Chp 1 Ratio, Proportion, Indices and Logarithms.

Ratio

Ratio is the comparison of two similar attributes in same units

Ratio

- multiplier
- as a bridging element
- Comparison

$\frac{a}{b}$ a ⇒ antecedent ⇒ first term; b ⇒ consequent ⇒ second term

Types of ratio :- [a:b]

1] Inverse b:a
2] Compound a:b & c:d ac:bd
3] Duplicate $a^2 : b^2$
4] Sub duplicate $\sqrt{a} : \sqrt{b}$.
5] Triplicate $a^3 : b^3$
6] Sub triplicate $\sqrt[3]{a} : \sqrt[3]{b}$
7] Continued a:b:c (a:b, b:c)
8] Commensurable
antecedent & consequent must be integers.
9] In-commensurable
antecedent & consequent are non-integers

Proportion

If two ratios are equal they are said to be in proportion.

Proportion

- Mean proportion
- third proportion
- Fourth proportion

$$\underset{\text{2nd proportion}}{\overset{\text{1st proportion}}{\frac{a}{b}}} : \frac{c}{d}$$

c → 3rd proportion, d → 4th proportion

Cross product rule :-
$ad = bc$
ad ↓ product of means
bc ↓ product of extremes

Properties of proportion :- [a:b]

1] $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$ (Cross product rule)

2] $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$ [Invertendo]

3] $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$ (Alternando)

4] $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} : \frac{c+d}{d}$
[Componendo]

5] $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} : \frac{c-d}{d}$
[Dividendo]

6] $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} : \frac{a+d}{c-d}$
[Componendo - Dividendo]

7] $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then

$\frac{a+c+e+\ldots}{b+d+f\ldots}$ [Addendo]

8] $\frac{a}{b} : \frac{c}{d} : \frac{e}{f}$ then

$\frac{a-c-e-}{b-d-f-\ldots}$ [Subtrahendo]

If a, b, c are in continued proportion then $\frac{a}{b} : \frac{b}{c}$ ie $b^2 = ac$.

↓ mean proportion, ↓ 1st proportion, ↓ 3rd proportion

$b = \sqrt{ac}$

Geometric mean of a & c

When three or more numbers are so related that ratio of first to second, the ratio of second to third and third to fourth etc are all equal the numbers are said to be in continued proportion

Eg:- $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$ then $\frac{a+b+c}{c}$

$\frac{3+4+7}{7} = \frac{14}{7} = 2$

Indices

- It is a power game.
- Factor which multiplies is called base & the no. of times it is multiplied is called power or Index.

Laws of indices:-

1] $a^m \times a^n = a^{m+n}$ [Base must be same]

2] $\frac{a^m}{a^n} = a^{m-n}$

3] $(a^m)^n = a^{mn}$

4] $a^0 = 1$

5] $(ab)^n = a^n . b^n$

6] $a^1 = a$

7] $a^{-m} = \frac{1}{a^m}$, $a^{m} = \frac{1}{a^{-m}}$

8] $a^x = a^y$ then $x = y$
$a \neq 0, 1, y$

9] $x^a = y^a$ then $x = y$
$x, y \neq 0, 1, -1$

10] $\sqrt[m]{a} = a^{\frac{1}{m}}$

11] $\left[\frac{a}{b}\right]^m = \frac{a^m}{b^m}$

12] $a^b = c \Rightarrow a = c^{\frac{1}{b}}$

Note:- $(a+b)^m \neq a^m + b^m$
$(a-b)^m \neq a^m - b^m$

Logarithm.

base value

If $\boxed{a^{x} = n}$ → exponential form : $n > 0,\ a > 0,\ a \neq 1$

↓

$\boxed{\log_a n = x}$ → logarithmic form

$n > 0,\ a > 0$ and $a \neq 1$

Natural no's only

a

Natural logarithm ← → Common logarithm base 10

e ↓

used for calculus used for calculation

e is universal constant

$e = 2.718284590$

Base remains same other two change their place.

Remarks:- 1] $a^0 = 1,\ \log_a 1 = 0$

2] $a^1 = a,\ \log_a a = 1$

Fundamental laws of logarithm:-

1] $\log_a mn = \log_a m + \log_a n$

2] $\log_a \frac{m}{n} = \log_a m - \log_a n$

3] $\log_a m^n = n \cdot \log_a m$

4] $\log_a m = \log_b m \times \log_a b$

$\therefore \log_b m = \frac{\log_a m}{\log_a b}$

[Change of Base]

Logarithm

Whole part → Characteristic

Decimal part → mantissa

5] $\log_b m = \frac{1}{\log_m b}$

6] $\log_b m \times \log_m b = 1$

7] $\log 1 = 0$

8] $\log_a a = 1$

9] $a^{\log_a x} = x$

[Inverse logarithm property]

10] $\log_a m = \frac{\log m}{\log a}$

[Base Changing formula]

11] $\log 10 = 1$

12] If base is understood base is taken as 10.

