CHAPTER

16 Theoretical Distribution

THEORETICAL DISTRIBUTION OF PROBABILITY

A probability distribution is a way of describing the probabilities of different outcomes in a random experiment or event. It is similar to a frequency distribution, where we distribute different frequencies over class intervals, but in a probability distribution, we distribute different probabilities.

1. In the case of a **Discrete random variable**, which takes on a finite or countable number of values, the probability distribution is typically represented by a probability mass function (PMF). The PMF gives the probability of each possible outcome.

   E.g.: Binomial Distribution and the Poisson Distribution

2. In the case of a **Continuous random variable**, which can take on any value within a certain range, the probability distribution is typically represented by a probability density function (PDF). The PDF gives the probability density at each point. The area under the PDF represents the probability of an event occurring within a certain range.

   E.g.: Normal Distribution

As a probability distribution exists only in theory, it is called the theoretical distribution of Probability.

WHY DO WE NEED TO STUDY?

- We generally do sampling by frequency distribution, and based on the same samples if we need to find some results. We need to learn probability distribution.
- For example: Finding the age of different tube lights
- Theoretical probability distribution will help you create future projections.
- For Example: Finding the projection of profitability of tube lights in next 10 years
- Statistical analysis is possible only on the basis of theoretical probability distribution.
- A probability distribution also possesses all the characteristics of an observed distribution.

Like finding mean (μ), median (μ), mode(μ_O), standard deviation (σ) etc. exactly the same way we have done earlier.

THINGS TO STUDY

- Discrete Probability Distributions:
  1. Binomial Distribution
  2. Poisson Distribution

- Continuous Probability Distributions:
  1. Normal Distribution

BINOMIAL DISTRIBUTION

- It is derived from a particular type of random experiment known as Bernoulli process named after the famous mathematician Bernoulli.
- When 'trial' is attempted to produce a particular outcome which is neither certain nor impossible.
- The characteristics of Bernoulli trials are stated below:
  (a) Trial is associated with two mutually exclusive and exhaustive outcomes
  (b) The occurrence of one of which is known as a 'success' and as such its non occurrence as a 'failure'.
  As an example, flipping a coin multiple times and counting the number of heads.
- The trial is independent.
- The probability of a success, usually denoted by p, and hence that of a failure, usually denoted by $q = 1 - p$, remain unchanged throughout the process
- The number of trials is a finite positive integer.
  Under such conditions, A discrete random variable x is defined to follow binomial distribution with parameters n and p, to be denoted by $x \sim B(n, p)$, if the probability mass function of x is given by
  $f(x) = P(X = x) = {}^nC_x (p)^x (q)^{n-x}$ or $x = 0, 1, 2, \ldots, n$

  $= 0$ otherwise

Example 1. What is the probability of getting 3 heads if 6 unbiased coins are tossed simultaneously? **(ICAI)**

(a) 0.50 (b) 0.25 (c) 0.3125 (d) 0.6875

Sol. (c) According to the question,

Number of trials: $n = 6$,

Specific outcome (getting 3 heads): $x = 3$

Probability of success: $p = \frac{1}{2}$,

Probability of failure: $q = 1 - \frac{1}{2} = \frac{1}{2}$

Binomial distribution is given by the formula,

$$P(X = x) = {}^nC_x (p)^x (q)^{n-x}$$

$$P(X = 3) = {}^6C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{6-3} \quad \left[\because {}^nC_r = \frac{n!}{r! \times (n-r)}\right]$$

$$= \frac{6!}{3! \times (6-3)!} \times \left(\frac{1}{8}\right)\left(\frac{1}{8}\right) = \frac{20}{64} = 0.3125$$

Therefore, the required probability is 0.3125.

Hence, the correct answer is option (c).

Example 2. What is the probability of making 3 correct guesses in 5 True – False answer type questions? **(ICAI)**

(a) 0.3125 (b) 0.5676 (c) 0.6875 (d) 0.4325

Sol. (a) According to the question,

Number of trials: $n = 5$,

Specific outcome (making 3 correct guesses): $x = 3$

Probability of success: $p = \frac{1}{2}$

Probability of failure: $q = 1 - \frac{1}{2} = \frac{1}{2}$

Binomial distribution is given by the formula,

$$P(X = x) = {}^{n}C_x (p)^x (q)^{n-x}$$

$$P(X = 3) = {}^{5}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} = \frac{5!}{2! \times 3!}\left(\frac{1}{8}\right)\left(\frac{1}{4}\right)$$

$$= \frac{10}{32} = \frac{5}{16} = 0.3125$$

Hence, the correct answer is option (a).

IMPORTANT POINTS IN CONNECTION WITH BINOMIAL DISTRIBUTION

1. As $n > 0$, $p, q, \geq 0$, it follows that $f(x) \geq 0$ for every x. Also, $\Sigma f(x) = f(0) + f(a) + f(b) + \ldots + f(n) = 1$
2. Binomial distribution is known as bi-parametric distribution as it is characterized by two parameters n and p. This means that if the values of n and p are known, then the distribution is known completely.
3. The mean of the binomial distribution is given by $\mu = np$
4. Depending on the values of the two parameters, binomial distribution may be unimodal or bi- modal μ_0, the mode of binomial distribution, is given by:

   μ_0 = the largest integer contained in:

   (a) $(n + 1)p$ if $(n + 1)p$ is a non-integer

   (b) $(n + 1)p$ and $(n + 1)p - 1$ if $(n + 1)p$ is an integer.
5. Variance of a binomial variable is always less than its mean. Since p and q are numerically less than or equal to 1 and Variance is maximum at $p = q = 0.5$ and the maximum value is $\frac{n}{4}$.
6. Additive property of binomial distribution. If X and Y are two independent variables such that $X \sim B(n_1, P)$ and $Y \sim B(n_2, P)$ then $(X + Y) \sim B(n_1 + n_2, P)$

APPLICATIONS OF BINOMIAL DISTRIBUTION:

- Binomial distribution is applicable when the trials are independent and each trial has just two outcomes: success and failure. It is applied in coin tossing experiments, sampling inspection plans, genetic experiments and so on.

Example 3. If in Binomial distribution, np = 9 and npq = 2.25 then q is equal to **(ICAI)**

(a) 0.25 (b) 0.75 (c) 1 (d) None

Sol. (a) Given: np = 9 and npq = 2.25

$\Rightarrow \frac{npq}{np} = \frac{2.25}{9}$

$\Rightarrow q = 0.25$

Therefore, the value of q is 0.25.

Hence, the correct answer is option (a) i.e. 0.25.

Example 4. X is a binomial variable with n = 20. What is the mean of X if it is known that X is symmetric? **(ICAI)**

(a) 5 (b) 10 (c) 2 (d) 8

Sol. (b) Given: Number of trials (n) = 20

We know that,

If X is symmetric then occurrence of success is equal to failure.

p = q = 0.5 where p is success, q is failure

Mean is given by the formula,

$\Rightarrow \bar{X} = np$

= 20(0.5)

= 10

Hence, the correct answer is option (b).

Example 5. What is the number of trials of a binomial distribution having mean and SD as 3 and 1.5 respectively? **(ICAI)**

(a) 2 (b) 4 (c) 8 (d) 12

Sol. (d) Given: Mean (X) = 3 Standard Deviation (σ) = 1.5

Mean is given by the formula,

$\bar{X} = np$

Standard Deviation is given by the formula,

$S.D(\sigma) = \sqrt{Var(X)} = \sqrt{npq}$

$\Rightarrow np = 3$ and $\sqrt{npq} = 1.5$

Put the value of np = 3 in $\sqrt{npq} = 1.5$

$\Rightarrow \sqrt{3q} = 1.5$

$\Rightarrow 3q = 2.25$

$\Rightarrow q = \frac{2.25}{3}$

$\Rightarrow q = 0.75$

As we know, $p = 1 - q$

$\Rightarrow p = 1 - 0.75$

$\Rightarrow p = 0.25$

Put $p = 0.25$ in $np = 3$,

$\Rightarrow n(0.25) = 3$

$\Rightarrow n = \frac{3}{0.25}$

$\Rightarrow n = 12$

Hence, the correct answer is option (d).

Example 6. If X is a binomial variate with parameter 15 and $\frac{1}{3}$, what is the value of mode of the distribution? **(ICAI)**

(a) 5 and 6 (b) 5 (c) 5.50 (d) 6

Sol. (b) Given: Number of trials (n) = 15

Success $(p) = \frac{1}{3}$

Here, $(n+1)p = \frac{16}{3}$, which is a non-integer

We know that,

Mode = the largest integer contained in $(n + 1)p$ if $(n + 1)p$ is a non-integer.

Here, $(n+1)p = \frac{16}{3} = 5\frac{1}{3}$

Thus, the largest integer is 5.

Therefore, the mode is 5.

Hence, the correct answer is option (b).

Example 7 If it is known that the probability of a missile hitting a target is $\frac{1}{8}$, what is the probability that out of 10 missiles fired, at least 2 will hit the target? **(ICAI)**

(a) 0.4258 (b) 0.3968 (c) 0.5238 (d) 0.3611

Sol. (d) Given: Number of trials (n) = 10

Success $(p) = \frac{1}{8}$

$\text{Failure}(q) = 1 - p = 1 - \frac{1}{8} = \frac{7}{8}$

The probability that at least 2 will hit the target will be given by,

= 1 – [Probability that none will hit the target + Probability that one will hit the target]

= 1 – [P (X = 0) + P(X = 1)]

Binomial distribution is given by the formula,

$$P(X = x) = {}^nC_x(p)^x(q)^{n-x} \qquad \left[\because {}^nC_r = \frac{n!}{r! \times (n-r)!}\right]$$

Required Probability $= 1 - [P(X = 0) + P(X = 1)]$

$$= 1 - {}^{10}C_0\left(\frac{1}{8}\right)^0\left(\frac{7}{8}\right)^{10-0} + {}^{10}C_1\left(\frac{1}{8}\right)^1\left(\frac{7}{8}\right)^{10-1}$$

$$= 1 - \frac{10!}{0! \times 10!}(1)\left(\frac{7}{8}\right)^{10} + \frac{10!}{1! \times 9!}\left(\frac{1}{8}\right)^1\left(\frac{7}{8}\right)^9$$

$$= 1 - \left(\frac{7}{8}\right)^9\left(\frac{7}{8} + \frac{10}{8}\right) = 1 - \left[\left(\frac{7}{8}\right)^9\left(\frac{17}{8}\right)\right]$$

= 1 – 0.6389 = 0.3611

Hence, the correct answer is option (d).

