Chapter-10

Probability

CA VINOD REDDY

1. Probability is the

→ Chance OR possibility of happening OR Non-happening of any event.

2. Classical Definition of Probability

→ Probability of any event $= \dfrac{\text{No. of outcomes in favour of that event}}{\text{No. of all possible equally likely outcomes}}$

3. **Coin** : 2 possible outcomes : Head (H), Tail (T)

Dice/Die : 6 possible outcomes : 1, 2, 3, 4, 5, 6 points

Card : 52 playing cards
- 26 Red cards
  - 13 Diamonds
  - 13 Hearts
- 26 Black cards
  - 13 spades
  - 13 clubs

13 cards : 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace

4. **A coin is tossed 2 times what is probability of getting**

2 heads	1 head	Atleast 1 head	Atmost 1 head
HH	HT, TH	HT, TH, HH	HT, TH, TT
$= \frac{1}{4} = 25\%$	$= \frac{2}{4} = \frac{1}{2} = 0.50$ $= 50\%$	$= \frac{3}{4} = 0.75$ $= 75\%$	$= \frac{3}{4} = 0.75$ $= 75\%$

All possible equally likely outcomes : HH, HT, TH, TT

unbiased = Fair

THT TTT HHT HTT

All possible outcomes THH TTH HHH HTH

5. An unbaised coin is tossed 3 times. Find the probability of getting

2 tails	Atleast 2 heads	No tails	Atmost 2 tails
TTH THT HTT $= \frac{3}{8}$ $= 37.50\%$	HHT HHH HTH THH $= \frac{4}{8}$ $= \frac{1}{2} = 50\%$	HHH $= \frac{1}{8}$ $= 12.50\%$	All except TTT $= \frac{7}{8} = 87.50\%$

6. An unbaised coin is tossed 4 times. What is the probability of getting

2 heads $= 6/16 = 3/8$	3 tails $= 4/16 = 1/4$	Atleast 3 tails $= 5/16$	Atmost 3 tails
HHTT TTHH HTHT THTH HTTH THHT	TTTH TTHT THTT HTTT	TTTH TTTT TTHT THTT HTTT	All except TTTT $= 15/16$

HHHH HHTH THHH THTH
HHHT HHTT THHT THTT
HTHH HTTH TTHH TTTH
HTHT HTTT TTHT TTTT

} All possible equally likely outcomes

7. A dice is rolled once. What is the probability of getting

3 Points	4 Points	1 Point	Atmost 3 Points	Atleast 5 Points	Odd Number as point	Prime Number as point
$= \frac{1}{6}$	$= \frac{1}{6}$	$= \frac{1}{6}$	$= \frac{3}{6}$	$= \frac{2}{6}$	$= \frac{3}{6}$	$= \frac{3}{6}$

8. A dice is rolled twice what is the probability of getting

→ **7 points as sum** $= \frac{6}{36} = \frac{1}{6}$

→ **8 points as sum** $= \frac{5}{36}$

→ **9 or more points** $= \frac{4+3+2+1}{36}$

$= 10/36$

→ **Atleast 3 points** $= 35/36$

→ **Odd points on both dice**

$= 9/36$

Sum=2	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
Sume=3	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
Sum=4	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
Sum=5	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
Sum=6	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
Sum=7	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)
	Sum =8	Sum =9	Sum =10	Sum =11	Sum =12	

→ **Odd points on atleast one dice**

$= 27/36$

→ **Even points on both dice** $= 9/36$

→ **5 or 7 points** $= \frac{4+6}{36} = \frac{10}{36}$

→ **Sum as prime number** $= \frac{1+2+4+6+2}{36} = \frac{15}{36}$

→ **Odd points on atleast one dice** $= 27/36$

→ **Sum as odd number** $= 18/36$

→ **Sum as even number** $= 18/36$

9. A card is drawn from a well shuffled pack of 52 cards. What is probability of getting :

a. A diamond = $(^{13}C_1 / ^{52}C_1) = 1/4$

b. A King = $(^{4}C_1 / ^{52}C_1) = 4/52 = 1/13$

c. A Black Card = $(^{26}C_1 / ^{52}C_1) = 1/2$

d. A Black Queen = $(^{2}C_1 / ^{52}C_1) = 1/26$

e. A Jack = $(^{4}C_1 / ^{52}C_1) = 4/52 = 1/13$

10. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A') = 1 - P(A)$

$P(B') = 1 - P(B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A-B) = P(A) - P(A \cap B) = P(A \cap B')$

$P(B-A) = P(B) - P(A \cap B) = P(B \cap A')$

$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$P(A \cup B') = 1 - P(B-A)$

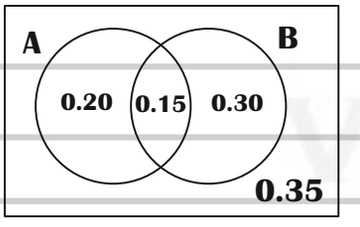
$P(B \cup A') = 1 - P(A-B)$

$P(A \triangle B) = P(A-B) + P(B-A) = P(A \cup B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$

11.



$P(A) = 0.35$

$P(B) = 0.45$

$P(A') = 1 - P(A) = 0.65$

$P(B') = 1 - P(B) = 0.55$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.65$

$P(A \cap B) = 0.15$

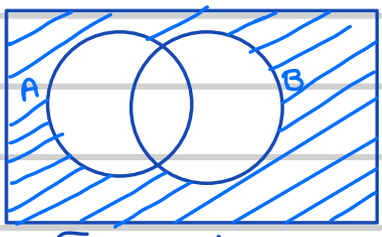
$P(A-B) = 0.20$

$P(B-A) = 0.30$

$P(A' \cap B') = 0.35$

$P(A \triangle B) = 0.20 + 0.30 = 0.50$

12. De-morgan's rule of probability (with diagram)

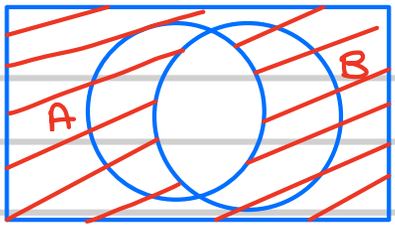


S = Sample space

$(A \cup B)' = (A' \cap B')$

$P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$



$(A \cap B)' = (A' \cup B')$

$P(A' \cup B') = P(A \cap B)'$

$= 1 - P(A \cap B)$

13. If 2 dice are rolled then

Sum of points on 2 dice	Probability
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

