CHAPTER-3

LINEAR INEQUALITIES

INEQUALITIES :

* Inequalities are statements where two quantities are unequal but a relationship exists between them.
* These type of inequalities occur in business whenever there is a limit on supply, demand, sales etc.
* For example, if a producer requires a certain type of raw material for his factory and there is an upper limit in the availability of that raw material, then any decision which he takes about production should involve this constraint also.

LINEAR INEQUALITIES :

* Any linear function that involves an inequality sign is a linear inequality.
* It may be of one variable or of more than one variable.
* Eg : $x > 0$, $x \le 0$, $3x + y < 6$.

SOLUTION SPACE :

* The values of the variables that statisfy an inequality are called solution space / SS / feasible region / solution sets / polygonal convex.

EXTREME POINTS OF THE SOLUTION SET :

* The objective function attains maximum or a minimum value at one of the corner points of the feasible solution known as extreme points of the solution set.

STEPS UNDER GRAPHICAL METHOD

* Formulate the linear programming problem, i.e., express the objective function & constraints in the standardised format.
* Plot the capacity constraints on the graph paper. For this purpose normally two terminal points are required. This is done by persuming simultaneously that one of the constraints is zero. When constraints concerns only one factor, then line will have only one origin point and it will run parallel to the other axis.

* Identify feasible region & coordinates of corner points. Mostly it is done by breading the graph, but a point can be identified by solving simultaneous equation relating to two lines which intersect to form a point on graph.
* Determine the region that satisfies the set of given inequalities.
* Ensure that the region is bounded. If the region is not bounded, either there are additional hidden conditions which can be used to bound the region or there is no solution to the problem.
* Construct the matrix E of the extreme points, and the column vector C of the objective function.
* Find the matrix product EC. For maximization determine the row in EC where the largest element appears; while for minization, determine the row in EC where the smallest element appears.
* The objective function is optimized corresponding to the same row elements of the extreme point matrix E.

NOTE :

* When we come across the words capacity, availability, atmost, maximum in word problems, we have to use $\leq$ sign.

* When we have the words minimum, atleast, we have to use $\geq$ sign.

IMPORTANT NOTE :

* If $a \leq b$, then $-a > -b$.

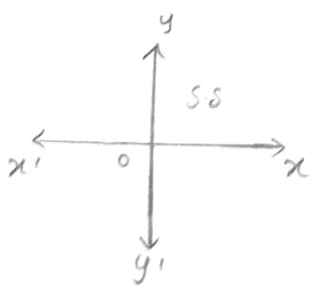
Eg: $2 < 3$ but $-2 > -3$.

* If $a > b$, then $\frac{1}{a} < \frac{1}{b}$.

Eg: $3 > 2$ but $1/3 < 1/2$

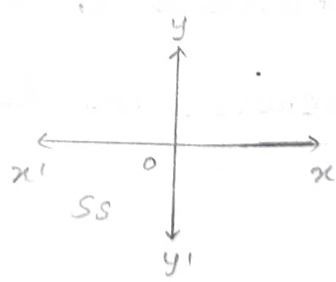
* The inequality sign changes when we multiply both sides of the equation by -1 or when we take reciprocals.

LINEAR INEQUALITIES WITH TWO VARIABLES :



