Chapter 5

5 Marks

PERMUTATIONS AND COMBINATIONS

TOPIC : FACTORIAL

FUNDAMENTAL PRINCIPLES OF COUNTING

PERMUTATIONS

COMBINATIONS

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FACTORIAL

- $0! = 1$
- $1! = 1$
- $2! = 2 x 1 = 2$
- $3! = 3x2x1 = 6$
- $4! = 4x3x2x1 = 24$
- $5! = 5x4x3x2x1 = 120$
- $6! = 6x5x4x3x2x1 = 720$
- $7!=7x6x5x4x3x2x1 = 5040$

FACTORIAL

$$n! = n\,(n-1)\,(n-2)\,...........3.2.1$$

- 4 ! = 4 x 3 x 2 x 1
- 3 ! = 3 x 2 x 1
- 2 ! = 2 x 1
- 1 ! = 1

FACTORIAL

For a natural number n

$n! = n(n-1)!$

$n! = n(n-1)(n-2)!$

$n! = n(n-1)(n-2)(n-3)!$

EXAMPLE

$5! = 5 X 4 X 3 X 2 X 1$

$5! = 5 X 4!$

$5! = 5 X 4 X 3!$

$5! = 5 X 4 X 3 X 2!$

FACTORIAL

RESULT :

$$(n + 1)! - n! = \Rightarrow n.n!$$

Example 1: Find 5!, 4! and 6!

Example 2: Find 9! / 6!; 10! / 7!.

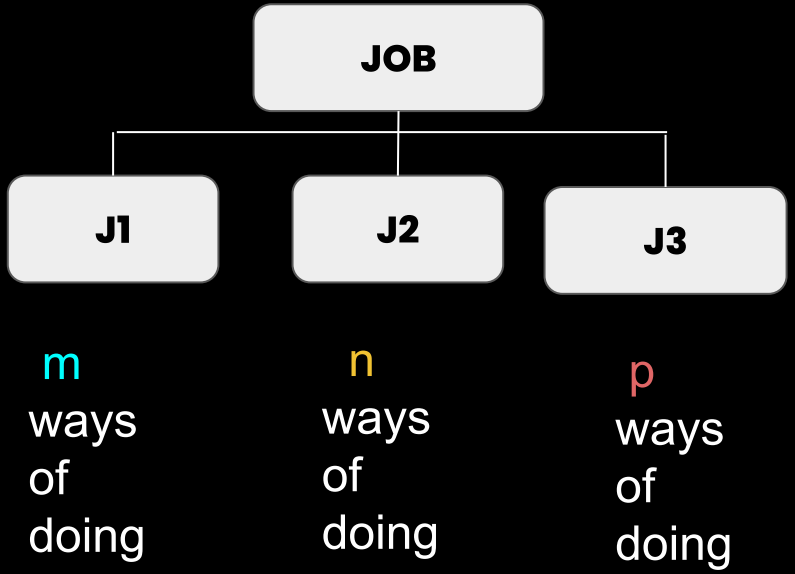
Example 3: Find x if $1/9! + 1/10! = x/11!$

Fundamental Principles of Counting

AND

Multiplication Principle

A job is divided into a number of sub- jobs which are unconnected to each other and the job is said to be performed if each sub -job is performed