Example 8. Assuming that one-third of the population is tea drinkers and each of 1000 enumerators takes a sample of 8 individuals to find out whether they are tea drinkers or not, how many enumerators are expected to report that five or more people are tea drinkers?

(a) 100 (b) 95 (c) 88 (d) 90 **(ICAI)**

Sol. (c) Given: 1000 enumerators i.e., N = 1000

Number of trials (n) = 8

$$\text{Success}(p) = \frac{1}{3}$$

$$\text{Failure}(q) = 1 - \frac{1}{3} = \frac{2}{3}$$

To find: enumerators are expected to report that five or more people are tea drinkers i.e., $P(X \geq 5)$

Binomial distribution is given by the formula,

$$P(X = x) = {}^nC_x(p)^x(q)^{n-x} \qquad \left[\because {}^nC_r = \frac{n!}{r! \times (n-r)!}\right]$$

Thus, $P(X \geq 5) = P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$

$$= {}^8C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^{8-5} + {}^8C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^{8-6} + {}^8C_7\left(\frac{1}{3}\right)^7\left(\frac{2}{3}\right)^{8-7} + {}^8C_8\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)^{8-8}$$

$$= 56\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^3 + 28\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^2 + 8\left(\frac{1}{3}\right)^7\left(\frac{2}{3}\right)^1 + \left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)^0$$

$$= \left(\frac{1}{3}\right)^5 \left[56\left(\frac{2}{3}\right)^3 + 28\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + 8\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^3\right]$$

$$= \left(\frac{1}{3}\right)^5 \left[\frac{448 + 112 + 16 + 1}{3^3}\right]$$

$$= \frac{1}{3^8}(577) = 0.08794$$

As N = 1000,

Thus, the required enumerators (the expected frequency)

$N \times P(X = x) = 0.08794 \times 1000 = 87.94 \approx 88$

Hence, the correct answer is option (c).

Example 9. In 10 independent rollings of a biased die, the probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times. What is the probability that an even number will appear twice when the die is rolled 8 times?

(ICAI)

(a) 0.0304 (b) 0.1243 (c) 0.2315 (d) 0.1926

Sol. (a) Given: Number of trials (n) = 10

Success (p) = Even number appear on dice

To find: Probability that an even number will appear twice when the die is rolled 8 times

Binomial distribution is given by the formula,

$P(X = x) = {}^nC_x(p)^x(q)^{n-x}$ $\left[{}^nC_r = \frac{n!}{r! \times (n-r)!}\right]$

n → Number of trials

x → number of times for a specific outcome within n trials

p → Success

q → Failure

According to the question,

$\Rightarrow P(X = 5) = 2P(X = 4)$

$$\Rightarrow {}^{10}C_5(p)^5(q)^{10-5} = 2\left[10_{C_4}(p)^4(q)^{10-4}\right]$$

$$\Rightarrow {}^{10}C_5(p)^5(q)^5 = 2\left[10_{C_4}(p)^4(q)^6\right]$$

$$\Rightarrow \frac{p}{5} = \frac{2q}{6}$$

$$\Rightarrow \frac{p}{5} = \frac{2q}{6}$$

$$\Rightarrow p = \frac{5q}{3}$$

Success and failures adds up to 1,

So, $p + q = 1$

Put the value of $p = \frac{5q}{3}$ in $p + q = 1$

$$\Rightarrow \frac{5q}{3} + q = 1$$

$$\Rightarrow \frac{5q + 3q}{3} = 1$$

$$\Rightarrow 8q = 3$$

$$\Rightarrow q = \frac{3}{8}$$

$$\Rightarrow p = 1 - q = 1 - \frac{3}{8} = \frac{5}{8}$$

Now,

P(X = 2)

$$\Rightarrow 8_{C_2}\left(\frac{5}{8}\right)^{ü}\left(\frac{3}{8}\right)^{-}$$

$$\Rightarrow \frac{8!}{2! \times 6!}\left(\frac{5}{8}\right)^{2}\left(\frac{3}{8}\right)^{6}$$

$$\Rightarrow 28\,\frac{5^2 \times 3^6}{8^8}$$

$\Rightarrow 0.03041 \approx 0.0304$

Hence, the correct answer is option (a).

PRACTICE QUESTIONS (PART A)

1. What are the parameters of binomial distribution?
   (a) n (b) p (c) Both n and p (d) None of these

2. The Standard Deviation of Binomial distribution is **(Dec 2022)**
   (a) npq (b) $\sqrt{npq}$ (c) np (d) $\sqrt{np}$

3. If X is a binomial variate with $p = \frac{1}{3}$ for the experiment of 90 trials, them the standard deviation is equal to **(July 2021)**
   (a) $-\sqrt{5}$ (b) $\sqrt{5}$ (c) $2\sqrt{5}$ (d) $\sqrt{15}$

4. There is a fire in a house, chances of catching fire to fireman is 25%. What is the probability that out of 5 workmen, 4 or more will be in the fire?
   (a) $\frac{15}{4^5}$ (b) $\frac{1}{64}$ (c) $\frac{1}{256}$ (d) None of these

5. An unbiased die is tossed 500 times. The mean of the number of 'sixes' in these 500 tosses is **(ICAI)**

(a) $\frac{50}{6}$ (b) $\frac{500}{6}$ (c) $\frac{5}{6}$ (d) None of these

6. If in a Binomial distribution if mean = 20, S.D. = 4, then n is equal to **(ICAI)**

(a) 80 (b) 100 (c) 90 (d) None of these

7. In Binomial distribution if n = 4 and $p = \frac{1}{3}$ then the value of variance is **(ICAI)**

(a) $\frac{8}{3}$ (b) $\frac{8}{9}$ (c) $\frac{4}{3}$ (d) None of these

8. For binomial distribution E(x) = 2, V(x) = $\frac{4}{3}$. Find the value of n.

(a) 3 (b) 4 (c) 5 (d) 6

9. If dates are selected at random, then the probability of getting two sundays is :

(a) 0.29 (b) 0.99 (c) 0.49 (d) 0.39

10. An experiment succeeds twice as often as it fails. What is the probability that in next five trials there will be at least three successes?

(a) $\frac{33}{81}$ (b) $\frac{46}{81}$ (c) $\frac{64}{81}$ (d) $e^{\frac{6}{7}}$

Answer Key									
1. (c)	2. (b)	3. (c)	4. (b)	5. (b)	6. (b)	7. (b)	8. (d)	9. (a)	10. (c)

POISSON DISTRIBUTION

- Poisson distribution is a theoretical discrete probability distribution which can describe many processes. Simon Denis Poisson of France introduced this distribution way back in the year 1837.

  Let us think of a random experiment under the following conditions:
- The probability of finding success in a very small time interval (t, t + dt) is kt, where k (> 0) is a constant.
- The probability of having more than one success in this time interval is very low.
- The probability of having success in this time interval is independent of t as well as earlier successes.
- The above model is known as the Poisson Model. The probability of getting x successes in a relatively long time interval T containing m small time intervals t i.e. T = kt is given by $\frac{e^{kt}}{x!}(kt)^x$ for $x = 0, 1, 2, \ldots$

Taking kt = m, the above form is reduced to $\frac{e^{-m}m^x}{x!}$ for $x = 0, 1, 2, \ldots$

DEFINITION OF POISSON DISTRIBUTION

- A random variable X is defined to follow Poisson distribution with parameter m, to be denoted by $X \sim P(m)$ if the probability mass function of x is given by

$$f(x) = P(X = x) = \frac{e^{-m}m^x}{x!} \text{ for } x = 0, 1, 2, \ldots\ldots\ldots$$

Here, e is a transcendental quantity with an approximate value as 2.71828.

Example 10. If 1.5 percent of items produced by a manufacturing units are known to be defective, what is the probability that a sample of 200 items would contain no defective item? **(ICAI)**

(a) 0.05 (b) 0.15 (c) 0.20 (d) 0.22

Sol. (a) Given: 1.5% defective items i.e. $(p) = \frac{1.5}{100} = 0.015$

Number of units (n) = 200

In Poisson Distribution,

$m = np = 200(0.015) \Rightarrow m = 3$

Probability mass function will be given as,

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

As we want 0 defective unit according to the question, then x = 0

$$\Rightarrow P(X = 0) = \frac{e^{-3}3^0}{0!}$$

$$= \frac{1}{e^3} = \frac{1}{(2.71828)^3} = 0.04978 \approx 0.05$$

Hence, the correct answer is option (a).

Example 11. X is a Poisson variate satisfying the following condition $9\ P(X = 4) + 90\ P(X = 6) = P(X = 2)$. What is the value of $P(X \le 1)$? **(ICAI)**

(a) 0.5655 (b) 0.6559 (c) 0.7358 (d) 0.8201

Sol. (c) As we know, For the poisson distribution with mean m

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

According to the question,

$$9P(X = 4) + 90P(X = 6) = P(X = 2)$$

Take e^{-m} and m^2 across the equation,

$$\Rightarrow 9\frac{m^2}{24} + 90\frac{m^4}{720} = \frac{1}{2}$$

$$\Rightarrow \frac{3m^2}{8} + \frac{m^4}{8} = \frac{1}{2}$$

$\Rightarrow m^4 + 3m^4 - 4 = 0$

$\Rightarrow m^4 + 4m^2 - m^2 - 4 = 0$

$\Rightarrow m^2 (m^2 + 4) - 1 (m^2 + 4) = 0$

$\Rightarrow (m^2 - 1) (m^2 + 4) = 0$

$\Rightarrow m^2 - 1 = 0$ and $m^2 + 4 = 0$

$\Rightarrow m = \pm 1$ and

$m^2 = -4$ which is not possible.

So, m = 1, as mean cannot be negative

Now, $P(X \le 1)$ will be given as,

$P(X \le 1) = P(X = 0) + P(X = 1)$ when m = 1

$$= \frac{e^{-1}1^0}{0!} + \frac{e^{-1}1^1}{1!} = \frac{1}{e} + \frac{1}{e}$$

$$= \frac{2}{e} = \frac{2}{2.71828}$$

$= 0.73579 \approx 0.7358$

Hence, the correct answer is option **(c)** i.e., 0.7358.

