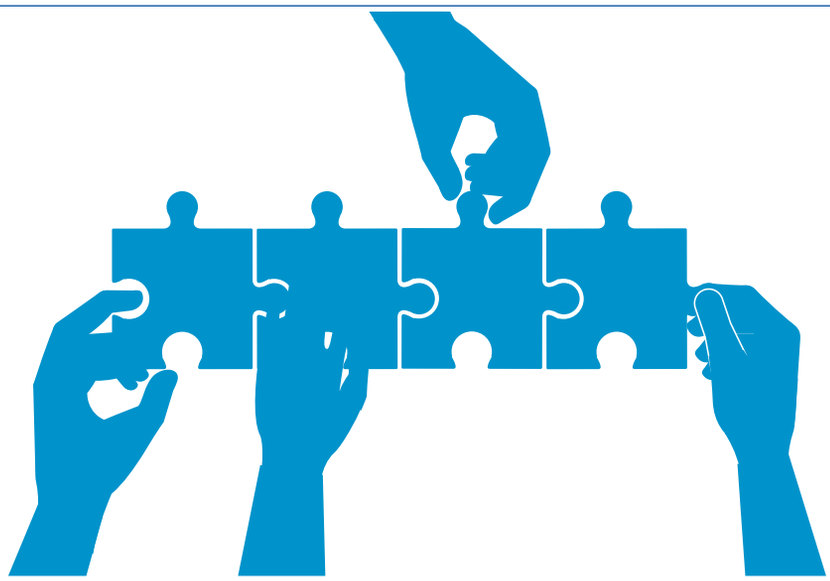
Chapter 5



Permutations & Combinations

CA VINOD REDDY

1. Permutation = Arrangement where order of objects is important.

ABC, BAC, BCA, ACB, CAB, CBA are 6 diff permutations

Combination = Selection where order is not important

ABC, BAC, BCA, ACB, CAB, CBA : This is only one selection.

2.

$0! = 1$

$1! = 1$

$2! = 2 \times 1 = 2$

$3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$5! = 5 \times 4! = 5 \times 4 \times 3! = 120$

$6! = 720$

$7! = 5040$

$8! = 40320$

$9! = 362880$

$10! = 3628800$

$11! = 11 \times 10!$

$12! = 12 \times 11 \times 10!$

a. $\frac{19!}{18!} = \frac{19 \times 18!}{18!} = 19$

b. $\frac{16!}{14!3!} = \frac{16 \times 15 \times 14!}{14! \times 6} = 40$

c. $\frac{x!}{(x-1)!} = \frac{x(x-1)!}{(x-1)!} = x$

d. $\frac{(x+3)!}{(x+2)!} = \frac{(x+3)(x+2)!}{(x+2)!} = (x+3)$

e. $\frac{(x-3)!}{(x-1)!} = \frac{(x-3)!}{(x-1)(x-2)(x-3)!} = \frac{1}{(x-1)(x-2)}$

3. In how many ways 3 students can stand in a line for a photograph?

→ ${}^3P_3 = 6$ ways where ${}^nP_r = \frac{n!}{(n-r)!}$

'r' students out of 'n' students can be arranged in ${}^nP_r$ ways

4. In how many ways 4 students can stand in a line for a photograph?

→ ${}^4P_4 = 24$ ways

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$${}^nP_r = n!/(n-r)!$$

5. There are 5 students A, B, C, D, E in how many ways 2 of them can be

Selected $= {}^5C_2 = 10$ ways

AB BE
AC CD
AD CE
AE DE
BC
BD

$${}^5C_2 \times 2! = {}^5P_2$$

$${}^nC_r \times r! = {}^nP_r$$

Arranged $= {}^5P_2 = 20$ ways

AB BE BA EB
AC CD CA DC
AD CE DA EC
AE DE EA ED
BC CB
BD DB

6. $${}^nP_r = \frac{n!}{(n-r)!}$$ where n = positive integer & $n \geq r \geq 0$

r = Non-Negative integer

${}^nP_r = n(n-1)(n-2) \ldots\ldots\ldots$ r tems	${}^{18}P_3 = 18 \times 17 \times 16 = 4896$
${}^nP_0 = 1$	${}^{100}P_2 = 100 \times 99 = 9900$
${}^nP_1 = n$	${}^{50}P_4 = 50 \times 49 \times 48 \times 47 = 55,27,200$
${}^nP_2 = n(n-1)$	${}^{25}P_1 = 25$
${}^nP_3 = n(n-1)(n-2)$	${}^{20}P_5 = 20 \times 19 \times 18 \times 17 \times 16 = 1860480$
${}^nP_4 = n(n-1)(n-2)(n-3)$	${}^{24}P_8 = 24 \times 23 \times 22 \times 21 \times 20 \times 19 \times 18 \times 17$
${}^nP_5 = n(n-1)(n-2)(n-3)(n-4)$	$= 29654190720$
${}^nP_n = n!$	

7. $$\frac{{}^{18}P_3 \times {}^{16}P_3}{{}^{19}P_4 \times {}^{17}P_2} = \frac{18 \times 17 \times 16 \times 18 \times 15 \times 14}{19 \times 18 \times 17 \times 16 \times 17 \times 16} = \left(\frac{210}{323}\right)$$

8. $$\frac{9!}{6!2!} \times {}^5P_2 = \frac{9 \times 8 \times 7 \times 6!}{6! \times 2} \times 5 \times 4 \times 2$$

$$= 5040 = 7!$$

9. **AND** ⟹ **Multiply**
   OR ⟹ **Add**

Golden rule

10. **n! can also be written as** $\lfloor n$

11. **How many different words can be formed by using letters of word :**

SQUARE : $6! = 720$ different words

HEXAGON : $7! = 5040$ different words

MISSISSIPPI : $\frac{11!}{4!4!2!} = 34650$ different words

I - 4 times
S - 4 times
P - 2 times

BOSTON : $\frac{6!}{2!} = 360$ different words

MANAGEMENT : $\frac{10!}{2!2!2!2!} = 2,26,800$ different words

M : 2 E : 2
A : 2 N : 2

PERMUTATION : $\frac{11!}{2!} = 1,99,58,400$ different words

BANANA :

$= \frac{6!}{3!2!} = 60$ different words

FOUNDATION : $= \frac{10!}{2!2!} = 9,07,200$ diff words

INTERMEDIATE : $= \frac{12!}{2!2!3!} = 1,99,58,400$ diff words

12. How many different words can be formed by using letters of word _____ if all vowels should be kept together.

BANANA : (A, A, A) B, N, N $= \frac{4!}{2!} \times \frac{3!}{3!} = 12$ diff words

PERCEPTION : (E, E, I, O) P, R, C, P, T, N $= \frac{7!}{2!} \times \frac{4!}{2!} = 30,240$ diff words

JAYARAMAN : (A, A, A, A) J, Y, R, M, N $= 6! \times \frac{4!}{4!} = 720$ diff words

STATISTICS : (A I I) S, T, T, S, T, C, S $= \frac{8!}{3!3!} \times$ Internal arrange.