14. A card is drawn from a well shuffled pack of 52 cards then what is probability that it is a -

a. Spade = $P(A) = 13/52$

b. Queen = $P(B) = 4/52$

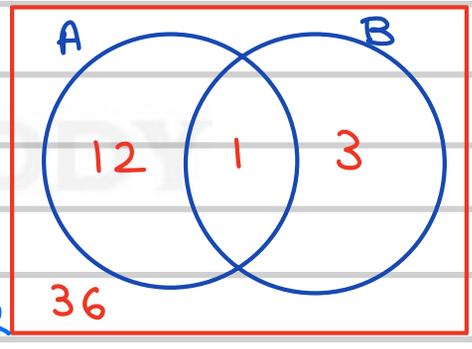
c. Spade and Queen = $P(A \cap B) = 1/52$

d. Spade or Queen = $P(A \cup B) = 16/52$

e. Spade but not Queen = $P(A - B) = P(A \cap B') = 12/52$

f. Queen but not Spade = $P(B - A) = P(B \cap A') = 3/52$

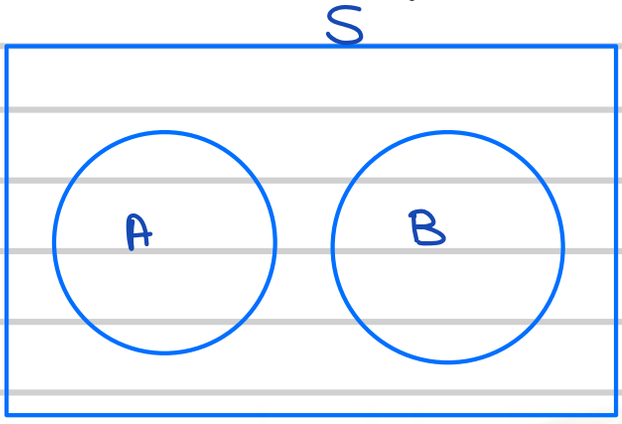
g. Neither Spade nor Queen = $P(A' \cap B') = 36/52$



A : event that card drawn is spade

B : event that card drawn is queen

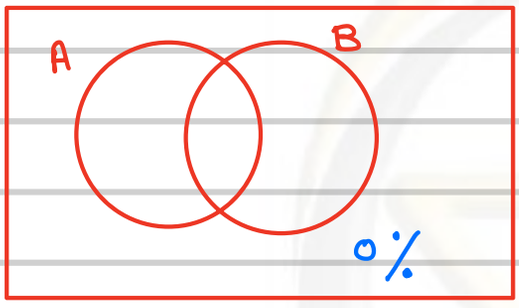
15. A, B are said to be mutually exclusive events then :



Here
$P(A \cap B) = 0$
$\therefore$ A, B are mutually exclusive events

occu. of A prevents occu. of B & vice versa

16. A, B are said to be mutually exhaustive events then :



Here $P(A \cup B) = 1.00 = 100\%$
i.e. $P(A' \cap B') = 0$
$\therefore$ A, B are mutually exhaustive events

When occu. of at least one event is compulsory then events are said to be mutually exhaustive

17. A, B are said to be independent events when :

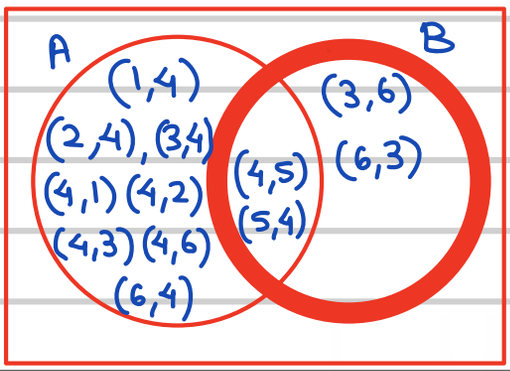
$P(A \cap B) = P(A) \times P(B)$

When occur. of one event doesn't affect occu. OR Non occur. of other event then events are said to be Independent events

18.

Events A & B are said to be	If
Mutually exclusive events	P (A∩B) = 0
Mutually exhaustive events	P (A∪B) = 1.00
Independent events	P (A∩B) = P(A) x P(B)
Equally likely events	P(A) = P(B)

19. 2 dice are rolled. It is observed that sum of points is 9. What is probability that 4 has appeared on one of the dice?



B : be the event that sum of points on 2 dice is 9

A : be the event that 4 points have appeared on one of the dice

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{4/36} = \frac{2}{4} = \frac{1}{2}$$

20.

$P(A/B) = \frac{P(A \cap B)}{P(B)}$ $\quad$ $P(A'/B) = \frac{P(A' \cap B)}{P(B)}$

$P(B/A) = \frac{P(B \cap A)}{P(A)}$ $\quad$ $P(A'/B') = \frac{P(A' \cap B')}{P(B')}$

$P(A/B') = \frac{P(A \cap B')}{P(B')}$ $\quad$ $P(B'/A) = P(B' \cap A)/P(A)$

$P(B/A') = \frac{P(B \cap A')}{P(A')}$ $\quad$ $P(B'/A') = P(B' \cap A')/P(A')$

21. If A, B are independent events then :

$$P(A \cap B) = P(A) \times P(B)$$

$P(A \cap B') = P(A) \times P(B')$

$P(B \cap A') = P(B) \times P(A')$

$P(A' \cap B') = P(A') \times P(B')$

$P(A/B) = P(A)$

$P(A/B') = P(A)$

$P(B/A) = P(B)$

$P(B/A') = P(B)$

$P(A'/B) = P(A')$

$P(A'/B') = P(A')$

$P(B'/A) = P(B')$

$P(B'/A') = P(B')$

22.

8 Red
6 White
5 Black

3 balls are drawn. What is probability of getting

2 Red balls

$= \frac{({}^8C_2 \times {}^{11}C_1)}{{}^{19}C_3}$

$= \frac{308}{969}$

$= 31.7853\%$

Atleast 2 white Balls

$= \frac{({}^6C_2 \times {}^{13}C_1) + ({}^6C_3 \times {}^{13}C_0)}{{}^{19}C_3}$

$= \frac{195 + 20}{969}$

$= \frac{215}{969}$

$= 22.1878\%$

Atmost 1 Black Ball

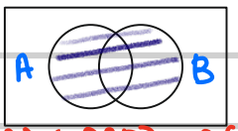
$= \frac{({}^5C_1 \times {}^{14}C_2) + ({}^5C_0 \times {}^{14}C_3)}{{}^{19}C_3}$

$= \frac{455 + 364}{969}$

$= \frac{819}{969} = 84.52\%$

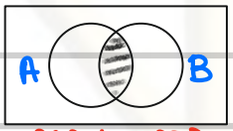
23.

