CA FOUNDATION - PAPER 3 - BUSINESS MATHEMATICS AND LOGICAL REASONING & STATISTICS

Often, we encounter news of price rise, GDP growth, production growth, etc. It is important for students of Chartered Accountancy to learn techniques of measuring growth/rise or decline of various economic and business data and how to report them objectively. After reading this capsule, students will be able to understand Purpose of constructing index number and its important applications in understanding rise or decline of production, prices, etc. different methods of computing index number.

CHAPTER 18: INDEX NUMBERS

INTRODUCTION

Index numbers are convenient devices for measuring relative changes of differences from time to time or from place to place. Just as the arithmetic mean is used to represent a set of values, an index number is used to represent a set of values over two or more different periods or localities.

The basic device used in all methods of index number construction is to average the relative change in either quantities or prices since relatives are comparable and can be added even though the data from which they were derived cannot themselves be added. For example, if wheat production has gone up to 110% of the previous year's production and cotton production has gone up to 105%, it is possible to average the two percentages as they have gone up by 107.5%. This assumes that both have equal weight; but if wheat production is twice as important as cotton, percentage should be weighted 2 and 1. The average relatives obtained through this process are called the index numbers.

- **Relatives:** One of the simplest examples of an index number is a price relative, which is the ratio of the price of single commodity in a given period to its price in another period called the base period or the reference period. It can be indicated as follows:

Price relative = $\frac{P_n}{P_o} \times 100$

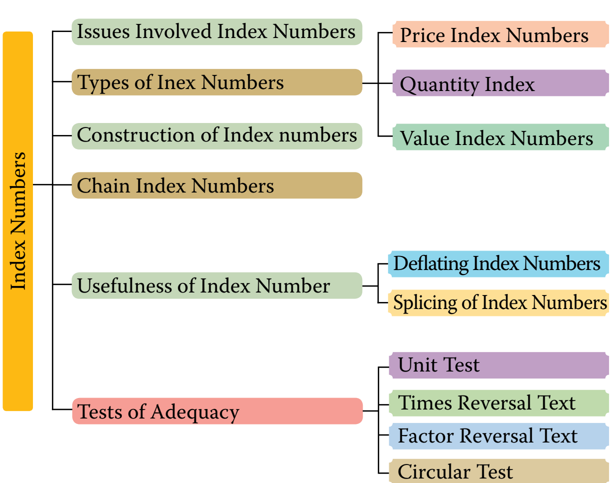
There can be other relatives such as of quantities, volume of consumption, exports, etc. The relatives in that case will be:

Quantity relative = $\frac{Q_n}{Q_o} \times 100$

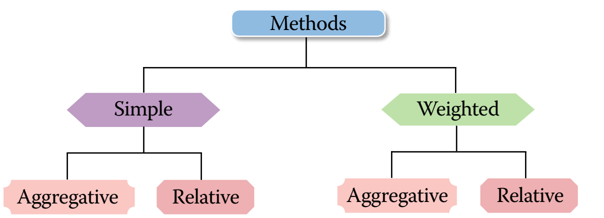
Similarly, there are value relatives:

Value relative = $\frac{V_n}{V_o} = \frac{P_n Q_n}{P_o Q_o} = \left(\frac{P_n}{P_o} \times \frac{Q_n}{Q_o}\right) \times 100$

Index number Overview



Methods of Index numbers:



Price Index numbers

(a) Simple aggregative price index = $\frac{\sum P_n}{\sum P_0} \times 100$

(b) Laspeyres' Index: In this Index base year quantities are used as weights:

Laspeyres Index = $\frac{\sum P_n Q_o}{\sum P_o Q_o} \times 100$

(c) Paasche's Index: In this Index current year quantities are used as weights:

Passche's Index = $\frac{\sum P_n Q_n}{\sum P_o Q_n} \times 100$

(d) The Marshall-Edgeworth index uses this method by taking the average of the base year and the current year

Marshall-Edgeworth Index = $\frac{\sum P_n (Q_o + Q_n)}{\sum P_o (Q_o + Q_n)} \times 100$

(e) Fisher's ideal Price Index: This index is the geometric mean of Laspeyres' and Paasche's.