Example 12. If 1 percent of an airline's flights suffer a minor equipment failure in an aircraft, what is the probability that there will be exactly two such failures in the next 100 such flights? **(ICAI)**

(a) 0.50 (b) 0.184 (c) 0.265 (d) 0.256

Sol. (b) Given: Probability that an airline's flights suffer a minor equipment failure (p) = 1%

$$= \frac{1}{100} = 0.01$$

Number of flights (n) = 100

Thus, m = np = 100(0.01)

$\Rightarrow m = 1$

Probability mass function will be given as,

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

According to the question,

For exactly 2 equipment failure i.e. x = 2

$$\Rightarrow P(X = 2) = \frac{e^{-1}1^2}{2!} = \frac{1}{2e}$$

$= 0.183932 \approx 0.184$

Hence, the correct answer is option (b).

It is wiser to remember the following important points in connection with Poisson distribution:

1. Since $e^{-m} = \frac{1}{e^m} > 0$, whatever may be the value of m, m > 0, it follows that f(x) ≥ 0 for every x. Also it can be established that f(x)=1

   i.e., f(0) + f(a) + f(b) + ... = 1
2. Poisson distribution is known as a uniparametric distribution as it is characterized by only one parameter m.
3. The mean of Poisson distribution is given by m i.e. $\mu = m$
4. The variance of Poisson distribution is given by $\sigma^2 = m$
5. Like binomial distribution, Poisson distribution could be also unimodal or bimodal depending upon the value of the parameter m.

   We have μ_0 =

   (a) The largest integer contained in m if m is a non-integer

   (b) m and m – 1 if m is an integer
6. Poisson approximation to Binomial distribution:

   The Poisson distribution can be used as an approximation to the binomial distribution under certain conditions. When the number of trials in a binomial distribution (n) is large and the probability of success (p) is small, the binomial distribution becomes computationally challenging. In such cases, the Poisson distribution provides a simpler and more convenient approximation.

   (a) The conditions for using the Poisson approximation to the binomial distribution are as follows:

   (b) The number of trials (n) is large, typically greater than 20.

   (c) The probability of success (p) is small, typically less than or equal to 0.05.

   (d) The events occur independently of each other.

   Then, B(n, p) ϑ P(m)
7. Additive property of Poisson distribution:

   The Poisson distribution has an additive property, which means that if we have two or more independent Poisson-distributed random variables, the sum of these variables will also follow a Poisson distribution.

   Mathematically, if X and Y are two independent Poisson-distributed random variables with parameters m_1 and m_2 respectively, then the sum Z = X + Y will be a Poisson-distributed random variable with parameter $m = m_1 + m_2$.

   i.e., If $X \sim P(m_1)$ and $Y \sim P(m_2)$ and X and Y are independent then

   $Z = X + Y \sim P(m_1 + m_2)$

Application of Poisson distribution:

Poisson distribution is applied when the total number of events is pretty large but the probability of occurrence is very small. Thus we can apply Poisson distribution, rather profitably, for the following cases:

1. The distribution of the number of printing mistakes per page of a large book.

2. The distribution of the number of road accidents on a busy road per minute.

3. The distribution of purchases of number of shoe laces per minute at shoe store

4. The distribution of the number of demands per minute for health centers and so on.

FITTING A POISSON DISTRIBUTION

As explained earlier, we can apply the method of moments to fit a Poisson distribution to an observed frequency distribution. Since Poisson distribution is uniparametric, we equate m, the parameter of Poisson distribution, to the arithmetic mean of the observed distribution and get the estimate of m.

i.e $\widehat{m} = \bar{x}$

The fitted Poisson distribution is then given by $\hat{f}(x) = \frac{e^{-m}(\hat{m})^x}{x!}$ for $x = 0, 1, 2, \ldots$

Example 13. If the standard deviation of a Poisson variate X is 2, what is $P(1.5 < X < 2.9)$?

(a) 0.231 (b) 0.158 (c) 0.15 (d) 0.146 **(ICAI)**

Sol. (d) Given : Standard Deviation (m) of a Poisson variate X is 2.

To find : $P(1.5 < X < 2.9)$

In Poisson Distribution, Mean$(\overline{X})$ = Variance (γ) = m

As we know, Standard deviation $(\sigma) = \sqrt{m}$

$\Rightarrow 2 = \sqrt{m}$

$\Rightarrow m = 4$

As X only takes discrete values, thus

For, $1.5 < X < 2.9$

$\Rightarrow X = 2$

Now, $P(X = x) = \frac{e^{-m} \times m^x}{x!}$

$P(X = 2) = \frac{e^{-4} \times 4^2}{2!}$

$= e^{-4} \times 8 = 0.146$

Hence, the correct answer is option (d) i.e., 0.146.

Example 14. For a Poisson variate X, P (X = 1) = P (X = 2). What is the mean of X? **(ICAI)**

(a) 1.00 (b) 1.50 (c) 2.00 (d) 2.50

Sol. (c) Given: P (X = 1) = P (X = 2)

Probability mass function will be given as,

$P(X = x) = \frac{e^{-m}m^x}{x!}$

$\Rightarrow \frac{e^{-m}m^1}{1!} = \frac{e^{-m}m^2}{2!} \Rightarrow 1 = \frac{m}{2}$

$\Rightarrow m = 2$

Therefore, the mean of X is 2.

Hence, the correct answer is option **(c)**.

Example 15. If X is a Poisson variate such that, P (X = 1) = 0.7, P (X = 2) = 0.3, then P (X = 0) **(July 2021)**

(a) $e^{\frac{6}{7}}$ (b) $e^{-\frac{6}{7}}$ (c) $e^{-\frac{2}{3}}$ (d) $e^{-\frac{1}{3}}$

Sol. (b) We know that,

Formula for poisson distribution for random variable $X, P(X) = \frac{e^{-m}m^x}{x!}$

Given P(x = 1) = 0.7, P(x = 2) = 0.3

So, $\frac{e^{-m}m^1}{1!} = me^{-m} = 0.7$

Also $\frac{e^{-m}m^2}{2!} = \frac{m^2e^{-m}}{2} = 0.3$

Dividing them, we get

Now, $\frac{m^2e^{-m}}{2me^{-m}} = \frac{0.3}{0.7}$

$\Rightarrow m = \frac{6}{7}$

Now, $P(x = 0) = \frac{e^{-m}m^0}{0!} = e^{-6/7}$

Hence, option (b) is correct.

Example 16. If for a Poisson variable X, f(2) = 3f(4), what is the variance of X ? **(ICAI)**

(a) 2 (b) 2 (c) $\sqrt{2}$ (d) 3

Sol. (a) Given: f(2) = 3f(4)

Probability mass function will be given as,

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

In Poisson Distribution, Mean = Variance = m

According to the question,

$\Rightarrow P(X = 2) = 3P(X = 4)$

$$\Rightarrow \frac{e^{-m}m^2}{2!} = 3\left(\frac{e^{-m}m^4}{4!}\right)$$

$\Rightarrow m^2 = 4 \Rightarrow m = \sqrt{4}$

$\Rightarrow m = 2$

Hence, the correct answer is option (a).

Example 17. A Company has two cars which it hires out during the day. The number of Cars demanded with mean 1.5. Then percentage of days on which only one car was in demand is equal to

(a) 23.26 (b) 33.47 (c) 44.62 (d) 46.40

Sol. (b) Let X denote the number of cars which are hired out per day.

For Poisson distribution, mean = m = 1.5

We know that, $P(x) = \frac{e^{-m} \cdot m^x}{x!}$

So, $P(X = 1) = \frac{e^{-1.5} \cdot (1.5)^1}{1!} = \frac{0.2231 \times 1.5}{1} = 0.33465 = 33.47\%$

Hence, the correct option is (b) i.e. 33.47%

Example 18. If the parameter of poisson distribution is m and mean + S.D. = $\frac{6}{25}$ then find.

(a) $\frac{3}{25}$ (b) $\frac{1}{25}$ (c) $\frac{4}{25}$ (d) $\frac{3}{5}$ **(Dec 2020)**

Sol. (b) We know that for poisson distribution the mean and variance is same.

Given, Mean + SD = $\frac{6}{25}$

Let mean = Vairance = m

Then, SD = $\sqrt{m}$

$\Rightarrow m + \sqrt{m} = \frac{6}{25}$

For the given options,

Option **(a)**: $\frac{3}{25} + \sqrt{\frac{3}{25}} \neq \frac{6}{25}$

Option **(b)**: $\frac{1}{25} + \sqrt{\frac{1}{25}} = \frac{1}{25} + \frac{1}{5} = \frac{6}{25}$ which is true

Option **(c)**: $\frac{4}{35} + \sqrt{\frac{4}{25}} = \frac{4}{25} + \frac{2}{5} = \frac{14}{25} \neq \frac{6}{25'}$

Option **(d)**: $\frac{3}{5} + \sqrt{\frac{3}{5}} \neq \frac{6}{25'}$

Hence, $m = \frac{1}{25}$, option (b) is correct.

Example 19. In a certain manufacturing process, 5% of the tools produced turn out to be defective. Find the probability that in a sample of tools, at most will be defective. Given e^{-2} = 0.135

(a) 0.555 (b) 0.932 (c) 0.785 (d) 0.675

Sol. (d) Given: 5% defective items i.e. $(p) = \frac{5}{100} = 0.05$

Number of units (n) = 40

In Poisson Distribution,

$m = np = 40(0.05)$

$\Rightarrow m = 2$

Probability mass function will be given as,

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

Now, P(at most 2 defective tools) is given by

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= \frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!}$$

$= e^{-2}(1 + 2 + 2) = 0.135 \times 5 = 0.675$

Hence, the correct answer is option (d) i.e., 0.675.

