020

If East is replaced by South-East, then West will be replaced by which replaced by which of the following directions?

(a) North East (b) North (c) East (d) North West

(d)

Question 8 – MTP June, 2021

One evening before sunset, two friends Ravi and Raj were talking to each other face to face. If Ravi's shadow was exactly to his left side, which direction was Raj facing?

(a) West (b) East (c) North (d) South

Solution

(c)

Question 9 – June, 2019; MTP June, 2021; MTP December, 2021

Ramu moved a distance of 75 meters towards North. He then turned to left and walked for about 25 m, turned left again and walks 80 m. Finally, he turned to the right at an angle of 45°. In which direction was he moving finally?

(a) South-East (b) South-West (c) North-West (d) North-East

Solution

(b)

Question 10 – December, 2020; January, 2021

A man is facing west. He turns 45° in the clockwise direction and then another 180° in the same direction and then 270° in the anti-clockwise direction. Which is the facing now?

(a) South-West (b) North-West (c) West (d) South

(a)

Question 11 – June, 2022

A person facing North moves 70º in clockwise direction. He again moved 300º in clockwise direction. In which direction is he facing now?

(a) North – West (b) South – East (c) North – East (d) South – West

Solution

(c)

Question 12 – ICAI SM

If A stands on his head with his face towards North, in which direction will his left hand point?

(a) North-East (b) North (c) East (d) North-West

Solution

(c)

Question 13 – July, 2021

One morning after sunrise, Vikram and Shailesh were standing in a lawn with their backs towards each other. Vikram's shadow fell exactly towards left hand side. Which direction was Shailesh facing?

(a) East (b) West (c) North (d) South

Solution

(d)

In the morning, the sun would be on the East. This implies that any shadow would fall on West. Since Vikram's shadow fell on his left, this means that his left is towards West, right is towards East, and face is towards North. Since Vikram and Shailesh are standing with their backs towards each other, Shailesh would face opposite to Vikram, i.e., South.

Question 14 – December, 2022

It is 3 o'clock in a watch. If the minute hand points towards the North-East, then the hour hand will point towards the:

(a) South (b) South-West (c) North-West (d) South-East

Solution

(d)

If the minute hand points towards North-East, the hour hand would point towards South-East as the angle between North-East and South-East is 90 degrees.

Question 15 – June, 2022 (Similar)

A direction pole was situated on the crossing. Due to an accident, the pole turned in such a manner that the pointer, which was showing East, started showing North. One traveller went to the wrong direction thinking it to be West. In what direction was he actually travelling?

(a) North (b) South (c) East (d) West

Solution

(b)

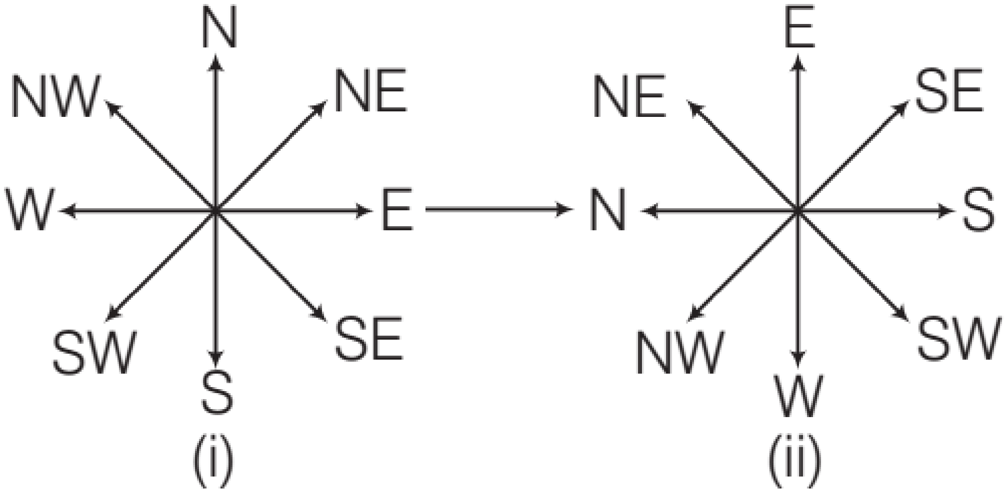


Fig. (i) is the original position and Fig (ii) is the position after the accident.

West direction shown by the pole in actual is South direction.

Question 16

Two ladies and two men are playing cards and are seated at North, East, South and West of a table. No lady is facing East. Persons sitting opposite to each other are not of the same gender. One man is facing South. Which directions are the ladies facing?

(a) East and West (b) South and East (c) North and East (d) North and West

Solution

(d)

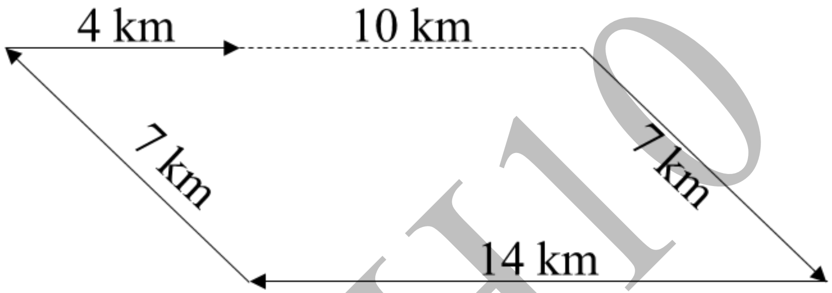
Question 17 – MTP June, 2023

Ram moves towards South-East a distance of 7 km, then he moves towards West and travels a distance of 14 km. From there he moves towards North–West for a distance of 7 km and finally he moves a distance of 4 km towards east. How far is he now from the starting point?

(a) 3 km (b) 4 km (c) 10 km (d) 11 km

Solution

(c)


4 km
10 km
7 km
7 km
14 km

Question 18 – MTP June, 2023

One morning a boy starts walking in a particular direction for 5 Km and then takes a left turn and walks another 5 Km. Thereafter, he again takes left turn and walks another 5 Km and at last he takes right turn and walks 5 Km. Now he sees his shadow in front of him. What direction he did start initially?

(a) South (b) North (c) East (d) West

Solution

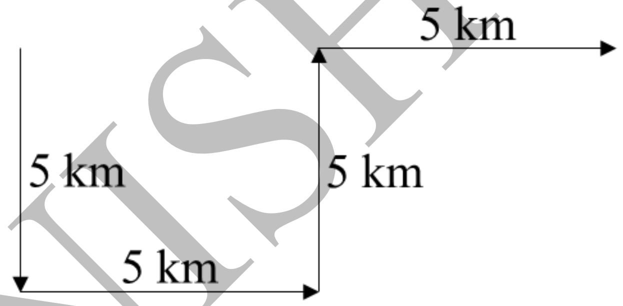
(b)

In the morning, the sun is in the East direction. Therefore, the shadow would be formed in the West direction. If the boy sees his shadow in front of him, he must be facing the west direction.

Now, try the options.

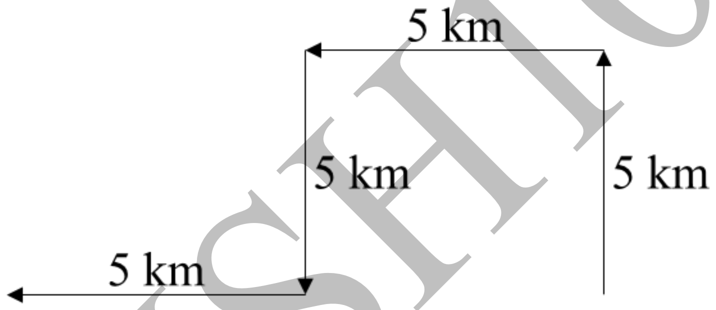
Option (a) → South

If the boy starts walking in the South direction, he would be facing the East Direction as shown below:



Option (b) → North

If the boy starts walking in the North direction, he would be facing the West direction as shown below:



Therefore, option (b) is the answer.

Question 19 – July, 2021

A and B start moving towards each other from two places 200 m apart. After walking 60 m, B turns left and goes 20 m, then he turns right and goes 40 m. He then turns right again and comes back to the road on which he had started walking. If A and B walk with the same speed, what is the distance between them now?

(a) 80 m (b) 70 m (c) 40 m (d) 60 m

Solution

(c)

Since both A and B are travelling at the same speed, the total distance covered by B will be equal to the total distance covered by A.

Total distance covered by B is 60 m + 20 m + 40 m + 20 m = 140 m

A has travelled the same distance, but on the same straight line.

Distance travelled by B on the same straight line is 60 + 40 = 100 m.

Therefore, the distance between A and B on the straight line is 140 m – 100 m = 40 m.

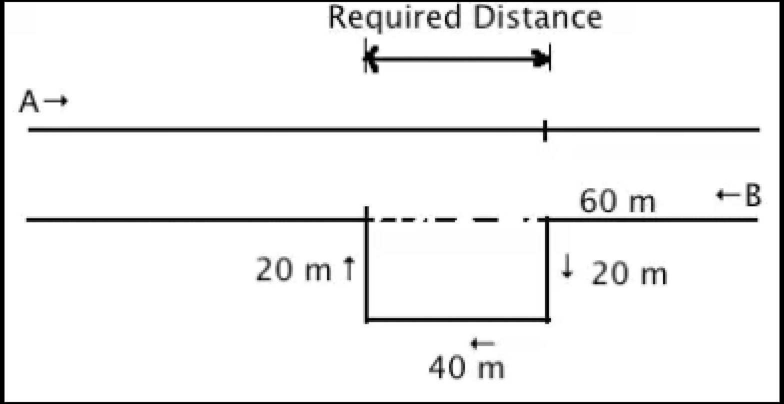
Correct option is C 40 m

Distance traveled by A on road $= 60 + 20 + 40 + 20 = 140$ m

Distance traveled by B on road $= 60 + 40 = 100$ m

Required difference $= 140 - 100 = 40$ m

Hence, option 'C' is correct.



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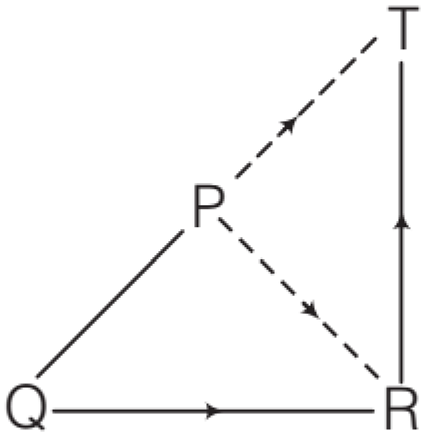
Question 20

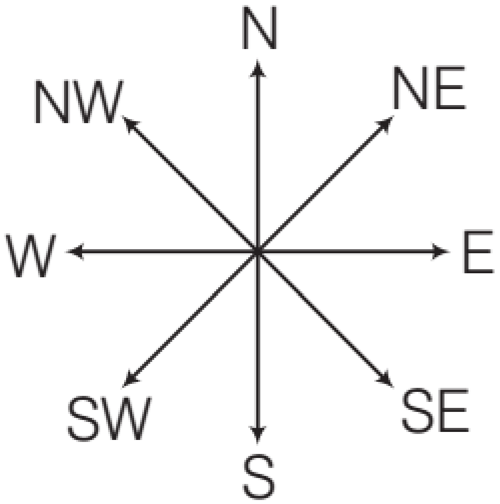
There are four towns P, Q, R, and T. Q is to the South-West of P, R is to the East of Q and South-East of P and T is to the North of R in line with QP. In which direction of P is T located?

(a) South-East (b) North (c) North-East (d) East

Solution

(c)





T is in the North-East of P.

Seating Arrangements

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Question 1 – January, 2021

Five friends A, B, C, D and E are sitting on a bench. A is sitting next to B; C is sitting next to D, D is not sitting with E; E is at the left end of bench. C is on second position from the right; A is on the right side of B who is the right side of E. A and C are sitting together. What is the position of B?

(a) Second from Right (b) Centre (c) Extreme Left (d) Second from Left

Solution

(d)

Question 2 – ICAI SM

Four Indians, A, B, C and D and four Chinese E, F, G and H are sitting in a circle around a table facing each other in a conference. No two Indians or Chinese are sitting side by side. C who is sitting between G and E is facing D, F is between D and A and facing G, H is to the left of B. Who is sitting left of A?

(a) E (b) F (c) G (d) H

Solution

(a)

Question 3 – January, 2021

A is seated between D and F at a round table. C is seated opposite to D. E is left to D. Who sits opposite to B?

(a) A (b) D (c) C (d) F

Solution

(a)

Question 5 – December, 2020

Eight friends I, J, K, L, M, N, O and P are sitting in a circle facing the centre. J is sitting between O and L; P is third to the left of J and second to the right of I; K is sitting between I and O; J & M are not sitting opposite to each other. Which of the following statements is NOT correct?

(a) K is sitting third to the right of L
(b) I is sitting between K and N
(c) L and I are sitting opposite to each other
(d) M is sitting between N and L

(d)

Question 6 – December, 2020

Five girls G, H, I, J, K are sitting in a row facing south not necessarily in the same order. H is sitting between G and K; I is immediate right to K; J is immediate left to G. Which of the following is true?

(a) J is third to the left of K
(b) G is second to the left of I
(c) H is to the right of K
(d) H is to the left of G

Solution

(a)

Question 7 – November, 2019

5 persons are standing in a line. One of the 2 persons at the extreme ends is a professor and the other is a businessman. An advocate is standing to the right of a student. An author is to the left of the business man. The student is standing between the professor and advocate. Counting from left, the author is at which place?

(a) 2^{nd} (b) 3^{rd} (c) 4^{th} (d) None

(c)

Question 8 – November, 2019

Six persons are sitting in a circle facing the center. Parikh is between Bablu and Narender; Ashok is between Chitra and Pankaj. Chitra is on the immediate left of Bablu. Who is on the immediate right of Bablu?

(a) Parikh (b) Pankaj (c) Narender (d) Chitra

(a)

Question 9 – November, 2018

Eight persons A, B, C, D, E, F, G and H are sitting in a line. E sits second right to D. H sits fourth left to D. C and F are immediate neighbours, but C is not immediate neighbour of A. G is not neighbour of E. Only two persons sit between A and E. The persons on left end and right end respectively are:

(a) G and E (b) B and E (c) H and E (d) G and B

(a)

Question 10 – November, 2018

Six flats on a floor in two rows facing North and South are allotted to P, Q, R, S, T and U. Q gets a North facing flat and it is not next to S. S and U get diagonally opposite flat. R next to U gets a South facing flat and T gets a North facing flat. Who's flat is between Q and S?

(a) T (b) U (c) R (d) P

(a)

Question 11 – MTP November, 2020

Five people A, B, C, D and E are seated about a round table. Every chair is spaced equidistant from adjacent chairs.

1. C is seated next to A
2. A is seated two seats from D
3. B is not seated next to A

Which of the following must be true?

1. D is seated next to B
2. E is seated next to A

Select the correct answer from the codes given below:

(a) Only 1 (b) Only 2 (c) Both (d) Neither

(c)

Question 12 – ICAI SM

Ten students are A to J are sitting in a row facing west.

1. B and F are not sitting on either of the edges.
2. G is sitting left of D and H is sitting to the right of J.
3. There are four persons between E and A.
4. I is the north of B and F is the south of D.
5. J is between A and D and G is in E and F.
6. There are two persons between H and C.

If G and A interchange their positions, then who become the immediate neighbours of E?

(a) G and F (b) Only F (c) Only A (d) J and H

Solution

(c)

E	G	F	D	J	A	H or C	B	I	C or H

Question 13 – July, 2021

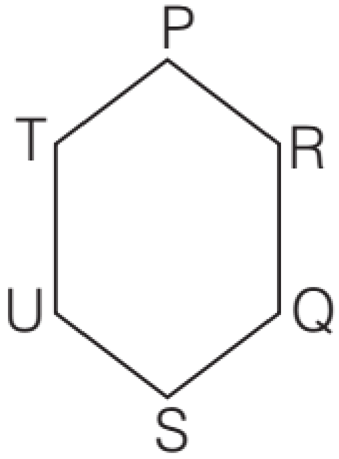
Six friends P, Q, R, S, T and U are sitting around a hexagonal table each at one corner and are facing the centre of the hexagon. P is second to the left of U. Q is the neighbour of R and S. T is second to the left of S.

Which one is sitting opposite to P?

(a) R (b) Q (c) T (d) S

Solution

(d)


P
T
R
U
Q
S

Question 14

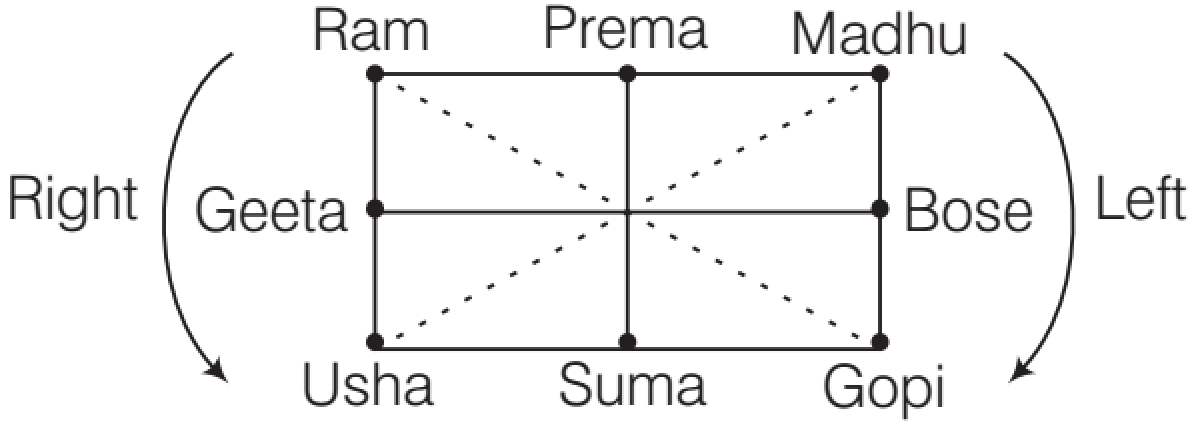
Four boys and four girls are sitting around a square facing the centre. One person is sitting at each corner and at the mid-point of each side of the square. Madhu is sitting diagonally opposite to Usha who is to the right of Geeta. Ram who is to the left of Geeta is diagonally opposite to Gopi who is to the left of Bose. Position of Suma is not to the right of Madhu but in front of Prema. Who is sitting opposite to Bose?

(a) Geeta (b) Prema (c) Getta (d) Madhu

Solution

(a)

Arrangement according to the question is as follows



Clearly, Geeta is sitting opposite to Bose.

Question 15

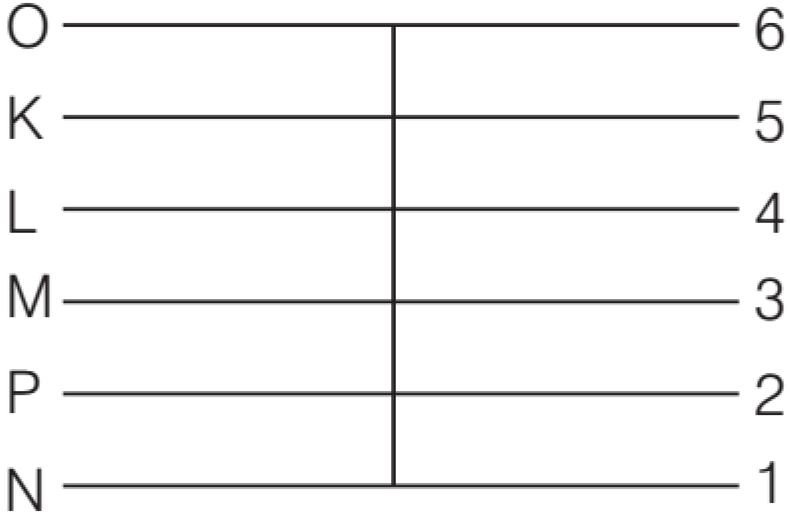
Six people – K, L, M, N, O and P live on six different floors of a building not necessarily in the same order. The lower most floor of the building is numbered 1, the one above is numbered 2 and so on till the top most floor numbered 6. L lives on an even numbered floor. L lives immediately below K's floor and immediately above M's floor. P lives immediately above N's floor. P lives on an even numbered floor. O does not live on floor number 4.

Three of the following four are alike in a certain way based on the given arrangement and hence form a group. Which of the following does not belong to that group?

(a) MN (b) OL (c) KM (d) PK

Solution

(d)



Question 16

A, B, C, D, E, F and G are sitting on a wall and all of them are facing East. C is on the immediate right of D. B is at an extreme end and has E as his neighbour. G is between E and F. D is sitting third from the South end.

Which of the following pairs of people is sitting at the extreme ends?

(a) AB (b) AE (c) CB (d) FB

Solution

(a)

B	E	G	F	D	C	A

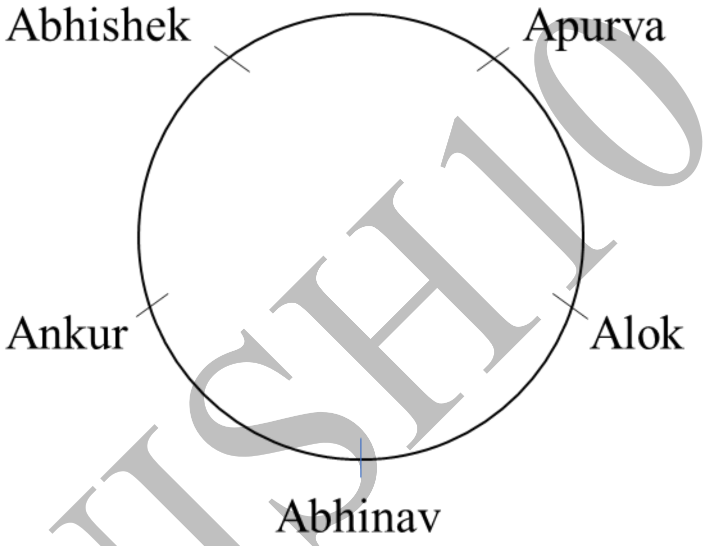
Question 17 – MTP June, 2023

Five students are standing in a circle. Abhinav is between Alok and Ankur. Apurva is on the left of Abhishek. Alok is on the left of Apurva. Who is sitting next to Abhinav on his right?

(a) Apurva (b) Ankur (c) Abhishek (d) Alok

Solution

(d)


Abhishek
Apurva
Ankur
Alok
Abhinav

Question 18 – MTP June, 2023

P, Q, R, S, and T are seated in a line facing west. R is sitting at north end and S is sitting at south end. T is neighbour of R and Q. P and Q are seated together, then who is sitting the middle?

(a) P (b) Q (c) R (d) S

Solution

(b)

If they are facing West, then North is to their right and South is to their left.

S	P	Q	T	R

Question 19 – MTP June, 2023

Eight friends A, B, C, D, E, F, G and H are sitting in a circle facing the Centre, B is sitting between G and D. H is third to the left of B and second to the right of A. C is sitting between A and G and B and E are not sitting opposite to each other. Which of the following statement is not correct ?

(a) D and A are sitting opposite to each other

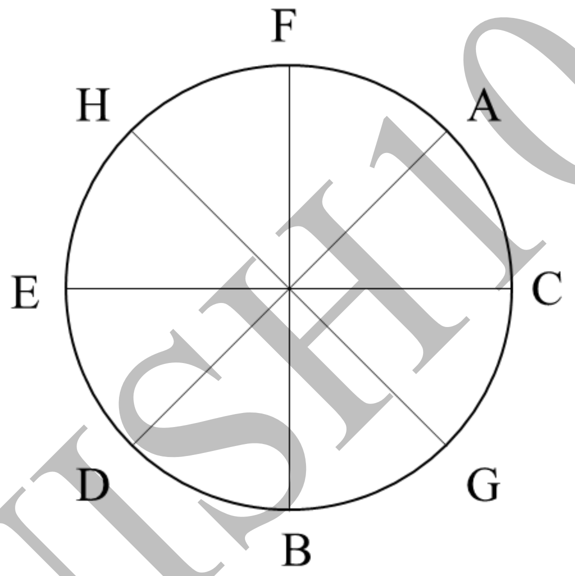
(b) C is third to the right of D

(c) E is sitting F and D

(d) A is sitting C and F

Solution

(c)



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Question 20 – MTP June, 2022

Study the following information carefully and answer the questions given below.

1. P, Q, R, S, T, U and V are sitting on a wall and all of them are facing West.
2. S is on the immediate left of R.
3. T is at an extreme end and has Q as his neighbour.
4. V is between Q and U.
5. S is sitting third from the north end.

Which of the following pairs of people are sitting at the extreme ends?

(a) QV (b) PR (c) TP (d) ST

Solution

(c)

T	Q	V	U	S	R	P

Blood Relations

Question 1 – December, 2020

Pointing to a lady, Sahil said, "She is the daughter of the woman who is the mother of the husband of my mother". What is the lady to Sahil?

(a) Aunt (b) Sister (c) Daughter (d) Sister-in-law

Solution

(a)

Question 2 – January, 2021

A girl introduced a boy as the son of the daughter of father of her uncle. The boy is girl's:

(a) Son (b) Brother (c) Son-in-law (d) Uncle

Solution

(b)

Question 3 – January, 2021

Pointing to a lady in a photograph, Ram said "Her son's father is the son in law of my mother". How is Ram related to the lady?

(a) Aunt (b) Cousin (c) Sister (d) Mother

Solution

(b)

Question 4 – January, 2021

P is the brother of Q and R, S is the mother of R. T is the father of P. Which of the following statements cannot be definitely true?

(a) S is the mother of P
(b) P is the son of S
(c) T is the husband of S
(d) Q is the son of T

Solution

(d)

Question 5 – December, 2020

Pointing to a lady, A said, “that woman is my nephew’s maternal grandmother”. How is that woman related to A’s sister who has no sister?

(a) Cousin (b) Son-in-law (c) Mother (d) Mother-in-law

Solution

(c)

Question 6 – June, 2019

A man said to a lady "your mother's husband's sister is my Aunt." How is the man related to the lady?

(a) Mother (b) Sister (c) Father (d) Brother

Solution

(d)

Question 7 – June, 2019

Pointing to an old man, Kailash said, “His son is my son’s uncle”. How is Kailash related to that old man?

(a) Brother (b) Either Son or Son-in-Law (c) Father (d) Grandfather

(b)

Question 8 – November, 2018

Pointing to a man in a photograph, a woman said, “the father of his brother is the only son of my grandfather”, how is the woman related to the man in the photograph?

(a) Mother (b) Aunty (c) Daughter (d) Sister

(d)

Question 9 – ICAI SM

A prisoner introduced a boy who came to visit him to the jailor as “Brothers and sisters I have none, he is my father’s son’s son”. Who is the boy?

(a) Nephew (b) Son (c) Cousin (d) Uncle

(b)

Question 10 – MTP June, 2021

In a family, there are six members A, B, C, D, E and F. A and B are a married couple, A being the male member. D is the only son of C, who is the brother of A. E is the sister of D. B is the daughter-in-law of F, whose husband has died. How is E related to C?

(a) Sister (b) Daughter (c) Cousin (d) Mother

Solution

(b)

Question 11 – MTP June, 2021

If A + B means A is brother of B, A – B means A is sister of B, and A × B means A is the father of B, which of the following means that C is the son of M?

(a) M – N × C + F (b) F – C + N × M (c) N + M – F × C (d) M × N – C + F

Solution

(d)

Question 12 – MTP May, 2020

Read the following information and answer the question:

‘A + B’ means ‘A is the daughter of B’

‘A × B’ means ‘A is the son of B’

‘A – B’ means ‘A is the wife of B’

If P × Q – S, which of the following is true:

(a) S is the wife of B
(b) S is the father of P
(c) P is the daughter of Q
(d) Q is the father of P

(b)

Question 13 – MTP November, 2019

Read the following information carefully to answer the questions that follow.

1. 'P + Q' means 'P is father of Q'
2. 'P – Q' means 'P is mother of Q'
3. 'P × Q' means 'P is brother of Q'
4. 'P ÷ Q' means 'P is sister of Q'

Which of the following means 'M' is maternal uncle of T?

(a) M ÷ K – T (b) M × K – T (c) M × K + T (d) M ÷ K + T

Solution

(b)

Question 14 – ICAI SM

Pointing to a lady in a photograph. Meera said. “Her father’s only son’s wife is my mother-in-law.” How is Meera’s husband related to that lady in the photo?

(a) Nephew (b) Uncle (c) Son (d) Father

(a)

Question 15 – MTP June, 2023

Suresh's sister is the wife of Ram. Ram is Rani's brother. Ram's father is Madhur. Sheetal is Ram's grandmother. Rema is Sheetal is daughter-in-law. Rohit is Rani's brother's son. Who is Rohit to Suresh?

(a) Brother in law (b) Son (c) Brother (d) Nephew

(d)

Let me be Rohit. I am Rani's brother's son; and Rani's brother is Ram; therefore, I'm Ram's son. Ram is my dad. Rani is my aunt. Suresh's sister is my dad's wife. This means Suresh is the brother of my mother. Therefore, Suresh is my uncle. So, I'm his nephew.

Question 16

If Neha says, “Amruta’s father Raj is the only son of my father-in-law Mahesh.” Then, how Bindu, who is the sister of Amruta, is related to Mahesh?

(a) Daughter (b) Wife (c) Daughter-in-law (d) Granddaughter

Solution

(d)

Question 17

If the uncle of the father of Rani is the grandson of the father of Anup and Anup is the only son of his father, then how Anup is related to Rani?

(a) Grandfather (b) Uncle (c) Maternal Uncle (d) Great Grandfather

Solution

(d)

Let's call Rani's father as Z; Z's uncle as Y; Anup's father as X.

	X – Great-Great-Grandfather
	Anup – Great-Grandfather
	Y – Grandfather

Z – Father	
Rani – Me	

Question 18

Madhu said, “My mother’s only son Ashok has no son”. Which of the following can be concluded?

(a) Ashok has only daughters
(b) Ashok is not married
(c) Ashok has only one sister
(d) None

Solution

(d)

If Ashok is my mother’s only son, then Ashok is my brother. My brother has no son. He could have daughter/s.

Question 19 – December, 2022

When Rani saw Vinit, she recollected that “He is the brother of my grandfather’s son”. How is Rani related to Vinit?

(a) Aunt (b) Daughter (c) Sister (d) Niece

Solution

(d)

Let me be Rani. My grandfather could be my Nanaji, or my Dadaji. If my grandfather is my Nanaji, his son would be my Mamaji, and his brother would again be my Mamaji. Hence, Vinit would be my Mamaji (Uncle), and I would be his Niece.

If my grandfather is my Dadaji, his son would be my Dad, and my dad’s brother would be my uncle, and I would still be his niece.

Question 20 – MTP June, 2023

A + B means, “A is the son of B”; A – B means, “A is the daughter of B”; A * B means, “A is the wife of B”; A $ B means, “A is the sister of B”. If A $ B – C * D is true, how is D related to B?

(a) Wife (b) Father (c) Grandmother (d) Grandfather

Solution

(b)

Let me be B. A $ B means A is the sister of B. Therefore, A is my sister. B – C means B is the daughter of C. Therefore, C is either my mom, or my dad. C * D means C is the wife of D. Therefore, C is my mom, and D is my dad.

Statistical Description of Data

CA NISHANT KUMAR

Question 1

Which of the following statements is false?

(a) Statistics is derived from the Latin word ‘Status’
(b) Statistics is derived from the Italian word ‘Statista’
(c) Statistics is derived from the French word ‘Statistik’
(d) None of these

Solution

(c)

Question 2

Statistics is concerned with:

(a) Qualitative information

(b) Qualitative information

(c) (a) or (b)

(d) Both (a) and (b)

Solution

(d)

Question 3

Statistics is defined in terms of numerical data in the:

(a) Singular Sense
(b) Plural Sense
(c) Either (a) or (b)
(d) Both (a) and (b)

Solution

(b)

Question 4

Statistics is applied in:

(a) Economics
(b) Business Management
(c) Commerce and Industry
(d) All these

Solution

(d)

Question 5

An attribute is:

(a) A Qualitative Characteristic
(b) A Quantitative Characteristic
(c) A Measurable Characteristic
(d) All these

Solution

(a)

Question 6

Nationality of a student is:

(a) An attribute
(b) A continuous variable
(c) A discrete variable
(d) (a) or (c)

Solution

(a)

Question 7

Drinking habit of a person is:

(a) An attribute

(b) A variable

(c) A discrete variable

(d) A continuous variable

Solution

(a)

Question 8

Marks of a student is an example of

(a) An attribute
(b) A discrete variable
(c) A continuous variable
(d) None of these

Solution

(b)

Question 9

Annual income of a person is

(a) An attribute
(b) A discrete variable
(c) A continuous variable
(d) (a) or (c)

Solution

(c)

Question 10

Age of a person is

(a) An attribute
(b) A discrete variable
(c) A continuous variable
(d) A variable

Solution

(c)

Question 11

The data collected on the height of a group of students after recording their heights with a measuring tape are

(a) Primary Data
(b) Secondary Data
(c) Discrete Data
(d) Continuous Data

Solution

(a)

Question 12

The primary data are collected by

(a) Interview Method
(b) Observation Method
(c) Questionnaire Method
(d) All these

Solution

(d)

Question 13

The quickest method to collect primary data is

(a) Personal Interview
(b) Indirect Interview
(c) Telephone Interview
(d) By observation

Solution

(c)

Question 14

The best method to collect data, in case of a natural calamity, is

(a) Personal Interview

(b) Indirect Interview

(c) Questionnaire Method

(d) Direct Observation Method

Solution

(a)

Question 15

In case of a rail accident, the appropriate method of data collection is by:

(a) Personal Interview
(b) Direct Interview
(c) Indirect Interview
(d) All these

Solution

(c)

Question 16

Which method of data collection covers the widest area?

(a) Telephone Interview Method
(b) Mailed Questionnaire Method
(c) Direct Interview Method
(d) All these

Solution

(b)

Question 17

The amount of non-responses is maximum in

(a) Mailed Questionnaire Method (b) Interview Method
(c) Observation Method (d) All these

Solution

(a)

Question 18

Data collected on religion from the census reports are

(a) Primary Data

(b) Secondary Data

(c) Sample Data

(d) (a) or (b)

Solution

(b)

Question 19

Some important sources of secondary data are

(a) Some important sources of secondary data are
(b) International and primary sources
(c) Private and primary sources
(d) Government sources.

Solution

(a)

Question 20

Internal consistency of the collected data can be checked when

(a) Internal data are given

(b) External data are given

(c) Two or more series are given

(d) A number of related series are given

Solution

(d)

Question 21

The accuracy and consistency of data can be verified by:

(a) Internal checking
(b) External checking
(c) Scrutiny
(d) Both (a) and (b)

Solution

(c)

Question 22

The mode of presentation of data are

(a) Textual, tabulation and diagrammatic (b) Tabular, internal and external
(c) Textual, tabular and internal (d) Tabular, textual and external

Solution

(a)

Question 23

For tabulation, ‘caption’ is:

(a) The upper part of the table
(b) The lower part of the table
(c) The main part of the table
(d) The upper part of a table that describes the column and sub-column

Solution

(d)

Question 24

‘Stub’ of a table is the:

(a) Left part of the table describing the columns
(b) Right part of the table describing the columns
(c) Right part of the table describing the rows
(d) Left part of the table describing the rows

Solution

(d)

Question 25

The entire upper part of a table is known as

(a) Caption (b) Stub

(c) Box Head (d) Body

Solution

(c)

Question 26

The unit of measurement in tabulation is shown in

(a) Box Head
(b) Body
(c) Caption
(d) Stub

Solution

(a)

Question 27

In tabulation source of the data, if any, is shown in the

(a) Footnote (b) Body
(c) Stub (d) Caption

Solution

(a)

Question 28

Which of the following statements is untrue for tabulation?

(a) Statistical analysis of data requires tabulation
(b) It facilitates comparison between rows and not columns
(c) Complicated data can be presented
(d) Diagrammatic representation of data requires tabulation

Solution

(b)

Question 29

The most accurate mode of data presentation is

(a) Diagrammatic Method
(b) Tabulation
(c) Textual Presentation
(d) None of these

Solution

(b)

Question 30

The chart that uses logarithm of the variable is known as:

(a) Line Chart
(b) Ratio Chart
(c) Multiple Line Chart
(d) Component Line Chart

Solution

(b)

Question 31

Multiple line chart is applied for:

(a) Showing multiple charts
(b) Two or more related time series when the variables are expressed in the same unit
(c) Two or more related time series when the variables are expressed in different unit
(d) Multiple variations in the time series

Solution

(b)

Question 32

Multiple axis line chart is considered when:

(a) There is more than one time series

(b) The units of the variables are different

(c) (a) or (b)

(d) (a) and (b)

Solution

(d)

Question 33

Horizontal bar diagram is used for

(a) Qualitative Data

(b) Data varying over time

(c) Data varying over space

(d) (a) or (c)

Solution

(d)

Question 34

Vertical bar diagram is applicable when

(a) The data are qualitative

(b) The data are quantitative

(c) When the data vary over time

(d) (b) or (c)

Solution

(d)

Question 35

In order to compare two or more related series, we consider:

(a) Multiple Bar Chart (b) Grouped Bar Chart
(c) (a) or (b) (d) (a) and (b)

Solution

(c)

Question 36

Divided bar chart is considered for:

(a) Comparing different components of a variable
(b) The relation of different components to the table
(c) (a) or (b)
(d) (a) and (b)

Solution

(d)

Question 37

Pie-diagram is used for:

(a) Comparing different components and their relation to the total
(b) Representing qualitative data in a circle
(c) Representing quantitative data in circle
(d) (b) or (c)

Solution

(a)

Question 38

Cost of sugar in a month under the heads raw materials, labour, direct production and others were 12, 20, 35 and 23 units respectively. What is the difference between the central angles for the largest and smallest components of the cost of sugar?

(a) 72° (b) 48° (c) 56° (d) 92°

Solution

(d)

Question 39

Hidden trend, if any, in the data can be noticed in:

(a) Textual presentation (b) Tabulation
(c) Diagrammatic Representation (d) All these

Solution

(c)

Question 40

Diagrammatic representation of data is done by

(a) Diagrams (b) Charts

(c) Pictures (d) All these

Solution

(d)

Question 41

The best method of presentation of data is

(a) Textual (b) Tabular
(c) Diagrammatic (d) (b) and (c)

Solution

(b)

Question 42

The most attractive method of data presentation is

(a) Tabular
(b) Textual
(c) Diagrammatic
(d) (a) or (b)

Solution

(c)

Question 43

The distribution of shares is an example of the frequency distribution of:

(a) A discrete variable

(b) A continuous variable

(c) An attribute

(d) (a) or (c)

Solution

(a)

Question 44

The number of accidents for seven days in a locality are given below:

No. of Accidents	0	1	2	3	4	5	6
Frequency	15	19	22	31	9	3	2

What is the number of cases when 3 or less accidents occurred?

(a) 56 (b) 6 (c) 68 (d) 87

Solution

(d)

Question 45

The distribution of profits of a blue-chip company relates to:

(a) A discrete variable
(b) A continuous variable
(c) An attribute
(d) (a) or (b)

Solution

(b)

Question 46

Mutually exclusive classification

(a) Excludes both the class limits
(b) Excludes the upper class limit but includes the lower class limit
(c) Includes the upper class limit but excludes the upper class limit
(d) Either (b) or (c)

Solution

(b)

Question 47

Mutually inclusive classification is usually meant for

(a) A discrete variable
(b) A continuous variable
(c) An attribute
(d) All these

Solution

(a)

Question 48

Mutually exclusive classification is usually meant for

(a) A discrete variable
(b) A continuous variable
(c) An attribute
(d) Any of these

Solution

(b)

Question 49

The LCB is

(a) An upper limit to LCL
(b) A lower limit to LCL
(c) (a) and (b)
(d) (a) or (b)

Solution

(b)

Question 50

The UCB is

(a) An upper limit to UCL

(b) A lower limit to LCL

(c) Both (a) and (b)

(d) (a) or (b)

Solution

(a)

Question 51

Length of a class is:

(a) The difference between the UCB and LCB of that class
(b) The difference between the UCL and LCL of that class
(c) (a) or (b)
(d) Both (a) and (b)

Solution

(a)

Question 52

For a particular class boundary, the less than cumulative frequency and more than cumulative frequency add up to:

(a) Total frequency
(b) Fifty per cent of the total frequency
(c) (a) or (b)
(d) None of these

Solution

(a)

Question 53

The following data relate to the incomes of 86 persons:

Income in ₹	500 – 999	1000 – 1499	1500 – 1999	2000 – 2499
No. of Persons:	15	28	36	7

What is the percentage of persons earning more than ₹1,500?