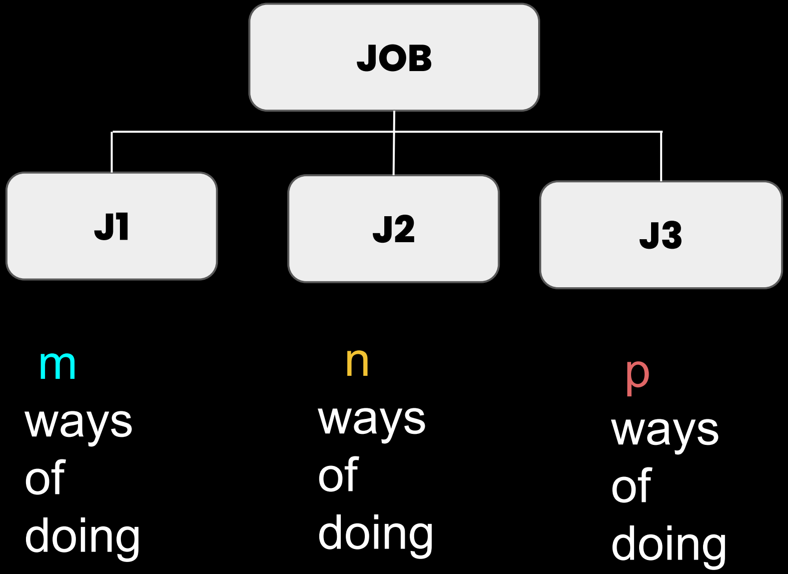
No . of ways of doing the job = m x n x p

Fundamental Principles of Counting

OR

Addition Principle

There are a number of independent jobs and we have to perform one of them . So the total number of ways of completing any one of the sub -jobs is the sum of the number of ways of completing each sub- jobs



No . of ways of doing the job = m + n + p

PERMUTATIONS

- A permutation is an arrangement in a definite order of a number of objects taken some or all at a time .

PERMUTATIONS

Arrangement made with the letters

a

b

c

Taking all at a time

- abc
- acb
- bac
- bca
- cab
- cba

PERMUTATIONS

Arrangement made with the letters

a

b

c

Taking two at a time

- ab
- ba
- ac
- ca
- bc
- cb

PERMUTATIONS

Theorem: **The number of permutations of n things when r are chosen at a time**

$$ {}^{n}P_{r} = n(n-1)(n-2)\ldots(n-r+1) $$

where the product has exactly r factors.

RESULTS :

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!}, 0 \le r \le n $$

EXAMPLE

- ${}^{5}P_{3}$
- ${}^{10}P_{2}$

RESULTS :

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!}, 0 \leq r \leq n $$

- ${}^{n}P_{n} = n!$

PERMUTATIONS WHEN ALL THE OBJECTS ARE DISTINCT

- The number of permutations of n **different** objects taken r at a time and objects do not repeat is denoted by ${}^{n}P_{r}$

$${}^{n}P_{r} = \frac{n!}{(n-r)!}, 0 \le r \le n$$

PERMUTATIONS WHEN ALL THE OBJECTS ARE DISTINCT

- The number of permutations of n **different** objects taken all at a time is denoted by $^{n}P_{n}$
- $$^{n}P_{n} = n!$$

Example How many words , with or without meaning can be formed by using all the letters of the word ' DELHI ' ,using each letter exactly once ?

PERMUTATIONS WHEN ALL THE OBJECTS ARE DISTINCT

- The number of permutations of n different objects taken r at a time, where repetition is allowed, is n^r

Example 1: Evaluate each of ${}^{5}P_{3}$, ${}^{10}P_{2}$, ${}^{11}P_{5}$.

Example 2: How many three letters words can be formed using the letters of the words

(a) SQUARE and (b) HEXAGON?

Example 3: In how many different ways can five persons stand in a line for a group photograph?

Example 4: First, second and third prizes are to be awarded at an engineering fair in which 13 exhibits have been entered. In how many different ways can the prizes be awarded?

PERMUTATIONS WHEN ALL THE OBJECTS ARE NOT DISTINCT

The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is

$$\frac{n!}{p_1!\,p_2!\ldots p_k!}.$$

PERMUTATIONS WITH RESTRICTIONS

- ***Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is***

$${}^{n-1}P_r$$

PERMUTATIONS WITH RESTRICTIONS

- *Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement*

$$r \cdot {}^{n-1}P_{r-1}$$

CIRCULAR PERMUTATIONS

- **The number of circular permutations of n different things chosen all at a time is**

  (n-1)!

- **The number of ways of arranging n persons along a round table so that no person has the same two neighbours is** $=\frac{1}{2}\underline{|n-1}$

- ***The number of necklaces formed with n beads of different*** $=\frac{1}{2}\underline{|n-1}$