$= 1120 \times \frac{3!}{2!} = 3,360$ diff words

COMPUTER :

(O, U, E) C, M, P, T, R $= 6! \times 3! = 4320$ diff words

CALCULATOR :

(A, U, A, O) C, L, C, L, T, R $= \frac{7!}{2!\,2!} \times \frac{4!}{2!} = 15,120$ diff words

TATED :

(A, E) T, T, D $= \frac{4!}{2!} \times 2! = 24$ diff words

13. In how many ways 'n' students can stand in a line for a photograph if r of them

Want to be always together?

$= (n - r + 1)! \times r!$

Want to be never together?

$=$ (All possible arrangement of n students $-$ Arrangements of n students where r of them are always together)

$= n! - (n - r + 1)!\, r!$

14. In how many ways 3 letter words can be formed by using letters of the word

SQUARE

$= {}^{6}P_3$

$= 120$ diff words

HEXAGON

$= {}^{7}P_3$

$= 210$ diff words

COMPUTER

$= {}^{8}P_3$

$= 336$ diff words

15. In how many ways 12 students can stand in a line for a photograph if

2 of the want to be always together?

$= 11! \times 2!$

$= 3,99,16,800 \times 2!$

$= 7,98,33,600$ diff photographs

2 of them want to be never together?

$= 12! - 7,98,33,600$

$= 39,91,68,000$ diff photographs

OR

$= {}^{10}P_{10} \times {}^{11}P_2$

$= 39,91,68,000$ diff ways

16. If $6 \times {}^{n}P_3 = 7 \times {}^{(n-1)}P_3$. Find n.

$6 \times n(n-1)(n-2) = 7 \times (n-1)(n-2)(n-3)$

$6n = 7n - 21$

$\therefore n = 21$

17. If ${}^{n}P_4 = 12 \times {}^{n}P_2$. then n = ?

$n(n-1)(n-2)(n-3) = 12\, n(n-1)$

$\boxed{n = 6}$

18. ${}^{n}P_3 : {}^{n}P_2 = 3:1$; then n = ?

$$\frac{n(n-1)(n-2)}{n(n-1)} = \frac{3}{1}$$

$\boxed{n = 5}$

19. ${}^{5}P_{r} = 60$; then r = ?

$5P_r = 60 = 5 \times 4 \times 3 = {}^{5}P_3$

$\therefore \quad r = 3$

20. The no. of ways in which letters of word 'TRIANGLE' can be arranged if word 'ANGLE' is always present.

T, R, I, (ANGLE)

$= 4! = 24$ diff words

21. In how many ways 5 students can form a

Line

$= {}^{5}P_5$

$= 5! = 120$ ways

Circle

$= \left(\frac{{}^{5}P_5}{5}\right)$

$= 4! = 24$ ways

22. In how many different ways 12 students can form a

Line

$= {}^{12}P_{12}$

$= 12!$

$= 47,90,01,600$

Circle

$= \left(\frac{{}^{12}P_{12}}{12}\right) = \frac{12!}{12}$

$= \frac{12 \times 11!}{12} = 11! = 3,99,16,800$

23. In how many ways _____ of 7 students can be formed out of 12 students

Line

$= {}^{12}P_7 = 3991680$ ways

Circle

$= \left(\frac{{}^{12}P_7}{7}\right) = 5,70,240$ ways

In how many ways 'r' students can be formed out of 'n' students can form

Line

$= {}^{n}P_r$

Circle

$= \left[{}^{n}P_r / r\right]$

24. The no. of ways in which 'n' diamonds can form a necklace.

$\Rightarrow \quad \frac{1}{2} \times (n-1)!$

The no. of ways in which 'n' students can form a circle are :

$(n-1)!$

25. The number of ways of arranging 'n' persons along a round table so that no person has the same 2 neighbours

$\frac{1}{2}(n-1)!$

26. No. of different necklaces can be formed with 'n' beads of different colours = ?

$= \frac{1}{2}(n-1)!$

27. Permutation of 'n' distinct things taken 'r' at a time when a particular object is

Always there?

$= {}^{n-1}P_{r-1} \times {}^{r}P_{1}$

$= r \cdot {}^{(n-1)}P_{(r-1)}$

Never there?

$= {}^{(n-1)}P_{r}$

28. How many 4 digit numbers can be formed by using 0,1,2,3,4,5 if repetition of digits is

Allowed

${}^{5}P_{1} \times {}^{6}P_{1} \times {}^{6}P_{1} \times {}^{6}P_{1}$

$= 1080$ Numbers

Not allowed

${}^{5}P_{1} \times {}^{5}P_{3}$

T H Tens O

$= 300$ Numbers

If question is silent about repetition of digits then

repetition of digits is NOT allowed.