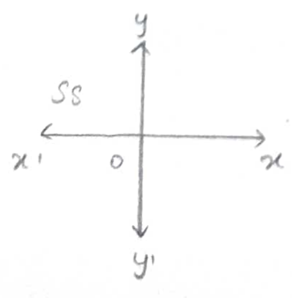
$x \geq 0$

$y \geq 0$



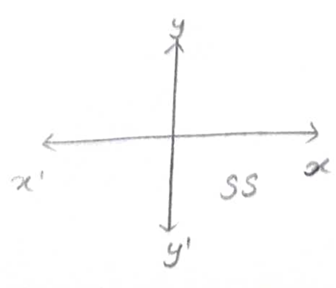
$x \leq 0$

$y \leq 0$



$x \leq 0$

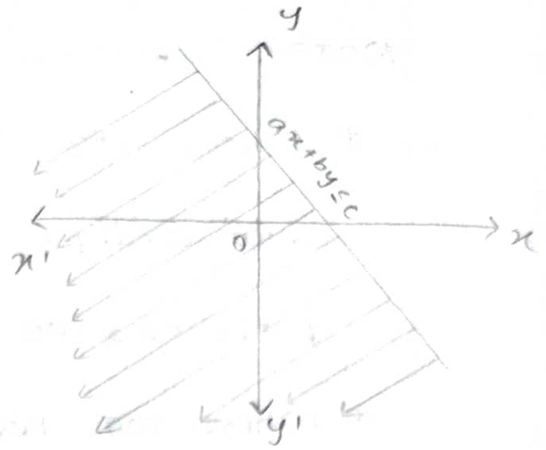
$y \geq 0$



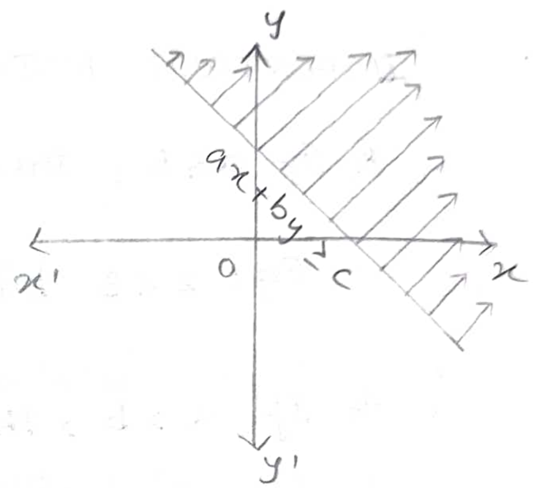
$x \geq 0$

$y \leq 0$

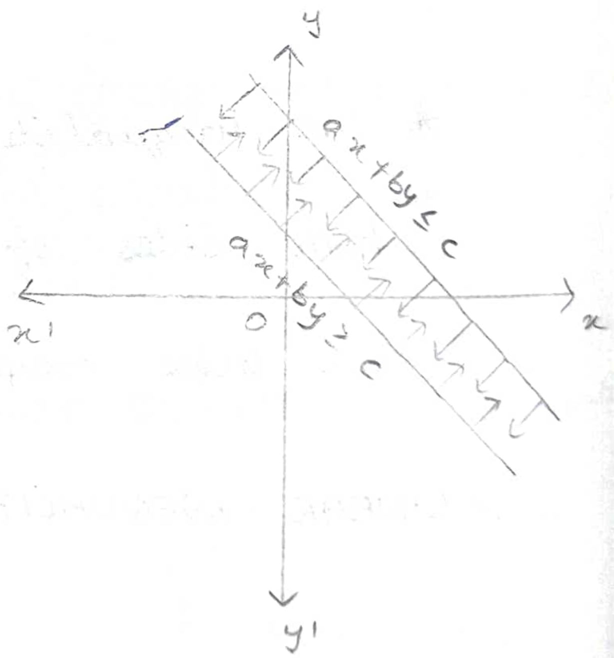
1, If all the inequalities have ≤ sign, then the solution space is bounded & towards the origin.



2, If all the inequalities have ≥ sign, then the solution space is unbounded & away from the origin.

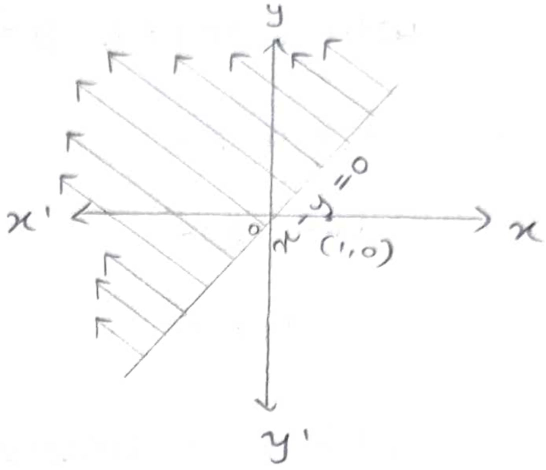


3, If the inequalities have both ≤ & ≥ sign, then the feasible region, if it exists will be bounded & will be between the lines.