Example 20. The number of accidents in a year attributed to taxi drivers in a locality follows Poisson distribution with an average 2. Out of 500 taxi drivers of that area, what is the number of drivers with at least 3 accidents in a year? **(ICAI)**

(a) 162 (b) 180 (c) 201 (d) 190

Sol. (a) Given,

Number of units (n) = 40

Total no. of taxi drivers (N) = 500

Mean (m) = 2

The probability of at least 3 accidents is given by $P(X \ge 3) = 1 - P(X < 3)$

We know that, $P(X = x) = \frac{e^{-m}m^x}{x!}$

$\Rightarrow P(X \ge 3) = 1 - P(X < 3)$

$\Rightarrow P(X \ge 3) = 1 - [P(0) + P(1) + P(2)]$

$$\Rightarrow P(X \ge 3) = 1 - \frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!}$$

$$\Rightarrow P(X \ge 3) = 1 - \frac{1}{(2.71828)^2}[1 + 2 + 2]$$

$$\Rightarrow (PX \ge 3) = 1 - \frac{1}{(2.71828)^2}[5] \Rightarrow P(X \ge 3) = 1 - 0.06767$$

$\Rightarrow P(X \ge 3) = 0.3233$

Now, the number of drivers with at least 3 accidents in a year = $N \times P(X \ge 3)$

$= 500 \times 0.3233$

$= 161.65 \approx 162$

Therefore, the number of drivers with at least 3 accidents in a year are 162.

Hence, the correct answer is option (a)

Example 21. A renowned hospital usually admits 200 patients every day. One percent of patients, on an average, require special room facilities. On one particular morning, it was found that only one special room was available. What is the probability that more than 3 patients would require special room facilities? **(ICAI)**

(a) 0.1428 (b) 0.1732 (c) 0.2235 (d) 0.3450

Sol. (a) Given: Number of patients (n) = 200

Success (p) = 1% = 1/100 = 0.01

Thus, it follows Poisson Distribution.

As we know, for the poisson distribution with mean m,

Mean = Variance = m

As, m = np

m = 200(0.1)

m = 2

According to the question,

$P(X > 3) = 1 - P(X \leq 3)$

$= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)]$

As we know that,

$$P(X = x) = \frac{e^{-m}m^{x}}{x!}$$

$$P(X > 3) = 1 - \left[\frac{e^{-2}2^{0}}{0!} + \frac{e^{-2}2^{1}}{1!} + \frac{e^{-2}2^{2}}{2!} + \frac{e^{-2}2^{3}}{3!}\right]$$

$$= 1 - \left(\frac{1}{(2.71828)^{2}} \times \frac{19}{2}\right) = 1 - 0.85712$$

$= 0.14288$

Hence, the correct answer is option (a).

Example 22. A random variable X follows Poisson distribution and its coefficient of variation is 50. What is the value of $P\left(\frac{x > 1}{x > 0}\right)$ **(ICAI)**

(a) 0.1876 (b) 0.2341 (c) 0.9254 (d) 0.8756

Sol.(c) Given: Coefficient of Variation (C.V) = 50

As we know, for the Poisson distribution with mean m

Mean = Variance = m and Standard Deviation (S.D) = $\sqrt{m}$

Coefficient of Variation (C.V) is given by the formula,

$$C.V. = \frac{S.D.}{m} \times 100$$

$$50 = \frac{\sqrt{m}}{m} \times 100$$

$$\sqrt{m} = 2$$

Squaring both sides,

m = 4

As we know that,

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

$$P\left(\frac{X > 1}{X > 0}\right) = \frac{1 - [P(X = 0) + P(X = 1)]}{1 - [P(X = 0)]}$$

Now, P(X > 1),

$$= 1 - \left[\frac{e^{-m}m^x}{x!} + \frac{e^{-m}m^x}{x!}\right]$$

$$= 1 - \left[\frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!}\right] = 1 - \frac{5}{(2.71828)^4}$$

= 0.9085

Now, P(X > 0),

$$= 1 - \left[\frac{e^{-4}4^0}{0!}\right]$$

$$= 1 - \frac{1}{(2.71828)^4}$$

= 0.981685

$$\text{Now, } P\left(\frac{x > 1}{x > 0}\right) = \frac{0.9085}{0.981685}$$

= 0.92544

Hence, the correct answer is option **(c)**.

Example 23. A car hire firm has 2 cars which are hired out everyday. The number of demands per day for a car follows Poisson distribution with mean 1.20. What is the proportion of days on which some demand is refused? (Given $e^{1.20}$ = 3.32) **(ICAI)**

(a) 0.25 (b) 0.3012 (c) 0.12 (d) 0.03

Sol. (d) According to the question,

As we know, For the poisson distribution with mean m

Mean = Variance = m = 1.20 (given)

As we know that,

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

The service will be refused if P(X > 3)

P(X > 3) will be given as,

$P(X > 3) = 1 - P(X \le 3)$

$= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)]$

$$= 1 - \left[\frac{e^{-m}m^x}{x!} + \frac{e^{-m}m^x}{x!} + \frac{e^{-m}m^x}{x!} + \frac{e^{-m}m^x}{x!}\right]$$

$$= 1 - \left[\frac{e^{-1.20}1.20^0}{0!} + \frac{e^{-1.20}1.20^1}{1!} + \frac{e^{-1.20}1.20^2}{2!} + \frac{e^{-1.20}1.20^3}{3!}\right]$$

$$= 1 - \frac{1}{e^{12}}(1 + 1.2 + 0.72 + 0.288) = 1 - 0.9663$$

$= 0.0337 \sim 0.03$

Hence, the correct answer is option (d).

PRACTICE QUESTIONS (PART B)

1. In Poisson distribution, which of the following is same ? **(Dec 2019)**
   (a) Mean and variance (b) Mean and SD
   (c) Both (d) None of these
2. Let X be a Poisson random variable with parameter λ. Then p(X) is equal to
   (a) $\frac{e^{\lambda} - e^{-\lambda}}{2}$ (b) $\frac{e^{\lambda} + e^{-\lambda}}{2}$ (c) $\frac{e^{2\lambda} - 1}{2}$ (d) $\frac{\lambda^x \cdot e^{-\lambda}}{x!}$ **(Dec 2019)**
3. Which one of the following has Poisson distribution? **(Dec 2020)**
   (a) The number of days to get a complete cure.
   (b) The number of defects per meter on long roll of coated polythene sheet.
   (c) The errors obtained in repeated measuring of the length of a rod.
   (d) The number of claims rejected by an insurance agency.
4. Mean of poisson distribution is 6 then variance is ________ .
   (a) 6 (b) $\sqrt{6}$ (c) 4 (d) 3
5. If the overall percentage of success in an exam is 60, what is the probability that out of a group of 4 students, at least one has passed? **(ICAI)**
   (a) 0.6525 (b) 0.9744 (c) 0.8704 (d) 0.0256
6. If 5% of the families in Kolkata do not use gas as a fuel, what will be the probability of selecting 10 families in a random sample of 100 families who do not use gas as a fuel. (Given $e^{-5} = 0.0067$)
   (a) 0.038 (b) 0.028 (c) 0.048 (d) 0.018
7. The probability that a random variable x following Poisson distribution would assume a positive value is $(1-e^{-2.7})$. What is the mode of the distribution?
   (a) 0 (b) 1 (c) 2 (d) None of these

8. Find the mean and standard deviation of x, where x is a Poisson variate satisfying the condition $P(x = 3) = P(x = 4)$.

(a) 4, 2 (b) 4, 4 (c) 2, 2 (d) None of these

9. For a Poisson distributed variable, we have, $P(X = 7) = 8P(X = 9)$ the mean of the distribution is **(Dec 2020)**

(a) 4 (b) 3 (c) 7 (d) 9

10. What is the standard deviation of the number of recoveries among 48 patients when the probability of recovering is 0.75? **(ICAI)**

(a) 36 (b) 81 (c) 9 (d) 3

Answer Key

1. (a) 2. (d) 3. (b) 4. (a) 5. (b) 6. (d) 7. (c) 8. (a) 9. (b) 10. (d)

NORMAL OR GAUSSIAN DISTRIBUTION

- The two distributions discussed so far, namely binomial and Poisson, are applicable when the random variable is discrete.
- In the case of a continuous random variable like height or weight, it is impossible to distribute the total probability among different mass points because between any two unequal values, there remains an infinite number of values.
- Thus, a continuous random variable is defined in term of its probability density function f (x), provided, of course, such a function really exists, f (x) satisfies the following condition: $f(x) \geq 0$ or for $x \in (-\infty, \infty)$ and $\int_{-\infty}^{\infty} f(x)dx = 1$
- It is also known as Gaussian Distribution as Karl Gauss was instrumental for deriving normal distribution and as such normal distribution but the contribution of De-Moivre, Laplace was significant,
- A continuous random variable x is defined to follow normal distribution with parameters μ and σ^2, to be denoted by $X \sim N(\mu, \sigma^2)$
- If the probability density function of the random variable x is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \text{ for } -\infty < x < \infty$$ where μ and σ are constants and $\sigma > 0$.

Example 11. The mean and the variance of a random variable having the probability density function $P(X = x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-4)^2}, -\infty < x < \infty$ is

(a) $4, \frac{1}{2}$ (b) $4, \frac{1}{\sqrt{2}}$ (c) 2, 2 (d) $2, \frac{1}{2}$

Sol. (a) Normal distribution density function is :

$$P(X = x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty$$

Given equation:

$$P(X = x) = \frac{1}{\sqrt{\pi}} e^{-(x-4)^2} = \frac{1}{\frac{1}{\sqrt{2}}\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-4}{\frac{1}{\sqrt{2}}}\right)^2}$$

Comparing given function with the standard form, we get

Mean (μ) = 4

$$\text{S. D. } (\sigma) = \frac{1}{\sqrt{2}}$$

$$\text{Variance } (\sigma^2) = \frac{1}{2}$$

Hence, the correct option is (a) i.e.. $4, \frac{1}{2}$

Example 24. What is the coefficient of variation of x, characterized by the following probability density function: $f(x) = \frac{1}{4\sqrt{2\pi}} e^{\frac{-(x-10)^2}{32}}$ for $-\infty < x < \infty$ **(ICAI)**

(a) 50 (b) 60 (c) 40 (d) 30

Sol (c) General form of probability density function is given as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Given in the question, $f(x) = \frac{1}{4\sqrt{2\pi}} e^{\frac{-(x-10)^2}{32}}$

where, σ is the standard deviation, μ is the mean

Comparing the coefficients,

$\sigma = 4, \mu = 10$

Coefficient of Variation is given by the formula,

$$\text{C. V.} = \frac{\sigma}{\mu} \times 100$$

$$\text{C. V.} = \frac{4}{10} \times 100$$

$\Rightarrow$ C. V. = 40

Hence, the correct answer is option **(c)** i.e., 40.

