

## Ratios

- Ratio is a comparison of two similar attributes in same units.

Ratio		
Multiplier	as a bridging element	Comparison element

- Types of ratios : ( a : b )

- Duplicate  $a^2 : b^2$
- Sub dupl.  $\sqrt{a} : \sqrt{b}$
- TriPLICATE  $a^3 : b^3$
- Sub Tripl.  $\sqrt[3]{a} : \sqrt[3]{b}$
- Compound  $(a : b, c : d) \rightarrow a \times c : b \times d$
- Continued  $\rightarrow a : b : c$  ( a : b, b : c )
- Inverse  $\rightarrow b : a$

## Proportion

- If two ratios are equal they are said to be in proportion.
- Each pair of ratio should have same units.

Proportion		
Mean Proportion	Third Proportion	Fourth Proportion

$$b^2 = \sqrt{ac} \quad \left| \quad \frac{a}{b} = \frac{b}{c} \right.$$

- Product of means = Prod of extremes

- Properties of Proportion {a:b}

- Invertendo  $\frac{b}{a} = \frac{d}{c}$
- Alternendo  $\frac{a}{c} = \frac{b}{d}$
- Componendo  $\frac{a+b}{b} = \frac{c+d}{d}$
- Dividendo  $\frac{a-b}{c} = \frac{c-d}{d}$
- Componendo & dividendo  $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

## Indices

- It is a power game..

- Properties :

- $\sqrt[b]{a} = a^{\frac{1}{b}}$
- $a^b a^c = a^{b+c}$
- $\frac{a^b}{a^c} = a^{b-c}$
- $(a \times b)^c = a^c \times b^c$   
 $(a+b)^c \neq a^c + b^c$   
 $(a-b)^c \neq a^c - b^c$
- $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$
- $a^b = c \rightarrow a = c^{\frac{1}{b}}$
- $a^b = a^c$  then Base same  
 $b = c$  power equate
- Power - same base - equate  
 $a^b = c^b \rightarrow a = c$
- $\frac{1}{a^{-b}} = a^b, \frac{1}{a^b} = a^{-b}$
- $(a)^0 = 1.$

## Logarithms

$$a^b = c$$

$$\downarrow$$

$$\log_a c = b$$

- Always assume base to be 10.
- $\log a + \log b = \log a \times b$
- $\log a - \log b = \log \frac{a}{b}$
- $\log 1 = 0$
- $m \log n = \log n^m$
- $\log_b^a = \frac{1}{\log_a^b}$
- $a^{\log_a x} = x$
- Shortcut :  
Type no.  
Type  $\sqrt{19}$  times  
Type  $-1 \times 227695$

# Equations

## General Form

- 1 variable  $ax + b = 0$
- 2 Variables  $ax + by + c = 0$
- 3 Variables  $ax + by + cz + d = 0$

To get unique solutions  
No. of equations = No. of Variables

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad (\text{Infinite Solution})$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad (\text{Unique Solution})$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad (\text{No Solution})$$

## Quadratic Equations

### General Form

- $ax^2 + bx + c = 0$
- if  $\alpha$  &  $\beta$  are roots then,

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\text{Sum of roots } (\alpha + \beta) = -\frac{b}{a}$$

$$\text{Product of roots } (\alpha\beta) = \frac{c}{a}$$

The roots can be found out using,

$$x = \frac{-b \pm \sqrt{b^2 - 4ca}}{2a}$$

Nature of Roots depends on  $D = b^2 - 4ac$

(a)  $D < 0$  Roots are imaginary

(b)  $D = 0$  Real and equal

(c)  $D > 0$  and perfect square number real,  
distinct (unequal) and Rational

(d)  $D > 0$  and NOT a perfect square real  
distinct and Irrational

## Cubic Equation

### General Form

$$ax^3 + bx^2 + cx + d = 0$$

$$\text{Sum of roots } (\alpha + \beta + \gamma) = -\frac{b}{a}$$

$$\text{Product of roots } (\alpha\beta\gamma) = -\frac{d}{a}$$

# Linear In-equations

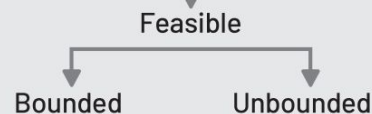
## Making the Inequation

- Use logics
- Signs of inequality  
 $\geq, \leq, >, <$
- Knock out in objective questions:  
Right values – satisfy  
Wrong values – not satisfy

## Solving the inequation

### Graphical Method\*\*\*

- S.01** Ignore the sign of inequality  
Put  $x = 0$  & calculate  $y$ , point  $(0, y)$   
Put  $y = 0$  & calculate  $x$ , point  $(x, 0)$
- S.02** Plot the points from S.01 on the graph,  
Draw straight lines.
- |   |   |
|---|---|
| <b>S.03</b> If line passes through $(0, 0)$<br>Put any point in the form $(x, 0)$<br>If satisfy –Shade towards<br>If not –Shade against | If it does not pass through $(0, 0)$ ,<br>Put $(0, 0)$<br>If satisfy –Shade towards<br>If not – Shade against |
|---|---|
- S.04** Shade the common region ; common area for all the inequations.



### Algebra

- Divide / Multiply with a negative no. ; change sign of inequality.
- Always change both sides
- Brackets

$( )$	$[ ]$	$\{ \}$
Open	Closed	Curly
$(2, 3)$	$[2, 3]$	$\{2, 3\}$
$\downarrow$	$2\checkmark$	$2\checkmark$
$2*$	$3\checkmark$	$3\checkmark$
$3*$	$2-3*$	$2-3*$
$2-3$		

# Time Value Of Money



## Simple Interest

- $S.I. = \frac{pxrxt}{100}$

- $A = P + S.I.$

1 S. I. is not बेवफा !

S.I. is always calculated on principal.

2 S.I. is constant for every year.

3 If Q. is चुपकी assume it to be of S.I.

4 Nature of r, t should be same

5 Time Scale में + P होगा

## Compound Interest

- Interest on Interest.

- $C.I. \geq S.I.$

- For the first period

$$C.I. = S.I.$$

- $A = P \times (1+i)^n$

- $C.I. = A - P$

- Nature of r & t is always same. Always focus on factor.