1. P(A∪B)



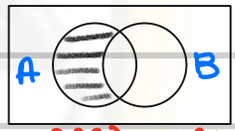
$= P(A) + P(B) - P(A \cap B)$

2. P(A∩B)



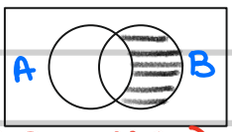
$= P(A) + P(B) - P(A \cup B)$

3. P(A∩B′)



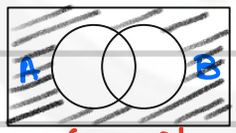
$= P(A) - P(A \cap B) = P(A - B)$

4. P(B∩A′)



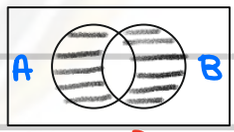
$= P(B) - P(A \cap B) = P(B - A)$

5. P(A′∩B′)



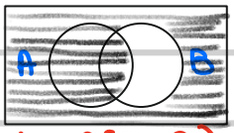
$= P(A \cup B)' = 1 - P(A \cup B)$

6. P(A△B)



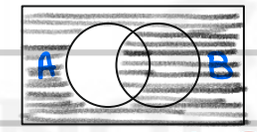
$P(A - B) + P(B - A) = P(A \cup B) - P(A \cap B)$

7. P(A∪B′)



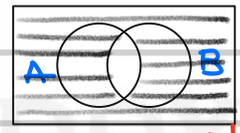
$= 1 - P(B - A)$

8. P(B∪A′)



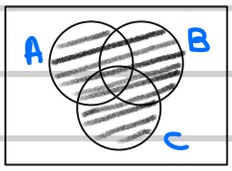
$= 1 - P(A - B)$

9. P(A′∪B′)



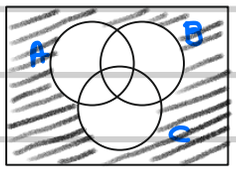
$= P(A \cap B)' = 1 - P(A \cap B)$

10. P(A∪B∪C)



$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

11. P(A′∩B′∩C′)



$= P(A \cup B \cup C)'$

$= 1 - P(A \cup B \cup C)$

24. If P(A) = 0.30, P(B) = 0.40, P(A∩B) = 0.15. Find

$P(A') = 1 - P(A) = 0.70$

$P(B') = 1 - P(B) = 0.60$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.55$

$P(A-B) = P(A) - P(A \cap B) = 0.15$

$P(B-A) = P(B) - P(A \cap B) = 0.25$

$P(A' \cap B') = 1 - P(A \cup B) = 0.45$

$P(A \cup B') = 1 - P(B-A) = 0.75$

$P(B \cup A') = 1 - P(A-B) = 0.85$

$P(A \triangle B) = P(A-B) + P(B-A) = 0.40$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.40} = 0.3750$

$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.15}{0.30} = 0.50$

$P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{0.15}{0.60} = 0.25$

$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{0.45}{0.60} = 0.75$

25. P(A) = 0.30, P(B) = 0.40, A, B are independent events, then find

$P(A \cap B) = P(A) \cdot P(B) = 0.12$

$P(A/B) = P(A) = 0.30$

$P(B/A) = P(B) = 0.40$

$P(A/B') = P(A) = 0.30$

$P(B/A') = P(B) = 0.40$

$P(A'/B') = P(A') = 0.70$

$P(B'/A') = P(B') = 0.60$

$P(A \cup B) = 0.30 + 0.40 - 0.12 = 0.58$

$P(A-B) = P(A \cap B') = P(A) \times P(B') = 0.30 \times 0.60 = 0.18$

$P(B-A) = P(B \cap A') = P(B) - P(A \cap B) = 0.28$

$P(A' \cap B') = P(A') \times P(B') = 0.70 \times 0.60 = 0.42$

$P(A' \cup B') = 1 - P(A \cap B) = 0.88$

$P(A \cap B) = P(A) \times P(B) = 0.12$

26. In a leap year selected at random what is probability of getting

53 Mondays	52 Mondays	Atleast 52 Mondays	54 Mondays
$= \frac{2}{7}$	$= \frac{5}{7}$	$= 100\%$	$= 0\%$

27. In a non-leap year selected at random what is probability of getting

53 Sundays	52 Sundays	Atleast 52 Sundays	54 Sundays
$= \frac{1}{7}$	$= \frac{6}{7}$	$= 100\%$	$= 0\%$

28. In a year selected at random what is the probability of getting

52 Tuesdays

$= \left(\frac{1}{4} \times \frac{5}{7}\right) + \left(\frac{3}{4} \times \frac{6}{7}\right)$

$= \frac{5}{28} + \frac{18}{28} = \left(\frac{23}{28}\right)$

53 Tuesdays

$= \left(\frac{1}{4} \times \frac{2}{7}\right) + \left(\frac{3}{4} \times \frac{1}{7}\right)$

$= \frac{2}{28} + \frac{3}{28} = \left(\frac{5}{28}\right)$

29. What is probability that 15th day of a randomly selected month is Sunday?

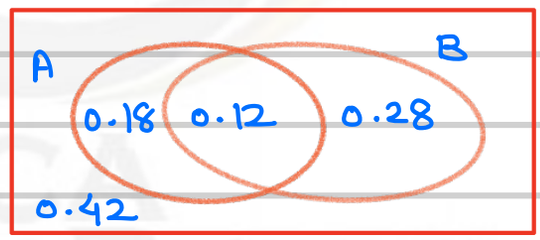
→ $\frac{1}{7}$

what is probability that 22nd day of a randomly selected month is not a saturday ? ⟹ 6/7

30. Probability of A passing exam is 0.30. and B passing exam is 0.40. What is the probability that

Here A, B are Indep. events $P(A) = 0.30,\ P(B) = 0.40$

Both will pass	Only A will pass	Only B will pass	Atleast one will pass	One & Only One will pass	Atleast one will fail
$= P(A \cap B)$	$= P(A - B)$	$= P(B - A)$	$= P(A \cup B)$	$= P(A \Delta B)$	$= P(A' \cup B')$
$= P(A) \cdot P(B)$	$= 0.18$	$= 0.28$	$= 0.58$	$= 0.46$	$= 0.88$
$= 0.30 \times 0.40$					
$= 0.12$					



