## CHAPTER 3

## LINEAR INEQUALITIES

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## INEQUALITIES

- Inequalities are statements where two quantities are unequal
but a relationship exists between them.

A statement involving variable(s) and the sign of inequality
$\rangle,<, \geq$ or $<$ is called an inequation or inequality

- An inequation may contain one or more variables.
- It may be linear or quadratic or cubic etc.

Example: Following are some examples of inequations:

- $3 x-2<0$
- $5 x+4 y \leq 3$

Inequalities in two variables

- $5 x-3>0$
- $4 x+5 \geq 0$
- $x^{2}+3 x+2<0$

Quadratic Inequalities

Inequalities in one variable

## SOLUTION SPACE OF AN INEQUATION

- A solution Space of an inequation is the value(s) of the variable(s)
that makes it a true statement.
- Consider the inequation $30 x \times 200$,x belong to whole number
- Let a be a non- zero real number and x be a variable. Then inequations of the form
- $a x+b<0$
- $a x+b \leq 0$
- $a x+b>0$
- $a x+b \geq 0$

Example

- $9 x+15>0$
- $2 x-3 \leq 0$

Rule 1: Equal numbers may be added to (or subtracted from) both sides of an inequality without affecting the sign of inequality

Rule 2: Both sides of an inequality can be multiplied (or divided) by the same positive number without changing the sign of inequality

Rule 3. when both sides are multiplied or divided by a negative number, then the sign of inequality is reversed.


Example Solve the following linear inequations:
(i) $2 x-4 \leq 0$

Example Solve the following linear inequations:
(ii) $4 x-12 \geq 0$

## LINEAR INEQUATIONS IN TWO VARIABLE

- Let $\mathbf{a}$, $\mathbf{b}$ be a non - zero real number and $x$, $y$ be variables. Then inequations of the form
- $a x+b y<c$
- $a x+b y>c$
- $\quad a x+b y \geq c$
- $a x+b y \leq c$
are known as linear inequations in two variables $x$ and $y$

Example
$2 x+3 y>6$
$2 x-39 y \leq 12$



## STEPS TO SOLVE LINEAR INEQUALITIES IN TWO VARIABLES

Step 1: Convert the given inequation, say $a x+b y \leq c$, into the equation which represents a straight line in $a x+b y=c$ in $x y$-plane.

Step 2: Put $y=0$ in the equation obtained in step 1 to get the point where the line meets with $x$-axis. Similarly, put $x=0$ to obtain a point where the line meets with $y$-axis .

Step 3: Join the points obtained in step 2 to obtain the graph of the line obtained from the given inequation. In case of a strict inequality i.e, $a x+b y+<c$ or $a x+b y>c$, draw the dotted line, otherwise make it thick line.

## STEPS TO SOLVE LINEAR INEQUALITIES IN TWO VARIABLES

Step 4: Choose a point, if possible $(0,0)$, not lying on this line: Substitute its coordinates in the inequation. If the inequation is satisfied, then shade the portion of the plane which contains the chosen point ; otherwise shade the portion which does not contain the chosen point .

Step 5: The shaded region obtained in step 4 represents the desired solution set.

Remark: In case of the inequalities $a x+b y \leq c$ and $a x+b y \geq c$ points on the line are also a part of the shaded region while in the case of inequalities $a x+b y<c$ and $a x+b y>c$ points on the line $a x+b y=c$ are not in the shaded region

## Solve the following:

- $3 x+y \leq 6$


## Solve the following:

- $x-y \leq-2$


## Solve the following:

- $y \leq x / 2$

EXAMPLE : Draw graphs of the following inequalities
$x \geq 0, y \geq 0, x \leq 6, y \leq 7, x+y \leq 12$ and shading the common region.