(a) 50 (b) 45 (c) 40 (d) 60

Solution

(a)

Question 54

The following data relate to the marks of a group of students:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
No. of Students	15	38	65	84	100

How many students got marks more than 30?

(a) 65 (b) 50 (c) 35 (d) 43

Solution

(c)

Question 55

Find the number of observations between 250 and 300 from the following data:

Value	More than 200	More than 250	More than 300	More than 350
No. of Observations	56	38	15	0

(a) 56 (b) 23 (c) 15 (d) 8

Solution

(b)

Question 56

A frequency distribution

(a) Arranges observations in an increasing order
(b) Arranges observation in terms of a number of groups
(c) Relates to a measurable characteristic
(d) All these

Solution

(b)

Question 57

The frequency distribution of a continuous variable is known as:

(a) Grouped Frequency Distribution
(b) Simple Frequency Distribution
(c) (a) or (b)
(d) (a) and (b)

Solution

(a)

Question 58

From the following data find the number class intervals if class length is given as 5.

73, 72, 65, 41, 54, 80, 50, 46, 49, 53

(a) 6 (b) 5 (c) 7 (d) 8

Solution

(d)

Question 59

Frequency density corresponding to a class interval is the ratio of:

(a) Class frequency to the total frequency
(b) Class frequency to the class length
(c) Class length to the class frequency
(d) Class frequency to the cumulative frequency

Solution

(b)

Question 60

Relative frequency for a particular class

(a) Lies between 0 and 1
(b) Lies between 0 and 1, both inclusive
(c) Lies between –1 and 0
(d) Lies between –1 to 1

Solution

(a)

Question 61

Mode of a distribution can be obtained from:

(a) Histogram

(b) Less than type Ogives

(c) More than type Ogives

(d) Frequency Polygon

Solution

(a)

Question 62

A comparison among the class frequencies is possible only in:

(a) Frequency Polygon
(b) Histogram
(c) Ogives
(d) (a) or (b)

Solution

(b)

Question 63

Frequency curve is a limiting form of

(a) Frequency Polygon
(b) Histogram
(c) (a) or (b)
(d) (a) and (b)

Solution

(d)

Question 64

Most of the commonly used frequency curves are

(a) Mixed
(b) Inverted J-shaped
(c) U-shaped
(d) Bell-shaped

Solution

(d)

Question 65

The distribution of profits of a company follows

(a) J-shaped frequency curve
(b) U-shaped frequency curve
(c) Bell-shaped frequency curve
(d) Any of these

Solution

(c)

Question 66

Median of a distribution can be obtained from

(a) Frequency Polygon
(b) Histogram
(c) Less than type Ogives
(d) None of these

Solution

(c)

Question 67

Out of 1000 persons, 25 per cent were industrial workers and the rest were agricultural workers. 300 persons enjoyed world cup matches on TV. 30 per cent of the people who had not watched world cup matches were industrial workers. What is the number of agricultural workers who had enjoyed world cup matches on TV?

(a) 260 (b) 240 (c) 230 (d) 250

Solution

(a)

No. of Industrial Workers = 25% of 1,000 = 250

No. of Agricultural Workers = 75% of 1,000 = 750

No. of persons who enjoyed world cup matches on TV = 300

No. of persons who had not watched world cup matches on TV = 1,000 – 300 = 700

Percentage of Industrial Workers who had not watched world cup matches on TV = 30%

∴ Percentage of Agricultural Workers who had not watched world cup matches on TV = 70%

∴ No. of Agricultural Workers who had not watched world cup matches on TV = 70% of 700 = 490.

∴ No. of Agricultural Workers who enjoy world cup matches on TV = Total No. of Agricultural Workers – No. of Agricultural Workers who had not watched world cup matches on TV = 750 – 490 = 260.

Question 68

A sample study of the people of an area revealed that total number of women were 40% and the percentage of coffee drinkers were 45 as a whole and the percentage of male coffee drinkers was 20. What was the percentage of female non-coffee drinkers?

(a) 10 (b) 15 (c) 18 (d) 20

Solution

(b)

Total Population = 100

Men = 60

Women = 40

Total Coffee Drinkers = 45

Male Coffee Drinkers = 20

Female Coffee Drinkers = 45 – 20 = 25

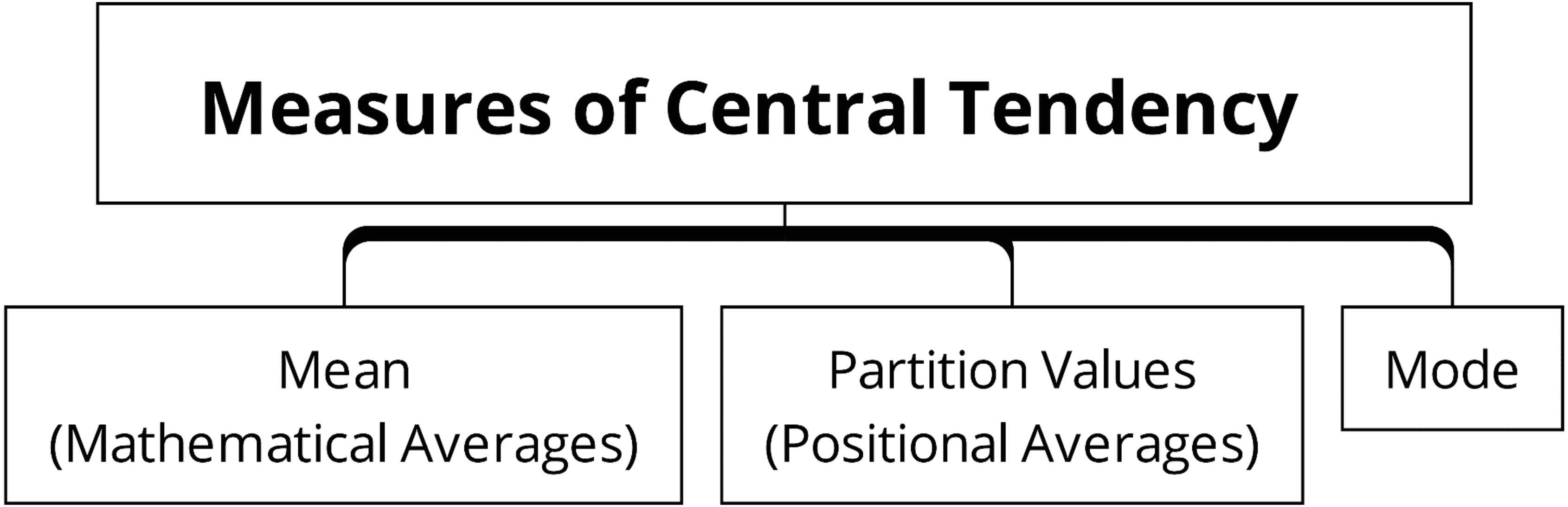
Female Non-Coffee Drinkers = 40 – 25 = 15

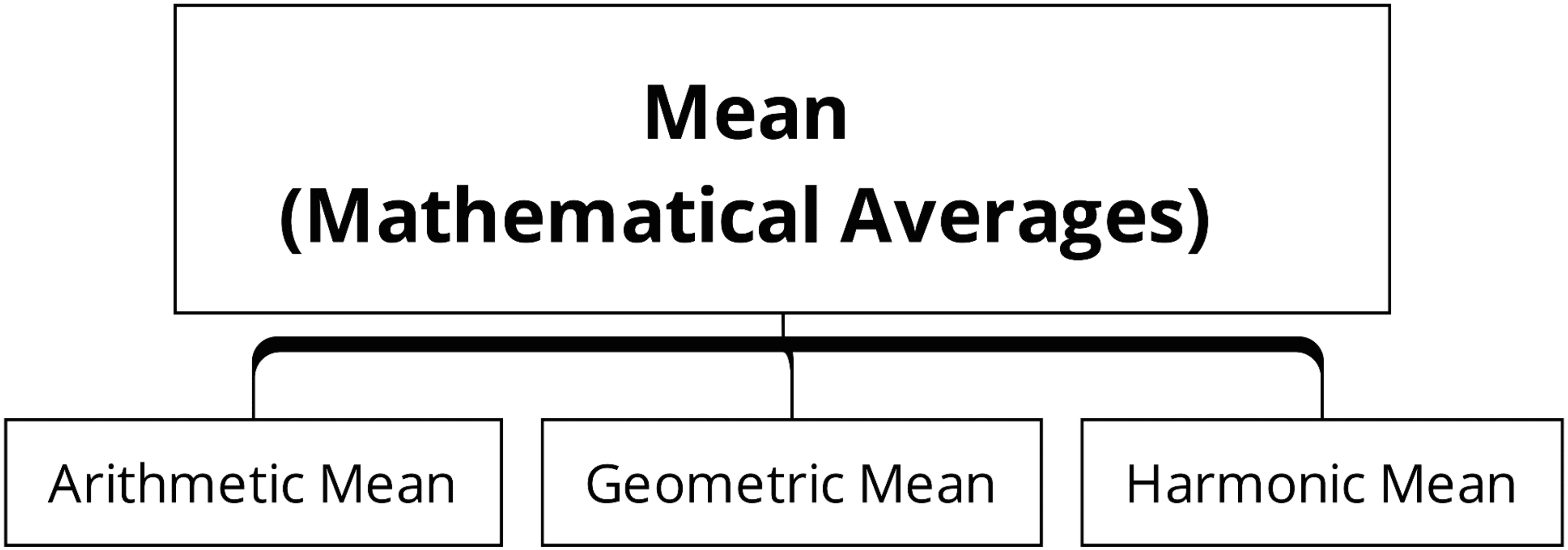
Chapter 14 – Measures of Central Tendency and Dispersion

CA NISHANT KUMAR

Measures of Central Tendency and Dispersion

- Measures of Central Tendency
- Measures of Dispersion


Measures of Central Tendency
Mean
(Mathematical Averages)
Partition Values
(Positional Averages)
Mode


Mean
(Mathematical Averages)
Arithmetic Mean
Geometric Mean
Harmonic Mean

Arithmetic Mean

Formula

- **Individual Series**
  $$\bar{x} = \frac{\sum x}{n}$$
- **Discrete Series**
  $$\bar{x} = \frac{\sum (fx)}{N}$$
  N = Total of Frequencies
- **Continuous Series**
  Same as Discrete. x is given by the mid point of the class intervals.

Properties

- AM for constant values is constant
- **Combined AM**
  $$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$
- Sum of Deviations from AM is always 0
- AM is affected due to change in origin as well as scale
  If x and y are two variables related as $y = a + bx$, and $\bar{x}$ is given, then $\bar{y} = a + b\bar{x}$.

CA NISHANT KUMAR

Questions Based on Arithmetic Mean

Question 1 – ICAI SM

Find the mean from the following data:

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50
No. of Students	5	13	23	27	30

(a) 19 (b) 20.45 (c) 21.12 (d) 22.33

Solution

(d)

Question 2 – ICAI SM

If there are 3 observations 15, 20, 25, then the sum of deviation of the observations from their AM is:

(a) 0 (b) 5 (c) –5 (d) None

Solution

(a)

Question 3 – ICAI SM

If the relationship between two variables u and v are given by $2u + v + 7 = 0$ and if the AM of u is 10, then the AM of v is:

(a) 17 (b) –17 (c) –27 (d) 27

Solution

(c)

Question 4 – ICAI SM

The average salary of a group of unskilled workers is ₹10,000 and that of a group of skilled workers is ₹15,000. If the combined salary is ₹12,000, then what is the percentage of skilled workers?

(a) 40%   (b) 50%   (c) 60%   (d) None

Solution

(a)

Question 5 – MTP November, 2021

At ABC ltd, the average age of employees is 36. Average age of male employees is 38 and that of females is 32. Find the ratio of female to male in the company.

(a) 1 : 3 (b) 2 : 1 (c) 1 : 2 (d) 3 : 1

Solution

(c)

Let the number of male employees be n_1 and their average age be $\bar{x}_1$; let the number of female employees be n_2 and their average age be $\bar{x}_2$.

Given $\bar{x} = 36;\ \bar{x}_1 = 38;\ \bar{x}_2 = 32$

Try the options.

Option (a) $\rightarrow$ 1 : 3

Therefore, $n_1 = 3;\ n_2 = 1$

$$\bar{x} = \frac{(3\times 38)+(1\times 32)}{3+1} = 36.5$$

Option (b) $\rightarrow$ 2 : 1

Therefore, $n_1 = 1;\ n_2 = 2$

$$\bar{x} = \frac{(1\times 38)+(2\times 32)}{1+2} = 34$$

Option (c) $\rightarrow$ 1 : 2

Therefore, $n_1 = 2;\ n_2 = 1$

$$\bar{x} = \frac{(2 \times 38) + (1 \times 32)}{2 + 1} = 36$$

Therefore, option (c) is the answer.

Question 6 – MTP November, 2018

If the mean of a set of observations $x_1, x_2, x_3, ..., x_n$ is $\overline{x}$, then the mean of the observations $x_i + ki$, where i = 1, 2, 3, …, n is:

(a) $\overline{x} + k(n+1)$ (b) $\overline{x} + kn$ (c) $\overline{x} + \frac{k}{n}$ (d) $\overline{x} + \frac{k}{2}(n+1)$

Solution

(d)

Let $x_1 = 1$; $x_2 = 2$; $x_3 = 3$; $k = 10$; $i = 1, 2, 3$

Now, the mean of x_1, x_2, x_3 = 2

The observations

$$x_1 + (k \times 1) = 1 + 10 = 11$$

$$x_2 + (k \times 2) = 2 + (10 \times 2) = 2 + 20 = 22$$

$$x_3 + (k \times 3) = 3 + (10 \times 3) = 3 + 30 = 33$$

The mean of these observations is 22.

Now, try the options.

Option (d) $\rightarrow \bar{x} + \frac{k}{2}(n+1)$

$$2 + \frac{10}{2}(3+1) = 22$$

Therefore, option (d) is the answer.

Question 7 – PYQ

Two years ago, a team of four persons had an average age of 14. Now, a new member is added to the team and the average age of the team is 17. What is the age of the new member?

(a) 17 (b) 19 (c) 21 (d) 23

Solution

(c)

Now, each person's age has increased by 2. Therefore, the new average of these 4 persons would also increase by 2. The mean is 14 + 2 = 16. This means that on an average, the age of each person is 16. Now, a new person is added to the group and the average age is

17. This means that the age of all the 5 persons is 17. Therefore, the age of the new person would be 17 + 1 + 1 + 1 + 1 = 21 years.

Question 8 – MTP June, 2013

The students of a class X^{th} have an average weight of 50 kg. The strength of the class is 49 students. On including the weight of the Principal, the average weight shoots up by 0.8 kg. Find the weight of the Principal.

(a) 75 (b) 90 (c) 85 (d) None

Solution

(b)

We know that the average weight of the 49 students in class X^{th} is 50kg.

Therefore, we have No. of Students (n) = 49; Average $\left(\overline{x}\right)$ = 50

We know that $\bar{x} = \frac{\sum x}{n}$

$$\Rightarrow \sum x = \bar{x} \times n = 50 \times 49 = 2,450$$

Therefore, the total weight of the students is 49 × 50 = 2450kg.

When the weight of the Principal is included, the average weight increases by 0.8 kg. This means the new average weight is 50 + 0.8 = 50.8 kg. Also, the total number of persons becomes 50.

Let the weight of the principal be *x*.

Putting the values in this formula $\bar{x} = \frac{\sum x}{n}$, we get:

$$50.8 = \frac{2,450 + x}{50}$$

$$\Rightarrow 50.8 \times 50 = 2,450 + x$$

$$\Rightarrow 2,540 = 2,450 + x$$

$$\Rightarrow x = 2,540 - 2,450$$

$$\Rightarrow x = 90$$

Question 9 – MTP June, 2013

The average of $(p+q)$ consecutive numbers starting from 1 is 'r'. If 's' is added to each of the numbers then the new average will be?

(a) $r+s$ (b) $r+(s/2)$ (c) $\{r+(p+q+s)\}/(p+q)$ (d) None

Solution

(a)

Question 10 – MTP June, 2013

The average weight of 40 people is increased by 2.4 kg when one man weight 73 kg is replaced by another man. Find the weight of the new man?

(a) 121 (b) 169 (c) 154 (d) 149

Solution

(b)

Initial Average $\left(\bar{x}\right) = \frac{\sum x}{n}$

$\Rightarrow \bar{x} = \frac{\sum x}{40}$

$\Rightarrow 40\bar{x} = \sum x$ …Eq. (1)

Let the weight of the replacing man be *y*. When the man (weighing *y* kgs) replaces the one weighing 73 kg, then new total becomes $\sum x - 73 + y$. New mean $(\bar{x}) = \bar{x} + 2.4$.

Putting these values in the formula for AM, we get:

$$\bar{x} + 2.4 = \frac{\sum x - 73 + y}{40}$$

$$\Rightarrow 40(\bar{x} + 2.4) = \sum x - 73 + y$$

$$\Rightarrow 40\bar{x} + 96 = \sum x - 73 + y$$

$$\Rightarrow 40\bar{x} + 96 + 73 - y = \sum x$$

$$\Rightarrow 40\overline{x} + 169 - y = \sum x \ldots \text{Eq. (2)}$$

Equating Eqs. (1) and (2), we get:

$$40\overline{x} = 40\overline{x} + 169 - y$$

$$\Rightarrow y = 169 + 40\overline{x} - 40\overline{x}$$

$$\Rightarrow y = 169$$

Alternatively: New weight = (2.4 kg × 40 persons) + 73 kg = 169

Question 11 – MTP June, 2013

The average salary of the whole employees in a company is ₹400 per day. The average salary of officers is ₹800 per day and that of clerks is ₹320 per day. If the number of officers is 40, then find the number of clerks in the company?

(a) 50 (b) 100 (c) 150 (d) 200

Solution

(d)

We have $\bar{x} = 400;\ n_1 = 40;\ \bar{x}_1 = 800;\ \bar{x}_2 = 320;\ n_2 = ?$

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\Rightarrow 400 = \frac{(40 \times 800) + (n_2 \times 320)}{40 + n_2}$$

Now, try the options.

Option (d) $\rightarrow$ 200

RHS: $\frac{(40 \times 800) + (200 \times 320)}{40 + 200} = 400$ = LHS

Question 12 – MTP June, 2013

The average of 6 numbers is 30. If the average of the first four is 25 and that of the last three is 35, the fourth number is:

(a) 25 (b) 30 (c) 35 (d) 40

Solution

(a)

Sum of the 6 numbers = 6 × 30 = 180

Sum of the first 4 numbers = 25 × 4 = 100

Therefore, the sum of last two numbers = 180 – 100 = 80

Sum of the last 3 numbers = $35 \times 3 = 105$

Therefore, the fourth term = $105 – 80 = 25$

Question 13 – MTP June, 2013

A student's marks were wrongly entered as 85 instead of 45. Due to that, the average marks for the whole class got increased by one-fourth. The no. of students in the class is:

(a) 80 (b) 160 (c) 40 (d) 20

Solution

(b)

Let the number of students in the class be *n*.

Since 85 was entered instead of 45, the total number of marks got increased by 40, and because of this, the average increased by ¼.

If the average is increased by ¼, we can say that every observation has increased by ¼; and this happened due to the total increase of 40 marks.

Therefore,

Per Student Increment × Number of Students = Total Increment

$\frac{1}{4} \times n = 40$

$\Rightarrow n = 40 \times 4 = 160$

Question 14 – MTP June, 2013

The mean of 100 observations is 50. If one of the observations which was 50 is replaced by 40, the resulting mean will be:

(a) 40 (b) 49.90 (c) 50 (d) None

Solution

(b)

Mean is given by $\bar{x} = \frac{\sum x}{n}$

$$50 = \frac{\sum x}{100}$$

$$\Rightarrow \sum x = 50 \times 100 = 5,000$$

Since one observation, which was 50, is replaced by 40, the revised $\sum x = 5,000 - 50 + 40 = 4,990$.

Therefore, revised mean: $\bar{x} = \frac{\sum x}{n} = \frac{4,990}{100} = 49.90$

Question 15 – MTP June, 2013

The mean annual salary of all employees in a company is ₹25,000. The mean salary of male and female employees is ₹27,000 and ₹17,000 respectively. Find the percentage of males and females employed by the company:

(a) 60% and 40% (b) 70% and 25% (c) 70% and 30% (d) 80% and 20%

Solution

(d)

We have $\bar{x} = 25{,}000;\ \bar{x}_1 = 27{,}000;\ \bar{x}_2 = 17{,}000$

Let the total number of employees be 100.

Now, try the options.

Option (a) → 60% and 40%.

It means that $n_1 = 60\% \times 100 = 60;\ n_2 = 40\% \times 100 = 40$

Using the values $n_1 = 60;\ n_2 = 40;\ \overline{x}_1 = 27,000;$ and $\overline{x}_2 = 17,000$, let's calculate the combined mean.

$$\overline{x} = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}$$

$$\Rightarrow \overline{x} = \frac{(60 \times 27,000) + (40 \times 17,000)}{60 + 40} = 23,000$$

Since the value of combined mean is given as 25,000, option (a) is not our answer.

Option (b) → 70% and 25%.

This doesn’t make any sense as the total is not 100%. Skip it.

Option (c) → 70% and 30%.

Now, we have $n_1 = 70;\ n_2 = 30;\ \overline{x}_1 = 27,000;\ \overline{x}_2 = 17,000$

$$\overline{x} = \frac{(70 \times 27,000) + (30 \times 17,000)}{70 + 30} = 24,000$$

Option (d) → 80% and 20%

Now, we have $n_1 = 80;\ n_2 = 20;\ \overline{x}_1 = 27,000;\ \overline{x}_2 = 17,000$

$$\overline{x} = \frac{(80 \times 27,000) + (20 \times 17,000)}{80 + 20} = 25,000$$

Geometric Mean

Formula

Individual Series

$$\left(x_1 \times x_2 \times \ldots \times x_n\right)^{1/n}$$

Discrete Series

$$\left(x_1^{f_1} \times x_2^{f_2} \times \ldots \times x_n^{f_n}\right)^{1/N}$$

Properties

- GM for constant values is constant
- $$\log G = \frac{1}{r}\sum \log x$$
  Here, r is the number of observations
- To calculate average RATE, use GM
- If z = xy, then GM (z) = GM (x) * GM (y)
- If z = x/y, then GM (z) = GM (x)/GM (y)

Questions Based on Geometric Mean

Question 16 – ICAI SM

What is the GM for the numbers 8, 24 and 40?

(a) 24 (b) 12 (c) $8.\sqrt[3]{15}$ (d) 10

Solution

(c)

Question 17 – ICAI SM

If GM of x is 10 and GM of y is 15, then the GM of xy is:

(a) 150 (b) log 10 × log 15 (c) log 150 (d) None

Solution

(a)

Question 18

If GM of *x* is 10, and GM of *y* is 15, then GM of *x*/*y* is:

(a) 150 (b) 2/3 (c) log 2/log 3 (d) None

Solution

(b)

Question 19 – MTP November, 2021

If the rates return from three different shares are 100%, 200% and 400% respectively. The average rate of return will be:

(a) 350% (b) 233.33% (c) 200% (d) 300%

Solution

(c)

Since percentages are given, we'll use Geometric Mean.

$$GM = (100 \times 200 \times 400)^{\frac{1}{3}}$$

$$\Rightarrow GM^3 = 8000000$$

Now, try the options.

Option (c) → 200

LHS = $200^3 = 8000000 = \text{RHS}$

Therefore, option (c) is the answer.

Question 20 – MTP November, 2018

The geometric mean of the series $1, k, k^2, ..., k^n$, where k is a constant is:

(a) $k^{(n+1)/2}$ (b) $k^{n+0.5}$ (c) k^{n+1} (d) $k^{n/2}$

Solution

(d)

Let the value of k be 10.

Then the series would be: 1, 10, 10^2, 10^3

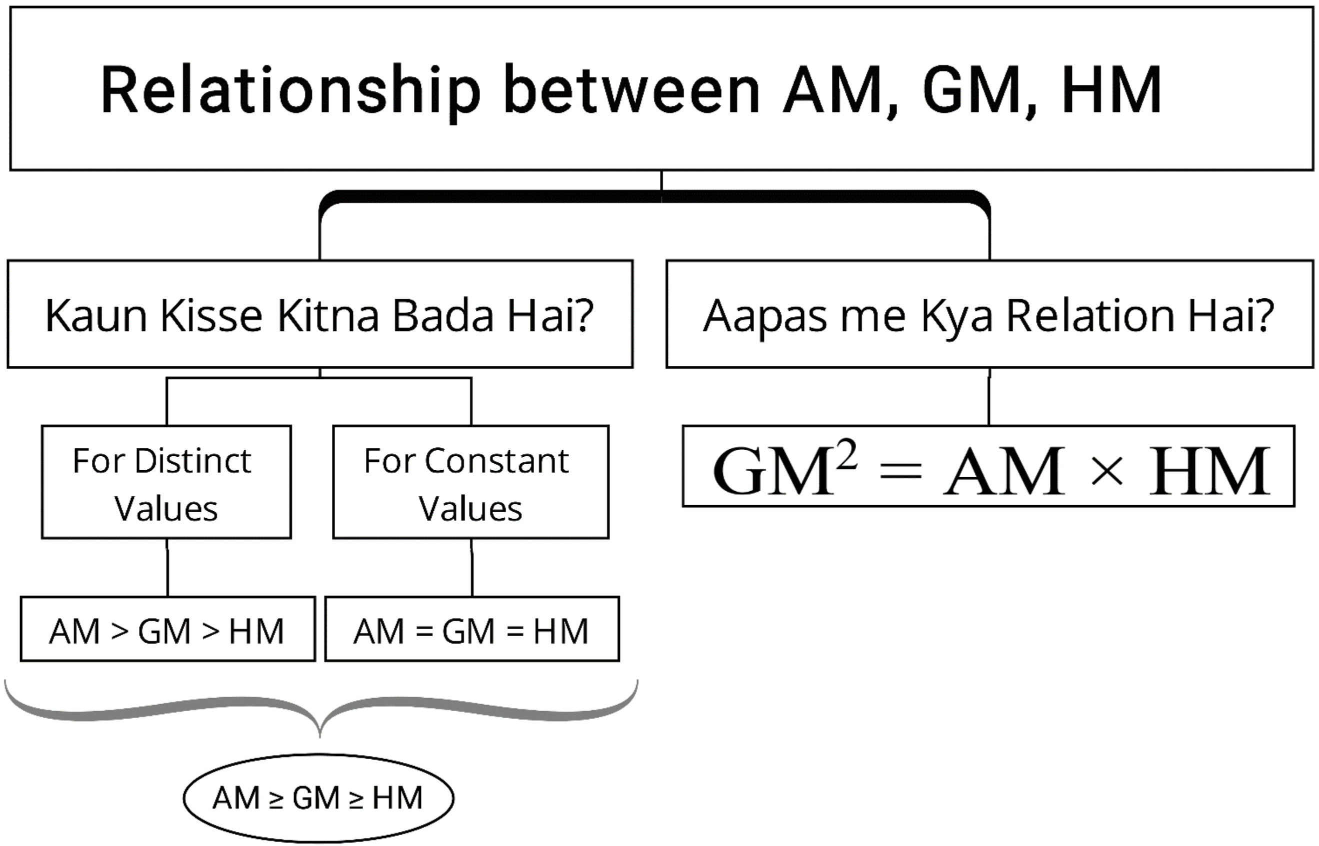
The geometric mean = $\sqrt[4]{1\times10\times100\times1000} = 31.62$

Now, try the options:

Option (d) $\rightarrow k^{n/2} = 10^{3/2} = 10^{1.5} = 31.62$

Harmonic Mean

- Formula
  - Individual Series
    $$HM = \frac{n}{\sum (1/x_i)}$$
  - Discrete Series
    $$HM = \frac{N}{\sum \left(\frac{f_i}{x_i} \right)}$$
- Properties
  - HM for constant values is constant
  - Combined HM
    $$\mathrm{HM} = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$$
  - The harmonic mean of 1, $\frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}$ is $\frac{2}{(n+1)}$.
  - Average Speed
    $$\frac{2xy}{x+y}$$


Relationship between AM, GM, HM
Kaun Kisse Kitna Bada Hai?
Aapas me Kya Relation Hai?
For Distinct Values
For Constant Values
$GM^2 = AM \times HM$
AM > GM > HM
AM = GM = HM
AM ≥ GM ≥ HM

Questions Based on Harmonic Mean

Question 21 – ICAI SM

Compute the HM for the numbers 6, 8, 12, 36.

(a) 9.93 (b) 5.77 (c) 6.77 (d) None

Solution

(a)

Question 22 – ICAI SM

If there are two groups with 75 and 65 as harmonic means and containing 15 and 13 observations then the combined HM is given by:

(a) 65 (b) 70.36 (c) 70 (d) 71

Solution

(c)

Question 23 – ICAI SM

An aeroplane flies from A to B at the rate of 500 km/hour and comes back from B to A at the rate of 700 km/hour. The average speed of the aeroplane is:

(a) 600 km per hour
(b) 583.33 km per hour
(c) $10\sqrt{35}$ km per hour
(d) 620 km per hour

Solution

(b)

Question 24 – ICAI SM

If the AM and HM for two numbers are 5 and 3.2 respectively then the GM will be

(a) 16.00 (b) 4.10 (c) 4.05 (d) 4.00

Solution

(d)

Question 25 – MTP June, 2013

AM and GM are both negative values, HM is equal to:

(a) $H = \frac{G}{A^2}$ (b) $H = \frac{G^2}{A}$ (c) $H = \frac{G^2}{\sqrt{A}}$ (d) None

Solution

(b)

Partition Values – Summary of Formulae

Partition Value	No. of Parts	No. of Partition Values	Symbol	Rank for Individual Series	Rank for Discrete Series	Rank for Continuous Series
Median	2	1	M	$\frac{n+1}{2}$	$\frac{N+1}{2}$	$\frac{N}{2}$
Quartile	4	3	Q_1 to Q_3	$Q_1 = \frac{n+1}{4}$, $Q_3 = \frac{3(n+1)}{4}$	$Q_1 = \frac{N+1}{4}$, $Q_3 = \frac{3(N+1)}{4}$	$Q_1 = \frac{N}{4}$, $Q_3 = \frac{3N}{4}$

Decile	10	9	D_1 to D_9	$D_1 = \frac{n+1}{10}$, $D_5 = \frac{5(n+1)}{10}$ and so on…	$D_1 = \frac{N+1}{10}$, $D_5 = \frac{5(N+1)}{10}$ and so on…	$D_1 = \frac{N}{10}$, $D_5 = \frac{5N}{10}$ and so on…
Percentile	100	99	P_1 to P_{99}	$P_1 = \frac{n+1}{100}$, $P_5 = \frac{5(n+1)}{100}$ and so on…	$P_1 = \frac{N+1}{100}$, $P_5 = \frac{5(N+1)}{100}$ and so on…	$P_1 = \frac{N}{100}$, $P_5 = \frac{5N}{100}$ and so on…

The formula for any partition value of a continuous series is $l + \frac{Rank - c}{f} \times i$

Property of a Median/Quartile/Decile/Percentile

If x and y are two variables related by $y = a + bx$ for any two constants a and b, then the median of y is given by $y_{me} = a + bx_{me}$.

Questions Based on Median

Question 26

What is the median for the following observations?

5, 8, 6, 9, 11, 4

(a) 6 (b) 7 (c) 8 (d) None

Solution

(b)

Question 27

Find the median of the following data:

Variable (x)	10	50	40	30	20
Frequency (f)	50	20	30	10	40

(a) 20 (b) 30 (c) 40 (d) None

Solution

(a)

As per the algorithm,

1. Arrange the series in ascending order of the variable:

Variable (*x*)	**Frequency (*f*)**
10	50
20	40
30	10
40	30
50	20
Total	150

2. Find the cumulative frequency:

Variable (*x*)	**Frequency (*f*)**	**Cumulative Frequency (*cf*)**
10	50	50
20	40	90
30	10	100
40	30	130
50	20	150

Total	150

3. Find out the rank using the formula $\left(\frac{N+1}{2}\right)$, where N is the total of frequencies

$$\text{Rank} = \frac{N+1}{2} = \frac{150+1}{2} = \frac{151}{2} = 75.5$$

Now, the first cumulative frequency is 50. We know that 75.5 doesn't come under 50. Therefore, we move on to the next cumulative frequency. The next cumulative frequency is 90. We know that 75.5 comes under 90. Now, the median is the variable corresponding to the cumulative frequency 90, i.e. 20. Therefore, 20 is the median.

Question 28 – ICAI SM

What is the value of median for the following data?

Marks	5 – 14	15 – 24	25 – 34	35 – 44	45 – 54	55 – 64
No. of Students	10	18	32	26	14	10

(a) 28 (b) 30 (c) 32.94 (d) 33.18

Solution

(c)

Question 29 – MTP May, 2020

Two variables *x* and *y* are given by $y = 2x - 3$. If the median of *x* is 20, what is the median of *y*?

(a) 20 (b) 40 (c) 37 (d) 35

Solution

(c)

Questions Based on Quartiles

Question 30 – ICAI SM

What is the value of the first quartile for observations 15, 18, 10, 20, 23, 28, 12, 16?

(a) 17 (b) 16 (c) 12.75 (d) 12

Solution

(c)

Question 31 – ICAI SM

The third quartile for the following data are:

Profits in '000 ₹	Less than 10	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
No. of Firms	5	18	38	20	9	2

(a) ₹33,500 (b) ₹33,000 (c) ₹33,600 (d) ₹33,250

Solution

(a)

Given data:

Profits in '000	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59

No. of Firms	5	18	38	20	9	2

Let's convert it into exclusive series:

CI	–0.5 – 9.5	9.5 – 19.5	19.5 – 29.5	29.5 – 39.5	39.5 – 49.5	49.5 – 59.5
f	5	18	38	20	9	2

Rank of Quartile $= \frac{N}{4} = \frac{5+18+38+20+9+2}{4} = 23$

Rank of $Q_3 = 3 \times 23 = 69$

Let's find out the cumulative frequency now:

CI	Frequency	Cumulative Frequency

–0.5 – 9.5	5	5
9.5 – 19.5	18	23
19.5 – 29.5	38	61
29.5 – 39.5	20	81
39.5 – 49.5	9	90
49.5 – 59.5	2	92

Clearly, the Rank of Q_3 comes under the cumulative frequency 81. Therefore, the Quartile Class Interval is 29.5 – 39.5.

Therefore, $l = 29.5$; $f = 20$; $i = 10$; $c = 61$

$$Q_3 = l + \frac{Rank - c}{f} \times i$$

$$Q_3 = 29.5 + \frac{69 - 61}{20} \times 10 = 33.5 = 33,500$$

Questions Based on Decile

Question 32 – ICAI SM

The third decile for the numbers 15, 10, 20, 25, 18, 11, 9, 12 is:

(a) 13 (b) 10.70 (c) 11 (d) 11.50

Solution

(b)

Question 33 – ICAI SM

Following distribution relates to the distribution of monthly wages of 100 workers. Compute D_7.

Profits in '000 ₹	Less than 500	500 – 699	700 – 899	900 – 1099	1100 – 1499	More than 1500
No. of Firms	5	23	29	27	10	6

(a) ₹1,032.83 (b) ₹1,048.96 (c) ₹995.80 (d) None

Solution

(c)

Questions Based on Percentile

Question 34 – ICAI SM

Following are the wages of the labourers: ₹82, ₹56, ₹90, ₹50, ₹120, ₹75, ₹75, ₹80, ₹130, ₹65. Find P_{82}.

(a) 62.75 (b) 81.20 (c) 120.20 (d) None

Solution

(c)

Question 35 – ICAI SM

The 65th percentile for the following data are:

Profits in '000 ₹	Less than 10	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
No. of Firms	5	18	38	20	9	2

(a) ₹29,000 (b) ₹28,680 (c) ₹29,184 (d) ₹29,250

Solution

(c)

Given data:

Profits in '000	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59

No. of Firms	5	18	38	20	9	2

Let's convert it into exclusive series:

CI	–0.5 – 9.5	9.5 – 19.5	19.5 – 29.5	29.5 – 39.5	39.5 – 49.5	49.5 – 59.5
f	5	18	38	20	9	2

Rank of $P_{65} = 65 \times \frac{N}{100} = \frac{65 \times 92}{100} = 59.8$

This comes under the cumulative frequency of 61 corresponding to the class interval 19.5 – 29.5.

Therefore, $l = 19.5;\ i = 10;\ f = 38;\ c = 23$

$$P_{65} = l + \frac{Rank - c}{f} \times i$$

$$P_{65} = 19.5 + \frac{59.8 - 23}{38} \times 10 = 29.184 = 29{,}184$$

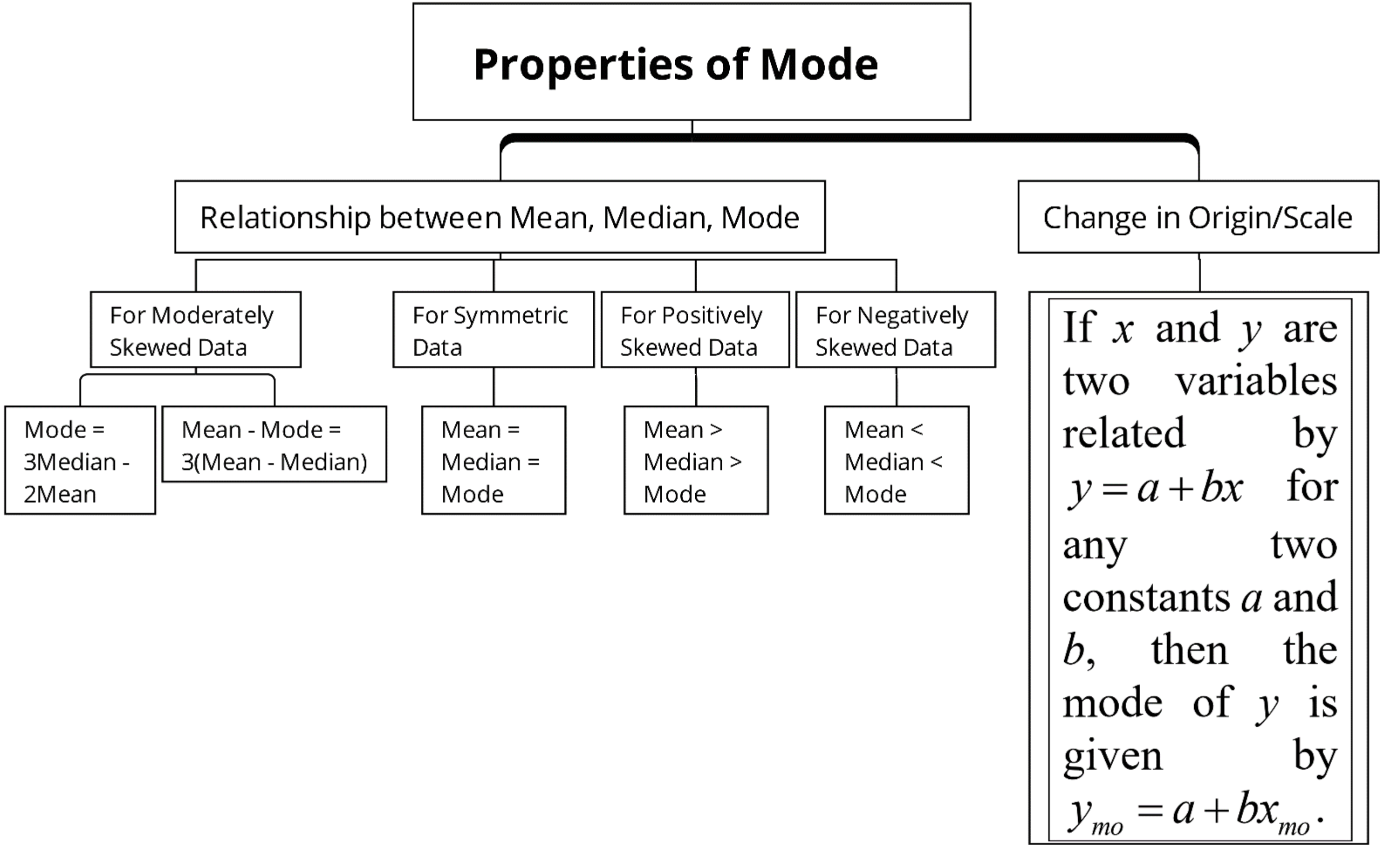
Mode

- **Meaning**
  - Largest Occurring Observation
- **Calculation**
  - Individual Series
    - Largest Occurring Observation
  - Discrete Series
    - Observation with Highest Frequency
  - Continuous Series
    - Formula

$$l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

Meaning of Symbols

l	Lower Limit of the Modal Class Interval
f1	Frequency of the Modal Class Interval
f0	Frequency of the Preceding Class Interval
f2	Frequency of the Succeeding Class Interval
i	Class Length



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Questions Based on Mode

Question 36 – ICAI SM

The modal profits for the following data is:

Profit in ₹ ’000	Below 5	Below 10	Below 15	Below 20	Below 25	Below 30
No. of Firms	10	25	45	55	62	65

(a) 11.50 (b) ₹11267 (c) ₹11667 (d) 11.67

Solution

(c)

Question 37 – ICAI SM

If $y = 2 + 1.50x$, and mode of x is 15, what is the mode of y?