31.

x	30	60	90	120	150
Prob. x	0.20	0.30	0.10	0.15	0.25

Find E(x), SD_x, Variance of x

E(x) = Expected value of x

x	P(x)	x.P(x)	x^2	$x^2.P(x)$
30	0.20		900	
60	0.30		3600	
90	0.10		8100	
120	0.15		14400	
150	0.25		22500	
		88.50		9855

$E(x) = \sum x.P(x) = 88.50$ = mean of x

variance of x $= E(x^2) - [E(x)]^2$

$= 9855 - 7832.25$

$= 2022.75$

SD of x = 44.975

32.

x	10	20	30	40	50
Prob. x	0.20	3k	5k	7k	k

Find E(x), SD_x, Variance of x

→ $0.20 + 3k + 5k + 7k + k = 1.00 = \sum P(x)$

$16k = 0.80$

$k = 0.05$

x	$P(x)$	x^2	$x.P(x)$	$x^2.P(x)$
10	0.20	100		
20	0.15	400		
30	0.25	900		
40	0.35	1600		
50	0.05	2500		
			29.00	990

$E(x)$ $E(x^2)$

$E(x) = \sum x.p(x) = 29$

vari. of $x = 990 - 29^2$

$= 149$

SD of $x = \sqrt{149}$

$= 12.20656$

$= E(x^2) - [E(x)]^2$

$= \sum x^2.P(x) - [\sum x.P(x)]^2$

33. If odds in favour of event A are 3 : 8. Find P(A), P(A')

→ $P(A) = \frac{3}{3+8} = \frac{3}{11}$

$P(A') = \frac{8}{11}$

34. If odds against event B are 8 : 13. Find P(B), P(B')

→ $P(B) = \frac{13}{21}$

$P(B') = \frac{8}{21}$

If $P(D) = 97/103$ then

odds in favour of event D are $\div$ 97 : 6

odds against event D are $\div$ 6 : 97

35. If odds in favour of event A are 3 : 11; Odds against event B are 2 : 15; A,B are independent events, then find :

$P(A) = \frac{3}{14}$

P(A) = $3/14$

$P(B) = \frac{15}{17}$

P(B) = $15/17$

P(A∩B) = $P(A) \times P(B) = (45/238)$

P(A∪B) = $\frac{3}{14} + \frac{15}{17} - \frac{45}{238} = \frac{261}{238} - \frac{45}{238} = \left(\frac{216}{238}\right)$

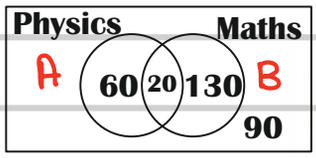
P(A′ ∩ B′) = $1 - \frac{216}{238} = (22/238)$

P(A-B) = $P(A \cap B') = P(A) \times P(B') = \frac{3}{14} \times \frac{2}{17} = \frac{6}{238}$

P(B-A) =

$P(B \cap A') = P(B) \times P(A') = \frac{15}{17} \times \frac{11}{14} = \frac{165}{238}$

36.



Find probability that a student likes

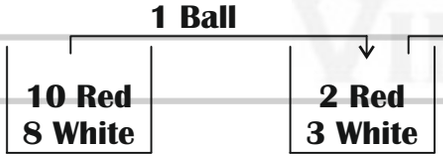
a. Maths if it is known that he likes physics = $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{20/300}{80/300}$

$= \frac{20}{80} = \frac{1}{4} = 0.25 = 25\%$

b. Physics if it is known that he doesn't likes maths =

$= P(A/B') = P(A \cap B')/P(B') = \frac{60/300}{150/300} = 60/150 = 0.40 = 40\%$

37.



1 ball is drawn.
What is the probability that it is a red ball?

Transfer

Red : $\frac{10}{18} \times \frac{3}{6} = \frac{30}{108}$

white : $\frac{8}{18} \times \frac{2}{6} = \frac{16}{108}$

Answer : $\frac{46}{108} = \frac{23}{54}$

$= 42.59259\%$

38. 2 Balls

5 Red, 6 White → 3 Red, 11 White

1 ball is drawn from it,
What is the probability that it is a white ball?

Transfer

2 Red : $\frac{5C_2}{11C_2} \times \frac{11}{16} = \frac{110}{880}$

2 white : $\frac{6C_2}{11C_2} \times \frac{13}{16} = \frac{195}{880}$

1R 1W : $\frac{5C_1 \times 6C_1}{11C_2} \times \frac{12}{16} = \frac{360}{880}$

Answer : $\frac{665}{880} = \frac{133}{176} = 75.5682\%$

39.

Information	Whether A,B are	
	Mutually Exclusive Events?	Mutually Exhaustive Events?
P(A) = 0.30; P(B) = 0.60 P(A ∩ B) = 0.10	NO	NO
P(A) = 0.60; P(B) = 0.50 P(A ∩ B) = 0.10	NO	Yes
P(A) = 0.30; P(B) = 0.40 P(A ∩ B) = 0	Yes	NO
P(A) = 0.65; P(B) = 0.35 P(A ∩ B) = 0	Yes	Yes

40. Two Broad divisions of Probability are

- Subjective Probability
  - Subjective Probability is basically dependent on personal judgement and experience.
    - It may be influenced by personal belief, attitude and bias.
- Objective Probability
  - It is not based upon personal judgement.

we are studying objective probability.

41. An experiment may be described as a performance that produces certain results.

The result or outcome of a random experiment are known as events.

42. **Events are of 2 types**

- **Simple or Elementary Event** → Getting Head when One Coin is tossed
- **Composite or Compound Event** → Getting Head when Two Coins are tossed

43. **Equally likely events are also known as Mutually Symmetric Events or Equi-probable events.**

If P(A) = 0.30, P(B) = 0.30 then A,B are equally likely events OR Equi-probable events OR Mutually Symmetric events

44. **If P(A) = 1.00 = 100% then event A is said to be a** SURE Event

45. **If P(B) = 0.00 = 0% then event B is said to be a** Impossible event

46.

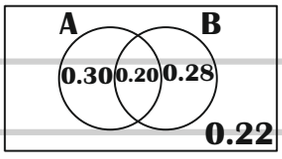
Wages in ₹	100-200	200-300	300-400	400-500
No. of workers	23	57	88	93

= 261

If a worker is selected at random, what is the probability that

1. He earns more than ₹ 300 = (181/261)
2. He earns more than ₹ 400 = (93/261)
3. He earns between ₹ 200 - ₹ 400 = (145/261)
4. He earns less than ₹ 300 = (80/261)

47.



