1. RATIO AND PROPORTION, INDICES, LOGARITHMS

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unit I : Ratio :

✱ Ratio :

Ratio is comparison of two or more quantities.

$$\boxed{3 : 4}$$

$\boxed{\text{antecedent}}$ $\boxed{\text{consequent}}$

Ex :

Q. which ratio is greater ?

① $2\left[\frac{1}{5}\right] : 3\left[\frac{1}{5}\right]$ OR $3.6 : 4.0$

$\frac{5\times2+1}{5} : \frac{5\times3+1}{5}$ $\qquad$ $\frac{36}{10} \quad \frac{40}{10}$

$\frac{11}{5} : \frac{16}{15}$ $\qquad$ $36 : 40$

$\frac{36}{4} \quad \frac{40}{4} = 9 : 10$

$11 : 16$

$\frac{11}{16} \quad \frac{9}{10} \Rightarrow \frac{110}{160} < \boxed{\frac{144}{160}}$

- **Inverse Ratio :**

Eg.→ $12 : 7$

$\boxed{7 : 12}$ ← Inverse Ratio

- Greater inequality → $a : b$ when $a > b$. Eg- $7 : 5$
- less inequality → $a : b$ when $a < b$ Eg- $5 : 7$
- Compounded Ratio

Eg→ $5 : 6$, $7 : 8$, $9 : 10$

compounded ratio will be :- $\boxed{21 : 32}$

$$\frac{\cancel{5}\times7\times\cancel{9}^{3}}{\cancel{6}\times8\times\cancel{10}} = \frac{21}{32}$$

- Duplicate Ratio (square) :

Eg → 2 : 3 ⟶ $\boxed{4 : 9}$

- Triplicate Ratio (cube) :

Eg → 2 : 3 ⟶ $\boxed{8 : 27}$

- Sub Duplicate Ratio : (squareroot)

Eg → 4 : 9 $\sqrt{4} : \sqrt{9} \Rightarrow \boxed{2 : 3}$

- Sub Triplicate Ratio : (cuberoot)

Eg → 27 : 64 $\Rightarrow (3^{\cancel{3}})^{\frac{1}{\cancel{3}}} : (4^3)^{\frac{1}{3}} \Rightarrow \boxed{3 : 4}$

- commensurable or Incommensurable :

Eg → i) 2.2 : 2.4 $\Rightarrow \frac{22}{10} \; \frac{24}{10} \Rightarrow \boxed{11 : 12}$ ← commensurable

ii) $\sqrt{7} : 8 \Rightarrow \sqrt{7} \times 8 \quad 8 \times \sqrt{7} \Rightarrow 7 : 8\sqrt{7}$ ← Incommensurable

- continued ratio :

Eg → 2 : 3 : 9 : 4

Unit II : prepositions.

- **properties of proportion**

① Invertendo → If $a : b = c : d$, then $b : a = d : c$ $\left(\frac{a}{b} \frac{c}{d}\right) \rightarrow$

② Alternendo → If $a : b = c : d$, then $a : c = b : d$ $\frac{a}{b}, \frac{c}{}$

③ componendo →
$a : b = c : d$ then
$(a+b) : b = (c+d) : d$

④ Dividendo → $a : b = c : d$
$(a-b) : b = (c-d) : d$

⑤ componendo & Dividendo →
$a : b = c : d$
$(a+b) : (a-b) = (c+d) : (c-d)$

⑥ Addendo → $a : b = c : d = e : f \ldots\ldots$ then,
$(a + c + e + \ldots) : (b + d + f + \ldots)$

⑦ subtrahendo → $(a - c - e - \ldots) : (b - d - f - \ldots)$

Unit - III : Indices

✱ **Indices** : The power, also known as the index, tells you how many times you have to multiply the number by itself and number will be ~~re~~called as base.

E.g :→ 2^4 , 3^3 , 18^2, etc.

- Cube Root : $\sqrt[3]{8} = (8)^{\frac{1}{3}} = (2\times2\times2)^{\frac{1}{3}}$ $(2^{\cancel{3}})^{\frac{1}{\cancel{3}}} = \underline{\underline{2}}$

- Square Root : $\sqrt[2]{4} = (4)^{\frac{1}{2}} = (2\times2)^{\frac{1}{2}}$ $(2^{\cancel{2}})^{\frac{1}{\cancel{2}}} = \underline{\underline{2}}$

- Negative powers : $2^{-3} = \frac{1}{2^3} = \frac{1}{2\times2\times2} = \underline{\underline{\frac{1}{8}}}$

▨ **Law 1** :

$$\boxed{a^m \times a^n = a^{m+n}}$$ (base must be same)

E.g :→ $2^3 \times 2^2 = 2^{3+2} = 2^5$

▨ **Law 2** :

$$\boxed{\frac{a^m}{a^n} = a^{m-n}}$$ (base must be same)

E.g :→

$$\frac{2^3}{2^2} = 2^{3-2} = 2^1$$

▨ **Law 3** :

$$\boxed{[a^m]^n = a^{m\times n}}$$

E.g :→ $[2^3]^2 = 2^{3\times2} = 2^6$

▨ Law 4 :

$$\boxed{[ab]^n = a^n \times b^n}$$

E.g :→ $(2\times3)^8 = 2^8 3^8$

✱ **Note** :

① $a^0 = 1$

② $a^{-m} = \frac{1}{a^m}$ & $\frac{1}{a^{-m}} = a^m$

③ $a^x = a^y$, then $x = y$

④ $x^a = y^a$, then $x = y$

⑤ $\sqrt[m]{a} = a^{\frac{1}{m}}$

Unit IV : LOGARITHM

Define :

For all real numbers n and all positive numbers a and x, Where $a \neq 1$.

* calculator Trick to find out Logarithm *

↓

Step 1 : Type number & press root for 15 times

↓

Step 2 : Substract 1 after step 1

↓

Step 3 : Divide number by 0.000070274 OR multiply by 14230

* **Law 1** :

$$\boxed{\log mn = \log_m + \log_n}$$

E.g: ① $\log_{(8\times7)} = \log_8 + \log_7$

ⓘⓘ $\log_{(1\times2\times3)} = \log_1 + \log_2 + \log_3$

* **Law 2** : $\boxed{\log\left(\frac{m}{n}\right) = \log m - \log n}$

E.g: ① $\log \frac{27}{5} = \log_{27} - \log_5$

ⓘⓘ $\log\left(\frac{5}{10}\right) = \log_5 - \log_{10}$

* **Law 3** :

$$\boxed{\log m^n = n.\log m}$$

E.g: ① $\log 81^3 = 3.\log 81$

ⓘⓘ $\log 2^2 = 2.\log 2$

* change of base :

$$\boxed{\log_b m = \log_a m \,/\, \log_a b}$$

E.g:→ $\log_2 8 = \dfrac{\log_{10} 8}{\log_{10} 2}$

* Antilogarithm Table : $\boxed{\log_a n = x}$

E.g:→ $\log_{10} x = 2.428$	$\log_{10} x = 2.428$
$\Rightarrow x = \text{antilog}\ (2.428)$	$\text{antilog}\ 2.428 = x = 267.80$

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* calculator trick to find Anti-logarithm.

