CHAPTER - 1

RATIO AND PROPORTION, INDICES, LOGARITHMS.

UNIT - 1

RATIO.

RATIO :

* A ratio is a comparison of the sizes of two or more quantities of the same kind by division.
* The ratio of a to b = a/b or a : b.
* The quantities a and b are called the terms of the ratio.
* a is called as the first term or antecedent.
* b is called as the second term or consequent.
* Both terms of a ratio can be multiplied or divided by the same (non-zero) number.
* Usually a ratio is expressed in lowest terms.
* The order of the terms is important.
* Ratio exists only between quantities of the same kind (same units).
* A ratio a : b is said to be greater inequality if a>b and of less inequality if a<b.

INVERSE RATIO :

* One ratio is the inverse of another if their product is 1.
* Inverse of a : b is b : a.

$a:b$
$c:d$
$ac:bd$

COMPOUND RATIO:

* If there are two ratios $a:b$ and $c:d$, then the compounded ratio is $ac:bd$.

DUPLICATE RATIO:

* A ratio compounded of itself is called duplicate ratio.
* Duplicate ratio of $a:b$ is $a^2:b^2$.

TRIPLICATE RATIO:

* The compounded ratio of three equal ratios is called triplicate ratio.
* Triplicate ratio of $a:b$ is $a^3:b^3$.

SUB-DUPLICATE RATIO:

* The sub-duplicate ratio of $a:b$ is $\sqrt{a}:\sqrt{b}$.

SUB-TRIPLICATE RATIO:

* The sub-triplicate ratio of $a:b$ is $\sqrt[3]{a}:\sqrt[3]{b}$

COMMENSURABLE QUANTITIES:

* The ratio of two similar quantities which can be expressed as a ratio of two integers is called as commensurable quantities.

* If a quantity increases or decreases in the ratio $a:b$ then new quantity = b of the original quantity $/a$. The fraction by which the original quantity is multiplied to get a new quantity is called the factor multiplying ratio.

INCOMMENSURABLE QUANTITIES:

* The ratio of two similar quantities which can cannot be expressed as a ratio of two integers is called incommensurable quantities.
* Eg: $\sqrt{3}, \sqrt{2}$

CONTINUED RATIO:

* Continued ratio is the relation (or compassion) between the magnitudes of three or more quantities of the same kind.
* $a:b:c:d$ is a continued ratio.

$t = d/s$.

UNIT - II

PROPORTIONS.

PROPORTIONS:

* An equality of two ratios is called a proportion.
* Four quantities a, b, c, d are said to be in proportion if $a:b = c:d$ $(a:b :: a:d)$.
* The quantities a, b, c, d are called terms of the proportion.
* a, b, c, d are called its first, second, third and fourth terms respectively.
* 1st & 4th terms are called extremes (or extreme term)
* 2nd & 3rd terms are called means (or middle terms)

CROSS PRODUCT RULE:

* Product of extremes = Product of means.
* i.e, $ad = bc$.

CONTINUOUS PROPORTIONS:

* When $a:b = b:c$, it is called as continuous proportions.
* Here, a is the first proportional.
* b is the mean proportional between a and c.
* c is the third proportional.
* $a:b = b:c \Rightarrow \frac{a}{b} = \frac{b}{c}$ i.e., $b^2 = ac \Rightarrow b = \sqrt{ac}$
* When a, b, c, d, e, f and g are in continuous proportion,

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} = \frac{f}{g}$$

NOTE:

* In a ratio $a:b$, both quantities must be of the same kind while in a proportion $a:b = c:d$, all the four quantities need not be of the same type.
* The first two quantities should be of the same kind and last two quantities should be of the same kind.

PROPERTIES OF PROPORTIONS:

* If $a:b=c:d$, then $ad=bc$.
* If $a:b=c:d$, then $b:a=d:c$ [INVERTENDO]
* If $a:b=c:d$, then $a:c=b:d$ [ALTERNENDO]
* If $a:b=c:d$, then $a+b:b=c+d:d$ [COMPONENDO]
* If $a:b=c:d$, then $a-b:b=c-d:d$ [DIVIDENDO]
* If $a:b=c:d$, then $a+b:a-b=c+d:c-d$

  [COMPONENDO & DIVIDENDO]
* If $a:b=c:d=e:f=\dots$, then each of these ratios is equal to $(a+c+e+\dots):(b+d+f+\dots)$ [ADDENDO]

  $(a-c-e-\dots):(b-d-f-\dots)$ [Subtrahendo]

UNIT - III

: INDICES:

INDICES:

* Theory of indices is a study of the behaviour of powers, if we multiply 2 or more terms, and we divide 2 terms.
* A factor which multiplies is called the "base".
* The number of times it is multiplied is called the "power" or the "index".

