PPA

Maths Formula

1 – Ratio Proportion, Logarithm and Indices

*** Ratio :-**

A ratio is a comparison of the sizes of two or more quantities of the same kind by division.

e.g. $\frac{a}{b}$ Or a : b

a = 1st term of antecedent.

b = 2nd term of consequent.

*** Remarks :-**

1) Both terms can be multiplied of divided by same (non zero) numbers.
2) A ratio is expressed in lowest form.
3) The order of the terms in a ratio is important. **e.g.** 3 : 4 is not same as 4 : 3
4) Ratio exists only between quantities of same kind. **e.g.** There is no ratio between Age and Weight of children.
5) Quantities to be compared (by division) must be in the same unit. **e.g.**

*** Inverse Ratio :-**

One ratio is the inverse of other if their product is 1.

e.g. The inverse ratio of a : b is b : a.

i.e. $\frac{a}{b} \times \frac{b}{a} = 1$

*** Compounded Ratio :-**

The Compounded ratio of a : b & c : d is ac : bd.

*** Duplicate Ratio :-**

A ratio compounded itself is called "Duplicate Ratio".

e.g. $a^2 : b^2$ is duplicate ratio of a : b.
Similarly triplicate ratio of a :b is $a^3 : b^3$

*** Sub-Duplicate Ratio :-**

Sub duplicate ratio of a : b is

$\sqrt{a} : \sqrt{b}$

*** Continued Ratio :-**

It is the comparison between the magnitude of three or more quantities of the same kind.

e.g. The continued ratio of Rs. 200, Rs. 400, Rs. 600 is 200:400:600 it means 1:2:3.

*** Proportion :-**

An equality of 2 ratios is called **"Proportion"**.

i.e. $\frac{a}{b} = \frac{c}{d}$ or a : b :: c:d

The number a, b, c and d are in proportion.

a and d are called extremes or extreme terms.

b and c are called means or middle terms.

Similarly, if 3 quantities a, b, c of the same kind are said to be in continuous proportion.

If a : b = b : c

i.e. $\frac{a}{b} = \frac{b}{c} \therefore b^2 = ac \therefore \sqrt{ac}$ also, when three or more (x, y, z, p, q, r) quantities are in continued proportion, then

$\frac{x}{y} = \frac{y}{z} = \frac{z}{p} = \frac{p}{q} = \frac{q}{r}$

*** Properties of Proportion :-**

1) If a : b = c : d then ad = bc
2) If a : b = c : d
then b : a = d : c (Invertendo)
3) If a : b = c : d then
a : c = b : d (Alternendo)
4) If a : b = c : d then
$\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)
5) If a : b = c : d then
$\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)
6) If a : b = c : d then
$\frac{a+b}{a-b} = \frac{c+d}{c+d}$ (C and D)
7) If a : b = c : d = e : f = ... then
Each ratio is equal to
$\frac{(a+c+e+\ldots)}{(b+d+f+\ldots)}$ .... (Addendo)
8) If a : b = c : d = e : f = ... then
$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{(a-c-e-\ldots)}{(b-d-f-\ldots)}$ ....
(Subtrahendo)

*** Indices :-**

4 x 4 x 4 x 4 x 4 = 4^5

In above case the number which multiplies i.e. 4 is called base and the number of times it multiplied i.e. 5 is called "index of power".

Also, $\sqrt[r]{a^1} = a^{1/r}$

*** Laws of Indices :-**

PPA

Maths Formula

1) $a^m \times a^n = a^{m+n}$
   e.g. $2^3 \times 2^2 = 2^{3+2} = 2^5$
2) $a^m \div a^n = a^{m-n}$
   e.g. $2^3 \div 2^2 = 2^{3-2} = 2^1 = 2$
3) $(a^m)^n = a^{m \times n}$
   e.g. $(2^3)^2 = 2^{3\times2} = 2^6$
4) $a^0 = 1$
   e.g. $2^0 = 1$
5) $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$
   e.g. $2^{-3} = \frac{1}{2^3}$ and $\frac{1}{2^{-3}} = 2^3$
6) If $a^x = a^y$ then $x = y$
7) If $x^a = y^a$ then $x = y$
8) $\sqrt[m]{a} = a^{1/m}$
   e.g. $\sqrt{4} = 4^{1/2}, \sqrt[3]{7} = (7)^{\frac{1}{3}}$

* Logarithm :-

If $a^x = n$, Then, x is said to be the logarithm of the number *n* to the base a.

i.e. $\log_a n = x$

e.g. i) $2^4 = 16 \Rightarrow \log_2 16 = 4$

ii) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$

i.e. Logarithm of 1000 to the base 10 is 3.

Note: Remember "Base will be base".

* Properties of Logarithm :-

1) $\log_a mn = \log_a m + \log_a n$
   e.g. $\log(2 \times 3) = \log 2 + \log 3$
2) $\log_a (m/n) = \log_a m = \log_a n$
   e.g. $\log 3/2 = \log 3 - \log 2$
3) $\log_a m^n = n \log_a m$
   e.g. $\log 3^2 = 2\log 3$
4) $\log_a a = 1$
   e.g. $\log_5 5 = 1$
5) $\log_a 1 = 0 \because a^0 = 1$
6) $\log_b a \times \log_a b = 1$
   e.g. $\log_3 2 \times \log_2 3 = 1$
7) $\log_b a \times \log_c b = \log_c a$
   e.g. $\log_3 2 \times \log_5 3 = \log_5 2$
8) $\log_b a = \frac{\log_c a}{\log_c b}$
   e.g. $\log_3 2 = \frac{\log 2}{\log 3}$

∗ How to find logarithm of a number using log tables:

The logarithm of a number consists of 2 part, the whole part or integral part is called characteristic and decimal part is called mantissa.

i.e. Characteristic – Whole part
Mantissa - Decimal part

* Characteristics :-

1) If number is greater than 1 then characteristic is +ve and it is **one less than** the number of digit to **the left of** the decimal point.

e.g.

Number	Characteristic
37	1
4623	3
6.2	0

2) If number is less than 1 then characteristic is –ve and it is **1 more than** the no of zeros on the right immediately after the decimal point

e.g.

Number	Characteristic
.8	-1 or $\bar{1}$
.07	-2 or $\bar{2}$
.00507	-3 or $\bar{3}$
.000670	-4 or $\bar{4}$

* Mantissa :-

It is the fractional part of the logarithm of a given number. Mantissa is always a +ve quantity.

e.g.

Number	Mantissa	Logarithm
Log 4597	=(0.6625)	3.6625
Log 459.7	=(0.6625)	2.6625
Log 45.97	=(0.6625)	1.6625
Log 4.597	=(0.6625)	0.6625
Log 0.4597	=(0.6625)	$\bar{1}.6625$
Log 0.497	=(0.6625)	$\bar{2}.6625$

* Antilogarithm :-

If χ is the logarithm of a given number *n* then '*n*' is called the Antilogarithm

i.e. if $\log_a n = \chi$ then n = antilog(x)

e.g. If log 61720 = 4.7904 then,
antilog 4.7904 = 61720.

2 – Equations

*** Equation :-**

It is defined to be a mathematical statement of equality.

e.g. $x^2 + 2x + 3 = 0$

$$\frac{x+2}{3} + \frac{x+3}{2} = 3$$

*** Conditional Equation :-**

If the equality is true for certain value of the variable involved, then eq^n is called conditional eq^n

e.g. $\frac{x+2}{3} + \frac{x+3}{2} = 3$ holds true only for x = 1

*** Identity :-**

If the equality is true for all value of the variable involved, the eq^n is called **identity.**

e.g. $\frac{x+2}{3} + \frac{x+3}{2} = \frac{5x+13}{6}$ is an identity.

The value of variable which satisfies the equation is called **solution** of the eq^n or **root** of eq^n.

*** Types of eq^n :-**

1) **Linear or simple eq^n** highest power 1
   e.g. $8x + 17(x-3) = 4(4x-9) + 12$
2) **Quadratic eq^n:** highest power 2
   e.g. $3x^2 + 5x + 6 = 0$
3) **Cubic eq^n:** highest power 3
   e.g. $4x^3 + 3x^2 + x – 7 = 1$
4) **Simultaneous eq^n:** 2 or more linear eq^n involving 2 or more variable.
   e.g. $x + 2y = 1; 2x + 3y = 2$

*** Simple equation :-**

Simple eq^n in 1 unknown 'x' is in the form of $\sigma_x + b = 0$ where a, b are constant and $a \neq 0$

There eq^ns can be solved by transposing the variable on 1 side and the constant on the other side.

*** Note :-**

From exam point of view solve these types of eq^ns by trial and error method

i.e. substituting options in given eq^n and balancing L.H.S and R.H.S.

Ex. -7 x + 1 = 5 – 3x will be satisfied for x equal to a) 2 b) -1 c) 1 d) nod.

Solution :- 1) Regular Method.

$-7x + 1 = 5 – 3x$

$\therefore -7x + 3x = 5 – 1$

$\therefore -4x = 4$

$\therefore x = -1$ Option(b)

2) Trial and Error Method

1) For option (a).

L.H.S. = $-7x + 1 = - 7(2) + 1$
$= -14 + 1 = -1$..(1)

R.H.S. = $5 – 3x = 5 – 3(2)$
$= 5 – 6 = -1$…..(2)

$-13 \neq -1$

$\therefore$a is not correct option

2) For option (b) i.e. (-1)

$-7x + 1 = 5 – 3x$

$-7(-1) + 1 = 5 – 3 (-1)$

$7 + 1 = 5 + 3$

$8 = 8$

$\therefore$b is correct option.

*** Simultaneous linear equation is 2 unknown :-**

General form: $ax + by + c = 0$

*** Method of Solution :-**

1) Elimination method :-

In this method, eliminate 1 variable by

i) Making their coefficient equal.
ii) Adding or substracting the eq^n depending upon their sign.
iii) Solving for the remaining variable.

e.g. Solve :-

$2x + 5y = 9$ (1)

$3x – y = 5$ (2)

Step 1 :-

Making their coe. Equal.

$2x + 5y = 9$ (1)

$3x – y = 5$ (2)

Multiply eq^n (2) by 5

$\therefore$ $5(3x – y = 5)$

$15x – 5y = 25$ (3)

Step 2 :-

Addind eq^n (1) and (3)

$2x + 5y = 9$

$15x – 5y = 25$

$17x = 34$

$\boxed{x = 2}$

Step 3 :-

Sub x = 2 in eq^n (2)

$3x – y = 5$

$3(2) – y = 5$

$6 – 5 = 4$

$\boxed{y = 1}$

2) **Cross-Multiplication Method :-**

If the eq^n are
a1 x + b1 y + c1 = 0
a2 x + b2 y + c2 = 0   Then

$$\frac{x}{(b_1c_2 - b_2c_1)} = \frac{y}{(a_1c_2 - a_2c_1)} = \frac{1}{(a_1b_2 - a_2b_1)}$$

And solve for x and y.

e.g. 2 x+5y = 9 and 3x – y = 5

Solution :- Convert them in std form

e.g. 2x + 5y – 9 = 0
3x – y – 5 = 0

Then ,

$$\frac{x}{(5)(-5)-(-9)(-1)} = \frac{-y}{(2)(-5)-(3)(-9)} = \frac{1}{(2)(-1)-(5)(3)}$$

$$\therefore \frac{x}{-25-9} = \frac{-y}{-10+27} = \frac{1}{-2+15}$$

$$\therefore \frac{x}{-34} = \frac{-y}{17} = \frac{1}{-17}$$

$$\therefore \frac{x}{-34} = \frac{1}{-17} \text{ and } \frac{-y}{17} = \frac{1}{-17}$$

$$x = \frac{-34}{-17} \text{ and } -y = \frac{17}{-17}$$

$$x = 2 \qquad y = 1$$

* **Quadratic Equation :-**

General form: $ax^2 + bx + c = 0$, x is called variable and the value of variable is called as root or solution of the eq^n.

Quadratic eq^n has 2 roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Let 1 root be α and another be β

$$\therefore \alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Also,

$$\alpha + \beta = -\frac{b}{a}$$

i.e. Sum of the roots = $-\frac{b}{a}$

$$\alpha.\beta = \frac{c}{a}$$

i.e. Product of the roots = $\frac{c}{a}$

* **Construction of quadratic eq^n :-**

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 - \frac{-b}{a}x + \frac{c}{a} = 0$$

i.e.
x^2 – (sum of the roots)x + (product of roots) = 0

* **Nature of Roots :- x =** $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Value	Nature of root
1) $b^2 - 4ac = 0$	Real and equal
2) $b^2 - 4ac > 0$	Real and unequal
3) $b^2 - 4ac < 0$	Imaginary
4) $b^2 - 4ac$ is perfect square	Real, rational and unequal
5) $b^2 - 4ac$ is not a perfect square	Real, Irrational and unequal

Ex. Examine the nature of the root
1) $x^2 - 8x + 16 = 0$
2) $3x^2 - 8x + 4 = 0$
3) $5x^2 - 4x + 2 = 0$
4) $2x^2 - 6x - 3 = 0$

Solution :-

1) a = 1, b = -8, c = 16
$\therefore b^2 - 4ac = (-8)^2 - 4(1)(16) = 0$
The roots are real and equal.

2) a = 3, b = -8, c = 4
$\therefore b^2 - 4ac = (-8)^2 - 4(3)(4) = 16 > 0$ and p.s.
The root are real, rational, unequal

3) a = 5, b = -4, c = 2
$b^2 - 4ac = (-4)^2 - 4 \times 5 \times 2 = -24 < 0$
Roots are imaginary and unequal

4) a = 2, b = -6, c = -3
$b^2 - 4ac = (-6)^2 - 4(2)(-3) = 60 > 0$ But not a p.s.
$\therefore$ **Roots are irrational and unequal.**

* **Solution of cubic eq^n :-**

For this type use trial and error method

i.e. substitute option in question and check.

