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MATHEMATICS AND LOGICAL REASONING

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MATHEMATICS INDEX

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1A RATIO, PROPORTION & PARTNERSHIP

THEORY

Ratio

- A ratio is a fraction (either proper or improper) which compares two or more quantities of similar kind, which enables us to understand as to how many times one quantity is involved in the other.

- If A : B ($\frac{A}{B}$) is a ratio, then the numerator A is called "Antecedent" and the denominator B is called the "Consequent".

- Ratios must be expressed in the simplest possible form and we can calculate ratios only when the quantities are commensurable (fully quantifiable).

- Two or more ratios can be bridged in order to have a continuous comparison between more than two variables.

- Rule for bridging more than two ratios :

  If ,a,b,c,d,e are five Quantities, and

$$\frac{a}{b}=\frac{N_1}{D_1};\frac{b}{c}=\frac{N_2}{D_2};\frac{c}{d}=\frac{N_3}{D_3};\frac{d}{e}=\frac{N_4}{D_4}$$

Then, a:b:c:d:e= $N_1N_2N_3N_4 : D_1N_2N_3N_4 : D_1D_2N_3N_4 : D_1D_2D_3N_4 : D_1D_2D_3D_4$

Let a : b is a ratio, then:

- $\frac{a}{b} > 1$ (Ratio of Greater Inequality)
- $\frac{a}{b} < 1$ (Ratio of Lesser Inequality)
- $\frac{a}{b} = 1$ (Ratio of Equality)

- $a^2 : b^2$ (Duplicate Ratio)
- $a^3 : b^3$ (Triplicate Ratio)
- $\sqrt{a} : \sqrt{b}$ (Sub-Duplicate Ratio)
- $\sqrt[3]{a} : \sqrt[3]{b}$ (Sub-Triplicate Ratio)
- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots\ldots$ If then the value of each ratio can be obtained by mean of any one of the following two operations;

a. Each ratio = $\frac{a+c+e+\ldots.}{b+d+f+\ldots.}$ (ADDENDO)

Or

b. Each ratio = $\frac{a-c-e-\ldots.}{b-d-f-\ldots.}$ (SUBTRANDENDO)

INVERSE RATIO:

- IR of a:b is b : a
- IR of a:b:c is bc : ac : ab
- IR of a:b:c:d is bcd : acd : abd : abc

COMPOUND RATIO:

The multiplying effect of all ratios given is known as compound ratio. If a:b and c:d are two ratios, then ac : bd is called the compounded ratio of the two.

Proportion

- Proportion is defined as the equality of two or more ratios. If $\frac{a}{b} = \frac{c}{d}$, in such a case the quantities a,b,c,d are said to be proportional, here 'd' is called the fourth proportional.
- If $\frac{a}{b} = \frac{b}{c}$, then a,b,c are said to be in continued proportion, where 'b' is called the mean proportional and 'c' is called third proportional.
- If $\frac{a}{b} = \frac{b}{c}$ or b^2 = ac $\therefore$ b = $\sqrt{ac}$

IF	THEN	PROPERTY
$\frac{a}{b} = \frac{c}{d}$	ad = bc	PRODUCT OF EXTREMES = PRODUCT OF MEANS
	$\frac{b}{a} = \frac{d}{c}$	INVERTENDO
	$\frac{a}{c} = \frac{b}{d}$	ALTERNENDO
	$\frac{a+b}{b} = \frac{c+d}{d}$	COMPONENDO
	$\frac{a-b}{b} = \frac{c-d}{d}$	DIVIDENDO
	$\frac{a+b}{a-b} = \frac{c+d}{c-d}$	COMPONENDO & DIVIDENDO
	$\frac{a-b}{a+b} = \frac{c-d}{c+d}$	DIVIDENDO & COMPONENDO

CLASSWORK SECTION

1. If x/2 = y/3 = z/7, then find the value of (2x − 5y + 4z) / 2y.
   a) 6/23 b) 23/6 c) 3/2 d) 17/6

2. The ratio of the number of 50 paise, Re. 1 and ₹ 5 coins with Mr. Zen is 5 : 2 : 1. If the amount with him is ₹ 38, then the number of Re. 1 coins with him is:
   a) 4 b) 8 c) 12 d) 16

3. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$. Then find the value of each ratio.
   a. 1 b. $\frac{1}{2}$ c. $\frac{1}{20}$ d. None of the above

4. An employer reduces the number of employees in the ratio of 19 : 16 and increases their wages in the ratio of 4 : 5. What is the ratio of the wage bill of the employer initially and now?
   a. 20 : 19 b. 17 : 16 c. 16 : 17 d. 19 : 20

Compound Ratio

5. Find the compounded ratio of 275 : 31, inverse of 729 : 1331, duplicate ratio of 2 : 5, triplicate ratio of 9 : 11, sub-duplicate ratio of 961 : 1296, sub-triplicate ratio of 729 : 1331.
   a. 1 : 1 b. 1 : 2
   c. 275 : 11 d. 31 : 25

Joint Ratio

6. If A : B = 2 : 3, B : C = 4 : 5 and C : D = 3 : 7, find A : B : C : D
   a) 4 : 6 : 15 : 35 b) 4 : 12 : 15 : 35
   c) 8 : 12 : 15 : 35 d) 8 : 16 : 25 : 35

7. If a : b = 3 : 5, b : c = 5 : 4, c : d = 2 : 3 and d is 50% more than e, find the ratio between a and e.
   a) 2 : 3 b) 3 : 4
   c) 3 : 5 d) 4 : 5

8. A man distributes his property of ₹ 6,00,000 among his three sons. The share of his first son is thrice that of the second son's share and the share of the second son is twice that of the third son. Find the ratio in which sons share the property.
   a) 1 : 2 : 6 b) 3 : 4 : 5
   c) 6 : 2 : 1 d) 2 : 4 : 6

9. What should be added to each of 3, 15, 38 and 134 so that the number become proportionate to each other.
   a) 3 b) 5 c) 7 d) 2

Mixtures and Alligation

10. In what proportion must rice @ ₹ 3.10/kg be mixed with rice @ ₹ 3.60/kg to make the mixture worth ₹ 3.25/kg?
   a. 3 : 5 b. 5 : 3
   c. 3 : 7 d. 7 : 3

PAST YEAR QUESTIONS

11. A box contains ₹ 56 in the form of coins of one rupee, 50 paise and 25 paise. The number of 50 paise coin is double the number of 25 paise coins and four times the numbers of one rupee coins. The numbers of 50 paise coins in the box is
   (a) 64 (b) 32 (c) 16 (d) 14

PARTNERSHIP

CLASSWORK SECTION

1. Simran Started a software business by investing Rs.50, 000 . After six months , Nanda joined her with capital of Rs. 80,000. After three years , they earned a profit of Rs.24,500. What was Simran's share in the profit ?

   (a) Rs.9423 (b) Rs.10500 (c) Rs.12,500 (d) Rs.14,000

1B INDICES, SURDS AND LOGARITHMS

THEORY

$a^x = N$

a = base

x = Power/Exponent/Index

N = Product

[But, $a \neq 0, 1, \pm\infty$]

Theory of Indices deals with the various changes in power, during various mathematical operations.

Basic Rules

1. $a^m \times a^n = a^{m+n}$
2. $\dfrac{a^m}{a^n} = a^{m-n}$
3. $\left(a^m\right)^n = a^{mn}$; m is added n times
4. $(ab)^m = a^m x b^m$
5. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
6. $a^0 = 1$
7. $a^{-n} = \dfrac{1}{a^n}$
8. If $a^m = a^n \Rightarrow m = n$; where, $a \neq 0, 1, -1, \pm\infty$

9. For $a^m = b^m$ if m $\neq$ 0 then

(i) $a = b$ (when m is odd)

(ii) $a = \pm b$ (when m is even)

10. $a^x = N$

$\Rightarrow a = N^{\frac{1}{x}} = \sqrt[x]{N}$

11. $(i) 0^a = 0$

$(ii) 1^a = 1$

$(iii) a^1 = a$

$(iv) a^0 = 1$

$(v) 0^0$ *has* no meaning

Basic Formulae

1. $(a+b)^2 = a^2 + 2ab + b^2$

2. $(a-b)^2 = a^2 - 2ab + b^2$

3. $a^2 - b^2 = (a+b)(a-b)$

4. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

5. $(a+b)^2 - (a-b)^2 = 4ab$

6. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

7. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$

8. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$

9. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

10. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

11. If $a+b+c=0$, then $a^3 + b^3 + c^3 = 3abc$

12. If $a^3+b^3+c^3=3abc, then\ \ either\ a+b+c=0$ or $a=b=c$
but both the results cannot hold true simultaneously

Rational Numbers, Irrational Numbers & Surds

- A Rational Number is a number which can be expressed in the form p/q, where q ≠ 0; p & q are integers and p and q are prime to each other, i.e., there is no common factor between p & q, other than 1.

- Any terminating and recurring decimals are rational numbers.

- Thus any non-recurring and non-terminating decimals are irrational numbers, and when the irrational numbers are expressed in radical form (root form), it is known as "Surds".

- Thus all the surds are irrational, but all irrational numbers are not surds.

- The numbers whose perfect root can be evaluated are rational quantities and numbers for which perfect roots cannot be evaluated are irrational quantities.

Order of Surds

If $\sqrt[k]{m}=(m)^{\frac{1}{k}}$ is a surd, then, it is said to be a surd of order "k".

Pure Surds and Mixed Surds

In case of pure surds, entire expression is kept within the radical sign. In mixed surds, it is expressed as a product of one rational and one irrational quantity.

Example:
$\sqrt{7}$ is a pure surd; $\sqrt{12}=\sqrt{4x3}=2\sqrt{3}$ is a mixed surd.

Conjugate of a Surd

If $\left(a+\sqrt{b}\right)$ or $\left(\sqrt{a}+\sqrt{b}\right)$ are surds, their respective conjugates would be given by,

$\left(a-\sqrt{b}\right)$ or $\left(\sqrt{a}-\sqrt{b}\right)$ and vice-versa.

Rationalization of Surds

Rationalization is a process, where we convert the irrational part of the surd into a rational quantity, with help of its conjugate.

Note: 1

- Rational + Rational = Rational
- Rational − Rational = Rational
- Rational x Rational = Rational
- Rational ÷ Rational = Rational

Note: 2

- Irrational + Irrational = Irrational
- Irrational − Irrational = Rational (only when the quantities are equal); otherwise −
- Irrational − Irrational = Irrational
- Irrational x Irrational = May be Rational or Irrational
- Irrational ÷ Irrational = May be Rational or Irrational

Note: 3

- Rational + Irrational = Irrational
- Rational − Irrational = Irrational
- Rational x Irrational = Irrational
- Rational ÷ Irrational = Irrational

Square Root of Surds

- The square root of a surd is always a surd.
- Every answer for square root must contain +ve or −ve sign and in the absence of +/- sign, "none of these" will be marked as answer.
- If the given surd, whose square root is to be evaluated is in the form $\left(a \pm \sqrt{b}\right)$, then the answer will also be in the form $\pm\left(x \pm \sqrt{y}\right)$.
- Square the options, in order to get the question back.

INDICES

1. $[\{(2)^{1/2}.(4)^{3/4}.(8)^{5/6}.(16)^{7/8}.(32)^{9/10}\}^4]^{3/25}$ is
   (a) A fraction (b) an integer
   (c) 1 (d) none of these

2. If $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, then the simplified form of
   $$\left[\frac{x^l}{x^m}\right]^{l^2+lm+m^2} \times \left[\frac{x^m}{x^n}\right]^{m^2+mn+n^2} \times \left[\frac{x^n}{x^l}\right]^{l^2+ln+n^2}$$
   (a) 0 (b) 1 (c) x (d) none of these

3. If $x = 3^{\frac{1}{3}} + 3^{\frac{-1}{3}}$, then $3x^3 - 9x$ is
   (a) 15 (b) 10
   (c) 12 (d) none of these

4. If $x^{1/p} = y^{1/q} = z^{1/r}$ and $xyz = 1$, then the value of $p + q + r$ is
   (a) 1 (b) 0
   (c) 1/2 (d) none of these

5. If $\frac{9^y.3^2.(3^{-y})^{-1} - 27^y}{3^{3x}.2^3} = \frac{1}{27}$ then $x - y$ is given by
   (a) −1 (b) 1 (c) 0 (d) none

6. If $(5.678)^x = (0.5678)^y = 10^z$ then
   (a) $\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 1$ (b) $\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = 0$
   (c) $\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = -1$ (d) None

7. If $ax^{2/3} + bx^{1/3} + c = 0$ then the value of $a^3x^2 + b^3x + c^3$ is given by
   (a) 3abcx (b) −3abcx (c) 3abc (d) −3abc

PAST YEAR QUESTIONS

8. Value of $(a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$ is:

(a) $a + \frac{1}{a}$ (b) $a - \frac{1}{a}$ (c) $a^2 + \frac{1}{a^2}$ (d) $a^2 - \frac{1}{a^2}$

9. If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$ then the value of $\left(\frac{a+b+c}{3}\right)^3$

(a) abc (b) 9abc

(c) $\frac{1}{abc}$ (d) $\frac{1}{9abc}$

LOGARITHMS

THEORY

If $a^x = N$, then $x = \log_a N$;   * $a \neq 0, 1, \pm\alpha$ and for the purpose of log, any negative quantity.

* x is called the logarithm of N (product) to the base "a".

Base "a"

- The base "a" of log can be any positive real number except 1.
- The base of log can be clearly divided into two parts:
- 0 < a < 1 (the proper fraction)
- a > 1 (positive integer / mixed fraction)
- Unless otherwise specified, the base of log is always taken to be 10 and this is known as Common Logarithm.
- For theoretical purpose, the base is always taken to be "e", where "e" is a constant and this is known as "Natural Logarithm".
- Common Logarithms are used for numerical calculations and Natural Logarithms are used in calculus.

Basic Rules

1. $\log_a mn = \log_a m + \log_a n$
2. $\log_a \frac{m}{n} = \log_a m - \log_a n$
3. $\log_a m^n = n \log_a m$
4. $\log_a a = 1$
5. $\log_a 1 = 0$
6. $\log_a 0 =$ Undefined
7. $\log_a -ve =$ Undefined

8. $\log_a m = \log_a n \Rightarrow m = n$

Change of Base in Logarithms

1. $\log_b a = \dfrac{\log_m a}{\log_m b}$ (m can be any common base) ($m \neq 0, 1, \pm\alpha$, -ve value)

2. $\log_a b = \dfrac{1}{\log_b a}$

3. $a^{\log_a x} = x$

Nature of Log Values

- All the values which are obtained from log tables are irrational numbers provided the numbers are not 10 or in the form of 10^n.
- $\log_b a$ is a rational quantity only when, $\dfrac{\log a}{\log b}$ is rational.
- If K is a number, then its log value, logK can be divided into two parts: a) Integral Part, b) Fractional Part.
- The integral part is called "Characteristics" and the fractional part is called "Mantissa".
- The integral characteristics part can be positive or negative or zero but not a fraction.
- The values of mantissa are always positive fractions.
- The values for mantissa are obtained from log tables.
- Characteristics are to be calculated before we evaluate mantissa from the log table.
- Value of characteristics = number of significant digits before decimal – 1

CLASSWORK SECTION

1. If $\log_{10}[98 + \sqrt{x^2 - 12x + 36}] = 2$, then $x =$
   a) 4
   b) 8
   c) 12
   d) 4, 8

2. If $\left(\frac{21}{10}\right)^x = 2$, then x = ?
   a) $\frac{\log 2}{\log 3 + \log 7 + 1}$
   b) $\frac{\log 2}{\log 3 + \log 7 - 1}$
   c) $\frac{\log 2}{\log 7 + \log 3 + 2}$
   d) None of the above

3. If $\log\left(\frac{x+y}{5}\right) = \frac{1}{2}(\log x + \log y)$, then $\frac{x}{y} + \frac{y}{x} =$
   a) 20 b) 23 c) 22 d) 21

4. If $\log_{0.5}(\log_x(\log_4 32)) = 2$, then x =
   a) 5/2 b) 625/16 c) 25/4 d) None of the above

5. $\log_2 \log_{\sqrt{2}} \log_3 81$ = ?
   a) 3 b) 2 c) 1 d) 0

6. If $\log_2 x + \log_4 x + \log_{16} x = 21/4$, these x is equal to
   (a) 8 (b) 4 (c) 16 (d) none of these

PAST YEAR QUESTIONS

7. The value of $\log(1^3 + 2^3 + 3^3 + \ldots n^3)$ is equal to:
   (a) 3 log 1 + 3 log 2 + . . . + 3 log n
   (b) 2 log n + 2 log (n + 1) − 2 log 2
   (c) log n + log (n + 1) + log (2n + 1) − log 6
   (d) 1

2 EQUATIONS

THEORY

Equations

An equation is defined as a mathematical statement of equality.