Main formulas:

1] $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

2] $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

3] $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

4] $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$

5] $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

6] $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

7] $(a+b)^2 - 4ab = (a-b)^2$

8] $(a+b)(a-b) = a^2 - b^2$

9] If $a+b+c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Chp 2 Equations

Linear Equation :-

Equation in which the highest power (degree) is 1.

General form :-
$ax + b = 0$; $a \neq 0$

It has only one root.

Simultaneous linear equations in two unknowns :-

General form :-
$ax + by + c = 0$; $a, b \neq 0$

Methods of solution :-
1] Elimination Method
2] Cross Multiplication Method :-

$$x = \frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1}$$

$$y = \frac{c_1 a_2 - c_2 a_1}{a_1 b_2 - a_2 b_1}$$

Note :- $b^2 - 4ac \geq 0 \Rightarrow$ Real

Quadratic Equation.

Equation in which the highest power (degree) is 2.

General form :-
$ax^2 + bx + c = 0$ $a \neq 0$,
$a, b, c \in R$

It has two roots

$b = 0$ is pure quadratic equation
$b \neq 0$ is affected quadratic

Methods of solution :-
1] Method of factorization
2] Method of substitution
3] Method of formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ = Determinant / Discriminant = Δ

Nature of roots :-
a) $b^2 - 4ac < 0$ Roots are Imaginary
b) $b^2 - 4ac = 0$ Roots are real and equal.
c) $b^2 - 4ac > 0$ Roots are real and unequal (distinct) and Rational & perfect sq.

Or. (Roots are conjugate)
Roots are real and unequal, irrational & not a perfect square.

Formation of a Quadratic Equation:-

$ax^2 + bx + c = 0$.

Let α & β be the roots.

Sum of roots $= \alpha + \beta = \frac{-b}{a}$

Product of roots $= \alpha\beta = \frac{c}{a}$.

$x^2 -$ (Sum of roots) + (Product of roots) $= 0$.

$x^2 - (\alpha+\beta) + (\alpha\beta) = 0$.

Note:-

1] α, β are roots then equation is $ax^2 + bx + c = 0$

2] $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots then equation is $cx^2 + bx + a = 0$.

3] Irrational root occurs in conjugate pairs

4] If one root is reciprocal to other root their product is 1 & so $\frac{\alpha}{\beta} = 1 \therefore \alpha = \beta$.

5] If one root is equal to other root but opposite in sign then their sum is 0 $\frac{\alpha}{\beta} = 0$ ie $\alpha = 0$.

Cubic Equations.

Equation in which the highest power (degree) is 3.

General form
$ax^3 + bx^2 + cx + d = 0$.

α, β, γ are roots of equation.

Sum of roots $= \alpha + \beta + \gamma = \frac{-b}{a}$

Sum of the roots taken two at a time:
$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$.

Product of roots $= \alpha\beta\gamma = \frac{-d}{a}$

To get unique solutions
No. of equations = No. of variables

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ [Infinite Solution]

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ [Unique Solution]

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ [No. Solution]

Chp 3 - Linear Inequalities.

Inequalities:-

Inequalities are statements where two quantities are unequal but a relationship exists between them.

- Linear Inequalities in one Variable and Solution space

Linear function involving inequality sign is a linear inequality.

$>$ ⇒ greater than
$<$ ⇒ less than
$\geq$ ⇒ minimum / atleast
$\leq$ ⇒ maximum / atmost.

$x \geq 0$ $x \leq 0$

Common region where all linear Inequalities are satisfied is called as feasible region or solution set or polygonal convex sets.

The objective function attains maximum or minimum value at one of the corner points of feasible solution known as extreme points of solution set.

Graphical Method.

Step 1: Ignore the inequality sign.
Put $x = 0$ & calculate y $(0, y)$
Put $y = 0$ & calculate x $(x, 0)$

Step 2: Plot the points from Step 1 on graph.
Draw straight lines

Step 3: Shade the region

$x \geq 0$	$x < 0$
then origin side shade inwards.	then non-origin side shade outwards.