Note :- Also, multiplication of roots = $-\frac{d}{a}$

* **Application of equations in co-ordinate geometry.**

1) Distance of a point = $\sqrt{x^2 - y^2}$ form origin p(x, y)

2) Distance between 2 points P(x,y) $Q(x_2y_2)$

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Maths Formula

*** Equations of straight line :-**

1) Slope intercept form
$y = mx + c$
Where, $m = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$
c = y intercept
2) Slope – point form
$(y - y_1) = m(x - x_1)$
3) Intercept form
$\frac{x}{a} + \frac{y}{b} = 1$

Note :-

1) If 2 lines are parallel then their slopes are equal.
2) If 2 lines are perpendicular then product of their slope = -1
3) If $ax + by + c = 0$ is eq^n of line. Then
i) eq^n of parallel line is $ax + by + k = 0$
ii) eq^n of perpendicular line is $bx - ay + k = 0$
4) If 2 lines interest at (x_1, y_1) then (x_1, y_1) satisfies the eq^n of line.
5) The eq^n of line passing through at (x_1, y_1) & (x_2, y_2) is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$
6) If, (x_1, y_1), (x_2, y_2), (x_3, y_3) are collinear then. $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

*** Note :-**

1) If 1 roots is reciprocal of the other them $c = a$
2) If one root is equal to other root but opp. In sign then $b = 0$
3) Irrational roots always occur in parts. i.e. $\{m + \sqrt{n}, m - \sqrt{n}\}$
4) If α is common root of the eq^n
$a1x^2 + b_1x + c_1 = 0$
$a2x^2 + b_2x + c_2 = 0$
Then,
$\alpha = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} = \frac{b_1c_2 - b_2c_1}{c_1a_2 - c_2a_1}$
And the condition is,
$(a_1b_2 - a_2b_1)\ (b_1c_2 - b_2c_1) = (c_1a_2 - ((2a_1)^2$

3 – Inequalities

*** <u>Inequalities</u> :-**

Inequalities are statements where 2 quantities are unequal but a relationship exist between them.

e.g. $3x + y < 6$; $x \geq$

$x - y \geq 2$; $y \leq \frac{x}{2}$

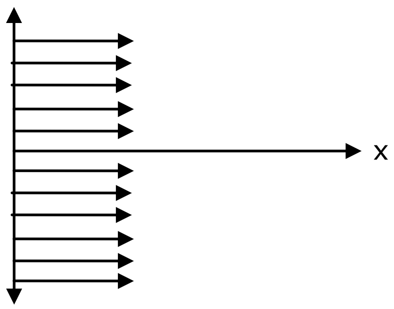
*** <u>Method of drawing the graph of inequality</u> :-**

Let the equation be $ax + by \geq c$

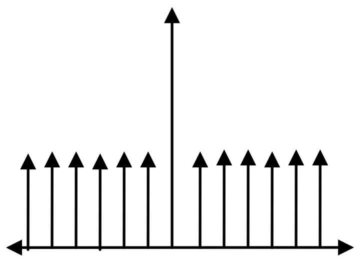
1) Replace the sing $\geq$ by equality i.e. take $ax + by = c$
2) Draw the graph of $ax + by = c$ which will be a straight line.
3) For the sign $\geq$ or $\leq$ the points on the line are included, and a thick line should be drawn
4) The line divides the plane is 2 regions. Now to identify which region satisfies the inequation plot any point. If this point satisfies the inequation, then the region containing the plotted point will be the desired region.

Ex. Draw the graph of following inequalities.

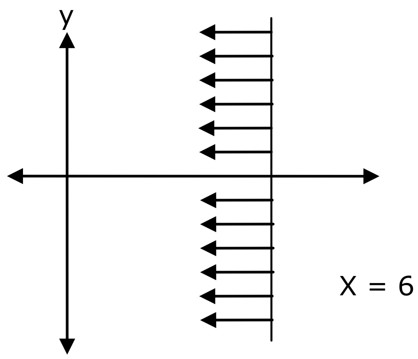
1) $x \geq 0$



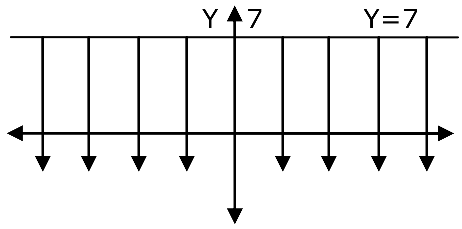
2) $y \geq 0$



3) $x \leq 6$, $x = 6$



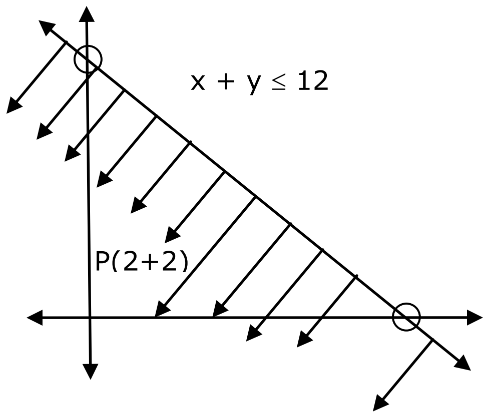
4) $x \leq 7$ **Solution :-**



5) $x + y \leq 12$

$x + y = 12$

x	0	12
y	12	0



4 – Simple and Compound Interest, Annuity

*** Interest :-**

It is the price paid by a borrow for the user of lenders money.

*** Why is Interest Paid :-**

1) Time value of money.
2) Opportunity cost.
3) Inflation.
4) Liquidity Preference.
5) Risk Factor.

*** Principle :-** It is the initial value of lending.

*** Rate of Interest :-**

The rate at which the interest is charged for a defined length of time for use of principle.

It is generally expressed in p.c.p.a.

*** Accumulated Amount (Balance) :-**

It is the final value of an investment. i.e. sum of the principle and interest earned.

*** Types of Interest :-**

1) **Simple Interest**
2) **Compound Interest**

1) Simple Interest :-

It is the interest computed on the principle for the entire period of borrowing.

It is calculated on the outstanding principle balance and **not on interest previously earned**.

As the interest is proportional to the money we borrow, period of time for which we keep the money and the rate of interest.

The simple interest can be computed as,

$$I = P \times I \times t$$

Where,

I = Amount of interest.
P = Principle.
i = Rate of Interest in decimal.
t = Time period in Years.

Also,

$$A = P + I$$
$$= P + Pit$$
$$A = P(1 + it)$$

Where,

A = Accumulated Amount

Ex. 1. How much interest will be earned on Rs. 2000 at 6% simple interest for 2 years?

Solution :-Given data

P = Rs. 2000
i = 6% = 0. 06
t = 2 years & To find **I**

Now, $I = P \times i \times t$

$= 2000 \times 0.06 \times 2$

$=$ **Rs. 240**

Ex. 2: Sachin deposited Rs. 100000 in his bank for 2 years at simple interest rate at 6%. How much would be the final value of deposited?

Solution: Given data

P = Rs. 100000
i = 6% = 0.06
t = 2 years

Now, **A = P(1 + it)**

$= 100000(1 + 0.06 \times 2)$

A = Rs. 112000

Ex. 3: Rahul invested Rs. 70000 in a bank at the rate of 6.5% pa simple interest rate. He received Rs. 85925 after the end of the term. Find the period for which sum was invested by Rahul

Solution: Given data

P = Rs. 70000
i = 6.5% pa
A = 85925

To find, t = ?

We know,

A = P (1 + it)

$85925 = 70000(1 + it)$

$$\frac{85925}{70000} = 1 + \frac{6.5}{100} \times t$$

t = 3.5 Years.

Ex. 4: Kapil deposited some amount in bank for 7½ years at the rate of 6% p.a. Simple interest. Kapil received Rs. 1,01,500 at the end of the term. Compute initial deposit of Kapil.

Solution: Given data

$t = 7\frac{1}{2}$ years $= \frac{15}{2}$ years
i = 6% = 0.06
A = Rs. 1,01,500

To find, P = ?

We know,

A = P(1 + it)

$$1{,}01{,}500 = P\left(1 + 0.06 \times \frac{15}{2}\right)$$

$\therefore$ **P = 70,000**

Ex. 5: A sum of Rs. 46,875 was lent out at simple interest and at the end of 1 year 8 months the total amount was Rs. 50,000. Find the rate of interest percent per annum.

Solution: Given data

P = Rs. 46,875

t = 1 year 8 months

8 Months = $\frac{8}{12}$ year

$\therefore$ t = (1 + $\frac{8}{12}$) year

= $\frac{20}{12}$ year

We know,

A = P(1 + it)

50,000 = 46,875 (1 + 1 x $\frac{20}{12}$)

i = 0.04

i = 4% pa

2) Compound Interest :-

If the interest is calculated on the principle and the **amount of previously earned interest,** then it is called as **"Compound Interest"**.

Now, $A_n = P(1+i)^n$ and

$I = P[(1+i)^n - 1]$

Where, An = Accrued Amount

P = Principle

$I = \frac{\text{Annual rate of interest}}{\text{No. of conversion periods / year}}$

n = No. of conversion periods

*** Conversion Period**:-

The period at the end of which the interest is compounded is called "Conversion Period".

Conversion Period	Description i.e. Compounded	No. of conv. Period
1 Day	Daily	365
1 Month	Monthly	12
3 Month	Quarterly	4
6 Month	Semi-Annually	2
1 Year	Annually	1

Ex. Rs. 2000 is invested at annual rate of interest of 10%. What is the amount after 2 years if compounding is done?

a) Annually b) Semi-Annually

c) Quarterly d) Monthly

Solution:- Given data

P = Rs. 2000

i = 10%

T = 2 Years.

a) **For annual compounding**

P = 2000, i = $\frac{10}{100}$, n = 2

$\therefore$ $A = P(1 + i)^n$

$\therefore$ $A = 2000\,(1+0.1)^2$

$\therefore$ A = **Rs. 2420**

b) **For semi-annual compounding**

P = 2000, i = $\frac{0.10}{100}$ =0.05, n = 4

$\therefore$ $A = P(1 + i)^n$

$\therefore$ $A = 2000\,(1+0.05)^4$

$\therefore$ A = **Rs. 2431**

c) **For Quarterly compounding**

P=2000, i=$\frac{0.1}{4}$=0.025, n=4x2=8

$A = P(1 + i)^n$

$A = 2000\,(1+0.025)^8$

A = **Rs. 2436.80**

d) **For Monthly Compounding**

P=2000, i=$\frac{0.1}{12}$=0.00833, n=2x12 = 24

$A = P(1 + i)^n$

$A = 2000\,(1+0.00833)^{24}$

A = **Rs. 2440.58**

Ex. A certain sum invested at 4% per annum compounded Semi-Annually amounts to Rs. 78030 at the end of 1 year. Find the sum.

Solution:- Given data

i = $\frac{0.04}{2}$ = 0.02

no of conversion period = 2 per year.

t = 1 year

n = 1 x 2 = 2

A = Rs. 78030

As, $A = P(1+i)^n$

$78030 = P(1+0.02)^2$

P = 75000

Ex. What annual rate of interest compounded annually doubles an investment in 7 years given that $2^{\frac{1}{7}}$ =1.10409

A = 2p, t = 7 years

n = 1 x 7 = 7

Now, $A = P(1+i)^n$

$2p = p(1+i)^7$

$2 = (1+i)^7$

$2^{\frac{1}{7}} = 1+i$

1.104090 – 1 = i

i = 0.10409

i = 10.41%

Ex. In what time will Rs. 8000 amount to Rs. 8820 @ 10% per annum interest compounded half yearly

Solution: Given data

No. of conversion period per year = 2

i = 0.1/2 = 0.05

Now, $A = P(1+i)^n$

$\therefore$ $8820 = 8000(1+i)^n$

$\therefore$ $\frac{8820}{8000} = (1.05)^n$

$\therefore$ $(1.05)2 = (1.05)^n$

$n = 2$

We know,

$n = t \times 2$

$\therefore$ $t = \frac{n}{2}$

$\therefore$ $t = \frac{2}{2}$

$\therefore$ $t =$ **1 year**

*** Effective Rate of interest(E) :-**

It is the equivalent annual rate of interest compounded annually if interest is compounded more than once a year.

$$\mathbf{E = (1+i)^n - 1}$$

Where, E = Effective interest rate

i = Actual interest rate in decimal

n = no of conversion period.

Ex. Find the effective rate of interest if an amount of Rs. 20,000 is deposited in a bank for 1 year at the rate of 8% p.a. Compounded Semi – annually.

Solution: Given data

P = Rs. 20,000

No. of conversion period = 2 per year.

$i = \frac{0.08}{2} = 0.04$

$n = 2 \times 1 = 2$

Now,

$E = (1+i)^n - 1$

$= (1+0.04)^2 - 1$

= **0.0816 or 8.06%**

Ex. Which is better investment 3% per year compounded monthly or 3.2 per year simple interest given that (1+0.0025)12 = 1.0304.

Solution:- Given data

No. of conversion period = 12 per year

$i = \frac{0.03}{12} = 0.0025$

$n = 12 \times 1 = 12$

Now,

$E = (1+i)^n - 1$

$= (1+0.0025)^{12} - 1$

$= 0.0304$

= **3.04%**

As effective rate of interest being less than 3.2%, the simple interest 3.2% per year is the better investment.

*** Annuity :-**

It is the sequence of periodic payments or receipts regularly over a specified period of time.

*** Perpetuity :-**

If the sequence of periodic payments or receipts taken place forever, it is called as perpetuity.

*** Condition to be Called Annuity :-**

1) Amount paid received must be constant over the period of annuity.
2) Time interval between 2 consecutive payments or receipts must be the same.

*** Types of Annuity :-**

1) Annuity Regular
2) Annuity due or Immediate

*** Annuity Regular :-**

In this, the first payment or receipt taken place at the end of 1st period **e.g. Rent of house**.

*** Annuity due or Immediate :-**

In this, the first payment or receipt is made today. **i.e.** At the beginning of the annuity.

e.g. Premium of LIC

*** Future Value :-**

It is the cash value of an investment at some time in the future i.e. it is tomorrow's value of today's money compounded at the rate of interest.

It can be calculated as,

$$\mathbf{F = C.F.(1+i)^n}$$

Where,

F = Future value

Cf = Cash flow

i = Interest rate

n = No of conversion period.

Ex. You invest Rs. 3000 in 2 year investment that pays you 12% per annum. Calculate the future value of the investment.

Solution: Given data

Cf = Rs. 3000

n = 2

i = 12% = 0.12

we know,

$f = cf(1+i)^n$

$= 3000(1+0.12)^2$

F = Rs. 3763.20

*** Future value of annuity regular.**

$$A(n, i) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Where,

A (n, i) = future value of annuity at the end of n year.
I = rate of interest in decimal

*** Future value of Annuity due or immediate:**
F = A (n, i) x (1 + i)

Ex. Find the future value of an annuity of Rs. 500 made annually for 7 year at interest rate of 14% compounded annually Given that $(1.14)^7$ = 2.5023
Solution: Given data
A = 500
N = 7
i = 14%
= 0.14

$$\therefore A(7,0.14) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 500\left[\frac{(1+0.14)^7 - 1}{0.14}\right]$$

A (7, 0.14) = Rs. 5365.35

Ex. Z invests Rs. 10,000 every year starting from today for next 10 years. suppose interest rest is 8% p.a. compound annually. Calculate future value of annuity. GT(1+0.08)10 = 2.158925.
Solution: Given data:
A = 10,000
i = 8% = 0.08
N = 10 Years

Now,

$$A(n, i) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$A(10,0.08)=10,000\left[\frac{(1+0.08)^{10} - 1}{0.08}\right]$$

A(10, 0.08) = Rs. 144865.625
But this is **e.g.** of Annuity due
$\therefore F_{AD} = A(n, i) \times (1+i)$
= (144865) x (1 + 0.08)
= **Rs. 156454.875**

*** <u>Present Value</u> :-**

It is today value of tomorrows money discounted at the interest rate.

$$P = \frac{A_n}{(1+i)^n}$$

Ex. What is the present value of Rs.1 to be received after 2 years compounded annually @ 10% interest rate.
Solution: Given data
N = 2 years
i = 0.1
An = Rs. 1

$$\text{Now, } P = \frac{A_n}{(1+i)^n}$$

$$= \frac{1}{(1+0.1)^2}$$

P = 0.83

*** <u>Present Value of Annuity regular</u> :-**

$$V = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

V = A. P(n, i) ________ (1)
Where,
V = present value of annuity
A = Annuity
From eq^n ______ (1)

$$A = \frac{V}{P(n,i)}$$

Ex. Sachin's mom promise him to give Rs. 10000 on very 31st Dec. for the next five year. Suppose today is 1st Jan.