1. multiply number by 0.000070274 / Divide by 14230

↓

2. Add 1 after step 1).

↓

3. press ' x = ' button for 15 times.

5/5

Equations

Coefficient of the variable

$$\overbrace{2x + 3}y = \underset{\downarrow}{3}$$

$$\underbrace{x + 3y}$$

Variable

constant

★ **Conditional Equation**

$$x + 3 = 0$$

↓

only particular value satisfy this equation.

★ **Identity**

$$(x+y)^2 = x^2 + 2xy + y^2$$

For all real values of x \$ y will satisfy.

★ **Solution / Root of the equation**

↓

Value of x.

Ex :- $x + 3 = 0$

$x = -3$ (Solution / Root)

★ **Linear Equation** (x^1)
or Simple Equation (Only 1 variable)

Ex :- $2x + 3$

★ **Linear Equation in two variables**

$$\boxed{ax + by = c}$$

METHODS

Elimination method | Cross Product Method

$a_1x + b_1y + c_1 = 0$ $\quad$ $a_2x + b_2y + c_2 = 0$

$$\frac{x}{\begin{matrix} b_1 & \times & c_1 \\ b_2 & & c_2 \end{matrix}} = \frac{y}{\begin{matrix} c_1 & \times & a_1 \\ c_2 & & a_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & \times & b_1 \\ a_2 & & b_2 \end{matrix}}$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \qquad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

* Linear Equation in Three variables.

$$\boxed{ax + by + cz = d}$$

Q. $2x + 3y + 4z = 0$, $\underset{(\times 2)}{x + 2y - 5z = 0}$ $\underset{(\times 10)}{}$, $10x + 16y - 6z = 0$

$$\begin{array}{r} 2x + 3y + 4z = 0 \\ 2x + 4y - 10z = 0 \\ \underline{- \quad - \quad + \quad} \\ -y + 14z = 0 \end{array} \qquad \begin{array}{r} 10x + 20y - 50z = 0 \\ 10x + 16y - 6z = 0 \\ \underline{- \quad - \quad + \quad} \\ 4y - 44z = 0 \end{array}$$

$$y = \frac{44z}{4} = 11z$$

* Quadratic Equation (max. power 2)

$$\boxed{ax^2 + bx + c = 0}$$

Factorizing method

$x^2 + 5x + 6 = 0$

$x^2 + 2x + 3x + 6 = 0$

$x(x+2) + 3(x+2) = 0$

$(x+2)(x+3) = 0$

$x = -2 \quad , \quad x = -3$

Quadratic Formula / Determinant method

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a) $b^2 - 4ac = 0$, roots are equal.

b) $b^2 - 4ac > 0$, roots are real no. and unequal.

c) $b^2 - 4ac < 0$, roots are imaginar ($\sqrt{-2}$)

d) $b^2 - 4ac$ is perfect square, rational

e) $b^2 - 4ac$ is not perfect square, irration

$$\underbrace{x = 2 + \sqrt{3} \qquad x = 2 - \overset{\sqrt{b^2-4ac}}{\sqrt{3}}}_{\text{conjugate pair}}$$

* Sum of roots :-

$$\alpha + \beta = -\frac{b}{a}$$

$$\boxed{x = \alpha, \beta} \downarrow \text{Roots}$$

* Product of roots

$$\alpha\beta = \frac{c}{a}$$

* If α and β are the roots of a equation, then

$$(x - \alpha)(x - \beta) = 0$$

* $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Q. If $p \neq q$ and $p^2 = 5p - 3$ and $q^2 = 5q - 3$, the equation having roots as $\frac{p}{q}$ and $\frac{q}{p}$ is

$p^2 - 5p + 3 = 0$ — p & q = x

$q^2 - 5q + 3 = 0$

$x^2 - 5x + 3 = 0$

Roots of equation is p & q.

$p + q = -5$

$pq = 3$

Ans. — $3x^2 - 19x + 3 = 0$

Q. The value of $2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \dots}}} = x$

$2 + \frac{1}{x} = x$ Ans. C) $1 + \sqrt{2}$

$2x + 1 = x^2$

$x^2 - 2x - 1 = 0$

~~$x^2 - x - x - 1 = 0$~~

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1(-1)}}{2 \times 1}$

$= \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

$x = 1 + \sqrt{2}$ (Addition में कभी भी negative नहीं आ सकता)

✱ Cubic Equation (max. power 2)

$$\boxed{ax^3+bx^2+cx+d=0}$$

Method :-

$$x^3+x^2-x-1=0$$
$$x^2(x+1)-(x+1)=0$$
$$(x+1)(x^2-1)=0$$
$$(x+1)(x-1)(x+1)=0$$
$$\Rightarrow (x+1)=0 \Rightarrow x=-1$$
$$\Rightarrow (x-1)=0 \Rightarrow x=1$$
$$\Rightarrow (x+1)=0 \Rightarrow x=-1$$

✱ Roots p, q, r.

$$p+q+r = \frac{-b}{a}$$

$$pq+qr+pr = \frac{c}{a}$$

$$pqr = \frac{-d}{a}$$

Q. If $\boxed{L+M+N=0}$ and L, M, N are rationals the roots of the equation $(M+N-L)x^2+(N+L-M)x+(L+M-N)=0$

$$(-L-L)x^2+(-M-M)x+(-N-N)=0$$
$$-2Lx^2+(-2M)x+(-2N)=0$$
$$-2(Lx^2+\cancel{2}Mx+N)=0$$
$$Lx^2+Mx+N=0$$

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$\Rightarrow b^2 - 4ac$

$\Rightarrow m^2 - 4NL$

$\Rightarrow (N+L)^2 - 4LN$ $\quad$ $[L+m+N=0$

$\Rightarrow (N^2 + L^2 + 2LN - 4LN)$ $\quad$ $-m = L+N$

$\Rightarrow (N^2 + L^2 - 2LN)$ $\quad$ $m^2 = (N+L)^2]$

$\Rightarrow (N-L)^2$

If N and L are rationals, then $(N-L)^2$ is also a rational and a perfect square.

So, $\boxed{\text{ans} - \text{real and rational}}$

LINEAR INEQUALITY

$2 \ \boxed{<} \ 3$

↓

inequality

$<$	Less than
$>$	Greater than
$\le$	Less than equal to
$\ge$	Greater than equal to

✱ Linear Inequation (One variable, max. power 1)

$\ge, <, >$

$$\boxed{ax + b \le 0}$$

* IMPORTANT

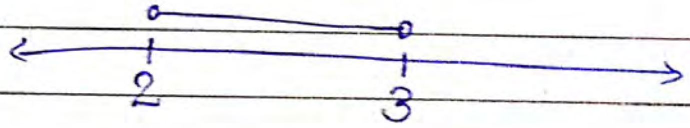
- When +ve or -ve value is subtracted or added, no change in inequality sign.
- When +ve value is divided / multiplied, no change.
- When -ve value is divided / multiplied, inequality sign will reverse.

