Chapter 6

4 Marks

SEQUENCE AND SERIES

TOPIC : SEQUENCE

SERIES

ARITHMETIC PROGRESSIONS

GEOMETRIC PROGRESSIONS

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- Teaches Maths in CA foundation and K-12
- Taught 30000+ students with highest score as 100

(1) 28, 2, 25, 27, —-

(2) 2,7, 11, 19, 31, 51, —-

Not Sequence

(3) 1, 2, 3, 4, 5, 6, —---

(4) 20, 18, 16, 14, 12, 10, -—--

Sequence

SEQUENCE

An ordered collection of numbers $a_1, a_2, a_3, a_4, \ldots\ldots\ldots a_n \ldots\ldots\ldots$ is a sequence if according to some definite rule or law, there is a definite value of a_n called the term or element of the sequence, corresponding to any value of the natural number n.

SEQUENCE

- a_1 is the 1st term of the sequence, a_2 is the 2nd term, a_n is the nth term.

- In the nth term a_n by putting n = 1, 2, 3..... successively, we get a_1, a_2 a_3, a_4,.....

- The nth term is also called the general term of the sequence.
- To specify a sequence, nth term must be known.

SEQUENCE

FINITE

- If the number of elements in a sequence is finite, the sequence is called finite sequence
- $a_1, a_2, a_3, a_4, \ldots\ldots\ldots, a_n$

  denoted by $\{a_i\}^n_{i=1}$
- **Example :** 2, 4, 6, 8, 10.

INFINITE

- If the number of elements is unending, the sequence is infinite
- $a_1, a_2, a_3, a_4, \ldots\ldots\ldots, a_n \ldots\ldots\ldots$

  denoted by $\{a_n\}^\infty_{n=1}$ or simply by $\{a_n\}$
- **Example :** 1, 3, 5, 7, ……

EXAMPLES

FINITE SEQUENCE

- **A sequence of even positive integers within 12 i.e., is**

  2, 4, 6, 8, 10.

- **A sequence of odd positive integers within 11 i.e., is**

  1, 3, 5, 7, 9.

EXAMPLES

INFINITE SEQUENCE

- The sequence $\{1/n\}$ is 1, 1/2, 1/3, 1/4.....
- The sequence $\{(-1)^n n\}$ is -1, 2, -3, 4, -5,.....
- The sequence $\{n\}$ is 1, 2, 3,.....
- The sequence $\{n / (n + 1)\}$ is 1/2, 2/3, 3/4, 4/5.....
- A sequence of even positive integers is 2, 4, 6, ...
- A sequence of odd positive integers is 1, 3, 5, 7, ...

SERIES

- An expression of the form $a_1 + a_2 + a_3 + ... + a_n + ...$

  which is the sum of the elements of the sequence $\{a_n\}$ is called a ***series.***

- Example : $1 + 3 + 5 + 7 + ...$
- If the series contains a finite number of elements, it is called a ***finite series***, otherwise called an ***infinite series***.

ARITHMETIC PROGRESSIONS (AP)

- 2 , 5 , 8 , 11 , 14 , 17,.....
- 15 , 13 , 11 , 9 , 7, 5 , 3 , 1, –1
- 2 , 2 , 2 , 2 , 2 .....

- A sequence $a_1, a_2, a_3, ... a_n$ is called an Arithmetic Progression (A.P.) when

  $a_2 - a_1 = a_3 - a_2 = a_n - a_{n-1}$

- A. P. is a sequence in which each term is obtained by adding a constant d to the preceding term.
- This constant 'd' is called the ***common difference*** of the A.P
- d can be positive , negative, zero

ARITHMETIC PROGRESSIONS (AP)

GENERAL FORM OF AP

- $a_1, a_2, a_3, a_4, a_5 \ldots\ldots$
- Here

$a_1 = a$

$a_2 = a + d$

$a_3 = a + 2d$

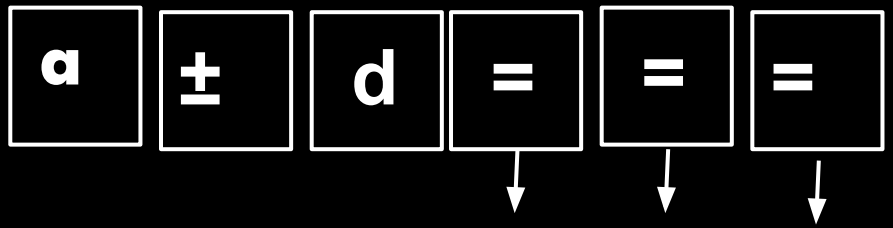
-
-
-

nth term of AP/ General term of AP

$$a_n = a + (n-1)\,d$$

Example 1 Find the 7th term of the A.P. 8, 5, 2, -1, -4,.....