LINES PASSING THROUGH ORIGIN:

1) $x - y \leq 0$



when $x = 1$ & $y = 0$,

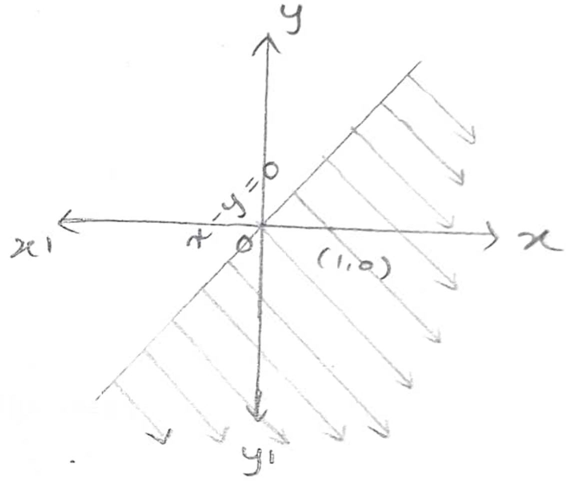
$x - y \leq 0$

$1 - 0 \leq 0$

$1 \leq 0$

This is mathematically false.

2) $x - y \geq 0$



when $x = 1$ & $y = 0$,

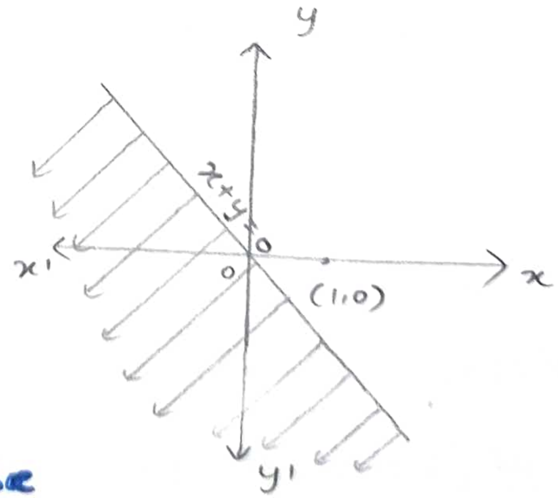
$x - y \geq 0$

$1 - 0 \geq 0$

$1 \geq 0$

This is mathematically true.

3) $x + y \leq 0$



when $x = 1$ & $y = 0$,

$x + y \leq 0$

$1 + 0 \leq 0$

$1 \leq 0$

This is mathematically false

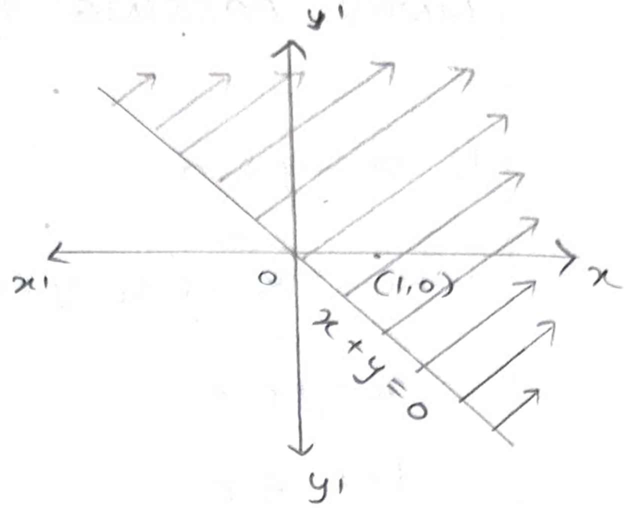
4, $x + y \geq 0$

when $x = 1$ & $y = 0$

$x + y \geq 0$

$1 + 0 \geq 0$

$1 \geq 0$



This is mathematically true.