What is the present value of this annuity if the interest rate is 10%.
Solution:

Year end	Gift amount	p.v. $A_n/(1+i)^n$
i	10,000	9090.91
ii	10,000	8264.46
iii	10,000	7513.15
iv	10,000	6830.13
v	10,000	6209.21
	P.V. = 37907.86	

OR

$$V = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 10,000\left[\frac{(1+0.1)^5 - 1}{1.1(1+0.1)^5}\right]$$

V = 37907.86

*** <u>Present Value of annuity due</u> :-**

<u>Steps</u>

i) Compute the present value of annuity as if it were a annuity regular for 1 period short.
ii) Add initial cash to the step 1.

Ex. In the previous example if Sachin's mom start giving the money from today i.e. 1st January what is the present value of annuity?
Solution:
Step I:

V $= A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

$= 10{,}000\left[\frac{(1+0.1)^4 - 1}{1.1(1+0.1)^4}\right]$

V = 31698.70

Step II: Add initial cash to step I

= 31698.70+10,000

P.V = Rs. 41698.70

*** <u>Sinking Fund</u> :-**

It is the fund credited for a specified purpose by Annuity at a specified interest rate.

It can be calculated by the formula of future value of annuity regular.

Ex. How much amount is required to be invested very year so as to accumulate Rs. 3,00,000 at the end of 10 years.

If interest is computed annually @ 10%

Solution: Given data

A(n,i) = 300,000

I = 10% = 0.1

n = 10

$A(n, i) = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

$300{,}000 = A\left[\frac{(1+0.1)^{10} - 1}{0.1}\right]$

A = Rs. 18823.62

*** <u>Notes</u> :-**

Difference between C.I. and S.I.

i) For 2 years = $p.i^2$

ii) For 3 years = $P \times i^2 (3 + i)$

<u>List of formula</u> :-

1) For simple interest

S.I. = p.i.t.

A = p(1+i t)

2) For compound interest

$A_n = p(1 + i)^n$

3) Effective rate of interest

$E = (1 + i)^n - 1$

4) Future value (f)

$F = C.F.\ (1 + i)^n$

5) Future value for

i) Annuity Regular

$A(n, i) = A\left[\frac{(1+i)^n - 1}{i}\right]$

ii) Annuity due

$F_{AD} = A(n, i) \times (1 + i)$

6) Present Value (p)

$P = \frac{A_n}{(1+i)^n}$

7) Present Value for

i) Annuity Regular

$V = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

ii) Annuity due

Present value of annuity regular for 1 period short + initial cash.

Maths Formula

5 – Permutation and Combination

* Permutation and Combination :-

A) Factorial Notation :-

1) 0! Or $\lfloor 0$ = 1
2) n! = n (n-1) (n-2)........2 x 1
   = n (n-1) (n-2)!
   = product of n to 1 natural number.

B) Fundamental Principles of Counting :-

1) AND $\rightarrow$ Multiplication
2) OR $\rightarrow$ Addition

C) Permutations

Arrangement of **r** objects out of **n** objects where **order of arrangement** is important

i.e. ^nP_r = n ** t

*** Types of Permutation :-**

1) Linear or row or straight line arrangement.

a) **For non-repetition :**

$$^nP_r = \frac{n!}{(n-1)!}$$

= **n (n-1) (n-2).......... (n-r+1)!** Total **r** factors

e.g.

$$^{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

= 10 x 9 x 8 x 7 ....4 Factors.

*** Remember :-**

1) $n \geq 0$
2) $n \geq r$

b) For Repetition Arrangement :-

$$^np_r = n^r$$

c) For no of permutation of n objects when p objects of 1st kind q object of 2nd kind, r object of 3rd kind and the rest are different then

$$^np_n = \frac{n!}{p! \times q! \times r!}$$

d) When particular object is always **included** in each arrangement

$$.r.\,^{n-1}p_{r-1}$$

e) When particular object is **never included** in each arrangement

$$= {}^{n-1}p_r$$

f) No. of possible arrangement in which **'n'** boys and **'n'** girls **
**

g) The No. of permutations of n different object taken al at a time when **2 Specified objects always come together** = 2(n-1)!

h) The No. of permutations of 'n' different objects taken all at a time. when **2 specified never comes together.**

= (n-2) (n-1)!

2) Circular Arrangements :-

a) The No. of circular permutation of **'n'** different object = (n-1)!

b) If clockwise and anticlockwise orders are not distinguish **i.e.** (same) then,

The No. of arrangement = $\frac{1}{2}$ (n -1)!

e.g. In necklace or garland

c) The No. of ways in which 'n' things of which P are same, can be arrange in circular order.

$$= \frac{(n-1)!}{p!}$$

Imp:-

1) Particular Total Never + always = arrange Included
2) $r = \sum_{1}^{n} r.\,^rp_r = {}^{n+1}p_{r+1} - 1$
3) $^np_n = n!$
4) $^np_1 = n$

*** Combination :-**

Selection of smaller or lesser number of things where the of order of selection is not important is called combination.

$$^nC_r = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Relation:- $r! \times {}^nc_r = {}^np_r$

* $^nc_o = 1$

* $^nc_n = 1$

*** Complementary Combination :-**

i.e. If $^nc_x = {}^nc_y$

then, either x = y

or x + y = n

i.e. $^nc_r = {}^nc_{n-r}$

Maths Formula

* **Pascals Law :-**

$$^{n}c_{r} + {}^{n}c_{r-1} = {}^{n+1}c_{r}$$

* **Atleast one selection: (1 or more)**

$$^{n}c_{1} + {}^{n}c_{2} + {}^{n}c_{3} \ldots + {}^{n}c_{n} = 2^{n} - 1$$

* **One or more in Alternative :-**

$$3^{n} - 1$$

* **Combination with Repeatation**

$$\{(p+1)\,(q+1)\,(r+1)\,..........\} - 1$$

* **Division into Groups :-**

1) The No. of ways in which **m + n + p** different things can be divided into 3 groups containing **m, n, p** things respectively.

$$\frac{(m+n+p)!}{m! \times n! \times p!}$$

2) **m = n = p**, then 3 groups are equal, hence required No. of ways of division,

$$= \frac{(3m)!}{3! \times (m!)^{3}}$$

3) If 3m things are distributed equally among 3 person

$$= \frac{(3m)!}{(m!)^{3}}$$

* **Combination in Geometry :-**

For **'n'** non – collinear

1) No. of straight line = $^{n}c_{2}$
2) No. of diagonal line = $^{n}c_{2} - n$
3) No. of triangle = $^{n}c_{3}$

* For **'n'** points, in which **'m'** points are collinear

1) No. of straight line = $^{n}c_{2} - {}^{m}c_{2} + 1$
2) No. of triangle = $^{n}c_{3} - {}^{m}c_{3}$

6 – Sequence and Series (A.P. and G.P.)

*** Sequence :-**

An ordered collection of numbers a_1, a_2, a_3 .... a_n is a sequence if according to some definite rule or law, there is a definite value or 'a_n' corresponding to any value of the natural number 'n'.

a_n is called term or element of the sequence.

*** Finite sequence :-**

If the number of element in a sequence is finite, the sequence is called finite sequence.

e.g. A sequence of even positive integer within 12 i.e. 2, 4, 6, 10.

It is denoted by $\{a_i\}_{i=1}^{n}$

*** In finite sequence :-**

If the no of element in a sequence is infinite the sequence is called infinite sequence.

e.g. The sequence $\left\{\frac{1}{n}\right\}$ is

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...$$

It is denoted by $\{a_i\}_{i=1}^{\infty}$ a or $\{a_n\}$

*** Series :-**

The sum of the all elements of the sequence {an} is called a series.

i.e. $a_1 + a_2 + a_3 + ... + a_n$

$$s_n = \sum_{r=1}^{n} a_r$$

Or

$$s_n = \sum a_n$$

Where,

S_n = sum of the 1st 'n' term

*** Arithmetic progression (AP) :-**

A sequence a_1, a_2, a_3, ... a_n is called an arithmetic progression (A.P.)

When

$a_2 - a_1 = a_3 - a_2 = a_n - a_{n-1}$

The means AP. is a sequence in which the difference between each term and its proceeding term is constant.

The constant 'd' is called the common difference of A.P.

e.g.

2, 5, 8, 11, 14 .... 17 is an A.P. in which d = 3

*** A.P. Series :-**

$a_n = a + (n-1)d$

1) Sum of 1st n terms of A.P.

$$S_n = \frac{n[2a + (n-1)d]}{2}$$

Or

$$S_n = \frac{n(a + l)}{2}$$

Where,

a = 1st term
l = last term
d = common difference

2) Sum of 1st n natural number

$$S = \frac{n(n+1)}{2}$$

3) Sum of 1st n odd number

$S = n^2$

4) Sum of the squares of the 1st n natural nos.

i.e. $\sum n^2$

$$S_n = \frac{n(n+1)(2n+l)}{6}$$

5) Sum of the cubes of the 1st n natural nos.

i.e. $\sum n^3$

$$S = \left\{\frac{n(n+1)}{2}\right\}^2$$

*** Geometric progression :-**

If in a sequence of terms, each term is constant multiple of the proceeding term, then the sequence is called a geometric progression.

That means in G.P. the ratio of the 2nd term to the 1st term is constant (r)

e.g.

1) 5, 15, 45 .... Common ratio = 3
2) 1, ½, ¼ .... Common ratio = ½

*** General form of an :- (A.P.)**

$a + ar + ar^2 + ...$

*** Geometric mean :-**

If a, b and c are in Gross Profit. **Then**,

$$\frac{b}{a} = \frac{c}{b}$$

i.e.

$b^2 = ac$ and $b = \sqrt{ac}$

Here, 'b' is called geometric mean between a and c

* **Formula :-**

1) n^{th} term = ar^{n-1}
2) sum of 1^{st} 'n' term of an G.P.

   $S_n = \dfrac{a(1-r^n)}{1-r}$ .... r < 1
3) $S_n = \dfrac{a(r^n-1)}{r-1}$ when, r > 1
4) Sum of infinite Geometric series:

   $S_\infty = \dfrac{a}{1-r}$ r < 1

7 – Set, Functions and Relation

*** Set :-**

A set is defined to be collection of well defined distinct object.

Each object is called as an **element** of set.

We denote set by capital letters and their elements by small letters.

e.g. A = {a, e, i, o, u}
b = {2, 4, 6, 8}

*** Method of describing a set:-**

1) Tabular method or Roster method or Braces form :-

In this we put **all the elements** of set within { }

e.g. A = Set of vowel in a alphabet
= **{a, e, i, o, u}**

2) Set-Builder form or Algebraic form or Rule method :-

In this we list the property or properties satisfied by the elements of the set.

e.g. B = The set of even number between 2 and 10 both inclusive.

$B = \{x : x = 2m,\ x \in \cancel{10 < m < 6}\}$ $m \in N$

Or = {~~x~~:x is even positive integer and x≤10} $m \leq 5$

*** Type of set :-**

1) Finite Set :-

It is a set consisting of finite number of elements.

e.g. A = {a, e, i, o, u}
B = {2, 4, 6, ...50}

2) Infinite Set :-

A set having an infinite number of element is called an infinite set.

e.g. A = {1, 2, 3, ...}
c = {x:x is a number of stars in the sky}

3) Null or empty or Void Set :-

A set having no elements in it is called as null set.

It is denoted by 'ϕ'

e.g.

A = {x:x is a perfect square of an integer, 5 < x < 8}

$\phi \neq \{0\}$ also $\phi \neq \{\phi\}$

4) Equal Set :-

If A and B have the same elements then they are called as equal set. (A = B)

e.g. A = {1, 2, 3, 4}, B = {3, 1, 2, 4}

5) Equivalent Set :-

If total number of elements of 1 set is equal to the number of element of another set, then the two sets are said to be equivalent set. $A \equiv B$

e.g. A = {1, 2, 3, 4}
B = { b, a, c, d}

6) Subset :-

If each element of set A belongs to set B, then A is said to be a subset of B. It is written as $A \subseteq B$

e.g. A = {1, 2, 3 }
B = {3, 1, 2, 4, 5} then A ** B
For 'n' elements there are 2^{nd} subsets.

7) Proper Subset :-

If P is a subset of Q but P is not equal to Q then P is called proper subset of Q.

It is written as $P \subset Q$.

A set having 'n' element has 2^{nd} 1 proper subset.

8) Power set :- P(A)

The set of all the subset of given set is known as power set.

e.g. A = {a, b}
then, P(A) = {{a}, {b}, {a, b}, ϕ}

9) Universal set :-

A fixed set under consideration is called as Universal set.

It may be finite or infinite.

e.g. A pack of cards may be taken as Universal set for set of diamond or spade.

10) Singleton set :-

A set containing 1 element is called singleton set.

e.g. A = {1}, B = {2}, C = {0}

11) Overlapping sets :-

If tow sets A and B have some common element then A and B are called as Overlapping set,

e.g. A = {2, 5, 7, 8}, B = {5, 6, 8}

12) Disjoint set :-

Two sets A and B are said to be disjoint of their intersection is empty.

i.e. no elements of set A belongs to set B

e.g. A = { 1, 3, 5,}, B = {2, 4}

$A \cap B = \phi$

*** Important Terms :-**

1) Cardinal number of set :-

Maths Formula

The number of distinct element set A is called its cardinal number and it is denoted by n(A).

e.g. R = {2, 3, 5, 7} then n(R) = 4

2) **<u>Union of set</u> :-**

For the 2 set A and B the set of all the elements of A and B is called union of A and B.

e.g. A = {1, 2, 3, 4}, B = {4, 5, 6}
then $A \cup B$ = {1, 2, 3, 4, 5, 6}

3) **<u>Intersection of set</u> :-**

For the 2 sets A and B the set of common elements of A and B is called intersection of set.

e.g. A = {1, 2, 3, 4} B = {3, 4, 7}
then $A \cap B$ = {3, 4}

4) **<u>Difference of 2 sets</u> :-**

For the 2 sets A and B the set containing all those elements of A which does not belong to B is known as difference of 2 sets. It is denoted.

$A \sim B$ or $A - B$ and read as A difference

e.g. A = {1, 2, 3, 4, 5}
B = {3, 5, 6, 7}
Then, $A \sim B$ = {1, 2, 4}

5) **<u>Complement of set</u> :-**

If U be the universal set and A be it's subset.