29. How many even numbers of 5 digits can be formed by using 2,3,4,5,6,7,8 if repetition of digits is

Not allowed	Allowed
$6P_4 \times 4P_1$	$7P_1 \times 7P_1 \times 7P_1 \times 7P_1 \times 4P_1$
= 1440 Numbers	= 9604 Numbers

30. How many 5 digit numbers greater than 23,000 can be formed by using 1,2,3,5,8,9

→ Starting with 2: $1P_1 \times 4P_1 \times 4P_3 = 96$

Starting with 3,5,8,9: $4P_1 \times 5P_4 = 480$

576 numbers

31. How many 4 digit numbers greater than 4700 can be formed by using 2,3,4,5,8 if repetition of digits is

Starting with	Allowed	Not allowed
4	$1P_1 \times 1P_1 \times 5P_1 \times 5P_1 = 25$	$1P_1 \times 1P_1 \times 3P_2 = 6$
5,8	$2P_1 \times 5P_1 \times 5P_1 \times 5P_1 = 250$	$2P_1 \times 4P_3 = 48$
	275 Numbers	54 Numbers

32. ${}^nC_r = \left(\frac{{}^nP_r}{r!}\right)$

No. of selections of r objects out of n $\times r! =$ No. of arrang. of r objects out of n

$${}^nC_r = \left[\frac{n!}{(n-r)!\,r!}\right]$$

$${}^nC_r \times r! = {}^nP_r$$

$${}^nC_r = \frac{n!}{r!\,(n-r)!}$$

where n = a positive integer, r = a Non-negative integer & $n \geq r \geq 0$

$${}^nC_r = \frac{n(n-1)(n-2)\ldots\ldots \text{r terms}}{r!}$$

33. Formulae on combinations

${}^nC_r = n! / (n-r)!\,r!$	${}^nC_3 = \frac{n(n-1)(n-2)}{3!}$
${}^nC_r = \frac{n(n-1)(n-2)\ \ldots\ r \text{ terms}}{r!}$ $= ({}^nP_r / r!)$	${}^nC_4 = \frac{n(n-1)(n-2)(n-3)}{4!}$
${}^nC_0 = 1$	${}^nC_n = 1$
${}^nC_1 = n$	
${}^nC_2 = \frac{n(n-1)}{2!}$	${}^nC_{n-r} = {}^nC_r$ ${}^{15}C_{11} = {}^{15}C_4$ ${}^{15}C_{11} = {}^{15}C_{15-11}$
${}^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r$	${}^{100}C_{98} = {}^{100}C_2$
${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots\ldots + {}^nC_n = (2)^n$	${}^nC_2 + {}^nC_3 + \cdots + {}^nC_n = 2^n - {}^nC_0 - {}^nC_1$ $= 2^n - 1 - n$
${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots\ldots + {}^nC_n = (2)^n - 1$	

34. ${}^{18}C_r = {}^{18}C_{r+2}$ then r = ?

$r + r + 2 = 18$

$2r = 16$

$r = 8$

cross check

${}^{18}C_8 = {}^{18}C_{10}$

35. ${}^{45}C_x = {}^{45}C_y$ then

$x = y$

OR

$x + y = n$

${}^{18}C_3 = {}^{18}C_x$

then $x = 3$ OR $x = 15$

36.

${}^{15}C_{11} = \frac{15!}{11!\,(15-11)!} = \frac{15!}{4!\,11!}$

${}^{15}C_4 = \frac{15!}{4!\,(15-4)!} = \frac{15!}{4!\,11!}$

$\therefore {}^{15}C_{11} = {}^{15}C_4$

${}^{15}C_{11} = {}^{15}C_{15-11}$

$\therefore {}^nC_r = {}^nC_{n-r}$

37. In how many ways 52 cards can be equally divided in 4 groups?

→ $\frac{52!}{13!\,13!\,13!\,13!}$ $= 52! / (13!)^4$

OR $\left({}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13} \right)$

38. In how many different ways 10 mangoes can be divided among 3 people such that they will get 2,3,5 mangoes

→ $\frac{10!}{2!\,3!\,5!}$

$= 2,520$ ways

$= {}^{10}C_5 \times {}^5C_3 \times {}^2C_2$

$= 252 \times 10 \times 1$

$= 2,520$ ways

39. $\frac{{}^nP_r}{{}^nC_r} = r!$

$\frac{{}^nC_r}{{}^nP_r} = \frac{1}{r!}$

${}^5P_r = \frac{5!}{(5-r)!}$

${}^5C_r = \frac{5!}{r!\,(5-r)!}$

$\therefore \frac{{}^5P_r}{{}^5C_r} = \frac{\frac{5!}{(5-r)!}}{\frac{5!}{r!\,(5-r)!}} = r!$

40. P (8, 3) = ${}^8P_3 = 336$

C (12, 4) = ${}^{12}C_4 = 495$

${}^{100}C_5 = 75287520$

${}^{90}P_3 = 704,880$

${}^{25}C_4 = 12,650$

${}^8P_5 = 6,720$

41. $\frac{{}^{18}P_3 \times {}^{17}C_2}{{}^{19}P_2 \times {}^{18}C_2} = \frac{18 \times 17 \times 16 \times 17 \times 16 \times 2}{2 \times 19 \times 18 \times 18 \times 17} = \frac{4352}{342} = \frac{2176}{171}$

$\frac{{}^{19}C_3 \times {}^{20}P_3 \times {}^{21}C_2}{{}^{22}P_3 \times {}^{20}P_2 \times {}^{21}C_3} = \frac{19 \times 18 \times 17 \times 20 \times 19 \times 18 \times 21 \times 20 \times 6}{6 \times 2 \times 22 \times 21 \times 20 \times 20 \times 19 \times 21 \times 20 \times 19}$

$= (459/1540)$

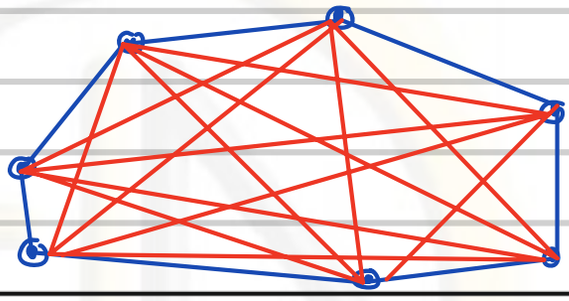
42. $\frac{{}^{20}P_3 \times {}^{21}P_4 \times {}^{22}C_4}{{}^{23}C_3 \times {}^{22}P_3 \times {}^{21}P_2} =$

$= \frac{20 \times 19 \times 18 \times 21 \times 20 \times 19 \times 18 \times 22 \times 21 \times 20 \times 19 \times 8}{21 \times 23 \times 22 \times 21 \times 22 \times 21 \times 20 \times 21 \times 20}$

$= \frac{19 \times 3 \times 19 \times 18 \times 5 \times 19}{23 \times 11 \times 7} = \left(\frac{1851930}{1771}\right)$

43. In a party of x people if everyone hand shakes with other. How many handshakes will take place

→ $^{x}C_2$

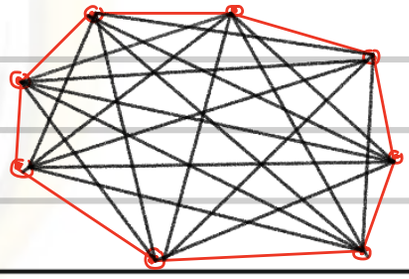


44. How many diagonals can be drawn in a polygon having :

7 sides = $^{7}C_2 - 7 = 21 - 7 = 14$ diagonals

8 sides = $^{8}C_2 - 8 = 20$ diagonals

10 sides = $^{10}C_2 - 10 = 35$ diagonals



45. In a group of 100 people, if everyone sends a greeting to other, How many cards will be used in total?

→ $^{100}P_2 = 9900$ cards

points are said to be collinear if a straight line can pass through all of them.