- Higher the compounding higher the amount.

- Time scale में  $\times$  होगा

- $WDV = H.V. \times (1 - i)$

- Doubling Period formula

$$T = 0.35 + \frac{69}{r}$$

- Tripling Period formula

$$T = 0.35 + \frac{111.111}{r}$$

## Effective Interest

$$E.I. = \{\text{एक साल का factor} - 1\} \times 100$$

- Always assume  $t = 1$  year

- Nature of r & t should be same.

### Three types of Questions

E.I. ✓

C.I. = ?

C.I. ✓

E.I. = ?

Compare

## Annuity

F.v.

P.v.

$$F.V. = I \times \left\{ \frac{(1+i)^n - 1}{i} \right\} \quad I \div \text{factor} = n \text{ times GT}$$

- Due : Starting from today  $\rightarrow$   
ans.  $\times (1+i)$

- Bulk amount

आज आया गया

P.v.

कल आया गया

F.v.

- 3 conditions for annuity.
  - Fixed Time Interval
  - Fixed Installments
  - Regular Payments

# Permutation & Combination

Events :  
And  $\rightarrow \times$   
Or  $\rightarrow +$

P & C

$$n \geq r$$

$$0! = 1$$

$n, r \rightarrow$  positive integers

## Permutation

It is all about arrangement, order matters.

$${}^n P_r = \frac{n!}{n-r!}$$

$n =$  no. of objects  
 $r =$  objects taken at a time

$${}^n P_r = n_{C_r} \times r!$$

### Restriction

जहाँ restriction हो पहले उसे भरने then focus on the rest.

### Always Together

- Always make a box
- Count box as one element & permute. Also arrange inside the box.

### Never together

For 2 elements: Total – always together

For > 2 elements: use logic

Or  
 $(n-1)(n-2)!$

### Repetition

- If objects are repeated then, जितनी बार एक object repeat उतने factorial से divide करो

### Circular

- $(n-1)!$  ways.
- Necklace etc.
- $\frac{1}{2}(n-1)!$

### Division

- Total objects! group ka!
- अगर group same तो उतने ! से divide.
- For distn  $\times$  person !

## Combination

It is about selection order does not matter

$${}^n C_r = \frac{n!}{(n-r)! \times r!} = \frac{n!}{r!}$$

### Properties

- $n_{C_r} = n_{C_{n-r}}$
- Pascal's law  
 $n_{C_r} + n_{C_{r-1}} = n_{C_r} + 1$
- $n_{C_1} = 1$
- $n_{C_0} = 1$
- $n_{C_{r_1}} = n_{C_{r_2}}$   
 $r_1 + r_2 = n$

### Geometry

- Diagonals  
 $= n_{C_2} - n$
- $\Delta$  can not be made from collinear points.

### All or None

$2^n$

Taking all or any no. of object i.e. r at a time



## AP / GP

### Arithmetic Progression

- It is about adding the constant no. to the first term & again.
- Every no. is A.M. of its previous & succeeding no.
- First Term = a    common difference = d
- Variety -1    series : given    value of term = ?  
 $T_n = a + (n-1)d$
- Variety -2    series : given    value = given     $n=?$
- Variety -3    series : given    sum = ?
- $S_n = \frac{n}{2}\{a+r\}$     or     $\frac{n}{2}\{2a+(n-r)d\}$
- Variety -4    series : given    sum = given    no. ?
- Variety -5    if two non consecutive term are given;  
 $d = \frac{T_m - T_n}{m - n}$
- Variety -6    Insertion of A.M. 's between two no.'s results in A.P.
- Variety -7    Sum's machine    =given Term = ?

### Geometric Progression

- It is about multiplying the constant no. again.
- Every no. is GM of its previous & succeeding terms.
- First term -a common ratio = r
- Variety no. 1    n = given     $T_n = ?$   
 $T_n = ar^{n-1}$
- Variety -2    n = ?     $T_n = ?$
- Variety -3    Series = given    sum = ?  
 $= s_n = \frac{ax(r^n - 1)}{r - 1}$     (r > 1)     $= s_n \frac{a(1 - r^n)}{1 - r}$   
If (r < 1)
- Variety -4    sum = given    n = ?
- Variety -5    calculation of r in two non consecutive;  
 $r = \left(\frac{T_m}{T_n}\right)^{\frac{1}{m-n}}$
- Variety -6    Insertion of GM's
- Variety -7    Sum of infinity series.  
 $S_\infty = \frac{a}{1 - r}$

# Sets, Function & Relations

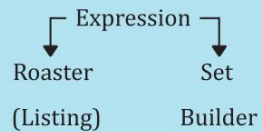
## Sets

## Relations

## Function

### Basics

"It is a well defined group of distinct objects."



• Cardinal no. = no. of elements in a set

No. of subsets =  $2^n$   
 Proper subsets =  $2^n - 1$

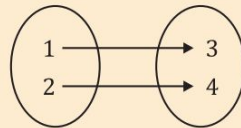
### Types

1. Universal Set : contains all the objects.
2. Subset : every element of A is in B.  $A \subset B$
3. Superset : every element of A is in B.  $B \supset A$
4. Null Set :  $\emptyset$ , 0 element.
5. Equal Set : Every element of A is in B & vice versa.
6. Equivalent Set :  $n(A) = n(B)$
7. Power Set : Set of all subsets.

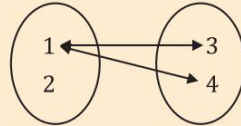
### Basics

Every subset of a Cartesian product of  $A \times B$  is called relation.