✱ $x \in R$ (x value is a real number)

↓

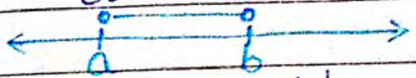
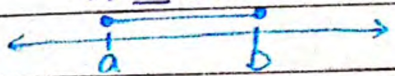
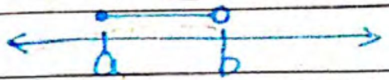
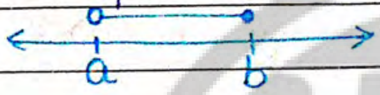
belongs to

* $2 < x < 3$



$x \in (2,3)$ [2 and 3 not included]

$x \in [2,3]$ [2 and 3 included]

	Inequality	Interval (Sol. Space)
a)	$a < x < b$ 	$x \in (a,b)$
b)	$a \leq x \leq b$ 	$x \in [a,b]$
c)	$a \leq x < b$ 	$x \in [2,3)$
d)	$a < x \leq b$ 	$x \in (a,b]$

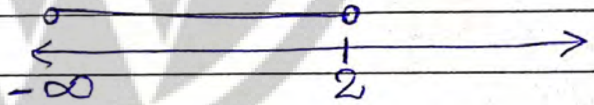
Q. Solve $7x - 1 < 5x + 3$ where x belong to real number?

$$7x - 5x < 3 + 1$$

$$2x < 4$$

$$\frac{2x}{2} < \frac{4}{2}$$

$$x < 2$$



$$x \in (-\infty, 2)$$

- ∞ में हमेशा Open Bracket.

✯ Two equations :-
- Common part is solution
- No common part — no solution

✯ Linear Inequation in two Variable

$$\boxed{ax + by \geq c}$$

- More than / Not less than $x > \square \,/\, x \geq \square$
- Less than / Not more than $x < \square \,/\, x \leq \square$

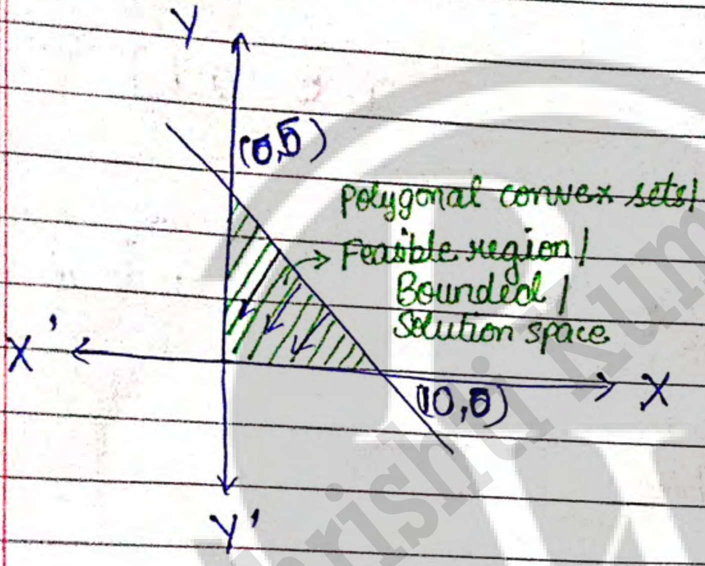
- Atleast / minimum $x \geq \square$
- Atmost / maximum $x \leq \square$

Graph

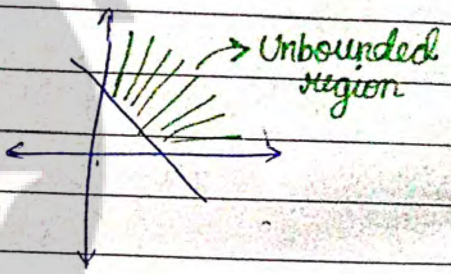
$x + 2y \leq 10$

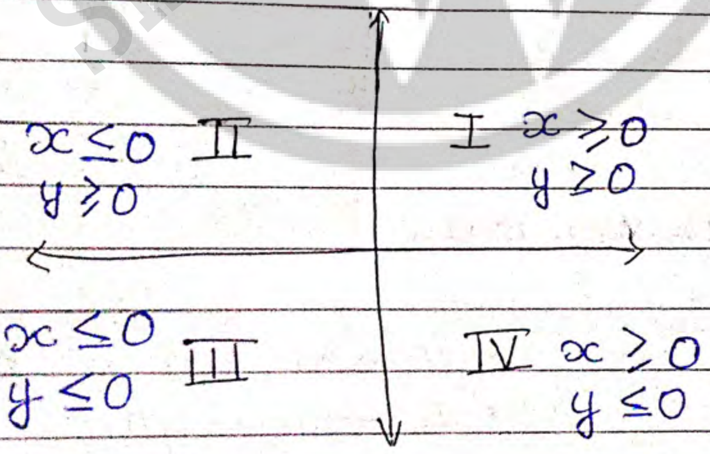
$0 + 2(0) \leq 10$

$0 \leq 10$ [TRUE]



x	0	10
y	5	0





* Value included नहीं हैं तो graph में dotted line.

* Optimal Solution

- Corners of a graph (extreme points)

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Q. The union however forbids him to employ less than 2 experienced person to each fresh person. The situation can be expressed as

$x \geq 2y$ $\left(\begin{matrix} \text{exp.} \rightarrow x \\ \text{fresh} \rightarrow y \end{matrix}\right)$

(fresh men के दुगने से कम नहीं होने चाहिए experienced)