CALCULATOR TRICK TO FIND nth TERM OF AN AP



2^{nd} term , 3^{rd} term , 4^{th} term , ...so on

- If d is positive take + or if d is negative take –

Example 2 Which term of AP $\frac{3}{\sqrt{7}}, \frac{4}{\sqrt{7}}, \frac{5}{\sqrt{7}}$ .....is $\frac{17}{\sqrt{7}}$?

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Example 3 If 5^{th} and 12^{th} terms of an A.P. are 14 and 35 respectively, find the A.P.

ARITHMETIC PROGRESSIONS (AP)

- nth term from the end of an AP — $\{l - (n - 1)\,d\}$
- THREE NUMBERS IN AP — $(a - d)\,,\ a\ ,\ (a + d)$
- FOUR NUMBERS IN AP — $(a - 3d)\,,\ (a - d)\,,\ (a + d)\,,\ (a + 3d)$

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Example 4 Divide 69 into three parts which are in A.P. and are such that the product of the first two parts is 483

ARITHMETIC PROGRESSIONS (AP)

- If 3 numbers a, b, c are in A.P., we say

  $b - a = c - b$ or $a + c = 2b$;

  b is called the arithmetic mean between a and c.

Example Find the arithmetic mean between 4 and 10.

ARITHMETIC PROGRESSIONS (AP)

SUM OF First n terms of AP

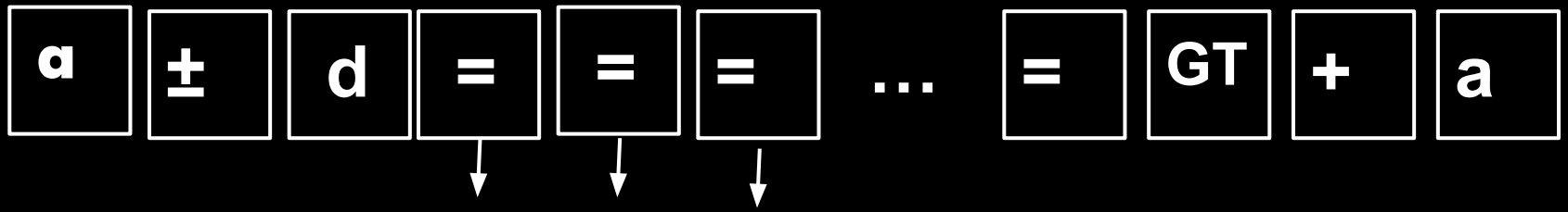
- Let S be the Sum,
- a be the 1st term and
- l be the last term of an A.P.
- If the number of term is n, then $a_n = l$.
- Let d be the common difference of the A.P.

$$S = \frac{n}{2}\{2a + (n-1)d\}$$

- $S = n(a + l)/2$

Find the sum of 23 terms of the AP 5, 9,13,17 ,.....

CALCULATOR TRICK TO FIND Sum of First n TERMs OF AN AP



- If d is positive take + or if d is negative take –

Find the sum of the series : 5 + 13 + +21 + ……. + 181

ARITHMETIC PROGRESSIONS (AP)

Result

$$a_n = S_n - S_{n-1}$$

RESULTS

$$1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}.$$

$$1 + 3 + 5 + \ldots + (2n - 1) = n^2$$

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

TRICKS

- How many 3 digit numbers are divisible by 7?

Ans : 128

STEP 1: Write first and last 3 digit number

100 and 999

Step 2 : Divide both by 7

100/7 = 14.2857

999/7 = 142.714

Step 3 : Avoid decimal and Subtract

142 - 14 = 128

JUNE 2023

Que. How many number between 74 and 25,556 are divisible by 5?