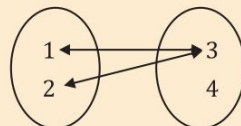
#### One to One



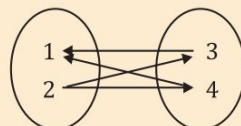
#### One to Many



#### Many to One



#### Many to Many



### Types

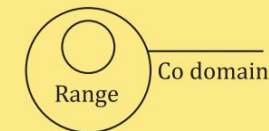
- Reflexive  
 $A = \{1, 2, 3\}$   
 $R = \{(1, 1), (2, 2), (3, 3)\}$   
 all  $a, a \in R$
- Symmetric  
 $A = \{1, 2, 3\}$   
 $R = \{(1, 2), (2, 1), (2, 3), (3, 2)\}$   
 $a, b \in R$  then  $b, a \in R$
- Transitive  
 $A = \{1, 2, 3\}$   
 $R = \{(1, 2), (2, 3), (1, 3)\}$   
 $a, b \in R$  &  $b, c \in R$  then,  $a, c \in R$
- $S \checkmark R \checkmark T \checkmark$   
 = Equivalence

### Basics

"Every R is not F but every F is a R."

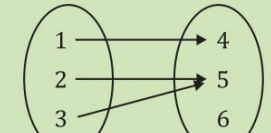
'No two ordered pairs should have same first element.'

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 Range = Image

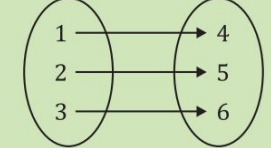


### Types

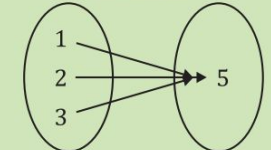
#### Into



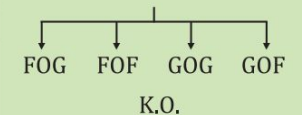
#### Onto



#### Constant



Inverse  
 $x \rightarrow y$   
 $y \rightarrow x$   
 Composite



# Differential Calculus

## Six Basic Rules of Differentiation

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \log_e a$$

$$\frac{d}{dx}(\text{constant}) = 0$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\text{Log } x) = \frac{1}{x}$$

Note:  $\frac{d}{dx}\{cf(x)\} = cf'(x)$   $c$  being constant.

## For Two Functions

$$h(x) = f(x) \pm g(x)$$

(Sum/Difference of function)

$$\frac{d}{dx}\{h(x)\} = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

$$h(x) = f(x) \cdot g(x)$$

(Product of functions)

$$\frac{d}{dx}\{h(x)\} = f(x) \frac{d}{dx}\{g(x)\} + g(x) \frac{d}{dx}\{f(x)\}$$

$$h(x) = \frac{f(x)}{g(x)}$$

(Quotient of function)

$$\frac{d}{dx}\{h(x)\} = \frac{g(x) \frac{d}{dx}\{f(x)\} - f(x) \frac{d}{dx}\{g(x)\}}{\{g(x)\}^2}$$

## Application of Differentiation

$$\text{Average cost (AC or } \bar{C}) = \frac{\text{Total Cost}}{\text{Out Put}} = \frac{C(X)}{X}$$

$$\text{Average variable cost (AVC)} = \frac{\text{Variable Cost}}{\text{Out Put}} = \frac{V(x)}{x}$$

$$\text{Average Fixed Cost (AFC)} = \frac{\text{Fixed Cost}}{\text{Out Put}} = \frac{F(x)}{x}$$

**Marginal Cost:** If  $C(x)$  the total cost producing  $x$  units then the increase in cost in producing one more unit is called marginal cost at an output level of  $x$  units and is given as  $\frac{dC}{dx}$

**Revenue Function:** Revenue,  $R(x)$ , gives the total money obtained (Total turnover) by selling  $x$  units of a product. If  $x$  units are sold at 'P per unit, then  $R(x) = P \cdot X$

**Marginal Revenue:** It is the rate of change in revenue per unit change in output. If  $R$  is the revenue and  $x$  is the output, then  $MR = \frac{dR}{dx}$

**Profit function:** Profit  $P(x)$ , the difference of between total revenue  $R(x)$  and total Cost  $C(x)$ .  
 $P(X) = R(x) - C(x)$

**Marginal Profit:** It is rate of change in profit per unit change in  $dP$  output i.e.  $\frac{dP}{dx}$

**Slope of Curve:** If  $y$  is any function then  $\frac{dy}{dx}$  represent the slope of tangent to the curve.



# Intergal Calculus

## 6 Basic Rules of Integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

$$\int dx = x + c, \text{ since } \int 1 dx = \int x^0 dx = \frac{x^1}{1} = x$$

$$\int e^x dx = e^x + c$$

$$\int e^{ax} dx = \frac{e^{ax}}{a} + c$$

$$\int \frac{dx}{x} = \log x + c$$

$$\int a^x dx = \frac{a^x}{\log_e a} + c$$

### Integration By parts

$$\int uv dx = u \int v dx - \int \left[ \frac{d(u)}{dx} \int v dx \right] dx$$

$$\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$$

$$\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$$

## Definite Integration

$$\int_a^b f(x) dx = F(b) - F(a)$$

'b' is called the upper limit and 'a' the lower limit of integration.

### Important Properties of Definite Intergal

$$\int_a^b f(x) dx = \int_b^a f(x) dx$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx, a < c < b$$

$$\int_0^b f(x) dx = \int_0^a f(a-x) dx$$

$$\text{when } f(x) = f(a+x) = \int_0^{na} f(x) dx = n \int_0^a f(x) dx$$

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx \text{ if } f(-x) = f(x)$$

$$\int_{-a}^a f(x) dx = 0 \text{ if } f(-x) = -f(x)$$

# Number Series & Coding-Decoding

## Learn by Heart

### Squares

$$1^2 = 1, 2^2 = 4, 3^2 = 9, \dots \text{upto } 25^2 = 625$$

### Cubes

$$1^3 = 1, 2^3 = 8, 3^3 = 27, \dots \text{upto } 15^3 = 3375$$

## What is "Method of Difference"?

Questions: 2, 5, 12, 27, 54, 97, ?

$$\text{Mod} \rightarrow 2, 5, 12, 27, 54, 97, \boxed{160}$$

$$3, 7, 15, 27, 43, \boxed{63}$$

$$4, 8, 12, 16, 20$$

Ans.  $\rightarrow$  160

## Alphabet Position Chart

1	2	3	4	5	6	7	8	9	10	11	12	13
A	B	C	D	E	F	G	H	I	J	K	L	M
Z	Y	X	W	V	U	T	S	R	Q	P	O	N
26	25	24	23	22	21	20	19	18	17	16	15	14



## Thumb Rule of Position

Alphabet Forward Position +  
Alphabet Backward Position = 27

**Example: Position of "I"**

Forward Position = 9.