46. In a plane of 20 non-collinear points

How many different straight lines can be drawn? = $^{20}C_2$

How many different triangles can be obtained? = $^{20}C_3$

47. In a plane there are 30 points out of which 8 are collinear

How many different straight lines can be drawn?

$= {}^{30}C_2 - {}^{8}C_2 + 1$

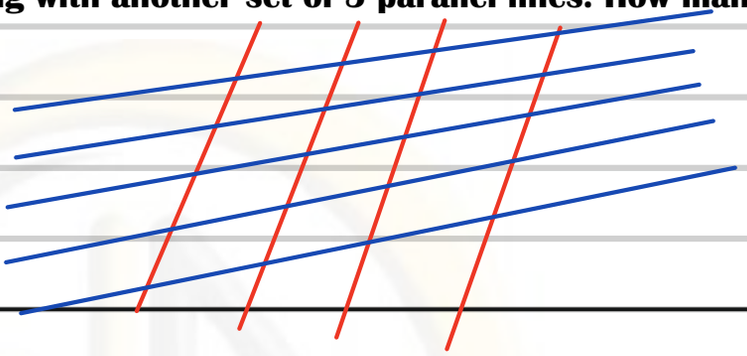
$= 435 - 28 + 1 = 408$ lines

How many different triangles can be obtained?

$= {}^{30}C_3 - {}^{8}C_3$

$= 4060 - 56 = 4004$ triangles

48. There are 4 parallel lines intersecting with another set of 5 parallel lines. How many parallelograms can be drawn?



$\rightarrow {}^{4}C_2 \times {}^{5}C_2$

$= 6 \times 10$

$= 60$ parallelograms

49. 8 Red; 3 Pink; 6 White Balls -

How many different selections of 3 balls are possible with

All Red balls

$= {}^{8}C_3 \times {}^{9}C_0$

$= 56$

2 Red balls

$= {}^{8}C_2 \times {}^{9}C_1$

$= 252$

Atleast 2 white balls

$= ({}^{6}C_2 \times {}^{11}C_1) + ({}^{6}C_3 \times {}^{11}C_0)$

$= 165 + 20$

$= 185$

No pink balls

$= {}^{3}C_0 \times {}^{14}C_3$

$= 364$

NO restri.

$= {}^{17}C_3$

$= 680$

50. 4 CA's; 3 Engineers; 8 Doctors -

How many ways a commitee of 4 members cane be formed with

Atleast 1 doctor

$= ({}^{8}C_1 \times {}^{7}C_3) + ({}^{8}C_2 \times {}^{7}C_2) + ({}^{8}C_3 \times {}^{7}C_1) + ({}^{8}C_4 \times {}^{7}C_0)$

$= 280 + 588 + 392 + 70$

$= 1330$ ways

OR

${}^{15}C_4 - ({}^{8}C_0 \times {}^{7}C_4) = 1365 - 35 = 1330$

Atleast 1 person of each profession

$= ({}^{4}C_2 \times {}^{3}C_1 \times {}^{8}C_1) + ({}^{4}C_1 \times {}^{3}C_2 \times {}^{8}C_1) + ({}^{4}C_1 \times {}^{3}C_1 \times {}^{8}C_2)$

$= 576$

51. There are 8 males & 11 females. In how many ways a committee of 5 members can be formed with

No restriction	Atleast 4 Females	Atmost 1 Female	3 Females
$= {}^{19}C_5$	$= ({}^{11}C_4 \times {}^{8}C_1) +$	$= ({}^{11}C_1 \times {}^{8}C_4) +$	$= {}^{11}C_3 \times {}^{8}C_2$
$= 11,628$ ways	$({}^{11}C_5 \times {}^{8}C_0)$	$({}^{11}C_0 \times {}^{8}C_5)$	$= 4,620$ ways
	$= 2640 + 462$	$= 770 + 56$	
	$= 3102$ ways	$= 826$ ways	

52. ${}^{n}P_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$ **a. True** **b. False**

$n = 5 \quad r = 3$

L.H.S. $= {}^{n}P_r = {}^{5}P_3 = 60$

R.H.S. $= {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1} = {}^{4}P_3 + 3 \times {}^{4}P_2$

$= 24 + 36 = 60$

53. A supreme court bench consist of 7 judges. In how many ways majority decision can be taken?

→ $= {}^{7}C_4 + {}^{7}C_5 + {}^{7}C_6 + {}^{7}C_7$

$= 35 + 21 + 7 + 1 = 64$ ways

54. A question paper has 8 questions. In how many ways atleast one question can be solved?

→ $2^8 - 1$

$= 255$ ways

OR ${}^{8}C_1 + {}^{8}C_2 + {}^{8}C_3 + \cdots + {}^{8}C_8$

$= 2^8 - {}^{n}C_0$

$= 255$

55. A question paper has 8 questions (each one has alternatives). In how many ways one or more questions can be solved?

→ $= 3^8 - 1$

$= 6560$ ways

56. No. of ways in which 9 things can be divided in 3 groups containing 2,3,4 things respectively.

→ $\frac{9!}{2!3!4!}$ OR $9C_2 \times 7C_3 \times 4C_4$

$= 1260$ ways $= 1260$ ways

57. Number of odd numbers greater than 500 can be formed by using 3, 1, 2, 8

→ 3 digit : $1P_1 \times 2P_1 \times 2P_1 = 4$

4 digit : $3P_3 \quad 2P_1 = 12$

16 Numbers

58. $\frac{{}^nP_r}{{}^{n-1}P_{r-1}} = \frac{n!/(n-r)!}{(n-1)!/(n-r+1)!} = \frac{n \times (n-1)!}{(n-r)!} \times \frac{(n-r)!}{(n-1)!}$

$= n$

59. A man has 12 friends in how many ways he can invite _____ for dinner

2 of them	Atleast 10 of them	5 of them	Atleast one of them	Atmost 10 of them
$= 12C_2$	$= 12C_{10} + 12C_{11} + 12C_{12}$	$= 12C_5$	$= 2^{12} - 1$	$= 2^{12} - 12C_{11} - 12C_{12}$
$= 66$ ways	$= 66 + 12 + 1$	$= 792$ ways	$= 4095$ ways	$= 4096 - 12 - 1$
	$= 79$ ways			$= 4083$ ways

60. In a paper there are 2 sections A, B containing 5, 8 questions respectively. In how many ways total 5 questions can be solved with atmost 3 questions of any one of the section.