Types of Equations

a) Linear equation in one variable.
b) Linear simultaneous equations in 2 or 3 variables.
c) Quadratic equations.
d) Cubic equations.
e) Bi-quadratic equations.
f) Exponential equations.

Quadratic Equations

- A quadratic equation is defined as polynomial equation of degree 2.
- A quadratic equation can be expressed in the following general form:

$$ax^2 + bx + c = 0;\ (a \neq 0)$$

- A quadratic equation can also be expressed in the factor form as follows:

$$a(x-\alpha)(x-\beta) = 0$$

Here, α and β are the roots or solutions of quadratic equations.

- The general solution of the quadratic equation can be obtained as follows:

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\text{Sum of roots} = \alpha + \beta = -\frac{b}{a}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a}$$

Structure of Quadratic Equations

If Sum (S) ($\alpha + \beta$) and Product (P) ($\alpha\beta$) of the roots are known, then the quadratic equation is

$$x^2 - Sx + P = 0$$

Sign of Roots of a Quadratic Equation

- When c=0, one root of the equation must be 0.
- When b and c are 0, then both the roots must be 0.
- If a, b, c all are of same sign, both roots are negative.
- If a and c are of same sign, opposite to that of b, then both the roots will be positive.
- If a and c are of opposite signs, one root is positive and another root is negative.

Nature of Roots

The expression "$b^2 - 4ac$" is called the "Discriminant (D)" of the quadratic equation.

- When D > 0, Roots are real and distinct.
- When D = 0, Roots are real and equal.
- When D < 0, Roots are imaginary.
- When D $\geq$ 0, Roots are real.
- When D is a perfect square, Roots are real, rational and unequal.
- When D is not a perfect square, Roots are real, irrational and unequal.
- If roots are equal use $b^2 = 4ac$.
- If roots are reciprocal of each other, use **a = c**
- If roots are equal but of opposite sign, use **b = 0**
- If roots are reciprocal but opposite in sign, use **c = -a**

Note

- Irrational roots will always appear in conjugate pairs.

$$\alpha = \left(a - \sqrt{b}\right) \text{ and } \beta = \left(a + \sqrt{b}\right)$$

- Imaginary roots will always appear in conjugate pairs

$$\alpha = (a - ib) \text{ and } \beta = (a + ib)$$

Cubic Equations

- A cubic equation is a polynomial equation of degree 3, and the general form is represented as follows:

$$ax^3 + bx^2 + cx + d = 0; \ (a \neq 0)$$

- The factor form of a cubic equation is given as follows:

$$a(x - \alpha)(x - \beta)(x - \gamma) = 0$$

Here, α, β, and γ are the roots or solutions of the cubic equation.

- Sum of roots = $\alpha + \beta + \gamma = -b/a$
- Product of the roots = $\alpha\beta\gamma = -d/a$

Bi-Quadratic Equations

- A bi-quadratic equation is a polynomial of degree 4, and the general form is represented as follows:

$$ax^4 + bx^3 + cx^2 + dx + e = 0; \ (a \neq 0)$$

- The factor form of a cubic equation is given as follows:

$$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta) = 0$$

Here, α, β, γ and δ are the roots or solutions of the bi-quadratic equation.

- Sum of roots = $\alpha + \beta + \gamma + \delta = -b/a$
- Product of the roots = $\alpha\beta\gamma\delta = e/a$

CLASSWORK

Choose the most appropriate option (a), (b), (c) or (d).

1. Ten years ago, the age of a father was four times of his son. Ten years hence, the age of the father will be twice that of his son. The present ages of the father and the son are.

   a) (50, 20) b) (60, 20)
   c) (55, 25) d) none of these

2. y is older than x by 7 years 15 years back, x's age was 3/4 of y's age. Their present ages are:

   a) (x=36, y=43) b) (x=50, y=43)
   c) (x=43, y=50) d) (x=40, y=47)

3. The sides of an equilateral triangle are shortened by 12 units 13 units and 14 units respectively and a right angle triangle is formed. The side of the equilateral triangle is

   a) 17 units b) 16 units c) 15 units d) 18 units

PAST YEAR QUESTIONS

1. Number of students in each section of a school is 36. After admitting 12 new students, four new sections are started. If total number of students in each section now is 30, then number of section initially were

   (a) 6 (b) 10 (c) 14 (d) 18

QUADRATIC EQUATIONS

1. If '−4' is a root of the equation $x^2 + ax - 4 = 0$ and the equation $x^2 + ax + b = 0$ has equal roots, the value of 'a' & 'b' are

   (a) $a = 2, b = \frac{5}{4}$ (b) $a = 3, b = \frac{9}{4}$

   (c) $a = \quad, b = \frac{5}{2}$ (d) none

2. If the equation $x^2 - (b + 4)x + 2b + 5 = 0$ has equal roots, then the values of 'b'

   (a) -2 (b) 2 (c) ± 2 (d) ± 1

3. If $p + q + r = 0$ and p, q, r are rational nos. the roots of equation $(q + r - p)x^2 + (r + p - q)x + (p + q - r) = 0$

   (a) real and irrational (b) real & equal

   (c) imaginary (d) real & rational

4. If the sum of the roots of the quadratyic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then $\frac{b^2}{ac} + \frac{bc}{a^2}$ is equal to

   a) 2 b) -2 c) 1 d) -1

PAST YEARS QUESTIONS

1. If roots of equation $x^2 + x + r = 0$ are α and β and $\alpha^3 + \beta^3 = -6$. Find value of r

   (a) $-\frac{5}{3}$ (b) $\frac{7}{3}$ (c) $-\frac{4}{3}$ (d) 1

2. If the ratio of the root of the equation $4x^2 - 6x + p = 0$ is 1 : 2 then the value of p is

   (a) 1 (b) 2 (c) -2 (d) -1

3. If difference between the roots of the equation $x^2 - kx + 8 = 0$ is 4 then the value of k is

   (a) 0 (b) ± 4 (c) $\pm 8\sqrt{3}$ (d) $\pm 4\sqrt{3}$

CUBIC EQUATION

Choose the most appropriate option (a), (b), (c) or (d)

1. The roots of the equation $x^3+7x^2-21x-27=0$ are

   a) (-3,-9,-1) b) (3,-9,-1) d) (3,9,1) e) (-3,9,1)

CONSISTENCY OF EQUATION

2. The system of equation $4x + 7y = 10$ and $10x + (35/2)y = 25$ have

   (a) unique solution (b) infinite solution

   (c) no solution (d) none

3 LINEAR INEQUALITIES

CLASS WORK

1. A car manufacturing company manufactures cars of two types A and B. Model A requires 150 man-hours for assembling, 50 man-hours for painting and 10 man-hours for checking and testing. Model B requires 60 man-hours for assembling, 40 man-hours for painting and 20 man-hours for checking and testing. There are available 30 thousand man-hours for assembling, 13 thousand man-hours for painting and 5 thousand man-hours for testing and checking. Let the company manufacture x units of type A model of car and y units of type B model of the car. Then, the inequalities are:

   a) $5x + 2y = 1000$, $5x + 4y \leq 1300$, $x + 2y \leq 500$, $x \geq 0$, $y \geq 0$

   b) $5x + 2y \leq 1000$, $5x + 4y \leq 1300$, $x + 2y \leq 500$, $x \geq 0$, $y \geq 0$

   c) $5x + 2y \leq 1000$, $5x + 4y = 1300$, $x + 2y = 500$, $x \geq 0$, $y \geq 0$

   d) $5x + 2y \leq 1000$, $5x + 4y \geq 1300$, $x + 2y \geq 500$, $x \geq 0$, $y \geq 0$

2. A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrient constituents, it is necessary to buy two additional products, say A and B. The contents of the various products (per unit) in nutrient constituent (eg., vitamins, proteins, etc.) is given in the following table:

Nutrient	Nutrient content in product		Minimum amount of Nutrient
	A	B	
M1	36	6	108
M2	3	12	36
M3	20	10	100

The last column of the above table gives the minimum amounts of nutrients constituents M1, M2 and M3 which must be given to the pigs. Express the above situation in terms of linear inequalities.

a) $\left.\begin{array}{l} 36x+6y\geq 108 \\ 3x+12y\geq 36 \\ 20x+10y\geq 100 \\ x\geq 0, y\geq 0 \end{array}\right\}$

b) $\left.\begin{array}{l} 6x+y\geq 18 \\ x+4y\geq 12 \\ x+0.5y\geq 5 \\ x\geq 0, y\geq 0 \end{array}\right\}$

c) $\left.\begin{array}{l} 36x+6y\leq 108 \\ 3x+12y\leq 36 \\ 20x+10y\leq 100 \\ x\geq 0, y\geq 0 \end{array}\right\}$

d) (a) & (b) both

3. The rules and regulations demand that the employers should employ not more than 5 experienced hands to 1 fresh one and this fact is represented by: (Taking experienced person as x and fresh person as y)

a) $y \geq x/5$

b) $y \geq x$

c) $y \leq x/5$

d) None of the above

4. The solution of the in-equality $\frac{(5-2x)}{3} \leq \frac{x}{6} - 5$ is:

a) $x \leq 8$ b) $x = 8$ c) $x \geq 8$ d) None of the above

5. If $\left|x+\frac{1}{4}\right| > \frac{7}{4}$, then which of the following is correct?

a) $x < -\frac{3}{2}$ or $x > 2$

b) $x < -2$ or $x > \frac{3}{2}$

c) $-2 < x < \frac{3}{2}$

d) None of the above

6. A company produces two types of leather belts, say A and B. Belt A is of superior quality and belt B is of lower quality. Each belt of type A requires twice as much time as required by a belt of type B. If all belts were of type B, the company could produce 1000 belts per day. But the supply of leather is sufficient only for 800 belts per day. Belt A requires fancy buckles and only 400 fancy buckles are available per day. For belt of type B only 700 buckles are available per day.

Constraints can be formulated by assuming that the company produce x units of belt A and y units of belt B as :

(a) $2x + y \le 1000$	(b) $2x + y \le 1000$	(c) $2x + y \ge 1000$
$x + y \ge 800$	$x + y \le 800$	$x + y \le 800$
$x \le 400$; $y \le 700$	$x \le 400$; $y \le 700$	$x \le 400$; $y \le 700$
$x \ge 0$; $y \ge 0$	$x \ge 0$; $y \ge 0$	$x \ge 0$; $y \ge 0$

d) None of these

4 TIME VALUE OF MONEY

Simple Interest

Simple interest is charged on the principal amount and hence it is same for every year.

A = Amount, P = principal, n = number of years, R = interest rate

$$SI = \frac{PTR}{100}$$

$$A = P + SI = P + \frac{PTR}{100} = P\left(1 + \frac{TR}{100}\right)$$

Notes:

- If rate of interest is known, then sum of money will double itself in 100/r years.
- If number of years is known, then sum of money will double itself @ 100/n %.
- A sum of money will become "n" times in $\frac{(n-1) \times 100}{R}$ years.

Example:

In how many years a sum of money @10% p.a. SI will become (a) double, (b) triple, (c) N times.

(a) Double	(b) Triple	(c) N times
$\frac{(2-1) \times 100}{10} = 10$ years	$\frac{(3-1) \times 100}{10} = 20$ years	$\frac{(N-1) \times 100}{10} = 10(N-1)$ years

- If the sum of money becomes "n_1" times in T_1 years and "n_2" times in T_2 years, then the ratio of their times is: $\frac{T_1}{T_2} = \frac{n_1 - 1}{n_2 - 1}$.

Compound Interest

- In case of compound interest, the interest is calculated on the amount of the succeeding years, i.e., principal keeps changing every year.
- Here interest on interest is also earned, thus money grow faster when Compounding is done

- If P is the principal, n = number of years for which interest is calculated and "i" (R/100) is the rate of interest, then, the amount A after n years will be given by:

$$A=P(1+i)^n$$

- In case of depreciation by diminishing balance method (WDV), if C = Cost of the machinery, I = rate of depreciation per annum and n = effective life of the machinery, then the depreciated value D after n years is :

$$D = C\,(1 - i)^n$$

  D is also known as the scrap value of the machinery.

- Compound Interest thus would be calculated as follows:

$$CI = A - P = P\left[(1+i)^n - 1\right]$$

- Depending upon the compounding style of interest rate, the effective formula for calculating Amount would be as follows:

Half Yearly or Semi Annually	Quarterly	Monthly
$A = P\left(1+\frac{i}{2}\right)^{2n}$	$A = P\left(1+\frac{i}{4}\right)^{4n}$	$A = P\left(1+\frac{i}{12}\right)^{12n}$

- When differential interest rates are charged (i_1, i_2, i_3,... ... i_n), then:

$$A = P(1+i_1)(1+i_2)(1+i_3)\ldots\ldots\ldots(1+i_n)$$

- Relationship between CI and SI
  a) For the first year, CI = SI, i.e. for the first year difference is zero.
  b) For two years, CI – SI = Pi^2
  c) For three years, CI – SI = $Pi^2(i + 3)$

Notes:

1. A sum of money will double itself in approximately 72/r years (known as Rule 72), where r is the rate of interest per annum.

2. A sum of money will triple itself in approximately 114/r (known as Rule 114), where r is the rate of interest per annum.

3. If a sum of money becomes "n" times in "t" years, then, it will become n^m times in "mt" years.
   Example: If sum of money doubles itself in 3 years, then it will be 8 times (2^3) in 3x3 = 9 years at CI.

Concept of Effective Rate of Interest

1. When the compounding is done more than once a year, then, the net annual rate of interest is found to be slightly higher than the given annual rate of interest.

2. This new rate of interest is known as the effective rate of interest and the given annual rate is called the nominal rate of interest.

3. Effective rate of interest is denoted by E and is given by the formula:

$$E = \left\{ (1+i)^n - 1 \right\} \text{ x } 100$$

   Where "i" is rate of interest, converted monthly, quarterly, half yearly and n is the number of conversion period per annum.

4. Effective rate of interest are particularly useful in making investment decisions when various options are given with differential interest rates.

5. Amongst various investment options, we shall choose that investment option, where effective rate of interest is maximum.

Concept of Present Value

Present Value is defined as the present worth of the money that would yield an amount A after n years at a specified rate of interest i.

$$\text{If } A = P(1+i)^n$$

$$\therefore\ P = PV = \text{Principal} = \frac{A}{(1+i)^n}$$

$$or, PV = A(1+i)^{-n}$$

Annuities

- Annuity is defined as a series of payments (usually equal) which are made at regular intervals of time (usually a year).

- The period for which the payment continues is called the status or the term of the annuity.

- Unless otherwise stated, the first payment will fall due at the end of every year. This is known as "Ordinary Annuity".

- When the payment falls due at the beginning of every year, i.e., immediately, it is called "Immediate Annuity".

- When the status or term of the annuity is not fixed, i.e., the payment is to be continued for an indefinite period, these are known as "Perpetual Annuity or Perpetuity".

- Hence forth, we shall maintain the following notation throughout. The regular annual payment i.e., annuity = P, rate of interest = "i" and the period for which payment is made = n (status or term of the annuity).

- The amount of the ordinary annuity is given by:

$$A=\frac{P}{i}\left\{(1+i)^{n}-1\right\}$$

- The amount of immediate annuity is obtained by multiplying amount obtained for ordinary annuity by (1 + i); hence the formula becomes:

$$A=\frac{P}{i}\left\{(1+i)^{n}-1\right\}(1+i)$$

- Note:
  1. When half yearly or quarterly or monthly payment is "P", in such a case change "i" to i/2 or i/4 or i/12 and change "n" to 2n or 4n or 12n respectively.

  2. When half yearly, quarterly or monthly rate of interest is "i", in such a case, change P to P/2, P/4 or P/12 and change n to 2n or 4n or 12n respectively.

- The present value of an annuity payable over a period of n years is defined as the sum of the present value of all the future payments.

- The present value of an ordinary annuity is represented by V and given as follows:

$$V = \frac{P}{i}\left\{1-(1+i)^{-n}\right\}$$

- If the term of the annuity is n years, then for evaluating the present value of the immediate annuity, first calculate the present value of the annuity for (n−1) years and then add to it the initial or first payment.

$$v = \frac{P}{i}\left\{1-(1+i)^{-n}\right\}(1+i)$$

- Present value of the perpetual annuity is given by,

V = P/i

Important concepts related to CA Inter and CA Final

Financial Management

1. Sinking Fund

It is the fund credited for a specified purpose by way of sequence of periodic payments over a time period at a specified interest rate. Interest is compounded at the end of every period. Size of the sinking fund deposit is computed from A = P.A(n, i) where A is the amount to be saved, P the periodic payment, n the payment period.

2. Leasing

Leasing is a financial arrangement under which the owner of the asset (lessor) allows the user of the asset (lessee) to use the asset for a defined period of time(lease period) for a consideration (lease rental) payable over a given period of time. This is a kind of taking an asset on rent.