Fisher's Index = $\sqrt{\frac{\sum P_n Q_o}{\sum P_o Q_o} \times \frac{\sum P_n Q_n}{\sum P_o Q_n}} \times 100$

(g) Weighted Average of Relative Method: $\frac{\sum \left(\frac{P_n}{P_o}\right) \times (P_o Q_o)}{\sum P_o Q_o} \times 100 = \frac{\sum P_n Q_o}{\sum P_o Q_o} \times 100$

(h) Chain Index = $\frac{\text{Link relative of current year X Chain Index of the previous year}}{100}$

Quantity Index Numbers

- Simple aggregate of quantities: $\frac{\sum Q_n}{\sum Q_0} \times 100$
- The simple average of quantity relatives: $\frac{\sum \frac{Q_n}{Q_0}}{N} \times 100$
- Weighted aggregate quantity indices:
  - (i) With base year price as weight : $\frac{\sum Q_n P_o}{\sum Q_o P_o} \times 100$ (Laspeyre's index)
  - (ii) With current year price as weight : $\frac{\sum Q_n P_n}{\sum Q_o P_n} \times 100$ (Paasche's index)
  - (iii) Geometric mean of (i) and (ii) : $\sqrt{\frac{\sum Q_n P_o}{\sum Q_o P_o} \times \frac{\sum Q_n P_n}{\sum Q_o P_n}} \times 100$ (Fisher's Ideal)
- Base-year weighted average of prices as relatives in Marshall-Edgeworth quantity index number.

  Weighted Relative method formula $\frac{\sum \left(\frac{Q_n}{Q_o} P_o Q_o\right)}{\sum P_o Q_o} \times 100$
- Value Indices $\frac{V_n}{V_o} = \frac{\sum P_n Q_n}{\sum P_o Q_o} \times 100$

THE CHAIN INDEX NUMBERS

So far we concentrated on a fixed base but it does not suit when conditions change quite fast. In such a case the changing base for example, 2018 for 2019, and 2019 for 2020, and so on, may be more suitable. If, however, it is desired to associate these relatives to a common base the results may be chained. Thus, under this method the relatives of each year are first related to the preceding year called the link relatives and then they are chained together by successive multiplication to form a chain index.

The formula is:

$$\text{Chain Index} = \frac{\text{Link relative of current year X Chain Index of the previous year}}{100}$$

Illustrations

Commodities	2018	2019	2020
Cheese (per 100 gms)	120	150	160
Egg (per piece)	30	36	40
Potato (per kg)	50	60	60
Aggregrate	200	246	260
Index	100	123	130

Simple Aggregative Index for 2019 over 2018 $= \frac{\sum P_n}{\sum P_o} = \frac{246}{200} \times 100 = 123$

and for 2020 over 2018 $= \frac{\sum P_n}{\sum P_o} \times 100 = \frac{260}{200} \times 100 = 130$

The above method is easy to understand but it has a serious defect. It shows that the first commodity exerts greater influence than the other two because the price of the first commodity is higher than that of the other two. Further, if units are changed then the Index numbers will also change. Students should independently calculate the Index number taking the price of eggs per dozen i.e., ` 36, ` 43.20, ` 39.60 for the three years respectively. This is the major flaw in using absolute quantities and not the relatives. Such price quotations become the concealed weights which have no logical significance.

Limitations of Index Numbers

So far we have studied various types of index numbers. However, they have certain limitations. They are:

1. As the indices are constructed mostly from deliberate samples, chances of errors creeping in cannot be always avoided.
2. Since index numbers are based on some selected items, they simply depict the broad trend and not the real picture.
3. Since many methods are employed for constructing index numbers, the result gives different values and this at times create confusion.

Usefulness of Index Numbers

In spite of its limitations, index numbers are useful in the following areas:

1. Framing suitable policies in economics and business. They provide guidelines to make decisions in measuring intelligence quotients, research etc.
2. They reveal trends and tendencies in making important conclusions in cyclical forces, irregular forces, etc.
3. They are important in forecasting future economic activity. They are used in time series analysis to study long-term trend, seasonal variations and cyclical developments.
4. Index numbers are very useful in deflating, i.e., they are used to adjust the original data for price changes and thus transform nominal wages into real wages.
5. Cost of living index numbers measure changes in the cost of living over a given period.