(a) ₹64.00 (b) 24.50 (c) ₹63.21 (d) ₹64.25

Solution

(b)

Question 38 – ICAI SM

For a moderately skewed distribution of marks in statistics for a group of 200 students, the mean mark and median mark were found to be 55.60 and 52.40. What is the modal mark?

(a) 20 (b) 13 (c) 46 (d) 23

Solution

(c)

Question 39 – PYQ

Given that for a distribution, the mean, median and mode are 23, 24, and 25.5. It is most likely that the distribution is __________ skewed.

(a) Positively (b) Symmetrically (c) Asymptotically (d) Negatively

Solution

(d)

For a positively skewed distribution, Mean > Median > Mode.

For a negatively skewed distribution, Mean < Median < Mode.

Question 40 – MTP June, 2013

Which of the following is the correct relation between mean, median and mode?

(a) Median = Mode + 2/3(Mean – Mode)
(b) 2Mean = Mode – 3Median
(c) 2Mean = Mode + 3Median
(d) Mode = 3Median + 2Mean

Solution

(a)

Mode = 3Median – 2Mean

Try the options.

Option (a) → Median = Mode + 2/3(Mean – Mode)

$$Median = Mode + \frac{2}{3}\left(Mean - Mode\right)$$

$$\Rightarrow Median = Mode + \frac{2Mean}{3} - \frac{2Mode}{3}$$

$$\Rightarrow Median = \frac{3Mode + 2Mean - 2Mode}{3}$$

$$\Rightarrow Median = \frac{3Mode + 2Mean - 2Mode}{3}$$

$$\Rightarrow 3Median = Mode + 2Mean$$

$$\Rightarrow Mode = 3Median - 2Mean$$

Question 41 – MTP June, 2013

If mean $\left(\bar{x}\right)$ is 10, and mode $\left(z\right)$ is 7, find out the value of median $\left(M\right)$.

(a) 9 (b) 17 (c) 3 (d) 4.33

Solution

(a)

We know that Mode = 3Median – 2Mean

Therefore, $\text{Median} = \dfrac{\text{Mode} + 2\text{Mean}}{3} = \dfrac{7+\left(2\times 10\right)}{3} = 9$

Measures of Dispersion

- Range
- Mean Deviation
- Standard Deviation
- Quartile Deviation

Range

- **Range**
  - Individual Series
    - Largest Observation - Smallest Observation
  - Continuous Series
    - Upper Most Class Boundary - Lower Most Class Boundary
- **Coefficient of Range**
  - $$\frac{L-S}{L+S}\times 100$$
- **Properties**
  - Range for constant observations is ZERO.
  - If x and y are two variables related as $y = a + bx$ for two constants a and b, then the range of y is given by $R_y = |b| \times R_x$

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Questions Based on Range

Question 42 – ICAI SM

What is the coefficient of range for the following wages of 8 workers?

₹80, ₹65, ₹90, ₹60, ₹75, ₹70, ₹72, ₹85.

(a) ₹30 (b) ₹20 (c) 30 (d) 20

Solution

(d)

Question 43 – ICAI SM

What is the coefficient of range for the following distribution?

Class Interval	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
Frequency	11	25	16	7	3

(a) 22 (b) 50 (c) 72.46 (d) 75.82

Solution

(c)

Question 44 – ICAI SM

If the relationship between x and y is given by $2x+3y=10$, and the range of x is ₹15, what would be the range of y?

(a) ₹20 (b) ₹5 (c) ₹15 (d) ₹10

Solution

(d)

Mean Deviation

Mean Deviation

- Individual Series: $\frac{1}{n}\sum\left|x_i - A\right|$
- Discrete & Continuous Series: $\frac{f\left|x - Median\right|}{N}$

Coefficient of Mean Deviation

$$\frac{\text{Mean Deviation about } A}{A} \times 100$$

Properties

- MD for constant observations is ZERO
- If x and y are two variables related as $y = a + bx$ for two constants a and b, then the MD of y is given by $MD_y = \left|b\right| \times MD_x$
- Sum of Absolute Deviations from Median is Minimum

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Questions Based on Mean Deviation

Question 45

The mean deviation about mode for the numbers

4/11, 6/11, 8/11, 9/11, 12/11, 8/11 is:

(a) 1/6 (b) 1/11 (c) 6/11 (d) 5/11

Solution

(a)

Standard Deviation

- **Standard Deviation**
  - Individual Series

    $$(SD) = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n}}$$

    $$SD = \sqrt{\frac{\sum (x_i^{\,2})}{n} - (\overline{x})^2}$$

  - Discrete & Continuous Series

    $$SD = \sqrt{\frac{\sum f_i (x_i - \overline{x})^2}{N}}$$

- **Coefficient of Variance**

  $$\frac{SD}{AM} \times 100$$

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Properties of Standard Deviation

- SD for constant observations is ZERO

- If x and y are two variables related as $y = a + bx$ for two constants a and b, then the SD of y is given by $SD_y = |b| \times SD_x$

- For any two numbers a and b, standard deviation is given by $\frac{|a-b|}{2}$

- For the first n natural numbers, standard deviation is given by $\sqrt{\frac{n^2-1}{12}}$.

- Combined Standard Deviation

$$SD = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

where,

$$d_1 = \bar{x}_1 - \bar{x}$$

$$d_2 = \bar{x}_2 - \bar{x}$$

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Questions Based on Standard Deviation

Question 47 – ICAI SM

What is the coefficient of variation of the following numbers?

53, 52, 61, 60, 64.

(a) 8.09 (b) 18.08 (c) 20.23 (d) 20.45

Solution

(a)

Coefficient of Variation $= \frac{SD}{AM} \times 100$

$$(SD) = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

Step 1 – Calculation of Mean

$$\bar{x} = \frac{53 + 52 + 61 + 60 + 64}{5} = 58$$

Step 2 – Calculation of Deviations

x	***x - Mean***	***(x - Mean)²***
53	-5	25
52	-6	36
61	3	9
60	2	4

64	6	36
		110

Step 3 – Calculation of Standard Deviation

$$(SD) = \sqrt{\frac{\sum\left(x_i - \overline{x}\right)^2}{n}}$$

$$(SD) = \sqrt{\frac{\sum\left(x_i - \overline{x}\right)^2}{n}} = \sqrt{\frac{110}{5}} = 4.69$$

Step 4 – Calculation of Coefficient of Variation

$$CV = \frac{SD}{AM} \times 100 = \frac{4.69}{58} \times 100 = 8.09$$

Question 48 – ICAI SM

A student computes the AM and SD for a set of 100 observations as 50 and 5 respectively. Later on, she discovers that she has made a mistake in taking one observation as 60 instead of 50. What would be the correct mean and SD if the wrong observation is replaced by the correct observation?

(a) 49.90; 6.91 (b) 49.40; 4.91 (c) 49.90; 4.90 (d) None

Solution

(c)

Question 49 – ICAI SM

If the SD of the first *n* natural numbers is 2, then the value of *n* must be:

(a) 2 (b) 7 (c) 6 (d) 5

Solution

(b)

Question 50 – ICAI SM

If AM and coefficient of variation of x are 10 and 40 respectively, what is the variance of $15 - 2x$?

(a) 8 (b) 64 (c) 74 (d) None

Solution

(b)

Question 51 – ICAI SM

If the mean and standard deviation of x are a and b respectively, then the SD of $\frac{x-a}{b}$ is:

(a) –1 (b) 1 (c) ab (d) a/b

Solution

(b)

Question 52 – ICAI SM

If x and y are related by $2x + 3y + 4 = 0$ and SD of x is 6, then SD of y is:

(a) 22 (b) 4 (c) $\sqrt{5}$ (d) 9

Solution

(b)

Question 53 – ICAI SM

If two samples of sizes 30 and 20 have means as 55 and 60 and variances as 16 and 25 respectively, then what would be the SD of the combined sample of size 50?

(a) 5.00 (b) 5.06 (c) 5.23 (d) 5.35

Solution

(b)

$$SD = \sqrt{\frac{n_1 s_1{}^2 + n_2 s_2{}^2 + n_1 d_1{}^2 + n_2 d_2{}^2}{n_1 + n_2}}$$

where,

$$d_1 = \overline{x}_1 - \overline{x}$$

$$d_2 = \overline{x}_2 - \overline{x}$$

$$\overline{x} = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}$$

$$\overline{x} = \frac{30 \times 55 + 20 \times 60}{30 + 20} = 57$$

$$d_1 = 55 - 57 = -2$$

$$d_2 = 60 - 57 = 3$$

$$SD = \sqrt{\frac{(30 \times 16) + (20 \times 25) + (30 \times 4) + (20 \times 9)}{30 + 20}} = 5.06$$

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Question 54 – ICAI SM

The standard deviation is independent of change of:

(a) Scale (b) Origin (c) Both (a) and (b) (d) None

Solution

(b)

Question 55 – ICAI SM

The best statistical measure used for comparing two series is:

(a) Mean Deviation (b) Range (c) Coefficient of Variation (d) Standard Deviation

Solution

(c)

Question 56 – MTP May, 2021

If sum of squares of the values = 3390, N = 30 and standard deviation = 7, find out the mean.

(a) 113 (b) 210 (c) 8 (d) None

Solution

(c)

$$SD = \sqrt{\frac{\sum(x)^2}{n} - (\bar{x})^2}$$

$$7 = \sqrt{\frac{3390}{30} - (\overline{x})^2}$$

$$(7)^2 = \left(\sqrt{\frac{3390}{30} - (\overline{x})^2}\right)^2$$

$$49 = \frac{3390}{30} - (\overline{x})^2$$

$$49 = 113 - (\overline{x})^2$$

$$(\overline{x})^2 = 113 - 49 = 64$$

$$\overline{x} = \sqrt{64} = 8$$

Question 57 – MTP May, 2021

The standard deviation of 10, 16, 10, 16, 10, 10, 16, 16 is:

(a) 4 (b) 6 (c) 3 (d) 0

Solution

(c)

Question 58 – MTP May, 2021

The of mean and SD of a series is $a + b$, if we add 2 to each observation of the series then the sum of the mean and SD will be:

(a) $a + b + 2$ (b) $6 - a + b$ (c) $4 + a - b$ (d) $a + b + 4$

Solution

(a)

Question 59 – MTP June, 2013

There are two startups in ecommerce sector struggling to acquire the market. Following data is for Mean and Standard Deviation of billing amount of bought items per month on their website:

Startup	No. of Customers/Month	Mean Billing Amount	SD of Billing Amount
A	40	₹2,500	₹10
B	30	₹2,200	₹11

Which startup has a better consistency when it comes to sales numbers?

(a) Startup A (b) Startup B (c) Both A and B (d) Need More Information

Solution

(a)

Better consistency would be of that startup whose coefficient of variation is less.

$$CV_A = \frac{SD_A}{AM_A} \times 100 = \frac{10}{2,500} \times 100 = 0.4$$

$$CV_B = \frac{SD_B}{AM_B} \times 100 = \frac{11}{2,200} \times 100 = 0.5$$

Therefore, Startup A is more consistent.

Question 60 – MTP June, 2013

If the coefficient of variation and standard deviation are 60 and 12 respectively, then the arithmetic mean of the distribution is:

(a) 40 (b) 36 (c) 20 (d) 19

Solution

(c)

$$CV = \frac{SD}{AM} \times 100$$

$$\Rightarrow AM = \frac{SD}{CV} \times 100 = \frac{12}{60} \times 100 = 20$$

Question 61 – MTP June, 2013

If the sum of square of the value equals to 3390, number of observations are 30 and Standard deviation is 7, what is the mean value of the above observation?

(a) 14 (b) 11 (c) 8 (d) 5

Solution

(c)

Given: $\sum x^2 = 3390;\ n = 30;\ SD = 7$

$$SD = \sqrt{\frac{\sum x^2}{n} - (\overline{x})^2}$$

$$\Rightarrow 7 = \sqrt{\frac{3390}{30} - \left(\bar{x}\right)^2}$$

Squaring both sides, we have:

$$\left(7\right)^2 = \left(\sqrt{\frac{3390}{30} - \left(\bar{x}\right)^2}\right)^2$$

$$\Rightarrow 49 = \frac{3390}{30} - \left(\bar{x}\right)^2$$

$$\Rightarrow \left(\bar{x}\right)^2 = \frac{3390}{30} - 49$$

$$\Rightarrow \bar{x} = \sqrt{\frac{3390}{30} - 49}$$

$$\Rightarrow \bar{x} = 8$$

Question 62 – MTP June, 2013

If the variance of random variable x is 18, then what is variance of $y = 2x + 5$?

(a) 34 (b) 39 (c) 68 (d) 72

Solution

(d)

When two variables x and y are related as $y = a + bx$, and SD_x is known, $SD_y = |b| \times SD_x$.

$$y = 2x + 5$$

$$\Rightarrow y = 5 + 2x$$

$$SD_y = |2| \times SD_x$$

$$\Rightarrow SD_y = 2 \times \sqrt{18}$$

$$\Rightarrow Var(y) = \left(2 \times \sqrt{18}\right)^2 = 72$$

Question 63 – MTP June, 2013

In a given set, if all data are of same value, then variance would be:

(a) 0 (b) 1 (c) –1 (d) 0.5

Solution

(a)

Quartile Deviation

Quartile Deviation

Individual, Discrete, Continuous Series

$$QD = \frac{Q_3 - Q_1}{2}$$

Coefficient of Quartile Deviation

$$\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

Properties

If x and y are two variables related as $y = a + bx$ for two constants a and b, then the QD of y is given by $QD_y = |b| \times QD_x$

Questions Based on Quartile Deviation

Question 64 – ICAI SM

The quartiles of a variable are 45, 52 and 65 respectively. Its quartile deviation is:

(a) 10 (b) 20 (c) 25 (d) 8.30

Solution

(a)

Question 65 – MTP June, 2013

In the equation $4x+2y=3$, quartile deviation for y is 3. Find the quartile deviation for x.

(a) 4.5 (b) 6 (c) 1.5 (d) None

Solution

(c)

$$4x+2y=3$$

$$\Rightarrow 4x=3-2y$$

$$\Rightarrow x=\frac{3}{4}-\frac{2}{4}y$$

$$QD_x = |b| \times QD_y$$

$$\Rightarrow QD_x = \left|-\frac{2}{4}\right| \times 3 = \frac{2}{4} \times 3 = 1.5$$

Question 66 – ICAI SM

The mean and SD for *a*, *b*, and 2 are 3 and $\frac{2}{\sqrt{3}}$ respectively. The value of *ab* would be?

(a) 5 (b) 6 (c) 11 (d) 3

Solution

(a)

We have 3 observations → *a*, *b*, and 2

Mean $\left(\bar{x}\right) = \frac{a+b+2}{3} = 3$

Therefore, $a+b+2 = 3\times3 = 9$

So, $a+b=9-2=7$

$$SD=\sqrt{\frac{\sum(x)^2}{n}-(\overline{x})^2}$$

$$\Rightarrow \frac{2}{\sqrt{3}}=\sqrt{\frac{a^2+b^2+2^2}{3}-(3)^2}$$

Squaring both sides, we have:

$$\left(\frac{2}{\sqrt{3}}\right)^2=\left(\sqrt{\frac{a^2+b^2+2^2}{3}-(3)^2}\right)^2$$

$$\Rightarrow \frac{4}{3} = \frac{a^2 + b^2 + 4}{3} - 9$$

$$\Rightarrow \frac{a^2 + b^2 + 4}{3} = \frac{4}{3} + 9$$

$$\Rightarrow a^2 + b^2 + 4 = 3\left(\frac{4}{3} + 9\right)$$

$$\Rightarrow a^2 + b^2 = \left[3\left(\frac{4}{3} + 9\right)\right] - 4 = 27$$

$$\Rightarrow a^2 + b^2 + 2ab - 2ab = 27$$

$$\Rightarrow (a+b)^2 - 2ab = 27$$

$$\Rightarrow (7)^2 - 2ab = 27$$

$$\Rightarrow 49 - 2ab = 27$$

$$\Rightarrow -2ab = 27 - 49$$

$$\Rightarrow -2ab = -22$$

$$\Rightarrow ab = \frac{-22}{-2} = 11$$

Question 67 – ICAI SM

Which one is an absolute measure of dispersion?

(a) Range (b) Mean Deviation (c) Standard Deviation (d) All these measures

Solution

(d)

Question 68 – MTP June, 2013

If Quartile deviation is 7, find the value of *x* from the arranged series: 2, *x*, 6, 7, 9, 16, 18.

(a) 5 (b) 2 (c) 8 (d) 6

Solution

(b)

Rank of $Q_1 = \frac{n+1}{4} = \frac{7+1}{4} = \frac{8}{4} = 2$

Therefore, Q_1 is the second term of the series, i.e., *x*.

Rank of $Q_3 = 3 \times 2 = 6$

Therefore, Q_3 is the 6th term of the series, i.e., 16.

$$QD = \frac{Q_3 - Q_1}{2}$$

$$\Rightarrow 7 = \frac{16 - x}{2}$$

$$\Rightarrow 7 \times 2 = 16 - x$$

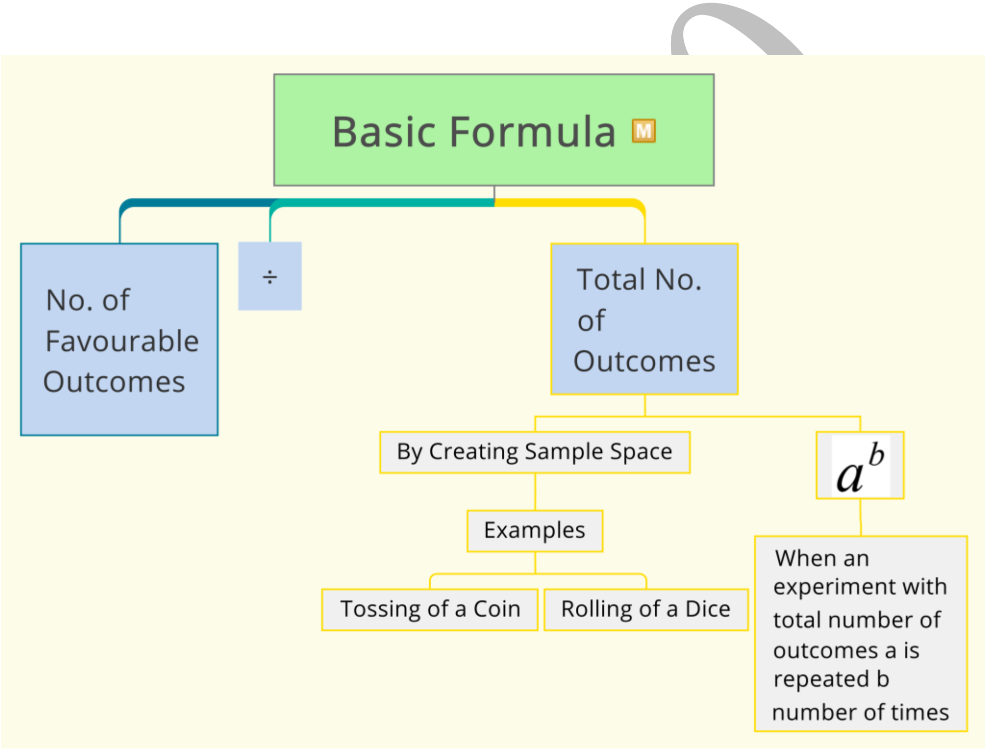
$$\Rightarrow 14 = 16 - x$$

$$\Rightarrow x = 16 - 14 = 2$$

Chapter 15 – Probability

CA NISHANT KUMAR

Introduction



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Question 1

A dice is rolled. Find the probability that even number appears.

(a) ½ (b) 2/3 (c) ¾ (d) None

Solution

(a)

No. of favourable outcomes = 3 (i.e., 2, 4, 6)

Therefore, Probability $= \frac{3}{6} = \frac{1}{2}$

Question 2

A coin is tossed three times. What is the probability of getting at least 2 heads?

(a) 1/2 (b) 2/3 (c) ¾ (d) None

Solution

(a)

We can see that no. of events containing at least two heads are 4, i.e., *HHH*, *HHT*, *HTH*, *THH*.

Therefore, Probability = 4/8 = 0.5.

Question 3

A dice is rolled twice. What is the probability of getting a difference of 2 points?

(a) 8/36 (b) 2/3 (c) ¾ (d) None

Solution

(a)

Here, the experiment is rolling of a dice, containing 6 outcomes. This experiment is repeated twice. Therefore, total number of outcomes is $6^2 = 36$.

Events in which the difference is of 2 points are {(1, 3), (2, 4), (3, 5), (4, 6), (3, 1), (4, 2), (5, 3), (6, 4)}. Therefore, total number of outcomes in favour of the event = 8.

Therefore, probability = 8/36 = 0.22.

Question 4

Two dice are thrown simultaneously. Find the probability that the sum of points on the two dice would be 7 or more.

OR

What is the chance of throwing at least 7 in a single cast with 2 dice?

(a) 5/12 (b) 7/12 (c) ¼ (d) 17/36

Solution

(b)

Two dice are thrown simultaneously is the same as one dice being thrown twice. Therefore, the total number of outcomes are $6^2 = 36$.

The question asks the probability of the sum being 7 or more. We know that the highest sum can be 12 when both the dice show 6. Now, a total of 7 or more, i.e., 7, or 8, or 9, or 10, or 11, or 12 can occur in the following combinations:

Condition	Events Corresponding to Condition	Total Events
Sum of 7	{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}	6
Sum of 8	{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}	5
Sum of 9	{(3, 6), (4, 5), (5, 4), (6, 3)}	4
Sum of 10	{(4, 6), (5, 5), (6, 4)}	3
Sum of 11	{(5, 6), (6, 5)}	2
Sum of 12	{(6, 6)}	1
Total		21

Therefore, probability = 21/36 = 0.58.

Question 5

A committee of 7 members is to be formed from a group comprising 8 gentlemen and 5 ladies. What is the probability that the committee would comprise of at least 2 ladies?

(a) 148/429 (b) 392/429 (c) 140/429 (d) None

Solution

(b)

There are 8 + 5 = 13 people in total. A committee of 7 members can be formed in $^{13}C_7$ = $\frac{13!}{7!(13-7)!} = \frac{13!}{7!6!} = 1{,}716$ ways.

Committee comprising of at least two ladies

No. of ways a committee can be formed comprising at least two ladies:

Combinations	**Ladies**	**Gents**		**Total**
2 ladies + 5 gents	${}^5C_2$	${}^8C_5$	${}^5C_2 \times {}^8C_5$	560
3 ladies + 4 gents	${}^5C_3$	${}^8C_4$	${}^5C_3 \times {}^8C_4$	700
4 ladies + 3 gents	${}^5C_4$	${}^8C_3$	${}^5C_4 \times {}^8C_3$	280
5 ladies + 2 gents	${}^5C_5$	${}^8C_2$	${}^5C_5 \times {}^8C_2$	28
Total				1,568

Therefore, the probability that the committee would comprise at least of 2 ladies = 1568/1716 = 0.9138

Question 6

There are three boxes with the following compositions:

Box/Colour	Blue	Red	White	Total
I	5	8	10	23
II	4	9	8	21
III	3	6	7	16

One ball drawn from each box. What is the probability that they would be of the same colour?

(a) 1052/7728 (b) 1053/7728 (c) 1054/7728 (d) None

Solution

(a)

Total Number of Outcomes = 23 × 21 × 16 = 7,728

Number of Favourable Outcomes:

We'll get happy if all of them are either blue, or red, or white.

Number of ways of selecting all three blue balls = 5 × 4 × 3 = 60

Number of ways of selecting all three red balls = 8 × 9 × 6 = 432

Number of ways of selecting all three white balls = 10 × 8 × 7 = 560

Total number of favourable outcomes = 60 + 432 + 560 = 1,052

Probability = 1,052/7,728

Question 7

A bag contains 15 one-rupee coins, 25 two-rupee coins and 10 five-rupee coins. If a coin is selected at random from the bag, then the probability of not selecting a one-rupee coin is:

(a) 0.30 (b) 0.70 (c) 0.25 (d) 0.20

Solution

(b)

Total no. of selections = 15 + 25 + 10 = 50

Total no. of selections of two-rupee coins and five-rupee coins = 25 + 10 = 35

Probability = 35/50 = 0.70

Question 8

If two letters are taken at random from the word HOME, what is the Probability that none of the letters would be vowels?

(a) 1/6 (b) ½ (c) 1/3 (d) ¼

Solution

(a)

Total no. of selections = ${}^{4}C_{2}$

No. of ways of selecting consonants = ${}^{2}C_{2}$

$$\text{Probability} = \frac{{}^{2}C_2}{{}^{4}C_2} = \frac{1}{6}$$

Question 9

Two balls are drawn from a bag containing 5 white and 7 black balls at random. What is the probability that they would be of different colours?

(a) 35/66 (b) 30/66 (c) 12/66 (d) None

Solution

(a)

Total Number of Outcomes = $^{12}C_2 = \frac{12 \times 11}{1 \times 2} = 66$

Number of Favourable Outcomes:

We'll get happy if one white and one black ball appears.

One white ball can be drawn in 5 ways, and one black ball can be drawn in 7 ways. Therefore, total number of ways = $5 \times 7 = 35$

Probability = 35/66

Question 10

What is the chance of getting at least one defective item if 3 items are drawn randomly from a lot containing 6 items of which 2 are defective items?

(a) 0.30 (b) 0.20 (c) 0.80 (d) 0.50

Solution

(c)

Total Number of Outcomes = $^{6}C_{3} = \frac{6\times5\times4}{1\times2\times3} = 20$

Number of Favourable Outcomes:

No. of ways of selecting at least one defective item = Total no. of ways – no. of ways of selecting no defective item = $20 - {}^{4}C_{3} = 20 - {}^{4}C_{1} = 20 - 4 = 16$

Probability = 16/20

Question 11

If two unbiased dice are rolled together, what is the probability of getting no difference of points?

(a) ½ (b) 1/3 (c) 1/5 (d) 1/6

Solution

(d)

Total number of outcomes = 36

Number of Favourable Outcomes = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} = 6

Probability = 6/36

Question 12

There are 10 balls numbered from 1 to 10 in a box. If one of them is selected at random, what is the probability that the number printed on the ball would be an odd number greater that 4?

(a) 0.50 (b) 0.40 (c) 0.60 (d) 0.30

Solution

(d)

Total number of outcomes = 10

Number of favourable outcomes = {5, 7, 9} = 3

Probability = 3/10

Question 13

What is the probability that a leap year selected at random would contain 53 Saturdays?

(a) 1/7 (b) 2/7 (c) 1/12 (d) ¼

Solution

(b)

A leap year has 366 days. Any year has 52 weeks = 52 × 7 = 364 days.

Since there are 52 weeks in any year, there are 52 Saturdays in any year.

Now, the remaining two days could be:

1. Sunday, Monday

2. Monday, Tuesday
3. Tuesday, Wednesday
4. Wednesday, Thursday
5. Thursday, Friday
6. Friday, Saturday
7. Saturday, Sunday

Therefore, the total number of outcomes = 7

Number of favourable outcomes = Outcomes containing a Saturday = 2

Probability = 2/7

Question 14

If two unbiased dice are rolled, what is the probability of getting points neither 6 nor 9?

(a) 0.25 (b) 0.50 (c) 0.75 (d) 0.80

Solution

(c)

Total number of outcomes = 36

Number of outcomes in which the points are either 6 or 9 = {(1, 5), (5, 1), (2, 4), (4, 2), (3, 3), (3, 6), (6, 3), (4, 5), (5, 4)} = 9

Number of favourable outcomes = 36 – 9 = 27

Probability = 27/36

Question 15

Find the probability that a four-digit number comprising the digits 2, 5, 6 and 7 would be divisible by 4.

(a) 4/13 (b) 5/13 (c) 8/24 (d) 7/13

Solution

(c)

Total number of 4 digit numbers that can be formed from these 4 digits = 4! = 24.

Any number is divisible by four if the number formed by the last two digits of that number is divisible by 4. For example, consider the number 45620. Now, the last two digits of this number are 2, and 0. The number formed from these two digits is 20, which is divisible by 4. Therefore, the number 45620 is also divisible by 4.

Now, we have 2, 5, 6, and 7. The two digit numbers that can be formed from these digits which are divisible by 4 are 52, 56, 72, and 76. Therefore, we know that the last two digits could either be 52, 56, 72, and 76. Therefore, there are 4 ways to fill the last two digits of a four-digit number. Now, after filling the last two spaces, we'll be left with the first two spaces and two digits to fill them. Hence, the first two digits can be filled in 2! ways.

Therefore, the no. of 4 digit numbers that can be formed from the digits 2, 5, 6, and 7, which are divisible by 4 is $2! \times 4 = 8$.

Therefore, probability = 8/24 = 0.33.

Question 16

A pair of dice is thrown together and the sum of points of the two dice is noted to be 10. What is the probability that one of the two dice has shown the point 4?

(a) ¾ (b) ½ (c) 2/3 (d) None

Solution

(c)

Total Number of Outcomes = {(4, 6), (6, 4), (5, 5)} = 3

Number of Favourable Outcomes = {(4, 6), (6, 4)} = 2

Probability = 2/3

Question 17

In a group of 20 males and 15 females, 12 males and 8 females are service holders. What is the probability that a person selected at random from the group is a service holder given that the selected person is a male?

(a) 12/20 (b) ½ (c) 2/3 (d) None

Solution

(a)

Since the selected person is a male, the total number of outcomes = 20.

Number of Favourable Outcomes = 12

Probability = 12/20

Question 18

It is given that a family of 2 children has a girl, what is the probability that the other child is also a girl?

(a) 0.50 (b) 0.75 (c) 1/3 (d) 2/3

Solution

(c)

Since it's given that one child is a girl, we cannot consider the outcome BB.

Total number of outcomes = {GG, BG, GB} = 3

Number of favourable outcomes = {GG} = 1

Probability = 1/3

Question 19

Two coins are tossed simultaneously. What is the probability that the second coin would show a tail given that the first coin has shown a head?

(a) 0.50 (b) 0.25 (c) 0.75 (d) 0.125

Solution

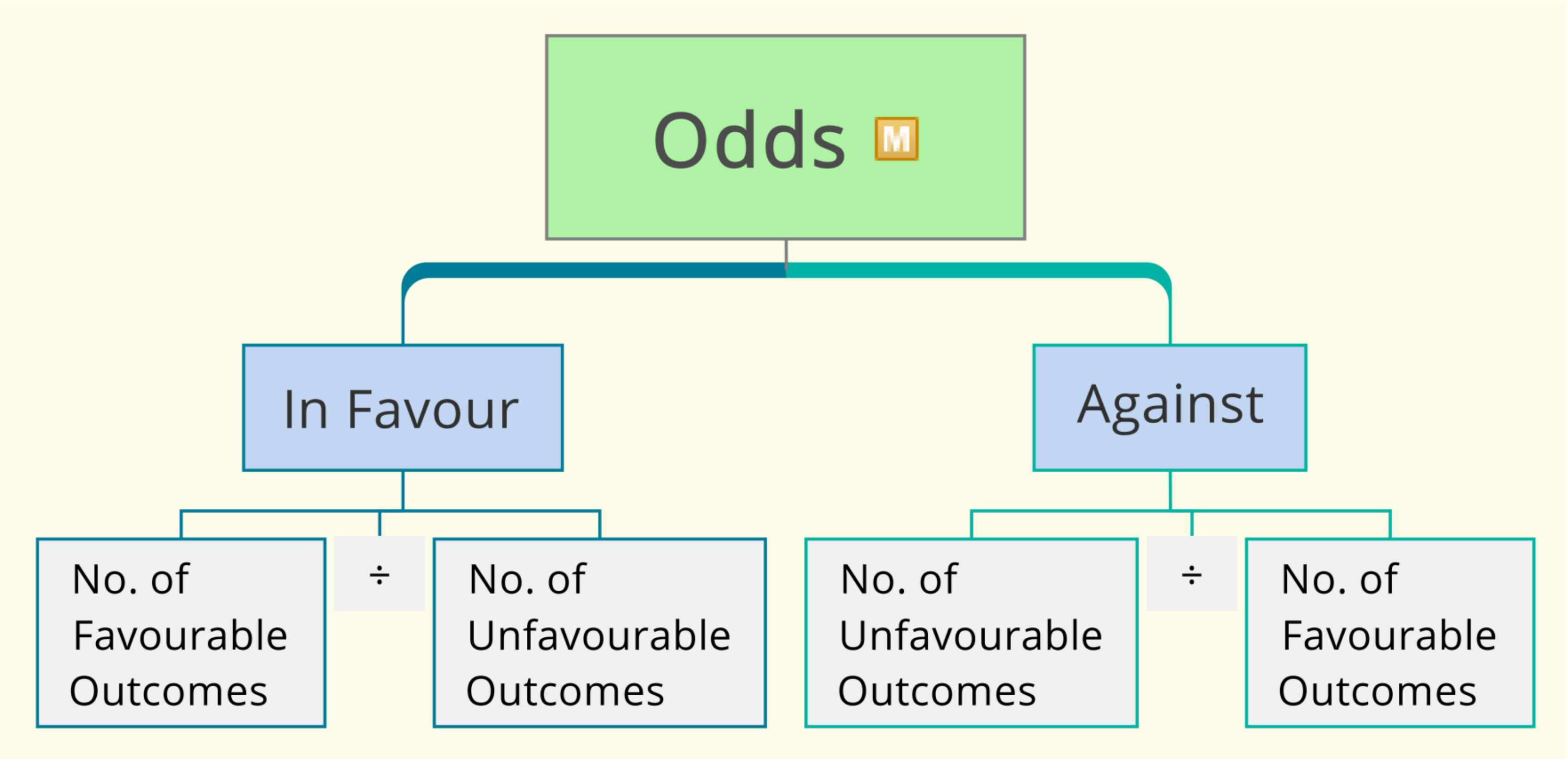
(a)

Since it is given that the first coin has shown a head, the total number of outcomes would be: {HH, HT} = 2

Number of favourable outcomes = {HT} = 1

Probability = ½

Odds


Odds
In Favour
Against
No. of Favourable Outcomes
÷
No. of Unfavourable Outcomes
No. of Unfavourable Outcomes
÷
No. of Favourable Outcomes

Question 20

If $p : q$ are the odds in favour of an event, then the probability of that event is:

(a) $\frac{p}{q}$ (b) $\frac{p}{p+q}$ (c) $\frac{q}{p+q}$ (d) None

Solution

(b)

Odds in Favour = $\frac{p}{q}$

Therefore, number of favourable outcomes = p

Number of unfavourable outcomes = q

Total number of outcomes = $p + q$

Therefore, probability = $\dfrac{p}{p+q}$

Question 21

If $P(A) = 5/9$, then the odds against the event A is:

(a) 5 : 9 (b) 5 : 4 (c) 4 : 5 (d) 5 : 14

Solution

(c)

Probability = 5/9

Therefore, number of favourable outcomes = 5

Total number of outcomes = 9

Therefore, number of unfavourable outcomes = 9 – 5 = 4

Odds Against Event = Number of unfavourable outcomes/Number of Favourable Outcomes

Therefore, Odds Against Event = 4/5 = 4 : 5

Question 22

If an unbiased die is rolled once, the odds in favour of getting a point which is a multiple of 3 is:

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

Solution

(a)

Total number of outcomes = 6

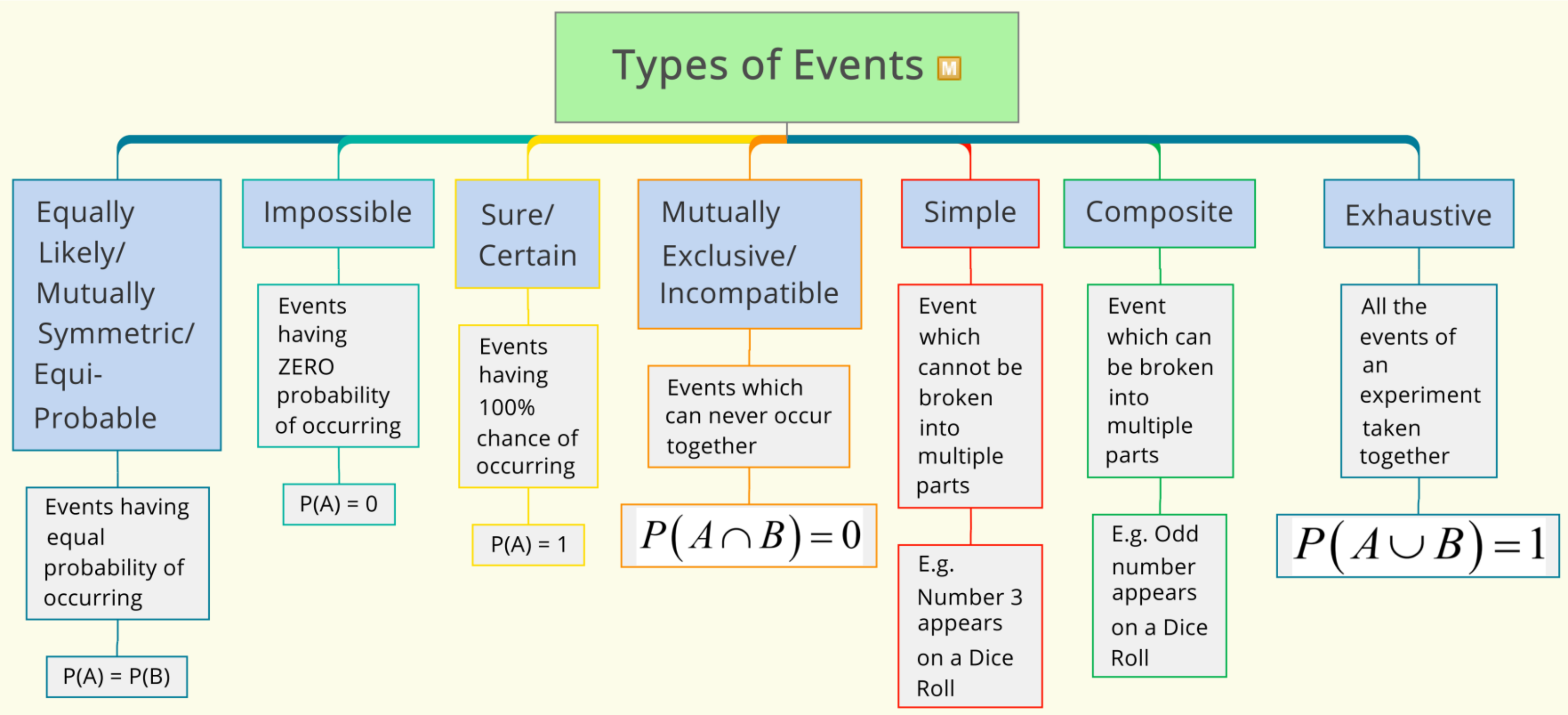
Number of favourable outcomes = {3, 6} = 2

Number of unfavourable outcomes = 6 – 2 = 4

Odds in Favour = Number of Favourable Outcomes/Number of Unfavourable Outcomes

Therefore, Odds in Favour = 2 : 4 = 1 : 2

Types of Events


Types of Events
Equally Likely/ Mutually Symmetric/ Equi-Probable
Events having equal probability of occurring
P(A) = P(B)
Impossible
Events having ZERO probability of occurring
P(A) = 0
Sure/ Certain
Events having 100% chance of occurring
P(A) = 1
Mutually Exclusive/ Incompatible
Events which can never occur together
$P(A \cap B) = 0$
Simple
Event which cannot be broken into multiple parts
E.g. Number 3 appears on a Dice Roll
Composite
Event which can be broken into multiple parts
E.g. Odd number appears on a Dice Roll
Exhaustive
All the events of an experiment taken together
$P(A \cup B) = 1$

Question 23

If $P(A) = P(B)$, then the two events A and B are:

(a) Independent (b) Dependent (c) Equally Likely (d) Both (a) and (c)

Solution

(c)

Question 24

If $P(A)=1$, then the event is known as:

(a) Symmetric Event (b) Dependent Event (c) Improbable Event (d) Sure Event

Solution

(d)

Question 25

Which of the following pairs of events are mutually exclusive?