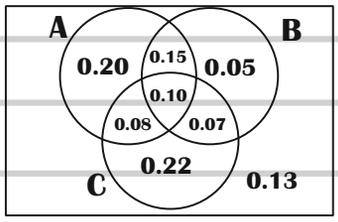
S = Sample Space = Set of all possible outcomes

From above diagram. Find

P(A) = 0.50
P(B) = 0.48
P(A′) = 0.50
P(B′) = 0.52
P(A∪B) = 0.78
P(A∩B′) = 0.30
P(B∩A′) = 0.28
P(AUB′) = 0.72

P(BUA′) = 0.70
P(A′UB′) = 0.80
P(A/B) = 0.20/0.48 = 0.4166666
P(B/A) = 0.20/0.50 = 0.40
P(A′/B′) = 0.22/0.52 = 0.42307692307
P(B′/A′) = 0.22/0.50 = 0.44
P(A′/B) = 0.28/0.48 = 0.5833333
P(B/A′) = 0.28/0.50 = 0.56

48.



From this Venn Diagram : Find

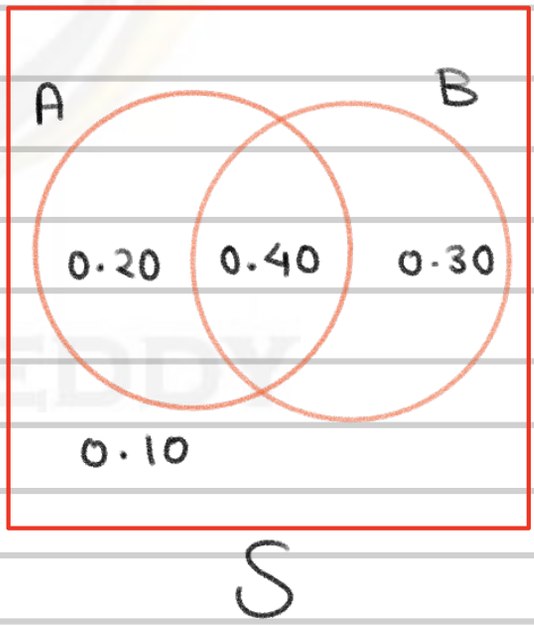
P(A) = 0.53
P(B) = 0.37
P(C) = 0.47
P(A′) = 0.47
P(B′) = 0.63
P(C′) = 0.53
P(A∩B) = 0.25
P(B∩C) = 0.17
P(A∩C) = 0.18
P(A∪B) = 0.65

P(B∪C) = 0.67
P(A∪C) = 0.82
P(A-B) = 0.28
P(B-A) = 0.12
P(A-C) = 0.35
P(C-A) = 0.29
P(B-C) = 0.20
P(C-B) = 0.30
P(A∪B∪C) = 0.87
P(A∩B∩C) = 0.10

P(A∩B′∩C′) = 0.20
P(B∩A′∩C′) = 0.05
P(C∩A′∩B′) = 0.22
P(A′∪B′) = 0.75
P(B′∪C′) = 0.83
P(A′∪C′) = 0.82
P(A′∩B′∩C′) = 0.13
P(A△B) = 0.40
P(B△C) = 0.50
P(A△C) = 0.64

49. **P(A-B) = 0.20, P(B-A) = 0.30, P(A′∩B′) = 0.10. Find**

P(A) = 0.60
P(B) = 0.70
P(A∪B) = 0.90
P(A∩B) = 0.40
P(A△B) = 0.50
P(A∪B′) = 0.70
P(B∪A′) = 0.80
P(A′∪B′) = 0.60
P(A′) = 0.40
P(B′) = 0.30



50. $P(A) = 0.30, P(B) = 0.20, P(C) = 0.60, P(A \cap B) = 0.10, P(B \cap C) = 0.15,$ $P(A \cap C) = 0.18, P(A \cap B \cap C) = 0.05$, Find $P(A \cup B \cup C)$ and $P(A' \cap B' \cap C')$, $P(A \cup B)$, $P(B \cup C)$ $P(A \cap C')$, $P(B \cup C')$

→ ① $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - n(A \cap C) + P(A \cap B \cap C)$

$= 0.72$

② $P(A' \cap B' \cap C') = 1 - P(A \cup B \cup C) = 0.28$

③ $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.40$

④ $P(B \cup C) = P(B) + P(C) - P(B \cap C) = 0.65$

⑤ $P(A \cap C') = P(A) - P(A \cap C) = 0.12$

⑥ $P(B \cup C') = 1 - P(C - B) = 1 - [0.60 - 0.15] = 0.55$

51. Odds in favour of an event are 2:3 and odds against another event are 3:7. Find the probability that only one of two events occurs. (2 events are independent of each other)

→ $P(A) = \frac{2}{5}$ $P(B) = \frac{7}{10}$ $P(A \cap B) = \frac{14}{50}$

$P(A \Delta B) = P(A \cup B) - P(A \cap B)$

$= \left(\frac{2}{5} + \frac{7}{10} - \frac{14}{50}\right) - \frac{14}{50} = \frac{20 + 35 - 14 - 14}{50} = \left(\frac{37}{50}\right)$

$= 74\ \%$

52. There are 3 boxed with composition of balls :

5 Red 8 Blue	6 Red 3 Blue	8 Red 2 Blue

If one box is selected at random and one ball is drawn, what is the probability that it is a red ball?

→ $= \left(\frac{1}{3} \times \frac{5}{13}\right) + \left(\frac{1}{3} \times \frac{6}{9}\right) + \left(\frac{1}{3} \times \frac{8}{10}\right)$

$= \left(\frac{5}{39} + \frac{6}{27} + \frac{8}{30}\right) = 61.7094\ \%$

53. In a business venture, a man can make profit of ₹ 50,000 or incur a loss of ₹ 10,000. The probability of making profit or incurring loss from past experience are known to be 0.75 and 0.25 respectively. What is his expected profit?

→

x	$P(x)$	$x \cdot P(x)$
50,000	0.75	37,500
−10,000	0.25	−2,500
	$E(x) =$	35,000

Expected profit = ₹ 35,000/-

54. Ashwat draws 2 balls from a bag containing 3 white and 5 red balls. He gets ₹ 500 if he draws a white ball and ₹ 200 if he draws a red ball. What is his expectation?