(a) 5090

(b) 5097

(c) 5095

(d) 5075

Ans : b

TRICKS

If the ratio between the sum of n terms of two AP is (7n +1) : (4n + 27) , find the ratio of their 11^{th} term

Sol : Ratio of their 11^{th} term is

$$\frac{7(21)+1}{4(21)+27} = \frac{148}{111}$$

we need to find ratio of 11^{th} term

Step 1 : multiply 11 by 2 and subtract 1

$2 \times 11 - 1 = 21$

Step 2 : In the given ratio substitute your n by 21 and get your answer

JUNE 2019

Que. The ratio of sum of n terms of the two AP's is (n+1) : (n-1) then the ratio of their mth term is

(a) (m+1) : 2m

(b) (m+1) : (m-1)

(c) (2m-1) : (m+1)

(d) m : (m-1)

Ans : d

TRICKS

If 9 times 9^{th} term of an AP is equal to 13 times the 13^{th} term then the 22^{nd} term of an AP is ?

Ans : 0

If m times the mth term of an AP is equal to n times the n^{th} term , then $(m + n)^{th}$ term of the AP is 0

$$m\,a_m = n\,a_n$$

$$a_{m+n} = 0$$

TRICKS

If $a_{64} = 13$ and $a_{13} = 64$, find $a_{101} = ?$

Ans : - 24

If m^{th} term of a given AP is n and its n^{th} term is m then its pth term is (n + m - p)

$$a_m = n$$

$$a_n = m$$

$$a_p = (n+m-p)$$

EXERCISE 6(A)

Choose the most appropriate option (a), (b), (c) or (d)

Que. 1 The nth element of the sequence 1, 3, 5, 7,..... is

(a) n

(b) 2n−1

(c) 2n +1

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 2 The nth element of the sequence −1, 2, −4, 8..... is

(a) $(-1)^n 2^{n-1}$

(b) 2^{n-1}

(c) 2^n

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 3 $\sum_{i=4}^{7} \sqrt{2i-1}$ can be written as

(a) $\sqrt{7} + \sqrt{9} + \sqrt{11} + \sqrt{13}$

(b) $2\sqrt{7} + 2\sqrt{9} + 2\sqrt{11} + 2\sqrt{13}$

(c) $2\sqrt{7} + 2\sqrt{9} + 2\sqrt{11} + 2\sqrt{13}$

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 4 The sum to ∞ of the series −5, 25, −125, 625, ... can be written as

(a) $\sum_{k=1}^{\infty} (-5)^k$

(b) $\sum_{k=1}^{\infty} 5^k$

(c) $\sum_{k=1}^{\infty} -5^k$

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 5 The first three terms of sequence when nth term t_n is $n^2 - 2n$ are

(a) −1, 0, 3

(b) 1, 0, 2

(c) −1, 0, −3

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 6 Which term of the progression −1, −3, −5, ... is −39

(a) 21^{st}

(b) 20^{th}

(c) 19^{th}

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 7 The value of x such that 8x + 4, 6x − 2, 2x + 7 will form an AP is

(a) 15

(b) 2

(c) 15/2

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 8 The m^{th} term of an A. P. is n and n^{th} term is m. The r^{th} term of it is

(a) m + n + r

(b) n + m − 2r

(c) m + n + r/2

(d) m + n − r

Choose the most appropriate option (a), (b), (c) or (d)

Que. 9 The number of the terms of the series $10 + 9\frac{2}{3} + 9\frac{1}{3} + 9 + ...$ will amount to 155 is

(a) 30

(b) 31

(c) 32

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 10 The nth term of the series whose sum to n terms is $5n^2 + 2n$ is

(a) 3n – 10

(b) 10n – 2

(c) 10n – 3

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 11 The 20^{th} term of the progression 1, 4, 7, 10 ..... is

(a) 58

(b) 52

(c) 50

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 12 The last term of the series 5, 7, 9, ... to 21 terms is

(a) 44

(b) 43

(c) 45

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 13 The last term of the A.P. 0.6, 1.2, 1.8, ... to 13 terms is

(a) 8.7

(b) 7.8

(c) 7.7

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 14 The sum of the series 9, 5, 1, ... to 100 terms is

(a) -18,900

(b) 18,900

(c) 19,900

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 15 The two arithmetic means between −6 and 14 is

(a) 2/3, 1/3

(b) 2/3, $7\frac{1}{3}$

(c) −2/3, $-7\frac{1}{3}$

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 16 The sum of three integers in AP is 15 and their product is 80. The integers are

(a) 2,8,5

(b) 8,2,5

(c) 2,5,8

(d) 8,5,2

Choose the most appropriate option (a), (b), (c) or (d)

Que. 17 The sum of n terms of an AP is $3n^2 + 5n$. The series is

(a) 8, 14, 20, 26

(b) 8, 22, 42, 68

(c) 22, 68, 114, ...

