Sequence And Series

a = First Term
d = common difference = $T_n - T_{n-1}$
l = Last Term = T_n
T_n = nth term
S_n = Sum of n terms

First term = $T_1 = a$
2^{nd} term = $T_2 = a + d$
3^{rd} term = $T_3 = a + 2d$
4^{th} term = $T_4 = a + 3d$
⋮
n^{th} term = $T_n = a + (n-1)d$

$T_{n} \rightarrow$ positive integer

$$n = \frac{l-a}{d} + 1$$

If an AP's m^{th} term is n and n^{th} term is m
then, (a) $(m+n)^{th}$ term is zero
(b) r^{th} term is $m+n-r$

i.e. $T_m = n$, $T_n = m$
$T_{m+n} = 0$, $T_r = m+n-r$

If $mT_m = nT_n$, Then $T_{m+n} = 0$
e.g. $20T_{20} = 30T_{30}$, $T_{50} = 0$.

Sum of n terms of an A.P.

$$S_n = \frac{n}{2}[a+l] \quad ; \quad l = a + (n-1)d$$

$$= \frac{n}{2}[2a + (n-1)d]$$

AP = difference remains constant

In an AP, If $S_m = S_n$ $(m \neq n)$

then $S_{m+n} = 0$.

e.g. $S_{20} = S_{40}$ $\therefore S_{60} = 0$.

In an AP, if $S_m = n$, $S_n = m$

$S_{m+n} = -(m+n)$

e.g., $S_{10} = 30$ $S_{30} = 10$ $\therefore S_{40} = -40$

In an AP $T_m = \frac{1}{n}$, $T_n = \frac{1}{m}$

1) $S_{mn} = \frac{1}{2}(mn+1)$

2) $T_n = S_n - S_{n-1}$ e.g. $T_{10} = S_{10} - S_9$

If $S_n = Pn^2 + Qn$

$a = P + Q$

$d = 2P$

Arithmetic Mean of a & b = $\frac{a+b}{2}$

Sum of n A.M. btw^n a & b = $\frac{n}{2}[a+b]$

Sum of first n natural numbers = $\Sigma n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Sum of first n even numbers = $n(n+1)$

Sum of first n odd numbers = n^2

Sum of squares of n natural nos. = $\Sigma n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Sum of cubes = $\left[\frac{n(n+1)}{2}\right]^2$

Geometric Progression
- A sequence in which the ratio between any two consecutive terms remain constant is called G.P.

To calculate 'r' devide any term by its preceeding term.

$$r = \frac{T_n}{T_{n-1}}$$

r = common ratio

$T_1 = a$
$T_2 = a \times r$
$T_3 = a \times r^2$
$T_4 = a \times r^3$

n^{th} term of G.P.

$$T_n = a \cdot r^{n-1}$$

T_{p-q}	T_p	T_{p+q}
m	$\sqrt{mn}$	n

Sum of n terms of G.P.

$$S_n = \frac{a[r^n - 1]}{r-1} \quad \text{when } r > 1$$

$$S_n = \frac{a[1 - r^n]}{1-r} \quad \text{when } r < 1$$

$$S_n = na \quad \text{when } r = 1$$

$$S_n = \frac{lr - a}{r-1} \quad \text{if } r > 1$$

$$S_n = \frac{a - lr}{1-r} \quad \text{if } r < 1$$

Sum of Infinite G.P.
when $|r| < 1$

$$S_\infty = \frac{a}{1-r}$$

Arithmetic Mean, Geometric Mean & Harmonic Mean of 2 positive numbers a and b

$$A.M = \frac{a+b}{2}$$

$$G.M = \sqrt{ab}$$

$$H.M. = \frac{2ab}{a+b}$$

1) $AH = G^2$
2) $A > G > H$ if a & b are unequal
3) $A = G = H$ if a & b are equal
4) $A \geq G \geq H$ if $a = b$ or $a \neq b$ is not specified.

Product of n terms of G.P = $(\text{middle term})^n$

If a, b, c are in GP i.e. $b^2 = ac$ and $a^p = b^q = c^r$ then
$p, q, r \rightarrow H.P$
$1/p, 1/q, 1/r \rightarrow A.P.$

If a, b, c are in G.P and $a^{1/x} = b^{1/y} = c^{1/z}$
then $1/x, 1/y, 1/z \Rightarrow HP.$
$x, y, z \longrightarrow A.P.$

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$$

= A.M of a & b if $\longrightarrow n = 0$

= G.M if $\longrightarrow n = \frac{-1}{2}$

= H.M if $\longrightarrow n = -1$

If the ratio between sum of n terms of two A.P. is $an+b : cn+d$ then ratio btwn their m^{th} term
$a(2m-1)+b : c(2m-1)+d$

If a^2, b^2, c^2 are in A.P

$\frac{a}{b+c}, \frac{b}{c+a}, \frac{1}{a+b}$ are in A.P.

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

If a, b, c are in A.P.

$$\left.\begin{array}{l} \frac{\sqrt{a}}{\sqrt{b}+\sqrt{c}}, \frac{\sqrt{b}}{\sqrt{c}+\sqrt{a}}, \frac{\sqrt{c}}{\sqrt{a}+\sqrt{b}} \\ \frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}} \end{array}\right\} \text{are in A.P.}$$