3. Capital Expenditure (investment decision)

Capital expenditure means purchasing an asset (which results in outflows of money) today in anticipation of benefits (cash inflow) which would flow across the life of the investment. For taking investment decision we compare the present value of cash outflow and present value of cash inflows. If present value of cash inflows is greater than present value of cash outflows decision should be in the favour of investment.

4. Valuation of Bond

A bond is a debt security in which the issuer owes the holder a debt and is obliged to repay the principal and interest. Bonds are generally issued for a fixed term longer than one year.

5. Perpetuity

Perpetuity is an annuity in which the periodic payments or receipts begin on a fixed date and continue indefinitely or perpetually. Fixed coupon payments on permanently invested (irredeemable) sums of money are prime examples of perpetuities.

The formula for evaluating perpetuity is relatively straight forward. Two points which are important to understand in this regard are:.

(a) The value of the perpetuity is finite because receipts that are anticipated far in the future have extremely low present value (today's value of the future cash flows).

(b) Additionally, because the principal is never repaid, there is no present value for the principal.

Therefore, the price of perpetuity is simply the coupon amount over the appropriate discount rate or yield.

Calculation of multi period perpetuity:

The formula for determining the present value of multi-period perpetuity is as follows:

$$PVA\infty = \frac{R}{(1+i)^1} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \ldots\ldots + \frac{R}{(1+i)} = \sum_{n=1}^{\infty} \frac{R}{(1+i)^n} = \frac{R}{i}$$

Where:

R = the payment or receipt each period

i = the interest rate per payment or receipt period

6. Calculation of Growing Perpetuity

A stream of cash flows that grows at a constant rate forever is known as growing perpetuity. The formula for determining the present value of growing perpetuity is as follows:

$$PVA = \frac{R}{(1+i)^1} + \frac{R(1+g)}{(1+i)^2} + \frac{R(1+g)^2}{(1+i)^3} + \ldots + \frac{R(1+g)^\infty}{(1+i)^\infty}$$

$$\sum_{n=1}^{\infty} \frac{R(1+g)^{n-1}}{(1+i)^n} = \frac{R}{i-g}$$

7. Net Present Value

Net present value = Present value of net cash inflow − Total net initial investment

Since it might be possible that some additional investment may also be required during the life time of the project then appropriate formula shall be:

Net present value = Present value of cash inflow − Present value of cash outflow

The steps to calculate net present value are:-

1. Determine the net cash inflow in each year of the investment.
2. Select the desired rate of return or discounting rate or Weighted Average Cost of Capital.
3. Find the discount factor for each year based on the desired rate of return selected.
4. Determine the present values of the net cash flows by multiplying the cash flows by respective the discount factors of respective period called Present Value (PV) of Cash flows
5. Total the amounts of all PVs of Cash Flows

Decision Rule:

If NPV > 0 Accept the Proposal

If NPV < 0 Reject the Proposal

8. Nominal Rate of Return

The nominal rate is the stated interest rate. If a bank pays 5% annually on a savings account, then 5% is the nominal interest rate. So if you deposit ₹ 100 for 1 year, you will receive ₹ 5 in interest.

However, that Rs. 5 will probably be worth less at the end of the year than it would have been at the beginning. This is because inflation lowers the value of money. As goods, services, and assets, such as real estate, rise in price.

The nominal interest rate is conceptually the simplest type of interest rate. It is quite simply the stated interest rate of a given bond or loan. It is also defined as a stated interest rate. This interest works according to the simple interest and does not take into account the compounding periods.

Real Rate of Return: The real interest rate is so named because it states the "real" rate that the lender or investor receives after inflation is factored in; that is, the interest rate that exceeds the inflation rate.

A comparison of real and nominal interest rates can therefore be summed up in this equation:

Nominal Rate of Return – Inflation = Real Rate of Return

Nominal Interest Rate = Real Interest Rate + Inflation

9. Compound Annual Growth Rate (CAGR)

Compound Annual Growth Rate (CAGR) is a business and investing specific term for the smoothed annualized gain of an investment over a given time periodit is not an accounting term, but remains widely used, particularly in growth industries or to compare the growth rates of two investments because CAGR dampens the effect of volatility of periodic returns that can render arithmetic means irrelevant. CAGR is often used to describe the growth over a period of time of some element of the business, for example revenue, units delivered, registered users, etc.

$$\text{CAGR}\,(t_0, t_n) = \left(\frac{V(t_n)}{V(t_0)}\right)^{\frac{1}{t_n - t_0}} - 1$$

Where $V(t_0)$ = Beginning Period ; $V(t_n)$ = End Period

CLASSWORK SECTION

SIMPLE INTEREST

1. Find rate of interest if the amount owed after 6 months is 2100, borrowed amount being Rs. 2000.
   (a) 10% (b) 8% (c) 9% d 11%

2. 46875 was lent out at SI and at the end of 1 yr 8 months, total amount was 50000. Find rate of int per annum?
   (a) 2% (b) 4% (c) 6% (d) 8%

3. Sum required to earn quarterly interest of 3600 at 18% p.a. is
   (a) 50,000 (b) 60,000 (c) 80,000 (d) none

4. A sum of 3402 amounts to 6804 on 20 yrs. What sum will amount to 5200 in 6 yrs at same rate?
   (a) 3000 (b) 4000 (c) 5000 (d) 600

5. A bike is purchased by making a down payment of 15000 and balance to be paid alongwith interest at 5% p.a. for 2 yrs. Total amount paid is 28200. Find cash price of the bike.
   (a) 28000 (b) 26000 (c) 27000 (d) 25000

PAST EXAM QUESTIONS

6. The rate of simple interest on a sum of money is 6% p.a. for first 3 years, 8% p.a. for the next five years and 10% p.a. for the period beyond 8 years. If the simple interest accrued by the sum for a period for 10 years is ₹ 1,560. The sum is:
   (a) ₹ 1,500 (b) ₹ 2,000 (c) ₹ 3,000 (d) ₹ 5,000

7. A sum of money doubles itself in 10 years. The number of years it would treble itself is :
   (a) 25 years (b) 15 years (c) 20 years (d) none

8. If a simple interest on a sum of money at 6% p.a. for 7 years is equal to twice of simple interest on another sum for 9 years at 5% p.a. The ratio will be

(a) 2 : 15 (b) 7 : 15 (c) 15 : 7 (d) 1 : 7

9. A sum of ₹ 44,000 is divided into three parts such that the corresponding interest earned after 2 years, 3 years and 6 years may be equal. If the rates of simple interest are 6% p.a., 8% p.a. and 6% p.a. respectively, then the smallest part of the sum will be :

(a) ₹ 4,000 (b) ₹ 8,000 (c) ₹ 10,000 (d) ₹ 12,000

COMPOUND INTEREST

10. Find present value of 10000 due in 2 yrs at 5% p.a. compound interest paid annually?

(a) 9050 (b) 9070 (c) 9080 (d) 9090

11. A machinery worth 10000 is depreciated at the rate of 10% p.a. for first 3 yrs. 8% p.a. for next 2 yrs. Find its value after 5 yrs.

(a) 5170.25 (b) 7170.25 (c) 6170.25 (d) 8170.25

PAST EXAM QUESTIONS

12. The difference between the simple and compound interest on a certain sum for 3 year at 5% p.a. is ₹ 228.75. The compound interest on the sum for 2 years at 5% p.a. is

(a) ₹ 3,175 (b) ₹ 3,075 (c) ₹ 3,275 (d) ₹ 2,975

13. A person deposited ₹ 5000 in a bank. The deposit was left to accumulate at 6% compounded quarterly for the first five years and at 8% compounded semi-annually for the next eight years. The compound amount at the end of 13 years is:

(a) ₹ 12621.50 (b) ₹ 12613.10 (c) ₹ 13613.10 (d) none

14. If compound interest on a sum for 2 years at 4% per annum is ₹ 102, then the simple interest on the same sum for the same period at the same rate will be

(a) ₹ 99 (b) ₹ 101 (c) ₹ 100 (d) ₹ 95

EFFECTIVE RATE OF INTEREST

15. Which is a better investment?
    (i) 9% p.a. compounded half yrly.
    (ii) 9.23% p.a. S.I.
    (a) (i) (b) (ii) (c) both (d) none

ANNUITY (FUTURE VALUE)

16. A company issued 10% cumulative debentures of Rs. 100 each, 5000 cumulative debentures are to be redeemed with 10% of interest for 5 yrs. For this a Sinking Fund is created and invested at 12% rate of C.I. Sum to be transferred every year to sinking fund is
    (a) 805500 (b) 126834.64 (c) 207382 (d) 126755

17. A machine costing 5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by new model at 25% higher cost after 25 years with a scrap value realization of 25000. What amount should be set aside every year if sinking fund investment at 3.5% C.I p.a.?
    (a) 16000 (b) 16500 (c) 16050 (d) 16005

ANNUITY (PRESENT VALUE)

18. Present value of an annuity which pays 200 at the end of each 3 months for 10 years, assuming money to be worth 5% p.a. converted quarterly.
    (a) 3809.20 (b) 3109.60 (c) 6265.38 (d) none

19. A man purchased house valued at 3,00,000 by making a payment of 2,00,000 at the time of purchase and agreed to pay balance with interest at 12% p.a. compounded half yearly in 20 equal half yearly installments. If first installment is paid after 6 months from the date of purchase then amount of each installment is
    (a) 8719 (b) 8679 (c) 7719 (d) 8769

PAST YEARS QUESTION

20. A company considering proposal of purchasing a machine either by, making full payment of ₹4000 or by leasing it for four years at an annual rate of ₹1,250. Which course of action is preferable, if the company can borrow money at 14% compounded annually?

[Given: $(1.14)^4 = 1.68896$]

(a) Leasing is preferable (b) Should be Purchased

(c) No difference (d) None of these

Typical Sums related to Important Concepts

21. If the cost of capital be 12% per annum, then the net present value (in nearest *) from the given cash flow is given as

Year	0	1	2	3
Operating profit (in lakh *)	(100)	60	40	50

(A) 31048 (B) 34185 (C) 21048 (D) 24187

5 PERMUTATION AND COMBINATION

THEORY

Permutation

- Permutation is defined as the arrangement of things by taking some or all at a time
- Permutation is order dependent
- Fundamental principle of counting;

  If one operation can be performed in 'm' ways and another operation can be performed in 'n' ways, then the total number of ways in which both the operation can be performed will be given by 'm n' ways
- Definition of Factorial 'n', i.e., n! or $\lfloor n$

  Factorial n (n!) is defined as the continued product of first n natural numbers or first n positivel integers and is expressed as n! = 1 x 2 x 3 x 4 ... . x n

- $\lfloor n = n \times \lfloor n-1 = n \times (n-1) \lfloor n-2 = \ldots\ldots$
-

1!	= 1	6!	= 720
2!	= 2	7!	= 5040
3!	= 6	8!	= 40320
4!	= 24	9!	= 362880
5!	= 120	10!	= 3628800

- Mathematical definition of Permutation (Repetition not allowed):

  Total number of arrangements of 'n' different things taking "r" at a time will be given by

  nPr or P (n, r) = $\frac{\lfloor n}{\lfloor n-r}$, where r $\leq$ n.

Note:

- When r = n, it is known as "all at a time"
- When r < n, it is known as "some at a time"
- r can never exceed n
- n and r must be positive integers

- ${}^{n}P_{1} = n$
- ${}^{n}P_{0} = 1$
- ${}^{n}P_{n} = n!$
- ${}^{n}P_{n} = {}^{n}P_{(n-1)} = n!$
- But ${}^{n}P_{r} \neq {}^{n}P_{(r-1)}$

- **Permutation or arrangements of 'n' different things in which few are alike (Repetition not allowed)**

  The total number of arrangements of n different things in which p are alike and of one kind, q are alike and a second kind, r are alike and yet of another kind and the rest are different, will be given by $\frac{\lfloor n}{\lfloor p \lfloor q \lfloor r}$

- **Permutation when repetitions are allowed**

  The total number of arrangements of n things taken r at a time when each thing may be repeated once, twice, thrice, ...to r number of times will be given by n^r

- **Rules for restricted Permutation**

a) Whenever the arrangements should begin or end or begin and end with a particular letter or object keep the objects fixed at the respective places and arrange the rest.

b) When in the arrangement of n things, r things are together, the total arrangements will be given by: $(n - r + 1)!\, r!$

c) When in the arrangements of n things, r things are together in a specified order, the total arrangement will be given by $(n - r + 1)!$

d) Total number of ways in which out of n things, r things are never together = total ways – number of ways when they are always together, i.e., $n! - (n - r + 1)!\, r!$

e) When the relative positions of few objects are to be kept unaltered it implies that the objects can be interchanged or arranged in their respective place only.

f) In problems involving re-arrangements always subtract 1 from the total arrangements.

g) When in the arrangement of n things, r alike things are together, then total number of arrangements will be given by $(n - r + 1)!$

Circular Permutation (When the things are arranged in a ring or circle)

- Total ways in which n things can be arranged in a ring or circle is $\lfloor n-1$

- Total ways in which n things can be arranged in a ring or circle with respect to any object will be given by $\lfloor n$

- When the clockwise or anti clock wise position cannot be disguised (for example: arrangements of different flowers in garland or arrangement of different beads in a necklace etc), in such a case the total number of circular arrangements will be given by (n – 1)! / 2

Arrangements of digits

- There are 10 random digits: 0, 1, 2, 3, 4, 5, 6, 7 ,8, 9

- 5 odd digits (1, 3, 5, 7, 9) and

- 5 even digits (0, 2, 4, 6, 8)

- Unless otherwise mentioned no number can start with '0'

- If there are 'n' different digits (0 is included) then the total number of n digit numbers not beginning with 0 will be given by: $\lfloor n$ - $\lfloor n-1$

- If there are n different digits (0 is included) and we are to form a number with r different digits then the total number of r digit numbers not beginning with 0 will be given by: ${}^{n}P_{r}$ - ${}^{n-1}P_{r-1}$

CLASSWORK SECTION

Fundamental Principle of Counting

1. There are 26 stations on a railway line. How many different kinds of tickets of class II must be printed in order that a passenger may go from any one station to another by purchasing a ticket.

   a) 65 b) 240 c) 650 d) 1300

Formula Pattern - ^nP_r

2. $^nP_3 : {}^nP_2 = 3:1$, then the value of n is:

   a) 4 b) 5 c) 6 d) 7

Alike Items – Repetition not allowed

3. INSTITUTION

   a) 554499 b) 445588 c) 554400 d) None of the above

In how many ways the results of:

4. In how many ways can 3 persons enter into 4 hotels if (i) repetition is allowed, and (ii) repetition is not allowed?

   a) $3^4, 3^4$ b) 4^3,P(4,3)

   c) 3^4,P(4,3) d) None of the above

Restricted Permutation

5. How many words can be formed of the letters in the word COSTING, the vowels being not separated?

   a) 144 b) 1440 c) 1280 d) 2880

All different words formed by the letters of the word BHARAT:

6. In how many different ways can the letters of the word “CONSTITUTION” be arranged? How many of these will have the letter N both at the beginning and at the end?

a) 9979200, 151200 b) 9989920, 152150
c) 9979000, 151000 d) None of the above

Circular Permutation

7. In how many ways can 7 persons be arranged at a round table so that 2 particular persons can be together?

a) 180 b) 240 c) 360 d) None of the above

8. In how many ways 8 stones of different colours be arranged on a ring? In how many of these arrangements red and yellow beads being separated?

a) 2520, 900 b) 2520, 1800
c) 1800, 2520 d) 1800, 1260

Problem Involving Digits

9. How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1, so that odd digits are at odd places?

a) 18 b) 19 c) 20 d) None

10. How many four digits numbers can be formed with the digits 3, 4, 5, 6? Find the sum of all the numbers thus formed.

a) 24, 1420 b) 24, 1520 c) 24, 4742 d) 24, 119988

11. How many even numbers greater than 300 can be formed with the digits 1, 2, 3, 4 and 5 (no digit being repeated)?

a) 121 b) 111 c) 222 d) 124

Miscellaneous

12. How many ways can 3 boys and 5 girls be arranged in row so that no 2 boys are together?

a) 14400 b) 604800 c) 2880 d) 28800

13. One arrange 10 letters taken 7 at a time. In how many of these 3 particular letters (i) always occur, and (ii) never occurs?

a) 720, 30240 b) 30240, 5040

c) 5040, 5040 d) 176400, 5040

14. Find rank of word 'ZENITH'.

(a) 613 (b) 614 (c) 615 (d) 616

COMBINATION

- Combination is the selection of different items from a given number of items
- Combination is order independent
- The total number of combinations or selections of r items from n different items will be given by;

$$^nC_r \text{ or } C(n, r) = \frac{\lfloor n}{\lfloor r \times \lfloor n-r} \quad \text{where } r \leq n$$