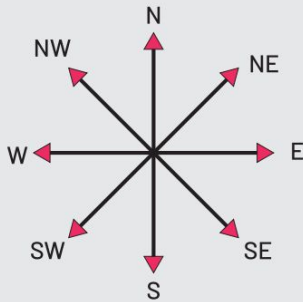
Backward Position =  $27 - 9 = 18$

What is N Backward Position?

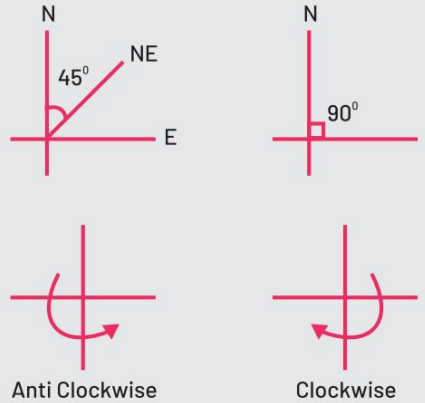
$27 - 14 = 13$

# Direction Test

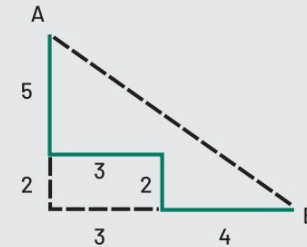
## Basic 8 Directions



## Angles & Direction



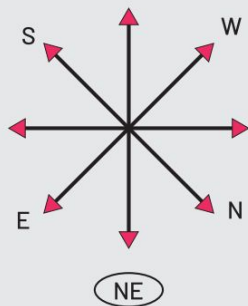
## How to Calculate Distance?



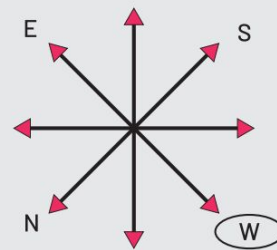
$$AB = \sqrt{(5 + 2)^2 + (3 + 4)^2}$$

$$B = \sqrt{49 + 49} = 7\sqrt{2}$$

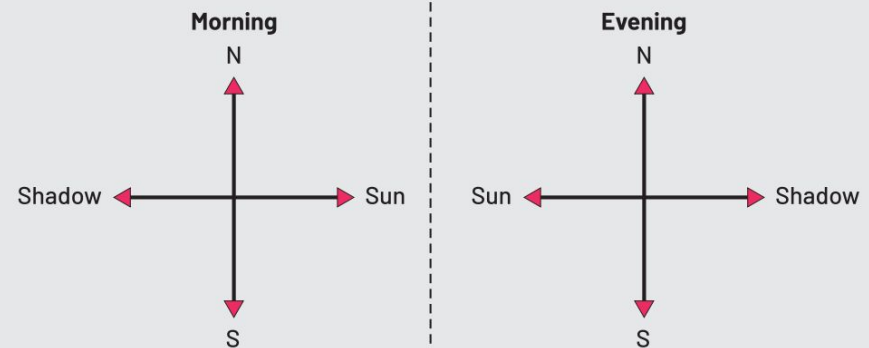
## If South Becomes NE



## If SE Becomes West

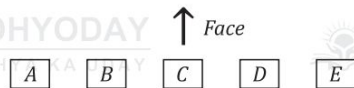


## Shadows base questions



# Seating Arrangement

## In Linear Arrangement People Sit Facing North



Back



### In the above arrangement

- B & A are to the left of C.
- D & E are to the right of C.
- B is immediate left of C.
- There are two persons between A & D.
- D is third to the right of A.

## Circular Arrangement

4 People



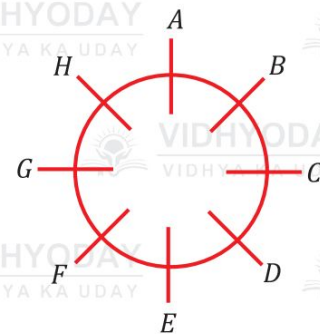
5 People



6 People



**Note :** Spacing between any two person should be same.



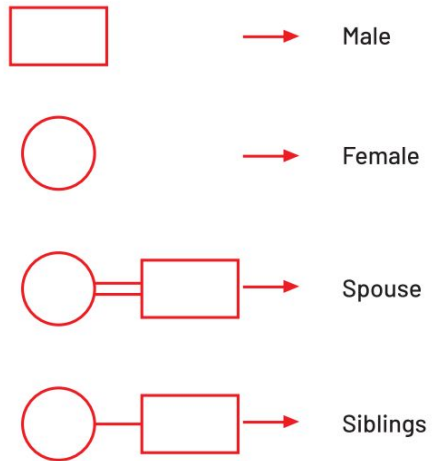
A is Diametrically opposite to E, H to D ..... and so on

G is second to left of E.

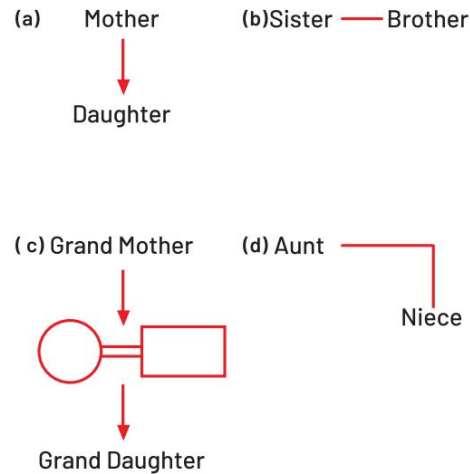
G is third to right of B.