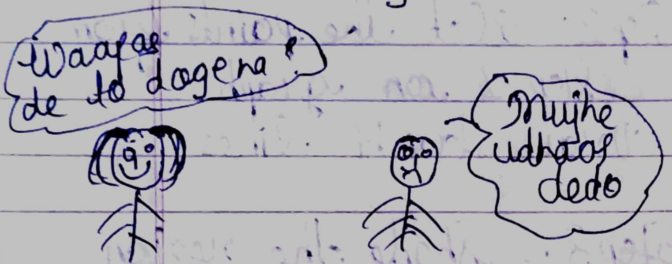
Step 4: Common region for all inequations is the feasible region. By using the points find the maximum or minimum value.

Note:-
When you divide / multiply equation with negative no, then sign of inequality changes.

Chp 4. - Mathematics of finance.

Finance :- Art & Science of managing money.

Interest :- cost of using other's money.



Ms X (Rich) ↓ Lender

Ms Y (Poor) ↓ Borrower.

Risk taker	↓ wants returns.
Reducing his liquidity	↓ wants reward
Time value of money	↓ wants compensation for maintaining same purchasing power
Opportunity cost	↓ wants atleast same profit

Because of all these
↓
Interest is charged

Some Important terms :-

1] Principal = Borrowed money or Invested money

2] Amount / Accumulated amount = Principal + Interest

Types of Interest

Simple Interest | Compound Interest

a] **Simple interest** :- no interest on interest.

Eg :- Ms X borrowed ₹10,000 @ 10% S.I. for 3 years.

Principal	10000
Interest 1st year	+ 1000
Amount 1st year	11000
Interest 2nd year	+ 1,000
Amount 2nd year	12,000
Interest 3rd year	1,000
Amount 3rd year	13,000

Principal = ₹10,000.
Interest for 3 years = ₹3000
Amount after 3 years = ₹13000

$SI = P \cdot r \cdot t$
P = Principal
r = Rate of interest
t = time.

Amount $= P + I$
$= P + Prt$
$= P \cancel{+} [1 + rt]$

In simple interest,
a) for double amount
$r = \frac{1}{t}$ & $t = \frac{1}{r}$

b) for triple amount
$r = \frac{2}{t}$ & $t = \frac{2}{r}$

c) When two time & two amounts are given.
$t_1 \quad t_2 \quad A_1 \quad A_2$

$$r = \frac{A_2 - A_1}{A_1 t_2 - A_2 t_1}$$

d) When two rates and two amounts are given.
$r_1 \quad r_2 \quad A_1 \quad A_2$

$$t = \frac{A_2 - A_1}{A_1 r_2 - A_2 r_1}$$

b) Compound Interest :-
Interest on interest is calculated.

Eg:- Ms X borrowed ₹2,00,000 @ 20% for 3 years.

Principle	₹ 2,00,000.
Interest 1st year	40,000
Amount 1st year	2,40,000
Interest 2nd year	48,000
Amount 2nd year	2,88,000
Interest 3rd year	57600
Amount 3rd year	3,45,600

Principal = ₹2,00,000.
Amount = ₹3,45,600.
Interest = ₹1,45,600.

$$\text{Amount} = P\left[1 + \frac{r}{m}\right]^{t \times m}$$

m = No. of conversion period in a year.
for annually ⇒ m = 1.
for semi annually ⇒ m = 2
for quarterly ⇒ m = 4
for monthly ⇒ m = 12
for daily ⇒ m = 365.

If direct Compound Interest then

$$CI = A - P$$
$$= P\left[1+\frac{r}{m}\right]^{t\times m} - P$$
$$= P\left[\left(1+\frac{r}{m}\right)^{t\times m} - 1\right.$$
$$CI = P\left[\left(1+\frac{r}{m}\right)^{t\times m} - 1\right]$$

Eg:- $P = 100$, $A = 200$
$r = 10\%$ annually find t

$$A = P\left[1+\frac{r}{m}\right]^{t\times m}$$
$$200 = 100\left[1+\frac{0.10}{1}\right]^{t\times 1}$$
$$2 = (1.10)^t$$ Since power is in variable

Put log on both sides
$$\log 2 = \log(1.10)^t$$
$$\log 2 = t.\log(1.10)$$
$$t = \frac{\log 2}{\log(1.10)}$$
$$t = \frac{0.3010}{0.0414}$$
$t = 7.27$ years.