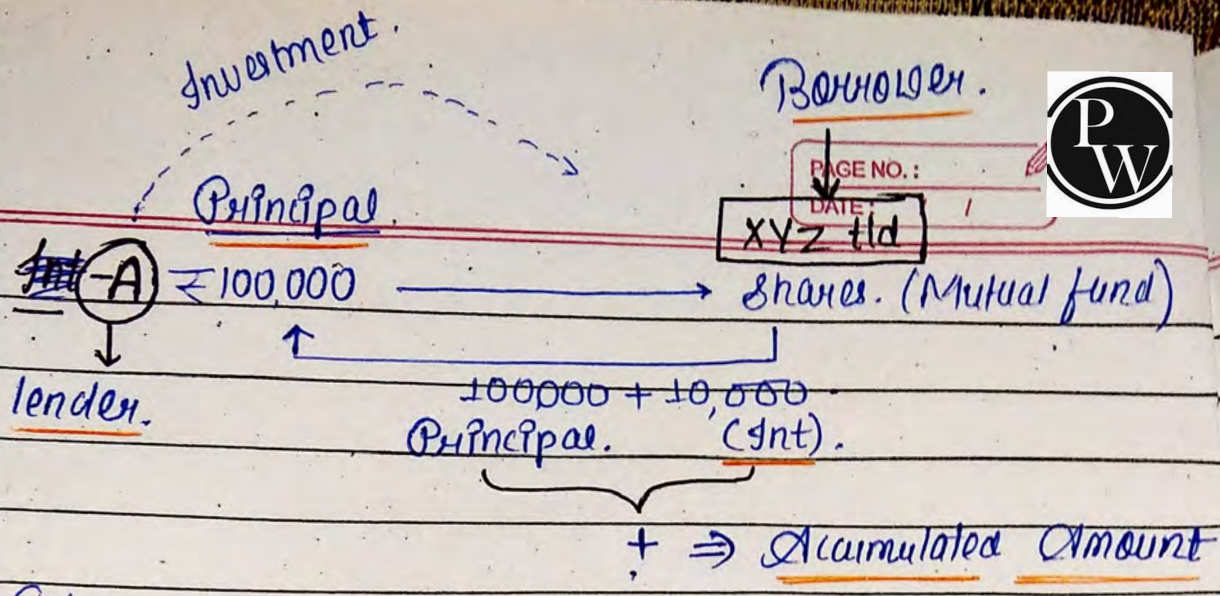
Points to Remember:-

* When origin is not on any line.

$ax + by \leq c$ } Towards origin

$ax + by \geq c$ } Away from origin

• c — Positive



- Extra money on principal = Interest.
- Accumulated amt = Principal + Interest.

  ↓

  given by Borrower.
- Principal amount = given by lender. at first.
- Rate of Int in %age (R). = $\frac{\text{Interest}}{\text{Principal}} \times 100$
- Time period = Time for which principal amount is invested (generally per annum. when not specified)

Type of Interest

- Simple Interest
- Compound Interest

ⓐ Simple Interest ⇒ $\boxed{\frac{P\times r\times t}{100}}$ [When r is in %]

$\boxed{P\times i\times t}$ [When r is in decimal]

here. [$i = \frac{r}{100}$]

(B). Accumulated Amount (A) in case of (SI) ⇒

$$A = P + (I) \rightarrow SI.$$

$$A = P + (SI) \longrightarrow \frac{P\times r\times t}{100}$$

$$A = (P) + \left(\frac{(P)\times r\times t}{100}\right)$$

$$\boxed{A = P\left[1 + \left(\frac{rt}{100}\right)\right]}$$

- Simple Interest on Calculator →

→ $\boxed{SI = P\times T\times r\,\%.}$ Trick.

- Accumulated Amount on Calculator -

→ $\boxed{A = (P \times T \times r\%) + P.}$ trick

(ii) Accumulated Amount for n^th year -

n^th year = $\boxed{A_n = P + n(SI_1)}$ → SI_1 = simple Interest for 1 year.

and, $SI_1 = \frac{prt}{100}.$

Example ⇒ $A_3 = A + 3(SI_1).$

$A_2 = P + 2(SI_1).$

Lecture 02

(b) Compound Interest - (CI)

- CI = For next year Principal = Accumulated amt of previous year.

Ex- for 1st year.

$P = 1000$, $t = 1$ yrs

$r = 10\%$

$SI_1 = \frac{P \times r \times t}{100}$

$= \frac{10\cancel{00} \times 10 \times 1}{\cancel{100}} = 100$ ₹

$A = P + CI.$

$= 1000 + 100$

$= 1100$

for next year (2nd year).

$P =$ Accumulated amount of 1st year.

$P = 1100$

$r = 10\%.$

$t = 1$ yr.

$CI_2 = \frac{P \times r \times t}{100} = \frac{11\cancel{0}\cancel{0} \times 10 \times 1}{1\cancel{0}\cancel{0}} = 110$ ₹.

and so on.

$$\boxed{CI = I_1 + I_2 \ldots}$$

(i) Accumulated Amount (A) in case of (CI) –

$$\boxed{A_n = P\left(1 + \frac{r}{100}\right)^n.}$$ nth year.

also, $\boxed{A = P + CI}$

then $CI = A - P$

$$CI = \boxed{A} - P \longrightarrow P\left(1+\frac{r}{100}\right)^n$$

$$\boxed{CI = P\left(1+\frac{r}{100}\right)^n - P}$$

- Derivation of Accumulated amount formula:-

for 1st year ⇒ SI = CI.

A_1 ⇒ Accummulated amt of SI = Accumulated amt of CI.

$$A_1 = P\left(1+\frac{r}{100}\right) \Rightarrow \text{as } t = 1$$

2nd year onwards

$$P_2 = A_1$$

3rd year

$$P_3 = A_2$$

⋮

$$n^{th} = P_n = A_{n-1}$$

for any year.

$$\boxed{A_n = P_n\left(1+\frac{r}{100}\right)^n}$$

- Accumulated Amount in case of CI on calculator-

$$\boxed{A = P + \underbrace{r\% + r\% + r\% \ldots}_{\text{Press } r\% \text{ for } t \text{ times.}}}$$ ⇐ Trick

- Compound Interest on calculator from Accumulated amount -

$$CI = A - P.$$

Trick ⇒ $$\boxed{CI = (P + \underbrace{r\% + r\% + r\% ..}_{\text{Press } r\% \text{ for } t \text{ times.}}) + \text{Principal.}}$$

- ## Conversion Period.

In previous situations time of annually so the P of next year was the accumulated amt of previous year.

But if we want to take the Principal of next year is according to annually, semi annually, quarterly, monthly or daily then,

Period	Conversion (C)	(no of conversion year) C.
1 yr	Annually (Jan → Jan)	1.
6 Month	Semi Annually (Jan → July → Jan)	2
3 Month	Quarterly (Jan → April → July → Oct → Jan)	4
Monthly	Every Month.	12
daily	Every day	365

- Accumulated Amount in case of conversion period.

$$\boxed{A = P\left(1+\frac{r}{c\times100}\right)^{t\times c}} \Big/ \boxed{A = P(1+i)^{n}}$$

$$n = t\times c,\ i = \frac{r}{100c}$$

- Compound Interest in case of Conversion Period

$$\boxed{CI = P\left[\left(1+\frac{r}{c\times100}\right)^{t\times c} - 1\right]}$$

Or

$$\boxed{CI = P\left[(1+i)^{n} - 1\right]}$$

$$i = \frac{r}{100c},\ n = t\times c$$

- Accumulated Amount in case of Conversion period on calculator →

If. conversion semiannually, quarterly etc. and r will be given according to p.a interest rate. in que. than we can convert the rate in semiannually rate of Interest as-

$$\frac{r \rightarrow (\text{annual rate of Interest}}{c \rightarrow (\text{conversion period})}$$

than Let as assume $\frac{r}{c}$ as R%.

$$R\% = \frac{r}{c}..$$

on Calculator.-

$$\boxed{\begin{array}{c} A = P + \underbrace{R\% + R\% + R\%}_{n \text{ times}} \\ \downarrow \\ (n = t \times c) \end{array}}$$

- Compound Interest in Case of Conversion period on calculator →

considering R% the same as above

$CI = A - P.$

$$\boxed{\begin{array}{c} CI = P + \underbrace{R\% + R\% + R\%}_{n \text{ times}} - P. \\ (n \overset{\downarrow}{=} t \times c) \end{array}}$$

Note :- If conversion period is not given assume that it is 1.