(a)	A: The student reads in School	B: He studies Philosophy
(b)	A: Raju was born in India	B: He is a fine Engineer
(c)	A: Ruma is 16 years old.	B: She is a good Singer
(d)	A: Peter is under 15 years of age	B: Peter is a voter of Kolkata

Solution

(d)

Question 26

An event that can be split into further events is known as:

(a) Complex Event (b) Mixed Event (c) Simple Event (d) Composite Event

Solution

(d)

Question 27

If an unbiased coin is tossed once, then the two events Head and Tail are:

(a) Mutually Exclusive (b) Exhaustive (c) Equally Likely (d) All

Solution

(d)

Question 28

If $P(A)=P(B)$, then

(a) A and B are the same events
(b) A and B must be same events
(c) A and B may be different events
(d) A and B are mutually exclusive events

Solution

(c)

Set Theoretic Approach to Probability

- Sample space represents the Universal Set, denoted by S or Ω.
- An event A is defined as a non-empty subset of S.
- Then, probability of event A is given by: $P(A)=\frac{n(A)}{n(S)}$, where, $n(A)$ is the cardinal number of the set A; and $n(S)$ is the cardinal number of the set S.

Set Theoretic Approach

Two Events

- **Mutually Exclusive**
  - $P(A \cup B) = P(A) + P(B)$
- **Others**
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Three Events

- **Mutually Exclusive**
  - $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
- **Others**
  - $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Other Formulas

- **Exhaustive Events**
  - Two Events
    - $P(A \cup B) = 1$
  - Three Events
    - $P(A \cup B \cup C) = 1$
- **Equally Likely**
  - $P(A) = P(B) = P(C)$
- **Only A Occurs**
  - $P(A - B) = P(A) - P(A \cap B)$
- **Only B Occurs**
  - $P(B - A) = P(B) - P(A \cap B)$

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Question 29

If $P(A \cap B) = 0$, then the two events A and B are:

(a) Mutually Exclusive (b) Exhaustive (c) Equally Likely (d) Independent

Solution

(a)

Question 30

If, for two events A and B, $P\left(A \cup B\right) = 1$, then A and B are:

(a) Mutually Exclusive (b) Equally Likely (c) Exhaustive (d) Dependent

Solution

(c)

Question 31

If A, B and C are mutually exclusive and exhaustive events, then $P(A)+P(B)+P(C)$ equals to:

(a) $\frac{1}{3}$ (b) 1 (c) 0 (d) any value between 0 and 1

Solution

(b)

Since these events are mutually exclusive, we have $P(A\cap B)=0$; $P(A\cap C)=0$; $P(B\cap C)=0$; $P(A\cap B\cap C)=0$

Since these events are exhaustive, $P(A \cup B \cup C) = 1$

We know that:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Therefore,

$$1 = P(A) + P(B) + P(C) - 0 - 0 - 0 + 0$$

$$\Rightarrow P(A) + P(B) + P(C) = 1$$

Question 32

If A denotes that a student reads in a school and B denotes that he plays cricket, then:

(a) $P(A \cap B) = 1$ (b) $P(A \cup B) = 1$ (c) $P(A \cap B) = 0$ (d) $P(A) = P(B)$

Solution

(c)

Question 33

Three events *A*, *B* and *C* are mutually exclusive, exhaustive, and equally likely. What is the probability of the complementary event of A?

(a) 6/11 (b) 3/11 (c) 1/6 (d) 2/3

Solution

(d)

Since these events are mutually exclusive, we have $P(A \cap B) = 0$; $P(A \cap C) = 0$; $P(B \cap C) = 0$; $P(A \cap B \cap C) = 0$

Since the events are exhaustive, we have $P(A \cup B \cup C) = 1$

Since the events are equally likely, we have $P(A)=P(B)=P(C)$

We know that:

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$$

Therefore,

$$1=P(A)+P(B)+P(C)$$

$$\Rightarrow P(A)+P(B)+P(C)=1$$

Since $P(A)=P(B)=P(C)$

$$\Rightarrow P(A)+P(A)+P(A)=1$$

$$\Rightarrow 3P\left(A\right)=1$$

$$\Rightarrow P\left(A\right)=\frac{1}{3}$$

$$\Rightarrow P\left(A'\right)=1-\frac{1}{3}=\frac{2}{3}$$

Question 34

A, B and C are three mutually exclusive and exhaustive events such that $P(A) = 2P(B) = 3P(C)$. What is $P(B)$?

(a) 6/11 (b) 3/11 (c) 1/6 (d) 1/3

Solution

(b)

Since these events are mutually exclusive, we have $P(A \cap B) = 0$; $P(A \cap C) = 0$; $P(B \cap C) = 0$; $P(A \cap B \cap C) = 0$

Since these events are exhaustive, $P(A \cup B \cup C) = 1$

We know that:

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$$

Therefore,

$$1=P(A)+P(B)+P(C)$$

$$\Rightarrow P(A)+P(B)+P(C)=1$$

Now, Since $P(A)=2P(B)=3P(C)$, $P(C)=\frac{2}{3}P(B)$

Therefore, $2P(B)+P(B)+\frac{2}{3}P(B)=1$

$$\Rightarrow \frac{6P(B)+3P(B)+2P(B)}{3}=1$$

$$\Rightarrow 6P(B)+3P(B)+2P(B)=3$$

$$\Rightarrow 11P(B)=3$$

$$\Rightarrow P(B)=\frac{3}{11}$$

Question 35

A number is selected at random from the first 1000 natural numbers. What is the probability that the number so selected would be a multiple of 7 or 11?

(a) 0.25 (b) 0.32 (c) 0.22 (d) 0.33

Solution

(c)

Total Number of Outcomes $n(S) = 1000$

Numbers divisible by 7 $n(7) = \frac{1000}{7} = 142.86 = 142$

Numbers divisible by 11 $n(11)=\frac{1000}{11}=90.90=90$

There could be some numbers divisible both by 7 as well as by 11. For this, we need to take the LCM of 7 and 11, i.e. 77. Any number divisible by 77 is divisible by 7 as well as by 11.

Numbers divisible by 77 $n(77)=\frac{1000}{77}=12.98=12$

Therefore, Numbers divisible by 7 and 11 $n(7\cap 11)=\frac{1000}{77}=12.98=12$

Probability that the number is divisible by 7 = $P(7)=\frac{n(7)}{n(S)}=\frac{142}{1000}$

Probability that the number is divisible by 11 = $P(11)=\frac{n(11)}{n(S)}=\frac{90}{1000}$

Probability that the number is divisible by both 7 and 11 = $P(7\cap 11)=\frac{n(7\cap 11)}{n(S)}=\frac{12}{1000}$

Probability that the number is divisible by 7 or 11 = $P(7\cup 11)=P(7)+P(11)-P(7\cap 11)$

$$P(7\cup 11)=\frac{142}{1000}+\frac{90}{1000}-\frac{12}{1000}=\frac{220}{1000}=0.22$$

Question 36

The probability that an Accountant's job applicant has a B. Com. Degree is 0.85, that he is a CA is 0.30 and that he is both B. Com. and CA is 0.25 out of 500 applicants, how many would be B. Com. or CA?

(a) 450 (b) 500 (c) 900 (d) 950

Solution

(a)

Let B. Com be denoted by B and CA be denoted by C.

We have $P(B) = 0.85;\ P(C) = 0.30;\ P(B \cap C) = 0.25;\ n(S) = 500$

We know that $P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$\Rightarrow P(B \cup C) = 0.85 + 0.30 - 0.25 = 0.90$

Also, we know that $P(B \cup C) = \frac{n(B \cup C)}{n(S)}$

$\Rightarrow n(B \cup C) = P(B \cup C) \times n(S) = 0.90 \times 500 = 450$

Question 37

If $P(A-B)=1/5$, $P(A)=1/3$ and $P(B)=1/2$, what is the probability that out of the two events *A* and *B*, only *B* would occur?

(a) 10/30 (b) 11/30 (c) 9/30 (d) None

Solution

(b)

$$P(A-B)=P(A)-P(A\cap B)$$

$$\Rightarrow \frac{1}{5}=\frac{1}{3}-P(A\cap B)$$

$$\Rightarrow P\left(A\cap B\right)=\frac{1}{3}-\frac{1}{5}=\frac{5-3}{15}=\frac{2}{15}$$

$$P\left(B-A\right)=P\left(B\right)-P\left(A\cap B\right)=\frac{1}{2}-\frac{2}{15}=\frac{15-4}{30}=\frac{11}{30}$$

Question 38

There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

(a) 0.30 (b) 0.90 (c) 0.45 (d) None

Solution

(b)

We have $P(A)=0.80$; $P(B)=0.60$; $P(C)=0.50$; $P(A\cap B)=0.46$; $P(B\cap C)=0.32$; $P(A\cap C)=0.48$; $P(A\cap B\cap C)=0.26$

We know that

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$$

$$\Rightarrow P(A\cup B\cup C)=0.80+0.60+0.50-0.46-0.48-0.32+0.26=0.90$$

Question 39

If a card is drawn at random from a pack of 52 cards, what is the chance of getting a Spade or an ace?

(a) 4/13 (b) 5/13 (c) 0.25 (d) 0.20

Solution

(a)

Let event *A* be drawing of a Spade, and event *B* be drawing of an Ace.

$$P(A)=\frac{13}{52};\ P(B)=\frac{4}{52};\ P(A\cap B)=\frac{1}{52}$$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Therefore, $P(A \cup B) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = 0.30769$

Question 40

If $P\left(\overline{A} \cup \overline{B}\right) = 5/6$, $P(A) = 1/2$ and $P\left(\overline{B}\right) = 2/3$, what is $P(A \cup B)$?

(a) 1/3 (b) 5/6 (c) 2/3 (d) 4/9

Solution

(c)

$$P(A' \cup B') = \frac{5}{6};\ P(A) = \frac{1}{2};\ P(B') = \frac{2}{3}$$

$$P(A' \cup B') = \frac{5}{6}$$

$$\Rightarrow P(A\cap B)' = \frac{5}{6}$$

$$\Rightarrow 1 - P(A\cap B) = \frac{5}{6}$$

$$\Rightarrow 1 - \frac{5}{6} = P(A\cap B)$$

$$\Rightarrow P(A\cap B) = \frac{1}{6}$$

$$P(B') = \frac{2}{3}$$

$$\Rightarrow 1 - P(B) = \frac{2}{3}$$

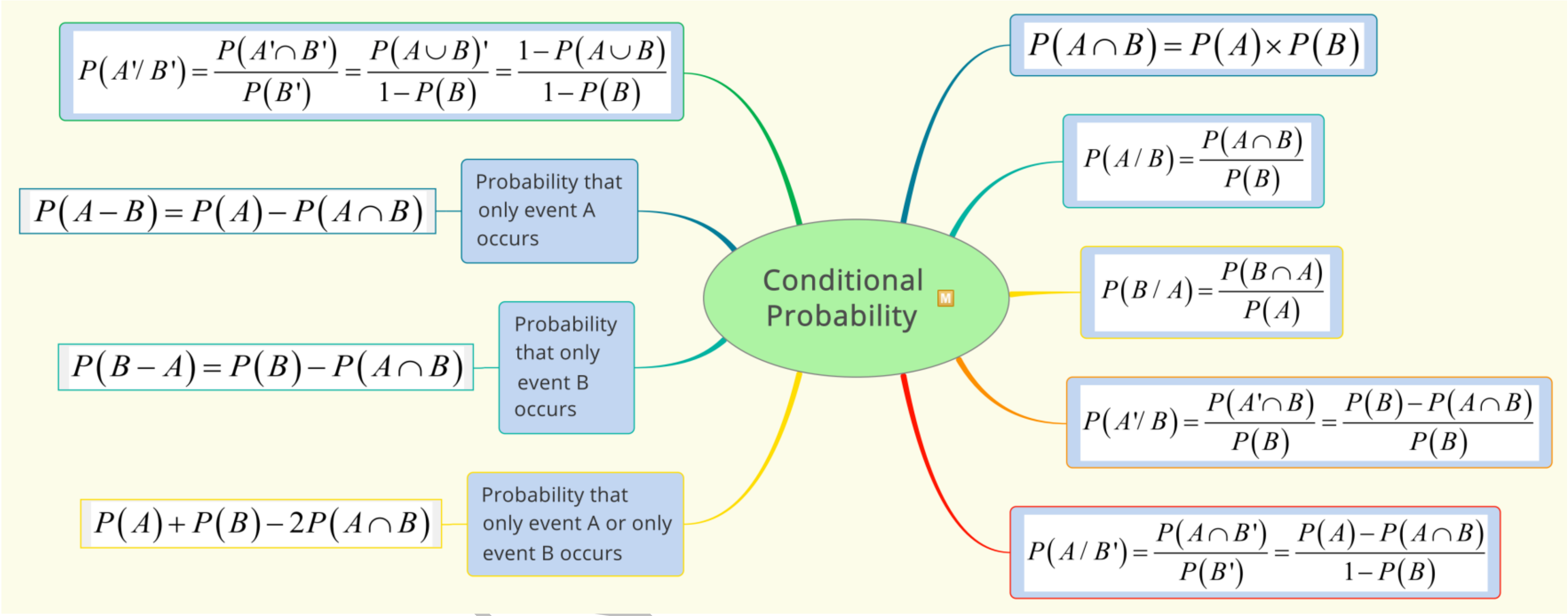
$$\Rightarrow 1 - \frac{2}{3} = P(B)$$

$$\Rightarrow P(B) = \frac{1}{3}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$$

Conditional Probability and Compound Theorem of Probability

CA NISHANT KUMAR



CA NISHANT KUMAR

Question 41

A box contains 5 white and 7 black balls. Two successive draws of 3 balls are made with replacement. The probability that the first draw would produce white balls and the second draw would produce black balls are respectively:

(a) 6/321 (b) 1/20 (c) 35/144 (d) 7/968

Solution

(d)

The balls are drawn "with replacement". This means that after the first draw of three balls is made, those three balls are returned to the bag, and thereafter another draw of three balls is made.

Total number of outcomes $= n(S) = {}^{5+7}C_3 = {}^{12}C_3 = \dfrac{12\times11\times10}{1\times2\times3} = 220$

Let event A be the event of first draw of three balls. Since we need all white balls, $n(A) = {}^{5}C_3 = \dfrac{5\times4}{1\times2} = 10$

Therefore, $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{10}{220}$

Let event B be the event of second draw of three balls. Since we need all black balls, $n(B) = {}^{7}C_3 = \dfrac{7\times6\times5}{1\times2\times3} = 35$

Therefore, $P(B)=\dfrac{n(B)}{n(S)}=\dfrac{35}{220}$

We need $P(A\cap B)$.

Since the number of favourable outcomes of event B won't change because of happening of event A, these events are independent, $P(A\cap B)=P(A)\times P(B)$

$$\Rightarrow P(A\cap B)=\frac{10}{220}\times\frac{35}{220}=0.00723$$

Question 42

A box contains 5 white and 7 black balls. Two successive draws of 3 balls are made without replacement. The probability that the first draw would produce white balls and the second draw would produce black balls are respectively:

(a) 3/926 (b) 1/30 (c) 35/108 (d) 5/264

Solution

(d)

The balls are drawn without replacement. This means that after the first draw of three balls is made, these balls are not returned to the box.

Let A be the event of first draw. Currently, there are 12 balls in the box. So, total number of outcomes $= n(S) = {}^{5+7}C_3 = {}^{12}C_3 = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220$

Since we need all white balls, $n(A) = {}^{5}C_3 = \frac{5 \times 4}{1 \times 2} = 10$

Therefore, $P(A) = \frac{n(A)}{n(S)} = \frac{10}{220}$

Let B be the event of second draw. Now, there are only 9 balls in the box since 3 balls have already drawn and not been returned. So, total number of outcomes $= n(S) = {}^{9}C_3 = \frac{9 \times 8 \times 7}{1 \times 2 \times 3} = 84$

Since we need all black balls, $n(B) = {}^7C_3 = \frac{7 \times 6 \times 5}{1 \times 2 \times 3} = 35$

Therefore, $P(B) = \frac{n(B)}{n(S)} = \frac{35}{84}$

We need $P(A \cap B)$.

Since the number of favourable outcomes of event *B* won't change because of happening of event *A*, these events are independent, $P(A \cap B) = P(A) \times P(B)$

$$\Rightarrow P(A \cap B) = \frac{10}{220} \times \frac{35}{84} = 0.018939$$

Question 43

There are two urns containing 5 red and 6 white balls and 3 red and 7 white balls respectively. If two balls are drawn from the first urn without replacement and transferred to the second urn and then a draw of another two balls is made from it, what is the probability that both the balls drawn are red?

(a) 7/20 (b) 35/88 (c) 65/726 (d) 3/20

Solution

(c)

Following three cases arise:

Case 1 – Both balls drawn from Urn I are Red

Case 2 – Both balls drawn from Urn I are White

Case 3 – One ball drawn from Urn I is Red and the other ball is White

Case 1 – Both balls drawn from Urn I are Red

Probability of drawing both red balls from Urn I = $\frac{{}^{5}C_{2}}{{}^{11}C_{2}} = \frac{10}{55}$

When these balls get transferred to Urn II, Urn II will have 5 Red and 7 White balls.

Probability of drawing both red balls from Urn II = $\frac{{}^{5}C_{2}}{{}^{12}C_{2}} = \frac{10}{66}$

Therefore, Probability of drawing both red balls from Urn II when both balls drawn from Urn I are red = $\left(\frac{10}{55}\times\frac{10}{66}\right)$

Case 2 – Both balls drawn from Urn I are White

Probability of drawing both white balls from Urn I = $\frac{{}^{6}C_2}{{}^{11}C_2}=\frac{15}{55}$

When these balls get transferred to Urn II, Urn II will have 3 Red and 9 White balls.

Probability of drawing both red balls from Urn II = $\frac{{}^{3}C_2}{{}^{12}C_2}=\frac{3}{66}$

Therefore, Probability of drawing both red balls from Urn II when both balls drawn from Urn I are red = $\left(\frac{15}{55} \times \frac{3}{66}\right)$

Case 3 – One ball drawn from Urn I is Red and the other ball is White

Probability of drawing one red and one white ball from Urn I = $\frac{{}^{5}C_1 \times {}^{6}C_1}{{}^{11}C_2} = \frac{30}{55}$

When these balls get transferred to Urn II, Urn II will have 4 Red and 8 White balls.

Probability of drawing both red balls from Urn II = $\frac{{}^{4}C_2}{{}^{12}C_2} = \frac{6}{66}$

Therefore, Probability of drawing both red balls from Urn II when both balls drawn from Urn I are red = $\left(\frac{11}{55}\times\frac{6}{66}\right)$

Therefore, Probability = $\left(\frac{10}{55}\times\frac{10}{66}\right)+\left(\frac{15}{55}\times\frac{3}{66}\right)+\left(\frac{30}{55}\times\frac{6}{66}\right)=0.0891$

Question 44

What is the probability of having at least one 'six' from 3 throws of a perfect die?

(a) $5/6$ (b) $\left(5/6\right)^3$ (c) $1-\left(1/6\right)^3$ (d) $1-\left(5/6\right)^3$

Solution

(d)

Question 45

Tom speaks truth in 30 percent cases and Dick speaks truth in 25 percent cases. What is the probability that they would contradict each other?

(a) 0.325 (b) 0.400 (c) 0.925 (d) 0.075

Solution

(b)

Probability that Tom speaks truth $= \frac{30}{100} = 0.30$

Probability that Tom does not speak truth = 1 – 0.30 = 0.70

Probability that Dick speaks truth = 0.25

Probability that Dick does not speak truth = 1 – 0.25 = 0.75

They both would contradict each other when one speaks the truth and the other does not. So, we'll have two cases –

1. Either Tom speaks the truth and Dick does not, or
2. Dick speaks the truth and Tom does not

Probability that Tom speaks the truth and Dick does not = 0.30 × 0.75 = 0.225

Probability that Dick speaks the truth and Tom does not = 0.25 × 0.70 = 0.175

Probability that both of them contradict each other = 0.225 + 0.175 = 0.4000

Question 46

There are three persons aged 60, 65 and 70 years old. The survival probabilities for these three persons for another 5 years are 0.7, 0.4 and 0.2 respectively. What is the probability that at least two of them would survive another five years?

(a) 0.425 (b) 0.456 (c) 0.392 (d) 0.388

Solution

(d)

Person	Probability of Survival	Probability of Non-Survival
A	0.7	0.3

B	0.4	0.6
C	0.2	0.8

$(0.7 \times 0.4 \times 0.8) + (0.7 \times 0.2 \times 0.6) + (0.4 \times 0.2 \times 0.3) + (0.7 \times 0.4 \times 0.2) = 0.388$

Question 47

A problem in probability was given to three CA students A, B and C whose chances of solving it are 1/3, 1/5 and 1/2 respectively. What is the probability that the problem would be solved?

(a) 4/15 (b) 7/8 (c) 8/15 (d) 11/15

Solution

(d)

Student	Probability of Solving	Probability of Non-Solving
A	1/3	2/3
B	1/5	4/5
C	½	½

$$1-\left(\frac{2}{3}\times\frac{4}{5}\times\frac{1}{2}\right)=0.733$$

Question 48

Rupesh is known to hit a target in 5 out of 9 shots whereas David is known to hit the same target in 6 out of 11 shots. What is the probability that the target would be hit once they both try?

(a) 79/99 (b) 77/88 (c) 88/150 (d) 11/15

Solution

(a)

$$1-\left(\frac{4}{9}\times\frac{5}{11}\right)=0.7979$$

Question 49

If 8 balls are distributed at random among three boxes, what is the probability that the first box would contain 3 balls?

OR

8 identical balls are placed at random in three bags. What is the probability that the first bag will contain 3 balls?

(a) 0.2731 (b) 0.3256 (c) 0.1924 (d) 0.3443

Solution

(a)

The first ball can be distributed to the 1st box or 2nd box or 3rd box, i.e., it can be distributed in 3 ways. Similarly, the second ball also can be distributed in 3 ways. Thus, the first two balls can be distributed in 3^2 ways. Proceeding this way, we find that 8 balls can be distributed to 3 boxes in 3^8 ways which is the total number of elementary events.

Let A be the event that the first box contains 3 balls which implies that the remaining 5 balls must go to the remaining 2 boxes which, as we have already discussed, can be done in 2^5 ways. Since 3 balls out of 8 balls can be selected in ${}^8C_3$ ways, the event can occur in ${}^8C_3 \times 2^5$ ways, thus we have:

$$P(A) = \frac{{}^8C_3 \times 2^5}{3^8} = \frac{56 \times 32}{6561} = \frac{1792}{6561}$$

Question 50

For two events A and B, $P(B)=0.3$, $P(A \text{but not } B)=0.4$, and $P(\text{not } A)=0.6$. The events A and B are:

(a) exhaustive (b) independent (c) equally likely (d) mutually exclusive

Solution

(d)

$$P(B)=0.3$$

$$P(A-B)=0.4$$

$$P(A')=0.6$$

$$P(A-B)=P(A)-P(A\cap B)$$

$$P(A')=1-P(A)$$

$$P(A)=1-P(A')=1-0.6=0.4$$

$$P(A-B)=P(A)-P(A\cap B)$$

$$P(A\cap B)=P(A)-P(A-B)$$

$$P(A\cap B)=0.4-0.4=0$$

Question 51

If, for two independent events *A*, and *B*, $P\left(A\cup B\right)=\frac{2}{3}$ and $P\left(A\right)=\frac{2}{5}$, what is $P\left(B\right)$?

(a) 4/15 (b) 4/9 (c) 5/9 (d) 7/15

Solution

(b)

$$P\left(A\cup B\right)=P\left(A\right)+P\left(B\right)-P\left(A\cap B\right)$$

$$\frac{2}{3}=\frac{2}{5}+P\left(B\right)-\left\{P\left(A\right)\times P\left(B\right)\right\}$$

$$\frac{2}{3}=\frac{2}{5}+P(B)-\left\{\frac{2}{5}\times P(B)\right\}$$

$$\frac{2}{3}=\frac{2}{5}+P(B)-\frac{2}{5}P(B)$$

$$\frac{2}{3}=\frac{2}{5}+\frac{3}{5}P(B)$$

$$\frac{3}{5}P(B)=\frac{2}{3}-\frac{2}{5}$$

$$P(B)=\left(\frac{2}{3}-\frac{2}{5}\right)\times\frac{5}{3}=\frac{4}{9}$$

Question 52

A packet of 10 electronic components is known to include 2 defectives. If a sample of 4 components is selected at random from the packet, what is the probability that the sample does not contain more than 1 defective?

(a) 1/3 (b) 2/3 (c) 13/15 (d) 3/15

Solution

(c)

Total Number of Outcomes = $^{10}C_4 = \frac{10\times9\times8\times7}{1\times2\times3\times4} = 210$

Number of Favourable Outcomes:

Case 1 – No defectives

No. of ways of selecting 4 components such that none is defective =

$$^{8}C_{4} = \frac{8\times7\times6\times5}{1\times2\times3\times4} = 70$$

Case 2 – Only 1 defective

We need one defective; this can be obtained in $^{2}C_{1} = 2$ ways.

We need the other three non-defectives; this can be obtained in $^{8}C_{3} = \frac{8\times7\times6}{1\times2\times3} = 56$ ways.

Therefore, only one defective in a sample of 4 can be obtained in 2 × 56 = 112 ways.

Therefore, number of favourable outcomes = 70 + 112 = 182

Probability = 182/210 = 0.867

Question 53

Mr. Roy is selected for three separate posts. For the first post, there are three candidates, for the second, there are five candidates and for the third, there are 10 candidates. What is the probability that Mr. Roy would be selected?

(a) 13/25 (b) 14/25 (c) 15/25 (d) None

Solution

(a)

Let Post 1 be denoted by *A*, Post 2 be denoted by *B*, and Post 3 be denoted by *C*.

Probability that Mr. Roy would get selected in *A* = $P\left(A\right)=\frac{1}{3}$;

Probability that Mr. Roy would get selected in B = $P\left(B\right)=\frac{1}{5}$;

Probability that Mr. Roy would get selected in C = $P\left(C\right)=\frac{1}{10}$

Probability that Mr. Roy would get selected in A or B or C = $P\left(A\cup B\cup C\right)$

If we subtract the probability of Mr. Roy not getting selected at all from the total probability, i.e., 1, we'll get the desired result.

Probability that Mr. Roy would not get selected at all = $P\left(A'\cap B'\cap C'\right)=P\left(A'\right)\times P\left(B'\right)\times P\left(C'\right)$

$$= \left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{5}\right)\times\left(1-\frac{1}{10}\right) = 0.48$$

Probability that Mr. Roy would get selected = 1 – 0.48 = 0.52

Question 54

X and *Y* stand in a line with 6 other people. What is the probability that there are 3 persons between them?

(a) 1/5 (b) 1/6 (c) 1/7 (d) 1/3

Solution

(c)

There are 8 people in total. They can be arranged in 8! ways.

Therefore, total number of outcomes = 8! = 40,320

We need to select 3 people out of 6 people (since we need to exclude *X* and *Y*). This can be done in ${}^6C_3$ ways. Now, let's consider them as one unit.

Now, we have *X*, these three, *Y*, and the remaining 3 persons.

Let's see in how many ways they can be arranged. Consider *X*, the selected three, and *Y* as 1 unit. Now, there are 4 distinct units:

1. *X*, the three selected, and *Y*
2. The remaining three persons

These four people can be arranged in 4! ways.

Also, the three selected can be arranged among themselves in 3! ways.

Also, *X* and *Y* can be arranged among themselves in 2! ways.

$\therefore$ Favourable number of outcomes = ${}^{6}C_{3} \times 4! \times 3! \times 2! = \dfrac{6\times5\times4}{1\times2\times3} \times 24 \times 6 \times 2 = 5,760$

Probability = 5,760/40,320 = 0.142857

Question 55

Given that $P\left(A\right)=1/2$, $P\left(B\right)=1/3$, $P\left(A\cap B\right)=1/4$, what is $P\left(A'/B'\right)$?

(a) ½ (b) 7/8 (c) 5/8 (d) 2/3

Solution

(c)

$$P\left(A'/B'\right)=\frac{P\left(A'\cap B'\right)}{P\left(B'\right)}=\frac{P\left(A\cup B\right)'}{1-P\left(B\right)}=\frac{1-P\left(A\cup B\right)}{1-P\left(B\right)}=\frac{1-\left[P\left(A\right)+P\left(B\right)-P\left(A\cap B\right)\right]}{1-P\left(B\right)}$$

$$P\left(A'/B'\right)=\frac{1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right]}{1-\frac{1}{3}}=0.625$$

Question 56

The odds in favour of an event is 2 : 3 and the odds against another event is 3 : 7. Find the probability that only one of the two events occurs.

(a) 25/50 (b) 27/50 (c) ½ (d) None

Solution

(b)

Odds in Favour of $(A) = \frac{2}{3}$

Therefore, number of favourable outcomes of $A = 2$

Number of unfavourable outcomes of $A = 3$

Total number of outcomes of $A = 3 + 2 = 5$

Therefore, $P(A) = \frac{2}{5}$

Odds Against $(B) = \frac{3}{7}$

Therefore, number of unfavourable outcomes of $B = 3$

Number of favourable outcomes of $B = 7$

Total number of outcomes of $B = 10$

$$P(B) = \frac{7}{10}$$

$$P(A\cap B)=P(A)\times P(B)=\frac{2}{5}\times\frac{7}{10}=\frac{7}{25}$$

Probability that only one of them occurs:

$$P(A)+P(B)-2P(A\cap B)=\frac{2}{5}+\frac{7}{10}-\left(2\times\frac{7}{25}\right)=0.54$$

Question 57

For a group of students, 30%, 40% and 50% failed in Physics, Chemistry, and at least one of the two subjects respectively. If an examinee is selected at random, what is the probability that he passed in Physics if it is known that he failed in Chemistry?

(a) ½ (b) 1/3 (c) ¼ (d) 1/6

Solution

(a)

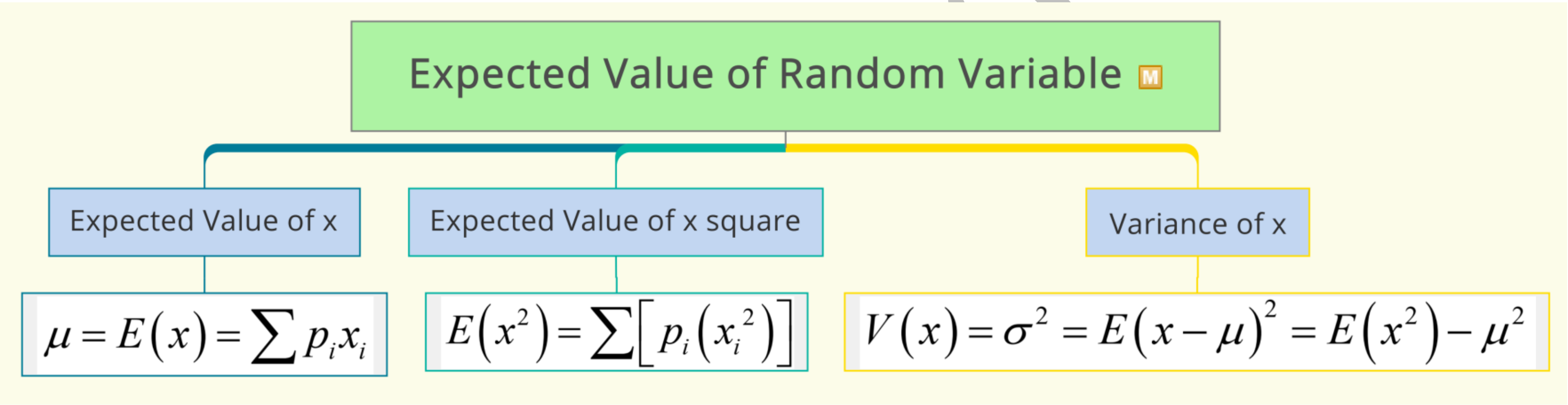
Given $P(P) = 0.30;\ P(C) = 0.40;\ P(P \cup C) = 0.50$

$$P(P \cap C) = P(P) + P(C) - P(P \cup C)$$

$$\Rightarrow P\left(P\cap C\right)=0.30+0.40-0.50=0.20$$

Therefore, $P\left(P'/C\right)=\dfrac{P\left(P'\cap C\right)}{P\left(C\right)}=\dfrac{P\left(C\right)-P\left(P\cap C\right)}{P\left(C\right)}=\dfrac{0.40-0.20}{0.40}=0.50$

Expected Value of a Random Variable



Question 58

In a business venture, a man can make a profit of ₹50,000 or incur a loss of ₹20,000. The probabilities of making profit or incurring loss, from the past experience, are known to be 0.75 and 0.25 respectively. What is his expected profit?

(a) ₹42,500 (b) ₹32,500 (c) ₹35,000 (d) None

Solution

(b)

Question 59

A bag contains 6 white and 4 red balls. If a person draws 2 balls and receives ₹10 and ₹20 for a white and red balls respectively, then his expected amount is:

(a) ₹25 (b) ₹26 (c) ₹29 (d) ₹28

Solution

(d)

Both White Balls = 20

Both Red Balls = 40

One White One Red = 30

$$= \left(\frac{^{6}C_2}{^{10}C_2} \times 20 \right) + \left(\frac{^{4}C_2}{^{10}C_2} \times 40 \right) + \left(\frac{^{6}C_1 \times {}^{4}C_1}{^{10}C_2} \times 30 \right)$$

$$= \left(\frac{15}{45} \times 20 \right) + \left(\frac{6}{45} \times 40 \right) + \left(\frac{24}{45} \times 30 \right) = 28$$

Question 60

Moidul draws 2 balls from a bag containing 3 white and 5 Red balls. He gets ₹500 if he draws a white ball and ₹200 if he draws a red ball. What is his expectation? If he is asked to pay ₹400 for participating in the game, would he consider it a fair game and participate?

(a) ₹625; No (b) ₹625; Yes (c) ₹450; Yes (d) ₹450; No

Solution

(b)

Question 61

A packet of 10 electronic components is known to include 3 defectives. If 4 components are selected from the packet at random, what is the expected value of the number of defective?

(a) 1.20 (b) 1.21 (c) 1.69 (d) 1.72

Solution

(a)

There could be 0 defectives, 1 defective, 2 defectives, or 3 defectives.

$$P\left(x=0\right)=\frac{{}^{7}C_{4}}{{}^{10}C_{4}}=\frac{{}^{7}C_{3}}{{}^{10}C_{4}}=\frac{\frac{7\times6\times5}{1\times2\times3}}{\frac{10\times9\times8\times7}{1\times2\times3\times4}}=\frac{35}{210}$$

$$P\left(x=1\right)=\frac{{}^{7}C_{3}\times{}^{3}C_{1}}{{}^{10}C_{4}}=\frac{35\times3}{210}=\frac{105}{210}$$

$$P\left(x=2\right)=\frac{{}^{7}C_{2}\times{}^{3}C_{2}}{{}^{10}C_{4}}=\frac{{}^{7}C_{2}\times{}^{3}C_{1}}{{}^{10}C_{4}}=\frac{21\times3}{210}=\frac{63}{210}$$

$$P\left(x=3\right)=\frac{{}^{7}C_{1}\times{}^{3}C_{3}}{{}^{10}C_{4}}=\frac{7\times1}{210}=\frac{7}{210}$$

$$E\left(x\right)=\sum px$$

$$E(x)=\left(\frac{35}{210}\times 0\right)+\left(\frac{105}{210}\times 1\right)+\left(\frac{63}{210}\times 2\right)+\left(\frac{7}{210}\times 3\right)=1.2$$

Question 62

The probability that there is at least one error in an account statement prepared by 3 persons *A*, *B* and *C* are 0.2, 0.3 and 0.1 respectively. If *A*, *B* and *C* prepare 60, 70 and 90 such statements, then the expected number of correct statements is:

(a) 170 (b) 176 (c) 178 (d) 180

Solution

(c)

The probability that there is at least one error in an account statement prepared by 3 persons *A*, *B* and *C* are 0.2, 0.3 and 0.1 respectively.

Therefore, probability that there is no error in an account statement prepared by 3 persons *A*, *B* and *C* are (1 – 0.2), (1 – 0.3) and (1 – 0.1) respectively.

A, *B* and *C* prepare 60, 70 and 90 such statements.

Therefore, probability that there is no error in an account statement prepared by 3 persons *A*, *B* and *C* are 0.80, 0.70, and 0.90 respectively.

Expected Number of Correct Statements = $\left(60\times 0.80\right)+\left(70\times 0.70\right)+\left(90\times 0.90\right)=178$

Question 63

A random variable *x* has the following probability distribution:

x	0	1	2	3	4	5	6	7
$P(x)$	0	$2k$	$3k$	k	$2k$	k^2	$7k^2$	$2k^2 + k$

Find the value of *k*.

(a) 0.10 (b) 0.50 (c) 0.40 (d) 0.31

Solution

(a)

$$\sum P(x) = 1$$

$$0 + 2k + 3k + k + 2k + k^2 + 7k^2 + 2k^2 + k = 1$$

$$10k^2 + 9k = 1$$

$$10k^2 + 9k - 1 = 0$$

$$10k^2 + 10k - k - 1 = 0$$

$$10k(k+1) - 1(k+1) = 0$$

$$(10k-1)(k+1) = 0$$

$$k = \frac{1}{10} = 0.10$$

Question 64

A random variable *x* has the following probability distribution:

x	0	1	2	3	4	5	6	7
$P(x)$	0	$2k$	$3k$	k	$2k$	k^2	$7k^2$	$2k^2 + k$

Find the value of $P(x<3)$.

(a) 0.10 (b) 0.50 (c) 0.40 (d) 0.31

Solution

(b)

$$P(x<3)=P(x=0)+P(x=1)+P(x=2)$$

$$P(x<3)=0+(2\times 0.10)+(3\times 0.10)=0.50$$

Question 65

A random variable has the following probability distribution:

x:	4	5	7	8	10
P:	0.15	0.20	0.40	0.15	0.10

Find $E\left[x-E\left(x\right)\right]^2$. Also, find $v\left(3x-4\right)$.

(a) 3.04; 29.36 (b) 3.04; 27.36 (c) 4.04; 27.36 (d) None

Solution

(b)

Formulas to be used:

$$\mu = E(x) = \sum px$$

$$E(x^2) = \sum px^2$$

$$\sigma^2 = E(x-\mu)^2 = E(x^2) - \mu^2$$

$$E(x) = \sum px = (0.15\times 4) + (0.20\times 5) + (0.40\times 7) + (0.15\times 8) + (0.10\times 10) = 6.60$$

$$E(x-E(x))^2 = E(x-\mu)^2 = E(x^2) - \mu^2$$

$$E(x^2) = \sum px^2 = (0.15\times 4^2) + (0.20\times 5^2) + (0.40\times 7^2) + (0.15\times 8^2) + (0.10\times 10^2) = 46.60$$

$$\mu^2 = (6.60)^2 = 43.56$$

$$E\left(x-E\left(x\right)\right)^2=E\left(x-\mu\right)^2=E\left(x^2\right)-\mu^2=46.60-43.56=3.04$$

This is the variance.