Example 25. What is the first quartile of X having the following probability density function?

$$f(x) = \frac{1}{\sqrt{72\pi}} e^{\frac{-(x-10)^2}{72}} \text{ for } -\infty < x < \infty$$ **(ICAI)**

(a) 4 (b) 5 (c) 5.95 (d) 6.75

Sol (c) General form of probability density function is given as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{(x-\mu)^2}{\sigma^2}\right)}$$

Given in the question,

$$f(x) = \frac{1}{\sqrt{72\pi}} e^{\frac{-(x-10)^2}{72}}$$

where, σ is the Standard deviation, μ is the mean

Comparing the coefficients,

$\Rightarrow 2\sigma^2 = 72$ and $\mu = 10$

$\Rightarrow \sigma = 6$ and $\mu = 10$

First Quartile is given as,

$Q_1 = \mu - 0.675(\sigma)$

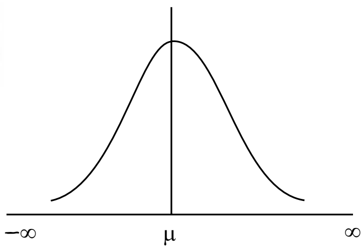
$= 10 - 0.675(6)$

$= 10 - 4.05 = 5.95$

Hence, the correct answer is option (c).

SOME IMPORTANT POINTS RELATING TO NORMAL DISTRIBUTION ARE LISTED BELOW

1. The name Normal Distribution has its origin some two hundred years back as the then mathematicians were in search for a normal model that can describe the probability distribution of most of the continuous random variables.
2. If we plot the probability function $y = f(x)$, then the curve, known as probability curve, takes the following shape:



- **Curve is symmetric**: If you look above curve is the symmetrical curve which is bell shaped and has one peak representing the mode of normal distribution.
- **The line of symmetry is $x = \mu$** which is similar to binomial distribution, which is symmetric about $p = 0.5$

Tails of the Curve: If we look at the two tails of the normal curve, it extends indefinitely on both sides of the curve and both the left and right tails never touch the horizontal axis. The total area of the normal curve or for that any probability curve is taken to be unity i.e. one. Since the vertical line drawn through x = divides the curve into two equal halves, it automatically follows that The area between $-\infty$ to μ = the area between μ to $\infty = 0.5$

When the mean is zero, we have

Area between $-\infty$ to 0 = Area between 0 to ∞ = 0.5

$\Rightarrow$ If we take $\mu = 0$ and $\sigma = 1$, we have

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \text{ for } -\infty < z < \infty$$

The random variable z is known as **Standard normal variate** (or variable) or standard normal deviate.

The probability that a standard normal variate X would take a value less than or equal to a particular value say X = x is given by

$\phi(x) = P(X \le x)$

$\phi(x)$ is known as the cumulative distribution function.

We also have $\phi(0) = P(X \le 0)$

= Area of the standard normal curve between $-\infty$ and 0

= 0.5

The normal distribution is known as biparametric distribution as it is characterized by two parameters μ and σ^2. Once the two parameters are known, the normal distribution is completely specified.

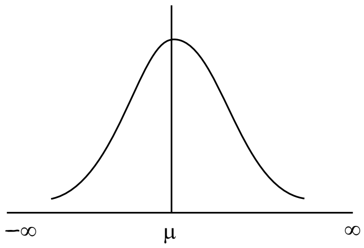
PROPERTIES OF NORMAL DISTRIBUTION

- Since $\pi = \frac{22}{7}, e^{-\theta} = \frac{1}{e^{\theta}} > 0$ whatever maybe, it follows that $f(x) \ge 0$ for every x.
- It can be shown that $\int_{-\infty}^{\infty} f(x) = 1$
- The mean of the normal distribution is given by μ. Further, since the distribution is symmetrical about $x = \mu$, it follows that the mean, median and mode of a normal distribution coincide, all being equal to μ.
- The standard deviation of the normal distribution is given by σ.
- Mean deviation of normal distribution is $\sigma\sqrt{\frac{2}{\pi}} = 0.8\sigma$
- The first and third quartiles are given by

  $Q_1 = m - 0.675\,\sigma$ and $Q_3 = m + 0.675\,\sigma$

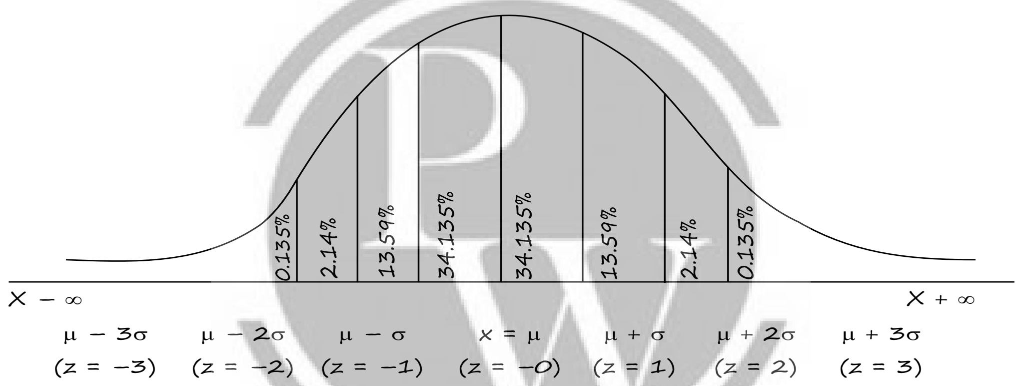
So, Quartile Deviation = 0.675σ

- The normal distribution is symmetrical about $x = \mu$. As such, its skewness is zero i.e. the normal curve is neither inclined to move towards the right (negatively skewed) nor towards the left (positively skewed).
- The normal curve $y = f(x)$ has two points of inflexion to be given by $x = \mu - \sigma$ and $x = \mu + \sigma$ i.e. at these two points, the normal curve changes its curvature from concave to convex and from convex to concave.



- If $x \sim N(\mu, \sigma^2)$ then $z = \frac{ü - \mu}{\sigma} \sim N(0, 1)$, $z \sim N(0, 1)$, z is known as standardized normal variate or normal deviate.

- We also have $P(z \le k) = \phi(k)$ which is known as the cumulative distribution function.
- The values of $\phi(k)$ for different k are given in a table known as "Biometrika." Because of symmetry, we have $\phi(-k) = 1 - \phi(k)$
  We can evaluate the different probabilities in the following manner:
- $P(x < a) = P\left[\frac{x-u}{\sigma} < \frac{a-\mu}{\sigma}\right] = P(z < k)$ where, $k = \frac{a-\mu}{\sigma} = \phi(k)$
- Also $P(x \le a) = P(x < a)$ as x is continuous.
- $P(x > b) = 1 - P(x < b) = 1 - \phi\left(\frac{b-\mu}{\sigma}\right)$
- $P(a < x < b) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$ ordinate at x = a is given by $\left(\frac{1}{\sigma}\right)\phi\left(\frac{a-\mu}{\sigma}\right)$
- Also, $\phi(-k) = \phi(k)$
- The value of $\phi(k)$ for different k are also provided in the Biometrika Table.
- Area under the normal curve is shown in the following figure :



- If x and y are independent normal variables with means and standard deviations as μ_1 and μ_2 and σ_1 and σ_2 respectively, then z = x + y also follows normal distribution with mean $(\mu_1 + \mu_2)$ and SD $= \sqrt{\sigma_1^2 + \sigma_2^2}$ respectively.

SOME IMPORTANT PROPERTIES OF STANDARD NORMAL VARIATE

If we take $\mu = 0$ and $\sigma = 1$,we have $f(x) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ for $-\infty < z < \infty$

1. z has mean, median and mode all equal to zero.
2. The standard deviation of z is 1. Also the approximate values of mean deviation and quartile deviation are 0.8 and 0.675 respectively.
3. The standard normal distribution is symmetrical about z=0.
4. The two points of inflexion of the probability curve of the standard normal distribution are −1 and 1.
5. The two tails of the standard normal curve never touch the horizontal axis.

6. The upper and lower p per cent points of the standard normal variable z are given by $P(Z > z_p) = p$ and $P(Z < z_{1-p}) = p$ i.e., $P(Z < z_p) = p$ respectively since for a standard normal distribution
$(z_{1-p} = -z_p)$
Selecting P = 0.005, 0.025, 0.01 and 0.05 respectively, We have
$z_{0.005} = 2.58$
$z_{0.025} = 1.96$
$z_{0.01} = 2.33$
$z_{0.05} = 1.645$

APPLICATION OF NORMAL DISTRIBUTION

The applications of normal distribution are not restricted to statistics only. Many science subjects, social science subjects, management, commerce etc. find many applications of normal distributions. Most of the continuous variables like height, weight, wage, profit etc. follow normal distribution. If the variable under study does not follow normal distribution, a simple transformation of the variable, in many cases, would lead to the normal distribution of the changed variable.

Example 26. If the two quartiles of $N(\mu, \sigma^2)$ are 14.6 and 25.4 respectively, what is the standard deviation of the distribution? **(ICAI)**

(a) 9 (b) 6 (c) 10 (d) 8

Sol. (d) Given: First Quartile $(Q_1) = 14.6$, Third Quartile $(Q_3) = 25.4$

To find: Standard Deviation (σ)

As we know,

Quartile deviation is given as, $Q_D = \frac{Q_3 - Q_1}{2}$

$$Q_D = \frac{25.4 - 14.6}{2}$$

$$Q_D = 5.4$$

Standard deviation is given as, S.D $(\sigma) = \frac{3 \times Q_D}{2}$

$$\sigma = \frac{3(5.4)}{2}$$

$$\sigma = \frac{16.2}{2}$$

$$\sigma = 8.1 \approx 8$$

Hence, the correct answer is option (d).

Example 27. If the mean deviation of a normal variable is 16, what is its quartile deviation?

(a) 10.00 (b) 13.50 (c) 15.00 (d) 12.05 **(ICAI)**

Sol. (b) Given: Mean Deviation = 16

To find: Quartile Deviation (Q_D)

Mean deviation is given by the formula,

Mean Deviation = 0.8 × σ [σ → Standard Deviation]

16 = 0.8 σ

$$\sigma = \frac{16}{ü}$$

σ = 20

Quartile deviation is given as,

$Q_D = 0.675 \times \sigma$

= 0.675(20)

= 13.50

Hence, the correct answer is option (b) i.e., 13.50.