Choose the correct answer/answers
Ques 1 (i) An employer recruits experienced (x) and fresh workmen $(y)$ for his firm under the condition that he cannot employ more than 9 people. $x$ and $y$ can be related by the inequality
a. $x+y \neq 9$
b. $x+y \leq 9 x \geq 0, y \geq 0$
c. $x+y \geq 9 x \geq 0, y \geq 0$
d. None of these

Choose the correct answer/answers
(ii ) On the average experienced person does 5 units of work while a fresh one 3 units of work daily but the employer has to maintain an output of at least 30 units of work per day. This situation can be expressed as
a. $\quad 5 \mathrm{x}+3 \mathrm{y} \leq 30$
b. $5 x+3 y>30$
c. $5 x+3 y \geq 30 x \geq 0, y \geq 0$
d. None of these

Choose the correct answer/answers
(iii) The rules and regulations demand that the employer should employ not more than 5 experienced hands to 1 fresh one and this fact can be expressed as
a. $y \geq x / 5$
b. $5 y \leq x$
c. $5 \mathrm{y} \geq \mathrm{x}$
d. None of these

Choose the correct answer/answers
(iv) The union however forbids him to employ less than 2 experienced person to each fresh person. This situation can be expressed as
a. $x \leq y / 2$
b. $y \leq x / 2$
c. $y \geq x / 2$
d. $x>2 y$

## Choose the correct answer/answers

( $v$ )The graph to express the inequality $x+y \leq 9$ is

c.

b.

d. None of these

## Choose the correct answer/answers

( vi ) The graph to express the inequality $5 x+3 y \geq 30$ is
©.

b.

C.

d. None of these

## Choose the correct answer/answers

( vii ) The graph to express the inequality $\mathrm{y} \leq\left(\frac{1}{2}\right) \mathrm{x}$ is indicated by
©.

b.

C.

d.


Choose the correct answer/answers

$$
\text { (viii) } L_{1}: 5 x+3 y=30 L_{2}: x+y=9 L_{3}: y=x / 3 L_{4}: y=x / 2
$$

The common region (shaded part) shown in the diagram refers to
a. $5 \mathrm{x}+3 \mathrm{y} \leq 30$
$x+y \leq 9$
$y \leq 1 / 5 x$ $\mathrm{y} \leq \mathrm{x} / 2$
b. $5 x+3 y \geq 30$
$x+y \leq 9$
$y \geq x / 3$
$y \leq x / 2$
$x \geq 0, y \geq 0$
c. $5 x+3 y \geq 30$
$x+y \geq 9$
$y \leq x / 3$
$y \geq x / 2$
$x \geq 0, y \geq 0$
d. None of these


Ques 2 A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is at least 9 units of vitamin $A, 7$ units of vitamin $B, 10$ units of vitamin C and 12 units of vitamin D. The vitamin content per Kg. of each food is shown below:

|  | A | B | C | D |
| :--- | :--- | :--- | :--- | :--- |
| Food I: | 2 | 1 | 1 | 2 |
| Food II: | 1 | 1 | 2 | 3 |

Assuming $x$ units of food lis to be mixed with $y$ units of food II the situation can be expressed as
d. $2 \mathrm{x}+\mathrm{y} \leq 9$
$x+y \leq 7$
$x+2 y \leq 10$
$2 x+3 y \leq 12$
$x>0, y>0$
b. $2 x+y \geq 30$

$$
x+y \leq 7
$$

$$
x+2 y \geq 10
$$

$$
x+3 y \geq 12
$$

c. $2 x+y \geq 9$
$x+y \geq 7$
$x+y \leq 10$
$x+3 y \geq 12$
d. $2 x+y \geq 9$
$x+y \geq 7$
$x+2 y \geq 10$
$2 x+3 y \geq 12$
$x \geq 0, y \geq 0$,

Ques 3 Graph of inequalities drawn below:
$L_{1}: 2 x+y=9 L_{2}: x+y=7 L_{3}: x+2 y=10 L_{4}: x+3 y=12$
The common region (shaded part) indicated on the diagram is expressed by the set of inequalities
a. $2 x+y \leq 9$
$x+3 y \geq 12$
b. $2 x+y \geq 9$
$x+y \leq 7$
$x+2 y \geq 10$
c. $2 x+y \geq 9$
$x+y \geq 7$
$x+2 y \geq 10$
$x+3 y \geq 12$
$x \geq 0, y \geq 0$
d. None of these


Choose the correct answer/answers
Que. 4 The common region satisfied by the inequalities $L_{1}: 3 x+y \geq 6, L_{2}$ : $x+y \geq 4, L_{3}: x+3 y \geq 6$, and $L_{4}: x+y \leq 6$ is indicated by

b.