DEFLATING TIME SERIES USING INDEX NUMBERS

Sometimes a price index is used to measure the real values in economic time series data expressed in monetary units. For example, GNP initially is calculated in current price so that the effect of price changes over a period of time gets reflected in the data collected. Thereafter, to determine how much the physical goods and services have grown over time, the effect of changes in price over different values of GNP is excluded. The real economic growth in terms of constant prices of the base year therefore is determined by deflating GNP values using price index.

Year	*Wholesale Price Index*	*GNP at Current Prices*	*Real GNP*
2019	113.1	7499	6630
2020	116.3	7935	6823
2021	121.2	8657	7143
2022	127.7	9323	7301

The formula for conversion can be stated as

$$\text{Deflated Value} = \frac{\text{Current Value}}{\text{Price Index of the current year}} \times 100$$

$$\text{Or Current Value} \times \frac{\text{Base Price } (P_0)}{\text{Current Price } (P_n)}$$

Shifting and Splicing of Index Numbers

These refer to two technical points: (i) how the base period of the index may be shifted, (ii) how two index covering different bases may be combined into single series by splicing.

Shifted Price Index

Year	Original Price Index	Shifted Price Index to base 2021
2011	100	71.4
2012	104	74.3
2013	106	75.7
2014	107	76.4
2015	110	78.6
2016	112	80.0
2017	115	82.1
2018	117	83.6
2019	125	89.3
2020	131	93.6
2021	140	100.0
2022	147	105.0

The formula used is, $\text{Shifted Price Index} = \frac{\text{Original Price Index}}{\text{Price Index of the year on which it has to be shifted}} \times 100$

Splicing two sets of price index numbers covering different periods of time is usually required when there is a major change in quantity weights. It may also be necessary on account of a new method of calculation or the inclusion of new commodity in the index.

Splicing Two Index Number Series

Year	Old Price Index [2010 = 100]	Revised Price Index [2015=100]	Spliced Price Index [2015 = 100]
2010	100.0		87.6
2011	102.3		89.6
2012	105.3		92.2
2013	107.6		94.2
2014	111.9		98.0
2015	114.2	100.0	100.0
2016		102.5	102.5
2017		106.4	106.4
2018		108.3	108.3
2019		111.7	111.7
2020		117.8	117.8

You will notice that the old series 2010 has to be converted shifting to the base. 2015 i.e, 114.2 to have a continuous series, even when the two parts have different weights

TEST OF ADEQUACY

There are four tests:

1. **Unit Test:** This test requires that the formula should be independent of the unit in which or for which prices and quantities are quoted. Except for the simple (unweighted) aggregative index all other formulae satisfy this test.

(ii) Time Reversal Test: It is a test to determine whether a given method will work both ways in time, forward and backward. The test provides that the formula for calculating the index number should be such that two ratios, the current on the base and the base on the current should multiply into unity. In other words, the two indices should be reciprocals of each other. Symbolically,

$$P_{01} \times P_{10} = 1$$

where P_{01} is the index for time 1 on 0 and P_{10} is the index for time 0 on 1.

You will notice that Laspeyres' method and Paasche's method do not satisfy this test, but Fisher's Ideal Formula does.

While selecting an appropriate index formula, the Time Reversal Test and the Factor Reversal test are considered necessary in testing the consistency.

Laspeyres: $P_{01} = \frac{\sum P_1Q_0}{\sum P_0Q_0}$ $\qquad P_{10} = \frac{\sum P_0Q_1}{\sum P_1Q_1}$

$$P_{01} \times P_{10} = \frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_0Q_1}{\sum P_1Q_1} \neq 1$$

Paasche's: $P_{01} = \frac{\sum P_1Q_1}{\sum P_0Q_1}$ $\qquad P_{10} = \frac{\sum P_0Q_0}{\sum P_1Q_0}$

$$P_{01} \times P_{10} = \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum P_0Q_0}{\sum P_1Q_0} \neq 1$$

Fisher's: $P_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}}$ $\qquad P_{10} = \sqrt{\frac{\sum P_0Q_1}{\sum P_1Q_1} \times \frac{\sum P_0Q_0}{\sum P_1Q_0}}$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum P_0Q_1}{\sum P_1Q_1} \times \frac{\sum P_0Q_0}{\sum P_1Q_0}} = 1$$