- formula for difference between CI & SI in general

$$CI - SI = P\left[\left(1 + \frac{r}{100}\right)^t - 1\right] - P.r.t$$

$$\left\{ CI - SI = P\left[\left(1 + \frac{r}{100}\right)^t - 1 - rt\right] \right\}$$

Or.

you can separately calculate CI with the calculator trick as-

$$CI = P + \underbrace{R\% + R\% + R\%}_{n \text{ times}} - P.$$

and $SI = P \times t \times r\%$

and

than simply subtract.

CI with SI.

Permutation & Combination - SUN

Permutation & Combination (4-6 m)

Mathematical Principle of counting →

* If there are m ways to do one task & n ways to do another.

Agar dono ek sath karna toh → $m \times n$ ways to do both the tasks simultaneously.
(and case)

Agar dono mai se ek karna hai → $m + n$ ways.
(or case)

* Factorial ($\lfloor 7$ or $7!$) → $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$0! = 1 \quad 1! = 1$

as $n! = n \times \underbrace{(n-1) \times (n-2) \cdots \times 1}$

$(n-1)! = (n-1) \times (n-2) \cdots 1$

$\therefore \quad n! = n \times (n-1)!$

* Permutation - Selection with arrangement → order matters

* Combination - Selection without arrangement → order doesn't matter

* Permutation - $3 \times 2 \rightarrow {}^3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1}$

3 mai se 2 ko nikal ke arrange →

Calc. trick → ${}^nP_n$ → n se lehe n no. of terms multiply

NOTE → • ${}^nP_n = n!$ • ${}^nP_0 = 1$

• ${}^nP_1 = n$ • if $2! = (5-n)!$
then $2 = 5-n$

In Linear arrangement, for n things & n places → possible arrangements $= {}^nP_n = n!$

* When units can stick together → Ex- vowels in 'FAILURE'

Condition 1 → when order of vowels can change stiching.

Step 1 → Consider all units as one → $F \overset{①}{L}\ \overset{②}{R}\ \overset{③}{\underbrace{A I U E}_{\downarrow}}{}^{④}$

Step 2 → find the possible arrangement in these 4 then multiply it with the possible arrangement in ④

i.e. $^{4}P_{4} \times {}^{4}P_{4}$

Condition 2 → when order does not change

As there is only one possibility in the stiching unit.

So, the possible arrangement of the 1, 2, 3 & 4 i.e. $^{4}P_{4}$ with be the answer.

* Can't Stick together →

* Condn = Condition
* P = Possibilities.

For n units with 2 unit not together

Never come together = Normal P without any condition (n!) − Come together P (n−1)!(2!) ← opposite of condn given in q

NOTE → Always clean the most plot twisting condn

* Circular Permutation → For n things & n places

$P = (n-1)! \rightarrow 1 \times {}^{n-1}P_{n-1} \rightarrow$ Cuz it doesn't matter where the 1st person sit / 1st unit is placed

- For not together → we come together P as $1 \times {}^{n-2}P_{n-2}\ ({}^{2}P_{2})$ and Normal P as $(n-1)!$
  ↓
  −1 for 2 units stiching together

*• Agar units mai kai fark naho → $P = \frac{(\overset{\text{no. of unit}}{n}-1)!}{2}$
(In circular)

* Theorem 1 → no. of units → n , no. of obj can be taken by n → r
& one unit didn't take r, then
↳ it can be more

$$P = {}^{n-1}P_r$$

* Theorem 2 → no. of units → n , no. of obj can be taken by n → r
& ek obj. toh noga hi Let it be ex. mean x

$$P = \underset{{}^rP_1}{r} \times {}^{n-1}P_{r-1}$$

cuz ek fix hai

• $\sum_{r=1}^{9} r \cdot {}^rP_r = 1(1!) + 2(2!) \dots 9(9!)$
$= 10! - 1! = 10! - 1$

$$\sum_{r=1}^{n} r \cdot {}^rP_n = (n+1)! - 1$$

• Sum of n digits without repetitions — $(n-1)!$ (Sum of digits) (111.. n times) *

* Combination → ${}^nC_r = \frac{n!}{(n-r!)(r!)} = \frac{{}^nP_n}{{}^rP_r}$ $\underbrace{\overset{\text{Combination}}{\text{Choose}} \text{ \& Arrange}}_{\text{Permutation}}$

${}^nC_n = 1$, ${}^nC_0 = 1$, $n \geq r \geq 0$ always.

***** ${}^nC_r = {}^nC_{n-r}$

*****

$^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$

$^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$

* Permutation when → repetition

1) In n things, n_1 are alike of one kind, n_2 of one kind ....

$$\frac{n!}{n_1!\,n_2!}$$

Ex → MISSISSIPPI, here I, S, P are identical
I = 4, S = 4, P = 2

∴ Possible arrangements = $\frac{11!}{4!\,4!\,2!}$

2) ~~A~~ n things can be arranged in r places with repetition →

$$n^r$$

Ex → No. of p in which 6 letters be posted in 4 letter box → 6^4 ×
4^6 ✓ → ~~coz letters can be repeated~~

l_1 l_2 l_3 l_4 l_5 l_6
4 4 4 4 4 4 = 4^6

3) Combination of different things taking some or all of n things at a time → kabhi 10 mai se 1, 10 mai se 2, ... 10 mai se 10,

$\cancel{{}^nC_0} + {}^nC_1 + {}^nC_2 + {}^nC_3 \cdots {}^nC_n = 2^n - 1$ → 1 for $^nC_0$

this is $^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 \cdots {}^{10}C_{10} = 2^{10} - 1$

Sub. in RHS,

Kyuki na choose karne wala scene nai hai wrong no.

4) Combination of n things taken some or all at a time with similarity (combo of 1 & 3), n_1 things are alike, n_2 are alike, etc.

Yeh specific n_1 se kuch nai liya

$$(n_1+1)(n_2+1)(n_3+1)\underbrace{-1}$$

sab n_1, n_2 & n_3 se kuch bhi nai liya

$$= \frac{{}^{1}P_1}{1!} + \frac{{}^{2}P_2}{2!} \dots \frac{{}^{n_1}P_{n_1}}{n_1!}$$

cuz identical.

Ex- 10 donuts, 6 waffles & 8 Pastries, different ways you can take

$$(10+1)(6+1)(8+1)-1 = 692$$

$$\frac{{}^{1}P_1}{1!} + \frac{{}^{2}P_2}{2!} + \frac{{}^{3}P_3}{3!} + \frac{{}^{4}P_4}{4!} + \frac{{}^{5}P_5}{5!} + \frac{{}^{6}P_6}{6!}$$

$\Rightarrow 1+1+1+1+1+1 = 6$

5) Combination when r_1 from n_1 & r_2 from n_2 ... things are selected.

$${}^{n_1}C_{r_1} \times {}^{n_2}C_{r_2}$$

Ex→ Ways in which 9 can be divided into group of 2, 4, 3 things.

$${}^{9}C_2 \times {}^{7}C_4 \times {}^{3}C_3$$

Here n after every group division is decreasing.