# Blood Relations

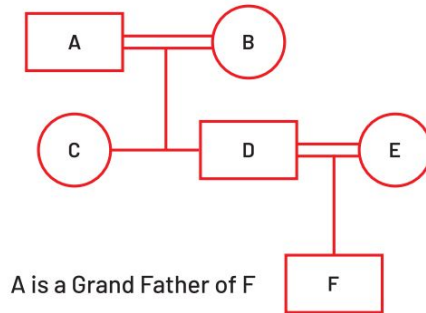
## Gender Representation



## Different Relationship Levels

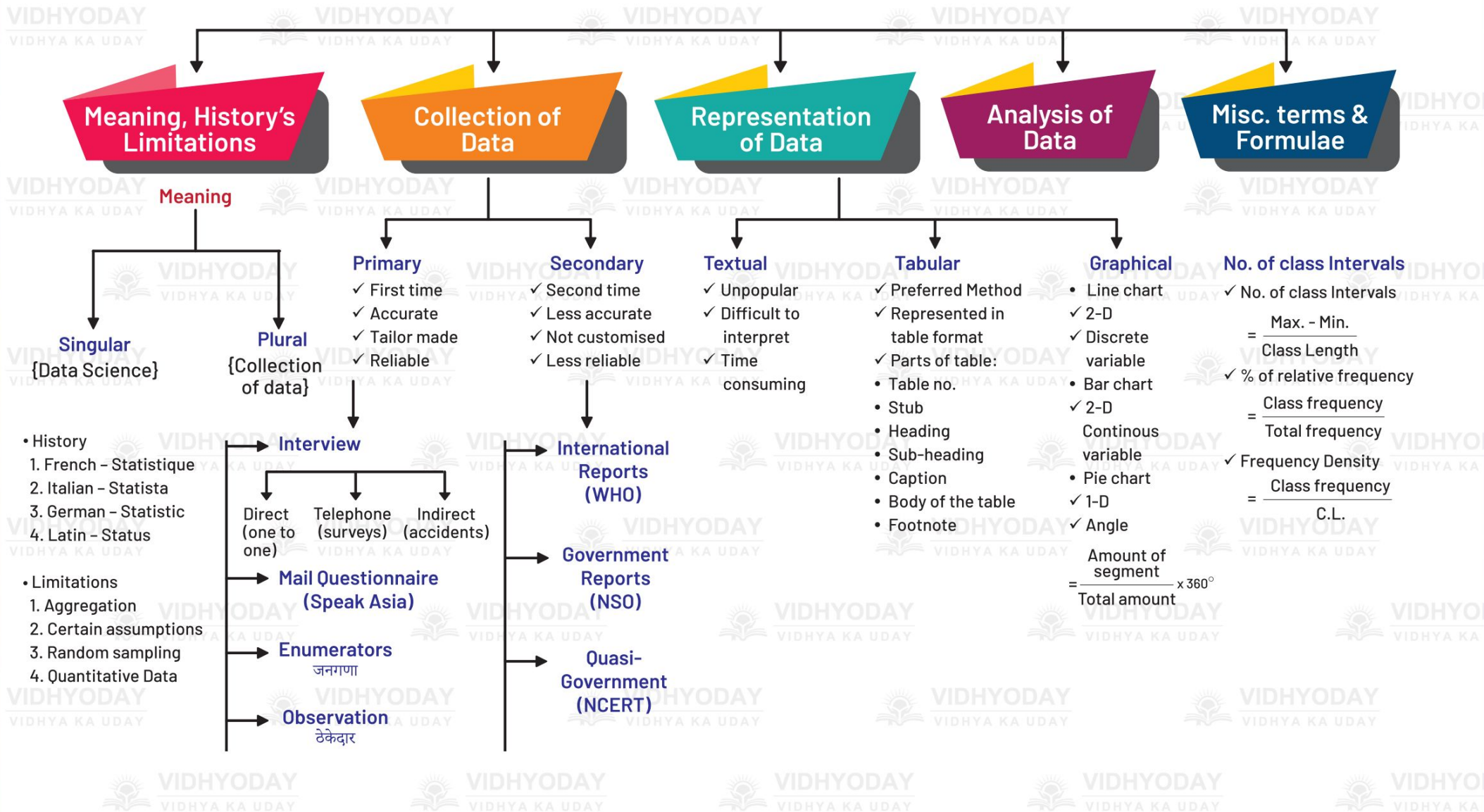


## Sample Family Tree



A is a Grand Father of F  
B is a Mother of C & D  
C is a Sister-in-Law of E  
F is a Nephew of C  
A is a Father-in-Law of E  
C & D are siblings  
D & E are Spouse

# Statistical Distribution of Data



# Central Tendency

## लवार्स Property

- Δ of origin ✓
- Δ of scale ✓
- Δ of sign ✓

## Quantitative Average

- $AM \geq GM \geq HM$
- $GM = \sqrt{AM \times HM}$

Relationship :  
**Mode :  $3md - 2\bar{X}$**   
 **$m_0 - \bar{X} = 3(md - \bar{X})$**

## Positional Average

### AM

- Average formula =  $\frac{\text{sum}}{\text{no}}$

$$\frac{\sum x}{n}, \frac{\sum fx}{\sum f}, \frac{\sum fm}{\sum f}$$

#### • Properties

1. A.M. is the most popular measure of CT.
2. Sum of deviations from A.M. is always 0.  
 $\sum X - \bar{X} = 0$

3. Combined A.M. can be calculated.

$$\bar{X}_{12} = \frac{\bar{X}_1 n_1 + \bar{X}_2 n_2}{n_1 + n_2}$$

4. Mean can be calculated using assumed mean formula

$$\bar{X} = A + \frac{\sum d}{n}$$

5. A.M. can not be represented graphically.

6.  $\sum (X - \bar{X})^2 = \text{minimum}$

### GM

- GM is best measure of CT for ratios & percentages.

- Formula Individual  
 $(axbac \dots) \frac{1}{n}$

#### Discrete

$$\left( X - x \frac{f^1}{2} \dots X \frac{f^n}{n} \right) \frac{1}{\sum n}$$

#### Continuos

$$(M_1^{f_1} M_2^{f_2} \dots) \frac{1}{\sum f}$$

### HM

- Used for variables having reciprocal relationship

- Formula Individual

$$HM = \frac{n}{\frac{1}{X_1} + \frac{1}{X_2} + \dots + \frac{1}{X_n}}$$

#### Discrete

$$HM = \frac{\sum f}{\frac{f_1}{X_1} + \frac{f_2}{X_2} + \dots + \frac{f_n}{X_n}}$$

$$HM = \frac{\sum f}{\frac{f_1}{m_1} + \frac{f_2}{m_2} + \dots + \frac{f_n}{m_n}}$$

- आवन जावन Q is imp
- HM is the reciprocal of AM

- Combined HM

$$= \frac{n_1 + n_2}{\frac{n_1}{HM_1} + \frac{n_2}{HM_2}}$$

### MEDIAN

- Individual  
 इधर से काटो, उधर से काटो, बीच में जो बचा वो median

- Discrete

$$S.O1 \frac{N}{2}$$

S.O1  $\frac{N}{2}$  को Locate करो in C.F.