  Eg: 4^5 => Here 4 is base & 5 is power or index.

LAW -1 [PRODUCT RULE] :

$$a^m \times a^n = a^{m+n}$$

LAW -2 [QUOTIENT RULE]:

$$\frac{a^m}{a^n} = a^{m-n}$$

LAW -3 [WHOLE POWER RULE]:

$$(a^m)^n = a^{mn}$$

LAW-4:

$$(ab)^n = a^n b^n$$

TRANSFORMATION OF POWER :

* If two terms are equal (left side & right side single term), then we can transfer the power from left to right and right to left.

Eg: $16^2 = 2^8$

$16 = 2^{8/2} = 2^4 = 16$.

NOTES:

* $a^0 = 1$

Eg: $2^0 = 1$, $3^0 = 1$.

* $a^{-m} = \frac{1}{a^m}$ & $\frac{1}{a^{-m}} = a^m$

Eg: $2^{-3} = \frac{1}{2^3}$, $\frac{1}{2^{-3}} = 2^3$

* $a^{1/m} = \sqrt[m]{a}$

Eg: $8^{1/3} = \sqrt[3]{8} = 2$

* $a^{m/n} = \sqrt[n]{a^m}$

Eg: $8^{2/3} = \sqrt[3]{8^2} = 4$

* If $a^x = a^y$, then $x = y$.

* If $x^a = y^a$, then $x = y$.

* $(a^m)^n \neq a^{m^n}$.

Eg: $(2^3)^4 \neq 2^{3^4}$

$(2^3)^4 = 2^{3\times4} = 2^{12} = 4096$.

$2^{3^4} = 2^{81} = 2.42\times10^{24}$

UNIT - IV

LOGARITHMS.

LOGARITHMS:

* The logarithm of a number to a given base is the index or the power to which the base must be raised to produce the number, i.e., to make it equal to the given number. If there are three

* If $a^x = n$, where $n>0$; $a>1$

then $\log_a n = x$.

i.e., the logarithm of n to the base 'a' is x.

Eg: $2^4 = 16$

$\Rightarrow \log_2 16 = 4$

FUNDAMENTAL LAWS OF LOGARITHM:

i> PRODUCT RULE:

$$\log_a mn = \log_a m + \log_a n$$

ii> QUOTIENT RULE:

$$\log_a \frac{m}{n} = \log_a m - \log_a n$$

iii> WHOLE POWER RULE:

$$\log_a m^n = n \log_a m$$

iv> RECIPROCAL RULE:

$$\log_n m = \frac{1}{\log_m n}$$

v> CHANGE OF BASE RULE:

$$\cancel{\log_a} m \times \log_n a = \log_n m$$

$$\Rightarrow \log_a m = \frac{\log_n m}{\log_n a}$$

CONFUSING SITUATIONS:

* $\log(m+n)$
* $\log(m-n)$
* $(\log m)^n$
* $\log m \times \log n$.

[There is no formula. They can't be amplified nor simplified. They exists as it is]

NOTES :

* The logarithm of 1 to any base is 0. This is because any number raised to the power zero is one.

$$a^0 = 1 \Rightarrow \log_a 1 = 0.$$

* The logarithm of any quantity to the same base is unity. This is because any quantity raised to the power 1 is that quantity only.

$$a^1 = a \Rightarrow \log_a a = 1.$$

* If power contains log & base of log and theory of indices bases are same, then we can apply this technique.

$$\cancel{a}^{\cancel{\log a} z} = z$$

$$a^{b \log_a z} = \cancel{a}^{\cancel{\log a} z^b} = z^b$$

* If the base of logarithm contain a^n form, we can apply this technique.

$$\log_{a^n} z = \frac{1}{n} \log_a z$$

* If the base of log is not specifies, we can assume bases are same.
* In some situations, we have to assume base as 10 in order to get the answer.
* To apply product rule, quotient rule and whole power rule bases are not needed. But, for applying reciprocal rule and change of base rule, bases must be given.

WHEN AND WHERE TO USE FUNDAMENTAL LAWS OF LOGARITHM:

* Product rule : When addition is noticed.
* Quotient rule : When subtraction is noticed.
* Reciprocal rule : When denominator is to be avoided.
* Change of base rule : When multiplication is noticed.

TYPES OF LOGARITHM :

There are two kinds of logarithm. They are :

* Common or Naperian logarithm.
  [Base of log is 10]
* Natural logarithm - used only in calculus.
  [Base of log is e]

LOGARITHM TABLE :

* The logarithm of a number consists of two parts, the whole part or the integral part is called Characteristics and the decimal part is called the mantissa.