Then the set of all the elements which are not in A is called as complement of A and denoted by A' or A^{-1}

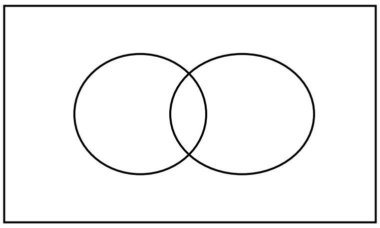
e.g. U = {1, 2, 3, ... 9}
A = {1, 3, 5, 7, 9}
Then, A' = {2, 4, 6, 8}

*** <u>Ven Diagram</u> :-**

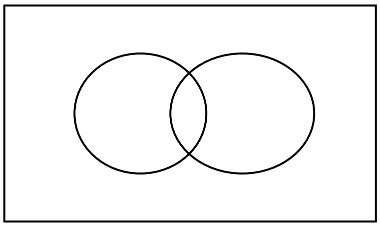
John Venn, an English logician invented this diagram to present pictorial representation, the diagram display operations on sets.

In Venn diagram, we denote universe (U) by a region enclosed within a rectangle and any subject of U will be shown by circle or closed curve.

e.g. i) $A \cup B$



ii) $A \cap B$



* **<u>Symmetric Difference</u> :-**

For the 2 sets A and B, the symmetric difference is $(A \sim B) \cup (B \sim A)$

And denoted by $A \Delta B$

*** <u>Laws in Algebra of sets</u> :-**

1) **<u>Commutative Laws</u> :-**

$A \cup B = B \cup A$ And $A \cap B = B \cap A$

2) **<u>Associative Laws</u> :-**

$A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

3) **<u>Idempotent Laws</u> :-**

$A \cup A = A$, $A \cap A = A$

4) **<u>Distributive Laws</u>** :-

$A \cup (B \cap C) = (A \cup B) \cap A \cup C$
$A \cap (B \cup C) = (A \cap B) \cup A \cap C$

5) **<u>Identify Laws</u> :-**

$A \cup \phi = A$; $A \cap \phi = \phi$
$A \cup U = U$; $A \cap = A$

6) **<u>Complement Laws</u> :-**

$A \cup A` = U$ ; $A \cap A` = \phi$
$\phi` = U$, $U` = \phi$. (A') = A

7) **<u>Debenture-Morgan's Laws</u> :-**

$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

* **<u>No of elements in a set</u> :-**

1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
For disjoint set $n(A \cap B) = 0$
$n(A \cup B) = n(A) + n(B)$

2) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
For disjoint set,
$n(A \cup B \cup C) = n(A) + n(B) + n(C)$

*** <u>Ordered Pair</u> :-**

Two elements a and b listed in specific order, form an ordered paid, denoted by (a, b).

Maths Formula

* **Cartesian Product of set :-**

The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called Cartesian product of set.

Where, A and B are two non empty sets.

e.g. A = {1, 2, 3}, B = {4, 5}, then

A x B = {(1, 4), (1,5), (2, 4), (2, 5), (3, 4), (3, 5)

Also, n(A x B) – n(A) x n(B) and

$(3, 5) \neq (5, 3)$

$A \times B \neq B \times A$

But, n(A x B) = n(B x A)

* **Function :-**

If x and y be 2 real variable related to some rule such that corresponding to every value of x, we get value of y then x is said to be Function.

Let A and B be 2 non empty set, then the rule f by which the very element of set is associated with element of B(f(x)) is called **Function or mapping** from A to B.

We Write $F : A \rightarrow B$

A is called domain of f and B is called co-domain or range of f.

* **Domain :-**

It is the set of the value that the independent variable x can take

* **Range :-**

It is the set of value that f(x) can take for all possible values of x

e.g. Let A = {1, 2, 3, 4}

B = {1, 4, 9, 16, 25}

The Rule $f(x) = x^2$

i.e. $f(1)=1$, $f(2) = 4$, $f(3) = 9$, $f(4) = 16$

Here, domain of f = {1, 2, 3, 4}

Range of f = {1, 4, 9, 16}

* **One – One Function :-**

Let F $A \rightarrow B$, if different elements in A have different images in B, then f is called as One–One or Injective Function.

e.g. Let A = {1, 2, 3, 4}

B = {2, 4, 6}

$F : A \rightarrow B$.

$f(x) = 2x$ Then,

$f(1) = 2$, $f(2) = 4$, $f(3) = 6$. Here, different element is A have different images is B, Hence, f is One – One.

2) **Onto or subjective Function :-**

Let $F : A \rightarrow B$. If every element of B has at least 1 pre-image in A, then f is said to be an onto function.

3) **Bijection Function :-**

A one-one and onto function is said to bijective

It is also known as one to one correspondence.

* **Identity function :-**

A bijective function with domain A and range A is known as identity function.

* **Into Function :-**

Let $F : A \rightarrow B$. If there exist even a single element in B having no pre image in A, then f is said to be an into function.

* **Constant Function :-**

Let $F : A \rightarrow B$. If all the elements in A have the same image n B then f is said to be a constant function.

Let $F : A \rightarrow B : f(x) = 5 \ \forall x \in A$

Then, all the elements in A have the same image namely 5 in B.

The range of constant function is singleton set.

* **Equal Function :-**

The two functions F and g are said to be equal, if they have some domain and they satisfy the condition $f(x) = g(x)$ for all x.

* **Inverse Function :-**

If $F : A \rightarrow B$ is one – one onto function, then $F^{-1} : A \rightarrow B : f^{-1}(y) = x$ is called an inverse function 'f^{-1}' is called inverse of f.

* **Relation :-**

If A and B are 2 non-empty set then, every subset A x B is a relation from A to B.

* **Domain of Relation :-**

If R is a relation from A to B then set of all first co-ordinates of elements of R is called the domain of R.

* **Range of Relation :-**

The set of all second co-ordinates of elements of R is called range of R.

e.g. Let A = {1, 2, 3}

B = {2, 4, 6}

AxB = {(1,2)(1,4)(1,6)(2,2)(2,4)(2,6)(3,2)(3,4)(3,6)}

If we consider the relation

A x B = {(1,2)(1,4)(3,2)(3,4)}

Then, Dom (R) = {(1,3)}

Range (R) = {(2, 4)}

* **Type of Relation :-**

1) **Reflexive Relation :-**

If R contains all the ordered paid of the form (a, a) the R is called reflexive.

2) Symmetric Relation :-

If R contains the all ordered pair of the form (a, b) as well as (b, a) then R is called symmetric relation.

3) Transistive Relation :-

If the relation R contains the (a, c) for every (a, b) $\in$ R and (b, c) $\in$ R then R is called transistive relation.

e.g. Let A = {1, 2, 3}

Then,

i) R_1 = {(1,1) (2,2) (3,3) (1,2)} is reflexive and transitive but no symmetric, since (1,2) $\in$ R but (2, 1) $\notin$ R_1

ii) R_2 = {(1,1) (2,2) (1,2) (2,1)} is symmetric and transitive but not reflexive, since (3, 3) $\notin$ R2.

iii) R_3 = {(1,)(2,2) (3,3) (1,2) (2,1) (2,3) (3,2)} is reflexive and symmetric but not transitive since (1,2) $\in$ R_3 and (2,3) $\in$ R but (1, 3) $\notin$ R_3.

8 – Limits and Continuity

*** Function :-**

If x and y be 2 variable related to some rule. Such that corresponding to every value of x we get a defined value of y, then is said to be function of x and it is written as

$$y = f(x)$$

*** Types of Function :-**

1) Even Function :-

If $f(-x) = f(x)$

Then, $f(x)$ is said to be an even function.

e.g. Let $f(x) = x^2 + 2x^4$

Now, $f(x) = (-x)^2 + 2(-x)^4$
$= x^2 + 2x^4$

$\therefore f(-x) = f(x)$

$\therefore$ The given function $f(x)$ is an even function.

2) Odd Function :-

If $f(-x) = -f(x)$

$f(x)$ is said to be an odd function.

e.g. Let $f(x) = 5x + 6x^3$

$f(-x) = 5(-x) + 6(-x)^3$
$= -5x - 6x^3$
$= -(5x + 6x^3)$

$f(-x) = -f(x)$

$\therefore f(-x)$ is an odd function.

3) Periodic Function :-

If the function $f(x)$ repeats after equal interval of time then the $f(x)$ is called as Periodic Function.

4) Inverse Function :-

If $y = f(x)$ then we can write $x = g(y)$ then the function $g(y)$ is called as the inverses of $f(x)$.

e.g. $x = y^2$ is the inverse function of

$$y = \sqrt{x}$$

5) Composite Function :-

If $y = f(x)$ and $x = g(y)$

Then $y = f[g(y)]$ is called composite function.

*** Concept of Limit :-**

1) Let $f(x) = 2x$

If x	then $f(x)$	
1.9	3.8	(Left
1.99	3.98	Hand
1.999	3.998	Limit)
2	4	

i.e. As the value of x approaches to 2 such that $x < 2$ the value of $f(x)$ approaches to $\lim_{x \to 2^-} 2x = 4$.

This, can be written as

2)

if x	Then $f(x)$	
2.0001	4.0002	(Right
2.001	4.002	Hand
2.01	4.02	Limit)
2	4	

i.e. As the value of x approaches to 2 such that $x > 2$ the value of $f(x)$ approaches to 4.

This, can be written as $\lim_{x \to 2} 2x = 4$.

*** Rules :-**

1) $\lim_{x \to a} [f(x) \pm g(x) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

2) $\lim_{x \to a} [f(x) \cdot g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

3) $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

4) $\lim_{x \to a} k = k$ .... k = constant

5) $\lim_{x \to a} k.\, f(x) = k.\, \lim_{x \to a} [f(x)]$

6) $\lim_{x \to a} + \dfrac{1}{x} = +\infty$

7) $\lim_{x \to a} + \dfrac{1}{x} = -\infty$

As is called infinity thus $\lim_{x \to 0} + \dfrac{1}{x}$ ***********

*** Note :-**

If the left hand limit and right hand limit are not equal then the limit does not exist.

*** Some Important Limits :-**

1) $\lim_{x \to a} \dfrac{e^x - 1}{x} = 1$

2) $\lim_{x \to a} \dfrac{a^x - 1}{x} = \log_e a$

3) $\lim_{x \to a} \dfrac{\log(1 + x)}{x} = 1$

4) $\lim_{x \to a} \left[1 + \frac{1}{x}\right]^x = e$

5) $\lim_{x \to a} [1 + x]^{\frac{1}{x}} = e$

6) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n.a^{n-1}$

7) $\lim_{x \to a} \frac{e^x - 1}{x} = n$

Where, e = exponential number
= 2.71828182
≈ 2.7183

*** <u>Continuity</u> :-**

A function f(x) is said to be continuous at x = a if and only if.

i) f(x) is defined at x = a

ii) $\lim_{x \to a} f+(x) = \lim_{x \to a} + f(x)$

iii) $\lim_{x \to a} - f(x) = f(a)$

*** <u>Useful Information</u>** :-

i) The sum difference product of 2 continuous functions is a continuous function.

ii) The quotient of 2 continuous function is a continuous function provided the denominator is not equal to zero.

9 – Differential and Integral Calculus

* **Differential Calculus :-**

Differentiation is defined as the limiting value of the ratio of the change in the function corresponding to small change in variable as the change in variable tends to zero.

i.e. derivative of f(x)

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

This is denoted by f'(x) or

$$\frac{dy}{dx} \text{ or } \frac{d}{dx}f(x)$$

The process of differentiation is called the first principle or definition.

* **Rules :-**

1) Sum Rule :-

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} + \frac{dv}{dx}$$

2) Product Rule:-

$$\frac{d}{dx}(u \,.\, v) = u\frac{dv}{dx} - v\frac{du}{dx}$$

3) Quotient Rule :-

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Where, u and v are functions of x

* **Formula :-**

1) $\frac{d}{dx}(x) = 1$

2) $\frac{d}{dx}(kx) = k$

3) $\frac{d}{dx}(x^n) = n\,x^{n-1}$

4) $\frac{d}{dx}\,a^x = a^x \,.\, \log_e a$

5) $\frac{d}{dx}\,e^x = e^x$

6) $\frac{d}{dx}\,k = 0$ .... k = constant

7) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

8) $\frac{d}{dx}\,\frac{1}{x} = -\frac{1}{x^2}$

* **Chain Rule :-**

If y = f(u) and u = f(x)

Then, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = f'(u) \times g(x)$

e.g. Differential $\log(1 + x^2)$ w.r.t.x

solution:- Let $(1 + x^2) = u$

$\therefore y = \log(1 + x^2) = \log u$

$\frac{dy}{dx} = \frac{d}{dy}(\log u)$

$\frac{dy}{du} = \frac{1}{u}$ (1) and

$U = 1 + x^2$

$\frac{du}{dx} = 0 + 2x$

$\frac{du}{dx} = 2x$ (2)

From eq^n (1) and (2)

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{u} \times 2x$

$= \frac{1}{1+x^2} \times 2x$

$\frac{dy}{dx} = \frac{2x}{1+x^2}$

* **Implicit Function :-**

The function where y cannot be directly defined as a function of x is called an implicit functions of x.

e.g. xy = 0

diff. w.r.t.x

$x.\frac{dy}{dx} + y = 0$ .... Product Rule

$\frac{dy}{dx} = \frac{-y}{x}$

* **Parametric Function :-**

When both the variables i.e. x and y are expressed in terms of a third variable then the equations are called parametric equation.

e.g. find $\frac{dy}{dx}$ if $x = at^3$, $y = \frac{1}{t}$

$\frac{dx}{dt} = 3at^2$, $\frac{dy}{dt} = -\frac{1}{t^2}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\frac{1}{t^2}}{3at^2}$$

$$\frac{dy}{dx} = \frac{1}{3at^4}$$

*** Logarithmic Differentiation**:-

The process of finding aut derivative by taking logarithm in the first instance is called Logarithmic Differentiation.

e.g. Find $\frac{dy}{dx}$ if = $y = x^x$

solution:- $y = x^x$

$\log y = x.\log x$

Diff. w.r.t.y.

$$\frac{1}{y}.\frac{dy}{dx} = x.\frac{1}{x} + \log x.1$$

$$\frac{dy}{dx} = y(1 + \log x)$$

*** Geometrical Interpretation :-**

The derivative of $f(x)$ at a point x represents the slope or gradient of the tangent to the curve $y = f(x)$ at point x.

e.g. Find the gradient of the curve $y = 3x^2 - 5x + 4$ at the point (1, 2)

Solution:- $y = 3x^2 - 5 + 4$

$$\frac{dy}{dx} = 6x - 5$$

$$\left(\frac{dy}{dx}\right)(t.2) = 6(1) - 5 = 1$$

Thus the gradient of the curve at (1, 2) is 1

*** Integration :-**

Integration is the inverse process of differentiation and is denoted by symbol "$\int$"

e.g. $\frac{d}{dx}\log x = \frac{1}{x}$

$$\therefore \int \frac{1}{x}\,dx = \log x$$

*** Rules:-**

1) $\int[f(x) \pm g(x)] = \int f(x)\,dx \pm \int g(x)dx$

2) $\int[k\,.\,fx]dx = k \pm \int f(x)\,dx$

*** Basic formulas :-**

i) $\int x_n\,dx = \frac{x^n + 1}{n+1} + c$ ….. $(n \neq -1)$

ii) $\int dx = x$

iii) $\int e^x\,dx = e^x + c$

iv) $\int e^{ax}\,dx = \frac{e^{ax}}{a} + c$

v) $\int \frac{dx}{x} = \log x + c$

vi) $\int a^x\,dx = \frac{a^x}{\log_e a} + c$

vii) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right) + c$

viii) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right) + c$

ix) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \left| x + \sqrt{x^2 + a^2} \right| + c$

x) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \left| x + \sqrt{x^2 - a^2} \right| + c$

xi) $\int e^x\,.\,(f(x) + f'(x))\,dx = e^x.\,f(x) + c$

xii) $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log(x + \sqrt{x^2 + a^2} + c$

xiii) $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log(x + \sqrt{x^2 + a^2}) + c$

xi) $\int \frac{f(x)}{f(x)}\,dx = \log[f(x)] + c$

*** Integration by Substitution :-**

It is some time possible by a change of independent variable to transform a function into another which can be readily integrated.