Example 28. If the quartile deviation of a normal curve is 4.05, then its mean deviation is

(a) 5.26 (b) 6.24 (c) 4.24 (d) 4.80 **(ICAI)**

Sol. (d) Given: Quartile Deviation (Q_D) = 4.05

We know that,

Quartile deviation is given as,

$\Rightarrow Q_D = 0.675 \times \sigma$

$\Rightarrow 4.05 = 0.675 \times \sigma$

$$\Rightarrow \sigma = \frac{4.05}{0.675}$$

$\Rightarrow \sigma = 6$

Now, Mean Deviation will be given as,

Mean Deviation = 0.8 × σ [σ → Standard Deviation]

= 0.8 × 6 = 4.8

Hence, the correct answer is option (d).

Example 29. If the First quartile and mean deviation about median of a normal distribution are 13.25 and 8 respectively, then the mode of the distribution is **(ICAI)**

(a) 20 (b) 10 (c) 15 (d) 12

Sol. (a) Given: First Quartile (Q_1) = 13.25

Mean Deviation = 8

Mean deviation is given by the formula,

Mean Deviation = 0.8 × σ [σ → Standard Deviation]

$\Rightarrow 8 = 0.8 \times \sigma$

$$\Rightarrow \sigma = \frac{8}{0.8}$$

$\Rightarrow \sigma = 10$

First Quartile is given as,

$\Rightarrow Q_1 = \mu - 0.675(\sigma)$ [μ → Mean]

$\Rightarrow 13.25 = \mu - 0.675(10)$

$\Rightarrow \mu = 13.25 + 6.75$

$\Rightarrow \mu = 20$

As we know, in Normal Distribution,

Mean = Median = Mode

So, Mode = 20

Hence, the correct answer is option (a).

Example 30. If the inflexion points of a normal distribution are 6 and 14..Find its standard deviation?

(a) 4 (b) 6 (c) 10 (d) 12

Sol. (a) Points of inflexion of a normal curve = Mean ± Standard Deviation

i.e. $x = \mu \pm \sigma$

Here, x values are 6 and 14

So, $\mu + \sigma = 14$ ...(i)

$\mu - \sigma = 6$ ...(ii)

Add equation (i) and (ii),

Substitute in eq. (i)

$\Rightarrow 10 + \sigma = 14$

$\Rightarrow \sigma = 14 - 10$

$\Rightarrow \sigma = 4$

Hence, the correct option is (a) i.e. 4.

Exampl 31. If the area of standard normal curve between z = 0 to z = 1 is 0.3413, then the value of φ(a) is **(ICAI)**

(a) 0.5000 (b) 0.8413 (c) -0.5000 (d) 1

Sol. (b) Given:

Area of standard normal curve between z = 0 to z = 1 is 0.3413

To find: $\varphi(1)$

$\varphi(1)$ will be given as, the area of standard normal curve between $-\infty$ to 1.

This will divided into 2 segments, i.e., ($-\infty$ to 0) and (0 to 1)

We know that,

Area between $z = -\infty$ to $z = 0$ is 0.5.

Now, area between $z = -\infty$ to $z = 1$

= 0.5 + 0.3413

= 0.8413

Therefore, $\varphi(1) = 0.8413$

Hence, the correct answer is option **(b)**.

Example 32. Area of the normal curve **(ICAI)**

(a) between $-\infty$ to μ is 0.50 (b) between μ to $-\infty$ is 0.50.

(c) between $-\infty$ to ∞ is 0.50. (d) both (a) and (b)

Sol. (d) Normal Distribution is also called Gaussian or bell curve. It is defined as probability distribution where the values of a random variable are distributed symmetrically. These values are equally distributed on the left and the right side of the central tendency.

It is formulated as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \text{ for } -\infty < x < \infty$$

As we can see in the formula x lies in between $-\infty$ to ∞.

So by definition, Area of normal curve is between $-\infty$ to μ is 0.50 and between μ to $-\infty$ is 0.50.

Hence, the correct answer is option (d).

Example 33. The cumulative distribution function of a random variable X is given by **(ICAI)**

(a) $F(x) = P(X \le x)$ (b) $F(x) = P(X < x)$

(c) $F(x) = P(X \ge x)$ (d) $F(x) = P(X = x)$

Sol. (a) The cumulative distribution function (CDF) calculates the cumulative probability for a given x-value.

CDF is used to determine the likelihood that a random observation taken from the population will be less than or equal to a particular value.

It is defined as $F(x) = P(X \le x)$.

Hence, the correct answer is option **(a)**.

Example 34. If X follows normal distribution with $\mu = 50$ and $\sigma = 10$, what is the value of $P\left(\frac{x \le 60}{x > 50}\right)$? **(ICAI)**

(a) 0.8413 (b) 0.6828 (c) 0.1587 (d) 0.7256

Sol. (b) Given: X follows normal distribution where $\mu = 50$ and $\sigma = 10$

To find: $P\left(\frac{x \le 60}{x > 50}\right)$

z is given by the formula, $z = \frac{x-\mu}{\sigma}$

$P\left(\frac{X \le 60}{X > 50}\right)$ this will be given as, $P\left(\frac{50 < X \le 60}{X > 50}\right)$

When $P(50 < X \le 60)$,

$$P\left(\frac{50-50}{10} < \frac{x-\mu}{\sigma} \le \frac{60-50}{10}\right)$$

$P(0 < z \le 1) = 0.3414$ (Using z score table)

When $P(X > 50)$

$$P\left(\frac{x-\mu}{\sigma} > \frac{50-50}{10}\right)$$

$P(z > 0) = 0.500$ (Using z score table)

Now, put the value of

$P(50 \langle X \leq 60)$ and $P(X \rangle 50)$ in $P\left(\frac{50 < x \leq 60}{x > 50}\right)$,

$= \frac{0.3414}{0.500} = 0.6828$

Hence, the correct answer is option (b).

Example 35. For a certain type of mobiles, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. A person owns one of those mobiles and wants to know the probability that length of time will be between 50 & 70 hours is

(Given) ϕ (1.33) = 0.9082, ϕ (0) = 0.5) **(July 2021)**

(a) -0.4082 (b) 0.5 (c) 0.4082 (d) -0.5

Sol. (c) We know that $z = \frac{X - \mu}{\sigma}$

Given that $\mu = 50$ hours, S. D. = $\sigma = 15$

So, $P(50 \leq X \leq 70) = P\left(\frac{50-50}{15} \leq z \leq \frac{70-50}{15}\right)$

$P(50 \leq X \leq 70) = P(0 \leq z \leq 1.33)$

$= \phi(1.33) - \phi(0)$

$= 0.9082 - 0.5$

$= 0.4082$

Hence, option (c) is correct.

PRACTICE QUESTIONS (PART C)

1. The probability density function of a normal variable is given by:

(a) $f(x) = \sigma \frac{1}{\sqrt{2\pi\sigma}} \cdot e^{\frac{(x-\lambda^2)}{2\sigma^2}}$ for $0 < x < \infty$

(b) $f(x) = \frac{1}{\sqrt{2\pi\sigma}} \cdot e^{\frac{(x-\mu^2)}{2\sigma^2}}$ for $-\infty < x < \infty$

(c) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$

(d) None of these

2. X and Y are two independent Normal variables, then the distribution of X + Y is ______

(a) Normal Distribution (b) t-distribution

(c) Chi-Square distribution (d) F-distribution

3. The Normal Curve is

(a) Positively Skewed (b) Negatively Skewed

(c) Symmetrical (d) All of these

4. If the two quartiles of N (μ,σ^2) are 47.30 and 52.70 respectively, what is the mean deviation about the median of the distribution?

(a) 9.0 (b) 6.5 (c) 3.20 (d) 8.45

5. Shape of Normal Distribution Curve

(a) Depends on its parameters (b) Does not depend on its parameters

(c) Either (a) or (b) (d) Neither (a) nor (b)

6. In Normal distribution 95% observation lies between _____ & ____ .

(a) $(\mu - 2\sigma, \mu + 2\sigma)$ (b) $(\mu - 3\sigma, \mu + 3\sigma)$

(c) $(\mu - 1.96\sigma, \mu + 1.96\sigma)$ (d) $(\mu - 2.58\sigma, \mu + 2.58\sigma)$

7. If x is a poisson variate with mean m then $z = \frac{x - m}{\sqrt{m}}$ follows _______ distribution.

(a) Normal (b) Binomial (c) Bernoulli (d) None of the above

Answer Key

1. (c) 2. (a) 3. (c) 4. (c) 5. (a) 6. (c) 7. (a)

Example 36. The mean of a binomial distribution with parameter n and p is **(ICAI)**

(a) $n(1 - p)$ (b) $np(1-p)$ (c) np (d) $\sqrt{np(1-p)}$

Sol. (c) Binomial distribution gives the probability that a discrete random variable will be exactly equal to a specific value. The probability mass function is only used for discrete random variables.

It is formulated as, $f(x) = {}^nC_x\, p^x q^{n-x}$

Where, $n \rightarrow$ Number of Trials

$x \rightarrow 0, 1, 2, 3,....$

$p \rightarrow$ Success

$q \rightarrow$ Failure

The mean or expected value (E) is given as multiplying the number of trials (n) and the probability of success (p).

$\Rightarrow (E) = np$

Hence, the correct answer is option (c).

Example 37. The variance of a binomial distribution with parameters n and p is **(ICAI)**

(a) $np^2(1 - p)$ (b) $\sqrt{np(1-p)}$ (c) $nq(1 - q)$ (d) $n^2p^2(1 - p)^2$

Sol. (c) Binomial distribution gives the probability that a discrete random variable will be exactly equal to a specific value. The probability mass function is only used for discrete random variables.

It is formulated as, $f(x) = {}^nC_x\, p^x q^{n-x}$

Where, $n \rightarrow$ Number of Trials

$x \rightarrow 0, 1, 2, 3, \ldots$

$p \rightarrow$ Success

$q \rightarrow$ Failure

Variance of the binomial distribution is a measure of the dispersion of the probabilities with respect to the mean value.

It is formulated by, $\sigma = npq$ where, $p = 1 - q$

Thus, $\sigma = nq(1-q)$

Hence, the correct answer is option (c) .