NOTE → If koi shape wala Q hai toh sides ka khayal rakhna

Imp. Questions → Lecture 2 → Q2 & 3
" 3 → Q13
Lecture 4 → Q1 & 9
" 5 → Q5 & 8 & 21.

Sequence & Series — 3/4 marks

$\downarrow$
pattern

Ex :- $\underbrace{2, 3}_{+1}, \underbrace{4}_{+1}, 5, 6, 7, \dots$

a) Sequence of natural number (n) : $1, 2, 3, 4, 5, 6, \dots n$

b) Sequence of square of natural numbers (n^2) : $1^2, 2^2, \dots n^2$

c) Sequence of $(1/n)$: $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \frac{1}{n}$

d) Sequence of Odd Positive number : $1, 3, 5, 7, \dots (2n-1)$

e) Sequence of Even Positive numbers : $1, 4, 6, 8, \dots (2n)$

$\therefore$ where (n = place 1, 2, 3)

SERIES

$\downarrow$
Adding elements of sequence.

$\Sigma n = 1 + 2 + 3 + 4 + \dots$
$\Sigma n^2 = 1^2 + 2^2 + 3^2 + 4^2 + \dots$

Two Types :-

i) **Finite Series**
- countable

Ex : $1 + 2 + 3 + 4 + \dots + 100$

ii) **Infinite Series**
- uncountable

Ex : $1 + 2 + 3 + 4 + \dots$

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$S_n = u_1 + u_2 + u_3 + u_4 + \dots + u_n$

$$\boxed{S_n = \sum_{i=1}^{n} u_i}$$

ARITHMETIC PROGRESSION (AP)

↓

Common diff. b/w consecutive terms

Ex :- 2, 4, 6, 8, 10, ...

AP (d) = 4 − 2 = 6 − 4 = 8 − 6 = 10 − 8 = 2

★ n^{th} term of AP

i) If 1^{st} term = a, common diff = d, then

n^{th} term : $a_1, a_2, a_3, a_4 \dots a_n$ } AP

$$\begin{array}{ccccc} 1^{st} & 2^{nd} & 3^{rd} & 4^{th} & 5^{th} \\ 3, & 6, & 9, & 12, & 15 \end{array}$$

Ex :-

↓

$a_1 = a$ $\quad d = 6 - 3 = 9 - 6 = 3$

$a_n = a + (n-1)d$

$= 3 + (n-1)3$

$= 3 + 3n - 3$

$= 3n$

AP = $a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \dots a + (n-1)d$

★ 3 terms in AP = $(a-d), a, (a+d)$

$d = d$

★ 4 terms in AP = $(a-3d), (a-d), (a+d) (a+8d)$

$d = 2d$

★ 5 terms in AP = $(a-2d), (a-d), (a), (a+d), (a+2d)$

$d = d$

★ 6 terms in AP = $(a-5d), (a-3d), (a-d), (a+d), (a+3d), (a+5d)$

$d = 2d$

Sum = $\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$

$S_n = \frac{n}{2}(a+l)$ ↳ last term → a_n term

ARITHMETIC MEAN

Three numbers, a, b, c are in AP with diff 'd'.

$AM = \frac{a+c}{2} = b$

$\overset{T_1}{a}, \overset{T_2}{a+d}, \overset{T_3}{a+2d}$ $AM = \frac{(a)+(a+2d)}{2}$

$= \frac{2a+2d}{2} = a+d$

Three AM insertion :-

$\underset{a}{2}, \underset{a+d}{AM_1}, \underset{a+2d}{AM_2}, \underset{a+3d}{AM_3}, \underset{a+4d}{6}$ } 5 terms are in AP

* Sum of 1^{st} n natural numbers $= \frac{n(n+1)}{2}$

* Sum of squares of 1^{st} n natural no. $= \frac{n(n+1)(2n+1)}{6}$

* Sum of cubes $= \frac{n^2(n+1)^2}{4}$

$\boxed{t_n = S_n - S_{n-1}}$

GEOMETRIC PROGRESSION (GP)

↓

Consecutive terms having same ratio

Ex :- 2, 4, 8, 16, 32, ...

$$r = \frac{2}{4} = \frac{8}{4} = \frac{16}{8} = \frac{32}{16} = 2 = \frac{a_n}{a_{n-1}}$$

GP :- $a, ar, ar^2, ar^3, \dots, ar^{n-1}$

$$a_n = ar^{n-1}$$

☆ Sum of GP :-

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

i) When $r > 1$

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$

ii) When $r < 1$

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right)$$

iii) When $r = 1$

$$S_n = a \times n$$

☆ Sum of Infinite Geometric Series $(a + ar + \dots + \infty)$

i) When $r \geq 1$ $\quad S_\infty = \infty$

ii) When $-1 \leq r < 1$ $\quad S_\infty = \frac{a}{1-r}$

		Common difference
★ 3 terms in GP	$\frac{a}{r}, a, ar$	$r = r$
★ 4 terms in GP	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$	$r = r^2$
★ 5 terms in GP	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$	$r = r$
★ 6 terms in GP	$\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar^3, ar^5$	$r = r^2$

3 terms are in GP, then $b^2 = ac$] GEOMETRIC MEAN
a, b, c

Insertion of 3 Geometric Means :-

$$\frac{1}{16},\ GM_1,\ GM_2,\ GM_3,\ 16 \rightarrow ar^{(\text{no. of GMs} + 1)}$$

$$a \quad ar \quad ar^2 \quad ar^3,\ ar^4$$

★ Sum of $1 + 11 + 111 + 1111 + \dots = \frac{1}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$

$$= \frac{1}{81}\left[10(10^n - 1) - 9n\right]$$

★ Three terms a, b, c   AP $\Rightarrow 2b = a + c$

GP $\Rightarrow b^2 = ac$

Chapter - 7 Sets, Relations and Functions (3/4 Marks)

- Notes

If we consider a collection of objects given in such a way that it is possible to tell beyond that whether a given object is in the collection under consideration or not then such a collection of objects is called well defined collection of objects.

Sets

A set is a collection of well defined distinct objects.

- Sets are generally denoted by capital letters.
- The elements of sets are generally denoted by small letters.
- Each object of the set is called an element or member of the set.
- Examples -

(1) $A = \{a, e, i, o, u\}$
2) $B = \{1, 2, 3, 4, 5\}$

Date
Page

Representations of Sets

Roster or Braces form
or
Tabular form

Set Builder form
or
Algebraic method
or
Rule method
or
property method

1. Roster Form

It is the method in which we list all the elements separating them by commas and enclosing these in curly braces.

For eg - (1) $\{2, 4, 6, 8, 10\}$

(2) $\{1, 3, 5, 7, 9, 11, 13, 15\}$

- Note -

(a) In this method, the order of elements does not matter.