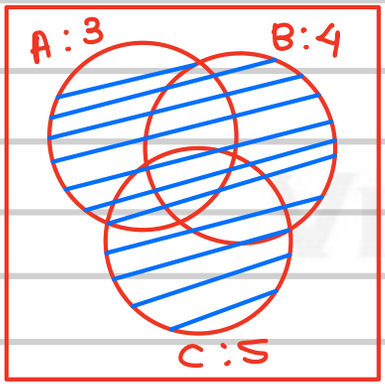
→

	x	$P(x)$	$x \cdot P(x)$
2 Red	₹ 400	$^5C_2 / ^8C_2 = 10/28$	
2 white	₹ 1000	$^3C_2 / ^8C_2 = 3/28$	
1 Red 1 white	₹ 700	$\frac{^3C_1 \times ^5C_1}{^8C_2} = 15/28$	

$E(x) =$ ₹ 625/-

55. A number is selected form first 1000 natural numbers, what is probability that number is divisible by 3 or 4 or 5.

→



$$P(A \cup B \cup C)$$
$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$
$$= \frac{333}{1000} + \frac{250}{1000} + \frac{200}{1000} - \frac{83}{1000} - \frac{50}{1000} - \frac{66}{1000} + \frac{16}{1000}$$
$$= (600/1000) = 60\%.$$

56. The probability of an event lies between 0 and 1, both inclusive.

$0 \leq$ Probability (Any event) ≤ 1.00

57. A : Vinod is a minor

B : Vinod is a major

Here A, B are mutually exclusive events as well as mutually exhaustive events as $P(A \cup B) = 1.00$, $P(A \cap B) = 0$

58. A : Ashwat is an Indian

B : Ashwat is an American

Here A, B are mutually exclusive events as $P(A \cap B) = 0$ but A, B are not mutually exhaustive.

59. All general Formulae at one place :

1. $P(A) = 1 - P(A')$
2. $P(B') = 1 - P(B)$
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
4. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
5. $P(A-B) = P(A) - P(A \cap B)$
6. $P(B-A) = P(B) - P(A \cap B)$
7. $P(A \cup B') = 1 - P(B-A)$
8. $P(B \cup A') = 1 - P(A-B)$
9. $P(A \triangle B) = P(A-B) + P(B-A)$
10. $P(A' \cap B') = 1 - P(A \cup B)$
11. $P(A' \cup B') = 1 - P(A \cap B)$
12. $P(A \cup B \cup C) = 1 - P(A' \cap B' \cap C')$
13. $P(A' \cap B' \cap C') = 1 - P(A \cup B \cup C)$
14. $P(A/B) = P(A \cap B) / P(B)$
15. $P(B/A) = P(B \cap A) / P(A)$
16. $P(A/B') = P(A \cap B') / P(B')$
17. $P(B/A') = P(B \cap A') / P(A')$
18. $P(A'/B) = P(A' \cap B) / P(B)$
19. $P(A'/B') = P(A' \cap B') / P(B')$
20. $P(B'/A') = P(B' \cap A') / P(A')$
21. $P(B'/A) = P(B' \cap A) / P(A)$

60. When A,B are mutually exclusive events

- $P(A \cap B) = 0$
- $P(A \cup B) = P(A) + P(B)$
- $P(A-B) = P(A)$
- $P(B-A) = P(B)$
- $P(A' \cup B') = 1.00$
- $P(A/B) = 0$
- $P(B/A) = 0$
- $P(A \triangle B) = P(A) + P(B)$
- $P(A \cup B') = P(B')$
- $P(B \cup A') = P(A')$

61. When A,B are mutually exhaustive events then :

$P(A \cup B) = 1.00$

$P(A' \cap B') = 0$

$P(A \cap B) = P(A) + P(B) - 1$

$P(A/B') = 1.00$

$P(B/A') = 1.00$

$P(A \triangle B) = 1 - P(A \cap B) = P(A \cap B)' = P(A' \cup B')$

62. When A,B are independent events then, $P(A \cap B) = P(A) \times P(B)$

$P(A \cap B') = P(A) \times P(B')$	$P(A/B') = P(A)$
$P(B \cap A') = P(B) \times P(A')$	$P(B/A') = P(B)$
$P(A' \cap B') = P(A') \times P(B')$	$P(A'/B) = P(A')$
$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$	$P(A'/B') = P(A')$
$P(A/B) = P(A)$	$P(B'/A) = P(B')$
$P(B/A) = P(B)$	$P(B'/A') = P(B')$

63. 2 dice are rolled, what is probability that points on first dice are more than points on second dice?

outcomes in favour : (2,1) (3,1) (3,2) (4,1) (4,2) (4,3) (5,1) (5,2) (5,3) (5,4) (6,1) (6,2) (6,3) (6,4) (6,5)

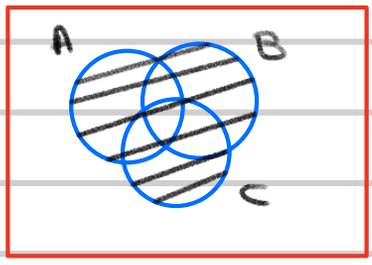
$= \frac{15}{36} = \frac{5}{12}$

64. A committee of 5 members is formed from 8 ladies and 9 gents. What is probability that ladies form the majority?

$$= \frac{({}^8C_3 \times {}^9C_2) + ({}^8C_4 \times {}^9C_1) + ({}^8C_5 \times {}^9C_0)}{{}^{17}C_5}$$

$$= \frac{2016 + 630 + 56}{6188} = \frac{2702}{6188} = 43.6652\%$$

65. A problem of maths was given to 3 students, chances of solving it are 1/3, 1/5, 1/2 respectively. What is the probability that problem gets solved?



→ (Independent events)

$P(A \cup B \cup C) = 1 - P(A' \cap B' \cap C')$

$= 1 - \left(\frac{2}{3} \times \frac{4}{5} \times \frac{1}{2}\right)$

$= 1 - \frac{8}{30} = \left(\frac{22}{30}\right) = \frac{11}{15}$

66. 8 identical balls are placed at random in 3 bags. What is the probability that first bag contains 3 balls?