Example 38. The maximum value of the variance of a binomial distribution with parameters n and p is **(ICAI)**

(a) $\frac{n}{2}$ (b) $\frac{n}{4}$ (c) $np(1 - p)$ (d) $2n$

Sol. (b) Binomial distribution gives the probability that a discrete random variable will be exactly equal to a specific value. The probability mass function is only used for discrete random variables.

It is formulated as, $f(x) = {}^nC_x\, p^x q^{n-x}$

Where, $n \rightarrow$ Number of Trials

$x \rightarrow 0, 1, 2, 3, \ldots$

$p \rightarrow$Success

$q \rightarrow$Failure

Variance of the binomial distribution is a measure of the dispersion of the probabilities with respect to the mean value.

It is formulated by, $\sigma = npq$

$\Rightarrow np(1 - p)$

When, $p = q = \frac{1}{2}$ (i.e., when the occurrence of success and failure is equal)

$\Rightarrow n \times \frac{1}{2} \times \left(1 - \frac{1}{2}\right)$

$\Rightarrow n \times \frac{1}{2} \times \frac{1}{2}$

$\Rightarrow \frac{n}{4}$

Hence, the correct answer is option (b).

Example 39. The method usually applied for fitting a binomial distribution is known as

(a) Method of least square **(ICAI)**

(b) method of moments

(c) method of probability distribution

(d) method of deviations

Sol. (b) Binomial distribution gives the probability that a discrete random variable will be exactly equal to a specific value. The probability mass function is only used for discrete random variables.

It is formulated as, $f(x) = {}^nC_x\, p^x q^{n-x}$

Where, $n \to$ Number of Trials

$x \to 0, 1, 2, 3, ...$

$p \to$ Success

$q \to$ Failure

The method usually applied for fitting a binomial distribution is known as the method of moments.

Hence, the correct answer is option (b).

Example 40. The normal curve is **(ICAI)**

(a) Positively skewed.
(b) Negatively skewed.
(c) Symmetrical
(d) All of these.

Sol. (c) Normal Distribution is also called Gaussian or bell curve. It is defined as probability distribution where the values of a random variable are distributed symmetrically. These values are equally distributed on the left and the right side of the central tendency.

It is formulated as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for } -\infty < x < \infty$$

The key points associated with Normal curve are listed below:

- It is symmetrical about the ordinate at the central point of the curve.
- The skewness for a normal distribution is zero and the data is symmetric.

From the above points we can clearly see that the normal curve is symmetric.

Hence, the correct answer is option (c).

Example 41. The mean and mode of a normal distribution **(ICAI)**

(a) May be equal
(b) May be different
(c) Are always equal
(d) (a) or (b)

Sol. (c) Normal Distribution is also called Gaussian or bell curve. It is defined as probability distribution where the values of a random variable are distributed symmetrically. These values are equally distributed on the left and the right side of the central tendency.

It is formulated as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for } -\infty < x < \infty$$

As normal distribution is symmetrical, i.e., mean, median and mode are all equal.

Hence, the correct answer is option (c).

Example 41. The interval (μ – 3σ, μ + 3σ) covers **(ICAI)**

(a) 95% area of a normal distribution

(b) 96% area of a normal distribution

(c) 99% area of a normal distribution

(d) all but 0.27% area of a normal distribution

Sol. (d) Probability density function defines the density of the probability that a continuous random variable will lie within a particular range of values.

It is formulated as, $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$

According to the question, the interval $(\mu - 3\sigma, \ \mu + 3\sigma)$ will cover all but 0.27% area of a normal distribution.

Hence, the correct answer is option (d).

Example 44. The probability mass function of binomial distribution is given by **(ICAI)**

(a) $f(x) = p^x q^{n-x}$

(b) $f(x) = {}^nC_x p^x q^{n-x}$

(c) $f(x) = {}^nC_x q^x p^{n-x}$

(d) $f(x) = {}^nC_x p^{n-x} q^x$

Sol. (b) Probability mass function of binomial distribution gives the probability that a discrete random variable will be exactly equal to a specific value. The probability mass function is only used for discrete random variables.

It is formulated as, $f(x) = {}^nC_x \ p^x q^{n-x}$

Where, n → Number of Trials

p → Probability of getting success

q → Probability of getting failure

Hence, the correct answer is option (b) i.e., $f(x) = {}^nC_x \ p^x q^{n-x}$.

Example 45. If X ~ P (m) and its coefficient of variation is 50, what is the probability that X would assume only non-zero values? **(ICAI)**

(a) 0.018 (b) 0.982 (c) 0.989 (d) 0.976

Sol. (b) Given: Coefficient of variation (C.V.) = 50

Coefficient of Variation is given by the formula,

$C.V = \frac{\sigma}{\bar{X}} \times 100$ [σ→ Standard deviation, $\bar{X}$→ Mean]

In Poisson Distribution, Mean $(\bar{X})$ = Variance (γ) = m

As we know, Standard Deviation $(\sigma) = \sqrt{\gamma} = \sqrt{m}$

Put the respective values,

$50 = \frac{\sqrt{m}}{m} \times 100$

$\Rightarrow \sqrt{m} = 2$

$\Rightarrow m = 4$

Probability mass function will be given as,

$$P(X = x) = \frac{e^{-m} m^x}{x!}$$

$$P(X = 0) = \frac{e^{-4} 4^0}{0!}$$

$$= \frac{1}{e^4} = \frac{1}{(2.71282)^4}$$

= 0.01846

Thus, the probability that X would assume only non-zero values i.e.

$P(X \geq 1) = 1 - P(X = 0)$

= 1 − 0.01846

= 0.98154 ≈ 0.982

Hence, the correct answer is option (b).

Example 46. If the points of inflexion of a normal curve are 40 and 60 respectively, then its mean deviation is

(a) 8 (b) 5 (c) 10 (d) 6

Sol. (a) Given: Points of inflexion of a normal curve are 40 and 60

Let σ be the standard deviation and μ be the mean, then

According to the question,

$\mu - \sigma = 40$ ...(i)

$\mu + \sigma = 60$ ...(ii)

Adding (i) and (ii)

$2\mu = 100$

$$\Rightarrow \mu = \frac{100}{2}$$

$\Rightarrow \mu = 50$

Put $\mu = 50$ in (i)

$\Rightarrow 50 - \sigma = 40$

$\Rightarrow \sigma = 50 - 40$

$\Rightarrow \sigma = 10$

Since, the distribution is symmetric about mean value

Thus, mean, median and mode will coincide.

Mean deviation will be given as,

Mean Deviation = $0.8\,\sigma = 0.8(10) = 8$

Hence, the correct answer is option (a).

Example 47. The quartile deviation of a normal distribution with mean 10 and standard deviation 4 is **(Dec 2020)**

(a) 54.24 (b) 23.20 (c) 0.275 (d) 2.70

Sol. (d) Given,

Mean = 10, Standard deviation = 4

We know that,

Quartile deviation is given as,

$\Rightarrow Q_D = 0.675 \times \sigma$

$= 0.675(4) = 2.70$

Hence, the correct answer is option (d).

Example 48. The salary of workers of a factory is known to follow normal distribution with an average salary of ₹ 10,000 and standard deviation of salary as ₹ 2,000. If 50 workers receive salary more than ₹ 14,000, then the total number of workers in the factory is

(a) 2193 (b) 2000 (c) 2200 (d) 2500 **(ICAI)**

Sol. (a) Given: $\mu = 10,000$ and $\sigma = 2,000$

Now, $P(x > 14,000)$

$$= P\left(\frac{x - 10000}{2000} > \frac{14000 - 10000}{2000}\right)$$

$= P(Z > 2)$

$= 0.5 - P(0 \le Z \le 2)$

$= 0.5 - 0.4772$

$= 0.0228$

Number of workers having salary more than 14,000 = 50

Therefore, $N \times P(X > 14000) = 50$

$\Rightarrow N \times 0.0228 = 50$

$$\Rightarrow N = \frac{50}{0.0228}$$

$\Rightarrow N = 2192.98$

$\Rightarrow N = 2193$

Therefore, the total number of workers in the factory is 2,193.

Hence, the correct answer is option **(a)**.

Example 49. 50 percent of a certain product has weight 60 kg or more whereas 10 per cent have weight 55 kg or less. On the assumption of normality, what is the variance of weight? Given $\varphi(1.28) = 0.90$ **(ICAI)**

(a) 15.21 (b) 9.00 (c) 16.00 (d) 22.68

Sol. (a) Given,

$P(X > 60) = 0.5$

$$\Rightarrow P\left(\frac{X-\mu}{\sigma} > \frac{60-\mu}{\sigma}\right) = 0.5$$

$$\Rightarrow P(Z>0) = 0.5$$

$$\Rightarrow \frac{60-\mu}{\sigma} = 0$$

$$\Rightarrow \mu = 0$$

Also, $\phi(1.28) = 0.90$

$$\Rightarrow P(-\infty < Z < 1.28) = 0.90$$

$$\Rightarrow P(0 < Z < 1.28) = 0.90 - 0.50$$

$$\Rightarrow P(0 < Z < 1.28) = 0.4$$

$$P(X < 55) = 0.10$$

$$\Rightarrow P\left(\frac{X-60}{\sigma} < \frac{55-60}{\sigma}\right) = 0.10$$

$$\Rightarrow P\left(Z < -\frac{5}{\sigma}\right) = 0.1$$

$$\Rightarrow P\left(-\frac{5}{\sigma} < 0\right) = 0.4$$

$$\Rightarrow -\frac{5}{\sigma} = -1.28$$

$$\Rightarrow \sigma = \frac{-5}{-1.28}$$

$$\Rightarrow \sigma^2 = \left(\frac{5}{1.28}\right)^2 = 15.258$$

Therefore, the variance of weight is 15.21 (approx.).

Hence, the correct answer is option (a).