S.O3 आगे वाला is median

- Continuos

S.O1 follow discrete

$$S.O2 M_c = 1 + \left\{ \frac{\frac{N}{2} - C}{F} \right\} x H$$

- Md is not affected by extremities of the observations

- Sum of absolute deviation from median is minimum.

$$\sum |x - x_{md}| = \text{minimum}$$

- Calculated through Ogive.

- Partition Values

$$\text{Value} = \left[ \text{order} X \left\{ \frac{n+1}{4/10/100} \right\} \right] \text{th term}$$

Quartiles      Deciles      Percentiles

- Best for open' end classification

### MODE

- Individual  
 Most repeated no.

- Discrete  
 No. with highest frequency

- Continuos  
 Find out model class & use.

#### Formula :

$$MO = 1 + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

- It is not uniquely defined.
- Calculated using Histogram.

•  $\overbrace{\text{Unimodel Bimodel Multimodel}}^{\text{Mode}}$

# Measures of Dispersion

[Measures of Dispersion] "Second order of averages"

लवणिस Property

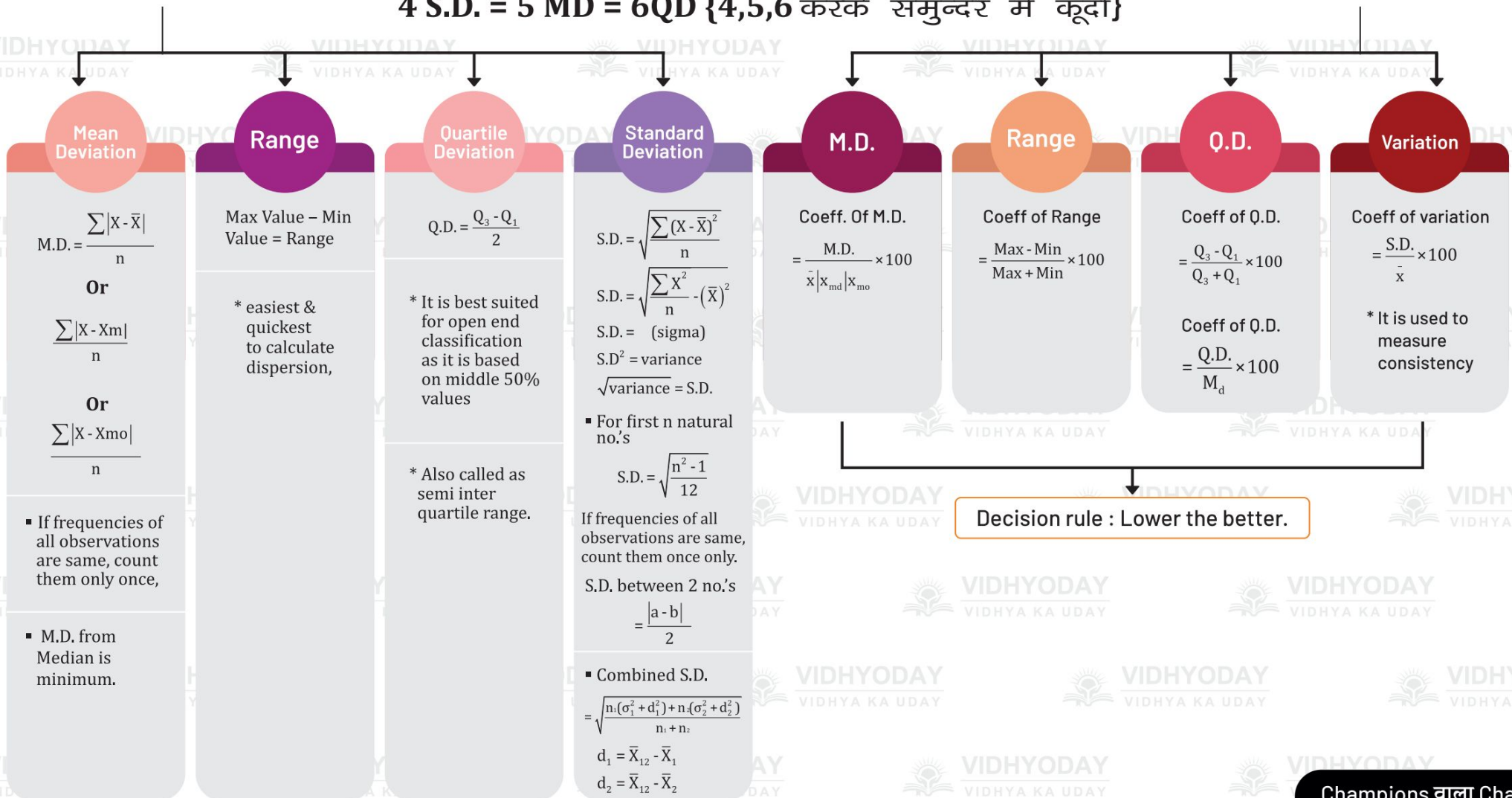
- Δ of origin ✗
- Δ of scale ✓
- Δ of sign ✗