Therefore, ${\sigma_x}^2=3.04$

$$y=3x-4$$

$$Variance_y=\left(\sigma_y\right)^2$$

$$Variance_y=\left(|b|\times\sigma_x\right)^2$$

$$Variance_y=\left(3\times\sqrt{3.04}\right)^2=27.36$$

Question 66

The following data relate to the distribution of wages of a group of workers:

Wages in ₹:	50-60	60-70	70-80	80-90	90-100	100-110	110-120
No. of Workers:	15	23	36	42	17	12	5

If a worker is selected at random from the entire group of workers, what is the probability that his wage would be more than ₹100?

(a) 0 (b) $\frac{37}{75}$ (c) $\frac{17}{150}$ (d) $\frac{19}{30}$

Solution

(c)

Theoretical Distributions

CA NISHANT KUMAR

Binomial Distribution

Binomial Distribution is used to find out the probability where the total no. of outcomes is huge. The probability is given by the following formula:

$$P(x) = {}^{n}C_{x} p^{x} q^{n-x}, \text{ for } x = 0, 1, 2, 3, \ldots, n$$

Here,

n = number of times the experiment is repeated

x = the requirement of the question

p = probability of success in each trial

q = probability of failure in each trial = $1 - p$

Sometimes, $P(x)$ is also written as $f(x)$. $f(x)$ is called "Probability Mass Function".

Conditions

Binomial distribution is applicable only if the following conditions are satisfied:

1. All the trials are independent, and
2. Each trial has only two outcomes.

Page 16.4 – Example 16.1 (i)

A coin is tossed 10 times. Assuming the coin to be unbiased, what is the probability of getting 4 heads?

(a) 107/512 (b) 105/512 (c) 106/512 (d) None

Solution

(b)

Here $n = 10$; $p = ½$; $q = ½$; $x = 4$

We know that $P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

Therefore, $P(4) = {}^{10}C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{10-4} = 0.20507$

Page 16.4 – Example 16.2

If 15 dates are selected at random, what is the probability of getting two Sundays?

(a) 0.29 (b) 0.48 (c) 0.56 (d) None

Solution

(a)

Here n = 15; p = 1/7; q = 1 – 1/7 = 6/7; x = 2

We know that $P(x) = {}^{n}C_{x} p^{x} q^{n-x}$

$$P(x=2) = {}^{15}C_{2}\left(\frac{1}{7}\right)^{2}\left(\frac{6}{7}\right)^{13} = 0.2888 \approx 0.29$$

Exercise – Set B – Question 6

What is the probability of getting 3 heads if 6 unbiased coins are tossed simultaneously?

(a) 0.50 (b) 0.25 (c) 0.3125 (d) 0.6875

Solution

(c)

Here, $n=6;\ p=0.5;\ q=0.5;\ x=3$

$$P(x)={}^{n}C_{x}p^{x}q^{n-x}$$

$$P(x=3)={}^{6}C_{3}(0.5)^{3}(0.5)^{6-3}=0.3125$$

Exercise – Set B – Question 8

What is the probability of making 3 correct guesses in 5 True – False answer type questions?

(a) 0.3125 (b) 0.5676 (c) 0.6875 (d) 0.4325

Solution

(a)

Here, $n = 5;\ p = 0.5;\ q = 0.5;\ x = 3$

$$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$P(x=3) = {}^{5}C_{3}(0.5)^{3}(0.5)^{5-3} = 0.3125$$

Page 16.4 – Example 16.1 (ii)

A coin is tossed 10 times. Assuming the coin to be unbiased, what is the probability of getting at least 4 heads?

(a) 848/1024 (b) 848/1025 (c) 849/1024 (d) None

Solution

(a)

Here $n = 10$; $p = ½$; $q = ½$; $x \geq 4$

We know that $P(x) = {}^{n}C_{x} p^{x} q^{n-x}$

$$P(x \geq 4) = P(x=4) + P(x=5) + P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$$

Or $P\left(x \geq 4\right) = 1 - P\left(x < 4\right)$

$$\Rightarrow P\left(x \geq 4\right) = 1 - \left[P\left(x=0\right) + P\left(x=1\right) + P\left(x=2\right) + P\left(x=3\right)\right]$$

$$\Rightarrow P\left(x \geq 4\right) = 1 - \left[{}^{10}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10} + {}^{10}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^9 + {}^{10}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^8 + {}^{10}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7\right]$$

$$\Rightarrow P\left(x \geq 4\right) = 1 - \left[0.171875\right] = 0.828125$$

Page 16.4 – Example 16.1 (iii)

A coin is tossed 10 times. Assuming the coin to be unbiased, what is the probability of getting at most 3 heads?

(a) 13/64 (b) 12/64 (c) 11/64 (d) None

Solution

(c)

Here $n = 10$; $p = ½$; $q = ½$; $x \le 4$

We know that $P(x) = {}^nC_x p^x q^{n-x}$

$$P(x \le 3) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$$

$$\Rightarrow P\left(x \le 3\right) = {}^{10}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10} + {}^{10}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^9 + {}^{10}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^8 + {}^{10}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7$$

$$\Rightarrow P\left(x \le 3\right) = 0.171875$$

Page 16.4 – Example 16.3

The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. What is the probability that out of 5 workmen, 3 or more will contract the disease?

(a) 0.0906 (b) 0.0086 (c) 0.8006 (d) None

Solution

(b)

Here $n = 5$; $p = 0.10$; $q = 1 – 0.10 = 0.90$; $x \geq 3$

We know that $P(x) = {}^nC_x p^x q^{n-x}$

$$P\left(x \geq 3\right) = P\left(x = 3\right) + P\left(x = 4\right) + P\left(x = 5\right)$$

$$\Rightarrow P\left(x \geq 3\right) = {}^{5}C_{3}\left(0.10\right)^{3}\left(0.90\right)^{2} + {}^{5}C_{4}\left(0.10\right)^{4}\left(0.90\right)^{1} + {}^{5}C_{5}\left(0.10\right)^{5}\left(0.90\right)^{0}$$

$$\Rightarrow P\left(x \geq 3\right) = 0.00856 \approx 0.0086$$

Exercise – Set B – Question 7

If the overall percentage of success in an exam is 60, what is the probability that out of a group of 4 students, at least one has passed?

(a) 0.6525 (b) 0.9744 (c) 0.8704 (d) 0.0256

Solution

(b)

Pass percentage is 60. This means 60%. This means that p = 0.6. Therefore, q = 0.4.

Therefore, we have, $n = 4;\ p = 0.6;\ q = 0.4;\ x \geq 1$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$P(x \geq 1) = 1 - P(x < 1)$$

$$= 1 - P(x = 0)$$

$$= 1 - {}^{4}C_{0}(0.6)^{0}(0.4)^{4-0}$$

$$= 1 - 0.0256$$

$$= 0.9744$$

Exercise – Set C – Question 1

If it is known that the probability of a missile hitting a target is 1/8, what is the probability that out of 10 missiles fired, at least 2 will hit the target?

(a) 0.4258 (b) 0.3968 (c) 0.5238 (d) 0.3611

Solution

(d)

We have $n = 10;\ p = \frac{1}{8} = 0.125;\ q = 1 - 0.125 = 0.875;\ x \geq 2$

$$P\left(x \geq 2\right) = 1 - P\left(x < 2\right)$$

$$=1-\left[P\left(x=0\right)+P\left(x=1\right)\right]$$

$$=1-\left[{}^{10}C_0\left(0.125\right)^0\left(0.875\right)^{10}+{}^{10}C_1\left(0.125\right)^1\left(0.875\right)^9\right]$$

$$=0.3611$$

Exercise – Set C – Question 6

In 10 independent rollings of a biased die, the probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times. What is the probability that an even number will appear twice when the die is rolled 8 times?

(a) 0.0304 (b) 0.1243 (c) 0.2315 (d) 0.1926

Solution

(a)

Since it is a biased die, we can't say that the probability of even number in every trial is 3/6. Therefore, $p \neq 3/6$. From the given information, we'll have to calculate the value of p first.

Clearly, $n = 10$; $p = p$; $q = 1 - p$

Given:

$$P\left(x=5\right)=2P\left(x=4\right)$$

$$\Rightarrow {}^{10}C_5 p^5 q^5 = 2\left[{}^{10}C_4 p^4 q^6\right]$$

$$\Rightarrow 252 p^5 q^5 = 2\times 210 p^4 q^6$$

$$\Rightarrow \frac{p^5}{p^4}=\frac{2\times 210}{252}\times\frac{q^6}{q^5}$$

$$\Rightarrow p=\frac{5}{3}q$$

$$\Rightarrow 3p = 5q$$

$$\Rightarrow 3p = 5(1 - p)$$

$$\Rightarrow 3p = 5 - 5p$$

$$\Rightarrow 3p + 5p = 5$$

$$\Rightarrow 8p = 5$$

$$\Rightarrow p = \frac{5}{8}$$

$$\Rightarrow q = 1 - \frac{5}{8} = \frac{3}{8}$$

Now, we have $n = 8$; $p = 5/8$; $q = 3/8$; $x = 2$

$$P(x=8) = {}^{8}C_{2}\left(\frac{5}{8}\right)^{2}\left(\frac{3}{8}\right)^{6} = 0.0304$$

Page 16.6 – Example 16.4

Find the probability of a success for the binomial distribution satisfying the following relation $4P(x=4)=P(x=2)$ and having the parameter *n* as six.

(a) 1 (b) 1/3 (c) 2/3 (d) None

Solution

(b)

$$4P(x=4)=P(x=2)$$

$$\Rightarrow 4\times {}^{6}C_{4}p^{4}q^{2}={}^{6}C_{2}p^{2}q^{4}$$

$$\Rightarrow 4 \times \frac{p^4}{p^2} = \frac{q^4}{q^2}$$

$$\Rightarrow 4p^2 = q^2$$

$$\Rightarrow 4p^2 = (1 - p)^2$$

Now try the options.

Page 16.9 – Example 16.8

An experiment succeeds thrice as after it fails. If the experiment is repeated 5 times, what is the probability of having no success at all?

(a) 1/1024 (b) 2/3 (c) 1/1025 (d) None

Solution

(a)

We have $n = 5$; $p = 3q$; $x = 0$

$$p = 3q$$

$$\Rightarrow p = 3(1 - p)$$

$$\Rightarrow p = 3 - 3p$$

$$\Rightarrow 4p = 3$$

$$\Rightarrow p = \frac{3}{4}$$

$$\Rightarrow q = \frac{1}{4}$$

$$P\left(x = 0\right) = {}^5C_0 \left(\frac{3}{4}\right)^0 \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$$

Important Points

1. Binomial Distribution is applicable when the random variable (x) is discrete.
2. As $n > 0,\ p, q > 0$, therefore, $f(x) \geq 0$ for every x.

   Also, $\sum f(x) = f(0) + f(1) + f(2) + f(3) + ... + f(n) = 1$
3. Binomial distribution is known as bi-parametric distribution as it is characterised by two parameters n and p. This means that if the values of n and p are known, then the distribution is known completely.
4. The mean of the binomial distribution is given by $\mu = np$.
5. A binominal distribution is symmetrical when $p = q$.

Exercise – Set B – Question 2

x is a binomial variable with $n = 20$. What is the mean of x if it is known that x is symmetric?

(a) 5 (b) 10 (c) 2 (d) 8

Solution

(b)

If x is symmetric, $p = q = 0.5$.

Therefore, mean = $np = 20 \times 0.5 = 10$

Exercise – Set C – Question 2

x is a binomial variable such that $2P(x=2)=P(x=3)$ and mean of x is known to be 10/3. What would be the probability that x assumes at most the value 2?

(a) 16/81 (b) 17/81 (c) 47/243 (d) 46/243

Solution

(b)

Mean = $np = \frac{10}{3}$

$$2P(x=2)=P(x=3)$$

$$2\times {}^{n}C_{2}p^{2}q^{n-2}={}^{n}C_{3}p^{3}q^{n-3}$$

$$2\times\frac{n!}{2!(n-2)!}\times p^{2}q^{n-2}=\frac{n!}{3!(n-3)!}\times p^{3}\times q^{n-3}$$

$$\frac{n!}{(n-2)!}\times p^{2}q^{n-2}=\frac{n!}{6(n-3)!}\times p^{3}\times q^{n-3}$$

$$\frac{1}{(n-2)!}\times q^{n-2}=\frac{1}{6(n-3)!}\times p\times q^{n-3}$$

$$\frac{(n-3)!}{(n-2)!}=\frac{pq^{n-3}}{6q^{n-2}}$$

$$\frac{(n-3)!}{(n-2)(n-3)!} = \frac{pq^{n-3-(n-2)}}{6}$$

$$\frac{1}{n-2} = \frac{pq^{n-3-n+2}}{6}$$

$$\frac{1}{n-2} = \frac{pq^{-1}}{6}$$

$$\frac{1}{n-2} = \frac{p}{6q}$$

$$6q = np - 2p$$

$$6(1-p) = np - 2p$$

$$6 - 6p = np - 2p$$

$$6 - np = 6p - 2p$$

$$6 - \frac{10}{3} = 4p$$

$$p = \frac{6 - \frac{10}{3}}{4} = \frac{\frac{18-10}{3}}{4} = \frac{8}{3} \div 4 = \frac{8}{3} \times \frac{1}{4} = \frac{2}{3}$$

$$q = \frac{1}{3}$$

$$n \times \frac{2}{3} = \frac{10}{3} \Rightarrow 2n = 10 \Rightarrow n = 5$$

$$P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= {}^5C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^5 + {}^5C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^4 + {}^5C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3$$

$$= 0.20987$$

Exercise – Set C – Question 4

If a random variable x follows binomial distribution with mean as 5 and satisfying the condition $10P(x=0)=P(x=1)$, what is the value of $P(x\geq 1/x>0)$?

(a) 0.67 (b) 0.56 (c) 0.99 (d) 0.82

Solution

(c)

Given Mean = $np=5$

Given $10P(x=0)=P(x=1)$

$\Rightarrow 10\times {}^{n}C_{0}p^{0}q^{n}={}^{n}C_{1}p^{1}q^{n-1}$

$$\Rightarrow 10\times1\times1\times q^{n}=n\times p\times q^{n-1}$$

$$\Rightarrow 10\times(1-p)^{n}=np(1-p)^{n-1}$$

$$\Rightarrow 10(1-p)^{n}=\frac{np(1-p)^{n}}{(1-p)}$$

$$\Rightarrow 10(1-p)^{n}(1-p)=np(1-p)^{n}$$

$$\Rightarrow 10(1-p)=np$$

$$\Rightarrow 10-10p=np$$

Putting the value of np = 5 above, we get:

$$10-10p=5$$

$$\Rightarrow 10p = 10 - 5$$

$$\Rightarrow 10p = 5$$

$$\Rightarrow p = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow q = \frac{1}{2}$$

Putting the value of *p* in *np* = 5, we get:

$$np = 5$$

$$\Rightarrow n \times \frac{1}{2} = 5$$

$$\Rightarrow n = 5 \times 2 = 10$$

$$P\left(x \geq 1 / x > 0\right) = \frac{P\left(x \geq 1\right) \cap P\left(x > 0\right)}{P\left(x > 0\right)} = \frac{1 - P\left(x < 1\right)}{1 - P\left(x = 0\right)} = \frac{1 - P\left(x = 0\right)}{1 - P\left(x = 0\right)} = 1$$

Important Points (Contd.)

6. Mode of a Binomial Distribution is given by $\mu_0 = (n+1)p$

   a. If the value of $(n+1)p$ is an integer (i.e., without decimal part), then the binomial distribution is said to have two modes. It is called a bi-modal binomial distribution. The two modes are given by:

      i. $(n+1)p$, and

      ii. $[(n+1)p]-1$

For example, if, in a binomial distribution, $n=11$, and $p=\frac{1}{2}$, then $(n+1)p$ = $(11+1)\times\frac{1}{2} = \frac{12}{2} = 6$ (Integer). Therefore, this binomial distribution will have two modes:

i. $(n+1)p = 6$

ii. $[(n+1)p]-1 = 6-1 = 5$

b. If the value of $(n+1)p$ is a fraction (i.e., with a decimal part), then the binomial distribution is said to have one mode. It is called a unimodal binomial distribution. Its mode is given by the largest integer contained in $(n+1)p$.

For example, if, in a binomial distribution, $n = 12$, and $p = \frac{1}{3}$, then $(n+1)p$ =

$(12+1) \times \frac{1}{3} = \frac{13}{3} = 4.33$

Since the answer is a fraction, this binomial distribution has only one mode. Its mode is given by the largest integer contained in $(n+1)p$. Therefore, the mode is 4.

Exercise – Set B – Question 4

If x is a binomial variate with parameter 15 and 1/3, what is the value of mode of the distribution?

(a) 5 and 6 (b) 5 (c) 5.50 (d) 6

Solution

(b)

Mode = $(n+1)p = (15+1)\times\frac{1}{3} = 5.33$

Since this is a fraction, mode is the highest integer, i.e., 5.

Important Points (Contd.)

7. The variance of the binomial distribution is given by $\sigma^2 = npq$.
   a. Variance of a binomial distribution is always less than its mean.
   b. If $p = q = 0.5$, variance is the maximum, and is given by $\frac{n}{4}$.
8. Standard Deviation of a binomial distribution is given by $\sigma = \sqrt{npq}$.

Exercise – Set B – Question 3

If $X \sim B(n, p)$, what would be the greatest value of the variance of x when $n = 16$?

(a) 2 (b) 4 (c) 8 (d) $\sqrt{5}$

Solution

(b)

In a binomial distribution, the value of the variance is maximum when $p = q = 0.5$.

Variance is given by $n/4 = 16/4 = 4$.

Exercise – Set B – Question 1

What is the standard deviation of the number of recoveries among 48 patients when the probability of recovering is 0.75?

(a) 36 (b) 81 (c) 9 (d) 3

Solution

(d)

Here, $n = 48$; $p = 0.75$

Standard Deviation of a Binomial Distribution = $\sigma = \sqrt{npq} = \sqrt{48 \times 0.75 \times 0.25} = 3$

Exercise – Set B – Question 5

What is the number of trials of a binomial distribution having mean and SD as 3 and 1.5 respectively?

(a) 2 (b) 4 (c) 8 (d) 12

Solution

(d)

Mean of a Binomial Distribution is given by $np = 3$

SD of a Binomial Distribution is given by $\sqrt{npq} = 1.5$

Putting the value of $np = 3$ above, we get $\sqrt{3q} = 1.5$

$$\Rightarrow \left(\sqrt{3q}\right)^2 = (1.5)^2$$

$$\Rightarrow 3q = 2.25$$

$$\Rightarrow q = \frac{2.25}{3} = 0.75$$

If $q = 0.75$, $p = 1 - 0.75 = 0.25$

Therefore, we have $n \times 0.25 = 3$

$$\Rightarrow n = \frac{3}{0.25} = 12$$

Page 16.6 – Example 16.5

Find the binomial distribution for which mean and standard deviation are 6 and 2 respectively.

(a) ${}^{18}C_x\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{18-x}$ (b) ${}^{20}C_x\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{18-x}$ (c) ${}^{22}C_x\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{18-x}$ (d) None

Solution

(a)

Mean = $np = 6$

Standard Deviation = $\sqrt{npq} = 2$

Squaring both sides, we get:

$$\left(\sqrt{npq}\right)^2 = 2^2$$

$$\Rightarrow npq = 4$$

Substituting the value of *np* = 6 in the above equation, we get:

$$6q = 4$$

$$\Rightarrow q = \frac{4}{6} = \frac{2}{3}$$

$$\Rightarrow p = 1 - \frac{2}{3} = \frac{1}{3}$$

Substituting the value of p from above in the equation *np* = 6, we get:

$$np = 6$$

$$\Rightarrow n \times \frac{1}{3} = 6$$

$$\Rightarrow n = 6 \times 3 = 18$$

Therefore, Binomial Distribution = ${}^{18}C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{n-x}$

Page 16.10 – Example 16.9

What is the mode of the distribution for which mean and SD are 10 and $\sqrt{5}$ respectively?

(a) 10 (b) 11 (c) 10 and 11 (d) None

Solution

(a)

Mean = $np = 10$

Standard Deviation = $\sqrt{npq} = \sqrt{5}$

Squaring both sides:

$$\left(\sqrt{npq}\right)^2 = \left(\sqrt{5}\right)^2$$

$$\Rightarrow 10q = 5$$

$$\Rightarrow q = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow p = \frac{1}{2}$$

Putting the value of *p* from above in the equation *np* = 10

$$n \times \frac{1}{2} = 10$$

$$\Rightarrow n = 2 \times 10 = 20$$

Mode is dependent on the value of $\left(n+1\right)p$

$$\left(n+1\right)p=\left(20+1\right)\times\frac{1}{2}=\frac{21}{2}=10.5$$

Since it is fractional, Mode is the largest integer contained in it.

Therefore, Mode = 10

9. Additive property of binomial distribution:

Let x and y be two independent binomial distributions where x has the parameters n_1 and p, and y has the parameters n_2 and p. Then $(x+y)$ will be a binomial distribution with parameters (n_1+n_2) and p.

Page 16.10 – Example 16.10

If x and y are 2 independent binomial variables with parameters 6 and ½ and 4 and ½ respectively, what is $P\left(x+y\geq 1\right)$?

(a) 1023/1024 (b) 1024/1023 (c) Both (d) None

Solution

(a)

We have $n_1=6;\ n_2=4;\ p=\dfrac{1}{2}$

Let $z=x+y$

The parameters of z will be: $n_1 + n_2 = 6 + 4 = 10$ and $p = \frac{1}{2}$

$$P(z \geq 1) = 1 - P(z < 1)$$

$$\Rightarrow P(z \geq 1) = 1 - P(z = 0)$$

$$\Rightarrow P(z \geq 1) = 1 - \left[{}^{10}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{10} \right]$$

$$\Rightarrow P(z \geq 1) = 1 - 0.0009765625 = 0.9990234375$$

10. Sometimes, Binomial Distribution is also written as $B(n, p)$. So, if, in a question you find something like "$X \sim B(5, 0.4)$", it means that $n = 5$, and $p = 0.4$. Here, X denotes the requirement of the question.

General Questions

Exercise – Set A – Question 9

The probability mass function of binomial distribution is given by:

(a) $f(x) = p^x q^{n-x}$ (b) $f(x) = {}^nC_x p^x q^{n-x}$ (c) $f(x) = {}^nC_x q^x p^{n-x}$ (d) $f(x) = {}^nC_x p^{n-x} q^x$

Solution

(b)

Exercise – Set A – Question 10

If x is a binomial variable with parameters n and p, then x can assume:

(a) any value between 0 and n

(b) any value between 0 and n, both inclusive.

(c) any whole number between 0 and n, both inclusive.

(d) any number between 0 and infinity.

Solution

(c)

Exercise – Set A – Question 12

The mean of a binomial distribution with parameter n and p is

(a) $n(1-p)$ (b) $np(1-p)$ (c) np (d) $\sqrt{np(1-p)}$

Solution

(c)

Exercise – Set A – Question 16

The mean of binomial distribution is:

(a) always more than its variance.
(b) always equal to its variance
(c) always less than its variance
(d) always equal to its standard deviation

Solution

(a)

Exercise – Set A – Question 15

For a binomial distribution, mean and mode

(a) are never equal
(b) are always equal.
(c) are equal when $q = 0.50$
(d) do not always exist

Solution

(c)

Exercise – Set A – Question 17

For a binomial distribution, there may be:

(a) one mode (b) two modes (c) Multi-mode (d) (a) or (b)

Solution

(c)

Exercise – Set A – Question 13

The variance of a binomial distribution with parameters *n* and *p* is:

(a) $np^2(1-p)$ (b) $\sqrt{np(1-p)}$ (c) $nq(1-q)$ (d) $n^2p^2(1-p)^2$

Solution

(c)

Exercise – Set A – Question 18

The maximum value of the variance of a binomial distribution with parameters n and p is:

(a) $n/2$ (b) $n/4$ (c) $np(1 - p)$ (d) $2n$

Solution

(b)

Poisson Distribution

Poisson Distribution is used to find out the probability where the total no. of outcomes is too huge and the probability of success is extremely small. The probability is given by the following formula:

$$P(x) = \frac{e^{-m} \times m^x}{x!}, \text{ for } x = 0, 1, 2, 3, \ldots, n$$

Here,

e = exponential constant = 2.71828

m = mean = np

x = the requirement of the question

Sometimes, $P(x)$ is also written as $f(x)$. $f(x)$ is called "Probability Mass Function".

Exercise – Set B – Question 14

If 1 per cent of an airline's flights suffer a minor equipment failure in an aircraft, what is the probability that there will be exactly two such failures in the next 100 such flights?

(a) 0.50 (b) 0.184 (c) 0.265 (d) 0.256

Solution

(b)

Here, we have $n = 100;\ p = \frac{1}{100} = 0.01$

Therefore, $m = np = 100 \times 0.01 = 1$

$$P\left(x=2\right)=\frac{\left(2.71828\right)^{-1}\left(1\right)^{2}}{2!}$$

$$P\left(x=2\right)=\frac{1}{2\times 2.71828}=0.1839$$

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Exercise – Set B – Question 12

If 1.5 per cent of items produced by a manufacturing unit are known to be defective, what is the probability that a sample of 200 items would contain no defective item?

(a) 0.05 (b) 0.15 (c) 0.20 (d) 0.22

Solution

(a)

Here $n = 200;\ p = \frac{1.5}{100} = 0.015$

Therefore, $m = np = 200 \times 0.015 = 3$

$$P\left(x=0\right)=\frac{\left(2.71828\right)^{-3}\times\left(3\right)^{0}}{0!}$$

$$P\left(x=0\right)=\frac{1}{\left(2.71828\right)^{3}}=0.0497$$

Page 16.14 – Example 16.15 (1)

Between 9 AM and 10 AM, the average number of phone calls per minute coming into the switchboard of a company is 4. Find the probability that during one particular minute, there will be no phone calls. (given $e^{-4} = 0.018316$)

(a) 0.018316 (b) 0.18416 (c) 0.018416 (d) None

Solution

(a)

$m = 4$

$$P(x) = \frac{e^{-m}.m^{x}}{x!}$$

$$P\left(x=0\right)=\frac{e^{-4}\left(4\right)^{0}}{0!}=\frac{0.018316\times 1}{1}=0.018316$$

Page 16.14 – Example 16.15 (2)

Between 9 AM and 10 AM, the average number of phone calls per minute coming into the switchboard of a company is 4. Find the probability that during one particular minute, there will be at most 3 phone calls. (given $e^{-4} = 0.018316$)

(a) 0.33 (b) 0.43 (c) 0.55 (d) None

Solution

(b)

$$P(x) = \frac{e^{-m}.m^{x}}{x!}$$

$$P(x \le 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$P\left(x\le 3\right)=\frac{e^{-4}\left(4\right)^{0}}{0!}+\frac{e^{-4}\left(4\right)^{1}}{1!}+\frac{e^{-4}\left(4\right)^{2}}{2!}+\frac{e^{-4}\left(4\right)^{3}}{3!}$$

$$P\left(x\le 3\right)=\frac{0.018316\times\left(4\right)^{0}}{0!}+\frac{0.018316\times\left(4\right)^{1}}{1!}+\frac{0.018316\times\left(4\right)^{2}}{2!}+\frac{0.018316\times\left(4\right)^{3}}{3!}$$

$$P\left(x\le 3\right)\approx 0.43$$

Page 16.15 – Example 16.16 (1)

If 2 per cent of electric bulbs manufactured by a company are known to be defectives, what is the probability that a sample of 150 electric bulbs taken from the production process of that company would contain exactly one defective bulb?

(a) 0.33 (b) 0.43 (c) 0.15 (d) None

Solution

(c)

Here, $n = 150;\ p = \frac{2}{100} = 0.02$

$m = np = 150 \times 0.02 = 3$

$$P\left(x\right)=\frac{e^{-m}.m^{x}}{x!}$$

$$P\left(x=1\right)=\frac{2.71828^{-3}\times 3^{1}}{1!}=\frac{3}{2.71828^{3}}\approx 0.15$$

Page 16.15 – Example 16.16 (2)

If 2 per cent of electric bulbs manufactured by a company are known to be defectives, what is the probability that a sample of 150 electric bulbs taken from the production process of that company would contain more than 2 defective bulbs?

(a) 0.33 (b) 0.58 (c) 0.15 (d) None

Solution

(b)

Here, $n = 150;\ p = \frac{2}{100} = 0.02$

$m = np = 150 \times 0.02 = 3$

$$P\left(x\right)=\frac{e^{-m}.m^{x}}{x!}$$

$$P\left(x>2\right)=1-P(x\leq 2)$$

$$P\left(x>2\right)=1-\left[P\left(x=0\right)+P\left(x=1\right)+P\left(x=2\right)\right]$$

$$P\left(x>2\right)=1-\left[\frac{2.71828^{-3}\times 3^{0}}{0!}+\frac{2.71828^{-3}\times 3^{1}}{1!}+\frac{2.71828^{-3}\times 3^{2}}{2!}\right]$$

$$P\left(x>2\right)\approx 0.58$$

Page 16.16 – Example 16.17

The manufacturer of a certain electronic component is certain that two per cent of his product is defective. He sells the components in boxes of 120 and guarantees that not more than two per cent in any box will be defective. Find the probability that a box, selected at random, would fail to meet the guarantee? Given that $e^{-2.40} = 0.0907$.

(a) 0.43 (b) 0.58 (c) 0.15 (d) None

Solution

(a)

Here, $n = 120;\ p = \dfrac{2}{100} = 0.02$

$m = np = 120 \times 0.02 = 2.40$

As per Poisson Distribution, $P(x) = \frac{e^{-m}.m^{x}}{x!}$

A box, selected at random would fail to meet the guarantee if more than 2.40 components turn out to be defective.

$$P(x > 2.40) = 1 - P(x \le 2.40)$$

$$P(x > 2.40) = 1 - \left[P(x = 0) + P(x = 1) + P(x = 2)\right]$$

$$P(x > 2.40) = 1 - \left[\frac{e^{-2.40}.(2.40)^{0}}{0!} + \frac{e^{-2.40}.(2.40)^{1}}{1!} + \frac{e^{-2.40}.(2.40)^{2}}{2!}\right]$$

$$P\left(x>2.40\right)=1-\left[\frac{0.0907\times 1}{1}+\frac{0.0907\times 2.40}{1}+\frac{0.0907.\left(2.40\right)^{2}}{2}\right]$$

$$P\left(x>2.40\right)\approx 0.43$$

Exercise – Set C – Question 11

A renowned hospital usually admits 200 patients every day. One per cent patients, on an average, require special room facilities. On one particular morning, it was found that only one special room is available. What is the probability that more than 3 patients would require special room facilities?

(a) 0.1428 (b) 0.1732 (c) 0.2235 (d) 0.3450

Solution

(a)

Here $n = 200;\ p = \frac{1}{100}$

Therefore, $m = np = 200 \times \frac{1}{100} = 2$

$$P(x) = \frac{e^{-m} m^x}{x!}$$

$$P(x > 3) = 1 - P(x \leq 3)$$

$$P(x > 3) = 1 - \left[P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) \right]$$

$$P(x > 3) = 1 - \left[\frac{e^{-2} \times 2^0}{0!} + \frac{e^{-2} \times 2^1}{1!} + \frac{e^{-2} \times 2^2}{2!} + \frac{e^{-2} \times 2^3}{3!} \right]$$

$$P\left(x>3\right)=1-\left[\frac{\left(2.71828\right)^{-2}\times 2^{0}}{0!}+\frac{\left(2.71828\right)^{-2}\times 2^{1}}{1!}+\frac{\left(2.71828\right)^{-2}\times 2^{2}}{2!}+\frac{\left(2.71828\right)^{-2}\times 2^{3}}{3!}\right]$$

$$P\left(x>3\right)=1-\left[\frac{1}{\left(2.71828\right)^{2}}+\frac{2}{\left(2.71828\right)^{2}}+\frac{4}{2\times\left(2.71828\right)^{2}}+\frac{8}{6\times\left(2.71828\right)^{2}}\right]$$

$$P\left(x>3\right)=1-\left[\frac{1}{\left(2.71828\right)^{2}}\left\{1+2+\frac{4}{2}+\frac{8}{6}\right\}\right]$$

$$P\left(x>3\right)=1-\left[0.8571\right]=0.1428$$

Exercise – Set C – Question 12 – Ambiguous

A car hire firm has 2 cars which are hired out every day. The number of demands per day for a car follows Poisson distribution with mean 1.20. What is the proportion of days on which some demand is refused? (Given $e^{1.20} = 3.32$)

(a) 0.25 (b) 0.3012 (c) 0.12 (d) 0.03

Solution

(d)

Here, $m = 1.20;\ x > 2$

$$P(x) = \frac{e^{-m} m^{x}}{x!}$$

$$P\left(x>2\right)=1-P\left(x\le 2\right)$$

$$P\left(x>2\right)=1-\left[P\left(x=0\right)+P\left(x=1\right)+P\left(x=2\right)\right]$$

$$P\left(x>2\right)=1-\left[\frac{e^{-1.20}\times 1.20^{0}}{0!}+\frac{e^{-1.20}\times 1.20^{1}}{1!}+\frac{e^{-1.20}\times 1.20^{2}}{2!}\right]$$

$$P\left(x>2\right)=1-\left[\frac{1}{e^{1.20}}+\frac{1.20}{e^{1.20}}+\frac{1.44}{2\times e^{1.20}}\right]$$

$$P\left(x>2\right)=1-\left[\frac{1}{e^{1.20}}\times\left\{1+1.20+\frac{1.44}{2}\right\}\right]$$

$$P\left(x>2\right)=1-\left[\frac{1}{3.32}\times\left\{1+1.20+\frac{1.44}{2}\right\}\right]=0.12$$

Exercise – Set C – Question 9

X is a Poisson variate satisfying the following condition $9P(X=4)+90P(X=6)=P(X=2)$. What is the value of $P(X\le 1)$?

(a) 0.5655 (b) 0.6559 (c) 0.7358 (d) 0.8201

Solution

(c)

$$9P(X=4)+90P(X=6)=P(X=2)$$

$$\Rightarrow 9\times\frac{e^{-m}m^4}{4!}+90\times\frac{e^{-m}m^6}{6!}=\frac{e^{-m}m^2}{2!}$$

$$\Rightarrow \frac{9e^{-m}m^4}{24} + \frac{90e^{-m}m^6}{720} = \frac{e^{-m}m^2}{2}$$

$$\Rightarrow \frac{3e^{-m}m^4}{8} + \frac{e^{-m}m^6}{8} = \frac{e^{-m}m^2}{2}$$

$$\Rightarrow \frac{3e^{-m}m^4 + e^{-m}m^6}{8} = \frac{e^{-m}m^2}{2}$$

$$\Rightarrow \frac{e^{-m}m^4\left(3+m^2\right)}{8} = \frac{e^{-m}m^2}{2}$$

$$\Rightarrow \frac{m^2\left(3+m^2\right)}{4} = 1$$

$$\Rightarrow m^2\left(3+m^2\right)=4$$

$$\Rightarrow 3m^2+m^4=4$$

$$\Rightarrow m^4+3m^2-4=0$$

Let $t=m^2$

Then $t^2+3t-4=0$

$$\Rightarrow t^2+4t-t-4=0$$

$$\Rightarrow t\left(t+4\right)-1\left(t+4\right)=0$$

$$\Rightarrow \left(t-1\right)\left(t+4\right)=0$$

$$\Rightarrow t=1;-4$$

Since t cannot be negative, therefore, t = 1.

$$m^2 = 1 \Rightarrow m = 1$$

$$P(X \le 1) = P(X = 0) + P(X = 1)$$

$$\Rightarrow P(X \le 1) = \frac{e^{-m} m^0}{0!} + \frac{e^{-m} m^1}{1!}$$

$$\Rightarrow P(X \le 1) = \frac{e^{-1} \times 1}{1} + \frac{e^{-1} \times 1}{1}$$

$$\Rightarrow P(X \le 1) = e^{-1} + e^{-1}$$

$$\Rightarrow P(X \le 1) = 2e^{-1} = \frac{2}{e} = \frac{2}{2.71828} = 0.7358$$

Page 16.17 – Example 16.18 (i)

A discrete random variable x follows Poisson Distribution. Find the value of $P(x = \text{at least } 1)$. You are given $E(x) = 2.20$ and $e^{-2.20} = 0.1108$

(a) 0.89 (b) 0.95 (c) 0.10 (d) None

Solution

(a)

$$P(x = \text{at least } 1)$$

$$P(x \geq 1) = 1 - P(x < 1)$$

$$P(x \geq 1) = 1 - P(x = 0)$$

$$P\left(x \geq 1\right) = 1 - \frac{e^{-2.20}.\left(2.20\right)^{0}}{0!}$$

$$P\left(x \geq 1\right) = 1 - \frac{0.1108 \times 1}{1} = 0.8892$$

Page 16.17 – Example 16.18 (ii)

A discrete random variable x follows Poisson Distribution. Find the value of $P\left(X \le 2 / X \ge 1\right)$. You are given $E\left(x\right)=2.20$ and $e^{-2.20}=0.1108$

(a) 0.89 (b) 0.58 (c) 0.10 (d) None

Solution

(b)

$$P\left(x \le 2 / x \ge 1\right)$$

$$P\left(x \le 2 / x \ge 1\right)=\frac{P\left[\left(x \le 2\right)\cap\left(x \ge 1\right)\right]}{P\left(x \ge 1\right)}$$

$P\left(x \le 2 / x \ge 1\right) = \dfrac{P\left(x=1\right)+P\left(x=2\right)}{1-P\left(x<1\right)}$ (Imagine a number line, and then imagine the common shaded region between $\left(x \le 2\right)$ and $\left(x \ge 1\right)$.

$$P\left(x \le 2 / x \ge 1\right) = \frac{P\left(x=1\right)+P\left(x=2\right)}{1-P\left(x=0\right)}$$

$$P\left(x \le 2 / x \ge 1\right) = \frac{\dfrac{e^{-2.20} \times 2.20^{1}}{1!} + \dfrac{e^{-2.20} \times 2.20^{2}}{2!}}{1-\dfrac{e^{-2.20} \times 2.20^{0}}{0!}}$$

$$P\left(x \le 2 / x \ge 1\right) = \frac{0.1108 \times 2.20 + \frac{0.1108 \times 2.20^2}{2}}{1 - 0.1108} \approx 0.58$$

Important Points

1. Poisson Distribution is applicable when the random variable (x) is discrete.
2. Since $e^{-m} = \frac{1}{e^m} > 0,$ whatever may be the value of m (>0), it follows that $f(x) > 0$ for every x.
   Also, $\sum f(x) = f(0) + f(1) + f(2) + f(3) + ... + f(n) = 1.$
3. Poisson distribution is known as a uniparametric distribution as it is characterised by only one parameter m.
4. The mean of Poisson distribution is given by m, i.e., $\mu = m = np.$
5. The variance of Poisson distribution is given by $\sigma^2 = m = np.$
6. The standard deviation of Poisson distribution is given by $\sigma = \sqrt{m} = \sqrt{np}.$

Exercise – Set B – Question 10

If the mean of a Poisson variable *x* is 1, what is *P* (*x* = takes the value at least 1)?

(a) 0.456 (b) 0.821 (c) 0.632 (d) 0.254

Solution

(c)

$$P(x) = \frac{e^{-m} \times m^{x}}{x!}$$

Here, we have $m = 1;\ x \geq 1$

$$P(x \geq 1) = 1 - P(x < 1)$$

$$=1-P\left(x=0\right)$$

$$=1-\frac{\left(2.71828\right)^{-1}\left(1\right)^{0}}{0!}$$

$$=1-\frac{1}{2.71828}$$

$$=0.6321$$

Exercise – Set B – Question 13

For a Poisson variate *x*, *P* (*x* = 1) = *P* (*x* = 2). What is the mean of *x*?

(a) 1.00 (b) 1.50 (c) 2.00 (d) 2.50

Solution

(c)

$$P\left(x=1\right)=P\left(x=2\right)$$

$$\frac{e^{-m}m^{1}}{1!}=\frac{e^{-m}m^{2}}{2!}$$

$$m = \frac{m^2}{2}$$

$$2m = m^2$$

$$m = 2$$

Page 16.12 – Example 16.11

Find the mean and standard deviation of x where x is a Poisson variate satisfying the condition $P(x=2)=P(x=3)$.