Eg:- $P = 100$, $A = 300$
$t = 8$ years, find r.

$$A = P\left[1+\frac{r}{m}\right]^{t\times m}$$
$$300 = 100\,[1+r]^{8}$$
$$3 = (1+r)^{8}$$ Since base is variable
$$\left(3^{\frac{1}{8}}\right) = 1+r$$
$$\left(3^{\frac{1}{8}}\right) - 1 = r$$
$$r = 1.1472 - 1$$
$$r = 0.1472$$
or
$$r = 14.72\%$$

Depreciation

↓

Decrease in value of asset.

↓

C = Cost
d = Rate of depreciation
t = Time period for which asset is used
S.V = Scrap value ie value of asset after 't' yrs.

$$S.V = C\,[1-d]^{t}$$

Nominal Rate

↓

Any rate which is compounded
⇒ Monthly
⇒ Quarterly
⇒ Semi-annually
⇒ Daily.

[Interest shown is less but interest received is high]

Effective Rate

↓

Any annual compounding rate

[Interest shown and received both are same]

Nominal rate can be converted to effective rate

$$r_e = \left[1+\frac{r}{m}\right]^m - 1$$

Eg:- 8% semi annually is nominal rate

⇓

It can be converted in annually

$$r_e = \left[1+\frac{r}{m}\right]^m - 1$$

$$= \left[1+\frac{0.08}{2}\right]^2 - 1$$

$$= (1.04)^2 - 1$$

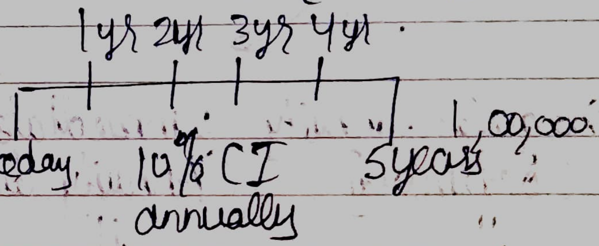
$$= 0.0816$$

or 8.16% Effective rate

Present value of an amount which is to be received in future

$$\text{Present value} = \frac{\text{Future value}}{\left[1+\frac{r}{m}\right]^{t \times m}}$$

Eg:-



After 5yrs you will receive ₹1,00,000 but if you want that money today you won't received ₹1,00,000 you will get less money.

$$P.V = \frac{1,00,000}{\left[1+\frac{0.10}{1}\right]^5}$$

$$= \frac{1,00,000}{1.61051}$$

$$= ₹62,092$$

↓

Today you will received this much.

Annuity.

↓

⇒ Sequence of payments (receipts)

⇒ Same payment (5000, 5000, ...)

⇒ Same time intervals between two payments (3m, 3m, ...)

Eg:- LIC premium.
House loan EMI.

Two types of Annuity.

Regular Annuity (Ordinary)	Immediate (Annuity due)
⇓	⇓
When regular payment are made at the end of period	When regular payments are made in beginning of each period.
⇓	
Eg:- House loan, EMI.	Eg:- LIC premium
[At end of every month, bank will deduct money from A/C]	[At start of every month, bank will deduct money from A/C]

i) Regular Annuity.

Future value :- When benefit of many periodic investments received in future

$$FV = R\left[\frac{(1+i)^{n}-1}{i}\right]$$

R — Regular periodic payment

$$i = \frac{r}{m}$$

$n = t \times m$ [Total payments]

Eg:- You started investing ₹5000 every year in a recurring deposit which gives 10% C.I. How much will you receive after 15 years?

Future value =

$$5000\left[\frac{(1+0.10)^{15}-1}{0.10}\right]$$

= ₹1,58,862

Present value: When monetary benefit is received today then it is repaid in installments.

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Eg:- Mr X. took a loan of ₹5,00,000 which is to be repaid in 5 yearly installments. Its rate of interest is 6% compounded annualley. Find the value of each instalement.

$$P.V =$$

$$5,00,000 = R\left[\frac{1-(1+0.06)^{-5}}{0.06}\right]$$

$$5,00,000 = R(4.2123)$$

$$R = \frac{5,00,000}{4.2123}$$

$$R = ₹1,18,700$$

b) Immediate Annuity:-

$$F.V = R(1+i)\left[\frac{(1+i)^n-1}{i}\right]$$

$$PV = R(1+i)\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Perpetuity.

⇓

Annuity for infinite period.