CHARACTERISTIC :

* If the given number has n digits, then its Characteristic is n-1.
* For decimal numbers, after decimal dot, no zero means, then the characterisitic is -1 & 1 zero means, then the characteristic is -2.

NUMBER	CHARACTERISTIC	
4623	3	→ [one less than the number of digits to the left of the decimal point]
37	1	
6·21	0	
0·8	-1	→ [one more than the number of zeros on the right immediately after the decimal point]
0·07	-2	
0·00004	-5	

MANTISSA :

* The mantissa is the fractional part of the logarithm of a given number.
* It is always positive & decimal form.
* It is taken from log table.

STEPS TO FIND MATISSA :

* Take 1st 4 digits of a number (regardless of decimal point). Eg: $\log_{10} \underline{15}.\underline{27}$.
* Now, by using log table, use 15th row & 2nd column.
* The corresponding value is 1818.
* Now, in mean difference, see 15th row and 7th column of mean difference.
* The corresponding value is 20.
* Add both the values.

i.e.,
$$\begin{array}{r} 1818 \\ (+)\ 20 \\ \hline 1838 \\ \hline \end{array}$$

* Therefore, 1838 is the mantissa part.

EXAMPLES:

NUMBER	CHARACTERISTIC	MANTISSA	LOGARITHM.
Log 1527	3	1838	3·1838
Log 152·7	2	1838	2·1838
Log 15·27	1	1838	1·1838
Log 1·527	0	1838	0·1838
Log ·1527	-1	1838	$\bar{1}$·1838
Log ·01527	-2	1838	$\bar{2}$·1838
Log ·001527	-3	1838	$\bar{3}$·1838.

POINTS TO REMEMBER:

* From the above example, we can understand that, for same figures, there will be difference only in the characteristics and mantissa always remains the same.

* It may be noted that $\bar{1}$·1838 is different from -1·1838, i.e., $\bar{1}$·1838 = -1+ 0·1838 = -0·8162

  => Log 0·1527 = $\bar{1}$·1838 = -0·8162

* Here, -1·1838 is a negative number whereas, in $\bar{1}$·1838, 1 is negative number & 0·1838 is a positive number.

ANTILOGARITHMS:

* If x is the logarithm of a given number n with a base, then n is called the antilogarithm (antilog) of x to that base.
* If $\log_a n = x$; then $n = \text{antilog } x$.
* Eg: $\log 61720 = 4.7904$

$\Rightarrow 61720 = \text{antilog } 4.7904$.

* Therefore, we conclude that, whole or integer part of antilog (left side of decimal point) plus one is equal to the number of digits of log.

SURDS:

* $\sqrt{2}, -\sqrt{3}, \sqrt{3}+5, 3\sqrt{5}$ are surds. i.e., when we can't simplify a number to remove a root, then it is called as surd.
* If difference between a^2 & b^2 is 1, then

$$\frac{1}{a-b} = a+b.$$

* Eg: $\frac{1}{3-\sqrt{8}} = 3+\sqrt{8}$ $\left\{\begin{matrix} a=3 ; & b=\sqrt{8} ; & a^2-b^2 = 9-8 \\ a^2=9 ; & b^2=8 ; & a^2-b^2=1 \end{matrix}\right\}$

$\frac{1}{\sqrt{2}-1} = \sqrt{2}+1$ $\left\{\begin{matrix} a=\sqrt{2} ; & b=1 ; & a^2-b^2 = 2-1 \\ a^2=2 ; & b^2=1 ; & a^2-b^2=1 \end{matrix}\right\}$.

$\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ $\left\{\begin{matrix} a=\sqrt{6} ; & b=\sqrt{5} ; & a^2-b^2 = 6-5 \\ a^2=6 ; & b^2=5 ; & a^2-b^2=1 \end{matrix}\right\}$

FORMULAE TO REMEMBER:

* Sum of 1st n terms $= \frac{n(n+1)}{2}$

* Sum of cubes of 1st n terms $= \left[\frac{n(n+1)}{2}\right]^2$

DIRECT PROPORTIONAL:

* When x increases, y will also increases.

$$x \propto y$$

$$\Rightarrow x = ky.$$

INVERSELY PROPORTIONAL:

* When x increases, y will decreases.

$$x \propto 1/y$$

$$\Rightarrow x = k/y.$$

CYCLICAL FORM:

* If the question is in cyclical form, then the answer is 1.

* In question, replace $\underline{a}$ by $\underline{b}$ & $\underline{b}$ by $\underline{c}$ & $\underline{c}$ by $\underline{a}$ and if we get the same question, then we can say question is in cyclical form and so the answer is 1.