Example. 50 If the weekly wages of 5000 workers in a factory follows normal distribution with mean and standard deviation as ₹ 700 and ₹ 50 respectively, what is the expected number of workers with wages between ₹ 660 and ₹ 720? **(ICAI)**

Given, $\Phi(0.8) = 0.2881$, $\Phi(0.4) = 0.1554$

(a) 2,050 (b) 2,200 (c) 2,218 (d) 2,300

Sol. (c) Given,

X follows normal distribution with $\mu = 700$, $\sigma = 50$ and $N = 5000$

Number of workers with wages between ₹ 660 and ₹ 720 =

$N \times P(660 < X < 720)$

Now, $P(660 < X < 720)$

$$\aleph P\left(\frac{660 \aleph 700}{50} \quad \frac{X \;\; 700}{50} \quad \frac{720 \;\; 700}{50}\right)$$

$= P(-0.8 < Z < 0.4)$

$= P(-0.8 < Z < 0) + P(0 < Z < 0.4)$

$= P(0 < Z < 0.8) + P(0 < Z < 0.4)$

$= 0.2881 + 0.1554$

$= 0.4435$

Number of workers with wages between ₹ 660 and ₹ 720

$\Rightarrow N \times P(660 < X < 720)$

$= 5000 \times 0.4435$

$= 2217.5 \approx 2218$

Hence, the correct answer is option (c).

Example 51. In a sample of 800 students, the mean weight and standard deviation of weight are found to be 50 kg and 20 kg respectively. On the assumption of normality, what is the number of students weighing between 46 kg and 62 kg ?

Given area of the standard normal curve between z = 0 to z = 0.20 = 0.0793 and area between z = 0 to z = 0.60 = 0.2257. **(ICAI)**

(a) 250 (b) 244 (c) 240 (d) 260

Sol. (b) Given, The weight of the student follows normal distribution with $\mu = 50$ and $\sigma = 20$

.

Number of students having weights between 46 and 62

$= N \times P(46 \le X \le 62)$

Now, $P(46 < X < 62)$

$$\Rightarrow P\left(\frac{46-50}{20} < \frac{X-50}{20} < \frac{62-50}{20}\right)$$

$\Rightarrow P(-0.2 < Z < 0.6)$

$\Rightarrow P(-0.2 < Z < 0) + P(0 < Z < 0.6)$

$\Rightarrow P(0 < Z < 0.2) + P(0 < Z < 0.6)$

$\Rightarrow 0.0793 + 0.2257$

$\Rightarrow 0.3050$

Number of students having weighing between 46 and 62

$= N \times P(46 \le X \le 62)$

$\Rightarrow 800 \times 0.3050$

$\Rightarrow 244$

Hence, the correct answer is option (b).

PRACTICE QUESTIONS (PART D)

1. Standard deviation of binomial distribution is: **(ICAI)**

(a) $(npq)^2$ (b) $\sqrt{npq}$ (c) $(np)^2$ (d) $\sqrt{np}$

2. The wages of workers of factory follows: **(June 2016)**

(a) Binomial distribution (b) Poisson distribution
(c) Normal distribution (d) Chi-square distribution

3. The normal curve is:

(a) Positively skewed (b) Negatively skewed
(c) Symmetrical (d) All of these

4. If for a Binomial distribution B(n, p); n = 4 and also P(x = 2) = P(x = 3) then the value of p is **(Dec 2019)**

(a) 9/11 (b) 1 (c) 1/3 (d) 1/9

5. In a discrete random variable X follows uniform distribution and assumes only the values 8, 9, 11, 15, 18, 20. Then $P(X \leq 15)$ is____

(a) 1/2 (b) 1/3 (c) 2/3 (d) 2/5

6. If x and y are independent normal variates with Mean and Standard Deviation as μ_1 and μ_2 and σ_1 and σ_2 respectively, then z = x + y also follows normal distribution with

(a) Mean = $\mu_1 + \mu_2$ and S.D. = 0
(b) Mean = 0 and S.D. = $\sigma_1^2 + \sigma_2^2$
(c) Mean = $\mu_1 + \mu_2$ and S.D. = $\sqrt{\sigma^2_1 + \sigma_2^2}$
(d) None of these.

7. In Poisson distribution μ_4 = 2, its variance i.e. μ_2 is

(a) 2/3 (b) 1/2 (c) 1/3 (d) 3/2

8. Name the distribution which has Mean = Variance.

(a) Binomial distribution (b) Poisson distribution
(c) Normal distribution (d) Chi-square

9. An example of a bi-parametric discrete probability distribution is: (Dec 2017)

(a) Binomial distribution (b) Poisson distribution
(c) Normal distribution (d) Both (a) and (b)

10. If X ~ N (50, 16), then which of the following is not possible? (June 2017)

(a) P(X > 60) = 0.30(b)P(X < 50) = 0.50
(c) P(X < 60) = 0.40(d)P(X > 50) = 0.50

11. If for a distribution mean = variance, then the distribution is said to be:

(a) Normal(b)Binomial
(c) Poisson(d)None of these

12. For a Binomial distribution if variance = $(\text{Mean})^2$, then the values of n and p will be:

(a) 1 and 1/2 (b) 2 and 1/2 (c) 3 and 1/2 (d) 1 and 1

13. In a normal distribution about 95 per cent of the observations lie between ____and____.
(a) $\mu - 2\sigma, \mu + 2\sigma$ (b) $\mu - 3\sigma, \mu + 3\sigma$
(c) $\mu - 1.96\sigma, \mu + 1.96\sigma$ (d) $\mu - 2.58\sigma, \mu + 2.58\sigma$

14. An example of a bi-parametric discrete probability distribution is
(a) Binomial distribution (b) Poisson distribution
(c) Normal distribution (d) Both (a) and (b)

15. The variance of a binomial distribution with parameters n and p is:
(a) $np^2(1-p)$ (b) $np(1-p)$ (c) $nq(1-q)$ (d) $n^2p^2(1-p)^2$

16. X is a Poisson variate satisfying the following condition:
$9P(X = 4) + 90(X = 6) = P(X = 2)$. What is the value of $P(X \le 1)$?
(a) 0.5655 (b) 0.6559 (c) 0.7358 (d) 0.8201

17. In normal distribution, Mean, Median and Mode are **(July 2021)**
(a) Zero (b) Not Equal (c) Equal (d) Null

18. Probability distribution may be
(a) discrete (b) continuous (c) infinite (d) (a) or (b)

19. Which one of the following is an uniparametric distribution? **(Jan 2021)**
(a) Poisson (b) Normal (c) Binomial (d) Hyper geometric

20. For a Poisson variate X, $P(X = 2) = 3P(X = 4)$, then the standard deviation of X is
(a) $\sqrt{2}$ (b) 3 (c) 4 (d) 5 **(Nov 2018)**

21. The mean of the Binomial distribution $B(4, \frac{1}{3})$ is equal to **(Nov 2018)**
(a) 3/5 (b) 4/3 (c) 8/3 (d) 3/4

22. If for a normal distribution $Q_1 = 54.52$ and $Q_3 = 78.86$, then the median of the distribution is **(Nov 2018)**
(a) 12.17 (b) 39.43 (c) 66.69 (d) None of these

23. What is the mean of X having the following density function?

$$f(x) = \frac{1}{4\sqrt{2\pi}} e^{-\frac{(x-10)^2}{32}} \text{ for } -\infty < x < \infty$$ **(Nov 2018)**

(a) 4 (b) 10 (c) 40 (d) None of the above

24. The probability that a student is not a swimmer is 1/5, then the Probability that out of five students four are swimmer is
(a) $(4/5)^4(1/5)$ (b) ${}^5C_1\,(1/5)^4(4/5)$
(c) ${}^5C_4\,(4/5)^1\,(1/5)^4$ (d) None of these

25. 4 coins were tossed 1600 times. What is the probability that all 4 coins do not turn head upward at a time? **(June 2019)**
(a) $1600\,e^{-100}$ (b) $1000\,e^{-100}$ (c) $100\,e^{-1600}$ (d) e^{-100}

26. If mean and variance are 5 and 3 respectively then relation between p and q is **(June 2019)**

(a) $p > q$ (b) $p < q$ (c) $p = q$ (d) p is symmetric

27. In a Poisson distribution if $P(x = 4) = P(x = 5)$ then the parameter of Poisson distribution is **(June 2019)**

(a) 4/5 (b) 5/4 (c) 4 (d) 5

28. Area between = 1.96 to + 1.96 in a normal distribution is: **(June 2019)**

(a) 95.45% (b) 95% (c) 96% (d) 99%

29. A coin with probability for head as is tossed 100 times. The standard deviation of the number of head 5 turned up is **(Jan 2021)**

(a) 3 (b) 2 (c) 4 (d) 6

30. If Poisson distribution is such that $P(X = 2) = P(X = 3)$ then the variance of the distribution is **(Dec 2022)**

(a) $\sqrt{3}$ (b) 3 (c) 6 (d) 9

Answer Key

1. (c)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (c)
11. (c)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (c)	**18.** (d)	**19.** (a)	**20.** (a)
21. (b)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (d)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (c)	**30.** (b)

SUMMARY

- **Binomial distribution:** Trials are independent and each trail has only two outcomes Success & failure. $P(X = x) = {}^nC_x\, p^x q^{n-x}$
  Denoted by $X \sim B(n, p)$
  mean : $\mu = np$
  variance : $\sigma^2 = npq$
  $p + q = 1$
- **Poisson distribution:** Trials are independent and probability of occurrence is very small in given time. $P(X = x) = \frac{e^{-\lambda}\ \lambda^x}{x!}$
  Denoted by $X \sim P(m)$
  mean : $\mu = m$
  variance: $\sigma^2 = m$
- **Normal distribution:** When distribution is symmetric $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
  Mean = Median = Mode = μ
  Variance = σ^2

- We note that 99.73 percent of the values of a normal variable lies between $\mu - 3\sigma$ & $\mu + 3\sigma$.
- If x and y are independent normal variables with means and standard deviations as μ_1 μ_1 and μ_2 and σ_1, and σ_2 respectively, then z = x + y also follows normal distribution with mean $\mu_1 + \mu_2$ and SD = $\sqrt{\sigma_1^2 + \sigma_2^2}$ respectively.
- If a continuous random variable z follows standard normal distribution, to be denoted by z ~ N (0, 1), then the probability density function of z is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\pi^2/2} \quad \text{for} \quad -\infty < z < \infty$$

- Some important properties of z are listed below :
  - (i) z has mean, median and mode all equal to zero.
  - (ii) The standard deviation of z is 1 . Also the approximate values of mean deviation and quartile deviation are 0.8 and 0.675 respectively.
  - (iii) The standard normal distribution is symmetrical about z = 0.
  - (iv) The two points of inflexion of the probability curve of the standard normal distribution are –1 and 1.

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