Ordinary Perpetuity

⇓

$$P.V = \frac{R}{i}$$

Immediate Perpetuity

⇓

$$P.V = \frac{R}{i}(1+i)$$

or

$$\frac{R}{i} + R$$

Eg:- How much you should invest now to receive ₹10,000 at the end of every year for indefinite period if rate of discount is 8% C.I annually?

$$PV = \frac{R}{i} = \frac{10000}{0.08} = 1,25,000$$

Eg:- How much you should invest now to receive ₹10,000 in the beginning of every year starting from today for indefinite period if rate of discount is 8% C.I annually?

$$PV = \frac{R}{i}[1+i] = \frac{10000(1+0.08)}{0.08}$$

$$= 1,35,000.$$

Growing Perpetuity.

⇓

When regular payment is received with growth for infinite period

Regular | Immediate

$$P.V = \frac{R}{i-g}$$

$$PV = \frac{R}{i-g}(1+I)$$

Sinking fund

⇓

Saving money out of profit so that you can purchase an asset in future or you can pay off a liability in future

⇓

saved amount is invested in future & Interest is earned

R = saving   F.V of regular annuity will be used

Benefit received in future

Eg:- Suppose you have to payoff a liability after 10 years for that you are required ₹ 50,000 after 10 years. How much you should invest in sinking fund which gives 12% p.a compounded annualy ~~so~~ that you can fulfill your future needs?

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$5,00,000 = R\left[\frac{(1.12)^{10} - 1}{0.12}\right]$$

$$5,00,000 = R(17.54)$$

$$R = ₹28,506.$$

Net Present Value

⇓

used for taking investment decision.

⇓

NPV = Present value of all cash inflow − Present value of all cash outflow

If NPV >0 accept proposal
If NPV <0 reject proposal

Eg:- In a project ₹50,000 spend today. It generated ₹30,000 after 1 yr and ₹60,000 after 4 yrs. Find NPV of project if discount rate is 10% annually.

$$NPV = \frac{30000}{(1.10)} + \frac{60,000}{(1.10)^2} - 50,000$$

$= 27272.72 + 49586.77$
$- 50,000$
$= 26859.49.$

Leasing

⇓

Taking Asset on rent
[For long period]

Owner of Asset ⇓ Lesser

User of Asset ⇓ Lessee

In this type of question purchase value of asset will be given.

But problem is we don't know how much should be annual rent?

⇓

Use P.V of Regular Annuity to find reasonable rent.

⇓

$$\text{Cost of Asset} = \text{Rent}\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Eg:- Mr X can purchase a machine for ₹50,000 today or he can take the machine on lease for 7 years in ₹9000 Annual rent. Which option is better? Purchasing or Leasing if money is worth 10% Annuity.

In this question find reasonable rent & then compare it with actual rent (ie 9000)

$$50,000 = \text{Rent}\left[\frac{(1-(1+i)^{-n}}{i}\right]$$

$$50,000 = R\left[\frac{1-(1.10)^{-7}}{0.10}\right]$$

$$50,000 = R[4.8684]$$

$$R = 10,270.$$

Reasonable rent is ₹10,270

But he is getting machine on lease in much more cheaper price (ie 9000) so he should go for leasing

Valuation of Bond.

$\Downarrow$

Price at which bond should be purchased = Present value of all future Interest + Present value of bond's maturity value

Eg:- Bond of ₹500 at which interest rate is 10% & maturity period is 4 years. At what price it should be purchased if investor wants 15% return.

$$\text{Price of bond} = \frac{50}{(1.15)} + \frac{50}{(1.15)^2} + \frac{50}{(1.15)^3} + \frac{50}{(1.15)^4} + \frac{500}{(1.15)^4}.$$

$= 43.47 + 37.80 + 32.87 + 28.58 + 285.87$

$= ₹428.59.$

CAGR (Compound Annual Growth Rate)

Eg:-

Time	2010	2011	2012	2013
Revenue	100	110	140	160.

$P = 100 \mid A = 160.$

$\text{Time} = 2013 - 2010 = 3\text{ yrs.}$

$A = P\{1+r\}^t$

$160 = 100[1+r]^3$

$\left(\frac{160}{100}\right) = (1+r)^3$

$\left(\frac{160}{100}\right)^{\frac{1}{3}} = 1+r.$

$r = \left(\frac{160}{100}\right)^{\frac{1}{3}} - 1$

or

$$CAGR = \left[\frac{V(t_n)}{V(t_0)}\right]^{\frac{1}{t_n - t_0}} - 1.$$

$= \left(\frac{160}{100}\right)^{\frac{1}{2013-2010}} - 1$

$= \left(\frac{160}{100}\right)^{\frac{1}{3}} - 1.$

$= 1.1696 - 1$

$= 0.1696$

or

$16.96\%.$