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 18 The number of numbers between 74 and 25,556 divisible by 5 is

(a) 5,090

(b) 5,097

(c) 5,095

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 19 The pth term of an AP is (3p – 1)/6. The sum of the first n terms of the AP is

(a) n (3n+1)

(b) n(3n+1)/12

(c) n/12 (3n–1)

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 20 The arithmetic mean between 33 and 77 is

(a) 50

(b) 45

(c) 55

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 21 The 4 arithmetic means between −2 and 23 are

(a) 3, 13, 8, 18

(b) 18, 3, 8, 13

(c) 3, 8, 13, 18

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 22 The first term of an A.P is 14 and the sums of the first five terms and the first ten terms are equal in magnitude but opposite in sign. The 3rd term of the AP is

(a) $6\frac{4}{11}$

(b) 6

(c) 4/11

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 23 The sum of a certain number of terms of an AP series −8, −6, −4..... is 52. The number of terms is

(a) 12

(b) 13

(c) 11

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 24 The first and the last term of an AP are −4 and 146. The sum of the terms is 7171. The number of terms is

(a) 101

(b) 100

(c) 99

(d) none of these

Choose the most appropriate option (a), (b), (c) or (d)

Que. 25 The sum of the series $3\frac{1}{2} + 7 + 10\frac{1}{2} + 14 + \ldots$ to 17 terms is

(a) 530

(b) 535

(c) $535\frac{1}{2}$

(d) none of these

Nov 2018

Que. If pth term of an AP is q and its qth term is p , then what will be the value of r th term?

(a) p + q +r

(b) p + q – r

(c) p – q – r

(d) p+q

Ans : b

Dec 2022

Que. If pth term of an AP is q and its qth term is p , then what will be the value of (p + q)th term?

(a) 0

(b) 1

(c) p+q−1

(d) 2(p+q−1)

Ans : a

Dec 2019

Que. The sum of five terms of AP is 75 . find the 3rd term

(a) 20

(b) 30

(c) 15

(d) None of these

Ans : c

June 2022

Que. The first and last terms of an AP are 5 and 905 . Sum of the terms is 45, 955 . The number of terms is

(a) 99

(b) 100

(c) 101

(d) 102

Ans : c

July 2021

Que. If the sum of ‘n ‘ terms of an AP is $2n^2$, the fifth term is

(a) 20

(b) 50

(c) 18

(d) 25

Ans : c

Jan 2021

Que. The nth terms of the series 3 + 7 + 13 + 21 + 31 + ... is

(a) 4n-1

(b) n^2 +2n

(c)n^2 + n + 1

(d) n^3 + 2

Ans : c

June 2023

Que. If 9th and 19th term of an AP are 35 and 75 , respectively , then its 20 term is

(a) 78

(b) 79

(c) 80

(d) 81

Ans : b

June 2022

Que. If nth term of the AP 9, 7, 5.... Is same as the nth term of the AP 15, 12, 9....., then n will be

(a) 7

(b) 9

(c) 15

(d) 11

Ans : a

Nov 2018

Que. The value of K , for which the terms 7K + 3 , 4K − 5 , 2K + 10 are in AP is

(a) −13

(b) −23

(c) 13

(d) 23

Ans : b

Dec 2015

Que. Three NO's a ,b , c are in AP find a - b + c

(a) a

(b) - b

(c) b

(d) c

Ans : c

May 2018

Que. Insert two Arithmetic means between 68 and 260

(a) 132 , 196

(b) 130 , 194

(c) 70 , 258

(d) none

Ans : a

DEC 2021

Que. The sum of series 7 + 14 + 21 + ... to 17th term is :

(a) 1071

(b) 971

(c) 1171

(d) 1271

Ans : a