e.g. $\int (2x + 3)^7\,dx$

Solution:-

Put $2x + 3 = t$

$\therefore 2 = \frac{dt}{dx}$

$\therefore dx = \frac{dt}{2}$

$\therefore I = \int t^7.\,\frac{dt}{2}$

$\therefore \frac{1}{2}\,.\,\frac{t^{7+1}}{7+1}$

$\therefore \frac{1}{2} \times \frac{t^8}{8}$

$\therefore \frac{t^8}{16}$

$\therefore \frac{(2x+3)^8}{16}$

*** Integration by Parts :-**

$\int u.v dx =$

$u \int v.dx - \int[\frac{d}{dx} u. \int vdx]\, dx$

Where,

u and v are 2 different functions of x

e.g. $\int x.\, e^x\, dx$

$= x. \int e^x.dx - \int[\frac{d}{dx}(x).\, e^x\, .\, dx]dx$

$= x.\, e^x - \int [1\, .\, e^x]\, dx$

$= x\, .\, e^x - e^x$

$= e^x (x - 1) + c$

* **Method of Partial Fraction** :-

1) Non-Repeated liner factor.

$$\frac{f(x)}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

2) Repeated linear factor.

$$\frac{f(x)}{(x+a)^2 + (x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{**}{(x+b)}$$

3) Irreducible quadratic factor.

$$\frac{f(x)}{ax^2+bx+c} = \frac{Ax+B}{ax^2+bx+c}$$

* **Definite Integration** :-

$$-\int_a^b f(x)dx = f(b) - f(a)$$

"b" is called upper limit and "a" is the lower limit of integration.

e.g. $\int_0^2 (x^5)dx$

Solution :-

$$\int_0^2 (x^5)dx = \left(\frac{x^6}{6}\right)_0^2$$

$= \frac{1}{6}(2^6 - 0) = \frac{64}{6} = \frac{32}{3}$

* In definite integration the constant "c" should not be added.

* **Important Properties** :-

1) $\int_a^b f(x)\, dx = \int_a^b f(t).\, dt$

2) $\int_a^b f(x)\, dx = - \int_b^a f(x).dx$

3) $\int_a^c f(x)\, dx = \int_a^b f(x)\, dx \quad \int_b^c f(x)\, dx$

4) $_0\int^a f(x)\, dx = {_0\int^a} f(a - x)\, dx$

5) When $f(x) = f(a + x)$

$\int_0^{na} f(x)\, dx = n\, .\, \int_0^a f(x)\, dx$

6) $_0\int^a f(x)\, dx = 2\, {_0\int^a} f(x)\, dx$ ….. If $f(x)$ is even

$= 0$ .... If $f(x)$ is odd

10 – Statistical Description of data

*** <u>Origin of word 'Statistics'</u> :-**

Latin –	Status (LS)
Italian –	Statista (TA)
German –	Statistik (GK)
French –	Statistique (FS)

*** <u>Definition of Statistic</u> :-**

1) Singular Sense :-

The **scientific method** that is employed for collecting, analyzing and presenting data, leading finally to drawing statistical interference

2) Plural Sense :-

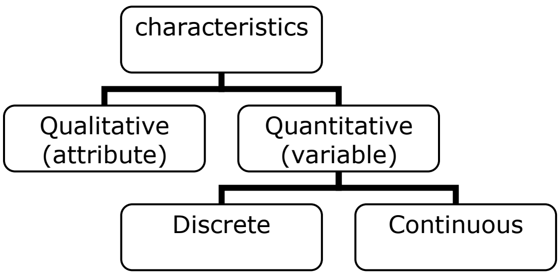
Data qualitative as well as quantitative, that are collected, usually with a view of having statistical analysis.

*** <u>Application of Statistics</u> :-**

1) Economics
2) Business Management
3) The theory of statistical sampling is depend upon rules of random sampling.

*** <u>Data</u> :-**

It is defined as the quantitative information about some particular characteristics under consideration.



*** <u>Discrete Variable</u> :-**

When a variable assumes a finite or a countable infinite number of isolated values it is known as discrete variable.
e.g. No of petals in flower.

*** <u>Continuous variable</u> :-**

If a variable assumes and value from the given interval then it is called as continuous variable.
e.g. Height, weight, sale, profit etc.

*** <u>Primary data</u> :-**

The data which are collected form the 1^{st} time by investigator or agency are known as primary data.
e.g. data collected for cencus.

*** <u>Secondary data</u> :-**

If the already collected data used by a different person or agency is called secondary data.
e.g. data collected from government and international sources.

*** <u>Collection of primary data</u> :-**

1) Interview method.
   a) Personal interview method.
   b) Indirect interview method.
   c) Telephone interview method.
2) Mailed questionnaire method.
3) Observation method.
4) Questionnaires filled and sent by enumerators.

*** <u>More Accurate Methods</u> :-**

1) Personal interview method and indirect method :-
   Covers Large / Wide area.
2) Mailed questionnaire method. Non responses are maximum in.
3) Telephone interview and Mailed questionnaire method :-
   Quickest method – Telephone interview method

*** <u>Sources of Secondary Data</u> :-**

1) International sources
2) Government sources
3) Private and quasi – Govt. sources
4) Unpublished research institute.

*** <u>Objective of Classification of Data</u> :-**

1) To make the data easily understoodable and interpretable.
2) To make comparison possible between various characteristics.
3) To make it good for statistical analysis.
4) To eliminate unnecessary details

*** <u>Classification of data</u> :-**

1) Chronological or Temporal or time series data.
2) Geographical to spatial series data.
3) Qualitative of ordinal data.
4) Quantitative or cardinal data.

* **Mode of presentation of data :-**

1) Textual Presentation :-

Presents data with the help of paragraph which describes the information.

It is not preferred, because it is dull, monotonous and comparison between diff. observation is not possible.

2) Tabular Presentation :-

A systematic presentation of data with the help of statistical table having rows and column

The statistical table contains 5 different things.

1) Caption :-

Upper part of column which describes the column.

2) Box – Head :-

Entire upper part of the tale which includes column, sub column along with caption

3) Stub :–

Left part of the table providing the describing of rows.

4) Body : –

It is the main part of the table that contains numerical figure.

5) Title and Footnote :–

* **Feature of Tabulation :–**

1) If facilitates comparison between rows and column.
2) Complicated data can also be represented.
3) It is must for diagrammatic presentation.
4) Without Tabulation, statistical analysis of data is not possible.

3) Diagrammatic representation of data :–

1) Line diagram or histogram
2) Bar diagram
3) Pie chart

1) Line diagram or histogram :-

In this we plot each pair of values (t_1, γ_1) and the plotted points are then successively joined by line segments and the resulting chart is known as line diagram.

2) Bar diagram :-

i) Horizontal Bar diagram :-

Qualitative data or data varying over space.

ii) vertical Bar diagram :-

Quantitative data or time series data.

* **Types :–**

1) **Multiple or grouped bar diagram :-** To compare related series.
2) **Component or sub divided bar diagram :-** To represent data which is divided into no of components.
3) **Divided bar chart or percentage bar diagram :-** To compare different component of variable. Relating of the components to the whole.

* **Frequency Distribution :–**

It is defined as a tabular representation of statistical data usually in ascending order.

The figure, signifying the number of times or how frequently a particular class occur is known as the Frequency.

When Tabulation is done in respect of **discrete random** variable, it is known as discrete or **ungrouped** or simple frequency distribution.

When the characteristic under consideration is a **Continuous variable** then it is known as grouped frequency distribution.

*** For continuous variable :-**

Range ≅ No. of class interval x class length

* **Important terms :-**

1) Class limit (CL) :-

It is the minimum value and maximum value of class interval.

The minimum value is called as Lower class limit (CL). The maximum value is called as Upper class limit (UCL).

2) Class Boundary :-

It is defined as the actual class limit of a class interval.

For overlapping classification

i.e. (20 – 30, 30 – 40)

LCL = LCB and,

UCL = UCB

For Non – overlapping classification.

i.e. (20 – 29, 30 – 39)

$$\therefore \quad LCB = LCL \ \frac{d}{2}$$

$$\therefore \quad LCB = 20 - \left(\frac{30-29}{2}\right)$$

$$= 19.5$$

And, $\text{UCB} = \text{UCL} = \frac{d}{2}$

$$= 29 + \left(\frac{30-29}{2}\right)$$

$$= 29.5$$

3) <u>Mid point or Mid value of Class mark</u> :-

It is defined as the total of the 2 class limits or class boundaries to be divided by 2.

$$\therefore \text{Mid – Point} = \frac{\text{LCL}+\text{UCL}}{2}$$

$$= \frac{\text{LCB}+\text{UCB}}{2}$$

4) <u>Width or size of class interval</u> :-

It is defined as the difference between the UCB and the LCB of that class interval.

5) <u>Cumulative Frequency</u> :-

It is defined as the No. of observations less than the value or less than or equal to the class boundary for less than type cumulative frequency.

For more than type it is defined as the No. of observations more than the value or equal to the or more than the class boundary.

6) <u>Frequency density</u> :-

It is defined as the ratio of the frequency of that class interval to the corresponding class length.

7) <u>Relative Frequency</u> :-

It is defined as the ratio of the frequency to the total frequency.

*** <u>Graphical Representation or a frequency distribution</u> :-**

1) Histogram or Area diagram
2) Frequency Polygon
3) Ogives or cumulative frequency polygon

1) <u>Histogram</u> :-

In order to draw histogram the class limits are first converted to the corresponding class boundaries and a series of adjacent rectangles, one against each class interval is drawn.

Histogram is used to find mode graphically.

2) <u>Frequency Polygon</u> :-

In order to plot frequency polygon we first plot the points for mid point of interval and class frequency and by joining this point by line segments we get frequency polygon.

3) <u>Ogives or cumulative frequency graph</u> :-

By plotting cumulative frequency against the respective class boundary we get ogives.

Ogives are used to get quartile and median.

*** <u>Frequency Curve</u> :-**

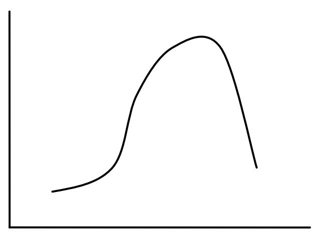
Frequency curve is a smooth curve for thick the total area is taken to be unity.

It is obtained by drawings a smooth the mid–points of the upper sides of a rectangles forming the histogram.

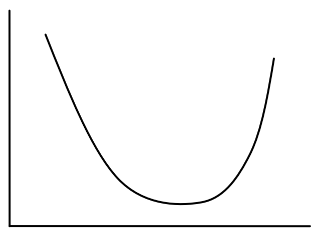
They are mainly 4 types.

1) Bel – shaped curve –
2) U – shaped curve
3) J – shaped curve
4) Mixed curve

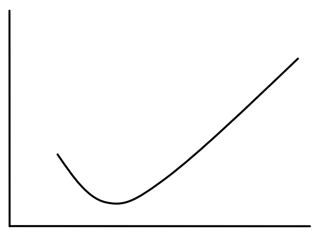
1)



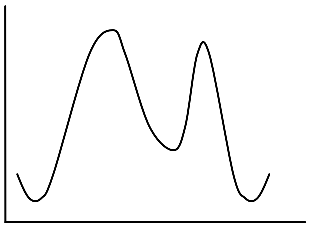
2)



3)



4)



11 – Measures of Central Tendency and Disperson

*** Central Tendency :-**

It may be defined as the tendency of a given set of observation to cluster around a single central or middle value.

And this single value known as Measure of Central Tendency or location or average.

*** Different Measures of Central tendency :-**

1) Arithmetic Mean (AM)
2) Median (Me)
3) Mode (Mo)
4) Geometric Mean (GM)
5) Hannonic Mean (HM)

*** Criteria for an Ideal Measure of central tendency :-**

i) It should be properly and unambiguously defined.
ii) It should be easy to comprehend.
iii) It should be simple to compute.
iv) It should based on all observations.
v) It should have certain desirable mathematical properties.
vi) It should be least affected by the presence of extreme observations.

*** Arithmetic Mean (AM) :–**

It is defined as the sum of observations to the number of observations.

$$\overline{X} = \frac{\Sigma x_i}{N}$$

*** Simple (Ungrouped) frequency distribution :-**

$$\overline{X} = \frac{\Sigma f_i x_i}{\Sigma x_i} = \frac{\Sigma f_i x_i}{N}$$

*** For Grouped Frequency distribution :-**

$$\overline{X} = A + \frac{\Sigma f_i d_i}{N} \times c$$

Where, $\overline{X}$ = AM
N = Total No. of observation
A = Assumed mean
$d_i = \frac{x_i - A}{c}$
c = Class Width

*** Properties of AM :-**

1) If all the observations are equal the AM is that number itself.
2) The algebric sum of deviation from AM is zero.
i.e. $\sum(x_i - \overline{X}) = 0$
and $\sum f_i (x_i - \overline{X}) = 0$
3) The two groups with n_1 and n_2 observations and $\overline{X}_1$ and $\overline{X}_2$ AM, then their combined mean
$$\overline{X} = \frac{n_1\overline{x} + n_2\overline{x}}{n_1 + n_2}$$

*** Median :-**

It is defined as the middle most value when the observations are arranged in ascending or descending order.

*** For Discrete Variable :-**

Median can be found out by inspection.

*** For Simple (Ungrouped) Frequency distribution :-**

Median can be found out by finding the $\left(\frac{N}{2}\right)^{th}$ observation.

*** For Grouped Frequency distribution :-**

Median can be find out by following formula

$$\textbf{Micro economics} = L_1 + \frac{\frac{N}{2} - f_1}{f_m}$$

Where,

l_1 = Lower limit of median class
N = Total Frequency
f_1 = c.f. of pre – median class
f_m = frequency of median class
c = length or width of median class

*** Partition Values :-**

It may be defined as values dividing a given set of observations into number of equal parts.

*** Quartiles :-**

These are the values which divides the given set of observation into 4 equal parts.

So, there are 3 Quartiles Q_1, Q_2 and Q_3 .

$$Q_p = (n + 1) \times \frac{p}{4}$$

*** Deciles :-**

These are the values which divides the given set of observations into 10 equal parts.

So, there are 9 deciles.

i.e. $D_1, D_2, D_3 \ldots\ldots\ldots\ldots D_9$

Maths Formula

$$Dp = (n + 1) \times \frac{P}{10}$$

*** Percentiles or Centiles :-**

These are the values which divides the given set of observations into 100 equal parts.