- No arrangement (Permutation) is possible without selection (Combination) but selection (Combination) process can take place independently
- Thus $^nC_r < {}^nP_r$, except when r=0 or 1
- Relation between ^nP_r and ^nC_r
  - $^nP_r = {}^nC_r \times \lfloor r$
  - $\frac{^nP_r}{^nC_r} = \lfloor r$
  - $^nC_0 = {}^nP_0 = 1$
  - $^nC_1 = {}^nP_1 = n$
  - $^nC_n = 1,\ {}^nP_n = \lfloor n$

Complementary Combination

- $^nC_r = {}^nC_{n-r}$ (Use this result, when $r > \frac{n}{2}$)
- If $^nC_x = {}^nC_y$ then either
  a. x = y or
  b. x+y=n or
  c. both the results can hold true simultaneously
- $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

- $$\frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}$$

Application of combinations

Case 1:

Total number of straight lines that can be formed out of 'n' points on a plane when no three of them are co-linear will be given by

$$^nC_2$$

Case 2:

Total number of triangles that can be formed out of these 'n' points is

$$^nC_3$$

Case 3:

Total Number of circles that can be formed out of these 'n' points is

$$^nC_3$$

Case 4:

Total number of lines that can be formed with 'n' points when p of them are collinear will be given

$$^nC_2 - {^pC_2} + 1$$

Case 5:

Total number of triangles that can be formed with 'n' points when p of them are collinear will be given by

$$^nC_3 - {^PC_3}$$

Case 6:

The total number of points of intersection that can be obtained from 'n' straight lines are $^nC_2$ when,

i. No two of them are parallel and

ii. No three of them are concurrent

Case 7:

To find the number of diagonals in a polygon having 'n' sides

No of diagonals = $^nC_2 - n$

Where $^nC_2$ = total number of lines by joining 2 vertices in pairs and 'n' number of sides = number of vertices

$$^{n}C_{2}-n$$

$$=\frac{n(n-1)}{2}-n=n\left(\frac{n-1}{2}-1\right)\Rightarrow n\left(\frac{n-1-2}{2}\right)\Rightarrow n\left(\frac{n-3}{2}\right)$$

Case 8:

Total number of selections or combinations of 'n' different things taking one or more at a time (i.e., at least 1) will be given by

$$^{n}C_{1}+^{n}C_{2}+^{n}C_{3}+\ldots\ldots\ldots+^{n}C_{n}=2^{n}-1$$

Case 9:

Combinations or selections of things which are alike.

Total number of combinations or selection of p, q, r items by taking one or more(atleast one) will be given by, (p+1)(q+1)(r+1)-1

When p are alike and of one kind, q are alike and of a second kind and r are alike and of yet of another kind.

Note:

Total number of selections of p alike, q alike and r different items by taking at least one will be given by $(p+1)(q+1)2^{r}-1$

Case 10: Division into groups

- The total number of ways in which (m+n) items can be divided into two distinct groups containing m & n items respectively will be given by:

$$\frac{\lfloor m+n}{\lfloor m \lfloor n}$$

- Total ways in which m+n+p items can be divided into 3 distinct groups containing m, n & p items respectively will be given by,

$$\frac{\lfloor m+n+p}{\lfloor m \lfloor n \lfloor p}$$

Case i :

When m = n or m=n=p then 2m or 3m items can be equally distributed into two or three distinct groups in,

$$\frac{\lfloor 2m}{(\lfloor m)^2} \text{ or } \frac{\lfloor 3m}{(\lfloor m)^3} \text{ ways}$$

Case ii:

When the identities of the groups are not distinct i.e, the groups are alike in such a case 2m or 3m items can be distributed equally into 2 or 3 identified groups in

$$\frac{\lfloor 2m}{(\lfloor m)^2} \times \frac{1}{2!} \text{ or } \frac{\lfloor 3m}{(\lfloor m)^3} \times \frac{1}{3!} \text{ ways}$$

CLASSWORK SECTION

Basic Meaning

1. If ${}^{13}C_6 + 2.\ {}^{13}C_5 + {}^{13}C_4 = {}^{15}C_x$, what is the value of X?

   a) 6 b) 9
   c) Either a) or b) d) Both a) and b)

2. If $\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{1}{4}$ and $\frac{{}^{n}C_{r}}{{}^{n}C_{r+1}} = \frac{1}{3}$ then find the value of n and r?

   a) 35,7 b) 53,8 c) 35,8 d) 19,4

"n" different things, "r" to be selected ($r \leq n$) – With Restrictions

3. The question paper on Mathematics and Statistics contains 10 questions divided into two groups of 5 questions each. In how many ways can an examinee select 6 questions taking at least two questions from each group?

   a. 200 b. 150 c. 100 d. 250

"n" different things, any number can be selected at a time

4. In how many ways a man can invite 5 friends to a dinner so that two or more of them remain present?

   a. 24 b. 25 c. 26 d. 32

A man has 5 German, 4 Spanish and 3 French friends. Find:

5. A Supreme Court Bench consists of five judges. In how many ways the bench can give a decision in majority?

   a. 16 b. 15 c. 31 d. 32

Application of Combination in Geometry

6. Find the number of straight lines formed by joining 10 different points on a plane, no three of them being collinear (with the exception of 4 points which are collinear).

   a. 41 b. 45 c. 39 d. 40

7. Find the number of triangles formed by joining 10 different points on a plane, no three of them being collinear (with the exception of 4 points which are collinear).

   a. 120 b. 116 c. 121 d. 126

8. A polygon has 44 diagonals. Find the number of its sides.

   a. 10 b. 11 c. 12 d. 14

How many selections can be made by taking any letters from the words

9. A person has in his bag 14 notes of Rs. 10 each, 9 notes of Rs. 5 each, 4 notes of Rs. 2 each and 7 notes of Re. 1 each. In how many different ways can he contribute to a charitable fund?

   a) 3000 b) 6000 c) 5999 d) 2999

Division into Groups – either distinct or alike

10. Divide 12 items in two groups so that each containing 8 and 4 items.

    a) $\frac{12!}{8!}$ b) $\frac{12!}{4!8!}$

    c) $\frac{8!4!}{12!}$ d) None of the above

Mixed Bag

11. In how many ways can the letters of the word FORECAST taken 3 at a time and the word MILKY taken 2 at a time be arranged?

    a) 62700 b) 67000 c) 68720 d) 67200

12. How many different factors can 2160 have?

    a) 40 b) 39 c) 37 d) 45

6 SEQUENCE AND SERIES

THEORY

- A sequence is defined as an array of numbers in such a manner so that there is a similarity in a given array, which enables us to determine the term or terms preceding or succeeding to such an array.

- A sequence can be categorized into 3 parts:
  a) Arithmetic Progression
  b) Geometric Progression
  c) Harmonic Progression

	Arithmetic Progression	Geometric Progression
Definition	Series which increases or decreases by a fixed quantity	Series which increases or decreases by a fixed proportion
First Term	a	a
Constant	Common Difference = d	Common Ratio = r
Last Term	$l = t_n = a + (n-1)d$	$l = t_n = a.r^{n-1}$
Sum	$S_n = \frac{n}{2}[2a+(n-1)d]$ $S_n = \frac{n}{2}(a+l)$	$S_n = a.\frac{1-r^n}{1-r}$ when $r<1$ $S_n = a.\frac{r^n-1}{r-1}$ when $r>1$

- If three numbers are in G.P., their Logarithms are always in A.P.

Infinite GP Series

$$a + ar + ar^2 + ar^3 + ...\alpha = \frac{a}{1-r} \quad \text{given } |r| < 1$$

Sum of Natural Numbers:

$$\sum n = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$

$$\sum n^2 = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6} n(n+1)(2n+1)$$

$$\sum n^3 = 1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}$$

Harmonic Progression(H.P)

- Three numbers are in H.P, If their reciprocals are in A.P
- a,b,c are in H.P , if $\frac{1}{a}$ $\frac{1}{b}$ $\frac{1}{c}$ are in A.P.
- H.P fails when one of the terms of the A. P is Zero.

$$t_n \text{ of HP} = \frac{1}{t_n \text{ of the crresponding A.P}}$$

Concept of A.M , G.M and H.M

If a & b are any unequal real positive numbers then,

	A.M(A)	G.M(G)	H.M(H)
Definition	$\frac{a+b}{2}$	$+\sqrt{ab}$	$\frac{2ab}{a+b}$
Relation	i) A > ii) $A \times H$	G > = G^2	H

Things to remember

- **The ratio of the sum of X number of A.Ms to the sum of Y number of A.Ms is always X : Y**
- **Two numbers can have more than one A.M/G.M/H.M**
- **A.Ms/G.Ms/ H.Ms are also the members of A.P/G.P/ H.P**

CLASSWORK SECTION

ARITHMETIC PROGRESSION

Choose the most appropriate option (a), (b), (c) or (d).

1. The a^{th} term of an AP is b and b^{th} term is a. Then c^{th} term of it is
   (a) $a + b + c$ (b) $b + a - 2c$
   (c) $a + b + c/2$ (d) $a + b - c$

2. Third term of an AP is 8 and the 17th term is 51/2. The 23rd term is
   (a) 37 (b) 33 (c) 41 (d) 31

3. The n^{th} term of the series whose sum to n terms $3n^2 + 2n$ is
   (a) $3n - 1$ (b) $8n - 2$ (c) $11n - 3$ (d) none of these

4. The sum of all numbers between 400 and 900 which are divisible by 13 is
   (a) 22504 (b) 29405 (c) 25402 (d) 25350

5. The 4 arithmetic means between – 2 and 23 are
   (a) 3, 13, 8, 18 (b) 18, 3, 8, 13
   (c) 3, 8, 13, 18 (d) none of these

6. The r^{th} term of AP is $(3r - 1)/6$. The sum of first p terms of the series is
   (a) $n(3p + 1)$ (b) $(p/12)(3p + 1)$
   (c) $(p/12)(3p - 1)$ (d) none of these

PAST YEARS QUESTIONS

7. On 1st January every year a person buys national saving certificates of value exceeding that of his last years purchase by Rs. 100. After 10 years he finds that the total value of the certificates purchased by him is Rs. 54500. Find the value of certificates purchased by him in the first year
   (a) 6000 (b) 4000 (c) 5000 (d) 5500

8. If in an AP, Tn represent nth term $t_7 : t_{10} = 5 : 7$ then $t_8 : t_{11}$ = _____
(a) 13 : 16 (b) 17 : 23 (c) 14 : 17 (d) 15 : 19

9. If sum of 3 arithmetic means between 'a' and 22 is 42 then 'a' = ____
(a) 14 (b) 11 (c) 10 (d) 6

GEOMETRIC PROGRESSION

10. If x, y, z are in GP, and xyz = 27/8. The value of y is
(a) 3/2 (b) 2/3 (c) 2/5 (d) none of these

11. A ball is dropped from a height of 48 m and rebounds two third of the distance it falls. It continued to fall and rebound in this way, how far will it travel before coming to rest
(a) 240 m (b) 260 m (c) 380 m (d) none

12. If x, y, z are pth, qth and rth terms of a GP then the value of $x^{q-r} y^{r-p} z^{p-q}$ is
(a) 0 (b) 1 (c) –1 (d) none of these

13. If the pth term of the series 16, 8, 4, . . . is $\frac{1}{2^{17}}$. The value of p is
(a) 25 (b) 22 (c) 23 (d) none of these

14. Given x, y, z are in GP, $x^p = y^q = z^\sigma$ then $\frac{1}{p}, \frac{1}{q}, \frac{1}{\sigma}$ are in
(a) AP (b) GP
(c) Both AP and GP (d) none of these

15. The value of S = 2/3 + 5/9 + 2/27 + 5/81 + ... to infinite terms is
(a) 11/8 (b) 8/11 (c) 3/11 (d) none of these

PAST YEARS QUESTIONS

16. If G be geometric mean between a and b then the value of $\frac{1}{G^2 - a^2} + \frac{1}{G^2 - b^2}$ is equal to
(a) G^2 (b) $3G^2$ (c) $1/G^2$ (d) $2/G^2$

17. A GP (Geometric Progression) consists of 2n terms. If the sum of the terms occupying the odd places is S_1 and that of the terms in even places is S_2. The common ratio of the progression is

(a) n (b) $2S_1$ (c) $\frac{S_2}{S_1}$ (d) $\frac{S_1}{S_2}$

SPECIAL SERIES

18. Sum of 'n' terms whose t_n is $n^2 + 2^n$

(a) $\frac{n(n+1)(2n+1)}{6} + 2(2^n - 1)$ (b) $\frac{(n+1)(2n+1)}{6} + 2(2^n - 1)$

(c) $\frac{n(n+1)^2}{6} + 2(2^n - 1)$ (d) None

MIXED BAG

19. If the sum of p terms of an AP is same as the sum of its q terms, then the sum of the first (p + q) terms is:

a) 0 b) p + q c) p – q d) None of the above

20. 300 trees are planted in a regular pattern in rows in the shape of an isosceles triangle, the numbers in the successive rows diminishing by one from the base to the apex. How many trees are there in the row, which forms the base of the triangle

a) 30 b) 21 c) 27 d) 24

21. If the sums of n, 2n and 3n terms of an AP be S_1, S_2 and S_3 respectively, then show that S_3 = ?

a) $3(S_2 - S_1)$ b) $(S_2 - S_1)$ c) $2(S_2 - S_1)$ d) $3(S_2 + S_1)$

22. $31^3 + 32^3 + 33^3 + ... + 50^3$

a) 2010000 b) 3025000 c) 2870000 d) 1409400

23. The sum of the first three terms of a G.P. is to the sum of the first six terms as 125:152. Find the common ratio of the G.P.

a. 0.40 b. 0.50 c. 0.75 d. 0.60

24. The first, tenth and twenty-eighth term of an AP are three successive terms of a GP. Find the common ratio of the GP. given that the sum of the first 28 terms of the AP is 210, find its first term.

a. 2, 2 b. 2, 3 c. 3, 2 d. − 3, 2

25. If a^2, b^2, c^2 are in AP, the $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in :

a. Geometric Progression
b. Arithmetic Progression
c. Both a) and b) above
d. None of the above

26. If a, b, c are in GP and x, y be the arithmetic means between a, b and b, c respectively, then which of the following/s is/are true?

(a) $\frac{a}{x}+\frac{c}{y}=2$
(b) $\frac{1}{x}+\frac{1}{y}=\frac{2}{b}$
(c) Both a) and b) above
(d) Neither a) nor b) is true

7A SET THEORY RELATION AND FUNCTIONS

SET THEORY RELATIONS

STANDARD NOTATIONS

1) $\cup$ $\Rightarrow$ OR (Union)
2) $\cap$ $\Rightarrow$ and (Intersection)
3) $\Rightarrow$ $\Rightarrow$ Implies
4) $\in$ $\Rightarrow$ belongs to
5) $\notin$ $\Rightarrow$ does not belong to
6) $\forall$ x $\Rightarrow$ for all x
7) : $\Rightarrow$ such that
8) / $\Rightarrow$ such that
9) $\subset$ $\Rightarrow$ Subset OR Proper Subset.
11) $\not\subset$ $\Rightarrow$ (not a proper subset)
12) $\supset$ $\Rightarrow$ (Superset)
13) ~ $\Rightarrow$ (Difference)
14) $\varnothing$ or { } $\Rightarrow$ (nullset)
15) U or S $\Rightarrow$ (Universal set)

2. SET THEORY (Concepts)

1. A set is a collection of well-defined and distinct object. The objects are called the elements of the set.

2. Sets are denoted by A, B, C, D etc and the elements are kept within brackets.

   e.g A = {a, b, c, d}

   A = {1, 2, 3, 4}

3. METHOD OF DESIGNATING A SET

i. ROSTER METHOD / TABULAR METHOD / ENUMERATION METHOD

ii. PROPERTY METHOD / SELECTOR METHOD / RULE METHOD/SET BUILDER NOTATION.

1) Under Roster or Enumeration method the set is defined by listing all the elements.

   e.g A = {a, e, i, o, u}

2) Under Property Method the sets are indicated by their common characteristics which an object must possess in order to its elements.

   e.g. A = { x : x is a vowel}

TYPES OF SETS

1) A set is said to be finite when the elements can be exhausted by counting.

   A = {4,5,6}

2) A set is said to be infinite when its elements can not be exhausted by counting.