→ A - 5 B - 8

$^5C_2 \times {}^8C_3 = 560$

$^5C_3 \times {}^8C_2 = 280$

840 ways

61. $\frac{{}^xP_2 \cdot {}^xP_3}{{}^xP_4 \cdot {}^xP_1} = \frac{x(x-1) \cdot x(x-1)(x-2)}{x(x-1)(x-2)(x-3)x} = \left(\frac{x-1}{x-3}\right)$

62. $\frac{{}^{10}P_3 \times {}^2P_1}{{}^{11}P_4} = \frac{10 \times 9 \times 8 \times 2}{11 \times 10 \times 9 \times 8} = \left(\frac{2}{11}\right)$

63. $\frac{{}^{18}P_2 \times {}^{20}P_3}{{}^{21}P_3 \times {}^{19}P_3} = \frac{{}^{6}18 \times 17 \times 20 \times 19 \times 18}{{}^{7}21 \times 20 \times 19 \times 19 \times 18 \times 17} = \left(\frac{6}{133}\right)$

64. In how many ways 10 students can be arranged in a line if 4 of them want to be always together?

→ $7! \times 4!$

$= 1,20,960$ ways

65. There are 9 students, in how many ways they can stand in a line if 2 of them want to be never together?

→ $= 9! - 8!2!$

$= 362880 - 80640 = 2,82,240$ ways

OR

$7! \times {}^8P_2$

$= 2,82,240$ ways

66. In how many ways letters of word 'DAUGHTER' can be arranged if all vowels should always be together?

→ (AUE) D, G, H, T, R

$= 6! \times 3! = 4320$ words

67. In how many ways letters of word 'CALCULATOR' can be arranged if all consonants should always be together?

→ (CLCLTR) A, U, A, O $= \frac{5!}{2!} \times \frac{6!}{2!2!}$

$= 10,800$ words

68. How many 3 digit numbers can be formed by using 1,2,3,4,5 if repetition of digits is

Allowed

$5P_1 \times 5P_1 \times 5P_1 = 125$ numbers

Not allowed

$5P_3 = 120$ numbers

Please Note

If question is silent about whether repetition of digits is allowed or not then REPETITION OF DIGITS IS NOT ALLOWED

69. How many 4 digit numbers greater than 5000 can be formed by using 3,5,8,2,1 if repetition of digits is

Not allowed

$2P_1 \times {}^4P_3 = 48$ numbers

Allowed

$2P_1 \times 5P_1 \times 5P_1 \times 5P_1 = 250$ Numbers

70. How many numbers greater than 8000 can be formed by using 1,2,7,8,9 if repetition of digits is

(4 digit OR 5 digit)

Not allowed

4 digit $2P_1 \times {}^4P_3 = 48$

5 digit $5P_5 = 120$

168

Allowed

4 digit $2P_1 \times 5P_1 \times 5P_1 \times 5P_1 = 250$

5 digit $5P_1 \times 5P_1 \times 5P_1 \times 5P_1 \times 5P_1 = 3125$

3375

71. How many 5 digit numbers divisible by 5 can be formed by using 0,2,3,4,5,8,9, if repetition of digits is

Not allowed

ending with 5

$5P_1 \times {}^{5}P_3 \times {}^{1}P_1 = 300$

ending with 0

${}^{6}P_4 \times {}^{1}P_1 = 360$

660

Allowed

$6P_1 \times 7P_1 \times 7P_1 \times 7P_1 \times 2P_1$

= 4116 Numbers

72. How many 5 digit numbers greater than 34,000 can be formed by using 3,1,2,7,8,0

→ starting with 3 : $1P_1 \times 2P_1 \times 4P_3 = 48$

starting with 7,8 : $2P_1 \times {}^{5}P_4 = 240$

288 Numbers

73. In how many ways 5 sisters & 6 brothers can stand in a line for a photograph if no 2 sisters or no 2 brothers should stand together?

→ B S B S B S B S B S B

$= (6P_6 \times 5P_5) = 86,400$ photographs

74. How many 2 digit numbers can be formed with atleast one digit as 7?

→ starting with 7

$1P_1 \times 10P_1 = 10$

ending with 7

$8P_1 \times 1P_1 = 8$

18 Number

17	70	7C
27	71	77
37	72	78
47	73	79
57	74	87
67	75	97

75. In how many ways 11 players out of 16 players can be selected if -

There is No restriction?	2 Particular players must be included?	3 Particular players must be excluded?	2 Particular players must be excluded & 4 particular players must be included?
$= {}^{16}C_{11}$	$= {}^{2}C_{2} \times {}^{14}C_{9}$	$= {}^{3}C_{0} \times {}^{13}C_{11}$	$= {}^{2}C_{0} \times {}^{4}C_{4} \times {}^{10}C_{7}$
$= 4368$ ways	$= 2002$ ways	$= 78$ ways	$= 120$ ways

76. $\dfrac{{}^{20}P_2 \times {}^{21}C_3}{{}^{18}C_5 \times {}^{21}P_2} = ?$ a. $\dfrac{1805}{12852}$ b. $\dfrac{1826}{18562}$ c. $\dfrac{1528}{17882}$ d. None of these

$$\frac{\cancel{20} \times 19 \times \cancel{21} \times \overset{5}{\cancel{20}} \times 19 \times \overset{2}{\cancel{12}}\overset{4}{\cancel{0}}}{\cancel{6} \times 18 \times 17 \times \overset{4}{\cancel{16}} \times \overset{3}{\cancel{15}} \times 14 \times \cancel{21} \times \cancel{20}} = \left(\frac{1805}{12852}\right)$$