So, there are 99 percentiles.

i.e. P_1, P_2, P_3 ....P_{99}.

$$PP = (n+ 1) \times \frac{P}{100}$$

Where,

n = Total observations

*** Mode :-**

It is defined as the value that occurs the maximum number of times.

Depending upon the observation values of modes may one or more or none.

*** For unclassified Data :-**

Mode can be find out by inspection.

*** For Frequency (Ungrouped) distribution :-**

Mode is the observation having maximum frequency.

*** For Frequency (grouped) Distribution :-**

Mode can be calculated as,

$$\text{Mode} = l_1 + \left(\frac{f_0 - f_1}{2f_0 - f_1 - f_2}\right) \times c$$

Where,

l_1 = LCB of modal class
f_0 = frequency of modal class
f_1 = frequency of pre–modal class
f_2 = frequency of post modal class
c = Class length.

Mean – Mode = 3(Mean – Median)

*** Geometric Mean :-**

It is defined as the n^{th} root of the product of the observation.

$G = (x_1, x_1, x_3 \ldots\ldots x_n)^{1/n}$ **For unclassified data**

$= \left(x_1^{f1}, x_2^{f2}, x_3^{f3}, \ldots\ldots x_n^{fn}\right)^{1/n}$ .... For frequency distribution

*** Properties :-**

1) Logarithm G for a set of observations is the AM of the logarithm of the observations.
2) If all the observations are equal (say K) then their GM is also K.
3) GM of the product of 2 variable is the product of their GM's if z = xy then (GM) of z = (GM) of x X (GM) of y.
4) GM of the ratio of 2 variables is the ratio of the GM's of the 2 variable.
i.e. If z = x/y then,
$$\text{(GM) of } z = \frac{\text{(GM) of } x}{\text{(GM) of } y}$$

*** Harmonic Mean (HM) :-**

It is defined as the reciprocal of the AM of the reciprocal of the observations.

$$H = \frac{n}{\Sigma[\frac{1}{x_i}]}$$

*** Frequency distribution :-**

$$H = \frac{N}{\Sigma[\frac{f_i}{x_i}]}$$

*** Properties :-**

1) If all the observations are equal (say k) then the HM is also k.
2) If there are 2 groups with n_1 and n_2 observations and H_1 and H_2 as respective HM's then the combined HM is given by
$$H = \frac{n_1 + n_2}{\frac{n_1}{H_2} + \frac{n_2}{H_2}}$$
3) Relation between AM, GM and HM.

For any set of +ve observations

AM ≥ GM ≥ HM

*** Properties of GM and HM :-**

1) Both posseses some mathematical properties.
2) Rigidly defined
3) Based on all observations
4) Difficult to comprehend
5) Difficult to Compute
6) Applications are limited like computation of average rates and ratios and such like things.

AM	Median	Mode
*****		

*** Dispersion :-**

It is defined as the amount of deviation of the observation, from an appropriate measure of central tendency

It may be classified into.

Maths Formula

1) **Absolute Measure of Dispersion**
2) **Relative Measure of Dispersion**

1) **Absolute Measure of Dispersion**
It again classified into
i) Range
ii) Mean Deviation
iii) Standard Deviation
iv) Quartile Deviation

2) **Relative measure of Dispersion**
It again classified into
i) Coefficient of Range
ii) Coefficient of Mean deviation
iii) Coefficient of Variation
iv) Coefficient of Quartile deviation

*** <u>IMP Points</u> :-**
i) Relative measures of dispersion are unit free / unit less.
ii) Relative measures are used to compare two or more distribution.
iii) Relative measures of dispersion are difficult to compute and comprehend.

*** <u>Range</u> :-**
It is defined as the difference between the largest and smallest observation.

$$\therefore \textbf{Range} = L - S$$

For grouped frequency distribution it is the difference between extreme class boundaries (Not Limit)

*** <u>Coefficient of Range</u> :-**

$$= \frac{L-S}{L+S} \times 100$$

* If y = a + bx and
R_x is range of x then range of y (R_y) is given by.

$$R_y = |b| \times R_x$$

*** <u>Mean Deviation (M.D.)</u> :-**
It is defined as the arithmetic mean of the absolute deviation of the observations from an appropriate measure of central tendency.
$\therefore$M.D. about 'A' is given by

$$(MD)_A = \frac{1}{n}\sum |x_i - A|$$

For grouped frequency

$$(MD)_A = \frac{1}{n}\sum |x_i - A| . f_t$$

*** Coefficient of Mean deviation :-**

$$= \frac{\text{mean deviation about A}}{A} \times 100$$

* If y = a + bx then
MD of y = $|b|$ x MD of x

*** <u>Mean Deviation (MD)</u>**
It is defined as the root mean square deviation when the deviations are taken from the AM of the observations.

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{\sum x_i^n}{n} - \bar{x}^2}$$

For grouped frequency :-

$$SD = \sqrt{\frac{\sum (x_i - \bar{x})^2 . f_i}{N}} = \sqrt{\frac{\sum f_i x_i^2}{N} - \bar{x}^2}$$

*** <u>Variance</u> :-**
The square of standard deviation is known as varience.
i.e. Varience = S_2

*** <u>Coefficient of Varience</u> :-**

$$\textbf{Cv} = \frac{SD}{AM} \times 100$$

*** <u>IMP Notes</u> :-**
1) If all the observations are constant i.e. equal then SD is **zero**.
2) If y = a + bx Then, $[SD]y = |b| x (SD)_x$
3) For 2 groups containing n_1 and n_2 observations, $\overline{x_1}$ and $\overline{x_2}$ as respective AM_S, s_1 and s_2 as respective SD_S. Then combined SD is given by.

$$S = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 s_2^2}{n_1 + n_2}}$$

Where,

$$d1 = \overline{x_1} - \bar{X}$$

$$d2 = \overline{x_2} - \bar{X}$$

And, $\bar{X} = \frac{n_1 \overline{x_1} + n_2 + n_2 \overline{x_2}}{n_1 + n_2}$ = combined AM

*** <u>Quartile Deviation (Qa)</u> :-**

$$Q_d = \frac{Q_3 - Q_1}{2}$$

*** <u>Coe of quartile variation</u> :-**

$$\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

*** <u>Standard Deviation (SD)</u> :-**
i) Best measure of dispersion
ii) Rigidly defined
iii) Based on all observations
iv) Easy to compute

v) Not much affected by sampling fluctuations.
vi) It has some desirable mathematical properties.
vii) Most widely and commonly used measure of dispersion.

* **Range** :-
i) Quickest to compute
ii) Applications in statistical quality control
iii) Too much affected by the presence of extreme observations.

* **Mean deviation** :-
i) Rigidly defined
ii) Based on all observation
iii) Not much affected by sampling fluctuation
iv) Difficult to compute and comprehend.
v) Does not possess mathematical properties.

* **Quartile Deviation** :-
i) Rigidly defined
ii) Easy to compute
iii) Not much affected by sampling fluctuations
iv) It is based on the central fifty percent observations.
v) It is not based on all the observations.
vi) It has no mathematical properties.
vii) It is best measure of dispersion for open end classification.

12 – Correlation and Regression

* **Bivariate Date :-**

When data are collected on 2 variable simultaneously, they are known as bivariate data.

The corresponding frequency distribution is known as bivariate frequency distribution or Two way classification or joint frequency distribution.

e.g. Prepare a Bivariate frequency Table for the following data Relating to the marks in statistics (X) and Mathematics (Y):

(15, 13), (1, 3), (2, 6), (8, 3), (15, 10), (3, 9), (13, 19), (10, 11), (6, 4), (18, 14), (10, 19), (12, 8), (11, 14), (13, 16), (17, 15), (18, 18), (11, 7), (10, 14), (14, 16), (16, 15), (7, 11), (5, 1), (11, 15), (9, 4), (10, 15), (13, 12), (14, 17), (10, 11), (6, 9), (13, 17), (16, 15), (6, 4), (4, 8), (8, 11), (9, 12), (14, 11), (16, 15), (9, 10), (4, 6), (5, 7), (3, 11), (4, 16), (5, 8), (6, 9), (7, 12), (15, 6), (18, 11), (18, 19), (17, 16), (10, 14)

Take class interval for both 0 – 4

X \ Y	0–4	4–8	8–12	12–16	16–20	Total
0–4	III(1)	I (1)	II (2)			4
4–8	I (1)	IIII (4)	~~IIII~~ (5)	I (1)	I (1)	12
8–12	I (1)	II (2)	IIII (4)	I (6)	~~IIII~~(5)	11
12–16		I (1)	III (3)	II (2)	III (3)	9
16–20			I (1)	~~IIII~~(5)		
Total	3	8	15	14	10	50

From the above distribution we can obtain 2 types of univariate distribution.

1) **Marginal Distribution**
2) **Conditional Distribution**

1) Marginal Distribution :-

It is the distribution of the any one of the variable under consideration.

e.g. The marginal distribution of marks in statistics is,

Marks	No. of students
0 – 4	1
4 – 8	12
8 – 12	14
12 – 16	11
16 – 20	9
Total	50

2) Conditional Distribution :-

The distribution of any of the variable under particular condition ** group is know as

e.g. The conditional distribution of marks in statistics for students having mathematics marks between 8 – 12.

Marks	No. of students
0 – 4	2
4 – 8	5
8 – 12	4
12 – 16	3
16 – 20	1
Total	15

* **Correlation Analysis :-**

There are 2 types of correlation.

1) Positive Correlation
2) Negative Correlation

1) Positive Correlation :-

If the 2 variables move in the same direction i.e. an increase in one variable introduce increase in another variable are known to be positively correlation.
e.g. Height and weight, profit and investment.

2) Negative Correlation :-

If the 2 variable mover in the opposite direction i.e. increase in 1 variable result a decrease in another variable, then the 2 variables are known to have a negative correlation e.g. The Price and Demand.
* The two variables are known to be uncorrelated if the movement on the part of 1 variable does not produce any movement on the other variable.
e.g. Shoe size and intelligence.

* **Measure of Correlation :-**

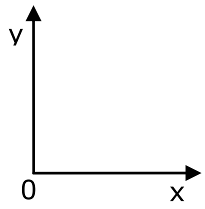
1) Scatter Diagram
2) Karl pearson's product moment correlation coefficient
3) Spearmans rank correlation coefficient
4) Coefficient of concurrent deviation.

1) Scatter Diagram :-

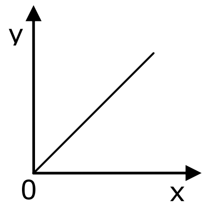
It is simple diagrammatic method used to establish relation between a pair of variable having linear or curvilinear relationship. **But It fails to measure the extent of relationship**.

Each data point (X_i Y_i) is represented on rectangular axes of coordinates. The set of all such points gives the scatter diagram.

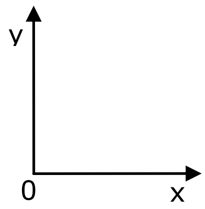
Maths Formula



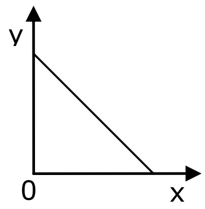
-ve Correlation
(0 < r < 1)



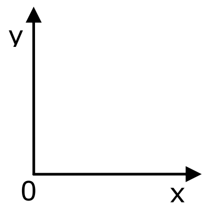
Perfect +ve
Correlation
(r = 1)



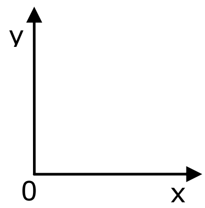
-ve Correlation
(-1 < r <0)



Perfect +ve
Correlation
(r = -1)



No Correlation
(r = 0)



Curvilinear
Correlation
(r = 0)

2) <u>Karl Pearson's Product Moment Correlation Coefficient (r)</u> :-

It is defined as the ratio of covariance between 2 variables to the product of the standard deviations of the 2 variables.

i.e. $r = y_{xy} = \frac{Cov(x,y)}{s_x \times s_y} = \frac{Cov(xy)}{6x.6y}$

Where,

$$cov(x, y) = \frac{\sum(X_i - \bar{x})^2(Y_i - \bar{y})}{n}$$

$$cov(x\ y) = \frac{\sum X_i Y_i}{n} - \bar{x}.\bar{y}$$

and, $S_x = \sqrt{\frac{\Sigma(X_i - \bar{x}^2}{n}} = \sqrt{\frac{\Sigma X_i^2}{n} - \bar{x}^2}$ ---(**)

$6y = S_y = \sqrt{\frac{\Sigma(Y_i - \bar{y}^2}{n}} = \sqrt{\frac{\Sigma Y_i^2}{n} - \bar{y}^2}$

A single formula for computing correlation coefficient is,

$$r = \frac{n\Sigma X_i Y_i - \Sigma X_i \times Y_i}{\sqrt{n\Sigma x_i^2 - (\Sigma x_i)^2}\sqrt{n\Sigma y_i^2 - (\Sigma y_i)^2}}$$

*** <u>In Case of Bivariate Distribution</u> :-**

$$cov(x, y) = \frac{\sum_{i,j} x_i y_i f_i j}{N} - \bar{x} X \bar{y}$$

$$S_x = \sqrt{\frac{\Sigma f_i x_i^2}{N} - \bar{x}^2}$$

$$S_y = \sqrt{\frac{\Sigma f_i y_i^2}{N} - \bar{y}^2}$$

Single formula is given by,

$$r = \frac{N\sum_{ij} fijuivj - \Sigma fjvj}{\sqrt{N\Sigma fiui^2 - (\Sigma fiui)^2}\cdot\sqrt{N\Sigma fivj^2 - (\Sigma fjvj)^2}}$$

*** <u>Properties</u> :-**

1) The coefficient of correlation is unit free measure.
2) The coefficient of correlation has no effect of change of origin.
3) If $u = \frac{x-a}{b}$ and $v = \frac{x-b}{d}$

Then,

$$r_{xy} = \frac{b \cdot d}{|b| \cdot |d|} \cdot r_{uv}.$$

4) The coefficient of correlation always lies between –1 and 1 including both the limit

$$-1 \le r \le 1$$

3) <u>Spearmon's Rank Correlation Coefficient</u> :-

It is used to find correlation between 2 qualitative characteristics and it can also be applied to find the level of agreement between 2 judges.

It is given by,

$$r_R = 1 - \frac{\sigma\sum di^2}{n(n^2-1)}$$

r_R = Rank correlation coefficient,
$-1 \le r \le 1$
$di = x_i - y_i$
= difference in ranks
n = No. of individuals.

In case 'u' indivisuals gets the same rank, the above formula becomes.

$$r_R = 1 - \frac{\sigma\left[\Sigma di^2 + \Sigma\frac{(tj^3 - tj)}{12}\right]}{n(n^2-1)}$$

Maths Formula

*** <u>Coefficient of Concurrent Deviation</u> :-**

This is very simple and casual method,

In this we attatch a +ve sign for a x – value (except the first) if this value is more than the previous value and assign a –ve value if this value is less than the previous value. This is done for the y – series as well.