   Eg. A = {1, 2, 3..........}

3) SINGLETON SET : A set which has only 1 element is called Singleton set

   e.g A = {2}

3. A FEW STANDARD INFINITE SETS

1. I^+ = Sets of Positive integers = N = Set of natural numbers

   = { 1, 2, 3.....}

2. W = Set of whole nos.

   = {0, 1, 2.....}

3. I^- = Sets of Negative integers

   = {-1, -2, -3.....}

4. I = Set of Integers

   = {0, ±1, ±2, ±3.........}

5. Q = Sets of Rational nos.

6. R = Set of real nos

• NULL SET / EMPTY SET / VOID SET

  It is a set having no element in it. It is denoted by $\varnothing$ or { }

  A = {x : x is a real no. whose square is negative}

4. EQUAL SETS

Two sets are said to be equal if all the elements of A belong to B and all the elements of B belong to A

A = { S, T, R, A, N, D}

B = { S, T, A, N, D, A, R, D}

Note : Order of arrangement or repetition of elements does not affect the property of equality.

5. EQUIVALENT SETS

If the total no. of elements of one set is equal to the total no. of elements of another set, then the two sets are said to be equivalent. The elements may or may not be same always.

A = {1, 2, 3, 4}

B = {b, l, u, e}

$A \equiv B$

6. SUB SET

If each element of set A is an element of set B, then A is said to be a subset of B or A is contained in B or B is the Superset of A.

Symbolically, $A \subseteq B$

If a set has n elements than the number of subset are 2^n .

e.g. If A = {1, 2, 3}

then the subsets of A are $\varnothing$, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}

Therefore the total number of subsets are $2^3 = 8$

Note 1. : If a set has n elements then

i. TOTAL NUMBER OF SUBSETS = 2^n

ii. TOTAL NUMBER OF NON- EMPTY SUBSETS = $2^n - 1$

iii. TOTAL NUMBER OF PROPER SUBSETS = $2^n - 1$

iv. TOTAL NUMBER OF NON- EMPTY PROPER SUBSETS = $2^n - 2$

Note 2. : i. Every set is a subset of itself

ii. Φ is a subset of every set

iii. In subset element may be equal

iv. If $A \subseteq B$ and $B \subseteq A$ $A = B$

7. PROPER SUB SET

If each element of set A is an element of set B but there is atleast 1 element in B which is not in A, in such a case A is said to be proper subset of B and is symbolically denoted by :

$A \subset B$: for example, A={1,2,3}

To the above e. g. the proper subsets of A are {1}, {2}, {3}, {1,2}, {1,3}, {2,3} & $\varnothing$

{1,2,3} is the improper subset because all the element are equal.

8. UNIVERSAL SET (U \ S)

Universal set or the universe is the set which contains all the elements under investigation in a particular content.

Eg. U = {1, 2, 3, 4, 5}

A = {2, 3}

B = {1, 3, 5}

C = {4, 5}, etc

Here A, B, C are all subsets of U.

9. POWER SET

It is defined as the set of all possible subsets in a particular investigations. If a set contains n elements, its power set will contain 2^n elements.

A = {2, 3, 4} Total elements in the Power set will be $2^3 = 8$

[there are 3 elements in set A]

P(A) = {∅, {2}, {3}, {4}, {2,3}, {2, 4}, {3, 4}, {2,3,4}}

e.g. The power set of A contains 128 elements. Find the no. of elements in set A Let there be n elements in Set A

$\therefore 2^n = 128$

Or $2^n = 2^7$

Or n = 7 ∴ Set A has 7 elements

10. CARDINAL NO. IN A SET: n(A)

If a set A contains "X" no. of elements, then the cardinal no. in set A will be given by:

n(A) = x.

e.g. A {2, 3, 4, 5}

n(A) = 4

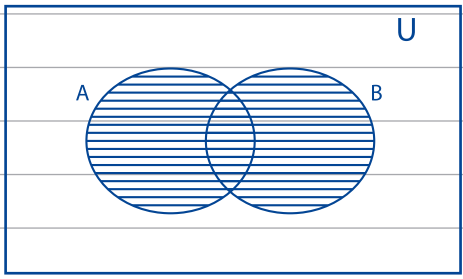
SET OPERATIONS

1. UNION OR JOIN OF 2 SETS

If A & B are 2 sets then the Union or Join of 2 sets is defined as, the set of all elements which belong either to A or to B or to both A & B.

Symbolically $A \cup B = \{ x : x \in A \text{ or } x \in B \}$

NOTE : Here 'UNION' ⇒ or



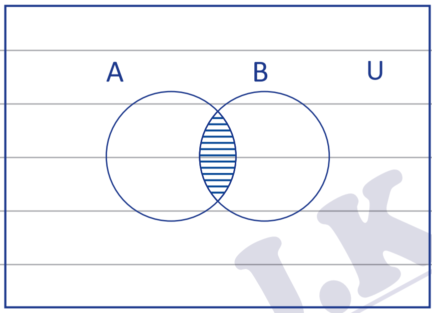
e.g

A = {1, 2, 3, 4, 5} : B = {2, 3, 5, 6, 7}

$A \cup B$ = {1,2,3,4,5,6,7}

2. INTERSECTION OF 2 SETS

If A & B are 2 sets, then the intersection of the sets A & B is the set of those elements which belong to both A & B and is denoted by $A \cap B$.



Symbolically,

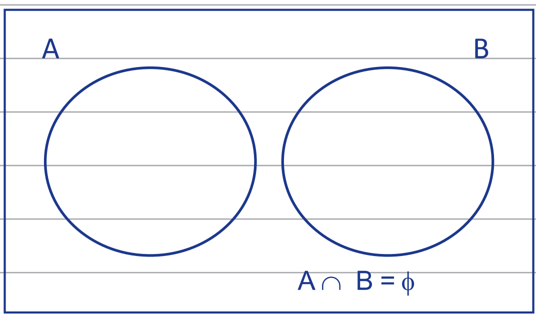
$A \cap B = \{ x : x \in A \text{ and } x \in B\}$

A = { 1, 2, 3, 4}   B = (3, 4, 5}

$A \cap B$ = { 3, 4}

3. DISJOINT SETS

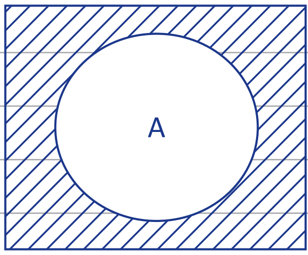
2 Sets are said to be disjoint when they have no elements in common i.e. their intersection is a Null Set.



e.g. If $A = \{1, 3, 5,\}$ $B = \{2, 4\}$ then $A \cap B = \phi$

therefore A & B are disjoints sets.

4. COMPLEMENT OR NEGATION SET



If U be the universal set and A be its subset, then the complement of set A in relation to U is the set whose element belong to U and not to A. This is denoted by :

$\hat{A}$ or A' or $A^c = (U - A)$

therefore $A^c = \{x : x \in U \text{ and } x \notin A\}$

e.g.

$U = \{1, 2, 3, 4, ...10\}$

$A = \{2, 3, 5, 7\}$

$B = \{1, 2, 9, 10\}$

$A^C = \{1, 4, 6, 8, 9, 10\}$

$B^C = (4, 3, 5, 7, 6, 8\}$

5. DIFFERENCE OF 2 SETS

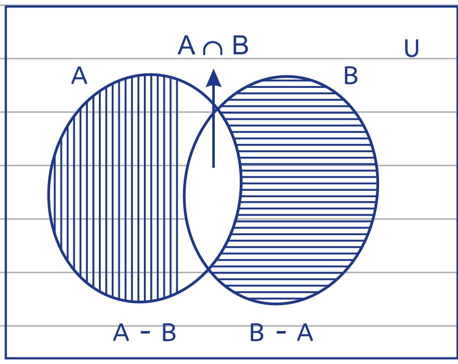
$A - B = \{x : x \in A \text{ and } x \notin B\}$ Or

A ~ B

$B - A = \{x : x \in B \text{ and } x \notin A\}$

Or

B ~ A



e. g. $A = \{1, 2, 3, 4, 5\}$

$B = \{3, 5, 6, 7\}$

$A - B = \{1, 2, 4\}$ and $B - A = \{6, 7\}$

6. CARTESIAN PRODUCT OF 2 SETS

If A and B are 2 sets, then the set of all ordered pairs (x, y) such that $x \in A$ and $y \in B$ is called Cartesian Product of A & B and it is denoted by A x B (read an A "cross" B)

Symbolically, $A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$

$A = \{1,2\}$ $B = (3, 4, 7\}$

$A \times B = \{(1, 3), (1, 4), (1, 7), (2,3), (2, 4), (2, 7)\}$

B x A = {(3, 1), (3, 2), (4, 1), (4,2), (7, 1), (7, 2)}

A x B $\neq$ B x A but A X B $\cong$ B x A since n (A x B)

= n (B x A)

Note : 1. If n(A) = m and n(B) = n then the total number of elements in A x B = m x n

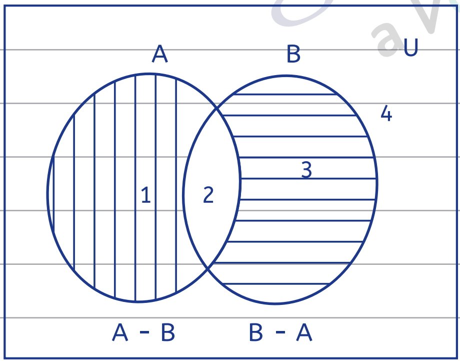
2. The total number of subsets of A x B = 2^{mn}

Notes :

1. $\phi' = U$
2. $U' = \phi$
3. $(A^c)^c = A$
4. $A \cup A' = U$
5. $A \cap A' = \phi$
6. $A \subset B$ then $B' \subset A'$
7. $A \cup \phi = A$
8. $A \cap \phi = \phi$
9. $A \cup U = U$
10. $A \cap U = A$

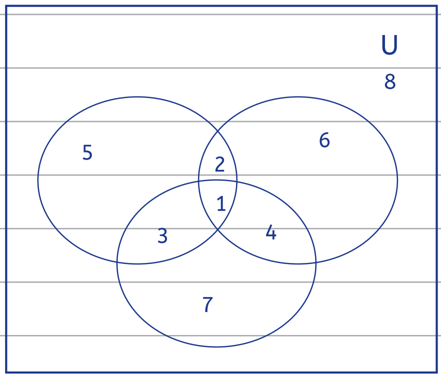
PARTITIONING OF SETS

Case 1



1. A-B or $A \cap B^c$ or A but not B = n(A) – n($A \cap B$)
2. ($A \cap B$) or (A and B)
3. B-A or $A^c \cap B$ or B but not A = n(B) – n($A \cap B$)
4. $A^c \cap B^c$ or neither A nor B or $n(A \cup B)^c$ or n(U) – n($A \cup B$)
5. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Case 2



1. $(A \cap B \cap C)$
2. $n(A \cap B \cap C^c) = n(A \cap B) - n(A \cap B \cap C)$
3. $n(A \cap B^c \cap C) = n(A \cap C) - n(A \cap B \cap C)$
4. $n(A^c \cap B \cap C) = n(B \cap C) - n(A \cap B \cap C)$
5. $n(A \cap B^c \cap C^c) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
6. $n(A^c \cap B \cap C^c) = n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$
7. $n(A^c \cap B^c \cap C) = n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
8. $n(A^c \cap B^c \cap C^c) = n(A \cup B \cup C)^c = n(U) - n(A \cup B \cup C)$
9. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Notes :

a) (2), (3), (4) are cases where only 2 items of the 3 are taken at a time.

b) (5), (6), (7) are cases where only 1 item of the 3 is taken at a time

c) (8) is the case where no item of the 3 are taken.

d) (1) is the case where all the items are taken i.e. the common part to all the 3.

LAWS

ASSOCIATIVE LAW

(a) $A \cup (B \cup C) = (A \cup B) \cup C$

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

DISTRIBUTIVE LAW

(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

DEMORGAN'S LAW

(a) $(A \cup B)^c = A^c \cap B^c$

(b) $(A \cap B)^c = A^c \cup B^c$

DEMORGAN'S LAW ON DIFFERENCE OF SETS

(a) $A - (B \cup C) = (A-B) \cap (A-C)$

(b) $A - (B \cap C) = (A-B) \cup (A-C)$

CARTESIAN PRODUCT

(a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(b) $A \times (B - C) = (A \times B) - (A \times C)$

RELATIONS

1. If A and B are two non empty sets, then any sub-set of A x B is called a relation from A to B. If R is a relation, then, $R \in A \times B$.

2. A = {1, 2, 3, 5} B = { 2, 4}
   Then, A x B = {(1, 2), (1, 4), (2, 2), (2, 4), (3, 4), (3, 2), (5, 2), (5, 4)}

3. If we consider the relation 'is less than' then the set of all ordered pairs R in A x B, where
   (i) R = {(1,2), (1, 4), (2, 4), (3, 4)} = $\{(x, y) : x \in A, Y \in B, X R Y\}$

   (ii) Let A = (1, 2, 3, 4 ....32) R be the relation "one fourth of A"
   R = { (1, 4), (2,8), (3, 12), (4, 16), (5, 20), (6, 24), (7, 28), (8, 32)}

4. Number of Relation

If A and B are 2 sets containing m and n items respectively, then A x B will have mn ordered pairs, Total number of subsets of mn ordered pairs = 2^{mn}

Since each relation is subset of A x B.

$\therefore$ Total Relation = 2^{mn}

e.g. if n(A) = 4, n(B) = 2

Total relations = 2^8 = 256.

5. Domain and Range of Relation

If A and B are 2 non-empty sets and R be the relation, then the set of first element in the ordered pair (x, y) is called the Domain of the relation and the set of second elements in the ordered pair is called the Range of the relation.

e.g. : A = { 1, 3, 4, 5, 7}

B = (2, 4, 6, 8)

And R is the relation 'is one less than' from

A to B, then

R = {(1, 2), (3, 4), (5, 6), (7, 8)}

Domain of R = { 1, 3, 5, 7}

Range of R = (2, 4, 6, 8}

Co-domain of R = (2, 4, 6, 8}

Range $\subseteq$ Co-domain

TYPES OF RELATIONS

1. Note : A relation R in set A is a subset of A x A

2. A relation R in set A is said to be "Reflexive", if (a, a) $\in$ R, for all a $\in$ A

   where 'a' is the element of set A

   e.g. : A = {2, 4, 7} then the relation R =

   {(2, 2), (4, 4), (7, 7)} is reflexive.

3. A relation R in set A is called "Symmetric"

   if (a, b) $\in$ R, then (b, a) $\in$ R.

   e.g. A = {2, 4, 7}

   R = {(2,4), (4, 2), (2, 7), (7, 2)} is a

   symmetric relation.

4. A relative R in Set A is called "Transitive" relation if (a, b), (b, c) $\in$ R, then (a, c) $\in$ R

   e.g. : R = {(2, 4), (4, 7), (2, 7)} is transitive

5. A relation which is reflexive, symmetric and transitive is called an "Equivalence" relation.

Note :

1. Inverse of Equivalence relation is also an Equivalence relation.
2. Intersection of two Equivalence relation is also Equivalence relation.

Inverse Relation

Let, R be the relation from set A to B, then the inverse relation of R is denoted by R^{-1} is a relation from B to A.

$\therefore$ If R is a subset of A x B.

