Quadratic Equation

$ax^2 + bx + c = 0$ where $\ \ = 0$

⇒ Roots of Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒ Nature of roots

1) If $b^2 - 4ac = 0$, Roots are real and equal.
2) If $b^2 - 4ac > 0$, Roots are real and unequal.
3) If $b^2 - 4ac < 0$, Roots are imaginary.
4) If $b^2 - 4ac > 0$ and perfect square, Roots are real, rational, unequal
5) If $b^2 - 4ac < 0$ but not perfect square, roots are real, irrational and unequal.

Sum of roots of quadratic equation

$$\alpha + \beta = \frac{-b+\sqrt{\Delta}}{2a} + \frac{-b-\sqrt{\Delta}}{2a}$$

$$= \frac{-b+\sqrt{\Delta}-b-\sqrt{\Delta}}{2a}$$

$$= \frac{-2b}{2a} = \frac{-b}{a}$$

$\therefore$ Sum of roots $= \frac{-b}{a}$

Product of Roots

$$\alpha\beta = \frac{-b+\sqrt{\Delta}}{2a} \times \frac{-b-\sqrt{\Delta}}{2a}$$

$$= \frac{b^2-\Delta}{4a^2} = \frac{b^2-(b^2-4ac)}{4a^2}$$

$$= \frac{\cancel{b^2}-\cancel{b^2}+4ac}{4a^2} = \frac{\cancel{4a}c}{\cancel{4a^2}} = \frac{c}{a}$$

Some Important points :-

1) If two roots are equal in magnitude but opposite in sign then sum $-b/a = 0$, $b=0$
2) If one root is reciprocal of another then product is 1
   $c/a = 1$, $c = a$
3) Irrational roots and imaginary roots always occur in conjugate pair.
   e.g. If one root is $2+\sqrt{3}$ the other will be $2-\sqrt{3}$
   If one root is $3+2i$, the other will be $3-2i$

$\Rightarrow$ Construction of quadratic equation with given roots.

x^2 - (sum of root) x + Product of roots = 0

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

1) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= \frac{b^2 - 2ac}{a^2}$$

2) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

3) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$.

a	b	c	Roots
+	−	+	Both roots positive
+	+	+	Both roots negative
+	±	−	One root positive & one negative.

CUBIC EQUATION

Standard Equation :-

$$ax^3 + bx^2 + cx + d = 0$$

Roots are α, β, γ

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$