→ $n = 8,\ p = \frac{1}{3},\ q = \frac{2}{3},\ x = 3$

$\text{prob}(x = 3) = {}^{8}C_{3}\left(\frac{1}{3}\right)^{3}\left(\frac{2}{3}\right)^{5} = \frac{56 \times 1 \times 32}{6561} = \frac{1792}{6561}$

$= 27.31291\%$

67. $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$, Find $P(A'/B')$

→ $P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{4}\right)}{\frac{2}{3}} = \frac{1 - \frac{6+4-3}{12}}{\frac{2}{3}}$

$= \left(\frac{1 - \frac{7}{12}}{\frac{8}{12}}\right) = \left(\frac{5/12}{8/12}\right) = \frac{5}{8} = 62.50\%$

68. The probability that there is atleast one error in an account statement prepared by 3 persons A,B,C are 0.20, 0.30, 0.10 respectively. If A, B, C prepare 60, 70, 90 such statements. Find expected number of correct statements.

a. 170 b. 176 ~~c. 178~~ d. 180

	x	$P(x)$	$x \cdot P(x)$
A	60	0.80	
B	70	0.70	
C	90	0.90	
		$E(x)$	178

69.

x	1	2	4	6	8
Prob. x	k	2k	3k	3k	k

Find Expected Value of x, SD of x, Variance of x.

→

$10k = 1.00$

$k = 0.10$

x	$P(x)$	x^2	$x.p(x)$	$x^2.p(x)$
1	0.10	1		
2	0.20	4		
4	0.30	16		
6	0.30	36		
8	0.10	64		
			4.30	22.90

$E(x) = \sum x.p(x) = 4.30$

$\text{vari. of } x = E(x^2) - [E(x)]^2$

$= 22.90 - 4.30^2 = 4.41$

$\text{SD of } x = \sqrt{4.41}$

$= 2.10$

70. 5 Red, 6 White, 4 Black → **4 Balls are drawn. What is the proabibility that there is atleast one ball of each colour?**

→

$$= \frac{(5C_1 \times 6C_1 \times 4C_2) + (5C_1 \times 6C_2 \times 4C_1) + (5C_2 \times 6C_1 \times 4C_1)}{15C_4}$$

$$= \frac{180 + 300 + 240}{1365} = \frac{720}{1365} = 52.7472\,5\,\%$$

71. 5 Red, 12 Blue, 3 Pink → **5 Balls are drawn. What is the proabibility that there is atleast one ball of each colour?**

→

$$= \frac{\left((5C_1 \times 12C_1 \times 3C_3) + (5C_1 \times 12C_3 \times 3C_1) + (5C_3 \times 12C_1 \times 3C_1) + (5C_2 \times 12C_2 \times 3C_1) + (5C_2 \times 12C_1 \times 3C_2) + (5C_1 \times 12C_2 \times 3C_2)\right)}{20C_5}$$

$$= \left(\frac{60 + 3300 + 360 + 1980 + 360 + 990}{15504}\right)$$

$$= (7050/15504) = 45.4721\,\%$$

72. The expected number of heads in 100 tosses of an unbaised coin is :

$= 100 \times 0.50 = 50$

73. A man can kill a bird once in 3 shots. The probability that bird is not killed is

a. 1/3 b. 2/3 c. 1.00 d. 0

74. If on an average 9 ships out of 10 return safely to the port, the probability that one ship returns to the port safely is

a. 1/10 b. 9/10 c. 8/10 d. None of these

75. A family has 2 children. The probability that both of them are boys if it is known that one of them is a boy is :

a. 1.00 b. 1/2 c. 3/4 d. None of these

76. Probability of throwing an odd number with an ordinary six faced die is?

a. 1/2 b. 1.00 c. -1/2 d. 1/6

77. When none of the outcomes is favourable to the event then event is said to be

a. Certain b. Sample c. Impossible d. None

78. What is probability that 4 children selected at random would have different birthdays?

a. 98.36% b. 100% c. 99.82% d. 0%

$\left(\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365}\right)$

79. For 2 independent events A, B, P(A∪B) = 2/3, P(A) = 2/5, P(B) = ?

a. 4/15 b. 4/9 c. 5/9 d. 7/18 e. None

$\frac{2}{3} = \frac{2}{5} + P(B) - \frac{2}{5} \times P(B) \quad \frac{2}{3} - \frac{2}{5} = P(B)\left[1 - \frac{2}{5}\right] \quad \frac{4}{15} \times \frac{5}{3} = P(B)$

80. What is chance of throwing atleast 7 in a single cast with 2 dice? $= 20/45 = 4/9$

a. 5/12 b. 7/12 c. 1/4 d. 17/36 e. None

$= \left(\frac{6+5+4+3+2+1}{36}\right) = 21/36 = 7/12$

81. Expected value of a random variable

a. Is always positive
b. May be positive or negative
c. May be positive, negative or zero
d. Can never be zero

82. P(A) = 8/17, then odds against event A is .

a. 8:17 b. 17:8 c. 8:9 d. 9:8

odds in favour = 8 : 9 odds against event A = 9 : 8

83. Initially probability was branch of

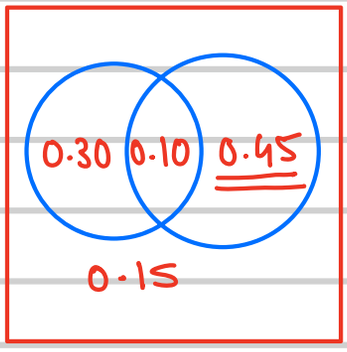
a. Physics b. Chemistry c. Statistics d. Mathematics

84. Subjective probability may be used in

a. Mathematics b. Statistics c. Management d. Biology

85. P(A-B) = 0.30, P(A∩B) = 0.10, P(A'∩B') = 0.15.

Find P(A), P(A∪B), P(A'∪B'), P(B), P(A△B), P(B-A), P(A/B), P(B'/A')



$P(A) = 0.40$
$P(A \cup B) = 0.85$
$P(A' \cup B') = 0.90$
$P(B) = 0.55$
$P(A \triangle B) = 0.75$
$P(B - A) = 0.45$

$P(A/B) = \frac{0.10}{0.55} = \frac{2}{11}$

$P(B'/A') = \frac{0.15}{0.60} = \frac{1}{4}$

86. P(A/B) is defined only when

a. B is a sure event
b. B is an impossible event
c. B is not a sure event
d. B is not an impossible event

$P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow$ this is defined only when $P(B) \neq 0$

87. P(A/B′) is defined only when

a. B is a sure event
b. B is an impossible event
~~c.~~ B is not a sure event
d. B is not an impossible event

$P(A/B') = \frac{P(A \cap B')}{P(B')}$ This is defined only when $P(B') \neq 0$ i.e. $P(B) \neq 1.00$