(b) while ~~th~~ writing the set in roster form all the elements are generally taken as distinct

2. Set - Builder Method

In this method we figure out the common

property which is there in every element of set and not there with any element outside the set.

For eg - (1) $A = \{x : x$ is a letter in the word SAMPURNA $\}$

(2) $B = \{x : x$ is an integer $x^2 \leq 9\}$

★.. Few important and widely used Sets

N - Set of all natural numbers

W - Set of whole numbers

Z - Set of integers

R - Set of real numbers

Z^+ - Set of positive integers

R^+ - Set of positive real numbers

Q - Set of rational numbers

Q^+ - Set of positive rational numbers.

★.. Some Special sets

(a) Finite and Infinite Sets

when the number of elements are countable it is called finite set.
For eg - {1, 4, 9, 16, 25}

when the number of elements are uncountable it is called infinite set.
For eg - {1, 2, 3, 4, 5 -------}

(b) Empty and Non-empty Sets

The sets which contains no elements are called Empty set / Null Set / Void Set. Usually denoted by { } or Φ
For eg - Set of prime numbers between 32 and 36.

The set which contains atleast one element is called non-empty set. For eg -
{1}, {2, 9}, {6, 5, 3, 8}

(c) Equal and Equivalent Set

Two sets A and B are said to be equal if every element of A is in B and every element of B is in A.

For eg. A = {1, 2, 4} and B = {1, 2, 4}
then A = B

Two sets are said to be equivalents if n(A) = n(B). For eg - {1, 2, 3} & {3, 9, 6}

Remark - Equal sets are equivalent but equivalent sets need not be equal.

(d) Singleton Set

The set containing only one element. Eg = {2}

(e) Universal Set

The set containing all the elements under consideration in a particular problem is called universal set.

(f) Disjoint Sets

When two sets have no element in common they are called disjoint sets.

★... Subsets

Set A will be the subset of B if every element of A is also an element of B or we can say $A \subset B$ if, whenever $a \in B$ then $a \in B$.

(a) Proper subset and Super Set

If $A \subset B$ but $B \not\subset A \Rightarrow A \neq B$
then A is a proper subset of B.
and B is the super set.
Remark - ϕ has no proper subset.

For eg - {3} is a proper subset of {2, 3, 5}

(b) Power set

The collection of all the possible subsets of a given set A is called the power set of A. It is denoted by P(A)

For eg - If $A = \{1, 2\}$
then, $P(A) = \{\{1\}, \{2\}, \{1, 2\}, \{\ \}\}$.

★... If a set has n elements then the number of subsets is 2^n.

★★... The number of proper subset is $2^n - 1$.

★★★... The number of distinct elements contained in finite set is called its cardinal number

- Subset of the set of Real Numbers

$N \subset Z \subset Q \subset R$

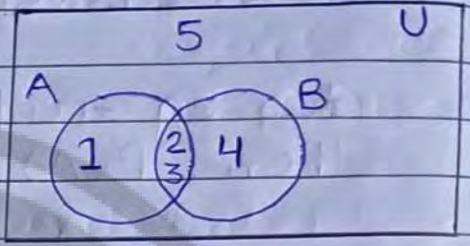
N - Set of natural no.s
Z - Set of integers
Q - Set of rational no.s
R - Set of real no.s

- Venn Diagram

These diagrams consists of rectangles and circles inside it. Universal set is a rectangle and other sets are circles.

For eg -

$U = \{1, 2, 3, 4, 5\}$
$A = \{1, 2, 3\}$
$B = \{2, 3, 4\}$



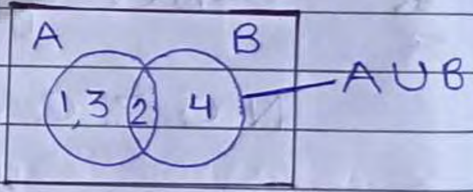
★... Operations on Sets

Union of Sets

$A \cup B = A$ or B

In set builder form = $A \cup B = \{x : x \in A$ or $x \in B\}$

For eg. $A = \{1, 2, 3\}$
$B = \{2, 4\}$
$A \cup B = \{1, 2, 3, 4\}$



2. Intersection of Sets

only common elements
$A \cap B = \{x : x \in A$ and $x \in B\}$

$A = \{1, 2, 3\}$
$B = \{2, 3, 4\}$

$A \cap B = \{2, 3\}$

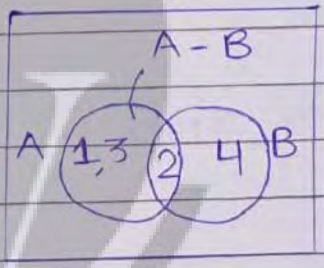
★★.. If $A \cap B = \phi$ the set is called the disjoint set.

3. Difference of Sets

In set builder form definition of A-B =

$A - B = \{x : x \in A \text{ and } x \notin B\}$

For eg. $A = \{1, 2, 3\}$
$B = \{2, 4\}$

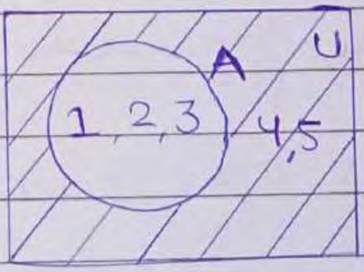


$A - B = \{1, 3\}$

4. Complement of Set

Complement of a set is denoted by A^c and A' and it includes all elements in universal set except of elements in A.

$U = \{1, 2, 3, 4, 5\}$
$A = \{1, 2, 3\}$
$B = \{4, 5\}$



Note -

(A) For two sets A and B -

Case 1 $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Case 2 $n(A \cup B) = n(A) + n(B)$ [if A & B are disjoint sets, ie $A \cap B = \phi$]

(B) For ~~two~~ three sets A, B and C -

Case 1 $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C]$

Case 2 $n(A \cup B \cup C) = n(A) + n(B) + n(C)$ [if A, B and C are disjoint sets]

(C) De Morgan's Law

(i) $(P \cup Q)' = P' \cap Q'$

(ii) $(P \cap Q)' = P' \cup Q'$

★★. Product Sets

(a) Ordered pair

Two elements a and b listed in specified order form an ordered pair denoted by (a, b).

(b) Cartesian Product of Sets

If A and B are two non empty sets, then the set of all ordered pairs (a, b) such that a belongs to A and b belongs to B is called the Cartesian product of A and B to be denoted by A × B

For eg. if $A = \{a, b\}$ and $B = \{p, q\}$
then, $A \times B = \{(a, p), (a, q), (b, p), (b, q)\}$

$B \times A = \{(p, a), (p, b), (q, a), (q, b)\}$

Points to be noted -

(a) If $A = \phi$ or $B = \phi$ then $A \times B = \phi$

(b) $n(A \times B) = n(A) \times n(B)$

(c) ordered pairs $(a, b) \neq (b, a)$

(d) Also, $A \times B \neq B \times A$

(e) But, $n(A \times B) = n(B \times A)$

Relations

Any subset of the product of A and B is called its subset.