(a) 3.00 (b) 1.50 (c) 2.00 (d) 2.50

Solution

(a)

$$P(x=2)=P(x=3)$$

$$\frac{e^{-m}.m^2}{2!}=\frac{e^{-m}.m^3}{3!}$$

$$\frac{m^2}{2} = \frac{m^3}{6}$$

$$6m^2 = 2m^3$$

$$\frac{6}{2} = \frac{m^3}{m^2}$$

$$m = 3$$

Therefore, mean = 3.

Standard Deviation $(\sigma) = \sqrt{m} = \sqrt{3}$

Page 16.13 – Example 16.13

The standard deviation of a Poisson variate is 1.732. What is the probability that the variate lies between –2.3 to 3.68?

(a) 3.00 (b) 1.50 (c) 0.65 (d) 2.50

Solution

(c)

$$\sigma = \sqrt{m} = 1.732$$

$$\Rightarrow m = (1.732)^2 = 3$$

$$P(-2.3 < x < 3.68) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$P\left(-2.3 < x < 3.68\right) = \frac{e^{-3}.3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!}$$

$$P\left(-2.3 < x < 3.68\right) = e^{-3}\left(\frac{1}{1} + \frac{3}{1} + \frac{9}{2} + \frac{27}{6}\right)$$

$$P\left(-2.3 < x < 3.68\right) = \frac{1}{\left(2.71828\right)^3} \times \left(\frac{1}{1} + \frac{3}{1} + \frac{9}{2} + \frac{27}{6}\right) = 0.647 \approx 0.65$$

Page 16.14 – Example 16.14

x is a Poisson variate satisfying the following relation:

$$P(x=2)=9P(x=4)+90P(x=6)$$

What is the standard deviation of x?

(a) 2 (b) 1 (c) 4 (d) 3

Solution

(b)

$$P(x=2)=9P(x=4)+90P(x=6)$$

$$\frac{e^{-m}.m^2}{2!} = 9\times\frac{e^{-m}.m^4}{4!} + 90\times\frac{e^{-m}.m^6}{6!}$$

$$\frac{e^{-m}.m^2}{2} = \frac{9e^{-m}.m^4}{24} + \frac{90e^{-m}.m^6}{720}$$

$$\frac{e^{-m}.m^2}{2} = \frac{3e^{-m}.m^4}{8} + \frac{e^{-m}.m^6}{8}$$

$$\frac{e^{-m}.m^2}{2} = \frac{3e^{-m}m^4 + e^{-m}m^6}{8}$$

$$\frac{e^{-m}m^2}{2} = \frac{e^{-m}m^4\left(3+m^2\right)}{8}$$

$$\frac{1}{2}=\frac{m^2\left(3+m^2\right)}{8}$$

$$m^2\left(3+m^2\right)=4$$

Now, try the options.

Option (a) $\rightarrow$ 2

If Standard Deviation is 2, then m = 4.

LHS: $4^2\left(3+4^2\right)\neq 4$

Therefore, option (a) cannot be the answer.

Option (b) $\rightarrow$ 1

If Standard Deviation is 2, then $m = 1$

LHS: $1^2\left(3+1^2\right)=4$

Therefore, option (b) is the answer.

Exercise – Set B – Question 9

If the standard deviation of a Poisson variate *x* is 2, what is *P* (1.5 < *x* < 2.9)?

(a) 0.231 (b) 0.158 (c) 0.15 (d) 0.144

Solution

(d)

Standard Deviation = $\sqrt{m} = 2 \Rightarrow m = 4$

We know that $P(x) = \dfrac{e^{-m} \times m^{x}}{x!}$, for $x = 0, 1, 2, 3, \ldots, n$

Since *x* can only take integral values, $1.5 < x < 2.9 \Rightarrow x = 2$.

Therefore, $P\left(x=2\right)=\dfrac{\left(2.71828\right)^{-4}\times 4^{2}}{2!}=\dfrac{16}{\left(2.71828\right)^{4}\times 2}=0.1465$

Exercise – Set B – Question 11

If $X \sim P(m)$ and its coefficient of variation is 50, what is the probability that X would assume only non-zero values?

(a) 0.018 (b) 0.982 (c) 0.989 (d) 0.976

Solution

(b)

$$CV = \frac{SD}{AM} \times 100$$

$$50 = \frac{\sqrt{m}}{m} \times 100$$

$$\frac{50}{100}=\frac{\sqrt{m}}{\sqrt{m}\sqrt{m}}$$

$$0.5=\frac{1}{\sqrt{m}}$$

$$0.5\sqrt{m}=1$$

$$\sqrt{m}=\frac{1}{0.5}$$

$$\left(\sqrt{m}\right)^2=\left(\frac{1}{0.5}\right)^2$$

$$m=4$$

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$$P(x>0) = 1 - P(x \le 0)$$

$$= 1 - P(x = 0)$$

$$= 1 - \frac{e^{-m} m^{x}}{x!}$$

$$= 1 - \frac{(2.71828)^{-4}(4)^{0}}{0!}$$

$$= 1 - \frac{1}{(2.71828)^{4}}$$

$$= 0.982$$

Exercise – Set B – Question 15

If for a Poisson variable x, $f(2)=3f(4)$, what is the variance of x?

(a) 2 (b) 4 (c) $\sqrt{2}$ (d) 3

Solution

(a)

$$f(2)=3f(4)$$

$$\Rightarrow P(x=2)=3P(x=4)$$

$$\Rightarrow \frac{e^{-m}m^2}{2!}=3\times\frac{e^{-m}m^4}{4!}$$

$$\Rightarrow \frac{1}{2} = 3 \times \frac{m^2}{24}$$

$$\Rightarrow 6m^2 = 24$$

$$\Rightarrow m^2 = \frac{24}{6}$$

$$\Rightarrow m^2 = 4$$

$$\Rightarrow m = \sqrt{4} = 2$$

Exercise – Set C – Question 10

A random variable *x* follows Poisson distribution, and its coefficient of variation is 50. What is the value of $P\left(x>1/x>0\right)$?

(a) 0.1876 (b) 0.2341 (c) 0.9254 (d) 0.8756

Solution

(c)

$$CV=\frac{SD}{AM}\times 100$$

In Poisson Distribution SD is square root of AM.

$$SD = \sqrt{AM}$$

Therefore, $CV = \frac{\sqrt{AM}}{AM} \times 100$

$$\Rightarrow CV = \frac{100}{\sqrt{AM}}$$

$$\Rightarrow 50\sqrt{AM} = 100$$

$$\Rightarrow \sqrt{AM} = \frac{100}{50} = 2$$

$$\Rightarrow AM\left(m\right) = 4$$

$$P\left(x>1/x>0\right)=\frac{P\left(x>1\right)\cap P\left(x>0\right)}{P\left(x>0\right)}$$

$$\Rightarrow P\left(x>1/x>0\right)=\frac{P\left(x>1\right)}{P\left(x>0\right)}$$

$$\Rightarrow P\left(x>1/x>0\right)=\frac{1-P\left(x\le 1\right)}{1-P\left(x=0\right)}$$

$$\Rightarrow P\left(x>1/x>0\right)=\frac{1-\left\{P\left(x=0\right)+P\left(x=1\right)\right\}}{1-P\left(x=0\right)}$$

$$\Rightarrow P\left(x>1/x>0\right)=\frac{1-\left\{\frac{e^{-m}m^{0}}{0!}+\frac{e^{-m}m^{1}}{1!}\right\}}{1-\frac{e^{-m}m^{0}}{0!}}$$

$$\Rightarrow P\left(x>1/x>0\right)=\frac{1-\left\{\frac{e^{-4}\times 4^{0}}{0!}+\frac{e^{-4}\times 4^{1}}{1!}\right\}}{1-\frac{e^{-4}\times 4^{0}}{0!}}$$

$$\Rightarrow P\left(x>1/x>0\right)=\frac{1-\left\{\frac{e^{-4}\times 1}{1}+\frac{e^{-4}\times 4}{1}\right\}}{1-\frac{e^{-4}\times 1}{1}}$$

$$\Rightarrow P\left(x>1/x>0\right)=\frac{1-\left\{e^{-4}+4e^{-4}\right\}}{1-e^{-4}}$$

$$\Rightarrow P\left(x>1/x>0\right)=\frac{1-5e^{-4}}{1-e^{-4}}$$

$$\Rightarrow P\left(x>1/x>0\right)=\frac{1-\frac{5}{e^{4}}}{1-\frac{1}{e^{4}}}$$

$$\Rightarrow P\left(x>1/x>0\right)=\frac{1-\frac{5}{\left(2.71828\right)^{4}}}{1-\frac{1}{\left(2.71828\right)^{4}}}=\frac{0.90842}{0.98168}=0.9254$$

Important Points (Contd.)

7. Like binomial distribution, Poisson distribution could be also unimodal or bimodal depending upon the value of the parameter *m*.
    a. If *m* is an integer, there are two modes:
        i. m
        ii. $m - 1$
    b. If *m* is a fraction, the mode is given by the largest integer contained in *m*.

Page 16.13 – Example 16.12

The probability that a random variable *x* following Poisson Distribution would assume a positive value is $\left(1-e^{-2.7}\right)$. What is the mode of the distribution?

(a) 2 (b) 3 (c) 4 (d) None

Solution

(a)

Given

$$P(x>0)=1-e^{-2.7}$$

$$1-P(x\leq 0)=1-e^{-2.7}$$

$$1-P(x=0)=1-e^{-2.7}$$

$$P(x=0)=e^{-2.7}$$

$$\frac{e^{-m}m^{0}}{0!}=e^{-2.7}$$

$$e^{-m}=e^{-2.7}$$

$$m=2.7$$

Since m is fractional, mode will be the largest value of integer contained in it. Therefore, mode = 2.

Important Points (Contd.)

8. Poisson approximation to Binomial distribution

   When n is rather large and p is rather small so that $m = np$ is moderate then $B(n, p) \cong P(m)$.

9. Additive property of Poisson distribution:

   Let x and y be two independent poisson distributions where x has the parameter m_1, and y has the parameter m_2. Then $(x + y)$ will be a poisson distribution with parameter $(m_1 + m_2)$.

Normal or Gaussian Distribution

$$P(x) = f(x) = \frac{1}{\sigma\sqrt{2\pi}}.e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}, \text{ for } -\infty < x < \infty$$

Here,

e = exponential constant = 2.71828

x = random variable

μ = mean of the normal random variable x

σ = standard deviation of the given normal distribution

Sometimes, $P(x)$ is also written as $f(x)$. $f(x)$ is called "Probability Density Function".

Page 16.24 – Example 16.20

For a random variable *x*, the probability density function is given by: $f\left(x\right)=\frac{e^{-\left(x-4\right)^2}}{\sqrt{\pi}}$, for $-\infty < x < \infty$. Identify the distribution and find its mean and variance.

(a) $\mu = 4;\ \sigma^2 = \frac{1}{2}$ (b) $\mu = 4;\ \sigma^2 = \frac{3}{2}$ (c) $\mu = 5;\ \sigma^2 = \frac{1}{2}$ (d) None

Solution

(a)

The standard normal density function is given by $P(x) = f(x) = \frac{1}{\sigma\sqrt{2\pi}}.e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}$.

Given: $f(x) = \frac{e^{-(x-4)^2}}{\sqrt{\pi}}$

The breakup of the power of *e* is analysed as follows:

Standard Formula	**Formula as per Question**
$\frac{-(x-\mu)^2}{2\sigma^2}$	$\frac{-(x-4)^2}{1}$
$\Rightarrow \frac{-1}{2\sigma^2}\times(x-\mu)^2$	$\Rightarrow \frac{-1}{1}\times(x-4)^2$

Comparing it with the standard formula, we have:

$$(x-\mu)^2 = (x-4)^2$$

Therefore, Mean $(\mu) = 4$

Also, we have $\dfrac{-1}{2\sigma^2} = \dfrac{-1}{1}$

$$\Rightarrow 2\sigma^2 = 1$$

$$\Rightarrow \sigma^2 = \frac{1}{2}$$

Therefore, mean is 4 and variance is ½.

Exercise – Set B – Question 16

What is the coefficient of variation of *x*, characterised by the following probability density function: $f(x) = \frac{1}{4\sqrt{2\pi}} e^{-(x-10)^2/32}$ for $-\infty < x < \infty$?

(a) 50 (b) 60 (c) 40 (d) 30

Solution

(c)

The standard form is $P(x) = f(x) = \frac{1}{\sigma\sqrt{2\pi}} . e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}$

Given: $f(x)=\frac{1}{4\sqrt{2\pi}}e^{-(x-10)^2/32}$

The breakup of the power of *e* is analysed as follows:

Standard Formula	**Formula as per Question**
$\frac{-(x-\mu)^2}{2\sigma^2}$	$\frac{-(x-10)^2}{32}$
$\Rightarrow \frac{-1}{2\sigma^2}\times(x-\mu)^2$	$\Rightarrow \frac{-1}{32}\times(x-10)^2$

Comparing the given equation with the standard form, we have $\mu=10$.

Also, we have $-\frac{1}{32} = -\frac{1}{2\sigma^2}$

$$2\sigma^2 = 32$$

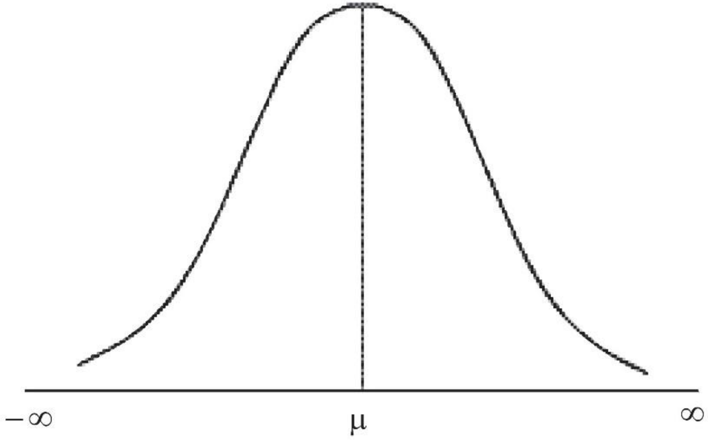
$$\sigma^2 = 16$$

$$\sigma = 4$$

$$CV = \frac{SD}{AM} \times 100 = \frac{4}{10} \times 100 = 40$$

Important Points

1. Normal Distribution is applicable when the random variable (x) is continuous.
2. If we plot the probability function $y = f(x)$, then the curve, known as probability curve, takes the following shape:



The area under this curve gives us the probability.

3. The area between $-\infty$ and μ = the area between μ and ∞ = 0.5

4. If $\mu = 0$, and $\sigma = 1$, we have $f(z) = \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-z^2}{2}\right)}$, for $-\infty < z < \infty$.

   The random variable z is known as standard normal variate (or variable) or standard normal deviate. It is given by $z = \frac{x - \mu}{\sigma}$.

5. Normal distribution is bell shaped.
6. It is unimodal.
7. The normal distribution is known as biparametric distribution as it is characterised by two parameters μ and σ^2. Once the two parameters are known, the normal distribution is completely specified.
8. Since the normal distribution is symmetrical about its mean (μ), Mean = Median = Mode.

9. Relationship between MD, SD, and QD → $4SD = 5MD = 6QD$

10. Mean Deviation = 0.8σ.

11. Quartile Deviation = 0.675σ.

Exercise – Set B – Question 18

If the two quartiles of $N(\mu, \sigma^2)$ are 14.6 and 25.4 respectively, what is the standard deviation of the distribution?

(a) 9 (b) 6 (c) 10 (d) 8

Solution

(d)

We know that Quartile Deviation = 0.675σ

Quartile Deviation = $\frac{Q_3 - Q_1}{2} = \frac{25.4 - 14.6}{2} = 5.4$

Therefore, $\sigma = \frac{QD}{0.675} = \frac{5.4}{0.675} = 8$

Exercise – Set A – Question 36

The quartile deviation of a normal distribution with mean 10 and SD 4 is:

(a) 0.675 (b) 67.50 (c) 2.70 (d) 3.20

Solution

(c)

QD = 0.675SD

QD = 0.675 × 4 = 2.70

Exercise – Set B – Question 19

If the mean deviation of a normal variable is 16, what is its quartile deviation?

(a) 10 (b) 13.5 (c) 15 (d) 12.05

Solution

(b)

We know that $4SD = 5MD = 6QD$

$5 \times 16 = 6QD$

$QD = 80 \div 6 = 13.33$

Exercise – Set B – Question 22

If the 1st quartile and mean deviation about median of a normal distribution are 13.25 and 8 respectively, then the mode of the distribution is

(a) 20 (b) 10 (c) 15 (d) 12

Solution

(a)

$Q_1 = 13.25$

MD = 8

Mean Deviation = 0.8σ

Quartile Deviation = 0.675σ

Standard Deviation = 8/0.8 = 10

Quartile Deviation = 0.675 × 10 = 6.75

$$QD = \frac{Q_3 - Q_1}{2}$$

$$6.75 = \frac{Q_3 - 13.25}{2}$$

$$Q_3 = (2 \times 6.75) + 13.25 = 26.75$$

$$\text{Median } = \frac{Q_1 + Q_3}{2} = \frac{13.25 + 26.75}{2} = 20$$

Exercise – Set A – Question 35

The mean deviation about median of a standard normal variate is:

(a) 0.675σ (b) 0.675 (c) 0.80σ (d) 0.80

Solution

(d)

A standard normal random variable is a normally distributed random variable with mean $\mu = 0$ and standard deviation $\sigma = 1$.

Therefore, mean deviation = $0.80 \times 1 = 0.80$

Exercise – Set B – Question 21

If the quartile deviation of a normal curve is 4.05, then its mean deviation is:

(a) 5.26 (b) 6.24 (c) 4.24 (d) 4.80

Solution

(d)

4SD = 5MD = 6QD

5MD = 6QD

MD = 6QD/5 = (6 × 4.05)/5 = 4.86

Page 16.25 – Example 16.21

If the two quartiles of a normal distribution are 47.30 and 52.70 respectively, what is the mode of the distribution? Also find the mean deviation about median of this distribution.

(a) 50; 3.20 (b) 100; 4.20 (c) 50; 4.20 (d) None

Solution

(a)

We know $Q_1 = \mu - 0.675\sigma$ and $Q_2 = \mu + 0.675\sigma$

Therefore,

$\mu - 0.675\sigma = 47.30 \ldots$Eq. (1)

$$\mu + 0.675\sigma = 52.70 \ldots \text{Eq. (2)}$$

Adding both the equations, we'll get:

$$2\mu = 100$$

$$\Rightarrow \mu = \frac{100}{2} = 50$$

In a normal distribution, Mean = Median = Mode; therefore, Mode = 50

Putting the value of $\mu = 50$ in Eq. (1), we'll get:

$$50 - 0.675\sigma = 47.30$$

$$\Rightarrow 0.675\sigma = 50 - 47.30 = 2.70$$

$$\Rightarrow \sigma = \frac{2.70}{0.675} = 4$$

We know that $MD = 0.8SD$

$$\Rightarrow MD = 0.8 \times 4 = 3.20$$

Important Points (Contd.)

12. Q_1 and Q_3 are equidistant from the median, therefore,

    i. $Q_1 = \mu - 0.675\sigma$, and

    ii. $Q_3 = \mu + 0.675\sigma$

Exercise – Set B – Question 17

What is the first quartile of *x* having the following probability density function?

$$f(x) = \frac{1}{\sqrt{72\pi}} e^{-(x-10)^2/72} \text{ for } -\infty < x < \infty$$

(a) 4 (b) 5 (c) 5.95 (d) 6.75

Solution

(c)

$$Q_1 = \mu - 0.675\sigma$$

The standard format is $P\left(x\right)=f\left(x\right)=\frac{1}{\sigma\sqrt{2\pi}}.e^{\left(\frac{-\left(x-\mu\right)^{2}}{2\sigma^{2}}\right)}$

Comparing this with the given expression, we have $\mu=10$.

Also, we have $-\frac{1}{72}=-\frac{1}{2\sigma^{2}}$

$$2\sigma^{2}=72$$

$$\sigma^{2}=36$$

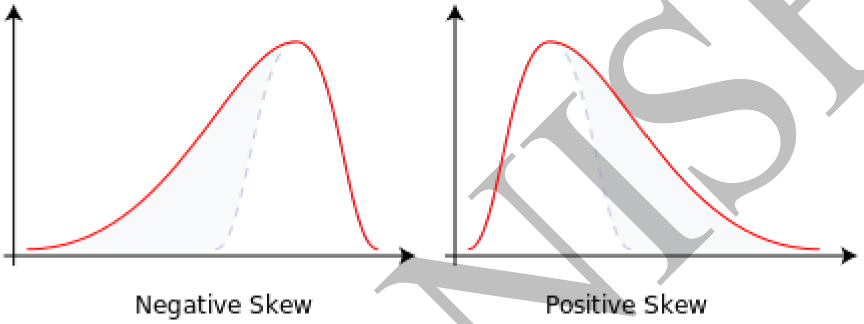
$$\sigma=6$$

$$Q_{1}=\mu-0.675\sigma=10-\left(0.675\times6\right)=5.95$$

Important Points (Contd.)

13. Median $- Q_1 = Q_3 -$ Median.

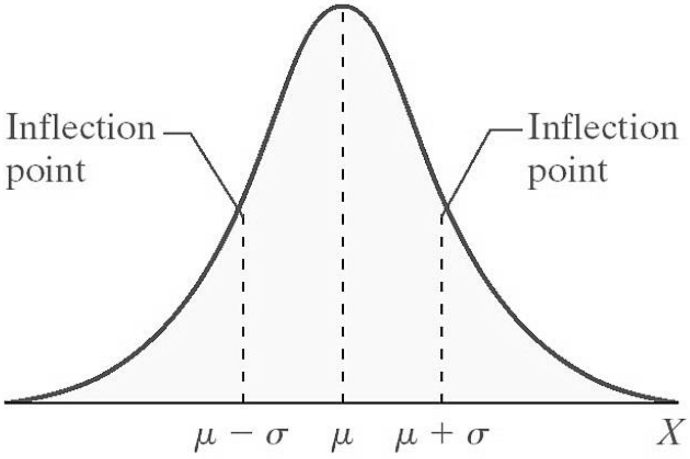
14. The normal distribution is symmetric about Therefore, its skewness is zero, i.e., the curve is neither tilted towards right (negatively skewed), nor towards left (positively skewed).



15. Points of inflexion – A normal curve has two inflexion points, i.e., the points where the curve changes its shape from concave to convex, and from convex to concave. These two points are given by:

i. $x = \mu - \sigma$, and

ii. $x = \mu + \sigma$



Exercise – Set A – Question 37

For a standard normal distribution, the points of inflexion are given by:

(a) $\mu - \sigma$ and $\mu + \sigma$ (b) $-\sigma$ and σ (c) –1 and 1 (d) 0 and 1

Solution

(c)

A standard normal random variable is a normally distributed random variable with mean $\mu = 0$ and standard deviation $\sigma = 1$.

Exercise – Set B – Question 20

If the points of inflexion of a normal curve are 40 and 60 respectively, then its mean deviation is:

(a) 8 (b) 45 (c) 50 (d) 60

Solution

(a)

Points of inflexion are given by $x=\mu-\sigma$, and $x=\mu+\sigma$.

Therefore, $40=\mu-\sigma$, and $60=\mu+\sigma$

Adding them, we get $100=2\mu \Rightarrow \mu=50$

Putting this value in equation 1, we get $\sigma = 50 - 40 = 10$

We know that $4SD = 5MD = 6QD$

4SD = 5MD

4 × 10 = 5MD

MD = 8

Page 16.25 – Example 16.22

Find the points of inflexion of the normal curve $f\left(x\right)=\frac{1}{4\sqrt{2\pi}}.e^{-\left(x-10\right)^2/32}$ for $-\infty < x < \infty$.

(a) 6 and 14 (b) 7 and 15 (c) 8 and 16 (d) None

Solution

(a)

The standard normal density function is given by $P\left(x\right)= f\left(x\right)=\frac{1}{\sigma\sqrt{2\pi}}.e^{\left(\frac{-\left(x-\mu\right)^2}{2\sigma^2}\right)}$.

Given: $f(x)=\frac{1}{4\sqrt{2\pi}}.e^{-(x-10)^2/32}$

The breakup of the power of *e* is analysed as follows:

Standard Formula	**Formula as per Question**
$\frac{-(x-\mu)^2}{2\sigma^2}$	$\frac{-(x-10)^2}{32}$
$\Rightarrow \frac{-1}{2\sigma^2}\times(x-\mu)^2$	$\Rightarrow \frac{-1}{32}\times(x-10)^2$

Comparing it with the standard formula, we have:

$$(x-\mu)^2=(x-10)^2$$

Therefore, Mean $(\mu)=10$

Also, we have $\frac{-1}{2\sigma^2} = \frac{-1}{32}$

$\Rightarrow 2\sigma^2 = 32$

$\Rightarrow \sigma^2 = \frac{32}{2} = 16$

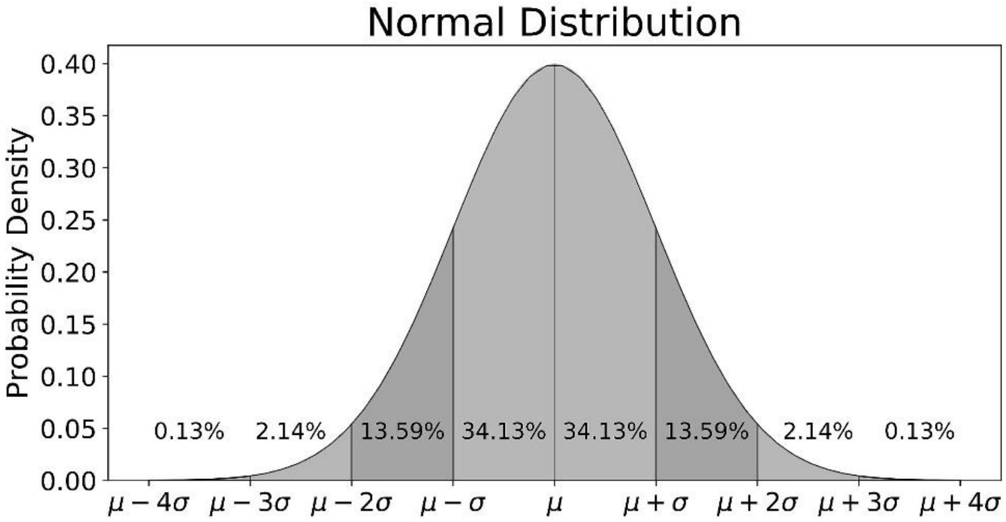
$\Rightarrow \sigma = \sqrt{16} = 4$

We know that points of inflexion are given by $\mu - \sigma$ and $\mu + \sigma$

Therefore, $\mu - \sigma = 10 - 4 = 6;\ \mu + \sigma = 10 + 4 = 14$

Important Points (Contd.)

16. In a normal distribution, $\mu \pm 1\sigma$ covers 68.27% of area, $\mu \pm 2\sigma$ covers 95.45% of area, and $\mu \pm 3\sigma$ covers 99.73% of area.



Exercise – Set A – Question 39

The interval $\left(\mu - 3\sigma,\ \mu + 3\sigma\right)$ covers:

(a) 95% area of a normal distribution
(b) 96% area of a normal distribution
(c) 99% area of a normal distribution
(d) all but 0.27% area of a normal distribution

Solution

(d)

Important Points (Contd.)

17. Under a normal distribution, the area enclosed between mean (μ) and 1σ is 0.34135; mean and 2σ is 0.47725; and mean and 3σ is 0.49865.

18. In case of normal distribution
    i. Highest Value = Mean + Half of Range, and
    ii. Lowest Value = Mean – Half of Range

19. Normal Distribution with $X = 0$, and $\sigma = 1$ is known as Standard Normal Distribution.

20. The height of normal curve is maximum at the Mean Value.

21. Additive Property: If there are two Independent Normal Distributions $x \sim N\left(\mu_1, \sigma_1^2\right)$ and $y \sim N\left(\mu_2, \sigma_2^2\right)$, then $z = x + y$ follows normal distribution with mean $\left(\mu_1 + \mu_2\right)$ and $SD = \sqrt{\sigma_1^2 + \sigma_2^2}$ respectively.

Page 16.32 – Example 16.32

x and y are independent normal variables with mean 100 and 80 respectively and standard deviation as 4 and 3 respectively. What is the distribution of (x + y)?

(a) 180; 5 (b) 190; 10 (c) 180; 10 (d) None

Solution

(a)

Mean of z = 100 + 80 = 180

$$SD = \sqrt{\sigma_1{}^2 + \sigma_2{}^2}$$

$$SD = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

Exercise – Set B – Question 24

If x and y are 2 independent normal variables with mean as 10 and 12 and SD as 3 and 4, then $(x + y)$ is normally distributed with:

(a) Mean = 22 and SD = 7
(b) Mean = 22 and SD = 25
(c) Mean = 22 and SD = 5
(d) Mean = 22 and SD = 49

Solution

(c)

Mean = 10 + 12 = 22

$$SD = \sqrt{\sigma_1{}^2 + \sigma_2{}^2}$$

$$SD = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

Problems on Finding Probability Through Graph

Page 16.26 – Example 16.24

x follows normal distribution with mean as 50 and variance as 100. What is $P(x \geq 60)$?
Given $\phi(1) = 0.8413$.

(a) 0.16 (b) 0.26 (c) 0.36 (d) None

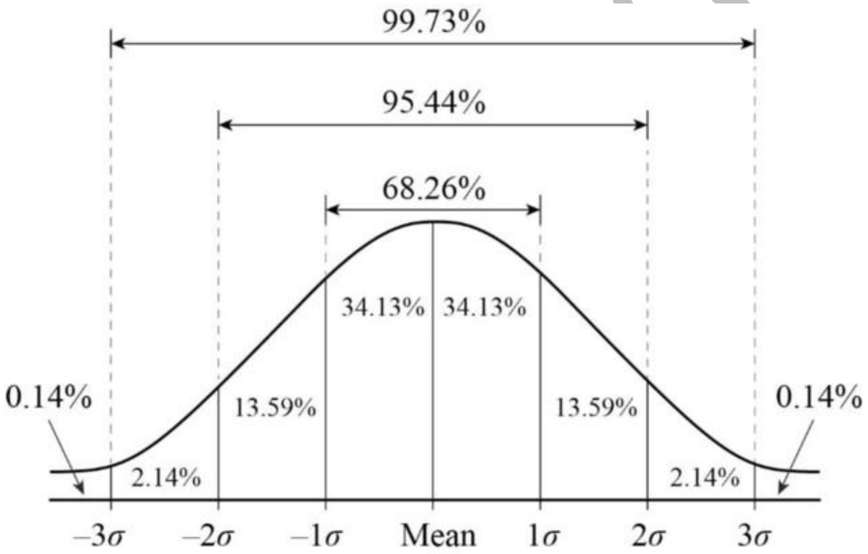
Solution

(a)

Given: $\mu = 50$; $\sigma = \sqrt{100} = 10$

For x = 60, $Z = \frac{x - \mu}{\sigma} = \frac{60 - 50}{10} = 1$

$$P(x \geq 60) = P(Z \geq 1)$$



$Z = -3$ $Z = -2$ $Z = -1$ $Z = 0$ $Z = 1$ $Z = 2$ $Z = 3$

As can be seen from the above diagram, the area to the right side of Z = 1 is 13.59 + 2.14 + 0.14 = 15.87% ≈ 16% or 0.16

However, this is not always so straightforward and simple. The $\phi(1) = 0.8413$ given in the question denotes the area from the left end to *Z* = 1; and we know that the total area of the graph is 1. So, if we subtract 0.8413 from 1, we'll get the desired area.

Therefore, 1 – 0.8413 = 0.1587 ≈ 0.16

Page 16.27 – Example 16.27 (i)

In a sample of 500 workers of a factory, the mean wage and SD of wages are found to be ₹500 and ₹48 respectively. Find the number of workers having wages more than ₹600. Given that $\phi(2.08) = 0.9812$

(a) 0.0188 (b) 9 (c) 10 (d) None

Solution

(b)

Given: $\mu = 500$; $\sigma = 48$

$$Z = \frac{x - \mu}{\sigma}$$

For $x = 600$; $Z = \dfrac{600 - 500}{48} = 2.08$

$$P(x > 600) = P(Z > 2.08) = 1 - \phi(2.08) = 1 - 0.9812 = 0.0188$$

Therefore, number of workers = $0.0188 \times 500 = 9.4 \approx 9$

Page 16.27 – Example 16.27 (ii)

In a sample of 500 workers of a factory, the mean wage and SD of wages are found to be ₹500 and ₹48 respectively. Find the number of workers having wages less than ₹450. Given that $\phi(1.04) = 0.8508$.

(a) 0.1492 (b) 75 (c) 10 (d) None

Solution

(b)

Given: $\mu = 500$; $\sigma = 48$

$$Z = \frac{x - \mu}{\sigma}$$

For $x = 450$; $Z = \dfrac{450-500}{48} = -1.04$

Since the graph is symmetrical, $\phi(-k) = 1 - \phi(k)$

$$P(x < 450) = P(z < -1.04) = \phi(-1.04) = 1 - \phi(0.14) = 1 - 0.8505 = 0.1492$$

Therefore, number of workers = $0.1492 \times 500 = 74.6 \approx 75$

Page 16.27 – Example 16.27 (iii)

In a sample of 500 workers of a factory, the mean wage and SD of wages are found to be ₹500 and ₹48 respectively. Find the number of workers having wages between ₹548 and ₹600. Given that $\phi(2.08) = 0.9812$; $\phi(1) = 0.8413$.

(a) 70 (b) 75 (c) 0.1399 (d) None

Solution

(a)

Given: $\mu = 500$; $\sigma = 48$

$$Z = \frac{x - \mu}{\sigma}$$

For x = 548; $Z = \frac{548 - 500}{48} = 1$

For x = 600; $Z = \frac{600 - 500}{48} = 2.08$

$$P(548 < x < 600) = P(1 < z < 2.08) = \phi(2.08) - \phi(1) = 0.9812 - 0.8413 = 0.1399$$

Therefore, number of workers = 0.1399 × 500 = 69.95 ≈ 70

Exercise – Set C – Question 15

In a sample of 800 students, the mean weight and standard deviation of weight are found to be 50 kg and 20 kg respectively. On the assumption of normality, what is the number of students weighing between 46 kg and 62 kg? Given area of the standard normal curve between $z = 0$ to $z = 0.20 = 0.0793$ and area between $z = 0$ to $z = 0.60 = 0.2257$.

(a) 250 (b) 244 (c) 240 (d) 260

Solution

(b)

Here, $n = 800$; $\mu = 50$; $\sigma = 20$

$$z = \frac{x - \mu}{\sigma}$$

For $x = 46$; $z = \frac{46 - 50}{20} = -0.2$

For $x = 62$; $z = \frac{62 - 50}{20} = 0.6$

$$P(46 < x < 62) = P(-0.2 < x < 0.6)$$

We need the area from –0.2 to 0, and then from 0 to 0.6. We are given the area from 0 to 0.2. This will be obviously the same as the area from –0.2 to 0.

Therefore, the area = 0.0793 + 0.2257 = 0.305

Therefore, number of students = 0.305 × 800 = 244

Page 16.29 – Example 16.28

The distribution of wages of a group of workers is known to be normal with mean ₹500 and SD ₹100. If the wages of 100 workers in the group are less than ₹430, what is the total number of workers in the group? Given $\phi(0.70) = 0.758$.

(a) 413 (b) 400 (c) 500 (d) None

Solution

(a)

Given: $\mu = 500$; $\sigma = 100$

$$Z = \frac{x - \mu}{\sigma}$$

For $x = 430$; $Z = \frac{430 - 500}{100} = -0.70$

We know that $\phi(-k) = 1 - \phi(k)$

$$P(x < 430) = P(z < -0.70) = \phi(-0.70) = 1 - \phi(0.70) = 1 - 0.758 = 0.242$$

Let the total number of workers be n.

Number of workers whose wages are less than 430 = $0.242 \times n$

Therefore, $100 = 0.242 \times n$

$n = 100 \div 0.242 = 413.22 \approx 413$

Exercise – Set C – Question 16

The salary of workers of a factory is known to follow normal distribution with an average salary of ₹10,000 and standard deviation of salary as ₹2,000. If 50 workers receive salary more than ₹14,000, then the total no. of workers in the factory is:

(a) 2,193 (b) 2,000 (c) 2,200 (d) 2,500

Solution

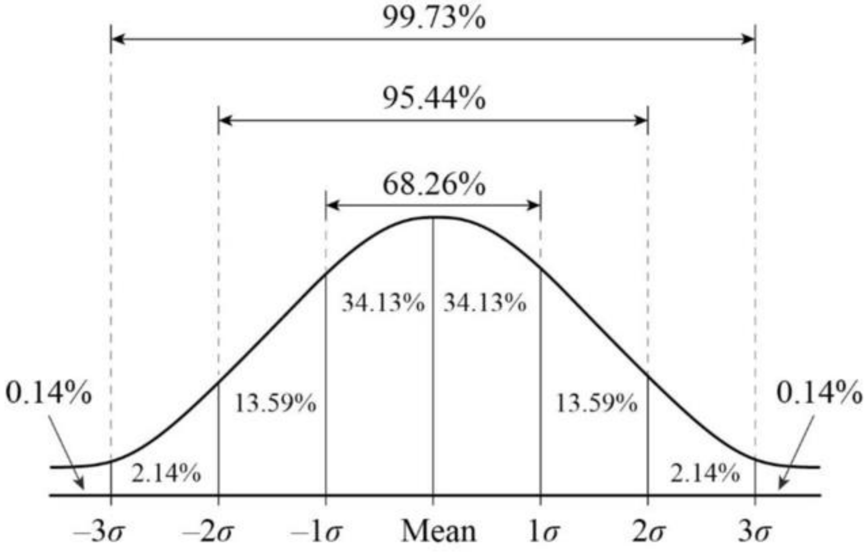
(a)

Given: $\mu = 10,000;\ \sigma = 2,000$

$$Z = \frac{x - \mu}{\sigma}$$

For x = 14,000; $Z = \dfrac{14,000 - 10,000}{2,000} = 2$

$$P\left(x > 14,000\right) = P\left(z > 2\right)$$



Therefore, $P(z > 2) = 0.0228$

Let the total number of workers be n.

Number of workers whose salary is more than 14,000 = 0.0228 × n

Therefore, 50 = 0.0228 × n

n = 50 ÷ 0.0228 = 2,192.98 ≈ 2,193

Page 16.30 – Example 16.30

The mean of a normal distribution is 500 and 16 per cent of the values are greater than 600. What is the standard deviation of the distribution? (Given that the area between $z = 0$ and $z = 1$ is 0.34; $\phi(1) = 0.84$)

(a) 50 (b) 100 (c) 200 (d) None

Solution

(b)

Given $\mu = 500$; $x = 600$

$$Z = \frac{x - \mu}{\sigma}$$

For $x = 600$; $Z = \frac{600-500}{\sigma} = \frac{100}{\sigma}$

$$P(x > 600) = P\left(z > \frac{100}{\sigma}\right) = 0.16$$

Now, draw the graph. From the graph, you'll find that $P(z > 1) = 0.16$

Then, $P\left(z > \frac{100}{\sigma}\right) = P(z > 1)$

$$\frac{100}{\sigma} = 1 \Rightarrow \sigma = 100$$

Exercise – Set C – Question 17

For a normal distribution with mean as 500 and SD as 120, what is the value of *k* so that the interval [500, *k*] covers 40.32 percent area of the normal curve? Given $\phi(1.30) = 0.9032$.

(a) 740 (b) 750 (c) 656 (d) 800

Solution

(c)

Given: $\mu = 500$; $\sigma = 120$

$$Z = \frac{x - \mu}{\sigma}$$

The interval given is [500, k]. 500 is the mean. Z = 0 at mean. Hume isse right side ka thoda sa area chahiye, jisse mean se le ke us area tak ka area 40.32% ho jaaye. Hume $\phi(1.30) = 0.9032$ diya hua hai. Isme se agar –∞ se z = 0 tak ka area minus karen, toh 0.4032 aayega…aur wohi toh hume chahiye.

Matlab, jo z aayega, wohi ϕ diya hua hai. Matlab z 1.30 aayega.

$$Z = \frac{x - \mu}{\sigma}$$

$$1.30 = \frac{k - 500}{120}$$

Try the options.