77. There are 8 men and 7 women, in how many ways a committee of 4 members can be formed :

Without any restriction	With 2 Men	With Atleast 3 Men	With Atmost 1 Woman
$= {}^{15}C_{4}$	$= {}^{8}C_{2} \times {}^{7}C_{2}$	$= ({}^{8}C_{3} \times {}^{7}C_{1}) + ({}^{8}C_{4} \times {}^{7}C_{0})$	$= ({}^{7}C_{1} \times {}^{8}C_{3}) + ({}^{7}C_{0} \times {}^{8}C_{4})$
$= 1365$ ways	$= 588$ ways	$= 392 + 70$	$= 392 + 70$
		$= 462$ ways	$= 462$ ways

78. 8 Red, 3 White, 4 Pink Balls - How many different selections of 4 balls are possible with atleast one ball of each colour?

$$= ({}^{8}C_{2} \times {}^{3}C_{1} \times {}^{4}C_{1}) + ({}^{8}C_{1} \times {}^{3}C_{2} \times {}^{4}C_{1}) + ({}^{8}C_{1} \times {}^{3}C_{1} \times {}^{4}C_{2})$$

$$= 336 + 96 + 144 = 576 \text{ ways}$$

79. There are 'm' points in a plane out of which 'k' are collinear

How many different straight lines can be drawn by joining them?

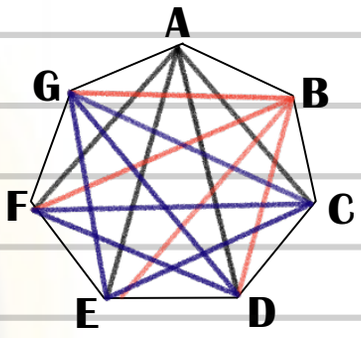
$= ({}^{m}C_2 - {}^{k}C_2 + 1)$

How many different triangles can be obtained by joining them?

$= ({}^{m}C_3 - {}^{k}C_3)$

80. How many diagonals can be drawn in a Heptagon?

(Heptagon = A polygon having 7 sides)



No. of diagonals that can be drawn in a heptagon

$=$ (No. of sides plus diagonals $-$ No. of sides)

$= {}^{7}C_2 - 7 = 14$ diagonals

81. A man has 13 friends. In how many ways he can invite _____ for dinner

Atleast one of them	4 of them	4 or 7 of them	Atmost 11 of them
$= {}^{13}C_1 + {}^{13}C_2 + \cdots$ $\cdots + {}^{13}C_{13}$ $= 2^{13} - 1$ $= 8191$ ways	$= {}^{13}C_4$ $= 715$ ways	$= {}^{13}C_4 + {}^{13}C_7$ $= 715 + 1716$ $= 2431$ ways	$= 2^{13} - {}^{13}C_{12} - {}^{13}C_{13}$ $= 8192 - 13 - 1$ $= 8178$ ways

82. There are 4 papers in an exam. in how many ways student can pass the exam if passing in all papers is compulsory to pass the exam?

a. 1 b. 15 c. 16 d. None of these

83. There are 5 papers in an exam. in how many ways a student can pass the exam if student passes the exam if he passes in atleast one paper?

a. 32 b. 31 c. 1 d. None of these

84. There are 5 multiple choice questions with 4 options each. How many different sequences of answer are possible?

→ $4C_1 \times 4C_1 \times 4C_1 \times 4C_1 \times 4C_1$

$= 4^5 = 1024$ diff sequences

85. There are 6 multiple choice questions. First 4 questions have 4 options each and last two questions have 5 options each. How many different sequences of answer are possible?

→ $= 4C_1 \times 4C_1 \times 4C_1 \times 4C_1 \times 5C_1 \times 5C_1$

$=$ 6400 diff sequences

86. There are 8 males and 5 females. In how many ways a committee of 5 members can be formed so that males are in majority?

→ $(8C_3 \times 5C_2) + (8C_4 \times 5C_1) + (8C_5 \times 5C_0)$

$= 560 + 350 + 56$

$=$ 966 ways

87. No. of arrangements of 'n' different things taken 'r' at a time in which a particular thing

Alway there

$= {}^{r}P_1 \times {}^{n-1}P_{r-1}$

$= r \cdot {}^{(n-1)}P_{(r-1)}$

Never there

$= {}^{(n-1)}P_r$

88. No. of selections of 'n' different things taken 'r' at a time in which a particular thing

Is alway there → $= {}^{1}C_{1} \times {}^{n-1}C_{r-1}$ → $= {}^{n-1}C_{(r-1)}$

Is never there → $= {}^{(n-1)}C_{r}$

89. Find sum of all 4 digit numbers formed by using 1,3,5,7

a. 1,06,656 b. 1,78,252 c. 1,78,282 d. None

$= \frac{24}{4}(1111 + 3333 + 5555 + 7777)$

$= 1,06,656/-$ (OR) $= \frac{24}{2}(1357 + 7531) = 1,06,656$

90. "P_r can also be written as :

a. $\frac{n!}{r!}$ b. ~~$\frac{\lfloor n}{\lfloor (n-r)}$~~ c. $\frac{\lfloor n}{\lfloor r \lfloor (n-r)}$ d. None

91. There are 6 Books on Eco, 3 on Maths, 2 on stats. In how many ways all books can be placed on a shelf if books on same subject are to be always together?

a. 1,06,656 b. 1,78,252 c. 51,840 d. None

(6) (3) (2)

$= 3! \times 6! \times 3! \times 2! = 51,840$ ways

92. The number of ways in which 7 girls can form a ring is :

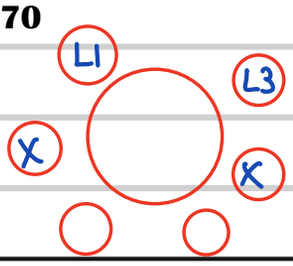
a. 700 b. 710 ~~c. 720~~ d. 360

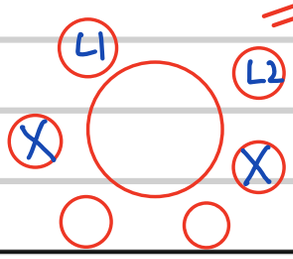
$= \frac{7!}{7} = 720$

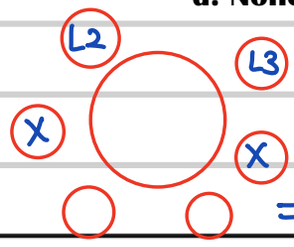
7 Girls can form a ring in $(7-1)!$ ways