R^{-1} is a subset of B x A which consists of all the ordered pairs which when reversed belongs to R.

e.g. A = (2, 3, 5, 7), B = (4, 6, 9, 10, 11)

R be the relation "is a divisior of" from A to B

then, R = {(2, 4), (2, 6), (2, 10), (3, 6), (3, 9), (5, 10)}

$\therefore$ R^{-1} is a relation from B to A will be given by;

R^{-1} in this relation "is divisible by"

Domain of R^{-1} = {4, 6, 10, 9} = Range of R

Range of R^{-1} ={2, 3, 5) = Domain of R

Note :

D (R^{-1}) = R (R)

R (R^{-1}) = D(R)

FUNCTIONS

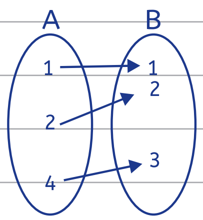
1. If A and B are 2 non-empty sets then, function is a rule or correspondence which associates every element 'X' of A to a unique element of 'Y' in B.
2. Symbolically we express it as $f : A \to B$

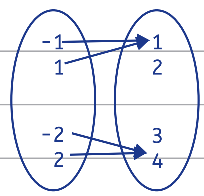
Note :

1. Set from which it is defined is called domain i.e. Set A
2. Set to which it is defined is called co-domain i.e. Set B
3. The set of images are the ranges of the function, Range $\subseteq$ Co-domain

Types of Functions

One – One (Injective) | Many – One | One – Many (does not exist)

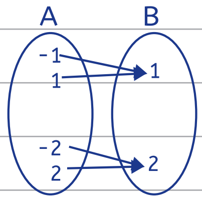




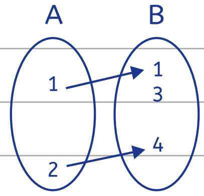
Each Element in A has only one image in B and each element in B has one pre-image in A

At least two elements in A has the same image in B and at least one element in B, has more than one pre-image in A

Onto Function
(Surjective Function)
Range = co-domain

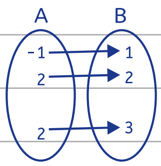


Into Function
Range $\subseteq$ co-domain

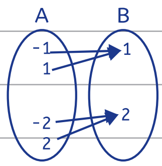


There should be at least one element in Set B which has no pre-image in A

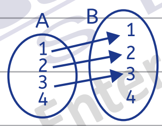
One one-onto
(Bijective Function)



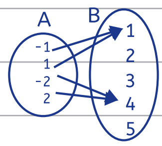
Many one-onto



One – one Into



Many Into



Domain : {-1, 1, -2, 2}
Co-Domain : {1, 2, 3, 4, 5}
Range : {1, 4)

CLASSWORK SECTION

(For Q. No. 1 to 6)

If A = { a, b, c, d, e}; B = { a, e, i, o, u} and C = {m, n, o, p, q, r, s, t, u}

1. If A and B are two sets containing 4 and 7 distinct elements respectively, find the minimum possible number and maximum possible number of elements $A \cup B$.

   a) 5, 10 b) 4, 12 c) 7, 11 d) 8, 13

2. If A = { 1, 2, 3}, B = {3, 4}, and C = {4, 5, 6} then $(A \times B) \cap (B \times C)$ is equal to :

   a) { } b) {(3, 4)}
   c) {(2, 3), (3, 2), (3, 4)} d) None of the above

3. The number of non - empty subsets of the set {8, 9, 10, 11, 15} is :

   a) 32 b) 31 c) 30 d) 33

4. Two finite sets have p and q number of elements. The total number of subsets of the first set is eight times the total number of subsets of the second set. Find the value of p − q.

   a) 2 b) 3 c) 4 d) None of the above

5. In a class of 65 students, 35 students have taken Mathematics, 40 have taken Statistics. Find the no. of students who have taken both. Find the no. of students who have taken Mathematics but not Statistics. (Assume that every student has to take atleast one of the two subjects.)

   (a) 10, 25 (b) 10, 10 (c) 10, 20 (d) 10, 30

6. In a City, there are three daily newspaper published X, Y, Z. 65% of the people of the city read X, 54% read Y, 45% read Z, 38% read X and Y, 32% read Y and Z, 28% read X and Z. 12% do not read any of the three papers. If 10,00,000 person live in the city. Find the number of persons who read all the three newspaper.

   (a) 220000 (b) 230000 (c) 120000 (d) 200000

7. If A = {a, b, c, d} and B= {p, q, r, s} then which of the following are relations from A to B?

a) R1 = {(a, p), (b, r),(c, s)}

b) R2 = {(q, b), (c, s),(d, r)}

c) R3 = {(a, p), (b, r),(c, r)(s, q)}

d) R4 = {(a, p), (b, s),(s, b)(q, a)}

8. If A = {1, 3, 5, 7} and B = {2, 4, 6, 8, 10} and R = {(1, 8), (3, 6), (5, 2), (1, 4)} be a relation from A to B, then Dom(R) = ?

a) {1, 5} b) { 1, 3, 5} c) (3, 5} d) None of the above

9. In the above question, what is the Range (R)?

a) {1, 3, 5} b) { 8, 6, 2, 4} c) (2, 4, 6} d) None of the above

10. What can be said about the relation R = {(a, a), (a, b), (a, c),(b, b), (b, c), (c, a), (c, b), (c, c)} defined on Set A = {a, b, c}?

a) Reflexive, Symmetric, Transitive

b) Non Reflexive, Symmetric, Transitive

c) Reflexive, Symmetric, Non Transitive

d) Reflexive, Non-Symmetric, Non Transitive

11. Find in each case the type of relation:

A = {1, 2, 3}

R_1 = {(1,1), (2,2), (3,3,), (1, 2)}

R_2 = {(1,1), (2,2), (1,2,), (2, 1)}

R_3 = {(1,1), (2,2), (3,3,), (1, 2), (2,1), (2, 3)(3,2)}

R_4 = {(1,1), (2,3), (3,2,)}

FUNCTIONS

1. The domain and range of $\{(x, y) : y = x^2\}$ where $x, y \in R$ is
   (a) (reals, natural numbers)
   (b) (reals, positive reals including zero)
   (c) (reals, reals)
   (d) none of these

2. If $f(x) = 1/1 - x$ and $g(x) = (x - 1)/x$, than $fog(x)$ is
   (a) x
   (b) $1/x$
   (c) $-x$
   (d) none of these

3. The inverse h^{-1} when $h(x) = \log_{10} x$ is
   (a) $\log_{10} x$
   (b) 10^x
   (c) $\log_{10} (1/x)$
   (d) none of these

7B LIMITS & CONTINUITY

Limits (THEORY)

Type I

$\underset{x\to a}{Lt}\, f(x)=f(a)$ $\quad \underset{x\to a}{Lt}\dfrac{f(x)}{g(x)}=\dfrac{f(a)}{g(a)}; if\ g(a)\neq 0$

Type II

$\underset{x\to a}{Lt}\dfrac{f(x)}{g(x)}\ \&\ g(a)=0$, then cancel the common terms from numerator and denominator using algebraic treatments.

The reduced form would be: $\underset{x\to a}{Lt}\dfrac{f(x)}{g(x)}=\underset{x\to a}{Lt}\dfrac{p(x)}{q(x)}=\dfrac{p(a)}{q(a)}$

Type III

$\underset{x\to \infty}{Lt}\dfrac{f(x)}{g(x)}$, Divide numerator and denominator by the highest power of x, and then put 1/x = 0.

Type IV(Standard Limits)

- $\underset{x\to 0}{Lt}\dfrac{e^x-1}{x}=1 \quad \underset{x\to 0}{Lt}\dfrac{e^{mx}-1}{x}=m \quad \underset{x\to 0}{Lt}\dfrac{e^{mx}-1}{mx}=1$
- $\underset{x\to 0}{Lt}\dfrac{a^x-1}{x}=\log_e a \quad \underset{x\to 0}{Lt}\dfrac{a^{mx}-1}{x}=m.\log_e a \quad \underset{x\to 0}{Lt}\dfrac{a^{mx}-1}{mx}=\log_e a$
- $\underset{x\to 0}{Lt}\dfrac{\log(1+x)}{x}=1 \quad \underset{x\to 0}{Lt}\dfrac{\log(1+mx)}{x}=m \quad \underset{x\to 0}{Lt}\dfrac{\log(1+mx)}{mx}=1$
- $\underset{x\to a}{Lt}\dfrac{x^n-a^n}{x-a}=n.a^{n-1} \quad \underset{x\to a}{Lt}\dfrac{x^n-a^n}{x^m-a^m}=\dfrac{n.a^{n-1}}{m.a^{m-1}}=\dfrac{n}{m}.a^{n-m}$
- $\underset{x\to \infty}{Lt}\left(1+\dfrac{1}{x}\right)^x=e \quad \underset{x\to \infty}{Lt}\left(1+\dfrac{a}{x}\right)^x=e^a$
- $\underset{x\to 0}{Lt}(1+x)^{\frac{1}{x}}=e; \ \underset{x\to \infty}{Lt}(1+x)^{\frac{a}{x}}=e^a; \ \underset{x\to 0}{Lt}(1+ax)^{\frac{1}{x}}=e^a$

CLASSWORK SECTION

Type II

1. $\underset{x \to 9}{\text{Lt}} \frac{\sqrt{x}-3}{x-9}$

a) 1/6 b) 1/5 c) 1/9 d) 1/7

2. $\underset{x \to 1}{\text{Lt}} \frac{x^7-2x^5+1}{x^3-3x^2+2}$

a) 0 b) 1 c) 2 d) 4

Type – III – Limits, When the variable tends to Infinity

3. $\underset{x \to \infty}{\text{Lt}} \frac{(1+2x^2)(3-x^4)}{(1+x^2)(5+x^4)}$

a) -2 b) 2 c) 1 d) −1

4. $\underset{x \to \infty}{\text{Lt}} \frac{1^2+2^2+3^2+...+x^2}{x^3}$

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{2}{5}$

Type – IV - Definition

For each of the following functions (from Q No. 20 to25),evaluate the following limit:

$\underset{h \to 0}{\text{Lt}} \frac{f(x+h)-f(x)}{h}$

5. $f(x) = \frac{1}{\sqrt{x}}$

a) $\frac{1}{2\sqrt{x}}$ b) $\frac{-1}{2\sqrt{x}}$ c) $\frac{-1}{2x\sqrt{x}}$ d) None of the above

Type – V – Standard Limits

6. $\underset{x \to 0}{Lt} \dfrac{e^{-2x} - 1}{x}$

a) 1 b) 2 c) -1 d) -2

7. $\underset{x \to 0}{Lt} \dfrac{e^{\alpha x} - e^{\beta x}}{x}$

a) $\alpha + \beta$ b) $\alpha . \beta$ c) $\alpha - \beta$ d) None of the above

8. $\underset{x \to 0}{Lt} \dfrac{6^x - 3^x - 2^x + 1}{x^2}$

a) $\log_e 3 + \log_e 2$ b) $\log_e 3 - \log_e 2$
c) $\log_e 6$ d) $\log_e 3 . \log_e 2$

9. $\underset{h \to 0}{Lt} \dfrac{(x+h)^n - x^n}{h}$

i) $n. x^{n-1}$ j) $n.a^{n-1}$ k) a^{n-1} l) $n^2.a^{n-1}$

10. $\underset{x \to \infty}{Lt} \left(\dfrac{x+6}{x+1}\right)^{(x+6)}$

a) e b) e^2 c) e^3 d) e^5

11. $\underset{x \to 1}{Lt} \dfrac{ax^2 + bx - 2}{x - 1} = 3$, find a & b.

a) 1, 1 b) 1, 2 c) 1, 3 d) 1, 4

CONCEPT OF CONTINUITY OF A FUNCTION

A function f (x) is said to be Continuous at a particular point, x =a , if it satisfy the following conditions:

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$$

Left hand = Right hand =Functional
Limit (LHL) Limit (RHL) Value

Note1: Equality of RHL and LHL is treated as a condition for existence of limit i.e, limit of a function will exist if LHL=RHL

Note2: For Continuity, equality of the functional value at that point is also necessary.

Note3: For all Continuous functions, limit must exist, but existence of limit, is not a sufficient condition for continuity of a function.

Note4: Sum, difference , product and quotient of all continuous functions are always continuous.

Note5: All polynomials are continuous.

Note6: If a given function is of the form $\frac{f(x)}{g(x)}$, where both f (x) and g(x) are polynomials in x, it will be everywhere continuous except at the points at which it is undefined i.e; points of discontinuity of such functions are the points where g(x) =0.

Example: In each of the following cases, discuss continuity of the functions at x=5

i) $f(x)=\frac{x^2-25}{x-5}$

Solution: LHL= $\lim_{x\to 5^-}\frac{x^2-25}{x-5}=\lim_{x\to 5^-}\frac{2x}{1}=2\times 5=10$

RHL= $\lim_{x\to 5^+}\frac{x^2-25}{x-5}=\lim_{x\to 5^+}\frac{2x}{1}=2\times 5=10$

$$f(5)=\frac{25-25}{5-5}=\frac{0}{0}(undifined)$$

since,LHL = RHL $\neq$ f (5), f (x)is discontinuous atx = 5,although the limit has existed.

ii) $f(x)=\frac{x^2-25}{x-5}, when \quad x\neq 5$

=10, when x=5

Solution: LHL=10=RHL taken from(i)

Given , f (5) =10 since, LHL=RHL= f (5), f (x) is continuous at x = 5

iii) $f(x)=\frac{x^2-25}{x-5}, when \quad x\neq 5$

=2, when x = 5

Solution: LHL=RHL=10 taken from(ii)

Given , f (5) = 2 since, LHL=RHL$\neq$ f (5), f (x) is discontinuous at x = 5

Example 2: Find the points of discontinuity of the function, $f(x)=\frac{(x^2-3x+2)}{(x^2-5x+6)}$

Solution: The given function will be continuous at all points, except at the points at which it is undefined i.e the points at which its denominator is 0. $(x^2-5x+6)=0$

Points of discontinuity are 2 and 3 $\Rightarrow(x-2)(x-3)=0$

$\Rightarrow x=2,3$

WORKING CODES for Q. No. 1 to 18

Mark C : if function is continuous at the given point

Mark D : if function is discontinuous at the given point

Mark X : if nothing can be said about the continuity of the function at the given point

Mark Y : if function is neither continuous nor discontinuous at the given point

CLASSWORK SECTION

12. f(x) = $\begin{cases} x+1, x \geq 1 \\ 2x+1, x<1 \end{cases}$, Check continuity x = 1

a) C b) X c) D d) Y

13. f(x) = $\left.\begin{matrix} x^2, x>2 \\ 4, x=2 \\ 2x, x<2 \end{matrix}\right\}$ Check continuity at x = 2

a) C b) D c) X d) Y

14. f(x) = $\left.\begin{matrix} 3+2x, -\frac{2}{3} \leq x<0 \\ 3-2x, 0 \leq x<\frac{3}{2} \\ -3-2x, \ x \geq \frac{3}{2} \end{matrix}\right\}$ Check continuity at x = $\frac{3}{2}$

a) C b) D c) X d) Y

15. f(x) = $\frac{3x^2-4x}{x^3+x^2-x-1}$

a) ± 1 b) 1 c) -1 d) None of the above

16. Given f(x) $\begin{cases} = x+1, \quad x \leq 1 \\ = 3-ax^2, x>1 \end{cases}$, for what value of a, will f(x) be continuous at x = 1?

a) 2 b) 1 c) 8 d) 6

BASIC CONCEPTS OF DIFFERENTIAL AND INTEGRAL CALCULUS

8 DIFFERENTIAL CALCULUS

THEORY

Let y = f(x) be a continuous function. Then, the value of y depends upon the value of x and it changes with a change in the value of x. We use the word increment to denote a small change, i.e., increase or decrease in the values of x and y.

Let Δy be an increment in y corresponding to an increment Δx in x.

Then, $\frac{dy}{dx} = \underset{h\to 0}{Lt} \frac{f(x+h)-f(x)}{h}$. This limit, if it exists finitely, is called the derivative or differential coefficient of y = f(x) with respect to x and is denoted by $\frac{dy}{dx}$ or $f'(x)$ or y_1 . The process of finding the derivative is known as differentiation.

Standard Derivatives

$\frac{d}{dx}x^n = n.x^{n-1}$	$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}x = 1$	$\frac{d}{dx}\frac{1}{x^n} = -\frac{n}{x^{n+1}}$
$\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$	$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$	$\frac{d}{dx}\frac{1}{\sqrt{x}} = -\frac{1}{2x\sqrt{x}}$	$\frac{d}{dx}e^x = e^x$
$\frac{d}{dx}e^{mx} = m.e^{mx}$	$\frac{d}{dx}a^x = .a^x.\log_e a$	$\frac{d}{dx}a^{mx} = m.a^{mx}.\log_e a$	$\frac{d}{dx}\log_e x = \frac{1}{x}$

Product and Quotient Rule

$$\frac{d}{dx}u.v = u.\frac{d}{dx}v + v.\frac{d}{dx}u \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$$

Parametric Functions

Sometimes x and y are given as function of another variable t. Then t is called a parameter. Let x = f(t) and y = g(t), then:

$$\frac{dy}{dx} = \frac{dy}{dt} \bigg/ \frac{dx}{dt}$$

Implicit Functions

When the variables x and y are not explicitly or clearly defined in terms of each other ,the function takes an implicit form. We differentiate both sides of the equation term wise, keeping in mind that $\frac{d}{dx}2y = 2.\frac{dy}{dx}$ & $\frac{d}{dt}m^2 = 2m.\frac{dm}{dt}$ and so on.

Function of a Function – Chain Rule

If $y = f(t)$ and $t = g(x)$, then $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$, and the rule can be further extended.

Logarithmic Differentiation – Log Rule

When the given function is a power of some expression or a product of expressions, we take logarithm on both sides and differentiate the implicit functions so obtained.

If $y = f(x)^{g(x)}$, then; $\log y = g(x).\log f(x)$......Then proceed.

Slope – Applied Differentiation

For $y = f(x)$, slope at any point (x_1, y_1) is given by $\frac{dy}{dx}_{at\ x_1, y_1}$

Higher Order Derivatives

Let y = f(x) be a differentiable function of x whose second and higher order derivatives exists. The first, second, third, ... and the nth derivatives of this function are denoted by;

dy/dx, d^2y/dx^2, d^3y/dx^3, ..., d^ny/dx^n or y_1, y_2, y_3, ... , y_n or $f'(x)$, $f''(x)$, ...