If both (x, y) the deviations are same, then it is known as “concurrent’ deviation.

The coefficient of concurrent deviation is given by, $rc = \pm\sqrt{\pm\frac{(2c-m)}{m}}$

c = No. of concurrent deviations
m = Total No. of deviations
If 2c – m < o we take +ve
Sign both inside and outside

And

If 2c – m < o we take –ve
Sign both inside and outside.

$$-1 \le rc \le 1$$

*** <u>Regression Analysis</u> :-**

In this we find or estimate the value of 1 variable for a given value of another variable. on the basis of an average mathematical relationship between 2 variables.

In case of simple regression of ‘y’ depends on ‘x’ then the regression line of ‘y’ on ‘x’ is given by

$$y = a + bx$$

where,

a, b = regression parameter

also, $a = \bar{y} - b\bar{x}$

And b = regression coefficient of y on x = b_{yx}

$$b_{yx} = \frac{cov(x,y)}{S_x^{\,2}}$$

$$b_{yx} = r \cdot \frac{s_y}{s_x}$$

$$b_{yx} = \frac{n\Sigma xiyi - \Sigma xi \cdot \Sigma yi}{n\Sigma xi^2 - (\Sigma xi)^2}$$

Similarly,

If x depends on, then the regression line of x on y is

$$x = z + b'y$$

$$b' = b_{xy} = \frac{cov(x,y)}{s_y^2} = r \cdot \frac{s_x}{s_y}$$

$$a' = \bar{x} - b'\bar{y}$$

also, $b_{xy} = \frac{n\Sigma xiyi - \Sigma xi \cdot \Sigma yi}{n\Sigma yi^2 - (\Sigma yi)^2}$

*** Properties of regression line :-**

1) The regression coefficients (b_{yx} and b_{xy}) remain unchanged due to shift of origin.
2) But changes due to shift of scale.

   If $u = \frac{x-a}{p}$, $v = \frac{y-b}{q}$

   Then, $b_{yx} = \frac{q}{p} X\, b_{vu}$

   $b_{xy} = \frac{p}{q} X\, b_{uv}$
3) 2 Lines of regression intersect at a point ($\bar{x}$, $\bar{y}$)
4) The coefficient of correlation between 2 variables x and y is the simple geometric mean of the 2 regression coefficients.

   $r = \pm\sqrt{byx \cdot bxy}$

   r takes the common sign of both.

*** <u>IMP Points</u> :-**

1) The best measure of correlation is provided by persons correlation coefficient.
2) Pearson’s correlation coefficient is only applicable to linear relationship between variables.
3) If x and y are independent then correlation coefficient is zero, but converse is not always true.
4) If in a relation the 2 variables are not causally related due to the existence of third variable, such a correlation is known as spurious correlation or non-sense correlation.
5) The square of the correlation coefficient is known as coefficient of determination.

   $r^2 = \frac{Explained Varience}{Total Varience}$
6) Coefficient of non-determination

   $= 1 - r^2$
7) If r = 1 or –1, the two lines of regression coinside
8) If r = 0, the regression lines are perpendicular to each other.

13 – Probability and Mathematical Expectation

*** Introduction :-**

Probability is the branch of Mathematics.

The 2 broad divisions of probability are
i) Subjective probability.
ii) Objective probability.

The subjective probability is basically dependent on personal judgment and experience and it can be used in field of decision making management.

*** Important Terms :-**

1) Experiment :-
It is the performance or act that produces certain result.

2) Random Experiment :-
It is the experiments whose results are depend on chance **i.e.** uncertain.
e.g. Tossing a coin, rolling a dice.

3) Events :-
The results or outcomes of random experiment are known as events.

4) Simple or Elementary Event :-
The event which cannot be decomposed into further events.
e.g. Tossing a coin once.

5) Composite or Compound Event :-
The event that can be decomposed into two or more events.
e.g. Getting head when coin tossed twice.

6) Mutually Exclusive or Incompatible Events :-
If the occurrence of 1 event implies the non occurrence of other event.
i.e. only 1 event can occur simultaneously, then such events are called mutually exclusive events.
e.g. If a coin is tossed we get 2 mutually exclusive event head and tail.

7) Exhaustive Events :-
The total number of possible outcomes of a random experiment is called exhaustive events.

8) Equally likely Events :-
Two or more events are said to equally likely if the chance of their happening is equal.
e.g. head and tail are equally likely events in tossing an unbiased coins.

9) Independent Event :-
If the occurrence of 1 event does not affect the occurrent of another event then the events are said to be independent event.

10) Dependent Events :-
If the occurrence of 1 event affects the occur rent of other subsequent event then the events are said to be dependent events.

11) Complementary Events :-
The complement of an event means non-occurrence of that event.
It is denoted by, A’, A^c or $\overline{A}$.

12) Favorable Cases :-
The number of outcomes which result in the happening of a desired event are called favorable cases.

*** Classical Definition of Probability (P) :-**

Probability of occurrence of the event A is defined as the ratio of the number of events favorable to A to the total No. of events.

$$P(A) = \frac{\text{No. of equally likely events favourable to A}}{\text{Total no. of equally likely events}}$$

*** Remarks :-**
1) $0 \le P(A) \le 1$
2) $P(A) + P(A') = 1$

*** Limitations :-**
1) It is applicable only when the total No. of events is finite.
2) It can be used only when the events are equally likely.
3) This definition is confined to the problems of games of chance.

*** Statistical Definition :-**

The probability of A is defined as the limiting value of the ratio of F_A to ‘n’ as ‘n’ tends to infinity.

$$\text{i.e. } P(A) = \lim_{n \to \infty} \frac{F_A}{n}$$

It is based on Relative frequency.

*** Set Theoretic Approach :-**

1) $P(A) = \dfrac{n(A)}{n(S)}$

2) If A and B are mutually exclusive then
i) $P(A \cap B) = O$
ii) $P(A \cup B) = P(A) + P(B)$

3) If A and B and C are mutually exclusive then
$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

4) 2 events A and B are exhaustive if
$P(A \cup B) = 1$

5) 3 events A, B and C are exhaustive if
$P(A \cup B \cup C) = 1$

6) If A, B and C are equally likely then
$P(A) = P(B) = P(C)$

*** Axiomatic or Modern Definition :-**

P (A) is defined as the probability of A i P satisfies the following axioms.

i) $P(A) \geq 0$ for every $A \subseteq S$
ii) $P(S) = 1$
iii) For any sequence of mutually exclusive events A_1, A_2, A_3 ....

$P(A_1 \cup A_2 \cup A_3 \cup ....) = P(A_1) + P(A_2) + P(A_3) + ...$

*** Addition theorems :-**

1) Theorem 1 :-

For 2 mutually exclusive events A and B
$P(A \cup B) = P(A) + P(B)$

2) Theorem 2 :-

For n mutually exclusive events $A_1, A_2, A_3, ... A_n$
$P(A_1 \cup A_2 \cup .... \cup A_n) = P(A_1) + ... P(A_n)$
Extension of theorem 1

3) Theorem 3 :-

For any 2 events A and B
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

4) Theorem 4 :-

For any 3 events A, B and C
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

*** Compound probability or Joint Probability :-**

The probability of occurrence of two events A and B simultaneously is known as compound probability. And is denoted by $P(A \cap B)$.

*** Dependent Events :-**

i) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

ii) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

Where,
A and B are dependent event.

$P\left(\frac{B}{A}\right)$ read as probability of event B given that the event A has already occurred.

*** For Independent Events :-**

i) $P\left(\frac{B}{A}\right) = P(B)$

ii) $P\left(\frac{A}{B}\right) = (A)$

iii) $P(A \cap B) = P(A) \times P(B)$
iv) $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$
v) If A and B are independent then following pairs are also independent.
a) A and B'
b) A' and B
c) A' and B'

*** Theorems of Compound Probability :-**

1) Theorem 1 :-

$P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right)$

2) Theorem 2 :-

$P(A \cap B \cap C) = P(A) \times P\left(\frac{B}{A}\right) \times P\left(\frac{C}{A \cap B}\right)$

*** Random Variable–Probability Distribution :-**

1) Random Variable or stochastic variable :-

It is denoted by capital latter.
e.g. If a coin is tossed 3 times and X denotes the No. of heads, then X is a random variable.
further it can be classified as
a) Discrete random
b) Continuous random

a) Discrete random variable :-

it is the random variable which assumes finite number or countably infinite number of values.
e.g. No. of petals in a flower

b) Continuous random :-

it is the random variable which assumes uncountably infinite number of values.
e.g. like Height, Weight etc.

* **Probability Distribution of Random Variable :-**

It is defined as a statement expressing the different values taken by random variable and the corresponding probabilities.
e.g. If a random variable X assumes a finite value, X_1, X_2, X_3, ... with corresponding probabilities P_1, P_2, P_3, ... P_n
Such that,
i) $Pi \geq 0$ ii) $\sum Pi = 1$

Then. probability distribution is given by

X	X_i	X_2	X_3 .... X_n	Total
P	P_1	P_2	P_3 ....P_n	1

* **Probability Mass Function :-**

If X is discrete variable and $f(x)$ really exists, then $f(x)$ is known as probability mass function (pmf) of x, $f(x)$ must satisfy.
i) $f(x) \geq 0$ for every x
ii) $\sum f(x) = 1$
Where, $f(x) = p(x) = x$

* **Probability Density Function :-**

If $f(x)$ is continuous random variable defined over an interval $[\propto, \beta]$ and $f(x)$ is exist in that interval, then it is called as probability density function (pdf) it must satisfies the following condition.
i) $f(x) \geq 0$ for $x \in [\propto, \beta]$
ii) $_{\propto}\int^{\beta} f(x) \, . \, dx = 1$

* **Expected Value or Expectation of Random Variable :-**

It is defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.
i.e. $\mu = E(x) = \sum pixi$

i) Expected value of x^2 is
$E(x^2) = \sum pixi^2$

ii) Variance value of x (σ^2)
$\sigma^2 = V(x) = E(x - \mu)^2$
$= E(x^2) - \mu^2$

iii) $\mu_y = a + b\mu_x$

iv) $\sigma_y = |b| \times \sigma_x$
Where, σ = Standard Deviation
μ = Expected Value

* **For Random Variable :-**

* **Proportion of Expected Value :-**

1) $E(k) = k$ k = Constant
2) $E(x + y) = E(x) + E(y)$
3) $E(k \, . \, x) = k \, . \, E(x)$
4) $E(x \, . \, y) = E(x) \, . \, E(y)$

14 – Theoretical Distribution

*** Theoretical Probability Distribution :-**

Distributing the total probability (i.e. one) to different mass point or different class interval is known as theoretical probability distribution.

This may be help in making statistical analysis.

This may be profitably employed to make short terms projections.

There are main 2 types of probability distribution.

I) Discrete Probability Distribution :-

The probability distribution of discrete variable is known as discrete probability distribution.

It again classified into

a) Binomial distribution.
b) Poisson distribution.

II) Continuous Probability Distribution :-

The probability distribution of continuous variable is known as continuous probability distribution

It against classified into.

a) Normal distribution
b) Chi-square Distribution
c) T-Distribution
d) F-Distribution

*** Binomial distribution :-**

It is the most important and frequently used probability distribution.

*** Trial :-**

It is an attempt to produce a particular outcome which is neither certain nor impossible.

*** Characteristics of Trial :-**

1) If trial contains 2 mutually exclusive and exhaustive outcomes. The occurrence of one is known as success and non occurrence is known as failure.
2) Trials are independent.
3) Probability of success usually denoted by 'p' hence that of failure by q = 1 – p
4) The number of trials 'n' is a finite positive integer.

Now, A discrete random variable 'r' is defined to follow binomial distribution with parameters n and p to be denoted by X ~ B (n,p) if the probability mass function of 'r' is given by

$$\mathbf{f(r) = P(X=r) = {}^{n}C_r . p^r . q^{n-r} .....}$$

r = 0, 1, 2 ......n, 1= 0 Otherwise

*** Important Points :-**

1) $f(r) \geq 0$ and $\sum f(r) = f(0) + f(1) + \ldots + fn = 1$
   * It is biparametric (n and p). It is parametric (n & P)
2) Mean (μ) n.p
3) Depending on the values of n and p. The B.D may be bi-modal or uni-modal.
4) Mode (μ_0) = The largest integer in (n+1) P.......... (n+1) p $\notin$ 1
   = (n + 1) P and (n+1) P – 1 .... (n + 1) p $\in$ 1.
5) Variance = σ^2 = npq

* Variance is always less than mean.

* Also it has its maximum value at p = 0.5 = q and it is n/4

*** Applications :-**

1) It is applied when the trials ae independent
2) Also each trials has just 2 outcomes
3) Such as coin tossing, sampling inspection plan, genetic experiment and so on.

*** Poisson distribution :-**

It is a theoretical discrete probability distribution which can describe many processes. A random Variable X is defined to follow Poisson distribution with parameter m, to be denoted by X ~ P (m) if the probability mass function of X is given by.

$$f(x) = P(X = x) = \frac{e^{-m} \cdot m^x}{x!}$$

x = 0, 1, 2 ....

*** Important Points :-**

1) $f(x) \geq 0$ and $\sum f(x) = f(1) + f(2) + \ldots = 1$
2) It is known uniparametric distribution (m)
3) Mean, μ = m
4) Varience σ^2 = m
5) Standard Deviation = $\sigma = \sqrt{m}$
6) It is also unimodal or bimodal.
   Mode No. = The largest integer in m, if m $\notin$ 1 m and m – 1 if m $\in$ 1.

7) If n = No. of independent trial of B.D. such that $n \to \infty$
And P = Probability of success, tends to zero.
Then, binomial distribution can be approximated by Poisson distribution with parameter (m = np)
i.e. $B(n, p) \cong p(m)$

* **Applications :-**

It is applied when the total No. of events are large but the probability of occurrence is very small.

1) The distribution of printing mistakes per page of large book.
2) The distribution of no of road accidents on a busy road per minute.
3) The distribution of no of radio active elements per minute in a fusion process.

* **Normal or Gaussian distribution :-**

To find the probability distribution continuous random variable we use ** distribution.

It is the most important and universal accepted continuous probability distribution.

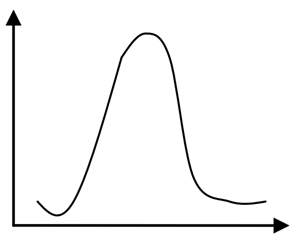
A continuous random variable x, ** defined to follow normal distribution with parameter μ and σ^2 to be denoted ** $X \sim N(\mu, \sigma^2)$ if the pdf of random variable ** given by

$$f(x) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(\bar{x}-\mu)^2}{2\sigma^2}}$$

f or $-\infty < x < \infty$

* **Important Points :-**

1) The probability curve of normal distribution takes the bell shape curve.