93. 3 Ladies and 3 gents are to be seated on a round table so that 2 and only 2 ladies should sit together. The number of arrangements are :

a. 70 b. 27 ~~c. 72~~ d. None of these







$= {}^2P_2 \times {}^2P_1 \times {}^3P_3$

$= 24$

$= 24 + 24 + 24$

94. In a group of boys the no. of arrangements of 4 boys is 12 times the number of arrangements of 2 boys. The no. of boys in the group is

a. 10 b. 8 ~~c. 6~~ d. None of these

$${}^nP_4 = 12 \times {}^nP_2$$

$$n(n-1)(n-2)(n-3) = 12 \times n(n-1) \qquad \boxed{n = 6}$$

95. $\sum_{r=1}^{r=10} r \cdot {}^rP_r = ?$ a. ${}^{11}P_{11}$ ~~b. ${}^{11}P_{11} - 1$~~ c. ${}^{11}P_{11} + 1$ d. None of these

$= 1 \times {}^1P_1 + 2 \times {}^2P_2 + 3 \times {}^3P_3 + 4 \times {}^4P_4 + \ldots\ldots\ldots + (10 \times {}^{10}P_{10})$

$= 1 + 4 + 18 + 96 + 600 + 4320 + 35280 + 322560 + 3265920 + 36288000$

$= 39916799$

96. There are 10 trains plying between Latur and Pune, The no. of ways in which a man can go from Latur to Pune and return by different train is

a. 99 ~~b. 90~~ c. 80 d. 100

$= {}^{10}P_1 \times {}^9P_1 = 90$

97. The number of ways in which six '+' and four '-' signs can be arranged in a line such that no '-' signs occur together is

a. 7!/3! b. (6! x 7!) / 3! ~~c. 35~~ d. None of these

$+ \; + \; + \; + \; + \; + \; +$

AAAAAA BBBB

$= \dfrac{6! \times {}^7P_4}{6! \, 4!}$

$= 35$ ways

98. The number of ways in which letters of word 'MOBILE' be arranged so that consonants always occupy the odd places is :

a. 36 b. 63 c. 30 d. None of these

1st 3rd 5th

$= {}^3P_3 \times {}^3P_3 = 36$ ways

99. 5 persons are sitting along a round table in such a way that tallest person is always to the immediate right of shortest person. The no. of such arrangements are :

a. 6 b. 8 c. 24 d. None of these

$= 3! = 6$ ways

100. In how many ways 17 balls can be arranged in a line if 7 of them are Black, 6 are Red and 4 are White?

$$\frac{17!}{7!\,6!\,4!} = 40,84,080$$

101. The number of different words that can be formed with 12 consonants and 5 vowels by taking 4 consonants and 3 vowels in each word are

a. ${}^{12}C_4 \times {}^5C_3$ b. ${}^{17}C_7$ c. 4950 x 7! d. None of these

$= \left({}^{12}C_4 \times {}^5C_3\right) \times 7!$

102. How many different words can be formed by using all letters of word 'ASSASSINATION' if all vowels should be together?

→ AAIAIO S, S, S, S, N, T, N

$= \frac{8!}{4!\,2!} \times \frac{6!}{3!\,2!} = 50,400$ diff. words

103. How many numbers greater than a million can be formed with the digits 0,4,4,5,5,5,3?

a. 420 ~~b. 360~~ c. 7! d. None of these

$= \frac{6P_1 \times 6P_6}{2!\,3!} = 360$ numbers

104. $4 \times {}^{n}P_3 = 5 \times {}^{(n-1)}P_3$; then value of 'n' is

a. 12 b. 13 c. 14 ~~d. 15~~

$4 \times n(n-1)(n-2) = 5 \times (n-1)(n-2)(n-3)$

$4n = 5n - 15$

$n = 15$

105. The number of ways in which 8 examination papers can be arranged so that best and worst paper never come together are :

~~a. 8! - 2!7!~~ b. 8! - 7! c. 8! d. None of these

= (All possible arrangements of 8 papers) − (Arrangements where Best & worst papers are together)

$= 8! - 2!\,7!$

106. How many 6 digit numbers can be formed out of 4,5,6,7,8,9 if no digit being repeated?

a. 6!-5! b. 6! c. 6!+5! d. None of these

$6P_1 \times 5P_1 \times 4P_1 \times 3P_1 \times 2P_1 \times 1P_1 = 6P_6$

107. There are 50 stations on a railway line, How many different kinds of tickets to be printed to enable a passenger to travel from one station to another station?

a. 2500 b. 2450 c. 2400 d. None of these

CST Andh: CGT

$50P_2 = 2450$ diff. tickets

108. In ^nP_r, ^nC_r; n is always

a. positive integer b. an integer c. zero d. None of these

$nP_r = \frac{n!}{(n-r)!}$ & $nC_r = \frac{n!}{r!(n-r)!}$

where $n \geq r \geq 0$

n = a positive integer

r = Non-Negative integer

109. If all permutations of word 'CHALK' are written in a dictionary sequence. the rank of word 'CHALK' is

a. 30 b. 31 c. 32 d. None of these

starting with 1-24

ACHKL

starting C - 25-48

CAHKL

25-26-27-28-29-30

31st CHAKL

32nd CHALK

110. How many words can be formed by using letter A thrice, letter B twice. letter C once.

a. 60 b. 120 c. 90 d. 6

A, A, A, B, B, C $= \frac{6!}{3!2!} = 60$ diff words

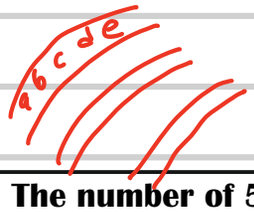
111. If different permutations of the word 'EXAMINATION' are listed in a dictionary, How many items are there in the list before the first word starting with E?