INTEGRAL CALCULUS

THEORY

Fundamental Integrals

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\int \frac{dx}{x^n} = \frac{1}{(1-n).x^{n-1}} + C$	$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$	$\int dx = x + C$
$\int \frac{dx}{x} = \log\lvert x\rvert + C$	$\int e^x dx = e^x + C$	$\int e^{mx} dx = \frac{e^{mx}}{m} + C$	$\int a^x dx = \frac{a^x}{\log_e a} + C$
$\int a^{mx} dx = \frac{a^{mx}}{m \log_e a} + C$			

Integration by Parts

$$\int u.v \, dx = u.\int v dx - \int \left\{ \frac{du}{dx} \int v dx \right\} dx$$

Standard Integrals

- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$, **Given** $(\lvert x\rvert > \lvert a\rvert)$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$, **Given** $(\lvert x\rvert > \lvert a\rvert)$
- $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log \left| (x + \sqrt{x^2 \pm a^2}) \right| + C$
- $\int \sqrt{x^2 + a^2} \, dx = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right| + C$
- $\int \sqrt{x^2 - a^2} \, dx = \frac{x\sqrt{x^2 - a^2}}{2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + C$

Definite Integrals:- Important Properties

$\int_a^b f(x)\,dx = \int_a^b f(z)\,dz$

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \quad (a<c<b)$

$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$\int_{-a}^{+a} f(x)\,dx = 0$, if f(x) is an odd function.

$\int_{-a}^{+a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f(x) is an even function.

$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$\int_a^b f(x)\,dx = \phi(b) - \phi(a)$

$\int_a^b x\,dx = \left.\frac{x^2}{2}\right]_a^b = \frac{b^2}{2} - \frac{a^2}{2} = \frac{b^2-a^2}{2}$

APPLICATION OF DERIVATIVE & INTEGRATION IN COMMERCE AND ECONOMICS

1. COST FUNCTION = C(x) = Fixed Cost + Variable Cost
$$a + bx$$

2. AVERAGE COST = AC = $\dfrac{\text{TOTAL COST}}{\text{QUANTITY}} = \dfrac{C(x)}{x}$

3. MARGINAL COST = MC = $\dfrac{d}{dx}$ (TOTAL COST) = $\dfrac{d}{dx}$ (C(x))

4. REVENUE FUNCTION = R (x) = Quantity x Price
R (x) = x.p

5. AVERAGE REVENUE = AR = $\dfrac{\text{TOTAL REVENUE}}{\text{QUANTITY}} = \dfrac{x.p}{x} = p$

6. MARGINAL REVENUE = MR = $\dfrac{d}{dx}$ (TOTAL REVENUE) = $\dfrac{d}{dx}$ (R (x))

7. PROFIT FUNCTION = P(x) = R(x) − C(x)

8. AVERAGE PROFIT = $\dfrac{P(x)}{x} = \dfrac{\text{TOTAL PROFIT}}{\text{QUANTITY}}$

9. MARGINAL PROFIT = MP = $\dfrac{d}{dx}$ (R(x) − C(x)) = $\dfrac{d}{dx}$ (P (x))

10. MAXIMUM PROFIT:
Level of output at which profit is maximum can be obtained through the following :
$$MR = MC$$

11. Consumption Function. The consumption function expresses a relationship between the total income (I) and the total national consumption (C). It is denoted by C = f(I).
Marginal Propensity to Consume. It is the rate of change of the consumption with respect to income.
∴ Marginal Propensity to Consume = MPC = $\dfrac{dC}{dI}$.
Marginal Propensity to Save
Let S denote the saving, then Saving S = (Total income − Total consumption) = I − C

∴ Marginal Propensity to Save = MPS = $\dfrac{dS}{dI}$.
It indicates how fast saving changes with respect to income.

12. At Equilibrium, Qd = Qs

On solving the demand and supply equation, we obtain the equilibrium Price and Quantity.

13. Total Cost $= \int$ Marginal Cost dx $= \int MC \; dx$

$= C(x) + k$

where k = fixed cost

i). $AC = \int MAC\,dx$

ii). $VC = \int MVC\,dx$

14. Total Cost for 'n' units $= \int_0^n MC \; dx$

15. Total Revenue $= \int$ Marginal Revenue dx

$= \int MR\,dx = R(x)$

16. Total Revenue for 'n' units $= \int_0^n MR \; dx$

MAXIMA AND MINIMA (EXTREME VALUE)

Given : $y = f(x)$

Steps for finding Maxima and Minima of a function.

1. Find $\frac{dy}{dx}$

2. Equate $\frac{dy}{dx} = 0$ to obtain the value/values of x

3. Find $\frac{d^2y}{dx^2}$ and put therein the values of x obtained from Step 2, and observe the result:

   (i) if $\frac{d^2y}{dx^2} < 0$, then the function attains its **Maximum Value,** at that point and the maximum value of the function can be obtained by putting the value in the original function.

   (ii) If $\frac{d^2y}{dx^2} > 0$, the function attains the **Minimum Value,** at that point and the minimum value of the function can be obtained by putting the value in the original function.

   (iii) If on putting the value of 'x' $\frac{d^2y}{dx^2} = 0$, but $\frac{d^3y}{dx^3} \neq 0$, then the function will have a Point of Inflexion, at a point.

   In other words, at Point of Inflexion, the curve changes its Curvature.

CLASS WORK

DIFFERENTIATION

1. If $y = a^x + x^a + a^a$, then $\frac{dy}{dx} =$

(a) $xa^{x-1} + ax^{a-1} + aa^{a-1}$ (b) $a^x \log a + ax^{a-1}$
(c) $a^x \log a + ax^{a-1} + aa^{a-1}$ (d) none

2. If $f(x) = x^2 - 6x + 5$, then $f'(2) - f'(5) =$

(a) $-3f'(2)$ (b) $3f'(2)$
(c) $2f'(2)$ (d) $4f'(2)$

3. $f(x) = a^x x^k$, then $f'(x)$

(a) $f(x)(a - \log a)$ (b) $f(x)(a + \log a)$
(c) $f(x)\left(\frac{k}{x} - \log a\right)$ (d) $f(x)\left(\frac{k}{x} + \log a\right)$

4. $f(x) = {}^xC_2$, then $f'(1) =$

(a) 1 (b) $\frac{1}{2}$
(c) $-\frac{1}{2}$ (d) $\frac{1}{6}$

5. Derivative of $\sqrt{x^2 + \sqrt{x}}$

(a) $\frac{1}{2\sqrt{x^2 + \sqrt{x}}}$ (b) $2x + \frac{1}{2\sqrt{x}}$
(c) $\frac{1}{2\sqrt{x^2 + \sqrt{x}}}\left(2x + \frac{1}{2\sqrt{x}}\right)$ (d) none

6. $y = \sqrt{x^2 + a^2}$, then $y\dfrac{dy}{dx} =$

(a) x (b) 2x (c) y (d) 2y

7. $x^2 + xy + y^2 = 0$, then $\dfrac{dy}{dx} =$

(a) $-\left(\dfrac{2x+y}{x+2y}\right)$ (b) $-\left(\dfrac{2x-y}{x+2y}\right)$

(c) $-\left(\dfrac{x+2y}{2x+y}\right)$ (d) $\dfrac{2x+y}{x+2y}$

8. $x^y = e^x$, then $\dfrac{dy}{dx}$ is

(a) $\dfrac{\log x - 1}{\log x}$ (b) $\dfrac{\log x - 1}{(\log x)^2}$

(c) $\dfrac{\log x + 1}{\log x}$ (d) $\dfrac{\log x + 1}{(\log x)}$

9. $y = 2at$; $x = at^2$, then $\dfrac{dy}{dx}$ at $t = 1$

(a) 1 (b) 0 (c) 2 (d) a

10. $y = x^{\log x}$, then $\dfrac{\frac{dy}{dx}}{y} =$

(a) $2x^{-1} \log x$ (b) $2x \log x$

(c) $x^{\log x}\left(\dfrac{2\log x}{x}\right)$ (d) none

11. $y = (3x+1)^{\frac{1}{4}}(4x+1)^{\frac{1}{5}}(5x+1)^{\frac{1}{6}}$, then $\dfrac{\frac{dy}{dx}}{y}$ =

(a) $\frac{3}{4}\left(\frac{1}{3x+1}\right)+\frac{4}{5}\left(\frac{1}{4x+1}\right)+\frac{5}{6}\left(\frac{1}{5x+1}\right)$

(b) $y\left[\frac{3}{4}\left(\frac{1}{3x+1}\right)+\frac{4}{5}\left(\frac{1}{4x+1}\right)+\frac{5}{6}\left(\frac{1}{5x+1}\right)\right]$

(c) $(x-3)^{-1}+\frac{1}{3}(x-4)^{-1}+\frac{1}{4}(x-5)^{-1}$

(d) none

12. $y = e^{k\log x} + e^{x\log k}$, then $\frac{dy}{dx}$ =

(a) $x^k + k^x$ (b) $kx^{k-1} + k^x \log k$

(c) $kx^{k-1} + xk^{x-1}$ (d) none

13. If $y = \left(x+\sqrt{x^2-4}\right)^m$, then $(x^2-4)\left(\frac{dy}{dx}\right)^2 - m^2y^2$ =

(a) 0 (b) 1 (c) 2 (d) none

PAST EXAM QUESTIONS

14. If $x^y = y^x$, then $\frac{dy}{dx}$ gives:

(a) $\frac{x(x\log y - y)}{y(y\log x - x)}$ (b) $\frac{x(y\log x - x)}{y(x\log y - y)}$

(c) $\frac{y(x\log y - y)}{x(y\log x - x)}$ (d) none of these

15. If $x^y = e^{x-y}$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{2\log x}{(1+\log x)^2}$ (b) $\frac{\log x}{(1+\log x)}$

(c) $\frac{\log x}{(1+\log x)^2}$ (d) none of the above

INTEGRATION

1. $\int(e^{2x} + e^{-3x})dx$

(a) $\frac{e^{2x}}{2} + \frac{e^{-3x}}{3} + c$ (b) $\frac{e^{2x}}{2} - \frac{e^{-3x}}{3} + c$

(c) $\frac{e^{2x}}{2} - \frac{e^{+3x}}{3}$ (d) none

2. $\int \frac{dx}{\sqrt{x+2}}$

(a) $\sqrt{x+2} + c$ (b) $2\sqrt{x+2} + c$

(c) $\frac{2}{\sqrt{x+2}} + c$ (d) none

3. $\int x^2 2^x \, dx$

(a) $x^2 \frac{2^x}{\log 2} - \frac{x\,2^x}{(\log 2)^2} + \frac{2\,2^x}{(\log 2)^3} + c$

(b) $\frac{x^2\,2x}{\log 2} + \frac{2x\,2^x}{(\log 2)^2} + \frac{2^x}{(\log 2)^3} + c$

(c) $\frac{x^2\,2^x}{\log 2} + \frac{2x\,2^x}{(\log 2)^2} + \frac{2^x}{(\log 2)^3} + c$

(d) none

4. $\int x^n \log x \, dx$

(a) $\frac{x^{n+1}}{(n+1)^2}[(n+1)\log x + 1] + c$ (b) $\frac{x^{n+1}}{(n+1)^2}[(n+1)\log x - 1] + c$

(c) $\frac{x^{n+1}}{n+1}[\log x - 1] + c$ (d) none

5. $\int \frac{3x+2}{(x-2)(x-3)}\,dx$

(a) $11 \log (x-3) - 8 \log |x-2| + c$ (b) $8 \log |x-3| - 11 \log |x-2| + c$

(c) $-11 \log |x-3| + 8 \log |x-2| + c$ (d) $-8 \log |x-3| + 11 \log |x-2| + c$

6. $\int \frac{7x^2}{(x^3+2)^3}\,dx$

(a) $\frac{7}{6}(x^3+2)^2 + c$ (b) $-\frac{7}{6}(x^3+2)^2 + c$

(c) $-\frac{7}{6}\left[\frac{1}{(x^3+2)^2}\right] + c$ (d) $\frac{7}{6}\left[\frac{1}{(x^3+2)^2}\right] + c$

7. $\int e^x(x^3 + 5x^2 + 4x)\,dx$

(a) $e^x(x^3 + x^2) + c$ (b) $e^x(x^3 + 2x^2) + c$

(c) $e^x(x^3 + 3x^2) + c$ (d) $e^x(x^3 + 4x^2) + c$

8. $\int e^x\left(\frac{2-x}{(1-x)^2}\right)dx$

(a) $e^x\left(\frac{1}{1-x}\right) + c$ (b) $e^x\left(\frac{-1}{1-x}\right) + c$

(c) $e^x\left(\frac{1}{2-x}\right) + c$ (d) none

9. $\int \sqrt{x^2+4}\,dx$

(a) $\frac{x}{2}\sqrt{x^2+4} + c$

(b) $\frac{x}{2}\sqrt{x^2+4} + 8 \log |x + \sqrt{x^2+4}| + c$

(c) $\frac{x}{2}\sqrt{x^2+4} + 2 \log |x + \sqrt{x^2+4}| + c$

(d) $\frac{x}{2}\sqrt{x^2+4} - 2 \log |x + \sqrt{x^2+4}| + c$

10. $\int_{4}^{5} f(x)\,dx - \int_{4}^{5} f(9-x)\,dx$

(a) 0 (b) 1
(c) −1 (d) none

11. $\int_{-3}^{3} (x^3 + x)dx$

(a) 0 (b) 3
(c) −3 (d) 1

12. Equation of the curve which passes through the point (1, 0) and $F^1(x) = 2x - 1$

(a) $y = x^2 - x - 1$ (b) $y = x^2 - x - 2$
(c) $y = x^2 - x$ (d) none

PAST EXAM QUESTIONS

13. The value of $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}\,dx$ is

(a) 0 (b) 3 (c) 2 (d) 1

14. $\int \frac{1}{x(x^5+1)}\,dx$

(a) $\log\left(\frac{x^5}{x^5-1}\right)+c$

(b) $\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right)+c$

(c) $\frac{1}{3}\log\left(\frac{x^5}{x^5+1}\right)+c$

(d) $\frac{1}{3}\log\left(\frac{x^5+1}{x^5}\right)+c$

J.K. SHAH CLASSES
a Veranda Enterprise

LOGICAL REASONING

1 NUMBER SERIES, CODING AND DECODING AND ODD MAN OUT

- Series is a sequential order of numbers, letters or both arranged in some specific rules.
- These Rules can be based on mathematical operations, place of letters in alphabetical order etc.

Different types of Series

1. Number Series
2. Letter Series
3. Alpha-Numeric Series
4. Continuous pattern Series

NUMBER SERIES

Number series is a logical sequence of more than one elements made of arithmetical digits.

Some Types of number series:

1. Same numbers addition or subtraction series.
2. Increasing order addition or subtraction series.
3. Same number multiplication or division series
4. Increasing order multiplication or division series
5. Same number multiplication and addition or subtraction series

6. Same number multiplication and addition or subtraction in increasing order series

7. Increasing order multiplication and same number addition or subtraction series.

8. Increasing order multiplication and increasing order addition or subtraction series

9. Multiplication and division series.

10. Square series

11. Cube series

12. Square addition series

13. Prime number series

14. Digital operation of number series

15. Mixed combination series

CLASS WORK SECTION

In the following series replace the question (?) with the suitable option.

1. 27, 32, 30, 35, 33, ?

   a) 28 b) 31 c) 36 d) 38

2. 24, 60, 120, 210, ?

   a) 300 b) 336 c) 420 d) 525

3. 198, 194, 185, 169, ?

   a) 92 b) 136 c) 144 d) 112

4. 6, 13, 38, ?, 532, 2675

   a) 129 b) 123 c) 172 d) 164

5. 45, 46, 70,141, ? , 1061.5

   a) 353 b) 353.5 c) 352.5 d) 352

ODD MAN OUT

1. 9, 14,19,25, 32, 40

   a) 14 b) 25 c) 32 d) 9

2. 4, 5, 12, 38, 160 805, 4836

   a) 12, b) 160 c) 38 d) 805

3. 7, 4, 5, 9, 20, 51, 160.5

   a) 4 b) 51 c) 9 d) 20

LETTER SERIES, ALPHA NUMERIC AND CONTINUOUS PATTERN SERIES

Letter series is a sequence of letters taken from English alphabet and such sequence follows a certain logical pattern

1. PMK, MPK, MKP, KMP, ?
   a) PMK b) KMP c) MPK d) KPM

2. P3, M8, ?, G24, D35
   a) K15 b) J13 c) I13 d) J15

3. Which of the following is odd one: (J-2019)
   a) CEHL b) KMPT c) OQTX d) NPSV

4. _sr_tr_srs_r_srst_
   a) ttssrr b) tsrtsr c) strtrs d) tstttr

CODING AND DECODING

Coding-Decoding is process of transmitting an information from one place to other using some suitable codes, so that it might reach to other person safely.