## Absolute MOD

## Relationship Between MD, QD & S.D.

## Relative MOD

4 S.D. = 5 MD = 6 QD {4,5,6 करके समुन्दर में कूदो}





# Probability

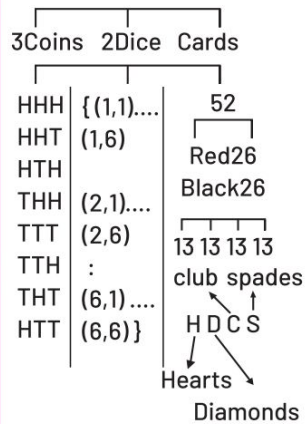
## Basics

$$P(A) = \frac{\text{Fav.}}{\text{Total}}$$

Odds in favour = m : n

Odds in against = n : m

$$P(A) = \frac{m}{m+n} \quad P(\bar{A}) = \frac{n}{m+n}$$



## Terminology

- Exp. = कीड़े करना
- Random Exp = outcome is not know
- Exhaustive = पूरी दुनिया  
Union = 1
- Equally likely = सब बराबर  $P(A) = P(B)$
- Mutually Exclusive  
मछली = 0  
 $P(A \cap B) = 0$
- Sure event  
 $P(A) = 1$
- Impossible Event  
 $P(A) = 0$
- Dependent = formula  
 $P(A \cap B) = P(A) \times P(B/A)$
- Independent  
मछली =  $P(A) \times P(B)$

## P & C

- Mostly combination (balls, cards, committee, geometry figures etc.)
- $\frac{\text{Fav.}}{\text{Total}}$
- Fav = with restrictions
- Total = w/o restriction

## Exp. Value

- Nothing but weighted avg.
- $E(x) = \sum px$
- Sum of probability = 1  
 $\sum p = 1$
- अगर bracket में x के अलावा expression है तो change x.
- Properties :  
 $E(x \pm y) = E(x) \pm E(y)$   
 $E(Kx) = KE(x)$   
 $E(x+y) = E(x) / E(y)$   
 $E(x \cdot y) = E(x) \times E(y)$
- Variance  
 $E[X - e(X)]^2$

## Venn Diagram

- $A \cup B = A + B - A \cap B$
- $$\begin{array}{c} A \cap B \\ \swarrow \quad \searrow \\ \text{Dependent} \quad \text{Independent} \\ \quad \quad \quad A \times B \end{array}$$
- Only A  
 $A - A \cap B$
- Only B  
 $B - A \cap B$
- $\overline{A \cap B}$   
 $= \bar{A} \cup \bar{B} = 1 - A \cup B$   
 $\bar{A} \cup \bar{B} = 1 - P(A \cup B)$
- $\overline{A \cup B}$   
 $\bar{A} \cap \bar{B} = 1 - P(A \cup B)$
- $A^c = 1 - A$
- $B^c = 1 - B$

## Conditional Probability

- Probability of A when B has already occurred  
 $P(A/B)$   
 $= \frac{P(A \cap B)}{P(B)}$
- For e.g.  
 $P(\bar{A}/\bar{B})$   
 $= \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$   
 $= \frac{1 - P(A \cap B)}{1 - P(B)}$

# Theoretical Distribution



## Binomial

Given by James Bournouli  
(Discrete Distribution)

**Formula** :  $P(x=r) = {}^n C_r p^r q^{n-r}$

**Applicability** : Future में IPM करेंगे

Finite Trials  
Independent Events  
Probability  
Mutually Exclusive Events

**Properties** : सप्तऋषि

1. Mean = np
2. Variance = npq
3. Mean = Variance
4. Biparametric B(n, P)
5. Max. Variance =  $\frac{n}{4}$
6. Mode = (n+1)p

Decimal Ignore Decimal; Unimodal Case

Integer Bimodal: Ans, Ans-1

7. Combined binomial Distribution  
 $P(x+y=r) = {}^{n_1+n_2} C_r p^{x_1} q^{n_1-x_1} p^{x_2} q^{n_2-x_2}$

## Poisson

Given by Simon Poisson  
(Discrete Distribution)

**Formula**  $P(x=r) = \frac{e^{-m} x^m}{r!}$

**Applicability** : n → ∞  
p → 0  
np → infinite  
{Flight Example}

**Properties** : Six - Sense

1. mean = np
2. variance = mean
3. Uniparametric (only m is parameter)
4. mode = nxp
5. B.D. can be approximated with P.D.
6. Combined Poisson Distribution

$$P(x+y=r) = \frac{e^{-(m_1+m_2)} (m_1+m_2)^r}{r!}$$

Decimal Ignore Decimal; Unimodal Case

Integer Bimodal: Ans, Ans-1

## Normal

Given by Carl Gauss (Continuous Distribution)  
Also called Gaussian distribution

**Formula** :  $P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$   
 $z = \frac{x-\mu}{\sigma}$

**Properties**

1. It is bi-parametric X-N ( $\mu, \sigma^2$ )
2. It is bell shaped curve.
3. It is symmetrical around mean.
4. Standard Results:  
 $\mu \pm \sigma = 68.30\%$ ,  $\mu \pm 2\sigma = 95.50\%$ ,  $\mu \pm 3\sigma = 97.70\%$   
Fix Rate free, Shine Life Life, Shine Shine Raven
5. Points of inflexion  
 $\mu + \sigma$   
 $\mu - \sigma$
6. 4, 5, 6 कके समुंदर में 'कूदो,  
4.S.D. = 5 M.D. = 6 Q.D.
7. Standard Normal Variate  
 $\mu=0$   $\sigma=1$
8. Quartiles  
 $Q_1 = \mu - 0.675\sigma$   
 $Q_2 = \mu$   
 $Q_3 = \mu + 0.675\sigma$



Normal Curve

# Correlation

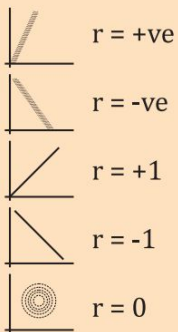
- Δ of origin ✗
- Δ of scale ✗
- Δ of sign ✓

## Correlation ( Measures the degree of linear relationship between two variables )

### Scatter Diagram

- It only tell us the nature of correlation & not degree of correlation

- Five Diagrams



- Correlation of Straight line is always +1 or -1. It depends upon the direction between x & y.
- Equation of a Straight Line is  $ax + by = c$
- Proportion --> decides 1 or not
- direction --> decides + or -

### Rank Correlation

$$r_0 = 1 - \frac{6\sum d^2}{n^3 - n}$$

n = no. of observations.  
d = difference of ranks

- Sum of difference of ranks is always 0.
- If ranks are exactly opposite then  $r = -1$ .
- Even if ranks are reversed, it remains same.