2) It is symmetrical about x = u
3) It has one peak i.e. unique mode
4) The total area of the curve is taken unity.
5) Area between $-\infty$ to μ = Area between μ to ∞
6) It is known as biparametric distribution μ and σ^2
7) Mean = median = mode = μ
8) Mean deviation $\cong 0.8\sigma$

Where, σ = Standard deviation

9) First Quartile $Q_1 = \mu - 0.6756$
10) Third Quartile $Q_3 = \mu + 0.675\,\sigma$
11) Quartile Deviation = 0.675σ
12) Its Skewness is zero.
13) **Point of Inflexion :-** The point at which the normal curve changes its curvature from concave to convex and vice versa
14) The normal curve has 2 point of inflexion at $x = u - \sigma$ and $x = u + \sigma$
15) If $x \sim N(u, \sigma^2)$ then $z = x - \frac{\mu}{\sigma} \sim N(0, 1)$, z is know as standardized normal variate or normal deviate.
16) If x & y are independent normal variable with means and SD at σ_1, σ_2 then z = x + y also follows normal distribution with mean $(u_1 + u_2)$ and SD = $\sqrt{6_1^2 + 6_2^2}$

* **Applications :-**

1) Many science subject, social science subjects, commerce management find many application of normal distribution.
2) When n No. of trials of a binomial distribution is large and 'p' the probability of success is moderate then the binomial distribution also tends to normal distribution.

* **CHI – Square Distribution :-**

If a continuous random variable x follows chi – square distribution with n degrees of freedom, to be denoted by $x \sim x_n^2$, then the probability density function of x is given by $f(x)$ $= kk \cdot e^{-\frac{x}{2}} \cdot x^{\frac{x}{2}-1}$

Where, K = const. for $o < x < \infty$

* **Important Properties :-**

i) Mean = u
ii) $\sigma = \sqrt{2n}$
iii) It is positively skewed. **i.e.** the probability curve of chi – square distribution is inclined more on the right

* **t-Distribution :-**

If a continuous random variable ** follows t – distribution with n df, then its pdf is given by

$$f(t) = k \,.\, \left(1 + \frac{t^2}{n}\right)^{-(n+1)/2}$$

*** Important Properties :-**

1) Mean = 0
2) Standard deviation = $\sqrt{\frac{n}{n-2}}$ n > 2
3) Symmetrical about t = o
4) For large n(> 30), t – distribution tends to Standard normal distribution.

*** F – Distribution :-**

It a continuous random variable f – Follows F – distribution with n_1 and n_2 degrees of freedom to be denoted by $f \sim f_{n1.n2}$ then its probability density function is given by

$$f(f) = k \,.\, f^{\frac{n_1}{2}-1}\left(1+\frac{n_1 f}{n_2}\right)^{\frac{-(n_1+n_2)}{2}}$$

*** Important Properties :-**

1) Mean = $\frac{n_2}{n_2-2}$
2) S.D. = $\frac{n_2}{n_2-2}\sqrt{\frac{2(n_1+n_2-2}{n_1(n_2-4)}}$ $n_2 > 4$
3) It has positive Skewness.
4) For large values of n_1 and n_2 F – distribution tends to normal distribution.

15 – Sampling Theory

*** Introduction :-**

It is used to know about the unknown universe on the basis of our knowledge from the known sample.

*** Some Important Terms :-**

1) Population or Universe :-

It may be defined as the aggregate of all the units under consideration and the No. of units belonging to population is known as population size (N).

2) Existent Population :-

A population consisting of real objects is known as existent population.

e.g. Population of Town, Population of lamps produced by a company etc.

3) Hypothetical Population :-

A population consisting of hypothetical objects is known as hypothetical population.

e.g. Population of heads when a coin is tossed infinitely.

4) Sample : - (n)

It is defined as the part of population so selected with a view, to represent the characteristics of population.

5) Parameter :-

It may be defined as a characteristic of a population based on all the units of the population.

e.g.

a) Population mean (μ)

$$\mu = \frac{\sum_{\alpha=1}^{n} x_d}{N}$$

Where,

$x^d = \propto^{th}$ member of population.

b) Population Proportion (P)

$$P = \frac{X}{N}$$

Where,

X = X people possessing attribute.

c) Population Varience (σ^2)

$$\therefore \quad \sigma^2 = \frac{\Sigma(x_\alpha - u)^2}{N}$$

$$\textbf{And SD} = \sigma = \frac{\Sigma(x_\alpha - u)^2}{N}$$

6) Sampling Distribution :-

The probability distribution of the all the values of the statistic (T) of different possible sample is known as sampling distribution.

7) Statistics :- (T)

It may be defined as a statistical measure of sample observations.

*** Standard Error (S.E.) :-**

The Standard deviation of the statistic (T) is known as the standard error (S.E.) of T.

S.E. can be regarded as a measure of precision achieved by sampling.

$$\text{S.E.}(\bar{x}) = \frac{\sigma}{\sqrt{n}} \text{ .... for SRS WR}$$

$$= \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} \text{ .... for SRS WOR}$$

Standard Error for proportion

$$\text{S.E. (p)} = \sqrt{\frac{pq}{n}} \text{ .... for SRS WR}$$

$$= \sqrt{\frac{pq}{n}} \cdot \sqrt{\frac{N-n}{N-1}} \text{ .... for SRS WOR}$$

Where,

SRS WR – simple Random Sampling with Replacement

SRS WOR – Simple Random Sampling without Replacement

$\sqrt{\frac{N-n}{N-1}}$ Finite population's correction (fpc)

*** Basic Principle of Sample Survey :-**

Sample survey is the study of the unknown population on the basis of a proper representative sample drawn from it.

1) Law of Statistical Regularity :-

A/c to this, the sample would posses the characteristic of that population.

2) Principle of Inertia :-

The results derived from a sample are likely to be more reliable accurate and precise as the sample size increases.

3) Principle of Optimization :-

It ensures that an optimum level of efficiency at minimum cost, with the selection of an appropriate sampling design.

4) **Principle of Validity :-**

It states that a sampling design is valid only if it is possible to obtain valid estimates.

We prefer sample survey to complete enumeration due to following factor.

a) **Speed :-**

Sample Survey conducted more quickly.

b) **Cost :-**

For each unit it is more in case of Sample Survey but incase of over all cos Sample Survey likely to be inexpensive.

c) **Reliability :-**

Sample Survey is more reliable due to trained person.

d) **Accuracy :-**

in Sample Survey the sampling error can be reduced to greater extent (these error are absent n complete enumeration). But the non – sampling error can not be controlled in case of complete enumeration.

e) **Necessity :-**

When it cames to destructive sampling, sampling becomes necessity.

*** Errors in Sample Survey ;-**

It may be defined as the deviation between the value of population parameters as obtained from sample normal distribution its observed value.

These are of 2 types.

1) Sampling Error
2) Non – sampling Error

1) **Sampling Error :-**

Since only a part of population is investigated in a sampling, very sampling design is subjected to this type of error.

The factor contributing to sampling errors are listed below.

a) Error arising due to defective sampling design
b) Error arising out due to substitution.
c) Error owing to faulty demarcation of units.
d) Error owing to wring choice of statistics.
e) Variability in the populations.

2) **Non – Sampling Error :-**

This type of errors happen both in sampling and complete enumeration.

Factors responsible for this are:

Lapse of memory, preference for certain digit, ignorance, psychological factors communication gap etc.

*** Type of Sampling :-**

1) **Probability sampling**
   a) Simple Random Sampling
   b) Stratified Sampling
   c) Multistage Sampling
2) **Non – Probability sampling**
   a) Judgment or purposive sampling.
3) **Mixed sampling**
   a) Systematic sampling

1) **Sampling Random Sampling (SRS) :-**

The units are selected independent of each other in such a way that each unit belonging to the population has an equal chance of being a part of the sample.

It is very simple and effective method if.

1) The population is not very large.
2) The sample is not very small
3) Te population under consideration is not heterogeneous.

Also, it is free from sampler's biases.

2) **Stratified Sampling :-**

When a population is large and heterogeneous, then we use stratified sampling.

In this we devide the population into No. of strata or sub-population such that there should be very little variation among the unit of some stratum, and maximum variation between diff. stratum.

*** Purpose :-**

1) To make representation of all the sub population.
2) To provide an estimate of parameter
3) Reduction in variability and thereby an increase in precision.
4) It is not advisable if,
   a) The population is not large.
   b) Some prior information is not available.
   c) There is not much heterogeneity amongst the unit of population.

There are two types of allocation of sample of size.

1) **Proportional allocation of Bowelys allocation :-**

Used when there is not much variation between strata variances. In this sample sizes are taken as proportional to the population size.

2) <u>Neyman's Allocation</u> :-

Used when there is much variation between strata variance. In this sample size vary jointly with population size and population standard deviation.

3) <u>Multi – stage Sampling</u> :-

In this type of complicated sampling, the population is composed of first stage sampling units, each of which in its turn is supposed to compose of second stage sampling units and so on till we reach the ultimate sampling unit.

1) The coverage is quite large.
2) It same computational labor
3) Cost effective
4) Adds flexibility in sampling but compared to stratified it is less accurate.

4) <u>Systematic Sampling</u> :-

It refers to a sampling scheme where te units constituting the sample are selected at regular interval after selecting the vary first unit at random **i.e.** with equal probability.

This type of systematic sampling is known as "liner systematic sampling".

1) Very convenient method
2) Less time consuming
3) Less expensive and simple
4) No statistical interference can be drawn about population parameter.

*** <u>Perposive or Judgment Sampling</u> :-**

The type of sampling is dependent on the samples will and he applies his own judgment based on his belief, prejustice, whims and interest to select sample.

- Non – probabilistic
- Varies from person to person
- No statistical hypothesis can be tested.

*** <u>Theory of Estimation</u> :-**

If the population under consideration is completely unknown and we find the population parameter from our knowledge about the sample observation. Then, this aspect is known as Estimation of population Parameter.

And if some information about population is already known to us and we want to verify that information on the basis of random sample this aspect is known as tests of significance.

1) The point estimation of population mean, varience and proportion are the corresponding sample statistics.

i.e. $\hat{u} = \overline{x}$

$$\sigma = \sqrt{\frac{\Sigma(x_i - \overline{x})^2}{n}} \text{ and ** } = p$$

1) Interval Estimation :-

In this we consider an interval of values which is supposed to contain the parameter i.e. the unknown which we want to find.

16 – Index Number

* **Index Number :-**

It is a ratio or an average of ratios expresses as a percentage.

These are of 2 types.

1) Simple :-

It is computed for 1 variable.

2) Composite :-

It is computed for 2 or more variable most index numbers are composite in nature.

* **Issues involved in the construction of Index Number :-**

1) Selection of data
2) Base period
3) Selection of Weights
4) Use of average
5) Choice of variables
6) Selection of formula

* **Construction of Index Number :-**

We denote the prices during n^{th} period by 'P_n'. And corresponding price during a base period by 'P_o'

And $\sum P_n$ = Summing of prices during n^{th} period for all the commodities.

* **Relatives :-**

It is the simplest example of Index number.

Price Relative = $\frac{P_n}{P_o}$

2) Link Relative :-

When successive price or quantities are taken, the relatives are called the link relative,

$$\frac{P_1}{P_0}, \frac{P_2}{P_1}, \frac{P_3}{P_2}, \ldots\ldots$$

3) Chain Relative :-

When relatives are in respect to a fixed base period these are called the chain relative.

$$\frac{P_1}{P_0}, \frac{P_2}{P_0}, \frac{P_3}{P_0}, \ldots\ldots$$

* **Method of Computing Index Number :-**

Methods

- Simple
  1) Aggregative
  2) Relative
- Weighted
  1) Aggregative
  2) Relative

1) Simple Aggregative Method :-

In this we express the total commodity prices in a given year as a percentage of total commodity prices in the base year.

i.e. $\frac{\Sigma P_n}{\Sigma P_0} \times 100$

2) Simple Average or Relative :-

In this method, we find the relatives of each variable for the base period. And the index number is the average of all such relatives.

* **Weighted Method :-**

1) Weighted Aggregative Method :-

In this method we weigh the price of each commodity of by a suitable factor (quantity or value weight sold during base year or any year).

Example.

a) Laspeyre's Index :-

In this base year quantities are used as weight.

$$\text{L.I.} = \frac{\Sigma P_n P_o}{\Sigma P_0 P_o} \times 100$$

b) Paasche's Index :-

In this current year quantities are used as weights.

$$\text{P.I.} = \frac{\Sigma P_n P_n}{\Sigma P_o P_n} \times 100$$

c) Method based on some typical period :-

In this any year 't' quantities are used weights.

$$= \frac{\Sigma P_n P_t}{\Sigma P_o P_t} \times 100$$

d) Marshall – Edgeworth Index :-

In this we uses average of base year and current year as weight.

$$= \frac{\Sigma P_n(Q_o + Q_n)}{\Sigma P_0(Q_o + Q_n)} \times 100$$

e) Fisher Ideal Index :-

It is geometric mean of Laspeyre's and Paasche's

$$F.I. = \sqrt{\frac{\Sigma P_n Q_o}{\Sigma P_0 Q_o} \times \frac{\Sigma P_n Q_n}{\Sigma P_o Q_n}} \times 100$$

f) Dorbish and Bowley's index No. :-

In this the weighted arithmetic mean is used.

*** Chain Index Number :-**

$$\frac{\text{Link Relative current year X Chain index of previous year}}{10}$$

*** Deflated Value** :-

$$\frac{\text{Current } x \text{ Value}}{\text{Price Index of the Current Year}}$$

*** Shifting Price Index :-**

$$\frac{\text{Original Price Index}}{\text{Price Index of year on which it has to be shifted}} \times 100$$

*** Test of adequacy :-**

1) Unit Test :-

This test requires that the formula should be independent of unit.

Only simple aggregative index does not satisfy this test.

2) Time Reversal Test :-

This method determines weather a given method will work both ways in time, forward and backward i.e. The two indices should be reciprocal of each other.

$$\mathbf{P_{01} \times P_{10} = 1}$$

Where,

P_{01} = Index for time 1 on 0

P_{10} = Index for time 0 on 1

Only Fisher's ideal formula satisfies this test.

For Fishers :-

$$P_{01} = \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0}} \text{ And}$$

$$P_{10} = \sqrt{\frac{\Sigma P_0 Q_1}{\Sigma P_1 Q_1} \times \frac{\Sigma P_0 Q_0}{\Sigma P_1 Q_0}}$$

And $P_{01} \times P_{10} = 1$

3) Factor Reversal Test :-

This holds when the product of price index and the quantity index should be equal to the corresponding value index.

i.e. $P_{01} \times Q_{01} = V_{01}$

this test also satisfies by fisher only and therefore it is called as ideal index number.

4) Circular Test :-

The test of shiftability of base is called the circular Test.

Only simple GM of price relative and the weighted aggregative with fixed weight meets this test.

*** Limitation of Index Number :-**

1) Errors cannot be always avoided.
2) It only depict the broad trend and not the real picture.
3) Different methods gives different value creating confusion.

*** Advantages :-**

1) Provide guidelines for decision making.
2) Reveal trends and tendencies in making important conclusion.
3) Used in forecasting future economic activity.
4) Useful in deflating.
5) Changes in the cost of living measured by index No.

*** Notes :-**

1) GM makes index No. time reversal
2) AM of group indices gives general index.
3) Real wages = $\frac{\text{Wages} \times 100}{\text{index Number}}$