Page 16.26 – Example 16.25

If a random variable x follows normal distribution with mean as 120 and standard deviation as 40, what is the probability that $P\left(x \le 150 / x > 120\right)$? Given that area of the normal curve between $Z = 0$ and $Z = 0.75$ is 0.2734.

(a) 0.55 (b) 0.96 (c) 0.26 (d) None

Solution

(a)

Given: $\mu = 120$; $\sigma = 40$

$$Z = \frac{x - \mu}{\sigma}$$

For $x = 150$; $Z = \dfrac{150-120}{40} = 0.75$

For $x = 120$; $Z = \dfrac{120-120}{40} = 0$

$$P\left(x \le 150 / x > 120\right) = \frac{P\left(x \le 150\right) \cap P\left(x > 120\right)}{P\left(x > 120\right)}$$

$$= \frac{P\left(z \le 0.75\right) \cap P\left(z > 0\right)}{P\left(z > 0\right)} = \frac{0.2734}{0.50} = 0.5468 \approx 0.55$$

Page 16.29 – Example 16.29

The mean height of 2000 students at a certain college is 165 cms and SD 9 cms. What is the probability that in a group of 5 students of that college, 3 or more students would have height more than 174 cm? Given $\phi(1) = 0.8413$.

(a) 0.1587 (b) 0.1857 (c) 0.03106 (d) None

Solution

(c)

Given $n = 2000;\ \mu = 165;\ \sigma = 9;\ x = 174$

First let's find out the probability that any student chosen at random has height more than 174 cms.

$$Z = \frac{x-\mu}{\sigma}$$

For x = 174; $Z = \frac{174-165}{9} = 1$

$$P(x > 174) = P(Z > 1) = 1 - \phi(1) = 1 - 0.8413 = 0.1587$$

Now, this problem will be solved further as a Binomial Distribution problem.

We have n = 5; $x \geq 3$; p = 0.1587; q = 0.8413

$$P(x \geq 3) = P(x = 3) + P(x = 4) + P(x = 5)$$

$$P(x \geq 3) = {}^5C_3 (0.1587)^3 (0.8413)^2 + {}^5C_4 (0.1587)^4 (0.8413)^1 + {}^5C_5 (0.1587)^5 (0.8413)^0$$

$$P(x \geq 3) = 0.03106$$

Exercise – Set A – Question 38

The symbol $\phi(a)$ indicates the area of the standard normal curve between:

(a) 0 to a (b) a to ∞ (c) –∞ to a (d) –∞ to ∞

Solution

(c)

Exercise – Set B – Question 23

If the area of a standard normal curve between $Z = 0$ to $Z = 1$ is 0.3413, then the value of $\phi(1)$ is:

(a) 0.5000 (b) 0.8413 (c) –0.5000 (d) 1

Solution

(b)

$\phi(1) =$ Area from left end till $Z = 1$.

$$= 0.5000 + 0.3413 = 0.8413$$

Exercise – Set A – Question 29

The total area of the normal curve is:

(a) one (b) 50 per cent (c) 0.50 (d) any value between 0 and 1

Solution

(a)

Chapter 17 – Correlation and Regression

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Correlation

Consider two variables *x* and *y*. If we need to find out the extent of relationship between these two variables, we take help of correlation. For example, the demand of a commodity in the market depends upon a lot of factors, such as price, number of consumers in the market, income of the people, changes in prices of related goods, and so on. If we need to find out the effect on demand due to a change in say, price, then we'll use correlation. Therefore, correlation is used to find out the extent to which **two variables** are related to each other.

Correlation is expressed using r. The value of correlation ranges from –1 to +1, both inclusive. Therefore, $-1 \le r \le 1$. If:

1. $r = -1$, it is called a perfect negative correlation
2. $-1 < r < 0$, it is called a negative correlation
3. $r = 0$, it is called no correlation
4. $0 < r < 1$, it is called a positive correlation
5. $r = +1$, it is called a perfect positive correlation

Question 1

Correlation analysis aims at

(a) Predicting one variable for a given value of the other variable
(b) Establishing relation between two variables
(c) Measuring the extent of relation between two variables
(d) Both (b) and (c)

Solution

(d)

Question 2

The correlation between shoe-size and intelligence is:

(a) Zero (b) Positive (c) Negative (d) None

Solution

(a)

Question 3

The correlation between the speed of an automobile and the distance travelled by it after applying the brakes is

(a) Negative (b) Zero (c) Positive (d) None

Solution

(a)

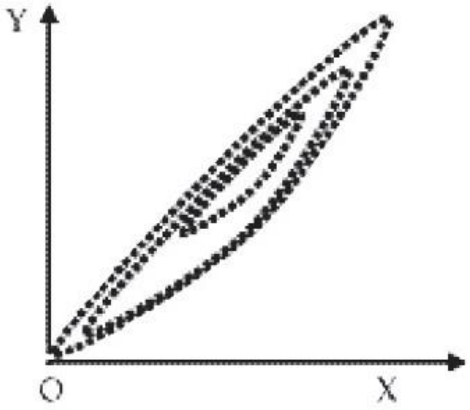
Measures of Correlation

1. Scatter Diagram
2. Karl Pearson’s Product Moment Correlation Coefficient
3. Spearman’s Rank Correlation Co-efficient
4. Co-efficient of Concurrent Deviations

Scatter Diagram

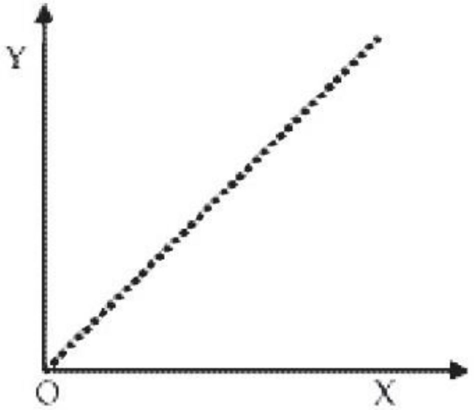
In this method, points a plotted on a graph paper for different values of x and y. Thereafter, the shape of the diagram on the graph determines the kind of correlation between those two variables.

Showing Positive Correlation



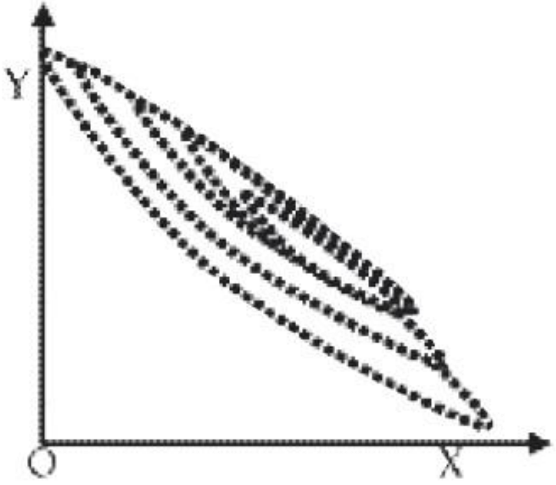
(0 < r <1)

Showing Perfect Correlation



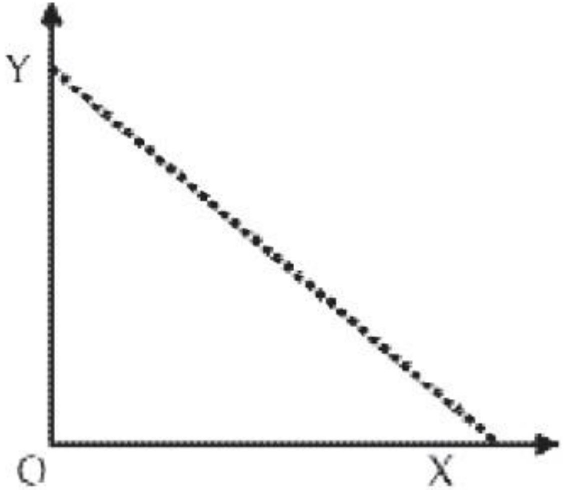
(r = 1)

Showing Negative Correlation



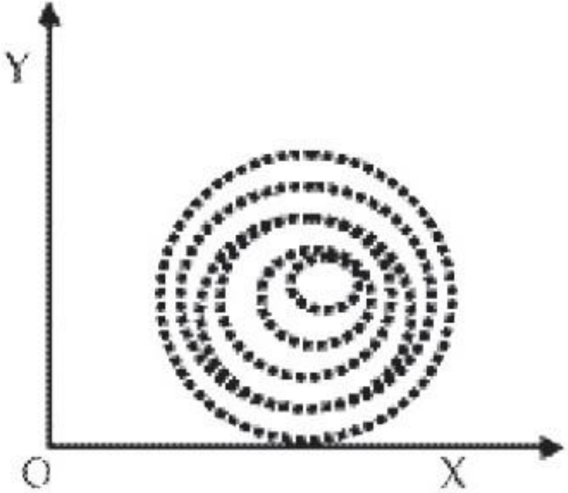
(−1 < r <0)

Showing Perfect Negative Correlation



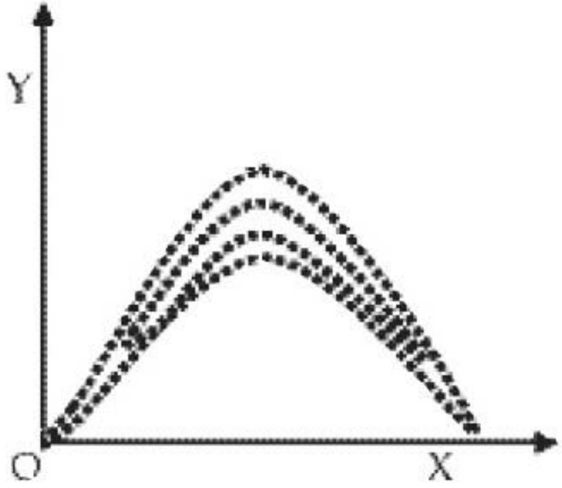
(r = −1)

Showing No Correlation



(r = 0)

Showing Curvilinear Correlation



(r = 0)

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Question 4

Scatter diagram is considered for measuring:

(a) Linear relationship between two variables
(b) Curvilinear relationship between two variables
(c) Neither (a) nor (b)
(d) Both (a) and (b)

Solution

(d)

Question 5

If the plotted points in a scatter diagram lie from upper left to lower right, then the correlation is:

(a) Positive (b) Zero (c) Negative (d) None

Solution

(c)

Question 6

If the plotted points in a scatter diagram are evenly distributed, then the correlation is:

(a) Zero (b) Negative (c) Positive (d) (a) or (b)

Solution

(a)

Question 7

If all the plotted points in a scatter diagram lie on a single line, then the correlation is:

(a) Perfect Positive (b) Perfect Negative (c) Both (a) and (b) (d) Either (a) or (b)

Solution

(d)

Question 8

Scatter diagram helps us to:

(a) Find the nature correlation between two variables
(b) Compute the extent of correlation between two variables
(c) Obtain the mathematical relationship between two variables
(d) Both (a) and (c)

Solution

(a)

Question 9

If the value of correlation coefficient is positive, then the points in a scatter diagram tend to cluster:

(a) From lower left corner to upper right corner
(b) From lower left corner to lower right corner
(c) From lower right corner to upper left corner
(d) From lower right corner to upper right corner

Solution

(a)

Question 10

When $r = 1$, all the points in a scatter diagram would lie:

(a) On a straight line directed from lower left to upper right
(b) On a straight line directed from upper left to lower right
(c) On a straight line
(d) Both (a) and (b)

Solution

(a)

Karl Pearson's Product Moment Correlation Coefficient

Correlation coefficient is given by $r = r_{xy} = \frac{Cov(x,y)}{S_x \times S_y} = \frac{Cov(x,y)}{\sigma_x \times \sigma_y}$

Here,

1. $Cov(x,y)$ means co-variance of x and y, and is given by:

$$Cov(x,y) = \frac{\sum (x-\overline{x})(y-\overline{y})}{n} = \frac{\sum xy}{n} - \overline{x}.\overline{y}$$

2. S_x means the standard deviation of x (σ_x), and is given by:

$$S_x = \sqrt{\frac{\sum (x-\overline{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \overline{x}^2}$$

3. S_y means the standard deviation of y $\left(\sigma_y\right)$, and is given by:

$$S_y = \sqrt{\frac{\sum\left(y-\bar{y}\right)^2}{n}} = \sqrt{\frac{\sum y^2}{n} - \bar{y}^2}$$

Alternatively, Correlation coefficient can also be directly calculated using the following formula:

Correlation coefficient is given by $r = \dfrac{n\sum xy - \sum x \times \sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$

Question 11

Find product moment correlation coefficient from the following information:

x	2	3	5	5	6	8
y	9	8	8	6	5	3

(a) 0.48 (b) 0.93 (c) –0.93 (d) None

Solution

(c)

Question 12

Compute the correlation coefficient between *x* and *y* from the following data:

$$n = 10;\ \sum xy = 220;\ \sum x^2 = 200;\ \sum y^2 = 262;\ \sum x = 40;\ \sum y = 50$$

(a) 0.91 (b) 0.92 (c) 0.93 (d) 0.94

Solution

(a)

Question 13

If for two variable *x* and *y*, the covariance, variance of *x* and variance of *y* are 40, 16 and 256 respectively, what is the value of the correlation coefficient?

(a) 0.01 (b) 0.625 (c) 0.4 (d) 0.5

Solution

(b)

$$r = \frac{Cov(x, y)}{\sigma_x \sigma_y} = \frac{40}{\sqrt{16}\sqrt{256}} = 0.625$$

Question 14

If the covariance between two variables is 20 and the variance of one of the variables is 16, what would be the variance of the other variable?

(a) $S_y^{\,2} \geq 25$ (b) More than 10 (c) Less than 10 (d) More than 1.25

Solution

(a)

$$r = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{20}{\sqrt{16} \times \sigma_y} = \frac{20}{4\sigma_y} = \frac{5}{\sigma_y}$$

Therefore, denominator should be more than 5.

$$\sigma_y \geq 5$$

Squaring both sides, we get:

$$\left(\sigma_y\right)^2 \geq \left(5\right)^2$$

$$\sigma_y{}^2 \geq 25$$

Question 15

For two variables x and y, it is known that Cov $(x, y) = 8$, $r = 0.4$, variance of x is 16 and sum of squares of deviation of y from its mean is 250. The number of observations for this bivariate data is:

(a) 7 (b) 8 (c) 9 (d) 10

Solution

(d)

Question 16

If $Cov(x, y) = 15$, what restrictions should be put for the standard deviations of x and y?

(a) No restriction
(b) The product of the standard deviations should be more than 15.
(c) The product of the standard deviations should be less than 15.
(d) The sum of the standard deviations should be less than 15.

Solution

(b)

Properties

1. Karl Pearson's method is the best method for finding correlation between two variables provided the relationship between the two variables is linear.
2. The Coefficient of Correlation is a unit-free measure.
3. The coefficient of correlation remains invariant under a change of origin and/or scale of the variables under consideration depending on the sign of scale factors.
   In other words, let there be two variables x and y. Let the correlation coefficient between them be r_{xy}. Now, if they are changed to another set of variables, say, u and v, then,
   $r_{uv} = r_{xy}$, if b and d have the same sign, or
   $r_{uv} = -r_{xy}$, if b and d have opposite signs.
   Here,

$$b = \frac{-\text{Coefficient of } u}{\text{Coefficient of } x}, \text{ and } d = \frac{-\text{Coefficient of } v}{\text{Coefficient of } y}$$

Question 17

If $u+5x=6$ and $3y-7v=20$ and the correlation coefficient between x and y is 0.58, then what would be the correlation coefficient between u and v?

(a) 0.58 (b) –0.58 (c) –0.84 (d) 0.84

(b)

$$u+5x=6$$

$$5x+u-6=0$$

$$b=\frac{-Coefficient\ of\ u}{Coefficient\ of\ x}=\frac{-(1)}{5}=-\frac{1}{5}$$

Similarly,

$$3y - 7v = 20$$

$$3y - 7v - 20 = 0$$

$$d = \frac{-Coefficient\, of\ v}{Coefficient\, of\ y} = \frac{-(-7)}{3} = \frac{7}{3}$$

Since *b* and *d* have opposite signs,

$$r_{uv} = -r_{xy} = -0.58$$

Question 18

Given that the correlation coefficient between *x* and *y* is 0.8, write down the correlation coefficient between *u* and *v* where $2u+3x+4=0$ and $4v+16y+11=0$.

(a) 0.8 (b) –0.8 (c) –0.9 (d) 0.9

Solution

(a)

Question 19

If the relation between x and u is $3x+4u+7=0$ and the correlation coefficient between x and y is –0.6, then what is the correlation coefficient between u and y?

(a) –0.6
(b) 0.8
(c) 0.6
(d) –0.8

Solution

(c)

We have $3x+4u+7=0$

$$b = \frac{-Coefficient\, of\; u}{Coefficient\, of\; x} = \frac{-4}{3} = -\frac{4}{3}$$

In this question, we are not given the equation in terms of *v* and *y*. Therefore, we cannot calculate *d*.

We'll use the following formula:

$$r_{xy} = \frac{bd}{|b||d|}.r_{uv}$$

In the above formula, we calculate *b* and *d* in the same manner. However, since we do not have the equation in terms of *v* and *y*, we cannot calculate *d*.

Therefore, the formula reduces to $r_{xy} = \frac{b}{|b|}.r_{uy}$.

Therefore, $r_{uy} = \frac{r_{xy} \times |b|}{b} = \frac{-0.6 \times \frac{4}{3}}{-\frac{4}{3}} = 0.6$

Question 20

From the following data:

x	2	3	5	4	7
y	4	6	7	8	10

The coefficient of correlation was found to be 0.93. What is the correlation between u and v as given below?

u	–3	–2	0	–1	2
v	–4	–2	–1	0	2

(a) –0.93
(b) 0.93
(c) 0.57
(d) –0.57

Solution

(b)

We need to find the relationship between *x* and *u*, and *v* and *y*.

For $x = 2$, it is given that $u = -3$. Also, for $x = 3$, it is given that $u = -2$.

Clearly, by observation we find that $x - 5 = u \Rightarrow x - u - 5 = 0$

$$b = \frac{-Coefficient\ of\ u}{Coefficient\ of\ x} = \frac{-(-1)}{1} = 1$$

Also, for $y = 4$, it is given that $v = -4$. Also, for $y = 6$, it is given that $v = -2$.

Clearly, by observation we find that $y - 8 = v \Rightarrow y - v - 8 = 0$

$$d = \frac{-Coefficient\, of\ v}{Coefficient\, of\ y} = \frac{-(-1)}{1} = 1$$

Since *b* and *d* have the same signs, $r_{uv} = r_{xy} = 0.93$

Question 21

Pearson's correlation coefficient is used for finding:

(a) Correlation for any type of relation
(b) Correlation for linear relation only
(c) Correlation for curvilinear relation only
(d) Both (b) and (c)

Solution

(b)

Question 22

Product moment correlation coefficient is considered for

(a) Finding the nature of correlation
(b) Finding the amount of correlation
(c) Both (a) and (b)
(d) Either (a) and (b)

Solution

(c)

Question 23

Product moment correlation coefficient may be defined as the ratio of:

(a) The product of standard deviations of the two variables to the covariance between them
(b) The covariance between the variables to the product of the variances of them
(c) The covariance between the variables to the product of their standard deviations
(d) Either (b) or (c)

Solution

(c)

Question 24

The covariance between two variables is:

(a) Strictly positive (b) Strictly negative
(c) Always 0 (d) Either positive or negative or zero.

Solution

(d)

Spearman's Rank Correlation Coefficient

This method is used to find the correlation between two qualitative characteristics, say, beauty and intelligence. Let's see the following questions to understand Spearman's Rank Correlation Coefficient.

Question 25

Compute the coefficient of rank correlation between sales and advertisement expressed in thousands of rupees from the following data:

Sales:	90	85	68	75	82	80	95	70
Advertisement:	7	6	2	3	4	5	8	1

(a) 0.65 (b) 0.90 (c) 0.92 (d) 0.95

Solution

(d)

Solution

Let sales be denoted by x, and advertisement be denoted with y. The largest value of x is given the rank 1, and thereafter, the next largest value is given the rank 2, and so on. Rank of x is denoted by R_x. Similarly, largest value of y is given the rank 1, and thereafter, the next largest value is given the rank 2, and so on. Rank of y is denoted by R_y. Thereafter, the difference between the two ranks is calculated, which is denoted by d.

Following table is prepared:

x	y	R_x	R_y	$d = R_x - R_y$	d^2
90	7	2	2	0	0
85	6	3	3	0	0
68	2	8	7	1	1
75	3	6	6	0	0
82	4	4	5	–1	1
80	5	5	4	1	1
95	8	1	1	0	0
70	1	7	8	–1	1
Total					4

Spearman's Rank Correlation Coefficient is given by $r_R = 1 - \frac{6\sum d^2}{n(n^2-1)}$.

Therefore, $r_R = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 4}{8(8^2-1)} = 1 - \frac{24}{8(64-1)} = 0.95$

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Question 26

Compute the coefficient of rank correlation between Eco. marks and stats. Marks as given below:

Eco Marks:	80	56	50	48	50	62	60
Stats Marks:	90	75	75	65	65	50	65

(a) 0.15 (b) 0.10 (c) 0.12 (d) 0.25

Solution

(a)

Let Economics Marks be denoted by x, and Statistics Marks be denoted with y. The largest value of x is given the rank 1, and thereafter, the next largest value is given the rank 2, and so on. Rank of x is denoted by R_x. Following table shows the arrangement of values of x in descending order and corresponding ranks:

x	R_x
80	1
62	2
60	3
56	4
50	5
50	6
48	7

We can see from the above table that the value 50 is present twice, corresponding to ranks 5 and 6. Now, since we can't discriminate between the same two values by giving one of them rank 5 and the other one rank 6, we would take the average of the ranks 5 and 6, i.e. 5.50, and give both the 50s the rank 5.50. All the other ranks would remain the same. Thus, the revised ranks are:

x	R_x
80	1
62	2
60	3
56	4
50	5.5
50	5.5
48	7

Similarly, largest value of y is given the rank 1, and thereafter, the next largest value is given the rank 2, and so on. Rank of y is denoted by R_y. Following table shows the arrangement of values of y in descending order and corresponding ranks:

y	R_y
90	1
75	2
75	3
65	4
65	5
65	6
50	7

Again, we can see from the above table that the value 75 is present twice, corresponding to ranks 2 and 3. Now, since we can't discriminate between the same two values by giving one of them rank 2 and the other one rank 3, we would take the average of the ranks 2 and 3, i.e. 2.50, and give both the 75s the rank 2.50.

Also, we can see from the above table that the value 65 is present thrice, corresponding to ranks 4, 5, and 6. Now, since we can't discriminate between the same three values, we would take the average of the ranks 4, 5, and 6, i.e., $\frac{4+5+6}{3}=5$, and give all the 65s the rank 5. All the other ranks would remain the same. Thus, the revised ranks are:

y	R_y
90	1
75	2.5
75	2.5
65	5
65	5

65	5
50	7

Therefore, our ultimate table is:

x	y	R_x	R_y	$d = R_x - R_y$	d^2
80.00	90.00	1.00	1.00	0.00	0.00
56.00	75.00	4.00	2.50	1.50	2.25
50.00	75.00	5.50	2.50	3.00	9.00
48.00	65.00	7.00	5.00	2.00	4.00
50.00	65.00	5.50	5.00	0.50	0.25
62.00	50.00	2.00	7.00	–5.00	25.00
60.00	65.00	3.00	5.00	–2.00	4.00
Total					44.50

Spearman's Rank Correlation Coefficient is given by $r_R = 1 - \frac{6\left[\sum d^2 + \frac{\sum\left(t_j^{\,3} - t_j\right)}{12}\right]}{n\left(n^2 - 1\right)}$.

Here, t_j represents the number of times a particular rank is repeated. In our question,

- the Rank of x (5.50) is repeated twice; therefore, $t_j = 2$. Hence $t_j^{\,3} - t_j = \left(2^3 - 2\right)$;
- the Rank of y (2.50) is repeated twice; therefore, $t_j = 2$. Hence $t_j^{\,3} - t_j = \left(2^3 - 2\right)$;
- the Rank of y (5.00) is repeated thrice; therefore, $t_j = 3$. Hence $t_j^{\,3} - t_j = \left(3^3 - 3\right)$.

Spearman's Rank Correlation Coefficient is given by:

$$r_R = 1 - \frac{6\left[\sum d^2 + \frac{\sum\left(t_j^{\ 3} - t_j\right)}{12}\right]}{n\left(n^2 - 1\right)}$$

$$= 1 - \frac{6\left[44.50 + \frac{\left(2^3 - 2\right) + \left(2^3 - 2\right) + \left(3^3 - 3\right)}{12}\right]}{7\left(7^2 - 1\right)}$$

$$= 1 - \frac{6\left[44.50 + \frac{6 + 6 + 24}{12}\right]}{7 \times 48}$$

$$= 1 - \frac{6\left[44.50 + 3\right]}{336}$$

$$= 0.15$$

Question 27

For a number of towns, the coefficient of rank correlation between the people living below the poverty line and increase of population is 0.50. If the sum of squares of the differences in ranks awarded to these factors is 82.50, find the number of towns.

(a) 9 (b) 10 (c) 8 (d) 11

Solution

(b)

Question 28

While computing rank correlation coefficient between profit and investment for the last 6 years of a company the difference in rank for a year was taken 3 instead of 4. What is the rectified rank correlation coefficient if it is known that the original value of rank correlation coefficient was 0.4?

(a) 0.3 (b) 0.2 (c) 0.25 (d) 0.28

Solution

(b)

Question 29

For finding correlation between two attributes, we consider

(a) Pearson’s correlation coefficient
(b) Scatter diagram
(c) Spearman’s rank correlation coefficient
(d) Coefficient of concurrent deviations

Solution

(c)

Question 30

For finding the degree of agreement about beauty between two Judges in a Beauty Contest, we use:

(a) Scatter diagram
(b) Coefficient of rank correlation
(c) Coefficient of correlation
(d) Coefficient of concurrent deviation

Solution

(b)

Question 31

If there is a perfect disagreement between the marks in Geography and Statistics, then what would be the value of rank correlation coefficient?

(a) Any value (b) Only 1 (c) Only –1 (d) (b) or (c)

Solution

(c)

Co-efficient of Concurrent Deviations

This method is used when we are not serious about the magnitude of the two variables, i.e., we just need to determine whether there exists a positive or a negative correlation. Let's understand it with the help of a question.

Question 32

What is the coefficient of concurrent deviations for the following data:

Supply	68	43	38	78	66	83	38	23	83	63	53
Demand	65	60	55	61	35	75	45	40	85	80	85

(a) 0.82 (b) 0.85 (c) 0.89 (d) –0.81

Solution

(c)

Supply	Sign of deviation from the previous figure (*a*)	Demand	Sign of deviation from the previous figure (*b*)	Product of deviation (*a* × *b*)
68		65		
43	–	60	–	+
38	–	55	–	+
78	+	61	+	+
66	–	35	–	+
83	+	75	+	+
38	–	45	–	+
23	–	40	–	+
83	+	85	+	+
63	–	80	–	+
53	–	85	+	–

From the above table, we can see that we have calculated the deviations 10 times. These are called the total number of deviations, denoted by the letter *m*. Therefore, $m = 10$.

It can also be seen that the last column (product of deviation) contains 9 positive signs. These are known as the concurrent deviations, denoted by the letter *c*. Therefore, $c = 9$.

The coefficient of concurrent deviation is given by:

$$r_c = \sqrt{\frac{(2c-m)}{m}}, \text{ if } (2c-m) > 0; \text{ or}$$

$$r_c = -\sqrt{-\frac{(2c-m)}{m}}, \text{ if } (2c-m) < 0.$$

In our question, we have $2c - m = 2(9) - 10 = 18 - 10 = 8$.

Since $\left(2c-m\right)>0$, the coefficient of concurrent deviation is given by:

$$r_c=\sqrt{\frac{\left(2c-m\right)}{m}}=\sqrt{\frac{8}{10}}=\sqrt{0.8}=0.89$$

Therefore, there is a positive correlation between Supply and Demand.

Question 33

For 10 pairs of observations, no. of concurrent deviations was found to be 4. What is the value of the coefficient of concurrent deviation?

(a) $\sqrt{0.2}$ (b) $-\sqrt{0.2}$ (c) $1/3$ (d) $-1/3$

Solution

(d)

We have m = 9; c = 4

$$2c - m = (2 \times 4) - 9 = 8 - 9 = -1$$

Since $(2c - m) < 0$, the coefficient of concurrent deviation is given by:

$$r_c = -\sqrt{-\frac{(2c-m)}{m}} = -\sqrt{-\frac{-1}{9}} = -\sqrt{\frac{1}{9}} = -1/3$$

Question 34

The coefficient of concurrent deviation for p pairs of observations was found to be $1/\sqrt{3}$. If the number of concurrent deviations was found to be 6, then the value of p is.

(a) 10 (b) 9 (c) 8 (d) None

Solution

(a)

We have $r_c = 1/3;\ c = 6$

Since r is positive, it is safe to assume that $2c - m > 0$

$$r_c = \sqrt{\frac{(2c - m)}{m}}$$

$$\Rightarrow \frac{1}{3} = \sqrt{\frac{(2 \times 6) - m}{m}}$$

$$\Rightarrow \frac{1}{3} = \sqrt{\frac{12 - m}{m}}$$

If there are *p* pairs of observations, the number of deviations (*m*), would be *p* – 1.

Let's try the options now.

Option (a) → 10

If $p = 10$, then $m = 10 - 1 = 9$

$$\text{RHS} = \sqrt{\frac{12-9}{9}} = \sqrt{\frac{3}{9}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

Therefore, option (a) is the answer.

Question 35

When we are not concerned with the magnitude of the two variables under discussion, we consider:

(a) Rank correlation coefficient
(b) Product moment correlation coefficient
(c) Coefficient of concurrent deviation
(d) (a) or (b) but not (c)

Solution

(c)

Question 36

What is the quickest method to find correlation between two variables?

(a) Scatter diagram
(b) Method of concurrent deviation
(c) Method of rank correlation
(d) Method of product moment correlation

Solution

(b)

Question 37

What are the limits of the coefficient of concurrent deviations?

(a) No limit
(b) Between –1 and 0, including the limiting values
(c) Between 0 and 1, including the limiting values
(d) Between –1 and 1, the limiting values inclusive

Solution

(d)

Regression

In regression analysis, we are concerned with the estimation of one variable for a given value of another variable on the basis of an average mathematical relationship between the two variables. For example, consider the following data:

x	y
2	5
4	9
6	13
8	17

We can see that the values of y are related to the values of x by $y = 2x + 1$. Using this, we can easily find out the value of y for any given value of x. Similarly, $y = 2x + 1 \Rightarrow 2x = y - 1 \Rightarrow x = (y - 1)/2$. Using this, we can easily find out the value of x for any given value of y.

When there are two variables *x* and *y* and if *y* is influenced by *x* i.e. if *y* depends on *x*, then we get a simple linear regression or simple regression. *y* is known as dependent variable or regression or explained variable and *x* is known as independent variable or predictor or explanator.

In case of a simple regression model:

1. if *y* depends on *x*, then the regression line of *y* on *x* is given by:
   a. either $y = a + bx$, where,
      i. $b = b_{yx} = \dfrac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2}$, or, $b_{yx} = r\dfrac{\sigma_y}{\sigma_x}$, or, $b_{yx} = \dfrac{Cov(x,y)}{\left(\sigma_x\right)^2}$, and
      ii. $a = a_{yx} = \bar{y} - \left(\bar{x} \times b_{yx}\right)$

   b_{yx} is known as the regression coefficient.

b. or, $\left(y-\overline{y}\right)=b_{yx}\left(x-\overline{x}\right)$

2. if *x* depends on *y*, then the regression line of *x* on *y* is given by:

a. either $x=a+by$, where,

i. $b=b_{xy}=\dfrac{n\sum xy-\left(\sum x\right)\left(\sum y\right)}{n\sum y^2-\left(\sum y\right)^2}$, or, $b_{xy}=r\dfrac{\sigma_x}{\sigma_y}$, or, $b_{xy}=\dfrac{Cov\left(x,y\right)}{\left(\sigma_y\right)^2}$, and

ii. $a=a_{xy}=\overline{x}-\left(\overline{y}\times b_{xy}\right)$

b_{xy} is known as the regression coefficient.

b. or, $\left(x-\overline{x}\right)=b_{xy}\left(y-\overline{y}\right)$

Question 38

The following data relate to the mean and SD of the prices of two shares in a Stock Exchange:

Share	Mean (in ₹)	SD (in ₹)
Company A	44	5.60
Company B	58	6.30

Coefficient of correlation between the share prices = 0.48.

Find the most likely price of share A corresponding to a price of ₹60 of share B and the most likely price of share B for a price of ₹50 of share A.

(a) 61.24; 44.85 (b) 44.85; 61.24 (c) 55.48; 44.85 (d) None

Solution

(b)

Let the share of Company A be denoted by x and the share of Company B be denoted by y.

We have $\bar{x} = 44;\ \sigma_x = 5.60;\ \bar{y} = 58;\ \sigma_y = 6.30;\ r = 0.48$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = 0.48 \times \frac{6.30}{5.60} = 0.54$$

$$a_{yx} = \bar{y} - b_{yx}\bar{x} = 58 - (0.54 \times 44) = 34.24$$

$$y = a_{yx} + b_{yx}x$$

$$\Rightarrow y = 34.24 + 0.54x$$

Therefore, when the share price of Company A (x) is ₹50, the share price of Company B (y) is given by: $y = 34.24 + (0.54 \times 50) = 61.24$.

Again,

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = 0.48 \times \frac{5.60}{6.30} = 0.4267$$

$$a_{xy} = \overline{x} - b_{xy}\overline{y} = 44 - (0.4267 \times 58) = 19.25$$

$$x = a_{xy} + b_{xy}y$$

$$\Rightarrow x = 19.25 + 0.4267y$$

Therefore, when the share price of Company B (y) is ₹60, the share price of Company A (x) is given by: $x = 19.25 + (0.4267 \times 60) = 44.85$.

Question 39

If the regression coefficient of *y* on *x*, the coefficient of correlation between *x* and *y* and variance of *y* are –3/4, $\frac{\sqrt{3}}{2}$, and 4 respectively, what is the variance of *x*?

(a) $2/\sqrt{3/2}$ (b) 16/3 (c) 4/3 (d) 4

Solution

(b)

We have $b_{yx} = -\frac{3}{4};\ r = \frac{\sqrt{3}}{2};\ \sigma_y{}^2 = 4$

$$b_{yx} = r \times \frac{\sigma_y}{\sigma_x}$$

$$\Rightarrow \sigma_x = r \times \frac{\sigma_y}{b_{yx}} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{4}}{-3/4}$$

$$\Rightarrow \sigma_x{}^2 = \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{4}}{-3/4} \right)^2 = 5.33$$

Properties of Regression Lines

The regression coefficients remain unchanged due to a shift of origin but change due to a shift of scale.

Question 40

If the relationship between two variables x and u is $u + 3x = 10$ and between two other variables y and v is $2y + 5v = 25$, and the regression coefficient of y on x is known as 0.80, what would be the regression coefficient of v on u?

(a) 7/75 (b) 8/75 (c) 9/75 (d) None

Solution

(b)

$$u + 3x = 10 \Rightarrow u = 10 - 3x$$

$$2y + 5v = 25 \Rightarrow v = \frac{25}{5} - \frac{2}{5}y$$

$$b_{vu} = \frac{Scale\ of\ v}{Scale\ of\ u} \times b_{yx} = \frac{-2/5}{-3} \times 0.80 = 0.1067$$

Now, try the options.

Option (b) → 8/75 = 0.1067

Therefore, option (b) is the answer.

Question 41

If $u = 2x + 5$ and $v = -3y - 6$ and regression coefficient of y on x is 2.4, what is the regression coefficient of v on u?

(a) 3.6 (b) –3.6 (c) 2.4 (d) –2.4

Solution

(b)

$$u = 2x + 5 \Rightarrow u = 5 + 2x$$

$$v = -3y - 6 \Rightarrow v = -6 - 3y$$

$$b_{vu} = \frac{Scale\ of\ v}{Scale\ of\ u} \times b_{yx} = \frac{-3}{2} \times 2.4 = -3.6$$

The two lines of regression intersect at the mean of *x* and mean of *y* where *x* and *y* are the variables under consideration

Question 42

For the variables x and y, the regression equations are given as $7x - 3y - 18 = 0$ and $4x - y - 11 = 0$. Find the arithmetic means of x and y.

(a) (3, 2)   (b) (3, 1)   (c) (4, 2)   (d) None

Solution

(b)

Try the options.

Question 43

If the regression line of y on x and of x on y are given by $2x + 3y = -1$ and $5x + 6y = -1$ then the arithmetic means of x and y are given by:

(a) (1, –1) (b) (–1, 1) (c) (–1, –1) (d) (2, 3)

Solution

(a)

Question 44

If $y = 3x + 4$ is the regression line of y on x and the arithmetic mean of x is –1, what is the arithmetic mean of y?

(a) 1 (b) –1 (c) 7 (d) None

Solution

(a)

The coefficient of correlation between two variables x and y is the simple geometric mean of the two regression coefficients. The sign of the correlation coefficient would be the common sign of the two regression coefficients.

Question 45

For the variables x and y, the regression equations are given as $7x – 3y – 18 = 0$ and $4x – y – 11 = 0$. Identify the regression equation of y on x.

(a) $7x – 3y – 18 = 0$ (b) $4x – y – 11 = 0$ (c) Can’t Say (d) None

Solution

(a)

Let $7x – 3y – 18 = 0$ be the regression equation of y on x, and $4x – y – 11 = 0$ be the regression equation of x on y.

$$7x - 3y - 18 = 0 \Rightarrow y = -\frac{18}{3} + \frac{7}{3}x \Rightarrow b_{yx} = \frac{7}{3}$$

$$4x - y - 11 = 0 \Rightarrow x = \frac{11}{4} + \frac{1}{4}y \Rightarrow b_{xy} = \frac{1}{4}$$

$$r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{\frac{7}{3} \times \frac{1}{4}} = 0.7638$$

Since *r* lies from –1 to 1, our assumption is correct.

Question 46

For the variables *x* and *y*, the regression equations are given as $7x - 3y - 18 = 0$ and $4x - y - 11 = 0$. Given the variance of *x* is 9, find the SD of *y*.

(a) 9.1647 (b) 9.1467 (c) 9.1764 (d) None

Solution

(a)

Let $7x - 3y - 18 = 0$ be the regression equation of *y* on *x*, and $4x - y - 11 = 0$ be the regression equation of *x* on *y*.

$$7x - 3y - 18 = 0 \Rightarrow y = -\frac{18}{3} + \frac{7}{3}x \Rightarrow b_{yx} = \frac{7}{3}$$

$$4x - y - 11 = 0 \Rightarrow x = \frac{11}{4} + \frac{1}{4}y \Rightarrow b_{xy} = \frac{1}{4}$$

$$r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{\frac{7}{3} \times \frac{1}{4}} = 0.7638$$

Since r lies from –1 to 1, our assumption is correct.

$$b_{yx} = r \times \frac{\sigma_y}{\sigma_x}$$

$$\Rightarrow \sigma_y = \frac{b_{yx} \times \sigma_x}{r} = \frac{\frac{7}{3} \times \sqrt{9}}{0.7638} = 9.1647$$

Question 47

If $4y - 5x = 15$ is the regression line of y on x and the coefficient of correlation between x and y is 0.75, what is the value of the regression coefficient of x on y?

(a) 0.45 (b) 0.9375 (c) 0.6 (d) None

Solution

(a)

$4y - 5x = 15$ is the regression equation of y on x.

In the standard form, it can be written as $y = \frac{15}{4} + \frac{5}{4}x \Rightarrow b_{yx} = \frac{5}{4}$

We know that $r = \sqrt{b_{yx} \times b_{xy}}$

$$\Rightarrow 0.75 = \sqrt{\frac{5}{4} \times b_{xy}}$$

Squaring both sides, we get:

$$(0.75)^2 = \frac{5}{4} \times b_{xy}$$

$$\Rightarrow b_{xy} = \frac{(0.75)^2 \times 4}{5} = 0.45$$

Probable Error and Standard Error

1. Probable Error (P.E.) is given by

   a. $P.E. = 0.674 \times \frac{1-r^2}{\sqrt{N}}$, or

   b. $P.E. = 0.6745 \times \frac{1-r^2}{\sqrt{N}}$, or

   c. $P.E. = 0.675 \times \frac{1-r^2}{\sqrt{N}}$

2. Limits of the correlation coefficient of the population is given by $p = r \pm P.E.$

3. Standard Error (S.E.) is given by $S.E. = \frac{1-r^2}{\sqrt{N}}$

4. $PE = \frac{2}{3} SE$

Question 48

Compute the Probable Error assuming the correlation coefficient of 0.8 from a sample of 25 pairs of items.