a. 9,06,200 ~~b. 9,07,200~~ c. 9,08,200 d. 9,05,200

No. of words starting with A $= \frac{10!}{2!\,2!} = 9,07,200$

112. A letter lock consist of 3 rings marked with 5 different letters. Number of maximum unsuccessful attempts to open the lock is :

~~a. 124~~ b. 125 c. 120 d. 75



Max No. of pass words possible $= 5P_1 \times 5P_1 \times 5P_1 = 125$

Max unsuccessful attempts $= 124$

113. The number of 5 letter words that can be formed using letters of word 'DELHI' which begin and end with vowel, when repetitions are allowed is

a. 125 b. 625 ~~c. 500~~ d. 1350

$2P_1 \times 5P_1 \times 5P_1 \times 5P_1 \times 2P_1 = 500$

114. The number of ways in which 20 persons be seated along a round table if there are 7 seats is :

a. $^{20}P_7$ b. $^{20}P_7 / 7!$ c. $^{20}C_7$ ~~d. $^{20}P_7 / 7$~~

$= \left(\frac{20P_7}{7}\right)$

No. of ways in which 'r' students out of 'n' can form a Line $= nPr$ / circle $= nPr/r$

115. $^{n}P_r = 120 \times {}^{n}C_r$, then r = ?

~~a. 5~~ b. 120 c. 24 d. 4

$\frac{nPr}{nCr} = 120 = 5! = r! \quad \therefore r = 5$

116. In how many ways letters of the word 'BALLOON' be arranged so that 2 L's do not come together is :

~~a. 900~~ b. 1200 c. 800 d. 600

$= \left[\text{All possible arrangements} - \text{arr. where 2 L's are together}\right] = \frac{7!}{2!\,2!} - \left(\frac{6!}{2!} \times \frac{2!}{2!}\right) = 1260 - 360$

117. $^{15}C_{11} / {}^{15}C_{10} = ?$

a. 15/11 b. 15/10 c. 5/10 d. None of these

$$\frac{{}^{15}C_{11}}{{}^{15}C_{10}} = \left[\frac{\frac{15!}{11!\,4!}}{\frac{15!}{10!\,5!}}\right] = \frac{10! \times 5 \times 4!}{11 \times 10! \times 4!} = 5/11$$

118. How many even numbers greater than 300 can be formed with digits 1,2,3,4,5.

No repetition being allowed

a. 112 b. 111 c. 113 d. 121

3 digit : Ending with 4 $\quad {}^2P_1 \times {}^3P_1 \times {}^1P_1 = 6$

Ending with 2 $\quad {}^3P_1 \times {}^3P_1 \times {}^1P_1 = 9$

4 digit : $\quad {}^4P_3 \times {}^2P_1 = 48$

5 digit : $\quad {}^4P_4 \times {}^2P_1 = 48$

111 Numbers

119. $^{43}C_{(r-6)} = {}^{43}C_{(3r+1)}$ **, then r = ?**

a. 12 b. 8 c. 6 d. 10

$${}^{43}C_{r-6} = {}^{43}C_{3r+1}$$

$$r - 6 + 3r + 1 = 43$$

$$4r - 5 = 43$$

$$4r = 48$$

$$r = 12$$

If $^nC_x = {}^nC_y$ then $x = y$ OR $x + y = n$

120. A committee of 3 ladies and 4 gents to be formed out of 8 ladies and 7 gents.

Mrs. X refuses to serve in a committee if Mr. Y is there. Number of such committees are :

a. 1530 b. 1500 c. 1520 d. 1540

$$= ({}^8C_3 \times {}^7C_4) - ({}^1C_1 \times {}^1C_1 \times {}^7C_2 \times {}^6C_3)$$

$$= 1960 - 420$$

$$= 1540$$

OR

Mr. Y is there : ${}^1C_1 \times {}^6C_3 \times {}^7C_3 = 700$

Mr. Y is Not there : ${}^1C_0 \times {}^6C_4 \times {}^8C_3 = 840$

1540

121. What is rank of word 'TALK' if all words by using letters of word are arranged in a dictionary sequence ?

a. 20 b. 18 c. 19 d. None of these

Total 24 words

1st word : A K L T

19th T A K L

20th T A L K

1 – 6 starting with A

7 – 12 starting with K

13 – 18 starting with L

122. How many 3 digit odd numbers can be formed by using 1,3,5, if repetition of digits is allowed?

a. 3^3 b. 3! c. (3x3x4) d. None of these

$3P_1 \times 3P_1 \times 3P_1 = 3^3 = 27$

123. $^{56}P_{(r+6)} : {}^{54}P_{(r+3)} = 30,800 : 1$; then r = ?

a. 42 b. 41 c. 45 d. None of these

$$\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{30,800}{1}$$

$$\frac{56!}{(56-r-6)!} \times \frac{(54-r-3)!}{54!} = 30,800$$

$$\frac{56 \times 55 \times 54!}{(50-r)!} \times \frac{(51-r)(50-r)!}{54!} = 30,800$$

$$51-r = 10$$

$$\therefore r = 41$$

124. There are 6 questions in section A and 7 in section B. In how many ways 8 questions can be attempted with atmost 6 questions from any section are :

a. 360 ~~b. 1281~~ c. 6 d. 42

A - 6 B - 7

$^6C_2 \times {}^7C_6 = 105$

$^6C_3 \times {}^7C_5 = 420$

$^6C_4 \times {}^7C_4 = 525$

$^6C_5 \times {}^7C_3 = 210$

$^6C_6 \times {}^7C_2 = 21$

1281 ways

125. How many words can be formed by using all letters of word 'REDDY'

a. 120 ~~b. 60~~ c. 240 d. None of these

$= \frac{5!}{2!} = 60$ diff words

126. Find sum of all 5 digit numbers formed by using 1, 2, 5, 8, 9

$\Rightarrow = \frac{120}{5}(11111 + 22222 + 55555 + 88888 + 99999)$

$= 66,66,600$

127 How many 5 digit numbers divisible by 4 can be formed by using 2, 1, 8, 6, 3

$\Rightarrow$ Ending with 12 : $^3P_3 \times {}^1P_1 \times {}^1P_1 = 6$

32 : $= 6$

28 : $= 6$

68 : $= 6$

16 : $= 6$

36 : $= 6$

Before you

Work SMART

You must

Work HARD

- CA VINOD REDDY -

YOU CANNOT BUILD A REPUTATION ON WHAT YOU ARE GOING TO DO