Different Types of coding and decoding:

1. Coding based on Rearrangement of Letters
2. Coding based on replacement of letters
3. Opposite letter coding
4. Coding of Letters by their Left and Right Letters
5. Number coding
6. Symbol coding based on Similarity
7. Coding by substitution or word replacement
8. Fictitious Language Coding
9. Coding by Comparison

CLASS WORK SECTION

1. In a certain code language, COMPUTRONE is written as PMOCTUENOR. How is ADVANTAGES written in that same code?
   a) ADVANSEGAS b) ADTANSEAG c) AVDANTAGES d) AVDATNSEGA

2. If in a certain code language SIMILAR is written as IZORNRH, how will NATURAL be written in that language?
   a) OZIFGZM b) OZIFGMZ c) OZIFZMG d) OZIFMZG

3. In a certain code RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code? (M-2018)
   a) 318826 b) 318286 c) 618826 d) 338816

4. In a certain language 'DEW' written as 1625529 'GET' is written as 4925400, then how will TWO be written in that language?
   a) 400529522 b) 400529225 c) 400925225 d) 400225925

5. If P = 16 and PUT = 6720 then PICK?
   a) 4137 b) 4590 c) 4032 d) 4752

6. In a certain code 256 means 'you are good' 637 means 'we are bad' and 358 mens 'good and bad'. Which of the following represents 'and' in that code? (M-2018)
   a) 2 b) 5 c) 8 d) 3

2 DIRECTION TESTS

Direction is a measurement of position of one thing with respect to another thing or a reference point.

Types :

1. Finding direction only

2. Find the distance only

3. Finding both the distance and direction.

CLASS WORK SECTION

1. At sunrise, Amit and Deepak are having conversation standing in front of each other. The shadow of Deepak is formed towards the right hand of Amit. What direction is Deepak facing?

   a) North-East b) South c) East d) North

2. Samar wants to go college which is situated in a direction opposite to that of a mall. He starts from his house, which is in the east and comes at four-way place. His left side road goes to the mall and straight in front is the railway station. In which direction is the college located?

   a) North b) North-East c) South d) East

3. Laxman went 15kms to North then he turned west and covered 10 kms. Then he turned South and covered 5 kms, finally turning to East he covered 10 kms. In which direction he is now moving? (M-2018)

   a) East b) West c) North d) South

4. A man is facing East, then he turns left and goes to 10 meter then turns right and goes 5 meter then goes 5meter to the south and from their 5meter to West. In which direction is he from his original place? (M-2018)

   a) East b) West c) North d) South

5. Surbhi is facing east, she turns 100degree in the clockwise direction and then 145 degree in the anti-clockwise direction. Which direction is she facing now?

   a) West b) North-East c) North d) South-West

6. A train runs 120km in West direction, then 30km in South direction and then 80 km in East direction before reaching the station. In which direction is the station from the train's starting point?

   a) South-West b) North-West c) South-East d) South

7. If X stands on his head with his face towards South, to which direction will his left hand point?

a) East b) West c) South d) North

8. Vinod Starts from his house and travels 4km in East direction, after that he turns towards left and moves 4km. Finally, he turns towards left and moves 4km. At what distance and in which direction he finally stands from his starting point?

a) North, 4km b) North-East 4km
c) South 12km d) West 4km

9. Two buses start from the opposite points of a main road, 150km apart. The first bus runs for 25km and takes a right turn and then runs for 15km. It then turns left and runs for another 25km and takes the direction back to reach the main road. In the meantime, due to the minor break down the other bus has run only 35km along the main road. What would be the distance between the two buses at this point?

a) 65km b) 80km c) 75km d) 85km

10. Raghu is at point A. He walks 3km to the North and then turns to his left. He walks, 4km in this direction. He turns left again and walks 6 km. If he wishes to reach point A again, in which direction should he be walking and what distance will he have to cover?

a) South-East, 5km b) South-East, 4 km
c) North-East, 5 km d) North-East, 4 km.

3 SEATING ARRANGEMENT

Sitting arrangement questions are based on the sitting sequence pattern, direction, facing outside or inside etc.

Different types of Questions covered.

1. Linear arrangement
2. Circular arrangement
3. Polygonal arrangement

CLASS WORK SECTION

LINEAR ARRANGEMENT

1. 5 friends are sitting on a bench. A is to the left of B but on the right of C. D is to the right of B but on the left of E. Who are at the extremes?

a) A, B b) A, D c) B, D d) C, E

2. Five children are sitting in a row. S is sting next to P but not T. K is sitting next to R, who is sitting on the extreme left and T is not sitting next to K. Who is/are adjacent to S? (M-2018)

a) K and P b) R and P c) only P d) P and T.

3. Five boys are standing in a row facing East, Pavan is to the left of Tavan, Vipin, Chavan. Tavan, Vipin and Chavan are to the left of Nakul. Chavan is between Tavan and Vipin. If Vipin is fourth from the left, then how far is Tavan from the right?

a) First b) Second c) Third d) Fourth

4. Five boys A, B, C, D and E are sitting on a stair in the following way

E is above A

D is under B

B is under A

D is between B and C

Who is at the lowest position of the stair?

a) A b) C c) E d) B

5. Eight persons A, B, C, D, E, F, G and H are sitting in a line.

E is second right to D.

H sits fourth left to D.

C and F are immediate neighbors, but C is not immediate neighbor of A.

G is not neighbor of E

Only two person sit between A and E.

The persons on left end and right end respectively are

a) G and E b) B and E c) H and E d) G and B

Directions: (Q. no. 6 to 10)

Read the following information carefully to answer the given questions.

A, B, C, D, E, F, G and H are seated in straight line facing North.

C sits fourth to left of G.

D sits seconds to right of G.

Only two people sit between D and A.

B and F are immediate neighbors of each other.

B is not an immediate neighbour of A.

H is not an immediate neighbour of D.

6. Who amongst the following sits exactly in the middle of the persons who sit fifth from the left and the person who sits sixth from the right?

a) C b) H c) E d) F

7. Who amongst the following sits third to the right of C?

a) B b) F c) A d) E

8. Which of the following represents persons seated at the two extreme ends of the line?

a) C, D b) A, B c) B, G d) D, H

9. What is the position of H with respect to F?

a) Third to the left b) Immediate right

c) Second right d) Fourth to left

10. How many persons are seated between A and E?

a) One b) Two c) Three d) Four

Circular Arrangement

1. Five persons are sitting facing centre of a circle. Pramod is sitting to the right of Rajan. Raju is sitting between Brejesh and Naveen. Raju is to the left of Brejesh and Rajan is to right of Brejesh. Who is sitting to the left of Naveen?

a) Pramod b) Raju c) Brejesh d) Rajan

Directions (Q. no.4 to 6)

Read the following information carefully to answer the question that follow:

Six girls are sitting in a circle.

Sonia is sitting opposite to Radhika.

Poonam is sitting right of Radhika but left of Deepti.

Monika is sitting left of Radhika

Kamini is sitting right of Sonia and left of Monika

Now, Deepti and Kamini, Monika and Radhika mutually exchange their positions.

1. Who will be opposite to Sonia?

a) Radhika b) Monika c) Kamini d) Sonia

2. Who will be sitting left of Kamini?

a) Poonam b) Deepti c) Radhika d) Sonia

3. Who will be sitting left of Deepti?

a) Sonia b) Monika c) Radhika d)Poonam

Polygonal Arrangement

1. Four boys and four girls are sitting around a square facing the centre. One person is sitting at each corner and at the midpoint of each side of the square. Madhu is sitting diagonally opposite to Usha who is to the right of Geeta. Ram who is to the left of Geeta is diagonally opposite to Gopi who is to the left of Bose. Position of Suma is not to the right off Madhu but in front of Prema. Who is sitting opposite to Bose?

a) Geeta b)Prema c) Suma d) Madhu

Directions (Q. no.2 to 5)

Read the following information carefully to answer the question that follow:

Six people A, B, C, D, E and F are sitting on the ground in a hexagonal shape. All the sides of hexagon so formed are of same length. A is not adjacent to B or C. D is not adjacent to C or E. B and C are adjacent. F in the middle of D and C.

2. Which of the following is not a correct neighbour pair?

a) A and F b) D and F c) B and E d) C and F

3. Who is at the same distance from D as E is from D?

a) B b) C c) D d) F

4. Which of the following group has the correct order of arrangement?

a) A,F, B b) F, A, E c) B, C, F d) D, A, B

5. If one neighbour of A is D, who is the other one?

a) B b) C c) E d) F

4 BLOOD RELATION

Blood relation between two individuals is defined as a relation between them by the virtue of their birth rather than by their marriage or any other reasons.

Different Types of Blood Relation questions

1. Blood relation based on Conversation
2. Blood relation based on Puzzles
3. Symbolically Coded Blood Relationship

CLASS WORK SECTION

Type 1. Blood relation based on Conversation

1. Vinod introduces Vishal as the son of the only brother of his father's wife. How is Vinod related to Vishal? (M-2018)

   a) Cousin b) Brother c) Son d) Uncle

2. Pointing to a picture, Summit said, she is the mother of my son's wife's daughter. How is lady related to the Summit. (J-2019)

   a) Uncle b) Cousin c) Daughter d) None

3. Pointing to a lady Rishi said, "The son of her brother is the brother of my wife". How is this lady related to Rishi?

   a) Mother-in Law b) Mother's sister
   c) Sister of Father-in Law d) None of the above.

4. Pointing towards a girl, Anurag says, "This girl is the daughter of the only child of my father". What is the relation of Anurag's wife with the girl?

   a) Sister b) Aunt c) Daughter d) Mother

Type 2. Blood relation based on Puzzle

Directions (Q. no.1 to 3)

Read the following information carefully to answer the question that follow:

There are six children playing football, namely, P, Q, R, S, T and U.

P and T are brothers,

U is the sister of T.

R is the only son of P's Paternal uncle,

Q and S are the daughters of the only brother of R's father.

1. Six persons are seen together in a group. They are A, B, C, D, E and F.
B is brother of D, but D is not brother of B,
F is brother of B.
C and A are married together.
F is son of C, but C is not mother of F.
E is brother of A.
The number of female member in the group is (N-2018)

a)1 b) 2 c) 3 d) 4

2. P's father is Q's son. M is the paternal uncle of P and N is the brother of Q. How is M related to N?

a) Nephew b) Cousin
c) Data inadequate d) None of these

3. In a family, there are seven persons comprising two married couple. T is the only son of M and the grandson of K. M is a widower. M and R are brothers and W is the daughter in law of J, who is the mother of R and the grandmother of D. How is D related to M?

a) Son b) Son in law c) Nephew or Niece d) Brother

Type 3. Symbolically Coded Blood Relation

Directions (Q. no.1 & 2)
Read the following information carefully to answer the question that follow:
'P x Q' means 'P is sister of Q'.
'P + Q' means 'P is mother of Q'
'P – Q' means 'P is father of Q'
'P ÷ Q' means 'P is brother of Q'

1. If P + Q means P is the mother of Q.
P ÷ Q means p is the father of Q.
P – Q means P is the sister of Q.
Then which of the following relationship shows that M is the daughter of R? (N-2018)

a) R ÷ M + N b) R + N ÷ M c) R – M ÷ N d) None of these

2. S x T means that S is the mother of T

   S + T means that S is father of T

   S − T means that S is the sister of T

   On the basis of this information, you have to select the option which shows that A is the grandfather of T?

   a) A + S + B − T b) A x B + C − T

   c) A + B − C x T d) (a) & (c) both

Directions (Q. no.1 to 5)

Read the following information carefully to answer the question that follow:

'A + B' means 'A is the father of B'

'A x B' means 'A is the sister of B'

'A $ B' means 'A is the wife of B'

'A % B' means 'A is the mother of B'

'A ÷ B' means 'A is the son of B.'

1. What should come in place of the question mark, to establish that J is the brother of T in the expression?

   J ÷ P % H ? T % L

   a) x b) ÷ c) $ d) Either + or x

2. Which among the given expression indicate that M is the daughter of D?

   a) L % R $ D + T x M

   b) L + R $ D + M x T

   c) L % R % D + T ÷ M

   d) L $ D ÷ R % M ÷ T

3. Which among the following options is true, if the expression 'I + T % J x L ÷ K' is definitely true?

   a) L is the daughter of T

   b) K is the son in law of I

   c) I is the grandmother of L

   d) J is the brother of L

4. Which among the following expressions is true, if Y is the son of X is definitely false?

a) W % L x T x Y ÷ X

b) W + L x T x Y ÷ X

c) X + L x T x Y ÷ W

d) W $ X + L + Y + T

5. What should come in the place of the question mark, to establish that T is the sister in law of Q in the expression

R % T x P ? Q + V

a) + b) % c) x d) $

APPENDIX

Table I - LOGARITHM

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	32	36	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0964	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1208	1239	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523								3	6	9	12	15	19	22	25	28
				1553	1584	1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818								3	6	9	11	14	17	20	23	26
				1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	6	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	3	5	8	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	17	19	21
						2672	2695	2718	2742	2765	2	4	7	9	11	14	16	19	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3023	3054	3075	3096	3116	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3909	3927	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	11	13	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	12	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	10	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	11	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12

32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	631	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6235	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6575	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6037	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

Example:

Log 2 = 0.3010: Log 20 = 1.3010: Log 200 = 2.3010: Log 2,000 = 3.3010 etc.

Log 2 = 0.3010 - 1 – (–) 0.699

Log 0.02 = 0.3010 - 2 – (–) 1.699

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7166	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7306	7314	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7358	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7452	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7848	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7958	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8158	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8428	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8673	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8738	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	2	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	2	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	2	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	2	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	2	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9450	9455	9460	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4

89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9945	9952	0	1	1	2	2	3	3	4	4
99	9958	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Table II - ANTILOGARITHM

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
100	1000	1002	1005	1007	1009	1012	1014	1016	1018	1021	0	0	1	1	1	1	2	2	2
101	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
102	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
103	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
104	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
105	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
106	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
107	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
108	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
109	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
110	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
111	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
112	1381	1321	1324	1327	1330	1334	1337	1340	1342	1348	0	1	1	1	2	2	2	2	3
113	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
114	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
115	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
116	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
117	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
118	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
119	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
120	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
121	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
122	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
123	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
124	1738	1742	1746	1750	1754	1758	1762	1768	1770	1774	0	1	1	2	2	2	3	3	4
125	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
126	1820	1824	1828	1832	1837	1841	1845	1849	1897	1858	0	1	1	2	2	3	3	3	4
127	1862	1866	1871	1875	1879	1884	1888	1892	1941	1901	0	1	1	2	2	3	3	3	4
128	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
129	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
130	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
131	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
132	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
133	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
134	2188	2193	2198	2203	2206	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
135	2239	2244	2249	2254	2259	2265	2270	2275	2280	2256	1	1	2	2	3	3	4	4	5

136	2291	2286	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
137	2344	2350	2355	2359	2366	2271	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
138	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
139	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
140	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
141	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
142	2630	2636	2642	2649	2655	2661	2667	2673	2679	2624	1	1	2	2	3	4	4	5	6
143	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
144	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
145	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
146	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
147	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
148	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
149	3090	3097	3105	3112	3118	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
150	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
151	3236	3243	3251	3258	3268	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
152	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
153	3388	3396	3404	3412	3420	3428	3436	3442	3451	3459	1	2	2	3	4	5	6	6	7
154	3467	3475	3483	3491	3499	3508	3516	3524	3532	2540	1	2	2	3	4	5	6	6	7
155	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	6	7
156	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
157	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
158	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
159	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	4	5	6	7	8
160	3981	3990	3999	4009	4018	4027	4036	4046	4055	4065	1	2	3	4	5	6	6	7	8
161	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
162	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
163	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
164	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
165	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
166	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
167	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
168	4788	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
169	4898	4909	4920	4932	4943	4955	4986	4977	4989	5000	1	2	3	5	6	7	8	9	10
170	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
171	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
172	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
173	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
174	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
175	5632	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
176	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
177	5858	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
178	6028	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
179	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
180	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
181	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
182	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
183	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
184	6918	6934	6950	6965	6982	6598	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
185	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
186	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
187	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
188	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16

189	7762	7780	7796	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
190	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
191	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
192	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
193	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
194	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
195	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
196	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
197	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
198	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
199	9772	9795	9817	9840	9836	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

Example:

If Log x = 0.301. then x = Antilog 0.301 = 2

If Log x = 1.301. then x = (Antilog 0.301) × 10 = 20

If Log x = 2.301. then x = (Antilog 0.301) × 100 = 200

If Log x = (–) 0.699, then we can write Log x = (– 1 + 0.301) : Thus x = Antilog (0.301) / 10 = 0.2

If Log x = (–) 1.699, then we can write Log x = (– 2 + 0.301) : Thus x = Antilog (0.301) / 100 = 0.02


Good Luck!