(a) 0.0486 (b) 0.4086 (c) 0.5076 (d) None

Solution

(a)

Given r = 0.8; n = 25

$$P.E. = 0.6745 \times \frac{1-r^2}{\sqrt{N}} = 0.6745 \times \frac{1-(0.8)^2}{\sqrt{25}} = 0.0486$$

Question 49

If r = 0.7; and n = 64 find out the probable error of the coefficient of correlation and determine the limits for the population correlation coefficient.

(a) 0.943; (0.743, 0.657) (b) 0.543; (0.743, 0.657) (c) 0.043; (0.743, 0.657) (d) None

Solution

(c)

Given r = 0.7; and n = 64

$$P.E. = 0.6745 \times \frac{1-r^2}{\sqrt{N}} = 0.6745 \times \frac{1-(0.7)^2}{\sqrt{64}} = 0.0430$$

$$p = r \pm P.E. \Rightarrow 0.7 \pm 0.043 \Rightarrow p = 0.7 + 0.043,\ 0.7 - 0.043 = (0.743,\ 0.657)$$

Important Points

1. If $r < P.E.$, there is no evidence of correlation.
2. If the value of r is more than 6 times of the probable error, then the presence of correlation coefficient is certain.
3. Since r lies between –1 and +1 ($-1 < r < 1$), the probable error is never negative, i.e., Probable Error is always a positive figure.
4. Coefficient of Determination (Also known as "Percentage of Variation Accounted for") $\left(r^2\right) = \dfrac{\text{Explained Variance}}{\text{Total Variance}}$
5. Coefficient of Non-Determination (Also known as "Percentage of Variation Unaccounted for") $= 1 - r^2$

6. The two lines of regression coincide, i.e., become identical when $r = -1$ or 1. In other words, if there is a perfect negative or positive correlation between the two variables under discussion, the two lines of regression coincide.
7. If $r = 0$, regression lines are perpendicular to each other.
8. If two variables x and y are independent or uncorrelated, then, obviously, the correlation coefficient between x and y is zero. However, the converse of this statement is not necessarily true, i.e., if the correlation coefficient, due to Pearson, between two variables comes out to be zero, then we cannot conclude that the two variables are independent. All we can conclude is that no linear relationship exists between the two variables. This, however, does not rule out the existence of some non-linear relationship between the two variables.

Question 50

If the coefficient of correlation between two variables is 0.7 then the percentage of variation unaccounted for is:

(a) 70% (b) 30% (c) 51% (d) 49%

Solution

(c)

Given $r = 0.7$

Percentage of Variation Unaccounted For = $1 - r^2 = 1 - (0.7)^2 = 0.51$

Question 51

If the coefficient of correlation between two variables is –0.9, then the coefficient of determination is:

(a) 0.9 (b) 0.81 (c) 0.1 (d) 0.19

Solution

(b)

Given $r = -0.9$

$$r^2 = (-0.9)^2 = 0.81$$

Chapter 19 – Index Numbers and Time Series

CA NISHANT KUMAR

Introduction

An index number is a ratio of two or more time periods, one of which is the base time period. The value at the base time period serves as the standard point of comparison.

The base time period is that time period from which the comparisons are to be made. For example, in 2009 the price of a McAloo Tikki burger was ₹20; in 2020, it's ₹40. Now, if I need to compare the price of 2020 with the price of 2009, 2009 will be the base time period, and 2020 will be current time period. The price in the base time period is denoted as P_0. The price in the current time period is denoted as P_1. The ratio of the price of the current period (2020, i.e., P_1) to the price of the base period (or reference period, i.e., 2009, i.e., P_0), is known as the Price Relative, and is denoted as P_{01}. Therefore, $P_{01} = \frac{P_1}{P_0}$.

Therefore, Price Relative $= \frac{P_n}{P_0}$. It is expressed as a percentage as follows: Price Relative

$$= \frac{P_n}{P_0} \times 100.$$

Question 1

Price-relative has been expressed in terms of:

(a) $P = \frac{P_n}{P_0}$ (b) $P = \frac{P_0}{P_n}$ (c) $P = \frac{P_n}{P_0} \times 100$ (d) $P = \frac{P_0}{P_n} \times 100$

Solution

(c)

Question 2

If the index number of prices at a place in 1994 is 250 with 1984 as base year, then the prices have increased on average by:

(a) 250% (b) 150% (c) 350% (d) None

Solution

(b)

Question 3

If the prices of all commodities in a place have increased 1.25 times in comparison to the base period, the index number of prices of that place now is:

(a) 125 (b) 150 (c) 225 (d) None

Solution

(c)

Question 4

If the prices of all commodities in a place have decreased 35% over the base period prices, then the index number of prices of that place is now:

(a) 35 (b) 135 (c) 65 (d) None

Solution

(c)

Question 5

The index number in wholesale prices is 152 for August 1999 compared to August 1998. During the year there is net increase in prices of wholesale commodities to the extent of:

(a) 45% (b) 35% (c) 52% (d) 48%

Solution

(c)

Question 6

If the price of all commodities in a place have increased 1.25 times in comparison to the base period prices, then the index number of prices for the place is now:

(a) 100 (b) 125 (c) 225 (d) None

Solution

(c)

Question 7

The prices of a commodity in the year 1975 and 1980 were 25 and 30 respectively taking 1980 as base year the price relative is:

(a) 109.78 (b) 110.25 (c) 113.25 (d) None

Solution

(d)

Question 8

The prices of a commodity in the years 1975 and 1980 were 25 and 30 respectively, taking 1975 as base year the price relative is:

(a) 120 (b) 135 (c) 122 (d) None

Solution

(a)

Simple Aggregative Method

$$\text{Simple Aggregative Price Index} = \frac{\sum P_n}{\sum P_0} \times 100$$

Question 9

From the following data

Commodity	Base Price	Current Price
Rice	35	42
Wheat	30	35
Pulse	40	38
Fish	107	120

The simple aggregative index is:

(a) 115.8 (b) 110.8 (c) 112.5 (d) 113.4

Solution

(b)

Simple Average of Price Relatives

$$\text{Index} = \frac{\sum\left(\frac{P_1}{P_0}\times 100\right)}{N}$$

Question 10

From the following table by the method of relative using Arithmetic mean the price Index number is:

Commodity	Wheat	Milk	Fish	Sugar
Base Price	5	8	25	6
Current Price	7	10	32	12

(a) 140.35 (b) 148.25 (c) 140.75 (d) None

Solution

(b)

Geometric Mean of Price Relatives

Index = Geometric Mean of Individual Years' Price Relatives

Question 11

From the following data:

Commodities	Base Year	Current Year
A	25	55
B	30	45

The index numbers from G.M. Method is:

(a) 181.66 (b) 185.25 (c) 181.75 (d) None

Solution

(a)

Commodities	Base Year	Current Year	Price Relative

A	25	55	55 ÷ 25 = 2.2
B	30	45	45 ÷ 30 = 1.5

Geometric Mean = $\sqrt{2.2 \times 1.5} = 1.81659$

Index = 1.81659 × 100 = 181.659 ≈ 181.66

Weighted Average Method

In this method, we assign a weight to the prices of the commodities. Thereafter, the average is calculated as follows:

$$General\ Index = \frac{Sum\ of\ Products}{Sum\ of\ Weights}$$

Question 12

From the following data for the 5 groups combined

Group	Weight	Index Number
Food	35	425
Cloth	15	235
Power & Fuel	20	215
Rent & Rates	8	115
Miscellaneous	22	150

The general index number is:

(a) 270 (b) 269.2 (c) 268.5 (d) 272.5

Solution

(b)

Group	Weight	Index Number	Product
Food	35	425	14,875
Cloth	15	235	3,525
Power & Fuel	20	215	4,300
Rent & Rates	8	115	920
Miscellaneous	22	150	3,300
Total	100		26,920

$$Weighted\ Average = \frac{26920}{100} = 269.20$$

The weights are usually the quantities of the commodities. These indices can be classified into two broad groups:

1. Weighted Aggregative Index
2. Weighted Average of Relatives

Weighted Aggregative Index

In this method, weights are assigned to the prices of the commodities. The weights are usually either the quantities or the value of goods, sold either during the base year, or the given year, or an average of some years. Various alternative formulae used are as follows:

1. Laspeyres' Index: In this Index, base year quantities are used as weights:

$$\text{Laspeyres Index} = \frac{\sum P_n Q_0}{\sum P_0 Q_0} \times 100$$

2. Paasche's Index: In this Index current year quantities are used as weights:

$$\text{Passche's Index} = \frac{\sum P_n Q_n}{\sum P_0 Q_n} \times 100$$

3. Methods based on some typical Period:

Index $= \frac{\sum P_n Q_t}{\sum P_0 Q_t} \times 100$, where t stands for some typical period of years, the quantities of which are used as weights.

The Marshall-Edgeworth index uses this method by taking the average of the base year and the current year.

$$\text{Marshall-Edgeworth Index} = \frac{\sum P_n \left(Q_0 + Q_n\right)}{\sum P_0 \left(Q_0 + Q_n\right)} \times 100$$

4. Bowley's Price Index: This index is the arithmetic mean of Laspeyres' and Paasche's.

$$\text{Bowley's Index} = \frac{\text{Laspeyres'} + \text{Paasche's}}{2}$$

5. Fisher's ideal Price Index: This index is the geometric mean of Laspeyres' and Paasche's.

$$\text{Fisher's Index} = \sqrt{\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times \frac{\sum P_n Q_n}{\sum P_0 Q_n}} \times 100$$

Laspeyre's Index

$$\text{Laspeyres Index} = \frac{\sum P_n Q_0}{\sum P_0 Q_0} \times 100$$

Question 13

If $\sum P_0Q_0 = 1360$, $\sum P_nQ_0 = 1900$, $\sum P_0Q_n = 1344$, $\sum P_nQ_n = 1880$, then Laspeyre's Index Number is:

(a) 0.71 (b) 1.39 (c) 1.75 (d) None

Solution

(b)

Laspeyre's Index Number = $\frac{\sum P_nQ_0}{\sum P_0Q_0} = \frac{1900}{1360} = 1.39$

Paasche's Index Number

$$\text{Passche's Index} = \frac{\sum P_n Q_n}{\sum P_0 Q_n} \times 100$$

Question 14

From the following data

Commodities		**A**	**B**	**C**	**D**
1992 Base Year	Price	3	5	4	1
	Quantity	18	6	20	14
1993 Current Year	Price	4	5	6	3
	Quantity	15	9	26	15

The Paasche's Price Index Number is:

(a) 146.41 (b) 148.25 (c) 144.25 (d) None

Solution

(a)

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Paasche's Index Number:

$$\frac{\sum P_n Q_n}{\sum P_0 Q_n} = \frac{(4\times15)+(5\times9)+(6\times26)+(3\times15)}{(3\times15)+(5\times9)+(4\times26)+(1\times15)}\times100 = \frac{306}{209}\times100 = 146.41$$

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Question 15

If the ratio between Laspeyre's Index Number and Paasche's Index Number is 28 : 27, then the missing figure in the following table P is:

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
X	L	10	2	5
Y	L	5	P	2

(a) 7 (b) 4 (c) 3 (d) 9

Solution

(b)

We know that $L = \frac{\sum P_n Q_0}{\sum P_0 Q_0} \times 100$, and $P = \frac{\sum P_n Q_n}{\sum P_0 Q_n} \times 100$

$$L = \frac{(2 \times 10) + (P \times 5)}{(L \times 10) + (L \times 5)} \times 100 = \frac{20 + 5P}{10L + 5L} \times 100 = \frac{5(4 + P)}{15L} \times 100 = \frac{4 + P}{3L} \times 100$$

$$P = \frac{(2 \times 5) + (P \times 2)}{(L \times 5) + (L \times 2)} \times 100 = \frac{10 + 2P}{5L + 2L} \times 100 = \frac{10 + 2P}{7L} \times 100$$

$$\frac{L}{P} = \frac{\frac{4 + P}{3L} \times 100}{\frac{10 + 2P}{7L} \times 100} = \frac{\frac{4 + P}{3L}}{\frac{10 + 2P}{7L}} = \frac{4 + P}{3L} \times \frac{7L}{10 + 2P} = \frac{4 + P}{3} \times \frac{7}{10 + 2P} = \frac{28 + 7P}{30 + 6P}$$

$$\frac{28}{27} = \frac{28 + 7P}{30 + 6P}$$

$$1.037 = \frac{28 + 7P}{30 + 6P}$$

Now, try the options.

Option (b) → 4

RHS = $\frac{28 + (7 \times 4)}{30 + (6 \times 4)} = 1.037$

Marshall Edgeworth Index

$$\text{Marshall-Edgeworth Index} = \frac{\sum P_n\left(Q_0 + Q_n\right)}{\sum P_0\left(Q_0 + Q_n\right)} \times 100$$

Question 16

From the following data:

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	7	17	13	25
B	6	23	7	25
C	11	14	13	15
D	4	10	8	8

The Marshall-Edgeworth Index Number is:

(a) 148.25 (b) 144.19 (c) 147.25 (d) 143.78

Solution

(b)

Marshall-Edgeworth Index

$$= \frac{\sum P_n\left(Q_0 + Q_n\right)}{\sum P_0\left(Q_0 + Q_n\right)} \times 100$$

$$= \frac{\{13(17+25)\} + \{7(23+25)\} + \{13(14+15)\} + \{8(10+8)\}}{\{7(17+25)\} + \{6(23+25)\} + \{11(14+15)\} + \{4(10+8)\}} \times 100 = \frac{1403}{973} \times 100 = 144.19$$

Bowley’s Price Index

$$\text{Bowley's Index} = \frac{\text{Laspeyres'} + \text{Paasche's}}{2}$$

Question 17

Bowley's Index Number is expressed in the form of:

(a) $\frac{Laspeyre's\ Index + Paasche's\ Index}{2}$

(b) $\frac{Laspeyre's\ Index \times Paasche's\ Index}{2}$

(c) $\frac{Laspeyre's\ Index - Paasche's\ Index}{2}$

(d) None

Solution

(a)

Question 18

The Bowley's Price index number is represented in terms of:

(a) A.M. of Laspeyre's and Paasche's Price index number
(b) G.M. of Laspeyre's and Paasche's Price index number
(c) A.M. of Laspeyre's and Walsh's price index number
(d) None

Solution

(a)

Fisher's Ideal Price Index

$$\text{Fisher's Index} = \sqrt{\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times \frac{\sum P_n Q_n}{\sum P_0 Q_n}} \times 100$$

Question 19

From the following data base year:-

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	4	3	6	2
B	5	4	6	4
C	7	2	9	2
D	2	3	1	5

Fisher’s Ideal Index is:

(a) 117.3 (b) 115.43 (c) 118.35 (d) 116.48

Solution

(a)

Fisher's Index

$$= \sqrt{\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times \frac{\sum P_n Q_n}{\sum P_0 Q_n}} \times 100$$

$$= \sqrt{\frac{(6\times 3)+(6\times 4)+(9\times 2)+(1\times 3)}{(4\times 3)+(5\times 4)+(7\times 2)+(2\times 3)} \times \frac{(6\times 2)+(6\times 4)+(9\times 2)+(1\times 5)}{(4\times 2)+(5\times 4)+(7\times 2)+(2\times 5)}} \times 100$$

$$= \sqrt{\frac{63}{52} \times \frac{59}{52}} \times 100 = 117.3$$

Question 20

If $\sum P_n Q_n = 249$, $\sum P_0 Q_0 = 150$, Paasche's Index Number = 150 and Drobiseh and Bowely's Index number = 145, then the Fisher's Ideal Index Number is:

(a) 75 (b) 60 (c) 145.97 (d) 144.91

Solution

(d)

Paasche's Index Number = 150

$$\frac{\sum P_n Q_n}{\sum P_0 Q_n} \times 100 = 150$$

$$\Rightarrow \frac{249}{\sum P_0Q_n} \times 100 = 150$$

$$\Rightarrow \frac{249}{\sum P_0Q_n} = \frac{150}{100}$$

$$\Rightarrow \frac{\sum P_0Q_n}{249} = \frac{100}{150}$$

$$\Rightarrow \sum P_0Q_n = \frac{100 \times 249}{150} = 166$$

Also, Bowley's Index Number = 145

$$B = \frac{L + P}{2}$$

$$\Rightarrow 145 = \frac{L + 150}{2}$$

$$\Rightarrow L = (145 \times 2) - 150 = 140$$

We know that $L = \frac{\sum P_n Q_0}{\sum P_0 Q_0} \times 100$

$$\Rightarrow 140 = \frac{\sum P_n Q_0}{150} \times 100$$

$$\Rightarrow \sum P_n Q_0 = \frac{140 \times 150}{100} = 210$$

Now, we know that $F = \sqrt{\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times \frac{\sum P_n Q_n}{\sum P_0 Q_n}} \times 100$

$$\Rightarrow F = \sqrt{\frac{210}{150} \times \frac{249}{166}} \times 100 = 144.91$$

Question 21

Bowley's index number is 150. Fisher's index number is 149.95. Paasche's index number is:

(a) 146.13 (b) 154 (c) 148 (d) 156

Solution

(a)

$$B = \frac{L+P}{2} = 150 \Rightarrow L = (150 \times 2) - P \Rightarrow L = 300 - P \ldots \text{ Eq. (1)}$$

$$F = \sqrt{L \times P} = 149.95 \Rightarrow L = \frac{(149.95)^2}{P} \ldots \text{ Eq. (2)}$$

From Equations (1) and (2), we have:

$$300 - P = \frac{(149.95)^2}{P}$$

Now, try the options.

Option (a) → 146.13

LHS = 300 – 146.13 = 153.87

RHS = $\frac{(149.95)^2}{146.13} = 153.87$

Weighted Average of Relatives

In this method, weighted arithmetic mean is used to calculate the index.

$$\text{Index} = \frac{\sum\left[\frac{P_n}{P_0}\times\left(P_0Q_0\right)\right]}{\sum P_0Q_0}\times 100$$

Question 22

Given below are the data on prices of some consumer goods and the weights attached to the various items. Compute price index number for the year 1985 (Base 1984 = 100)

Items	Unit	1984	1985	Weight
Wheat	Kg.	0.50	0.75	2
Milk	Litre	0.60	0.75	5
Egg	Dozen	2.00	2.40	4
Sugar	Kg.	1.80	2.10	8
Shoes	Pair	8.00	10.00	1

Then weighted average of Price Relative Index is:

(a) 125.43 (b) 123.3 (c) 124.53 (d) 124.52

Solution

(b)

Items	Unit	1984	1985	Price Relative	Weight	Products
Wheat	Kg.	0.50	0.75	1.50	2.00	3.00
Milk	Litre	0.60	0.75	1.25	5.00	6.25
Egg	Dozen	2.00	2.40	1.20	4.00	4.80
Sugar	Kg.	1.80	2.10	1.17	8.00	9.33
Shoes	Pair	8.00	10.00	1.25	1.00	1.25
Total					20.00	24.63

$$Weighted\ Average\ of\ Price\ Relative = \frac{24.63}{20} \times 100 = 123.15$$

The Chain Index Numbers

Till now, we have been taking a fixed base; however, when conditions change rapidly, the fixed base does not suit the required needs. In such a case, changing base is more suitable. For example, the base for the year 1999 could be 1998; the base for the year 2000 could be 1999 (not 1998), the base for the year 2001 could be 2000 (neither 1998, nor 1999), and so on. If it is desired to associate these relatives to a common base, the results are chained. Thus, under this method the relatives of each year are first related to the preceding year, called the link relatives, and then they are chained together by successive multiplication to form a chain index.

$$\text{Chain Index} = \frac{\text{Link Relative of the Current Year} \times \text{Chain Index of the Previous Year}}{100}$$

For example,

Year	Price	Link Relatives (Taking Previous Year as Base Year)	Chain Indices (Taking 1991 as Base Year)
1991	50	100.00	100.00
1992	60	$\frac{60}{50}\times100=120.00$	$\frac{120}{100}\times100=120.00$
1993	62	$\frac{62}{60}\times100=103.33$	$\frac{103.33}{100}\times120=124.00$
1994	65	$\frac{65}{62}\times100=104.84$	$\frac{104.84}{100}\times124=129.90$

Question 23

From the following data:

Year	1992	1993	1995	1996	1997
Link Index	100	103	105	112	108

(Base 1992 = 100) for the years 1993–97. The construction of chain index is:

(a) 103, 100.94, 107, 118.72
(b) 103, 108.15, 121.3, 130.82
(c) 107, 100.25, 104, 118.72
(d) None

Solution

(b)

Year	Link Relatives	Chain Indices

	(Taking Previous Year as Base Year)	(Taking 1992 as Base Year)
1992	100.00	100.00
1993	103.00	$\frac{103.00 \times 100.00}{100.00} = 103.00$
1995	105.00	$\frac{105.00 \times 103.00}{100.00} = 108.15$
1996	112.00	$\frac{112.00 \times 108.15}{100.00} = 121.128$
1997	108.00	$\frac{108.00 \times 121.128}{100.00} = 130.82$

Quantity Index Numbers

1. Simple Aggregate of Quantities

$$\text{Index} = \frac{\sum Q_n}{\sum Q_0} \times 100$$

2. Simple Average of Quantity Relatives:

$$\text{Index} = \frac{\frac{\sum Q_n}{\sum Q_0}}{n} \times 100$$

3. Weighted Aggregate Quantity Indices:
   a. With base year weight (Laspeyre's index)

$$\text{Index} = \frac{\sum Q_n P_0}{\sum Q_0 P_0} \times 100$$

b. With current year weight (Paasche's index)

$$\text{Index} = \frac{\sum Q_n P_n}{\sum Q_0 P_n} \times 100$$

c. Fisher's Ideal (Geometric mean of the above)

$$\text{Index} = \sqrt{\frac{\sum Q_n P_0}{\sum Q_0 P_0} \times \frac{\sum Q_n P_n}{\sum Q_0 P_n}} \times 100$$

4. Base-year weighted average of quantity relatives

$$\text{Index} = \frac{\sum\left[\frac{Q_n}{Q_0}\times\left(P_0Q_0\right)\right]}{\sum P_0Q_0}\times 100$$

Question 24

From the following data:

Commodities	Q_0	P_0	Q_1	P_1
A	2	2	6	18
B	5	5	2	2
C	7	7	4	24

Then the fisher's quantity index number is:

(a) 87.34 (b) 85.24 (c) 87.25 (d) 78.93

Solution

(d)

Value Indices

Value = Price × Quantity

$$\text{Value Index} = \frac{\sum V_n}{\sum V_0} = \frac{\sum P_n Q_n}{\sum P_0 Q_0}$$

Question 25

From the following data:

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	4	3	6	2
B	5	4	6	4
C	7	2	9	2
D	2	3	1	5

Then the value ratio is:

(a) $\frac{59}{52}$ (b) $\frac{49}{47}$ (c) $\frac{41}{53}$ (d) $\frac{47}{53}$

Solution

(a)

Commodity	Base Year			Current Year		
	Price	**Quantity**	**Value**	**Price**	**Quantity**	**Value**
A	4	3	12	6	2	12
B	5	4	20	6	4	24
C	7	2	14	9	2	18
D	2	3	6	1	5	5
Total			52			59

Therefore, the Value Index = $\frac{59}{52}$

Question 26

The total value of retained imports into India in 1960 was ₹71.5 million per month. The corresponding total for 1967 was ₹87.6 million per month. The index of volume of retained imports in 1967 composed with 1960 (= 100) was 62.0. The price index for retained inputs for 1967 our 1960 as base is:

(a) 198.61 (b) 197.61 (c) 198.25 (d) None

Solution

(b)

"Volume" is just another word for "Quantity".

We are given the values of 1960 as well as of 1967.

$$\text{Value Index} = \frac{V_{1967}}{V_{1960}} = \frac{87.6}{71.5} \times 100 = 122.52$$

Quantity Index = 62

We know that Value Index = Price Index × Quantity Index

$$\text{Therefore, Price Index} = \frac{\text{Value Index}}{\text{Quantity Index}} = \frac{\frac{87.6}{71.5} \times 100}{62} \times 100 = 197.61$$

Limitations and Usefulness of Index Numbers

Limitations

1. As the indices are constructed mostly from deliberate samples, chances of errors creeping in cannot be always avoided.
2. Since index numbers are based on some selected items, they simply depict the broad trend and not the real picture.
3. Since many methods are employed for constructing index numbers, the result gives different values and this at times creates confusion.

Usefulness

1. Framing suitable policies in economics and business: They provide guidelines to make decisions in measuring intelligence quotients, research etc.

2. They reveal trends and tendencies in making important conclusions in cyclical forces, irregular forces, etc.
3. They are important in forecasting future economic activity. They are used in time series analysis to study long-term trend, seasonal variations and cyclical developments.
4. Index numbers are very useful in deflating i.e., they are used to adjust the original data for price changes and thus transform nominal wages into real wages.
5. Cost of living index numbers measure changes in the cost of living over a given period.

Deflating Time Series Using Index Numbers

$$Deflated\ Value = \frac{Current\ Value}{Price\ Index\ of\ the\ Current\ Year}$$

or

$$Current\ Value = \frac{Base\ Price\ (P_0)}{Current\ Price\ (P_n)}$$

$$Real\ Wages = \frac{Actual\ Wages}{Cost\ of\ Living\ Index} \times 100$$

Year	Wholesale Price Index	Gross National Product at Current Prices	Real Gross National Product
1970	113.1	7499	$\frac{7499}{113.1}\times 100 = 6630$
1971	116.3	7935	$\frac{7935}{116.3}\times 100 = 6823$
1972	121.2	8657	$\frac{8657}{121.2}\times 100 = 7143$
1973	127.7	9323	$\frac{9323}{127.7}\times 100 = 7301$

Question 27

In 1980, the net monthly income of the employee was ₹800/- p.m. The consumer price index number was 160 in 1980. It rises to 200 in 1984. If he has to be rightly compensated, the additional D.A. to be paid to the employee is:

(a) ₹175/- (b) ₹185/- (c) ₹200/- (d) ₹125/-

Solution

(c)

Consumer Price Index = 160 → Salary = ₹800

Consumer Price Index = 200 → Salary = ?

$$\text{Salary} = \frac{200 \times 800}{160} = ₹1,000$$

Therefore, additional dearness allowance = ₹1,000 – ₹800 = ₹200/-

Question 28

Consumer price index number goes up from 110 to 200 and the Salary of a worker is also raised from ₹325 to ₹500. Therefore, in real terms, to maintain his previous standard of living he should get an additional amount of:

(a) ₹85 (b) ₹90.91 (c) ₹98.25 (d) None

Solution

(b)

Consumer Price Index = 110 → Salary = ₹325

Consumer Price Index = 200 → Salary = ?

$$\text{Salary} = \frac{200 \times 325}{110} = ₹590.91$$

However, he is getting salary of ₹500.

Therefore, additional allowance = ₹590.91 – ₹500 = ₹90.91

Question 29

Net monthly salary of an employee was ₹3000 in 1980. The consumer price index number in 1985 is 250 with 1980 as base year. If the has to be rightly compensated, then, the dearness allowance to be paid to the employee is:

(a) ₹4,800.00 (b) ₹4,700.00 (c) ₹4,500.00 (d) None

Solution

(c)

Consumer Price Index = 100 → Salary = ₹3,000

Consumer Price Index = 250 → Salary = ?

$$\text{Salary} = \frac{250 \times 3,000}{100} = ₹7,500$$

Therefore, additional dearness allowance = ₹7,500 – ₹3,000 = ₹4,500

Question 30

Net Monthly income of an employee was ₹800 in 1980. The consumer price Index number was 160 in 1980. It is rises to 200 in 1984. If he has to be rightly compensated. The additional dearness allowance to be paid to the employee is:

(a) ₹240 (b) ₹275 (c) ₹250 (d) None

Solution

(d)

Consumer Price Index = 160 → Salary = ₹800

Consumer Price Index = 200 → Salary = ?

$$\text{Salary} = \frac{200 \times 800}{160} = ₹1{,}000$$

Therefore, additional dearness allowance = ₹1,000 – ₹800 = ₹200

Question 31

Consumer Price index number for the year 1957 was 313 with 1940 as the base year. The Average Monthly wages in 1957 of the workers into factory be ₹160/-. Their real wages is:

(a) ₹48.40 (b) ₹51.12 (c) ₹40.30 (d) None

Solution

(b)

Question 32

During the certain period the C.L.I. goes up from 110 to 200 and the Salary of a worker is also raised from 330 to 500, then the real terms is:

(a) loss by ₹50 (b) loss by ₹75 (c) loss by ₹90 (d) None

Solution

(a)

The language is very weird.

They want to ask us the difference between the real wages of the two time periods.

Real wage is the wage as per the base index price (i.e., 100)

So, real wage when the index was 110:

When the index is 110, the wages are 330

Therefore, when the index was 100, the wages were $\frac{100 \times 330}{110} = 300$

Also,

When the index is 200, the wages are 500

Therefore, when the index was 100, the wages were $\frac{100 \times 500}{200} = 250$

Therefore, clearly, there is a loss of ₹50 in real terms.

Question 33

During a certain period the cost of living index number goes up from 110 to 200 and the salary of a worker is also raised from ₹330 to ₹500. The worker does not get really gain. Then the real wages decreased by:

(a) ₹45.45 (b) ₹43.25 (c) ₹100 (d) None

Solution

(c)

Clearly, it's the same question as before. However, ICAI has solved it as follows:

For index 110, the wages were ₹330.

For index 200, the wages should have been (200 × ₹330) ÷ 110 = ₹600

Clearly, there's a loss of ₹100.

Shifting and Splicing of Index Numbers

Shifting of Index Numbers

$$Shifted\ Price\ Index = \frac{Original\ Price\ Index}{Price\ Index\ of\ the\ Year\ on\ which\ it\ has\ to\ be\ shifted} \times 100$$

Year	Original Price Index	Shifted Price Index to Base 1990
1988	125	$\frac{125}{140}\times 100 = 89.3$
1989	131	$\frac{131}{140}\times 100 = 93.6$
1990	140	$\frac{140}{140}\times 100 = 100.0$
1991	147	$\frac{147}{140}\times 100 = 105.0$

Splicing of Index Numbers

Splicing means combining two index covering different bases into a single series. Splicing two sets of price index numbers covering different periods of time is usually required when there is a major change in quantity weights. It may also be necessary on account of a new method of calculation or the inclusion of new commodity in the index.

Year	**Old Price Index [1900 = 100]**	**Revised Price Index [1995 = 100]**	**Spliced Price Index [1995 = 100]**
1990	100.0		$\frac{100}{114.2}\times 100 = 87.6$
1991	102.3		$\frac{102.3}{114.2}\times 100 = 89.6$
1992	105.3		$\frac{105.3}{114.2}\times 100 = 92.2$
1993	107.6		$\frac{107.6}{114.2}\times 100 = 94.2$
1994	111.9		$\frac{111.9}{114.2}\times 100 = 98.0$
1995	114.2	100.0	100.0

1996		102.5	102.5
1997		106.4	106.4
1998		108.3	108.3
1999		111.7	111.7
2000		117.8	117.8

Test of Adequacy

There are four tests:

1. **Unit Test –**
    a. This test requires that the formula should be independent of the unit in which (or, for which) prices and quantities are quoted.
    b. All the formulae satisfy this test, except for the simple (unweighted) aggregative index.
2. **Time Reversal Test –**
    a. It is a test to determine whether a given method will work both ways in time, forward and backward.

b. The test provides that the formula for calculating the index number should be such that two ratios, the current on the base and the base on the current should multiply into unity.

c. In other words, the two indices should be reciprocals of each other. Symbolically, $P_{01} \times P_{10} = 1$, where, $P_{01} = \frac{P_1}{P_0}$, and $P_{10} = \frac{P_0}{P_1}$.

d. Check of Different Methods

   i. Laspeyres' method

$$P_{01} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0}, \; P_{10} = \frac{\sum P_0 Q_0}{\sum P_1 Q_0}$$

$$P_{01} \times P_{10} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_0 Q_0}{\sum P_1 Q_0} \neq 1$$

Therefore, Laspeyres' Method does not satisfy this test.

ii. Paasche's method

$$P_{01} = \frac{\sum P_1 Q_n}{\sum P_0 Q_n},\ P_{10} = \frac{\sum P_0 Q_n}{\sum P_1 Q_n}$$

$$P_{01} \times P_{10} = \frac{\sum P_1 Q_n}{\sum P_0 Q_n} \times \frac{\sum P_0 Q_n}{\sum P_1 Q_n} \neq 1$$

Therefore, Paasche's Method does not satisfy this test.

iii. Fisher's Ideal

$$P_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}},\ P_{10} = \sqrt{\frac{\sum P_0 Q_1}{\sum P_1 Q_1} \times \frac{\sum P_0 Q_0}{\sum P_1 Q_0}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}} \times \sqrt{\frac{\sum P_0Q_1}{\sum P_1Q_1} \times \frac{\sum P_0Q_0}{\sum P_1Q_0}} = 1$$

Therefore, Fisher's Idea does satisfy this test.

3. **Factor Reversal Test –**

   a. This states that the product of price index and the quantity index should be equal to the corresponding value index, i.e., $\frac{\sum P_1Q_1}{\sum P_0Q_0}$.

   b. Symbolically, $P_{01} \times Q_{01} = V_{01}$.

   c. Check for Fisher's Method

$$P_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}},\ Q_{01} = \sqrt{\frac{\sum Q_1P_0}{\sum Q_0P_0} \times \frac{\sum Q_1P_1}{\sum Q_0P_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}} \times \sqrt{\frac{\sum Q_1P_0}{\sum Q_0P_0} \times \frac{\sum Q_1P_1}{\sum Q_0P_1}}$$

$$= \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum Q_1P_0}{\sum Q_0P_0} \times \frac{\sum Q_1P_1}{\sum Q_0P_1}}$$

$$= \sqrt{\frac{\left(\sum P_1Q_1\right)^2}{\left(\sum P_0Q_0\right)^2}}$$

$$= \frac{\sum P_1Q_1}{\sum P_0Q_0}$$

Therefore, Fisher’s Method satisfies this test as well.

d. While selecting an appropriate index formula, the Time Reversal Test and the Factor Reversal test are considered necessary in testing the consistency.

e. Since Fisher's Index number satisfies both the tests (Time Reversal, as well as Factor Reversal), it is called an Ideal Index Number.

4. **Circular Test –**

a. As per this test, $P_{01} \times P_{12} \times P_{20} = 1$.

b. Therefore, this property enables us to adjust the index values from period to period without referring to the original base every time.

c. The test of this shiftability of base is called the circular test.

d. This test is not met by Laspeyres, or Paasche's or the Fisher's ideal index.

e. The simple geometric mean of price relatives and the weighted aggregative with fixed weights meet this test.

Question 34

If the 1970 index with base 1965 is 200, and 1965 index with base 1960 is 150, the index 1970 on base 1960 will be :

(a) 700 (b) 300 (c) 500 (d) 600

Solution

(b)

Let the year 1960 be P_0, the year 1965 be P_1, and the year 1970 be P_2.

We need to find out the index of 1970 (P_2), on base 1960 (P_0). Therefore, we need to find P_{02}.

As per the question,

- the 1970 index with base 1965 is 200. This means that $P_{12} = 200$.
- the 1965 index with base 1960 is 150. This means that $P_{01} = 150$.

As per the circular test, we know that $P_{01} \times P_{12} \times P_{20} = 1$.

Therefore, $150 \times 200 \times P_{20} = 1 \Rightarrow P_{20} = \frac{1}{150 \times 200} \times 100 = \frac{1}{300}$.

Therefore, $P_{02} = \frac{1}{P_{20}} = 300$.

Miscellaneous Questions

Question 35

When the cost of Tobacco was increased by 50%, a certain hardened smoker, who maintained his formal scale of consumption, said that the rise had increased his cost of living by 5%. Before the change in price, the percentage of his cost of living was due to buying Tobacco is:

(a) 15% (b) 8% (c) 10% (d) None

Solution

(c)

The language of this question is not correct. They want to ask what percentage of his Cost of Living his Tobacco consumption is. (Tobacco consumption iski Cost of Living ka kitna percent hai?)

Let x be Cost of Living Index and y be Tobacco Price.

Increase in x = 5% $\Rightarrow$ $0.05x$; increase in y = 50% $\Rightarrow$ $0.5y$

$0.05x = 0.5y$

$$\frac{y}{x} = \frac{0.05}{0.5} = 0.10$$

Question 36

The consumer price Index for April 1985 was 125. The food price index was 120 and other items index was 135. The percentage of the total weight given to food index is:

(a) 66.67 (b) 68.28 (c) 90.25 (d) None

Solution

(a)

Let the weight of the food price index be x. Then the weight of the other index would be $(100 - x)$.

Weighted Average = Sum of Products/Sum of Weights

$$\text{Consumer Index} = \frac{120x + 135(100 - x)}{100}$$

$$125 = \frac{120x + 135(100 - x)}{100}$$

$$12500 = 120x + 13500 - 135x$$

$$12500 = 13500 - 15x$$

$$15x = 13500 - 12500$$

$$15x = 1000$$

$$x = \frac{1000}{15} = 66.67$$

Question 37

From the following data with 1966 as base year:

Commodity	Quantity Units	Values (₹)
A	100	500
B	80	320
C	60	150
D	30	360

The price per unit of commodity A in 1966 is:

(a) ₹5 (b) ₹6 (c) ₹4 (d) ₹12

Solution

(a)

Question 38

The price of a commodity increases from ₹5 per unit in 1990 to ₹7.50 per unit in 1995 and the quantity consumed decreases from 120 units in 1990 to 90 units in 1995. The price and quantity in 1995 are 150% and 75% respectively of the corresponding price and quantity in 1990. Therefore, the product of the price ratio and quantity ratio is:

(a) 1.8 (b) 1.125 (c) 1.75 (d) None

Solution

(b)

Base Year (1990)		**Current Year (1995)**	
Price (P_0) (₹)	**Quantity (Q_0)**	**Price (P_1) (₹)**	**Quantity (Q_1)**
5	120	7.50	90

Price Ratio = 7.50/5

Quantity Ratio = 90/120

Product = 7.50/5 × 90/120 = 1.125

Question 39

If the price index for the year, say 1960 be 110.3 and the price index for the year, say 1950 be 98.4, then the purchasing power of money (Rupees) of 1950 in 1960 is:

(a) ₹1.12 (b) ₹1.25 (c) ₹1.37 (d) None

Solution

(a)

Think of it in this way. A McAloo Tikki burger costed ₹20 in 2009, and ₹40 in 2020. This means that means that ₹40 had the power to purchase 2 burgers back in 2009. Therefore, the purchasing power of the money of 2009 in 2020 is 2.

Similarly, in this question, something which could be bought for ₹98.4 back in 1950 is costing ₹110.3 now, in 1960. Therefore, the purchasing power of the money of 1950 in 1960 = 110.3 ÷ 98.4 = 1.12.

Question 40

In 1996 the average price of a commodity was 20% more than in 1995 but 20% less than in 1994; and more over it was 50% more than in 1997 to price relatives using 1995 as base (1995 price relative 100) Reduce the data is:

(a) 150, 100, 120, 80 for (1994–97)
(b) 135, 100, 125, 87 for (1994–97)
(c) 140, 100, 120, 80 for (1994–97)
(d) None

Solution

(a)

- In 1996 the average price of a commodity was 20% more than in 1995 → this means that if the index in 1995 was 100, then the index in 1996 would be 120.

- but 20% less than in 1994 → this means that 1996 index is 20% less than the index of 1994. Let the index of 1994 be x. Then the 1996 index = 80% of x. This means $0.8x = 120 \Rightarrow x = 120 \div 0.8 = 150$
- and more over it was 50% more than in 1997 → this means that 1996 index is 50% more than the index of 1997. Let the index of 1997 be x. $1.5x = 120 \Rightarrow x = 120 \div 1.5 = 80$

So, we have:

1994	150
1995	100
1996	120
1997	80