(iii) Factor Reversal Test: This holds when the product of price index and the quantity index should be equal to the corresponding value index, i.e.,

Symbolically: $P_{01} \times Q_{01} = V_{01}$

Fishers' $P_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}}$ $\qquad Q_{01} = \sqrt{\frac{\sum Q_1P_0}{\sum Q_0P_0} \times \frac{\sum Q_1P_1}{\sum Q_0P_1}}$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum Q_1P_0}{\sum Q_0P_0} \times \frac{\sum Q_1P_1}{\sum Q_0P_1}} = \sqrt{\frac{\sum P_1Q_1}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_0}} = \frac{\sum P_1Q_1}{\sum P_0Q_0} = V_{01}$$

Thus Fisher's Index satisfies Factor Reversal test. Because Fisher's Index number satisfies both the tests in (ii) and (iii), it is called an Ideal Index Number.

(iv) Circular Test: It is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base. For example, if the 1970 index with base 1965 is 200 and 1965 index with base 1960 is 150, the index 1970 on base 1960 will be 300. This property therefore enables us to adjust the index values from period to period without referring each time to the original base. The test of this shiftability of base is called the circular test.

- This test is not met by Laspeyres, or Paasche's or the Fisher's ideal index. **The simple geometric mean of price relatives and the weighted aggregative with fixed weights meet this test.**

Example 1: Compute Fisher's Ideal Index from the following data:

Commodities	*Base Year*		*Current Year*	
	Price	*Quantity*	*Price*	*Quantity*
A	4	3	6	2
B	5	4	6	4
C	7	2	9	2
D	2	3	1	5

Show how it satisfies the time and factor reversal tests.

Solution:

Commodities	P_0	Q_0	P_1	Q_1	P_0Q_0	P_1Q_0	P_0Q_1	P_1Q_1
A	4	3	6	2	12	18	8	12
B	5	4	6	4	20	24	20	24
C	7	2	9	2	14	18	14	18
D	2	3	1	5	6	3	10	5
					52	63	52	59

Fisher's Ideal Index: $P_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}} \times 100 = \sqrt{\frac{63}{52} \times \frac{59}{52}} \times 100$

$= \sqrt{1.375} \times 100 = 1.172 \times 100 = 117$

Time Reversal Test:

$P_{01} \times P_{10} = \sqrt{\frac{63}{52} \times \frac{59}{52} \times \frac{52}{59} \times \frac{52}{63}} = \sqrt{1} = 1$

∴ Time Reversal Test is satisfied.

Factor Reversal Test:

$P_{01} \times Q_{01} = \sqrt{\frac{63}{52} \times \frac{59}{52} \times \frac{52}{59} \times \frac{52}{63}} = \sqrt{\frac{59}{52} \times \frac{59}{52}} = \frac{59}{52}$

Since, $\frac{\sum P_1Q_1}{\sum P_0Q_0}$ is also equal to $\frac{59}{52}$, the Factor Reversal Test is satisfied.

Example 2: If the ratio between Laspeyre's index number and Paasche's Index number is 28 : 27. Then the missing figure in the following table P is :

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
X	L	10	2	5
Y	L	5	P	2

Solution: So Laspeyre's index number = ∑ PxQo / ∑ PoQo
= 2 x 10 + P x 5 / L x 10 + L x 5
= 20 + 5P / 15 L
= 5 (4 + P) / 15 L
= 4 + P / 3L

Now for the Paasche's index number we have, ∑ Px Qx / ∑ Po Qx
= 2 x 5 + P x 2 / L x 5 + L x 2
= 2P + 10 / 7L
= 2 (P + 5) / 7L

Given Ratio = L : P = 28:27

So 4 + p / 3L / 2 (P + 5) / 7L = 28/27
or 7 (4 + P) / 6(P + 5) = 28/27
or 9(4 + P) = 8(P + 5)
36 + 9P = 8P + 40
or P = 40 – 36
or P = 4

Example 3: The consumer price index for 2006 on the basis for 2006 on the basis of 2005 from the following data is:

Commodities in 2005	Quantities consumed in 2005	Prices in 2005	Prices in 2006
A	6	5.75	6.00
B	6	5.00	8.00
C	1	6.00	9.00
D	6	8.00	10.00
E	4	2.00	1.50
F	1	20.00	15.00

Solution:

Commodities in 2005	Quantities consumed in 2005(q_0)	Price in 2005 (P_0)	Prices in 2006 (P_1)	P_1q_0	P_0q_0
A	6	5.75	6.00	36	34.50
B	6	5.00	8.00	48	30.00
C	1	6.00	9.00	9	6.00
D	6	8.00	10.00	60	48.00
E	4	2.00	1.50	6	8.00
F	1	20.00	15.00	15	20.00
				$\sum P_1q_0=174$	$\sum P_0q_0=146.5$

Consumer Price Index = $\frac{\sum P_1q_0}{\sum P_0q_0}\times 100 = \frac{174}{146.5}\times 100 = 118.77$

Example 4: Net monthly salary of an employee was ₹30,000 in 2000. The consumer price index 2015 is 250 with 2000 as base year, if he has to rightly compensated, then dearness allowance to be paid to the employee is:

Solution: The consumer price index number in 2015 is 250 with 2000 as base year.

if in 2000 = 100 then in 2015 = 250

if in 2000 = 1 then in 2015 = 250/100 = 2.5

if in 2000 = 30000 then 250/100* 30000 = 75,000

additional dearness allowance to be paid to the employee is = 75000 – 30000 = ₹45000

additional dearness allowance to be paid to the employee = ₹45,000

Example 5: Consumer price index number goes up from 110 to 200 and the Salary of a worker is also raised from ₹330 to ₹500. Therefore, in real terms, to maintain his previous standard of living, he should get an additional amount of:

Solution: Cost of Living Index in base year = 110,

Cost of Index in current year = 200

Salary of worker in base year = 330

Salary of worker in current year = 500

$$Real\ wages\ (for\ base\ year) = \frac{\text{Money wages in base year}}{\text{Cost of living index in base year}}\times 100$$

$$= \frac{330}{110}\times 100$$

$$= 300$$

$$Real\ wages\ (for\ current\ year) = \frac{\text{Money wages in current year}}{\text{Cost of living index number in current year}}\times 100$$

$$= \frac{300}{200}\times 100 = 250$$

Thus, we can say that even though money wage of the worker had increased from ₹330 to ₹500, his real wage has fallen from ₹300 to ₹250. This implies a loss of ₹50 in real terms.

Example 6: If the price of a commodity in a place have decreased by 30% over the base period prices, then the index number of that place is:

Solution: Base price of any commodity = 10 decreased price = 30% of 100 = 30

Index number of that place now = 100-30 =70

Example 7: If with an increase of 10% in prices, the rise in wages is 20% then the real wages has increased by

$$\text{Solution : Real wages} = \frac{\text{Real wage of current year}}{\text{Real wage of base year}}\times 100$$

$$= \frac{120}{100}\times 100$$

=120-100=20%

Example 8: In the year 2010, the monthly salary of clerk was ₹24,000. The consumer price index was 140 in the year 2010, which rises to 224 in 2016. If he has to be rightly compensated, what additional monthly salary to be paid to him?

Solution

Year	CPI	Salary
2010	140	24000
2016	224	X

= 140/224 = 24000/x

$$x = \frac{24000\times 224}{140} = 38,400$$

DA=38,400-24,000

=14,400

Important Points:

1. Circular Test is an extension of time reversal test.
2. Time Reversal Test and Factor Reversal Test is satisfied by: Fisher's Ideal Index.
3. The ratio of price of single commodity in a given period to its price in the preceding year price is called the relative price.
4. Fisher's Ideal Formula does not satisfy Circular test.
5. The best average for constructing an index number is: Geometric Mean.
6. The time reversal test is satisfied by Fisher's index number.
7. The factor reversal test is satisfied by : Fisher's index.
6. The circular test is satisfied by Simple GM price relative.
7. Fisher's index number is based on Geometric mean of Laspeyre's and Paasche's index numbers.
8. Paasche index is based on: Current year quantities.
9. Fisher's Ideal Formula does not satisfy Circular test.
10. Purchasing Power of Money is the Reciprocal of price index number and inverse relationship between Purchasing power of money and price index number.