88. P(X/Y) is defined only when Y is not an impossible event.

P(X/Y′) is defined only when Y is not a sure event

89. If A,B,C are 3 mutually exclusive and exhaustive events such that P(A) = 2.P(B) = 3.P(C) then P(B) = ?

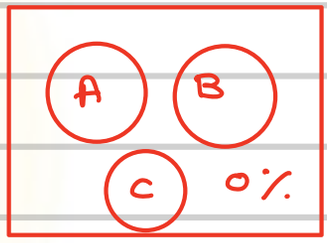
a. 6/11 ~~b.~~ 3/11 c. 1/6 d. 1/3

$1.00 = P(A) + P(B) + P(C) - 0 - 0 - 0 + 0$

$1 = 2 \cdot P(B) + P(B) + \frac{2}{3} P(B)$

$1 = P(B) \left[3 + \frac{2}{3} \right]$

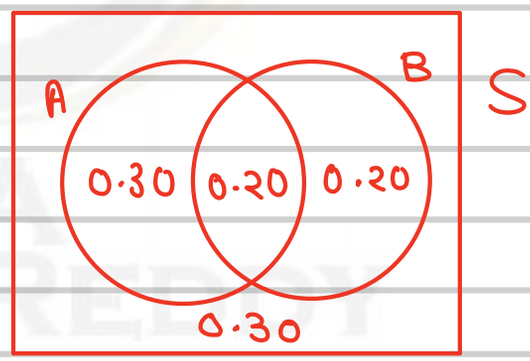
$1 = P(B) \times \frac{11}{3}$ $\therefore P(B) = \frac{3}{11}$



90. P(A-B) = 0.30, P(A△B) = 0.50, P(A′∪B′) = 0.80

Find P(A′ ∩ B′)

→ $P(A' \cap B') = 0.30$



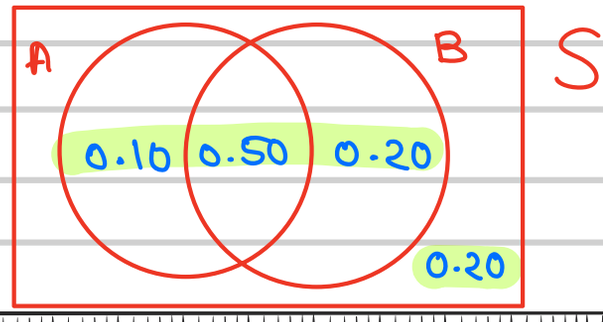
91. P(A) = 0.60, P(B) = 0.70, P(A′∩B′) = 0.20

Find P(A-B), P(B-A), P(A∩B)

→ $P(A-B) = 0.10$

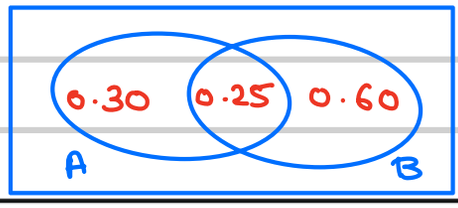
$P(B-A) = 0.20$

$P(A \cap B) = 0.50$



92. P(A-B) = 0.30, P(B-A) = 0.60, P(A) = 0.55 Find P(AUB)

a. 1.15 b. 0.15 c. 0.85 ~~d.~~ Wrong data



93. 2 dice are rolled, what is probability that sum of points is a prime number?

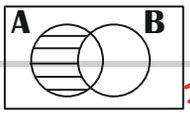
$$= \frac{1+2+4+6+2}{36} = \frac{15}{36} = \frac{5}{12}$$

94. One card is drawn from each of 2 packs of 52 cards. What is probability that atleast one of them is an ace?

a. 8/104 b. ${}^{8}C_2/{}^{104}C_2$ c. 25/169 d. 1/169 e. None

$$\text{prob(atleast one ace)} = 1 - \text{prob(Both cards are Non-ace)}$$

$$= 1 - \left(\frac{48}{52} \times \frac{48}{52}\right) = 1 - \frac{144}{169} = 25/169$$

95. 

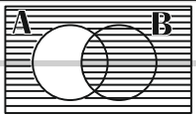
Shaded area represents

~~a.~~ (A-B)

b. (B-A)

c. (AUB′)

d. (A′UB′)

96. 

Shaded area represents

a. (A-B)

b. (A+B)

c. (AUB′)

~~d.~~ (BUA′)

97. A number is selected from first 100 natural numbers, what is the probability that

It is a perfect square?

$= \frac{10C_1}{100C_1}$

$= \frac{10}{100} = 10\%$

It is a perfect cube?

$= \frac{4C_1}{100C_1}$

$= 4\%$

It is an odd number?

$= \frac{50C_1}{100C_1}$

$= 50\%$

98. 2 cards are drawn one after other from a pack of 52 cards, what is the probability that both cards are kings if cards are drawn

Without Replacement

$= \left(\frac{4}{52} \times \frac{3}{51}\right) = \frac{1}{13} \times \frac{1}{17}$

$= 1/221$

With Replacement

$= \left(\frac{4}{52} \times \frac{4}{52}\right) = \frac{1}{13} \times \frac{1}{13}$

$= 1/169$

99. 2 numbers are selected from first 50 natural numbers, find the probability that both are divisible by 3?

$= \frac{16C_2 \times 34C_0}{50C_2} = \frac{120}{1225} = 9.7959\%$

100. Mr. A says to Mr. B "If it rains today I will give you ₹ 50,000 but if it doesn't rain today you have to pay me ₹ 80,000". Find expected gain / (loss) for Mr. B if probability of raining is 0.20

	x	$P(x)$	$x \cdot P(x)$
Rain	50,000	0.20	10,000
No Rain	−80,000	0.80	−64,000

$\sum x \cdot prob(x) = -54,000$

Expected Gain for B = −54,000

i.e. Expected Loss for B = 54,000

101. A and B tossed 3 coins each. What is probability that both of them will get same number of heads?

→

No. of heads	A prob(x)	B prob(x)	
0	1/8	1/8	1/64
1	3/8	3/8	9/64
2	3/8	3/8	9/64
3	1/8	1/8	1/64
			20/64

Answer : $\frac{20}{64} = \frac{5}{16} = 31.25\%$.

Every good or Bad Moment of Your life is a part of your life, It's not your LIFE!

If you are not willing to learn, No one can help you. If you are determined to learn, No one can stop you!

- CA VINOD REDDY -

All the late nights and Early mornings will pay off.

- CA VINOD REDDY -

Education is the key to unlock the golden door of FREEDOM

Every student can learn, just not on the same day!