### Concurrent Deviation

$$r_c = \pm \sqrt{\pm \frac{2c - m}{m}}$$

- If there is negative no inside the root, r is going to be negative.

m = no. of observations compared = n - 1.

c = no. of concurrent deviation (+'s)

### Karl Pearson

$$r = \frac{\text{Cov}(x,y)}{\sigma_x \sigma_y}$$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2} \sqrt{n\sum y^2 - (\sum y)^2}}$$

$$\text{Cov}(x,y)$$

$$= \frac{\sum (X - \bar{X})(Y - \bar{Y})}{n}$$

- Cov (X,Y) desides the nature of correlation -

$$-1 \leq r \leq +1$$

- n is a pure no. (unit free).

- Coefficient of determination =  $1 - r^2$  (Unexplained variance)

$$\text{P.E.} = \frac{0.675\sqrt{1 - r^2}}{n}$$

- Product Moment correlation (nick name)
- Useful for variables having only linear relationship.

# Regression

## Regression

### लवार्सिस Property

- Δ of origin ✗
- Δ of scale ✓
- Δ of sign ✓

### Regression Equations

- Unlike  $r$ , it tells us the exact increase in price of  $y$  if  $x$  is increased or vice versa.
- There are two equations :
  1.  $Y$  on  $x \rightarrow$  to calculate  $y$ .
  2.  $X$  on  $y \rightarrow$  to calculate  $x$ .
- $Y$  on  $x \rightarrow Y - \bar{Y} = b_{yx}(X - \bar{X})$   
 $X$  on  $y \rightarrow X - \bar{X} = b_{xy}(Y - \bar{Y})$
- $b_{yx} = r \frac{\sigma_Y}{\sigma_X}$  (जो पीछे हे वो नीचे हे।)  
 $b_{xy} = r \frac{\sigma_X}{\sigma_Y}$

#### Popular Questions :

- 1 Lines = given slope = ?
- 2 Lines = given  $r$  = ?
- 3 Slopes = given  $r$  = ?
- 4 Lines = given mean = ?
- 5 पहचान कौन ?

### Properties

- 1) Regression lines intersect each other at  $(\bar{X} - \bar{Y})$  i.e. mean. (K.O.)
- 2) Correlation coefficient is GM of regression coefficient.  
 $r = \sqrt{b_{yx} \times b_{xy}}$
- 3) The product of regression coefficients should be  $\leq 1$ .  
 $b_{yx} \times b_{xy} \leq 1$
- 4) If one coefficient is greater than unity the other should be less than unity.
- 5)  $b_{yx}$ ,  $b_{xy}$  &  $r$  are all of same sign.
- 6) Regression lines are made using least squares deviation method.
- 7) लवार्सिस Property : Δ of origin ✗ Δ of scale ✓ Δ of sign ✓
- 8)  $r = 0$  regression lines are perpendicular, if  $r \pm 1$ , lines will coincide.
- 9) पहचान कौन ?
  - S.01 Calculate  $b_{yx}$  &  $b_{xy} \leq 1$   
By assuming one line as  $Y$  on  $x$  & another as  $x$  on  $y$ .
  - S.02 Check if  $\sqrt{b_{yx} \times b_{xy}} \leq 1$
  - S.03 Yes  $\rightarrow$  assumption is true.  
No  $\rightarrow$  opposite is true.

# Index No.



CY value, when B.Y value is assumed to be 100. E.g.  
Sensex (1978-79) Index no. is a pure no.

## Simple Method

### Aggregative

$$P_{01} = \frac{\sum P_1}{\sum P_0} \times 100$$

### Relative

$$I = \frac{\sum I_R}{n}$$

$$I_R = \frac{P_1}{P_0} \times 100$$

## Weighted Method

$$\left[ \frac{\sum P_1 W}{\sum P_0 W} \right]$$

$$\text{Laspeyres} = \frac{\sum P_1 q_0}{\sum P_0 q_0}$$

(Base year Q.)

$$\text{Passche} = \frac{\sum P_1 q_1}{\sum P_0 q_1}$$

(Current year Q.)

\*\*\* (Ideal)

$$\text{Fishers} = \sqrt{L_a \times P_a}$$

$$\text{Dorbish \& Bowley} = \frac{L_a \times P_a}{2}$$

Marshall Edgeworth

$$P_{01} = \frac{\sum P_1 \left( \frac{q_0 + q_1}{2} \right)}{\sum P_0 \left( \frac{q_0 + q_1}{2} \right)}$$

$$\text{Walsh} \quad \left( \sqrt{P_{01}} \right) = \frac{\sum P_1 \sqrt{q_0 q_1}}{\sum P_0 \sqrt{q_0 q_1}}$$

$$\text{Weight} = \sqrt{q_0 q_1}$$

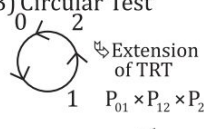
$$\text{(kelly)} = \frac{\sum P_1 q}{\sum P_0 q}$$

## Special Points

### Test

1) Unit test unit free satisfied by all.

2) Time - Reversal Test  $P_{01} \times P_{10} = 1$   
Kelly, MEW, Fishers Simple aggregative Satisfy TRT

3) Circular Test  
  
= 1

Fisher ×  
Kelley ✓  
aggregative ✓

4) Factor Reversal  
 $P_{01} \times q_{01} = v_{01}$

$$v_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_0} \times 100$$

### Inflation Deflation

(Index No - 100) = Inflation %

Deflated value mean B.Y. dh value

$$= \frac{\text{C.Y. Value}}{\text{C.Y. Index}} \times 100$$

### Base Shifting & CBI

CBI

$$= \frac{LR \times PYCBI}{100}$$

$$LR = \frac{\text{CY Price}}{\text{Prev. Price}} \times 100$$

### Salary Q.'s

C.Y. Salary  
C.Y. Index  
B.Y. Index

= B.Y. की Salary

B.Y. Salary  
B.Y. Index  
C.Y. Index

= C.Y. की Salary

• Today' salary - should have been = Real gain.

• Should have been -Today's salary = D.A.