C.

d. None of these

Choose the correct answer/answers
Que. 5 The region indicated by the shading in the graph is expressed by inequalities
a. $x_{1}+x_{2} \leq 2$
$2 x_{1}+2 x_{2} \geq 8$
b. $x_{1}+x_{2} \leq 2$
$x_{2} x_{1}+x_{2} \leq 4$

$$
x_{1} \geq 0, x_{2} \geq 0,
$$

c. $x_{1}+x_{2} \geq 2$
$2 x_{1}+2 x_{2} \geq 8$
d. $x_{1}+x_{2} \leq 2$
$2 x_{1}+2 x_{2}>8$


Choose the correct answer/answers
Que. 6 (i) The inequalities $x_{1} \geq 0, x_{2} \geq 0$, are: represented by one of the graphs shown below:
a.

b.

c.


Choose the correct answer/answers
(ii)The region is expressed as
a. $x_{1}-x_{2} \geq 1$
b. $x_{1}+x_{2} \leq 1$
c. $x_{1}+x_{2} \geq 1$
d. None of these


Choose the correct answer/answers
(iii) The inequality $-x_{1}+2 x_{2} \leq 0$ is indicated on the graph as
C.

c.

b.

d. None of these

## Choose the correct answer/answers

Que. 7 The common region indicated on the graph is expressed by the set of five inequalities
a. L1: $x_{1} \geq 0$
L2: $x_{2} \geq 0$
L3: $x_{1}+x_{2} \leq 1$
L4: $x_{1}-x_{2} \geq 1$
b. L1: $x_{1} \geq 0$
L2: $x_{2} \geq 0$
L3: $x_{1}+x_{2} \geq 1$
L4: $x_{1}-x_{2} \geq 1$
L5: $-\mathrm{x}_{1}+2 \mathrm{x}_{2} \leq 0$
L5: $-x_{1}+2 x_{2} \leq 0$

L1: $x_{1} \leq 0$
C. L2: $x_{2} \leq 0$

L3: $x_{1}+x_{2} \geq 1$
d. None of these


L4: $x_{1}-x_{2} \geq 1$
L5: $-x_{1}+2 x_{2} \leq 0$

## Choose the correct answer/answers

Que. 8 A firm makes two types of products: Type A and Type B. The profit on product $A$ is $₹ 20$ each and that on product $B$ is $₹ 30$ each. Both types are processed on three machines $M_{1} M_{2}$ and $M_{3}$. The time required in hours by each product and total time available in hours per week on each machine are as follows:

| Machine | Product A | Product B | Available Time |
| :---: | :---: | :---: | :---: |
| M1 | 3 | 3 | 36 |
| M2 | 5 | 2 | 50 |
| M3 | 2 | 6 | 60 |

The constraints can be formulated taking $\mathrm{x}_{1}=$ number of units A and $x_{2}=$ number of unit of $B$ as

$$
x_{1}+x_{2} \leq 12
$$

$$
3 x_{1}+3 x_{2} \geq 36
$$

C. $5 x_{1}+2 x_{2} \leq 50$

$$
2 x_{1}+6 x_{2} \leq 60
$$

c. $3 x_{1}+3 x_{2} \leq 36$

$$
\text { b. } \begin{aligned}
& 5 x_{1}+2 x_{2} \leq 50 \\
& 2 x_{1}+6 x_{2} \geq 60 \\
& x_{1} \geq 0, x_{2} \geq 0
\end{aligned}
$$

d. None of these

$$
\begin{aligned}
& 5 x_{1}+2 x_{2} \leq 50 \\
& 2 x_{1}+6 x_{2} \leq 60 \\
& x_{1} \geq 0, x_{2} \geq 0
\end{aligned}
$$

## Choose the correct answer/answers

Ques. 9 The set of inequalities L1: $x_{1}+x_{2} \leq 12$, L2: $5 x_{1}+2 x_{2} \leq 50$, L3: $x_{1}+3 x_{2} \leq 30, x_{1} \geq 0$, and $x_{2} \geq 0$ is represented by
d.

b.
c.


## d. None of these

## Choose the correct answer/answers

Que. 10 The common region satisfying the set of inequalities $x \geq 0, y \geq 0, L 1: x+y \leq 5, L 2: x+2 y \leq 8$ and L3: $4 x+3 y \geq 12$ is indicated by

b.

c.

d. None of these