If we have two non-void or null or empty A and B then the relation R from set A to Set B is represented by a R b where a is the set of elements belonging to A and b is the set of elements belonging to Set B.

Relation $\subseteq$ cartesian product

$R : A \to B \subseteq A \times B$

- Domain and Co-domain of Relation

For a relation from Set A to Set B ie a R b,

Domain (R) = $\{a : (a, b) \in R\}$
Codomain (R) = Set B
Range (R) = $\{b : (b, a) \in R\}$

- Types of Relation

1. Reflexive relation

If R contains all the ordered pairs of form (a, a) in $S \times S$ then R is called Reflexive relation
Relation = is equal to
In reflexive relation a is related to itself.

For eg - $A = \{1, 2, 3\}$
then $R = \{(1,1), (2,2), (3,3)\}$ is reflexive.

2. Symmetric Relation

If $(a,b) \in R$, $(b,a) \in R$ for every $a,b \in S$ the R is called the symmetric relation.

3. Transitive Relation

If $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$ for every $a, b, c \in S$ then R is called transitive relation.

Equivalence Relation

If a relation is reflexive, symmetric as well as transitive it is called an equivalence relation.

Functions

If we take two sets A and B, then relation f from set A to set B will be function only if every element of A has a unique image in B.

The element $f(x)$ of B is called the image of x while x is called the pre-image of $f(x)$

- Domain, Co-domain and Range of a Function

For a function from set A to set B ie $f(a)=b$ where $a \in A$ and $b \in B$ -

Domain = Set A
Co-Domain = Set B
Range $\leq$ Co-domain [Range is set of y, $y \in B$ & $y = f(x)$]

- Types of Functions

One-One or Injective Function.

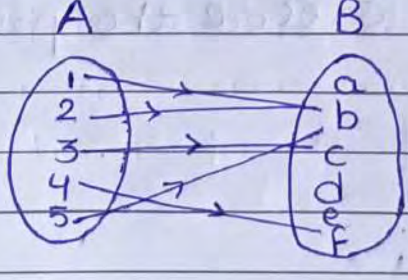
Let $f : A \rightarrow B$
If different elements in A have different ~~elements~~ images in B then f is said to be one-one or injective function or mapping.

Eg. $A = \{1, 2, 3\}$ $B = \{2, 4, 6\}$ $F : A \rightarrow B : f(x) = 2x$

2. Many one function

Functions for which we can match more than one element of the set A to the same element of set B are said to be many-one function

Eg.



Note -

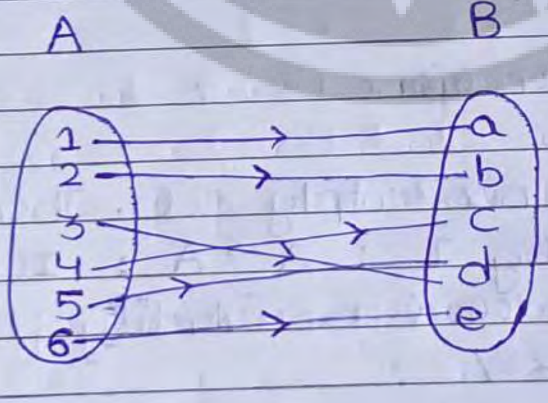
(a) If we have straight line than it is one-one function.

(b) If we have greater than sign from set A to set B then it is many-one function.

(c) If we have less than sign from set A to set B then relation is not a function.

3. Onto or Surjective functions

Let $f: A \rightarrow B$
If every element in B has atleast one pre image in A then f is said to be an onto function.
f is onto if and only if range of f = B

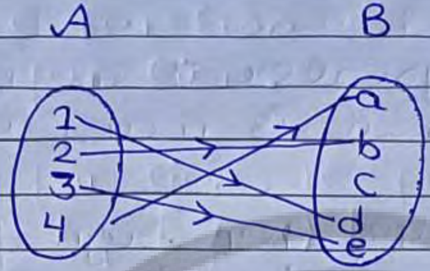


4. Into functions

Let $f: A \rightarrow B$

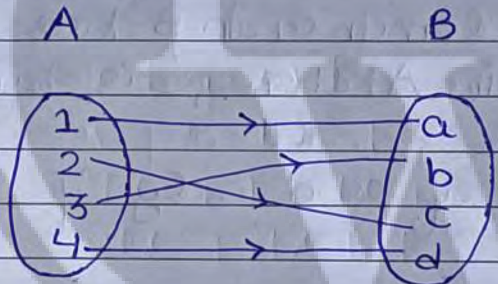
There exists even a single element in B having no pre-image in A then f is said to be an onto function.

Eg.



5. Bijective Function

A one and onto function is bijective.



6. Identity function

Let A be a non empty set. Then the function I defined by $I : A \to A : I(x) = x$ for all $x \in A$ is called an identity function on A.

$y = f(x) = x$

$y = x$

Eg. $A = \{1, 2, 3, 4\}$

$f = \{(1,1), (2,2), (3,3), (4,4)\}$

7. Constant Function

Let $f : A \to B$ defined in such a way that all elements in A have the same image in B then f is said to be a constant function.

$f(x) = C$

Eg. => $f(x) = 3$

$f : A \to A$

$A = \{1, 2, 3, 4\}$

$f = \{(1,3)(2,3), (3,3), (3,4)\}$

8 Equal Function

Two functions f and g are said to be equal if they have the same domain and they satisfy the condition $f(x) = g(x)$ for all x.

9. Inverse Function

Let y be an arbitrary element of B, so we may define a function f which is called as the inverse of f denoted by f^{-1} as follows-

If f be a one-one onto function from A to B
For $f(x) = y$, $f^{-1} : B \to A$

$f : A \to B$, $x \in A$, $y \in B$
$f(x) = y$, this is bijective function.
$f^{-1} = B \to A$ $\quad f^{-1}(y) = x$

10. Composite Function

A composite function is a function that depends on another function. It is created when one function is substituted into another function.

Let two functions be $f(x)$ and $g(x)$

$$fog = f[g(x)]$$

$$gof = g[f(x)]$$

Example - $f(x) = x^2+6$
$g(x) = 2x-1$

$$\begin{aligned}(fog)x &= f[g(x)] \\ &= (2x-1)^2+6 \\ &= 4x^2+1+4x+6 \\ &= 4x^2+4x+7\end{aligned}$$

$$\begin{aligned}(gof)x &= g[f(x)] \\ &= 2(x^2+6)-1 \\ &= 2x^2+12-1 \\ &= 2x^2+11\end{aligned}$$

★... Eg. of Inverse function

$f(x) = 2x$, f be a one-one onto function

$f'(x) = ?$ $\quad f^{-1}(y) = x = \frac{y}{2}$

$F(x) = 2x$

$f(x) = y$ $\quad f^{-1}(x) = \frac{x}{2}$

$y = 2x$

$x = y/2$